

Unemployment Risks and Intra-Household Insurance ^{*}

(Preliminary draft)

Javier Fernández-Blanco [†]

February 13, 2017

Abstract

We consider an economy with incomplete markets and intra-household risk sharing, where households are formed by a job-seeker and an employed spouse and differ by the productivity of the spouse. We study the constrained efficient private provision of insurance within the household through the labor supply of the spouse, and what unemployment risks should be publicly insured away. Unlike the spouse's total income, neither productivity nor labor supply is observed. We characterize the directed search equilibrium, and show that the spouse's labor supply is negatively affected by unemployment benefits regardless of the search outcome of the worker in line with the empirical evidence. We also show that the optimal unemployment benefits are contingent on the household's total income as it affects the trade-off between consumption-smoothing and job search incentives. Moreover, we numerically explore the welfare gains of implementing a household-income-based unemployment insurance.

Keywords: Unemployment Risks, Intra-Household Risk-sharing, Directed Search, Efficient Private Insurance

JEL Codes: J08, J22, J64, J65

^{*}We thank Nezih Guner, Claudio Michelacci and Aysegul Sahin for their feedback. We are also grateful to numerous participants at the SEA (2015), ENTER Jamboree (2016), Istanbul SaM Workshop (2016), the Annual SaM Conference (2016) and the ESEM (2016). Javier acknowledges financial support from the Spanish Ministry of Science and Technology under Grant No. ECO2015-67602, and from the Spanish Ministry of Economy and Competitiveness through the Severo Ochoa Programme for Centres of Excellence in R&D (SEV-2015-0563).

[†]Universitat Autònoma de Barcelona, MOVE and Barcelona GSE

1 Introduction

The primary goal of this paper is to study the constrained efficient private intra-household insurance against unemployment risks and, hence, to determine what risks should be publicly insured away. Two facts motivate this question.

First, significant consumption smoothing takes place at the household level. [Heathcote, Storesletten, and Violante \(2014\)](#) find that only does a 40% of individual permanent wage fluctuations pass through to household consumption.¹ Furthermore, [Blundell, Pistaferri, and Saporta-Eksten \(2016\)](#) estimate that, on average, adjusting the wife’s labor supply accounts for approximately 50% of the consumption insurance against a permanent fall in the husband’s earnings, with assets and transfers accounting for 20% each.² The analysis of the effects of the husband’s income on his wife’s labor supply goes back at least to [Mincer \(1962\)](#), who showed informal evidence that wives work more if their husbands are unemployed. [Blau and Kahn \(2007\)](#) estimate the average cross-wage elasticity of wife’s hours worked at -0.2.³ [Cullen and Gruber \(1996\)](#) estimate a 6% increase in the wife’s hours worked in the short run after the displacement of her husband.

Second, there appear to be sizable crowding-out effects of publicly-provided insurance on private provision. Two estimates of this distortion are provided by [Gruber \(1997\)](#) and [Cullen and Gruber \(2000\)](#).⁴ The former finds that a 10 percent increase in the replacement rate is associated with a reduction in the consumption drop of 2.8 percentage points. The latter find that hours worked of wives increase by 30% during an unemployment spell of their husbands

¹[Dynarski and Gruber \(1997\)](#) estimate the average elasticity of total consumption with respect to male’s earnings at 0.24, which implies that 76 cents are smoothed away for each dollar in male’s earnings.

²In contrast, and because of the gender gap in labor supply elasticities, permanent shocks on wives’ wages are primarily smoothed away through savings and government transfers.

³The evidence on cross-wage elasticity is quite limited and it is often conditioned on a positive annual hours worked of the husband. [Blau and Kahn \(2007\)](#) find that wage-cross elasticity is negative and significant at both the extensive and intensive margins, increases with education and is twice as high for married women with children under 6. Furthermore, there is no significant difference when including cohabitation. Likewise, [Hyslop \(2001\)](#) estimates that a \$1 increase in the husband’s hourly wages reduces wife’s annual earnings by \$300 and her labor supply by 35 annual hours. **We provide more references to the empirical evidence on this source of private insurance in the literature section.**

⁴As summarized by [Heathcote, Storesletten, and Violante \(2009\)](#), there is a number of private insurance sources. [Engen and Gruber \(2001\)](#) estimate that the negative percentage effect of UI on asset holdings is twice as large for singles as for married unemployed workers. However, as pointed out by [Chetty and Finkelstein \(2012\)](#), the magnitude of the effect of UI on private savings is modest, and the median wealth holdings are very low as reported by [Engen and Gruber \(2001\)](#) and [Chetty \(2008\)](#). See also [Kolsrud, Landais, Nilsson, and Spinnewijn \(2015\)](#) for Sweden. [Kaplan \(2012\)](#) documents that a large number of low-skilled youth males move back to their parents’ place after job loss. Moreover, using Canadian survey data, [Browning and Crossley \(2001\)](#) estimate a much smaller effect of the replacement rate on household total expenditure, and a significant effect on married workers whose spouse was unemployed.

in the absence of UI, but each dollar of UI reduces the wife's earnings by 36-73 cents.⁵

We investigate the optimal allocation of unemployment risks in a static economy with directed search and intra-household risk sharing. Households are formed by a job-seeker and an employed spouse, and differ by the productivity of the spouse. Partial insurance against unemployment (or consumption) risks is privately arranged by pooling income and adjusting the spouse's labor supply. In line with [Chetty and Saez \(2010\)](#), we assume no moral hazard is generated as a result of such an arrangement. The optimal design of the public unemployment insurance scheme factors in the intra-household risk-sharing mechanism, and, hence, takes into account the household's total income. Therefore, there is a trade-off between the distortions on job creation generated by the taxes necessary to finance the public provision and the forgone leisure stemming from the private arrangements of insurance. Moreover, the optimal design of the public provision of insurance is limited by the heterogeneity across households and the informational frictions: unlike the spouse's income, neither her working time nor her productivity is observed.

We first characterize the *laissez-faire* equilibrium. We show that the labor supply of the spouses married to unemployed workers is larger than the one of those married to employed workers, which is in good accordance with the empirical evidence referred to above. We also find that an increase in unemployment benefits reduces the labor supply of spouses of both unemployed and employed workers in equilibrium. The former refers to the crowd-out effects of the public provision of insurance, whereas the general equilibrium (income) effects through wages explain the latter, which is also consistent with the evidence reported by [Cullen and Gruber \(2000\)](#).

We then address the normative questions: What is the constrained efficient allocation of risks in this economy? Can it be decentralized in the market economy? We first show that the insurance level in the equilibrium allocation falls short of the optimal level as welfare gains are obtained from redistributing resources from households with the two members employed to households with one unemployed worker. Because of the lack of redistributive instruments, two sources of private insurance are excessively at work: the intensive margin of the labor supply of the spouse and the job creation margin in the labor markets.

In the case of ex-ante homogeneous households, we show that the planner's allocation can be decentralized in the market economy. This implementation requires three fiscal

⁵The precise estimate is much larger for instrumented than for potential UI. [Cullen and Gruber \(1996\)](#) also find that the effects of UI on the hours conditional on working, but not their employment likelihood, are large and significant for wives of employed husbands with high unemployment risk, but not for the unemployed group. This suggests that households anticipate those risks. The responsiveness to UI also varies over the life cycle. For example, the crowd-out effects are much larger in household with small children and for young couples.

instruments: unemployment benefits financed by lump sum and a proportional income tax on newly employed workers.

In Section 5, we quantitatively illustrate the welfare gains of moving from the current system to the planner’s solution. Relative to the present-day system, two outcomes are of interest. First, the welfare gains differ significantly over the distribution of households. Second, productivity gains also take place since the labor supply increases (reduces) for more (less) productive spouses. Finally, we explore how far away it is from the planner’s solution a simple policy consisting of a replacement rate and a dependency allowance contingent on the spouse being unemployed. As of 2015, nine states of the U.S. provide such an allowance, and its amount varies across states.⁶

In our analysis of the optimal unemployment insurance, we abstract from the persistent effects of unemployment on earnings by focusing on unemployment risks and the short run, thereby lessening the effects on the spouse’s labor supply.⁷ Consistent to the short run approach, no frictions to adjustments in the labor supply of the spouse have been modeled. We also abstract from moral hazard problems and monitoring as well as administrative costs, features that would augment the relative costs of public insurance.

The paper proceeds as follows. After a brief summary of the related literature, Section 2 describes the economy. In Section 3, we study the market equilibrium. Section 4 analyzes the planner’s solution. In Section 5, we undertake a numerical exercise, and Section 5 concludes. All proofs are relegated to the Appendix.

1.1 Related Literature

This paper contributes to several branches of the labor literature. First, in the search literature, several attempts have been undertaken to examine the optimal level of unemployment benefits under various sources of private insurance. For example, in a random search model, [Krusell](#),

⁶The states are Connecticut, Washington D.C, Illinois, Iowa, Maine, Michigan, New Jersey, Ohio and Pennsylvania. See the Department of Labor documentation: <http://www.unemploymentinsurance.doleta.gov/unemploy/pdf/uilawcompar/2015/monetary.pdf>

⁷[Stevens \(1997\)](#) finds that earnings remain approximately 9% below expected levels 6 years after job loss. Using PSID data, [Stephens \(2002\)](#) documents that husbands’ earnings remain about 20% lower 3-4 years after displacement, and wives’ working hours keep increasing during this period. He estimates a 11% increase on average in annual working hours of wives, which includes both the intensive and extensive margins and offsets over 25% of their husbands’ lost earnings, which is in line with the figure estimated by [Morissette and Ostrovsky \(2008\)](#) for Canada in the 1990s. [Dynarski and Gruber \(1997\)](#) also find a large response in wives’ earnings following their husbands’ job loss using CEX data for high-school and college graduates, but not significant using PSID data. Using CPS data, [Mankart and Oikonmou \(2014\)](#) estimate that wives are 7.7% more likely to participate in the labor market in the month in which their husband becomes unemployed, which is almost as high as the overall participation probability of wives.

Mukoyama, and Şahin (2010) find that the sizable negative effects on job creation limit significantly the generosity of the optimal public provision of insurance in an economy where workers can insure themselves through savings. Although not focused on the optimal unemployment insurance, Acemoglu and Shimer (1999) show that private markets offer insurance against unemployment risks to job-seekers who can direct their search. In the search literature, Burdett and Mortensen (1978) were the first in stating that the participation decision of a household member depends on the employment state of the other members. To the best of our knowledge, no attempt has been done to introduce households in this framework to study the optimal insurance.⁸

Second, Ortigueira and Siassi (2013) quantitatively assess unemployment insurance with couples in the Aiyagari-Hugget framework, in which households are hit by exogenous employment shocks. Instead, in our setting, the public insurance scheme as well as the private provision of insurance affect the search decisions and job opportunities of the unemployed. Attanasio, Low, and Sánchez-Marcos (2005) estimate larger welfare costs of uncertainty in the husband’s earnings in the absence of the ability of their wives to adjust the labor supply.

Our work is also closely related to the optimal income taxation literature, starting from Mirrlees (1971). Boone and Bovenberg (2004) and Hungerbühler, Lehmann, Parmentier, and Van der Linden (2006) analyze optimal taxation in a frictional labor market. While in the latter, the demand side and wages are exogenous, workers are risk neutral and unemployment benefits are constant in the latter. None of these deals with two-member households. The optimal income taxation with couples is the focus of Kleven, Kreiner, and Saez (2009). In an economy with heterogeneity in spouse’s wage rate and worker’s participation costs, they find that optimal tax rates on an individual’s income differ by the earnings of the spouse. In contrast to our setting, in their economy, unemployment is voluntary, wages are exogenous and constant across types, and the effects on the demand side of the market are overlooked, and the utility function is quasi-linear in consumption, which eliminates the income effects on labor supply of the spouse. Chetty and Saez (2010) also examine the optimal design of (income and health) insurance programs in the presence of private insurance.

The crowding-out effects of public intervention have also been analyzed in different settings. Krueger and Perri (2011) investigate the optimal degree of progressivity in the income tax as a public risk-sharing device in the presence of limited private insurance markets. They and

⁸As Guler, Guvenen, and Violante (2012) point out, there has been no continuation of their work until very recently. They analyze the case of couples jointly searching for jobs in different locations. As in our setting, wage dispersion arises in equilibrium in those models as the spouse’s income affects the reservation value of the job-seekers when there is perfect income pooling. They do not pay attention, however, to the efficient distribution of unemployment risks.

[Attanasio and Ríos-Rull \(2000\)](#) show that the introduction of mandatory public insurance may indeed backfire and reduce total insurance. As these two papers show, the specifics of the private insurance scheme matter for the net gains of the public provision. For example, the crowd-out effects of the public UI would have been larger if we had modeled potential inabilities of the households to smooth consumption in the short run due to e.g. habit formation or consumption commitments.

Finally, we model heterogeneity across households in the spouse's income, but other dimensions of heterogeneity are also potentially key for the design of the public provision of insurance. For example, differences across workers result from their unemployment duration and over the lifecycle as studied by [Hopenhayn and Nicolini \(1997\)](#) and [Michelacci and Ruffo \(2015\)](#), respectively.

[To be completed]

2 Benchmark Model

Consider an economy populated by a measure one of two-member households and a large continuum of risk-neutral firms. The mass of active firms is pinned down by free entry. Households are formed by an unemployed worker and her spouse endowed with market productivity $x \in [\underline{x}, \bar{x}]$, with $\underline{x} > 0$.⁹ The former searches for a job, whereas the spouse chooses her labor supply $\ell \in [0, 1]$. Let $F(x)$ denote the measure of households with type below level x , and it is assumed to be a differentiable cdf. Following the optimal taxation literature since [Mirrlees \(1971\)](#), productivity and labor supply (either hours worked or effort) are private information, whereas an individual's total earnings are observable by the government and the planner.¹⁰ It is convenient to think in terms of observable variables, and, hence, we model the spouse as deciding total earnings y instead of labor supply $\ell = y/x$.¹¹

To begin with, we follow [Guler, Guvenen, and Violante \(2012\)](#) and assume that households are the decision-making units, and consumption is a public good within the household.¹² They derive utility from consumption c and leisure of the spouse.¹³ We impose the following assumptions on the utility function $v(c, y/x)$ that describes the preferences of a couple:

- A1. v is thrice continuously differentiable.
- A2. v is increasing in consumption and leisure: $v_c > 0$, $v_\ell < 0$
- A3. v is strictly concave: $v_{\ell\ell}, v_{cc} < 0$, and $v_{\ell\ell}v_{cc} - v_{c\ell}^2 > 0$.
- A4. Weak complementarity: $v_{c\ell} \leq 0$.
- A5. $\lim_{\ell \rightarrow 0} v_\ell < \lim_{\ell \rightarrow 0} v_c$ and $\lim_{\ell \rightarrow 1} v_c < \lim_{\ell \rightarrow 1} v_\ell$.

The first three conditions are fairly standard. We also assume that the cross-partial derivative is non-positive, meaning complementarity between consumption and leisure, which includes

⁹Alternatively, x can be interpreted as ability or hourly wage. This latter interpretation neglects general equilibrium effects. The assumption of a positive lower bound is made for expositional reasons. Single-earner households can be thought of as the case with the spouse's productivity x being arbitrarily small.

¹⁰As pointed out by [Salanie \(2011\)](#), if labor supply were interpreted as hours worked, the government could force employers to report them.

¹¹For simplicity, we make the assumption that spouses work in a frictionless labor market and decide their labor supply at a given productivity. According to CPS data, not-in-the-labor-force wives amount to 30 percent of married women. To abstract from the underlying reasons of their non-participation decision -caring of children and elderly, etc.-, we model their market productivity at zero. Similarly, single primary earners are assumed to be married to a zero-productivity spouse. The empirical literature has not unraveled whether the increase in wives' hours results from increasing working time at the same job or from job-switching.

¹²In Section 3.3.2, we extend the analysis to a cooperative model of the household.

¹³We abstract from whether leisure of the household members are substitutes or complements by assuming indivisible labor supply of the unemployed worker.

the case of additive separability between consumption and leisure.¹⁴ The last condition ensures the existence of an interior solution in the household's problem.

There are four stages. In stage one, unemployed workers direct their search. That is, they choose a submarket and place an application at cost κ .¹⁵ A submarket or location is defined by a set of job characteristics. In stage two, firms decide on the submarket to place their vacancies, and incur cost k when posting a vacancy. Market productivity of newly employed workers is normalized to 1. As usual in the search literature, each recruiting firm holds a single vacancy. Meetings take place in stage three as described below. Some workers become employed, whereas some other workers remain unemployed and produce z at home. In stage four, spouses decide their total earnings y , and both production and consumption take place.

To ensure existence of equilibrium and that all jobless workers search for a job, we make the following two assumptions. First, there is a gap between net market productivity and vacancy creation costs, $1 - z > k$, to ensure that vacancy creation is a profitable activity. Second, cost κ is sufficiently small so that $\max_y v(y + 1, y/\underline{x}) - \max_y v(y + z, y/\underline{x}) > \kappa$.

Matching Rates. Meetings are bilateral. Workers find a job at submarket ω with probability $\nu(q)$, where q denotes the expected queue length or ratio of job-seekers to vacancies, whereas firms fill their vacancies with probability $\eta(q)$. Since the mass of newly employed workers equals the mass of newly filled vacancies in any given submarket w , it must be the case that $\nu(q) = \frac{\eta(q)}{q}$. We assume that ν is a decreasing function to capture the intuition that it is harder to find a job in tighter labor markets, and, hence, η is assumed to be increasing. Likewise, the following limit conditions are necessary to ensure existence of equilibrium and planner's allocations: $\lim_{q \rightarrow 0} \nu(q) = \lim_{q \rightarrow \infty} \eta(q) = 1$ and $\lim_{q \rightarrow \infty} \nu(q) = \lim_{q \rightarrow 0} \eta(q) = 1$. Let $\gamma(q) \equiv \frac{q\eta'(q)}{\eta(q)}$ denote the elasticity of the job-filling rate, which is assumed to be a decreasing function.¹⁶

3 Market Economy

In this section, we analyze an economy in which agents make decisions in a decentralized way to maximize their utility. There are potentially infinitely many submarkets. Each submarket is defined by a single-wage offer w . Whereas firms decide whether to create a vacancy and what wage to commit to, the household's decision is twofold. First, it chooses a submarket

¹⁴There is empirical evidence provided e.g. by Attanasio and Weber (1995) and Meghir and Weber (1996) pointing to complementarity in utility between consumption and leisure.

¹⁵Labor market participation is costly to make the social planner's problem analyzed in Section 4 non-trivial.

¹⁶These properties are satisfied for the usual matching functions, e.g. the Cobb-Douglas and urn-ball ones.

to submit a job application. Then, after learning the search outcome, it decides the labor supply of the spouse. We start detailing this last stage and, then, proceed backwards.

Stage Four. Let w denote the income of the job-seeker at the end of the period, with $w = z$ if unemployed. We denote the household's indirect utility function by V_x , which is defined as the maximand of the following program

$$V_x(w) \equiv \max_y v(y + w, y/x) \quad (1)$$

The Weierstrass theorem together with Assumption A1 ensures that V_x is well-defined. Notice that the first order necessary condition is also sufficient because of Assumption A3. Moreover, Assumption A5 ensures the existence of an interior solution for the first order condition. Therefore, the following equations uniquely determine the contingent earnings of the spouse, $y_x^e(w)$ and y_x^u .

$$v_c(y + w, y/x)x = -v_\ell(y + w, y/x) \quad (2)$$

$$v_c(y + z, y/x)x = -v_\ell(y + z, y/x) \quad (3)$$

As can be anticipated from the observation of the household's problem, the indirect utility function is central in the analysis of the market equilibrium. In the following lemma, we establish properties of function V_x and of the optimal total earnings of the spouse, which are inherited from the assumptions on the utility function v and follow from using repeatedly the Implicit Function Theorem to the first order conditions. In particular, the labor supply of the spouse is larger if married to an unemployed worker than to an employed one in line with the evidence reported in the Introduction. This is because labor supply is set to equate the marginal utility of leisure and the marginal utility of consumption, which is lower for households with the two members employed, together with the complementarity between consumption and leisure.

Lemma 3.1 *Function V_x is twice continuously differentiable, strictly increasing and concave. Furthermore, the optimal solution $y_x^e(\cdot)$ is twice continuously differentiable, and strictly decreasing in wages. In particular, $y_x^e(w) < y_x^u$ for all $w > z$.*

Moreover, this lemma states that the household utility $V_x(w)$ and its derivative $V'_x(w)$ are increasing and decreasing in the household type x , respectively.

Lemma 3.2 *Function V_x increases and its derivative V'_x decreases with the spouse's wage rate x .*

Stage Two. There is entry of firms in all submarkets as long as expected profits are positive in and out of equilibrium. That is, the following condition must hold both for all $w \in [z, 1]$:

$$\eta(q)(1 - w) \leq k, \text{ and } q \leq \infty, \text{ with complementary slackness.} \quad (4)$$

Intuitively, when the larger the wage, the lower the mass of vacancies posted. In the limit, no positive mass of firms commit to a wage equal to market productivity of workers.

Stage One. Job-seekers rationally anticipate the optimal behavior of firms in the second stage, and trade off a higher wage and a higher job-finding probability. The expected utility of a household with the spouse's productivity x amounts to $(1 - \nu(q(x)))V_x(z) + \nu(q(x))V_x(w)$, where $q(x)$ denotes the expected queue length in submarket w .

3.1 Equilibrium.

We now turn to the definition of equilibrium.

Definition 1 *A directed search equilibrium consists of, for all $x \in [\underline{x}, \bar{x}]$, household values U_x , earnings $y_x^e : [z, 1] \rightarrow \mathcal{R}_+$ and $y_x^u \in \mathcal{R}_+$, wages w_x , and a queue length function $Q : [z, 1] \rightarrow \mathcal{R}_+$ such that:*

i) Households optimally direct their search and choose earnings. For all $x \in [\underline{x}, \bar{x}]$,

$$(a) \quad \nu(Q(w))(V_x(w) - V_x(z)) + V_x(z) \leq U_x, \forall w \in [z, 1], \text{ and}$$

$$\nu(Q(w_x))(V_x(w_x) - V_x(z)) + V_x(z) = U_x$$

(b) For all $w \in [z, 1]$, $y_x^e(w)$ and y_x^u solve the respective household's problem, and, hence, satisfy conditions (2) and (3), respectively.

ii) Free entry of firms:

$\eta(Q(w))(1 - w) \leq k, \forall w \in [z, 1]$, and $Q(w) \leq \infty$, with complementary slackness. In particular, the first inequality is an equality for all w_x .

The first equilibrium condition is self-explanatory. The second condition determines the ratio of job-seekers to vacancies both on and off the equilibrium path. Workers form rational expectations about firms' decisions in stage 2. Specifically, they expect the ratio of job-seekers to firms in any submarket to be determined by the zero-profit condition. Thus,

the household's problem at the beginning of the period is¹⁷

$$\begin{aligned} \max_{q \geq 0, w \in [z, 1]} \quad & V_x(z) + \nu(q)(V_x(w) - V_x(z)) \\ \text{s. to} \quad & \text{condition (4)} \end{aligned} \tag{5}$$

3.2 Equilibrium Characterization

The following proposition states that there exists a unique equilibrium, and characterizes it. Equilibrium condition (6) is the first order condition of the household's problem. It equates the costs of creating a vacancy to the expected profits, which amount to the probability of filling a vacancy times the share $1 - \gamma(q)$ of the joint value of the firm-worker pair adjusted by the marginal utility of the household. Equilibrium equation (7) is the zero-profit condition expressed in terms of wages. For notational simplicity, we denote hereafter the equilibrium queue length at wage w_x as $q_x \equiv Q(w_x)$.

Proposition 3.3 *There exists a unique equilibrium. For any given household type $x \in [\underline{x}, \bar{x}]$, the equilibrium pair (q_x, w_x) is characterized by the following system of equations*

$$k = \eta(q)(1 - \gamma(q)) \left(\frac{V_x(w) - V_x(z)}{V'_x(w)} + 1 - w \right) \tag{6}$$

$$k = \eta(q)(1 - w) \tag{7}$$

The equilibrium is generically separating. Put differently, there is wage dispersion in equilibrium even though workers are equally productive and firms are also homogeneous. This is because the attitudes towards unemployment risks differ across households because of the private insurance arrangement. This poses a source of heterogeneity which has been overlooked when examining wage dispersion using a Mincerian regression. The optimal search strategy depends on the spouse's wage rate. Similarly to [Acemoglu and Shimer \(1999\)](#), households of higher types apply to higher-wage jobs if the absolute risk aversion of the indirect utility function, $\frac{-V''_x(w)}{V'_x(w)}$, decreases with the wage rate of the spouse x .¹⁸ Consider the case of additively separable, CRRA preferences, $v(c, \ell) = \frac{c^{1-\sigma}}{1-\sigma} + \frac{(1-\ell)^{1-\sigma}}{1-\sigma}$. Because absolute risk aversion is decreasing in x , job-seekers married to more productive spouses apply to higher-wage jobs, which are harder to get.

¹⁷For expositional simplicity, we omit the participation decision because of the assumption on sufficiently small search costs.

¹⁸Despite the differences, it is not surprising that we obtain a similar result to theirs as job-seekers can insure themselves through savings in their setting. This condition involves assumptions on the third derivative of the utility function v .

Proposition 3.4 *If $\frac{-V_x''(w)}{V_x(w)}$ is decreasing in x , then w_x and q_x are increasing in x . If it is constant, so are w_x and q_x .*

The following lemma states that the public provision of insurance crowds out private insurance as labor supply of spouses married to unemployed workers decreases with z , the parameter capturing home productivity and unemployment benefits. Furthermore, as is common in search models, wages increase and job-finding rates decrease with z . This result together with the negative relationship between the wage of the primary earner and the labor supply of the secondary earner stated in Lemma 3.1 implies that the labor supply of the spouse married to an employed worker also decreases with z . These direct and indirect negative effects of unemployment benefits on the spouse's labor supply are consistent with the empirical evidence reported by Cullen and Gruber (2000). They estimate that each \$100 in potential benefits lowers the working hours of wives of employed and unemployed workers by 5.2 and 22.7 per month, respectively. The general equilibrium effect through wages of unemployment benefits on the labor supply of the spouse in households with the two members employed has two components. While the positive macro effects of benefits on wages are supported by Hagedorn, Karahan, Manovskii, and Mitman (2015), the second component of the mechanism is in line with the negative cross-elasticities estimated by e.g. Hyslop (2001) and Blau and Kahn (2007) as reported in the Introduction.

Lemma 3.5 *Comparative Statics.*

1. *Wages and queue lengths increase with z .*
2. *The spouse's labor supply decreases with z regardless of the employment state of the worker.*

3.3 Extensions

In this section we examine two particular cases that have been considered in the literature: namely, quasi-linear preferences and a cooperative model of the household.

3.3.1 Quasi-linear Preferences.

Consider first a quasi-linear utility function in consumption, $v(c, y/x) = c + \phi(y/x)$, where function ϕ is twice continuously differentiable, decreasing and concave. Then, the indirect utility function V_x is linear in wages. As is well known, labor supply of the spouse is insensitive to the income of the job-seeker and, in particular, to unemployment benefits;

hence, household's consumption increases one-to-one with benefits. The equilibrium conditions are

$$k = \eta(q_x)(1 - \gamma(q_x))(1 - z), \quad \forall x \in [\underline{x}, \bar{x}] \quad (8)$$

$$k = \eta(q_x)(1 - w_x), \quad \forall x \in [\underline{x}, \bar{x}] \quad (9)$$

$$\phi'(y_x^j/x) = -x, \quad \forall x \in [\underline{x}, \bar{x}], j \in \{u, e\} \quad (10)$$

Notice that the first two equations are the counterparts of conditions (6) and (7), whereas the last one is the first order condition of the household's problem (2). It follows from the first two equilibrium conditions that all workers search in the same market regardless of their spouse's wage rate.¹⁹ The third condition shows no income effects on the labor supply of the spouse, and an increasing income y_x^j in x .

Consider next quasi-linear preferences in leisure: $v(c, y/x) = \psi(c) - y/x$, where ψ is a twice continuously differentiable, increasing and concave function. The equilibrium conditions are the same as before, except for the last one, which is replaced by

$$\psi'(w_x + y_x^e), \psi'(z + y_x^u) = 1/x \quad (11)$$

There is full insurance because the labor supply of the spouse adjusts to make consumption invariant to the search outcome. Furthermore, all workers search in the same market and, hence, wages are constant in productivity x . It follows that consumption increases with the spouse's productivity because so do the spouse's income, and the difference $y_x^u - y_x^e$ is also constant in x .

3.3.2 Cooperative Model

The benchmark economy hosts a unitary model of the household, in which households are the decision-making units that maximize a utility function subject to a budget constraint. This modeling has been questioned on empirical and theoretical grounds. See [Chiappori and Donni \(2009\)](#) for a survey. Therefore, it is worth checking the robustness of our results in a cooperative model of the household. In such models, each member of the household has their own preferences, and the decision-making process is usually not made explicit. Consumption is no longer a public good. Instead, the two members of the household pool income and decide on their individual consumption and labor supply. Importantly for our analysis, cooperative models ensure Pareto efficient intra-household outcomes.

¹⁹Notice that the absolute risk aversion of the indirect utility function V_x is zero.

For notational simplicity, let m and f denote the index of the two members of the household. Likewise, α stands for the Pareto weight on the utility of the first member, and captures the m 's relative power within the household. To abstract from the interaction between policy and intra-household power distribution, we assume that the Pareto weights do not depend on income, and in particular, they are insensitive to wage w and productivity x as well as unemployment benefits z .²⁰ The indirect utility function of a household of type x is

$$\begin{aligned} V_x(w) = & \max_{c^f, c^m, y} \alpha v^m(c^m, \underline{\ell}) + (1 - \alpha) v^f(c^f, y/x) \\ \text{s. to } & c^f + c^m = y + \mathcal{I}_e w + (1 - \mathcal{I}_e) z \end{aligned} \quad (12)$$

where v^f and v^m satisfy properties A1-A5, labor supply $\underline{\ell}$ is exogenous, and $\mathcal{I}_e$ is an indicator function that values one if the worker is employed and zero otherwise.

Notice that the first order conditions establish the following risk-sharing policy, which depends on the Pareto weights, $\alpha v_c^m = (1 - \alpha) v_c^f$. That is, the marginal utility must be equal across members after adjusting for the weight distribution within the household. Likewise, the household marginal gains of an increase in wages must be equal to the marginal gains from an equivalent increase in the spouse's leisure, $V'_x(w) = \frac{-(1-\alpha)}{x} v_\ell^f$.

The following lemma states that we obtain the same results as in the benchmark case.

Lemma 3.6 *Function V_x is twice continuously differentiable, strictly increasing and concave. The optimal solution y_x^e is twice continuously differentiable, and strictly decreasing in the wage. In particular, $y_x^e(w) < y_x^u$ for all $w > z$ and x . Furthermore, the spouse's hours worked decrease with z regardless of the employment status of the worker. The equilibrium allocation is determined by equations (6) and (7).*

4 Constrained Efficiency

The main result of this section is that constrained efficiency cannot be attained in the market economy because private provision of insurance against consumption risks is inefficiently limited, and the insurance mechanisms, in the labor market through vacancy creation and within the household through the spouse's labor supply, are excessively used. We first characterize the constrained efficient allocation.

²⁰In a *collective* model of the household instead, the Pareto weights may depend on the relative earnings, total income and other called *distribution factors* out of the model.

4.1 The Planner's problem

As usually assumed in the search literature, the social planner maximizes a utilitarian welfare function. It sets a mass of vacancies, assigns search strategies and transfers to workers, and faces the same coordination frictions as agents encounter in the market economy. Moreover, the planner cannot observe the type of the households and the spouse's labor supply; however, both the spouse's income y and the worker's employment state are observable.

More specifically, the planner designs a symmetric incentive compatible revelation mechanism that consists of a menu of contracts $\{(q_x, c_x^e, c_x^u, y_x^e, y_x^u) | x \in [\underline{x}, \bar{x}]\}$ indexed by the household's announcement of its type. The mechanism is symmetric in the sense that all households reporting a given type are treated identically. For any reported type x , the mechanism specifies a location where to submit an application and the associated job-finding probability, $\nu(q_x)$, consumption as well as the spouse's income contingent on the search outcome, (c_x^e, c_x^u) and (y_x^e, y_x^u) .

We say that a mechanism is *feasible* if total consumption promises do not exceed total output net of vacancy creation costs, i.e. if the following resource constraint holds

$$\int_{\underline{x}}^{\bar{x}} \frac{k}{q_x} dF(x) = \int_{\underline{x}}^{\bar{x}} \left(\nu(q_x)(1 + y_x^e - c_x^e) + (1 - \nu(q_x))(z + y_x^u - c_x^u) \right) dF(x) \quad (\text{RC})$$

The ex-ante utility of a household of type x reporting type $\hat{x}$ can thus be written as

$$\mathcal{U}_x(\hat{x}) \equiv \nu(q_{\hat{x}})v(c_{\hat{x}}^e, y_{\hat{x}}^e/x) + (1 - \nu(q_{\hat{x}}))v(c_{\hat{x}}^u, y_{\hat{x}}^u/x) \quad (13)$$

To simplify notation, let us denote $\mathcal{U}_x \equiv \mathcal{U}_x(x)$. The mechanism must be compatible with agents' incentives. This implies that job-seekers must truthfully reveal their types. That is, the following incentive compatibility constraints must hold.

$$\mathcal{U}_x \geq \mathcal{U}_x(x'), \quad \forall x, x' \in [\underline{x}, \bar{x}] \quad (\text{ICC}_x)$$

Furthermore, the value of job-search must exceed the application cost to ensure that participating in the market is desirable. That is, the following set of participation conditions must also hold.

$$\mathcal{U}_x \geq \kappa + v(c_x^u, y_x^u/x) \quad (\text{PC}_x)$$

We will refer as the constrained efficient allocation to the feasible incentive-compatible

mechanism that solves the planner's problem, which can be written as

$$\begin{aligned} \text{Planner's problem:} \quad & \max \int_{\underline{x}}^{\bar{x}} \mathcal{U}_x dF(x) \\ & \text{s. to } (\text{PC}_x), (\text{ICC}_x) \text{ and } (\text{RC}) \text{ for all } x \end{aligned}$$

To understand the importance of each one of these constraints, let us consider what allocation would be obtained if either one were subtracted. Obviously, resources are limited. If the participation conditions were eliminated, the planner would promise equal bundles regardless of the search outcome, and this allocation could trivially not be decentralized. If the incentive compatibility constraint were eliminated instead because types were observable and preferences were additively separable, then the planner's allocation could be decentralized in the market economy as we will comment later. However, consumption would be constant across types and higher types would produce more, yielding a declining expected utility over productivity levels. This result is well-known in the optimal taxation literature, see e.g. [Mankiw, Weinzierl, and Yagan \(2009\)](#). To see that incentive compatibility ensures that households with more productive spouses obtain higher values notice that

$$\mathcal{U}_x \geq \mathcal{U}_x(x') = \nu(q_{x'})v(c_{x'}^e, y_{x'}^e/x) + (1 - \nu(q_{x'}))v(c_{x'}^u, y_{x'}^u/x) > \mathcal{U}_{x'}, \quad \text{for } x' < x$$

where the first inequality is condition (ICC_x) , and the second inequality results from function v being strictly increasing in the household's type. The intuition underlying this result is that if the sign of the inequality were reversed, higher-type households would have incentives to misreport their type. The following lemma states existence of the planner's solution as a straightforward result of Weierstrass Theorem.

Lemma 4.1 *There exists a solution to the planner's problem. Furthermore, the expected utility in the constrained efficient allocation is monotonic over household types.*

To characterize the planner's solution, it is generally convenient to reduce the dimensionality of the problem by replacing the incentive-compatibility condition (ICC_x) by a first- and

a second-order condition²¹

$$\begin{aligned}\dot{\mathcal{U}}_x &= -\nu(q_x)v_\ell^e \frac{y_x^e}{x^2} - (1 - \nu(q_x))v_\ell^u \frac{y_x^u}{x^2}, & (FOC - ICC_x) \\ -\nu'(q_x)\dot{q}_x \frac{v_\ell^e y_x^e - v_\ell^u y_x^u}{x^2} - \nu(q_x) \frac{v_{c\ell}^e \dot{c}_x^e y_x^e + \dot{y}_x^e (v_{\ell\ell}^e y_x^e/x + v_\ell^e)}{x^2} \\ & - (1 - \nu(q_x)) \frac{v_{c\ell}^u \dot{c}_x^u y_x^u + \dot{y}_x^u (v_{\ell\ell}^u y_x^u/x + v_\ell^u)}{x^2} \geq 0 & (SOC - ICC_x)\end{aligned}$$

where, for notational simplicity, $v^j \equiv v(c_x^j, y_x^j/x)$ for $j \in \{u, e\}$ and for all x , and $\dot{n} \equiv \frac{dn}{dx}$ denote the derivative of variable n with respect to x . These necessary conditions are local. The following lemma states that they are also sufficient.

Lemma 4.2 *The above necessary conditions are also sufficient.*

4.2 Efficiency in the market economy

The following proposition states that constrained efficiency is not achieved in the *laissez-faire* equilibrium. It is easy to see that the equilibrium allocation belongs to the feasible set of the planner's problem. That is, the resource and participation constraints hold, and the equilibrium allocation is also incentive compatible. However, efficiency gains can be obtained by redistributing resources to increase consumption of the households with an unemployed member. Put differently, the private provision of insurance against consumption risks falls short of the constrained efficient level. Moreover, inefficiency may also result from the ex-ante heterogeneity across households and the market's inability of redistributing resources among them.

Proposition 4.3 *Under Assumptions A1-A5, the equilibrium allocation is not constrained efficient.*

Next, we examine the inefficiency result by looking at several particular cases.

²¹The FOC of the (ICC_x) indeed says that the total differential with respect to $\hat{x}$ is zero at $\hat{x} = x$,

$$\left. \frac{d\mathcal{U}_x(\hat{x})}{d\hat{x}} \right|_{\hat{x}=x} = \frac{\partial \mathcal{U}_x(\hat{x})}{\partial q_{\hat{x}}} \dot{q}_{\hat{x}} + \frac{\partial \mathcal{U}_x(\hat{x})}{\partial c_{\hat{x}}^e} \dot{c}_{\hat{x}}^e + \frac{\partial \mathcal{U}_x(\hat{x})}{\partial y_{\hat{x}}^e} \dot{y}_{\hat{x}}^e + \frac{\partial \mathcal{U}_x(\hat{x})}{\partial c_{\hat{x}}^u} \dot{c}_{\hat{x}}^u + \frac{\partial \mathcal{U}_x(\hat{x})}{\partial y_{\hat{x}}^u} \dot{y}_{\hat{x}}^u \Big|_{\hat{x}=x} = 0,$$

which is equivalent to the expression above. To obtain the second order condition at $\hat{x} = x$, we differentiate again with respect to $\hat{x}$. To simplify the second derivative, we benefit from this condition to hold for all x . After some tedious, but straightforward calculations, we obtain expression $(SOC-ICC_x)$.

4.3 Quasi-linear preferences in consumption

We first consider the case of a quasi-linear utility function, $v(c, y/x) = c + \phi(y/x)$, where function ϕ is decreasing and concave as assumed in Section 3.3.1. The following proposition states that the equilibrium allocation is constrained efficient. That is, the planner asks all job-seekers to search in the same location and the labor supply of the spouse is not affected by the search outcome. Importantly, worker types are separated costlessly, and the marginal rate of substitution is 1 for all households.

Proposition 4.4 *Constrained efficiency is attained in the market economy.*

Why are these preferences of interest? Certainly, these preferences do not satisfy Assumption A3. However, this case constitutes the benchmark for two different literatures. First, in the optimal income taxation literature, it has been common to assume such preferences because it greatly simplifies the problem and allows for a closed-form solution.²² See [Salanie \(2011\)](#) for a summary, and [Kleven, Kreiner, and Saez \(2009\)](#), for the optimal taxation of couples. Furthermore, this assumption is in fair agreement with the evidence on the income elasticity of labor supply for primary earners, mostly men. In contrast, when referring to secondary earners, mostly wives, labor supply elasticity is much larger, and the empirical evidence reported in the Introduction shows significant income effects of earnings and unemployment benefits on wife's hours of work.

Second, in the standard directed search model, agents are assumed to be risk neutral. Interestingly, the equilibrium allocation is constrained efficient because wages price waiting time. See [Moen \(1997\)](#). Therefore, although the focus of our analysis is on secondary earners and intra-household insurance, the economy with quasi-linear preferences in consumption is also of interest as a benchmark.

The following lemma establishes that constrained efficiency is also attained in equilibrium when household members have their own (quasi-linear) preferences and cooperate when making their decisions.

Lemma 4.5 *Consider a cooperative model of the household as the one described in Section 3.3.2 in which each member has quasi-linear preferences. Then, the equilibrium is constrained efficient.*

²²To make the problem nontrivial, a motive for redistribution is assumed for example by either giving a lower weight to higher-earnings agents or assuming a concave transformation of agent's utility function.

4.4 Economy with ex-ante homogeneous households

We now examine an economy with a degenerate distribution F . Recall that, in the market economy, the equilibrium pair (w, q) is determined by conditions (6) and (7), as stated in Proposition 3.3. For expositional purposes, we assume that neither productivity nor labor supply nor output is observable to the planner, and, hence, incentive compatibility must be taken into account.²³ The planner's problem can be written as follows

$$\begin{aligned} \max \quad & \nu(q)V(m^e) + (1 - \nu(q))V(m^u) \\ \text{s. to} \quad & \frac{k}{q} = \nu(q)(1 - m^e) + (1 - \nu(q))(z - m^u) \\ & \kappa \leq \nu(q)(V(m^e) - V(m^u)) \end{aligned} \tag{14}$$

where function V resembles expression (1). As stated in Proposition 4.3, the equilibrium is not constrained efficient. This is because of the inability of the market economy to efficiently insure the consumption risks away. Indeed, the income of the spouse is above the constrained efficient level as private intra-household insurance tries to compensate for the lack of redistribution across households.

Nonetheless, the planner's solution can be implemented in the market economy by setting an unemployment insurance system funded by lump sum and proportional income taxes. In order not to distort the labor supply of spouses, which provides intra-household insurance, their income is not taxed.²⁴ Furthermore, income taxes are necessary to convey the proper search incentives to job-seekers, and this is done through wages in a directed search economy. Notice that this is the case regardless of whether the spouse's income is observed or not because the planner factors in the spouse's optimal decision by equating the marginal utility from an additional consumption unit and an additional unit of leisure. It is worth underscoring that the planner's solution makes the publicly-provided insurance be based on the intra-household insurance and, hence, on household's total potential income.

We question again whether the implementation of the planner's solution still holds when considering a cooperative, in lieu of a unitary, model of the household. The answer is yes. Notice that the planner's problem written as in (14) allows for a straightforward interpretation of function V as in expression (12) in Section 3.3.2. The proof is straightforward; hence, omitted. Importantly, the specific optimal policy is contingent on the Pareto-weights of the

²³We obtain the same results if they were observable as would be the case from our set of assumptions.

²⁴This not taxing on agents with a high income elasticity is along the lines of [Alesina, Ichino, and Karabarbounis \(2011\)](#).

household model as so is the planner's allocation.

Proposition 4.6 *If households are ex-ante identical, then the spouse's income is excessively large in equilibrium. Constrained efficiency can be attained in the market economy through the implementation of a public unemployment insurance financed by a proportional income tax on newly employed workers and a lump sum tax. Furthermore, this result also holds with a cooperative model of the household.*

If workers' productivity were observable, the result of the decentralization of the planner's allocation could be extended to an economy with a non-degenerate cdf F by means of productivity-contingent tax rates. Otherwise, as assumed here, an incentive scheme, based on realized spouse's earnings, must be set to elicit information on actual intra-household insurance. We next analyze scenarios with ex-ante heterogeneous households averse to consumption risks.

4.5 Quasi-linear preferences in leisure

We start by considering quasi-linear preferences in leisure: $v(c, \ell) = \psi(c) - \ell$, where function ψ is increasing and concave. The following proposition characterizes the planner's solution.

Proposition 4.7 *The planner's allocation is characterized by the following conditions*

$$k = \eta(q_x)(1 - \gamma(q_x))(1 - z), \text{ and } c_x^e = c_x^u = c_x, \quad \forall x \quad (15)$$

The participation condition (PC_x) is never binding. There exists a subset of positive mass in which $\dot{c}_x > 0$, $\nu(q)\dot{y}_x^e + (1 - \nu(q))\dot{y}_x^u > 0$, and $\psi'(c_x) > \frac{1}{x}$. The equilibrium allocation is not constrained efficient. Furthermore, there may be bunching: $\dot{c}_x = \nu(q)\dot{y}_x^e + (1 - \nu(q))\dot{y}_x^u = 0$ within a subset of $[\underline{x}, \bar{x}]$.

Condition (15) states two outcomes: there is full intra-household insurance, and a single labor market is active. Furthermore, the constrained efficient vacancy creation is the one that maximizes output, which is inherently associated with the previous result. As full insurance is provided within the household, job-seekers behave as though risk-neutral agents, and, as a result, the planner aims at maximizing output to redistribute across ex-ante different households. Notice that both full private insurance and output maximization also take place in equilibrium, but there is no redistribution across households. Indeed, the equilibrium consumption level exceeds the planner's consumption, implying that labor supply of the

spouse is inefficiently large. Furthermore, there may exist a subset of individuals applying to contracts which specify the same consumption level and expected income of the spouse. Following the optimal taxation literature, we refer to this result as *bunching*.

Lemma 4.8 *Suppose there is no bunching in the planner's solution. Then, it can be decentralized through the implementation of a tax on household's total income.*

Proof of Lemma 4.8.

Let T denote a tax on household's total income. For the tax-distorted equilibrium to coincide with the planner's solution, efficiency condition (15) as well as $\psi'(c_x) > \frac{1}{x}$ must hold.

The counterpart of equilibrium equation (8), together with the zero-profit condition, resulting from the household's problem and program (1) are

$$\begin{aligned} (1 - \gamma(q)) \left(\frac{V(w) - V(z)}{v_c^e(1 - T'(w + y_x^e))} + 1 - w \right) &= \frac{k}{\eta(q)} \\ v_c^e(1 - T'(w + y_x^e)) &= v_c^u(1 - T'(z + y_x^u)) = \frac{1}{x} \Rightarrow T' < 1 \\ k &= \eta(q)(1 - w) \end{aligned}$$

For the consumption levels to be independent of the employment status, it must be the case that

$$c_x = w + y_x^e - T(w + y_x^e) = z + y_x^u - T(z + y_x^u)$$

Furthermore, the government balances its budget, and, hence,

$$\int_{\underline{x}}^{\bar{x}} \left(\nu(q)T(w + y_x^e) + (1 - \nu(q))T(z + y_x^u) \right) f(x) dx = 0$$

First, given the planner's queue length, the tax-distorted equilibrium wage w is determined by the zero-profit condition (9). Second, by comparing the second equilibrium condition with its planner's counterpart, we conclude that T must be an increasing function. Next, by combining the first two equations to replace the marginal utility and imposing the equal-consumption condition, we can rewrite the first equilibrium equation as

$$(1 - \gamma(q)) \left(1 - z - T(w + y_x^e) + T(w + y_x^u) \right) = \frac{k}{\eta(q)}$$

Therefore, the implementation of the planner's solution implies $T(w + y_x^e) = T(w + y_x^u)$, which leads to a pre-tax household's total income independent of the employment state, $w + y_x^e = z + y_x^u$. As w is invariant in x so is the difference $y_x^u - y_x^e$.

Finally, it is straightforward to combine the zero-profit condition and the balanced-budget constrained of the government to obtain the resource constraint of the planner. ||

[To be completed]

5 Quantitative Exploration

In this section, we quantitatively explore the welfare gains that would obtain from moving from the U.S. economy to another one with the optimal household-income-based insurance system. Consistently with our previous work, our benchmark hosts a unitary model of the household, and we also make the comparison with a cooperative model. In order to have the tax burden realistically distributed, we extend the model to have a mass μ of households with the two members employed at the beginning of the period, and re-scale the mass of households with one member unemployed at the beginning of the period to $1 - \mu$. Employed primary-earners become unemployed at the beginning of the period with probability λ . The unemployment rate at the end of the period amounts to $(1 - \nu(q))(\mu\lambda + 1 - \mu)$.

To discipline our parametrization, we use data for individuals aged 30 to 55 from Current Population Survey (CPS) from 1994:1 to 2015:12 and Job Openings and Labor Turnover Survey (JOLTS) for vacancies from 2000:12 to 2015:12.²⁵ We largely identify primary earners with men. Table 5 summarizes our parametrization. We set $\mu = 0.9400$ so that the unemployment rate matches the average monthly male unemployment rate, 5.4763%. Preferences are assumed to be $v(c, \ell) = \ln c + \sigma \ln(1 - \ell)$, with $\sigma > 0$, which is set to match the gender gap in hours worked, 0.8826.²⁶ The matching technology is assumed to be urn-ball, $\eta(q) = 1 - e^{-\alpha q}$. Parameters α and k are set to match the average monthly transition rate from unemployment to employment for males and the average vacancy-unemployment ratio.²⁷ The former amounts to 0.31, whereas the second is 1.3113. For this economy to be consistent with a constant unemployment rate, we will assume that a mass of employed workers become unemployed and collect benefits. That is, the job-separation rate for males is 0.0198.

²⁵We limit the age range to avoid early retirement issues and the effects on demographics on household formation. Approximately 69% of males in the labor force are married in the period considered. According to the National Survey of Family Growth of the U.S. Department of Health and Human Services, approximately 14% of adults aged 15-44 cohabite in 2013, and more than 50% have ever cohabited. Likewise, 30% of the first premarital cohabitations remain intact within the first three years, while over 40% transit to marriage.

²⁶The total hours worked per week amount to 43.3573 hours on average for males, and 38.2629 for females.

²⁷We assume no-gender/age-discriminating firms, and, hence, the actual vacancy-unemployment ratio needs not be adjusted given our constant-returns-to-scale matching technology. We retrieve the job opening rate from JOLTS, which is defined as the ratio of vacancies to the sum of vacancies and employment. Let us call this ratio r_v , and r_u denote the unemployment rate. Then, we have that the ratio of total vacancies to total unemployment amounts to $\frac{r_v(1-r_u)}{r_u(1-r_v)}$.

Table 1: Calibration

Parameter	Description	Value		Target	
		Degenerate F	Non-deg F	Degenerate F	Non-deg F
μ	mass starts employed	0.9315		male unemployment rate	
λ	job-separation rate	0.0198		male EU rate	
σ	preferences	0.0610		gender gap of hours worked	
α	matching tech.	0.3537		male monthly UE rate	
k	vacancy costs	0.0223		labor market tightness	
z	home productivity	0			
b	unemp. benefits	0.3623		replacement rate (Shimer, 2005)	
x	spouse's productivity	0.8208		gender wage gap	female dist. of earnings
κ	search costs	0.6968		for (PC) to be binding	
τ	income tax rate	0.0256		govt balanced budget	

We assume that $z = 0$, and, following [Shimer \(2005\)](#), we set the replacement rate at 0.4.²⁸ That is, unemployed workers obtain 40% of the average wage of primary earners. A proportional income tax is levied to finance the unemployment insurance system, and is set to balance the government's budget.²⁹ We set κ for the participation condition of the lowest HH type to be binding.

To have a first glance, we consider the scenario with a degenerate cdf F . Productivity x is set to match the average gender wage gap at 0.7281 for total earnings **so, this is not hourly wage gap!**. Table 5 shows the results in this case. The second column is the calibrated economy, while the first one corresponds to the laissez faire equilibrium and the last one to the constrained efficient allocation. We observe that the two sources of insurance play a role in the laissez faire equilibrium. The labor supply of secondary earners is larger in equilibrium than in the planner's allocation since there is no transfers across households. This private provision of insurance is partial, and, hence, risk-averse job-seekers apply to low-wage jobs, which are easier to obtain. As in [Acemoglu and Shimer \(1999\)](#), private markets respond to workers' preferences by posting a large mass of such jobs. As a result, the equilibrium unemployment rate is lower than its counterpart in the planner's allocation. According to our calibration, the public provision of insurance is excessive in the baseline economy, and crowds out the private insurance within the household. However, the crowding out effect on labor supply of the spouses is fairly limited, less than 1%. Consumption of the unemployed is inefficiently high, but the unemployment rate does not differ much from the laissez faire level, and total output net of vacancy creation costs is approximately 1% lower than optimal.

²⁸The US average UI reciprocity rate is 29% in the last quarter of 2015.

²⁹Notice that here we are imposing that females pay no taxes.

Table 2: Counterfactual Exercises with a Degenerate cdf F

Variable	Laissez Faire Equilibrium	Calibrated Equilibrium	Planner's Allocation
ℓ^e	0.8813	0.8806	0.8760
ℓ^u	0.9425	0.9171	0.9235
c^e	1.5967	1.6055	1.6679
c^u	0.7736	1.1151	1.0283
w	0.8733	0.9058	(0.9489, 0.2702)
q	0.5466	0.7626	2.533
Job-finding rate	0.3216	0.3100	0.2336
Unemp. rate	5.89%	6.0%	6.66%
Total output (net)	1.5808 (1.5401)	1.5786 (1.5494)	1.5690 (1.5601)

Because welfare gains are likely to differ over the distribution of households, we now turn to the general case with a non-degenerate distribution. We take into account that 33.66% of men are single as well as the unemployment rate of married women is 0.0389. Therefore, we assign an (arbitrarily) small productivity to the spouses of a mass of primary earners equal to 0.3755. Note that to be consistent with the planner's goal of maximizing welfare in a population comprised of men and women at similar percentages, this mass of primary earners with basically no private insurance also includes households formed by single women.³⁰ Then, we target the CPS distribution of (CPI-adjusted) weekly earnings -before deductions- for married employed females, which go from 0 to 2884.61. We construct a 1000-point grid, and use variable *earnwt* for weights. The earnings distribution is displayed in Figure 5.

We next assess how far away from the optimal level a simple policy that combines replacement rate and a dependency allowance is. This may be interesting not only because of its simplicity, but also because of the reduced amount of information (e.g. in terms of heterogeneity in preferences) that the government has access to.

We have assumed in the calibration exercise that secondary earners do not pay taxes to finance the UI system as they do not face unemployment risks. However, the planner is allowed to transfer resources from their income across households. The previous exercise is also a test to see whether this is the mechanism underlying the welfare gains,

[To be completed]

³⁰Notice that we do not make a number of adjustments. In particular, we do not factor in the lower marriage rate of women, the percentage of households in which women are primary earners and the extensive margin in the labor supply of married unemployed.

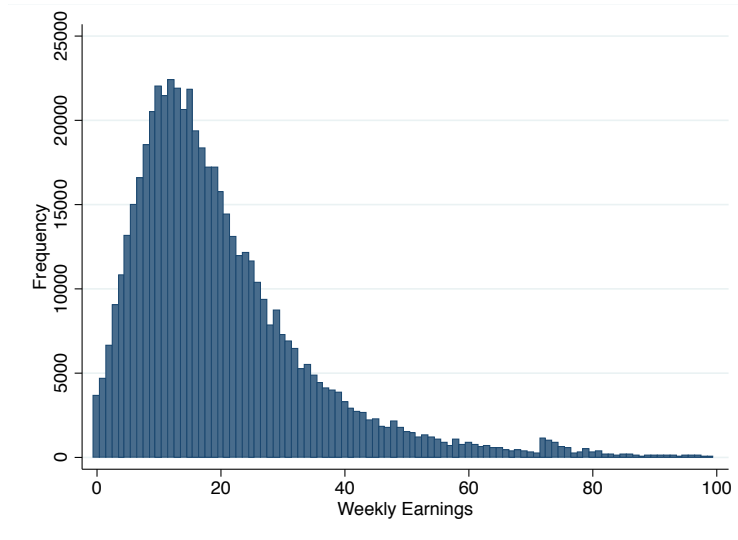


Figure 1: Actual Distribution of Weekly Earnings for Married Employed Women

6 Conclusions

There are many aspects that are left unexamined for the sake of the theoretical analysis. Important points are the extent of leisure complementarity and assortative mating by skill level (see e.g. [Boskin and Sheshinski \(1983\)](#)), the policy implications on family formation (see e.g. [Gayle and Shephard \(2016\)](#)) and inequality within the household (see e.g. the latter and [Alesina, Ichino, and Karabarbounis \(2011\)](#)). We believe that for short average duration and incidence of unemployment like the ones in the U.S. economy, the policy effects on these dimensions is arguably negligent for a large proportion of the labor force. Instead, assortative mating may have a first order effect on redistribution across households in the system-financing burden.

In the trade-off examined between public and private provision of insurance, we have assumed that the only private costs amount to forgone leisure. However, there might be others such as

References

ACEMOGLU, D., AND R. SHIMER (1999): “Efficient Unemployment Insurance,” *Journal of Political Economy*, 107(5), 893–928.

- ALESINA, A., A. ICHINO, AND L. KARABARBOUNIS (2011): “Gender-based taxation and the division of family chores,” *American Economic Journal: Economic Policy*, 3(2), 1–40.
- ATTANASIO, O., H. LOW, AND V. SÁNCHEZ-MARCOS (2005): “Female labor supply as insurance against idiosyncratic risk,” *Journal of the European Economic Association*, 3(2-3), 755–764.
- ATTANASIO, O., AND J.-V. RÍOS-RULL (2000): “Consumption smoothing in island economies: Can public insurance reduce welfare?,” *European Economic Review*, 44(7), 1225–1258.
- BLAU, F. D., AND L. M. KAHN (2007): “Changes in the Labor Supply Behavior of Married Women: 1980-2000,” *Journal of Labor Economics*, 25, 393–438.
- BLUNDELL, R., L. PISTAFERRI, AND I. SAPORTA-EKSTEN (2016): “Consumption Inequality and Family Labor Supply,” *American Economic Review*, 106(2), 387–435.
- BOONE, J., AND L. BOVENBERG (2004): “The optimal taxation of unskilled labor with job search and social assistance,” *Journal of Public Economics*, 88(11), 2227–2258.
- BOSKIN, M. J., AND E. SHESHINSKI (1983): “Optimal tax treatment of the family: Married couples,” *Journal of Public Economics*, 20(3), 281–297.
- BROWNING, M., AND T. F. CROSSLEY (2001): “Unemployment insurance benefit levels and consumption changes,” *Journal of public Economics*, 80(1), 1–23.
- BURDETT, K., AND D. MORTENSEN (1978): *Research in Labor Economics* vol. 2, chap. Labor Supply under Uncertainty. JAI Press.
- CHETTY, R. (2008): “Moral Hazard versus Liquidity and Optimal Unemployment Insurance,” *Journal of Political Economy*, 116(2), 173–234.
- CHETTY, R., AND A. FINKELSTEIN (2012): “Social insurance: Connecting theory to data,” Discussion paper, National Bureau of Economic Research.
- CHETTY, R., AND E. SAEZ (2010): “Optimal Taxation and Social Insurance with Endogenous Private Insurance,” *American Economic Journal. Economic Policy*, 2(2), 85.
- CHIAPPORI, P.-A., AND O. DONNI (2009): “Non-unitary models of household behavior: A survey of the literature,” .

- CULLEN, J. B., AND J. GRUBER (1996): “Spousal Labor Supply as Insurance: Does Unemployment Insurance Crowd Out the Added Worker Effect?,” Discussion paper, National Bureau of Economic Research.
- (2000): “Does unemployment insurance crowd out spousal labor supply?,” *Journal of Labor Economics*, 18(3), 546–572.
- DYNARSKI, S., AND J. GRUBER (1997): “Can families smooth variable earnings?,” *Brookings Papers on Economic Activity*, pp. 229–303.
- ENGEL, E. M., AND J. GRUBER (2001): “Unemployment insurance and precautionary saving,” *Journal of Monetary Economics*, 47(3), 545–579.
- FUDENBERG, D., AND J. TIROLE (1991): “Game theory, 1991,” *Cambridge, Massachusetts*, 393.
- GAYLE, G.-L., AND A. SHEPHARD (2016): “Optimal Taxation, Marriage, Home Production, and Family Labor Supply,” *mimeo*.
- GOLOSOV, M., P. MAZIERO, AND G. MENZIO (2013): “Taxation and Redistribution of Residual Income Inequality,” *Journal of Political Economy*, 121(6), 1160–1204.
- GRUBER, J. (1997): “The Consumption Smoothing Benefits of Unemployment Insurance,” *The American Economic Review*, 87(1), pp. 192–205.
- GULER, B., F. GUVENEN, AND G. L. VIOLANTE (2012): “Joint-search theory: New opportunities and new frictions,” *Journal of Monetary Economics*, 59(4), 352–369.
- HAGEDORN, M., F. KARAHAN, I. MANOVSKII, AND K. MITMAN (2015): “Unemployment Benefits and Unemployment in the Great Recession: The Role of Macro Effects,” .
- HEATHCOTE, J., K. STORESLETTEN, AND G. VIOLANTE (2014): “Consumption and Labor Supply with Partial Insurance: An Analytical Framework,” *THE AMERICAN ECONOMIC REVIEW*, 104(7), 1–52.
- HEATHCOTE, J., K. STORESLETTEN, AND G. L. VIOLANTE (2009): “Quantitative macroeconomics with heterogeneous households,” Discussion paper, National Bureau of Economic Research.
- HOPENHAYN, H. A., AND J. P. NICOLINI (1997): “Optimal unemployment insurance,” *Journal of political economy*, 105(2), 412–438.

- HUNGERBÜHLER, M., E. LEHMANN, A. PARMENTIER, AND B. VAN DER LINDEN (2006): “Optimal redistributive taxation in a search equilibrium model,” *The Review of Economic Studies*, 73(3), 743–767.
- HYSLOP, D. R. (2001): “Rising US earnings inequality and family labor supply: The covariance structure of intrafamily earnings,” *American Economic Review*, pp. 755–777.
- KAPLAN, G. (2012): “Moving back home: Insurance against labor market risk,” *Journal of Political Economy*, 120(3), 446–512.
- KLEVEN, H. J., C. T. KREINER, AND E. SAEZ (2009): “The optimal income taxation of couples,” *Econometrica*, 77(2), 537–560.
- KOLSRUD, J., C. LANDAIS, P. NILSSON, AND J. SPINNEWIJN (2015): “The Optimal Timing of Unemployment Benefits: Theory and Evidence from Sweden,” Discussion paper, Society for Economic Dynamics.
- KRUEGER, D., AND F. PERRI (2011): “Public versus private risk sharing,” *Journal of Economic Theory*, 146(3), 920–956.
- KRUSELL, P., T. MUKOYAMA, AND A. ŞAHİN (2010): “Labour-market matching with precautionary savings and aggregate fluctuations,” *The Review of Economic Studies*, 77(4), 1477–1507.
- MANKART, J., AND R. OIKONMOU (2014): “Household search and the aggregate labor market,” *mimeo*.
- MANKIW, N. G., M. WEINZIERL, AND D. YAGAN (2009): “Optimal Taxation in Theory and Practice,” *The Journal of Economic Perspectives*, pp. 147–174.
- MICHELACCI, C., AND H. RUFFO (2015): “Optimal life cycle unemployment insurance,” *The American Economic Review*, 105(2), 816–859.
- MINCER, J. (1962): “Labor force participation of married women: A study of labor supply,” in *Aspects of labor economics*, pp. 63–105. Princeton University Press.
- MIRRLEES, J. A. (1971): “An exploration in the theory of optimum income taxation,” *The review of economic studies*, pp. 175–208.
- MOEN, E. R. (1997): “Competitive search equilibrium,” *Journal of Political Economy*, 105(2), 385–411.

- MORISSETTE, R., AND Y. OSTROVSKY (2008): “How Do Families and Unattached Individuals Respond to Layoffs,” *Evidence from Canada.(PDF) Statistics Canada Catalogue no. 11F0019MIE*, (304).
- ORTIGUEIRA, S., AND N. SIASSI (2013): “How important is intra-household risk sharing for savings and labor supply?,” *Journal of Monetary Economics*, 60(6), 650–666.
- SALANIE, B. (2011): *The economics of taxation*. MIT press.
- SHIMER, R. (2005): “The Cyclical Behavior of Equilibrium Unemployment and Vacancies,” *American Economic Review*, 95(1), 25–49.
- STEPHENS, M. (2002): “Worker displacement and the added worker effect,” Discussion paper, Journal of Labor Economics.
- STEVENS, A. H. (1997): “Persistent effects of job displacement: The importance of multiple job losses,” *Journal of Labor Economics*, pp. 165–188.

7 Appendix

7.1 Appendix. Proofs of Section 3

Proof of Lemma 3.1.

Consider the first order condition (2). Let $f(w, y) \equiv v_c(y + w, y/x)x + v_\ell(y + w, y/x)$. Notice that $\frac{\partial f(w, y)}{\partial y} < 0$ due to Assumptions A3 and A4. Therefore, the Implicit Function Theorem ensures that there exists a unique function $y_x(w)$ such that $f(w, y_x(w)) = 0$ in an open neighborhood of w . Indeed, y_x is twice continuously differentiable since so is f because of assumption A1.

To show that y_x is a strictly decreasing function, we differentiate equation (2) with respect to w , and obtain

$$\begin{aligned} (v_{cc}x + v_{\ell c})\left(\frac{dy_x}{dw} + 1\right) + v_{c\ell}\frac{dy_x}{dw} + v_{\ell\ell}\frac{dy_x}{dw}\frac{1}{x} &= 0 \\ \Leftrightarrow \frac{dy_x}{dw} &= -\frac{v_{cc}x + v_{\ell c}}{v_{cc}x + 2v_{c\ell} + v_{\ell\ell}/x} < 0. \end{aligned} \tag{16}$$

Moreover, if $z < w$, then total earnings are lower if married with an employed worker, $y_x^e(w) < y_x^u$.

We now make use of these results to prove that function V_x is twice continuously differentiable. We can rewrite it as a composite function of twice continuously differentiable functions, $V_x(w) = v(y_x^e(w) + w, y_x^e(w)/x)$, and, hence, so is it.

To show that function V_x is strictly increasing and concave, we compute the first and second derivatives.

$$\begin{aligned} V'_x(w) &= v_c > 0 \\ V''_x(w) &= v_{cc} \left(\frac{dy_x^e}{dw} + 1 \right) + v_{cl} \frac{dy_x^e}{dw} \frac{1}{x} = \frac{v_{cc}v_{\ell\ell} - v_{cl}^2}{v_{cc}x^2 + 2xv_{cl} + v_{\ell\ell}} < 0 \end{aligned}$$

The first derivate is determined using the Envelope Theorem. To compute the second derivative, we have used expression (16). Assumptions A3 and A4 ensure that the second derivative is negative.||

Proof of Lemma 3.2.

Let $V_x(w) \equiv \max_y v(y + w, y/x)$. The first order condition of this maximization problem is $v_c x + v_\ell = 0$. Then,

$$\frac{\partial V_x}{\partial x} = -v_\ell \frac{y}{x^2} > 0$$

Thus, V_x is increasing in x .

Moreover, we obtain from the first order condition

$$\frac{\partial y_x}{\partial x} = \frac{-v_c + v_{cl}y_x + v_{\ell\ell}\frac{y_x}{x}}{x^2v_{cc} + 2xv_{cl} + v_{\ell\ell}}$$

Recall that $V'_x(w) = v_c$. Therefore,

$$\begin{aligned} \frac{\partial V'_x}{\partial x} &= v_{cc} \frac{\partial y_x}{\partial x} + v_{cl} \left(\frac{\partial y_x}{\partial x} \frac{1}{x} - \frac{y_x}{x^2} \right) = \\ &= \frac{(v_{cc} + v_{cl}/x) \left(-v_c + v_{cl}y_x + v_{\ell\ell}\frac{y_x}{x} \right) - \frac{y_x}{x^2} v_{cl} (x^2v_{cc} + 2xv_{cl} + v_{\ell\ell})}{x^2v_{cc} + 2xv_{cl} + v_{\ell\ell}} \\ &= \frac{(v_{cc}v_{\ell\ell} - v_{cl}^2)\frac{y_x}{x} - v_c(v_{cc} + v_{cl}/x)}{x^2v_{cc} + 2xv_{cl} + v_{\ell\ell}} < 0 \end{aligned}$$

where the last expression results after some simplifications.||

Proof of Proposition 3.3.

Consider problem (5) of a household of type x . Notice that the constraint establishes a positive relationship between w and q . Therefore, the household's problem can be rewritten

only in terms of the wage w . Since the resulting objective function is continuous in w and the domain $[z, 1]$ is compact, the Weierstrass Theorem ensures the existence of a solution. The first derivative becomes

$$-\nu(q)\eta(q)\frac{1-\gamma(q)}{\gamma(q)}\frac{V_x(w)-V_x(z)}{k}+\nu(q)V'_x(w)$$

When, $w = z$, the first term of the derivative is 0, while the second term is strictly positive. Likewise, the derivative is negative at $w = 1$. Furthermore, the objective function is nonnegative and values $V_x(z)$ at the two extremes of the domain. These two results together imply that the wage solution must be an interior point. The first order condition becomes

$$\text{(FOC): } \frac{V_x(w)-V_x(z)}{V'_x(w)} = \frac{k}{\eta(q)} \frac{\gamma(q)}{1-\gamma(q)}$$

The right hand side of this expression is decreasing in q and, hence, also in w . The derivative of the left hand side is

$$1 - \frac{(V_x(w)-V_x(z))V''_x(w)}{V'_x(w)^2} > 0$$

This expression is positive because function V_x is concave as stated in Lemma 3.1. Therefore, the solution of the first order condition must be unique, and it is also a sufficient condition.

As said in the text, equations (2) and (3) have a unique solution because of the concavity of the utility function. Therefore, there exists an equilibrium, and it is unique. ||

Proof of Proposition 3.4

We follow closely the proof of Proposition 2 in Acemoglu and Shimer (1999). Consider types x and x' . Given that $V_x(w)$ is strictly increasing in w , there exists an inverse function V_x^{-1} , which is also differentiable. Define function $f(s) \equiv \max_y v(y + V_j^{-1}(s), y/x)$, which is twice continuously differentiable. Notice that $V_{x'}(w) = f \circ V_x(w)$. By differentiating with respect to w , we obtain

$$V'_{x'}(w) = f'(V_x(w))V'_x(w), \text{ and } V''_{x'}(w) = f''(V_x(w))V'_x(w)^2 + f'(V_x(w))V''_x(w)$$

Using the first equality, we can rewrite the second expression as

$$f''(V_x(w))V'_x(w)^2 = V''_{x'}(w) - V''_x(w) \frac{V'_{x'}(w)}{V'_x(w)}$$

Therefore, f is a concave function, and, hence, $V_{x'}(w)$ is a concave (convex) transformation

of $V_x(w)$ if and only if $\frac{-V''_x(w)}{V'_x(w)}$ is greater (lower) than $\frac{-V''_x(w)}{V'_x(w)}$. Then, to show that sign of the wage difference it suffices to follow the remaining steps in the proof of Proposition 2 in [Acemoglu and Shimer \(1999\)](#). Finally, the difference in queue lengths is of the same sign as the wage difference since the equilibrium zero-profit condition establishes a positive relationship between q and w .||

Proof of Lemma 3.5.

1. The proof of wages and queue lengths as increasing functions of z is analogous to its counterpart in [Acemoglu and Shimer \(1999\)](#). Hence, it is omitted.
2. Similarly to the previous case, by differentiating equation (3) with respect to z , we obtain

$$\begin{aligned} (v_{cc}x + v_{cl}) \left(\frac{\partial y_x^u}{\partial z} + 1 \right) + (v_{cl} + v_{\ell\ell}/x) \frac{\partial y_x^u}{\partial z} &= 0 \\ \Leftrightarrow \frac{\partial \ell_x^u}{\partial z} &= \frac{\partial y_x^u}{\partial z} / x = \frac{-xv_{cc} - v_{cl}}{v_{cc}x^2 + 2xv_{cl} + v_{\ell\ell}}, \end{aligned}$$

which is negative.

Note that $\frac{\partial \ell_x^e}{\partial z} = \frac{d\ell_x^e}{dw} \frac{\partial w}{\partial z} \leq 0$ because the first factor is negative according to Lemma 3.1, while the second one was shown above to be non-negative.||

Proof of Lemma 3.6.

Consider the first order condition of the household problem (12), for a given x . After replacing c^m using the budget constraint, the first order conditions with respect to c^f and y are

$$f_1(w, c^f, y) = -\alpha v_c^m + (1 - \alpha)v_c^f = 0 \quad (17)$$

$$f_2(w, c^f, y) = \alpha v_c^m x + (1 - \alpha)v_\ell^f = 0 \quad (18)$$

Let $f(w, c^f, y) = (f_1(w, c^f, y), f_2(w, c^f, y))$. The Jacobian matrix of f is invertible since

$$\begin{aligned} |\mathbf{J}| &= \begin{vmatrix} \frac{\partial f_1}{\partial c^f} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial c^f} & \frac{\partial f_2}{\partial y} \end{vmatrix} = \begin{vmatrix} \alpha v_{cc}^m + (1 - \alpha)v_{cc}^f & -\alpha v_{cc}^m + (1 - \alpha)v_{cl}^f/x \\ -\alpha v_{cc}^m x + (1 - \alpha)v_{cl}^f & \alpha v_{cc}^m x + (1 - \alpha)v_{\ell\ell}^f/x \end{vmatrix} \\ &= ((1 - \alpha)\alpha v_{cc}^m (v_{cc}^f x^2 + v_{\ell\ell}^f + 2xv_{cl}^f) + (1 - \alpha)^2 (v_{cc}^f v_{\ell\ell}^f - (v_{cl}^f)^2)) / x > 0 \end{aligned}$$

Therefore, the Implicit Function Theorem ensures that, for a given x , there exists unique functions $c_x^f(w)$, $c_x^m(w)$ and $y_x(w)$ such that $f(w, c_x^f(w), y_x(w)) = 0$. Indeed, y_x is twice continuously differentiable since so is f because of assumption A1.

To show that y_x is a strictly decreasing function, we differentiate the two first order conditions (17) and (18) with respect to w , and obtain a system of two equations with unknowns $\frac{dy_x}{dw}$ and $\frac{dc_x^f}{dw}$. By manipulating them, we obtain

$$\frac{dy_x}{dw} = \frac{-\alpha v_{cc}^m (v_{c\ell}^f + x v_{cc}^f)}{\alpha v_{cc}^m (x v_{cc}^f + 2 v_{c\ell}^f + v_{\ell\ell}^f/x) + (1 - \alpha) (v_{cc}^f v_{\ell\ell}^f - (v_{c\ell}^f)^2)/x} < 0$$

Moreover,

$$\frac{dc_x^f}{dw} = -\frac{dy_x}{dw} \frac{v_{\ell\ell}^f/x + v_{c\ell}^f}{v_{c\ell}^f + x v_{cc}^f} > 0 \quad (19)$$

We now make use of these results to prove that function V_x is twice continuously differentiable. We can rewrite it as a composite function of twice continuously differentiable functions, $V_x(w) = \alpha v^m(y_x(w) + w - c_x^f(w), \underline{\ell}) + (1 - \alpha) v^f(c_x^f(w), y_x(w)/x)$, and, hence, so is it.

To show that function V_x is strictly increasing and concave, we compute the first and second derivatives.

$$\begin{aligned} V'_x(w) &= \alpha v_c^m \left(\frac{dy_x}{dw} + 1 - \frac{dc_x^f}{dw} \right) + (1 - \alpha) \left(v_c^f \frac{dc_x^f}{dw} + v_\ell^f \frac{dy_x}{dw} \frac{1}{x} \right) \stackrel{(17),(18)}{=} \alpha v_c^m > 0 \\ V''_x(w) &= \alpha v_{cc}^m \left(\frac{dy_x}{dw} + 1 - \frac{dc_x^f}{dw} \right) \stackrel{(19)}{=} \alpha v_{cc}^m \left(\frac{dy_x}{dw} \left(\frac{x v_{cc}^f + v_{\ell\ell}^f/x + 2 v_{c\ell}^f}{v_{c\ell}^f + x v_{cc}^f} \right) + 1 \right) \\ &\stackrel{(19)}{=} \alpha v_{cc}^m \frac{(1 - \alpha) (v_{cc}^f v_{\ell\ell}^f - (v_{c\ell}^f)^2)}{\alpha v_{cc}^m (x^2 v_{cc}^f + 2 x v_{c\ell}^f + v_{\ell\ell}^f) + (1 - \alpha) (v_{cc}^f v_{\ell\ell}^f - (v_{c\ell}^f)^2)} < 0 \end{aligned}$$

Assumptions A3 and A4 ensure that the second derivative is negative.

Let ℓ_x^e and ℓ_x^u denote hours worked by the spouse of an employed and unemployed worker, respectively. The proof of $\frac{\partial \ell_x^u}{\partial z} < 0$ is analogous to expression (19). Finally, $\frac{\partial \ell_x^e}{\partial z} = \frac{d\ell_x^e}{dw} \frac{\partial w}{\partial z} < 0$ since the second factor is positive as stated in Lemma 3.5. ||

7.2 Appendix. Proofs of Section 4

Proof of Lemma 4.2.

The proof is by contradiction.³¹ Suppose that such conditions are not sufficient, and, hence, there exists $\hat{x}$ and x such that $\mathcal{U}_x(\hat{x}) > \mathcal{U}_x$. Let us suppose without loss of generality that $\hat{x} > x$. That is, $\int_x^{\hat{x}} \frac{\partial U_x(a)}{\partial a} da > 0$. This integral can be developed to obtain

$$\begin{aligned} \int_x^{\hat{x}} \frac{\partial U_x(a)}{\partial a} da &= \int_x^{\hat{x}} \left(\nu'(q_a) \dot{q}_a (v(c_a^e, y_a^e/x) - v(c_a^u, y_a^u/x)) \right. \\ &\quad + \nu(q_a) \left(v_c(c_a^e, y_a^e/x) \dot{c}_a^e + v_\ell(c_a^e, y_a^e/x) \frac{\dot{y}_a^e}{x} \right) \\ &\quad \left. + (1 - \nu(q_a)) \left(v_c(c_a^u, y_a^u/x) \dot{c}_a^u + v_\ell(c_a^u, y_a^u/x) \frac{\dot{y}_a^u}{x} \right) \right) da \leq \\ &\int_x^{\hat{x}} \left(\nu'(q_a) \dot{q}_a (v(c_a^e, y_a^e/a) - v(c_a^u, y_a^u/a)) \right. \\ &\quad + \nu(q_a) \left(v_c(c_a^e, y_a^e/a) \dot{c}_a^e + v_\ell(c_a^e, y_a^e/a) \frac{\dot{y}_a^e}{a} \right) \\ &\quad \left. + (1 - \nu(q_a)) \left(v_c(c_a^u, y_a^u/a) \dot{c}_a^u + v_\ell(c_a^u, y_a^u/a) \frac{\dot{y}_a^u}{a} \right) \right) da = 0, \end{aligned}$$

where the inequality results from the integrand being an increasing function in x because of the (local) second order condition, and the last equality is the necessary first order condition. This is a contradiction, and, hence, the local conditions are also sufficient. ||

Proof of Proposition 4.3

Let (q_x, w_x) denote the unique equilibrium pair of distributions of queue lengths and wages. We first show that it belongs to the feasible set of the planner's problem. The resource constraint (RC) holds in equilibrium because the zero-profit condition is satisfied in all submarkets. The participation condition for all households holds with strict inequality because cost κ is sufficiently small by assumption. Likewise, utility-maximizing households of type x prefer pair (q_x, w_x) than $(q_{x'}, w_{x'})$ for any x' in equilibrium; hence, the equilibrium allocation is incentive compatible.

Consider now the following alternative allocation: $c_x^e = w_x + y_x^e - \epsilon$, $c_x^u = z + y_x^u + \delta$, such that it is resource-neutral, i.e. $\int_{\underline{x}}^{\bar{x}} \left(-\nu(q_x)\epsilon + (1 - \nu(q_x))\delta \right) dF(x) = 0$. Notice that this allocation yields a strictly higher value than the equilibrium one due to the concavity of the utility function v . It also belongs to the feasible set as the resource constraint holds. It is straightforward to see that the participation condition (PC _{x}) also holds because the participation cost κ is sufficiently small. The incentive compatibility conditions (ICC _{x}) also

³¹We closely follows Fudenberg and Tirole (1991, Ch. 7, p. 261).

hold for ϵ arbitrarily small because Proposition 3.3 ensures that the problem of type x households has a unique solution and the pair $(q_x, w_x - \epsilon)$ satisfies its inequality constraint.³² Therefore, the equilibrium allocation is not solution of the planner's problem.||

Proof of Proposition 4.4

Consider the following program

$$\max \int_{\underline{x}}^{\bar{x}} \left(\nu(q_x)(1 + y_x^e + \phi(y_x^e/x)) + (1 - \nu(q_x))(z + y_x^u + \phi(y_x^u/x)) - \frac{k}{q_x} \right) dF(x)$$

This is the planner's unconstrained problem after replacing the consumption values using the resource constraint. Then, we are to show that the equilibrium allocation is feasible and incentive compatible, and is a solution of the unconstrained problem, and, hence, must coincide with the planner's solution.

The first order conditions of the planner's unconstrained problem are, for all x ,

$$\begin{aligned} \frac{k}{\eta(q)} &= (1 - \gamma(q))(1 - z), & \text{where } q \equiv q_x \\ \phi'(y_x/x) &= -x, & \text{where } y_x \equiv y_x^e = y_x^u \end{aligned}$$

Notice that these equations coincide with the equilibrium conditions (8)-(10), and there exists a unique solution to this system of equations. Therefore, the equilibrium allocation is the solution of the planner's unconstrained problem. We define consumption levels as $c_x^e \equiv w + y_x/x$ and $c_x^u \equiv z + y_x/x$, where $w = 1 - \frac{k}{\eta(q)}$ is the equilibrium wage. For this allocation to be the planner's solution, it remains to show that all constraints of the planner's problem hold. First, the participation rate holds for all types as it does in equilibrium. Second, the equilibrium allocation is obviously incentive compatible. Therefore, the equilibrium allocation is constrained efficient.||

Proof of Lemma 4.5

The proof follows very closely the one for Proposition 4.4. First, for notational simplicity, let $\tilde{\alpha} \equiv \alpha \mathcal{I}_{\alpha \geq 0.5} + (1 - \alpha)(1 - \mathcal{I}_{\alpha \geq 0.5})$.

The equilibrium allocation is determined by condition (8) and (9), whereas the equilibrium condition (10) is replaced by $\phi'(y_x/x) = -x \frac{\tilde{\alpha}}{1 - \alpha}$, which leads to $y_x^e = y_x^u$.

As for the unitary model of the household, using the resource constraint, the planner's

³²The household's problem can be equivalently formalized with an inequality instead of an equality constraint.

objective function can be rewritten as

$$\int_{\underline{x}}^{\bar{x}} \left(\nu(q_x)(\tilde{\alpha}(1 + y_x^e) + (1 - \alpha)\phi(y_x^e/x)) + (1 - \nu(q_x))(\tilde{\alpha}(z + y_x^u) + (1 - \alpha)\phi(y_x^u/x)) - \frac{k}{q_x} \right) dF(x)$$

The necessary conditions of the unconstrained problem are, for all x ,

$$\begin{aligned} \frac{k}{\eta(q)} &= (1 - \gamma(q))(1 - z), & \text{where } q \equiv q_x \\ \phi'(y_x/x) &= -x \frac{\tilde{\alpha}}{1 - \alpha}, & \text{where } y_x \equiv y_x^e = y_x^u \end{aligned}$$

Since the equilibrium and efficiency conditions are the same, and there exists a unique solution to this system of equations, the equilibrium allocation is the solution of the planner's problem. ||

Proof of Proposition 4.6.

The first order conditions of the planner's problem (14) are

$$\begin{aligned} \nu'(q) \left((V(m^e) - V(m^u))(1 + \xi_2) + \xi_1(1 - m^e - z + m^u) \right) &= -\xi_1 \frac{k}{q^2} \\ V'(m^e)(1 + \xi_2) &\leq \xi_1 \text{ and } m^e \geq z, & \text{with comp. slackness} \\ V'(m^u) \left(1 - \xi_2 \frac{\nu(q)}{1 - \nu(q)} \right) &\leq \xi_1 \text{ and } m^u \geq z, & \text{with comp. slackness} \end{aligned}$$

where ξ_1 and ξ_2 are the Lagrange multipliers of the first and second constraints, respectively. Since $m^e > m^u > z$, we can use the last two conditions to replace the multipliers in the first equation to obtain

$$(1 - \gamma(q)) \left(\frac{V(m^e) - V(m^u)}{V'(m^e)} + 1 - m^e - z + m^u \right) = \frac{k}{\eta(q)} \quad (20)$$

Notice that $\xi_2 > 0$ because $m^e = m^u$ otherwise, and constraint (PC) would not hold. Therefore, the participation condition is binding. Let $(\hat{m}^e, \hat{m}^u, \hat{q})$ denote the planner's solution, which satisfies this necessary condition as well as the two constraints of the planner's problem with equality.

Consider the following set of fiscal instruments (b, τ, T) , where the first element stands for unemployment benefits, the second one is a proportional income tax rate for newly employed workers, and the last instrument is a lump sum tax paid per household. For the tax-distorted

equilibrium (w, q) to be constrained efficient, the following conditions must hold:

$$\hat{m}^u = z + b - T \quad (21)$$

$$\hat{m}^e = w(1 - \tau) - T \quad (22)$$

$$\tau \frac{k}{\eta(\hat{q})} = (1 - \gamma(\hat{q}))(\tau + b) \quad (23)$$

$$T + \nu(\hat{q})\tau w = (1 - \nu(\hat{q}))b$$

The first two conditions ensure that consumption in the decentralized economy equals its level in the planner's allocation, where the wage w is determined by equation (7) evaluated at $q = \hat{q}$. The third condition is necessary and sufficient for the equilibrium and the efficiency first order conditions (6) and (20) to be equal to one another. Finally, the last condition is the government's budget constraint. Notice that the government's equation is the same as the planner's resource constraint (14) after replacing wages using the zero-profit condition (7). Therefore, to show the implementation of the planner's solution, we need to determine a solution of the system of equations (21)-(23). We can eliminate T and write b in terms of τ by subtracting (21) from (22) to obtain

$$b = \frac{\left(1 - \frac{k}{\eta(\hat{q})}\right)(1 - \tau) - z}{\hat{m}^e - \hat{m}^u}.$$

Notice that b is strictly positive as both the numerator and denominator must be strictly positive. Then, we can obtain τ from equation (23) after replacing b as

$$\tau = \frac{1 - \frac{k}{\eta(\hat{q})} - z}{(\hat{m}^e - \hat{m}^u) \left(1 - \frac{k}{\eta(\hat{q})(1 - \gamma(\hat{q}))} - \frac{1 - \frac{k}{\eta(\hat{q})}}{\hat{m}^e - \hat{m}^u}\right)},$$

and then the values of b and T are uniquely determined. Finally, notice that $\tau \neq 0$ because otherwise $b = 0$ according to equation (23) and $T < 0$ due to condition (21), and, as a result, the government's budget constraint (24) would fail to hold.||

Proof of Proposition 4.7.

We first rewrite the planner's problem

$$\begin{aligned}
& \max \int_{\underline{x}}^{\bar{x}} \mathcal{U}_x f(x) dx \\
& \text{s. to} \quad \mathcal{U}_x = \nu(q_x) v(c_x^e, y_x^e/x) + (1 - \nu(q_x)) v(c_x^u, y_x^u/x) \\
& \quad \dot{A}_x = \left(\nu(q_x) (1 + y_x^e - c_x^e) + (1 - \nu(q_x)) (z + y_x^u - c_x^u) - \frac{k}{q_x} \right) f(x) \quad (24) \\
& \quad A_{\underline{x}}, A_{\bar{x}} = 0 \\
& \quad \mathcal{U}_x \geq \kappa + v(c_x^u, y_x^u/x) \quad (25) \\
& \quad \dot{\mathcal{U}}_x = \nu(q_x) \frac{y_x^e}{x^2} + (1 - \nu(q_x)) \frac{y_x^u}{x^2} \quad (26) \\
& \quad 0 \leq -\nu'(q_x) p_x^q (y_x^u - y_x^e) + \nu(q_x) p_x^e + (1 - \nu(q_x)) p_x^u \quad (27) \\
& \quad \dot{q}_x = p_x^q \\
& \quad \dot{y}_x^e = p_x^e \\
& \quad \dot{y}_x^u = p_x^u
\end{aligned}$$

The control variables are c_x^e , c_x^u , p_x^q , p_x^e , and p_x^u , whereas $\mathcal{U}$, q_x , y_x^e and y_x^u are the state variables. As usual in optimal control problems, we transform the resource constraint (RC) into differential equation (24) along with two boundary constraints. Inequality (25) is the participation condition (PC_x). The incentive compatibility conditions (FOC-ICC_x) and (SOC-ICC_x) become (26) and (27), respectively. The last three differential equations are state equations.

The Hamiltonian is defined as

$$\begin{aligned}
\mathcal{H} = & \mathcal{U}_x f(x) + \lambda_x^1 \left(\nu(q_x) (1 + y_x^e - c_x^e) + (1 - \nu(q_x)) (z + y_x^u - c_x^u) - \frac{k}{q_x} \right) f(x) \\
& + \lambda_x^2 \left(\nu(q_x) \frac{y_x^e}{x^2} + (1 - \nu(q_x)) \frac{y_x^u}{x^2} \right) + \lambda_x^3 \left(\mathcal{U}_x - \kappa - v_x^u \right) \\
& + \lambda_x^4 \left(\mathcal{U}_x - \nu(q_x) v_x^e - (1 - \nu(q_x)) v_x^u \right) \\
& + \lambda_x^5 \left(-\nu'(q_x) p_x^q (y_x^u - y_x^e) + \nu(q_x) p_x^e + (1 - \nu(q_x)) p_x^u \right) \\
& + \lambda_x^6 p_x^q + \lambda_x^7 p_x^e + \lambda_x^8 p_x^u
\end{aligned}$$

where λ_x^1 , λ_x^2 , λ_x^6 , λ_x^7 and λ_x^8 are the respective co-state variables, and the multipliers λ_x^3 , $\lambda_x^5 \geq 0$. To simplify notation, we denote $v_x^j \equiv v(c_x^j, y_x^j/x)$, for $j \in \{u, e\}$.

The following necessary conditions must be satisfied:

$$\begin{aligned} \frac{\partial \mathcal{H}}{\partial q_x} = -\dot{\lambda}_x^6 \Leftrightarrow & \lambda_x^1 f(x) \left(\nu'(q_x) (1 + y_x^e - c_x^e - z - y_x^u + c_x^u) + \frac{k}{q_x^2} \right) \\ & - \lambda_x^2 \nu'(q_x) (y_x^u - y_x^e) / x^2 - \lambda_x^4 \nu'(q_x) (v_x^e - v_x^u) \\ & + \lambda_x^5 \left(-\nu''(q_x) p_x^q (y_x^u - y_x^e) + \nu'(q_x) (p_x^e - p_x^u) \right) = -\dot{\lambda}_x^6 \end{aligned} \quad (28)$$

$$\frac{\partial \mathcal{H}}{\partial c_x^e} = 0 \Leftrightarrow \lambda_x^4 = -\frac{\lambda_x^1 f(x)}{v_c^e} \quad (29)$$

$$\frac{\partial \mathcal{H}}{\partial c_x^u} = 0 \Leftrightarrow \lambda_x^3 v_c^u = -(1 - \nu(q_x)) (\lambda_x^4 v_c^u + \lambda_x^1 f(x)) \quad (30)$$

$$\frac{\partial \mathcal{H}}{\partial y_x^e} = -\dot{\lambda}_x^7 \Leftrightarrow \lambda_x^1 \nu(q_x) f(x) + \lambda_x^2 \nu(q_x) / x^2 + \lambda_x^4 \nu(q_x) / x + \lambda_x^5 \nu'(q_x) p_x^q = -\dot{\lambda}_x^7 \quad (31)$$

$$\begin{aligned} \frac{\partial \mathcal{H}}{\partial y_x^u} = -\dot{\lambda}_x^8 \Leftrightarrow & \lambda_x^1 (1 - \nu(q_x)) f(x) + \lambda_x^2 (1 - \nu(q_x)) / x^2 + \lambda_x^3 / x + \lambda_x^4 (1 - \nu(q_x)) / x \\ & - \lambda_x^5 \nu'(q_x) p_x^q = -\dot{\lambda}_x^8 \end{aligned} \quad (32)$$

$$\frac{\partial \mathcal{H}}{\partial A_x} = -\dot{\lambda}_x^1 \Leftrightarrow \lambda_x^1 = \lambda^1, \forall x \quad (33)$$

$$\frac{\partial \mathcal{H}}{\partial p_x^q} = 0 \Leftrightarrow \lambda_x^6 = \lambda_x^5 \nu'(q_x) (y_x^u - y_x^e) \quad (34)$$

$$\frac{\partial \mathcal{H}}{\partial p_x^e} = 0 \Leftrightarrow \lambda_x^7 = -\lambda_x^5 \nu(q_x) \quad (35)$$

$$\frac{\partial \mathcal{H}}{\partial p_x^u} = 0 \Leftrightarrow \lambda_x^8 = -\lambda_x^5 (1 - \nu(q_x)) \quad (36)$$

$$\frac{\partial \mathcal{H}}{\partial \mathcal{U}_x} = -\dot{\lambda}_x^2 \Leftrightarrow f(x) + \lambda_x^3 + \lambda_x^4 = -\dot{\lambda}_x^2 \quad (37)$$

$$\lambda_x^3 \geq 0, \quad \text{and} \quad 0 = \lambda_x^3 \left(\mathcal{U}_x - \kappa - v_x^u \right) \quad (38)$$

$$\lambda_x^5 \geq 0, \quad \text{and} \quad 0 = \lambda_x^5 \left(-\nu'(q_x) p_x^q (y_x^u - y_x^e) + \nu(q_x) p_x^e + (1 - \nu(q_x)) p_x^u \right) \quad (39)$$

and since there are neither initial nor final conditions for $\mathcal{U}$, q_x , y_x^e and y_x^u , the following transversality conditions hold

$$\lambda_{\underline{x}}^2 = \lambda_{\bar{x}}^2 = 0, \lambda_{\underline{x}}^6 = \lambda_{\bar{x}}^6 = 0, \lambda_{\underline{x}}^7 = \lambda_{\bar{x}}^7 = 0, \lambda_{\underline{x}}^8 = \lambda_{\bar{x}}^8 = 0$$

Because the inequality constraints are concave in the control variables, the inequality constraint qualification holds leading to conditions (38) and (39).

We first show that $\lambda^1 > 0$. Using equation (37), we can write $\lambda_x^2 = -\int_{\underline{x}}^x (f(t) + \lambda_t^3 + \lambda_t^4) dt$

because of the transversality condition $\lambda_{\underline{x}}^2 = 0$. Likewise, the transversality condition implies

$$\lambda_{\underline{x}}^2 = 0 = -1 - \int_{\underline{x}}^{\bar{x}} (\lambda_t^3 + \lambda_t^4) dt \Leftrightarrow \lambda^1 = \frac{1}{\int_{\underline{x}}^{\bar{x}} \frac{1}{v_c^e} \left(1 - \frac{v_c^u - v_c^e}{v_c^u} (1 - \nu(q_x)) \right) f(x) dx} > 0,$$

where the sum $\lambda_t^3 + \lambda_t^4$ obtains from equations (29) and (30). The co-state variable is positive as it can easily be shown that $c_x^u \leq c_x^e$ and due to the concavity of function v .

From equations (35) and (36), we obtain $\lambda_x^7(1 - \nu(q_x)) = \lambda_x^8 \nu(q_x)$. By differentiating this expression with respect to x , we obtain

$$\dot{\lambda}_x^7(1 - \nu(q_x)) - \dot{\lambda}_x^8 \nu(q_x) = -\lambda_x^5 \nu'(q_x) p_x^q$$

We then subtract equation (32) multiplied by $\nu(q_x)$ from (31) times $1 - \nu(q_x)$ and use this last equality to obtain $\lambda_x^3 = 0$ for all x . We obtain that consumption does not vary with the employment state of the worker as it follows from equations (29) and (30) that

$$\lambda_x^3 v_c^u = \lambda^1 f(x) (1 - \nu(q_x)) \left(1 - \frac{v_c^u}{v_c^e} \right) \xRightarrow{\lambda_x^3=0, \lambda^1 \neq 0} c_x^e = c_x^u = c_x, \forall x \xRightarrow{(PC_x)} y_x^e < y_x^u, \forall x$$

Now, we differentiate equations (34) and (36) with respect to x

$$\begin{aligned} \dot{\lambda}_x^6 &= \dot{\lambda}_x^5 \nu'(q_x) (y_x^u - y_x^e) + \lambda_x^5 \nu''(q_x) p_x^q (y_x^u - y_x^e) + \lambda_x^5 \nu'(q_x) (p_x^u - p_x^e), \\ \dot{\lambda}_x^7 &= -\dot{\lambda}_x^5 \nu(q_x) - \lambda_x^5 \nu'(q_x) p_x^q, \end{aligned}$$

and substitute out $\dot{\lambda}_x^6$ and $\dot{\lambda}_x^7$ in equations (28) and (31), respectively, to obtain

$$\begin{aligned} (\dot{\lambda}_x^5 - \lambda_x^2/x^2) \nu'(q_x) (y_x^u - y_x^e) &= \lambda^1 f(x) \left(\nu'(q_x) \left(\frac{y_x^e - y_x^u}{x v_c^e} - 1 - y_x^e + z + y_x^u \right) - \frac{k}{q_x^2} \right) \\ \dot{\lambda}_x^5 - \lambda_x^2/x^2 &= \lambda^1 f(x) \left(1 - \frac{1}{x v_c^e} \right) \end{aligned} \quad (40)$$

By combining these two equations, we obtain expression (15), which implies $p_x^q = \dot{q}_x = 0$.

We now prove, by contradiction, that there exists a subset of positive mass for which the FOC is also sufficient. Suppose that there is no subset of positive mass within $(\underline{x}, \bar{x})$ such that $\dot{c}_x > 0$ or equivalently, due to the FOC of the (ICC- x) together with $\dot{q}_x = 0$, $\nu(q) p_x^e + (1 - \nu(q)) p_x^u > 0$. This implies that $\lambda^1 = v_c^e$, which is constant in x , and, hence, $\lambda_x^4 = -f(x)$, because of condition (29), and $\lambda_x^2 = 0$, due to condition (37) and the transversality conditions, for all $x \in (\underline{x}, \bar{x})$ except for a zero mass set. Then, equation (40) can be rewritten

as $\dot{\lambda}_x^5 = \lambda^1 f(x) \left(1 - \frac{1}{xv_c^e}\right)$. Since the last term is increasing in x and $\lambda_{\underline{x}}^5 = 0$ and $\dot{\lambda}_{\underline{x}}^5 \geq 0$, then $\lambda_{\bar{x}}^5 > 0$, which is a contradiction because of the transversality conditions. Therefore, there exists a subset $\mathcal{S} \subset [\underline{x}, \bar{x}]$ of positive mass in which both c_x and $\nu(q)y_x^e + (1 - \nu(q))y_x^u$ strictly increase with x .

We turn to show that for all $x \in \mathcal{S}$, $\frac{1}{xv_c^e} < 1$. By definition, $\nu(q)p_x^e + (1 - \nu(q))p_x^u > 0$ and $\dot{c}_x > 0$ for all $x \in \mathcal{S}$. Hence, by continuity, these inequalities hold within an open neighborhood of $x \in \mathcal{S}$. Therefore, $\lambda_x^5 = 0$ and $\dot{\lambda}_x^5 = 0$ because of condition (39). Equation (40) implies that $\frac{1}{xv_c^e} < 1$ if $\lambda_x^2 < 0$. To see this strict inequality, notice that $\dot{\lambda}_x^2 = \lambda^1 f(x) \int_{\underline{x}}^{\bar{x}} \left(\frac{1}{v_c^e(c_x, y_x^e/x)} - \frac{1}{v_c^e(c_t, y_t^e/t)} \right) f(t) dt$. As fraction $\frac{1}{v_c^e}$ is an increasing function in x , the derivative of the last term with respect to x is positive and, hence, $\dot{\lambda}_x^2$ changes sign at most once and is almost always different from 0. Since $\lambda_{\underline{x}}^2 = \lambda_{\bar{x}}^2 = 0$, we have that either $\lambda_x^2 = 0$ for all $x \in [\underline{x}, \bar{x}]$ or λ_x^2 is negative and convex in $(\underline{x}, \bar{x})$. However, the former case cannot be because it would imply $\dot{c}_x = 0$ within $(\underline{x}, \bar{x})$ from conditions (29) and (37).

Finally, it is immediate to see that the equilibrium allocation is not constrained efficient as equilibrium condition (11) holds neither in $\mathcal{S}$ nor out of it where $\dot{c}_x = 0$.