

Knowledge Diffusion, Trade and Innovation across Countries and Sectors

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Abstract

Sectors and countries are heterogeneous in their innovation, diffusion and production patterns. Standard models of trade and innovation do not model explicitly this heterogeneity. We develop and quantify a multi-country and multi-sectoral model of innovation, knowledge diffusion and trade and show that sectoral heterogeneity is important for welfare gains from openness. Using data on cross-country and cross-sector patent citations, input-output linkages, R&D intensity, and trade flows, we calibrate the model and perform several counterfactual experiments. Decreases in trade costs induce a reallocation of innovation towards sectors in which the country has a comparative advantage, as well as sectors with strong knowledge spillovers. This reallocation caused by sectoral heterogeneity has implications for the distribution of aggregate innovation and welfare. We find that, after a trade liberalization: (i) the cross-country distribution of welfare is more dispersed when we allow for cross-sectoral heterogeneity in production linkages, and (ii) the distribution shifts to the right when we allow for heterogeneity in R&D intensity and knowledge flows. Different from previous trade model of innovation, changes in trade costs have a non-negligible effect on both aggregate innovation and welfare when we introduce sectoral heterogeneity and knowledge spillovers.

Keywords: Technology Diffusion; R&D; Patent Citations; International Trade

JEL Classification: F12, O33, O41, O47

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1 Introduction

The world has increasingly become a highly interconnected network of countries and sectors which not only trade goods and services between each other, but at the same time, exchange ideas with one another. Recently, a growing strand of the trade literature has examined how the benefits of trade liberalization may spread across sectors, through production input-output linkages (Caliendo and Parro 2015). However, sectors are also linked along a different dimension—knowledge flows. Indeed, technological advances never happen in isolation (David 1990; Rosenberg 1982). Knowledge in one sector can be used to enhance innovation in another, and much alike the cross-sectoral production input-output linkages, the knowledge spillovers across sectors are far from uniform. Therefore, in a world with multiple sectors, when changes in trade costs alter the knowledge composition of the economy, the latter also conditions trade patterns and aggregate growth (as shown in the empirical research by Hausmann, Hwang, and Rodrik 2007; Hidalgo, Klinger, Barabási, and Hausmann 2007). Furthermore, although trade flows often serve as a vehicle for knowledge diffusion (Alvarez, Buera and Lucas, 2014), they do not necessarily follow the same pattern across sectors and countries. The literature so far has either treated these two as separate issues or has modeled them together as one channel (e.g. more trade necessarily implies more knowledge spillovers). Sectors are also heterogeneous in their R&D intensity. Accounting for these sources of heterogeneity across sectors is important to study the effects of openness on reallocation of R&D and production across sectors, and ultimately on the cross-country distribution of welfare across countries.

We start by documenting two empirical findings: (i) sectors and countries are highly heterogeneous in their strength of knowledge flows, and (ii) countries and sectors are heterogeneous in their R&D intensity and in the patterns of comparative advantage. In particular, we find that rich countries have a comparative advantage in those sectors that are more R&D intensity. Standard models of trade and innovation do not account for this heterogeneity in production and innovation, and do not model the effect of knowledge flows in shaping the patterns of comparative advantage and R&D intensity. In these models, changes in trade costs do not have a significant effect on innovation or welfare (Atkeson and Burstein 2010, Eaton and Kortum 1996 and Eaton and Kortum 1999)

To understand how trade plays a role in directing R&D and transferring ideas among multiple sectors, we need a structural framework of trade, innovation and knowledge spillovers with realistic features of intersectoral linkages both on the knowledge spillover dimension and on the production side. We develop a multi-sector and multi-country model of Ricardian trade in which technology evolves through a process of endogenous innovation and technology diffusion. The model is a multi-sector version of Eaton and Kortum 2002, as the one developed in Caliendo and Parro 2015. The production side of the model features input-output linkages, sectoral heterogeneity and trade in intermediate goods. This production structure delivers a gravity equation at the sector level that can be estimated to obtain both trade barriers and the level of technology of a sector-country pair. Both shape the comparative advantage of that sector-country (Levchenko and Zhang 2016). The level of technology reflects the stock of knowledge of the sector-country. Technology evolves

over time through two channels: (i) Innovators in each sector invest final output to come up with a new idea, which if successful can be used to produce an intermediate good. The innovation process is also affected by an externality: the larger the stock of knowledge of a sector-country, the larger the efficiency of innovation in that sector-country.; (ii) Ideas diffuse both across sectors and countries according to an exogenous process of diffusion. The novel feature of this model is that sectors and countries are connected not only through trade in intermediate goods but also through knowledge spillovers. These two channels interact in a way that allows us to explain differences in innovation across sectors and countries, and hence differences in productivity.

The model is solved in two stages. Given the probability distribution of firm's productivity together with trade barriers at the country-pair and sector level, we solve for a static competitive equilibrium for the world economy. The equilibrium is static in that we take as given the technology level that determines the patterns of trade. We then allow for the technology profile to evolve endogenously due to a process of innovation and diffusion. The second stage allows us to determine the characteristics of the innovation process that drives the endogenous evolution of comparative advantage. A similar approach has been used in Alvarez, Buera, and Lucas Jr 2008. Different from their paper, our diffusion channel produces a Frechet distribution of productivity, as in Eaton and Kortum 1999. Furthermore, Alvarez, Buera, and Lucas Jr 2008 abstract from innovation, which is a key channel in our model.

We calibrate the model to data on intersectoral patent citations, investment in R&D and international trade to match our novel empirical facts. Following Eaton and Kortum 2002, we derive a theoretical gravity equation, which we then estimate to uncover the trade cost parameters. The estimation procedure adds fixed effects which, together with data on wages, allow us to recover the technology parameters (Santacreu 2015 and Levchenko and Zhang 2016). We use cross-country intersectoral patent citations to discipline the direction and intensity in which knowledge in a particular sector is utilized in the innovation of other sectors. This allows us to directly uncover the intersectoral knowledge input-output relationships. Data on R&D intensity at the sector-country level together with and data on international patent citations allow us to calibrate the parameters that govern the evolution of technology.

We then perform a trade liberalization and examine its implications for the reallocation of R&D across sectors, and for welfare. In contrast to Eaton and Kortum 1996 and Eaton and Kortum 1999 , Buera and Oberfield 2016, and Atkeson and Burstein 2010, changes in trade costs in our model have an impact on R&D intensity at the country level. After a trade liberalization, R&D is reallocated toward sectors in which the country has a comparative advantage.¹ We find that this reallocation is more important in sectors with stronger knowledge spillovers.

Our quantitative framework has implications for welfare gains from trade. A trade liberalization strengthens a country's comparative advantage, and hence the static gains from trade. In addition,

¹A recent paper by Somale 2014 obtains the same findings through targeted research. Different from his model, however, we consider international technology diffusion as an additional source of technological progress. Furthermore, we provide a quantitative analysis of the effect of trade and technology diffusion on the reallocation of R&D across industries.

there are dynamic gains from trade due to higher R&D investment in those sectors in which the country has comparative advantage. Moreover, knowledge diffusion has two opposite effects on welfare. On the one hand, it enables faster productivity convergence and makes countries that are more similar, which dampens the static gains from trade. On the other hand, it provides strong dynamic gains, because countries can innovate with access to a larger foreign knowledge pool.

We find that, after a trade liberalization: (i) the cross-country distribution of welfare is more disperse when we allow for cross-sectoral heterogeneity in production linkages, and (ii) the distribution shifts to the right when we allow for heterogeneity in R&D intensity and knowledge flows. Different from previous trade model of innovation, changes in trade costs have a non-negligible effect on both aggregate innovation and welfare when we introduce sectoral heterogeneity and knowledge spillovers.

Despite of its complexity, the model comes with the benefit of tractability, as we build upon the Ricardian trade model of Eaton and Kortum 2002 with Bertrand Competition (Bernard, Eaton, Jensen, and Kortum 2003). The innovation and international technology diffusion processes are modeled in a similar fashion as in Eaton and Kortum 1996 and Eaton and Kortum 1999. The diffusion lags—backed by empirical observations—have an exponential distribution. All these features allow us to estimate the set of parameters based on observables in trade and citation data from steady-state relationships.

Related Literature Our paper merges and extends several strands of existing literature. The first is the literature on innovation, diffusion and international trade. Eaton and Kortum 1996 and Eaton and Kortum 1999 posit technological innovations and their international diffusion through trade as potential channels of embodied technological progress. Santacreu 2015 develops a model in which trade allows countries to adopt innovation developed abroad, and thus diffusion does not take place without trade. Our main departure from these previous works is that we allow knowledge diffusion and trade to operate separately, even though common economic forces may contribute to the development of both and diffusion and trade may benefit and reinforce each other. In addition, we extend these studies into multi-sector environment in which sectors interact both in the product space and in the technology space.

The second is the multi-sector trade literature which extends Eaton and Kortum 2002 trade model to multiple sectors (Chor 2010; Costinot, Donaldson, and Komunjer 2012). A recent growing body of research in this area also explores the trade and growth implications of interdependence across different sectors through intermediate input-output relationships (Eaton, Kortum, Neiman, and Romalis 2016, Caliendo and Parro 2015). Our paper differs in several dimensions. First, our focus is on innovation and knowledge diffusion. Second, besides the factor demand linkages, this paper also simultaneously consider the intrinsic interconnections of technologies embodied in different sectors, which turns out to be significant and relevant when studying innovation and diffusion. Related to the current work, Cai and Li 2016 study knowledge spillovers across sectors within a country and how trade costs affect the distribution of endogenous knowledge accumulation

across sectors. Different from our paper, however, cross-sector knowledge diffusion is not considered across countries and material input demand linkages across sectors are absent. Levchenko and Zhang 2016 provide evidence of relative productivity convergence across 72 countries over 5 decades: productivity grew systematically faster in initially relatively less productive sectors. These changes have had a significant impact on trade volumes and patterns, and a modest negative welfare impact.

Led by Hidalgo, Klinger, Barabási, and Hausmann 2007, several papers have shown that producing goods with strong synergy with each other can improve growth, as it is easier to adapt existing ideas and enter new sectors (e.g. Hausmann and Klinger 2007, Kali, Reyes, McGee, and Shirrell 2013, Hausmann and Klinger 2007). However, these studies mostly adopt the regression based approach which is hard to establish causality and to examine the general equilibrium implications of changing trade structure. Moreover, none of these studies consider at the same time the product complementarity along the intermediate input-output dimension.

The rest of the paper proceeds as follows. Section 3 presents the model. Section 4 describes the steady state and Section 5 presents the calibration and the counterfactual exercise. Finally section 7 concludes.

2 Motivating Facts

This section presents two empirical findings documenting the existence of the sector heterogeneity along the lines considered in the paper: (i) sectors and countries are highly heterogeneous in their strength of knowledge flows, and (ii) countries and sectors are heterogeneous in their R&D intensity and in the patterns of comparative advantage. In particular, we find that rich countries have a comparative advantage in those sectors that are more R&D intensity.

2.1 Cross-Country Cross-Sector Knowledge Diffusion

We start by documenting that sectors and countries are heterogeneous in the strength of knowledge spillovers, which we measure as the speed of diffusion between two country-sector pairs.

Building on Jaffe et al.’s (2000) finding that citations represent an indicator of knowledge spillovers albeit with some degree of noise, we use the U.S. Patent and Trade Office (USPTO) patent citation database (2016 edition) to trace knowledge flows across countries and sectors.² For each patent granted over the period of 1962-2006, the database details every patent that it cites. In the dataset, patents are organized by their technical features and each patent belongs to a technology field according to the International Patent Classification (IPC), which is then mapped into an ISIC-revision 3 industry sector.

To construct the diffusion speed between any country-sector pairs, we first calculate, for a given patent in sector k of country i , the citation lag (diffusion lag) of the *first* citation made from sector

²The updated NBER patent database is available at: <https://sites.google.com/site/patentdatapoint/Home>. It contains detailed patent and citation information, including the patent application year, grant year, the technological area to which it belongs, the nationality of patent inventors, the patent assignee, the citations made and received and by each patent, etc.

j in country n . We then calculate the inverse of the average citation lag of all first citations made from country n sector j to country i sector k over the thirty year period of 1976–2006, denoted by ε_{ni}^{jk} . Under the assumption that knowledge diffusion lag follows an exponential distribution—which is broadly consistent with the data— ε_{ni}^{jk} is the sample estimate for the rate parameter of the distribution, characterizing the diffusion speed. Figure 1 shows that the distribution diffusion speed is highly heterogeneous and skewed.

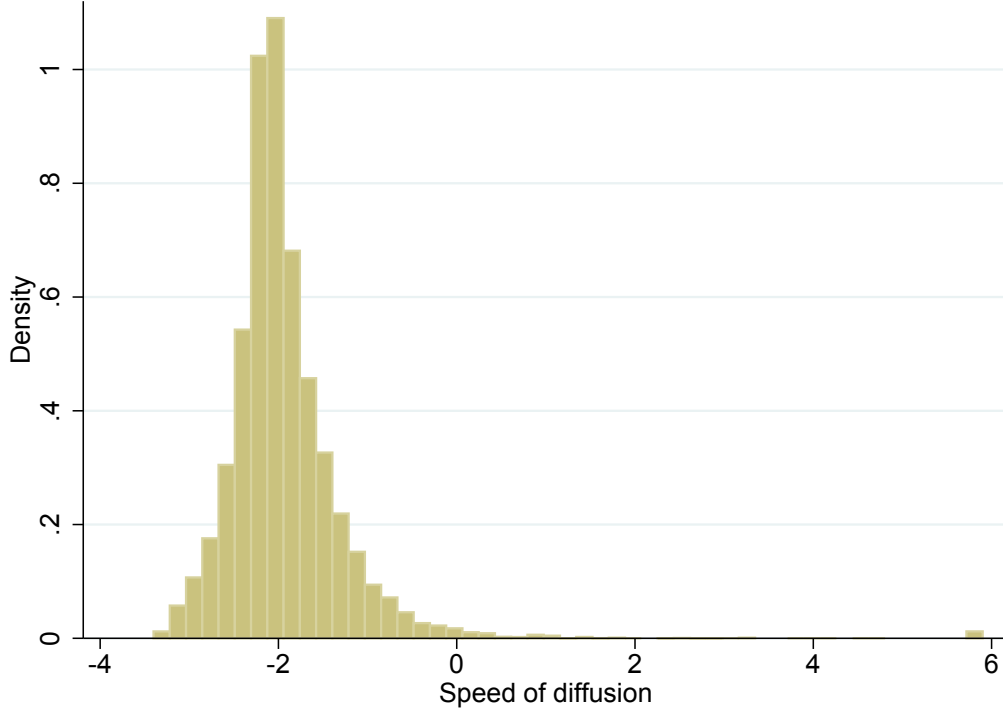


Figure 1: Speed of diffusion

The determinants of knowledge spillovers

Table 1 examines the determinants of cross-country-sector knowledge diffusion speed by estimating a gravity equation extended to include measures of linguistic and religious distance as well as common history variables that potentially affect effectiveness of interaction and communication, all obtained from CEPII. We also investigate whether trade plays any role in driving the diffusion speed once distances and historical variables are controlled for. Citing and cited country fixed effects are included to control for country-specific characteristics such as size, level of development, and geography. Since we are interested in not only cross-country but also cross-sector knowledge diffusion, we also include patent stock in citing and cited country-sector, and directional sector-pair fixed effect to capture the innate knowledge spillover relationship between different technologies (Cai and Li, 2016) that are independent of the source and destination countries.

Column (1) to (3) show that knowledge in sectors of country i diffuses faster to country n when

the two countries are geographically or linguistically closer to each other, they share a border, they both belong to the same regional free trade agreement (FTA), were ever in a colonial relationship but not after 1945, have shared a common colonizer, different in terms of latitude, or do not share the same currency. Somewhat surprisingly, trade linkages—that is exports between any country-sector pair combinations—are no longer play a significant role once the set of measures of geographic distance is included (while they are significantly positive when no gravity-factors are controlled for).

About half of the country-sector pairs (the nj - ik cells) have no observation of citations between one another, and thus no direct measure of diffusion speed is available. Three-quarters of these zero citation flows happen because either nj or ik has never filed for patents in the U.S. The other quarter appear when patents exist in both country-sector combinations but they do not cite one another or citation only exists in one direction. However, whether sector j of country cites patent from sector k in country i is not random. It is first related to the patent profile of nj and ik . For these that have patented in the U.S., it is also likely that underlying factors (such as cultural and geographic distance) affect whether country-sector pairs cite each other, which should account for much of the missing citation data. To handle the selection bias, we adopt the two-stage Heckman selection approach.

Column (5) reports the results from the first-stage Probit selection equation for the presence of a knowledge flow relationship using the same explanatory variables as the specification in Column (2) (with citing, cited country fixed effects, and citing-cited sector-pair fixed effects). The size of patent stock of both citing and cited country-sector pairs can be valid excluded variables for the second stage, as they indicated whether the country-sector has patented in the U.S. and hence affect the likelihood of whether they cite one another, but do not seem to affect the citation speed as demonstrated by Column (3). We also included the similarity of patent profiles between nj and ik in the second stage. The results show that most variables have the expected sign. In addition, the patent stock in both citing and cited country-sectors contribute positively to the likelihood of citation flows between them. Also interesting is the significant positive coefficients associated with trade flows in all directions. Trade increases the possibility of knowledge flow even though not its speed, and through both import and export.

2.2 R&D Allocation, Comparative Advantage and Income

The second empirical finding that we document is that rich countries have a comparative advantage in sectors that are more R&D intensive.

We obtain bilateral trade at the industry level and sector R&D data are from OECD STAN dataset. Based on sectoral exports, we calculate the standard Revealed Comparative Advantage (RCA) as $RCA_i^j = \frac{X_i^j}{\sum_j X_i^j} / (\frac{X_w^j}{\sum_j X_w^j})$, where X_i^j denotes the export in sector j by country i and by the world (comprising the 31 countries in our sample), respectively. We then define the R&D bias of a country's trade pattern by the cross-sector correlation between sectoral R&D intensity and the

Table 1: Determinants of Knowledge Diffusion Speed across Countries and Sectors

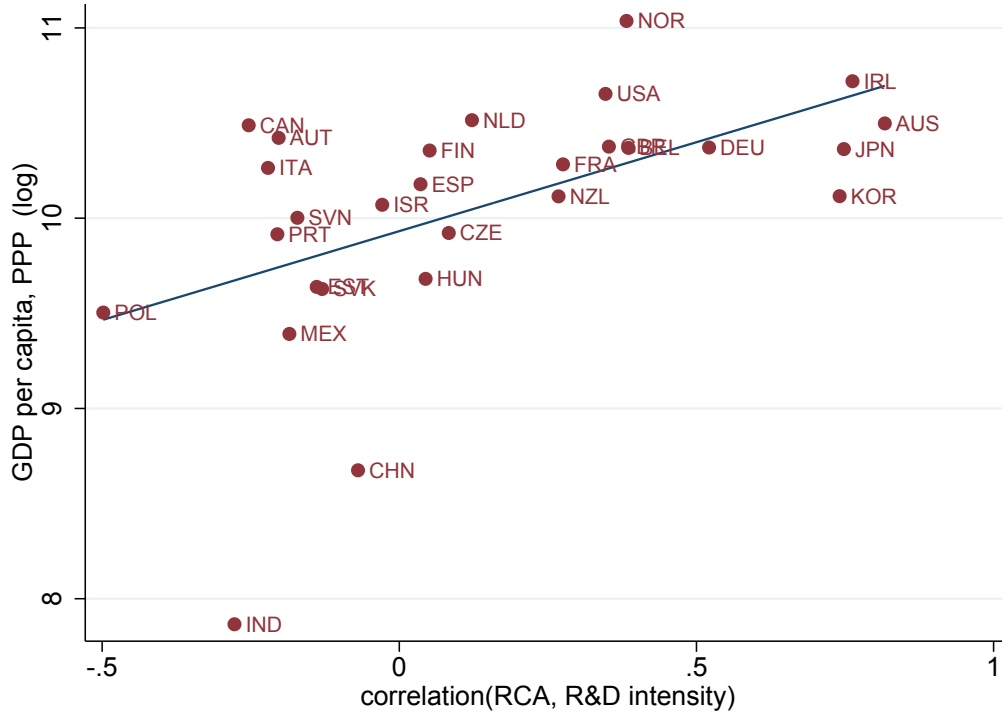
Dependent variable: $\log \varepsilon_{in}^{jk}$	OLS	OLS	OLS	Heckman	
	(1)	(2)	(3)	2nd Stage	Probit
	(1)	(2)	(3)	(4)	(5)
$\log(\text{Geographic Distance}_{ni})$	-0.047*** (-10.00)	-0.049*** (-10.04)	-0.048*** (-9.79)	-0.048*** (-9.82)	-0.061*** (-4.09)
Border_{ni}	0.023** (3.00)	0.023** (3.03)	0.023** (3.05)	0.023** (3.08)	0.218*** (8.48)
FTA_{ni}	0.034*** (4.13)	0.035*** (4.20)	0.035*** (4.24)	0.035*** (4.23)	0.032 (1.68)
$\text{Currency Union}_{ni}$	-0.029*** (-3.82)	-0.030*** (-3.84)	-0.028*** (-3.66)	-0.028*** (-3.63)	0.095*** (4.69)
$\text{Common language}_{ni}$	-0.044*** (-6.82)	-0.044*** (-6.73)	-0.043*** (-6.62)	-0.042*** (-6.59)	0.085*** (4.72)
landlocked_{ni}	-0.015 (-0.34)	-0.013 (-0.30)	-0.011 (-0.26)	-0.017 (-0.39)	-0.144** (-2.96)
$\text{absolute distance in latitude}_{ni}$	0.001*** (3.64)	0.001*** (3.63)	0.001*** (3.52)	0.001*** (3.51)	-0.001 (-1.95)
$\text{absolute distance in longitude}_{ni}$	0.001 (1.51)	0.001 (1.53)	0.001 (1.43)	0.001 (1.52)	0.002*** (11.40)
$\text{Linguistic Distance}_{ni}$	-0.087*** (-4.72)	-0.088*** (-4.74)	-0.088*** (-4.76)	-0.085*** (-4.59)	-0.840*** (-9.96)
$\text{Religious Distance}_{ni}$	0.039 (1.65)	0.038 (1.64)	0.041 (1.71)	0.037 (1.61)	-0.108* (-2.14)
Colony_{ni}	0.055*** (8.03)	0.056*** (8.11)	0.056*** (8.08)	0.054*** (7.93)	-0.087* (-2.49)
$\text{Common Colonizer}_{ni}$	0.089* (2.46)	0.090* (2.48)	0.090* (2.50)	0.090* (2.48)	0.03 (0.49)
$\text{Colony after 1945}_{ni}$	-0.146*** (-11.31)	-0.146*** (-11.33)	-0.148*** (-11.29)	-0.144*** (-11.22)	0.029 (-0.45)
$\log X_{ni}^j$		-0.001 (-1.30)	-0.001 (-1.28)	-0.001 (-1.13)	0.013*** (5.73)
$\log X_{ni}^j$		0.001 (0.65)	0.001 (0.55)	0.001 (0.81)	0.015*** (6.40)
$\log X_{in}^k$		0.000 (-0.24)	0.000 (-0.21)	0.000 (-0.05)	0.018*** (8.01)
$\log X_{in}^k$		-0.001 (-1.06)	-0.001 (-1.08)	-0.001 (-0.93)	0.010*** (4.54)
$\log \text{Patent Stock}_n^j$			0.001 (0.55)		0.018*** (7.58)
$\log \text{Patent Stock}_i^k$			0.000 (-0.28)		0.015*** (6.44)
Similiarity_{ni}			0.015 (0.84)		0.018 (0.50)
Citing Country FEs	Yes	Yes	Yes	Yes	Yes
Cited Country FEs	Yes	Yes	Yes	Yes	Yes
Sector-pair FEs	Yes	Yes	Yes	Yes	Yes
R^2	0.183	0.183	0.183		0.67
Num of obs	145,489	145,489	128,542	240,688	

RCA of the respective sectors:

$$\rho_i = \text{Corr}\left\{RCA_i^j, \frac{R\&D_i^j}{GDP_i}\right\}$$

Figure shows that countries with comparative advantage in R&D intensive sectors have higher income per capita. Although not reported here, this significantly positive relationship between the R&D bias of a country's trade pattern and income is largely unchanged, even after controlling for standard development accounting variables, such as human capital, and capital-output ratio and the natural resource rent.

Figure 2: Richer countries have comparative advantage in sectors with higher R&D intensity



3 The Model

We develop a general equilibrium model of trade in intermediate goods, with sector heterogeneity and input-output linkages, in which technology evolves endogenously through a process of innovation and cross-country cross-sector diffusion. The model builds upon the Ricardian trade model of Eaton and Kortum 2002 with Bertrand Competition (Bernard, Eaton, Jensen, and Kortum 2003). The innovation and international technology diffusion processes are modeled as Eaton and Kortum 1996 and Eaton and Kortum 1999.

There are N countries and J sectors. Countries are denoted by i and n and sectors are denoted by j and k . Labor is the only factor of production and we assume it to be mobile across sectors within a country but immobile across countries. In each country, there is a representative consumer

who consumes a non-traded final good and saves. A perfectly competitive final producer combines a composite output of all J sectors in the domestic economy with a Cobb-Douglas production function.

In each sector there is a producer of a composite good that operates under perfect competition and sells the good to the final producer and to intermediate producers from all sectors in that country. Intermediate producers use labor and composite goods to produce varieties that are traded and are used by the composite producer of that sector, either domestic or foreign. These firms operate under Bertrand competition and are heterogeneous in their productivity. Trade is Ricardian.

Finally, the technology of each sector-country pair evolves endogenously through a process of innovation and international and intersectoral technology diffusion. The innovation process follows the quality-ladders literature in that the new innovations increase the quality of the product in a given country-sector. Trade is balanced in every period.

3.1 Consumers

In each country there is a measure of L_n representative households who choose consumption optimally to maximize their life-time utility

$$U_{nt} = \sum_{t=0}^{\infty} \beta^t u(C_{nt}),$$

where $\beta \in (0, 1)$ is the stochastic discount factor, and C_{nt} represents consumption of country n at time t . The households own all the firms and finance R&D activities by the entrepreneurs.

3.2 Final production

Domestic final producers use a composite output from every domestic sector j in country n at time t , Y_{nt}^j , to produce a non-traded final output Y_{nt} according to the following Cobb-Douglas production function

$$Y_{nt} = \prod_{j=1}^J \left(Y_{nt}^j\right)^{\alpha_n^j}, \quad (1)$$

with α_n^j the share of sector production on total final output, and $\sum_{j=1}^J \alpha_n^j = 1$.

Final producers operate under perfect competition. Their profits are given by:

$$\Pi_{nt} = P_{nt}Y_{nt} - \sum_j P_{nt}^j Y_{nt}^j,$$

where P_{nt} is the price of the final produce, and P_{nt}^j is the price of the composite good produced in sector j from country n .

Under perfect competition, the price charged by the final producers to the consumers is equal

to their marginal cost, that is

$$P_{nt} = \prod_{j=1}^J \left(\frac{P_{nt}^j}{\alpha_n^j} \right)^{\alpha_n^j}.$$

The demand by final producers for the sector composite good is given by:

$$\alpha_n^j P_{nt} \frac{Y_{nt}}{Y_{nt}^j} = P_{nt}^j.$$

3.3 Intermediate producers

In each sector j there is a continuum of intermediate producers indexed by $\omega \in [0, 1]$ that use labor, $l_{nt}^j(\omega)$, and a composite intermediate good from every other sector k in the country, $m_{nt}^{jk}(\omega)$ to produce a variety ω according to the following constant returns to scale technology³

$$q_{nt}^j(\omega) = z_n^j(\omega) [l_{nt}^j(\omega)]^{\gamma_n^j} \prod_{k=1}^J [m_{nt}^{jk}(\omega)]^{\gamma_n^{jk}}, \quad (2)$$

with $\gamma_n^j + \sum_{k=1}^J \gamma_n^{jk} = 1$. Here γ_n^{jk} is the share of materials from sector k used in the production of intermediate ω is sector j , and γ_n^j is the share of value added. Firms are heterogeneous in their productivity $z_n^j(\omega)$.

The cost of producing each intermediate good ω is

$$c_{nt}^j(\omega) = \frac{c_{nt}^j}{z_n^j(\omega)},$$

where c_n^j denotes the cost of input bundle. In particular, given constant returns to scale:

$$c_n^j = \mathcal{R}_n^j W_{nt}^{\gamma_n^j} \prod_{k=1}^J (P_n^k)^{\gamma_n^{jk}},$$

with $\mathcal{R}_n^j = \prod_{k=1}^J (\gamma_n^{jk})^{-\gamma_n^{jk}} (\gamma_n^j)^{-\gamma_n^j}$ and W_{nt} is the nominal wage rate. Intermediate producers operate under Bertrand competition.

3.4 Composite intermediate goods (Materials)

Each sector j produces a composite good combining domestic and foreign varieties from that sector. Composite producers operate under perfect competition and buy intermediate products ω from the minimum cost supplier.

The production for a composite good in sector j in country n is given by the CES function, as

³The notation in the paper is such that every time there are two subscripts or two superscripts, the right one corresponds to the source country and the left one corresponds to the destination country.

in Ethier 1982,

$$Q_{nt}^j = \left(\int r_{nt}^j(\omega)^{1-1/\sigma^j} d\omega \right)^{\sigma^j/(\sigma^j-1)}, \quad (3)$$

where $\sigma^j > 0$ is the elasticity of substitution across intermediate good from sector j , and $r_{nt}^j(\omega)$ is the demand of intermediate goods from the lowest cost supplier in sector j .

The demand for each intermediate good ω is given by

$$r_{nt}^j(\omega) = \left(\frac{p_{nt}^j(\omega)}{P_{nt}^j} \right)^{-\sigma^j} Q_{nt}^j,$$

where

$$P_{nt}^j = \left(\int p_{nt}^j(\omega)^{1-\sigma^j} d\omega \right)^{\frac{1}{1-\sigma^j}}, \quad (4)$$

The sector composite producer uses varieties from its own sector, but only from the lower cost producer, since there is perfect competition.

Composite intermediate goods are used as materials for the production of the intermediate goods and final goods in the final production.

$$Q_{nt}^j = Y_{nt}^j + \sum_{k=1}^J \int m_{nt}^{kj}(\omega) d\omega.$$

3.5 International trade

We follow Bernard, Eaton, Jensen, and Kortum 2003 and assume Bertrand competition. Trade in goods is costly. In particular, there are iceberg transport costs from shipping a good in sector j from country i to country n , d_{ni}^j . The p 'th most efficient producer of variety ω from sector j in country i can deliver a unit of good to country n at the cost:

$$c_{pni}^j(\omega) = d_{ni}^j \frac{c_i^j}{z_{pi}(\omega)},$$

With Bertrand competition, as with perfect competition, composite producers in each sector and country buy from the lowest cost supplier. The cost of a good ω in country n is given by

$$c_{1n}^j(\omega) = \min_i \left\{ c_{1ni}^j(\omega) \right\},$$

In addition, Bertrand competition implies that the price charged by the producer will be the production cost of the second lowest producer

$$c_{2n}^j(\omega) = \min \left\{ c_{2ni^*}^j(\omega), \min_{i \neq i^*} \{ c_{1ni}^j(\omega) \} \right\},$$

where i^* satisfies $c_{1ni^*}^j(\omega) = c_{1n}^j(\omega)$. The low cost supplier will not want to charge a mark-up above

$\bar{m}^j = \sigma^j / (\sigma^j - 1)$. Hence,

$$p_n^j(\omega) = \min \left\{ c_{2n}^j(\omega), \bar{m}^j c_{1n}^j(\omega) \right\}.$$

Ricardian motives for trade are introduced as in Eaton and Kortum 2002, since productivity is allowed to vary by sector and country. The productivity of producing intermediate good ω in country i and sector j is drawn from a Frechet distribution with parameter T_i^j and shape parameter θ . A higher T_i^j implies a higher average productivity of that sector-country pair, while a lower θ implies more dispersion of productivity across varieties.

$$F(z_i^j) = \Pr \left[Z \leq z_i^j \right] = e^{-T_i^j z_i^{-\theta}},$$

and,

$$\Pr \left[p_{ni,t}^j < p \right] = 1 - e^{-T_{it}^j (d_{ni}^j c_{it}^j / p)^{-\theta}}.$$

Without loss of generality, we make the following assumption:

$$T_i^j = A_i^j T_{p,i}^j. \quad (5)$$

where A_i^j can be interpreted as a measure of the quality of ideas in country i and sector j —or the component of country i and sector j productivity that is explained by innovation and diffusion—and $T_{p,i}^j$ can be interpreted as the productivity of production. In the next sections, we determine how the productivity of innovation evolves over time.⁴ Throughout the paper, we assume that $T_{p,i}^j$ remains constant over time, but A_i^j evolves through innovation and diffusion.

Because each sector j in country n buys goods from the second cheapest supplier, the cost of good ω in sector j and country n is $p_{nt}^j(\omega) = \min \left\{ p_{nit}^j(\omega) \right\}$. Then, $c_{nt}^j(\omega)$ are realizations from G_n

$$G_n^j(p) = 1 - \prod_{i=1}^N \left(\Pr \left[p_{nit}^j > p \right] \right) = 1 - \prod_{i=1}^N e^{-T_{it}^j (d_{ni}^j c_{it}^j / p)^{-\theta}} = 1 - e^{-\Phi_{nt}^j p}$$

with $\Phi_{nt}^j = \sum_{i=1}^N T_{it}^j (d_{ni}^j c_{it}^j)^{-\theta}$ each country n and sector j accumulated technology. From here, we can obtain the distribution of prices of goods in sector j in country n as

$$P_{nt}^j = B^j \left(\Phi_{nt}^j \right)^{-1/\theta}, \quad (6)$$

with $B^j = \left[\frac{1+\theta-\sigma^j+(\sigma^j-1)(\bar{m}^j)^{-\theta}}{1+\theta-\sigma^j} \Gamma \left(\frac{2\theta+1-\sigma^j}{\theta} \right) \right]^{1/(1-\sigma^j)}$ and assuming $\sigma^j < (1 + \theta)$.

⁴This formulation is similar to the one introduced in Arkolakis, Ramondo, Rodríguez-Clare, and Yeaple 2013.

3.6 Expenditure shares

The probability that country i is the low cost supplier of a good in sector j that is to be exported to sector j in country n is

$$\pi_{nit}^j = \frac{T_{it}^j \left(c_{it}^j d_{ni}^j \right)^{-\theta}}{\Phi_{nt}^j}, \quad (7)$$

with $\Phi_{nt}^j = \sum_{i=1}^M T_{it}^j \left(c_{it}^j d_{ni}^j \right)^{-\theta}$. This is the fraction of goods that country n buy from country i in sector j . That probability is also the fraction of goods that sector j in country i sells to any sector in country n . In particular, the share country n spends from country i in sector j is

$$\pi_{nit}^j = \frac{X_{nit}^j}{X_{nt}^j}. \quad (8)$$

Therefore,

$$X_{nit}^j = \pi_{nit}^j X_{nt}^j,$$

with X_{nit}^j the expenditures of country n from sector j in country i and X_{nt}^j the total expenditures of country n in sector j . Substituting the expression for π_{nit}^j we have

$$X_{nit}^j = \frac{T_{it}^j \left(c_{it}^j d_{ni}^j \right)^{-\theta}}{\Phi_{nt}^j} X_{nt}^j$$

3.7 Endogenous growth: Innovation and international technology diffusion

We model the innovation process within each industry j as in Kortum 1997. Innovation follows the quality-ladders literature, in that a blueprint (i.e., an idea) is needed to produce an intermediate good. Ideas are developed with effort and they increase the efficiency of production of an intermediate good. In each sector j and country n , there are entrepreneurs that invest final output to come up with an idea. Within each sector, research efforts are targeted at any of the continuum of intermediate goods. In each country n and sector j , ideas are drawn at the Poisson rate λ_{nt}^j . That is, if a fraction of final output s_{nt}^j is invested into R&D by the entrepreneur, then ideas are created at the rate

$$\lambda_{nt}^j \left(s_{nt}^j \right)^{\beta_r}$$

with $\lambda_{nt}^j = \lambda_n^j A_{nt}^j$ and λ_n^j a scaling parameter that captures how productive are researchers in in sector j of country n in doing innovation, and $\beta_r \in (0, 1)$ a parameter of diminishing returns to investing into R&D. This process is microfounded in Eaton and Kortum 1996 and Eaton and Kortum 1999 and it ensures that there is a balanced growth path without scale effects.

Note that the productivity of innovation varies across sectors and countries. Innovators belong to a particular sector j but, within each sector, their research effort is targeted at any of the goods

in the continuum.

Ideas from sector j and country n may become an intermediate product in that sector/country. The efficiency $q^j(\omega)$ with which it enables good ω to be produced in sector j is drawn from the Pareto distribution $H(q) = 1 - q^{-\theta}$. An idea applies to only one good in the continuum. The good ω to which it is associated is drawn from the uniform distribution $[0, 1]$;

In equilibrium, only the best idea for each input in each country and sector it is actually used to produce an intermediate good in any sector and/or country. The efficient technology $z_n^j(\omega)$ for producing good ω in country n is the best idea for producing it yet discovered. A new idea is never adopted unless it surpasses the current state of the art $z_n^j(\omega)$. Following Eaton and Kortum 2006, the best technologies available in a country are realizations of a random variable that has a Frechet distribution:

$$F_n^j(z) = \exp[-A_n^j z^{-\theta}]$$

That is, the quality distribution of successful ideas inherit the distribution of productivity of the intermediate goods produced in a country. We elaborate more on this point later, when we introduce the incentives of an innovation.

Once an idea has arrived in sector j and country n there is no forgetting. New ideas created in each sector j and country n increase its average productivity, A_n^j . Ideas may also diffuse and exogenously to other sectors and/or countries. If an idea is discovered at time t in country i and sector k , then it diffuses to country n and sector j at time $t + \tau_{ni}^{jk}$. We assume that the diffusion lag τ_{ni}^{jk} has an exponential distribution with parameter ε_{ni}^{jk} (this is the speed of diffusion), so that $Pr[\tau_{ni}^{jk} \leq x] = 1 - e^{-\varepsilon_{ni}^{jk}x}$.

Through diffusion, technology in a country/sector is composed by the technologies developed in all sectors and countries. That is, $A_n^j = \sum_i \sum_k A_{ni}^{jk}$. Therefore, the flow of ideas diffusing to country n and sector j is given by the accumulation of the past research effort of each sector k in country i that has already been diffused, according to

$$\dot{A}_{nt}^j = \sum_{i=1}^N \sum_{k=1}^J \varepsilon_{ni}^{jk} \int_{-\infty}^t e^{-\varepsilon_{ni}^{jk}(t-s)} \lambda_{is}^k \left(s_{is}^k\right)^{\beta_r} ds, \quad (9)$$

with $\lambda_{is}^k = \lambda_i^k A_{is}^k$. If $\varepsilon_{ni}^{jk} \rightarrow \infty$, then there is instantaneous diffusion. If $\varepsilon_{ni}^{jk} \rightarrow 0$, then there is no diffusion. The growth of the stock of knowledge in a particular sector j and country n at time t depends on the past research effort that has been done by each other sector k in country i up to time t , and that has diffused at the rate ε_{ni}^{jk} .

3.7.1 The incentives to innovate

There is free entry into innovation. Entrepreneurs finance R&D issuing equity claims to the households. These claims pay nothing if the entrepreneur is not successful in introducing a new technology in the market, and it pays the stream of future profits from selling the good in a particular sector

either domestically or abroad, if the innovation succeeds. The price for a research success by an entrepreneur in a particular sector is the expected flow of profits that will last until a new success or a foreign producer may produce the good at a lower cost. Because of the probabilistic distribution of productivity, entrepreneurs will be indifferent on what product ω to devote its efforts, since in expectation, all products with a sector deliver the same expected profit. As in the quality ladders literature, we focus on a situation in which all products within an industry are targeted with the same intensity. Following the quality-ladders literature, a new idea will interact with the set of existing technologies in a particular sector and country if $Q > Z_n^j$, which occurs with probability

$$Pr[Q > Z_n^j] = \int_0^\infty Pr[Q > z] dF_n^j(z) = 1/A_n^j$$

This introduces a competitive effect, by which the larger the number of existing technologies in a sector/country, the lower the probability that the new idea lowers the cost there.

Then, the distribution of Q conditional on $Q > Z_n^j$ is

$$Pr[Q \leq q | Q > Z_n^j] = e^{-A_n^j q^{-\theta}}$$

Therefore, conditional on joining the set of best technologies, the quality of a new idea has the same distribution of the quality of existing technologies.

A local innovator will lower cost in sector j of country n if:

$$c_n^j(\omega)/q \leq \min_i \left\{ c_i^j(\omega) d_{ni}^j / z_i^j(\omega) \right\}$$

Therefore, using the results of the international trade section, the profits of an innovator in sector j in country n are

$$\Pi_{nt}^j = \sum_{i=1}^M \frac{\pi_{int}^j X_{it}^j}{(1 + \theta) A_{nt}^j}$$

Rearranging, we obtain

$$\Pi_{nt}^j = \frac{1}{(1 + \theta) A_{nt}^j} \sum_{i=1}^M \pi_{int}^j X_{it}^j, \quad (10)$$

The value of an idea that has been developed in country n and sector j is the expected present discounted value of the stream of future profits

$$V_{nt}^j = \int_t^\infty \left(\frac{P_{nt}^j}{P_{ns}^j} \right) e^{-\rho(s-t)} \Pi_{ns}^j ds, \quad (11)$$

Note that the incentive to innovate depends on the value of an innovation, which depends on: (i) the probability of the new technology lowering the cost of production there, $\frac{1}{A_n^j}$, and (iii) the

expected profits from selling the good to each potential country-sector, $\frac{X_{in}^j}{1+\theta}$.

The first order condition for innovation is:

$$\beta_r \lambda_{nt}^j V_{nt}^j (s_{nt}^j)^{\beta_r - 1} = P_{nt} Y_{nt}, \quad (12)$$

Therefore, the optimal R&D investment is a positive function of the value of an innovation, V_n^j and the productivity of innovation λ_n^j .⁵

3.8 Trade Balance

WE assume that trade is balanced every period. Total imports in country n are given by:

$$\sum_{i=1}^M \sum_{k=1}^J X_{nit}^k = \sum_{i=1}^M \sum_{k=1}^J \pi_{nit}^k X_{nt}^k = \sum_{k=1}^J X_{nt}^k \sum_{i=1}^M \pi_{nit}^k, \quad (13)$$

Then,

$$IM_{nt} = \sum_{k=1}^J X_{nt}^k \sum_{i=1}^M \pi_{nit}^k$$

Total exports in country n are given by:

$$EX_{nt} = \sum_{i=1}^M \sum_{k=1}^J X_{int}^k = \sum_{i=1}^M \sum_{k=1}^J \pi_{int}^k X_{it}^k$$

Trade balance implies

$$EX_{nt} = IM_{nt}$$

4 Endogenous growth along the balanced growth path

In our model, all countries grow at the same rate along the unique balanced growth path (BGP). Furthermore, because of diffusion, the level of technology A_n^j , and hence T_n^j , grow at the same rate g_A across countries and sectors. In what follows, we rewrite the equations of the model after normalizing the endogenous variables so that they grow at a zero rate along the BGP. Most variables are normalized by $W_M (T_M^J)^{1/\theta}$. When they are normalized by something else, we say it explicitly in the text. Note that variable grow at the same rate in steady-state in each country, but there are differences in growth rates across sectors that are driven by the sector specific parameters γ^{jk} , γ^j , λ_n^j , ε_{ni}^{jk} , and α^j .

⁵The optimization problem of the innovator is as follows. Innovators choose the amount of final output to be allocated into R&D. In our model, s_n^j is the fraction of final output that is spent into R&D activity. Therefore, innovators choose $S_n^j = s_n^j Y_n$ to maximize

$$\dot{A}_n^j V_n^j - P_n S_n^j$$

subject to equation (9).

(1) Probability of imports

$$\pi_{ni}^j = \hat{T}_i^j \frac{\left(\hat{c}_i^j d_{ni}^j\right)^{-\theta}}{\hat{\Phi}_n^j}, \quad (14)$$

where $\hat{T}_n^j = \frac{T_n^j}{T_M^j}$ and $\hat{\Phi}_n^j = \frac{1}{T_M^j} \frac{\Phi_n^j}{\left(W_M \prod_{k=1}^J (T_M^J)^{\Lambda_{jk}}\right)^{-\theta}}$

(2) Import shares

$$\hat{X}_{ni}^j = \pi_{ni}^j \hat{X}_n^j, \quad (15)$$

(3) Cost of production

$$\hat{c}_n^j = \mathcal{I}_n^j \hat{W}_n^j \prod_{k=1}^J (\hat{P}_n^k)^{\gamma_n^{jk}}, \quad (16)$$

(4) Intermediate good prices in each sector

$$\hat{P}_n^j = B^j \left(\hat{\Phi}_n^j\right)^{-1/\theta}, \quad (17)$$

(5) Cost distribution

$$\hat{\Phi}_n^j = \sum_{i=1}^M \hat{T}_i^j \left(d_{ni}^j \hat{c}_i^j\right)^{-\theta}, \quad (18)$$

(6) Price index

$$\hat{P}_n = \prod_{j=1}^J \left(\frac{\hat{P}_n^j}{\alpha_n^j}\right)^{\alpha_n^j}, \quad (19)$$

(7) Labor market clearing condition

$$\hat{W}_n L_n = \sum_{i=1}^J \gamma_n^j \sum_{i=1}^M \pi_{in}^j \hat{X}_i^j, \quad (20)$$

(8) Sector production

$$\hat{X}_n^j = \sum_{k=1}^J \gamma_n^{kj} \sum_{i=1}^M \pi_{in}^k \hat{X}_i^k + \alpha_n^j \hat{Y}_n, \quad (21)$$

where $\hat{Y}_n = \frac{P_n Y_n}{W_M}$.

(9) Final production

$$\hat{Y}_n = \hat{W}_n L_n + \sum_{j=1}^J \sum_{i=1}^M \pi_{in}^j \hat{X}_i^j, \quad (22)$$

(10) Resource constraint

$$\hat{Y}_n = \hat{C}_n + \sum_{k=1}^J s_n^k \hat{Y}_n, \quad (23)$$

(11) R&D expenditures

$$\beta_r \lambda_n^j \hat{V}_n^j (s_n^j)^{\beta_r - 1} = \hat{Y}_n, \quad (24)$$

(12) Value of an innovation

$$\hat{V}_{nt}^j = \int_t^\infty \left(\frac{P_{nt}^j}{P_{ns}^j} \right) e^{-\rho(s-t)} \hat{\Pi}_{ns}^j ds, \quad (25)$$

4.1 The mechanism

The model generates endogenously differences in innovation and income per capita across sectors and countries. Technology evolves endogenously through innovation and international technology diffusion. Changes in technology have an effect on the pattern of trade in the country, which changes the incentives to innovate through the effect on expected future profits. Therefore, countries and sectors in which technology diffusion is faster or in which international trade costs are lower have higher incentives to innovate, hence higher productivity and income per capita. The static trade model interacts with the dynamic part to identify the stock of knowledge, wages and growth rates. Heterogeneity in the production side at the country and sector level together with international and intersectoral heterogeneity in the knowledge linkages drives heterogeneity in the stock of knowledge in steady-state. This translates into variation in income per capita across countries.

To understand how knowledge diffusion and international trade can have an effect on the reallocation of R&D across sectors, we now derive the steady-state of the model, in which all variables grow at a constant rate. International and intersectoral diffusion guarantees that A_n^j growth at a common rate across sectors and countries. Denote the common growth rate as $\frac{\dot{A}_n^j}{A_n^j} = g_T$. Note that because of our assumption 5, and since we assume that $T_{p,n}^j$ is constant over time, T_n^j in the Frechet distribution also grows at rate g_T . From the resource constraint in equation (23), the fraction of final output that is invested into R&D, s_n^j , is constant in steady-state. This result and the expression for the value of an innovation implies that, in steady-state

$$\hat{V}_n^j = \frac{\hat{\Pi}_n^j}{\rho + g_T}$$

since by the definition of profits in steady-state,

$$\hat{\Pi}_n^j = \frac{\sum_{i=1}^M \pi_{in}^j \hat{X}_i^j}{\hat{Y}_n} \frac{\hat{Y}_n}{(1 + \theta) \hat{A}_n^j}$$

with $\hat{\Pi}_n^j = \frac{\Pi_n^j A_M^J}{W_M}$ and $\hat{Y}_n = \frac{P_n Y_n}{W_M}$. Note that trade has a positive effect on the value of an innovation

because now the innovator can access a larger market.⁶

We can now use the expression for the value of an innovation together with the optimal investment into innovation to obtain an expression for R&D intensity in steady-state:

$$\hat{V}_n^j = \frac{1}{(1+\theta)\hat{A}_n^j} \frac{\sum_{i=1}^M \pi_{in}^j \hat{X}_i^j}{\hat{Y}_n} \frac{1}{\rho + g_T}$$

$$\beta_r \lambda_n^j \frac{1}{(1+\theta)} \frac{1}{\rho + g_T} \frac{\sum_{i=1}^M \pi_{in}^j \hat{X}_i^j}{\hat{Y}_n} = s_n^{j 1-\beta_r}$$

Then,

$$s_n^j = \left(\beta_r \lambda_n^j \frac{1}{(1+\theta)} \frac{1}{\rho + g_T} \frac{\sum_{i=1}^M \pi_{in}^j \hat{X}_i^j}{\hat{Y}_n} \right)^{\frac{1}{1-\beta_r}}$$

Trade affects optimal investment into R&D at the sector level to the extent that it affects the reallocation of production into particular sectors. This result differs from previous papers in the literature that find that trade has no impact on R&D intensity. In our paper, R&D reallocates towards sectors in which the country has comparative advantage, through $\hat{Y}_n^j$.

Substituting into the growth rate of technologies of new technologies in equation (53)

$$g_T = \sum_{i=1}^N \sum_{k=1}^J \frac{\varepsilon_{ni}^{jk}}{g_T + \varepsilon_{ni}^{jk}} \lambda_i^k \frac{\hat{A}_i^k}{\hat{A}_n^j} \left(s_i^k \right)^{\beta_r}$$

$$1 = \sum_{i=1}^N \sum_{k=1}^J \frac{\varepsilon_{ni}^{jk}/g_T}{g_T + \varepsilon_{ni}^{jk}} \lambda_i^k \frac{\hat{A}_i^k}{\hat{A}_n^j} \left(\frac{1}{\rho + g_T} \beta_r \lambda_i^k \frac{1}{(1+\theta)} \frac{\sum_{n=1}^M \pi_{ni}^k \hat{X}_n^k}{\hat{Y}_n} \right)^{\frac{\beta_r}{1-\beta_r}}$$

Rearranging, we obtain an expression for the growth rate of the stock of knowledge in steady state,

$$g_T \hat{A}_n^j = \sum_{i=1}^N \sum_{k=1}^J \frac{\varepsilon_{ni}^{jk}}{g_T + \varepsilon_{ni}^{jk}} (\lambda_i^k)^{\frac{1}{1-\beta_r}} \hat{A}_i^k \left(\frac{1}{\rho + g_T} \beta_r \frac{1}{(1+\theta)} \frac{\sum_{n=1}^M \pi_{ni}^k \hat{X}_n^k}{\hat{Y}_n} \right)^{\frac{\beta_r}{1-\beta_r}} \quad (26)$$

The steady-state growth rate of the stock of knowledge depends positively on the speed of diffusion, the expected profits (note that it depends on trade costs through their effect on trade shares in the equation for profits) and negatively on the dispersion parameter. Following Eaton and Kortum 1999, the Frobenius theorem guarantees that there is a unique balanced growth path in which all countries and sectors grow at the same rate g_T . The expression for the growth rate can be expressed in matrix form as:

$$g_T A = \Delta(g_T) A$$

⁶Note that V_n^j and Π_n^j are normalized by $P_n Y_n$.

If the matrix $\Delta(g_T)$ is definite positive, then there exists a unique positive balanced growth rate of technology $g_T > 0$ given research intensities. Associated with that growth rate is a vector A (defined up to a scalar multiple), with every element positive, which reflects each country/sector relative level of knowledge along that balanced growth path.

5 Welfare Analysis: Gains from Trade

Welfare in our model is determined by the real wage. We can obtain an expression for the real wage in country i as

$$\frac{W_i}{P_i} \propto \prod_{j=1}^M \left(\frac{W_i}{P_i^j} \right)^{\alpha_i^j}$$

Using the first order conditions for prices and import shares, it can be shown that

$$\frac{W_i}{P_i^j} = \left(\frac{T_i^j}{\pi_{ii}^j} \right)^{1/\theta} \frac{W_i}{c_i^j} \propto \left(\frac{T_i^j}{\pi_{ii}^j} \right)^{1/\theta} \prod_{k=1}^j \left(\frac{W_i}{P_i^k} \right)^{\gamma_i^{jk}}$$

Therefore,

$$\frac{W_i}{P_i} \propto \prod_{j=1}^M \left(\left(\frac{T_i^j}{\pi_{ii}^j} \right)^{\alpha_i^j/\theta} \prod_{k=1}^j \left(\frac{W_i}{P_i^k} \right)^{\alpha_i^j \gamma_i^{jk}} \right) \quad (27)$$

Note that this formula resembles the standard welfare formula in Arkolakis, Costinot, and Rodríguez-Clare 2012. In a one sector version of our model, in which $j = 1$ and, $\gamma_i^{jk}=0$, $\alpha_i^j = 1$, equation 27 becomes

$$\frac{W_i}{P_i} \propto \left(\frac{T_i}{\pi_{ii}} \right)^{1/\theta} \quad (28)$$

This is the standard formula for welfare gains from trade that has been used in the literature and it depends on the aggregate productivity, the home trade shares and the trade elasticity.

Our formula for welfare in equation 27 is dynamic. Dynamics are driven by the evolution of the stock of ideas captures in T_i^j . In this sense, our formula is the multi-sector version of the one derived in Buera and Oberfield 2016.

6 Quantitative Analysis

We now perform a comparative statics exercise to shed light on the role of all the sources of heterogeneity of our model in explaining differences in R&D across countries and for welfare. In particular, we study the role of three sources of heterogeneity: (i) production linkages, (ii) efficiency of doing innovation, and (iii) knowledge spillovers. We first introduce a trade reform that consists

on a uniform reduction of trade barriers of 55%. We compare steady-states and analyze how R&D and comparative advantage reallocates across sectors, depending on the strength of knowledge spillovers. This reallocation may induce changes in the cross-country distribution of R&D and welfare gains from trade. We then analyze in more detail the role of each source of heterogeneity by performing the same uniform trade liberalization in a one sector model in which we shut down all sectoral heterogeneity, and two additional models in which we shut down heterogeneity across sectors one at a time: (i) a model with the same diffusion parameters across sectors and countries, that is $\varepsilon_{in}^{jk} = \varepsilon$, (ii) a model with the same intersectoral linkages in production across sectors, (iii) a model with no intersectoral linkages in production. We find that, in a model with royalties, changes in trade costs have an effect on R&D even in a one sector economy. What we find is that some countries increase their R&D whereas other countries decrease it after a trade liberalization. If we eliminate royalties, a trade liberalization has no effect on R&D at the country level.

6.1 Calibration

In Appendix C we describe the calibration approach that we follow to recover all the parameters of interest. Here we explain in more detail the calibration of the technology parameters, T_i^j , the diffusion parameters, ε_{in}^{jk} and the parameters that affect the innovation process, that is: (i) the elasticity of innovation, β_r and (ii) the productivity of innovation, λ_i^j .

6.1.1 Estimation of T_i^j : Gravity equation at the sector level

Following the methodology in Levchenko and Zhang 2016. we run gravity equations at the sector level to estimate the average productivity T_i^j .

We start from the trade shares in (8):

$$\pi_{ni}^j = \frac{X_{ni}^j}{X_n^j} = \frac{T_i^j \left(c_i^j d_{ni}^j \right)^{-\theta}}{\Phi_n^j}. \quad (29)$$

where $\Phi_n^j = \sum_{i=1}^N T_i^j \left(c_i^j d_{ni}^j \right)^{-\theta}$. Dividing the trade shares by their domestic counterpart and assuming $d_{nn}^j = 1$ generate

$$\frac{\pi_{ni}^j}{\pi_{nn}^j} = \frac{X_{ni}^j}{X_{nn}^j} = \frac{T_i^j \left(c_i^j d_{ni}^j \right)^{-\theta}}{T_n^j \left(c_n^j \right)^{-\theta}}. \quad (30)$$

Taking logs of both hand sides, we have

$$\log \left(\frac{X_{ni}^j}{X_{nn}^j} \right) = \log \left(T_i^j \left(c_i^j \right)^{-\theta} \right) - \log \left(T_n^j \left(c_n^j \right)^{-\theta} \right) - \theta \log(d_{ni}^j). \quad (31)$$

The log of the trade frictions can be expressed as

$$\log(d_{ni}^j) = \beta_0^j D_{ni}^k + \beta_1^j B_{ni} + \beta_2^j CU_{ni} + \beta_3^j RTA_{ni} + ex_i^j + \nu_{ni}^j \quad (32)$$

Following Eaton and Kortum 2002, D_{ni}^k is the distance dummy which equals one if the distance between country n and country i falls into the k^{th} interval as in $[0, 350]$, $[350, 750]$, $[750, 1500]$, $[1500, 3000]$, $[3000, 6000]$, $[6000, \text{maximum}]$ (in miles). The other control variables include dummies for common border, B_{ni} , currency union, CU_{ni} and regional trade agreement, RTA_{ni} , between country n and country i . ex_i^j is an exporter fixed effect, and ν_{ni}^j is the error term.

Substituting (32) back to (31) results in a gravity equation at the sector level as follows

$$\log\left(\frac{X_{ni}^j}{X_{nn}^j}\right) = \log\left(T_i^j (c_i^j)^{-\theta}\right) - \theta ex_i^j - \log\left(T_n^j (c_n^j)^{-\theta}\right) - \theta(\beta_0^j D_{ni}^k + \beta_1^j B_{ni} + \beta_2^j CU_{ni} + \beta_3^j RTA_{ni} + \nu_{ni}^j). \quad (33)$$

Define $\hat{F}_i^j = \log\left(T_i^j (c_i^j)^{-\theta}\right) - \theta ex_i^j$ and $F_n^j = \log\left(T_n^j (c_n^j)^{-\theta}\right)$. We then estimate the following equation using fixed effects

$$\log\left(\frac{X_{ni}^j}{X_{nn}^j}\right) = \hat{F}_i^j - F_n^j - \theta(\beta_0^j D_{ni}^k + \beta_1^j B_{ni} + \beta_2^j CU_{ni} + \beta_3^j RTA_{ni} + \nu_{ni}^j). \quad (34)$$

Once exporter and importer fixed effects are estimated, we recover the productivity of tradable sector j in country n relative to U.S., $\frac{T_n^j}{T_{US}^j}$, from the estimated importer fixed effects based on

$$S_n^j = \frac{\exp(F_n^j)}{\exp(F_{US}^j)} = \frac{T_n^j}{T_{US}^j} \left(\frac{c_n^j}{c_{US}^j}\right)^{-\theta} \quad (35)$$

in which the relative cost component can be computed by rewriting (16) as

$$\frac{c_n^j}{c_{us}^j} = \left(\frac{W_n}{W_{US}}\right)^{\gamma^j} \prod_{k=1}^{J-1} \left(\frac{P_n^k}{P_{US}^k}\right)^{\gamma^{jk}} \left(\frac{P_n^J}{P_{US}^J}\right)^{\gamma^{jJ}}, \quad (36)$$

where J indicates the nontradable sector. Using data on wages (in USD), estimates of price levels in the tradable sector and the nontradable sector relative to the United States, we can back up the relative cost. The nontradable relative price is obtained from the International Comparison Program (ICP). To compute the relative price of the tradable sector, we combine (14), (15) and (17) and get the following expression for relative prices of tradable goods

$$\frac{P_n^j}{P_{US}^j} = \left(\frac{X_{nn}^j/X_n^j}{X_{US,US}^j/X_{US}^j} \frac{S_{US}^j}{S_n^j}\right)^{\frac{1}{\theta}}. \quad (37)$$

To compute the relative productivity in nontradable sectors, we combine (17), (18) and set

$d_{ni}^J \rightarrow \infty$, for all i and n . From equation (18) in sector J , we have $\Phi_n^J = T_n^J (c_n^J)^{-\theta}$. Substituting this expression into (17), we obtain the nontradable good price as

$$p_n^J = \frac{c_n^J}{(T_n^J)^{1/\theta}} \quad (38)$$

The relative technology in nontradable sector can then be constructed based on

$$\frac{T_n^J}{T_{US}^J} = \left(\frac{c_n^J}{c_{US}^J} \frac{P_{US}^J}{P_n^J} \right)^\theta \quad (39)$$

Again, the cost ratios are calculated following (36).

To estimate the level of technology in each sector in all countries, we need to estimate the technology level in the U.S. First, we estimate the empirical sectoral TFP for the tradable sectors in the U.S. as the Solow residual (without capital in the production function)

$$\ln Z_{US}^j = \ln Y_{US}^j - \gamma^j \ln L_{US}^j - \sum_{k=1}^J \gamma^{jk} \ln M_{US}^{jk} \quad (40)$$

where Z_{US}^j denotes the measured productivity in U.S. in sector j , Y^j is the output, L^j is the labor input and M^{jk} is the intermediate input from sector k . The data are from OECD industry account.

Finicelli et al. (2013) show that trade and competition introduce selection in the TFP level, and the relationship between TFP and the level of technology T_{US}^j is given by

$$T_{US}^j = \left(Z_{US}^j \right) \theta \left[1 + \sum_{i \neq US} S_i^j \left(d_{US,i}^j \right)^{-\theta} \right]^{-1} \quad (41)$$

where S_i^j and $d_{US,i}^j$ are estimated based on the gravity equation. Specifically, the exporter fixed effect in trade friction, ex_i^j , is estimated using the importer and exporter fixed effects from Gravity equation (34): $ex_i^j = (F_i^j - \hat{F}_i^j)/\theta$. Lastly, we normalize the nontradable technology in the U.S. to one, and express all T_{US}^j relative to T_{US}^J as $\hat{T}_{US}^j = \left(Z_{US}^j / Z_{US}^J \right) \theta \left[1 + \sum_{i \neq US} S_i^j \left(d_{US,i}^j \right)^{-\theta} \right]^{-1}$.

For the purpose of this first calibration, we assume that θ is common across countries and set it equal to 8.28. Figure 3 shows, for a few sectors, that there is a strong positive relation between the relative productivity of innovation with respect to the United States, $\left(\hat{A}_i^j \right)$, and both R&D intensity and the stock of patents.

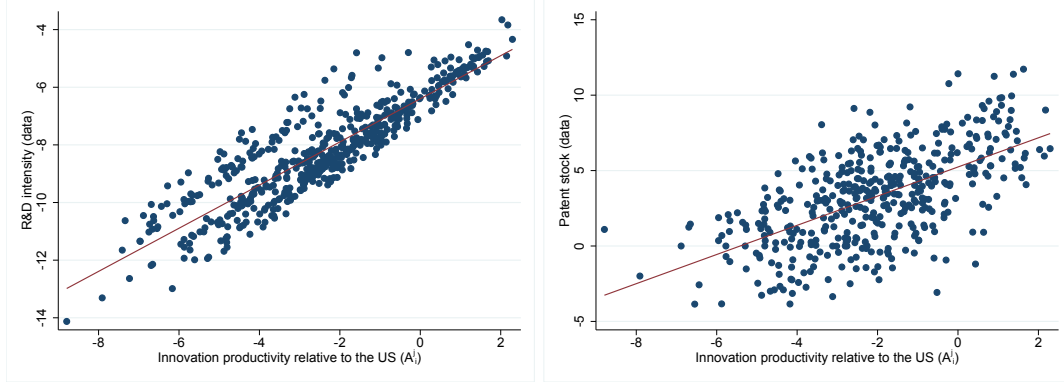


Figure 3: Technology and innovation

Figure 4 shows that, in all countries in our sample, the larger the R&D intensity, the larger the relative innovation productivity. Hence, our technology measure reflects the strength of innovation of the country.

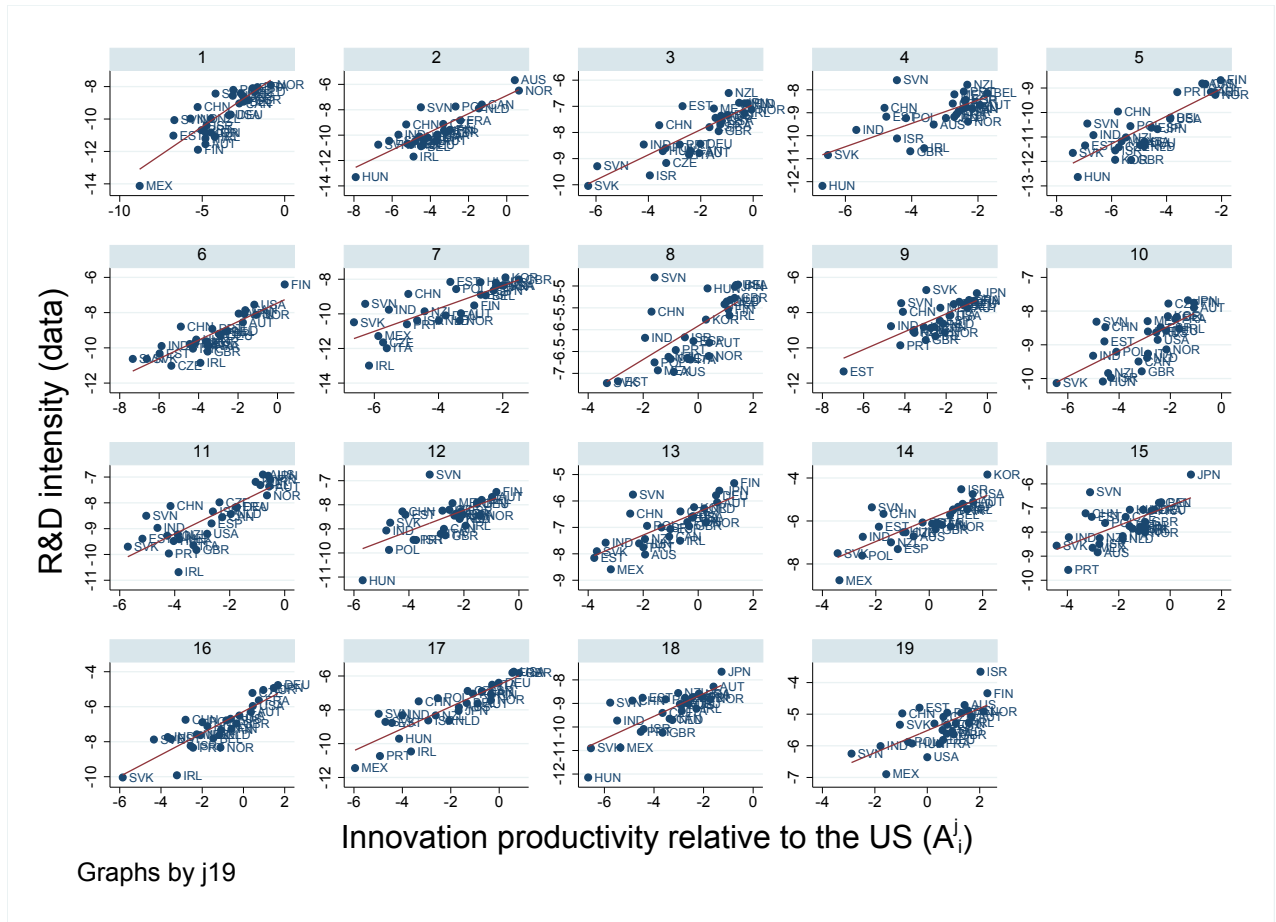


Figure 4: Technology and R&D across sectors

When we compare R&D intensity with total average productivity relative to the United States,

the relationship is not as strong (see figure 5). The weak correlation between R&D intensity and productivity has been documented in the literature. The reason is that the productivity of production, $T_{p,n}^j$ is not correlated with R&D intensity.

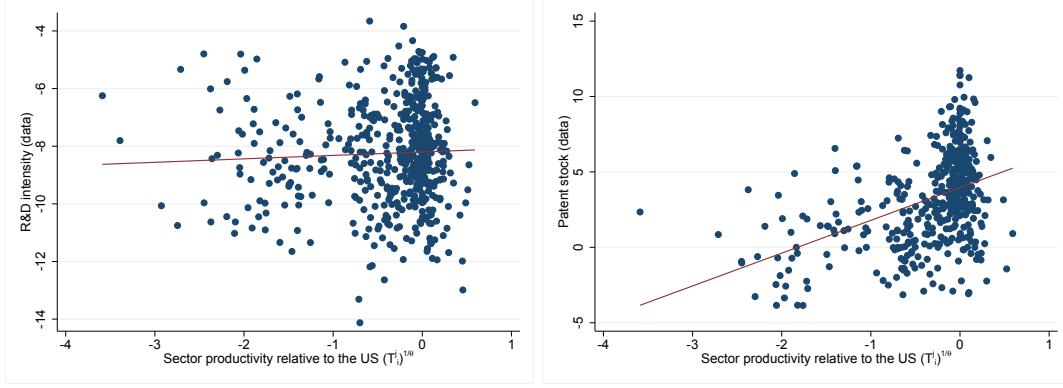


Figure 5: Relative productivity and innovation

We have also run our gravity equation at the sector level using $\theta = 4$ and a sector specific θ from Caliendo and Parro 2015. We find that the technology parameters estimated under different θ are highly correlated, as it has been documented in Levchenko and Zhang 2016.⁷

In particular, the calibration of technology parameters for $\theta = 4$ and $\theta = 8.28$ is 0.98, whereas the correlation of the technology parameter when θ is common and when we use the θ from Caliendo and Parro 2015 is 0.8. Figure 6 plots the Kernel Density of the technology parameters under the three values of θ .

⁷We are now looking into retail price data from the ICP 2011 program from the United Nations to obtain a sector specific dispersion parameter that does not rely on tariff data. Our plan is to follow Simonovska and Waugh 2014 to obtain an estimate for θ .

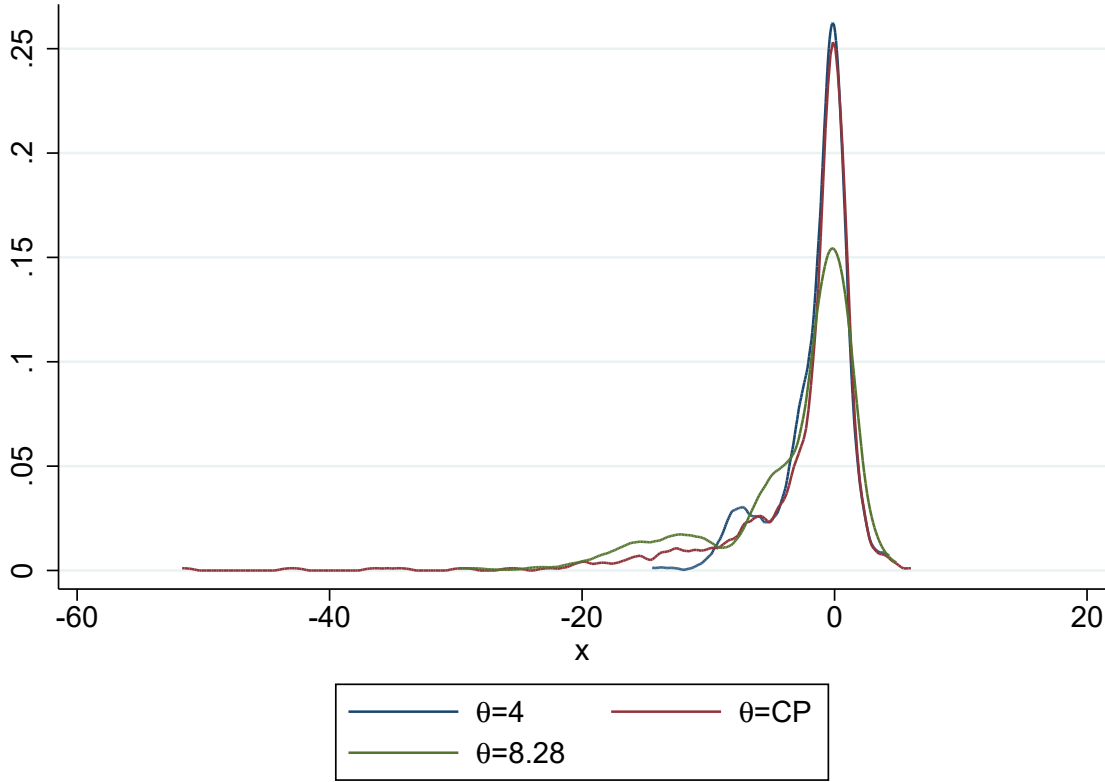


Figure 6: Technology at the Sectoral Level (different θ)

6.1.2 The speed of knowledge diffusion

We discipline the speed of knowledge diffusion, ε_{ni}^{jk} , using citation time lags from the U.S. NBER Patent Citation Database. Around 50% of patent applications to USPTO are from countries other than the U.S.

About half of the country-sector pairs (the nj - ik cells) have no observation of citations between one another, and thus no direct measure of diffusion speed is available. Three-quarters of these zero citation flows happen because either nj or ik has never filed for patents in the U.S. The other quarter appear when patents exist in both country-sector combinations but they do not cite one another or citation only exists in one direction.

To fill the missing values in our calibration, we first decompose those observed ε_{ni}^{jk} into four 3-dimension dummies F_{inj} , F_{ink} , F_{nij} and F_{nik} . We then estimate the missing values as $\hat{\varepsilon}_{ni}^{jk} = F_{inj} + F_{ink} + F_{njk} + F_{ijk}$. In some cases, the 3-dimension dummies are missing as well. For instance, when country pairs are too distant from each others. In those cases, we replace the 3-dimension dummies with 2-dimension dummies. For example $\hat{F}_{inj} = F_{in} + F_{nj} + F_{ij}$.

The following figure shows the histogram distribution of mean diffusion lag, $1/\varepsilon_{ni}^{jk}$, with missing values replaced by $\hat{\varepsilon}_{ni}^{jk}$. The distribution of the speed of diffusion is heterogeneous and highly skewed.

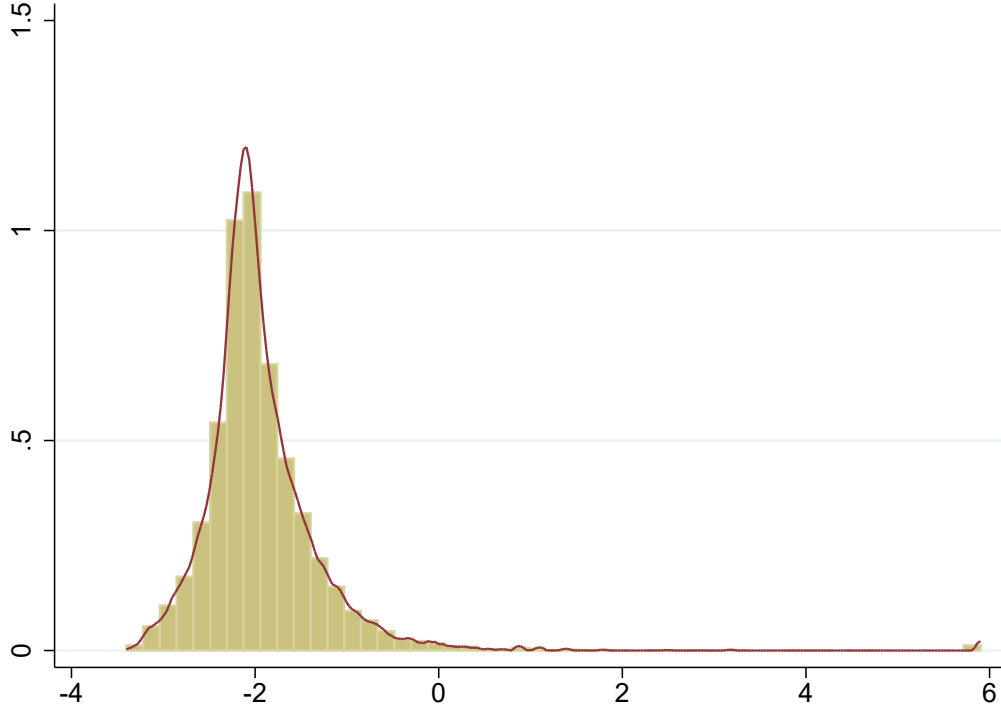


Figure 7: Mean Diffusion Lag

6.1.3 Parameters of innovation

We simulate the model in steady-state using the calibrated parameters on technology, trade barriers, production input-output linkages and the speed of diffusion. All the parameters up to the speed of technology diffusion allow us to obtain relative wages, costs, prices and trade shares in steady-state. Once we have obtained these variables we can use data on R&D intensity at the country-sector level, together with the expression for the growth rate of the economy in steady-state, equation 26. By assuming that all countries reach a growth rate of 2% in steady-state, which corresponds to a steady-state growth rate for the stock of knowledge of $g_T = \theta g_y = 16.56$, we can apply the Frobenius theorem and obtain a value for the productivity of innovation, λ_i^k , and the elasticity of innovation, β_r . In our estimation, we find that $\beta_r = 0.0185$. Figure 8 plots our estimated λ_i^k against both R&D intensity and the stock of patents. As the figure shows, there is a strong positive relationship between the productivity of innovation and both R&D intensity and the stock of patents.

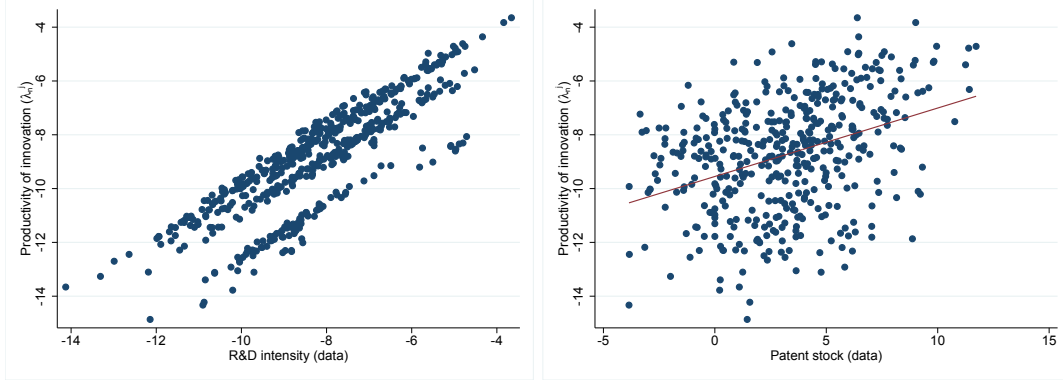


Figure 8: The productivity of innovation, R&D intensity and the stock of patents

We also observe a large heterogeneity in the productivity of innovation across countries and sectors. The mean is 0.0007727, and the standard deviation is 0.0020696, that is $\lambda_i^k \in [3.51 * 10^{-7} 0.025992]$.

With all the calibrated parameters, we can simulate wages and trade flows in our model. Figure 9 shows that our calibration strategy produces wages and measures for revealed comparative advantage that are consistent with those observed in the data.

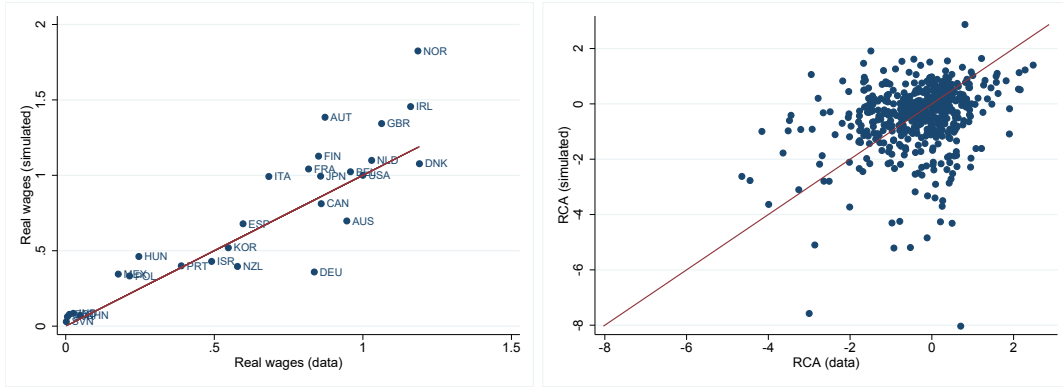


Figure 9: Validation

The algorithm

The calibration of the parameters of innovation, $\{\lambda_n^j, \beta_r, A_n^j\}$ follows a recursive algorithm. First, knowing $\{\gamma^j, \gamma^{jk}, \alpha^j, \sigma, T_n^j, d_{in}^j\}$, we use the trade structure of the model to obtain wages, prices, expenditures, trade shares, and output, from equations 14, 15, 16, 17, 18, 20, 21, 22, and 23.

Then, knowing $\{\varepsilon_{in}^{jk}, g_T\}$ we iterate over equation 26 to obtain $\{\lambda_n^j, \beta_r\}$. We do this in an iterative process in which, we guess over λ_n^j , and β_r we use R&D data, s_n^j , and we keep iterating until $g_T = 16.56$. We use 24 and 25 and the Frobenius theorem. The Frobenius theorem guarantees that there is a unique balanced growth path in which all countries and sectors grow at the same rate g_T . The expression for the growth rate can be expressed in matrix form as:

$$g_T A = \Delta(g_T) A$$

If the matrix $\Delta(g_T)$ is definite positive, then there exists a unique positive balanced growth rate of technology $g_T > 0$ given research intensities. Associated with that growth rate is a vector A (defined up to a scalar multiple), with every element positive, which reflects each country/sector relative level of knowledge along that balanced growth path. We update β_r so that $g_T = 16.56$ and we update λ_n^j so that R&D intensity matches the data. Then, we obtain A_n^j from the eigenvector associated to $\Delta(g_T = 16.56)$. Knowing T_n^j from the gravity regressions, and A_n^j from the Frobenius theorem, we can obtain $T_{p,n}^j$ from 5.

6.2 Counterfactual

We consider a uniform reduction of trade barriers, for each country pair and sector, of 55%. We then report the effect of this policy experiment on aggregate R&D, sectoral R&D across countries and welfare.

The algorithm

When we calibrated the parameters of the model, we were taking T_n^j as given from the gravity regressions. However, T_n^j will change across counterfactuals due to changes in A_n^j . Our algorithm to solve for the counterfactuals is as follows. Taking $\{\gamma^j, \gamma^{jk}, \alpha^j, \sigma, T_{p,n}^j, d_{in}^j, \beta_r, \lambda_n^j\}$ as given, we use the Frobenius theorem and the equations of the model to obtain the new g_T and A_n^j . We do this by iterating over equation 26 until $g_T(t-1) = g_T(t)$.

R&D intensity, revealed comparative advantage and diffusion

Contrary to the prediction of most models of international trade and innovation, trade has an effect in R&D intensity in our model. The effect is asymmetric across countries and sectors, and it depends both on the R&D intensity across sectors within the county, the input-output linkage structure, and the relative comparative advantage (RCA).

As Table 2 shows, R&D intensity moves towards sectors that experience a larger increase in RCA. We then, collapse our 4 dimensional diffusion parameter into two 2 dimensional parameters, once representing cited country-sector pair and one representing citing country-sector pair. R&D intensity increases in those country-sectors pairs that are more cited, and it decreases in those citing country-sector pairs. The main reason is that these sectors have to pay royalties, and there is an incentive to do more R&D in the highly cited sectors that receive royalty-payments.

Table 2: R&D, RCA and diffusion after a trade liberalization (country n and sector k)

	(1)
	R&D (% changes in the counterfactual)
R&D intensity (t=0)	-1.137*** (-3.61)
RCA (% changes in the counterfactual)	0.0226*** (6.32)
ε (cited country-sector)	7.905*** (8.91)
ε (citing country-sector)	-7.062*** (-4.33)
N	530

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Welfare Analysis

Finally, interactions between trade flows, knowledge spillovers and R&D have implication for welfare analysis. After a trade liberalization, welfare gains will be determined by two factors: (i) changes in revealed comparative advantage, and (ii) changes in R&D intensity. The cross-country distribution of welfare gains from trade is heterogeneous, as figure 10 shows. All countries gain from a trade liberalization. This heterogeneity is driven by heterogeneity of both the production side and the technology (R&D and diffusion) side of the model.

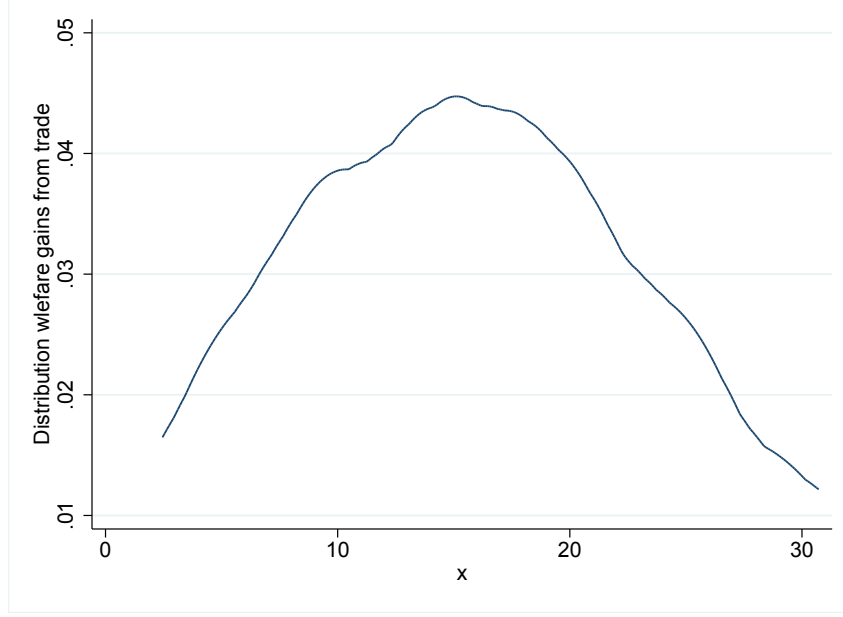


Figure 10: Welfare gains from trade

Figure 11 shows that, when we remove all sources of heterogeneity in the model, welfare gains from trade are smaller than when we allow for heterogeneity, and they are more similar across countries. To isolate the effect of different sources of sectoral heterogeneity on aggregate R&D and welfare. First, we consider a model in which we shut down all sources of heterogeneity across sectors. That is $\gamma^{jk} = \gamma_1$ and $\gamma^j = \gamma_2$, for all j and k , $\lambda_i^j = \lambda_i$ for all i and $\varepsilon_{in}^{jk} = \varepsilon_{in}$ for all j and k .

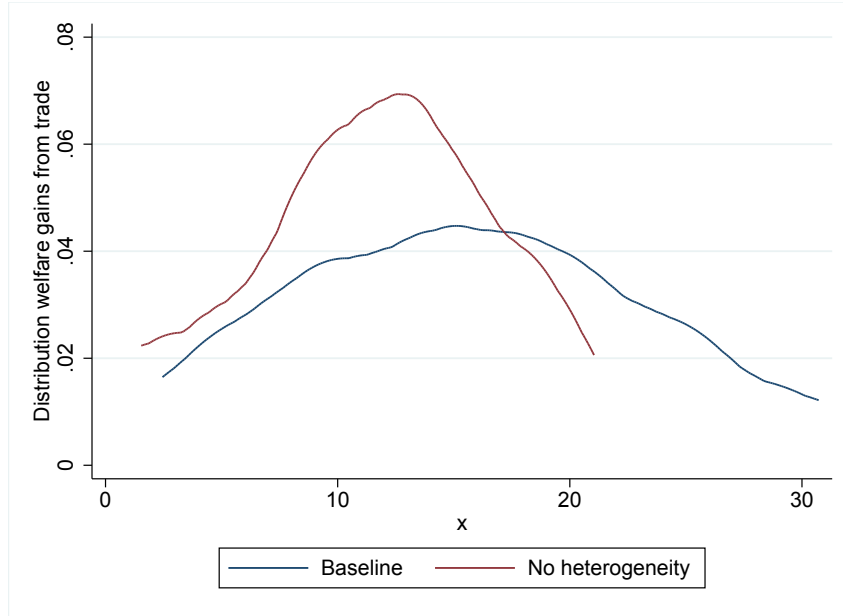


Figure 11: Welfare gains from trade

Differences in the cross-country distribution of the two models in figure

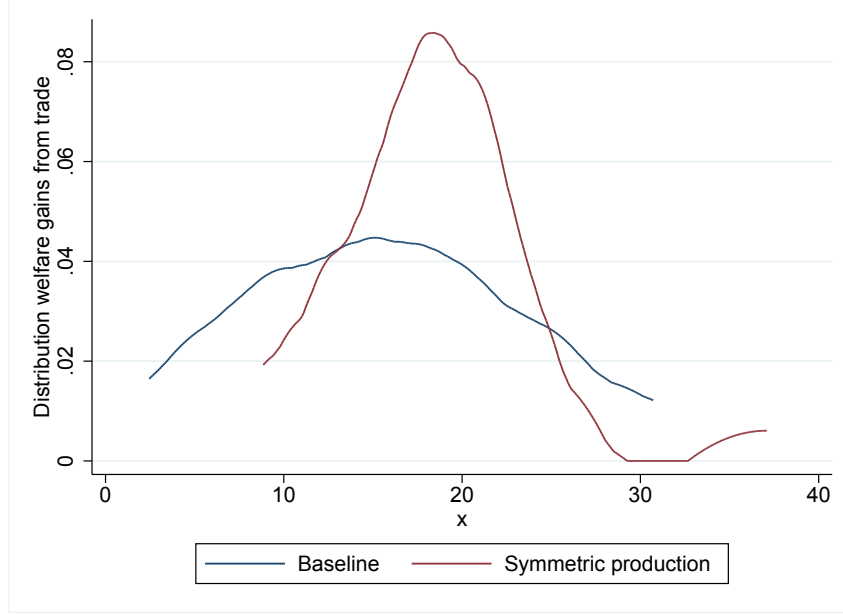


Figure 12: Welfare gains from trade

In this case, the dispersion in welfare gains is reduced, but the gains are still large. The figure shows that heterogeneity in production generates dispersion in welfare gains across countries, but it is heterogeneity in innovation that generates larger gains from trade after a trade liberalization. The reason why with both R&D and diffusion the gains are more concentrated across countries is because as trade costs go down, diffusion implies convergence in the stock of knowledge across countries. Because of evolving comparative advantage forces, a convergence in technology reduces the gains from trade as the forces of comparative advantage become more similar. Figure 13 shows that in a model with almost zero diffusion, the cross country distribution of welfare gains from trade is wider and shifted to the right.

Finally, we study the role of diffusion in our model. To do that, we recalibrate a model in which the diffusion parameters are set to a very small value of 0.0001, except for ε_{nn}^{jj} where we assume instantaneous diffusion. We then perform the same trade liberalization exercise as in the baseline model and evaluate the effect that a reduction of 55% in trade barriers has on R&D intensity and welfare in this model. Figure 13 shows that the welfare gains from trade in a model with very low diffusion are larger than in a model where we allow for diffusion. We then explore how the effect of trade on R&D intensity can shed light on the effect on welfare.

In table 3, we find that, in a model with very low diffusion, R&D moves towards sectors in which the country experiences a larger increase in comparative advantage, more so than in the baseline case presented in table 2. Furthermore, R&D moves towards sectors with initially large R&D intensity, which are also sectors with larger λ_n^j . That is, with very low diffusion, sectors with higher productivity of innovation experience substantial gains from a trade liberalization and sectors with lower productivity of innovation experience substantial losses in terms of R&D. We illustrate this point further in table 4.

Table 3: R&D, RCA and diffusion after a trade liberalization (Very low diffusion): (country n and sector k)

	(1)
	R&D (% changes in the counterfactual)
R&D intensity (initial)	2.919*** (5.79)
RCA(% changes in the counterfactual)	0.171*** (7.33)
N	532

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 4 shows that in the baseline model with diffusion, R&D does not move towards sectors with higher productivity of innovation. The reallocation happens especially towards sectors with higher knowledge spillovers, as table 2 showed.

Table 4: R&D and the productivity of innovation (country n and sector k)

	(1)	(2)
	R&D change (very low diff.)	R&D change (baseline)
λ_n^j	2.332*** (4.84)	-1.082*** (-15.74)
N	532	532

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Finally, we evaluate the role of diffusion in explaining welfare gains from trade. As before, we compare the effect of a trade liberalization in our baseline model where we allow for diffusion and in an equivalent model in which we have removed diffusion. We find that in a model in which we do not allow for diffusion, a trade liberalization generates larger welfare gains from trade for most countries than in our baseline model in which diffusion is allowed. The reason is that the stronger R&D specialization that takes place after a liberalization in a model without diffusion drives R&D towards industries with larger comparative advantage. In turn, and these industries benefit more from the reduction in trade barriers, which implies larger gains from trade.

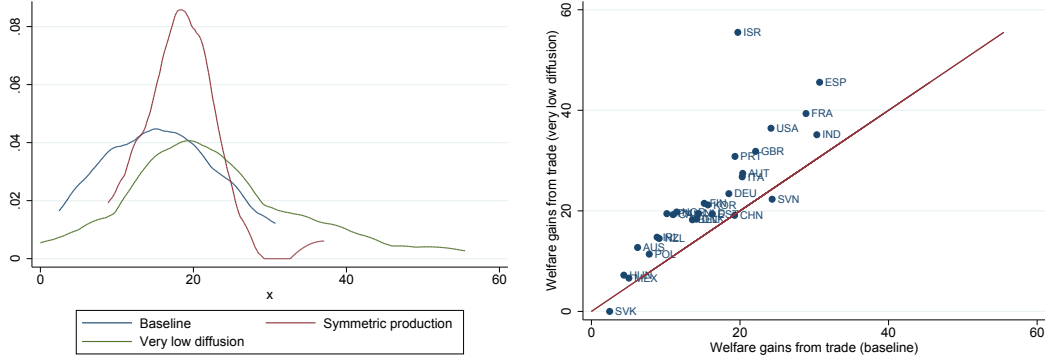


Figure 13: Welfare gains from trade

7 Conclusion

We develop a quantitative framework to study the interconnections between international trade, knowledge flows and input-output linkages. In our model, changes in trade barriers have an effect on innovation at the country level. The mechanism is that changes in trade frictions induce a reallocation of R&D toward sectors in which the country has a comparative advantage. The effect on aggregate innovation is stronger of the countries have a comparative advantage in highly applicable sectors.

Our results have implications for welfare analysis. Welfare gains from trade after trade liberalization result from reallocation of resources across sectors that are driven by comparative advantage and knowledge spillovers.

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Appendix

A Model Equations

There are 14 endogenous variables and we need 14 equations. The endogenous variables are

$$\{\pi_{in}^j, T_i^j, c_i^j, W_i, P_n^j, X_{ni}^j, X_n^j, P_n, Y_n, \Phi_n^j, C_n, s_n^j, V_n^j, A_n^j\}$$

The corresponding equations are:

(1) Probability of imports

$$\pi_{ni}^j = T_i^j \frac{\left(c_i^j d_{ni}^j\right)^{-\theta}}{\Phi_n^j}, \quad (42)$$

with

$$T_i^j = A_i^j T_{p,i}^j, \quad (43)$$

(2) Import shares

$$X_{ni}^j = \pi_{ni}^j X_n^j, \quad (44)$$

(3) Cost of production

$$c_n^j = \gamma_n^j W_{nt}^{\gamma_n^j} \prod_{k=1}^J (P_n^k)^{\gamma_n^{jk}}, \quad (45)$$

(4) Intermediate good prices in each sector

$$P_n^j = A^j (\Phi_n^j)^{-1/\theta}, \quad (46)$$

(5) Cost distribution

$$\Phi_n^j = \sum_{i=1}^M T_i^j \left(d_{ni}^j c_i^j\right)^{-\theta}, \quad (47)$$

(6) Price index

$$P_n = \prod_{j=1}^J \left(\frac{P_n^j}{\alpha_n^j}\right)^{\alpha_n^j}, \quad (48)$$

(7) Labor market clearing condition

$$W_n L_n = \sum_{i=1}^J \gamma_n^j \sum_{i=1}^M \pi_{in}^j X_i^j, \quad (49)$$

(8) Sector production

$$X_n^j = \sum_{k=1}^J \gamma_n^{kj} \sum_{i=1}^M X_i^k \pi_{in}^k + \alpha_n^j P_n Y_n, \quad (50)$$

(9) Final production

$$P_n Y_n = W_n L_n + \sum_{j=1}^J \sum_{i=1}^M \pi_{in}^j X_i^j, \quad (51)$$

(10) Resource constraint

$$Y_n = C_n + \sum_{k=1}^J s_n^k Y_n, \quad (52)$$

(11) Innovation

$$\dot{A}_{nt}^j = \sum_{i=1}^N \sum_{k=1}^J \varepsilon_{ni}^{jk} \int_{-\infty}^t e^{-\varepsilon_{ni}^{jk}(t-s)} \alpha_{is}^k \left(s_{is}^k\right)^{\beta^k} ds, \quad (53)$$

(12) R&D expenditures

$$\beta^j \lambda_{nt}^j V_{nt}^j \left(s_{nt}^j\right)^{\beta^j-1} = P_{nt}, \quad (54)$$

(13) Value of an innovation

$$V_{nt}^j = \int_t^{\infty} \left(\frac{P_{nt}^j}{P_{ns}^j}\right) e^{-\rho(s-t)} \Pi_{ns}^j ds, \quad (55)$$

with

$$\Pi_{nt}^j = \frac{1}{(1+\theta)A_{nt}^j} \sum_{i=1}^M X_{it}^j \pi_{int}^j. \quad (56)$$

B Data

This appendix describes the data sources and construction for the paper. 30 countries are included in our analysis based on data availability: Australia, Austria, Belgium, Canada, China, Czech Republic, Denmark, Estonia, Finland, France, Germany, Hungary, India, Ireland, Italy, Japan, Korea, Mexico, Netherlands, New Zealand, Norway, Poland, Portugal, Slovenia, Spain, Sweden, Switzerland, United Kingdom, and United States. When we use R&D data at the sector level, we

need to drop China, Sweden, Switzerland, Denmark, and India. The model is calibrated for the year 2005. There are 20 tradable sectors and one aggregate nontradable sector under consideration, which correspond to those in Caliendo and Parro 2015 and are reported in Table ??.

Bilateral trade flows at the sectoral level Bilateral trade data at sectoral level Data for expenditure by country n of sector j goods imported from country i (X_{ni}^j) are obtained from the OECD STAN Bilateral Trade Dataset. Values are reported in thousand U.S. dollars at current prices. Sectors are recorded at the ISIC (rev. 3) 2-3 digit level and were mapped into 2-digit tradable 20 sectors as listed in Table ?. We use the importer reported exports in each sector as the bilateral trade flows as it is generally considered to be more accurate than the exporter reported exports.

Value added and gross production Domestic sales in sector j , X_{nn}^j is calculated as $X_{nn}^j = Y_n^j - \sum_{i \neq n}^N X_{in}^j$, where both gross production of country n in sector j , Y_n^j and the total exports from n to i in sector j , $\sum_{i \neq n}^N X_{in}^j$, are obtained from OECD STAN Database for Structural Analysis. The database contains data at ISIC 2-digit level that can be easily mapped into our 21 sectors, at current prices and in national currencies. We use the exchange rates provided by OECD to convert the values into U.S. dollar. However, data are missing for China and India, for which we use the INDSTAT (2016 version) provided by United Nations Industrial Development Organization (UNIDO). This database is available for 4-digit ISIC (rev. 3) sectors and we aggregate them into 2-digit ISIC sectors to be consistent with the rest of the countries.

Trade barriers and gravity equation variables Data for variables related to trade costs and used in gravity equations at the country-pair level are obtained from CEPII database at www.cepii.fr/CEPII/en/bdd.modele/bdd.asp.

Wages Average annual wages is reported by OECD Labour statistics at current price in local currency. They are translated into U.S. dollars at the 2005 exchange rates to obtain the variable w_n in the model. However, wage data for China, India, and New Zealand are missing in this database, and are obtained from International Labor Organization (ILO).

R&D data R&D expenditures at the country-sector level are obtained from the database of OECD STAN by ISIC Revision 3 industries. Sectoral R&D data for all sectors in China, India and Sweden and a few sectors in other countries are missing, and we estimate the fitted value using the following approach. First, we run a regression using existing country-sector specific R&D and patent data from USPTO:

$$\log(R_{ij}) = \beta \log(PS_{ij}) + \mu_i + \gamma_j, \quad (57)$$

where R_{ij} is the R&D expenditure of country i in sector j and PS_{ij} is the patent stock of country i in sector j of year 2005. μ_i and γ_j are country and sector fixed effects. This relation is built on the

observations that (a) at steady state, R&D expenditure should be a constant ratio of R&D stock, and (b) innovation input (R&D) is significantly positively related to innovation output (patent). Assuming that (57) also holds for China, India and Sweden, we can obtain the fitted value of their sectoral level R&D expenditure.

$$\log(\hat{R}_{ij}) = \hat{\beta} \log(P_{ij}) + \hat{\mu}_i + \hat{\gamma}_j$$

For these three countries, we have information on all the right-hand-side variables except for the country fixed effect, $\hat{\mu}_i$. This allows us to compute the *share* of R&D in a given sector for each country,

$$\hat{s}_{ij} = \frac{\hat{R}_{ij}}{\sum_k \hat{R}_{ik}} = \frac{PS_{ij}^{\hat{\beta}} \exp(\hat{\mu}_i) \exp(\hat{\gamma}_j)}{\sum_j PS_{ij}^{\hat{\beta}} \exp(\hat{\mu}_i) \exp(\hat{\gamma}_j)} = \frac{PS_{ij}^{\hat{\beta}} \exp(\hat{\gamma}_j)}{\sum_j PS_{ij}^{\hat{\beta}} \exp(\hat{\gamma}_j)}.$$

We then obtain the aggregate R&D expenditure as percentage of GDP, s_i^{WB} , for country i from the World Bank World Development Indicator Database. The country-sector specific R&D can then be estimated as $\tilde{s}_{ij} = \hat{s}_{ij} s_i^{WB}$. For the countries with only one or two sectors missing, we estimate the fitted value using the same procedure. To maintain consistency across countries, we correct the OECD data generated total R&D with the World Bank total R&D.

$$\tilde{s}_{ij} = s_i^{WB} * R_{ij}^{OECD} / \sum_k R_{ik}^{OECD}$$

Finally, $\tilde{s}_{ij}$ is the R&D intensity parameters in Equations (62) and (9) that we use in the calibration and counterfactual simulation for country i and sector j .

C Calibration

In this section, we describe the procedure that we follow to calibrate all the relevant parameters of our model.

- θ : For the dispersion parameter, we try three different values: Following ?, we use $\theta = 4$, $\theta = 8.28$ and θ taken from Table A.1 in Caliendo and Parro 2015. The technology parameters estimated under different θ are highly correlated, as in ?.
- σ^j : The elasticity of substitution parameter is taken from Broda and Weinstein (2006) for the United States (this parameter is sector specific but not country-specific. We matched SITC rev 3 into ISIC rev 3 and take the mean σ^i of SITC sectors that belong to the same ISIC sector. Data is based on their estimates for period 1990-2001. We do not need this parameter for any of our results.
- γ_n^j and γ_n^{jk} from the I/O tables. Given our production function, the labor share = value added share (as we don't have capital). So γ_n^j is calculated as value added/gross output V_n^j/Y_n^j for each country-sector, γ_n^{jk} is input value of sector k (row sectors) to the gross output of sector j

Table 5: List of Industries

Sector	ISIC	Industry Description
1	C01T05	Agriculture, Hunting, Forestry and Fishing
2	C10T14	Mining and Quarrying
3	C15T16	Food products, beverages and tobacco
4	C17T19	Textiles, textile products, leather and footwear
5	C20	Wood and products of wood and cork
6	C21T22	Pulp, paper, paper products, printing and publishing
7	C23	Coke, refined petroleum products and nuclear fuel
8	C24	Chemicals and chemical products
9	C25	Rubber and plastics products
10	C26	Other non-metallic mineral products
11	C27	Basic metals
12	C28	Fabricated metal products, except machinery and equipment
13	C29	Machinery and equipment, nec
14	C30T33X	Computer, Electronic and optical equipment
15	C31	Electrical machinery and apparatus, nec
16	C34	Motor vehicles, trailers and semi-trailers
17	C35	Other transport equipment
18	C36T37	Manufacturing n.e.c. and recycling
19	C40T95	Nontradables

(column sectors) for country n or the share of intermediate consumption of sector j in sector k over the total intermediate consumption of sector k times $1 - \gamma_n^j$.

- β^j is the elasticity of innovation and we can assume that is the same across countries and sectors.

The remaining parameters that we need to calibrate are d_{in}^j , and T_n^j , and the growth rate of the economy.

1. We use bilateral trade gravity equation to estimate the country-sector specific competitiveness and productivity. We follow as close as possible to Caliendo and Parro (2014) with the same set of countries and sectors. In the production side, sectors are connected by Input-output linkages and trade flows, but service is non-tradable. For robustness, we try two methods to estimate country-sector specific productivity level and distance parameters.

- – Method 1

First, we run sector specific gravity equations with constraints on the importer n and export i fixed effects ($\sum_i S_i^j = 1$ and $\sum_n S_n^j = 1$), to obtain importer-exporter-sector specific distance $D_{ni}^j = \sum_k \rho_k^j \log(D_k)$ and country-sector fixed effects $\{S_i^j\}$ and $\{S_n^j\}$.

$$\log \left(\frac{X_{nit}^j}{X_{nnt}^j} \right) = S_i^j - S_n^j - D_{ni}^j \quad (58)$$

$$= S_i^j - S_n^j - \sum_{k=1}^{10} \rho_k^j D_k \quad (59)$$

where D_1 to D_6 are distance dummy variables equal to one if the population weighted distance countries n and i is between 0 and 375 kilometers, 375 and 750 kilometers, 750 and 1500 kilometers, 1500 and 3000 kilometers, 3000 and 6000 kilometers, and above 6000 kilometers; D_7 to D_{10} are dummy variables indicating if countries n and i share common language, common border, belong to the same free trade agreement and costumes union. When $X_{nit}^j = 0$, we enter $\log \left(\frac{X_{nit}^j}{X_{nnt}^j} \right)$ as $\log \left(\frac{X_{nit}^j * 1000 + 1}{X_{nnt}^j * 1000} \right)$.

ρ_k^j is the sensitivity of sector j 's trade flow to the k^{th} trade barrier. By allowing sector specific sensitivities, trade liberalization in the counterfactual simulation will cause production structural change effect, pushing low distance sensitive sectors to remote countries and nontradable service sectors to central countries.

Second, armed with the S_i^j , S_n^j and D_{ni}^j from gravity equations, we then combine Equation (32) to (34) to obtain the country-sector specific cost c_i^j and productivity T_i^j for three different sets of $\{\theta\}$: (I) $\theta = 4$ for all non-service sectors, (ii) $\theta = 8.28$ for all non-service sectors, and (iii) $\{\theta\}$ from Caliendo and Parro (2014).

- – Method 2

We compute the D_{ni}^j using the sector specific version of Equation (12) in Eaton and Kortum (2002) and P_i^j on the right hand side of the equation from World Bank International Consumer Price dataset for 24 countries, using different sets of $\{\theta\}$ as in Method 1. Then we calculate c_i^j using (32), and substitute c_i^j into (11) to derive T_i^j , also under different sets of $\{\theta\}$.

1. Once we have a value for the fixed effects at the exporter level F_n^k we can plug them into equation (5) to obtain Φ_n^j which is a measure of technology progress in a county.
2. Then, we can use (1) to obtain π_{in}^j
3. Then we can use equation (4) and obtain P_n^j .
4. We then plug this into equation (6) to obtain P_n .
5. We now follow Caliendo and Parro and guess a vector of wages and use (7), (8) and (9) to obtain wages, expenditure X_n^j and Y_n^j . We guess vector of wages and update using the labor market clearing condition.

6. Then we can obtain the profits and the value of an innovation using (13)
7. Then use (12) to obtain s_n^j
8. Then, use equation (11) to obtain g and T_n^j using the Frobenius theorem

D Extended Model: Royalty Payments

In addition, the innovator gets royalties from the technologies that have diffused to other countries and sectors. Royalty payments are proportional to the profits that intermediate goods in other sectors and countries obtain from using that technology. The expected royalty payment is

$$\chi_{in,t}^{kj} \frac{\Pi_{it}^k}{A_{it}^k} = \chi_{in,t}^{kj} \frac{1}{(1 + \theta^k) A_{it}^k} \sum_{m=1}^M X_{mt}^k \pi_{mit}^k, \quad (60)$$

where $\chi_{in,t}^{kj}$ is the expected fraction of technologies that sector k in country i is using from sector j in country n . Note that $\chi_{nn}^{jj} = 1$.

The value of an idea that has been developed in country n and sector j is the expected present discounted value of the stream of future profits

$$V_{nt}^j = \sum_{i=1}^M \sum_{k=1}^J \int_t^\infty \left(\frac{P_{it}^k}{P_{is}^k} \right) \chi_{in,s}^{kj} e^{-\rho(s-t)} \frac{\Pi_{is}^k}{A_{is}^k} ds, \quad (61)$$

Note that the incentive to innovate depends on the value of an innovation, which depends on: (i) the probability of the new technology lowering the cost of production there, $\frac{1}{A_n^j}$, and (iii) the expected profits from selling the good to each potential country-sector, $\frac{X_{in}^j}{1+\theta}$.

The first order condition for innovation is:

$$\beta_r \lambda_{nt}^j V_{nt}^j \left(s_{nt}^j \right)^{\beta_r - 1} = P_{nt} Y_{nt}, \quad (62)$$

D.1 Balance of payments

The current account balance equals the trade balance plus the net foreign income derived from net royalty payments. Total imports in country n are given by:

$$\sum_{i=1}^M \sum_{k=1}^J X_{nit}^k = \sum_{i=1}^M \sum_{k=1}^J \pi_{nit}^k X_t^k = \sum_{k=1}^J X_{nt}^k \sum_{i=1}^M \pi_{nit}^k, \quad (63)$$

Then,

$$IM_{nt} = \sum_{k=1}^J X_{nt}^k \sum_{i=1}^M \pi_{nit}^k$$

Total exports in country n are given by:

$$EX_{nt} = \sum_{i=1}^M \sum_{k=1}^J X_{int}^k = \sum_{i=1}^M \sum_{k=1}^J \pi_{int}^k X_{it}^k$$

Net royalty payments are given by

$$RP_{nt} = \sum_{j=1}^J RP_{nt}^j$$

and

$$RP_{nt}^j = \sum_{i=1}^M \sum_{k=1}^J \left(\chi_{in}^{kj} \Pi_i^k - \chi_{ni}^{jk} \Pi_n^j \right)$$

The balance of payments implies

$$EX_{nt} = IM_{nt} - RP_{nt}$$