Long-Term Finance and Economic Development

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Abstract

What are the linkages between maturity of corporate debt, liquidity of financial markets and the real economy? Relative to developed countries, firms on developing economies borrow at shorter maturities and assets are traded in markets with larger frictions. I study how firms choose and finance long-term projects in a production economy subject to an over-the-counter trading friction in financial markets. The liquidity spread increases with the maturity of the asset, generating an upward sloping yield curve. As a result, improvements in liquidity of financial markets flattens the yield curve, improves borrowing terms for long-term projects, and induce firms to invest at longer horizons. I calibrate the model to match the US corporate debt market. Counterfactual exercises show that the liquidity of the secondary market can account for a large fraction of the variations in maturity choices across countries. Finally, I propose policies to improve liquidity and maturity and increase welfare.

JEL Classifications: E44, G30, O16.

Keywords: Debt maturity, Over-the-counter market, Liquidity, Secondary markets.

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1 Introduction

Access to long-term finance is an important determinant of economic development according to many policy makers[1]. How can we stimulate the use of long-term finance? Firms' liabilities are traded in frictional markets which are substantially less liquid in developing countries, relative to advance ones. This paper argues that improvements in the liquidity of the financial market can reduce borrowing costs, particularly at longer horizons, as long-term assets rely on the possibility of being traded in secondary markets. This is a new channel trough which financial development—an increase in market's liquidity—can promote economic development.

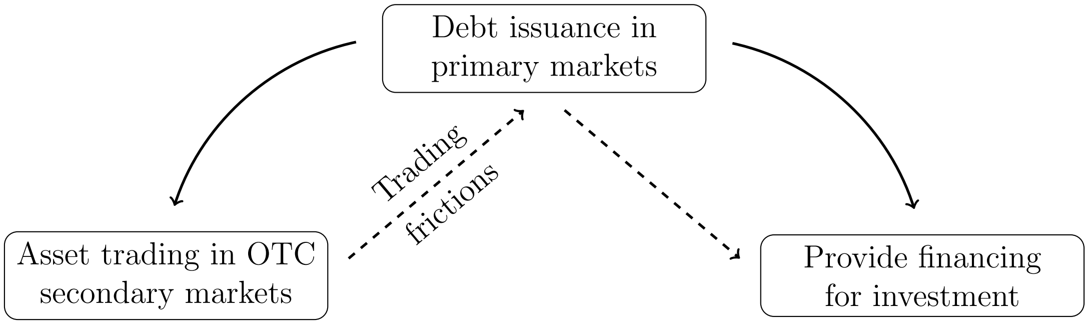
Section 2 explores the empirical evidence on corporate debt maturity and the liquidity of the secondary market across countries. First, it is well known that firms borrow at shorter maturities in developing countries. For example the median maturity is 5 years in developing countries, while it is 10 years in advance ones (Cortina Lorente, Didier, and Schmukler, 2016). Second, using data about asset trading in secondary markets for the US and Argentina, we show that indeed financial markets are substantially less liquid in Argentina than in the US. For example, the median turnover is 12% in Argentina while it is 57% in the US.

To understand the linkages between maturity of corporate debt, liquidity of financial markets and the real economy we study a production economy with an over-the-counter (OTC) trading friction in the corporate bonds market à la Duffie, Gârleanu, and Pedersen (2005). Investors choose projects indexed by their back-loaded profile such that longer periods of investment are associated with higher returns. Figure 1 shows a schematic representation of financial markets. Firms finance their investments projects by tapping the primary market and issuing corporate bonds. They take the yield curve as given and choose the bond's maturity. Then, these assets are traded in OTC markets. We use this framework to study how trading frictions in the secondary market affect the yield curve and financial and investment decisions of firms.

Section 4 analyze the lenders' problem and derive two important properties about how trading frictions affect the interest rate schedule. First, the endogenous credit spread due to liquidity is increasing in maturity. Intuitively, an investor can exit a long position by selling the asset on the secondary market, or waiting until maturity. Hence, longer-term assets rely more on the possibility of being traded. As a result, the trading friction delivers an upward sloping yield curve which is consistent with the data. Second, liquidity is more important for long-term

[1] For example, see World Economic Forum (2011); European Comission (2013); OECD (2013); Group of Thirty (2013); World Bank (2015) among others.

Figure 1: **Schematic representation of financial markets.**



assets, in the sense that improvements in liquidity not only reduce the liquidity spread but also reduce its slope with respect to maturity. Finally, we show that there is a feedback loop between default and liquidity. The liquidity spread increases with the default probability.

Section 5 study how firms choose and finance investment projects. There is a continuum of potential plans differentiated by their back-loaded profile such that longer periods of investment are associated with future higher returns. We provide two microfoundations that generate this return function, one based in a quality ladder model, and one based on time-to-build capital. However, the qualitative results holds under a general return function with back-loaded profiles. In the benchmark model firms issue debt to finance investment costs, with a maturity that matches the duration of the project.[2] The steepness of the yield curve plays an important role on the investment decision. When the term premium—the difference between borrowing long- and short-term—is high, long-term projects are too costly to finance, inducing firms to choose shorter-term projects.

Section 6 solves the equilibrium of lenders and borrowers decisions and study the effect of financial development on credit spreads and maturity choices. In particular, we evaluate the consequences of an increase in the liquidity of the OTC market. First, when the secondary market is more liquid, there is a *level* effect and the credit spread diminish for all maturities. More importantly, there is also a *slope* effect and the yield curve flattens because long-term assets rely more on the secondary market than shorter-term assets. Hence, financial development reduces the interest rate schedule and its slope with respect to maturity. Regarding the maturity choice, back-loaded profiles imply that in order to produce a more profitable firm, an entrepreneur needs borrowing for a longer period of time. Hence, when borrowing costs at longer horizons decline, firms invest in more profitable longer-term projects.

[2] In Section 10 we extend the model and allow firms to finance long-term projects with rollover short-term debt.

Section 7 estimates the model to match moments from the corporate debt market in the U.S. We compute the difference between the yield curve for Treasuries and the High Quality Market of corporate bonds (rated A and above). We show that this spread increases with maturity and use it to identify the liquidity spread in the model. We then use the estimated model to assess if variation in trading frictions can account for the relationship between maturity and GDP observed in the data. We choose trading frictions across countries to match the maturity of debt and then evaluate the performance on real variables such as GDP. The model predicts a relationship between maturity and GDP quite similar to the data counterpart for developed economies, and can account for about half of the GDP differences for less-developed countries.

In Section 8 we explore the effect of two policies designed to increase the liquidity of financial markets. First, we propose a simple tax-subsidy scheme in which the government provides a subsidy to search costs. Second, we consider the effects of *Government-Sponsored Intermediaries* (GSIs) that intermediate in secondary markets by buying bonds at a higher price than in private meetings. In both cases we consider a proportional profit tax to finance the intervention. Both policies are able to increase liquidity and maturity. Welfare gains are larger in economies with more frictions and GSIs are more efficient than subsidies to improve welfare.

We perform several extensions and robustness of the model. First, in the benchmark model, assets of different maturities are traded in a single secondary market. A potential concern could be that many assets of short maturities, with small gains from trade, preclude the entry of buyers to the secondary market. In Section 9 we consider an alternative specification such that secondary markets are segmented by the time to maturity of the asset. The main take-away of this exercise is that even though market tightness (ratio of sellers-to-buyers) for short term bonds increases, the tightness for long-term assets remains similar to the benchmark model with a single market. Hence, the secondary market in the baseline case is effectively a market for long-term assets.

In the benchmark model firms are able to issue bonds only at the beginning of the project, they are not allowed to rollover short-term contracts to finance long-term projects. However, one important question regarding the importance of long-term finance is why firms do not use short-term debt to finance their investment needs. Previous empirical studies have shown that firms tend to match the maturity of their assets and liabilities. They often use long-term debt to make long-term investments, such as purchases of fixed assets or equipment, while they use short-term debt to finance working capital such as payroll and inventory.[3] Section 10

[3]Studies for developed and developing countries find evidence that firms do match the maturity of their assets and liabilities (e.g., Stohs and Mauer (1996) for the United States; Schiantarelli and Sembenelli (1997)

extends the model and allow entrepreneurs to rollover short-term contracts to finance long-term projects. In this framework, the financial cost has two components: issuance and illiquidity costs. We show that this extension features the same trade-offs as the benchmark model. When secondary markets are more liquid the total financial cost reduces and firms decide to rollover less frequently, issue at longer maturities, and start projects with longer time-to-build.

Finally, Section 10 conducts robustness across different assumptions of the model. First, we show that when the default risk decreases, the firm invest in longer term projects. Second, when we allow for rollover the technology choice in a model with and without rollover are quite similar and changes in the liquidity of the secondary market play a similar role in both the model with and without rollover. One potential concern about the exercises performed in this paper is that the liquidity of the secondary market and the rollover cost can be correlated. In the model the liquidity cost is endogenous while the rollover cost is exogenous and fixed, so they do not respond to changes to the liquidity of the market, and a potential *Lucas critique* may apply. However, when the secondary market becomes more liquid, the issuance and rollover costs should diminish. Hence, the assumption that the issuance cost are fixed is a conservative one. In the model, when the liquidity of the secondary market improves, the firm adopt a project of longer maturity. Similarly, when the issuance cost diminish, the firm choose a longer-term project. Hence, if the two effect are present—an increase in liquidity and a reduction of the issuance cost—the firm will increase the maturity of the project even more than with constant issuance costs.

The model abstracts from alternative sources of borrowing. For example, bank lending is an important source of credit in developing economies. However, the maturity of bank loans tend to be shorter than the maturity of corporate bonds (e.g. Cortina Lorente, Didier, and Schmukler, 2016) and financial systems generally become more market-based during the process of economic development (Demirgüç-Kunt, Feyen, and Levine, 2013). In a model with bank lending, banks will need to rise capital to extend long term loans. Hence, the frictions analyzed in this paper are likely to also affect bank's borrowing generating higher rates on firm's loans. The financial architecture on this paper can also be interpreted as venture capital or private equity funds. These are investment vehicles that provide long-term finance, in particular for start-ups that need financing for back-loaded projects. However, these markets are smaller than corporate debt and there is little empirical evidence to quantify the model. Hence, we use data on corporate

for Italy and the United Kingdom; Schiantarelli and Srivastava (1997) for India; and Schiantarelli and Jaramillo (2002) for Ecuador). Additionally, Graham and Harvey (2001) use survey data to show that chief financial officers of U.S. companies consider matching the maturity of their firm's debt with the life of its assets as the most important factor affecting their choice between short- and long-term debt.

bonds in the quantitative section of the paper.

Related literature First, this paper is related to the literature on financial frictions and development (e.g. Greenwood, Sanchez, and Wang, 2010; Buera, Kaboski, and Shin, 2011; Moll, 2014; Midrigan and Xu, 2014). However, most of these papers focus on short-term (one period) contracts and the financial friction is usually a collateral constraint. One notable exception is Cole, Greenwood, and Sanchez (2016) that studies the optimal long-term contract in a production economy. The contribution of this paper to this literature is to consider OTC trading frictions and study how variations in bonds' maturity is related to economic development.

Second, this paper is also related to the literature on OTC markets developed by Duffie, Gârleanu, and Pedersen (2005). Some papers (e.g. Chen, Xu, and Yang, 2012; He and Milbradt, 2014; Chen, Cui, He, and Milbradt, 2017) study OTC trading frictions for bonds markets. The contribution to this strand of the literature is to study how trading frictions affect the yield curve and the maturity choices. Another contribution is that we introduce this friction in a production economy to analyze the consequences on investment and output.

Third, there is a literature that analyze the term structure of interest rates through the lens of the consumption based capital asset pricing model (see Grkaynak and Wright, 2012, for a recent review). These papers extend the expectation hypothesis, which posits that long-term interest rates are expectations of future average short-term rates. This paper is closer to an old literature that attributes the shape of the yield curve to liquidity considerations. Geromichalos, Herrenbrueck, and Salyer (2016) propose a monetary-search model with assets of different maturities to rationalize the shape of the yield curve. This paper use a simpler search framework, based on Duffie, Gârleanu, and Pedersen (2005), to recover the yield curve and take it to the data.

Fourth, there are some papers (e.g. Bruche and Segura, 2016; Arseneau, Rappoport, and Vardoulakis, 2016) that study how the liquidity of the secondary market is linked to the type of assets issued in the primary market. This paper contributes by studying the implications of this feedback for non-financial firms and how it affect the real sector of the economy.

Finally, some papers study the choice of maturity of projects (e.g. Majd and Pindyck, 1987) and how to finance those technology choices (e.g. Manuelli and Sanchez, 2016). The contribution is to study these choices in an equilibrium model subject to trading frictions.

The rest of the paper is organized as follows. Section 2 describe the empirical findings. Section 3 describes the economic environment. Section 4 and 5 solve the problem of lenders and borrowers in partial equilibrium. Section 6 solves the general equilibrium and study the effect of financial development. Section 7 estimate the model and presents the numerical results. Next, in Section 8 we perform the normative analysis of the economy. Sections 9 and 10 perform extensions and robustness of the model. Finally, Section 11 concludes. Proofs are gathered in the Appendices.

2 Empirical Facts

We document two empirical facts about corporate debt markets across countries. First, borrowing data from Cortina Lorente, Didier, and Schmukler (2016), we show that the maturity of corporate debt is positively correlated with the development of the economy. One characteristic about these assets, both in developed and developing countries, is that they are traded in over-the-counter markets. However, this markets are quite different across countries. Using data of trading over the counter for the US and Argentina, we show that secondary debt markets are substantially more illiquid in Argentina. These facts motivate the model presented in Section 3.

2.1 Corporate debt maturity

There is a large empirical literature showing that firms in developing countries borrow at shorter maturities than in advanced economies (e.g., Caprio and Demirgüç-Kunt, 1998; Demirgüç-Kunt and Maksimovic, 1998, 1999; Booth, Aivazian, Demirguc-Kunt, and Maksimovic, 2001; Fan, Titman, and Twite, 2012, among others). Most of th empirical evidence is based on the so called "short- and long-term debt" (below and above one year) gathered from firm-level balance sheet information. Using this type of evidence, studies find that the ratio of long-term debt (maturity greater than one year) to total liabilities is typically lower in developing countries than in developed ones. Similar arguments have been made using bank-level balance sheet data, which distinguish total bank credit granted to corporations, aggregated in broad maturity buckets. This paper focus on corporate debt as bank lending is usually associated with shorter term borrowing.

Cortina Lorente, Didier, and Schmukler (2016) compile a large dataset of corporate bond issuances in domestic markets for 1991-2014.[4] The top left panel of Figure 2 shows the empirical distribution of corporate debt maturities for advanced and developing countries. In developing countries, the median maturity is about 5 years, while it is about 10 years in advanced ones. The top right figure shows that maturity and GDP per capita relative to US are positively correlated across regions. For example, in the US the average maturity is 12.2 years. However, for Latin America average maturity is about 7.6 and GDP per capita is about 30% relative to US. Of course, both debt maturity and GDP are endogenous variables which are affected by several factors. The objective of this paper is to study how variations in liquidity of corporate debt markets are able to explain differences in debt maturity and GDP.

2.2 Liquidity of financial markets

Previous empirical studies on the development of the financial sector focus on the size of the market, by looking at moments like debt-to-gdp ratios (e.g. Borensztein, Cowan, Eichengreen, and Panniza, 2008). Levine and Zervos (1998) studies the turnover of stock market as a measure of financial development (see also De la Torre, Gozzi, and Schmukler, 2007). In this section we focus on a different aspect of financial development. In particular, how easy is to undo a financial position in corporate bonds markets for lenders, i.e., the liquidity of the market. We estimate different measures related to liquidity for the U.S. and Argentina.

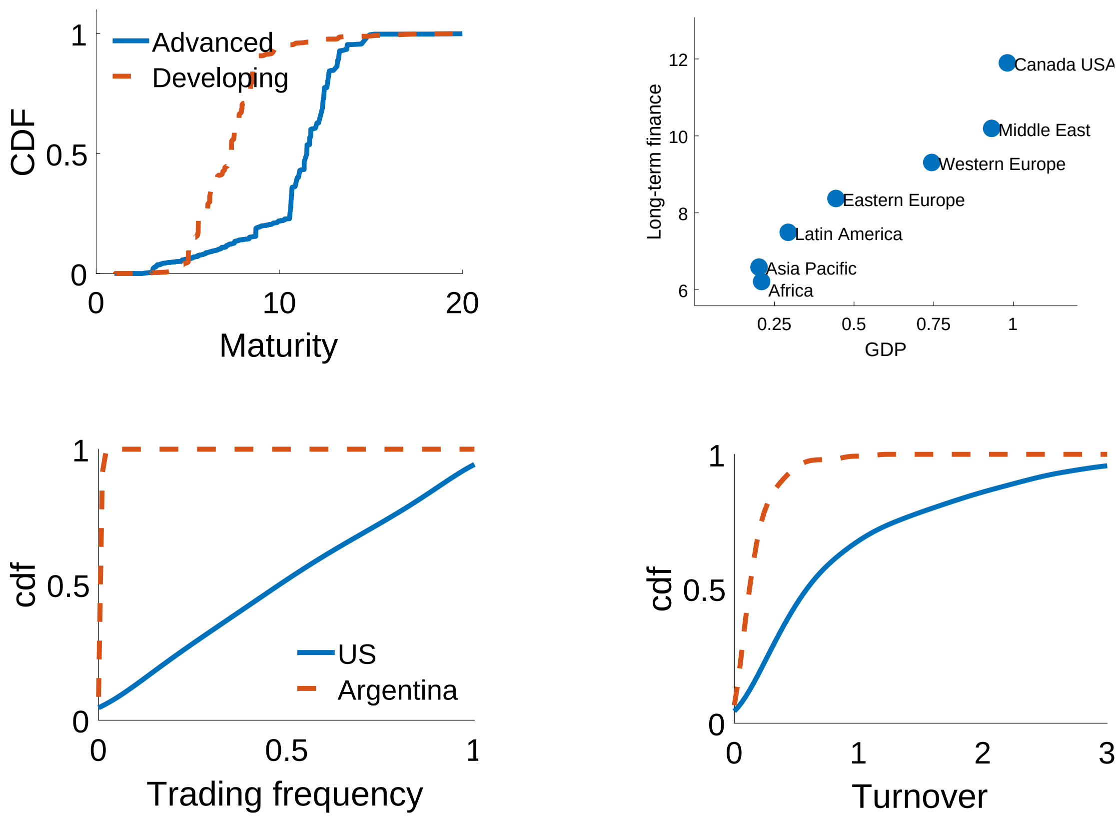
For the US we use data from Trade Reporting and Compliance Engine (TRACE) that provides transaction-level information of corporate bonds in secondary markets and has been widely used to study the liquidity of this market (e.g. Edwards, Harris, and Piwowar, 2007; Bao, Pan, and Wang, 2011, among others). For Argentina, we use a novel data set borrowed from *Mercado Abierto Electronico* (MAE, the over-the-counter exchange in Argentina). We got access to transaction-level information of corporate bonds (*Obligaciones Negociables*) which provides similar information than the TRACE database.[5] Appendix F.2 describes both datasets and their respective cleaning. We compare the liquidity of the US and Argentinean markets during the year 2012.[6]

[4] I thanks the authors for sharing their data, for details see Appendix F.1.

[5] I thanks Argentina's Central Bank to provide access to the data. To the best of my knowledge, this is the first paper to use this database. Borensztein, Cowan, Eichengreen, and Panniza (2008) documents some aggregate facts about the corporate debt market in Argentina using an aggregate version of this data.

[6] Results are similar for different time periods.

Figure 2: **Corporate bonds markets**



Note: Maturity is computed as the weighted average maturity (in years) of corporate bonds at issuance in domestic markets. Trading frequency is the frequency of days with at least one trade. Source: Cortina Lorente, Didier, and Schmukler (2016), TRACE and MAE. See Appendix F for details.

The first measure of liquidity is the trading frequency, defined as the frequency of days with at least one trade for each asset. The bottom left panel of Figure 2 shows the distribution of this measure in US and Argentina. In the US, the median of the trading frequency is 47.8%—i.e., assets are traded at least once every two days. On the other hand, in Argentina the median is 0.4%—i.e., assets are traded a bit more than once a year. Second, we measure liquidity with the annualized turnover ratio. The right bottom panel of Figure 2 shows that this measure is also much lower in Argentina than in US. The median turnover is 57% in the US whereas it is 11.9% in Argentina.

We conclude that secondary markets are substantially more illiquid in Argentina. It has been shown that the liquidity of the secondary market affects asset prices in the US (e.g. He and Milbradt, 2014). The evidence presented so far suggest that their consequences can be much severe in developing countries. In the next section we present a model in which the liquidity of the secondary market affects the maturity of corporate debt and reconciles the evidence presented in this section.

3 Theory: Environment

Time is continuous, starts at $t = 0$, and goes on forever. The economy is populated by a financial sector, a production sector, and a representative household. Firms choose from a continuous menu of back-loaded investment projects characterized by their maturity. Longer periods of investment are associated with higher future returns. To finance investment, firms borrow from the financial sector with issuance of corporate bonds. These securities are then traded by financiers in the over-the-counter (OTC) secondary market.

We study how trading frictions in the secondary market affect financial and investment decisions of firms. Section 4 studies the problem of lenders in partial equilibrium and describe how trading frictions affect the yield curve. Section 5 analyze the borrowers problem in partial equilibrium to understand how firms choose maturity taking as given the yield curve. Section 6 characterize the equilibrium by the interaction of lenders and borrowers decisions and study the effects of financial development.

4 Lenders

The financial sector trades corporate bonds in OTC markets à la Duffie, Gârleanu, and Pedersen (2005). Financiers take the distribution of securities in the economy as given. These are zero-coupon bonds with maturity τ and face value equal to one. Default occurs at a Poisson arrival rate λ^D. Upon default bond holders receives a payoff equal to zero. Every period a mass μ^0 of bonds with maturity τ are issued. Therefore, the mass of assets with time-to-maturity t that did not default are given by $\mu(t) = \mu^0 e^{-\lambda^D(\tau-t)}$.

Financiers can have either zero or one asset. An investor without the asset has three alternatives: (i) go to the primary market and get an *on-the-run* bond (i.e. a newly issued bond); (ii) search in the secondary market for an *off-the-run* bond; or (iii) stay inactive. Because there is a large inflow of financiers, primary and secondary markets are competitive and financiers are indifferent between going to these markets or staying inactive.

When a financier buys an asset he starts *unconstrained*. However, he faces an idiosyncratic liquidity risk of becoming *constrained*. With Poisson intensity λ^H he has to pay a holding cost h per unit of time. The financier has three ways of stop paying the holding cost: (i) Sell the asset in the secondary market; (ii) Wait until maturity; or (iii) Liquidity by default. Note that the idiosyncratic risk generates motives for trade in secondary markets.

Secondary market Sellers in the secondary market are the constrained financiers holding the asset. For each time-to-maturity t, there are $\mu^C(t)$ sellers. The total mass of sellers is $\mu^C = \int_0^\tau \mu^C(t)dt$. Buyers are financiers searching in the secondary market, with mass μ^B. There is random matching across different maturities (i.e., assets of different time-to-maturity are pulled together) and the matching function has constant returns to scale $M\left(\mu^C, \mu^B\right) = A\left(\mu^C\right)^\alpha \left(\mu^B\right)^{1-\alpha}$. Section 9 extends the model such that secondary markets are segmented by the time-to-maturity of the assets and shows that the main results are also present in that model.

Define market tightness as the ratio of sellers-to-buyers, $\theta = \frac{\mu^C}{\mu^B}$. A seller finds a counterpart at rate $\lambda^S = A\theta^{\alpha-1}$ and a buyer finds a counterpart with an asset of maturity t at rate $\lambda^B(t) = A\theta^\alpha \frac{\mu^C(t)}{\mu^C}$. Upon a meeting, the seller and the buyer bargain over the price of the asset, $P^S(t)$.

4.1 Distribution of financiers

Lemma 1 shows the steady-state distribution of constrained and unconstrained financiers with an asset.

Lemma 1. *The distribution of financiers with an asset is given by*

$$\mu^U(t) = \frac{\mu^0 \lambda^H}{\lambda^H + \lambda^S}\left(e^{\lambda^D(t-\tau)}\frac{\lambda^S}{\lambda^H} + e^{\left(\lambda^H+\lambda^S+\lambda^D\right)(t-\tau)}\right) \tag{1}$$

$$\mu^C(t) = \frac{\mu^0 \lambda^H}{\lambda^H + \lambda^S}\left(e^{\lambda^D(t-\tau)} - e^{\left(\lambda^H+\lambda^S+\lambda^D\right)(t-\tau)}\right) \tag{2}$$

At every moment, an asset can be hold by an unconstrained agent, a constrained agent, or be defaulted. Recall that default is an absorbing state while financiers transit from unconstrained to constrained and vice-versa. When the asset is just issued (i.e., time-to-maturity equal to τ), all the assets are hold by unconstrained agents. However, as time evolves, some of these agents become constrained and some of the assets enter into default. When the secondary market is well functioning (the selling intensity, λ^S, is relatively high), the mass of constrained agents μ^C is small. However, when λ^S diminish, the secondary market is more illiquid and the mass of constrained agents increases.

4.2 Value functions

Let $(D^U(t), D^C(t))$ be the value of unconstrained and constrained agent holding an asset with time-to-maturity t and let D^S be the value of search in secondary markets. Then

$$\rho D^S = -c + \int_0^\tau \lambda^B(t) \left(D^U(t) - D^S - P^S(t) \right) dt, \tag{3}$$

where ρ is the discount factor and c is the search cost. The discounted value of search is equal to the search cost and the expected gains from trade. With intensity $\lambda^B(t)$ the buyer is matched with a seller of a bond with time to maturity t, and the gains from trade are $D^U(t) - D^S - P^S(t)$. Note that the agent is searching across assets of maturities $t \in [0, \tau]$. Free entry in the secondary market implies $D^S \leq 0$.

The values of financiers holding an asset are

$$\rho D^U(t) = -\dot{D}^U(t) + \lambda^H \left(D^C(t) - D^U(t) \right) + \lambda^D \left(0 - D^U(t) \right) \tag{4}$$

$$\rho D^C(t) = -h - \dot{D}^C(t) + \lambda^S \left(P^S(t) - D^C(t) \right) + \lambda^D \left(0 - D^C(t) \right) \tag{5}$$

At maturity, $t = 0$, both investors recover the principal value of the asset, hence $D^U(0) = D^C(0) = 1$. Equation (4) defines the value of unconstrained agents. The left-hand side is the required return from holding the bond. The first term on the right-hand-side represents the change in value due to being closer to maturity. The second term captures the liquidity shocks that transform the investor into a constrained agent, which occurs at intensity λ^H. The third term captures the risk of default of the bond. Equation (5) captures the value of constrained agents and follows a similar explanation than the previous equation. The two differences are on the right-hand side: a constrained investor incurs a holding cost h, and the third term reflects the value of the secondary market. The investor meets a counterpart at rate λ^S and sell his bond at price $P^S(t)$.

4.3 Asset prices

The price in the secondary market, $P^S(t)$, is the outcome of Nash Bargaining between the seller and the buyer

$$\max_{P^S(t)} \left(P^S(t) - D^C(t)\right)^{\gamma} \left(D^U(t) - P^S(t)\right)^{(1-\gamma)}$$

where γ is the bargaining power of the seller. The solution is

$$P^S(t) = D^C(t) + \gamma\left(D^U(t) - D^C(t)\right). \tag{6}$$

They split the gains from trade $\left(D^U(t) - D^C(t)\right)$ according to the bargaining power γ.

The price in the primary market is given by the free-entry condition. Hence, $P(\tau,\Lambda) = D^U(\tau)$. Lemma 2 solves equations (4) and (5) and characterizes the asset price in the primary market.

Lemma 2. *The asset price in the primary market is*

$$P(\tau,\Lambda) = e^{-(\rho+\lambda^D)\tau} - \mathcal{L}(\tau,\Lambda), \tag{7}$$

where the illiquidity cost $\mathcal{L}(\tau,\Lambda)$ is

$$\mathcal{L}(\tau,\Lambda) = h \quad \frac{\lambda^H}{\lambda^H+\Lambda} \quad \int_0^{\tau} e^{-\left(\rho+\lambda^D\right)t}\left(1-e^{-\left(\lambda^H+\Lambda\right)t}\right)dt. \tag{8}$$

and liquidity is defined by $\Lambda := \lambda^S\gamma$. The illiquidity cost satisfies the following properties:

1. *$\mathcal{L}(\tau,\Lambda)$ is positive.*
2. *Sensitivity with respect to maturity τ:*
   (a) *It is increasing in τ;*
   (b) *It has a finite limit, $\lim_{\tau\to\infty}\mathcal{L}(\tau,\Lambda) = h\frac{\lambda^H}{(\rho+\lambda^D)(\rho+\lambda^D+\lambda^H+\gamma\lambda^S)}$.*
3. *Sensitivity with respect to liquidity shocks λ^H:*
   (a) *If there are no liquidity shocks, $\lambda^H = 0$, then $\mathcal{L}(\tau,\Lambda) = 0$;*
   (b) *If $\lambda^H \to \infty$ (i.e., always has to pay the cost h) then*

$$\lim_{\lambda^H\to\infty}\mathcal{L}(\tau,\Lambda) = h\frac{1-e^{-(\rho+\lambda^D)t}}{\rho+\lambda^D}.$$

4. *Sensitivity with respect to liquidity* Λ*:*

   (a) *It is decreasing in* Λ*;*

   (b) *If there are no secondary markets,* $\Lambda = 0$*, the liquidity term only represents the expected holding costs, i.e.,*

$$\mathcal{L}\left(\tau, 0\right) = h \int_0^\tau e^{-\left(\rho+\lambda^D\right)t}\left(1 - e^{-\lambda^H t}\right) dt;$$

   (c) *If secondary markets are totally liquid (i.e.,* $\Lambda \to \infty$*) then* $\mathcal{L}\left(\tau, \Lambda\right) = 0$*.*

5. *Liquidity is more important for long-term assets:* $\frac{\partial^2 \mathcal{L}(\tau,\Lambda)}{\partial\tau\Lambda} \leq 0$*.*

The price $P(\tau, \Lambda)$ has two terms. The first one represent the *frictionless* solution. That is, the value of a promise to pay one unit in τ periods when the discount rate is ρ and the default intensity is λ^D. Note that absent the second term, the expectation hypothesis holds: long-term interest rate are equivalent to the average of short-term rates.

The second term, $\mathcal{L}(\tau, \Lambda)$, represents the *illiquidity cost*. It is equivalent to the expected discounted time that the asset will be in the *constrained* state times the respective holding cost h. When this term is different from zero the expectation hypothesis does not hold and borrowing at longer horizons is more expensive than the average of short-term rates. We refer to $\Lambda = \lambda^S\gamma$ as the liquidity of the secondary market. Recall that λ^S is the selling intensity in the secondary market (an equilibrium object). When λ^S increases, it is easier to sell in the secondary market. Similarly, γ is the bargaining power of the seller in the secondary market (a parameter). when γ increases, the seller keeps a larger fraction of the gains from trade. Therefore, we say that the secondary market is more liquid if Λ increases. Lemma 2 also characterizes the properties of the illiquidity cost.

Figure 3a summarize three important properties of the illiquidity cost presented in Lemma 2. First, the illiquidity cost is increasing in maturity. As the life of the asset increases, the asset spend more time in the market and therefore the expected time in the constrained state increases. Second, the illiquidity cost is decreasing in liquidity. If the secondary market is more liquid, then the asset spend less time in the constrained state and so the illiquidity cost decreases. Third, the cross partial derivative with respect to maturity and liquidity is negative. This fact reflect that the liquidity of the secondary market is more important for long-term assets. Recall that an investor that wants to exist a long position on a bond can either sell it on the secondary market or wait until maturity. Hence, the role of the secondary market is more important for long-term assets and as a result the cross partial derivative is negative: an increase in liquidity benefits more long-term assets.

4.4 Yield curve

We can translate the price of the asset into a compound interest rate defined as $r(\tau,\Lambda)$ that solves $P(\tau,\Lambda) = e^{-r(\tau,\Lambda)\tau}$, i.e., the value of the asset conditional on it being held to maturity without default or re-trading. From Equation (7), the interest rate for a bond of maturity τ is

$$r(\tau,\Lambda) = \rho + \lambda^D + \frac{1}{\tau}\log\left(\frac{1}{1-e^{(\rho+\lambda^D)\tau}\mathcal{L}(\tau,\Lambda)}\right). \tag{9}$$

The first term, ρ, is the risk-free rate, while the remainder two terms are the credit spread that can be decomposed into a default and a liquidity component, following He and Milbradt (2014). Consider a marginal investor with no idiosyncratic liquidity risk. Such investor requires an interest rate equal to $\rho+\lambda^D$. Therefore, the credit spread due to default is λ^D. Finally, we can define the credit spread due to liquidity by subtracting the default component. Hence, the third term correspond to the liquidity spread

$$cs^{liq}(\tau,\Lambda) = \frac{\log\left(1-e^{(\rho+\lambda^D)\tau}\mathcal{L}(\tau,\Lambda)\right)}{\tau}$$

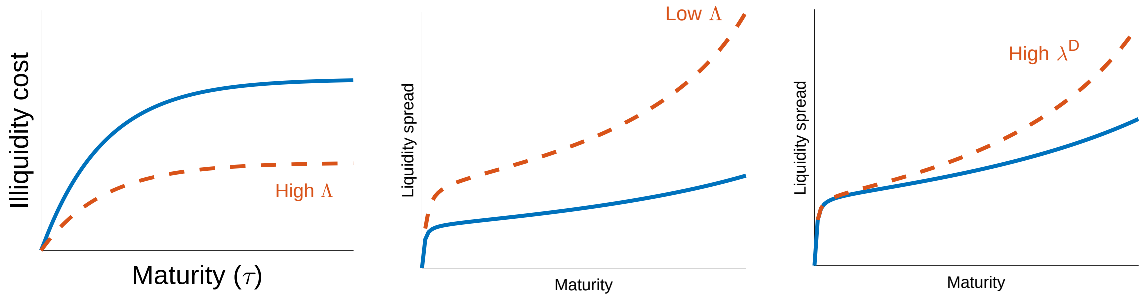
We define the term premium as the difference between the short-term rate $r(0)$ and the long-term rate $r(\tau)$, i.e., $\mathrm{tp}(\tau,\Lambda) = r(\tau) - r(0)$. That is $\mathrm{tp}(\tau,\Lambda) = cs^{liq}(\tau,\Lambda)$. Note that in the model the term premium is equivalent to the credit spread due to liquidity. This implies that variations in interest rates across maturities are given by differences in the liquidity component of the asset.

There is a feedback-loop between default and liquidity. The default rate λ^D affects the credit spread due to liquidity. As in this model the default decision is simple, liquidity does not affect default.

Lemma 3. *The liquidity spread is increasing in maturity, decreasing in liquidity and increasing in the default intensity. Moreover, the proportional bid-ask spread is increasing in maturity.*

Lemma 3 establish that the liquidity spread (and the term premium) is increasing in maturity and decreasing in liquidity Λ. Figure 3b shows these properties. Importantly, note that the

Figure 3: **Liquidity cost and spread.**



(a) Liquidity cost (b) Liquidity spread (c) Liquidity spread

Note: Illiquidity cost $\mathcal{L}(\tau, \Lambda)$ is increasing in maturity and decreasing in liquidity. The liquidity spread is increasing in maturity, decreasing in the liquidity, and increasing with default. The parameter values are discussed in Section 7.

liquidity of the secondary market affects more the long end of the curve. Recall that this result was also present in $\mathcal{L}(\tau, \lambda^S)$ and the yield curve preserves the property.

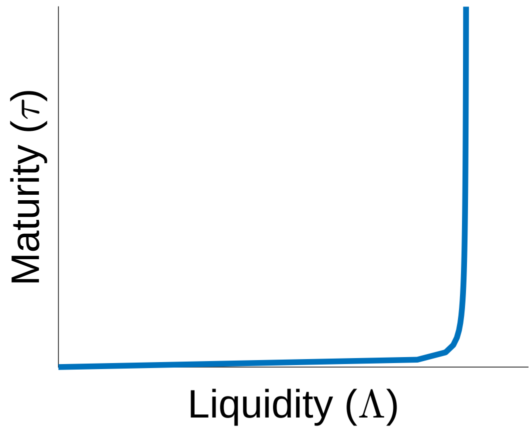
Lemma 3 also shows that the liquidity spread is increasing in the default intensity Λ. There are three forces at play. The first two trade-offs are in partial equilibrium (i.e. holding fixed the liquidity λ^S) and the last one is related to the general equilibrium (when the liquidity of the secondary market changes). First, as the default probability increases there is more 'liquidity by default' (i.e., the constrained financier stop paying the holding cost due to default), $\mathcal{L}(\tau)$ decreases and the term premium diminish. Second, the discount factor increases and for the same illiquidity cost $\mathcal{L}(\tau)$, the investor requires a higher term premium. Lemma 3 shows that the second effect dominates. Finally, in general equilibrium there are less gains from trades, λ^S decreases, and this effect amplifies the increase in the term premium. Figure 3c shows this feedback-loop between default and liquidity.

Finally, we can define the proportional bid-ask spread as the gains from trade normalized by the mid-price

$$BA(t) = \frac{D^U(t) - D^C(t)}{1/2\left(D^U(t) + D^C(t)\right)}$$

Lemma 3 shows that the proportional bid-ask spread is increasing in maturity. When we consider an asset of longer maturity, there are more gains from trade. As a result, the bid-ask spread increases with maturity. This result is consistent with empirical evidence (e.g., Edwards, Harris, and Piwowar, 2007).

Figure 4: **Free-entry in secondary markets**



Note: Free entry condition (10)*. The parameter values are discussed in Section 7.*

4.5 Free entry

The free-entry condition in the secondary market (3) implies a market-tightness, or equivalent a liquidity $\Lambda(\tau)$ for each possible maturity $\tau \geq 0$ characterized by

$$c = (1-\gamma) \int_0^{\tau} \lambda^B(t) \left(D^U(t) - D^C(t) \right) dt \tag{10}$$

Lemma 4 characterizes Λ as a function of τ. It shows that when assets are of zero maturity, there are no gains from trade, hence no entry in secondary markets and therefore $\Lambda = 0$. As τ increases, gains from trade also increase, and as a result there is more entry of potential buyers which causes an increase in liquidity. Moreover, when τ goes to infinity, the gains from trade are bounded which implies that Λ converges to a finite number. Interestingly, as we will see in the quantitative Section 7, this bound is approach quite quickly, for maturities above 5 years. Figure 4 shows the relationship between Λ and τ.

Lemma 4. $\Lambda(\tau)$ *is increasing in* τ *and* $\Lambda(\tau) \mapsto [0, \overline{\Lambda}]$.

5 Borrowers

In this section we derive a demand for long-term borrowing. At every moment a new cohort of identical firms enters the economy and choose a new project. There is a continuum of projects differentiated by their back-loading strategy. In this section we provide a microfoundation based on a quality ladder model. This interpretation will be useful for the estimation of the model. Appendix B.3 propose an alternative specification with a model of time-to-build capital that also generates a similar demand for long-term bonds.[7] More generally, the qualitative results hold under a general formulation for back-loaded projects that require long-term financing.

5.1 Quality ladder model

We follow the standard setup with a continuum of intermediate goods producers with monopolistic power and final goods producers in perfect competition (e.g., Akcigit and Kerr, 2017). The final good is produced with labor and a continuum of intermediate goods $j \in [0,1]$ with the production technology

$$Y = \frac{L^{\beta}}{1-\beta} \int q_j^{\beta} k_j^{1-\beta} dj$$

where L is the labor input, k_j is the quantity of intermediate good j, and q_j is its quality. We normalize the price of the final good Y to be one in every period without loss of generality. The final good is produced competitively with input prices taken as given. The inverse demand of intermediate inputs reads $p_j = L^{\beta} q_j^{\beta} k_j^{-\beta}$ and the labor demand is $L = \frac{\beta Y}{w}$.

Each intermediate good j is produced with a linear technology $k_j = \overline{q} l_j$ where $\overline{q} = \int_0^1 q_j dj$ is the average quality, and l_j is the labor input. Intermediate producers take the inverse demand and maximize static profits

$$\pi(q) = \max_{k_j \geq 0} p_j k_j - \frac{w}{\overline{q}} k_j \qquad \text{s.t. } p_j = L^{\beta} q_j^{\beta} k_j^{-\beta}$$

There is an exogenous labor supply normalized to one. Labor market clearing implies $L + \int_0^1 l_j = 1$. Lemma 5 shows that static profits are linear in quality and aggregate output is linear in the average quality of the economy.

Lemma 5. *Static profits are linear in quality and output is linear in average quality*

$$\pi(q) = \underline{\pi} q_j \qquad Y = \underline{Y}\overline{q} \tag{11}$$

[7] Rioja and Valev (2004) finds that finance boosts growth in rich countries primarily by speeding-up productivity growth, while finance encourages growth in poorer countries primarily by accelerating capital accumulation. In B.3 we propose an alternative microfoundation for back-loaded projects based on time-to-build capital.

where the constants $\underline{\pi}$ *and* $\underline{Y}$ *are*

$$\underline{\pi} := \beta L \left(\frac{\overline{q}}{w}\right)^{\frac{1-\beta}{\beta}} (1-\beta)^{\frac{1-\beta}{\beta}} \qquad L = \frac{\beta}{(1-\beta)^2 + \beta} \qquad \underline{Y} := \frac{\beta^{\beta}(1-\beta)^{(1-2\beta)}}{(1-\beta)^2+\beta}.$$

Life cycle of intermediate goods producers The goal is to derive a demand for long-term finance. There are two main empirical facts that motivates the assumptions about the back-loaded profile of investment projects. First, the *Arrow's replacement effect* establish that small young firms are more innovative than large and old firms (e.g., Arrow, 1962; Itenberg, 2015; Akcigit and Kerr, 2017). Second, small firms are more financially constrained, and in particular for research and development (Itenberg, 2015). Based on these facts, we assume that a newborn firm choose a project maturity τ such that she is *young* for $t \leq \tau$ and *mature* otherwise.

A young firm invest in research and development to improve the quality of the product. First, as it is standard in the literature, we assume that quality make jumps of size δ_2 that arrive at Poisson rate λ^Q. Second, as we want to abstract from strategic default, we assume a deterministic component on quality such that at maturity the value of the firm is such that she always want to repay the debt.[8] Hence, the evolution of quality is given by $\dot{q} = \delta_1 + \lambda^Q \delta_2$. Doing research is costly. The firm has to pay κ per unit of time doing research. Moreover, the firm is financially constrained when young, so she has to borrow to finance the investment project, generating a demand for long-term finance.

At age τ the firm becomes mature, stop doing R&D and start production with quality $q(\tau, N)$, where N is a counting process with the number of jumps on quality before τ. Static profits are linear in quality $\pi(q) = \underline{\pi} q$. Hence, the net present value of a mature firm with quality q is given by

$$F(q) = \frac{\pi(q)}{\rho + \lambda^E}$$

where λ^E is the Poisson arrival rate of an exogenous exit shock.

[8] Appendix XXX extends the model to allow for default ... XXX

5.2 Financing and choice of project's maturity

A newborn firm choose the maturity of the project τ. To finance the costs of research and development the firm issue zero coupon bonds. A bond of maturity y has a fix cost of issuance Φ and an interest rate $r(y)$. Let J be the number of times that a young firm issue bonds in financial markets. Given the fix cost of issuance we know that the firm wants to tap the bonds market only a finite number of times and not issue infinitesimal short-term debt. Moreover, we assume that once the firm becomes mature she does not need to borrow any longer.[9] As the interest rate is higher than the discount factor of the firm, a mature firm will not tap the bonds market. This implies that only young firms will be borrowing in this economy.

Consider a young firm of age x that is building a project with maturity $\tau \geq x$, has received N jumps on the quality, and has to rollover outstanding debt B. Its value is defined by

$$\begin{aligned} E(\tau, x, N, B) &= \max_{y \leq \tau - x} e^{-\left(\rho+\lambda^D\right)y} \mathbb{E}\left[E(\tau, x+y, N', B')\right] \\ B' &= e^{r(y)y}\left(B + \Phi + I(y)\right) \\ E(\tau, \tau, N, B) &= F(q) - B \end{aligned}$$

The firm chooses the maturity of the new issuance $y \in [0, \tau - x]$ and borrow B' to rollover existing debt, B, pay the fix cost of issuance, Φ, and borrow money to finance research for the next y periods. $I(y)$ is the money needed to finance the research costs in a windows of length y. The firm deposit this money in a bank account with risk-free rate ρ and withdraw κ per units of time, so $I(y) = \kappa \frac{1-e^{-\rho\tau}}{\rho}$.

Lemma 6 shows that the firm's value can be decomposed in two terms: (i) The net present value of the project, and (ii) Financial cost.

Lemma 6. *A new firms solves*

$$\max_{\tau} e^{-\left(\rho+\lambda^D\right)\tau} \mathbb{E}\left[F(N, \tau)\right] - FIN^{COST}(\tau)$$

where the financial cost is

$$FIN^{COST}(\tau) = e^{-\left(\rho+\lambda^D\right)\tau} \min_{J, \{y_j\}_{j=1}^{J}} \sum_{i=1}^{J} \left(\Phi + I(y_i)\right) e^{\sum_{s=i}^{J} r_s y_s} \qquad s.t. \sum_{j=1}^{J} y_j = \tau$$

[9]The project becomes *liquid* and the firm can repay outstanding debt with the value of the project.

The financial cost for a project of maturity τ consist of choosing the amount of issuances J and the maturity of each bond $\{y_j\}_{j=1}^J$ to minimize the net present value of issuances costs Φ and investment needs $I(y_j)$. There are two financial frictions that shape the financial decisions: (i) Issuance cost, and (ii) Liquidity spread.

First, the top panel of Figure 5 shows the number of issuances and the financial cost for different issuance cost Φ. Naturally, as the issuance cost increases, the number of issuances decreases and the financial cost increases. Note that if Φ is sufficiently large, the firm optimally choose to issue only one time, matching the maturity of the project and the liabilities. In the next section we focus in this special case to derive a sharper analytical characterization of the effects of secondary markets liquidity on the project's choice.

Second, recall that as the secondary market becomes more liquid—higher Λ—the credit spread due to liquidity diminish, generating a flatter yield curve. The bottom panel of Figure 5 shows the number of issuances and the financial cost under different values of the liquidity of secondary markets for a given project τ. As Λ^S increases, firms decide to rollover less often, borrow at longer maturities, and reduce the financial cost. Hence, the financial cost for a given τ diminish. In the next section we study how this change in financial cost affects the decision of which project to implement.

Finally, a newborn firm choose the maturity of the project to solve $\max_\tau E(\tau, 0, 0, 0)$.

5.3 Financial wedge

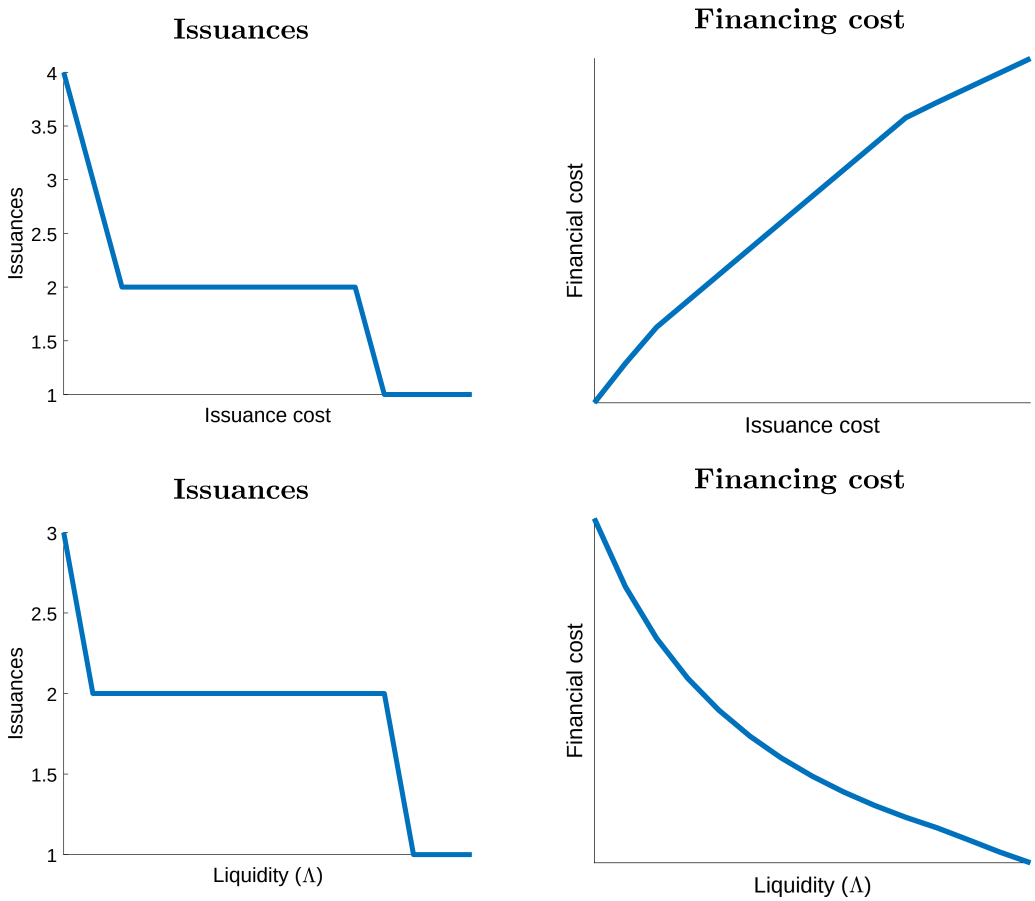
Empirical evidence shows that firms tend to match the maturity of assets and liabilities.[10] Hence, from now on we assume that firms match the maturity of the project and the bond making only one issuance, $J = 1$. In section 10 we relax this assumption and show that the main mechanisms are also present for lower values of the issuance cost when the firm decides to issue debt more than one time.

The firm's problem is

$$\max_{\tau} \quad e^{-\left(\rho+\lambda^D\right)\tau}\mathbb{E}\left[F\left(\tau, N\right)\right] - \left(\Phi + I\left(\tau\right)\right)e^{\mathrm{cs}^{\mathrm{liq}}(\Lambda,\tau)\tau}. \tag{12}$$

[10] See Stohs and Mauer (1996); Guedes and Opler (1996); Frederick C. Scherr (2001); Dew-Becker (2012) for the U.S. and Schiantarelli and Sembenelli (1997); Schiantarelli and Srivastava (1997); Schiantarelli and Jaramillo (2002) for international evidence. Myers (1977) provides a theory for maturity matching.

Figure 5: **Financial cost with rollover**

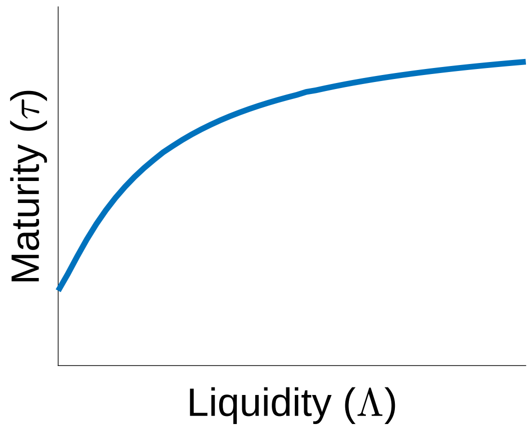


Note: Financial cost for a given maturity τ and different issuance cost Φ and liquidity Λ. Parameter values are discussed in Section 7.

The firm discount factor is $\rho + \lambda^D$ while the interest is $r(\tau) = \rho + \lambda^D + \mathrm{cs}^{\mathrm{liq}}(\Lambda, \tau)$. The liquidity spread is a friction on the lender's problem that creates a wedge between lenders and borrowers discount factors. Importantly, recall that this wedge is increasing in maturity. The financial cost increases with τ for two reasons: (i) The firm has to pay the cost for a longer period, and (ii) The liquidity spread increases. The optimal maturity solves

$$\begin{aligned}\frac{\partial \mathbb{E}\left[F\left(\tau, N\right)\right]}{\partial \tau} = &\left(\rho + \lambda^D\right) \mathbb{E}\left[F\left(\tau, N\right)\right] + \frac{\partial I\left(\tau\right)}{\partial \tau} e^{r(\Lambda,\tau)\tau} \\ &+ \left(\Phi + I\left(\tau\right)\right) e^{r(\Lambda,\tau)\tau} cs^{liq}\left(\Lambda, \tau\right)\left(1 + \epsilon_{cs^{liq},\tau}\right)\end{aligned} \tag{13}$$

Figure 6: **Optimal maturity**



Note: Optimal maturity $\tau(\Lambda)$. The parameter values are discussed in Section 7.

where $\epsilon_{cs^{liq},\tau}$ is the elasticity of the liquidity spread to maturity is

$$\epsilon_{cs^{liq},\tau} = \frac{\partial cs^{liq}(\Lambda,\tau)}{\partial \tau}\frac{\tau}{cs^{liq}(\Lambda,\tau)}$$

Consider a marginal increase in τ. The left hand side of Equation (13) represents the benefits of operating a firm with higher quality and hence higher returns. The right hand side captures three associated costs. First, a more back-loaded project will require more time to become profitable which implies higher discounting. Second, a larger firm requires more investment. Note that even without financial frictions (i.e. constant interest rates, $r(\tau) = \rho + \lambda^D$ and $\Phi = 0$) we have an interior solution for τ.[11] Intuitively, there is an interior solution because as the firm choose a larger τ it is both more costly and it takes more time to complete. Hence, the size of the firm can be determined even with constant returns to scale.

The third term of Equation (13) captures the two terms of the financial cost.First, as maturity increases, the firm has to pay the liquidity spread for a longer period. Second, the liquidity spread increases with maturity. These two forces induced by the financial frictions makes the optimal maturity to be shorter than in the frictionless economy. Recall that the liquidity spread is characterized by liquidity, Λ. We can characterize the interest rate schedule both when secondary markets are fully liquid ($\Lambda = \infty$) and when there are no secondary markets ($\Lambda = 0$). It is easy to show that $\underline{\tau} = \tau(0) < \lim_{\Lambda\to\infty} \tau(\Lambda) = \overline{\tau} < \infty$. Figure 6 shows that $\tau(\Lambda)$ is increasing. As Λ increases, the liquidity spread decreases, and the firm issue at longer horizons.

[11] Note that this is also true even if we have no default, $\lambda^D = 0$.

5.4 General model of back-loaded project

We can derive a demand for long-term debt under a general specification of back-loaded projects. Consider a reduced-form formulation for the value of the project $F(\tau)$ with F increasing and either linear or concave. Then, the demand for long-term debt will be analogous to the one derived in this section. The advantage of the quality ladder model is that we can easily take it to the data using moments of the R&D literature.

6 Equilibrium and Financial Development

We characterize the equilibrium of the model as the intersection of the entry decision of lenders in financial markets from Section 4 and the issuance choice of borrowers from Section 5. We study the steady-state search and matching equilibrium of this economy.

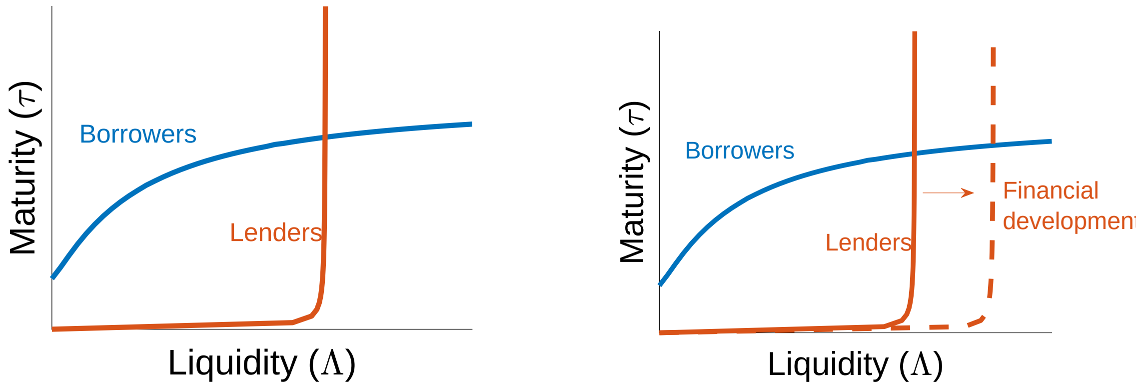
Definition 1. *A steady-state search and matching equilibrium is characterized by the liquidity of secondary markets Λ, an interest rate schedule $r(\Lambda, \tau)$, and maturity τ such that:*

1. *Liquidity Λ solves the free-entry condition of financiers (10), taking the assets as given;*
2. *Maturity τ solves the firm's optimal maturity problem (12), taking the interest rate schedule $r(\Lambda, \tau)$ as given.*

The equilibrium is characterized by liquidity Λ and maturity τ given by (10) and (13). The left panel of Figure 7 shows the intersection of these two curves and characterize the equilibrium. Lemma 4 establish that $\Lambda(\tau)$ is increasing in τ and $\Lambda(\tau) \in [0, \overline{\Lambda}]$. In Section 5 we quantitatively solve for $\tau(\Lambda)$ as an increasing function with bounded support $\tau(\Lambda) \in [\underline{\tau}, \overline{\tau}]$. As a result, there exists a pair (Λ^*, τ^*) such that $\tau(\Lambda^*) = \tau^*$ and $\Lambda(\tau^*) = \Lambda^*$.[12]

[12] A sufficient condition for a unique solution is that $\tau(\Lambda)$ to be increasing and concave and $\Lambda(\tau)$ to be increasing and concave. We cannot derive an analytical characterization of these properties. However, by visual inspection on the numerical example on Figure 7a we can see that these properties are satisfied. Therefore, for the parameter values used in the numerical example there exist a unique equilibrium and we conjecture that this property holds for a wide range of the parameter space.

Figure 7: **Equilibrium characterization**



(a) Equilibrium

(b) Financial development

Note: Equilibrium characterization and financial development. The parameter values are discussed in Section 7.

6.1 Financial development

We interpret financial development as an increase in the matching efficiency in the secondary market, A. First, note that the optimal maturity, characterized by the curve $\tau(\Lambda)$, depends on the equilibrium Λ, but not directly on A. Hence, the curve $\tau(\Lambda)$ does not move. However, the curve $\Lambda(\tau)$ depends directly on A. First, given a maturity and a market tightness, the selling intensity increases with the matching efficiency. Hence, the curves moves to the right. Second, the gains from trade are larger when the matching efficiency increases. This implies more entry of buyers. As a result, the curves moves further to the right. The red-dashed line in Figure 7b shows the new locus for the equilibrium condition. When the matching efficiency increases, both the selling intensity and maturity increase. Hence, financial development—an increase of matching efficiency of secondary markets—causes an increase of debt maturity. In the next section we evaluate this mechanism quantitatively.

The search costs in the secondary market, c also capture the trading frictions. This parameter has an effect similar to an increase in the matching efficiency A. When the entry cost diminish, more potential buyers enter into the OTC market and the selling intensity increases. As a result, the term premium diminish and firms issue at longer horizon and invest in more profitable projects.

What is financial development? First, there is literal interpretation as the technology to execute trades. For example, in developed markets there are clearing houses such as Euroclear or Clearstream. However, in developing countries the time to execute a trade is delayed by technological constraints. For example, in Argentina investors liquidate securities in one place, but make payments in a different bank.[13] Second, a broader interpretation is to think about the participants in the market. In developed financial markets there are mutual funds which are agents that trade more frequent than the rest of the market participants. However, these funds are either small or inexistent in developing countries. As a result, this could imply a lower liquidity in developing countries. Finally, Bethune, Sultanum, and Trachter (2017) shows that private information on the valuation of the securities create informational rents and reduce trading. Hence, their model predicts that markets will be more illiquid when there are larger informational rents which might be the case of developing countries, for example due to weak credit bureaus.

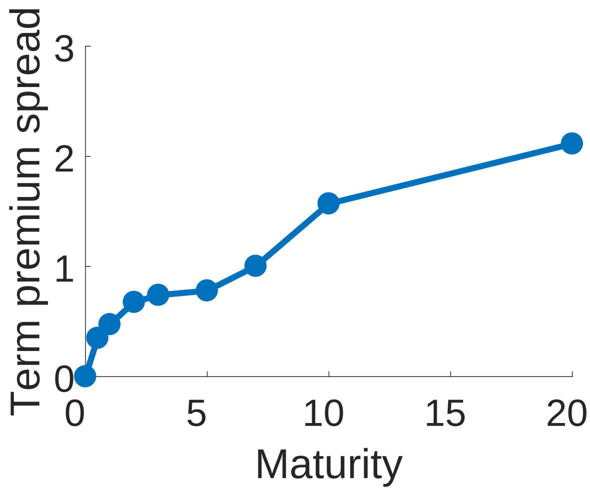
7 Quantitative Exploration

The model is calibrated to match moments for the U.S. corporate debt market. Some parameters can be calibrated "externally", while others must be calibrated "internally" from the simulation of model. Table 1 summarize the parameter values.

Liquidity spread The most important moment to calibrate the model is the term premium driven by the liquidity spread. Let $r^{\text{treasury}}(t)$ and $r^{\text{corporate}}(t)$ be the zero-coupon yield curve for treasuries and corporate debt respectively. For corporate debt we focus on the High quality market (bonds with rating A and above) such that we can abstract from default risk. The U.S. Treasury provides synthetic estimates of these two yield curves. We define the liquidity spread as $S(t) = r^{\text{corporate}}(t) - r^{\text{treasury}}(t)$. The identification assumption is that all the variations in the spread between corporate and treasury rates is given by differences in the liquidity of the market in which those securities are traded. Note that this spread is positive and increases with maturity. Figure 8 shows the estimated liquidity spread. We target only this spread at the equilibrium maturity, i.e., one point of this yield curve. Next, we verify the performance of the model in replicating the data at different maturities.

[13] Recently in Argentina, BYMA—the local exchange market—is trying to unify these operations to increase the liquidity of the market.

Figure 8: **Liquidity spread.**



The figure shows the spread between the yield curve for treasuries and corporate bonds. Source: U.S. Treasury.

Financial sector To discipline the search cost c and the holding costs h we target the term premium at maturity of $\tau = 12.2$ which is equal to 1.6% and the expected time to sell which is equal to 2 weeks (He and Milbradt, 2014). The intensity of the liquidity shocks λ^H influence the turnover rate. Note that turnover is equal to $\left(\left(\lambda^H\right)^{-1} + \left(\lambda^S\right)^{-1}\right)^{-1}$. Hence, we can directly calibrate λ^H without simulating the model.[14] The annual turnover rate is target to 57% (He and Milbradt, 2014; Chen, Cui, He, and Milbradt, 2017).

Maturity For the maturity of corporate bonds we target a debt maturity at issuance of 12.2 years (Cortina Lorente, Didier, and Schmukler, 2016). This moment provides information on the shape of the back-loaded projects. We normalize the constant $\underline{\pi} = 1$ and internally calibrate the investment cost κ to match this target.

Matching For the secondary market we assume a constant-return-to-scale Cobb-Douglass matching function with efficiency parameter normalized to one, and symmetric loadings on sellers and buyers. We further assume symmetric bargaining power, $\gamma = 0.5$.

We assume a discount factor of $\rho = 0.2$ (He and Milbradt, 2014). For the default and exit rate we assume $\lambda^D = \lambda^E = 3\%$ to match the default rate of speculative grade firms (Moody, 2015).

[14]The expected time to sell of two weeks implies that $\lambda^S = 26$.

Quality ladder We normalize the measure of newborn firms $\mu^0 = 1$. Recall that the growth of quality has a deterministic component to guarantee that there is no default at maturity. Hence, we calibrate δ_1 such that $F(\delta_1 \tau, 0) \geq I(\tau)e^{r(,\Lambda,\tau)\tau}$. Next, we calibrate δ_2 and λ^Q to match the growth rate with age of quality and the standard deviation of log quality (Hsieh and Klenow, 2009, 2014).

Table 1: **Parameters**

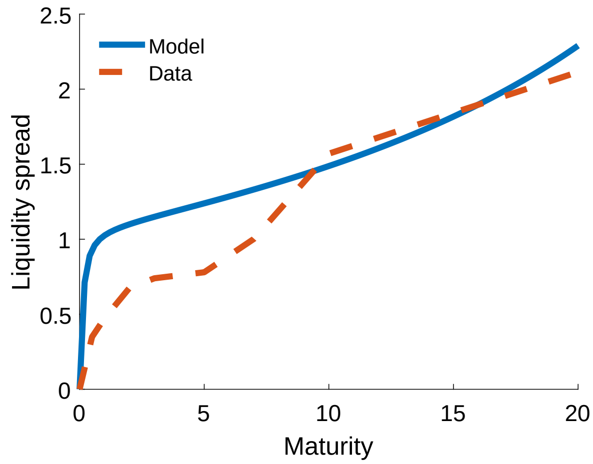
Parameter	**Value**
Poisson intensity for ideas (λ^Q)	0.3707
Holding costs (h)	0.2506
Search cost (c)	0.0352
TFP matching technology (A)	10.0000
Discount factor (ρ)	0.0200
Exit shock (λ^E)	0.0300
Matching secondary markets (α^s)	0.5000
Constant profits ($\underline{\pi}$)	1.0000
Bargaining power of seller in secondary markets (γ)	0.5000
Mass of entrepreneurs (μ^0)	1.0000
Default rate (λ^D)	0.0300
Intensity h (λ^H)	0.5828
Investment cost (κ)	0.2379
TFP deterministic (δ_1)	0.0236
TFP stochastic (δ_2)	0.1162

Table 2 shows that the model is able to match the seven targeted moments. Even though the seven parameters are jointly chosen to match the seven targeted moments, we can derive some intuition on the identification of the parameters. Appendix E.4 shows that the chosen moments provide useful information to estimate the parameters of the model. We now move to evaluate the model outside the target moments.

Table 2: **Moments.**

	Data	**Model**
Turnover	0.57	0.57
Maturity	12.20	11.89
Credit Spread Liquidity	1.60	1.60
Expected time to sell	2.00	2.00
Standard deviation log TFP	0.85	0.89
Expected TFP growth	0.07	0.07
Default at maturity	0.00	0.00

Figure 9: **Liquidity spread: Data vs Model**



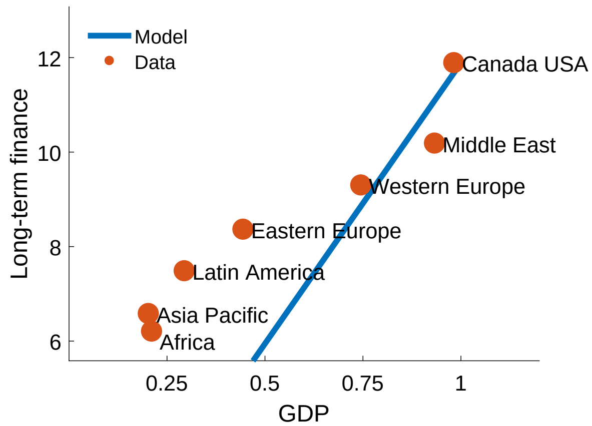
7.1 Validation

We target the liquidity spread at $\tau = 12.2$. Figure 9 shows that the model performs well in replicating the liquidity spread for maturities between zero and 20 years. The model overshoot a bit at short maturities. However, this should not be relevant for this model as the object of study is long-term finance.

There is some indirect empirical evidence that is in line with the mechanism of the paper: When financial markets are more liquid, firms choose to issue at longer maturities. First, Saretto and Tookes (2013) compares issuance of firms with and without credit default swaps (CDS). The argument is that securities of firms with CDS trade in more liquid financial markets. The paper finds that firms with CDS increase the maturity between 0.68 and 1.79 years. Second, Cortina Lorente, Didier, and Schmukler (2016) studies firms in developing countries issuing bonds in foreign markets which are more liquid than domestic financial markets. The paper finds that maturity increases by 1.6 years when a firm decides to issue abroad. Hence, the empirical evidence support the mechanism that when the liquidity of financial markets improves, firms choose to issue at longer maturities.

The second mechanism of the paper argues that when the term premium increases, firm choose to invest in shorter-term projects. Dew-Becker (2012) finds empirical evidence on this mechanism. Using data on the U.S., he finds that when the term premium increases the duration of investment decreases. Also, Yamarthy (2016) shows that when firms shift their long-term debt ratio to longer maturities, profitability and investment rates are higher.

Figure 10: **Model versus Data.**



Note: The model shows the equilibrium allocations of maturity and GDP under different frictions in the secondary market. The data is the same as in Figure 2.

7.2 Quantitative results: Model vs data

We explore the role of trading frictions on the real economy. For sovereign debt markets, Broner, Lorenzoni, and Schmukler (2013) shows that it is more expensive to borrow long-term in developing countries than in advanced ones. However, there is very little evidence on the yield curve for corporate bonds. To overcome this problem, we use the model to infer the trading frictions that are consistent with the data on maturity choices across countries on Figure 2. The exercise consist of finding the frictions in financial markets that are consistent with the debt's maturity observed in the data, and then evaluate the performance of the model on replicating the effects on the real economy. Figure 10 shows that the model predicts a relationship between maturity and GDP quite similar to the data counterpart. Note that the model has a high explanatory power for developed countries. However, for less-developed economies trading frictions explains about half of the GDP differences.

8 How to increase the liquidity of financial markets?

While it is outside the scope of this paper to fully characterize optimal policy in this environment, in this section we explore the effects of two policies designed to increase the liquidity of financial markets. The objective of the government is to maximize the value of intermediate goods producers as it is the only sector generating positive profits in equilibrium.

First, we propose a simple tax-subsidy scheme in which the government subsidies a fraction of the search costs c. To finance the policy, the government charges a proportional profit tax on intermediate good producers.

The second policy consist of creating an public institution called *Government-Sponsored Intermediaries* (GSIs) that intermediate in secondary markets by buying bonds at a higher price than in private bilateral meetings. Again, to finance this institution, the government also charges a proportional profit tax on intermediate good producers.

8.1 Government-Sponsored Intermediaries

The policy consists of creating a government agency to intermediate assets in secondary markets to promote liquidity. To preserve the same technological constraints as private agents, assume that government agents are also subject to idiosyncratic risk of holding costs so they act both as buyers and sellers in the secondary market. However, government buys at a higher price, giving more gains from trade to the private sector. In particular, the government buys at the maximum price such that only constrained private agents wants to sell the asset. By providing this higher price, the government will run negative profits. The resources (and therefore the size of GSIs) is limited by the taxes leived to the corporate sector.

Figure 11 shows a schematic representation of the model when we introduce GSIs. Note that private sellers can now sell both to private and government agents. Moreover, private buyers can now buy in the secondary markets from either private or government sellers. In this section we describe the main characteristics of the model with GSIs. Appendix C describes the full solution of the model.

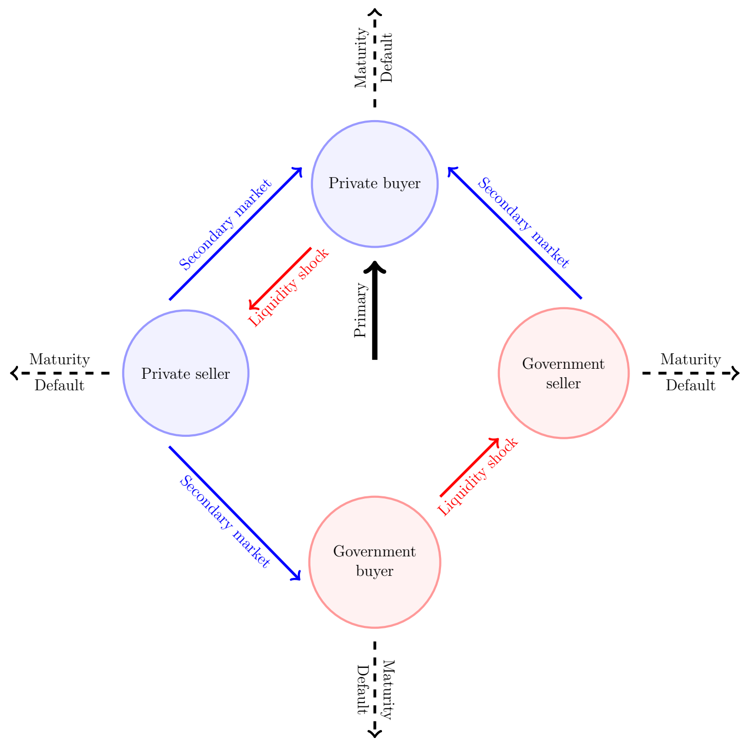
Asset Prices There are three type of meetings in secondary markets. In a meeting between a private buyer and either a private seller or a government seller the price is determined by Nash Bargaining. However, when the government act as a buyer, the price is the maximum value such that only constrained private agents want to sell

$$P^{S,P-P}(t) = D^C(t) + \gamma(D^U(t) - D^C(t)) \tag{14}$$

$$P^{S,G-P}(t) = G^C(t) + \gamma(D^U(t) - G^C(t)) \tag{15}$$

$$P^{S,P-G}(t) = D^U(t) \tag{16}$$

Figure 11: **Model with GSIs.**



where $G^C(t)$ is the value of a constrained government agent and $(P^{S,P-P}, P^{S,G-P}, P^{S,P-G})$ are the prices in private meetings, government acting as a seller, and government acting as a buyer, respectively.

Matching There is random matching between sellers and buyers as in the benchmark model. The total mass of sellers, μ^C, is composed by private and government sellers. Private sellers are $\mu^{C,P} = \int_0^\tau \mu^{C,P}(s)ds$ where $\mu^{C,P}(s)$ is the measure of private agents holding an asset and willing to sell. Similarly, government sellers are $\mu^{C,G} = \int_0^\tau \mu^{C,G}(s)ds$, where $\mu^{C,G}(s)$ is the measure of government agents holding an asset and willing to sell.

The total measure of buyers is composed by private buyers $\mu^{B,P}$, determined by a free-entry condition, and government buyers $\mu^{B,G}$ which is the policy instrument decided by the government.

The market tightness is $\theta = \frac{\mu^C}{\mu^B}$ and characterizes the buying and selling intensities $\lambda^B = A\theta^\alpha$, and $\lambda^S = A\theta^{\alpha-1}$. Note that both private and government sellers can meet with a private buyer at intensity $\lambda^{S,P} = \lambda^S \frac{\mu^{B,P}}{\mu^B}$ and a government buyer with intensity $\lambda^{S,G} = \lambda^S \frac{\mu^{B,G}}{\mu^B}$. Both private and government buyers meet with a private seller at intensity $\lambda^{B,P}(t) = \lambda^B \frac{\mu^{C,P}(t)}{\mu^C}$ and a government seller at intensity $\lambda^{B,G}(t) = \lambda^B \frac{\mu^{C,G}(t)}{\mu^C}$. Lemma 7 characterize the distribution of private and government agents that are unconstrained and constrained holding the asset.

Lemma 7. *Let $F(t) = \left[\mu^{P,U}(t), \mu^{P,C}(t), \mu^{G,U}(t), \mu^{G,C}(t)\right]$. The distribution is characterized by*

$$F_j(t) = \sum_{i=1}^{4} e^{r_i(t-\tau)} B_i V(j,i)$$

where r and V are the eigenvalues and eigenvectors of A, $B = V^{-1}F(\tau)$ with $F(\tau) = [\mu^0, 0, 0, 0]$ and

$$A = \begin{bmatrix} \lambda^H + \lambda^D & -\lambda^{S,P-P} & 0 & -\lambda^{S,G-P} \\ -\lambda^H & \lambda^D + \lambda^{S,P-P} + \lambda^{S,P-G} & 0 & 0 \\ 0 & -\lambda^{S,P-G} & \lambda^H + \lambda^D & -\lambda^{S,G-G} \\ 0 & 0 & -\lambda^H & \lambda^D + \lambda^{S,G-P} + \lambda^{S,G-G} \end{bmatrix}$$

Private values The value of holding an asset for an unconstrained agent is equivalent to the benchmark model. However, the value of a constrained agent is different as he is now able to sell it both to a private and government buyer. Note that the price that the government offer is equivalent to the price in private meetings but in which the seller has all the bargaining power. Hence, the value of a constrained agent is equivalent to the benchmark model with an augmented selling intensity: $\Lambda = \lambda^{S,P}\gamma + \lambda^G$.

The free entry condition for the private sector changes as now they can buy both from private and government sellers

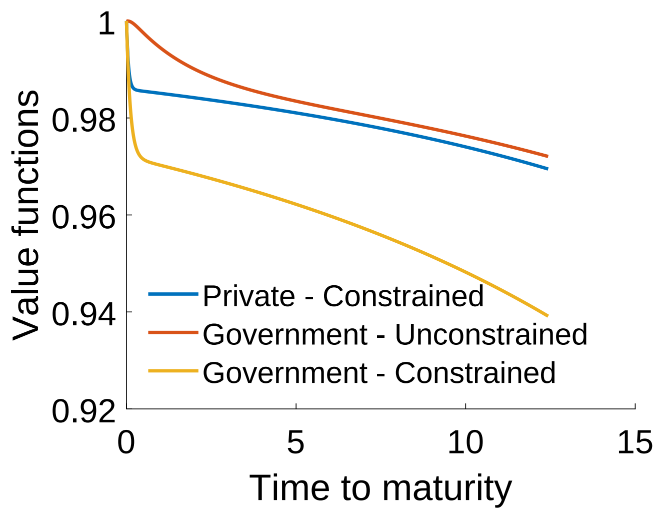
$$c \geq \int_0^\tau \lambda^{B,P}(t)\left(D^U(t) - D^S - P^{S,P}(t)\right)dt + \int_0^\tau \lambda^{B,G}(t)\left(D^U(t) - D^S - P^{S,G}(t)\right)dt$$

Government values The values for unconstrained and constrained government agents are given by

$$\rho G^U(t) = -\dot{G}^U(t) + \lambda^H\left(G^C(t) - G^U(t)\right) + \lambda^D\left(0 - G^U(t)\right) \tag{17}$$
$$\rho G^C(t) = -h - \dot{G}^C(t) + \lambda^{S,P}\left(P^{S,G-P}(t) - G^C(t)\right) + \lambda^D\left(0 - G^C(t)\right) \tag{18}$$

Note that the value for an unconstrained government agent is similar than for a private agent, and a constrained government agent has to option to sell the asset to a private buyer.

Figure 12: **GSI: Value functions.**



Note: Value functions relative to the value of an unconstrained agent in the private sector.

The value of entry for government agents is

$$\rho G^S = -c + \int_0^\tau \left[\lambda^{B,P}(t)\left(G^U(t) - G^S - P^{S,P-G}(t)\right)\right] dt \tag{19}$$

As the government is buying at a relatively expensive price, the value of entry is negative. Figure 12 shows the values relative to an unconstrained private agent. Note that the value of entry for government agents, G^S, is negative as G^U is below D^U (the price at which government buys).

Lemma 8 characterize the solution of the value functions as exponential polynomials, and solves for the free entry condition and the value of entry for the government.

Lemma 8. *The value functions for private and government agents are*

$$D^U(t) = A_0^{DU} + \sum_{j=1}^{2} A_j^{DU} e^{-B_j t} \qquad D^C(t) = A_0^{DC} + \sum_{j=1}^{2} A_j^{DC} e^{-B_j t}$$

$$G^C(t) = A_0^{GC} + \sum_{j=1}^{3} A_j^{GC} e^{-B_j t} \qquad G^U(t) = A_0^{GU} + \sum_{j=1}^{4} A_j^{GU} e^{-B_j t}$$

where $B_1 = \rho + \lambda^D$, $B_2 = \rho + \lambda^D + \lambda^H + \Lambda$, $B_3 = \rho + \lambda^D + \lambda^{S,G-P}\gamma^{G-P}$, and $B_4 = \rho + \lambda^D + \lambda^H$. Constants A_j^i are defined in the Appendix. Analytical solutions for free-entry conditions are also provided in the Appendix.

Cost of GSIs The cost of running GSIs is captured by the value of entry G^S. If the government choose a measure $\mu^{B,G}$, the total cost is given by $\mu^{B,G}B(\tau)G^S$. Hence, the government's budget constraint reads

$$\mu^{B,G}B(\tau)G^S = x^F\mu^F\mathbb{E}\left[f\left(\tau, N\right)\right]$$

where x^F is the proportional tax on profits for the intermediate good producers and μ^F is the measure of firms in the production stage.

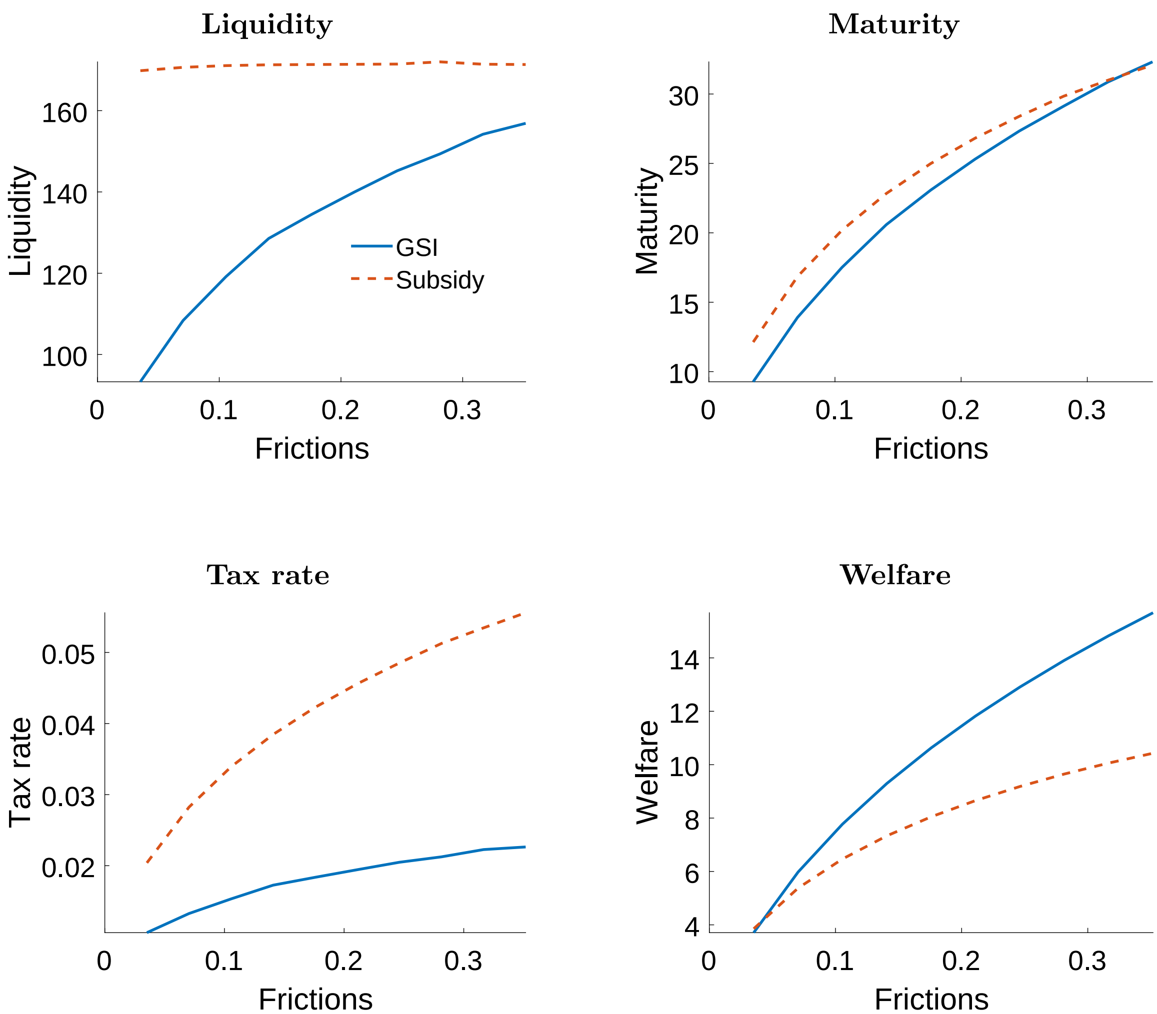
8.2 Results: Optimal policies

We compare the optimal subsidies scheme and GSIs for economies with different trading frictions (i.e., differences in entry costs, c). The top panel of Figure 13 shows the change in liquidity and maturity, relative to the laissez faire economy. The subsidy scheme is able to achieve higher increase in liquidity and maturity than GSIs. Note that under the subsidy scheme the increase in liquidity is almost constant in the size of the trading frictions. However, GSIs are able to achieve higher increases in liquidity in markets with more frictions.

The bottom panel of Figure 13 shows that the tax rate needed to implement the optimal subsidy scheme is higher than the tax rate needed for GSIs. As a result, the welfare increases under GSIs are larger than the welfare increases under the subsidy scheme. The objective of the government is to maximize the value of intermediate goods producers. Both policies have a direct and an indirect effect. First, as the government is charging a corporate tax, the direct effect reduces welfare. However, the indirect effect is the increase in liquidity and, therefore, the reduction in credit spreads. The results in Figure 13 show that the indirect effect is larger than the direct effect as welfare increase in those economies.

To conclude, note that in both the subsidy scheme and the GSIs welfare gains are larger in economies with larger frictions. Moreover, GSIs are more efficient than subsidies to improve the welfare of the economy.

Figure 13: **Government interventions.**



9 Extension: Segmented Markets

In the benchmark model of Section 4 assets of different maturities are traded in a single secondary market. A potential concern could be that assets of short maturities, with small gains from trade, preclude the entry of buyers to the secondary market. In this section we consider an alternative specification such that secondary markets are segmented by the time-to-maturity of the asset. The main take-away of this section is that even though the market tightness for short term bonds increases, the market tightness for long-term assets remains similar to the case with only one market. Hence, the secondary market in the benchmark model is effectively a market for long-term assets. We describe the main characteristics of the model in the main text and relegate to Appendix D the full characterization of this extension.

Lets τ be the initial maturity and consider the case in which secondary markets are segmented in N markets. Let $0 = \tau_1 < \cdots < \tau_{N+1} = \tau$ so each market $j = 1, \ldots, N$ trade assets with time to maturity $t \in [\tau_j, \tau_{j+1}]$.

Matching and distribution of agents Let $\mu^j(t) = \left[\mu^{U,j}(t), \mu^{C,j}(t)\right]$ be the measure of unconstrained and constrained agents holding an asset with time to maturity t. To solve for the distribution of agents in each market we start by the last market. The boundary condition is that $\mu^N(\tau) = [\mu^0, 0]$. Next, we can iterate towards markets of shorter maturities. The boundary condition for market $j = 1, \ldots, N-1$ is $\mu^j(\tau_{j+1}) = \mu^{j+1}(\tau_{j+1})$. Lemma 9 characterize the distribution of agents in each market.

Lemma 9. *The measure of agents for markets $j = 1, \ldots, N$ is given by the following backward recursion*

$$\begin{bmatrix} \mu^{U,N+1}(\tau) \\ \mu^{C,N+1}(\tau) \end{bmatrix} = \begin{bmatrix} \mu^0 \\ 0 \end{bmatrix}$$

and

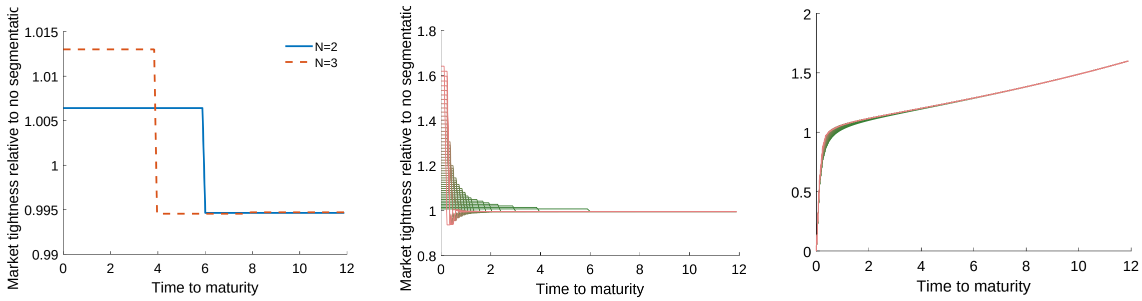
$$\begin{aligned}
\mu^{U,j}(t) =& \frac{\lambda^H}{\lambda^H + \lambda^{S,j}} \left[\frac{\lambda^{S,j}}{\lambda^H} e^{\lambda^D (t-\tau_{j+1})} \left(\mu^{U,j+1}(\tau_{j+1}) + \mu^{C,j+1}(\tau_{j+1}) \right) \right. \\
& \left. - e^{\left(\lambda^H + \lambda^{S,j} + \lambda^D\right)(t-\tau_{j+1})} \left(-\mu^{U,j+1}(\tau_{j+1}) + \lambda^{S,j} \mu^{C,j+1}(\tau_{j+1}) \right) \right] \\
\mu^{C,j}(t) =& \frac{\lambda^H}{\lambda^H + \lambda^{S,j}} \left[e^{\lambda^D (t-\tau_{j+1})} \left(\mu^{U,j+1}(\tau_{j+1}) + \mu^{C,j+1}(\tau_{j+1}) \right) \right. \\
& \left. + e^{\left(\lambda^H + \lambda^{S,j} + \lambda^D\right)(t-\tau_{j+1})} \left(-\mu^{U,j+1}(\tau_{j+1}) + \lambda^{S,j} \mu^{C,j+1}(\tau_{j+1}) \right) \right]
\end{aligned}$$

Value functions To solve for the value functions we start with the first market, where we have the boundary condition that at maturity the value is equal to one, and iterates towards longer-term markets. Let $D^j(t) = \left[D^{U,j}(t), D^{C,j}(t)\right]$ be the value functions of unconstrained and constrained agents holding an asset with time to maturity t in market $j = 1, \ldots, N$. The boundary condition for market 1 is $D^1(\tau_1) = [1, 1]$. Value matching for market $j = 2, \ldots, N$ implies $D^j(\tau_j) = D^{j-1}(\tau_j)$. The Hamilton-Jacobi-Bellman equations are the same as before.

Free entry Free entry in each market implies

$$c = (1-\gamma) \int_{\tau_j}^{\tau_{j+1}} \lambda^{B,j}(t) \left(D^{U,j}(t) - D^C(t) \right) dt$$

Figure 14: **Segmented markets.**



Market tightness | Market tightness | Liquidity spread

Note: The first (second) figure shows the market tightness relative to no segmentation for $N = 2, 3$ ($N = 2, \ldots, 50$). The third figure shows the liquidity spread for $N = 1, \ldots, 50$.

Appendix D.2 provides the solution of the value functions and the free entry condition.

9.1 Results

The first two panels of Figure 14 shows the market tightness relative to the case of only one market. Market segmentation makes markets for short-term assets more tight (more sellers to buyers) as there are fewer gains from trade in those markets. However, for markets of long-term bonds, the market tightness is similar to the case of no segmentation. The second figure repeats this exercise under different degrees of segmentation (up to $N = 50$). Note that even with 50 different markets, the tightness for markets with maturity above 4 years are almost identical to the case of no segmentation.

The third panel of Figure 14 shows the liquidity spread for different models with $N = 1$ to $N = 50$. As the market tightness after four years is identical in all these models, the implied liquidity spread is also the same. For short-term assets (maturities up to 4 years), there are some differences in the market tightness, but generating very little variation in the yield curve.

Therefore, we conclude that the secondary market in the benchmark model with $N = 1$ is effectively a market for long-term assets.

10 Robustness: Results with rollover

We solve the model under different specifications to learn how they affect the equilibrium choices. We consider versions of the model with and without default; with a secondary market that can be centralized, OTC, or without secondary market (referred as a *shut-down* of the market); with and without the possibility to rollover shorter-term debt to finance long-term projects; and with different values for the issuance cost (Φ).

The most important equilibrium variable is the interest rate. Lemma 3 shows three results about the yield curve: (i) Increasing in maturity, (ii) Decreasing in liquidity, and (iii) Increasing in the default probability. We now perform a quantitative evaluation of these forces and study how they affect the maturity choice.

Figure 15 shows the number of issuances, the average debt maturity, the project's maturity, and the credit spreads under different parameter values. First, when the fix cost of issuance increases, the firm chooses to issue lest frequent and increase the debt maturity. However, as the financial cost increases with the issuance cost, the firm chooses to reduce the project's maturity. Hence, when the frictions in the primary market captured by Φ increases, firms choose projects of lower returns.

Second, when the liquidity of secondary markets increases, firms choose to issue less frequent and increase the debt's maturity. Note that now the firm invests in longer-term projects as the financial cost reduces with the liquidity of the market. Hence the result from the benchmark model—i.e., that firms invest in longer-term projects when the liquidity spread decreases—is also present when we allow the firm to rollover short-term debt.

One potential concern about the exercises performed in the paper is that the liquidity of the secondary market and the rollover cost can be correlated. In the model, liquidity costs are endogenous while rollover costs are exogenous and fixed, so they do not respond to changes in the liquidity of secondary markets, and a potential *Lucas critique* may apply. However, when the secondary market becomes more liquid, both issuance and rollover costs should diminish. Hence, the assumption that issuance costs are fixed is a conservative one. As we have shown in Figure 15, when liquidity of secondary markets improves, the firm adopts a project of longer maturity. Similarly, when the issuance cost diminish, the firm chooses longer-term projects. Hence, if the two effect are present (an increase in liquidity and a reduction of the issuance cost), the firm will increase the maturity of the project even more than with constant issuance costs.

11 Conclusion

This paper studies the linkages between maturity of corporate debt, liquidity of financial markets and the real economy. When corporate debt markets are more liquid, the financing cost for firms diminish, and entrepreneurs choose to enter in projects of longer maturities. Hence, firms invest in better technologies which promotes economic development.

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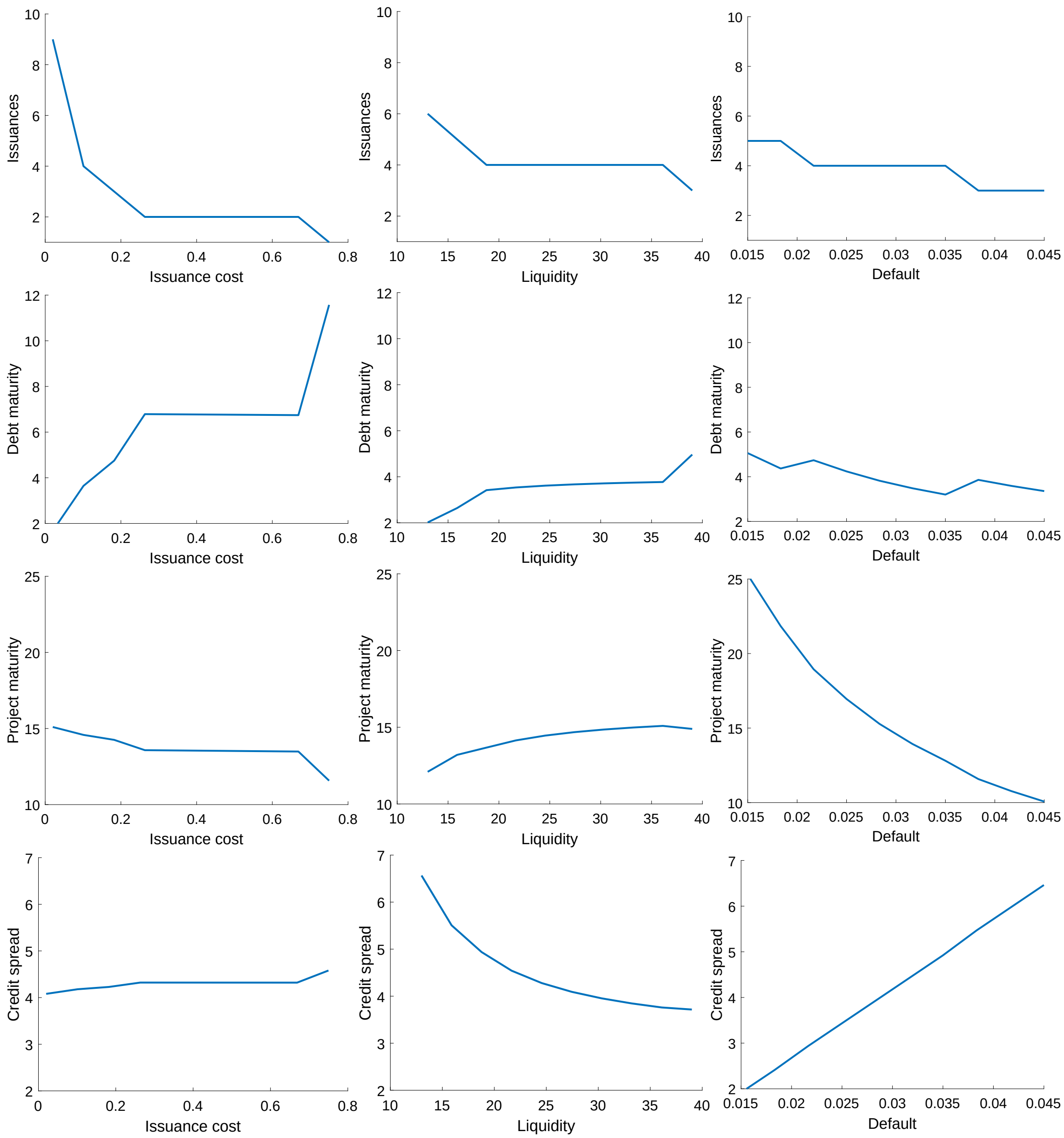
Figure 15: **Solution under alternative specifications**



Note: Solution of the model under different specifications for the issuance cost, liquidity, and default rates.

Appendix

A Omitted proofs: Lenders

In this section we provide the proofs of the results for lenders.

A.1 Distribution of financiers

Proof of Lemma 1. The law of motions are

$$-\dot{\mu}^U(t) = -\left(\lambda^H + \lambda^D\right)\mu^U(t) + \lambda^B(t)\,\mu^B$$
$$-\dot{\mu}^C(t) = \lambda^H \mu^U(t) - \left(\lambda^D + \lambda^S\right)\mu^C(t)$$

and matching implies $\mu^B \lambda^B(t) = \lambda^S \mu^C(t)$. Let $\mu(t) = \left[\mu^U(t), \mu^C(t)\right]$, then $\dot{\mu}(t) = A\mu(t)$ where

$$A = \begin{bmatrix} \lambda^D + \lambda^H & -\lambda^S \\ -\lambda^H & \lambda^D + \lambda^S \end{bmatrix}$$

with boundary condition $\mu(\tau) = [\mu^0, 0]$. Let V be the matrix with eigenvectors and R the vector with eigenvalues of A

$$V = \begin{bmatrix} -1 & \frac{\lambda^S}{\lambda^H} \\ 1 & 1 \end{bmatrix} \qquad R = \begin{bmatrix} \lambda^H + \lambda^S + \lambda^D \\ \lambda^D \end{bmatrix}$$

and the boundary condition defines $B = (V)^{-1}\mu(\tau)$, that is

$$B = \frac{\lambda^H \mu^0}{\lambda^H + \lambda^S}\begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Then

$$\mu^U(t) = \sum_{i=1}^{2} e^{R_i(t-\tau)} B_i V(1,i)$$
$$\mu^C(t) = \sum_{i=1}^{2} e^{R_i(t-\tau)} B_i V(2,i)$$

Hence

$$\mu^U(t) = \frac{\mu^0 \lambda^H}{\lambda^H + \lambda^S}\left(e^{\lambda^D(t-\tau)}\frac{\lambda^S}{\lambda^H} + e^{\left(\lambda^H+\lambda^S+\lambda^D\right)(t-\tau)}\right)$$
$$\mu^U(t) = \frac{\mu^0 \lambda^H}{\lambda^H + \lambda^S}\left(e^{\lambda^D(t-\tau)} - e^{\left(\lambda^H+\lambda^S+\lambda^D\right)(t-\tau)}\right)$$

□

A.2 Value functions

Proof of Lemma 2. Replace the price of the asset in the secondary market $P^S(t)$ in (5) so

$$\left(\rho+\lambda^D\right)D^U(t) = -\dot{D}^U(t) + \lambda^H\left(D^C(t) - D^U(t)\right)$$
$$\left(\rho+\lambda^D\right)D^C(t) = -h - \dot{D}^C(t) + \lambda^S\gamma\left(D^U(t) - D^C(t)\right)$$

Let $H(t) = D^U(t) - D^C(t)$ then

$$\left(\rho+\lambda^D+\lambda^H+\lambda^S\gamma\right)H(t) = h - \dot{H}(t)$$

Let $c_1 = \rho+\lambda^D+\lambda^H+\lambda^S\gamma$ and guess and verify that $H(t) = Ae^{-c_1 t} + B$ Hence, $B = \frac{h}{c_1}$. The boundary condition implies that $H(0) = 0$, so $A = -\frac{h}{c_1}$ and $H(t) = h\frac{1-e^{-c_1 t}}{c_1}$

Next, we can solve for $D^U(t)$ as

$$\left(\rho+\lambda^D\right)D^U(t) = -\dot{D}^U(t) - \lambda^H h\frac{1-e^{-c_1 t}}{c_1}$$

We can guess and verify that $D^U(t) = A + Be^{-\left(\rho+\lambda^D\right)t} + Ce^{-c_1 t}$.

$$A = -\frac{1}{\rho+\lambda^D}\frac{\lambda^H h}{c_1}$$
$$C = -\frac{1}{\lambda^H+\lambda^S\gamma}\frac{\lambda^H h}{c_1}$$

The boundary condition implies $1 = A + B + C$, so

$$B = 1 + \frac{\lambda^H h}{\left(\lambda^H+\lambda^S\gamma\right)\left(\rho+\lambda^D\right)}$$

A few lines of algebra deliver

$$\begin{aligned}
D^U(t) =& e^{-\left(\rho+\lambda^D\right)t} - \mathcal{L}\left(t,\lambda^S\right)\\
\mathcal{L}\left(t,\lambda^S\right) =& \frac{\lambda^H h}{\lambda^H+\lambda^S\gamma}\left(\frac{1-e^{-\left(\rho+\lambda^D\right)t}}{\rho+\lambda^D} - \frac{1-e^{-\left(\rho+\lambda^D+\lambda^H+\lambda^S\gamma\right)t}}{\rho+\lambda^D+\lambda^H+\lambda^S\gamma}\right)\\
\mathcal{L}\left(\tau,\lambda^S\right) =& h \quad \frac{\lambda^H}{\lambda^H+\lambda^S\gamma} \quad \int_0^\tau e^{-\left(\rho+\lambda^D\right)t}\left(1-e^{-\left(\lambda^H+\lambda^S\gamma\right)t}\right)dt
\end{aligned}$$

Finally, we can compute the value of a constrained agent as $D^C(t) = D^U(t) - H(t)$, that is

$$D^C(t) = e^{-\left(\rho+\lambda^D\right)t} - h\frac{1-e^{-\left(\rho+\lambda^D\right)t}}{\rho+\lambda^D} + \frac{\gamma\lambda^S}{\lambda^H}\mathcal{L}(t)$$

Properties of the illiquidity cost:

1. $\mathcal{L}(\Lambda,\tau)$ is positive as $\rho+\lambda^D+\Lambda+\lambda^H \geq \rho+\lambda^D$.
2. Sensitivity with respect to maturity τ:
   (a) Increasing in τ:

   $$\frac{\partial\mathcal{L}(\tau,\Lambda)}{\partial\tau} = h \quad \frac{\lambda^H}{\lambda^H+\Lambda} \quad e^{-\left(\rho+\lambda^D\right)\tau}\left(1-e^{-\left(\lambda^H+\Lambda\right)\tau}\right) \geq 0$$

   (b) Limit:

   $$\begin{aligned}
   \lim_{\tau\to\infty}\mathcal{L}(\tau,\Lambda) &= h\frac{\lambda^H}{\lambda^H+\Lambda}\left(\frac{1}{\rho+\lambda^D} - \frac{1}{\rho+\lambda^D+\lambda^H+\Lambda}\right)\\
   &= h\frac{\lambda^H}{\left(\rho+\lambda^D\right)\left(\rho+\lambda^D+\lambda^H+\Lambda\right)}
   \end{aligned}$$

3. Sensitivity with respect to liquidity shocks λ^H:
   (a) If there are no liquidity shocks ($\lambda^H = 0$), then $\mathcal{L}(\tau,\Lambda) = 0$.
   (b) If $\lambda^H \to \infty$ (i.e., always has to pay the cost h) then

   $$\lim_{\lambda^H\to\infty}\mathcal{L}(\tau,\Lambda) = h\frac{1-e^{-(\rho+\lambda^D)t}}{\rho+\lambda^D}$$

4. Sensitivity with respect to liquidity of the secondary market Λ:

   (a) It is decreasing in Λ. Note that the illiquidity cost is

$$\mathcal{L}(\tau,\Lambda) = \lambda^H h \left(\frac{1}{(\rho+\lambda^D)(\rho+\lambda^D+\lambda^H+\Lambda)} - \frac{e^{-\left(\rho+\lambda^D\right)\tau}}{(\lambda^H+\Lambda)(\rho+\lambda^D)} \right) + \lambda^H h \frac{e^{-\left(\rho+\lambda^D+\lambda^H+\Lambda\right)\tau}}{(\lambda^H+\Lambda)(\rho+\lambda^D+\lambda^H+\Lambda)}$$

so

$$\begin{aligned}\frac{\partial\mathcal{L}(\tau,\Lambda)}{\partial\Lambda} &= \lambda^H h \left(-\frac{1}{(\rho+\lambda^D)(\rho+\lambda^D+\lambda^H+\Lambda)^2} + \frac{e^{-\left(\rho+\lambda^D\right)\tau}}{(\rho+\lambda^D)(\lambda^H+\Lambda)^2} \right) \\ &- \lambda^H h \left(\frac{\tau e^{-\left(\rho+\lambda^D+\lambda^H+\Lambda\right)\tau}}{(\lambda^H+\Lambda)(\rho+\lambda^D+\lambda^H+\Lambda)} + \frac{e^{-\left(\rho+\lambda^D+\lambda^H+\Lambda\right)\tau}}{(\lambda^H+\Lambda)^2(\rho+\lambda^D+\lambda^H+\Lambda)} \right) \\ &- \lambda^H h \frac{e^{-\left(\rho+\lambda^D+\lambda^H+\Lambda\right)\tau}}{(\lambda^H+\Lambda)(\rho+\lambda^D+\lambda^H+\Lambda)^2}\end{aligned}$$

That is

$$\frac{\partial\mathcal{L}(\tau,\Lambda)}{\partial\Lambda} = \lambda^H h \left(-\frac{1}{b(a+b)^2} + \frac{e^{-b\tau}}{ba^2} \right) - \lambda^H h \frac{e^{-(a+b)\tau}}{a(a+b)} \left(\tau + \frac{2a+b}{a(a+b)} \right)$$

where $a = \lambda^H + \Lambda$ and $b = \rho + \lambda^D$. Note that $\frac{\partial\mathcal{L}(0,\Lambda)}{\partial\Lambda} = 0$ and

$$\lim_{\tau\to\infty} \frac{\partial\mathcal{L}(\tau,\Lambda)}{\partial\Lambda} = -\frac{\lambda^H h}{(\rho+\lambda^D)(\rho+\lambda^D+\lambda^H+\Lambda)^2} \leq 0$$

Hence, $\frac{\partial\mathcal{L}(\tau,\Lambda)}{\partial\Lambda} \leq 0$ if and only if

$$\lambda^H h \left(-\frac{1}{b(a+b)^2} + \frac{e^{-b\tau}}{ba^2} \right) - \lambda^H h \frac{e^{-(a+b)\tau}}{a(a+b)} \left(\tau + \frac{2a+b}{a(a+b)} \right) \leq 0$$

So

$$\begin{aligned}\lambda^H h \left(-\frac{1}{b(a+b)^2} + \frac{e^{-b\tau}}{ba^2} \right) &\leq \lambda^H h \frac{e^{-(a+b)\tau}}{a(a+b)} \left(\tau + \frac{2a+b}{a(a+b)} \right) \\ \frac{e^{-b\tau}}{ba^2} &\leq \frac{1}{b(a+b)^2} + \frac{e^{-(a+b)\tau}}{a(a+b)} \left(\tau + \frac{2a+b}{a(a+b)} \right)\end{aligned}$$

Define

$$
\begin{aligned}
L(\tau) &= \frac{e^{-b\tau}}{ba^2} \\
R(\tau) &= \frac{1}{b(a+b)^2} + \frac{e^{-(a+b)\tau}}{a(a+b)}\left(\tau + \frac{2a+b}{a(a+b)}\right)
\end{aligned}
$$

and note that $L(\tau) = 0$ and $R(\tau) \geq 0$. Hence, it is sufficient to show that the slope of $L(\tau)$ is more negative than that of $R(\tau)$ for all τ

$$
\frac{\partial L(\tau)}{\partial \tau} = -\frac{e^{-b\tau}}{a^2}
$$

and

$$
\begin{aligned}
\frac{\partial R(\tau)}{\partial \tau} &= -\frac{e^{-(a+b)\tau}}{a}\left(\tau + \frac{2a+b}{a(a+b)}\right) + \frac{e^{-(a+b)\tau}}{a(a+b)} \\
&= -\frac{e^{-(a+b)\tau}}{a}\left(\tau + \frac{2a+b}{a(a+b)} - \frac{1}{a+b}\right) \\
&= -\frac{e^{-(a+b)\tau}}{a}\left(\tau + \frac{1}{a}\right)
\end{aligned}
$$

So, the slope of L is more negative than the slope of R if

$$
\begin{aligned}
-\frac{e^{-b\tau}}{a^2} &\leq -\frac{e^{-(a+b)\tau}}{a}\left(\tau + \frac{1}{a}\right) \\
\frac{1}{a} &\geq e^{-a\tau}\left(\tau + \frac{1}{a}\right) \\
e^{a\tau} &\geq a\tau + 1 \\
a\tau &\geq \log(a\tau + 1)
\end{aligned}
$$

which holds for all $a\tau \geq 0$. [15]

(b) If there are no secondary markets, i.e., $\Lambda = 0$, then the illiquidity cost represents the *expected holding costs, i.e.,*

$$
\mathcal{L}(\tau, 0) = h \int_0^{\tau} e^{-\left(\rho + \lambda^D\right)t}\left(1 - e^{-\lambda^H t}\right) dt
$$

(c) If secondary markets are totally liquid (i.e., $\Lambda \to \infty$) then $\mathcal{L}(\tau, \Lambda) = 0$.

[15] See https://en.wikipedia.org/wiki/Natural_logarithm.

5. Liquidity is more important for long-term assets. Recall that

$$\frac{\partial \mathcal{L}(\tau,\Lambda)}{\partial \tau} = h\frac{\lambda^H}{\lambda^H+\Lambda}e^{-\left(\rho+\lambda^D\right)\tau}\left(1-e^{-\left(\lambda^H+\Lambda\right)\tau}\right)$$
$$\frac{\partial \mathcal{L}(\tau,\Lambda)}{\partial \tau} = \lambda^H h e^{-\left(\rho+\lambda^D\right)\tau}\int_0^\tau e^{-\left(\lambda^H+\Lambda\right)t}dt$$

therefore

$$\frac{\partial \mathcal{L}(\tau,\Lambda)}{\partial \tau\Lambda} = -\lambda^H h e^{-\left(\rho+\lambda^D\right)\tau}\int_0^\tau t e^{-\left(\lambda^H+\Lambda\right)t}dt \leq 0$$

□

A.3 Liquidity spread

Proof of lemma 3. 1. **The liquidity spread $cs^{liq}(\tau,\Lambda)$ is increasing in maturity τ**

$$cs^{liq}(t,\Lambda) = -\frac{1}{t}\log\left(1-e^{\left(\rho+\lambda^D\right)t}\mathcal{L}(t,\Lambda)\right)$$

so

$$\frac{\partial cs^{liq}(t,\Lambda)}{\partial t} = \frac{1}{t^2}\log\left(1-e^{\left(\rho+\lambda^D\right)t}\mathcal{L}(t,\Lambda)\right) + \frac{e^{-\left(\rho+\lambda^D\right)t}}{t}\frac{\left(\rho+\lambda^D\right)\mathcal{L}(t,\Lambda)+\frac{\partial \mathcal{L}(t,\Lambda)}{\partial t}}{1-e^{-(\rho+\lambda^D)t}\mathcal{L}(t,\Lambda)}$$

Recall that $\log(x) \geq \frac{x-1}{x}$. Hence

$$\log\left(1-e^{\left(\rho+\lambda^D\right)t}\mathcal{L}(t,\Lambda)\right) \geq \frac{-e^{\left(\rho+\lambda^D\right)t}\mathcal{L}(t,\Lambda)}{1-e^{(\rho+\lambda^D)t}\mathcal{L}(t,\Lambda)}$$

Which implies that

$$\frac{\partial cs^{liq}(t,\Lambda)}{\partial t} \geq \frac{1}{t^2}\frac{-e^{\left(\rho+\lambda^D\right)t}\mathcal{L}(t,\Lambda)}{1-e^{(\rho+\lambda^D)t}\mathcal{L}(t,\Lambda)} + \frac{1}{t}\frac{e^{-\left(\rho+\lambda^D\right)t}}{1-e^{-(\rho+\lambda^D)t}\mathcal{L}(t,\Lambda)}\left(\left(\rho+\lambda^D\right)\mathcal{L}(t,\Lambda)+\frac{\partial \mathcal{L}(t,\Lambda)}{\partial t}\right)$$
$$\frac{\partial cs^{liq}(t,\Lambda)}{\partial t} \geq \frac{1}{t^2}\frac{e^{-\left(\rho+\lambda^D\right)t}\mathcal{L}(t,\Lambda)}{1-e^{-(\rho+\lambda^D)t}\mathcal{L}(t,\Lambda)}\left(t\left(\rho+\lambda^D\right)+\frac{\partial \mathcal{L}(t,\Lambda)}{\partial t}\frac{t}{\mathcal{L}(t,\Lambda)}-1\right)$$

Let

$$\varepsilon_{\mathcal{L},\tau} = \frac{\partial \mathcal{L}(t,\Lambda)}{\partial t}\frac{t}{\mathcal{L}(t,\Lambda)}$$

then a sufficient condition for $\frac{\partial cs^{liq}(t,\Lambda)}{\partial t} \geq 0$ is

$$t\left(\rho+\lambda^D\right)+\varepsilon_{\mathcal{L},\tau} \geq 1$$

Recall that

$$\begin{aligned}
\varepsilon_{\mathcal{L},\tau} &= \frac{\tau}{\mathcal{L}(\tau,\Lambda)}\frac{\partial \mathcal{L}(\tau,\Lambda)}{\partial \tau} \\
\mathcal{L}(\tau,\Lambda) &= h\frac{\lambda^H}{\lambda^H+\Lambda}\left[\frac{1-e^{-\left(\rho+\lambda^D\right)\tau}}{\rho+\lambda^D}-\frac{1-e^{-\left(\rho+\lambda^D+\lambda^H+\Lambda\right)\tau}}{\rho+\lambda^D+\lambda^H+\Lambda}\right]
\end{aligned}$$

Hence

$$\varepsilon_{\mathcal{L}} = \tau\left[e^{-\left(\rho+\lambda^D\right)\tau}-e^{-\left(\rho+\lambda^D+\lambda^H+\Lambda\right)\tau}\right]\left[\frac{1-e^{-\left(\rho+\lambda^D\right)\tau}}{\rho+\lambda^D}-\frac{1-e^{-\left(\rho+\lambda^D+\lambda^H+\Lambda\right)\tau}}{\rho+\lambda^D+\lambda^H+\Lambda}\right]^{-1}$$

Let $a=\rho+\lambda^D$ and $b=\lambda^H+\Lambda$ and define

$$E(\tau,a,b)=\tau\left(a+\left[e^{-a\tau}-e^{-(a+b)\tau}\right]\left[\frac{1-e^{-a\tau}}{a}-\frac{1-e^{-(a+b)\tau}}{a+b}\right]^{-1}\right)-1$$

First, we can show that $E(0,a,b)=1$ for all a,b. Moreover, it is easy to show numerically that $E(\tau,a,b)\geq 0$ for all $\tau,a,b\geq 0$. Hence, the liquidity spread is increasing in maturity. Finally, it is straight forward to see that the liquidity spread is decreasing in liquidity Λ.

2. **The liquidity spread is increasing in the default intensity λ^D:**

$$\begin{aligned}
e^{\left(\rho+\lambda^D\right)\tau}\mathcal{L}(\tau,\Lambda) &= \frac{\lambda^H}{\lambda^H+\Lambda}\int_0^\tau e^{\left(\rho+\lambda^D\right)(\tau-t)}\left(1-e^{-\left(\lambda^H+\Lambda\right)t}\right)dt \\
\frac{\partial\left(e^{\left(\rho+\lambda^D\right)\tau}\mathcal{L}(\tau,\Lambda)\right)}{\partial\lambda^D} &= \frac{\lambda^H}{\lambda^H+\Lambda}\int_0^\tau(\tau-t)\,e^{\left(\rho+\lambda^D\right)(\tau-t)}\left(1-e^{-\left(\lambda^H+\Lambda\right)t}\right)dt>0
\end{aligned}$$

3. **The proportional bid-ask spread is increasing in maturity:** The mid-price is

$$\frac{1}{2}\left(D^{U}(t)+D^{C}(t)\right)=\frac{1}{2}\left(e^{-\left(\rho+\lambda^{D}\right)t}-\mathcal{L}(t)+e^{-\left(\rho+\lambda^{D}\right)t}-h\frac{1-e^{-\left(\rho+\lambda^{D}\right)t}}{\rho+\lambda^{D}}+\frac{\lambda^{S}\gamma}{\lambda^{H}}\mathcal{L}(t)\right)$$
$$=e^{-\left(\rho+\lambda^{D}\right)t}-\frac{1}{2}\left(h\frac{1-e^{-\left(\rho+\lambda^{D}\right)t}}{\rho+\lambda^{D}}+\left(1-\frac{\lambda^{S}\gamma}{\lambda^{H}}\right)\mathcal{L}(t)\right)$$
$$=e^{-\left(\rho+\lambda^{D}\right)t}-\frac{1}{2}\left(h\frac{1-e^{-\left(\rho+\lambda^{D}\right)t}}{\rho+\lambda^{D}}+\left(\frac{\lambda^{H}-\lambda^{S}\gamma}{\lambda^{H}}\right)\mathcal{L}(t)\right)$$

where

$$\left(\frac{\lambda^{H}-\lambda^{S}\gamma}{\lambda^{H}}\right)\mathcal{L}(t)=h\frac{\lambda^{H}-\lambda^{S}\gamma}{\lambda^{H}+\gamma\lambda^{S}}\left(\frac{1-e^{-\left(\rho+\lambda^{D}\right)t}}{\rho+\lambda^{D}}-\frac{1-e^{-\left(\rho+\lambda^{D}+\lambda^{H}+\lambda^{S}\gamma\right)t}}{\rho+\lambda^{D}+\lambda^{H}+\lambda^{S}\gamma}\right)$$

so

$$h\frac{1-e^{-\left(\rho+\lambda^{D}\right)t}}{\rho+\lambda^{D}}+\left(\frac{\lambda^{H}-\lambda^{S}\gamma}{\lambda^{H}}\right)\mathcal{L}(t)$$
$$=h\left(\frac{1-e^{-\left(\rho+\lambda^{D}\right)t}}{\rho+\lambda^{D}}+\frac{\lambda^{H}-\lambda^{S}\gamma}{\lambda^{H}+\gamma\lambda^{S}}\frac{1-e^{-\left(\rho+\lambda^{D}\right)t}}{\rho+\lambda^{D}}-\frac{\lambda^{H}-\lambda^{S}\gamma}{\lambda^{H}+\gamma\lambda^{S}}\frac{1-e^{-\left(\rho+\lambda^{D}+\lambda^{H}+\lambda^{S}\gamma\right)t}}{\rho+\lambda^{D}+\lambda^{H}+\lambda^{S}\gamma}\right)$$
$$=h\left(\frac{1-e^{-\left(\rho+\lambda^{D}\right)t}}{\rho+\lambda^{D}}\left(\frac{2\lambda^{H}}{\lambda^{H}+\gamma\lambda^{S}}\right)-\frac{\lambda^{H}-\lambda^{S}\gamma}{\lambda^{H}+\gamma\lambda^{S}}\frac{1-e^{-\left(\rho+\lambda^{D}+\lambda^{H}+\lambda^{S}\gamma\right)t}}{\rho+\lambda^{D}+\lambda^{H}+\lambda^{S}\gamma}\right)$$
$$=\frac{h}{\lambda^{H}+\gamma\lambda^{S}}\left(2\lambda^{H}\frac{1-e^{-\left(\rho+\lambda^{D}\right)t}}{\rho+\lambda^{D}}-\left(\lambda^{H}-\lambda^{S}\gamma\right)\frac{1-e^{-\left(\rho+\lambda^{D}+\lambda^{H}+\lambda^{S}\gamma\right)t}}{\rho+\lambda^{D}+\lambda^{H}+\lambda^{S}\gamma}\right)$$

and the mid-price is

$$\frac{1}{2}\left(D^{U}(t)+D^{C}(t)\right)=e^{-\left(\rho+\lambda^{D}\right)t}-\frac{h}{\lambda^{H}+\gamma\lambda^{S}}\left(\lambda^{H}\frac{1-e^{-\left(\rho+\lambda^{D}\right)t}}{\rho+\lambda^{D}}-\frac{\left(\lambda^{H}-\lambda^{S}\gamma\right)}{2}\frac{1-e^{-\left(\rho+\lambda^{D}+\lambda^{H}+\lambda^{S}\gamma\right)t}}{\rho+\lambda^{D}+\lambda^{H}+\lambda^{S}\gamma}\right)$$

The proportional bid-ask spread is

$$BA(t)=\frac{h\frac{1-e^{-\left(\rho+\lambda^{D}+\lambda^{H}+\lambda^{S}\gamma\right)t}}{\rho+\lambda^{D}+\lambda^{H}+\lambda^{S}\gamma}}{e^{-(\rho+\lambda^{D})t}-\frac{h}{\lambda^{H}+\gamma\lambda^{S}}\left(\lambda^{H}\frac{1-e^{-(\rho+\lambda^{D})t}}{\rho+\lambda^{D}}-\frac{(\lambda^{H}-\lambda^{S}\gamma)}{2}\frac{1-e^{-(\rho+\lambda^{D}+\lambda^{H}+\lambda^{S}\gamma)t}}{\rho+\lambda^{D}+\lambda^{H}+\lambda^{S}\gamma}\right)}$$

Define the gains from trade as

$$GT\left(t\right)=h\frac{1-e^{-\left(\rho+\lambda^{D}+\lambda^{H}+\lambda^{S}\gamma\right)t}}{\rho+\lambda^{D}+\lambda^{H}+\lambda^{S}\gamma}$$

so

$$\begin{aligned}
BA\left(t\right)&=\frac{GT\left(t\right)}{e^{-(\rho+\lambda^{D})t}-\frac{1}{\lambda^{H}+\gamma\lambda^{S}}\left(h\lambda^{H}\frac{1-e^{-(\rho+\lambda^{D})t}}{\rho+\lambda^{D}}-\frac{(\lambda^{H}-\lambda^{S}\gamma)}{2}GT\left(t\right)\right)}\\
BA\left(t\right)&=GT\left(t\right)\left[e^{-\left(\rho+\lambda^{D}\right)t}-\frac{1}{\lambda^{H}+\gamma\lambda^{S}}\left(h\lambda^{H}\frac{1-e^{-\left(\rho+\lambda^{D}\right)t}}{\rho+\lambda^{D}}-\frac{\left(\lambda^{H}-\lambda^{S}\gamma\right)}{2}GT\left(t\right)\right)\right]^{-1}\\
BA\left(t\right)&=\left[\frac{e^{-\left(\rho+\lambda^{D}\right)t}}{GT\left(t\right)}-\frac{1}{\lambda^{H}+\gamma\lambda^{S}}\left(\lambda^{H}\frac{h\frac{1-e^{-\left(\rho+\lambda^{D}\right)t}}{\rho+\lambda^{D}}}{GT\left(t\right)}-\frac{\left(\lambda^{H}-\lambda^{S}\gamma\right)}{2}\right)\right]^{-1}\\
BA\left(t\right)&=\left[\frac{e^{-\left(\rho+\lambda^{D}\right)t}}{GT\left(t\right)}-h\frac{\lambda^{H}}{\lambda^{H}+\gamma\lambda^{S}}\frac{\frac{1-e^{-\left(\rho+\lambda^{D}\right)t}}{\rho+\lambda^{D}}}{GT\left(t\right)}+\frac{1}{2}\frac{\lambda^{H}-\lambda^{S}\gamma}{\lambda^{H}+\gamma\lambda^{S}}\right]^{-1}
\end{aligned}$$

Note that $\frac{e^{-\left(\rho+\lambda^{D}\right)t}}{GT(t)}$ is decreasing in t as $e^{-\left(\rho+\lambda^{D}\right)t}$ is decreasing and $GT\left(t\right)$ is increasing in t. Note that $\frac{\frac{1-e^{-\left(\rho+\lambda^{D}\right)t}}{\rho+\lambda^{D}}}{GT(t)}$ is increasing in t as the discount in GT is larger than in the numerator. Hence, with the negative sign it is decreasing. Therefore, all the square bracket is decreasing in t, and as it is to the power of -1, the $BA\left(t\right)$ is increasing in t.

□

A.4 Free entry

Proof of Lemma 4. We start with the free entry of financiers. Recall that the gains from trade are

$$D^{U}\left(t\right)-D^{C}\left(t\right)=h\frac{1-e^{-c_{1}t}}{c_{1}}\qquad c_{1}=\rho+\lambda^{D}+\lambda^{H}+\lambda^{S}\gamma$$

The buyer gets $(1-\gamma)$ of the gains from trade. Hence, the free entry condition reads

$$c = (1-\gamma)\int_0^{\tau} \lambda^B(t)\, h \frac{1-e^{-c_1 t}}{c_1} dt$$

Next, recall that the matching intensity is

$$\lambda^B(t) = A\theta^{\alpha}\frac{\mu^C(t)}{\mu^C} \qquad \theta = \frac{\mu^C}{\mu^B}$$

where

$$\mu^C(t) = \mu^0 \frac{\lambda^H}{\lambda^H+\lambda^S}\left(e^{-(\tau-t)\lambda^D} - e^{-\left(\lambda^H+\lambda^D+\lambda^S\right)(\tau-t)}\right)$$
$$\mu^C = \int_0^{\tau}\mu^C(t)\,dt = \mu^0\frac{\lambda^H}{\lambda^H+\lambda^S}\left(\frac{1-e^{-\tau\lambda^D}}{\lambda^D} - \frac{1-e^{-\left(\lambda^H+\lambda^D+\lambda^S\right)\tau}}{\lambda^H+\lambda^D+\lambda^S}\right)$$

Hence, the free entry condition is

$$c = \frac{(1-\gamma)h}{c_1}A\theta^{\alpha}\int_0^{\tau}\frac{\mu^C(t)}{\mu^C}\left(1-e^{-c_1 t}\right)dt$$
$$c = \frac{(1-\gamma)h}{c_1}A\theta^{\alpha}\left(1-\frac{\int_0^{\tau} e^{-c_1 t}\mu^C(t)\,dt}{\int_0^{\tau}\mu^C(t)\,dt}\right)$$

Define $c_2 = \lambda^H+\lambda^D+\lambda^S$. Then it is easy to show that

$$\int_0^{\tau} e^{-c_1 t}\mu^C(t)\,dt = \mu^0\frac{\lambda^H}{\lambda^H+\lambda^S}\left(\frac{e^{-c_1\tau}-e^{-\lambda^D\tau}}{\lambda^D-c_1} - \frac{e^{-c_1\tau}-e^{-c_2\tau}}{c_2-c_1}\right)$$

As a result, the ratio of integrals in the free entry condition reads

$$\frac{\int_0^{\tau} e^{-c_1 t}\mu^C(t)\,dt}{\int_0^{\tau}\mu^C(t)\,dt} = \left(\frac{e^{-c_1\tau}-e^{-\lambda^D\tau}}{\lambda^D-c_1} - \frac{e^{-c_1\tau}-e^{-c_2\tau}}{c_2-c_1}\right)\left(\frac{1-e^{-\tau\lambda^D}}{\lambda^D} - \frac{1-e^{-\left(\lambda^H+\lambda^D+\lambda^S\right)\tau}}{\lambda^H+\lambda^D+\lambda^S}\right)^{-1} \tag{20}$$

And the free-entry condition boils down to

$$c=\frac{(1-\gamma)\,h}{c_1}A\theta^{\alpha}\left(1-\left(\frac{e^{-c_1\tau}-e^{-\lambda^D\tau}}{\lambda^D-c_1}-\frac{e^{-c_1\tau}-e^{-c_2\tau}}{c_2-c_1}\right)\left(\frac{1-e^{-\tau\lambda^D}}{\lambda^D}-\frac{1-e^{-c_2\tau}}{c_2}\right)^{-1}\right)$$

First, note that it is easy to show that Equation (20) is increasing in τ. Next, consider $\tau=0$. Note that the ratio of integrals in the free entry condition is equal to one as

$$\begin{aligned}
&\lim_{\tau\to0}\left(\frac{e^{-c_1\tau}-e^{-\lambda^D\tau}}{\lambda^D-c_1}-\frac{e^{-c_1\tau}-e^{-c_2\tau}}{c_2-c_1}\right)\left(\frac{1-e^{-\lambda^D\tau}}{\lambda^D}-\frac{1-e^{-c_2\tau}}{c_2}\right)^{-1}\\
&=\lim_{\tau\to0}\left(\frac{-c_1e^{-c_1\tau}+\lambda^De^{-\lambda^D\tau}}{\lambda^D-c_1}-\frac{-c_1e^{-c_1\tau}+c_2e^{-c_2\tau}}{c_2-c_1}\right)\left(\frac{\lambda^De^{-\tau\lambda^D}}{\lambda^D}-\frac{c_2e^{-c_2\tau}}{c_2}\right)^{-1}\\
&=\lim_{\tau\to0}\left(\frac{(c_1)^2e^{-c_1\tau}-\left(\lambda^D\right)^2e^{-\lambda^D\tau}}{\lambda^D-c_1}-\frac{(c_1)^2e^{-c_1\tau}-(c_2)^2e^{-c_2\tau}}{c_2-c_1}\right)\left(\frac{-\left(\lambda^D\right)^2e^{-\tau\lambda^D}}{\lambda^D}-\frac{-(c_2)^2e^{-c_2\tau}}{c_2}\right)^{-1}\\
&=\left(\frac{(c_1)^2-\left(\lambda^D\right)^2}{\lambda^D-c_1}-\frac{(c_1)^2-(c_2)^2}{c_2-c_1}\right)\left(\frac{-\left(\lambda^D\right)^2}{\lambda^D}-\frac{-(c_2)^2}{c_2}\right)^{-1}\\
&=\left(\frac{\left(c_1+\lambda^D\right)\left(c_1-\lambda^D\right)}{\lambda^D-c_1}-\frac{(c_1+c_2)(c_1-c_2)}{c_2-c_1}\right)\left(c_2-\lambda^D\right)^{-1}\\
&=\left(-\left(c_1+\lambda^D\right)+(c_1+c_2)\right)\left(c_2-\lambda^D\right)^{-1}=\left(c_2-\lambda^D\right)\left(c_2-\lambda^D\right)^{-1}=1
\end{aligned}$$

where we applied L'Hopital's rule in the second and third line. As a result, the free-entry condition is satisfied if and only if $\lim_{\tau\to0}\theta=\infty$. Hence, $\lim_{\tau\to0}\lambda^S=0$. That is, $\lambda^S(0)=0$.

Next, consider the case of $\tau\to\infty$. The ratio of integrals in the free entry condition is equal to zero. Hence

$$c=\frac{h}{c_1}(1-\gamma)\,A\theta^{\alpha}$$

Recall that $c_1=\rho+\lambda^D+\lambda^H+\lambda^S\gamma$ and $\lambda^S=A\theta^{\alpha-1}$. Hence,

$$\rho+\lambda^D+\lambda^H+\gamma A\theta^{\alpha-1}=\frac{h\,(1-\gamma)}{c}A\theta^{\alpha}$$

As $\alpha\in(0,1)$ the left hand side is decreasing in θ and the right hand side is increasing in θ. As a result, there exists a unique $\theta\in\mathbb{R}_+$. That is, $\lim_{\tau\to\infty}\lambda^S(\tau)=\overline{\lambda^S}\in\mathbb{R}_+$. $\square$

B Omitted proofs: Borrowers

In this section we provide the proofs of the results for borrowers.

B.1 Quality ladder model

Proof of Lemma 5. For intermediate good producers, the first order condition implies

$$k_j = \left(\frac{(1-\beta)\,\overline{q}}{w}\right)^{\frac{1}{\beta}} Lq_j$$

so

$$p_j = \frac{w}{(1-\beta)\,\overline{q}}$$

Hence, static profits are

$$\pi(q) = \beta L \left(\frac{\overline{q}}{w}\right)^{\frac{1-\beta}{\beta}} (1-\beta)^{\frac{1-\beta}{\beta}} q_j$$
$$\pi(q) = \underline{\pi} q_j$$

And aggregate output is

$$Y = (1-\beta)^{\frac{1-2\beta}{\beta}} \left(\frac{\overline{q}}{w}\right)^{\frac{1-\beta}{\beta}} L\overline{q}$$

Labor demand of final good producers is $L = \frac{\beta Y}{w}$, hence

$$w = \overline{q}\beta^{\beta}(1-\beta)^{1-2\beta}$$
$$w = \overline{q}\tilde{\beta}$$

Replacing the wage, the constant in profit reduces to

$$\pi = L\beta^{\beta}(1-\beta)^{(1-2\beta)}(1-\beta)$$
$$\pi = L\tilde{\beta}(1-\beta)$$

Hence, aggregate output is

$$
\begin{aligned}
Y &= L\frac{w}{\beta} \\
Y &= L\overline{q}\frac{(1-\beta)^{(1-2\beta)}}{\beta^{1-\beta}}
\end{aligned}
$$

Finally, labor market clearing implies $\int l_j + L = 1$ and $k_j = \overline{q} l_j$ so

$$
\begin{aligned}
1 &= \int \left(\frac{1}{\overline{q}} \left(\frac{(1-\beta)\,\overline{q}}{w} \right)^{\frac{1}{\beta}} L q_j \right) + L \\
1 &= L \left(\left(\frac{(1-\beta)}{\tilde{\beta}} \right)^{\frac{1}{\beta}} + 1 \right)
\end{aligned}
$$

and

$$
L = \frac{\beta}{(1-\beta)^2 + \beta}
$$

Hence, output is linear in average quality

$$
Y = \overline{q}\frac{\beta^{\beta}(1-\beta)^{(1-2\beta)}}{(1-\beta)^2 + \beta}
$$

□

B.2 Rollover

Proof. Consider a sequence $\{y_i\}_{i=1}^{J}$ such that $\sum_{i=1}^{J} y_i = \tau$. Then

$$
E(\tau, 0, 0, 0) = e^{-\left(\rho+\lambda^D\right)\tau}\mathbb{E}\left[F(N,\tau) - B_J\right]
$$

where for $j = 1, \ldots, J$

$$
B_j = e^{r(y_j)y_j}\left(\Phi + B_{j-1} + I(y_j)\right)
$$

and $B_0 = 0$. Hence

$$
E(\tau, 0, 0, 0) = e^{-\left(\rho+\lambda^D\right)\tau}\mathbb{E}\left[F(N,\tau)\right] - e^{-\left(\rho+\lambda^D\right)\tau}B_J
$$

Note that for the financial choices there is no uncertainty. Hence, we can define the financial cost as

$$\text{FIN}^{\text{COST}}(\tau) = \min_{J,\{y_j\}_{j=1}^{J}} e^{-\left(\rho+\lambda^D\right)\tau} B_J$$

and

$$E(\tau,0,0,0) = \max_{\tau} e^{-\left(\rho+\lambda^D\right)\tau} \mathbb{E}\left[F(N,\tau)\right] - \text{FIN}^{\text{COST}}(\tau)$$

Note that we can iterate on B_j forward to get

$$\begin{aligned}
B_0 &= 0 \\
B_1 &= e^{r_1 y_1}\left(\Phi + I(y_1)\right) \\
B_2 &= e^{r_2 y_2}\left(\Phi + I(y_2)\right) + e^{r_1 y_1 + r_2 y_2}\left(\Phi + I(y_1)\right) \\
B_3 &= e^{r_3 y_3}\left(\Phi + I(y_3)\right) + e^{r_2 y_2 + r_3 y_3}\left(\Phi + I(y_2)\right) + e^{r_1 y_1 + r_2 y_2 + r_3 y_3}\left(\Phi + I(y_1)\right)
\end{aligned}$$

that is

$$B_j = \sum_{i=1}^{j} \left(\Phi + I(y_i)\right) e^{\sum_{s=i}^{j} r_s y_s}$$

So

$$\text{FIN}^{\text{COST}}(\tau) = e^{-\left(\rho+\lambda^D\right)\tau} \min_{J,\{y_j\}_{j=1}^{J}} \sum_{i=1}^{J} \left(\Phi + I(y_i)\right) e^{\sum_{s=i}^{J} r_s y_s}$$

$$\begin{aligned}
E(\tau,0,0,0) &= \max_{\tau} e^{-\left(\rho+\lambda^D\right)\tau} \mathbb{E}\left[F(N,\tau)\right] - \text{FIN}^{\text{COST}}(\tau) \\
\text{FIN}^{\text{COST}}(\tau) &= e^{-\left(\rho+\lambda^D\right)\tau} \min_{J,\{y_j\}_{j=1}^{J}} \sum_{i=1}^{J} \left(\Phi + I(y_i)\right) e^{\sum_{s=i}^{J} r_s y_s}
\end{aligned}$$

□

B.3 Time-to-build capital

Rioja and Valev (2004) finds that finance boosts growth in rich countries primarily by speeding-up productivity growth, while finance encourages growth in poorer countries primarily by accelerating capital accumulation. In this section we propose an alternative microfoundation for back-loaded projects based on time-to-build capital.

At every moment a new cohort of identical firms μ^0 enters the economy and choose a project to implement. There is a continuum of projects differentiated by the time-to-build $\tau \geq 0$. For $t \in [0, \tau)$ the firm is *young* and is investing. For $t \geq \tau$ the firm is *mature* and is producing.

A young firm starts with no capital, $K(0) = 0$ and investment is subject to a time-to-build constraint à la Majd and Pindyck (1987) such that $dK = idt$ and $i \leq \kappa$. The investment rate i per unit of time cannot exceed κ. Given the linearity, it is optimal to build at maximum capacity. Hence, a firm with a project of duration τ concludes its investment stage with capital equal to $K = \tau\kappa$. This firm is subject to an exogenous exit shock that arrives at Poisson intensity λ^D in which case the project is destroyed and the firm defaults.

A mature firm use the capital to produce and profits are given by $\pi = zK^\sigma$, where z is the productivity and σ are the returns to scale. This firm is subject to an exogenous exit shock that arrives at Poisson intensity λ^E. The net present value for a mature firm with project τ is

$$F(\tau) = \frac{zK^\sigma}{\rho + \lambda^E}. \tag{21}$$

Note that the return on the project F is increasing in the time spent on investment. Hence, time-to-build is an alternative formulation for back-loaded projects.

C Government-Sponsored Intermediaries

C.1 Distribution of financiers

Proof of Lemma 7. Total assets with time to maturity t is $\mu(t) = \mu^0 e^{-\lambda^D t}$. These assets are hold by 4 type of agents: private and government; constrained and unconstrained, $\mu(t) = \mu^{U,P}(t) + \mu^{C,P}(t) + \mu^{U,G}(t) + \mu^{C,G}(t)$.

The law of motions for private sector are

$$-\dot{\mu}^{U,P}(t) = -\left(\lambda^H + \lambda^D\right)\mu^{U,P}(t) + \left(\lambda^{B,P-P}(t) + \lambda^{B,G-P}(t)\right)\mu^{B,P}$$
$$-\dot{\mu}^{C,P}(t) = \lambda^H \mu^{U,P}(t) - \left(\lambda^D + \lambda^{S,P-P} + \lambda^{S,P-G}\right)\mu^{C,P}(t)$$

with boundary conditions $mu^{U,P}(\tau) = \mu^0$ and$\mu^{C,P}(\tau) = 0$.

The law of motions for Government are

$$-\dot{\mu}^{U,G}(t) = -\left(\lambda^{H}+\lambda^{D}\right)\mu^{U,G}(t) + \left(\lambda^{B,P-G}(t)+\lambda^{B,G-G}(t)\right)\mu^{B,G}$$
$$-\dot{\mu}^{C,P}(t) = \lambda^{H}\mu^{U,G}(t) - \left(\lambda^{D}+\lambda^{S,G-P}+\lambda^{S,G-G}\right)\mu^{C,G}(t)$$

with boundary conditions $\mu^{U,G}(\tau) = \mu^{C,G}(\tau) = 0$.

Note that matching implies

$$\mu^{B,P}\lambda^{B,P-P}(t) = \mu^{C,P}(t)\lambda^{S,P-P}$$
$$\mu^{B,P}\lambda^{B,G-P}(t) = \mu^{C,G}(t)\lambda^{S,G-P}$$
$$\mu^{B,G}\lambda^{B,P-G}(t) = \mu^{C,P}(t)\lambda^{S,P-G}$$
$$\mu^{B,G}\lambda^{B,G-G}(t) = \mu^{C,G}(t)\lambda^{S,G-G}$$

Define

$$F(t) = \left[\mu^{U,P}(t), \mu^{C,P}(t), \mu^{U,G}(t), \mu^{C,G}(t)\right]$$

Boundary condition

$$F(\tau) = \left[\mu^{0}, 0, 0, 0\right]$$

System

$$\dot{F}(t) = AF(t)$$

where

$$A = \begin{bmatrix} \lambda^{H}+\lambda^{D} & -\lambda^{S,P-P} & 0 & -\lambda^{S,G-P} \\ -\lambda^{H} & \lambda^{D}+\lambda^{S,P-P}+\lambda^{S,P-G} & 0 & 0 \\ 0 & -\lambda^{S,P-G} & \lambda^{H}+\lambda^{D} & -\lambda^{S,G-G} \\ 0 & 0 & -\lambda^{H} & \lambda^{D}+\lambda^{S,G-P}+\lambda^{S,G-G} \end{bmatrix}$$

Solution is of the form

$$F(t) = \sum_{j=1}^{4} c_j e^{r_j t}\mathbf{v}_j$$

where r_j and $\mathbf{v}_j$ are the eigenvalues and eigenvectors of A, c_j are constant pin down by the boundary condition $F(\tau) = [\mu^{0}, 0, 0, 0]$

Let R be a diagonal matrix with the eigenvalues in the diagonal, and let V be the matrix with the eigenvalues in the columns. Then let $B = e^{R\tau}c$ where we use the matrix exponential (i.e. it is a diagonal matrix) so

$$\begin{aligned} F(\tau) &= VB \\ B &= V^{-1}F(\tau) \end{aligned}$$

Hence, as $e^{R\tau}$ is diagonal,

$$c = e^{-R\tau}B$$

Hence

$$\begin{aligned} F_j(t) &= \sum_{i=1}^{4} e^{r_i t}e^{-r_i\tau}B_iV(j,i) \\ F_j(t) &= \sum_{i=1}^{4} e^{r_i(t-\tau)}B_iV(j,i) \\ \int_0^{\tau} F_j(t) &= \sum_{i=1}^{4} \frac{B_iV(j,i)}{r_i}\left(1-e^{-r_i\tau}\right) \end{aligned}$$

□

C.2 Value functions

Proof of Lemma 8. Structure of the proof:

1. Solve for $D^C(t)$ and $D^U(t)$ as sum of exponential functions
2. Solve for $G^C(t)$ as sum of exponential functions
3. Solve for $G^U(t)$ as sum of exponential functions
4. Solve for free entry in private sector
5. Solve for G^S

Private sector The solution of the private sector is the same as before, hence

$$\begin{aligned} D^U(t) &= \sum_{j=1}^{2} A_j^{DU} e^{-B_j^{DU} t} + A_3^{DU} \\ D^C(t) &= \sum_{j=1}^{2} A_j^{C} e^{-B_j^{C} t} + A_3^{C} \end{aligned}$$

with

$$\begin{aligned} B_1^{DU} &= B_1^{DC} = \rho + \lambda^D \\ B_2^{DU} &= B_2^{DC} = \rho + \lambda^D + \lambda^H + \lambda^S \end{aligned}$$

and

$$\begin{aligned} A_1^{DU} &= 1 + \frac{h\lambda^H}{\left(\lambda^H + \lambda^S\right)\left(\rho + \lambda^D\right)} \\ A_2^{DU} &= -\frac{h\lambda^H}{\left(\lambda^H + \lambda^S\right)\left(\rho + \lambda^D + \lambda^H + \lambda^S\right)} \\ A_3^{DU} &= -h\frac{\lambda^H}{\lambda^H + \lambda^S}\left(\frac{1}{\rho + \lambda^D} - \frac{1}{\rho + \lambda^D + \lambda^H + \lambda^S}\right) \end{aligned}$$

and

$$\begin{aligned} A_1^{DC} &= 1 + \frac{h\lambda^H}{\left(\lambda^H + \lambda^S\right)\left(\rho + \lambda^D\right)} \\ A_2^{DC} &= \frac{h\lambda^S}{\left(\lambda^H + \lambda^S\right)\left(\rho + \lambda^D + \lambda^H + \lambda^S\right)} \\ A_3^{DC} &= -\frac{h}{\lambda^H + \lambda^S}\left(\frac{\lambda^H}{\rho + \lambda^D} + \frac{\lambda^S}{\rho + \lambda^D + \lambda^H + \lambda^S}\right) \end{aligned}$$

Government Constrained Recall that

$$\left(\rho + \lambda^D + \lambda^{S,G-P}\gamma^{G-P}\right) G^C(t) = -h - \dot{G}^C(t) + \lambda^{S,G-P}\gamma^{G-P} D^U(t)$$

where $D^U(t) = \sum_{j=1}^{2} A_j^{DU} e^{-B_j^{DU} t} + A_3^{DU}$. Let

$$\begin{aligned}
a_j &= \lambda^{S,G-P} \gamma^G A_j^{DU} \text{ for } j = 1, 2 \\
a_3 &= \rho + \lambda^D + \lambda^{S,G-P} \gamma^{G-P} \\
a_4 &= -h + \lambda^{S,G-P} \gamma^{G-P} A_3^{DU} \\
B_j^{GC} &= A_j^{DU} \text{ for } j = 1, 2 \\
B_3^{GC} &= a_3
\end{aligned}$$

Then

$$a_3 G^C(t) = -\dot{G}^C(t) + \sum_{j=1}^{2} a_j e^{-B_j^{GC} t} + a_4$$

Guess a solution of the form

$$G^C(t) = \sum_{j=1}^{3} A_j^{GC} e^{-B_j^{GC} t} + A_4^{GC}$$

And verify that

$$a_3 \left(\sum_{j=1}^{3} A_j^{GC} e^{-B_j^{GC} t} + A_4^{GC} \right) = \sum_{j=1}^{3} B_j^{GC} A_j^{GC} e^{-B_j^{GC} t} + \sum_{j=1}^{2} a_j e^{-B_j^{GC} t} + a_4$$

Therefore

$$\begin{aligned}
A_j^{GC} &= \frac{a_j}{a_3 - B_j^{GC}} \text{ for } j = 1, 2 \\
A_4^{GC} &= \frac{a_4}{a_3}
\end{aligned}$$

And the boundary condition implies

$$1 = \sum_{j=1}^{4} A_j^{GC}$$

$$
\begin{aligned}
B_3^{GC} &= a_3 \\
B_j^{GC} &= A_j^{DU} \text{ for } j = 1, 2 \\
A_j^{GC} &= \frac{\lambda^{S,G-P}\gamma^{G}A_j^{DU}}{\rho + \lambda^{D} + \lambda^{S,G-P}\gamma^{G-P} - B_j^{GC}} \text{ for } j = 1, 2 \\
A_4^{GC} &= \frac{-h + \lambda^{S,G-P}\gamma^{G-P}A_3^{DU}}{\rho + \lambda^{D} + \lambda^{S,G-P}\gamma^{G-P}} \\
A_3^{GC} &= 1 - \sum_{j=1}^{2} A_j^{GC} - A_4^{GC}
\end{aligned}
$$

Government unconstrained Recall that

$$\left(\rho + \lambda^{D} + \lambda^{H}\right) G^{U}(t) = -\dot{G}^{U}(t) + \lambda^{H} G^{C}(t)$$

with $G^{C}(t) = \sum_{j=1}^{3} A_j^{GC} e^{-B_j^{GC} t} + A_4^{GC}$. Let

$$
\begin{aligned}
a_j &= \lambda^{H} A_j^{GC} \text{ for } j = 1, 2, 3 \\
a_4 &= \rho + \lambda^{D} + \lambda^{H} \\
a_5 &= \lambda^{H} A_4^{GC}
\end{aligned}
$$

then

$$a_4 G^{U}(t) = -\dot{G}^{U}(t) + \sum_{j=1}^{3} a_j e^{-B_j^{GC} t} + a_5$$

Guess that

$$G^{U}(t) = \sum_{j=1}^{4} A_j^{GU} e^{-B_j^{GU} t} + A_5^{GU}$$

with

$$
\begin{aligned}
B_j^{GU} &= B_j^{GC} \text{ for } j = 1, 2, 3 \\
B_4^{GU} &= a_4
\end{aligned}
$$

then

$$a_4 \left(\sum_{j=1}^{4} A_j^{GU} e^{-B_j^{GU} t} + A_5^{GU} \right) = \sum_{j=1}^{4} B_j^{GU} A_j^{GU} e^{-B_j^{GU} t} + \sum_{j=1}^{3} a_j e^{-B_j^{GC} t} + a_5$$

so

$$\begin{aligned} A_j^{GU} &= \frac{a_j}{a_4 - B_j^{GU}} \text{ for } j = 1,2,3 \\ A_5^{GU} &= \frac{a_5}{a_4} \end{aligned}$$

And the boundary condition implies

$$1 = \sum_{j=1}^{5} A_j^{GU}$$

Hence

$$\begin{aligned} B_j^{GU} &= B_j^{GC} \text{ for } j = 1,2,3 \\ B_4^{GU} &= \rho + \lambda^D + \lambda^H \\ A_j^{GU} &= \frac{\lambda^H A_j^{GC}}{\rho + \lambda^D + \lambda^H - B_j^{GU}} \text{ for } j = 1,2,3 \\ A_5^{GU} &= \frac{\lambda^H A_4^{GC}}{\rho + \lambda^D + \lambda^H} \\ A_4^{GU} &= 1 - \sum_{j=1}^{3} A_j^{GU} - A_5^{GU} \end{aligned}$$

Free entry implies Free entry implies that

$$c = (1-\gamma)\int_0^\tau \lambda^{B,P-P}(t)\left(D^U(t) - D^C(t)\right)dt + \int_0^\tau \lambda^{B,G-P}(t)\left(D^U(t) - G^C(t)\right)dt$$

where both value functions and matching intensities are sum of exponential functions, so it is simple to solve the integral.

Start with the first term, and recall that they share the same roots, so

$$D^U(t) - D^C(t) = \sum_{j=1}^{2}\left(A_j^{DU} - A_j^{DC}\right)e^{-B_j^{DU}t} + \left(A_3^{DU} - A_3^{DC}\right)$$

Regarding the matching intensity, recall that

$$\begin{aligned}\lambda^{B,P-P}(t) &= \lambda^{B}\frac{\mu^{C,P}(t)}{\mu^{C}}\\ \mu^{C,P}(t) &= \sum_{i=1}^{4} e^{r_i(t-\tau)}B_iV(2,i)\end{aligned}$$

so $\int_0^\tau \lambda^{B,P-P}(t)\left(D^U(t)-D^C(t)\right)dt$ is equal to

$$\begin{aligned}&\frac{\lambda^B}{\mu^C}\int_0^\tau\left[\sum_{i=1}^{4}e^{r_i(t-\tau)}B_iV(2,i)\left(\sum_{j=1}^{2}\left(A_j^{DU}-A_j^{DC}\right)e^{-B_j^{DU}t}+\left(A_3^{DU}-A_3^{DC}\right)\right)\right]dt\\ =\ &\frac{\lambda^B}{\mu^C}\sum_{i=1}^{4}e^{-r_i\tau}B_iV(2,i)\left(\sum_{j=1}^{2}\left(A_j^{DU}-A_j^{DC}\right)\int_0^\tau e^{\left(r_i-B_j^{DU}\right)t}dt+\left(A_3^{DU}-A_3^{DC}\right)\int_0^\tau e^{r_it}dt\right)\\ =\ &\frac{\lambda^B}{\mu^C}\sum_{i=1}^{4}e^{-r_i\tau}B_iV(2,i)\left(\sum_{j=1}^{2}\left(A_j^{DU}-A_j^{DC}\right)\frac{e^{\left(r_i-B_j^{DU}\right)\tau}-1}{r_i-B_j^{DU}}+\left(A_3^{DU}-A_3^{DC}\right)\frac{e^{r_i\tau}-1}{r_i}\right)\\ =\ &\frac{\lambda^B}{\mu^C}\sum_{i=1}^{4}B_iV(2,i)\left(\sum_{j=1}^{2}\left(A_j^{DU}-A_j^{DC}\right)\frac{e^{-B_j^{DU}\tau}-e^{-r_i\tau}}{r_i-B_j^{DU}}+\left(A_3^{DU}-A_3^{DC}\right)\frac{1-e^{-r_i\tau}}{r_i}\right)\end{aligned}$$

Similarly, recall that the firs two roots of G^C are the same as D^U, so

$$D^U(t)-G^C(t)=\sum_{j=1}^{2}\left(A_j^{DU}-A_j^{GC}\right)e^{-B_j^{DU}t}-A_3^{GC}e^{-B_3^{GC}t}+\left(A_3^{DU}-A_4^{GC}\right)$$

and

$$\begin{aligned}\lambda^{B,G-P}(t) &= \lambda^{B}\frac{\mu^{C,G}(t)}{\mu^{C}}\\ \mu^{C,G}(t) &= \sum_{i=1}^{4} e^{r_i(t-\tau)}B_iV(4,i)\end{aligned}$$

Hence $\int_0^\tau \lambda^{B,G-P}(t)\left(D^U(t) - G^C(t)\right)dt$ is equal to

$$\begin{aligned}
& \frac{\lambda^B}{\mu^C}\left[\int_0^\tau \sum_{i=1}^4 e^{r_i(t-\tau)} B_i V(4,i)\left(\sum_{j=1}^2\left(A_j^{DU} - A_j^{GC}\right)e^{-B_j^{DU}t} - A_3^{GC}e^{-B_3^{GC}t} + \left(A_3^{DU} - A_4^{GC}\right)\right)dt\right] \\
= \; & \frac{\lambda^B}{\mu^C}\sum_{i=1}^4 e^{-r_i\tau} B_i V(4,i)\left(\sum_{j=1}^2\left(A_j^{DU} - A_j^{GC}\right)\int_0^\tau e^{\left(r_i - B_j^{DU}\right)t}dt\right. \\
& \left. -A_3^{GC}\int_0^\tau e^{\left(r_i - B_3^{GC}\right)t}dt + \left(A_3^{DU} - A_4^{GC}\right)\int_0^\tau e^{r_i t}dt\right) \\
= \; & \frac{\lambda^B}{\mu^C}\sum_{i=1}^4 e^{-r_i\tau} B_i V(4,i)\left(\sum_{j=1}^2\left(A_j^{DU} - A_j^{GC}\right)\frac{e^{\left(r_i - B_j^{DU}\right)\tau} - 1}{r_i - B_j^{DU}}\right. \\
& \left. -A_3^{GC}\frac{e^{\left(r_i - B_3^{GC}\right)\tau} - 1}{r_i - B_3^{GC}} + \left(A_3^{DU} - A_4^{GC}\right)\frac{e^{r_i\tau} - 1}{r_i}\right) \\
= \; & \frac{\lambda^B}{\mu^C}\sum_{i=1}^4 B_i V(4,i)\left(\sum_{j=1}^2\left(A_j^{DU} - A_j^{GC}\right)\frac{e^{-B_j^{DU}\tau} - e^{-r_i\tau}}{r_i - B_j^{DU}}\right. \\
& \left. -A_3^{GC}\frac{e^{-B_3^{GC}\tau} - e^{-r_i\tau}}{r_i - B_3^{GC}} + \left(A_3^{DU} - A_4^{GC}\right)\frac{1 - e^{-r_i\tau}}{r_i}\right)
\end{aligned}$$

Finally

$$\begin{aligned}
c \; = \; & \frac{\lambda^B(1-\gamma)}{\mu^C}\sum_{i=1}^4 B_i V(2,i)\left(\sum_{j=1}^2\left(A_j^{DU} - A_j^{DC}\right)\frac{e^{-B_j^{DU}\tau} - e^{-r_i\tau}}{r_i - B_j^{DU}} + \left(A_3^{DU} - A_3^{DC}\right)\frac{1 - e^{-r_i\tau}}{r_i}\right) \\
& + \frac{\lambda^B\left(1-\gamma^{G-P}\right)}{\mu^C}\sum_{i=1}^4 B_i V(4,i)\left(\sum_{j=1}^2\left(A_j^{DU} - A_j^{GC}\right)\frac{e^{-B_j^{DU}\tau} - e^{-r_i\tau}}{r_i - B_j^{DU}}\right. \\
& \left. -A_3^{GC}\frac{e^{-B_3^{GC}\tau} - e^{-r_i\tau}}{r_i - B_3^{GC}} + \left(A_3^{DU} - A_4^{GC}\right)\frac{1 - e^{-r_i\tau}}{r_i}\right)
\end{aligned}$$

Entry of government agents The value of entry for the government is

$$\rho G^S = -c + \int_0^\tau \lambda^{B,P-G}(t)\left(G^U(t) - G^S - P^{S,P-G}(t)\right)dt$$

recall that the first two roots of G^U, D^U and D^C are the same, hence

$$\begin{aligned}
G^S\left(\tau+\int_0^\tau \lambda^{B,P-G}(t)\,dt\right) &= -c+\int_0^\tau \lambda^{B,P-G}(t)\left(G^U(t)-D^C(t)-\gamma^{P-G}\left(D^U(t)-D^C(t)\right)\right)dt \\
G^S\left(\tau+\lambda^B\frac{\mu^{C,P}}{\mu^C}\right) &= -c+\int_0^\tau \lambda^{B,P-G}(t)\left(G^U(t)-D^C(t)-\gamma^{P-G}\left(D^U(t)-D^C(t)\right)\right)dt
\end{aligned}$$

where

$$\begin{aligned}
& G^U(t)-D^C(t)-\gamma^{P-G}\left(D^U(t)-D^C(t)\right)=G^U(t)-\gamma^{P-G}D^U(t)-\left(1-\gamma^{P-G}\right)D^C(t) \\
= & \sum_{j=1}^{2}\left(A_j^{GU}-\gamma^{P-G}A_j^{DU}-\left(1-\gamma^{P-G}\right)A_j^{DC}\right)e^{-B_j^{DU}t} \\
& +\sum_{j=3}^{4}A_j^{GU}e^{-B_j^{GU}t}+\left(A_5^{GU}-\gamma^{P-G}A_3^{DU}-\left(1-\gamma^{P-G}\right)A_3^{DC}\right)
\end{aligned}$$

so $\int_0^\tau \lambda^{B,P-G}(t)\left(G^U(t)-D^C(t)-\gamma^{P-G}\left(D^U(t)-D^C(t)\right)\right)$ is equal to

$$\begin{aligned}
& \frac{\lambda^B}{\mu^C}\left[\int_0^\tau \sum_{i=1}^{4}e^{r_i(t-\tau)}B_iV(2,i)\left(\sum_{j=1}^{2}\left(A_j^{GU}-\gamma^{P-G}A_j^{DU}-\left(1-\gamma^{P-G}\right)A_j^{DC}\right)e^{-B_j^{DU}t}\right.\right. \\
& \left.\left.+\sum_{j=3}^{4}A_j^{GU}e^{-B_j^{GU}t}+\left(A_5^{GU}-\gamma^{P-G}A_3^{DU}-\left(1-\gamma^{P-G}\right)A_3^{DC}\right)\right)\right]dt \\
= & \frac{\lambda^B}{\mu^C}\sum_{i=1}^{4}e^{-r_i\tau}B_iV(2,i)\left(\sum_{j=1}^{2}\left(A_j^{GU}-\gamma^{P-G}A_j^{DU}-\left(1-\gamma^{P-G}\right)A_j^{DC}\right)\int_0^\tau e^{\left(r_i-B_j^{DU}\right)t}dt\right. \\
& \left.+\sum_{j=3}^{4}A_j^{GU}\int_0^\tau e^{\left(r_i-B_j^{GU}\right)t}dt+\left(A_5^{GU}-\gamma^{P-G}A_3^{DU}-\left(1-\gamma^{P-G}\right)A_3^{DC}\right)\int_0^\tau e^{r_it}dt\right) \\
= & \frac{\lambda^B}{\mu^C}\sum_{i=1}^{4}e^{-r_i\tau}B_iV(2,i)\left(\sum_{j=1}^{2}\left(A_j^{GU}-\gamma^{P-G}A_j^{DU}-\left(1-\gamma^{P-G}\right)A_j^{DC}\right)\frac{e^{\left(r_i-B_j^{DU}\right)\tau}-1}{r_i-B_j^{DU}}\right. \\
& \left.+\sum_{j=3}^{4}A_j^{GU}\frac{e^{\left(r_i-B_j^{DU}\right)\tau}-1}{r_i-B_j^{DU}}+\left(A_5^{GU}-\gamma^{P-G}A_3^{DU}-\left(1-\gamma^{P-G}\right)A_3^{DC}\right)\frac{e^{r_i\tau}-1}{r_i}\right) \\
= & \frac{\lambda^B}{\mu^C}\sum_{i=1}^{4}B_iV(2,i)\left(\sum_{j=1}^{2}\left(A_j^{GU}-\gamma^{P-G}A_j^{DU}-\left(1-\gamma^{P-G}\right)A_j^{DC}\right)\frac{e^{-B_j^{DU}\tau}-e^{-r_i\tau}}{r_i-B_j^{DU}}\right. \\
& \left.+\sum_{j=3}^{4}A_j^{GU}\frac{e^{-B_j^{DU}\tau}-e^{-r_i\tau}}{r_i-B_j^{DU}}+\left(A_5^{GU}-\gamma^{P-G}A_3^{DU}-\left(1-\gamma^{P-G}\right)A_3^{DC}\right)\frac{1-e^{-r_i\tau}}{r_i}\right)
\end{aligned}$$

Hence

$$G^S = \left(\rho+\lambda^B\frac{\mu^{C,P}}{\mu^C}\right)^{-1}\left(-c+\frac{\lambda^B}{\mu^C}\sum_{i=1}^{4}B_iV(2,i)\left(\sum_{j=1}^{2}\left(A_j^{GU}-\gamma^{P-G}A_j^{DU}-\left(1-\gamma^{P-G}\right)A_j^{DC}\right)\frac{e^{-B_j^{DU}\tau}-e^{-r_i\tau}}{r_i-B_j^{DU}}\right.\right.$$
$$\left.\left.+\sum_{j=3}^{4}A_j^{GU}\frac{e^{-B_j^{DU}\tau}-e^{-r_i\tau}}{r_i-B_j^{DU}}+\left(A_5^{GU}-\gamma^{P-G}A_3^{DU}-\left(1-\gamma^{P-G}\right)A_3^{DC}\right)\frac{1-e^{-r_i\tau}}{r_i}\right)\right)$$

□

D Segmented markets

D.1 Distributions of financiers

Proof of Lemma 9. Total assets are $\mu^j(t)=e^{(t-\tau)\lambda^D}\mu^0$. The evolution for unconstrained and constrained agents in market j are

$$-\dot{\mu}^{U,j}(t)=-\left(\lambda^H+\lambda^D\right)\mu^{U,j}(t)+\mu^{B,j}\lambda^{B,j}(t)$$
$$-\dot{\mu}^{C,j}(t)=\lambda^H\mu^{U,j}(t)-\left(\lambda^D+\lambda^{S,j}\right)\mu^{C,j}(\tau_j)$$

Recall that matching implies that $\mu^{B,j}\lambda^{B,j}(t)=\mu^{C,j}(t)\lambda^{S,j}$. Hence the system is

$$\dot{\mu}^j(t)=\begin{bmatrix}\lambda^H+\lambda^D & -\lambda^{S,j}\\ -\lambda^H & \lambda^D+\lambda^{S,j}\end{bmatrix}\begin{bmatrix}\mu^{U,j}(t)\\ \mu^{C,j}(t)\end{bmatrix}$$

With eigenvalues λ^D and $\lambda^H+\lambda^{S,j}+\lambda^D$. Define V^j to be the matrix with the eigenvectors and R^j the diagonal matrix with the eigenvalues so

$$V^j = \begin{bmatrix}-1 & \frac{\lambda^{S,j}}{\lambda^H}\\ 1 & 1\end{bmatrix}$$
$$R^j = \begin{bmatrix}\lambda^H+\lambda^{S,j}+\lambda^D & \\ & \lambda^D\end{bmatrix}$$
$$B^j = \left(V^j\right)^{-1}\mu^{j+1}(\tau_{j+1})$$

where

$$\left(V^j\right)^{-1} = \frac{\lambda^H}{\lambda^H + \lambda^{S,j}} \begin{bmatrix} -1 & \lambda^{S,j} \\ 1 & 1 \end{bmatrix}$$

Then

$$\begin{aligned} \mu^{U,j}(t) &= \sum_{i=1}^{2} e^{R^j(i)(t-\tau_{j+1})} B^j(i) V^j(1,i) \\ \mu^{C,j}(t) &= \sum_{i=1}^{2} e^{R^j(i)(t-\tau_{j+1})} B^j(i) V^j(2,i) \end{aligned}$$

For $j = 1, \dots, N-1$ we have that

$$\begin{aligned} \mu^{U,j}(t) =& \frac{\lambda^H}{\lambda^H + \lambda^{S,j}} \left[\frac{\lambda^{S,j}}{\lambda^H} e^{\lambda^D(t-\tau_{j+1})} \left(\mu^{U,j+1}(\tau_{j+1}) + \mu^{C,j+1}(\tau_{j+1}) \right) \right] \\ & - \frac{\lambda^H}{\lambda^H + \lambda^{S,j}} \left[-e^{\left(\lambda^H + \lambda^{S,j} + \lambda^D\right)(t-\tau_{j+1})} \left(-\mu^{U,j+1}(\tau_{j+1}) + \lambda^{S,j} \mu^{C,j+1}(\tau_{j+1}) \right) \right] \\ \mu^{C,j}(t) =& \frac{\lambda^H}{\lambda^H + \lambda^{S,j}} \left[e^{\lambda^D(t-\tau_{j+1})} \left(\mu^{U,j+1}(\tau_{j+1}) + \mu^{C,j+1}(\tau_{j+1}) \right) \right] \\ & + \frac{\lambda^H}{\lambda^H + \lambda^{S,j}} \left[+e^{\left(\lambda^H + \lambda^{S,j} + \lambda^D\right)(t-\tau_{j+1})} \left(-\mu^{U,j+1}(\tau_{j+1}) + \lambda^{S,j} \mu^{C,j+1}(\tau_{j+1}) \right) \right] \end{aligned}$$

□

D.2 Value functions

Let $D^{H,j}(t) = D^{U,j}(t) - D^{C,j}(t)$ and $c_j = \rho + \lambda^D + \lambda^H + \gamma\lambda^{S,j}$, then

$$c_j D^{H,j}(t) = h - \dot{D}^{H,j}(t)$$

Solution $D^{H,j}(t) = A^{H,j} e^{-c_j t} + B^j$ so

$$c_j \left(A^{H,j} e^{-c_j t} + B^j \right) = h + c_j A^{H,j} e^{-c_j t}$$

so

$$B^j = \frac{h}{c_j}$$

and the boundary condition pin downs A^j.

Note that for $j=1$ the boundary condition is $D^{U,1}(\tau_1)=D^{C,1}(\tau_1)=1$ so

$$\begin{aligned} 0 &= A^{H,1}+B^1 \\ A^{H,1} &= -\frac{h}{c_1} \end{aligned}$$

and

$$D^{H,1}(t)=h\frac{1-e^{-c_1 t}}{c_1}$$

For $j=2,\ldots,N$ we have that $D^{H,j}(\tau_j)=D^{H,j-1}(\tau_j)$. Where

$$\begin{aligned} D^{H,j-1}(\tau_j) &= A^{H,j-1}e^{-c_{j-1}\tau_j}+\frac{h}{c_{j-1}} \\ D^{H,j}(\tau_j) &= A^{H,j}e^{-c_j\tau_j}+\frac{h}{c_j} \end{aligned}$$

so

$$\begin{aligned} A^{H,j}e^{-c_j\tau_j}+\frac{h}{c_j} &= A^{H,j-1}e^{-c_{j-1}\tau_j}+\frac{h}{c_{j-1}} \\ A^{H,j} &= e^{c_j\tau_j}\left(A^{H,j-1}e^{-c_{j-1}\tau_j}+\frac{h}{c_{j-1}}-\frac{h}{c_j}\right) \end{aligned} \tag{22}$$

Hence

$$D^{H,j}(t)=A^{H,j}e^{-c_j t}+\frac{h}{c_j}$$

where A^j are pin down by the recursion (22) with initial condition $A^{H,1}=-\frac{h}{c_1}$.

Next, we can solve for the value of constrained and unconstrained agents using $D^{H,j}$ and the initial conditions.

Unconstrained

$$\begin{aligned} \left(\rho+\lambda^D\right)D^{U,j}(t) &= -\dot{D}^{U,j}(t)-\lambda^H D^{H,j} \\ \left(\rho+\lambda^D\right)D^{U,j}(t) &= -\dot{D}^{U,j}(t)-\lambda^H\left(A^{H,j}e^{-c_j t}+\frac{h}{c_j}\right) \end{aligned}$$

Guess $D^{U,j}(t) = A^{U,j} + B^{U,j} e^{-\left(\rho+\lambda^{D}\right)t} + C^{U,j} e^{-c_j t}$ then

$$\left(\rho + \lambda^{D}\right)\left(A^{U,j} + B^{U,j} e^{-\left(\rho+\lambda^{D}\right)t} + C^{U,j} e^{-c_j t}\right) = \left(\rho + \lambda^{D}\right) B^{U,j} e^{-\left(\rho+\lambda^{D}\right)t} + c_j C^{U,j} e^{-c_j t} - \lambda^{H}\left(A^{H,j} e^{-c_j t} + \frac{h}{c_j}\right)$$

hence

$$\begin{aligned} A^{U,j} &= -\frac{\lambda^{H} h}{\left(\rho + \lambda^{D}\right) c_j} \\ C^{U,j} &= \frac{-\lambda^{H} A^{H,j}}{\left(\rho + \lambda^{D}\right) - c_j} = \frac{\lambda^{H} A^{H,j}}{\lambda^{H} + \gamma \lambda^{S,j}} \end{aligned}$$

And the boundary condition pin down $B^{U,j}$.

For $j = 1$ we have that $D^{U,1}(\tau_1) = 1$ and $\tau_1 = 0$ so

$$B^{U,1} = 1 - A^{U,j} - C^{U,j}$$

For $j = 2, \ldots, N$ we have that

$$D^{U,j}(\tau_j) = D^{U,j-1}(\tau_j)$$

so

$$\begin{aligned} &A^{U,j} + B^{U,j} e^{-\left(\rho+\lambda^{D}\right)\tau_j} + C^{U,j} e^{-c_j \tau_j} = A^{U,j-1} + B^{U,j-1} e^{-\left(\rho+\lambda^{D}\right)\tau_j} + C^{U,j-1} e^{-c_{j-1}\tau_j} \\ &B^{U,j} = e^{\left(\rho+\lambda^{D}\right)\tau_j}\left(A^{U,j-1} - A^{U,j} + B^{U,j-1} e^{-\left(\rho+\lambda^{D}\right)\tau_j} + C^{U,j-1} e^{-c_{j-1}\tau_j} - C^{U,j} e^{-c_j \tau_j}\right) \end{aligned}$$

which defines a recursion in $B^{U,j}$

Free entry

Free entry in each market:

$$c_j = (1-\gamma)\int_{\tau_j}^{\tau_{j+1}} \lambda^{B,j}(t)\left(D^{U,j}(t) - D^{C}(t)\right) dt$$

where

$$
\begin{aligned}
\lambda^{B,j}(t) &= A\left(\theta^{j}\right)^{\alpha}\frac{\mu^{C,j}(t)}{\mu^{C,j}} \\
\mu^{C,j}(t) &= \frac{\lambda^{H}}{\lambda^{H}+\lambda^{S,j}}\Big[e^{\lambda^{D}(t-\tau_{j+1})}\left(\mu^{U,j+1}(\tau_{j+1})+\mu^{C,j+1}(\tau_{j+1})\right)+e^{\left(\lambda^{H}+\lambda^{S,j}+\lambda^{D}\right)(t-\tau_{j+1})}\left(-\mu^{U,j+1}(\tau\right. \\
\mu^{C,j} &= \int_{\tau_j}^{\tau_{j+1}}\mu^{C,j}(t)\,dt \\
D^{U,j}(t)-D^{C}(t) &= A^{H,j}e^{-c_j t}+\frac{h}{c_j}
\end{aligned}
$$

Note that

$$
\begin{aligned}
\mu^{C,j} &= \int_{\tau_j}^{\tau_{j+1}}\mu^{C,j}(t)\,dt \\
\mu^{C,j} &= \frac{\lambda^{H}}{\lambda^{H}+\lambda^{S,j}}\left[\left(\mu^{U,j+1}(\tau_{j+1})+\mu^{C,j+1}(\tau_{j+1})\right)\int_{\tau_j}^{\tau_{j+1}}e^{\lambda^{D}(t-\tau_{j+1})}dt+\left(-\mu^{U,j+1}(\tau_{j+1})+\lambda^{S,j}\mu^{C,j+1}(\tau_{j+1})\right)\right. \\
\mu^{C,j} &= \frac{\lambda^{H}}{\lambda^{H}+\lambda^{S,j}}\left[\left(\mu^{U,j+1}(\tau_{j+1})+\mu^{C,j+1}(\tau_{j+1})\right)\frac{1-e^{\lambda^{D}(\tau_j-\tau_{j+1})}}{\lambda^{D}}+\left(-\mu^{U,j+1}(\tau_{j+1})+\lambda^{S,j}\mu^{C,j+1}(\tau_{j+1})\right)\frac{1-}{}\right.
\end{aligned}
$$

And

$$
\begin{aligned}
\int_{\tau_j}^{\tau_{j+1}}\lambda^{B,j}(t)\left(D^{U,j}(t)-D^{C}(t)\right)dt &= \int_{\tau_j}^{\tau_{j+1}}A\left(\theta^{j}\right)^{\alpha}\frac{\mu^{C,j}(t)}{\mu^{C,j}}\left(A^{H,j}e^{-c_j t}+\frac{h}{c_j}\right)dt \\
&= \frac{A\left(\theta^{j}\right)^{\alpha}}{\mu^{C,j}}A^{H,j}\int_{\tau_j}^{\tau_{j+1}}\mu^{C,j}(t)\,e^{-c_j t}dt+\frac{h}{c_j}A\left(\theta^{j}\right)^{\alpha}\frac{\int_{\tau_j}^{\tau_{j+1}}\mu^{C,j}(t)\,dt}{\mu^{C,j}} \\
&= \frac{A\left(\theta^{j}\right)^{\alpha}}{\mu^{C,j}}A^{H,j}\int_{\tau_j}^{\tau_{j+1}}\mu^{C,j}(t)\,e^{-c_j t}dt+\frac{h}{c_j}A\left(\theta^{j}\right)^{\alpha}
\end{aligned}
$$

where

$$
\begin{aligned}
\int_{\tau_j}^{\tau_{j+1}}\mu^{C,j}(t)\,e^{-c_j t}dt &= \frac{\lambda^{H}\left(\mu^{U,j+1}(\tau_{j+1})+\mu^{C,j+1}(\tau_{j+1})\right)}{\lambda^{H}+\lambda^{S,j}}\int_{\tau_j}^{\tau_{j+1}}e^{-c_j t+\lambda^{D}(t-\tau_{j+1})}dt \\
&\quad+\frac{\lambda^{H}\left(-\mu^{U,j+1}(\tau_{j+1})+\lambda^{S,j}\mu^{C,j+1}(\tau_{j+1})\right)}{\lambda^{H}+\lambda^{S,j}}\int_{\tau_j}^{\tau_{j+1}}e^{-c_j t+\left(\lambda^{H}+\lambda^{S,j}+\lambda^{D}\right)(t-\tau_{j+1})}dt
\end{aligned}
$$

that is

$$\int_{\tau_j}^{\tau_{j+1}} \mu^{C,j}(t)\, e^{-c_j t} dt = \frac{\lambda^H \left(\mu^{U,j+1}(\tau_{j+1}) + \mu^{C,j+1}(\tau_{j+1})\right)}{\lambda^H + \lambda^{S,j}} \frac{e^{-c_j \tau_{j+1}} - e^{-c_j \tau_j + \lambda^D (\tau_j - \tau_{j+1})}}{\lambda^D - c_j}$$
$$+ \frac{\lambda^H \left(-\mu^{U,j+1}(\tau_{j+1}) + \lambda^{S,j} \mu^{C,j+1}(\tau_{j+1})\right)}{\lambda^H + \lambda^{S,j}} \frac{e^{-c_j \tau_{j+1}} - e^{-c_j \tau_j + \left(\lambda^H + \lambda^{S,j} + \lambda^D\right)(\tau_j - \tau_{j+1})}}{\lambda^H + \lambda^{S,j} + \lambda^D - c_j}$$

E Numerical solution

E.1 Rollover

To solve the model with rollover use the following algorithm

1. Guess τ
   (a) Solve FIN^{COST}:
      i. Start with $J = 1$ and solve for $\text{FIN}^{\text{COST}}(\tau, J)$
      ii. For $J = J + 1$:
         A. Solve $\text{FIN}^{\text{COST}}(\tau, J)$. Initial guess $y = \frac{\tau}{J}$, + fminsearch
         B. Compute FIN^{COST}, if lower, iterate, otherwise, stop.
      iii. Compute valfun at τ, and fminsearch

E.2 GSIs

Let $\mu^{B,G} = \xi \mu^{B,P}$ so $\mu^B = \mu^{B,P}(1+\xi)$, $\frac{\mu^{B,P}}{\mu^B} = \frac{1}{1+\xi}$ and $\frac{\mu^{B,G}}{\mu^B} = \frac{\xi}{1+\xi}$. Algorithm:

1. Guess tax rate
2. Solve equilibrium:
   (a) Guess market tightness θ
   (b) Given θ, and ξ, solve for optimal maturity τ

(c) Given θ, ξ, and τ: Solve for measure of agents, coefficient of value functions, and value of entry for private agents

(d) Check free-entry condition, adjust θ and iterate

3. Compute the value of entry for government G^S

4. Compute gov Budget constraint, adjust taxes and iterate

E.3 Segmented markets

1. Guess $\{\theta^j\}_{j=1}^N$
2. Solve for measures and respective coefficients
3. Solve for value functions and respective coefficients
4. Check the free-entry conditions
5. Update $\{\theta^j\}_{j=1}^N$ and iterate until convergence.

E.4 Identification

F Data

F.1 Corporate debt Maturity across countries

The data for maturities in Figure 2 was shared by Cortina Lorente, Didier, and Schmukler (2016). The source is Thompson Reuters Security Data Corporation Platinum database and details about the data can be found in Cortina Lorente, Didier, and Schmukler (2016). Table **??** reports the list of countries along with each region, income classification and average maturity of corporate debt. For GDP we use GDP per capita relative to US for 2014 downloaded from the World Bank database (GDP per capita PPP, constant 2011 international \$, series NY.GDP.PCAP.PP.KD).

Region	Country	Maturity

Africa	Egypt	5.6
Africa	South Africa	6.3
Asia Pacific	Australia *	10.0
Asia Pacific	Bangladesh	52.5
Asia Pacific	China	5.7
Asia Pacific	Hong Kong *	6.1
Asia Pacific	India	8.5
Asia Pacific	Indonesia	6.0
Asia Pacific	Japan	7.8
Asia Pacific	Malaysia	8.8
Asia Pacific	New Zealand *	7.3
Asia Pacific	Pakistan	5.7
Asia Pacific	Philippines *	6.8
Asia Pacific	Singapore *	6.1
Asia Pacific	South Korea *	3.8
Asia Pacific	Taiwan *	5.9
Asia Pacific	Thailand	5.9
Asia Pacific	Vietnam	6.3
Canada USA	Canada *	12.3
Canada USA	United States *	11.9
Eastern Europe	Croatia *	2.5
Eastern Europe	Czech Republic *	5.7
Eastern Europe	Hungary *	2.0
Eastern Europe	Poland *	7.1
Eastern Europe	Romania	5.0
Eastern Europe	Russia	8.4
Eastern Europe	Turkey	2.5
Latin America	Argentina	4.9
Latin America	Bolivia	4.5
Latin America	Brazil	6.7
Latin America	Chile	15.7
Latin America	Colombia	8.0
Latin America	Costa Rica	6.8
Latin America	Ecuador	2.5
Latin America	El Salvador	2.4

Latin America	Mexico	6.5
Latin America	Panama	6.8
Latin America	Peru	6.1
Latin America	Venezuela	2.6
Middle East	Israel *	9.6
Middle East	Oman *	4.0
Middle East	Saudi Arabia *	17.0
Middle East	United Arab Emirates *	5.4
Western Europe	Austria *	7.7
Western Europe	Belgium *	9.7
Western Europe	Denmark *	5.2
Western Europe	Finland *	6.4
Western Europe	France *	8.9
Western Europe	Germany *	8.5
Western Europe	Greece *	3.8
Western Europe	Ireland-Rep *	7.0
Western Europe	Italy *	7.1
Western Europe	Luxembourg *	6.9
Western Europe	Netherlands *	8.9
Western Europe	Norway *	9.1
Western Europe	Portugal *	8.3
Western Europe	Spain *	7.1
Western Europe	Sweden *	4.9
Western Europe	Switzerland *	7.7
Western Europe	United Kingdom *	10.8

*Note: This table presents the list of countries that constitute the different regions, their classification by income level and the average maturity.Countries are classified as developed or developing based on the World Bank income level classification of 2012 * means the country is classified as developed.*
Source: Cortina Lorente, Didier, and Schmukler (2016).

F.2 OTC trading data

For the US we use data from Trade Reporting and Compliance Engine (TRACE) that provides transaction-level information of corporate bonds in secondary markets and has been widely used to study the liquidity of this market (e.g. Edwards, Harris, and Piwowar, 2007; Bao, Pan, and Wang, 2011, among others). I clean the data following Dick-Nielsen (2009) and merge with bond characteristics provided by Mergent Fixed Income Securities Database (FISD). We access to this database trough Wharton Research Data Services.

For Argentina, we use a novel data set borrowed from *Mercado Abierto Electronico* (MAE, the over-the-counter exchange in Argentina). We got access to transaction-level information of corporate bonds (*Obligaciones Negociables*) which provides similar information than the TRACE database.[16] We compare the liquidity of the US and Argentinean markets during the year 2012.[17]

F.3 Treasury data

For interest rates we use estimates from the US Treasury for the synthetic zero-coupon bonds yield curve. The daily treasury data was retrieved from `https://www.treasury.gov/resource-center/data-chart-center/interest-rates`. The corporate yield curve corresponds to the High quality market (bonds with rating above A) and is available at `https://www.treasury.gov/resource-center/economic-policy/corp-bond-yield/`. We define the treasury and corporate yield curve as the daily average for the year 2016.

[16] I thanks Argentina's Central Bank to provide access to the data. To the best of my knowledge, this is the first paper to use this database. Borensztein, Cowan, Eichengreen, and Panniza (2008) documents some aggregate facts about the corporate debt market in Argentina using an aggregate version of this data.

[17] Results are similar to different time periods.

Figure 16: **Identification.**