Dynamic Financial Constraints: Which Frictions Matter for Corporate Policies?

Boris Nikolov[†] Lukas Schmid[‡] Roberto Steri[§]

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Abstract

We build, solve, and estimate a range of dynamic models of corporate investment and financing. We focus on limited enforcement, moral hazard, and tradeoff models. All models share a common technology structure, but differ in the friction generating financial constraints. Using panel data on Compustat firms for the period 1980-2015 and a more recent dataset on private firms from Orbis, we determine which features of the observed data allow to distinguish among the models, and we assess which model performs best at rationalizing observed corporate investment and financing policies across various samples. Our tests, based on empirical policy function benchmarks, favor limited commitment models for larger compustat firms, and moral hazard models for private firms.

Keywords: Moral hazard, Limited enforcement, tradeoff, structural estimation, corporate policies

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[†]Faculty of Business and Economics at the University of Lausanne and Swiss Finance Institute. E-mail: boris.nikolov@unil.ch

[‡]Fuqua School of Business, Duke University & CEPR. E-mail: lukas.schmid@duke.edu

[§]Faculty of Business and Economics at the University of Lausanne. E-mail: roberto.steri@unil.ch

1 Introduction

Corporate finance revolves around the study of financial frictions. Indeed, as pointed out forcefully by Modigliani and Miller, in the absence of frictions restricting firms' access to external financing, corporations financing decisions are irrelevant to their valuations and real policies. But what are the sources of financial frictions? While there is little disagreement about the relevance of financing constraints, their nature is much more debated. Is firms' access to external finance restricted because tax advantages make it attractive to firms to issue loans that they might not be able to pay back after adverse shocks, as in a trade-off model? Is it because firms cannot commit to honor their obligations, as in a limited commitment model? Or is it because firms might misreport their effective performance to lenders and divert cash flows, as moral hazard models have suggested? And, are these frictions equally important across firms, if at all?

In this paper, we propose to take a step towards providing guidance regarding these important questions by structurally estimating a host of dynamic financing models as proposed in the literature, and comparing their fit across different data samples. In every estimation, we ask: Which of the proposed models provides the best description of the actual behavior of a given set of companies, if any? Do our data allow us to discriminate between the relevance of these models for a particular set of firms?

Our approach relies on recent advances in computation and estimation. On the computational side, the linear programming approach to dynamic programming, introduced by Trick and Zin (1993) and recently extended for the purposes of corporate finance models in Nikolov, Schmid, and Steri (2016), enables us to efficiently solve a large number of dynamic models of firm financing, such as trade-off, limited commitment and dynamic moral hazard models, that are computationally challenging because of the high-dimensionality of the set of choice variables. Regarding estimation, we adopt a novel approach to stuctural estimation, empirical policy function estimation, introduced by Bazdresch, Kahn, and Whited (2016), that identifies novel empirical benchmarks and allows to succinctly trace out firms' dynamic behavior[1].

More specifically, we start by laying out in a unified environment a triplet of models of dynamic firm financing that have received attention in both the empirical and theoretical literature. The

[1]see Gala and Gomes (2016) for a related approach

first is a standard trade-off model, similar to Hennessy and Whited (2005), in which tax advantages of debt encourage firms to issue loans, leading to uncontingent commitments in the future. The relevance of tax considerations for the determination of firms' capital structures has long been highlighted in the literature. In more recent years, in contrast, the theoretical literature has emphasized the the role of financial contracting in determining firm policies and dynamics.[2] Financial contracts arise to mitigate incentive conflicts between firms' insiders and outsiders and affect corporations' financial structures, investment policies and valuations. We account for these developments by considering, first, a model in which an optimal lending contract between lenders and shareholders determine firm financing when the latter cannot commit to honor their obligations, as in Albuquerque and Hopenhayn (2004) or more recently in Rampini and Viswanathan (2010) and Rampini and Viswanathan (2013), and second, a model in which firms' access to external financing is curtailed by moral hazard in the presence of asymmetric information about cash flows, so that shareholders can divert cash flows from lenders, similar to Clementi and Hopenhayn (2006), Quadrini (2004) or Verani (2016).

Our strategy is to evaluate all of these models by estimating their policy functions on a variety of samples and assess their relative fit. To that end, we consider full samples and subsamples drawn from both the standard Compustat universe, as well as a more recent dataset on US private firms from Orbis. Our data thus allow us to evaluate model fit across small and large firms, public and private firms, (relatively) constrained and unconstrained firms, among others. Following Bazdresch, Kahn, and Whited (2016), we estimate empirical policy functions on each of these samples, so that our estimator picks model parameters that minimizes the distance between policy functions recovered from simulations and the empirical benchmarks. Our implementation is based on investment policy functions. Policy function estimation is an attractive approach to model comparison in this context, as the procedure has excellent power to detect misspecification.

Our preliminary estimation results favor limited commitment models as best rationalizations of the behavior of large public firms. When it comes to smaller public firms, trade-off and limited commitment model provide similarly adequate descriptions of corporate investment. The relative fit of trade-off and limited commitment models across samples captures the relative importance of

[2]See e.g. the surveys of Harris and Raviv (1991)and Zingales (2000).

state-contingent financing instruments for corporate policies. In this sense, our results highlight how such financing options are reflected in large firms' investment behavior. Indeed, our results are thus consistent with the recent literature documenting that state-contingent instruments such as credit lines or derivatives positions are more prevalent for larger firms, as in Nikolov, Schmid, and Steri (2016) or Rampini and Viswanathan (2010, 2013). Smaller firms need to rely more on loans so that for those firms, trade-off models provide an equally accurate description of their behavior.

Intriguingly, our estimation procedure strongly favors moral hazard models over all others in the case of private firms. In our dataset, private firms tend to be smaller than public firms, they tend to invest more, be more more profitable, and significantly more levered. Our results suggests that cash flow diversion is highly relevant for this class of firms. Realistically, cash flow diversion is likely to be interpreted not narrowly as outright stealing of profits, but more broadly perhaps as conflicts of interest about the proper use of funds in firms that are less transparent lacking the scrutiny of the public spotlight. One way to interpret our results then is that leverage arises as an effective device to discipline such conflicts.

Our structural estimation procedure also yields point estimates of the relevant parameters across models, and thsu also allow us to gauge the magnitudes of the financial frictons and agency costs necessary to rationalize observed firm behavior, both in the full sample as well as across subsamples. The parameter estimates of particular concern relate to the unobserved agency conflicts that drive financial structure and investment, such as the degree of cash flow diversion in moral hazard models, and the amount of capital firms can abscond with in limited commitment models. Regarding cash flow diversion, our results indicate that in order to rationalize observed corporate policies, firm owners need to be able to divery abut 30 cents on the dollar of profits. On the other hand, consistent with earlier results in Nikolov, Schmid, and Steri (2016), firms can collateralize about 50 percent of their capital stock in limited commitment models.

Literature Review Our paper belongs to a small, but growing literature which tries to estimate, or quantitatively evaluate, the empirical implications of the literature on dynamic agency conflicts across a variety of economic environments. In our attempt to distinguish and discriminate across different models of dynamic financial constraints, we take the lead from the important work by

Karaivanov and Townsend (2014) and Karaivanov and Wright (2011), who use similar numerical techniques and estimation methods to distinguish among different sources of financing constraints that inhibit risk sharing among households in thai villages, and international capital flows, respectively. Our work is the first attempt, to the best of our knowledge to follow this agenda in the context of the literature of financial contracting in dynamic corporate finance.

A number of recent papers have used structural estimation to evaluate models based on a single source of agency conflicts for firm dynamics and financing. For example, Li, Whited, and Wu (2016) investigate a dynamic limited commitment model to gauge the relative importance of tax advantages and agency conflicts for firm financing. Relatedly, Ai, Kiku, and Li (2016) estimate a dynamic moral hazard model in general equilibrium to assess the severity of the agency conflicts arising from effort provision for the real economy. We differ from these papers by our focus on model comparison, and estimation technique. Verani (2016) presents and estimates a model combining moral hazard and limited commitment and estimates it on macroeconomic data from Colombia, while we focus on firm-level panel data.

Regarding structural estimation and quantitative evaluation in dynamic corporate finance, our work builds on a growing and maturing literature that estimates and gauges the magnitude of financial frictions, as in Gomes (2001), Cooley and Quadrini (2001), Hennessy and Whited (2005, 2007), Moyen (2004, 2007), Hackbarth and Mauer (2012), Taylor (2010), Taylor (2013), Korteweg (2010), Glover (2015), Morellec, Nikolov, and Schurhoff (2012), and Morellec, Nikolov, and Schurhoff (2016).

In our implementation of a limited commitment model, we follow the work of Albuquerque and Hopenhayn (2004), and especially Rampini and Viswanathan (2010, 2013), Ai and Li (2016), Zhang (2016), and Sun and Zhang (2016). Perhaps unsuprisingly, our approach is closest to our recent companion paper, Nikolov, Schmid, and Steri (2016), which emphasizes the implementation of a limited commitment model with state-contingent debt with a mixture of real world securities such as straight loans and credit lines.

Our implementation of a discrete time moral hazard model follows the work of Clementi and Hopenhayn (2006) and especially, Quadrini (2004), who quantitatively examines a dynamic moral hazard model when shocks can be persistent. Doepke and Townsend (2006) develop computational techniques to deal with the challenging case when persistent shocks are privately observed only. In contrast, we assume that persistent shocks are publicly observable. For the case with privately observed persistent shocks, Fu and Krishna (2016) develop an implementation of the corresponding cash flows by means of real world securities.

2 A Triplet of Models

In this section, we present the models that we take to the data and attempt to empirically discriminate between. The models themselves are fairly standard and have been widely used in the literature to address a variety of questions in corporate finance, growth and development, among others. To facilitate comparison, we present them in a unified setup that emphasizes similarities, and readily identifies differences.

The models are i) a standard trade-off model where tax advantages encourages financing with one-period debt which is limited by an endogenous collateral constraint, ii) a limited commitment model which retains the tax advantages of debt financing, but allows for a more flexible implementation of debt structure by means of state-contingent debt repayment schedules, which may be thought of straight debt and credit lines or derivatives, as well as iii) a model where external financing from lenders is limited by asymmetric information, in that lenders do not observe shock realizations and financial contracts thus need to be structured such that borrowing firms are induced to revealing the truth in the presence of moral hazard.

We keep the specification of technology identical across models, so that differences in observed policies stem exclusively from different financial frictions. We outline the technology first, and then describe in detail the different sources of frictions. Perhaps slightly deviating from the standard formulations of the models familiar from the literature, we cast them as firm rather than equity value maximization problems, to facilitate comparison. We establish the equivalence of these formulations in the appendix.

2.1 Technology and Investment

We consider the problem of value-maximizing firms in a perfectly competitive environment. Time is discrete. We assume that all agents are risk-neutral, so that the one period interest rate r is constant.

After-tax operating profits for firm i in period t depend upon the capital stock k_{it} and shocks z_{it} and η_{it}, respectively, and are given by

$$\pi(k_{it}, z_{it}, \eta_{it}) = (1-\tau)((z_{it}+\eta_{it})k_{it}^{\alpha} - f), \tag{1}$$

where $0 < \tau < 1$ denotes the corporate tax rate, $0 < \alpha < 1$ is the capital share in production, and $f > 0$ is a fixed cost incurred in the production process. Note that a capital share less than unity captures decreasing returns to scale. The variable z_{it} reflects shocks to demand, input prices, or productivity and follows a stochastic process with bounded support $Z = [\underline{z}, \overline{z}]$, with $-\infty < \underline{z} < \overline{z} < \infty$, and described by a transition function $Q_z(z_{it}, z_{it+1})$[3]. Finally, η_{it} is an iid disturbance, which takes values η with probability p and $-\eta$ with proability $1-p$. The shock η_{it} only plays a role in the context of dynamic moral hazard.

At the beginning of each period the firm is allowed to scale its operations by choosing its next period capital stock k_{it+1}. This is accomplished through investment i_{it}, which is defined by the standard capital accumulation rule

$$k_{it+1} = k_{it}(1-\delta) + i_{it}, \tag{2}$$

where $0 < \delta < 1$ is the depreciation rate of capital. Given our modeling of corporate taxation, we account for a depreciation tax allowance in the form of $\tau\delta k_{it}$.

Investment is subject to capital adjustment costs. As in Bolton, Chen, and Wang (2011), for example, we follow the neoclassical literature (Hayashi, 1982) and consider convex adjustment costs for simplicity. We parameterize capital adjustment costs with the functional form

$$\Psi(k_{it+1}, k_{it}) \equiv \frac{1}{2}\psi\left(\frac{i_{it}}{k_{it}}\right)^2 k_{it}, \tag{3}$$

where the parameter ψ governs the severity of the adjustment cost.

2.2 Trade-off I

Our first model is a standard trade-off model in which firms aim at exploiting the tax advantage of debt financing available in the US tax code. In the context of this model, we assue that $\eta_{it} \equiv 0$, so that $\pi_{it} \equiv \pi(k_{it}, z_{it})$.

[3]In our empirical work, we parameterize z_{it} so as to provide a discrete approximation to a continuous AR(1) process with persistence ρ_z and conditional volatility σ_z.

Financing At time t, firms have to the option to issue one-period bonds b_{it+1}, that are due at the beginning of the next period, with interest. Limited liability implies that there are states in which firms will be unable to fully repay their debt obligations at time $t+1$. This is because internal funds after a sequence of bad shocks are so low that they are not sufficient to cover repayments. In such states, shareholders default on their commitments, creditors take over and recover a fraction of firms cash flows and assets net of bankruptcy costs. In anticipation of such states, creditors adjust the yields on debt so as to break even in expectation. In other words, they will charge a default premium Δ_{it+1} above the risk-free rate to be compensated for potential losses in default, so that the effective interest rate on bonds amounts to $r + \Delta_{it+1}$. We determine Δ_{it+1} endogenously below.

In line with the US tax code, we assume that interest payments are tax deductible, so that the effective repayment due in period $t+1$ amounts to $(1 + (1-\tau)(r + \Delta_{it+1})b_{it+1}$ only. The amount $\tau(r + \Delta_{it+1})b_{it+1}$ therefore represents a tax shield.

We assume that firms have to repay debt commitments at the beginning of the period, after realization of the shocks, and before issuing new debt and taking investment decisions. At that point, the firm is solvent if and only if

$$(1-\tau)\pi(z_{it}, k_{it}) + (1-\delta)k_{it} + \tau\delta k_{it} - (1 + (r + \Delta_{it})(1-\tau))\, b_{it} \geq 0$$

which is equivalent to the condition $z_{it} \geq \bar{z}_{it} \equiv \bar{z}(k_{it}, b_{it}, \Delta_{it})$, with

$$\bar{z}_{it} = \frac{(1 + (r + \Delta_{it})(1-\tau))b_{it} - (1-\delta)k_{it} - \tau\delta k_{it}}{(1-\tau)\pi(z_{it}, k_{it})}.$$

In other words, whenever the profitability shock falls below $\bar{z}_{it}$, shareholders will default and creditors take over. Since shocks are persistent, creditors will anticipate these states, and determine the default premium accordingly. Given risk neutrality, creditors break even in expectation if

$$r + \Delta_{it+1} = \int_{\bar{z}_{t+1}}^{\infty} r d\phi(z_{t+1}|z_t) + \xi \int_{-\infty}^{\bar{z}_{t+1}} (1-\tau)\pi(z_{it+1}, k_{it+1}) + (1-\delta)k_{it+1} + \tau\delta k_{it+1} d\phi(z_{t+1}|z_t),$$

where ξ denote deadweight costs incurred in bankruptcy. Note that the default premium Δ_{it+1} is a function of k_{it+1}, b_{it+1} and z_{it} only, and can be determined ex ante. In this context, using this timing convention, it requires solving a nonlinear equation for each grid point, but outside the optimization loop.

With the default premium at hand, we can determine firms' payouts. Debt and internal resources can be used to fund investment expenditures, or distributions d_{it} to shareholders. Given limited liability, seasoned equity offerings are effectively precluded. While this may initially appear restrictive, in the data equity issuances are often employee-initiated issues[4]. Employee-initiated issues are not part of our model and characterize the exercise of stock options With this caveat in mind, then, we have that

$$d_{it} \equiv (1-\tau)\pi(z_{it}, k_{it}) - k_{it+1} + (1-\delta)k_{it} - \Psi(k_{it+1}, k_{it}) + \tau\delta k_{it} - (1 + (r+\Delta_{it})(1-\tau))\, b_{it} + b_{it+1} \geq 0.$$

Firm problem Investment and fnancing policies are set to maximize firm value. Capital accumulation and financing needs reflect the persistent profitability shocks z_{it}, while debt policies additionally exploit the tax advantage, and constraints. More formally, firm value $W(k_{it}, b_{it}, z_{it})$ satisfies the following Bellman equation:

$$\begin{aligned} W(k_{it}, b_{it}, z_{it}) \equiv & \max_{k_{it+1}, b_{it+1}} (1-\tau)\pi(k_{it}, z_{it}) - k_{it+1} + (1-\delta)k_{it} - \Psi(k_{it+1}, k_{it}) + \tau\delta k_{it} \\ & + \tau(r+\Delta_{it}) b_{it} \mathcal{I}_{z_{it} \geq \bar{z}_{it}} - \xi((1-\delta)k_{it} + \tau\delta k_{it})\mathcal{I}_{z_{it} < \bar{z}_{it}} \\ & \qquad + \tfrac{1}{1+r} E_t[W(k_{it+1}, b_{it+1}, z_{it+1})] \end{aligned}$$

subject to

$$(1-\tau)\pi(z_{it}, k_{it}) - k_{it+1} + (1-\delta)k_{it} - \Psi(k_{it+1}, k_{it}) + \tau\delta k_{it} - (1 + (r+\Delta_{it})(1-\tau))\, b_{it} + b_{it+1} \geq 0,$$

$$r + \Delta_{it+1} = \textstyle\int_{\bar{z}_{t+1}}^{\infty} r d\phi(z_{t+1}|z_t) + \xi \int_{-\infty}^{\bar{z}_{t+1}} (1-\tau)\pi(z_{it+1}, k_{it+1}) + (1-\delta)k_{it+1} + \tau\delta k_{it+1} d\phi(z_{t+1}|z_t).$$

2.3 Trade-off II

Our first model is a standard trade-off model in which firms aim at exploiting the tax advantage of debt financing available in the US tax code. In the context of this model, we assue that $\eta_{it} \equiv 0$, so that $\pi_{it} \equiv \pi(k_{it}, z_{it})$.

[4] McKoen (2015) documents the empirical relevance of employee-initiated equity issuances.

Financing At time t, firms have to the option to issue one-period bonds, b_{it+1} that are due at the beginning of the next period, with interest. In line with the US tax code, we assume that interest payments are tax deductible, so that the effective repayment due in period $t+1$ amounts to $(1+(1-\tau)r)b_{it+1}$ only. The amount $\tau r b_{it+1}$ therefore represents a tax shield.

In order to credibly commit to honor debt obligations, debt has to be fully collateralized by the amount of capital that lenders could recover in case of a liquidation, so that it is effectively riskfree. We denote by $0 < \theta < 1$ the fraction of capital that can be pledged as collateral. We interpret $\theta < 1$ as an indication that some capital may be intangible and cannot be pledged as collateral. An alternative interpretation is that some capital may be sold off in fire sales at discounted prices. Formally, this requires that

$$(1+r(1-\tau))b_{it+1} \leq \theta(1-\delta)k_{it+1}. \tag{4}$$

Debt and internal resources can be used to fund investment expenditures, or distributions d_{it} to shareholders. We assume that shareholders have limited liability, which effectively precludes seasoned equity offerings. While this may initially appear restrictive, in the data equity issuances are often employee-initiated issues[5]. Employee-initiated issues are not part of our model and characterize the exercise of stock options With this caveat in mind, then, we have that

$$d_{it} \equiv (1-\tau)\pi(z_{it},k_{it}) - k_{it+1} + (1-\delta)k_{it} - \Psi(k_{it+1},k_{it}) + \tau\delta k_{it} - (1+r(1-\tau))\,b_{it} + b_{it+1} \geq 0.$$

Firm problem Investment and fnancing policies are set to maximize firm value. Capital accumulation and financing needs reflect the persistent profitability shocks z_{it}, while debt policies additionally exploit the tax advantage, and constraints. More formally, firm value $W(k_{it}, b_{it}, z_{it})$

[5]McKoen (2015) documents the empirical relevance of employee-initiated equity issuances.

satisfies the following Bellman equation:

$$W(k_{it}, b_{it}, z_{it}) \equiv \max_{k_{it+1}, b_{it+1}} (1-\tau)\pi(k_{it}, z_{it}) - k_{it+1} + (1-\delta)k_{it} - \Psi(k_{it+1}, k_{it}) + \tau\delta k_{it} + \tau r b_{it} + \frac{1}{1+r} E_t[W(k_{it+1}, b_{it+1}, z_{it+1})]$$

subject to

$$(1-\tau)\pi(z_{it}, k_{it}) - k_{it+1} + (1-\delta)k_{it} - \Psi(k_{it+1}, k_{it}) + \tau\delta k_{it} - (1 + r(1-\tau))\, b_{it} + b_{it+1} \geq 0,$$

$$(1 + r(1-\tau))b_{it+1} \leq \theta(1-\delta)k_{it+1}.$$

2.4 Limited Commitment

We now relax the perhaps slightly stark assumption that firms only source of external financing is one-period debt. This assumption immediately precludes any instruments with a more state-contingent flavor such as credit lines, derivatives or even external equity. We relax this in the context of a limited commitment model, in which, formally, we allow the payoffs of the securities available to outside investors to be contingent on the realization of the profitability shock z_{it}. While we do not take a stand on the precise implementation of this instrument by means of real world securities for the sake of this paper, we refer to our companion paper Nikolov, Schmid, and Steri (2016), or the recent work by Rampini and Viswanathan (2013) or Li, Whited, and Wu (2016) for examples and estimation evidence. In the context of this model, we continue to assume that $\eta_{it} \equiv 0$, so that $\pi_{it} \equiv \pi(k_{it}, z_{it})$.

Financing Formally, in every period firms can sell a portfolio of securities whose payoffs $b_{it+1}(z_{it+1})$ to investors are contingent on the realization of next periods profitability shock z_{it+1}. Selling such a portfolio at time t thus raises an amount $b_{it+1} \equiv E_t[b_{it+1}(z_{it+1})]$. For the sake of our analysis here, we think of these state contingent payments as repayments to a lender, which need to be fully collateralized. That is, we require that

$$(1 + r(1-\tau))b_{it+1}(z_{it+1}) \leq \theta(1-\delta)k_{it+1}, \qquad \forall z_{it+1}.$$

Similar to the case of straight debt considered in the trade-off model above, these state-contingent debt instruments can be used to fund investment expenditures and distributions to shareholders, d_{it}, jointly with internal resources. We retain the assumption of limited liability on the shareholders' side, which requires that

$$\begin{aligned} d_{it} \quad \equiv \quad & (1-\tau)\pi(z_{it}, k_{it}) - k_{it+1} + (1-\delta)k_{it+1} - \Psi(k_{it+1}, k_{it}) + \tau\delta k_{it} \\ & - (1 + r(1-\tau))\, b_{it}(z_{it}) + E_t[b_{it+1}(z_{it+1})] \geq 0 \end{aligned}$$

Firm problem The firm's problem is then to choose investment and state-contingent financing plans so as to maximize firm value, subject to constraints. Formally, firm value $W(k_{it}, b_{it}, z_{it})$ satisfies the following Bellman equation:

$$\begin{aligned} W(k_{it}, b_{it}, z_{it}) \equiv \max_{k_{it+1}, b_{it+1}(z_{it+1})} & (1-\tau)\pi(k_{it}, z_{it}) - k_{it+1} + (1-\delta)k_{it} - \Psi(k_{it+1}, k_{it}) + \tau\delta k_{it} + \tau r b_{it}(z_{it}) \\ & + \tfrac{1}{1+r} E_t[W(k_{it+1}, b_{it+1}(z_{it+1}), z_{it+1})] \end{aligned}$$

subject to

$$(1-\tau)\pi(z_{it}, k_{it}) - k_{it+1} + (1-\delta)k_{it+1} - \Psi(k_{it+1}, k_{it}) + \tau\delta k_{it} - (1 + r(1-\tau))\, b_{it}(z_{it}) + E_t[b_{it+1}(z_{it+1})] \geq 0,$$

$$(1 + r(1-\tau))b_{it+1}(z_{it+1}) \leq \theta(1-\delta)k_{it+1}.$$

2.5 Moral Hazard

We now embed a dynamic moral hazard problem into our setup by assuming that lendera cannot observe the realization of all shocks and therefore has to rely on shareholders' report. Naturally, all else equal, shareholders have an incentive to underreport realized shocks as it allows them to pocket a larger share of realized cash flows by repaying less debt. An incentive compatible contract therefore designs a repayment schedule such that shareholders are always better off truthfully reporting. This can be achieved by implementing state-contingent repayments satisfying a number of constraints as detailed below.

Given that shocks are unobservable to lenders, some care must be taken with respect to the timing of decisions. Further, as previously, we realistically want to allow for serially correlated shocks z_{it}. To keep the analysis transparent, we assume, as before, that z_{it} follows a Markov Chain, which is publicly observable[6], especially for the lender. However, conditional on a particular realization of z_t, realized cash flows are also impacted by the iid disturbance η_{it}. To give rise to a meaningful dynamic moral hazard problem, critically, we assume that η_{it} is observable by shareholders, but *unobservable by* lenders. Accordingly, we now return to our initial specification that $\pi \equiv \pi(k_{it}, z_{it}, \eta_{it})$.

In this context, a lending contract amounts to a sharing rule that splits a firm's resources between payments to the lender, p_{it}, and payments to the shareholder, that is, dividends, d_{it} in a fully state-contingent manner. An optimal contract between shareholders and lenders maximizes the firm value W_{it} , subject to incentive constraints, promise keeping, as well as limited liability constraints. In this context, the incentive constraints amount to requiring that under the contract shareholders are always better off sticking to the contract and revealing the true realization of η_{it}, rather then underreporting realized cash flows and diverting cash flows. At this stage, we capture the cash flows that can be diverted by misreporting $\hat{\eta}_{it}$ rather than the true η_{it} by the general 'diversion function' $\mathcal{D}(k_{it}, z_{it}, \eta_{it}, \hat{\eta}_{it})$. We will discuss economically motivated functional forms below.

In this setting with dynamic moral hazard, it is convenient to use the equity value of the firm, V_{it}, as a state variable. Clearly, then, the value of debt can be recovered, from $b_{it} = W_{it} - V_{it}$. Similarly, the contract picks state-contingent dividend payments d_{it} as controls, so that the payments to the lender p_{it} are recovered from the resource constraint, as detailed below.

Regarding timing, let us denote the firm value at the end of period $t-1$ (that is, after the realization of all the $t-1$ and *before* the t shocks) as $W(k_{it-1}, V_{it-1}, z_{it-1})$. The state variable z_{it-1} is informative about the conditional distribution of z_{it} in period t, which affects the expected returns to capital. For tractability, we assume that shareholders decide at the end of period t about their investment expenditures at the beginning of period t, which entails adjustment costs as before and which leaves them with a depreciated capital stock after production. Finally, we accommodate tax deductability of interest on debt, $\tau r b_{it} = \tau r(W_{it} - V_{it})$, which translates into an adjusted discount

[6]See Doepke and Townsend, etc. for analyses of privately observable persistent shocks

rate for the firm $(1/(1+(1-\tau)r)$ rather than $1(1+r))$ and a penalty for foregone tax deductions on debt for the amount of $\tau r V_{it}$.

More formally, the firm value function satisfies

$$\begin{aligned} W(k_{it-1}, V_{it-1}, z_{it-1}) &= \max_{k_{it}, V_{z_{it},\eta_{it}}, d_{z_{it},\eta_{it}}} \frac{1}{1+(1-\tau)r}[-k_{it} - \Psi(k_{it}, k_{it-1}) + (1-\delta)k_{it} + \tau\delta k_{it} - r\tau V_{it-1} \\ &\quad + E_{t-1}[(1-\tau)\pi(k_{it}, z_{it}, \eta_{it}) + W(k_{it}, V_{z_{it},\eta_{it}}, z_{it})]] \end{aligned}$$

subject to

$$V_{it-1} = \frac{1}{1+r} E_{t-1}[d_{z_{it},\eta_{it}} + V_{z_{it},\eta_{it}}], \tag{5}$$

$$d_{z_{it},\eta_{it}} + V_{z_{it},\eta_{it}} \geq d_{z_{it},\hat{\eta}_{it}} + V_{z_{it},\hat{\eta}_{it}} + \mathcal{D}(k_{it}, z_{it}, \eta_{it}, \hat{\eta}_{it}), \quad \forall z_t, \forall \hat{\eta}_{it}, \tag{6}$$

$$d_{z_{it},\eta_{it}} \geq 0, \quad \forall z_{it}, \forall \eta_{it}, \tag{7}$$

$$V_{z_{it},\eta_{it}} \geq 0, \quad \forall z_{it}, \forall \eta_{it}, . \tag{8}$$

Here, V_{it-1} is the equity value at the end of period $t-1$, and the promise keeping constraint states that the (state-contingent) dividend payments to shareholders and equity value at the end of period t have to add up to V_{t-1} in expectation. The incentive constraints state that shareholders are always better reporting the true realization of the iid shock η_{it} and receiving a dividend $d_{z_{it},\eta_{it}}$ and continuation value $V_{z_{it},\eta_{it}}$, rather than misreporting $\hat{\eta}_{it}$ and pocketing the diverted cash flow, captured by the diversion function $\mathcal{D}(k_{it}, z_{it}, \eta_{it}, \hat{\eta}_{it})$, as well as the dividends and continuation values under the misreported cash flows, $d_{z_{it},\hat{\eta}_{it}} + V_{z_{it},\hat{\eta}_{it}}$. The diversion function, in its most starightforward specification, is just $\lambda(\pi(k_{it}, z_{it}, \eta_{it}) - \pi(k_{it}, z_{it}, \hat{\eta}_{it}))$, where $1-\lambda$ captures potential losses in cash flow diversion. Finally, in every state, dividend payments and equity values have to be non-negative, reflecting shareholders' limited liability.

Given our timing assumption, all the cash flows accrue intra-period, so that payments p_{it} to the lender are simply the mirror image of contractually designed dividend payments to shareholders. In other words, we must have that these respective payments exhaust available resources, so that, state by state,

$$p_{it} = -k_{it} - \Psi(k_{it}, k_{it-1}) + (1-\delta)k_{it} + \tau\delta k_{it} + \tau r(W_{it} - V_{it}) + \pi(k_{it}, z_{it}, \eta_{it}) - d_{it}.$$

This formulation emphasizes the trade-off between payments to shareholders and lenders, which must respect promise keeping and incentive constraints laid out above.

2.6 Determinants of Corporate Policies across Models

Before we proceed to the estimation, we provide some qualitative guidance to the determinants of corporate policies across models by inspecting their first order conditions. We relegate a detailed derivation of the complete set of optimality conditions to the appendix and focus on the most instructive ones in this sections.

Investment We start by inspecting the Euler equation for investment across the models. In case of the trade-off model, we recover a standard Euler equation

$$1 + \Psi(k_{it+1}, k_{it}) = \frac{1}{1+r} E_t[W_k(k_{it+1}, b_{it+1}, z_{it+1})] - \lambda_{LL,TO}(1 + \Psi(k_{it+1}, k_{it})) + \lambda_{BOR}\theta(1-\delta)$$

with envelope condition

$$\begin{aligned} W_k(k_{it}, b_{it}, z_{it}) &= (1-\tau)\pi_k(z_{it}, k_{it}) + (1-\delta) - \Psi_k(k_{it+1}, k_{it}) + \tau\delta \\ &\quad -\lambda_{LL}((1-\tau)\pi_k(z_{it}, k_{it}) + (1-\delta) - \Psi_k(k_{it+1}, k_{it}) + \tau\delta). \end{aligned}$$

Here $\lambda_{BOR,TO}$ is the lagrange multiplier on the collateral constraint, (4), and $\lambda_{LL,TO}$ is the multiplier on the limited liability constraint (5). The Euler equation has a straightforward interpretation in that the marginal costs of additional capital including adjustment costs $(1 + \Psi(k_{it+1}, k_{it}))$ are equal to the marginal benefits, which include the marginal value of additional debt capacity $(\lambda_{BOR}\theta(1-\delta))$ net of the impact of pressure on the limited liability constraint $(\lambda_{LL,TO}(1 + \Psi(k_{it+1}, k_{it})))$, as additional investment reduces resources that can be distributed to shareholders.

With limited commitment, the optimality condition for investment is similar in that the first-order condition with respect to capital becomes

$$\begin{aligned} 1 + \Psi(k_{it+1}, k_{it}) &= \frac{1}{1+r} E_t[W_k(k_{it+1}, b_{it+1}(z_{it+1}), z_{it+1})] - \lambda_{LL}(1 + \Psi_{k_{it+1}}(k_{it+1}, k_{it})) \\ &\quad + E_t[\lambda_{BOR}(z_{it+1})\theta(1-\delta)] \end{aligned}$$

Here $\lambda_{BOR}(z_{it+1})$ are the Lagrange multipliers on the set of state-contingent collateral constraints, (5). The envelope conditon coincides with the corresponding expression in case of the trade-off model. Formally, thus, and perhaps unsurprisingly, the only way in which access to state contingent securities affects corporate financing manifests itself is through the appearance of the expectation of marginal values of of additional borrowing capacity across states. Differences in investment policies with respect to the trade-off model thus depend on the dispersion and distribution of shocks, and the marginal values of additional borrowing capacity or in other words, on financing constraints. It is in this context that the additional financial flexibility of state-contingent financing instruments is reflected in real outcomes.

Due to the slightly different timing assumption, the investment Euler equation with moral hazard looks somewhat differently, but captures similar economic trade-offs. More specifically, we obtain

$$E_{t-1}[(1-\tau)\pi_k(k_{it}, z_{it}, \eta_{it})] = \delta(1-\tau) + \Psi_{k_{it}}(k_{it}, k_{it-1}) +$$
$$\tfrac{1}{1+(1-\tau)r} E_{t-1}\left[\Psi_{k_{it}}(k_{it+1}, k_{it})\right] + \tfrac{E_{t-1}[\mathcal{D}_k(k_{it}, z_{it}, \eta_{it}, \widehat{\eta}_{it})\lambda^{IC}_{z_{it},\eta_{it}}]}{1+(1-\tau)r},$$

where $\lambda^{IC}_{z_{it},\eta_{it}}$ are the (scaled) multipliers on the incentive compatibility constraints (6). The expected higher marginal profits from additional capital are not only matched with the costs of adjusting the capital stock, but also by its effects on the expected marginal value of diversion, $\mathcal{D}_k(k_{it}, z_{it}, \eta_{it}, \widehat{\eta}_{it})\lambda^{IC}_{z_{it},\eta_{it}}$, across states. While in the limited commitment case, investing results in more resources shareholders can potentially abscond with, in the moral hazard case investing potentially creates additional cash flows shareholders can misreport and divert. It is through these agency conflicts that these frictions can have real impact. Our estimation will provide us with guidance about the magnitude of their effects across firms.

Financing The preceding paragraph showed how financial conditions affect real outcomes by distorting the investment Euler equations across models. We next examine the optimality conditions that determine financial positions directly.

The first order condition with respect to debt on the trade-off model is and

$$\lambda_{BOR,TO}(1 + r(1-\tau)) \quad = \quad \frac{1}{1+r} E_t[W_b(k_{it+1}, b_{it+1}, z_{it+1})] + \lambda_{LL,TO}$$

with envelope conditon

$$W_b(k_{it}, b_{it}, z_{it}) = r\tau - \lambda_{LL,TO} \left(1 + r(1-\tau)\right).$$

Debt financing thus trades a tightening in the collateral constraint $(\lambda_{BOR}(1 + r(1-\tau)))$ with expected tax savings $(r\tau)$ and the intertemporal effects of additional debt on the limited liability constraints. The current limited liability constraint is relaxed, which is a benefit (λ_{LL}), but debt service due next period tightens future limited liability constraints, which is a cost $(-\lambda_{LL}\left(1 + r(1-\tau)\right))$. Clearly, the relevant multipliers also appear in the investment Euler equation, which shows formally that investment and financing must be jointly determined. In other words, as is well-known, the Modigliani-Miller propositions do not apply here.

In case of limited commitment, state-contingent financing instruments now require a first order condition for each $b_{it+1}(z_{it+1}), \forall z_{it+1}$, which reads

$$\lambda_{BOR,LC}(z_{it+1})(1 + r(1-\tau)) \;\; = \;\; \frac{1}{1+r} W_b(k_{it+1}, b_{it+1}(z_{it+1}), z_{it+1}) + \lambda_{LL,LC}.$$

The envelope condition coincides with the trade-off case. Firms can now trade-off the tightness of borrowing constraints against tax savings and dynamic effects of limited liability constraints state by state. This additional financial flexibility effectively allows firms to choose repayment schedules across states in optimal ways. In other words, they can design a repayment schedule which reduces debt service in states in which they anticipate limited liability constraints to be especially tight, which could arise in response to both good cash flow shocks that may trigger investment funding needs, but also to bad cash flow shocks, which reduce available funds and valuations. Our estimation procedure infers the relative strength of these channels directly from observed policies.

The Euler conditions for financing in the moral hazard case comprise both an optimality condition for future equity values, and for divident payments to shareholders. These policies jointly implicitly determine the optimal payments to lenders, p_{it} and the value of debt b_{it}. These payments depend on the realized state characterized by z_{it} and η_{it}, which requires the Euler equations to hold state by state. In case of continuation values, we have, $\forall z_{it}, \eta_{it}$,

$$\overbrace{\lambda^{IC}_{z_{it},\eta_{it}}}^{\text{benefit: truth telling (IC)}} + \overbrace{\lambda^{LL,V}_{z_{it},\eta_{it}}}^{\text{benefit: outside option (LL)}}$$

$$= \overbrace{\frac{r\tau}{1+(1-\tau)r}}^{\text{cost: lost tax shield on debt}} + \overbrace{\lambda^{PK}\left(\frac{1}{1+(1-\tau)r} - \frac{1}{1+r}\right)}^{\text{cost: promising 1\$ less debt to the firm (PK)}},$$

while for dividend payouts, we get

$$\overbrace{\lambda^{IC}_{z_{it},\eta_{it}}}^{\text{benefit: truth telling (IC)}} + \overbrace{\lambda^{LL,D}_{z_{it},\eta_{it}}}^{\text{benefit: outside option (LL)}} = \frac{\overbrace{\lambda^{PK}}^{\text{cost: need to pay more to keep the promise (PK)}}}{1+r}$$

where $\lambda^{LL,D}_{z_{it},\eta_{it}}$ and $\lambda^{LL,V}_{z_{it},\eta_{it}}$ are the multipliers on the limited liability constraints for dividends , (7), and continuation values, (8), respectively. λ^{PK} is the multiplier on the promise keeping constraint, (5). While raising promised equity values to shareholders encourages truth-telling and relaxes the limited liability constraints, it reduces debt values, and therefore tax savings that accrue to the firm. The latter channel is manifested in a direct cash flow reductions ($\frac{r\tau}{1+(1-\tau)r}$), and in value losses due to discount rate adjustments ($\lambda^{PK}\left(\frac{1}{1+(1-\tau)r} - \frac{1}{1+r}\right)$). Similarly, promising higher dividends relaxes incentive and limited liability constraints, but tightens the promise keeping constraint.

3 Model Computation and Estimation

In this section, we describe the two key steps to bring the models we laid out in Section 2 to the data. Section describes the numerical solution method we use for model computation. Section presents the structural estimation procedure and the statistical tests we use for model comparison.

3.1 Solution Method: Linear Programming

The triplet of models we laid out in Section 2 has no closed-form solution. Thus, we solve the dynamic programs all of them numerically. It is immediate to verify that the Bellman operators for all three models are contraction mappings because Blackwell's sufficient conditions are satisfied, and theorems 9.6 and 9.8 in Stokey and Lucas (1989) apply. In addition, the numerical solution of the limited commitment and moral hazard models is computationally challenging. The presence of state-contingent policies introduces a large number of control variables that makes the curse of dimensionality excessively severe for standard iterative computational methods.[7]

We overcome this difficulty by adopting the linear programming (LP) representation of dynamic programming problems with infinite horizon (Ross (1983)), building on Trick and Zin (1993), and Trick and Zin (1997). We exploit and extend linear programming methods to efficiently solve for the value and policy functions. Linear programming methods, while common in operations research, have been introduced into economics and finance in Trick and Zin (1993, 1997). We follow **?** to extend the LP approach to setups common in dynamic corporate finance. More specifically, we exploit a separation oracle, an auxiliary mixed integer programming problem, to deal with large state spaces and find efficient implementations of Trick and Zin's constraint generation algorithm.

To start with, any finite dynamic programming problem with infinite horizon can be equivalently formulated as a linear programming problem (LP). The LP representation associates every feasible decision at each grid point on the state space with a constraint. Specifically, the three models can be formulated as LP problems as follows:

[7] Because of the presence of several occasionally non-binding collateral constraints, the models cannot be solved numerically by interior point methods. In principle, all models can be solved on a discrete grid by standard iterative methods as value and policy function iteration. However, as discussed above, the application of these methods to the contracting models in 2 is computationally problematic.

$$\min_{W_{k,u,z}} \sum_{k=1}^{nk} \sum_{u=1}^{nu} \sum_{z=1}^{nz} W_{k,u,z} \tag{9}$$

$$s.t.$$

$$W_{k,u,z} \geq R_{k,u,z,a} + \sum_{z'=1}^{nz} \beta Q(z'|z) W_{k'(a),u'(a),z'} \qquad \forall k, u, z, a, \tag{10}$$

where u denotes the promised utility variable, namely b_{it} for the tradeoff and the limited commitment model and v_{it} for the moral hazard model; nk, nu, and nz are the number of grid points on the grids for $k_{i,t}$, $u_{i,t}$, and $z_{i,t}$ respectively; $W_{k,u,z}$ is the value function on the grid point indexed by k, u and z, a is an index for a *feasible* action on the grid for both capital, promised utility, and payouts, and $R_{k,u,z,a}$ denotes the return function corresponding to the action a starting from the state indexed by k, u and z; β is the appropriate discount rate; $Q(z'|z)$ is the transition matrix of the Markov chain driving profitability shocks; $k'(a)$ and $u'(a)$ denote the future values for the state variables given the current firm's decisions. For a formal proof, we refer to Ross (1983).

The solution of the LP above would require to store an extremely large matrix, because state-contingent decisions render the number of constraints in the problem enormous. Precisely, the set of feasible actions a is a highly dimensional object for both the limited committment model (due to state-contingent debt repayments) and the moral hazard model (due to state-contingent dividends and promised equity values). Computational requirements would therefore be excessive. Thus, we implement constraint generation, a standard operation research technique to attack problems with a large number of constraints. First, we solve a relaxed problem with the same objective function. Second, we use the current solution to identify the constraints it violates. Third, we add one of the violated constraints, namely the most violated one, to the relaxed problem. We iterate the procedure until all constraints are satisfied.

To practically implement the constraint generation procedure, we need to deal with another computational issue. The selection of the most violated constraint involves searching over an extremely large vector of grid points for all the state-contingent control variables. The computational burden would still be excessive for the two contracting models we solve. To do so, we implement a separation oracle, an auxiliary mixed-integer programming problem to identifies the most violated constraint[8]. Appendix B outlines the constraint-generation algorithm and the separation oracle for the three models.

We implement the codes with Matlab® and CPLEX® as a solver for the linear and mixed-integer programming problems. Our workstation has a CPU with 24 cores and 124GB of RAM. The models are solved with five grid points for the idiosyncratic shock, 21 grid points for capital, 17 grid points for current promised utility. All control variables are choses on a continuous grid up to CPLEX numerical precision, which is 1e-6.

3.2 Estimation Method: Empirical Policy Functions

The dynamic corporate finance literature relies typically on two estimation methods: the Simulated Method of Moments or SMM, (Hennessy and Whited, 2005, 2007, Taylor, 2010) and the Simulated Maximum Likelihood or SML (Morellec, Nikolov, and Schuerhoff, 2012, 2016). We depart from the literature and rely on an alternative method that is Indirect Inference or II (Gourieroux, Monfort, and Renault, 1993).

Unlike, SMM or SML, II relies on an auxiliary model for estimating the structural parameters. The auxiliary model is an approximation of the true data generating process. Typically, this approach is suitable in cases where the likelihood is not in closed form and is computationally infeasible. In our case, we choose II as it constitutes a natural framework for comparing models. In particular, for the auxiliary model, we choose the model policy functions. The competing models that we consider do not share the same parameters, but their policy functions are common. We can thus compare in this framework both nested and non-nested models. Below we further motivate our choice of estimation method and describe its implementation.

3.2.1 Empirical Policy Functions

A policy function[9] is an association between an optimal choice of the firm, for example investment or financing, and its currently observable state. In accordance with this definition, we write the policy function as

[8] Separation oracles are standard tools in operation research, as described in Schrijver (1998) and Vielma and Nemhauser (2011).

[9] The exposition follows Bazdresch, Kahn, and Whited (2016).

$$\mathbf{w} = P(\mathbf{x}) \tag{11}$$

where $\mathbf{x}$ is the vector of state variables and $\mathbf{w}$ is the vector of policy variables of the model. For example, in our models k_{it}, b_{it}, and z_{it} re the state variables and k_{it+1}, b_{it+1}, and d_{it+1} are the control variables, so we have $\mathbf{x}_{it} = \{k_{it}, b_{it}, z_{it}\}$ and $\mathbf{w}_{it} = \{k_{it+1}, b_{it+1}, d_{it+1}\}$.

One challenge when working with policy functions is that some state or control variables are unobservable. We tackle this challenge by working with observable transformations of these variables. For example, the state variable z is unobservable. In this case, we use zk^{α}/k, firm profitability, that is observable.

We now characterize the empirical counterpart of the policy function $\mathbf{w} = P(\mathbf{x})$. One way to do so is linear approximation. This approach however will fail to capture the non linearities embeded in our models. We select then a semiparametric approach.

We consider the following specification for each control variable

$$\mathbf{w}_{it}^{n} = P^{n}(\mathbf{x}_{it}) + \mathbf{u}_{it}^{n} \tag{12}$$

where n is the n^{th} element of the policy vector $\mathbf{w}_{it}^{n}$, u_{it}^{n} is the specification error with $E[\mathbf{u}_{it}^{n} \mid \mathbf{x}_{it}] = 0$. We estimate the function $P(\mathbf{x}_{it})$

We use a series approximation functions $p_j(\mathbf{x}_{it})$ where $j = 1, ...J$ to estimate the policy function $P(\mathbf{x}_{it})$. In particular, as $J \to \infty$, the expected mean square difference between the $P(\mathbf{x}_{it})$ and a linear combination of $p_j(\mathbf{x}_{it})$ approaches zero, that is

$$\lim_{J \to \infty} E\left(\sum_{j=1}^{J} h_j p_j(\mathbf{x}_{it}) - P(\mathbf{x}_{it})\right)^2 . \tag{13}$$

We find that a power series with linear, quadratic, and all cross-products perfroms well. We have experimented with several alternative series functions. We observe that our results are imune to the particular choice of the series functions.

3.2.2 Structural estimation: Indirect inference

We use Indirect Inference to structurally estimate our set of models. The Indirect Inference method relies on an auxiliary model. While the auxiliary model is an approximation of the true data generating process, it captures the most important features of the data. Empirical policy functions constitute a natrual candidate for an auxiliary model as they characterize the solution of the model. Below we detail the estimation procedure.

We define the vector of observed data $\mathbf{v}_{it} \equiv (\mathbf{w}_{it}, \mathbf{x}_{it})$, where $i = 1, ..., N$ indexes firms and $t = 1, ..., T$ indexes time. Similarly, we define the vector of simulated data $\mathbf{v}_{it}^s$, where $s = 1, ...S$ is the number of times we simulate the model. The simulated data vector, $\mathbf{v}_{it}^s(\beta)$, depends on the vector of structural parameters β. We define the estimating equation as

$$g(\mathbf{v}_{it}, \beta) = \frac{1}{nT} \sum_{i=1}^{n} \sum_{t=1}^{T} \left[\mathbf{h}(\mathbf{v}_{it}) - \frac{1}{S} \sum_{s=1}^{S} \mathbf{h}(\mathbf{v}_{it}^s(\beta)) \right] \tag{14}$$

where $\mathbf{h}(.)$ is the parameter vector from (13) defining the empirical policy functions. The dimension of $\mathbf{h}$ is larger than the dimension of the vector of structural parameters β. The Indirect Inference estimator for β is given by

$$\widehat{\beta} = \arg\min_{\beta} g(\mathbf{v}_{it}, \beta)' \widehat{W}_{nT} g(\mathbf{v}_{it}, \beta)$$

where $\widehat{W}_{nT}$ is a positive definite weighting matrix that converges in probability to a deterministic postive definite matrix W.

3.2.3 Testing and Model Selection

We now build a set of tests that help us evaluate the relative performance of the competing models.[10] We consider two separate estimations for which we use the same set of data moments, $\mathbf{h}(\mathbf{v}_{it})$, but that use different economic models, so that the simulated data differ across estimations. Let the parameter vector from the first estimation be β_1, and let the parameter vector from the

[10]This Section is from Nikolov and Whited (2014) with minor notational modifications.

second estimation be β_2. We want to test the null hypothesis that $E(g(\mathbf{v}_{it}, \beta_1) - g(\mathbf{v}_{it}, \beta_2)) = 0$. Equation (14) implies that this test is equivalent to the test of the null that

$$H(\beta_1, \beta_2) \equiv \mathbf{h}\left(\mathbf{v}_{it}^s(\beta_2)\right) - \mathbf{h}\left(\mathbf{v}_{it}^s(\beta_1)\right) = 0. \tag{15}$$

Because the expression $H(\beta_1, \beta_2)$ in (15) is a function of (β_1, β_2), we can use a standard Wald test. The hurdle is calculating the asymptotic variance of $H(\beta_1, \beta_2)$ because it contains two parameter vectors that are estimated separately.

We use the influence function technique from Erickson and Whited (2002) To calculate the asymptotic variance of $H(\beta_1, \beta_2)$, we note that it equals, by definition, the asymptotic variance of the influence function of $H(\beta_1, \beta_2)$. To calculate this influence function we use the delta method as follows.

Let $G(\mathbf{v}_i, \beta)$ be the Jacobian of $g(\mathbf{v}_i, \beta)$. The influence function for observation i for β is given by

$$\psi_\beta = (G(\mathbf{v}_i, \beta)' W G(\mathbf{v}_i, \beta))^{-1} G(\mathbf{v}_i, \beta)' W g(\mathbf{v}_i, \beta).$$

Therefore, the delta method gives the influence function for (15) as

$$\begin{aligned} \psi_{H(\mathbf{v}_i, \beta_1, \beta_2)} &= G(\mathbf{v}_i, \beta_1)(G(\mathbf{v}_i, \beta_1)' W G(\mathbf{v}_i, \beta_1))^{-1} G(\mathbf{v}_i, \beta_1)' W g(\mathbf{v}_i, \beta_1) && (16) \\ &- G(\mathbf{v}_i, \beta_2)(G(\mathbf{v}_i, \beta_2)' W G(\mathbf{v}_i, \beta_2))^{-1} G(\mathbf{v}_i, \beta_2)' W g(\mathbf{v}_i, \beta_2). && (17) \end{aligned}$$

To calculate the variance of $H(\beta_1, \beta_2)$, we simply need to calculate the covariance of $\psi_{H(\mathbf{v}_i, \beta_1, \beta_2)}$. To simplify this calculation, we note that under the null, the influence functions for $g(\mathbf{v}_i, \beta_1)$ and $g(\mathbf{v}_i, \beta_2)$ both equal $\psi_{h(\mathbf{v}_i)}$, which is the influence function for the data moment vector $h(\mathbf{v}_i)$. Making this substitution we have

$$\begin{aligned} \psi_{H(\mathbf{v}_i, b_1, b_2)} &= \big(G(\mathbf{v}_i, \beta_1)(G(\mathbf{v}_i, \beta_1)' W G(\mathbf{v}_i, \beta_1))^{-1} G(\mathbf{v}_i, \beta_1)' W && (18) \\ &- G(\mathbf{v}_i, \beta_2)(G(\mathbf{v}_i, \beta_2)' W G(\mathbf{v}_i, \beta_2))^{-1} G(\mathbf{v}_i, \beta_2)' W\big)\, \psi_{h(\mathbf{v}_i)}. && (19) \end{aligned}$$

The variance of $H(\beta_1, \beta_2)$ can then be obtained by covarying the influence function $\psi_{H(\mathbf{v}_i,\beta_1,\beta_2)}$ with itself,

$$\text{avar}(H(\mathbf{v}_i, \beta_1, \beta_2)) = \frac{1}{n}\left(1 + \frac{1}{K}\right) E\left[\psi_{H(\mathbf{v}_i,\beta_1,\beta_2)}\psi'_{H(\mathbf{v}_i,\beta_1,\beta_2)}\right],$$

where the term $\left(1 + \frac{1}{K}\right)$ accounts for simulation error. Finally, the Wald test for the null that any particular element of $H(\mathbf{v}_i, \beta_1, \beta_2)$ is zero can be constructed as the ratio of that element to the corresponding element of $\text{avar}(H(\mathbf{v}_i, \beta_1, \beta_2))$. This statistic is distributed as a χ^2 with one degree of freedom.

A Wald statistic for the test that $\beta_1 - \beta_2 = 0$ can be derived analogously. As above, the delta method gives us

$$\text{avar}(\beta_1 - \beta_2) = \frac{1}{n}\left(1 + \frac{1}{K}\right) E\left[\left(\psi_{b_1} - \psi_{b_2}\right)\left(\psi_{b_1} - \psi_{b_2}\right)'\right].$$

The construction of the Wald tests proceeds exactly as above.

4 Empirical Results

4.1 Qualitative Analysis

4.2 Data and Sample Splits

Our main sample is drawn from the Compustat database for the 1965 to 2015 period. We exclude firms that do not have a stock exchange code (EXCHG) equal to 10 or 11. We remove firms that operate in the financial sector (four-digit SIC code between 4900 and 4999) or in regulated sectors (four-digit SIC code between 6000 and 6999 or between 9000 and 9999). We also drop firms with less than two consecutive year of data to be able to compute growth rates when required. Our resulting sample includes 59,143 firm/year observations.

We then compute the following variables. Investment is CAPX/AT; net book leverage is (DLTT+DLC-CHE)/AT; profitability is OIBDP/AT; dividends are (DVT+PSSTKC-PSTKRV)/AT; log size is the natural logarithm of PPENT; market-to-book is (DLTT+DLC+PRCCF x CHSO)/AT. Following, for example, Hennessy and Whited (2005) and Hennessy and Whited (2007), for the estimation procedure we remove firm fixed effect of each state and control variable. To reconcile the average levels of the control variables in the data and in the model, we add the sample mean into each variable after removing fixed effects.

To quantify the importance of the different types of frictions behind each model, we estimate the three models on different subsamples of firms. These subsamples are formed by splitting the sample in two on each on the (rescaled) state variables, namely (log) size, profitability, and leverage, and market-to-book[11]. Firms in the top and bottom 20th percentiles are respectively assigned to the subsamples with high and low values for the sorting variable.

In addition, we also consider a subsample of private firms. We ascertain data on US private firms from the Orbis database in the period from 2003 to 2012, as described in Kalemli-Ozcan, Sorensen, Villegas-Sanchez, Volosovych, and Yesiltas (2015). We remove observations with missing, zero, or negative values for total assets. Due to limited data availability, data on dividends are not

[11]We consider the sample split on book-to-market equity because the market value of equity appears as a state variable in the moral hazard model.

available. In addition, market-to-book ratios cannot be computed for private firms in that there are no market share prices for them. We proxy investment as the growth rate of total assets, leverage as the ratio of the difference between total assets and shareholder funds and total assets, log size as the natural logarithm of total assets, and profitability as the ratio between profit and losses before taxes and total assets. We drop firm-year observations with missing values for investment, leverage, and profitability. We are left with an unbalanced panel with 277,074 firm/year observations.[12]

Table 1 reports the summary statistics of the aforementioned variables in the model. We winsorize all variables at the one percent level. In line with several existing studies, as a fraction of total assets, the average profitability of our Compustat sample is around 15 %, investment around 7.5 %, net book leverage around 16 %, payouts around 2.2 %. The average log size is slightly above 5, while the market-to-book equity is around 1.3. Compared to this benchmark, some patterns are worth noting. In particular, the table suggests that large firms are more mature than small ones. This is reflected in a lower profitable, a higher leverage in line with the observation of Gomes and Schmid (2010), they pay more dividends and have a lower market-to-book ratio. More profitable firms, possibly because of persistence in investment opportunities, are typically smaller, invest more, are less levered, pay more dividends, and have higher market-to-book ratios. Highly levered firms invest more, are larger, less profitable and have lower market-to-book ratios than low leverage firms. Finally, in comparison to public firms, private firms are more profitable, significantly smaller, invest more and are far more levered, with a remarkable debt-to-asset ratio around 60 %. While this figure appears extremely high, it is in line with Huynh, Paligorova, and Petrunia (2012), who discuss that it is mainly driven by higher short-term leverage than public firms. In addition, private firms display a high heterogeneity across firms, as a standard deviation around thirty percent suggests.

[Insert Table 1 Here]

[12]Not surprisingly, private firms are numerous in comparison to public firms in a given time period.

4.3 Estimation Results

4.3.1 Compustat Sample

4.3.2 Small vs Large Firms

4.3.3 High Leverage vs Low Leverage Firms

4.3.4 High Dividend vs Low Dividend Firms

4.3.5 Private vs Public Firms

5 Conclusion

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Appendix

A. Equivalence Between Firm and Equity Value Maximization

In this appendix, we provide a formal proof of the equivalence between firm value maximization (FVM) and the equity value maximization (EVM) formulation of the trade-off and limited commitment models perhaps more common in the literature, along with a derivation of the first order conditions of all three models that we referred to in the main text. For tractability, we use the recursive formulations of the respective models throughout.

Tradeoff Model

The EVM formulation of the tradeoff model is the following:

$$\begin{aligned} V(k,b,z) \equiv \max_{k',b'} & (1-\tau)\pi(z,k) - k' + (1-\delta)k - \Psi(k,k') + \tau\delta k \\ & - (1+r(1-\tau))\,b + b' \\ & + \tfrac{1}{1+r} E_t[V(k',b',z')] \end{aligned} \qquad \text{[Equity Value]}$$

subject to

$$(1-\tau)\pi(z,k) - k' + (1-\delta)k - \Psi(k,k') + \tau\delta k - (1+r(1-\tau))\,b + b' \geq 0, \qquad \text{[Limited Liability] } [\lambda_{LL}]$$

$$(1+r(1-\tau))b' \leq \theta(1-\delta)k'. \qquad \text{[Borrowing] } [\lambda_{BOR}]$$

The first-order conditions with respect to capital and debt are respectively

$$\begin{aligned} 0 \;=\; & -1 - \Psi(k,k') + \frac{1}{1+r} E_t[V_k(k',b',z')] \\ & -\lambda_{LL}(1+\Psi_{k'}(k,k')) \\ & +\lambda_{BOR}\theta(1-\delta) \end{aligned}$$

and

$$\begin{aligned} 0 \;=\; & 1 + \frac{1}{1+r} E_t[V_b(k',b',z')] \\ & +\lambda_{LL} \\ & -\lambda_{BOR}(1+r(1-\tau)). \end{aligned}$$

The envelope conditions with respect to the current capital and debt stock are

$$\begin{aligned} V_k(k,b,z) \;=\; & (1-\tau)\pi_k(z,k) + (1-\delta) - \Psi_k(k,k') + \tau\delta \\ & -\lambda_{LL}((1-\tau)\pi_k(z,k) + (1-\delta) - \Psi_k(k,k') + \tau\delta) \end{aligned}$$

and

$$
\begin{aligned}
V_b(k,b,z) &= -\left(1+r(1-\tau)\right) \\
&\quad -\lambda_{LL}\left(1+r(1-\tau)\right).
\end{aligned}
$$

Below, we report the FVM formulation in equations (x)-(y) for convenience:

$$
\begin{aligned}
W(k,b,z) \equiv \max_{k',b'}&(1-\tau)\pi(z,k) - k' + (1-\delta)k - \Psi(k,k') + \tau\delta k \\
&-\left(1+r(1-\tau)\right)b + b' && && \text{[Firm Value]} \\
&+\left(1+r\right)b - b' && \text{[Debt Payout]} \\
&+\tfrac{1}{1+r}E_t[W(k',b',z')]
\end{aligned}
$$

subject to

$$
(1-\tau)\pi(z,k) - k' + (1-\delta)k - \Psi(k,k') + \tau\delta k - \left(1+r(1-\tau)\right)b + b' \geq 0, \qquad \text{[Limited Liability] } [\lambda_{LL}]
$$

$$
(1+r(1-\tau))b' \leq \theta(1-\delta)k'. \qquad \text{[Borrowing] } [\lambda_{BOR}]
$$

The first-order conditions with respect to capital and debt are respectively

$$
\begin{aligned}
0 &= -1 - \Psi(k,k') + \frac{1}{1+r}E_t[W_k(k',b',z')] \\
&\quad -\lambda_{LL}(1+\Psi_{k'}(k,k')) \\
&\quad +\lambda_{BOR}\theta(1-\delta)
\end{aligned}
$$

and

$$
\begin{aligned}
0 &= \frac{1}{1+r}E_t[W_b(k',b',z')] \\
&\quad +\lambda_{LL} \\
&\quad -\lambda_{BOR}(1+r(1-\tau)).
\end{aligned}
$$

The envelope conditions with respect to the current capital and debt stock are

$$
\begin{aligned}
W_k(k,b,z) &= (1-\tau)\pi_k(z,k) + (1-\delta) - \Psi_k(k,k') + \tau\delta \\
&\quad -\lambda_{LL}((1-\tau)\pi_k(z,k) + (1-\delta) - \Psi_k(k,k') + \tau\delta)
\end{aligned}
$$

and

$$
W_b(k,b,z) = r\tau - \lambda_{LL}\left(1+r(1-\tau)\right).
$$

The Euler equations for both investment and financing are equivalent for the EVM and for the FMV formulation. Specifically, the investment Euler equation for the EVM specification is

$$
\begin{aligned}
0 &= -1-\Psi(k,k')+\frac{1}{1+r}E_t\big[\big(1-\lambda_{LL}(z')\big)\left((1-\tau)\pi_k(z',k')+(1-\delta)-\Psi_k(k',k'')+\tau\delta\right)\big] \\
&\quad -\lambda_{LL}(1+\Psi_{k'}(k,k')) \\
&\quad +\lambda_{BOR}\theta(1-\delta),
\end{aligned}
$$

which coincides with the one for the FVM formulation, that is

$$
\begin{aligned}
0 &= -1-\Psi(k,k')+\frac{1}{1+r}E_t\big[\big(1-\lambda_{LL}(z')\big)\left((1-\tau)\pi_k(z',k')+(1-\delta)-\Psi_k(k',k'')+\tau\delta\right)\big] \\
&\quad -\lambda_{LL}(1+\Psi_{k'}(k,k')) \\
&\quad +\lambda_{BOR}\theta(1-\delta).
\end{aligned}
$$

The financing Euler equation for the EVM is

$$
\begin{aligned}
0 &= 1-\frac{1}{1+r}E_t[(1+\lambda_{LL}(z'))\,(1+r(1-\tau))] \\
&\quad +\lambda_{LL} \\
&\quad -\lambda_{BOR}(1+r(1-\tau)),
\end{aligned}
$$

which also coincides with the following one for the FVM formulation

$$
\begin{aligned}
0 &= \frac{1}{1+r}E_t[r\tau-\lambda_{LL}(z')\,(1+r(1-\tau))] \\
&\quad +\lambda_{LL} \\
&\quad -\lambda_{BOR}(1+r(1-\tau))
\end{aligned}
$$

because

$$
1-\frac{1+r(1-\tau)}{1+r}-E_t[\lambda_{LL}(z')\,(1+r(1-\tau))]=\frac{r\tau}{1+r}-E_t[\lambda_{LL}(z')\,(1+r(1-\tau))].
$$

This implies that the EVM and the FVM models are solved by the same optimal policies and establishes the equivalence between the two formulations.

Limited Commitment Model

The EVM formulation of the tradeoff model is the following:

$$\begin{aligned} V(k,b,z) \equiv \max_{k',b(z')} & (1-\tau)\pi(z,k) - k' + (1-\delta)k - \Psi(k,k') + \tau\delta k \\ & -(1+r(1-\tau))\,b + E_t[b(z')] \\ & + \tfrac{1}{1+r} E_t[V(k',b(z'),z')] \end{aligned} \qquad \text{[Equity Value]}$$

subject to

$$(1-\tau)\pi(z,k) - k' + (1-\delta)k - \Psi(k,k') + \tau\delta k - (1+r(1-\tau))\,b + E_t[b(z')] \geq 0, \qquad \text{[Limited Liability] } [\lambda_{LL}]$$

$$(1+r(1-\tau))b(z') \leq \theta(1-\delta)k'. \qquad \forall z' \qquad \begin{matrix}\text{[Borrowing]} \\ [p(z,z')\lambda_{BOR}(z')]\end{matrix}$$

The first-order conditions with respect to capital and state-contingent debt are respectively

$$\begin{aligned} 0 &= -1 - \Psi(k,k') + \frac{1}{1+r} E_t[V_k(k',b(z'),z')] \\ &\quad -\lambda_{LL}(1+\Psi_{k'}(k,k')) \\ &\quad +E_t[\lambda_{BOR}(z')\theta(1-\delta)] \end{aligned}$$

and, $\forall z'$,

$$\begin{aligned} 0 &= 1 + \frac{1}{1+r} V_b(k',b(z'),z') \\ &\quad +\lambda_{LL} \\ &\quad -\lambda_{BOR}(z')(1+r(1-\tau)). \end{aligned}$$

Below, we report the FVM formulation in equations (x)-(y) for convenience:

$$\begin{aligned} W(k,b,z) \equiv \max_{k',b(z')} & (1-\tau)\pi(z,k) - k' + (1-\delta)k - \Psi(k,k') + \tau\delta k \\ & -(1+r(1-\tau))\,b + E_t[b(z')] \\ & +\ (1+r)\,b - E_t[b(z')] \qquad \text{[Debt Payout]} \\ & + \tfrac{1}{1+r} E_t[W(k',b(z'),z')] \end{aligned} \qquad \text{[Firm Value]}$$

subject to

$$(1-\tau)\pi(z,k) - k' + (1-\delta)k - \Psi(k,k') + \tau\delta k - (1+r(1-\tau))\,b + E_t[b(z')] \geq 0, \qquad \text{[Limited Liability] } [\lambda_{LL}]$$

$$(1+r(1-\tau))b(z') \leq \theta(1-\delta)k'. \qquad \forall z' \qquad \begin{matrix}\text{[Borrowing]} \\ [p(z,z')\lambda_{BOR}(z')]\end{matrix}$$

The first-order conditions with respect to capital and state-contingent debt are respectively

$$\begin{aligned}0 \;=\;& -1-\Psi(k,k')+\frac{1}{1+r}E_t[W_k(k',b(z'),z')]\\ & -\lambda_{LL}(1+\Psi_{k'}(k,k'))\\ & +E_t[\lambda_{BOR}(z')\theta(1-\delta)]\end{aligned}$$

and, $\forall z'$,

$$\begin{aligned}0 \;=\;& \frac{1}{1+r}W_b(k',b(z'),z')\\ & +\lambda_{LL}\\ & -\lambda_{BOR}(z')(1+r(1-\tau)).\end{aligned}$$

The envelope conditions for both the EVM and the FVM coincide with those of the tradeoff model.

The Euler equations for both investment and financing are equivalent for the EVM and for the FMV formulation. Specifically, the investment Euler equation for the EVM specification is

$$\begin{aligned}0 \;=\;& -1-\Psi(k,k')+\frac{1}{1+r}E_t[\left(1-\lambda_{LL}(z')\right)\left((1-\tau)\pi_k(z',k')+(1-\delta)-\Psi_k(k',k'')+\tau\delta\right)]\\ & -\lambda_{LL}(1+\Psi_{k'}(k,k'))\\ & +E_t[\lambda_{BOR}(z')\theta(1-\delta)]\end{aligned}$$

which coincides with the one for the FVM formulation, that is

$$\begin{aligned}0 \;=\;& -1-\Psi(k,k')+\frac{1}{1+r}E_t[\left(1-\lambda_{LL}(z')\right)\left((1-\tau)\pi_k(z',k')+(1-\delta)-\Psi_k(k',k'')+\tau\delta\right)]\\ & -\lambda_{LL}(1+\Psi_{k'}(k,k'))\\ & +E_t[\lambda_{BOR}(z')\theta(1-\delta)]\end{aligned}$$

The financing Euler equations for the EVM are, $\forall z'$,

$$\begin{aligned}0 \;=\;& 1-\frac{1}{1+r}(1+\lambda_{LL}(z'))\,(1+r(1-\tau))\\ & +\lambda_{LL}\\ & -\lambda_{BOR}(z')(1+r(1-\tau)),\end{aligned}$$

which also coincide, $\forall z'$, with the following for the FVM formulation

$$\begin{aligned}0 \;=\;& \frac{1}{1+r}\left(r\tau-\lambda_{LL}(z')\,(1+r(1-\tau))\right)\\ & +\lambda_{LL}\\ & -\lambda_{BOR}(z')(1+r(1-\tau))\end{aligned}$$

because, $\forall z'$,

$$1-\frac{1}{1+r}(1+\lambda_{LL}(z'))\,(1+r(1-\tau))=\frac{1}{1+r}\left(r\tau-\lambda_{LL}(z')\,(1+r(1-\tau))\right).$$

This implies that the EVM and the FVM models are solved by the same optimal policies and establishes the equivalence between the two formulations.

Moral Hazard Model

Firm problem

$$\begin{aligned} W(k_{it-1},V_{it-1},z_{it-1}) \quad = \quad & \max_{k_{it},V_{z_{it},\eta_{it}},d_{z_{it},\eta_{it}}} \frac{1}{1+(1-\tau)r}\{-k_{it}-\Psi(k_{it},k_{it-1})+(1-\delta)k_{it}+\tau\delta k_{it}-r\tau V_{it-1} \\ & +E_{t-1}[(1-\tau)\pi(k_{it},z_{it},\eta_{it})+W(k_{it},V_{z_{it},\eta_{it}},z_{it})]\} \end{aligned}$$

subject to

$$V_{it-1}\geq\frac{1}{1+r}E_{t-1}[d_{z_{it},\eta_{it}}+V_{z_{it},\eta_{it}}] \qquad \left[PK:\frac{\lambda^{PK}}{1+(1-\tau)r}\right]$$

$$d_{z_{it},\eta_{it}}+V_{z_{it},\eta_{it}}\geq d_{z_{it},\widehat{\eta}_{it}}+V_{z_{it},\widehat{\eta}_{it}}+\mathcal{D}(k_{it},z_{it},\eta_{it},\widehat{\eta}_{it})\ \forall z_{it},\forall\widehat{\eta}_{it} \qquad \left[IC:\ \frac{Q(z_{it}|z_t)Q(\eta_{it})\lambda^{IC}_{z_{it},\eta_{it}}}{1+(1-\tau)r}\right]$$

$$d_{z_{it},\eta_{it}}\geq 0,\ V_{z_{it},\eta_{it}}\geq 0\ \ \forall z_{it},\forall\eta_{it} \qquad \left[LL:\ \frac{Q(z_{it}|z_t)Q(\eta_{it})\lambda^{LL,V}_{z_{it},\eta_{it}}}{1+(1-\tau)r};\frac{Q(z_{it}|z_t)Q(\eta_{it})\lambda^{LL,D}_{z_{it},\eta_{it}}}{1+(1-\tau)r}\right]$$

Envelope Conditions

[ENV k_{it-1}]

$$W_k(k_{it-1},V_{it-1},z_{it-1})=-\frac{1}{1+(1-\tau)r}\Psi_{k_{it-1}}(k_{it},k_{it-1})$$

[ENV V_{it-1}]

$$W_V(k_{it},V_{z_{it},\eta_{it}},z_{it})=\frac{-r\tau}{1+(1-\tau)r}+\frac{\lambda^{PK}}{1+(1-\tau)r}$$

FOC Investment

[FOC k_{it}]

$$E_{t-1}[(1-\tau)\pi_k(k_{it}, z_{it}, \eta_{it}) + W_k(k_{it}, V_{z_{it},\eta_{it}}, z_{it})] = \\ \Psi_{k_{it}}(k_{it}, k_{it-1}) + \delta(1-\tau) + \tfrac{E[\mathcal{D}_k(k_{it}, z_{it}, \eta_{it}, \widehat{\eta}_{it})\lambda^{IC}_{z_{it},\eta_{it}}]}{1+(1-\tau)r}$$

[EUL k_{it}]

$$E_{t-1}[(1-\tau)\pi_k(k_{it}, z_{it}, \eta_{it})] = \delta(1-\tau) + \Psi_{k_{it}}(k_{it}, k_{it-1}) + \\ \tfrac{1}{1+(1-\tau)r} E_{t-1}\left[\Psi_{k_{it}}(k_{it+1}, k_{it})\right] + \tfrac{E_{t-1}[\mathcal{D}_k(k_{it}, z_{it}, \eta_{it}, \widehat{\eta}_{it})\lambda^{IC}_{z_{it},\eta_{it}}]}{1+(1-\tau)r}$$

FOC Promised Equity Value

[FOC $V_{z_{it},\eta_{it}}$]

$$\tfrac{1}{1+(1-\tau)r} Q(z_{it}|z_t)Q(\eta_{it})W_V(k_{it}, V_{z_{it},\eta_{it}}, z_{it}) - \tfrac{\lambda^{PK}}{1+(1-\tau)r}\tfrac{1}{1+r} Q(z_{it}|z_t)Q(\eta_{it}) \\ + \tfrac{Q(z_{it}|z_t)Q(\eta_{it})\lambda^{IC}_{z_{it},\eta_{it}}}{1+(1-\tau)r} + \tfrac{Q(z_{it}|z_t)Q(\eta_{it})\lambda^{LL,V}_{z_{it},\eta_{it}}}{1+(1-\tau)r} = 0$$

that is

$$\frac{-r\tau}{1+(1-\tau)r} + \frac{\lambda^{PK}}{1+(1-\tau)r} - \lambda^{PK}\frac{1}{1+r} + \lambda^{IC}_{z_{it},\eta_{it}} + \lambda^{LL,V}_{z_{it},\eta_{it}} = 0$$

[EUL $V_{z_{it},\eta_{it}}$]

$$\overbrace{\lambda^{IC}_{z_{it},\eta_{it}}}^{\text{benefit: truth telling (IC)}} + \overbrace{\lambda^{LL,V}_{z_{it},\eta_{it}}}^{\text{benefit: outside option (LL)}} \\ = \overbrace{\frac{r\tau}{1+(1-\tau)r}}^{\text{cost: lost tax shield on debt}} + \overbrace{\lambda^{PK}\left(\frac{1}{1+(1-\tau)r} - \frac{1}{1+r}\right)}^{\text{cost: promising 1\$ less debt to the firm (PK)}}$$

FOC Payouts

[FOC $d_{z_{it},\eta_{it}}$]

$$-\frac{\lambda^{PK}}{1+(1-\tau)r}\frac{1}{1+r}Q(z_{it}|z_t)Q(\eta_{it}) + \frac{Q(z_{it}|z_t)Q(\eta_{it})\lambda^{IC}_{z_{it},\eta_{it}}}{1+(1-\tau)r} + \frac{Q(z_{it}|z_t)Q(\eta_{it})\lambda^{LL,D}_{z_{it},\eta_{it}}}{1+(1-\tau)r} = 0$$

that is

$$\overbrace{\lambda^{IC}_{z_{it},\eta_{it}}}^{\text{benefit: truth telling (IC)}} + \overbrace{\lambda^{LL,D}_{z_{it},\eta_{it}}}^{\text{benefit: outside option (LL)}} = \overbrace{\frac{\lambda^{PK}}{1+r}}^{\text{cost: need to pay more to keep the promise (PK)}}$$

B. Solution by Mixed-Integer Programming

The following constraint generation algorithm converges to the unique fixed point of our Bellman problems.

1. solve the problem in (9) with an initial random subset of constraints for each state (k, u, z);
2. if all constraints $a \in \Gamma^n(k, u, z)$, for all (k, u, z), are satisfied, terminate the algorithm (where $\Gamma^n(k, u, z)$ is the set of feasible actions at iteration n);
3. for each state (k, u, z), add the constraint $a \in \Gamma^n(k, u, z)$ that generates the highest violation in (10) with respect to the current solution $W^n(k, u, z)$;
4. solve the problem with the current set of constraints;
5. go back to step 2.

The separation oracles fot the three problems are specified as follows.

Definition 1 (Separation Oracle - Tradeoff)

$$\max_{a=\{k',b',d\}} d_{k,b',z} + \sum_{z'=1}^{nz} Q(z'|z)\frac{1}{1+r}W_{k'(a),b'(a),z'} - W_{k,b,z} \tag{A1}$$

$s.t.$

$$0 \leq b' \leq \frac{\theta k'(1-\delta)}{1+r} \quad \forall z' \tag{A2}$$

$$0 \leq p(i_k) \leq 1 \qquad \forall i_k = 1, ..., n_k \tag{A3}$$

$$\sum_{i_k=1}^{n_k} p(i_k) = 1 \tag{A4}$$

$$k' = \sum_{i_k=1}^{n_k} p(i_k)k^G(i_k) \tag{A5}$$

$$d_{k,b',z} = (1-\tau)\pi(z,k) - k' + (1-\delta)k - \Psi(k',k) + \tau\delta k - (1 + r(1-\tau))\, b + b' \tag{A6}$$

$$d_{k,b',z} \geq 0$$

Definition 2 (Separation Oracle - Limited Enforcement)

$$\max_{a=\{k',b(z'),d\}} d_{k,b(z'),z} + \sum_{z'=1}^{nz} Q(z'|z)\frac{1}{1+r}W_{k'(a),b'(a),z'} - W_{k,b,z} \tag{A7}$$

$s.t.$

$$0 \leq b(z') \leq \frac{\theta k'(1-\delta)}{1+r} \quad \forall z' \tag{A8}$$

$$0 \leq p(i_k) \leq 1 \qquad \forall i_k = 1, ..., n_k \tag{A9}$$

$$\sum_{i_k=1}^{n_k} p(i_k) = 1 \tag{A10}$$

$$k' = \sum_{i_k=1}^{n_k} p(i_k)k^G(i_k) \tag{A11}$$

$$d_{k,b(z'),z} = (1-\tau)\pi(z,k) - k' + (1-\delta)k - \Psi(k',k) + \tau\delta k - (1 + r(1-\tau))\, b + \sum_{z'=1}^{nz} Q(z'|z)b(z') \tag{A12}$$

$$d_{k,b(z'),z} \geq 0$$

Equations (A2) and (A8) define the bounds for debt; equations (A3) , (A4), (A9) and (A10) define the variables $p(i_k)$ that have the role to select a grid point for capital on the grid $k^G(i_k)$ and linearize the term k'^{α} in the adjustment cost function; equations (A5) and (A11) pick the grid point for the chosen capital stock from $k^G(i_k)$; equations (A6) and (A12) defines dividends. The computation of the law of motion for future debt is obtained by interpolation with the logarithmic formulation of Vielma and Nemhauser (2011).

C. Estimation Procedure

D. Variable Definition

Table 1
Descriptive Statistics

The table reports summary statistics for firms characteristics across the subsamples of firms in this paper. Data of public firms are from Compustat and cover an unbalanced panel of 59,143 firm/year observations from 1965 to 2015. Data on private firms are from Orbis and cover an unbalanced panel of 277,074 firm/year observations from 2003 to 2012. Subsamples of small vs large, profitable vs unprofitable, and high vs low leverage firms are obtained from Compustat. Firms are split according to firms characteristic in correspondence to the 20th and 80th percentile respectively. Size is measured as the value of properties, plants and equipment in million dollars, and profitability, dividends, and investment are scaled by total assets. μ denotes means, and σ denotes standard deviations. All variables are winsorized at the 1 percent level.

	Profitability		Investment		Leverage		Dividends		Log(Size)		Market/Book	
	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ
COMPUSTAT	0.152	0.061	0.075	0.044	0.162	0.148	0.022	0.048	5.320	0.980	1.357	0.677
Large	0.141	0.060	0.067	0.044	0.191	0.156	0.030	0.047	6.562	0.491	1.304	0.659
Small	0.163	0.073	0.081	0.055	0.135	0.172	0.011	0.052	3.878	0.745	1.427	0.923
Profitable	0.234	0.042	0.088	0.048	0.125	0.162	0.035	0.056	5.072	1.056	1.749	0.908
Unprofitable	0.071	0.044	0.062	0.050	0.191	0.170	0.009	0.049	5.429	1.091	1.050	0.633
High Leverage	0.138	0.066	0.078	0.052	0.364	0.099	0.020	0.056	5.458	1.073	1.288	0.689
Low Leverage	0.166	0.070	0.069	0.043	-0.039	0.095	0.024	0.050	5.084	1.098	1.496	0.842
Private	0.168	0.362	0.121	0.373	0.605	0.299	N/A	N/A	3.291	2.814	N/A	N/A

Table 2
Estimation: Full Sample

Panel A reports parameter estimates and standard errors for the tradeoff, limited commitment, and moral hazard models using empirical policy functions. All models are estimated on the full sample of Compustat firms. J-statistics are reported in Panel A, with p-values in parentheses. α denotes the curvature of the production function, f the fixed production cost, ρ_z the persistence of the profitability shock, σ_z the volatility of the profitability shock, δ the depreciation rate, ψ the capital adjustment cost parameter, θ the tangibility parameter in the tradeoff and limited commitment model, λ the diversion parameter in the moral hazard model, p the probability of a high realization of the private information shock in the moral hazard model, and η the corresponding spread in profits. Panel B reports data and simulated moments for the Compustat sample and for the three models. Compustat data cover an unbalanced panel of 59,143 firm/year observations from 1965 to 2015.

Panel A: Parameters			
	Tradeoff	Limited Commitment	Moral Hazard
α	0.789	0.761	0.623
	(0.002)	(0.000)	(0.213)
f	39.172	65.574	1.602
	(0.004)	(1.691)	(0.263)
ρ_z	0.623	0.594	0.879
	(0.004)	(0.008)	(0.006)
σ_z	0.255	0.228	0.269
	(0.005)	(0.708)	(0.174)
δ	0.135	0.104	0.627
	(0.004)	(0.653)	(0.005)
ψ	0.467	0.454	0.023
	(0.006)	(2.654)	(0.000)
θ	0.454	0.467	
	(0.002)	(0.362)	
λ			0.285
			(0.789)
p			0.552
			(0.382)
η			0.112
			(0.260)
J-Test χ^2	96.378	65.162	160.617
	(0.000)	(0.000)	(0.000)

Panel B: Moments				
	Data	Tradeoff	Limited Commitment	Moral Hazard
Mean Profitability	0.152	0.142	0.108	0.165
St. Dev. Profitability	0.061	0.002	0.001	0.186
Autocorrelation Profitability	0.442	0.560	0.559	0.447
Mean Investment	0.075	0.133	0.102	0.107
St. Dev. Investment	0.044	0.004	0.003	0.051
Autocorrelation Investment	0.328	0.282	0.274	0.745
Mean Leverage	0.162	0.219	0.311	0.576
St. Dev. Leverage	0.148	0.006	0.002	0.035
Autocorrelation Leverage	0.510	0.853	0.833	0.140
Mean Payout	0.022	0.024	0.016	0.012
St. Dev. Payout	0.048	0.001	0.000	0.031
Autocorrelation Payout	0.340	0.641	0.645	0.001

Table 3
Model Comparison

The table reports the results of two-way and three-way statistical Wald tests to compare the tradeoff, limited commitment, and moral hazard model on different subsamples of firms. TO denotes the tradeoff model, LC denotes the limited commitment model, and MH the moral hazard model. "Tie" indicates a non-statististically distinguishable performance between models. Data of public firms are from Compustat and cover an unbalanced panel of 59,143 firm/year observations from 1965 to 2015. Data on private firms are from Orbis and cover an unbalanced panel of 277,074 firm/year observations from 2003 to 2012. Subsamples of small vs large, profitable vs unprofitable, and high vs low leverage firms are obtained from Compustat. Firms are split according to firms characteristic in correspondence to the 20th and 80th percentile respectively. Size is measured as total assets in million dollars, and profitability, dividends, and investment are scaled by total assets. All variables are winsorized at the 1 percent level.

	Comparison			
	TO vs LC	TO vs MH	LC vs MH	Best Fit
COMPUSTAT	LC	TO	LC	LC
Small	Tie	TO	LC	TO/LC
Large				
Profitable				
Unprofitable				
Low Leverage				
High Leverage				
Private	Tie	MH	MH	MH