

The Public and Private Provision of Safe Assets*

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Abstract

We develop a theory of optimal government debt in which publicly-issued and privately-issued safe assets are substitutes. While government bonds are backed by future tax revenues, privately-issued safe assets are backed by the future repayment of pools of defaultable private loans. We find that a higher supply of public debt crowds out privately-issued safe assets less than one for one and reduces the interest spread between borrowing and deposit rates. Our main result is that the optimal level of public debt does not fully crowd out private lending and maintains a positive interest spread. Moreover, the optimal level of public debt responds positively to an increase in the volatility of idiosyncratic shocks, to a decrease in the cost of default, and to an increase in financial repression.

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1 Introduction

Government bonds directly compete with privately-issued safe assets. This view is supported by empirical evidence suggesting that publicly- and privately- issued safe assets are substitutes, and that private safe asset issuance declines whenever public safe asset issuance rises.¹ This view is also the basis for theoretical work which focuses on the benefit of public liquidity in times when private liquidity is limited (e.g., Woodford, 1990, Holmstrom and Tirole, 1998).

If government debt and privately-issued safe assets are substitutes, what is the optimal public provision of safe assets? In this paper, we address this theoretical question in a general equilibrium model in which publicly-issued and privately-issued safe assets co-exist. Our starting premise is that while government bonds are backed by future tax revenues, privately-issued safe assets are backed by the future repayment of pools of defaultable private loans. Under these conditions, we ask the following questions: How does the supply of public debt affect the supply of private-issued safe assets? How does it affect the spread between deposit rates and borrowing rates in the economy? And what drives the optimal choice of public debt?

To answer these questions, we extend the economy of Huggett (1993) to allow for a study of the interaction between public debt and privately-issued safe assets. The economy is composed of heterogeneous households subject to idiosyncratic endowment shocks. Households can buy government bonds or privately-issued safe assets which are backed by pools of loans to other households. The government is committed to repaying its debt, and it does so with uniform lump sum taxes. In contrast, each borrowing household can default in any period. Under default, a household loses any accumulated assets, and it pays an exogenous and stochastic resource cost. As such, default is occasional and occurs whenever the cost of repaying debt exceeds the exogenous cost of default. Because the cost of default is idiosyncratic across households, pools of private loans to households are risk-free for savers. This market structure implies the existence of a single risk-free deposit rate equal to the interest rate on government bonds and privately-issued safe assets. We assume that households borrow anonymously and face a single risky borrowing rate which incorporates a premium tied to the aggregate amount of private defaults in the economy. Therefore, the spread between the borrowing rate and the deposit rate is a negative function of the aggregate recovery rate in the economy.

In our economy, a high enough supply of public debt fully crowds out privately-issued safe assets. In this case, the deposit rate and lending rate—which equal each other—are

¹See Gorton, Lewellen, and Metrick (2012) and Carlsson et al. (2016).

sufficiently high that no household borrows, there are no defaults, and all households hold positive levels of public debt. A version of Ricardian Equivalence holds since local changes in public debt have no effect on consumption allocations or interest rates. If public debt rises today, households experience lower taxes today and anticipate higher taxes in the future, and they respond by increasing their public debt holdings without changing consumption. The opposite occurs in response to a decrease in public debt.

The interesting role for public debt arises if public debt is sufficiently low. In this case, privately-issued safe assets are no longer zero since some households borrow, there are defaults, and a positive interest spread exists between the borrowing rate and the deposit rate. Under this circumstance, a local increase in public debt crowds out privately-issued safe assets less than one for one and reduces the interest spread between the borrowing rate and the deposit rate.

To understand the logic for this channel, suppose default risk were zero so that the borrowing rate and the deposit rate were equal. In this case, crowding out would be one for one, since a borrowing household experiencing lower taxes today in response to a public debt increase would anticipate higher taxes in the future and would respond by reducing its private borrowing one for one with the lower taxes (and the public debt increase) without changing consumption. Now if there is default risk, the borrowing rate faced by a household exceeds the deposit rate at which the government borrows. As a consequence, a public debt increase cannot mechanically have a neutral impact on a borrower's consumption; if private borrowing were reduced one for one with public debt, a borrowing household would maintain the same consumption today but increase its consumption tomorrow. Since the change in government policy is perceived by a borrowing household as a slackening of financial constraints, the borrowing household reduces its private borrowing less than one for one. Furthermore, the decline in private borrowing results in a decline in aggregate defaults in the economy, since default is only optimal if the amount due exceeds the resource cost of default. Therefore, higher public debt results in a lower interest spread.

The quantitative magnitude of this channel is significant. We find that an increase in steady state public debt reduces privately-issued safe assets and the interest spread. This result is in line with the negative empirical relationship between public debt and private safe asset issuance and the negative relationship between public debt and credit spreads in the data.²

²Carlsson et al. (2016) find less than one for one crowd out from public debt to private safe asset issuance. Cortes (2003) and Krishnamurthy and Vissing-Jorgensen (2012) find evidence that higher government debt is associated with lower levels of credit spreads.

The main result of our paper is that the optimal level of public debt induces a positive interest spread and does not fully crowd out private safe asset issuance. Even though full crowd out reduces financial market inefficiencies by slackening financial constraints and reducing the interest spread and the prevalence of costly defaults, it also increases the overall supply of safe assets and the deposit rate which increases inequality by allowing wealthy households to reap higher returns on their savings. Consequently, the optimal level of public debt equalizes the marginal cost of higher inequality with the marginal benefit of a more efficient financial market; these dual considerations imply an optimal policy which induces some private safe asset creation and a positive interest spread.

Finally, we highlight factors which impact the optimal level of public debt. We find that the optimal level of public debt rises if the volatility of idiosyncratic shocks rises or if the cost of default falls. If the volatility of idiosyncratic shocks rises, then the marginal benefit of a more efficient financial market rises, since unlucky households face a greater need to borrow during bad times. Analogously, if the cost of default falls, more defaults occur in equilibrium, which increases credit spreads and increases borrowing rates for unlucky households. In both of these circumstance, the government responds to the change in the environment by increasing public debt as a means of reducing credit spreads and making financial markets more efficient.

Our paper builds on several literatures. First, we contribute to the literature on the role of public debt in economies with limited private credit.³ This literature considers the impact of public debt when privately-issued safe assets are either non-existent or are backed by loans which never default in equilibrium. In contrast, we consider an environment where public debt competes directly with privately-issued safe assets backed by pools of defaultable loans to households. This allows us to analyze the relationship between public debt and the spread between the borrowing rate and the deposit rate and to determine the implications for optimal policy.⁴ Second, we contribute to the literature on optimal public debt management going back to the work of Barro (1979) and Lucas and Stokey (1983).⁵ In contrast to this literature, we allow for lump transfers, which removes

³For example, see Woodford (1990), Holmstrom and Tirole (1998), Aiyagari and McGrattan (1998), Azzimonti, de Francisco, and Quadrini (2014), and Angeletos, Collard, and Dellas (2016), among others. Our result that the optimal level of public debt does not fully crowd out the private lending market is in line with results in Yared (2013) and Azzimonti and Yared (2016).

⁴Our work is complementary to the work of Carapella and Williamson (2015) who also study the relationship between public debt and defaultable private debt. Whereas in their context public debt facilitates private lending by providing collateral, in our setting, public debt crowds out private lending by directly competing with it.

⁵See also Aiyagari, Marcet, Sargent, and Seppala (2002), Werning (2007), and Bhandari, Evans, Golosov, and Sargent (2016), among others.

the role of public debt for smoothing taxes and allows us to focus on public debt's role in providing liquidity. Third, our paper contributes to the literature which studies the consequences of safe asset shortages in economies with aggregate risk.⁶ In contrast to this literature, we focus on the optimal supply of safe assets in steady state and focus on the tradeoff between the public and private provision of safe assets. Finally, our paper contributes to the literature on default and credit spreads in heterogeneous agent economies by considering the implications of public debt on credit market conditions.⁷

The paper proceeds as follows. Section 2 illustrates our qualitative results in a two-period example. Section 3 describes the equilibrium of an infinite horizon economy. Section 4 summarizes the results from the quantitative exercise on the infinite horizon economy. Section 5 concludes and the Appendix includes additional results not in the main text.

2 Two-Period Example

We present a simple two-period economy with $t = 0, 1$ with non-defaultable public debt and defaultable private debt. This example illustrates our main results. We show that an increase in the supply of government bonds crowds out privately-issued safe assets less than one for one and reduces the spread between the borrowing rate and the deposit rate. Moreover, we characterize the optimal policy, and we show that the optimal supply of government debt does not fully crowd out private safe asset issuance.

2.1 Environment

2.1.1 Households

There is a continuum of two types of households indexed by $i = \{L, H\}$, each of size $1/2$. Each household's welfare is

$$\mathbb{E} \sum_{t=0,1} \log c_t^i \tag{1}$$

where c_t^i represents the consumption of household of type i at date t . Households have an endowment y_t^i , where we let $y_0^L = 1 - \delta$ and $y_0^H = 1 + \delta$ for $\delta \in (0, 1)$, and we let $y_1^L = y_1^H = 1$. We therefore refer to H -type households as "rich" households and L -type

⁶For example, see Caballero and Krishnamurthy (2009), Maggiori (2013), Caballero and Farhi (2015), and He, Krishnamurthy, and Milbardt (2016).

⁷See for example Athreya (2002, Livshits, Igor, James MacGee, and Michèle Tertilt (2007), and Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007), among others.

households as "poor" households.

The resource constraint of the economy is

$$c_t^L + c_t^H \leq y_t^L + y_t^H = 1 \quad (2)$$

for $t = 0, 1$ so that the aggregate endowment in the economy is constant.

The government levies a lump sum tax $\tau_t \geq 0$ uniformly across the population at both dates. At date 0, a household i chooses a quantity of assets $a^i \geq 0$ to buy at price q^n , and these assets pay off a^i with certainty at date 1. In addition, at date 0 a household can sell defaultable bonds $b^{i,d} \geq 0$ at price $q^d < q^n$. At date 1, a household receives an idiosyncratic cost of default shock $\gamma \geq 0$ and can decide whether or not to default on these bonds by choosing $D^i = \{0, 1\}$. If $D^i = 0$, the household repays $b^{i,d}$, and if $D^i = 1$, the household does not repay debt $b^{i,d}$, it loses assets a^i , and it suffers a cost to its endowment of size γ . We let γ be determined after decisions are taken at date 0 and before decisions are taken at date 1. The distribution of the shock γ is idiosyncratic across the population of households and is determined according to an exponential probability distribution $f(\gamma) = \exp^{-\gamma}$.

Each household faces the following budget constraints at $t = 0$ and $t = 1$, respectively:

$$c_0^i = y_0^i - \tau_0 - q^n a^i + q^d b^{i,d}, \text{ and} \quad (3)$$

$$c_1^i = y_1^i - \tau_1 + (1 - D^i)(a^i - b^{i,d}) - D^i \gamma, \quad (4)$$

where γ is heterogeneous across the population and stochastically determined.

Clearly, given a default cost γ , the household at date 1 defaults if $\gamma \leq b^{i,d} - a^i$ and it repays its debt $b^{i,d}$ if $\gamma > b^{i,d} - a^i$. This implies that

$$c_1^i = \begin{cases} y_1^i - \tau_1 - \gamma & \text{if } \gamma \leq b^{i,d} - a^i \\ y_1^i - \tau_1 + a^i - b^{i,d} & \text{if } \gamma \geq b^{i,d} - a^i \end{cases} \quad (5)$$

The household's problem is to choose c_0^i , c_1^i , a^i , and $b^{i,d}$ which maximize (1) given (3) and (5). After substitution of (3) and (5) into the household's welfare (1), this means that the household's maximization problem can be written as:

$$\max_{a^i \geq 0, b^{i,d} \geq 0} \left\{ \log(y_0^i - \tau_0 - q^n a^i + q^d b^{i,d}) + \int_0^\infty \log(y_1^i - \tau_1 - \min\{\gamma, b^{i,d} - a^i\}) f(\gamma) d\gamma \right\} \quad (6)$$

In this economy, we refer to $1/q^n$ as the deposit rate and $1/q^d$ as the borrowing rate

and the interest spread as corresponding to the difference between the two. Moreover, the total supply of safe assets equals

$$A = \sum_{i=L,H} a^{i,n}$$

2.1.2 Government

The government chooses taxes τ_0 and τ_1 and government debt $B \geq 0$ to satisfy its dynamic budget constraints at date 0 and 1:

$$0 = \tau_0 + q^g B \text{ and} \tag{7}$$

$$0 = \tau_1 - B \tag{8}$$

where q^g is the price of government bonds. Note that in contrast to households, the government is committed to always repaying its debt.

2.1.3 Financial Intermediation

There is a set of perfectly competitive financial intermediaries in the economy. These financial intermediaries sell safe assets to households at price q^n , and they buy non-defaultable bonds from the government at price q^g and defaultable bonds from the private sector at price q^d . The bonds purchased from the private sector are all pooled together, independently of how much each individual borrower borrows, which is why all households sell their private bonds at the same price q^d . No arbitrage in financial intermediation thus requires that

$$q^g = q^n \tag{9}$$

since the rate of riskless lending from intermediaries to the government must equal the rate of riskless lending from households to financial intermediaries, and this follows from the fact that intermediaries are competitive. Furthermore, no arbitrage requires that

$$q^d = q^n \frac{\sum_{i=L,H} (1 - F(b^{i,d} - a^i)) b^{i,d}}{\sum_{i=L,H} b^{i,d}} \tag{10}$$

so that an intermediary can achieve the same return—taking default risk into account—by buying non-defaultable government bonds versus buying defaultable private bonds. Note that the probability of being repaid by a household borrowing $b^{i,d}$ is $1 - F(b^{i,d} - a_i)$, since default occurs if the default cost γ is below $b^{i,d} - a_i$. Furthermore, if $\sum_{i=L,H} b^{i,d} = 0$

(or equivalently $b^{i,d} = 0$), then $q^d = q^g$, since in this case, any additional infinitesimally small level of borrowing would be valued as risk-free.⁸

2.2 Competitive Equilibrium

2.2.1 Definition of Competitive Equilibrium

Let us substitute (7) – (9) into (3) and (5) to achieve

$$c_0^i = y_0^i + q^n b^{i,n} + q^d b^{i,d}, \text{ and} \quad (11)$$

$$c_1^i = \begin{cases} y_1^i - B - \gamma & \text{if } \gamma < b^{i,d} + b^{i,n} - B \\ y_1^i - b^{i,n} - b^{i,d} & \text{if } \gamma \geq b^{i,d} + b^{i,n} - B \end{cases}, \quad (12)$$

where $b^{i,n} = B - a^i \leq B$. This means that the household's problem in (6) can be rewritten as:

$$\max_{b^{i,n} \leq B, b^{i,d} \geq 0} \left\{ \log(y_0^i + q^n b^{i,n} + q^d b^{i,d}) + \int_0^\infty \log(y_1^i - \min\{B + \gamma, b^{i,d} + b^{i,n}\}) f(\gamma) d\gamma \right\} \quad (13)$$

Moreover, substitution of (11) into the resource constraint (2) at $t = 0$ means that the resource constraint can be rewritten as

$$q^n (b^{L,n} + b^{H,n}) + q^d (b^{H,d} + b^{L,d}) = 0. \quad (14)$$

Equation (14) emerges from the fact that in a closed economy the overall amount of borrowing and lending (in nominal terms) across the population nets out to zero. Equations (13) and (14) are very useful since they illustrate how our economy is analogous to an economy in which there is no government and households exchange defaultable and non-defaultable bonds with each other. The level of government bonds B serves as a limit on the level of non-defaultable debt $b^{i,n}$ which a household can issue at higher bond price $q^n > q^d$. This level of debt can be negative since households can save. If households want to borrow above and beyond this threshold B , they can issue defaultable debt at price q^d . This analogy to an economy with no government is useful to keep in mind for understanding the mechanics of the model.

⁸This follows from L'Hopital's rule using our assumption of an exponential distribution.

Given a government policy B , a competitive equilibrium corresponds to levels of borrowing $\{b^{i,n}, b^{i,d}\}_{i=L,H}$ and bond prices $\{q^n, q^d\}$ which satisfy the following conditions:

1. $\{b^{i,n}, b^{i,d}\}_{i=L,H}$ maximize (13) for $i = L, H$ given bond prices q^n and q^d and debt limit B ,
2. $\{b^{i,d}\}_{i=L,H}$ and $\{q^n, q^d\}$ satisfy the no arbitrage condition between defaultable and non-defaultable debt (10) for $a^i = B - b^{i,n}$, and
3. $\{b^{i,n}, b^{i,d}\}_{i=L,H}$ and bond prices $\{q^n, q^d\}$ satisfy the resource constraint (14).⁹

2.2.2 Characterization of Competitive Equilibrium

In considering the household's problem in (13), note that that when $q^d < q^n$, it is sub-optimal for any household to choose interior non-defaultable debt $b^{i,n} < B$ and positive defaultable debt $b^{i,d} > 0$. In other words, a household does not simultaneously buy government bonds and borrow privately. Any such allocation is dominated by choosing instead non-defaultable debt equal to $b^{i,n} + \epsilon$ and defaultable debt $b^{i,d} - \epsilon$ for $\epsilon > 0$ arbitrarily small since such a perturbation strictly increases date 0 consumption while keeping date 1 consumption fixed. It follows then that $b^{i,d} > 0$ only if $b^{i,n} = B$, otherwise $b^{i,d} = 0$. This implies that default only occurs if $b^{i,n} = B$ and $b^{i,d} > \gamma$. A household therefore only borrows in the private market if it chooses not to buy government bonds.

The household's first order conditions in the solution to (13) are therefore:

$$q^n \frac{1}{y_0^i + q^n b^{i,n}} \geq \frac{1}{y_1^i - b^{i,n}} \text{ if } b^{i,d} = 0 \quad (15)$$

which holds with equality if $b^{i,n} < B$ and

$$q^d \frac{1}{y_0^i + q^n B + q^d b^{i,d}} = (1 - F(b^{i,d})) \frac{1}{y_1^i - B - b^{i,d}} \text{ if } b^{i,d} > 0. \quad (16)$$

It is clear in this economy that the rich save more than the poor because of a consumption smoothing incentive. Moreover, because the economy is closed, this means that only the poor may be constrained by the fact that their level of non-defaultable bonds cannot exceed B . The below lemma states this formally.

Lemma 1 *In a competitive equilibrium, $b^{H,n} < b^{L,n} \leq B$.*

⁹The resource constraint (2) at date 0 and the household's dynamic budget constraint (12) at date 1 together imply that the resource constraint (2) holds at date 1, and can therefore be ignored.

An implication of Lemma 1 is that the defaultable bond price is determined by the behavior of the poor households. Therefore, (10) becomes

$$q^d = q^n (1 - F(b^{L,d})) . \quad (17)$$

We can now characterize the economy if the level of government debt B is sufficiently large.

Proposition 1 (*High Public Debt*) *If $B \geq B^* = \delta/2$, then $A = B$, so that government debt fully crowds out privately-issued safe assets. Moreover, $q^n = q^d = 1$, so that there is no interest rate spread, and $b^{L,n} = \delta/2$, $b^{H,n} = -\delta/2$, and $b^{L,d} = b^{H,d} = 0$.*

This proposition states that if the level of public debt is large enough, then neither rich nor poor households feel constrained in their implicit borrowing limit on non-defaultable bonds, which means that they do not borrow privately. Consequently, the interest spread captured by the difference between q^n and q^d is zero since there is no private borrowing and no default.

More broadly, for high level of public debt, the level of public debt does not affect consumption allocations on the margin. This result is a reflection of the Ricardian Equivalence. If the government issues more debt to finance tax cuts, the private sector will utilize the additional resources from those tax cuts to buy the government debt without changing consumption allocations. This result ceases to hold once government debt becomes sufficiently low that poor households become constrained and decide to resort to private borrowing.

Proposition 2 (*Low Public Debt*) *If $B < B^* = \delta/2$, then*

1. *A rises less than one for one with B ,*
2. *$b^{L,n} = B$ and $b^{H,n}$ is decreasing in B ,*
3. *$b^{L,d} > 0$ and is decreasing in B and $b^{H,d} = 0$,*
4. *q^n is decreasing in B , and*
5. *$q^n/q^d > 1$ and is decreasing in B .*

Proposition 2 provides the first main result of this paper. Starting from low levels of government debt, poor households are borrowing constrained in the sense that they would like to borrow more at the deposit rate but are unable to. Consequently, they

participate in the private borrowing market and occasionally default. The proposition states that, starting from low levels of government debt, as government debt rises, the price of non-defaultable bonds declines. This result follows from the fact that a higher risk-free interest rate induces rich households to buy more of the newly issued government bonds. Moreover, as the level of government bonds rises, the credit spread captured by q^n/q^d decreases. This is because the more the government borrows, the less constrained are the poor households, and the less likely they are to borrow in the private market. This in turn means that default risk in the overall economy declines, which reduces the spread between the non-defaultable and the defaultable bond price.

2.3 Optimal Policy

Let us now consider the problem of a utilitarian government optimally choosing government debt taking into account its effects on the interest rate on defaultable and non-defaultable bonds. Without loss of generality, we can focus our attention on levels of debt $B \leq B^*$, since the choice of debt $B \geq B^*$ is associated with the same allocation and same welfare by Proposition 1.

Using (17) and Propositions 1 and 2, one can write social welfare as equal to:

$$\begin{aligned} & \log(1 + \delta + q^n b^{H,n}) + \log(1 - b^{H,n}) \\ & + \log(1 - \delta + q^n (B + (1 - F(b^{L,d})) b^{L,d})) + \left\{ \int_0^\infty \log(1 - B - \min\{\gamma, b^{L,d}\}) f(\gamma) d\gamma \right\} \end{aligned} \quad (18)$$

where the first line corresponds to the welfare of the rich and the second line corresponds to the welfare of the poor. Note that q^n , $b^{H,n}$, and $b^{L,d}$ are all implicit functions of B , where these necessarily satisfy the resource constraint (14) which can be rewritten as

$$B + b^{H,n} + (1 - F(b^{L,d})) b^{L,d} = 0 \quad (19)$$

and the Euler equations of the rich and the poor households which bind:

$$\frac{1}{1 + \delta + q^n b^{H,n}} = \frac{1}{q^n} \frac{1}{1 - b^{H,n}}, \text{ and} \quad (20)$$

$$\frac{1}{1 - \delta + q^n (B + (1 - F(b^{L,d})) b^{L,d})} = \frac{1}{q^n} \frac{1}{1 - B - b^{L,d}}, \quad (21)$$

where we have substituted in for q^d using (10).

Maximization of (18) taking into account that q^n , $b^{H,n}$, and $b^{L,d}$ are all implicit func-

tions of B defined by (19) – (21) and taking into account the Euler equations yields the following first order conditions to the government's program:

$$\begin{aligned} \frac{\partial q^n}{\partial B} b^{H,n} \left(\frac{1}{c_0^H} - \frac{1}{c_0^L} \right) + \left(q^n \frac{1}{c_0^L} - \int_0^\infty \frac{1}{1 - B - \min\{\gamma, b^{L,d}\}} f(\gamma) d\gamma \right) - \\ \frac{\partial b^{L,d}}{\partial B} f(b^{L,d}) q^n b^{L,d} \frac{1}{c_0^L} \geq 0 \end{aligned} \quad (22)$$

which holds with equality unless $B = B^*$.

This first order condition provides some insight regarding the forces determining the government's optimal choice of debt. On the one hand, a higher level of debt increases the riskless borrowing rate since this is the only way to induce additional lending from the rich. Mathematically, $b^{H,n} < 0$ and $\frac{\partial q^n}{\partial B} < 0$, so higher interest rates benefit the rich who save but hurts the poor who borrow, and because the poor have a higher marginal utility, this effect is negative on the margin, and this is reflected by the negative sign of the first term in (22) (since $c_0^H > c_0^L$). Higher levels of government debt also relaxes the poor's borrowing constraint vis a vis non-defaultable debt and this is captured by the second term in (22), and this effect is positive. Finally, higher public debt levels have an impact on credit spreads. Each individual poor household does to internalize the impact of its borrowing on the aggregate credit spread. A higher level of public debt helps to reduce the level of private debt (since $\frac{\partial b^{L,d}}{\partial B} < 0$) which reduces the level of default and the credit spread in the economy. This benefits the poor households and this is captured by the positive sign on the third term in (22).

The optimal policy thus equalizes the marginal benefit of additional debt to its marginal cost.

Proposition 3 (*optimal policy*) *The optimal policy which maximizes (18) admits $B < B^*$, which implies that $q^d/q^n > 1$ is optimal.*

Proposition 3 states that the optimal policy involves an interior level of public debt with a positive credit spread. If $B = B^*$, then the credit spread is zero and the poor are not borrowing constrained. The cost of lower public debt and increasing the credit spread and the constraints on the poor is second order since the second and third terms in (22) are both zero. In contrast, the benefit of reducing the overall borrowing costs to poor households by tightening borrowing constraints is first order, since the first term in (22) is negative if $B = B^*$. For these reasons, the optimal level of public debt allows some private borrowing and defaults and maintains a positive credit spread.

3 Infinite Horizon Model

In order to analyze the quantitative implications of the theoretical results derived in the two-period model of the previous section, we extend our analysis to an infinite horizon. We build on the heterogeneous agent model of Huggett (1993) by introducing government debt and private defaults. We proceed by describing the environment and we provide a recursive representation of the steady state equilibrium.

3.1 Environment

There are time periods $t = 0, \dots, \infty$. There is a continuum of mass 1 of households labeled by $i \in [0, 1]$. A household i has preferences

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t U(c_t^i), \beta \in (0, 1) \quad (23)$$

where U is continuously differentiable, strictly increasing, and strictly concave. The household's t -period endowment y_t^i and cost of default γ_t^i are stochastic. y_t^i follows a first-order Markov process with c.d.f. G , and γ_t^i is i.i.d. with c.d.f. F . There is no aggregate risk in the economy and the distribution of endowment shocks and default cost shocks is the same across time.

The dynamic budget constraints of the household and the government are identical to those of the two-period economy, which means that we can substitute the government's budget constraint into the household's budget constraint and represent the household's budget constraint (the analog of (11) and (12)) as:

$$c_t^i = y_t^i - \min \left\{ B_t + \gamma, b_t^{i,n} + b_t^{i,d} \right\} + q_t^n b_{t+1}^{i,n} + q_t^d b_{t+1}^{i,d}. \quad (24)$$

$b^{i,n}$ represents non-defaultable debt (i.e., government debt minus a household's safe assets), $b^{i,d}$ represents defaultable debt, and q_t^n and q_t^d represent the prices of non-defaultable and defaultable debt, respectively. By definition, the level of non-defaultable debt cannot exceed the level of government debt:

$$b_{t+1}^{i,n} \leq B_{t+1}.^{10} \quad (25)$$

To close the model, we require the following resource constraint of the economy at any

¹⁰Bond prices and the level of debt are independent of shocks since shocks are idiosyncratic across the population.

date t which is analogous to that in the two-period model:

$$q_t^n \int b_{t+1}^{i,n} d_i + q_t^d \int b_{t+1}^{i,d} d_i = 0. \quad (26)$$

The total net lending in the economy must be zero since the economy is closed. Moreover, no arbitrage in financial market requires that the price of defaultable debt equals the price of non-defaultable debt multiplied by the aggregate recovery rate:

$$q_t^d = q_t^n \frac{\int (1 - D_{t+1}^i) b_{t+1}^{i,d} d_i}{\int b_{t+1}^{i,d} d_i}, \quad (27)$$

where $D_{t+1}^i = 1$ only if $b_{t+1}^{i,n} + b_{t+1}^{i,d} > B_{t+1} + \gamma_{t+1}$.

3.2 Recursive Representation of Steady State

The household's problem is therefore to choose a stochastic sequence $\{c_t^i, b_{t+1}^{i,n}, b_{t+1}^{i,d}\}_{t=0}^{\infty}$ given initial debt levels $\{b_0^{i,n}, b_0^{i,d}\}$, a stochastic shock sequence $\{y_t^i, \gamma_t^i\}$ and a deterministic price and policy sequence $\{q_t^n, q_t^d, B_t\}$ which maximizes (23) subject to (24) and (25).

Let us consider a steady state in which bond prices are constant and equal to q^n and q^d and government policy is constant and equal to B . In this circumstance, the recursive problem of a household can be represented as:

$$\begin{aligned} V(b^n, b^d, y, \gamma) = \max_{b^{n'}, b^{d'}} & \left\{ U(y - \min\{B + \gamma, b^{i,n} + b^{i,d}\}) + q^n b^{n'} + q^d b^{d'} \right\} \\ & + \beta \int_{y'} \int_{\gamma'} V(b^{n'}, b^{d'}, y', \gamma') g(y'|y) f(\gamma') dy' d\gamma' \\ \text{s.t.} & \\ b^{n'} & \leq B \end{aligned}$$

The same reasoning as in the two-period economy implies that since $q^n > q^d$, $b^{d'} > 0$ only if $b^{n'} = B$. Therefore, households always maximize borrowing in the non-defaultable debt market before borrowing in the defaultable debt market. This means that default occurs only if $b^{d'} \geq \gamma$. We can use this observation to reduce the number of state variables in this program. Let b represent the overall borrowing of the household:

$$b = b^n + \frac{q^d}{q^n} b^d.$$

With some abuse of notation, we can rewrite the recursive program of the household with

a single choice variable:

$$V(b, y, \gamma) = \max_{b'} \left\{ \begin{aligned} &U(y - \min(b, B) - \min\{\gamma, b^d(b)\} + q^n b') \\ &+ \beta \int_{y'} \int_{\gamma'} V(b', y', \gamma') g(y'|y) f(\gamma') dy' d\gamma' \end{aligned} \right\} \quad (28)$$

$$\text{for } b^d(b) = \max \left\{ 0, \frac{q^n}{q^d} (b - B) \right\}$$

This program takes into account that if $b \leq B$, then households always repay b and default never occurs. If instead $b > B$, then under no default, households repay $B + (b - B) q^n / q^d$. In other words, they repay non-defaultable bonds, plus the defaultable bonds, which were borrowed at a premium. In contrast, under default, households repay B plus the default cost γ .

Let $g(b, y, \gamma)$ denote the optimal choice of b' that solves the maximization problem (28) and $\psi(b, y, \gamma)$ be the stationary joint distribution of the state in the population. The resource constraint (26) implies that

$$\int_{b, y, \gamma} g(b, y, \gamma) d\psi(b, y, \gamma) = 0 \quad (29)$$

and the no arbitrage condition (27) implies that at $b' = g(b, y, \gamma)$

$$q^d = q^n \frac{\int [1 - D'] b^d(b') d\psi}{\int b^d(b') d\psi} \text{ for } D' = \begin{cases} 0 & \text{if } b^d(b') < \gamma \\ 1 & \text{if } b^d(b') > \gamma \end{cases}. \quad (30)$$

The steady state consists of a distribution $\psi(b, y, \gamma)$, bond prices $\{q^n, q^d\}$, and government policy B , where b' solves (28) given $\{b, y, \gamma\}$ and (29) and (30) are satisfied.

4 Quantitative Exercise

In this section, we parameterize our benchmark model in order to analyze how interest rate spreads depend on government debt to output ratios.

The numerical method used to compute the steady-state is based on a discretization of the state-space. The computational algorithm is standard, with the exception that—relative to a Huggett model—we have to find two prices: q^n and q^d . These prices must clear the markets for risky and risk-free debts. In solving for the equilibrium, we make use of the theoretical properties of our model. For example, we know that the price of risk-free debt must lie in a bounded interval, $q^n \in [\beta, 1]$. In addition, we know that q^d is bounded above by q^n . This suggests a simple procedure to find the steady-state: starting from

an initial guess satisfying those properties, update prices using deviations from market clearing conditions. When there is excess demand for risk-free debt, reduce the guessed price and iterate using a bisection method. When lenders are making losses, increase the price of risky debt q^d . The algorithm is described in more detail in Appendix 7.1

4.1 Parametrization

To be completed

5 Conclusion

To be completed

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7 Appendix

Proof of Lemma 1. We first establish that $b^{H,n} < B$. Suppose by contradiction that $b^{H,n} = B$. From (14) and the fact that optimality requires of $b^{L,d} > 0$ only if $b^{L,n} = B$, it follows that $b^{L,n} < 0$ and $b^{L,d} = 0$. (15) for $i = L$ thus requires

$$q^n \frac{1}{1 - \delta + q^n b^{L,n}} = \frac{1}{1 - b^{L,n}}. \quad (31)$$

There are two cases to consider. In the first case, consider if $b^{H,d} = 0$. In this case, (14) implies that $b^{L,n} = -B$. (15) for $i = H$ requires

$$q^n \frac{1}{1 + \delta + q^n B} \geq \frac{1}{1 - B}. \quad (32)$$

However, (32) contradicts (31).

In the second case, consider if instead $b^{H,d} > 0$, then (16) for $i = H$ requires

$$q^d \frac{1}{1 + \delta + q^n B + q^d b^{H,d}} = (1 - F(b^{H,d})) \frac{1}{1 - B - b^{H,d}}$$

and substitution of (10) implies

$$q^n \frac{1}{1 + \delta + q^n B + q^d b^{H,d}} = \frac{1}{1 - B - b^{H,d}}. \quad (33)$$

Given (14), (31) and (33) imply

$$\frac{1}{1 - B - b^{H,d}} < \frac{1}{1 - b^{L,n}}$$

which implies that $b^{L,n} > B + b^{H,d} \geq 0$, which contradicts the fact that $b^{L,n} < 0$. Therefore, $b^{H,n} < B$.

Given that $b^{H,n} < B$, suppose that $b^{L,n} \leq b^{H,n} < B$. Optimality thus requires that $b^{H,d} = b^{L,d} = 0$, so that (31) characterizes the first order condition for $i = L$ and below is the first order condition for $i = H$

$$q^n \frac{1}{1 - \delta + q^n b^{H,n}} = \frac{1}{1 - b^{H,n}}. \quad (34)$$

Given (14), (34) implies that (31) cannot hold, which is a contradiction. Therefore, $b^{L,n} > b^{H,n}$. ■

Proof of Proposition 1. Suppose that $B \geq B^*$. Let us assume and later establish that the constraint that $b^{L,n} \leq B^*$ does not bind. This means that $b^{L,d} = b^{H,d} = 0$. Therefore, $q^n = q^d$ from (17) and there is no default so that (2) binds. (15) becomes

$$\frac{1}{1 + \delta + q^n b^{H,n}} = \frac{1}{q^n} \frac{1}{1 - b^{H,n}} \quad (35)$$

$$\frac{1}{1 - \delta + q^n b^{L,n}} = \frac{1}{q^n} \frac{1}{1 - b^{L,n}} \quad (36)$$

and the summation of these two first order conditions taking into account (14) implies that $q^n = 1$. Substitution of $q^n = 1$ into (35) and (36) implies that $b^{H,n} = -\delta/2$ and $b^{L,n} = \delta/2$.

We are left to establish that $b^{L,n} \leq B^*$ does not bind. Suppose that it did bind so that $b^{L,n} \geq \delta/2$. From (14), this implies that $b^{H,n} \leq -\delta/2$, and (35)–which must continue to hold–implies that $q^n \leq 1$. There are two cases to consider. Suppose first that $b^{L,d} = 0$. In this case, $b^{L,n} = -b^{H,n} = B^*$. The fact that the constraint that $b^{L,n} \leq B^*$ binds implies that

$$\frac{1}{1 - \delta + q^n B^*} > \frac{1}{q^n} \frac{1}{1 - B^*},$$

but this implies that $q^n > 1$, which is a contradiction. If instead $b^{L,d} > 0$, then (16) taking into account (17) implies

$$\frac{1}{1 - \delta + q^n B^* + q^d b^{L,d}} = \frac{1}{q^n} \frac{1}{1 - B^* - b^{L,d}}$$

but since $b^{L,d} > 0$, this also implies that $q^n > 1$, which is a contradiction. This establishes that $b^{L,n} \leq B^*$ cannot bind. ■

Proof of Proposition 2. Suppose that $B < B^*$. From the proof of Proposition 1, it cannot be the case that the constraint that $b^{L,n} \leq B^*$ does not bind since in that situation, $q^n = 1$, $b^{H,n} = -\delta/2$, and $b^{L,n} = \delta/2$, but this violates the constraint that $b^{L,n} < B^*$. Therefore, $b^{L,n} = B^*$ (which establishes the first statement in part (iii) of the proposition) and the economy is characterized by the Euler equation (35) and the Euler equation (16) for $i = L$, which can be rewritten as

$$\frac{1}{1 - \delta + q^n (B + (1 - F(b^{L,d})) b^{L,d})} \leq \frac{1}{q^n} \frac{1}{1 - B - b^{L,d}} \quad (37)$$

where we have taken into account (17).

Equation (37) binds whenever $b^{L,d} > 0$. We first establish that (37) must bind. If it

does not bind, then $b^{L,d} = 0$ which means that (37) becomes

$$\frac{1}{1 - \delta + q^n B} \leq \frac{1}{q^n} \frac{1}{1 - B}. \quad (38)$$

Moreover, (14) implies that $b^{H,n} = -B$. However, if that is the case, (38) implies that (35) is violated. Therefore, (37) holds with equality.

(14) taking into account (17) can be rewritten as

$$\begin{aligned} \frac{1}{1 + \delta + q^n b^{H,n}} &= \frac{1}{q^n} \frac{1}{1 - b^{H,n}} \\ \frac{1}{1 - \delta + q^n (B + (1 - F(b^{L,d})) b^{L,d})} &= \frac{1}{q^n} \frac{1}{1 - B - b^{L,d}} \\ b^{H,n} + B + (1 - F(b^{L,d})) b^{L,d} &= 0 \end{aligned} \quad (39)$$

(35), (37) which binds, and (39) provides a system of three equations and three unknowns $\{q^n, b^{H,n}, b^{L,d}\}$ for a given B . (35) implies that $b^{H,n}$ rises as q^n rises. Substitution of (35) and (39) into (37) which binds implies that

$$2\delta = (1 + \delta) F(b^{L,d}) b^{L,d} - 4b^{H,n}$$

which means that $b^{L,d}$ rises as $b^{H,n}$ rises. Since $b^{H,n}$ rises when q^n rises, this means that $b^{L,n}$ rises also when q^n rises. Now consider (39). Note that $(1 - F(b^{L,d})) b^{L,d} = \exp^{-b^{L,d}} b^{L,d}$ is rising in $b^{L,d}$ as long as $b^{L,d} < 1$, which must hold to guarantee $c_1^L > 0$. Therefore, by our above reasoning, $b^{H,n} + (1 - F(b^{L,d})) b^{L,d}$ rises as q^n rises, which from (39) means that B must decline as q^n rises.

These observations imply that as B rises, q^n declines, which leads to a decline in $b^{H,n}$ and $b^{L,d}$. Since $b^{L,d}$ declines, q^n/q^d also declines from (17). ■

Proof of Proposition 3. Suppose that the solution admits $B = B^*$. From the proof of Proposition 1, $b^{L,d} = 0$ and (??) equals zero in this case. Moreover, since $b^{L,d} = 0$, (??) also equals zero. However, given that (??) is negative, it follows that the first order condition to the government's program is violated. ■

7.1 Algorithm

To compute the model, we use a discrete approximation to the state space by creating an evenly spaced grid with 1300 points for b between $\underline{b} = -20$ and $\bar{b} = \bar{B} + B$. The

lower bound $\underline{b}$ corresponds to the maximum amount of assets available to agents in this economy. Its value is chosen such that there is an insignificant mass at that point across all the possible values of B considered. The upper bound $\bar{b}$ is larger than the theoretical upper bound on defaultable debt described in the previous section, $\bar{B}\frac{q^d}{q^n} + B$, because $q^d < q^n$. As long as the upper bound is below the natural debt limit, this is without loss of generality. The default shocks are approximated by a four-point distribution, as described in Section ???. The algorithm is described below.

1. Guess the price of risk-free bonds $q_s^n = \frac{q_s^l + q_s^h}{2}$, with $q_1^l = \beta$ and $q_1^h = 1$ in the first iteration.
2. Guess the price of risky bonds $q_s^d = q_s^n$.
3. For each state $\{b, y, \gamma\}$, solve the agent's problem using value function iteration. This determines the policy function $b' = g(b, y, \gamma)$ and the value function $V(b, y, \gamma)$.
4. Given an arbitrary distribution $\psi^s(b, y, \gamma)$, find the stationary distribution as a fixed point to the law of motion

$$\psi_{t+1}^s(b', y_j, \gamma_w) = \sum_{k=1}^{nb} \sum_{v=1}^{ng} \sum_{j=1}^{ny} \pi_{ij} p_w I_{\{b'=g(b_k, y_i, \gamma_v)\}} \psi_t^s(b_k, y_i, \gamma_v)$$

where

$$I_{\{b'=g(b_k, y_i, \gamma_v)\}} = \begin{cases} 1 & \text{if } b' = g(b_k, y_i, \gamma_v) \\ 0 & \text{otherwise.} \end{cases}$$

5. Check market clearing for risky debt

$$|q_s^d - q_s^n[1 - D'/L]| < \epsilon^d$$

where D' is the amount defaulted next period

$$D' = \sum_{\gamma_w} p_w \left[\sum_{b_k, \gamma_v, y_i} b^d(b') I_{\{b^d(b') > \gamma_w\}} \psi^s(b_k, y_i, \gamma_v) \right],$$

given

$$b' = g(b_k, y_i, \gamma_v) \quad \text{and} \quad b^d(b') = \max\{0, \frac{q_s^n}{q_s^d}(b' - B)\},$$

and L denotes total borrowing made from private markets in period t

$$L = \sum_{b_k, \gamma_v, y_i} b^d(b'^s(b_k, y_i, \gamma_v)).$$

- (a) If market clears, move to step 6.
- (b) If market does not clear, update

$$q_{s+1}^d = q_s^n [1 - D'/L]$$

and repeat steps 2 to 5 until convergence of q_s^d .

6. Check market clearing for risk-free debt

$$\left| \sum_{b_k, \gamma_v, y_i} g(b_k, y_i, \gamma_v) \psi^s(b_k, y_i, \gamma_v) \right| < \epsilon^n$$

- (a) If market clears, stop.
- (b) If market does not clear, then use the bisection method to update the guess for q_s^n

i. Set $q_{s+1}^l = q_s^n$ and $q_{s+1}^n = \frac{q_{s+1}^l + q_s^h}{2}$ when

$$\sum_{b_k, \gamma_v, y_i} g(b_k, y_i, \gamma_v) \psi^s(b_k, y_i, \gamma_v) < 0$$

ii. Set $q_{s+1}^h = q_s^n$ and $q_{s+1}^n = \frac{q_s^l + q_{s+1}^h}{2}$ otherwise.