Modeling House Prices*

Damien Gaumont
Université Panthéon-Assas (Paris II), CRED and TEPP

Randall Wright
UW-Madison, FRB Chicago, FRB Minneapolis and NBER

Yu Zhu
Bank of Canada

February 14, 2017

Abstract

This paper develops some novel models of housing markets based on price posting, using both noisy search and directed search. The models generate price dispersion and price stickiness – i.e., they are consistent with the observation that some sellers do not adjust listed prices when market conditions change, something commentators say is *puzzling* and *challenging* for standard economic theory. We also contribute to the literature on price dispersion, in general, by considering big-ticket durable goods and agents that trade infrequently. The framework is tractable: comparative statics are characterized and parametric examples are solved explicitly. Using English housing data, we show the empirical relevance of the directed search model. This contributes to the literature of directed-search by showing how to identify and estimate a directed-search model.

Key words: Housing Markets, Search, Sticky Prices, Price Dispersion

JEL classification nos: D4, R3

*For input we thank Jim Albrecht, Chao Gu, Peter Norman, Allechar Serrano. Wright acknowledges support from the Ray Zemon Chair in Liquid Assets at the Wisconsin School of Business. The usual disclaimers apply.

"I don't think anyone wants to argue that house prices are sticky." Stephen Williamson, *Sticky Prices and the Keynesian Narrative* (New Monetarist Economics blog, May 8, 2011).

"Home prices are subject to inertia and are sticky downward." Karl E. Case, *The Central Role of Home Prices in the Current Financial Crisis: How Will the Market Clear?* (BPEA Fall 2008).

1 Introduction

This paper develops some novel dynamic models of housing markets based on price posting by sellers and different types of search by buyers. One version has *noisy search*, which is defined in the literature to mean that buyers sample a random number of sellers each period. Another version has *directed search*, which means that buyers see all the prices listed by all sellers before deciding where to look, but can visit only one house, each period. Both approaches generate price dispersion, and given this, they can easily generate price stickiness – i.e., they are consistent with the observation that some sellers do not adjust their listed prices when market conditions change. The paper also contributes to the literature on price dispersion, in general, by incorporating big-ticket durable goods and agents that trade infrequently.

The fact that these models can generate price stickiness is relevant because it is often asserted that this is observed in the data. The empirical work of Genesove and Mayer (2001), e.g., leads them to conclude reservation prices are inflexible downward. As they put it, "sellers may be unable or unwilling to accept market prices for property in the down part of the cycle." They find this puzzling, and challenging for standard economic theory, but suggest that "loss aversion" along the lines of Tversky and Kahneman (1991) can account for it: when the market declines, "Owners who are averse to losses will have an incentive to attenuate that loss by deciding upon a reservation price that exceeds the level they would set in the absence of a loss, and so set a higher asking price, spend a longer time on the market, and receive a higher transaction price upon a sale." They also describe sellers as "overly optimistic."[1]

Emmerling et al. (2015) express similar views, and say seller behavior is in line with "regret theory," where the utility derived from accepting an offer depends upon past and future forgone offers. Case et al. (2003) say sellers are reluctant to trade below the "psychological price" at which the property was purchased after demand falls. Fabozzi et al. (2009) propose a theory based on "unrealistic expectations." See also Levitt and Syverson (2005), Buisson (2014) and Eyster et al. (2015). In general, these papers can be described as saying the following: house prices look sticky; this is a puzzle; and to understand it we must move away from standard theory with its restrictive assumptions like rational expectations and expected utility maximization. Our goal is to provide a counterpoint. While house prices may well look sticky, we show that this is not inconsistent with the discipline of rational expectations and expected utility maximization given the realistic frictions captured by rudimentary search theory.

Our argument does not rely on "menu costs," which seem to us too small to significantly influence the pricing of such big-ticket items. On that, Merlo et al. (2015) use transaction records for properties in England to study models with fixed costs of changing prices. They say "One of the most striking features of the Merlo and Ortalo-Magne [2004] data is that housing list prices appear to be highly (though not completely) sticky."[2] Moreover, "this finding presents a challenge, since the conventional wisdom is that traditional, rational, forward-

[1] To give some detail, they examine data from Boston between 1990 and 1997, where they find that sellers subject to losses set prices that are higher by 25% to 35% of the difference between the expected selling price and their purchase price, and end up with selling at prices 3% to 18% of this difference.

[2] To given some more detail, 77% of sellers in the UK sample never changed their listed price prior to selling, 18% changed only once, 4% twice, and 1% thrice. Knight (2002) finds something similar for US data.

looking economic theories are unable to explain extreme price stickiness of this sort, unless there are large menu costs associated with price revisions... [but] this cost is unlikely to be large." Interestingly, they claim a small menu cost "is sufficient to generate the high degree of list price stickiness observed in the data." Whatever one thinks of this claim, it is surely interesting to explore alternative explanations.[3]

One candidate explanation is learning. However, an analysis of learning in labor markets by Burdett and Vishwanath (1988), where workers do not know the distribution of potential offers in the market, implies reservation wages fall over time rather than stay rigid, so it is not clear how learning helps explain stickiness. Taylor (1999) models information frictions in housing markets explicitly, where buyers must make inferences about houses, and sellers try to manipulate the process by pricing strategically.[4] While this is interesting, again it is not clear how it explains stickiness. Yet we certainly agree with Taylor that modeling the sale of a single big-ticket item like a house differs in interesting ways from producers and consumers trading repeatedly in the same market. Therefore, we consider durable goods and agents that trade infrequently, which requires amending off-the-shelf models of search theory.

[3]Merlo et al. (2015) acknowledge that listed prices might be just a starting point for bargaining – i.e., they might be cheap talk – but do not model this explicitly (for that, see Menzio 2007 or Albrecht et al. 2014). Yet posted prices must mean something, otherwise it would not be much of a puzzle that they are sticky. Han and Strange (2014) find that houses do not always sell at posted prices, but a notable fraction do: "although it would be incorrect to model a home seller's asking price as a simple posted price (as with a good), it would also be incorrect to seethe asking price as being meaningless."

[4]On the one hand, if buyers see the history of a house on the market, "a prospective buyer who walks away from a high-priced house conveys little information concerning quality to subsequent consumers... but a prospective buyer who walks away from a low-priced house transmits a very strong signal regarding his assessment of quality." On the other hand, when the history is unobservable, "high initial prices promote rather than discourage herding. In this case, the seller's optimal policy is to post a low initial price in an effort to sell the house quickly, i.e. before herding commences." This is related to, e.g., Wolinsky (1983) or Bagwell and Riordan (1991), where pricing in general is a signal.

While our formal modeling approach is novel, it is consistent with much discussion in the housing literature. In a recent survey, Davis and Van Nieuwerburgh (2015) argue this market has some unique features, inlcuding the fact that "housing is infrequently traded and trades are subject to search frictions and large transaction costs." Similarly, in their survey Han and Strange (2015) argue that models designed to study other markets cannot be used for housing without modification. They go on to suggest three aspects of housing are particularly important: "First, houses are heterogeneous... which means that housing markets can be thin. Second, housing transactions take place under uncertainty... [where] buyers and sellers must search for each other... Third, there are important market frictions. Search is a costly activity. In addition, the exchange of housing has important transaction costs, including brokerage fees, transaction taxes and moving costs. In this situation, the housing market clears through price and time." Our approach tries to be consistent with all these ideas.

To summarize the motivation, here is the question: While loss aversion, regret theory, psychological prices and unrealistic expectations may or may not be features of reality, do we *need* them to explain house prices? An alternative that we think should be part of the conversation is search theory, which has been successful in related applications. However, our setup differs from existing search-based housing models.[5] In many such models, sellers of houses, like workers looking for a job, sample offers sequentially until deciding to stop. In our model with noisy search, sellers are more like the firms in Burdett and Judd (1993) or Burdett and Mortensen (1998), posting prices and waiting for someone to accept. In the version with directed search, they post prices to attract specific types of buyers, as

[5]There is a long literature on search models of housing, going back to Wheaton (1990), and indeed, going back further to Simon (1955). Maury and Tripier (2014), Albrecht et al. (2016), and Moen et al. (2016) are recent examples especially relevant from our perspective. The abovementioned surveys provide many more citations.

in Moen (1997) or Shimer (1996). These models capture some salient features of housing markets, but to focus on essentials, abstract from others, including real estate agents and bargaining, although these are both discussed briefly below.

Noisy search models have an endogenous distribution of listed prices $F(p)$, with density $f(p)$ and support $\mathcal{F} = [\underline{p}, \bar{p}]$, where the payoff of a seller is the same $\forall p \in \mathcal{F}$. This is not a knife-edge case; it is the unique equilibrium and $F(p)$ is determined endogenously so that sellers posting higher p get greater profits but take longer to sell. This can lead to stickiness: when market conditions change, $F(p)$ and $\mathcal{F}$ respond, but as long as the new and old supports overlap, some sellers can leave p the same. Suppose $\mathcal{F}_t$ shifts down to $\mathcal{F}_{t+1}$. Then a seller that was low in the support $\mathcal{F}_t$ can stick to the same p, making him high in the new $\mathcal{F}_{t+1}$, because while his probability of a sale falls *there is no profitable deviation* to a different price. The directed search model has heterogeneous buyers, each with a valuation u, distributed with density $g(u)$ and support $\mathcal{G} = [\underline{u}, \bar{u}]$. Different sellers cater to different buyer types by listing different prices, inducing a distribution $F(p)$. Again, high price sellers earn greater profits but take longer to transact, and when market conditions change, a long as the new and old supports overlap, some sellers can leave p the same because *there is no profitable deviation.*

This demonstrates that sticky house prices do not require deviating from rational expectations, abandoning expected utility maximization, or adopting devices like menu costs. A related point has been made using a version of Burdett-Judd by Head et al. (2013), although their focus was on generating sticky nominal prices in monetary economies (see also Caplin and Spulber 1987 and Eden 1994). While this is similar to our qualitative point, as mentioned, we amend the standard Burdett-Judd specification in several ways to better capture key feeatures of housing markets. In particular, when one buys or sells a house in the model, one takes oneself off the market, implying an opportunity cost of trade. More original is the setup with directed search. While there are other models with heterogeneity, posting and directed search, including Moen (1997), Mortensen and Wright (2002), Shi (2002,2006) or Shimer (2005), they have not been used in this way.

To finalize these introductory remarks, we emphasize that our interest is in *residual* price dispersion. It is no surprise that big houses in good neighborhoods cost more than small houses in bad neighborhoods. This is analogous to residual wage dispersion, where after controlling for observable characteristics, similar workers can be paid differently (Mortensen 2005a). Empirical analyses of housing find non-negligible residual price dispersion across observationally similar houses. Leung et.al. (2006) document variation in prices in Hong Kong after controlling for quality. Merlo and Ortalo-Magne (2004) find in UK data that houses with higher listed prices sell at higher prices but take longer to sell. De Wit and van der Klaauw (2013) find something similar in Dutch data. Maury and Tripier (2014) suggest that sellers' strategies trade off price against time to sell and this leads to residual dispersion. These findings are consistent with our theories.

The rest of the paper is organized as follows. Section 2 describes features of the environment that are common to both models. Sections 3 and 5 present the noisy and directed search versions, respectively. Section 6 discusses the implications for sticky prices. Section 8 concludes. Technical results are relegated to Appendices, which also contain a model with bargaining, instead of posting, for comparison.

2 Environment

Time is discrete and continues forever. In the city, there is a continuum of houses with measure 1 owned by homeowners. Among the homeowners, H are happy with their houses and S want to migrate to another city and put their houses on market. Each homeowner has only one house. There is a continuum of buyers with measure B who wants to buy one house. Once a buyer buys a house, he becomes a happy homeowner until he receives a relocation shock μ. In this event, he wants to move to another city and becomes a seller. After selling his house, the seller moves to a different city. What matters is the seller-buyer ratio, $\tau = S/B$. All agents discount the future with factor $\beta = 1/(1+r)$, $r > 0$. Houses are homogeneous ex ante, but buyers have idiosyncratic tastes. For simplicity, a buyer either likes a house or does not; in the former case he would get a flow utility u_H as a happy homeowner if he bought it, and in the latter case he would get flow utility 0. In the language of the literature, these are match-specific valuations – it is not the case that some houses are better than others, but one buyer might like house 1 and not house 2, while another likes house 2 but not house 1. In the directed search model, u_H varies across buyers; in the noisy search model it is the same for all buyers. Buyers and sellers enjoy flow utility u_B and u_S with $u_H > u_S \geq u_B \geq 0$.

After trading, the seller moves to another identical city and an identical buyer migrates into the city. This formation keeps population fixed.[6] To motivate this specification, consider the classic search-and-bargaining theory of middlemen by Rubinstein and Wolinsky (1987). They also assume buyers and sellers exit after trade and are replaced by new agents. Nosal et al. (2015) extend that framework to allow buyers and sellers to stay or exit probabilistically after each trade. The case where they stay forever, trying to trade each period, is easier because agents have no opportunity cost of continued search when executing a trade. It is true here, too, that things are easier if they stay in the market and trade each period, but that would not be natural in a theory of housing. In fact, once one sells or

[6]This means we do not have to worry about whether an individual should buy first or sell first, which is realistic but generates complications, including multiple steady states and dynamic equilibria. See Moen et al. (2016) and references therein.

buys a house, at some point in the future one may transact again, but since such trades are infrequent, our modeling choice is to have agents exit after trade as a reasonable approximation.

At each date t, every seller lists a price p_t for his house. In general, buyers observe a sample from the price distribution. In the directed search model, as in Moen (1997), Shimer (196) and many other papers, buyers see all the listings and choose to visit one seller per period. In the noisy search model, as in Burdett and Judd (1983) and many other papers, each period they sample a random number n of listings for houses that they like, each an i.i.d. draw from the equilibrium CDF, $F_t(p)$. Denote the probability a buyer observes n such prices by b_n, with $\Sigma_{n=0}^{\infty} b_n = 1$ and $\Sigma_{n=0}^{\infty} b_n n = \mathbb{E}n < \infty$. We show in the Appendix how to make n a choice, but for now it is exogenous. Of course, if he sees $n > 1$ prices, a buyer buys at the lowest – given the obvious result that (in equilibrium) no seller posts a p that buyers find unacceptable. For simplicity, although a buyer can sample more than one seller, a seller is not sampled by more than one buyer at a time, which implies the parameter restriction $\mathbb{E}n \leq \tau$.[7]

Let s_n denote the probability a given seller has a buyer that sees n houses he likes that period, including that of the given seller. It is not hard to see that $\tau s_n = n b_n$. The seller gets a flow utility u_S from his house while it is on the market, and gets a surplus $p - c$ after a sale, where $c \geq 0$ is a transaction cost, e.g., transfer taxes, realtor fees or moving expenses. We could alternatively impose this cost on buyers, or on both, but as in elementary tax-incidence theory that would not change the payoffs, since the cost is passed through in prices. A buyer gets a flow utility normalized to 0 while searching, and a surplus $u - p$ after

[7] This means we do not have to worry about multiple buyers trying to get the same house. If that were to happen, sellers generally would not want to commit to posted prices, but would run auctions, as discussed in different contexts by Julien et al. (2000,2008), and in the housing context by Albrecht et al. (2004). On this, we sacrifice realism for tractability.

a purchase. It is worth emphasizing that sellers can change prices each period at no cost. Burdett and Menzio (2014) study a version of Burdett-Judd with a menu cost to changing p, which is much more complicated, so we ignore it. Moreover, our point is to show how stickiness can emerge without such costs.

3 Model I: Noisy Search with Homogenous Agents

We begin with an extension/application of Burdett and Judd (1983). Given that a buyer accepts the lowest price he sees, as long as it is as least as good as the return to continued search, his value function is

$$V_t^B = u_B + \beta \sum_{n=1}^{\infty} b_n \int \max(V_{t+1}^H - V_{t+1}^B - p, 0) dF_{n,t}(p) + \beta V_{t+1}^B \tag{1}$$

where $F_{n,t}(p) = 1 - [1 - F_t(p)]^n$ is the distribution of the lowest price among n draws from the distribution of listed prices $F_t(p)$. The max operator in (1) indicates a buyer only buys if the best price satisfies $V_{t+1}^H - V_{t+1}^B \geq p$, which defines his reservation price[8]

$$R_t = V_{t+1}^H - V_{t+1}^B. \tag{2}$$

A happy home owner's value function is

$$V_t^H = u_H + \mu\beta \left(V_{t+1}^S - V_{t+1}^H\right) + \beta V_{t+1}^H$$

Sellers choose prices to maximize their payoff, as described by

$$V_t^S = u_S + \beta \max_{p \in [c, R_t]} \left\{ \sum_{n=1}^{\infty} s_{n,t} [1 - F_t(p)]^{n-1} (p - c - V_{t+1}^S) \right\} + \beta V_{t+1}^S \tag{3}$$

where $[1 - F_t(p)]^{n-1}$ in the summation is the probably that the listed p beats the other $n - 1$ prices seen by a buyer with n observations. The number of sellers in each period S_t follows

$$S_{t+1} = S_t + \mu H_t - b_0 B_t$$

[8]Notice $R_t < u$ because buyers exiting the market after trading; if they were to stay and continue searching, they would accept any $p \leq u$, which would be easier but not realistic.

where $B_t = B$ by construction.

In equilibrium, the highest listed price must be $\bar{p}_t = R_t$, because $\bar{p}_t < R_t$ implies the highest price seller can profitably deviate to $\bar{p}_t = R_t$. Moreover, all other prices in the support $\mathcal{F}_t$ must yield the same profit, which for $\bar{p}_t = R_t$ is

$$V_t^S = \beta s_{1,t}(R_t - c - V_{t+1}^S) + \beta V_{t+1}^S, \tag{4}$$

because by definition the highest price in the support never beats another price. Equating (3) and (4) and rearranging, we get

$$\frac{s_1(R_t - c - V_{t+1}^S)}{p - c - V_{t+1}^S} = \sum_{n=1}^{\infty} s_n \left[1 - F_t(p)\right]^{n-1}. \tag{5}$$

Since $F_t(\underline{p}_t) = 0$, (5) can be solved for $\underline{p}_t$ as a weighted average of the buyers' reservation price $R_t = V_{t+1}^H - V_{t+1}^B$ and the sellers' transaction plus opportunity cost,

$$\underline{p}_t = \eta \left(V_{t+1}^H - V_{t+1}^B\right) + (1 - \eta)\left(c + V_{t+1}^S\right), \tag{6}$$

where $\eta = s_1 / \sum_{n=1}^{\infty} s_n$ is the probability that when a seller gets a buyer the latter currently has no other house that he likes. Notice that this would be the outcome in a bargaining model where η is the sellers' bargaining power. In any case, we present the following without proof, as it is standard:[9]

Lemma 1 *If $b_0 + b_1 = 1$, there is one price and it gives all the surplus to sellers; if $b_1 = 0$, there is one price and it gives all the surplus to buyers; otherwise there is price dispersion.*

Lemma 2 *Given R_t and V_{t+1}^S, there is a unique solution to (5), $F_t(p)$, which is continuous on $\mathcal{F}_t = [\underline{p}_t, \bar{p}_t]$ and differentiable on $(\underline{p}_t, \bar{p}_t)$.*

[9] See Burdett et al. (2015) for a recent presentation, and for more details on the parametric examples presented below.

These results are also intuitive. For instance, it is easy to see why $\mathcal{F}_t$ is an interval with no gaps or mass points. If there were a gap between p' and p'', e.g., a seller posting p' could profitably deviate to $p' + \varepsilon$, as this does not lower his probability of a sale. And if there were a mass point at p', a seller posting p' could profitably deviate to $p' - \varepsilon$, as this increases his probability of a sale discretely while only sacrificing ε on the price.

In Appendix B we reduce the above equations to a forward-looking dynamical system in $\left(V_t^B, V_t^H, V_t^S, \tau_t\right)$ where $F_t(p)$ has been conveniently eliminated:

$$\begin{aligned}
V_t^B &= u_B + \beta(1 - b_0 - b_1)(V_{t+1}^H - V_{t+1}^B - V_{t+1}^S - c) + \beta V_{t+1}^B && (7)\\
V_t^H &= u_H + \mu\beta V_{t+1}^S + (1-\mu)\beta V_{t+1}^H, && (8)\\
V_t^S &= u_S + \beta\frac{b_1}{\tau_t}(V_{t+1}^H - V_{t+1}^B - V_{t+1}^S - c) + \beta V_{t+1}^S, && (9)\\
\tau_{t+1} &= (1-\mu)\tau_t + \frac{\mu}{B} + b_0 - 1 && (10)
\end{aligned}$$

Any sequence $\left(V_t^B, V_t^H, V_t^S, \tau_t\right)$ solving this system constitutes an equilibrium, as long as it is nonnegative and $\lim_{T\to\infty}\beta^T V_T^j = 0$.[10] Given a solution (V_t^B, V_t^H, V_t^S), we can construct sequences for $F_t(p)$ and other endogenous variables using the above conditions. It turns out what matters is the differences of the value functions $V_t^H - V_t^B = R_{t-1}$ and $V_t^H - V_t^S$. A stationary equilibrium (or steady state) is one where the variables are constant over time. We verify in Appendix D the following:

Proposition 1 *There is a unique stationary equilibrium. As long s $b_1 > 0$ and $b_0 + b_1 < 1$, it entails nondegenerate price dispersion. The endogenous variables*

[10] Nonnegativity is simply a participation requirement, while the limiting condition is standard in dynamic models; see Rocheteau and Wright (2013) for a recent discussion of the technical issues, but it is intuitively obvious that discounted payoffs must be bounded.

$\langle F(p), \bar{p}, \underline{p}, V^B, V^H, V^S \rangle$ *are given by* $\tau = \frac{1}{B} - \frac{1-b_0}{\mu}$ *and*

$$\sum_{n=1}^{\infty} n b_n [1 - F(p)]^{n-1} = b_1 \frac{V^H - V^B - V^S - c}{p - V^S - c} \tag{11}$$

$$V^H - V^B - V^S - c = \Sigma = \frac{r\left[\frac{1+r}{r+\mu}(u_H - u_S) - \frac{1+r}{r} u_B - c\right]}{r + 1 - b_0 - b_1 + \frac{r}{r+\mu}\frac{b_1}{\tau}}, \tag{12}$$

$$V^B = \frac{1+r}{r} u_B + \frac{1 - b_0 - b_1}{r} \Sigma, \tag{13}$$

$$V^H = \frac{1+r}{r+\mu} u_H + \frac{\mu}{r+\mu}\left(\frac{1+r}{r} u_S + \frac{b_1}{\tau}\frac{\Sigma}{r}\right), \tag{14}$$

$$V^S = \frac{1+r}{r} u_S + \frac{b_1}{\tau}\frac{\Sigma}{r} \tag{15}$$

$$\bar{p} = R = V^H - V^B, \tag{16}$$

$$\underline{p} = \frac{\mathbb{E}n - b_1}{\mathbb{E}n}\left(V^S + c\right) + \frac{b_1}{\mathbb{E}n}\left(V^H - V^B\right). \tag{17}$$

Proposition 1 yields closed-form solutions for all variables except $F(p)$, which in general is characterized by (11) as the solution to an polynomial of infinite order. However, we can still say a lot about $F(p)$, in general, and solve it explicitly for some parametric specifications of the matching probabilities b_n. As a simple example, consider first the minimal case that avoids the degenerate outcomes in Lemma 1, where $b_1 > 0$, $b_2 > 0$ and $b_n = 0 \ \forall n > 2$. In Appendix C we show this is an endogenous outcome for a certain specification of costly search, as in the original Burdett-Judd (1983) paper, but here we take it as a primitive.

In any case, when $b_n = 0 \ \forall n > 2$ (11) is linear in $F(p)$, and easily solves for

$$F(p) = 1 - \frac{b_1}{2b_2}\left(\frac{V^H - V^B - V^S - c}{p - V^S - c} - 1\right).$$

One can use (12) and (15) to obtain a closed form solution for $F(p)$. It is straightforward to show $F''(p) < 0$, so the density is decreasing. If $b_1 = \pi$ and $b_2 = 1 - \pi$, then π is a measure of sellers' market power used below for comparative statics.

As another example, consider a Poisson distribution for n with parameter $\lambda > 0$, i.e., $b_n = \lambda^n e^{-\lambda}/n!\ \forall n$. Notice $\mathbb{E}n = \lambda$. One can show using standard formulae that

$$\begin{aligned} F(p) &= 1 - \frac{1}{\lambda} \log \frac{V^H - V_B - V^S - c}{p - V^S - c} \\ \underline{p} &= \left(1 - e^{-\lambda}\right)\left(V^S + c\right) + e^{-\lambda}\left(V^H - V^B\right). \end{aligned}$$

One can also check $F''(p) < 0$, so again the density is decreasing. A Poisson distribution is interesting, among other reasons, because it makes it relatively easy to endogenize search intensity in these kinds of models (Mortensen 2005a,b).

Finally, consider a logarithmic distribution with parameter $\omega \in (0,1)$, i.e., $b_0 = 0$ and $b_n = -\omega^n / n \log(1-\omega)\ \forall n > 0$. Now the mean number of houses buyers see and like is $\mathbb{E}n = -\omega/(1-\omega)\log(1-\omega)$, and an increase in ω puts higher probabilities on bigger n. One can solve for

$$\begin{aligned} F(p) &= 1 - \frac{1}{\omega}\left(1 - \frac{p - V^S - c}{V^H - V^B - V^S - c}\right) \\ \underline{p} &= \omega\left(V^S + c\right) + (1-\omega)\left(V^H - V^B\right) \end{aligned}$$

Here $F(p)$ is linear – i.e., the p distribution is uniform. See Figure 1 (all Figures are at the end). We conclude from these examples that, after some routine algebra, the theory delivers nice, clean results.

	B	u_B	u_H	u_S	c	μ	π	ω
$F(p)$	$-$	$+$	$-$	$-$	$\pm$	$+$	$-$	$+$
$\bar{p}$	$+$	$-$	$+$	$+$	$\pm$	$-$	$+$	$-$
$\underline{p}$	$+$	$-$	$+$	$+$	$+$	$-$	$+$	$-$
Δ	$-$	$-$	$+$	$-$	$-$	$\pm$	$+$	$-$
τ	$-$	0	0	0	0	$+$	0	0
V^B	$-$	$+$	$+$	$-$	$-$	$\pm$	$-$	$+$
V^H	$+$	$-$	$+$	$+$	$-$	$-$	$+$	$-$
V^S	$+$	$-$	$+$	$+$	$-$	$-$	$+$	$-$

Table 1: Effects of Parameters with Noisy Search

Table 1 reports the effects of parameters on the endogenous variables, in general, and in the examples (see Appendix C for details). These accord well with intuition. An increase in the seller-buyer ratio τ, e.g., increases $F(p)\ \forall p$, and decreases $\underline{p}$ and $\bar{p}$. Hence, a fall in demand or rise in supply along the extensive margin, captured by higher τ, shifts the density $F'(p)$ to the left and increases the spread $\Delta \equiv \bar{p}-\underline{p}$. Similarly, a fall in demand along the intensive margin, captured by lower u, shifts $F'(p)$ left, but in this case decreases the spread Δ. And an increase in transaction cost c shifts the density $F'(p)$ right while decreasing Δ. In the example with $b_1 = \pi$ and $b_2 = 1-\pi$, higher π raises sellers' market power, and shifts the density $F'(p)$ right. In the logarithmic example, higher ω makes it more likely that buyers see more houses they like, which raises their market power, and shifts $F'(p)$ left.[11] Figure 1 are the resulting price densities when b_n follows Three-point, Poisson and logarithmic distributions, respectively.

Figure1 about here

4 Model II: Noisy Search with Heterogenous Buyers

Although the noisy search model with homogenous buyers generates residual price dispersion, trade-offs between price and time on market, and sticky house prices, the resulting price density is usually non-increasing. This is very different from the bell shape residual price density observed from data. One natural extension that may solve this problem is to have heterogenous buyers who value houses different. This section explore this possibility. Now let the buyers' value u follows a distribution $G(u)$, with density $g(u)$ and support $\mathcal{G} = [\underline{u}, \bar{u}]$. We assume that

[11] As another example, Moen et al. (2016) say it is a "challenge" for search-based models to generate negative comovement between the stock of houses for sale and transactions volume (see also Diaz and Jerez 2013). It does not seem hard in this model, as a decrease in demand on the extensive margin yields fewer transactions and leaves more houses on the market.

all sellers obtain the same utility after moving to a new city although they were heterogenous as buyers or happy homeowners. The rest of the environment is exactly the same as in Model I. In this section, we focus on the steady state equilibrium. Let V_u^B and V_u^H be the value function of buyers and happy homeowners whose value for owning is u, respectively.

$$
\begin{aligned}
V_u^B &= u_B + \beta \sum_{n=1}^{\infty} b_n \int \max\left(V_u^H - V_u^B - p, 0\right) dF_n(p) + \beta V_u^B, && (18) \\
V_u^H &= u + (1-\mu)\beta V_u^H + \mu\beta V^S, && (19) \\
V^S &= u_S + \beta \frac{b_1}{\tau} \max_p \left(p - c - V^S\right)\left[1 - G\left(R^{-1}(p)\right)\right] + \beta V^S. && (20)
\end{aligned}
$$

where $R(u) = V_u^H - V_u^B$ is the reservation price for type u buyer and $F_n(p) = 1 - (1 - F(p))^n$. Firms maximize their profit by choosing prices. The highest price that is charged in the equilibrium is

$$
\bar{p} = \max_p \left(p - c - V^S\right)\left[1 - G\left(R^{-1}(p)\right)\right]. \tag{21}
$$

A sellers who charges the highest price understands that a buyer will buy from him if and only if he has seen only one listing and his reservation value is above the price charged. Therefore, the highest price is chosen to balance between the revenue upon a sale and the probability of selling.Then any price that gives the same profit is charged in equilibrium which means

$$
\frac{b_1\left(\bar{p} - c - V^S\right)\left[1 - G\left(R^{-1}(\bar{p})\right)\right]}{\left[1 - G\left(R^{-1}(p)\right)\right]\left(p - c - V^S\right)} = \sum_{n=1}^{\infty} n b_n (1 - F(p))^{n-1} \tag{22}
$$

Notice compared to (5), the selling probability is adjusted by $1 - G\left(R^{-1}(p)\right)$. This reflects the fact that a seller may meet a buyer whose reservation value is below the listed price. Lastly, the law of motion of sellers implies that in steady state

$$
\tau = \frac{1}{B} - \frac{\sum_{n=1}^{\infty} b_n \int F_n(R(u))\, dG(u)}{\mu}. \tag{23}
$$

Notice when G is degenerate, we get back the equations in Section 3. A steady state equilibrium of a list $\left\langle \tau, \bar{p}, F, V_u^B, V_u^H, V^S \right\rangle$ that satisfies (18)-(23).

Due to the heterogeneity, it is difficult, if not impossible, to solve for the equilibrium. So we resort to numeric methods. The algorithm we use contains a nested fixed point problem:

- First, fix a $\tau_0 > \mathbb{E}n$. Come up with intitial guess for V_u^B, V_u^H, V^S denoted by $V_{u,0}^B$, $V_{u,0}^H, V_0^S$ where $V_{u,0}^B$, $V_{u,0}^H$ are both positive increasing functions and $R_0 = V_{u,0}^H - V_{u,0}^B$ is positive and strictly increasing.

- Compute the highest price $\bar{p}_0 = \arg\max_p \left(p - c - V_0^S\right)\left[1 - G\left(R_0^{-1}(p)\right)\right]$. Then use $\bar{p}_0, V_{u,0}^B$, $V_{u,0}^H, V_0^S$ to substitue $\bar{p}, V_u^B$, V_u^H, V^S in (22) and obtain F_0.

- Update using

$$
\begin{aligned}
V_{u,1}^B &= u_B + \beta \sum_{n=1}^{\infty} b_n \int \max\left(R_0 - p, 0\right) dF_{n,0}(p) + \beta V_{u,0}^B, \\
V_{u,1}^H &= u + (1-\mu)\beta V_{u,0}^H + \mu\beta V_0^S, \\
V_1^S &= u_S + \beta \frac{b_1}{\tau}\left(\bar{p}_0 - c - V^S\right)\left[1 - G\left(R_0^{-1}(p)\right)\right] + \beta V_0^S.
\end{aligned}
$$

- Repeat the above process until convergence.

- Compute implied τ according to (23). If $\tau > \tau_0$ increase τ_0. Otherwise, decrease τ_0.

- Iterate on the above steps until $\tau = \tau_0$.

The above algorithm works very well and convergence is fast. We experiment different combinations of g and b_n distribution in hope to generate bell shape price densities. The results are presented in Figure 2. Unfortunately, even with this additional heterogeneity, we could not find an example in which bell-shape price density arises. Therefore, it seems rather difficult, if impossible for the noisy search model to generate bell-shape price density.

Figure 2 about here

5 Model III: Directed Search with Heterogenous Buyers

Following Moen (1997) and Shimer (1996), suppose buyers see all prices, and then decide where to search, but there is still random matching. Define a submarket as a set of sellers with measure S listing the same price and a set of buyers directing their search towards them. In a submarket, the total number of buyer-seller meetings is given by $\mu = \mu(B, S)$, where μ is increasing, concave and satisfies constant returns. For a buyer, the probability of contacting a relevant counterparty is $\mu(B, S)/B = \mu(1, S/B) \equiv \alpha(\tau)$ where $\tau = S/B$. For a seller, the relevant probability is $\mu(B, S)/S = \mu(1, S/B)/(S/B) = \alpha(\tau)/\tau$. As is standard, assume $0 \leq \alpha(\tau), \alpha(\tau)/\tau \leq 1$, $\lim_{\tau\to 0} \alpha(\tau) = 0$, $\lim_{\tau\to\infty} \alpha(\tau)/\tau = 0$, $\alpha(\tau)$ is increasing, and $\alpha(\tau)/\tau$ is decreasing. Matching here is bilateral – buyers are never in simultaneous contact with multiple sellers or vice versa.[12]

Competitive search equilibrium works like this. When buyers and sellers are homogeneous, equilibrium is found by maximizing sellers's payoff wrt (p, τ) subject to buyers receiving their *market utility.* One can also solve the dual, or one can let third parties called *market makers* set up submarkets by posting (p, τ) to attract both buyers and sellers; the outcome is the same here, but the latter

[12]The large literature on directed search is surveyed by Julien et al. (2016). Some of these models have finite markets and take limits; for simplicity, we start with the continuum economy. Some of them have searchers locate a counterparty with probability 1 although they still may not trade, e.g., if multiple buyers show up at the same seller who can only serve a limited number; others, like the one here, have searchers locate counterparties stochastically according to a bilateral matching technology.

interpretation as attractive. As in Mortensen and Wright (2002), one can imagine market makers profiting by charging fees, but these are driven to 0 by free entry. Hence, they are stylized real estate agents. In any case, market utility is taken as given by individuals, but determined endogenously in equilibrium.

The generalization to heterogeneous buyers is that there is a submarket for each type. Let the value to acquiring a house be distributed across buyers according to CDF $G(u)$, with density $g(u)$ and support $\mathcal{G} = [\underline{u}, \bar{u}]$. For each u, there is a submarket with price p_u, seller-buyer ratio τ_u, a matching probability for buyers $\alpha(\tau_u)$ and a matching probability for sellers $\alpha(\tau_u)/\tau_u$. In equilibrium, all type u buyers go to submarket u and get payoff V^u, while sellers are endogenously distributed across submarkets so that V^S is the same everywhere. Nomalize $u_S = 0$ and let k be the utility of the seller after they move to another city.

$$
\begin{aligned}
V_u^B &= u_B + \alpha(\tau_u)\beta\left(V_u^H - V_u^B - p_u\right) + \beta V_u^B. \\
V_u^H &= u + \mu\beta V^S + (1-\mu)\beta V_u^H, \\
V^S &= \frac{\alpha(\tau_u)}{\tau_u}\beta\left(p_u - c + k - V^S\right) + \beta V^S, \\
\tau &= \frac{1}{B} - \frac{1}{\mu}\int \alpha(\tau_u) g(u)\, du
\end{aligned}
$$

To determine (p_u, τ_u), solve

$$
rV_u^B = (1+r) u_B + \max_{(p_u,\tau_u)} \left\{\alpha(\tau_u)\left(V_u^H - V_u^B - p_u\right)\right\} \tag{24}
$$

$$
\text{st } rV^S = \frac{\alpha(\tau_u)}{\tau_u}\left(p_u - c + k - V^S\right). \tag{25}
$$

The objective function is payoff to visiting this submarket; the constraint says sellers get their market utility. Using the constraint to eliminate p_u and ignore the term $(1+r) u_B$, we reduce the problem to

$$
\max_{(p_u,\tau_u)} \left\{\alpha(\tau_u)\left(V_u^H - V_u^B + k - c - V^s\right) - \tau_u r V^S\right\}.
$$

As is standard, there is a unique solution $\tau > 0$, and it satisfies the FOC

$$\alpha'(\tau_u)\left(V_u^H - V_u^B + k - c - V^s\right) = rV^S.$$

Given this, the price in submarket u is a weighted average, similar to (6),

$$p_u = \varepsilon_u\left(V_u^H - V_u^B\right) + (1-\varepsilon_u)\left(c - k + V^S\right),$$

where $\varepsilon_u \equiv \tau_u\alpha'(\tau_u)/\alpha'(\tau_u)$ is the elasticity of matching wrt the seller-buyer ratio.[13] Moreover, combining the above conditions we get

$$V^S = \frac{\alpha'(\tau_u)\left[\frac{1+r}{r}\left(\frac{r}{r+\mu}u - u_B\right) + k - c\right]}{\frac{r\alpha'(\tau_u)}{r+\mu} + r + \alpha(\tau_u) - \tau_u\alpha'(\tau_u)}.$$

This defines a demand for sellers in submarket τ_u as a function of their cost V^S, say $\tau_u = \rho_u\left(V^S\right)$, and it is straightforward to check $\partial\rho_u/\partial V^S < 0$. To clear the market, aggregate across submarkets and equate total demand to the fixed supply in the population:

$$\int \rho_u\left(V^S\right) dG(u) = \tau$$

In addition, in the steady state the outflow of sellers equals the inflow of sellers

$$\tau = \frac{1}{B} - \frac{1}{\mu}\int \alpha \circ \rho_u\left(V^S\right) dG(u).$$

Hence V^S is uniquely determined by market clearing and the stationarity condition through

$$\int \rho_u\left(V^S\right) dG(u) + \frac{1}{\mu}\int \alpha \circ \rho_u\left(V^S\right) dG(u) = \frac{1}{B}$$

Summarizing, we have the following result:

[13] This is a classic competitive search result: the total surplus is split with a share going to sellers equal to the elasticity of matching wrt their participation. In bargaining models, efficiency obtains if the sellers' bargaining power θ equals the relevant elasticity, which is the well-known Hosios (1990) condition. It is often said that competitive search delivers this condition, and hence efficiency, endogenously.

Proposition 2 *With directed search, there is a unique stationary equilibrium. As long as* $\bar{u} > \underline{u}$*, there is price dispersion. The endogenous variables* $\left\langle \tau_u, p_u, V_u^B, V_u^H, V^S \right\rangle$ *are given by:*

$$\frac{1}{B} = \int \rho_u \left(V^S\right) dG(u) + \frac{1}{\mu} \int \alpha \circ \rho_u \left(V^S\right) dG(u) \tag{26}$$

$$V^S = \frac{\alpha'(\tau_u) \left[\frac{1+r}{r} \left(\frac{r}{r+\mu} u - u_B \right) + k - c \right]}{\frac{r\alpha'(\tau_u)}{r+\mu} + r + \alpha(\tau_u) - \tau_u \alpha'(\tau_u)} \tag{27}$$

$$V_u^B = \frac{1+r}{r} u_B + \frac{\alpha(\tau_u)(1-\varepsilon_u)}{\alpha'(\tau_u)} V^S, \tag{28}$$

$$V_u^H = \frac{(1+r)u + \mu V^S}{r+\mu}, \tag{29}$$

$$p_u = \varepsilon_u \left(V_u^H - V_u^B\right) + (1-\varepsilon_u)\left(c - k + V^S\right). \tag{30}$$

Given parameters, across submarkets higher u *is associated with higher* τ_u*,* p_u*,* V_u^B *and* V_u^H*.*

These results follow directly from the discussion in the text, except for the last statement. For that, differentiate (27)-(29) to verify $\partial V_u^B/\partial u > 0$, $\partial V_u^H/\partial u > 0$ and $\partial \tau_u/\partial u > 0$. Then the constraint in (24) suggests that $\partial p_u/\partial u > 0$. Of course V^S is constant across submarkets. To be clear, (26) uniquely determines V^S, market utility of sellers; given V^S, (27) pins down τ_u, the seller-buyer ratio in each submarket as a function of u; then (28) pins down V_u^B , (29) pins down V_u^H and (30) pins down p_u in each submarket. In terms of economics, equilibrium segmentation is such that a higher valuation buyers search in submarkets with higher prices and arrival rates. Of course a higher arrival rate for buyers means a lower arrival rate for sellers, for which they are compensated by higher prices. This is the essence of competitive search equilibrium – here all agents trade off price against the probability of trade, similar to the way sellers trade off these variables in the noisy search model.

In Table 4, the first column shows the effects of changing G in the sense of first-order stochastic dominance. This raises $\bar{p}$ and $\underline{p}$, with an ambiguous effect on the spread Δ. It decreases $\bar{\tau}$ and decreases $\underline{\tau}$, tightness in the highest and lowest u submarkets. The second column is a special case, where changing σ corresponds to a translation of G (increasing every u to $u+\sigma$ with $\sigma > 0$), which again raises $\bar{p}$ and $\underline{p}$, but this time unambiguously lowers Δ. In the third column, changing ϕ corresponds to a scale transformation of G (increase every u to ϕu for $\phi > 1$). For the effects on $\bar{p}$ and $\underline{p}$ we impose a nonincreasing elasticity of the matching function, $\varepsilon'(\tau) \leq 0$, and for Δ we need to impose $\varepsilon'(\tau) = 0$, as is common in directed search. All other effects are general.

	G	σ	ϕ	c	B
$\bar{p}$	$+$	$+$	$+$	$+$	$+$
$\underline{p}$	$+$	$+$	$+$	$+$	$+$
$\bar{\tau}$	$-$	$-$	$-$	$+$	$-$
$\underline{\tau}$	$-$	$+$	$+$	$-$	$-$
Δ	$+$	$-$	?	$+$	
V_u^B	$-$	$-$	$-$	$\pm$	
V_u^H	$+$				
V^S	$+$	$+$	$+$	$-$	

Table 2: Effects of Parameters with Directed Search

We can rewrite (29), after eliminating the endogenous V^S and V^u, to get price as a weighted average of $U = (1+r)\,u/\,(r+\mu) - (1+r)\,u_B/r$ and $c-k$:

$$p_u = \frac{\left[r + \frac{\alpha(\tau_u)}{\tau_u}\right]\varepsilon_u U + \left[r + \alpha\left(\tau_u\right) - \frac{\alpha(\tau_u)}{\tau_u}\frac{\mu}{r+\mu}\frac{\varepsilon_u}{1-\varepsilon_u}\right]\left(1-\varepsilon_u\right)\left(c-k\right)}{\left[r + \frac{\alpha(\tau_u)}{\tau_u}\right]\varepsilon_u + \left[r + \alpha\left(\tau_u\right) - \frac{\alpha(\tau_u)}{\tau_u}\frac{\mu}{r+\mu}\frac{\varepsilon_u}{1-\varepsilon_u}\right]\left(1-\varepsilon_u\right)} \tag{31}$$

Notice U is the utility gain if a buyer becomes a happy home owner. He gets to enjoy a flow utility u every period if he does not receive the relocation shock and gives up the utility from being a buyer. If we compare this to static models of directed search, the results are similar, except here the weight involves the elasticity of the matching function adjusted by some additional terms, which capture the value of waiting and that buyers endogenize the fact that they will become a seller at some point. If the seller has a high matching probability in the future, e.g., he effectively has higher market power, and this move p_u closer to U. Moreover, notice p_u is increasing in

$$\frac{\varepsilon_u}{1-\varepsilon_u} \frac{r+\frac{\alpha(\tau_u)}{\tau_u}}{r+\alpha\left(\tau_u\right)-\frac{\alpha(\tau_u)}{\tau_u}\frac{\mu}{r+\mu}\frac{\varepsilon_u}{1-\varepsilon_u}}$$

Since the term $\alpha\left(\tau_u\right)/\tau_u$ is decreasing in τ_u, it is immediate that p_u is decreasing in τ_u if $\varepsilon' < 0$.

Figure 3 shows for examples of the exogenous density $g\left(u\right)$ and the endogenous density $f\left(p\right)$ induced by $g\left(u\right)$. What we emphasize is that market segmentation is such that in equilibrium higher valuation buyers search in submarkets with higher prices and higher arrival rates. Of course, a higher arrival rate for buyers means a lower arrival rate for sellers, but they are compensated for that by higher prices. This is the essence of competitive search – here all agents face a trade off between price and the probability of trade, similar to the way sellers face this trade off in the noisy search model. But unlike the noisy search model, the directed search model can generate price densities. In particular, if the g is bell shaped, f can be bell shaped.

Figure 3 about here

6 Sticky Prices

We now return to the substantive issue with which we began – sticky house prices. Figure 4 shows the model with random search, where $f\left(p\right)$ is the equilibrium price density before, and $f\left(p\right)'$ after, a change in fundamental market conditions, e.g., an increase in τ, u or c, as shown in Table 1. The supports $[\underline{p}, \bar{p}]$ and $[\underline{p}', \bar{p}']$ overlap, as must be the case whenever the change in conditions is not too big. Any seller posting $p \in [\underline{p}, \underline{p}')$ before the change must raise his price, since p is not in the new Burdett-Judd support; any seller posting $p \in [\underline{p}', \bar{p}]$ has no incentive to change – he could change to another $p \in [\underline{p}', \bar{p}']$ but it would not be a profitable deviation, since it would be offset exactly by a response in the probability of a sale. Obviously the argument is similar for a change in conditions that causes $f(p)$ to shift in the other direction.

Figure 5 shows how similar results emerge with directed search. There are submarkets for each buyer type $u \in [\underline{u}, \bar{u}]$, with p_u increasing in u. A change in market conditions is shown that increases p_u to p'_u for all u. As long as the supports $[\underline{p}, \bar{p}]$ and $[\underline{p}', \bar{p}']$ overlap, exactly as in the other model, sticky prices can emerge. Again, any seller posting $p \in [\underline{p}, \underline{p}')$ before the change must change his price, since p is not in the new support, but any seller posting $p \in [\underline{p}', \bar{p}]$ has no incentive to change – he could change to another $p \in [\underline{p}', \bar{p}']$ but it would not be a profitable deviation, since it would be offset exactly by an offsetting probability of a sale. Again the argument is similar for a change in the other direction. The bottom line is that there is *no puzzle* when some sellers stick to their prices after changes in market conditions.

Figure 4 and 5 about here

7 Empirical Application

The directed search housing model is suitable for estimation using micro-level data. To illustrate this, we show how to estimate it using English housing data. For the i-th house on the market, we observe $(\tilde{p}_i, T_i, X_i)$ where $\tilde{p}_i$ is the listing price, T_i is the time on market and X_i is a vector of housing characteristics. Notice the listed price $\tilde{p}_i$ takes into account the heterogeneity of houses, X_i. Throughout the section, we assume that it is possible to take out the house heterogeneity X_i from $\tilde{p}_i$ and obtain the residual prices p_i. This can be achieved usually through a Hedonic price estimation and take p_i to be the residuals from the regression.[14]

7.1 Identification

Before starting estimation of the model, it is important to ask the question that whether it is identified from the data. In other words, if we observe the distribution of (p_i, T_i) which is generated by the directed search model with primitives $(G, \alpha, c, k, \beta, \mu, B)$, can we ping down these primitives? Unfortunately, the answer is no in general. The reason is quite intuitive: there is no way to distinguish the effect of market seller-buyer ratio and that of the matching function from seller's waiting probability. Similarly, β and G cannot be separately identified.

To proceed, we assume that there are auxiliary data which can help us to ping down β, μ, B. In addition, since there is no way to disentangle c and k. We normalize $k = 0$ and hence c captures the difference between the cost and benefit of selling. Now the unknown model primitives reduce to (G, α, c). But even in this case, the model is still not non-parametrically identified. The key difficulty here is that we only observe sellers' waiting time but not buyers'. As a result, in principle, we can have a pretty good idea about sellers' cost but cannot say too much about the buyers' type distribution. More specifically, it is hard to distinguish whether a buyer pay for certain price due to low matching probability tomorrow or due to high value from the house. One solution is to parametrize α and seek semi-parametric identification. With a little abuse of notation, we parametrize $\alpha(\cdot)$ by $\alpha(\cdot, A)$ where A is a one-dimensional parameter such that $\alpha(\tau, A)/\tau$ is strictly monotone in A for every $\tau > 0$. In addition, $d\alpha(\tau, A)\, d\tau$ vanishes smoothly as $\tau \to \infty$. We can only allow for A to be one-dimensional because the model

[14]We do observe transanction prices in this data, which are usually slightly different from the listed prices. This may suggest that there are bargaining and bidding in this market. However, since we focus on a model with posting and commitment, we ignore this feature of the data and focus on the listed prices.

restriction provides only one equation to identify the matching function if G is left to be non-parametric. Even under this parametrization, proving identification is challenging. This is again because only the seller related information is observed. Especially, we need to recove G from the distribution of listed price, which is a complicated nonlinear functional of G. Nevertheless, Proposition 3 shows the model is identified. More remarkably, the proof of identification is constructive and leads naturally to an estimation procedure of the model.

Proposition 3 *The primitives of the directed search housing model* (G, A, c) *are point-identified from the joint distribution of* (p_i, T_i) *if* B, μ, β *are known and all buyers are served.*

Proof. We provide a constructive proof of this proposition. We start with identifying seller's cost. Let τ_i be the seller-buyer ratio that seller i faces and $m_i = \alpha(\tau_i, A)/\tau_i$ be his matching probability. Notice $m_i = 1/\mathbb{E}(T_i|p_i)$ which can be directly calculated from the joint distribution of (p_i, T_i) and hence is known. Then by the equal profit condition of the sellers,

$$p_i = c + \left(1 + \frac{r}{m_i}\right) V^S$$

The two unknown V^S and c are identified if m_i takes more than one values. This is guaranteed because the model suggests that $\mathbb{E}(T_i|p_i)$ is linear in p_i because

$$\frac{p_i - c - V^S}{\mathbb{E}(T_i|p_i)} = \frac{\alpha(\tau_i, A)}{\tau_i}\left(p_i - c - V^S\right)$$

and the RHS is constant in p_i.

Next, we move on to identify A and τ_i. Given θ_1, $\alpha(\tau, A)/\tau$ is monotonic in τ, one can invert it to obtain τ_i i.e.

$$\tau_i(A) = \alpha^{-1}(m_i, A).$$

Next, recall that

$$\frac{1}{B}=\int\tau_u dG\left(u\right)+\frac{1}{\mu}\int\tau_u\frac{\alpha\left(\tau_u\right)}{\tau_u}dG\left(u\right).$$

Because B and μ are known, if the right-hand side can be expressed as a monotonic function of A, the above equality identifies A. To achieve this, first define $G_S\left(u\right)$ to be the cumulative distribution of sellers who serve submarket with buyers' utility smaller or equal to u and g_S be its density

$$\int\tau_u dG\left(u\right)=\frac{S}{B}=\frac{1}{B/S}=1/\int\frac{1}{\tau_u}g_S\left(u\right)du$$

Let p_u and τ_u be the equilibrium price and seller-buyer ratio schedules. Notice that p_u is monotonically increasing in u and τ_u is monotonically decreasing in u. We can index the submarkets by price, i.e. write $\tau_{p_u}=\tau_u$. And $G_S\left(u\right)=F\left(p_u\right)$ where F is the marginal distribution of p_i with density f. Then the density of G_S, $g_S\left(u\right)=f\left(p_u\right)dp_u/du$. Therefore,

$$\int\frac{g_S\left(u\right)}{\tau_u}du=\int\frac{f\left(p_u\right)}{\tau_u}\frac{dp_u}{du}du=\int\frac{f\left(p_u\right)}{\tau_{p_u}}dp_u=\mathbb{E}\frac{1}{\tau_i}.$$

Given A, corresponding $\int\tau_u dG\left(u\right)=1/\mathbb{E}\frac{1}{\tau_i(A)}$ which can be directly computed from data. Next, notice that

$$F\left(p\right)=\int1\left(p_u\leq p\right)\tau_u g\left(u\right)du/\int\tau_u dG\left(u\right),$$

which suggests that

$$f\left(p_u\right)=\frac{\tau_u g\left(u\right)}{\frac{dp_u}{du}\int\tau_u dG\left(u\right)}$$

Therefore,

$$\begin{aligned}\int\tau_u\frac{\alpha\left(\tau_u\right)}{\tau_u}dG\left(u\right) &= \int\tau_u dG\left(u\right)\int f\left(p_u\right)\frac{\alpha\left(\tau_u\right)}{\tau_u}\frac{dp_u}{du}du\\ &= \int\tau_u dG\left(u\right)\int\frac{\alpha\left(\tau_{p_u}\right)}{\tau_{p_u}}dF\left(p_u\right).\end{aligned}$$

Given A, this last quantity is $\mathbb{E}m_i/\mathbb{E}\left[1/\tau_i\left(A\right)\right]$. Therefore, we can solve

$$\frac{1}{B}=\left(1+\frac{1}{\mu}\mathbb{E}m_i\right)\frac{1}{\mathbb{E}\left[1/\tau_i\left(A\right)\right]}$$

to obtain A. Since $\alpha^{-1}\left(m_i,A\right)$ is monotonic in A for every m_i, there is a unique solution to the above equation. This suggests that A is identified.

Lastly, because for each p its corresponding τ is identified from $\alpha^{-1}\left(\cdot,A\right)$. Therefore, its corresponding u is identified from (27). Then the whole schedue p_u is identified. Therefore, G is identified from the following equation.

$$G\left(u\right)=\mathbb{E}\left[\frac{1}{\tau_i\left(A\right)}\right]\mathbb{E}\left[\frac{1\left(p\leq p_u\right)}{\tau_i\left(A\right)}\right].$$

■

7.2 Estimation

There are multiple ways to estimate the model. Because the model predicts the joint density of listed price and time on market, one obvious method is to use a sieve maximum likelihood estimator or a sieve simulated method of moments. However, there are two disadvantages of these approaches. First, to implement these methods, one needs to compute the equilibrium of the model under a large number of parameters which is very time consuming. Second, both method involves solving potentially high-dimensional optimization problems, which is difficult to handle. This is especially true for the sieve maximum likelihood estimator because the problem is irregular in the sense that the support of price density depends on the unknown parameters. This can complicates the optimization due to the fact that the likelihood function may take value $-\infty$ for a wide range of parameters. One may be able to alleviate this by using Bayes method, which is still time consuming.

In light of this computational concern, we develop a direct method which exploits our constructive identification strategy. Direct estimation method is widely used in structural estimation, see Guerre, Perrigne and Vuong (2000) in first-price auctions and Bajari, Benkard and Levin (2007) in dynamic games. It is convenient due to the fact no equilibrium needs to be compute to implement the method. In addition, it does not involve complicated nonlinear optimization problem. Likewise, our estimator presented below involves no equilibrium computation and it only needs to solve a one-dimensional optimization problem

Our estimation strategy takes three steps. In the first step, we estimate the seller's cost and value function. Notice that from the equal-profit condition of the sellers,

$$p_i = c + \left[1 + \frac{r}{m_i}\right] V^S$$

where m_i is seller i's matching probability, which equals to $1/\mathbb{E}\left(T_i|p_i\right)$. Since T_i and p_i are both observed, $\mathbb{E}\left(T_i|p_i\right)$ can be estimate by a linear regression of T_i on p_i because it is linear in p_i. Denote the estimate of $\mathbb{E}\left(T_i|p_i\right)$ by $\hat{\mathbb{E}}\left(T_i|p_i\right)$. Next we can regress p_i on $1 + r\hat{\mathbb{E}}\left(T_i|p_i\right)$ and a constant to obtain estimates for V^S and c denoted as $\hat{c}$ and $\hat{V}^S$.

In the second step, we estimate A and the seller-buyer ratio for each seller. Recall the market clearing condition

$$\frac{1}{B} = \left(1 + \frac{1}{\mu}\mathbb{E}m_i\right) \frac{1}{\mathbb{E}\left[1/\alpha^{-1}\left(m_i, A\right)\right]}$$

Because m_i can be estimated by $\hat{m}_i = 1/\hat{\mathbb{E}}\left(T_i|p_i\right)$. Given A, $\mathbb{E}\left[1/\alpha^{-1}\left(m_i, A\right)\right]$ can be estimated by $\frac{1}{N}\sum_i 1/\alpha^{-1}\left(\hat{m}_i, A\right)$. Therefore, we arrive at the following equation in A.

$$\frac{1}{B} = \left(1 + \frac{1}{\mu}\frac{1}{N}\sum \hat{m}_i\right) \frac{1}{\frac{1}{N}\sum_i 1/\alpha^{-1}\left(\hat{m}_i, A\right)}$$

Therefore, $\hat{A}$ that solves the above equation is an estimate for A. And then τ_i can be estimated by $\hat{\tau}_i = \alpha^{-1}\left(\hat{m}_i, \hat{A}\right)$.

Lastly, we estimate G. One can invert (27) to obtain $\hat{u}_i$

$$\hat{u}_i = \frac{r+\mu}{1+r}\left[\frac{r}{r+\mu} + \frac{r+\alpha\left(\hat{\tau}_i, \hat{A}\right)}{\alpha'\left(\hat{\tau}_i, \hat{A}\right)} - \hat{\tau}_i\right]\hat{V}^S + \hat{c}.$$

Then

$$\hat{G}(u) = \frac{\sum_i 1\left(\hat{u}_i \leq u\right)/\hat{\tau}_i}{\sum_i 1/\hat{\tau}_i}.$$

To estimate the density $\hat{g}$, one can use a weighted kernel estimates. Specifically, let $K(\cdot)$ be a standard kernel function which is positive, symmetric and satisfies $\int K(x)\,dx = 1$ and $\int xK(x)\,dx = 0$. Let h be a bandwidth. Then $g(u)$ can be estimated by the following weighted kernel density estimate

$$\hat{g}(u) = \sum_i \frac{w_i}{h}K\left(\frac{\hat{u}_i - u}{h}\right),$$

where $w_i = \frac{1}{\hat{\tau}_i}/\sum_i \frac{1}{\hat{\tau}_i}$ is the weight for u_i. This captures the fact the distribution of p_i needs to be adjusted to obtain g. The following proposition shows the consistency of the estimators.

Proposition 4 *As $N \to \infty$, $\hat{c} \to^p c$, $\hat{A} \to^p A$ and $\hat{V}^S \to^p V^S$. In addition,*

$$\sup_u \left|\hat{G}(u) - G(u)\right| \to^p 0$$

and $\hat{g}(u) \to^p g(u)$ if u is in the interior of $\mathcal{G}$ and $h \to 0$, $N^{1/4}h \to \infty$.

It is not difficult to show that $\hat{c}$, $\hat{A}$, $\hat{V}^S$ and $\hat{G}(u)$ are asympotically normal following the standard approach. But for the interest of space, we do not pursue that in this paper. However, we do want to briefly discuss how to choose the bandwidth h because it is well-known the performance of kernel density estimates depends crucially on it. We consider two ways to choose h. The first one is the rule-of-thumb choice following Wang and Wang (2007)

$$h_N = 0.9N^{-1/5} \min\left(\hat{\sigma}_u, IQR_u\right)$$

where $\hat{\sigma}_u$ is the standard error of u_i under weights w_i and IQR_u is the interquartile range. The second choice is obtained using least-squares cross-valiadation. Define h_{lscv} to be the minimizer of the ISE (integrated squared error),

$$\begin{aligned} h_{lscv} &= \arg\min_h \sum_i \sum_j \int \frac{w_j w_i}{h^2} K\left(\frac{\hat{u}_i - u}{h}\right) K\left(\frac{\hat{u}_j - u}{h}\right) du \\ &\quad -2 \sum_i w_i \sum_{j \neq i} \frac{w_j}{h^2\left(1-w_i\right)} K\left(\frac{\hat{u}_i - \hat{u}_j}{h}\right). \end{aligned}$$

To assess finite sample performance of the estimators, we conduct Monte Carlo experiments which are shown in Table 3. It reports the mean and standard erros for $\hat{A}$ and $\hat{c}$, whose true values are 0.5 and 0.1 respectively. We also report the Mean Integrated Squared Error (MISE) for $\hat{g}$, which is defined as $\mathbb{E} \int \left(\hat{g}\left(u\right) - g\left(u\right)\right)^2 du$. These results are obtained from 1000 simulated samples. It justifies our theory because as N increases, the estimates get closer and closer to the true parameter. In addition, it shows that the rule-of-thumb choice for the bandwidth works very well and slightly better than the least-squares cross-validation bandwidth although the latter is computationally more intensive. For more details about the Monte Carlo Experiment please refer to Section XXX in the Appendix.

Sample Size	$\hat{A}$ (True Value 0.5)		$\hat{c}$ (True Value 0.1)		MISE of $\hat{g}$	
	mean	std	mean	std	h_N	h_{lscv}
500	0.5017	0.0136	0.0973	0.0127	0.1843	0.1943
1000	0.5008	0.0092	0.0993	0.0760	0.0824	0.0880
2000	0.5000	0.0063	0.0996	0.0049	0.0410	0.0440

Table 3: Summary Statistics of the Estimates

7.3 Data Application

Before we move on to the structural estimation, we would first like to test implications of the model. There are two important testable implications that come out of the sellers' listing decisions. First, the higher the listed price, the longer the house stays in market. Second and more importantly, the average time on market is linear in the listed price. We can test the first implication by simply regress time on market on the listed price and check whether the coefficient on listed price is significant positive. To do this test, we need to control for the heterogeneity in houses and also the time trend. Hence, we control for a rich set of house characteristics including size, number of bed room, number of bathrooms etc. and the appraisal value of the house. To test the second implication, we only need to add nonlinear terms of the listed price into the regression and check whether they are significant.

The regression results are presented in Table 3. We omit coefficient on time trend and house characteristics except the appraisal value for the interest of space. The numbers without brackets are point estimates and those in brackets are standard errors. The second column of the test reports the results with only linear term of listed price while the third column and forth column reports result with quadratic and both quadratic and cubic term. In the linear regression, the coefficient on the listed price is positive and significant at 5% level which is consistent with the model implication. If we add the quadratic term, this coefficient becomes significantly positive at 1% level while the coefficient on the squared listed price is not significant at any level. Once we add the cubic term, all coefficients become insignificant possibly due to the multicollinearity introduced by the cubic term. If we introduce higher order terms, similar thing happens. Nevertheless, the regression results are consistent with the model implications.

Lastly, we want to point out that the coefficient on appraisal price is always significantly negative. This is consistent with our intuition as a better house can be sold faster given other things are equal. However, this also shows that there is heterogeneity of the houses which are not captured by simple characteristics such as size, number of bedroom etc. So it is important to control for the appraisal price.

Time on Market			
Listed Price	0.001193**	0.0016973***	0.0010766
	(0.0005118)	(0.0006368)	(0.000922)
Listed Price^2	N/A	$-1.2E-9$	$4.2e-9$
		$(7.83E-10)$	$(5.61e-9)$
Listed Price^2	N/A	N/A	$-1.22e-14$
			$(1.18e-14)$
Appraisal Price	−0.0012245**	−0.0014244***	−0.0015113***
	(0.0005281)	(0.0005381)	(0.0005486)

*** significant at 1%, ** significant at 5%, * significant at 10%

Table 4: Regression Result

Now we move on to structural estimation. First, let $\alpha(\tau)/\tau = 1 - e^{-\frac{A}{\tau}}$ where $A > 0$. In other words, we assume the urn ball matching function. We assume that $B = 0.06$, i.e. the number of active buyers is 6% of the total housing stock. In addition, assume $\mu = 0.0083$ which implies that a household stays in a house for approximately 10 years on average. In the data, we observe daily information on houses but we aggregate it to the monthly level. We also set $\beta = 0.98$, which corresponds to a yearly discount rate of 0.78. We first regress p_i on X_i . Then we take the residuals to be the esimated residual price $\hat{p}_i$. Then we apply the direct method developed in the last section to $(\hat{p}_i, T_i)$.

Figure 7(a) plots the estimated distribution of buyers' value of an average house in the sample, i.e. $U = u(r+\mu)/(1+r) + \bar{X}\beta$ where $\bar{X}$ is the average characteristics of the houses in the data. As a reference point, the price predicted by the Hedonic model for this average house is 95100 pounds. One can see that most people value the house between $100,000$ pounds to $400,000$ pounds. Buyers' values are quite heterogeneous.

Figure7(b) plots changes in price densities predicted by the model under the estimated parameters. The blue curve is the price density predicted by the model under the parameters estimated from the data. The red curve is the price density when the number of buyers increaes by 1%. This shifts the distribution of housing prices to the right. And the effect is more than one-to-one. The average prices listed increases by about 3.4%. At the same time, the supports of the two distribution overlaps on a wide region, indicating that a lot of sellers do not need to change their prices although the fundamental of the market changes. The number of sales also increases. But due to the friction in the market, it only increases slightly by 0.44%.

Figure 7(c) and (d) shows the model predicted price distributions and densities and the empirical price distribution and density. It shows that under the estimated parameters, the model predicted price distriution is almost identical to the data. The difference in densities is due to the slight difference in estimating these two densities. We use a kernel density estimator for the data. While for the model, we approximate the cumulative distribution by a flexible B-splines. Then then density is obtained by taking the derivative of the B-spline approximation. This highlights the fact that the model is just identified. All the information from the data is used to fit the model.

Figure 7 about here

8 Conclusion

We analyzed models of housing markets based on price posting by sellers with either random noisy search or directed search, and showed that some sellers can stick to their listed prices after various types of shocks. While we are not necessarily unsympathetic to the idea that loss aversion, regret theory, psychological prices or unrealistic expectations may be features of reality, the goal was to demonstrate that we do not need them to explain the observations that others consider puzzling. Nor do we not need menu costs. Price stickiness follows from price dispersion which follows from rudimentary search theory.

While this was the narrow objective of the exercise, the models may have applications in housing economics that go beyond discussing price dynamics, and there are certainly many relevant extensions that one could entertain. The results may also have implications that go well beyond real estate economics. Genesove and Mayer (2001), e.g., suggest the following: the observation that sellers of such an important asset exhibit loss aversion gives added credence to the finding of such behavior in experimental settings. We are not sure about credence, but it would be incorrect to say that sticky house prices constitute definitive evidence of loss aversion or related phenomena. They follow easily from search frictions.

Appendix A: Alternative Specification

Here we present a model with housing construction and destruction. Thus, households sometimes re-enter the market looking for a new house, when their old one is destroyed, and sellers are builders who are always on the market with newly built homes. Let the measure of households be 1, and the measure of builders N. If the measure of households with a house is η, then $B = 1 - \eta$ is the measure of buyers currently houseless. They sample as in the baseline model, where information containing n price quotes arrives at Poisson rate b_n. The rate at which houses are destroyed is δ.[15] This implies $\dot{B} = \eta\delta - (1-\eta)(1-b_0) = 0$, and in steady state $B = \delta/(\delta + 1 - b_0)$, while the identity $Ns_n = Bnb_n$ implies

$$s_n = \frac{\delta n b_n}{N(\delta + 1 - b_0)}. \tag{32}$$

For buyers,

$$rV_0 = u_0 + \sum b_n \int^R (V_1 - V_0 - p) d\left\{1 - [1 - F(p)]^n\right\} \tag{33}$$

$$rV_1 = u_1 + \delta(V_1 - V_0). \tag{34}$$

The reservation price is $R = V_1 - V_0$, which implies

$$R = \frac{u_1 - rV_0}{r + \delta}. \tag{35}$$

For sellers posting p,

$$rV_f(p) = \sum s_n [1 - F(p)]^n (p - c),$$

where c now includes the transactions cost of a sale plus construction cost of a new unit (in steady state, after each sale the seller builds a new house). In particular, at $\bar{p} = R$

$$rV_f(R) = s_1(R - c). \tag{36}$$

[15] One can call δ the depreciation, but depreciation here is on the extensive margin – it happens all at once, as when it burns down. Note also that we can ignore fire insurance since agents are risk neutral.

Similar to the baseline model, equal profit implies

$$s_1(R-c)=\sum s_n[1-F(p)]^n(p-c). \tag{37}$$

As usual, eliminating the summation from (33) using (37), we have

$$rV_0=u_0+(1-b_0-b_1)(R-c) \tag{38}$$

Now (38) and (35) imply

$$R=\frac{u_1-u_0+(1-b_0-b_1)c}{r+\delta+1-b_0-b_1} \tag{39}$$

Given R, the rest is easy. From (37) and (32), the distribution solves

$$\sum b_n n[1-F(p)]^n=\frac{b_1(R-c)}{p-c} \tag{40}$$

Also, (36) for the firm and (34)-(35) for the household yield payoffs

$$rV_f = \frac{\delta b_1}{N(\delta+1-b_0)}(R-c) \tag{41}$$

$$rV_0 = u_1-(r+\delta)R \tag{42}$$

$$rV_1 = u_1-\delta R \tag{43}$$

Steady state equilibrium is a list $\{R, F(p), V_f, V_0, V_1\}$ solving (39)-(43). Also,

$$\underline{p}=\frac{b_1R+[\mathbb{E}n-b_1]c}{\mathbb{E}n},$$

while the seller buyer ratio is $N/B=N\delta/(\delta+1-b_0)$. In fact, even out of steady state, $F(p)$ is constant (similar to Pissarides, although for a slightly different reason).

Appendix B: Bargaining

Here is a model with bargaining, using the same matching technology as Section 3: a buyer can meet n sellers, but a seller can meet at most 1 buyer. When a buyer meets a single seller, he pays $p=\bar{p}$ and gets a share $1-\theta$ of the total surplus, where θ is sellers' bargaining power. When he meets multiple sellers, he picks one, pays $p = \underline{p} < \bar{p}$ and gets all of the total surplus. Here $\bar{p}$ comes from any standard bargaining solution, e.g., generalized Nash, while $\underline{p}$ is the usual outcome of Betrand competition. Thus, $u - \bar{p} - V^B = (1-\theta)\left(u - c - V^B - V^S\right)$ with probability b_1 and $u - \underline{p} - V^B = u - c - V^B - V^S$ with probability $1 - b_0 - b_1$.

In steady state,

$$\begin{aligned} rV^B &= (1 - b_0 - b_1\theta)\left(u - c - V^B - V^S\right) \\ rV^S &= (b_1/\tau)\theta\left(u - c - V^B - V^S\right). \end{aligned}$$

It is routine to solve for

$$\begin{aligned} \bar{p} &= \frac{\theta\left(r + b_1/\tau\right)u + \left[(1-\theta)\left(r + b_1\right) + 1 - b_0 - b_1\right]c}{r + b_1\left(1-\theta\right) + 1 - b_0 - b_1 + \theta b_1/\tau} \\ \underline{p} &= \frac{\theta b_1 u/\tau + r + \left(1-\theta\right)b_1 + 1 - b_0 - b_1}{r + b_1\left(1-\theta\right) + 1 - b_0 - b_1 + b_1\tau\theta}, \end{aligned}$$

as well as $\Delta = p_1 - p_2$, V^B and V^S.

Appendix C: Dynamics

To prove Proposition 1, first the law of motion for S_t satisfies

$$S_{t+1} = S_t + (1 - S_t)\,\mu - B\,(1 - b_0)$$

Divide both side by B and arrange to obtain (10). Notice τ_t is determined indepently by (10), whose only solution is

$$\tau_t = \tau = \frac{1}{B} + \frac{1 - b_0}{\mu}.$$

Next, we combine (1)-(5) to obtain a system of difference equations in $\left(V_t^B, V_t^H, V_t^S\right)$ given τ. To begin, rewrite (1) as

$$\begin{aligned} V_t^B &= u_B + \beta\sum_{n=1}^{\infty} b_n(V_{t+1}^H - V_{t+1}^B) + \beta V_{t+1}^B - \beta\sum_{n=1}^{\infty} b_n \int p dF_{n,t}(p) \\ &= u_B + \beta\sum_{n=1}^{\infty} b_n(V_{t+1}^H - V_{t+1}^B - V_{t+1}^S - c) + \beta V_{t+1}^B - \beta\sum_{n=1}^{\infty} b_n \int \left(p - c - V_{t+1}^S\right) dF_{n,t}(p) \\ &= u_B + \beta\left(1 - b_0\right)(V_{t+1}^H - V_{t+1}^B - V_{t+1}^S - c) + \beta V_{t+1}^B - \beta\Phi_t, \end{aligned}$$

where

$$\begin{aligned}
\Phi &= \sum_{n=1}^{\infty} n b_n \int \left(p - c - V_{t+1}^S\right) \left[1 - F_t\left(p\right)\right]^{n-1} f_t\left(p\right) dp \\
&= \tau \int \left(p - c - V_{t+1}^S\right) f_t(p) \sum_{n=1}^{\infty} s_n \left[1 - F_t(p)\right]^{n-1} dp \\
&= \left(R_t - c - V_{t+1}^S\right) \tau s_1 \int f_t(p) dp = b_1 (V_{t+1}^H - V_{t+1}^B - V_{t+1}^S - c)
\end{aligned}$$

Therefore, we have

$$V_t^B = u_B + \beta \left(1 - b_0 - b_1\right) \left(V_{t+1}^H - V_{t+1}^B - V_{t+1}^S - c\right) + \beta V_{t+1}^B,$$

using $R_t = u - V_{t+1}^B$. Also, Lemma 2 implies

$$V_t^S = u_S + \beta \frac{b_1}{\tau} (V_{t+1}^H - V_{t+1}^B - V_{t+1}^S - c) + \beta V_{t+1}^S.$$

Putting these together and letting $\mathbf{V}_t = (V_t^B, V_t^H, V_t^S)$, we have a linear system $\mathbf{V}_t = \Psi(\mathbf{V}_{t+1})$, given by

$$\begin{bmatrix} V_t^B \\ V_t^H \\ V_t^S \end{bmatrix} = \begin{bmatrix} u_B - \left(1 - b_0 - b_1\right) \beta c \\ u_H \\ u_S - \frac{b_1}{\tau} \beta c \end{bmatrix} + \beta \begin{bmatrix} b_0 + b_1 & 1 - b_0 - b_1 & -\left(1 - b_0 - b_1\right) \\ 0 & 1 - \mu & \mu \\ -\frac{b_1}{\tau} & \frac{b_1}{\tau} & 1 - \frac{b_1}{\tau} \end{bmatrix} \begin{bmatrix} V_{t+1}^B \\ V_{t+1}^H \\ V_{t+1}^S \end{bmatrix}.$$

The matrix on the RHS has eigenvalues $\lambda_1 = \beta$ and

$$\begin{aligned}
\lambda_2 &= \beta \frac{1 + b_0 - \mu - \frac{b_1}{\tau} + b_1 + \sqrt{\left(1 - b_0 - \mu + \frac{b_1}{\tau} - b_1\right)^2 + 4\mu \frac{b_1}{\tau}}}{2}, \\
\lambda_3 &= \beta \frac{1 + b_0 - \mu - \frac{b_1}{\tau} + b_1 - \sqrt{\left(1 - b_0 - \mu + \frac{b_1}{\tau} - b_1\right)^2 + 4\mu \frac{b_1}{\tau}}}{2}.
\end{aligned}$$

One can check that $\lambda_2/\beta \leq 1$ is if $4\mu\left(b_0 + b_1 - 1\right) < 0$ which holds obviously. In addition, $\lambda_3/\beta \geq -1$ if $2\frac{b_1}{\tau} - b_0 - b_1 - 1 \leq 0$. Because $\tau \geq \mathbb{E}n \geq b_1 + 2\left(1 - b_0 - b_1\right)$,

$$2\frac{b_1}{\tau} - b_0 - b_1 - 1 < 2\frac{b_1}{b_1 + 2\left(1 - b_0 - b_1\right)} - b_0 - b_1 - 1$$

The RHS is no more than 0 if $2b_0^2 + 3b_0 b_1 + b_1^2 + b_1 - 2 \leq 0$. One can show this holds because b_0 and b_1 are non-negative and $b_1 + b_0 \leq 1$. Therefore, the only solution to the system moving forward in real time, $\mathbf{V}_{t+1} = \Psi^{-1}(\mathbf{V}_t)$, satisfying $\lim_{T\to\infty} \beta^T \mathbf{V}_T = 0$ is the steady state. It is routine to solve for steady state as given in Proposition 1. ■

Appendix D: Comparative Statics

Here we derive the results in Table 1, letting "$x \simeq y$" mean "x and y take the same sign." Here we only present derivation for the effects of u_H and μ. The rest can be derived using the same approach.

We first look at the effect of u_H. The equilibrium condition can be rewritten as

$$\begin{aligned} M \begin{bmatrix} V^B \\ V^H \\ V^S \end{bmatrix} &= \begin{bmatrix} (1+r)\,u_B - (1-b_0-b_1)\,c \\ (1+r)\,u_H \\ (1+r)\,u_S - \frac{b_1}{\tau}c \end{bmatrix}, \\ M &= \begin{bmatrix} r-b_0-b_1+1 & b_0+b_1-1 & 1-b_1-b_0 \\ 0 & r+\mu & -\mu \\ \frac{b_1}{\tau} & -\frac{b_1}{\tau} & r+\frac{b_1}{\tau} \end{bmatrix}. \end{aligned}$$

Differentiate both sides to obtain

$$M \begin{bmatrix} \frac{dV^B}{du_H} \\ \frac{dV^H}{du_H} \\ \frac{dV^S}{du_H} \end{bmatrix} = \begin{bmatrix} 0 \\ 1+r \\ 0 \end{bmatrix}.$$

Notice the determinant of M is positive. Therefore, one can use the cramer's rule to obtain

$$\begin{aligned}
\frac{dV^B}{du_H} &\simeq r\left(1+r\right)\left(1-b_0-b_1\right)>0,\\
\frac{dV^H}{du_H} &\simeq r\left(1-b_0-b_1+\frac{b_1}{\tau}+r\right)\left(1+r\right)>0,\\
\frac{dV^S}{du_H} &\simeq r\frac{b_1}{\tau}\left(1+r\right)>0,\\
\frac{dR}{du_H} &= \frac{dV^H}{du_H}-\frac{dV^B}{du_H}\simeq r\frac{b_1}{\tau}+r^2>0,\\
\frac{d\underline{p}}{du_H} &= \frac{\mathbb{E}n-b_1}{\mathbb{E}n}\frac{dV^S}{du_H}+\frac{b_1}{\mathbb{E}n}\frac{dR}{du_H}>0,\\
\frac{d\Delta}{du_H} &\simeq \frac{d\Sigma}{du_H}=\frac{dR-dV^S}{du_H}\simeq r\left(1-\frac{b_1}{\tau}\right)>0,\\
\frac{d}{du_H}\left(\frac{\Sigma}{p-V^S-c}\right) &\simeq \frac{d\Sigma}{du_H}\left(p-V^S-c\right)+\frac{dV^S}{du_H}\Sigma>0.
\end{aligned}$$

The last inequality suggests that the RHS of (5) is increasing in u_H. Because the LHS of of (5) decreases in $F\left(p\right)$, $dF\left(p\right)/du_H<0$.

Next, we investigate the effect of μ. This is more involved as it has a direct effect on homeowners' value and has an indirect effect through changing equilibrium τ. Nevertheless, following the same steps

$$M\begin{bmatrix}\frac{dV^B}{d\mu}\\ \frac{dV^H}{d\mu}\\ \frac{dV^S}{d\mu}\end{bmatrix}=\begin{pmatrix}0\\ -\left(V^H-V^S\right)\\ -\frac{b_1}{\tau}\frac{1-b_0}{\tau\mu^2}\Sigma\end{pmatrix}.$$

Use Cramer's rule to obtain

$$\begin{aligned}
\frac{dV^B}{d\mu} &\simeq -\left(V^H - V^S\right)\tau\mu^2 + \frac{b_1}{\tau}\Sigma\left(1-b_0\right) \gtreqqless 0,\\
\frac{dV^H}{d\mu} &\simeq -\left(V^H - V^S\right)\left(r - rb_0 - rb_1 + r\frac{b_1}{\tau} + r^2\right) + \frac{b_1}{\tau}\frac{1-b_0}{\tau\mu^2}\Sigma\mu\left(b_0 - r - 1 + b_1\right) <\\
\frac{dV^S}{d\mu} &\simeq -r\left(V^H - V^S\right) - \frac{1-b_0}{\tau\mu^2}\Sigma\left(r+\mu\right)\left(r - b_0 - b_1 + 1\right) < 0,\\
\frac{dR}{d\mu} &\simeq -\left(V^H - V^S\right)r\left(r + \frac{b_1}{\tau}\right) - \frac{b_1}{\tau}\frac{1-b_0}{\tau\mu^2}\Sigma\mu\left[r\left(1-b_0-b_1\right) + \left(1-b_0-b_1+r\right)\right] <\\
\frac{dp}{d\mu} &= \frac{\mathbb{E}n - b_1}{\mathbb{E}n}\frac{dV^S}{d\mu} + \frac{b_1}{\mathbb{E}n}\frac{dR}{d\mu} < 0,\\
\frac{d\Delta}{d\mu} &\simeq \frac{d\Sigma}{d\mu} \simeq -\left(V^H - V^S\right)r^2\tau^2\mu^2 + 2b_1\left(1-b_0\right)\left(r+\mu\right)\left(2 - 2b_0 - 2b_1 + r\right)\Sigma \gtreqqless 0,\\
\frac{d}{d\mu}\left(\frac{\Sigma}{p - V^S - c}\right) &\simeq \frac{d\Sigma}{d\mu}\left(p - V^S - c\right) + \frac{dV^S}{d\mu}\Sigma.
\end{aligned}$$

From the last expression, if $d\Sigma/d\mu < 0$, then $\frac{d}{d\mu}\left(\frac{\Sigma}{p-V^S-c}\right) < 0$. If $d\Sigma/d\mu > 0$

$$\frac{d\Sigma}{d\mu}\left(p - V^S - c\right) + \frac{dV^S}{d\mu}\Sigma \leq \frac{d\Sigma}{d\mu}\Sigma + \frac{dV^S}{d\mu}\Sigma = \frac{dR}{d\mu}\Sigma < 0.$$

Therefore, the RHS of (5) is decreasing in μ. This suggests that $dF\left(p\right)/d\mu > 0$.

Directed-Search Model: Use the recursive equations to eliminate V_u^B and V_u^H from FOC to obtain

$$\left[\frac{r\alpha'\left(\tau_u\right)}{r+\mu} + r + \alpha\left(\tau_u\right) - \tau_u\alpha'\left(\tau_u\right)\right]V^S = \alpha'\left(\tau_u\right)\left[\frac{1+r}{r}\left(\frac{r}{r+\mu}u - u_B\right) + k - c\right] \tag{44}$$

The above equation uniquely pins down τ_u given Σ_s if α is strictly concave. Then, V^S can be pinned down by

$$\int \rho_u\left(V^S\right)dG\left(u\right) + \frac{1}{\mu}\int \alpha\circ\rho_u\left(V^S\right)dG\left(u\right) = \frac{1}{B} \tag{45}$$

Because $\frac{\partial\tau_u}{\partial V^S} < 0$, $\int \tau_u g\left(u\right)du$ is monotonically decreasing in V^S. Therefore, the equilibrium is unique.

Comparative Statics:

Comparative statics wrt G : Now consider an increase in G in the sense of first-order stochastic dominance. For this part, we assume the support of G is unchanged. Because $\rho_u\left(V^S\right)$ is increasing in u, the LHS of (45) V^S increases for any given V^S. To satisfy (45), V^S needs to increase as well. Consequently, V_u^H increases, τ_u decreases and p_u increases if $\varepsilon' < 0$ for every u. Notice so far we only use the fact that G increases in the sense of first-order stochastic dominance.

Because $\bar{u}$ and $\underline{u}$ are unchanged, $\bar{\tau}$ and $\underline{\tau}$ decrease, and $\bar{p}$ and $\underline{p}$ increase. Next, one can use the constraint in (24) to write

$$\Delta = r\left[\frac{\bar{\tau}}{\alpha\left(\bar{\tau}\right)} - \frac{\underline{\tau}}{\alpha\left(\underline{\tau}\right)}\right]V^S.$$

Then

$$\frac{\partial\Delta}{\partial G} \simeq V^S\frac{\alpha\left(\bar{\tau}\right) - \bar{\tau}\alpha'\left(\bar{\tau}\right)}{\alpha\left(\bar{\tau}\right)^2}\frac{\partial\bar{\tau}}{\partial V^S} - V^S\frac{\alpha\left(\underline{\tau}\right) - \underline{\tau}\alpha'\left(\underline{\tau}\right)}{\alpha\left(\underline{\tau}\right)^2}\frac{\partial\underline{\tau}}{\partial V^S} + \frac{\bar{\tau}}{\alpha\left(\bar{\tau}\right)} - \frac{\underline{\tau}}{\alpha\left(\underline{\tau}\right)}. \tag{46}$$

Differentiate (44) to obtain that

$$\frac{\partial\tau_u}{\partial V^S} = \frac{\alpha'\left(\tau_u\right)}{\alpha''\left(\tau_u\right)}\frac{r + \alpha\left(\tau_u\right) + \frac{r}{r+\mu}\alpha'\left(\tau_u\right) - \tau_u\alpha'\left(\tau_u\right)}{\left[r + \alpha\left(\tau_u\right)\right]V^S} < 0. \tag{47}$$

If $\alpha\left(\tau\right) = A\tau^{\varepsilon}$, one can eliminate $\partial\bar{\tau}/\partial V^S$ using (47) to obtain that

$$\frac{\partial\Delta}{\partial G} \simeq \frac{\bar{\tau} - \frac{r}{r+\mu}}{r + A\bar{\tau}^{\varepsilon}} - \frac{\underline{\tau} - \frac{r}{r+\mu}}{r + A\underline{\tau}^{\varepsilon}} > 0.$$

The last inequality holds because $\underline{\tau} < \bar{\tau}$.

Comparative statics wrt σ: First notice that

$$\frac{\partial\tau_u}{\partial u} = -\frac{\alpha'\left(\tau_u\right)^2}{\alpha''\left(\tau_u\right)}\frac{\frac{1+r}{r+\mu}}{\left[r + \alpha\left(\tau_u\right)\right]V^S} > 0, \tag{48}$$

After some algebra, one can show that

$$\int\left[1 + \frac{\alpha'\left(\tau_u\right)}{\mu}\right]\frac{\partial\tau_u}{\partial u}g\left(u\right)du + \frac{dV^S}{d\sigma}\int\left[1 + \frac{\alpha'\left(\tau_u\right)}{\mu}\right]\frac{\partial\tau_u}{\partial V^S}g\left(u\right)du = 0. \tag{49}$$

Substitute in the expressions for $\partial\tau_u/\partial u$ and $\partial\tau_u/\partial V^S$ to obtain

$$\begin{aligned}
\frac{dV^S}{d\sigma} &= \frac{\frac{1+r}{r+\mu}\int\left[1 + \frac{\alpha'\left(\tau_u\right)}{\mu}\right]\frac{\alpha'\left(\tau_u\right)^2}{\alpha''\left(\tau_u\right)}\frac{g(u)}{r+\alpha(\tau_u)}du}{\int\left[1 + \frac{\alpha'\left(\tau_u\right)}{\mu}\right]\frac{\alpha'\left(\tau_u\right)}{\alpha''\left(\tau_u\right)}\frac{r+\alpha(\tau_u)+\frac{r}{r+\mu}\alpha'(\tau_u)-\tau_u\alpha'(\tau_u)}{r+\alpha(\tau_u)}du} > 0, \\
\frac{dV_u^H}{d\sigma} &\simeq \frac{dV^S}{d\sigma} > 0,\ \frac{d\tau_u}{d\sigma} = \frac{\partial\tau_u}{\partial V^S}\frac{dV^S}{d\sigma} < 0.
\end{aligned}$$

Now we move on to $d\bar{\tau}/d\sigma$ which can be written as

$$\begin{aligned}\frac{d\bar{\tau}}{d\sigma} &= \frac{\partial\bar{\tau}}{\partial\sigma}+\frac{\partial\bar{\tau}}{\partial V^S}\frac{dV^S}{d\sigma}\simeq\alpha'(\bar{\tau})-\left[r+\alpha(\bar{\tau})+\frac{r\alpha'(\bar{\tau})}{r+\mu}-\bar{\tau}\alpha'(\bar{\tau})\right]D\\ D &= \frac{\int\left[1+\frac{\alpha'(\tau_u)}{\mu}\right]\frac{\alpha'(\tau_u)^2}{\alpha''(\tau_u)}\frac{g(u)}{r+\alpha(\tau_u)}du}{\int\left[1+\frac{\alpha'(\tau_u)}{\mu}\right]\frac{\alpha'(\tau_u)}{\alpha''(\tau_u)}\frac{r+\alpha(\tau_u)+\frac{r}{r+\mu}\alpha'(\tau_u)-\tau_u\alpha'(\tau_u)}{r+\alpha(\tau_u)}du}\end{aligned}$$

It is not difficult to verify that $D<1$ given $c-k>0$. Next notice the function

$$\phi(\tau)=\alpha'(\tau)-\left[r+\alpha(\tau)+\frac{r\alpha'(\tau)}{r+\mu}-\tau\alpha'(\tau)\right]D$$

is decreasing in τ and (49) implies that

$$\int\left[1+\frac{\alpha'(\tau_u)}{\mu}\right]\frac{\alpha'(\tau_u)}{\alpha''(\tau_u)}\frac{\phi(\tau)}{[r+\alpha(\tau_u)]V^S}g(u)\,du=0.$$

This suggests $\phi(\tau)$ takes positive values for some τ and negative values for other τ. Because $\phi'(\tau)<0$, it must be that there exists $\tilde{\tau}$ such that $\phi(\tau)<0$ if $\tau>\tilde{\tau}$ and $\phi(\tau)>0$ otherwise. This means that $d\bar{\tau}/d\sigma<0$. Similarly, one can show that $d\underline{\tau}/d\sigma>0$. It is immediate from (31) and $d\bar{\tau}/d\sigma<0$ that $\bar{p}$ increases. In addition, $\underline{p}=c-k+rV^S\underline{\tau}/\alpha(\underline{\tau})$ is increasing in σ because both V^S and $\underline{\tau}/\alpha(\underline{\tau})$ are increasing in σ.

Comparative statics wrt c: First notice

$$\frac{\partial\tau_u}{\partial c}=-\frac{\partial\tau_u}{\partial u}=\frac{\alpha'(\tau_u)^2}{\alpha''(\tau_u)}\frac{1}{[r+\alpha(\tau_u)]V^S}<0.$$

Combine it with (**??**) and (**??**) to obtain

$$\frac{dV^S}{dc}=-\frac{\int\frac{\alpha'(\tau_u)^2}{\alpha''(\tau_u)}\frac{g(u)}{[r+\alpha(\tau_u)]}du}{\int\frac{\alpha'(\tau_u)^2}{\alpha''(\tau_u)}\frac{(u-c)g(u)}{[r+\alpha(\tau_u)]V^S}du}<0.$$

In addition, notice

$$\frac{d\frac{d\tau_u}{dc}}{du}\simeq\frac{\int\frac{\alpha'(\tau_u)^2}{\alpha''(\tau_u)}\frac{g(u)}{[r+\alpha(\tau_u)]}du}{\int\frac{\alpha'(\tau_u)^2}{\alpha''(\tau_u)}\frac{(u-c)g(u)}{[r+\alpha(\tau_u)]V^S}du}>0$$

This fact and $\int\frac{d\tau_u}{dc}g(u)\,du=0$ implies that there must exist a threshold $\tilde{u}$ which is in the interior of the support of G such that

$$\frac{d\tau_u}{dc}\gtreqless 0\text{ if }u\gtreqless\tilde{u}$$

Therefore, $\frac{d\bar{\tau}}{dc} > 0$ and $\frac{d\underline{\tau}}{dc} < 0$. Next, following a similar logic as for $d\bar{p}/dv > 0$ and $d\underline{p}/dv > 0$, one can show that $d\bar{p}/dc > 0$ and $d\underline{p}/dc > 0$.

Now use (??) and ?? to obtain

$$\frac{dV^u}{dc} \simeq -\frac{\alpha(\tau_u)/\tau_u}{[r+\alpha(\tau_u)/\tau_u]} + \frac{\int \frac{\alpha'(\tau_u)}{\alpha''(\tau_u)} \frac{\alpha'(\tau_u)g(u)}{r+\alpha(\tau_u)} du}{\int \frac{\alpha'(\tau_u)}{\alpha''(\tau_u)} \frac{[r+\alpha(\tau_u)+\alpha'(\tau_u)-\tau_u\alpha'(\tau_u)]g(u)}{r+\alpha(\tau_u)} du}$$

which can be positive when τ_u is high enough.

Example 1 *Now we present a numeric example in which some buyers' value increases as the cost of selling c increases. Figure 5 plots* $\frac{dV^u}{dc}$ *with* $r = 0.2$, $c = 1$, $\alpha(\tau_u) = 0.01\tau_u^{0.1}$, $\tau = 6$ *and*

$$G(u) = \begin{cases} 0 & \textit{if } u < 3 \\ \left[\frac{u-3}{22}\right]^{0.001} & \textit{if } u \in [3, 25] \\ 1 & \textit{if } u > 25 \end{cases}.$$

One can see that when u is small, $\frac{dV^u}{dc} < 0$, *When u becomes sufficiently large,* $\frac{dV^u}{dc}$ *becomes positive. In this example, most buyers have very low value and there are very few buyers have very high value. An increase in c causes a slightly decrease in* τ_u *for low value buyers which has to be compensated by a huge increase in* τ_u *for high value buyers. This increase in* τ_u *is so large such that it dominates the effect of increasing prices.*

Under constant elasticity matching,

$$\frac{d\Delta}{dc} \simeq -\frac{1}{r+\bar{\tau}^a} + \frac{1}{r+\underline{\tau}^a} - \left[\frac{\bar{\tau}-1}{[r+\bar{\tau}^a]} - \frac{\underline{\tau}-1}{r+\underline{\tau}^a}\right]\frac{dV^S}{dc} > 0$$

Comparative Statics wrt τ: Next, consider comparative statics with respect to τ. Following a similar logic,

$$\frac{dV^S}{d\tau} = 1\Big/\int \frac{\alpha'(\tau_u)^2}{\alpha''(\tau_u)} \frac{u-c}{[r+\alpha(\tau_u)](V^S)^2} g(u)\,du < 0,$$

$$\frac{d\tau_u}{d\tau} = \frac{\frac{\alpha'(\tau_u)^2}{\alpha''(\tau_u)} \frac{u-c}{[r+\alpha(\tau_u)]}}{\int \frac{\alpha'(\tau_u)^2}{\alpha''(\tau_u)} \frac{u-c}{[r+\alpha(\tau_u)]} g(u)\,du} > 0,$$

$$\frac{d\Delta}{d\tau} \simeq \frac{-\bar{\tau}+1}{r+\alpha(\bar{\tau})} - \frac{-\underline{\tau}+1}{[r+\alpha(\underline{\tau})]} < 0$$

In addition, notice by (**??**) and the fact that τ_u increases, p_u decreases. The buyers enjoy higher meeting probability and lower prices. Therefore, V^u goes up.

Comparative Statics wrt r and A: Next, we derive comparative statics wrt $\kappa = A/r$. The comparative statics wrt A and r follows naturally.

$$\begin{aligned}
\frac{\partial \tau_u}{\partial V^S} &= -\frac{\tau_u}{1-a} \frac{1+\kappa\alpha\tau_u^a - \tau_u \kappa a \tau_u^{a-1} + \kappa a \tau_u^{a-1}}{\left[1+\kappa a \tau_u^a\right] V^S} < 0, \\
\frac{\partial \tau_u}{\partial \kappa} &= \frac{\tau_u}{1-a} \frac{1}{\left[1+\kappa a \tau_u^a\right] V^S} > 0
\end{aligned}$$

One can show

$$\frac{dV^S}{d\kappa} > 0, \frac{dV^S/\kappa}{d\kappa} < 0$$

Now we investigate how τ_u changes with κ. First, notice that

$$\frac{d\tau_u}{d\kappa} \simeq -\left[1+\kappa\alpha\tau_u^a - \tau_u \kappa a \tau_u^{a-1} + \kappa a \tau_u^{a-1}\right] \frac{dV^S}{d\kappa} + 1$$

and

$$\frac{d\left[1+\kappa\alpha\tau_u^a - \tau_u \kappa a \tau_u^{a-1} + \kappa a \tau_u^{a-1}\right]}{d\tau_u} = \kappa\left(1-\tau_u\right) a\left(a-1\right) \tau_u^{a-2}$$

which is negative if $\tau_u < 1$ and positive if $\tau_u > 1$. Therefore, $d\tau_u/d\kappa$ is increasing in u when τ_u is small and decreasing in u when τ_u is large. Notice τ_u is increasing in τ. Therefore, when τ is sufficiently small, $\bar{\tau} < 1$ and $\frac{d\tau_u}{d\kappa}$ is always increasing in u. Because $\int \frac{d\tau_u}{d\kappa} du = 0$, we must have

$$\frac{d\bar{\tau}}{d\kappa} > 0 \text{ and } \frac{d\underline{\tau}}{d\kappa} < 0.$$

As τ increases, $\bar{\tau}$ goes above 1 and we will continue to have $\frac{d\bar{\tau}}{d\kappa} > 0$ until τ reaches τ_1. At τ_1, $\bar{\tau}$ is sufficiently large and $\underline{\tau} < 1$. Therefore,

$$\frac{d\bar{\tau}}{d\kappa} < 0 \text{ and } \frac{d\underline{\tau}}{d\kappa} < 0.$$

As τ further increases to τ_2, $\frac{d\tau_u}{d\kappa}$ is decreasing for most u, then to guarantee $\int \frac{d\tau_u}{d\kappa} du = 0$,

$$\frac{d\bar{\tau}}{d\kappa} < 0 \text{ and } \frac{d\underline{\tau}}{d\kappa} > 0.$$

Notice in this case, when τ is just above τ_2, we continue to have $\underline{\tau} < 1$.

One can also show that

$$\frac{dV^u}{d\kappa} \simeq a - \left[1 + \kappa a \tau_u^{a-1}\right] \frac{\int \frac{\tau_u}{1-a} \frac{g(u)}{[1+\kappa a \tau_u^a]} du}{\int \frac{\tau_u}{1-a} \frac{\left(1+\kappa \alpha \tau_u^a - \tau_u \kappa a \tau_u^{a-1} + \kappa a \tau_u^{a-1}\right) g(u)}{[1+\kappa a \tau_u^a]} du}$$

which can be positive or negative depending on τ_u. If τ_u is small, $\frac{dV^u}{d\kappa} < 0$. Otherwise, $\frac{dV^u}{d\kappa} > 0$.

Now let us turn to the price schedule. Notice

$$\frac{u - p_u}{p_u - c} = \left[\frac{1}{a} - 1\right] \frac{1 + \kappa \tau_u^a}{1 + \kappa \tau_u^{a-1}}$$

Therefore,

$$d\left(\frac{u-p}{p-c}\right) / d\kappa \simeq \left[\kappa a \tau_u^{a-1} + \kappa (1-a) \tau_u^{a-2} + \kappa^2 \tau_u^{2a-2}\right] \frac{d\tau_u}{d\kappa} + \tau_u^{a-1} (\tau_u - 1).$$

Therefore, if $\tau_u < 1$ and $\frac{d\tau_u}{d\kappa} < 0$, $\frac{dp_u}{d\kappa} > 0$ and if $\tau_u > 1$ and $\frac{d\tau_u}{d\kappa} > 0$, $\frac{dp_u}{d\kappa} < 0$. This immediately implies that when $\tau < \tau_1$, $\frac{d\underline{p}}{d\kappa} > 0$. And if $\tau \in (\tau_1, \tau_2)$, $\frac{d\underline{p}}{d\kappa} > 0$. And if $\tau > \tau_2$, $\frac{d\bar{p}}{d\kappa} < 0$. The following proposition summarizes the above analysis. Since $\kappa = A/r$, the comparative statics with respect to A and r can be derived easily.

Lastly, we move on to $d\Delta/d\kappa$. Following a similar method as before, we obtain

$$\frac{d\Delta}{d\kappa} = a\left[\frac{\underline{\tau} - 1}{1 + \kappa \underline{\tau}^a} - \frac{\bar{\tau} - 1}{1 + \kappa \bar{\tau}^a}\right] \frac{dV^S}{\kappa d\kappa} - \left[\frac{\bar{\tau}}{[1 + \kappa \bar{\tau}^a]} - \frac{\underline{\tau}}{[1 + \kappa \underline{\tau}^a]}\right] \frac{V^S}{\kappa^2} < 0$$

Example 2 (Numeric Example) *Figure* **??** *plots the comparative statics of the whole schedules with respect to r as a function of u. It is constructed under $r = 0.05$, $c = 1.1$, $\alpha(\tau_u) = 2\sqrt{\tau_u}$ and u follows a uniform distribution on $[3, 4]$. Three values of τ are considered: 0.35, 1 and 5. Notice these comparative statics should have the opposite sign of those with respect to κ. By the previous proposition, when τ is small, $\frac{d\bar{\tau}}{d\kappa} > 0$, $\frac{d\underline{\tau}}{d\kappa} < 0$ which suggests that $\frac{d\bar{\tau}}{dr} < 0$, $\frac{d\underline{\tau}}{dr} > 0$. This is shown in Figure* **??***(a). As τ increases, according to the previous proposition, $\frac{d\bar{\tau}}{dr} > 0$, $\frac{d\underline{\tau}}{dr} > 0$. This is shown in Figure* **??***(d). If τ further increases,*

$\frac{d\bar{\tau}}{dr} > 0$, $\frac{d\underline{\tau}}{dr} < 0$ as shown in Figure **??***(g). In addition, dp_u/dr may be negative or positive depending on τ_u. Lastly, the sign of dV^u/dr is also not determined. As predicted above, when τ_u is sufficiently low, dV^u/dr may be positive. This is shown in Figure* **??***(c)*

■

Appendix E: More on the Monte Carlo Experiment

In the Monte Carlo experiment, we consider the following data generating process. Agents have a discount factor 0.99 . The number of buyers are about 10% of the total housing stock and the reallocation shock comes to happy home-owners with probability 9%. Buyers and sellers match with the urn ball matching function. Therefore, a seller's matching probability in submarket serving type u buyers is $\alpha(\tau_u)/\tau_u = 1 - \exp(-A/\tau_u)$. Obviously, given τ_u, this probability is strictly monotone in τ_u and hence satisfies our identification assumption. In this Monte Carlo, $A = 0.5$. The selling cost of the seller is set to be $c = 0.1$. An affine transformation of the buyers value distribution follows a beta distribution. The first two rows of Figure 6 shows the value distribution and density along with their estimates. The last two rows show the histogram of the estimates for A and c. They are constructed using 1000 random samples with sample size $N = 500, 1000, 2000$. Consistent with Table 3, as N increases, the estimates gets closer to their true values. It is not hard to see that the histograms of $\hat{A}$ and $\hat{c}$ are approximately normal.

Figure 6 about here

References

[1] J. Albrecht, A. Anderson, E. Smith and S. Vroman (2007) "Opportunistic Matching in the Housing Market," *IER* 48, 641-664.

[2] J. Albrecht, P. Gautier and S. Vroman (2016) "Directed Search in the Housing Market," *RED* 19, 218-231.

[3] K. Bagwell and M. Riordan (1991) "High and Declining Prices Signal Product Quality," *AER* 81, 224-239.

[4] F. Buisson (2014) "Prospect Theory and Loss Aversion in the Housing Market," mimeo.

[5] K. Burdett and K. Judd (1983) "Equilibrium Price Dispersion," *Econometrica* 51, 955-70.

[6] K. Burdett and G. Menzio (2014) "The (Q,S,s) Pricing Rule," mimeo.

[7] K. Burdett and D. Mortensen (1998) "Wage Differentials, Employer Size, and Unemployment," *IER* 39, 257-73.

[8] K. Burdett, A. Trejos and R. Wright (2015) "A New Suggestion for Simplifying the Theory of Money," mimeo.

[9] K. Burdett and T. Vishwanath (1988) "Declining Reservation Wages and Learning," *RES* 55, 655-665.

[10] A. Caplin and D. Spulber (1987) "Menu Costs and the Neutrality of Money," *QJE* 102, 703-725.

[11] M. Davis and S. Van Nieuwerburgh (2015) "Housing, Finance and the Macroeconomy," in *Handbook of Urban and Regional Economics*, vol 5, ed. G. Duranton, J. Henderson and W. Strange, 753-811.

[12] P. Diamond (1971) "A Model of Price Adjustment," *JET* 3, 156-168.

[13] A. Diaz and B. Jerez (2013) "House Prices, Sales, and Time on the Market: A Search Theoretic Framework," *IER* 54, 837-872.

[14] E. De Wit and B. van der Klaauw (2013) "Asymmetric Information and List-Price Reductions in the Housing Market," *Reg Sci & Urban Econ.* 43, 507-520.

[15] B. Eden (1994) "The Adjustment of Prices to Monetary Shocks when Trade is Uncertain and Sequential," *JPE* 102, 493-509.

[16] T. Emmerling, A. Yavas and Y. Yildirim (2015) "To Accept or Not to Accept: Optimal Strategy for Sellers in Real Estate," mimeo.

[17] F. Fabinozzi, R. Shiller and R. Tunaru (2009) "Hedging Real Estate Risk," *J Portfolio Manag*, 92-103.

[18] D. Genesove and C. Mayer (2001) "Loss Aversion and Seller Behavior: Evidence from the Housing Market," *QJE* 116, 1233-1260.

[19] L. Han and W. Strange (2014). "What is the role of the asking price for a house?" mimeo.

[20] L. Han and W. Strange (2015) "The Microstructure of Housing Markets: Search, Bargaining, and Brokerage," prepared for *Handbook of Regional And Urban Economics*, vol. 5, ed. G. Duranton, J. Henderson and W. Strange.

[21] C. He, R. Wright and Y. Zhu (2015) "Housing and Liquidity," *RED* 18, 435-455.

[22] A. Head, L. Liu, G. Menzio and R. Wright (2011) "Sticky Prices: A New Monetarist Approach," *JEEA* 10, 939-973.

[23] A. Hosios (1990) "On the Efficiency of Matching and Related Models of Search and Unemployment," *RES* 57, 279-98.

[24] B. Julien, J. Kennes, and I. King (2000) "Bidding for Labor," *RED* 3, 619-649.

[25] B. Julien, J. Kennes, and I. King (2008) "Bidding for Money," *JET* 142, 196-217.

[26] B. Julien, P. Kircher and R. Wright (2016) "Directed Search: A Guided Tour," mimeo.

[27] J. Knight (2002) "Listing Price, Time on Market, and Ultimate Selling Price: Causes and Effects of Listing Price Changes," *Real Estate Econ* 30, 213-37.

[28] C. Leung, Y. Leong and S. Wong (2006) "Housing Price Dispersion: An Empirical Investigation," *J. Real Estate Fin and Econ* 32, 357-385.

[29] S. Levitt and C. Syverson (2008) "Market Distortions When Agents are Better Informed: The Value of Information in Real Estate Transactions," *REStat* 90, 599-611.

[30] G. Menzio (2007) "A Theory of Partially Directed Search," *JPE* 115, 748-769.

[31] G. Menzio and S. Shi (2010) "Block Recursive Equilibria for Stochastic Models of Search on the Job," *JET* 145, 1453-1494.

[32] T. Maury and F. Tripier (2014) "Search Strategies on the Housing Market and their Implications on Price Dispersion," *J Housing Econ* 26, 55-80.

[33] A. Merlo and F. Ortalo-Magne (2004) "Bargaining over Residential Real Estate: Evidence from England," *J Urban Econ* 56, 192-216.

[34] A. Merlo, F. Ortalo-Magne and J. Rust (2015) "The Home Selling Problem: Theory and Evidence," *IER* 56, 457-484.

[35] E. Moen (1997) "Competitive Search Equilibrium," *JPE* 105, 385-411.

[36] E. Moen, P. Nenovz and F. Sniekers (2016) "Buying First or Selling First in Housing Markets," mimeo.

[37] D. Mortensen (2005*a*) *Wage Dispersion: Why Are Similar Workers Paid Differently?* MIT Press.

[38] D. Mortensen (2005*b*) "A Comment On 'Price Dispersion, Inflation, And Welfare' by A. Head And A. Kumar," *IER* 46, 573-578.

[39] D. Mortensen and R. Wright (2002) "Competitive Pricing and Efficiency in Search Equilibrium," *IER* 43, 1-20.

[40] R. Ngai and Keven D. Sheedy (2014) "Moving House," mimeo.

[41] R. Ngai and Keven D. Sheedy (2015) "The Ins and Outs of Selling Houses," mimeo.

[42] R. Novy Marx (2009) "Hot and cold markets," Real Estate Economics," 37, 122.

[43] C. Pissarides (2000) *Equilibrium Unemployment Theory*. Cambridge, MIT Press.

[44] G. Rocheteau and R. Wright (2005) "Money in Search Equilibrium, in Competitive Equilibrium, and in Competitive Search Equilibrium," *Econometrica* 73, 175-202.

[45] G. Rocheteau and R. Wright (2013) "Liquidity and Asset Market Dynamics," *JME* 60, 275-94.

[46] S. Shi (2002) "A Directed Search Model of Inequality with Heterogeneous Skills and Skill-Biased Technology," *RES* 69(2), 467-491.

[47] S. Shi (2006) "Wage differentials, Discrimination and Efficiency," *EER*, 275, 83-115.

[48] R. Shimer (1996) "Contracts in Frictional Labor Markets," mimeo.

[49] R. Shimer (2005) "The Assignment of Workers to Jobs in an Economy with Coordination Frictions," *JPE* 113, 996-1025.

[50] H. Simon (1955) "A Behavioral Model of Rational Choice," *QJE* 69, 99-118.

[51] C. Taylor (1999) "Time-on-the-Market as a Sign of Quality," *RES* 66, 555-578.

[52] A. Tversky and D. Kahneman (1991) "Loss Aversion in Riskless Choice: A Reference Dependent Model," *QJE* 61, 1039-1061.

[53] W. Wheaton (1990) "Vacancy, Search, and Prices in a Housing Market Matching Model," *JPE* 98, 1270-1292.

[54] A. Wolinsky (1983) "Prices as Signals of Product Quality," *RES* 50, 647-658.