Changing Business Cycle Dynamics in the US: The Role of Women's Employment

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Abstract

I examine the effects of female labor market behavior on the dynamics of aggregate employment and hours in the trend and over the business cycle in the US. I argue that the steep increase in women's labor force participation throughout the 1970s and 1980s, and its flattening out since the 1990s can contribute to explain three puzzling phenomena experienced by the US: the non-stationarity of aggregate hours, the decrease business cycle volatility of labor market variables before the great recession, and the recent jobless recoveries. To develop the analysis, I first construct an aggregate time series for hours by gender similar to the series used in in aggregate business cycle analysis, and provide descriptive empirical evidence supporting the hypothesis proposed in this paper. I then develop and estimate a dynamic stochastic general equilibrium model that allows for gender differences in hours and wages to assess the implications of the changing trend in female participation on the behavior of aggregate variables. I find that female specific shocks explain a substantial fraction of the volatility of aggregate outcomes, both at the business cycle frequency and in the longer run. Using a number of counterfactuals, I show that the trend in female participation and its variation over time can rationalize the three phenomena of interest. These findings have implications for the behavior of aggregate labor market variables in other advanced economies where female labor force participation is experiencing secular changes.

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1 Introduction

The rise in women's market work is one of the most notable economic developments in the post-war period in the United States. Female participation rose from 37% in 1960 to a peak of 61% in 1997, and then flattened out since then (figure ??). This phenomenon led to a sharp rise in the share of female hours, contributing substantially to the rise in aggregate hours per person in the US in the 1970s and 1980s (figure ??). While a large literature has studied the determinants of this rise, the implications of this phenomenon for aggregate business cycle dynamics have been left largely unexplored.

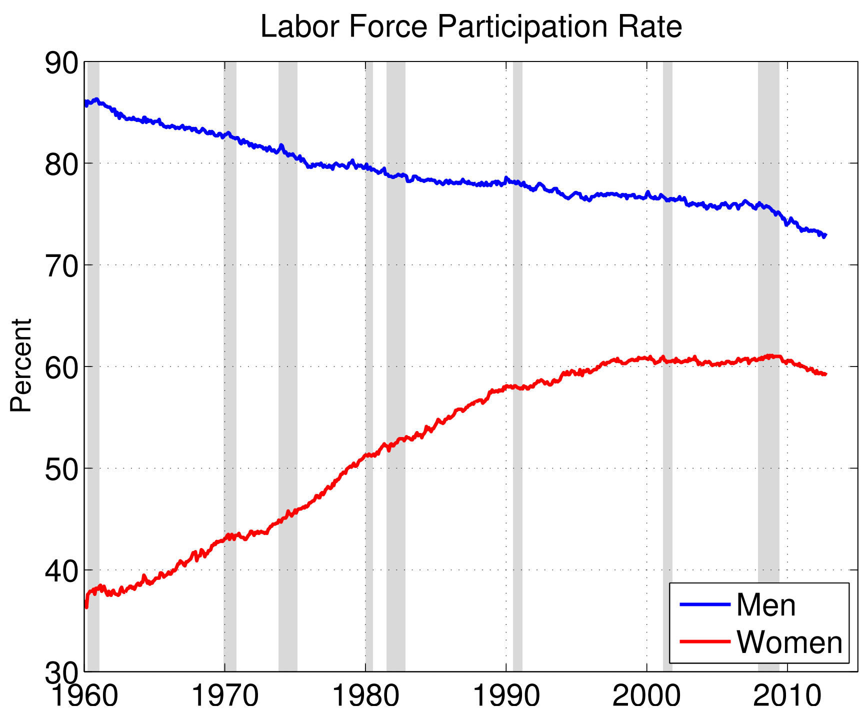

FIGURE 1: Labor force participation rate by gender. Source: Current Population Survey.

In this paper, we seek to link the historic rise in women's labor supply and its flattening out since the early 1990s to several important phenomena in US business cycles.

2 Hypotheses

Decline in the Business Cycle Volatility of Aggregate Hours Our first hypothesis is that the great moderation[1] - the decreased cyclical volatility observed over through the later part of the century- can be explained in part by the sharp increase in the hours worked by women throughout the 1970s and 1980s. The hours of women are less volatile than those of men at a cyclical frequency and exhibit a strong positive trend until the mid-1990s, as can be seen in figure ??, which plots

[1] See [?] for an excellent review of the evidence.

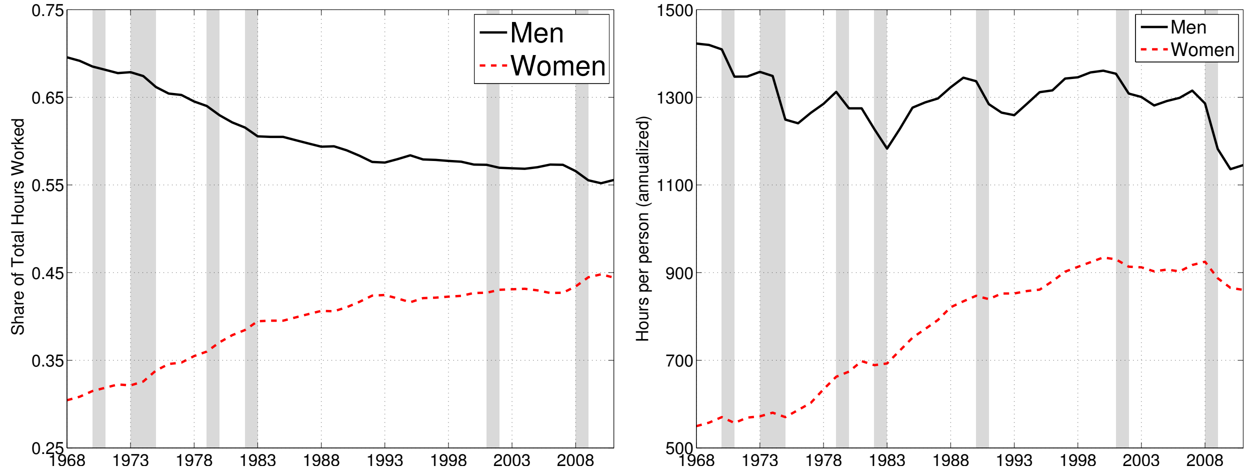

FIGURE 2: Left: The share of hours worked by gender. Right: hours per person. Source: CPS March Supplement.

the cyclical and trend component of female and male hours per capita. The increase in female participation precipitated the rise in the fraction of hours coming from women, thereby reducing the sensitivity of aggregate hours to business cycle fluctuation throughout this period.

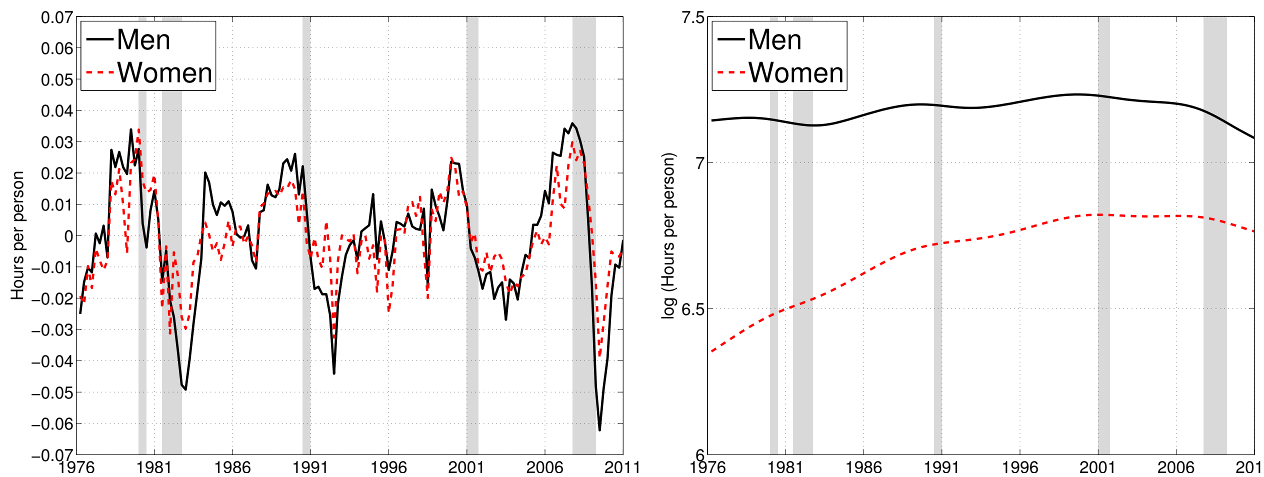

FIGURE 3: Cyclical component (left) and trend component (right) of quarterly hours by sex. Source: Monthly CPS.

As shown in Table 1, the ratio of the standard deviation of the cyclical component of male hours to the standard deviation of the cyclical component of real GDP is 1.15 in 1969-1984, while it is only 0.76 for women. In 1985-2005, these ratios are 1.41 for men and 0.83 for women, respectively, suggesting an increase in the relative volatility of hours for men at the business cycle frequency across the two periods. The standard deviations of aggregate hours and aggregate employment drop by more than 50% in 1985-2005 relative to 1969-1984, suggesting that the rise in the share of female hours played an important role in the decline in the cyclical sensitivity of aggregate hours, and possibly in the decline in the standard deviations of real GDP. Men's hours also display higher contemporaneous correlation with GDP throughout the sample period, which is consistent with a weaker cyclical sensitivity of female hours.

Table 1: Business Cycle Volatility Statistics

Cyclicality and Volality of Hours, Aggregate and by Gender

	1969-1984		1985-2005	
	std / std GDP	Corr. With GDP	std / std GDP	Corr. With GDP
GDP	1.00	1.00	1.00	1.00
hours total	0.99	0.89	1.10	0.83
hours men	1.15	0.91	1.41	0.84
hours women	0.76	0.77	0.83	0.61
hours men (fitted)[1]	1.21	0.90	1.30	0.85
hours women (fitted)[1]	0.67	0.81	0.81	0.72

Relative Volatility of female and Male hours

	1969-1984		1985-2005	
	hours	hours (fitted)[1]	hours	hours (fitted)[1]
std female/std male	0.66	0.55	0.59	0.63

Moderation in amplitude of business cycle fluctuations

	GDP	Aggregate Hours (Macro Data)	Aggregate Hours (Micro Data)	Total E/P
Std. Dev. For 1985-2005 over 1969-84	0.47	0.68	0.52	0.56

[1]The fitted hours series are the fitted values of a regression of gender specific hours on GDP and the aggregate version of hours. This specification produces a series for hours with variation coming solely from business cycle movements.

Note: All calculations above are performed on the hp-filtered detrended data using $\lambda = 6.25$.

Jobless Recoveries Our second hypothesis is that the recent jobless recoveries ([?] and [?]) can also be explained at least in part by the large rise in female hours through the 1970s and 1980s and their subsequent slowdown. The strong upward trend in women's labor force participation through the 1970 and 1980s was mirrored in their employment growth and hours per worker, which masked the relatively weak recoveries in the employment and hours of men for cycles in that period. When female participation stopped growing in the early 1990s, the cyclical variation in female hours started to resemble that of men, resulting in very weak growth in aggregate employment and hours in cyclical recoveries.

Evidence supporting this hypothesis is displayed in figure **??**. These charts plot the cumulative changes in the *log* of hours per capita from the unemployment through of the preceding expansion, ending three years after the unemployment peak. As figure **??** shows, in the 1970, 1974-75 and 1981-82 recessions, as women were experiencing a strong positive trend in labor force participation, hours per capita for women drop only slightly if at all during the recession, and experience a strong growth in the recovery. The 1990-91 cycle marks a change in this pattern, as the trend growth in female participation slowed substantially. While the decline in hours for women during the recession is still modest compared to men's, the recovery is weaker than in previous cycles, and also weaker than men's for the 1990-91 cycle. In the 2001 and 2007-09 cycles—when the upward trend in female participation had completely stopped—the behavior of female hours per capita is very similar to men's, except for a more modest amplitude of the fluctuation.[2] Figure **??** also shows that the behavior of male hours was very similar in all these cycles, with the only variation driven by the severity of the recession. Hours per capita always declined for men during recessions and did not regain pre-recession values except for the 1981-82 cycle, with the recovery particularly sluggish in the most recent cycles. These figures complement what we have seen in figure **??** and show that the changing trend in female participation is reflected in a change in the cyclical behavior for hours for women starting in early 1990s, while for men both the trend and the cyclical behavior of participation has been similar since 1970.

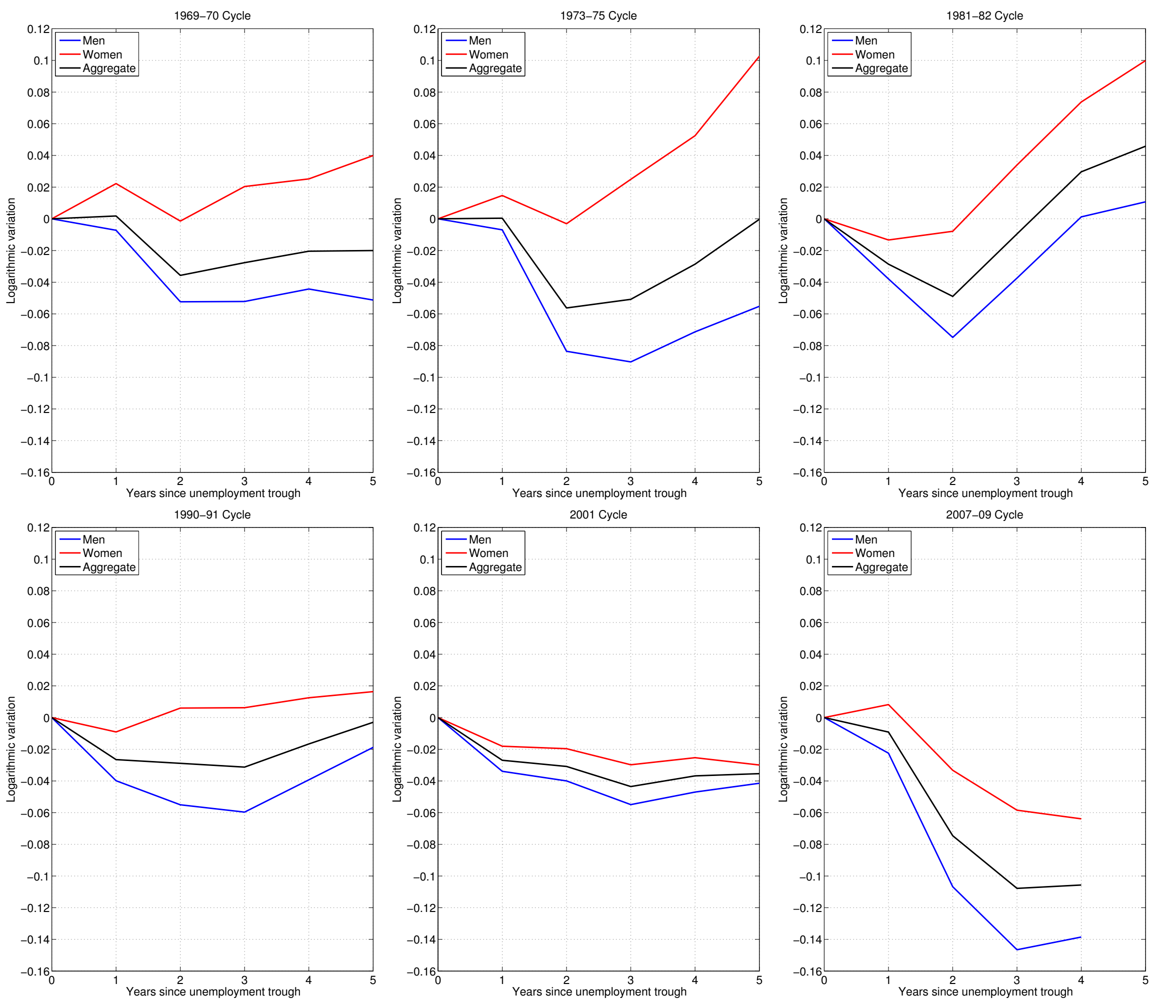

FIGURE 4: Log changes in hours per capita in the aggregate and by gender. Source: Bureau of Labor Statistics.

To assess the effects of the changing behavior of female hours on aggregate hours we now present

[2] [**?**] show that the smaller decline in female hours during the 2007-2009 recession is mostly accounted for by gender differences in the industry distribution.

some simple counterfactuals. Specifically, we compute counterfactual *aggregate* hours per capita for recent cycles, by replacing the *female* growth rate of hours per capita in each of the last three cycles with the average growth rate of female per capita hours in the early cycles at each date. We keep the evolution of male hours per capita as it is in the data, and compute the counterfactual aggregate using period specific population weights. The results are presented in Figure **??**. The scaled counterfactual multiplies the average change in female hours per capita in the early cycles by a factor to match the size of the drop in female per capita hours at the peak of the recession in each of the recent cycles, while the unscaled counterfactual omits this transformation.

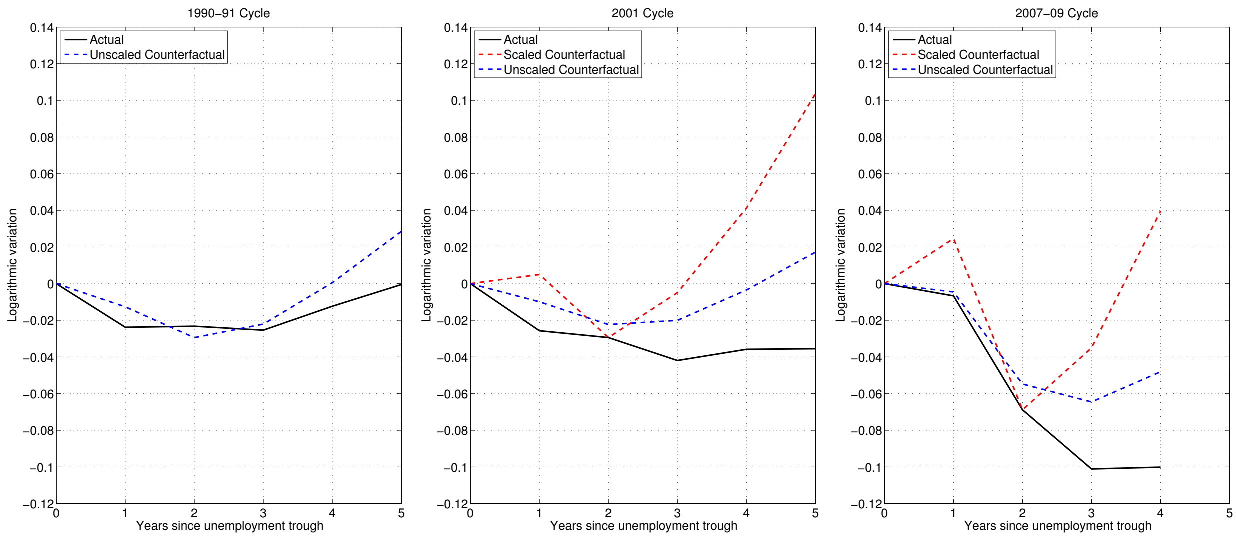

FIGURE 5: Female hours per capita counterfactual: Female hours per capita replaced with average for early recessions.

These counterfactuals reveal that the change in female hours behavior is an important factor in driving aggregate hours during recoveries, especially for the 2001 and 2007-09 cycles. For the 1990-91 cycle, the growth in counterfactual employment is approximately 3 percentage higher than the actual at the end of the cycle.[3] For the 2001 cycle, the unscaled counterfactual is 5 percentage points higher, while the scaled counterfactual is 13 percentage points higher, reflecting the very mild decline in employment during the recession for this cycle. For the 2007-2009 cycle, scaled counterfactual aggregate hours per capita is 4 percentage points above pre-recession levels by the end of the window, and the unscaled counterfactual is 5 percentage points above the actual by the end of the window. Thus, if female hours growth had continued to exhibit the behavior seen for the early cycles, recoveries from the the last three recessions would not have been jobless.

The changes are solely due to the variation in behavior of female hours. The same exercise with male hours show no difference between actual and counterfactual values of aggregate hours per capita, as seen in figure **??**.[4]

[3]The scaled counterfactual cannot be computed for the 1990-91 cycle as female hours per capita rise during the recession for this cycle.

[4]The 2007-09 cycle has the unique feature that hours per capita and other employment indicators continued to decline well after the unemployment rate peaked. This explains the earlier pick up in hours in the counterfactual

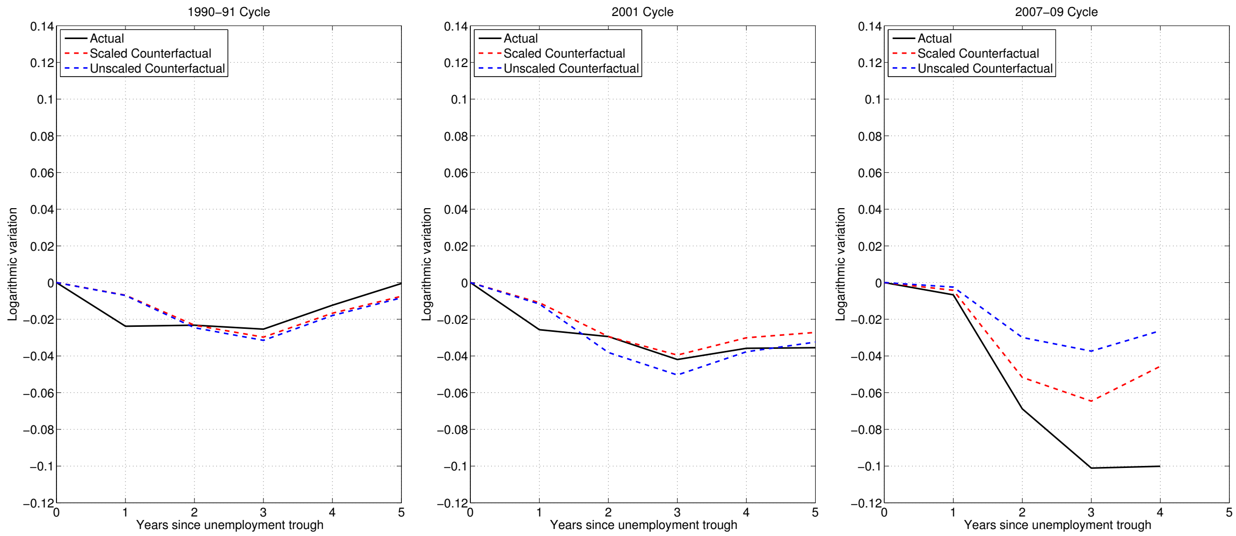

FIGURE 6: Male hours per capita counterfactual: Male hours per capita replaced with average for early recessions.

Non-stationarity of Aggregate Hours Our third hypothesis is related to the stationarity of hours in the post-war period. There is some debate on the stationarity of aggregate hours per capita in the empirical macroeconomics literature, because this property of hours is key in determining their response to technology shocks (see [**?**] and [**?**] and [**?**]). Figure **??** shows that male hours per capita have been stationary throughout the sample period. In light of the growth in female hours per capita and the rising share of female hours in total hours, it is not surprising that aggregate hours per capita should display trend like growth in the 1970s and 1980s. However, in the period since 1990, when the major changes in the labor market behavior of women had concluded, the female hours per capita also appear to be stationary.

To examine the role of increased female labor supply on aggregate hours, we resort again to a simple counterfactual, displayed in figure **??**. Hours per capita are computed as:

$$\frac{H_t^j}{P_t^j} = h_t^j \times w_t^j \times E_t^j / P_t^j, \tag{1}$$

where h_t^j denotes average hours worked per week and w_t^j average weeks worked per year for gender $j = f, m$. The evolution over time of each of these components is displayed in figure **??**.

The solid black line in figure **??** represents aggregate hours per capita. We impose that share of female hours in the total (Intensive Margin, red dashed line) and the share of female employment (Extensive Margin, blue dashed line) in the total remain constant at their 1968 values and compute the resulting path for aggregate hours. Clearly, the growth in female hours on the intensive margin, and especially the growth in female employment contributed substantially to the trend growth in aggregate hours per capita from the mid 1970s to the mid 1990s. Combined (Both, black dashed

series for both the female and male counterfactuals, and the smaller cumulative drop in aggregate hours per capita for the counterfactual relative to the actual.

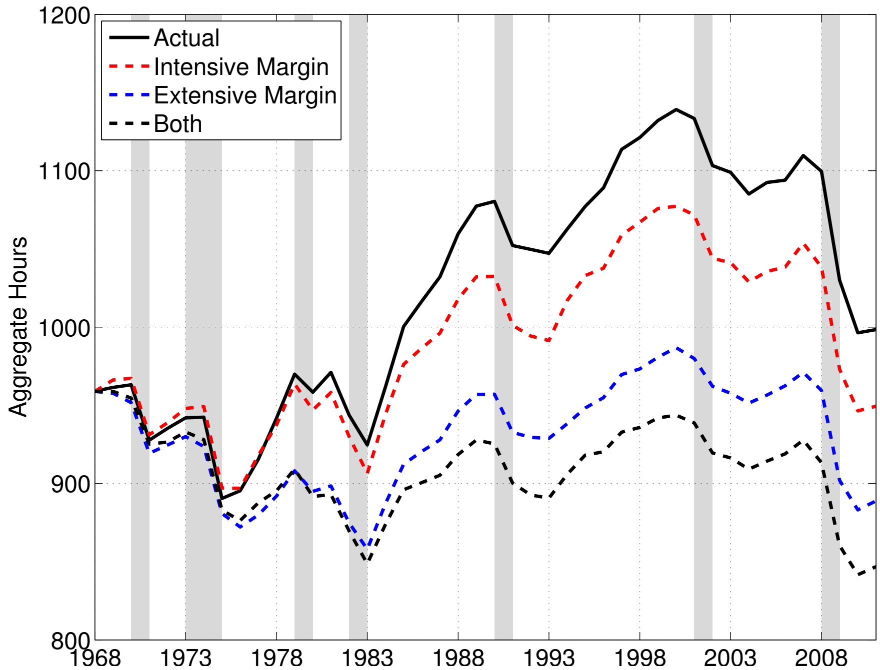

FIGURE 7: Counterfactual evolution of aggregate hours per person. Source: CPS March Supplement.

line) they can account for virtually all the trend in aggregate hours over that period, making the counterfactual series stationary.

The large contribution of the extensive margin is not surprising, given that employment has registered the largest degree of convergence across genders, as shown in figure **??**.

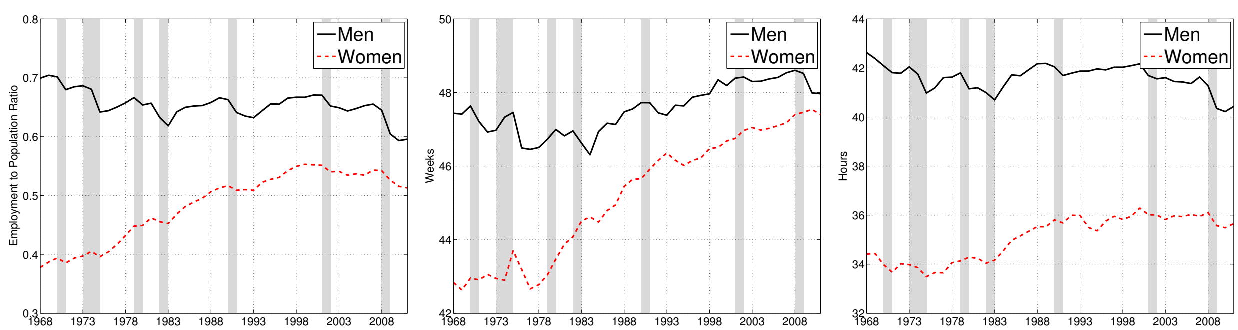

FIGURE 8: Left: Extensive margin, E/P. Center: Intensive margin, average weeks worked per year. Right: Intensive margin, average number of hours worked per week. Source: CPS March Supplement.

3 Contributions and Relation to the Existing Literature

There is an extensive literature on the rise in women's market work and the increase in their wages relative to men. Many of these explanations have focussed on the role of technological advances in increasing both the supply and the demand of female labor. On the supply side, Goldin and Katz (2002) [?] show that the diffusion of oral contraceptives reduced the costs and increased the returns to women's education, contributing to a rise in their participation and wages. Greenwood, Seshadri and Yorugoklu (2005) [?] argue that advances in home appliances increased female participation by reducing the time required for home production. Albanesi and Olivetti (2009) [?] show that improvements in maternal health and the introduction of infant formula are key to explaining the rise in participation of married women with children, and the rise in women's education and wages relative to men.[5]

On the demand side, Galor and Weil (1996) [?] attribute the rise in women's market work and the growth in their relative wages to technological innovations that increase the returns to intellectual rather than physical skills, in which women have a biological comparative advantage. Empirical evidence also supports this notion.[6] Rendall (2010) [?] shows that, as job requirements have shifted from more physical to more intellectual attributes, women, who always worked in occupations with relatively low physical requirements, also shifted from occupations with low to high intellectual requirements.

Compared to the macroeconomic literature on the rise in female labor supply and the decline in the gender wage gap, which attempts to spell out the detailed mechanisms that fueled the shift of female labor supply from the home to the market and the technological determinants of the rise in female relative wages, our approach takes as given the differences in productivity and disutility of labor between men and women and focuses instead on quantifying their relative role in producing the observed outcomes. Our estimation procedure, therefore, can be thought of as a device to back out the processes for the relative productivity and disutility of labor between men and women that rationalize both the evolution in female labor supply and of the gender wage gap in the last 40 years. Our results can be then used to examine the link between these phenomena and the changing behavior of aggregate labor market outcomes in the trend and at the cyclical frequency.

The objective of our analysis is two-fold. First, we wish to document the extent to which a business cycle model that ignores the distinction between female and male hours can be misleading, even as a tool to interpret standard aggregate macroeconomic facts. In particular, jobless recoveries in recent cycles and the great moderation are seen as puzzling, but they are in part a reflection of the historic path in the growth of female participation. Similarly, the possible non-stationarity of hours has been much debated in the literature, but it is hard to account for this phenomenon over the last fifty years without thinking about the contribution coming from the rise in female participation. Second, the model allows us to decompose the observed patterns in hours worked

[5] Little work has been done on the flattening out of female participation in the 1990s. Albanesi and Prados (2011) [?] show that the flattening out is driven by a decline in participation of prime age women with college degrees, and those married to high earning husbands. They identify the acceleration of the rise in the skill premium in the 1990s, and the corresponding sharp rise in top incomes for men, as the main contributor the the decline in the growth of female participation. Fernandez (2012) [?] and Fogli and Veldkamp (2012) [?] explore learning models in which women's participation rises as the perceived costs of women's work fall over time. These learning models also predict a flattening out of the rise in participation at the end of the learning process.

[6] Black and Juhn (2000) [?] argue that the rising demand for skilled workers may have contributed to the rise in participation of skilled women, and to the increase in the fraction of women in professional and managerial occupations, which were traditionally male. Additionally, Black and Spitz-Oener (2007) [?], find that women have witnessed relative increases in non-routine analytic tasks and non-routine interactive tasks, associated with higher skill levels.

and the relative wage into components due to demand and supply factors, both in the trend and at the cyclical frequency. This decomposition can provide a useful reference point for the more detailed modeling of the underlying drivers of the increased labor market participation and relative wages of women, and, perhaps most importantly, it allows us to better understand and predict aggregate responses of hours, employment and wages to macroeconomic shocks and government policies.

4 Research Outline

Our analysis is based on a rigorous empirical assessment of these hypotheses, using newly constructed monthly aggregate hours and wage series by gender based on micro-level data from the Current Population Survey. The resulting overall hours and employment series are consistent with the measures of aggregate hours and employment typically used in business cycle analysis.

To study the macroeconomic implications of the changing behavior of female hours over time, and to gain some further insight into their underlying drivers, we estimate a real dynamic stochastic general equilibrium (DSGE) model, in which we distinguish between male and female hours worked. Unlike in most of the business cycle literature, men and women's hours are not perfect substitutes in our model, and instead enter the production function as a CES aggregate with different marginal productivities, which fluctuate over time. Moreover, we assume that men and women differ in their marginal disutility of labor, both because of different Frisch elasticities of labor supply—a well documented empirical fact—as well as because of a different "taste" for market work, which we model as an exogenous stochastic process.

We use standard methods to estimate the model. Since female hours exhibit a strong positive trend up until the early 1990s, when they become stationary, in our estimation we take the period between 1995 and 2004 to correspond to a balanced growth path where only aggregate variables display a trend component, while we allow for gender specific trends in prior years.

We provide a brief description of the model in Section **??** and discuss its main qualitative properties. In Section **??**, we present estimation results for the model estimated for quarterly data from 1984 to 2004. These preliminary results highlight the importance of the novel features introduced in the environment, namely, the gender specific shocks, to account for business cycle variation in the aggregate variables. In ongoing work, we are separately estimating the model on the 1984-1994 (pre female labor force participation flattening) and post 1994, to identify structural changes in the estimated processes for the gender specific shocks. We well use these results to run counterfactuals intended to quantify and illustrate the role of changing female labor force participation for the properties of aggregate business cycles in the US.

4.1 Framework

We adapt a real variant of the business cycle model with investment specific productivity shocks and variable capital utilization in [**?**]. The economy is populated by a representative household and perfectly competitive representative firms. Households have female and male members, who share consumption but exhibit gender specific disutility for supplying labor. Female and male labor are both used in production, as well as capital. There are time varying gender specific productivity parameters, in addition to aggregate total factor productivity growth.

Households The economy is populated by a representative household comprising a continuum of unit measure of agents of different gender. A fraction p_t^j of the population is of gender $j = f, m$, where $\sum_{j=f,m} p_t^j = 1$. The household utility function is:

$$E_0 \sum_{t=0}^{\infty} \beta^t b_{t+s} \left[log\left(C_t - \eta C_{t-1}\right) - \sum_j p_t^j \varphi_t^j \frac{\left(H_t^j\right)^{1+\nu^j}}{1+\nu^j} \right],$$

where C_t is per capita consumption, η is the degree of habit formation and b_t is a shock to the discount factor, which follows the stochastic process:

$$\log b_t = \rho_b \log b_{t-1} + \varepsilon_{b,t},$$

with $\varepsilon_{b,t} \sim i.i.d.\, N(0, \sigma_b^2)$. H_t^j is per capita hours for agents of gender j and φ_t^j is a shock to the disutility of working, which is gender specific. These shocks follow a stochastic process described in section **??**.

This formulation of household utility reflects the assumption that there is risk sharing within the household, so that all its members, males and females, consume the same amount. The disutility from working, on the other hand, accrues to each member individually, so that members who work fewer hours have a lower marginal disutility, at least for given $\tilde{\varphi}_t^j$ and inverse Frisch elasticity ν^j.

The household's budget constraint is:

$$C_t + I_t + T_t \leq \sum_j p_t^j W_t^j H_t^j + r_t^k u_t \bar{K}_{t-1} - a(u_t)\bar{K}_{t-1},$$

where T_t is lump-sum taxes, w_t^j is the real wage for gender j and r_t^k is the rental rate of effective capital. Effective capital is:

$$K_t = u_t \bar{K}_{t-1},$$

where u_t is a utilization rate set by the household, at cost $a(u_t)$ per unit of physical capital $\bar{K}_t$. In steady state, $u = 1$, $a(1) = 0$ and $\chi \equiv \frac{a''(1)}{a'(1)}$. In the log-linear approximation of the model solution this curvature is the only parameter that matters for the dynamics.

The physical capital accumulation equation is:

$$\bar{K}_t = (1-\delta)\bar{K}_{t-1} + \mu_t \left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right) I_t,$$

where δ is the depreciation rate and the investment shock μ_t follows the stochastic process:

$$\log \mu_t = \rho_\mu \log \mu_{t-1} + \varepsilon_{\mu,t},$$

with $\varepsilon_{\mu,t}$ $i.i.d.N(0,\sigma_\mu^2)$. The function S captures the presence of adjustment costs in investment, as in [?]. In steady state, $S = S' = 0$ and $\varsigma \equiv S'' > 0$.

Firms The production function, also expressed in per capita terms, is:

$$Y_t = K_t^\alpha \left(\tilde{A}_t \tilde{L}_t\right)^{1-\alpha},$$

where Y_t is output and $\tilde{L}_t$ is a CES aggregator of the form:

$$\tilde{L}_t = \left[\omega^f \left(\tilde{L}_t^f\right)^\rho + \omega^m \left(\tilde{L}_t^m\right)^\rho\right]^{1/\rho}.$$

The parameter $\rho \in (-\infty, 1]$, determines the substitution elasticity between female and male hours, with $\rho = 1$ representing perfect substitution, $\rho \to -\infty$ representing a Leontieff production function and $\rho \to 0$ representing the Cobb-Douglas case. The labor inputs $\tilde{L}_t^j$ are measured in efficiency units per capita:

$$\tilde{L}_t^j = a_t^j p_t^j \frac{H_t^j}{H^j}$$

where a_t^j is a gender specific productivity index and hours worked are normalized by their steady state value. This normalization scales the parameters so that they represent the female and male shares of labor income in steady state.

Given the homogeneity of the production structure, we can only identify the relative efficiency of the two labor inputs. Therefore, it is useful to normalize the production function as:

$$\begin{aligned} Y_t &= K_t^\alpha \left(A_t L_t\right)^{1-\alpha} \\ L_t &= \left[\omega^f \left(\tilde{a}_t^f \frac{H_t^f}{H^f}\right)^\rho + \omega^m \left(\frac{H_t^m}{H^m}\right)^\rho\right]^{1/\rho} \end{aligned}$$

with

$$\begin{aligned} L_t &= \frac{\tilde{L}_t}{a_t^m p_t^m}, \\ A_t &= \tilde{A}_t a_t^m p_t^m, \\ \tilde{a}_t^f &= \frac{a_t^f}{a_t^m} \frac{p_t^f}{p_t^m}. \end{aligned}$$

This normalization of hours implies that the steady state value of the effective labor input L_t is also normalized to $L = 1$, since the distribution parameters will be equal to the income shares in steady state, so that $\omega^f + \omega^m = 1$.

The relative efficiency of women follows the stationary stochastic process:

$$\begin{aligned}
\log \tilde{a}_t^f &= \log \tilde{a}_t^{fT} + \log \tilde{a}_t^{fC}, \\
\log \tilde{a}_t^{fT} &= \rho_{a^{fT}} \log \tilde{a}_{t-1}^{fT} + \varepsilon_{a^{fT},t}, \\
\log \tilde{a}_t^{fC} &= \rho_{a^{fC}} \log \tilde{a}_{t-1}^{fC} + \varepsilon_{a^{fC},t},
\end{aligned}$$

with $\varepsilon_{a^{Xf},t}$ i.i.d. $N(0, \sigma^2_{a^{Xf}})$ for $X = T, C$. This decomposition of the relative efficiency into a trend (T) and a cycle (C) component, which will be distinguished by the priors on the autocorrelation and on the volatility of the innovations, is an attempt to isolate the slow-moving component of this process, without at the same time overly restricting its higher frequency behavior.[7]

The aggregate technology factor A_t, which compounds total factor and male-specific productivity, follows a stationary AR(1) process in its growth rate $z_t \equiv \Delta \log A_t$:

$$z_t = (1 - \rho_z)\gamma + \rho_z z_{t-1} + \varepsilon_{z,t},$$

with $\varepsilon_{z,t}$ distributed $i.i.d.N(0, \sigma_z^2)$. This specification implies that the level of technology is non stationary.

Resource Constraint The aggregate resource constraint is:

$$C_t + I_t + G_t + a(u_t)\bar{K}_{t-1} = Y_t,$$

where government spending is a time-varying fraction of GDP,

$$G_t = \left(1 - \frac{1}{g_t}\right) Y_t,$$

and the government spending shock g_t follows the stochastic process:

$$\log g_t = (1 - \rho_g) \log g + \rho_g \log g_{t-1} + \varepsilon_{g,t},$$

with $\varepsilon_{g,t} \sim i.i.d.N(0, \sigma_g^2)$.

4.1.1 Implications for Wages and Income Shares

The households' and firms' first order necessary conditions for optimality are derived in Appendix **??**. The labor supply equations in (**??**) imply:

$$\frac{W_t^f}{W_t^m} = \frac{\varphi_t^f}{\varphi_t^m} \frac{\left(H_t^f\right)^{\nu^f}}{\left(H_t^m\right)^{\nu^m}},$$

[7] A more parsimonious alternative would be to assume that there is no cyclical component in the relative efficiency of the two genders, and that all cyclical fluctuations come from the aggregate component. An intermediate case would be to add to the trend component just an i.i.d. process, rather than an AR(1).

from which we can recover the *relative* disutility of effort $\tilde{\varphi}_t^f \equiv \frac{\varphi_t^f}{\varphi_t^m}$ from observations on wages and hours worked, given the Frisch elasticities $\nu^f$and ν^m. In particular

$$\log \tilde{\varphi}_t^f = \log W_t^f - \log W_t^m - \nu^f \log H_t^f + \nu^m \log H_t^m.$$

From the mens' labor supply function:

$$W_t^m = \frac{\varphi_t^m \left(H_t^m\right)^{\nu^m}}{\Lambda_t},$$

it is then possible to recover the level of φ_t^m, although this is conditional on the process for the marginal utility Λ_t implied by the model's solution.

This identification strategy suggests to model women's relative disutility and men's absolute disutility of labor as independent processes, as follows:

$$\begin{aligned}
\log\left(\tilde{\varphi}_t^f/\tilde{\varphi}^f\right) &= \log \tilde{\varphi}_t^{fT} + \log \tilde{\varphi}_t^{fC}, \\
\log \tilde{\varphi}_t^{fT} &= \rho^T_{\tilde{\phi}^{fT}} \log \tilde{\varphi}_{t-1}^{fT} + \varepsilon_{\tilde{\varphi}^{fT},t}, \\
\log \tilde{\varphi}_t^{fC} &= \rho^C_{\tilde{\phi}^{fC}} \log \tilde{\varphi}_{t-1}^{fC} + \varepsilon_{\tilde{\varphi}^{fC},t}, \\
\log\left(\varphi_t^m/\varphi^m\right) &= \rho_{\varphi^m} \log \varphi_{t-1}^m + \varepsilon_{\varphi^m,t},
\end{aligned}$$

with $\varepsilon_{\tilde{\varphi}^{fX},t}$ i.i.d. $N(0,\sigma^2_{\tilde{\varphi}^{fX}})$ for $X = T, C$ and $\varepsilon_{\varphi^m,t}$ i.i.d. $N(0,\sigma^2_{\varphi^m})$.[8] This parametrization parallels the one adopted for the gender specific productivities. Similar considerations hold in terms of the trend/cycle decomposition.[9]

The labor demand equations above immediately imply the labor shares of income:

$$\begin{aligned}
\frac{W_t^f H_t^f}{Y_t} &= (1-\alpha)\,\omega^f \left(\frac{\tilde{a}_t^f \left(H_t^f/H^f\right)}{L_t}\right)^{\rho}, \\
\frac{W_t^m H_t^m}{Y_t} &= (1-\alpha)\,\omega^m \left(\frac{\left(H_t^m/H^m\right)}{L_t}\right)^{\rho},
\end{aligned}$$

and relative labor incomes:

$$\frac{W_t^f H_t^f}{W_t^m H_t^m} = \frac{\omega^f}{\omega^m}\left(\tilde{a}_t^f \frac{H_t^f}{H_t^m}\right)^{\rho}.$$

These equations imply that the *relative* paths for gender specific productivity can be easily recovered

[8]The averages of the φ processes pin down the levels of hours for men and women in steady state, but have no effect on the model's dynamics. However, we do not exploit this restriction, since the hours used in estimation are in deviation from their steady state value. See section **??** for more details.

[9]One difference is that, unlike for the productivities, whose level is not separately identified from that of the share parameters ω, the steady state levels of the disutility of work φ could be identified from information on the level of hours and wages by gender. However, we only retain information on the relative levels of hours and wages, since we have some doubts on the reliability of their absolute values. From this information, we pin down the relative disutility in steady state$\tilde{\varphi}^f$.

from observations on hours and wages, given the CES parameter ρ. In particular:

$$\log \tilde{a}_t^f = \frac{1}{\rho}\left[\log W_t^f - \log W_t^m - \log \frac{\omega^f}{\omega^m}\right] + \left(\frac{1}{\rho} - 1\right)\left[\log H_t^f - \log H_t^m\right].$$

Consumption insurance within the household, together with the separability of utility, implies equalization of the marginal utility of income:

$$\frac{(H_t^m)^{\nu^m}}{W_t^m} = \frac{\tilde{\varphi}_t^f \left(H_t^f\right)^{\nu^f}}{W_t^f},$$

which combined with the firm's optimality conditions implies:

$$\frac{(H_t^m)^{1+\nu^m-\rho}}{\frac{\omega^m}{(H^m)^\rho}} = \frac{\tilde{\varphi}_t^f \left(H_t^f\right)^{1+\nu^f-\rho}}{\frac{\omega^f}{(H^f)^\rho}\left(\tilde{a}_t^f\right)^\rho}.$$

This equation clearly shows that both the efficiency and the disutility factor have an effect on hours in equilibrium. This would not be the case under a Cobb-Douglas specification, in which $\rho = 0$, since with that specification income and substitution effects of the increase in gender-specific productivity (and hence wages) cancel out.

4.1.2 Solution

Total factor productivity A_t is not stationary. Therefore, we define normalized stationary variables and then characterize the solution of the model in terms of these rescaled variables. The details of the characterization are described in Appendix **??**. Here, we present the normalized steady state equations and the log-linearized state equations used for estimation. The derivation of these equations is also presented in Appendix **??**.

Steady State and Calibration The steady state for the rescale version of the model is characterized by the following system of equations, using the normalization $L = 1$:

$$\begin{aligned} 1 &= \beta e^\gamma r^k + (1-\delta)\,\beta e^\gamma \\ r^k &= \frac{e^\gamma}{\beta} - (1-\delta) \end{aligned}$$

$$k = \left(\frac{\alpha}{r^k}\right)^{\frac{1}{1-\alpha}}$$

$$y = k^\alpha$$

$$\begin{aligned}\overline{k} &= ke^{\gamma}\\ i &= \left[1-(1-\delta)e^{-\gamma}\right]\overline{k}\end{aligned}$$

$$c=\frac{y}{g}-i$$

$$\lambda=\frac{1}{c}\frac{e^{\gamma}-\beta\eta}{e^{\gamma}-\eta}.$$

In addition, equations (**??**), (**??**) and (**??**) provide four conditions to pin down wages and hours, with two extra degrees of freedom represented by the steady state values of the disutility of labor φ^j:

$$\begin{aligned}w^f &= (1-\alpha)\frac{y}{H^f}\omega^f,\\ w^m &= (1-\alpha)\frac{y}{H^m}\omega^m,\\ w^j &= \frac{\varphi^j\left(H^j\right)^{\nu^j}}{\lambda},\end{aligned}$$

for $j=f,\, m$.

Since we think of the steady state as corresponding to averages over the period 1995 to 2004, rather than to averages over the entire sample, we will compute the observables in deviation from their steady state value and not include any constant in the observation equations.[10] As a result, a value for the φ's need not be computed for the solution of the model. However, it might be of interest to compute a value for the relative disutility of work that is compatible with the observed relative hours and wages. Given an observed value for average relative hours $\tilde{H}^f$, and normalizing $H^m=1$, we have:

$$\frac{w^f}{w^m}=\frac{\omega^f}{\omega^m}\left(\tilde{H}^f\right)^{-1},$$

from which we can finally pin down the relative disutility,

$$\tilde{\varphi}^f=\frac{w^f}{w^m}\left(\tilde{H}^f\right)^{-\nu^f}.$$

The value of φ^mcompatible with the normalization $H^m=1$ is:

$$\varphi^m=(1-\alpha)\,y\omega^m\lambda.$$

We use the steady state equation to calibrate certain parameters of the model for which independent evidence is available.

Standard estimates (e.g. [**?**]) suggest that women's Frisch elasticity of labor supply is approxi-

[10]In most DSGE applications, it would be of interest to estimate the steady state level of hours for the entire sample period. Thus, this system of equations could be used to parametrize the steady state in terms of hours and use the first two equations to determine the wages, and the last two to recover the φ^j's compatible with the initial choice of hours.

Parameter	Value
α	0.33
β	0.99
δ	0.025
g	$\frac{1}{1-0.15}$
ν^f	0.21
ν^m	0.63
ρ	0.75
ω^f	0.375
ω^m	0.625

TABLE 1: Calibrated parameters.

mately three times that of men. Assuming an "average" macro elasticity of 2.84 from [?], and given women's share in total hours of 0.4, we have:

$$0.4\frac{3}{\nu^m} + 0.6\frac{1}{\nu^m} = 2.84$$

or $1/\nu^m = 1.58$ and $1/\nu^f = 4.73$.

The substitution elasticity between female and male labor is $1/(1-\rho)$. Plausible estimates based on [?] range between 3.2 and 4.2. We take 4 as a benchmark, which implies $\rho = 0.75$.

The observed average relative labor share of women over the period 1995 to 2004, which is what we take as the steady state, is 0.60. This implies:

$$\begin{aligned} \frac{\omega^f}{\omega^m} &= 0.60 \\ \omega^f + \omega^m &= 1 \end{aligned}$$

from which we get $\omega^m = 0.625$ and $\omega^f = 0.375$.

Finally, the steady state share of government spending in GDP $(1 - 1/g)$ is calibrated at 0.15, which roughly corresponds to the average share of G+NX in GDP over the period 1995 to 2005.

4.2 Estimation

We log linearize the model and the resulting state equations are derived in Appendix **??**.

We estimate the model according to standard procedures. The priors follow [?] for the parameters that are identical in the two models. For the persistence of the trend components of the relative technology and taste factors, i.e. $\rho_{\tilde{a}^{fT}}$ and $\rho^T_{\tilde{\varphi}^{fT}}$, we center the prior tightly around 0.98. Given the high persistence of these processes, the prior on their standard deviation is smaller than the one for the other shocks. For the cyclical components, we center the prior on the autocorrelation around 0.4, the same as for the growth rate of the technology shock, reflecting the view that the persistence of this component should be significantly smaller than that of the trend.

We first estimate the model on quarterly data from 1984Q1 to 2004Q4. The estimated parameters are presented in Table 2.

Table 2: Estimation Results

	Prior				Posterior					
	Dist	5%	Median	95%	Mode	Mean	SE	5%	Median	95%
γ	N	0.4507	0.5000	0.5493	0.5003	0.4995	0.0300	0.4498	0.4996	0.5483
χ	G	3.4764	4.9335	6.7505	6.0077	6.2231	1.0317	4.6095	6.1795	7.9986
η	B	0.3351	0.5000	0.6649	0.4535	0.4402	0.0822	0.3045	0.4402	0.5764
ζ	G	1.2907	2.5776	4.5271	5.9967	6.4239	1.0959	4.8021	6.3226	8.3815
ρ_b	B	0.2486	0.6143	0.9024	0.0365	0.0556	0.0318	0.0150	0.0496	0.1168
ρ_μ	B	0.2486	0.6143	0.9024	0.3599	0.3507	0.1032	0.1762	0.3524	0.5172
ρ_z	B	0.0976	0.3857	0.7514	0.2195	0.2776	0.1406	0.0705	0.2657	0.5282
ρ_g	B	0.2486	0.6143	0.9024	0.8425	0.7974	0.1090	0.5841	0.8188	0.9344
$\rho_{\tilde{a}^{fT}}$	B	0.9507	0.9836	0.9971	0.9904	0.9831	0.0110	0.9617	0.9853	0.9965
$\rho_{\tilde{a}^{fC}}$	B	0.1718	0.5000	0.8282	0.6294	0.6147	0.1709	0.3135	0.6271	0.8727
$\rho_{\tilde{\varphi}^{fT}}$	B	0.9507	0.9836	0.9971	0.9896	0.9816	0.0121	0.9587	0.9840	0.9961
$\rho_{\tilde{\varphi}^{fC}}$	B	0.1718	0.5000	0.8282	0.5959	0.5934	0.1734	0.2936	0.6016	0.8637
ρ_{φ^m}	B	0.2486	0.6143	0.9024	0.9698	0.9565	0.0242	0.9110	0.9603	0.9885
σ_b	IG1	0.0327	0.0679	0.2488	0.0111	0.0116	0.0013	0.0097	0.0114	0.0139
σ_μ	IG1	0.1756	0.3552	1.2108	0.1074	0.1146	0.0180	0.0889	0.1125	0.1470
σ_z	IG1	0.4077	0.7751	2.2455	0.1124	0.1143	0.0083	0.1015	0.1137	0.1288
σ_g	IG1	0.2038	0.3875	1.1227	0.0558	0.0569	0.0042	0.0505	0.0567	0.0642
$\sigma_{\tilde{a}^{fT}}$	IG1	0.0661	0.1367	0.4959	0.0335	0.0347	0.0040	0.0286	0.0344	0.0416
$\sigma_{\tilde{a}^{fC}}$	IG1	0.1370	0.2798	0.9784	0.0501	0.0511	0.0044	0.0444	0.0508	0.0587
$\sigma_{\tilde{\varphi}^{fT}}$	IG1	0.0685	0.1399	0.4892	0.0347	0.0362	0.0043	0.0298	0.0359	0.0437
$\sigma_{\tilde{\varphi}^{fC}}$	IG1	0.1370	0.2798	0.9784	0.0510	0.0522	0.0049	0.0447	0.0519	0.0607
σ_{φ^m}	IG1	0.1756	0.3552	1.2108	0.0578	0.0597	0.0052	0.0517	0.0593	0.0689

Quarterly data. 1984Q1 to 2004Q4.

It is interesting to examine the estimated path of the gender specific processes introduced in the model, $\tilde{a}_t^f$, $\tilde{\phi}_t^f$ and ϕ_t^m. The trend components of these shocks are displayed in figure **??**.

There is a clear and precisely estimated trend for all these processes until the mid-1990s. The path of $\tilde{a}^{fT}$ is increasing between 1984 and 1997, and virtually flat after that date. By contrast, the path of $\tilde{\phi}^{fT}$ and ϕ^m is declining between 1984 and 1998, and flat after that date. The decline in ϕ^m for the first part of the sample is likely driven by demographics, in particular the baby boom generation entering prime age and reaching their peak lifetime hours. In addition to this, there is a more pronounced increase in female hours, which is captured in the model by the estimated path of ϕ^f. At the same time, there is a growth in the path of female relative productivity which is also very pronounced.

These gender specific shocks are introduced to account for the heterogeneity across gender in the trend in wages and hours. They also matter for aggregate variables. This can be seen, first of

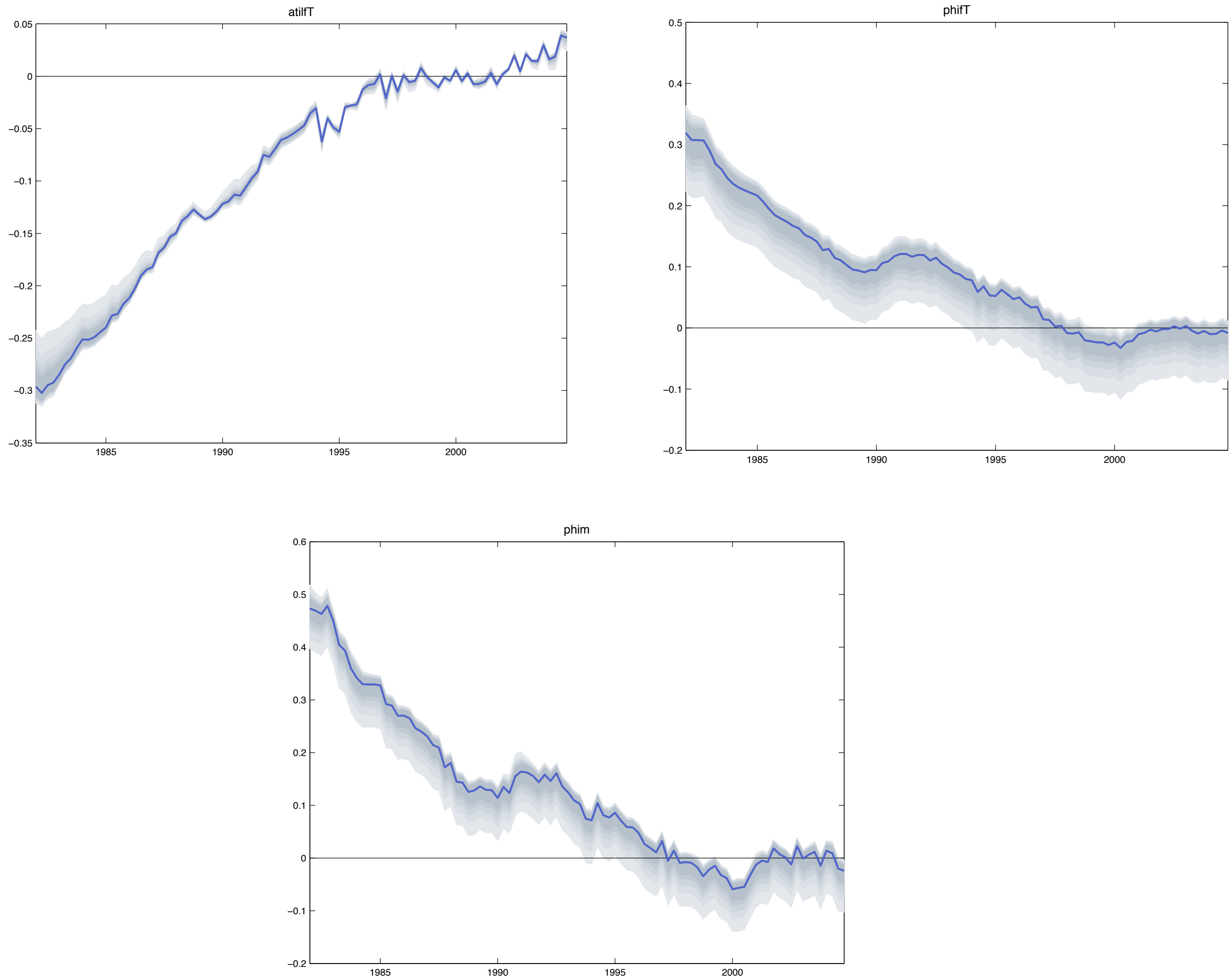

FIGURE 9: Estimated Paths of Gender Specific Parameters $\tilde{a}^{fT}$ (left), $\tilde{\phi}^{fT}$ (right) and ϕ^m (bottom)
Quarterly data. 1984Q1 to 2004Q4.

all, from the variance decomposition. The results are displayed in figure **??**, where the horizontal axis denotes the horizon in quarters. The gender specific shocks account for a sizable fraction of the variance of output, aggregate labor and investment at horizons greater than 4 quarters. Specifically, $\tilde{a}^{fT}$ (ocre area) and ϕ^m (graphite area) account for approximately 15% of the variance of output and aggregate labor at horizons 4-10 quarters, and just under 10% of the variance of investment. For $\tilde{phi}^{fT}$ (lighter green area) the fractions are somewhat smaller, but of a similar order of magnitude. Interestingly, at horizons greater than 4, these three shocks combined explain more of the variance of output and aggregate labor than the aggregate productivity shock. For investment, these three shocks combined explain about half of the variance explained by the aggregate productivity shock at horizons greater than 4 quarters.

We now display the impulse response functions to the gender specific shocks. The responses for the observed variables to a 1 percentage point rise in each of the gender specific variables are

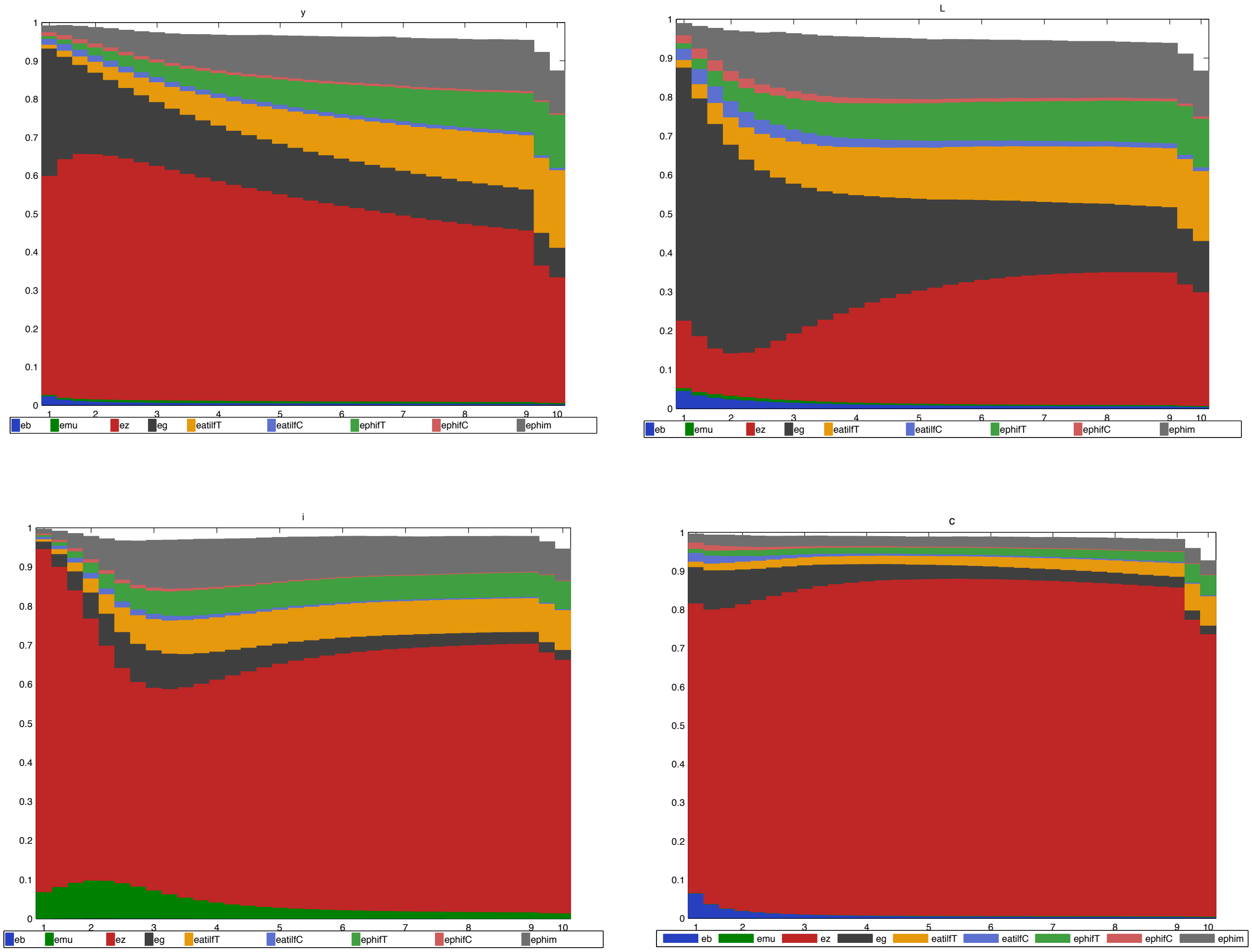

FIGURE 10: Variance Decomposition. Output (top left), aggregate labor (top right), investment (bottom left), consumption (bottom right). Quarterly data. 1984Q1 to 2004Q4.

displayed in figures **??** to **??**. For comparison, in figure **??**, we also present the response to a 1 percentage rise in z, the aggregate productivity shock.

The figures show a persistent response of both gender specific and aggregate outcomes. For the $\tilde{a}^{fT}$ shock, the female to male hours ratio (center left) rises by approximately 0.4 percentage points at horizons greater than 8 quarters, while male hours (center right) decline by approximately 0.1 percentage points. The female male wage ratio (bottom right) rises by 0.015 percentage points. Aggregate output (top left) and labor (center left) also rise, but only in the short run. The impact response for these variables is less than 0.1 percentage points.

The ϕ^m and $\tilde{\phi}^{fT}$ shocks generate impulse responses that are nearly mirror images of each other, in the intuitive direction. Again, the magnitude of the responses is sizable and persistent only for the gender specific variables, whereas for the aggregate variables the response is short lived and small. Comparing these results to the impulses responses to an aggregate productivity shock, it

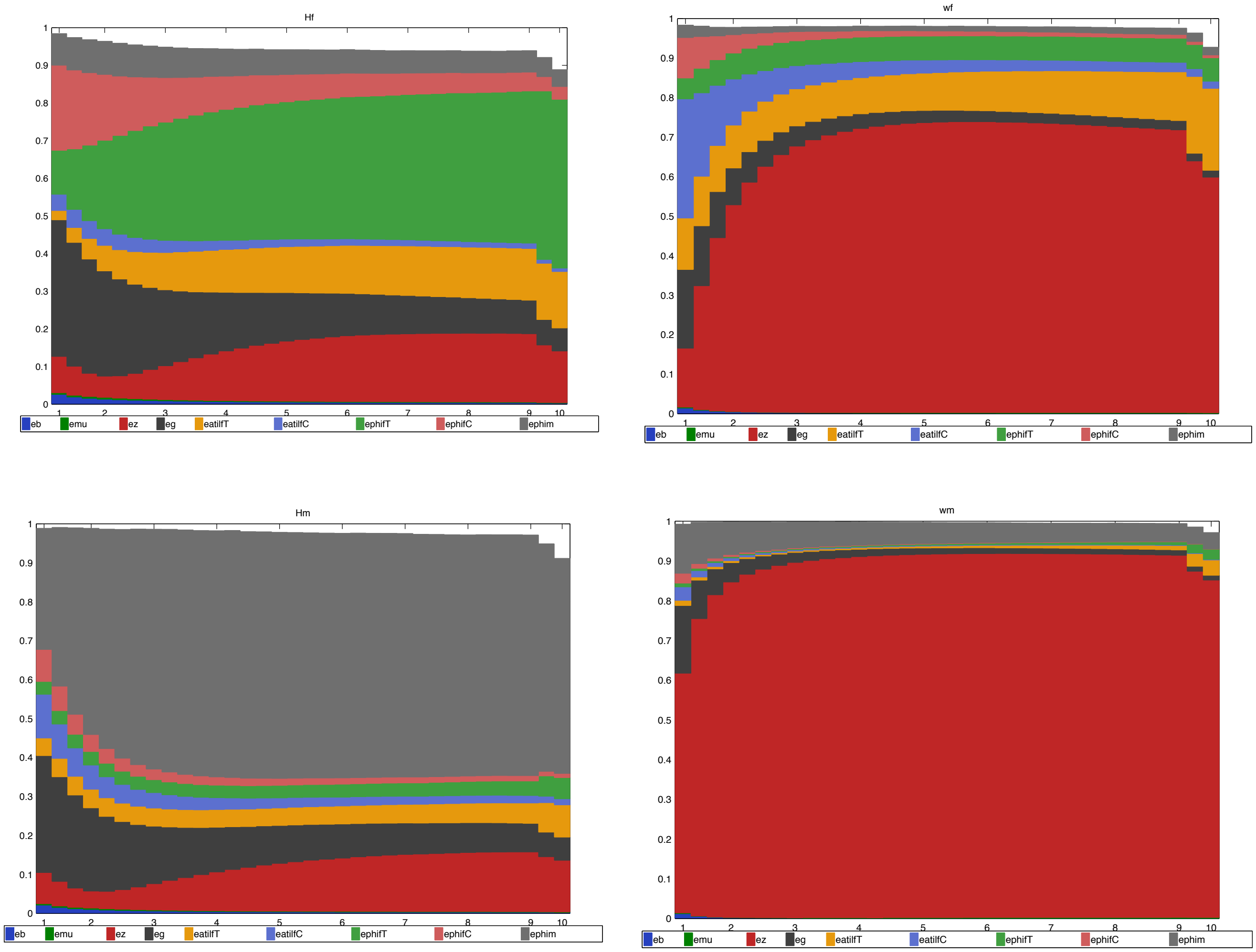

FIGURE 11: Variance Decomposition. H^f (top right), w^f top left, H^m (bottom left), w^m (bottom right). Quarterly data. 1984Q1 to 2004Q4.

is clear that the gender specific variables have much more sizable responses to the gender specific shocks. The persistence of the responses of the gender specific variables is much greater than for the aggregate variables, but the responses eventually die out.

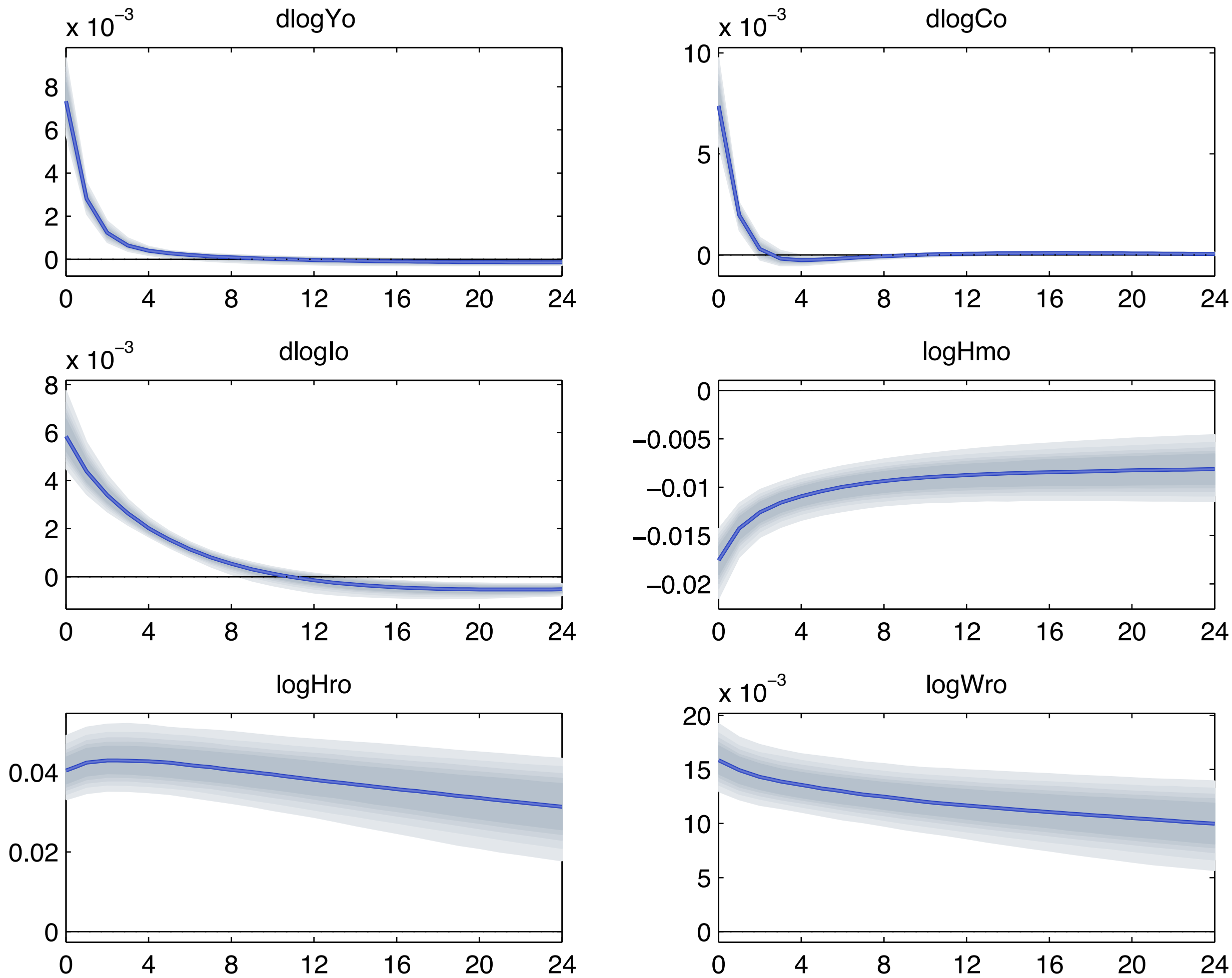

FIGURE 12: Impulse Responses to $\tilde{a}^{fT}$. Quarterly data. 1984Q1 to 2004Q4.

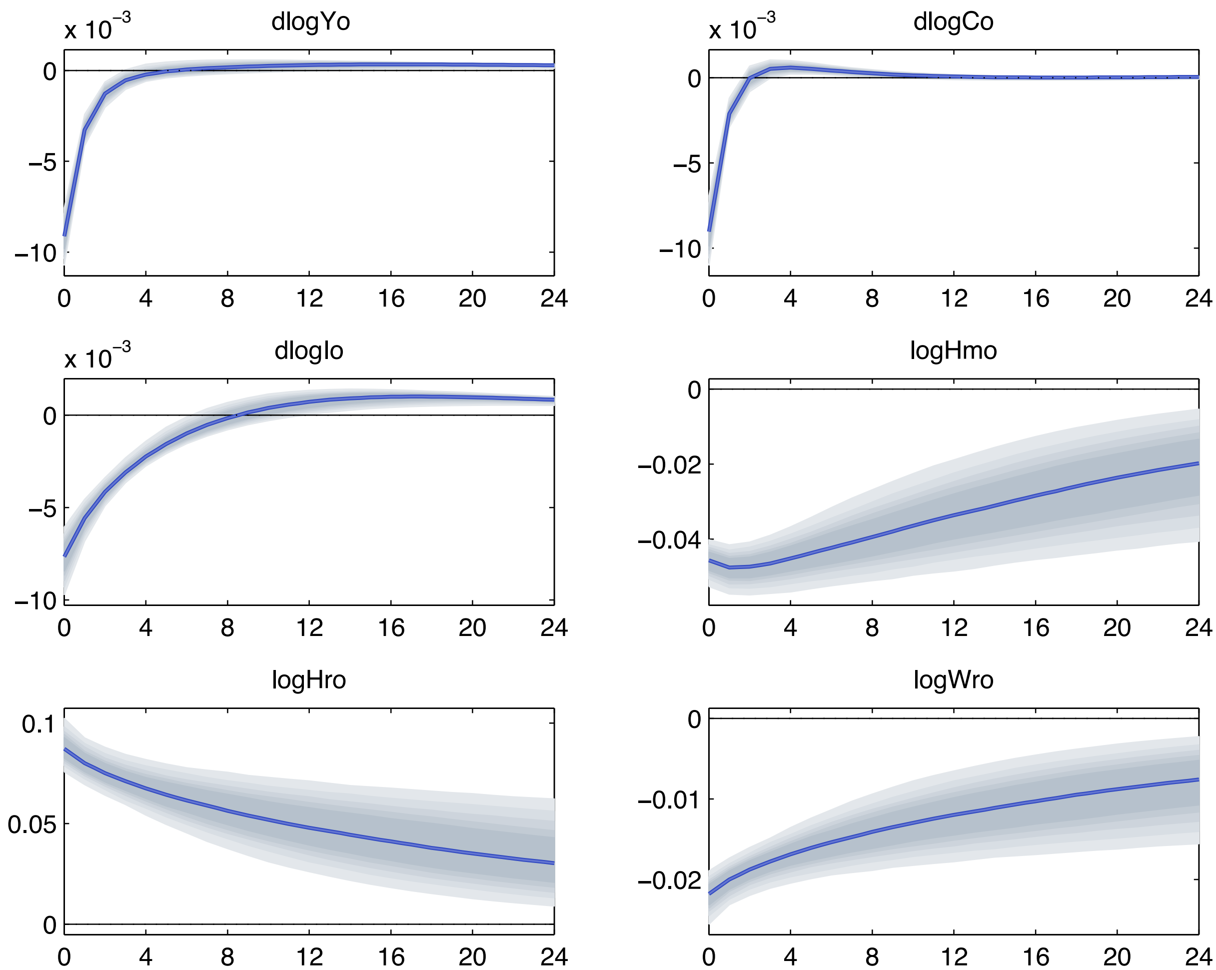

FIGURE 13: Impulse Responses to ϕ^m. Quarterly data. 1984Q1 to 2004Q4.

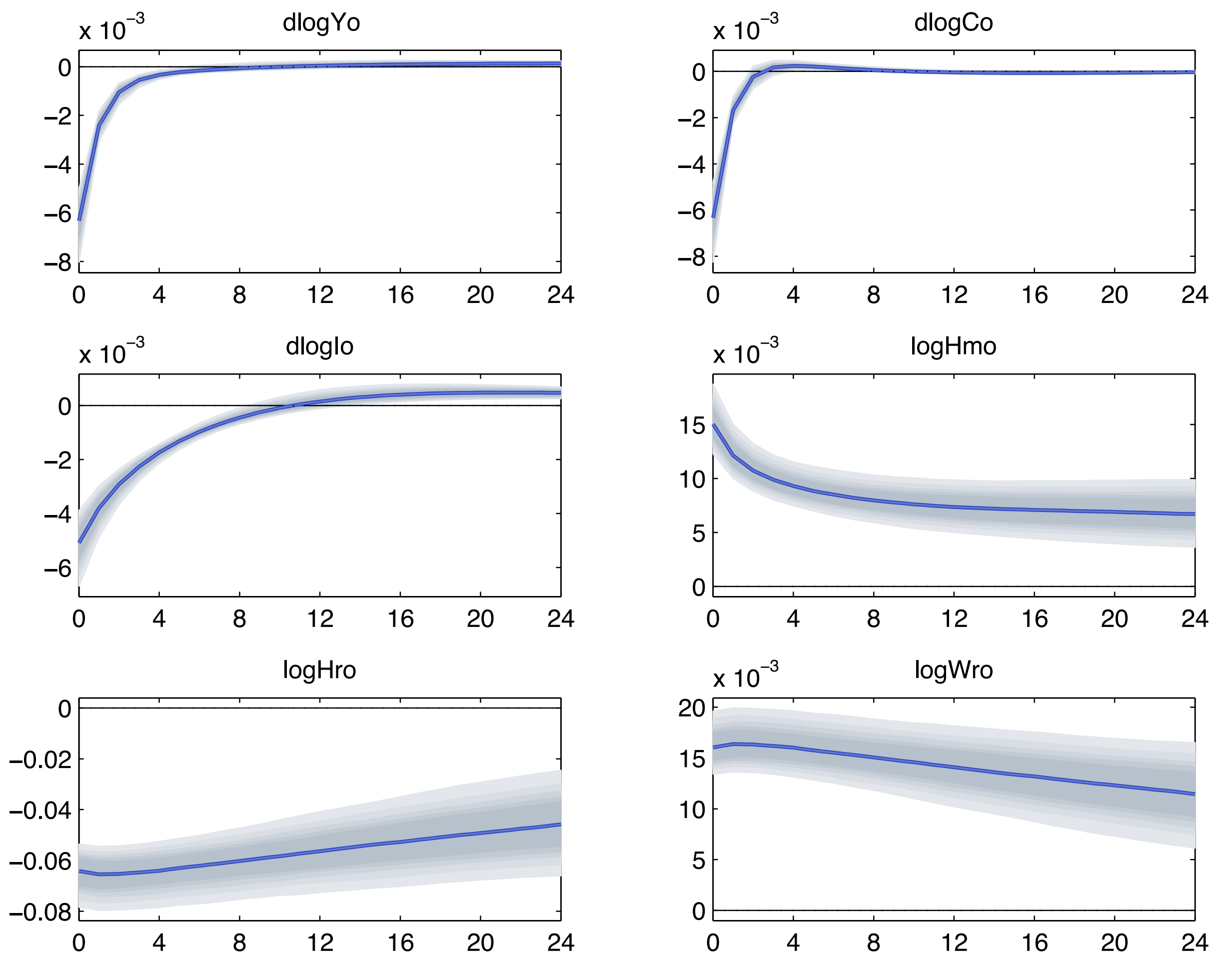

FIGURE 14: Impulse Responses to $\tilde{\phi}^{fT}$. Quarterly data. 1984Q1 to 2004Q4.

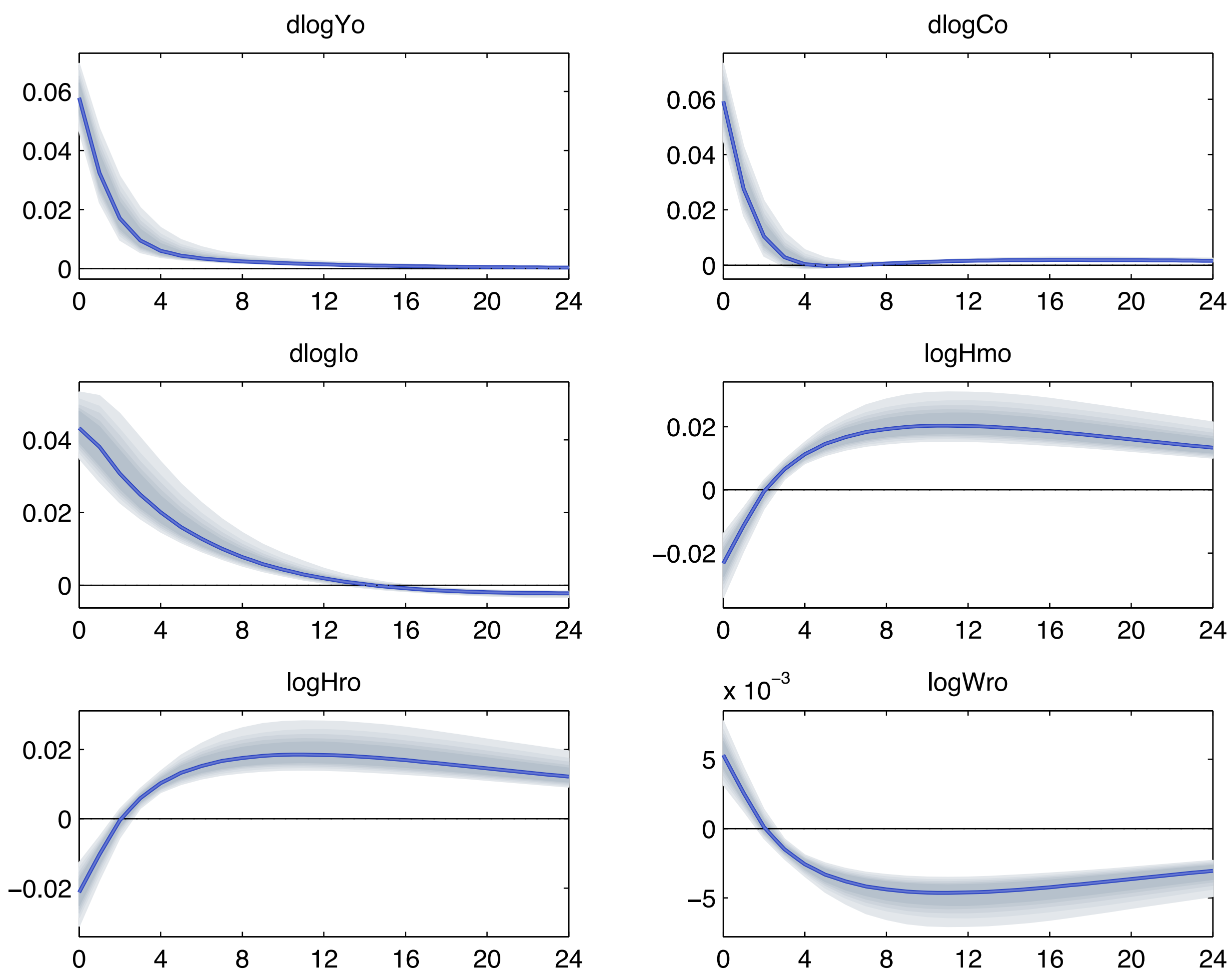

FIGURE 15: Impulse Responses to z. Quarterly data. 1984Q1 to 2004Q4.

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A Model Solution

A.1 Household and Firm Optimization

The household's first order necessary conditions for optimality are as follows:

- Consumption:

$$\Lambda_t = \frac{b_t}{C_t - \eta C_{t-1}} - \beta\eta E_t \frac{b_{t+1}}{C_{t+1} - \eta C_t}$$

  where $\Lambda_t$is the multiplier of the budget constraint;

- Physical capital $\left(\bar{K}_t\right)$:

$$\Phi_t = \beta E_t \left[\Lambda_{t+1}\left(r^k_{t+1} u_{t+1} - a(u_{t+1})\right)\right] + (1-\delta)\beta E_t \Phi_{t+1}$$

- Investment:

$$\Lambda_t = \Phi_t \mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - \frac{I_t}{I_{t-1}} S'\left(\frac{I_t}{I_{t-1}}\right)\right] + \beta E_t \left[\Phi_{t+1}\mu_{t+1}\left(\frac{I_{t+1}}{I_t}\right)^2 S'\left(\frac{I_{t+1}}{I_t}\right)\right]$$

- Utilization:

$$r^k_t = a'(u_t)$$

- Labor supply (H^j_t for $j = f, m$):

$$W^j_t = \frac{\varphi^j_t \left(H^j_t\right)^{\nu^j}}{\Lambda_t}. \tag{2}$$

The firms' first order necessary conditions are given by the following system of equations.

- Effective capital (K_t):

$$r^k_t = \alpha A^{1-\alpha}_t \left(\frac{L_t}{K_t}\right)^{1-\alpha};$$

- Demand for female labor $\left(H^f_t\right)$

$$W^f_t = (1-\alpha)\frac{Y_t}{H^f_t}\omega^f \left(\frac{\tilde{a}^f_t \left(H^f_t / H^f\right)}{L_t}\right)^\rho$$

- Demand for male labor (H^m_t):

$$W^m_t = (1-\alpha)\frac{Y_t}{H^m_t}\omega^m \left(\frac{\left(H^m_t / H^m\right)}{L_t}\right)^\rho$$

A.2 Characterizing the Solution for the Normalized Model

We adopt the following normalization to make the model stationary:

$$
\begin{aligned}
y &= Y/A \\
k &= K/A \\
\bar{k} &= \bar{K}/A \\
c &= C/A \\
i &= I/A \\
w^j &= W^j/A \\
\lambda &= \Lambda A \\
\phi &= \Phi A.
\end{aligned}
$$

We now rewrite the equilibrium conditions in terms of the normalized variables.

Households

- Consumption

$$\lambda_t = \frac{b_t e^{z_t}}{e^{z_t} c_t - \eta c_{t-1}} - \beta \eta E_t \frac{b_{t+1}}{e^{z_{t+1}} c_{t+1} - \eta c_t} \tag{3}$$

- Physical capital $\left(\bar{K}_t\right)$

$$\phi_t = \beta E_t \left\{ e^{-z_{t+1}} \lambda_{t+1} \left[r^k_{t+1} u_{t+1} - a\left(u_{t+1}\right) \right] \right\} + (1-\delta)\, \beta E_t \left(\phi_{t+1} e^{-z_{t+1}} \right) \tag{4}$$

- Investment

$$
\begin{aligned}
\lambda_t = \phi_t \mu_t &\left[1 - S\left(\frac{i_t}{i_{t-1}} e^{z_t} \right) - \frac{i_t}{i_{t-1}} e^{z_t} S'\left(\frac{i_t}{i_{t-1}} e^{z_t} \right) \right] \\
&+ \beta E_t \left[\phi_{t+1} e^{-z_{t+1}} \mu_{t+1} \left(\frac{i_{t+1}}{i_t} e^{z_{t+1}} \right)^2 S'\left(\frac{i_{t+1}}{i_t} e^{z_{t+1}} \right) \right]
\end{aligned} \tag{5}
$$

- Utilization

$$r^k_t = a'(u_t) \tag{6}$$

- Definition of effective capital

$$k_t = u_t \bar{k}_{t-1} e^{-z_t} \tag{7}$$

- Physical capital accumulation

$$\bar{k}_t = (1-\delta) e^{-z_t} \bar{k}_{t-1} + \mu_t \left[1 - S\left(\frac{i_t}{i_{t-1}} e^{z_t} \right) \right] i_t \tag{8}$$

- Labor supply (H_t^j for $j = f, m$)

$$w_t^j = \frac{\varphi_t^j \left(H_t^j\right)^{\nu^j}}{\lambda_t} \tag{9}$$

Firms

- Production function

$$y_t = k_t^\alpha (L_t)^{1-\alpha} \tag{10}$$

$$L_t = \left[\omega^f \left(\tilde{a}_t^f \frac{H_t^f}{H^f}\right)^\rho + \omega^m \left(\frac{H_t^m}{H^m}\right)^\rho\right]^{1/\rho} \tag{11}$$

- Effective capital (K_t)

$$r_t^k = \alpha \left(\frac{L_t}{k_t}\right)^{1-\alpha} \tag{12}$$

- Female labor demand $\left(H_t^f\right)$

$$w_t^f = (1-\alpha)\left(\frac{k_t}{L_t}\right)^\alpha (L_t)^{1-\rho} \frac{\omega^f}{H^f}\tilde{a}_t^f \left(\tilde{a}_t^f \frac{H_t^f}{H^f}\right)^{\rho-1}$$

$$w_t^f = (1-\alpha)\frac{y_t}{H_t^f}\omega^f \left(\frac{\tilde{a}_t^f \left(H_t^f/H^f\right)}{L_t}\right)^\rho \tag{13}$$

- Male labor demand (H_t^m)

$$w_t^m = (1-\alpha)\left(\frac{k_t}{L_t}\right)^\alpha (L_t)^{1-\rho} \frac{\omega^m}{H^m} \left(\frac{H_t^m}{H^m}\right)^{\rho-1}$$

$$w_t^m = (1-\alpha)\frac{y_t}{H_t^m}\omega^m \left(\frac{(H_t^m/H^m)}{L_t}\right)^\rho \tag{14}$$

Resource Constraint and Summary

- Resource constraint

$$c_t + i_t + a(u_t)e^{-z_t}\bar{k}_{t-1} = \frac{y_t}{g_t} \tag{15}$$

$$c_t + i_t + \frac{a(u_t)}{u_t}k_t = \frac{y_t}{g_t} \tag{16}$$

These are 14 dynamic equations in 14 unknowns$\left[\lambda, c, \phi, r^k, u, i, k, \bar{k}, w^f, w^m, H^f, H^m, y, L\right]$.

A.3 Log Linear Approximation

We can now derive the model's log-linear approximation. Log-linear deviations from steady state are defined as follows, for a generic variable ζ_t

$$\hat{\zeta}_t \equiv \log \zeta_t - \log \zeta,$$

except for $\hat{z}_t \equiv z_t - \gamma$.

Households

- Consumption

$$\begin{aligned}(e^\gamma - \eta\beta)(e^\gamma - \eta)\hat{\lambda}_t = \eta\beta e^\gamma E_t \hat{c}_{t+1} - \left(e^{2\gamma} + \eta^2\beta\right)\hat{c}_t + \eta e^\gamma \hat{c}_{t-1} \\ + \eta e^\gamma (\beta\rho_z - 1)\hat{z}_t + (e^\gamma - \eta\beta\rho_b)(e^\gamma - \eta)\hat{b}_t \quad (17)\end{aligned}$$

- Physical capital $\left(\bar{K}_t\right)$

$$\hat{\phi}_t = (1-\delta)\beta e^{-\gamma} E_t\left(\hat{\phi}_{t+1} - \hat{z}_{t+1}\right) + \left(1 - (1-\delta)\beta e^{-\gamma}\right) E_t\left[\hat{\lambda}_{t+1} - \hat{z}_{t+1} + \hat{r}^k_{t+1}\right]$$

- Investment

$$\hat{\lambda}_t = \hat{\phi}_t + \hat{\mu}_t - e^{2\gamma}\varsigma(\hat{\imath}_t - \hat{\imath}_{t-1} + \hat{z}_t) + \beta e^{2\gamma}\varsigma E_t\left[\hat{\imath}_{t+1} - \hat{\imath}_t + \hat{z}_{t+1}\right]$$

- Utilization

$$\hat{r}^k_t = \chi \hat{u}_t$$

- Definition of effective capital

$$\hat{k}_t = \hat{u}_t + \hat{\bar{k}}_{t-1} - \hat{z}_t$$

- Physical capital accumulation

$$\hat{\bar{k}}_t = (1-\delta)e^{-\gamma}\left(\hat{\bar{k}}_{t-1} - \hat{z}_t\right) + \left(1 - (1-\delta)e^{-\gamma}\right)(\hat{\mu}_t + \hat{\imath}_t)$$

- Labor supply ($H^j_t$for $j = f, m$)

$$\hat{w}^j_t = \hat{\varphi}^j_t + \nu^j \hat{H}^j_t - \hat{\lambda}_t$$

Firms

- Production function

$$\begin{aligned}\hat{y}_t &= \alpha\hat{k}_t + (1-\alpha)\hat{L}_t \\ \hat{L}_t &= \omega^f\left(\hat{\tilde{a}}^f_t + \hat{H}^f_t\right) + \omega^m \hat{H}^m_t\end{aligned}$$

- Effective capital (K_t)

$$\hat{r}_t^k = (1-\alpha)\left(\hat{L}_t - \hat{k}_t\right)$$

- Female labor demand $\left(H_t^f\right)$

$$\hat{w}_t^f = \hat{y}_t + (\rho - 1)\hat{H}_t^f + \rho\hat{\tilde{a}}_t^f - \rho\hat{L}_t$$

- Male labor demand (H_t^m)

$$\hat{w}_t^m = \hat{y}_t + (\rho - 1)\hat{H}_t^m - \rho\hat{L}_t$$

Resource Constraint

- Resource constraint

$$\frac{c}{y}\hat{c}_t + \frac{i}{y}\hat{\imath}_t + \frac{r^k k}{y}\hat{u}_t = \frac{1}{g}\hat{y}_t - \frac{1}{g}\hat{g}_t$$

A.3.1 Shocks

Following [?], we normalize the intertemporal preference shock as:

$$\hat{b}_t^* = \frac{(e^\gamma - \eta)(e^\gamma - \eta\beta\rho_b)(1-\rho_b)}{e^{2\gamma} + \eta^2\beta + \eta e^\gamma}\hat{b}_t$$

so as to make the coefficient on consumption in the Euler equation equal to one. The Euler equation pricing a real one-period bond with interest rate r_t, which we did not consider explicitly, reads:

$$\hat{\lambda}_t = E_t\left[\hat{\lambda}_{t+1} - \hat{z}_{t+1}\right] + \hat{r}_t$$

and substituting equation (**??**), we get

$$\begin{aligned} \frac{e^{2\gamma} + \eta^2\beta + \eta e^\gamma}{(e^\gamma - \eta\beta)(e^\gamma - \eta)}\hat{c}_t + (...) &= (...) + \frac{(e^\gamma - \eta\beta\rho_b)(1-\rho_b)}{e^\gamma - \eta\beta}\hat{b}_t \\ \hat{c}_t + (...) &= (...) + \frac{(e^\gamma - \eta)(e^\gamma - \eta\beta\rho_b)(1-\rho_b)}{e^{2\gamma} + \eta^2\beta + \eta e^\gamma}\hat{b}_t. \end{aligned}$$

Using this normalization, we have the following set of equations governing the exogenous shocks in the model:

$$
\begin{aligned}
\hat{b}_t^* &= \rho_b \hat{b}_{t-1}^* + \varepsilon_{b,t} \\
\hat{\mu}_t &= \rho_\mu \hat{\mu}_{t-1} + \varepsilon_{\mu,t} \\
\hat{z}_t &= \rho_z \hat{z}_{t-1} + \varepsilon_{z,t} \\
\hat{g}_t &= \rho_g \hat{g}_{t-1} + \varepsilon_{g,t} \\
\hat{\tilde{a}}_t^f &= \hat{\tilde{a}}_t^{fT} + \hat{\tilde{a}}_t^{fC} \\
\hat{\tilde{a}}_t^{fT} &= \rho_{\tilde{a}^{fT}} \hat{\tilde{a}}_{t-1}^{fT} + \varepsilon_{\tilde{a}^{fT},t} \\
\hat{\tilde{a}}_t^{fC} &= \rho_{\tilde{a}^{fC}} \hat{\tilde{a}}_{t-1}^{fC} + \varepsilon_{\tilde{a}^{fC},t} \\
\hat{\tilde{\varphi}}_t^f &= \hat{\tilde{\varphi}}_t^{fT} + \hat{\tilde{\varphi}}_t^{fC} \\
\hat{\tilde{\varphi}}_t^{fT} &= \rho_{\tilde{\varphi}^{fT}}^T \hat{\tilde{\varphi}}_{t-1}^{fT} + \varepsilon_{\tilde{\varphi}^{fT},t}^T \\
\hat{\tilde{\varphi}}_t^{fC} &= \rho_{\tilde{\varphi}^{fC}}^T \hat{\tilde{\varphi}}_{t-1}^{fC} + \varepsilon_{\tilde{\varphi}^{fC},t}^C \\
\hat{\varphi}_t^m &= \rho_{\varphi^m} \hat{\varphi}_{t-1}^m + \varepsilon_{\varphi^m,t}.
\end{aligned}
$$