Twin Peaks*

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October 28, 2016

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Abstract

This paper highlights a new stylized fact by documenting a great degree of heterogeneity in the distribution of export intensity across countries. Contrary to received wisdom from studies focusing on a single country, we show that export intensity distributions tend to exhibit modes at both ends of the support, a phenomenon that we refer to as 'twin peaks'. Using a standard trade model with firm-destination specific demand shifters that are distributed Fréchet with the same shape parameter across markets, we show that i) this feature directly results from heavy-tailed sale distributions ii) this feature (statistically) vanish for countries with large or small domestic market compared to their export market. Using publicly available firm-level data drawn from the World Bank Enterprise Surveys, we show that the model's structural parameters are easily estimated and can account for approximately 89% of cross-country variation in the distribution of export intensity. Our finding have many applications, notably with regards to volatility of firm's sales.
Keywords: Exports; Export Intensity Distribution; Heavy Tails

JEL classification: F12, F13, O47.

*We thank Julian di Giovanni, Keith Head, Emanuel Ornelas, Veronica Rappoport and Dani Rodrik and seminar participants at LSE/CEP for helpful comments. All remaining errors are our own.

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1 Introduction

Received wisdom over the last two decades of research on the role of firm heterogeneity on international trade suggests that the vast majority of exporters in a country only sell a relatively small fraction of their output abroad (Bernard et al., 2003; Brooks, 2006; Bernard et al., 2007; Arkolakis, 2010; Eaton et al., 2011). In this paper we show that this stylized fact is much less robust than initially thought. Using publicly available cross-country firm-level data drawn from the World Bank's Enterprise Surveys, we show that the distribution of export intensity, the share of sales accounted for exports, for exporting firms exhibits a remarkable degree of heterogeneity across countries which, to the best of our knowledge, has not been identified previously.

This point is vividly illustrated in Figure 1. Countries like Argentina, Brazil, Russia and South Africa follow the pattern identified in previous studies; firms with export intensity below 0.2 constitute more than half of exporters, while high-intensity ones (i.e. firms selling more than 80% of their output abroad) account for less than 10% of exporting firms. In Bangladesh, the Philippines and Thailand, on the other hand, we observe the opposite — firms that export almost all their output comprise 70% or more of exporters. A third group of countries (e.g. China, India, Uruguay and Turkey) exhibits what we denote in this paper 'twin peaks', a high concentration of firms on both ends of the export intensity distribution.

Figure 2 shows that bimodal export intensity distributions tend to be more the rule rather than the exception — for about two thirds of the 71 countries in our sample we reject the null hypothesis of unimodality.

This paper has three main objectives: first, we seek to understand what country characteristics determine the wide variety of shapes that we observe across countries. Second, we show that augmenting an otherwise standard trade model with independent firm-destination-specific demand shifters drawn from a heavy-tailed distribution, relative market size differences explain a remarkably high share of the cross-country variation in the distribution of export intensity. When we assume that demand shifters are distributed Fréchet with the

Figure 1: Export Intensity Distribution Across Countries: Selected Examples

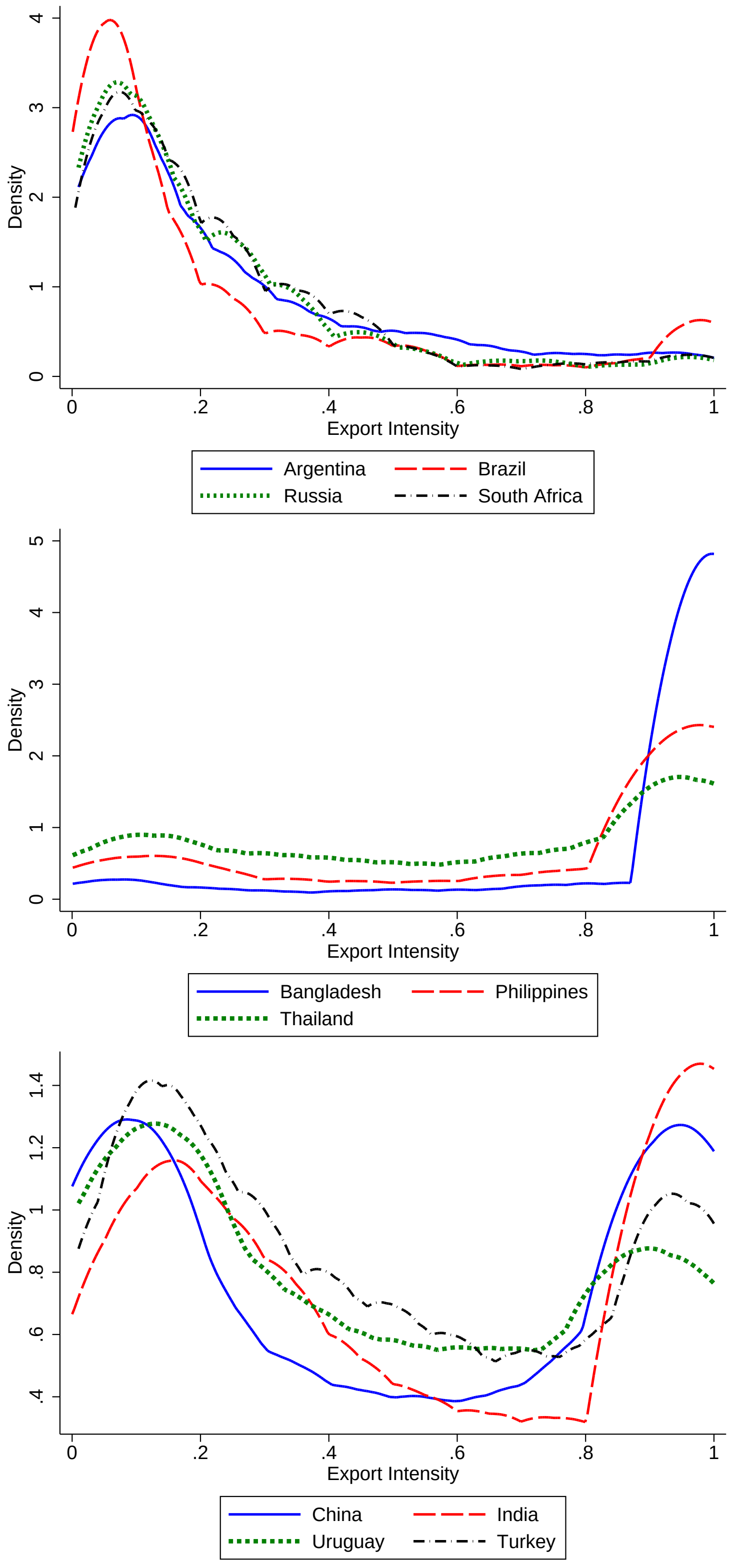
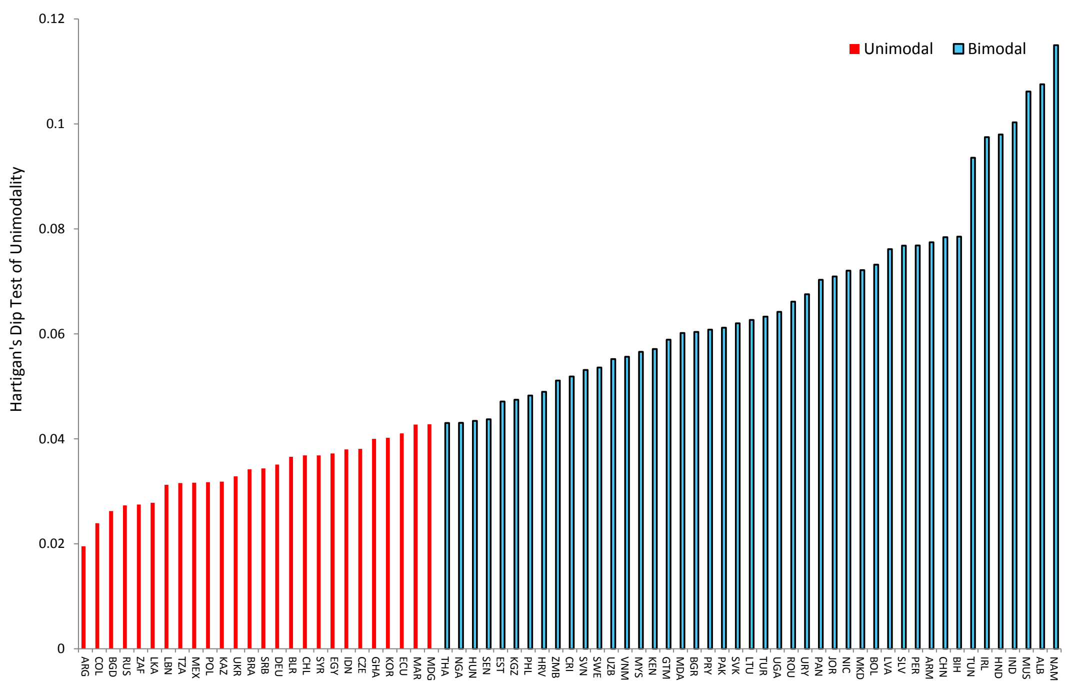


Figure 2: Dip-test on unimodality for 71 countries



same shape parameter across countries, the structural parameters governing the distribution of export intensity can be easily estimated. Relaxing this assumption by allowing the Fréchet shape parameters to vary across destination markets and countries or utilizing a lognormal distribution yields similar results. Third, by embedding our simple model into a small economy version of a general equilibrium model of trade with heterogeneous firms developed by Demidova and Rodríguez-Clare (2009, 2013), we study how different types of trade liberalization reforms affect the shape of the export intensity distribution.

We show a simple partial equilibrium model with constant-elasticity demand functions across markets in which firms face Fréchet-distributed firm-destination specific demand shifters, we can characterize the distribution of export intensity in closed form when shocks across markets have the same shape parameter. More importantly, we show that when the shape parameter governing the distribution of demand shifters is less than 1, i.e. when the underlying distribution of demand shifters is heavy-tailed, the distribution of export intensity becomes bimodal with modes at 0 and 1. The 'height' of the modes, in turn, is a function of the relative size of export demand vis-à-vis the domestic market. A further advantage of our model is that its structural parameters, i.e. the shape and scale parameters of the Fréchet distribution have economical meaning. While the shape parameter indicates how heavy-tailed the distribution is, the scale parameter corresponds to a country's aggregate demand shifters that incorporate total expenditure and the price index in the general equilibrium version. Finally, can be easily estimated these two parameters relying only on data on the median export intensity and the share of exporters in different export intensity bins across countries. We show that our model can account for 89% of the cross-country variation in export intensity distributions and our inferred relative export demand shifters are also strongly correlated with Gravity-based measures of external market potential.

Firm's export-intensity is usually used to evaluate the impact of export policies, both by researchers (Hauser, and Werner, 2009) or by export promotion agencies themselves. However, very little is known about the characteristics of export-intensity distribution across countries. Hence, understanding the moments, and fundamentals governing its shape is crucial. See Volpe discussion in Economic Policy.

Related Work. Previous work by Bernard et al. (2003), Brooks (2006), Arkolakis (2010) and Eaton et al. (2011) found that in the U.S., Colombia and France respectively, the vast majority of exporters sell the lion's share of their output domestically, with only very few firms fully devoted to exporting. Thus, this has been characterized as one of the key stylized facts characterizing the export behavior of firms by Bernard et al. (2007) and Melitz and Redding (2014) in their review articles.

The bimodal pattern of the export intensity distribution was firstly identified in the case of China by Lu (2010) and Defever and Riaño (2014). In this paper we find that the bimodality of the export intensity distribution is not specific to China, and we also show that this type of distribution does not only arise in countries that provide subsidies that target firms exporting the majority of their output such as duty drawback schemes and special economic zones.

Our use of heavy-tail demand shifters to explain the Twin Peaks phenomenon in the export intensity distribution contributes to the growing literature that seeks to understand the microeconomic origins of aggregate fluctuations (Gabaix, 2011; Acemoglu et al., 2015) and the role played by trade openness (di Giovanni et al., 2011; di Giovanni and Levchenko, 2012), particularly in small developing countries.

The rest of the paper is organized as follows. Section 2 presents a simple model that can successfully reproduce the observed cross-country heterogeneity of export distributions when fitted to the data. Section 9 extends our model to a small economy general equilibrium environment and discusses our calibration strategy. Section **??** presents the results of our policy experiments, and Section 11 concludes.

2 Theoretical Framework

We use a standard trade model with heterogeneous firms as proposed by Melitz (2003) as our benchmark to which we introduce Fréchet-distributed, firm-destination-specific demand shifters. Consider an industry populated by a large number of firms. Firms can sell their output in the domestic market (d) or export it to the rest of the world (x). Firms' demand in each market is subject to idiosyncratic, destination-specific demand shifters, $z_i, \quad i \in \{d, x\}$. Market i's size (which is common to all firms) is denoted by $A_i > 0$; thus, a firm that draws a vector of demand sifters (z_d, z_x) faces an iso-elastic demand function in market i (in terms of sales) given by:

$$r_i = z_i A_i p_i^{1-\sigma}, \tag{1}$$

where $\sigma > 1$ is the elasticity of demand.[1]

[1]Iso-elastic demand function can be obtained from a representative consumer with CES preferences:

$$\mathcal{U} = \left(\int_{\omega \in \Omega_d} [z_d(\omega)]^{\frac{1}{\sigma}} [q_d(\omega)]^{\frac{\sigma-1}{\sigma}} d\omega + \int_{\omega \in \Omega_m} [z_m(\omega)]^{\frac{1}{\sigma}} [q_m(\omega)]^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}},$$

where $q_d(\omega)$ and $q_m(\omega)$ denote consumption of domestically-produced and imported varieties, Ω_d and Ω_m represent respectively the set of available domestic and imported varieties and $\sigma > 1$ is the elasticity of substitution. The terms $z_d(\omega)$ and $z_m(\omega)$ are variety-destination-specific demand shifters.

Monopolistically competitive firms pay a fixed cost wf_e (where w denotes the wage at Home) to enter the market and draw their idiosyncratic productivity φ from a distribution $G_\varphi(\varphi)$. After observing its productivity, but before knowing the realizations of domestic and export demand shifters, a firm chooses whether to operate or not and what markets to serve (i.e. only produce for the domestic market, or sell both at home and abroad). Domestic production involves a fixed cost wf_d, while firms that choose to export need to pay an additional cost wf_x.

Firms choose their mode of production $k \in \{D, X, \varnothing\}$, where D denotes domestic production, X, exporting and $\varnothing$ indicates a firm that does not produce at all, by solving the following problem:

$$\max_{D,X,\varnothing} \Big\{ E[\pi_d] - wf_d, E[\pi_d] + E[\pi_x] - w(f_d + f_x), 0 \Big\}. \tag{2}$$

Firms' cost function $c(q)$ is assumed to be weakly increasing and convex in total output. The profit-maximizing prices chosen by a firm are $p_d = \left(\frac{\sigma}{\sigma-1}\right) c(q)$ and $p_x = \tau p_d$, where $\tau \geqslant 1$ is an iceberg transport cost. Hence, a firm's export intensity, E, defined as the share of total sales shipped to the rest of the world, is:

$$E(Z_d, Z_x) \equiv \frac{r_x}{r_d + r_x} = \frac{\tau^{1-\sigma} A_x z_x}{A_d z_d + \tau^{1-\sigma} A_x z_x} = \frac{Z_x}{Z_d + Z_x}. \tag{3}$$

There are two points worth remarking about (3). First, without the firm-destination specific demand shifters, the distribution of export intensity would be degenerate with all the mass concentrated at $\tau^{1-\sigma} A_x / (A_d + \tau^{1-\sigma} A_x)$. Second, in a model including an agricultural sector as in Helpamn and Krugman (1985), the export intensity would be degenerate with all the mass concentrated at $\tau^{1-\sigma}/(1 + \tau^{1-\sigma})$.

Fréchet distributions We assume that the demand shifters are the realization of an extreme value distribution. More specifically, firm draws the value of z_i for a given location i from the Fréchet distribution with cdf $G(z_i) = \exp[-z_i^{-\alpha_i}]$ for $z_i > 0$, where $\alpha_i > 0$ is the shape parameter.

Note that $Z_d \equiv A_d z_d$ and $Z_x \equiv \tau^{1-\sigma} A_x z_x$ are themselves Fréchet distributed with shape parameter α_d and α_x, and scale parameter $s_d = A_d$ and $s_x = \tau^{1-\sigma} A_x$ respectively.[2]

$$Zd \sim Frechet(s_d, \alpha_d) \ \text{ with } \ s_d = A_d \tag{4}$$

$$Zx \sim Frechet(s_x, \alpha_x) \ \text{ with } \ s_x = \tau^{1-\sigma} A_x \tag{5}$$

However, $E = Z_d/(Z_d + Z_x)$ is not distributed Fréchet. By imposing the assumption that the shape parameter is the same for both domestic and foreign demand shifters, i.e. $\alpha_d = \alpha_x = \alpha$, we can characterize the distribution of export intensity by relying on the following theorem by Nadarajah and Kotz (2006):

Theorem 1 (**Nadarajah and Kotz, 2006**)**.** *Suppose that Z_d and Z_x are distributed Fréchet with shape parameter $\alpha > 0$ and scale parameter $s_d > 0$ and $s_x > 0$ respectively. Then the cdf of $Z \equiv \frac{Z_d}{Z_x}$ is given by:*

$$F(z) \equiv Prob[Z \leqslant z] = \frac{\left[\left(\frac{s_x}{s_d}\right) z\right]^{\alpha}}{1 + \left[\left(\frac{s_x}{s_d}\right) z\right]^{\alpha}}. \tag{6}$$

Using theorem 1 as our stating point, we can state our first proposition as follows:

Proposition 1. *Assume that demand shifters $Z_i, \quad i \in \{d, x\}$ follow a Fréchet distribution with shape parameter α and scale parameter $s_d = A_d$ and $s_x = \tau^{1-\sigma} A_x$, then the cdf of the export intensity distribution is given by:*

$$H(e) \equiv Prob[E \leqslant e] = \frac{1}{1 + \left[\left(\frac{s_x}{s_d}\right) \frac{1-e}{e}\right]^{\alpha}}, \tag{7}$$

[2]The cdf for Z_i is therefore given by $G(Z_i) = \exp\left[-\left(\frac{Z_i}{s_i}\right)^{-\alpha}\right]$.

and the pdf is:

$$h(e) \equiv H'(e) = \frac{\alpha(s_d s_x)^\alpha e^{\alpha-1}(1-e)^{\alpha-1}}{[(s_d e)^\alpha + (s_x(1-e))^\alpha]^2}. \tag{8}$$

Proof. See Appendix. ■

3 Tails of the distribution and bimodality

No clear definition of Heavy tailed distribution: For the purpose of the paper, we provide a clear shape parameter generating a distribution which tail is fat enough to generate a bimodal export-intensity distribution.

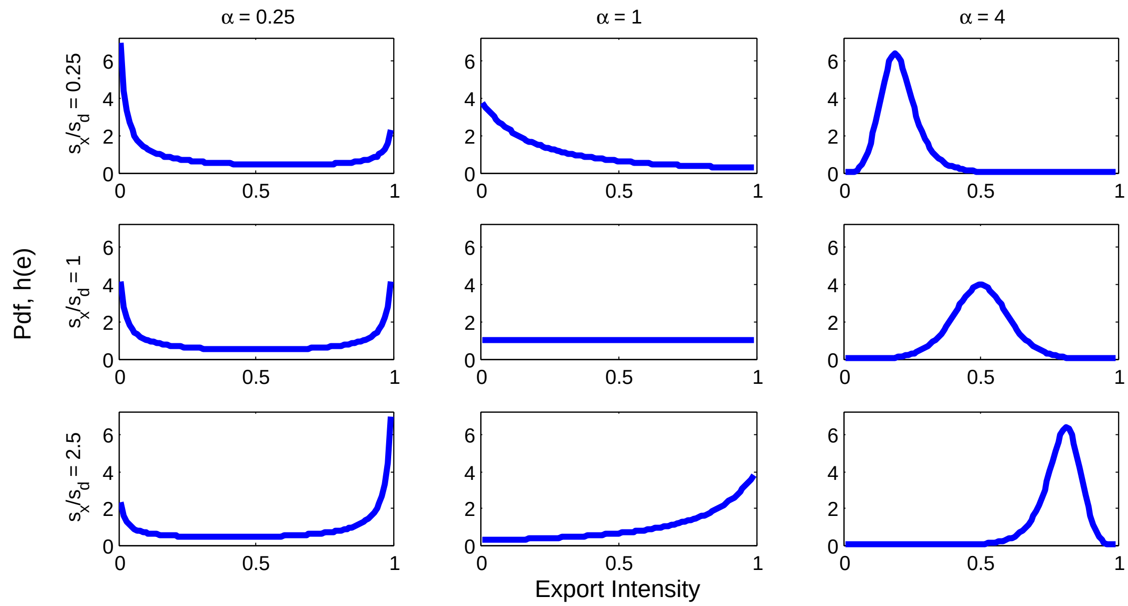
In continuous distributions, unimodality can be defined through the behavior of the cumulative distribution function (cdf). If the cdf is convex for x ¡ m and concave for x ¿ m, then the distribution is unimodal, m being the mode. Note that under this definition the uniform distribution is unimodal. See. Ushakov, N.G. (2001), "Unimodal distribution", in Hazewinkel, Michiel, Encyclopedia of Mathematics, Springer

Proposition 2. *When $\alpha < 1$, the distribution of export intensity is bimodal with modes at 0 and 1. The major mode is located at 0 when $s_x/s_d < 1$ and at 1 in the converse case. If $s_d = s_x$, then the distribution is symmetric.*

Proof. See Appendix. ■

Figure 3 illustrates the predictions of Proposition 2 by presenting the pdf of export intensity for different combinations of the Fréchet shape parameter (columns) and relative market size (rows). When $\alpha < 1$, the major mode of the export intensity distribution shifts from 0 to 1 as s_x/s_d increases above 1. Similarly, the concentration of mass at the major mode increases with the difference between the domestic and foreign market sizes. When $\alpha < 1$, a reduction in α shifts mass from the middle of the distribution towards 0 and 1, while changes in the scale parameters alter the magnitude of the modes in opposite directions —

Figure 3: Export Intensity Distribution as a function of α and s_x/s_d



e.g. when s_x increases, mass moves from the lower bound of the distribution towards 1, while the opposite happens when s_d increases. On the other hand, when $\alpha > 1$, the distribution becomes unimodal and more concentrated as the shape parameter increases. When $\alpha = 1$ and the domestic and foreign market have the same size, the export intensity distribution is uniform.

When α falls below 1, the underlying Fréchet distribution of demand shifters becomes more disperse and its tail becomes fatter.[3] As extreme realizations of demand shifters become more likely, and since demand shifters are independent across markets, the number of firms selling the majority of their output in either the domestic or export market increases, thereby generating twin peaks in the export intensity distribution. In Panel B of Figure D.1, we show that the quantile coefficient of kurtosis for the export intensity distribution $H(e)$ falls below that of a Uniform distribution when $\alpha < 1$. This means that more mass is concentrated

[3]Although the moment of order q for the Fréchet distribution is finite if and only if $\alpha > q$, we can use the quantile-based statistics proposed by Moors (1988) to characterize the properties of both the distribution of demand shifters and export intensity, because the quantile function for these two distributions is well defined. In Appendix D we show how the quantile-based alternatives for the first four moments of the above distributions behave as a function of the shape parameter α. Panel A of Figure D.1 shows that both the inter-quantile range and quantile coefficient of kurtosis for a Fréchet distribution increase when α falls.

around the 'shoulders' of the distribution, i.e. around one standard deviation from the mean — $\overline{E} \pm \sigma_E$ (Moors, 1988).

It is important to note that twin peaks do not only arise when demand shifters are distributed Fréchet. In Appendix E we present Monte Carlo simulations of export intensity distributions based on different underlying demand shifter distributions that also produce twin peaks (see Figure E.1). The main advantage of assuming that the firm-destination specific demand shifters z_i are drawn from a Fréchet distribution, is that we can characterize the cdf of the random variable $\frac{\tau^{1-\sigma} A_x z_x}{A_d z_d + \tau^{1-\sigma} A_x z_x}$ in closed form, which in turn enables us to easily estimate the shape and scale parameters of the export intensity distribution as we discuss below.

4 Identification Strategy

Corollary 1. *The median export intensity, μ, is given by:*

$$\mu = \frac{s_x}{s_d + s_x}, \tag{9}$$

which is independent of α.

Proof. Invert the cdf (7) to obtain the quantile function, $Q(p) \equiv H^{-1} = \frac{s_x p^{\frac{1}{\alpha}}}{s_x p^{\frac{1}{\alpha}} + s_d (1-p)^{\frac{1}{\alpha}}} = \frac{1}{1 + (\frac{1-p}{p})^{\frac{1}{\alpha}} (\frac{Sd}{Sx})}$, and evaluate it at $p = 0.5$ ■

Identification Strategy: Our identification strategy to back out the shape parameter of the export-intensity distribution rely on a very specific properties of the ratio distribution both in the case of the Frechet and Log normal distribution. Both for Frechet and lognormal sale distributions, the median export-intensity give us information on the relative country size. We use the result in Corollary 1 to recover the ratio of scale parameters $\frac{s_x}{s_d} = \frac{\tau^{1-\sigma} A_x}{A_d} = \frac{\mu}{1-\mu}$ that appears directly in the cdf in equation (7). Table 1 provides summary statistics for this measure of size of foreign relative to domestic demand (net of transport costs).

To summarize, the median export-intensity is unrelated to the shape of the underlying sale distributions. Our identification relies on how much exporters do you observe around the median. If the median is the anti-mode of the export-intensity, then the sales follow a heavy-tailed distribution. [4]

To the best or our knowledge, only the lognormal distribution (discussed in section 5) would have a similar property.

5 Estimating the Export Intensity Parameters

Data: The main source of firm-level information to be used in the project are the WBES/BEEPS surveys compiled by EBRD and the World Bank over the period 2000-2014, which cover 135 countries. The project will use a sample of 71 developing and transition countries with at least 100 exporting rms (the list of countries is provided in Appendix F). The following variables are constructed for each firm in the sample: Firm's total sales and its export intensity, which allows us to identify a firms export status, as well as its sales in the domestic and abroad. Information on firm's ownership status (domestic/foreign) and the share of imported inputs in the total value of intermediate inputs used by the firm are also available and is used to build some of our robustness checks.

Table 1: Inferred Relative Foreign Market Size

	Mean	Median	Min	Max
$\widehat{s_x/s_d}$	0.903	0.538	0.111	4.000

For the median country in our sample, the export market accounts for 54% of the size of the domestic market. There is a substantial amount of cross-country variation in $\widehat{s_x/s_d}$, with

[4] We try to identify such a properties in other distributions, such as Pareto, Beta, Gamma..etc.. and none of them share this feature.

the magnitude of the external market ranging from about one tenth of domestic demand for Brazil and Russia to four times as large for Albania and Vietnam. *Notice that 7 countries have a median export-intensity equal to one. For this countries, the ratio S_d/S_x cannot be identified, and we will have to exclude these countries from our estimation.*

Estimation: With our country-level relative size measure at hand, we can use the cross-country data on firm-level export intensity distributions available in the World Bank Enterprise Surveys to estimate the shape parameter α of the cdf $H(e)$. Since the surveys ask firms to provide the share of total sales accounted for by exports (conditional on exporting) using discrete bins, let B_{cq} denote the share of exporters in country c with export intensity in bin $q = 1, \ldots, 5$ observed in the data.[5] Thus, given the vector $\widehat{(s_x/s_d)}_c$, $\hat{\alpha}$ solves the following non-linear least squares problem:[6]

$$\min \sum\nolimits_{c=1}^{65} \sum\nolimits_{q=1}^{5} \left[B_{cq} - \left(H\left(\frac{q}{5}\right) - H\left(\frac{q-1}{5}\right) \middle| \widehat{(s_x/s_d)}_c \right) \right]^2, \tag{10}$$

Table 2 shows that the shape parameter is precisely estimated, and is significantly smaller than 1, thus generating bimodal export intensity distributions as described in Proposition 2 with mass concentrated around 0 and 1. Our proposed distribution arising from primitive Fréchet-distributed demand shifters explains approximately 89% of the cross-country variation in the export intensity distribution. Figure 4 vividly illustrates the goodness of fit of our model — notice that with the exception of six countries, the majority of the mass of the export intensity distribution is always concentrated in either the first or fifth export intensity bin.

Excluding foreign affiliates: We exclude all firms with at least 50 percent of their capital

[5] Bin 1 groups firms with an export intensity below 0.2, bin 2 groups firms with export intensity between 0.2 and 0.4, and so on.

[6] Bangladesh, Madagascar, Morocco, Pakistan, The Philippines and Sri Lanka are dropped of the estimation because they all have a median export intensity of 1, which implies that $s_x/s_d \to \infty$.

Table 2: Estimated Shape Parameter Export Intensity Distribution

	Full sample	Exclude foreign	Exclude processing	Countries without ESR	OECD countries	Non-OECD countries	Exclude Pure Exp.
Using the CDF with 5 bins							
α	0.839	0.883	0.888	0.854	0.847	0.838	0.967
	(0.019)	(0.021)	(0.021)	(0.026)	(0.048)	(0.021)	(0.018)
# Countries	65	67	67	35	12	53	71
# Observations	325	335	335	175	60	265	355
R^2	0.893	0.882	0.886	0.897	0.886	0.895	0.916
Using the quantile function with 8 quantiles							
α	0.681	0.684	0.675	0.752	0.782	0.665	0.832
	(0.012)	(0.012)	(0.012)	(0.017)	(0.032)	(0.012)	(0.011)
# Countries	65	67	67	35	12	53	71
# Observations	520	536	536	280	96	424	568
R^2	0.978	0.974	0.972	0.977	0.981	0.978	0.977

owned by a foreign company

Excluding processing plants: Processing activities, where inputs are imported and integrated all together in order to produce a final good, usually mainly designed to be exported. Unfortunate, the WBES does not provide information on processing trade. Instead, we rely on the share of intermediate inputs used by that firms that have been imported. More precisely, a firm is classified as a processing exporter if the share of imported inputs exceeds 90% of total expenditure in intermediate inputs.

Excluding firms located in Export Zones:

Excluding countries with Export share Requirements Since our objective is to infer the export intensity distribution that would have prevailed in China in the absence of subsidies with ESR, it is appropriate to consider only highly populated countries. Countries are classied as providing subsidies with ESR based on information gleaned from the

US Department of States Investment Climate Statements. Twelve countries in our sample employ subsidies with ESR; Appendix H provides a list of the subsidy programs featuring ESR in each of them. The information on subsidies with ESR is found in the sections on Performance Requirements and Incentives and Foreign Trade Zones/Free Trade Zones in the ICS. We nd evidence of subsidies with ESR being in place in 35 of the 71 countries in our sample.

After excluding these countries, we obtain a value for α only slightly bigger than the one obtained for the full sample. It may be surprising that such a restriction only marginally affect the estimation of the shape parameter estimated. However, the use of export requirements may affect both the prevalence of export-oriented firms and the median export intensity, which statistic is used to recover the market size ratio Sd/Sx. In other words, such policies may be affect both the estimation of the shape parameters itself, but also the estimations of the fundamental parameter describing the economy.

Excluding pure exporters: Consider that instead of a fixed cost of producing, top-up by another one for exporting, let us consider the case where firms face country specific fixed cost only. One in order to enter the domestic market (f_d) and another one in order to sale in the export market (f_x). Some firms may then prefer to only export, without selling anything on the domestic market. This raise the possibility of 'pure exporters', driven by selection into market. When recovering the parameters of the demand shocks, it is assumed that the firms have paid the fixed cost for both market. Hence, as a robustness we drop all the 'pure exporters' of our sample, restimate the median export intensity for $EI =]0,1[$.

Firms choose their mode of production $k \in \{D, X, \varnothing\}$, where D denotes domestic firms, X, exporters also selling in the domestic market, P pure exporting with no sales on the domestic market and $\varnothing$ indicates a firm that does not produce at all, by solving the following

Table 3: Estimated Shape Parameter Export Intensity Distribution
by sector

	Leather & textile	Metals & machinery	Food & beverage	Non-metal product	Electric product	Paper & furniture	Chemicals & pharma	Other manuf.
α	0.878	0.924	0.915	0.991	0.819	0.843	1.020	0.893
	(0.023)	(0.025)	(0.027)	(0.040)	(0.035)	(0.038)	(0.297)	(0.032)
# Countries	42	41	52	24	8	19	26	29
# Obs.	210	205	260	120	40	95	130	145
R^2	0.916	0.908	0.867	0.889	0.950	0.889	0.956	0.897

problem:

$$\max_{D,X,P,\varnothing} \Big\{ E[\pi_d] - wf_d, E[\pi_d] + E[\pi_x] - w(f_d + f_x), E[\pi_x] - wf_x, 0 \Big\}. \tag{11}$$

Sectoral analysis: Textiles Garments Leather account for 31.10 percent of the firm-level data All together, the 8 sectors represent about 98 percent of all the manufacturing firms in the WBES.

Figure 4: Model Fit Export Intensity Distribution

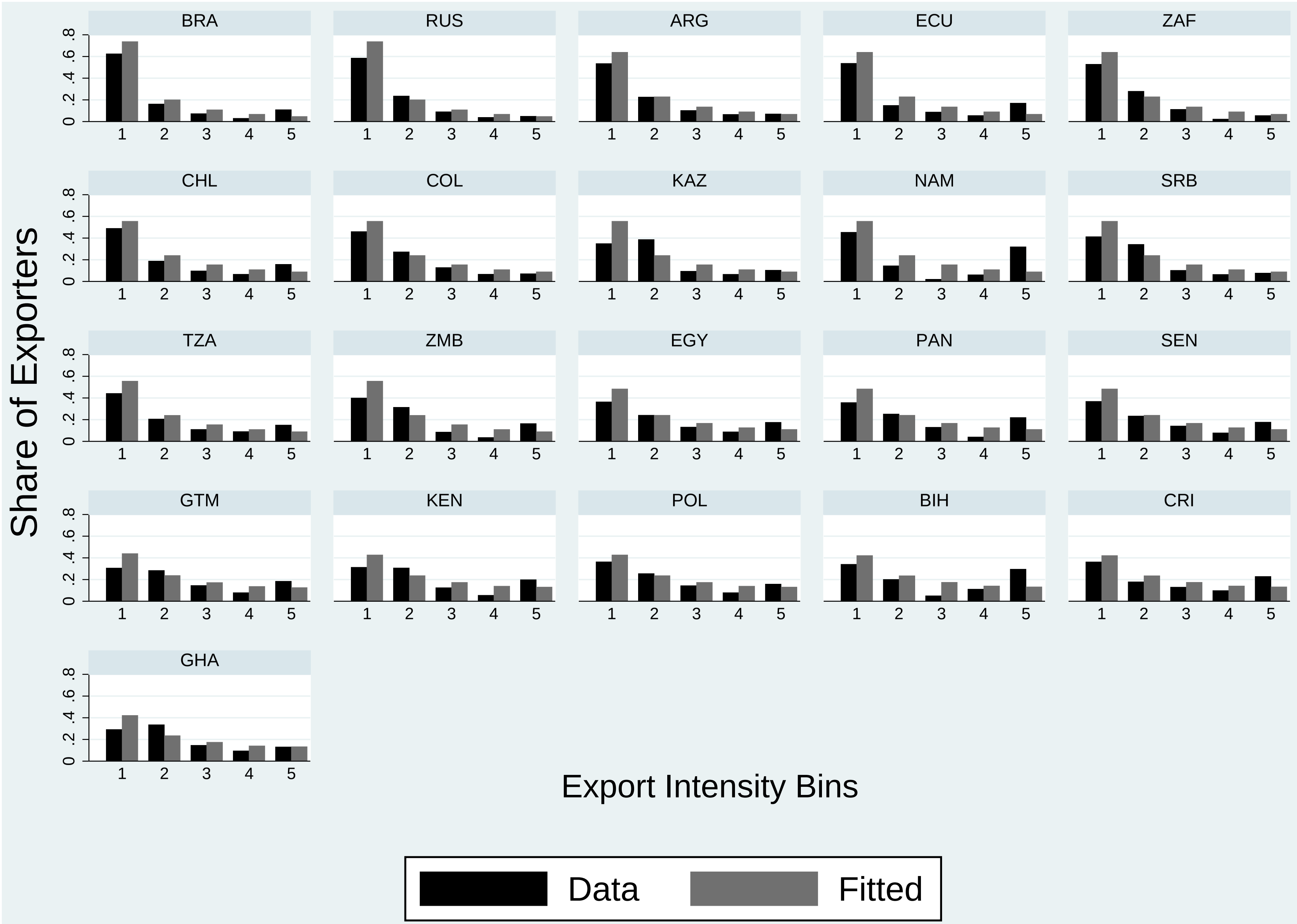


Figure 6: Model Fit Export Intensity Distribution, Continued

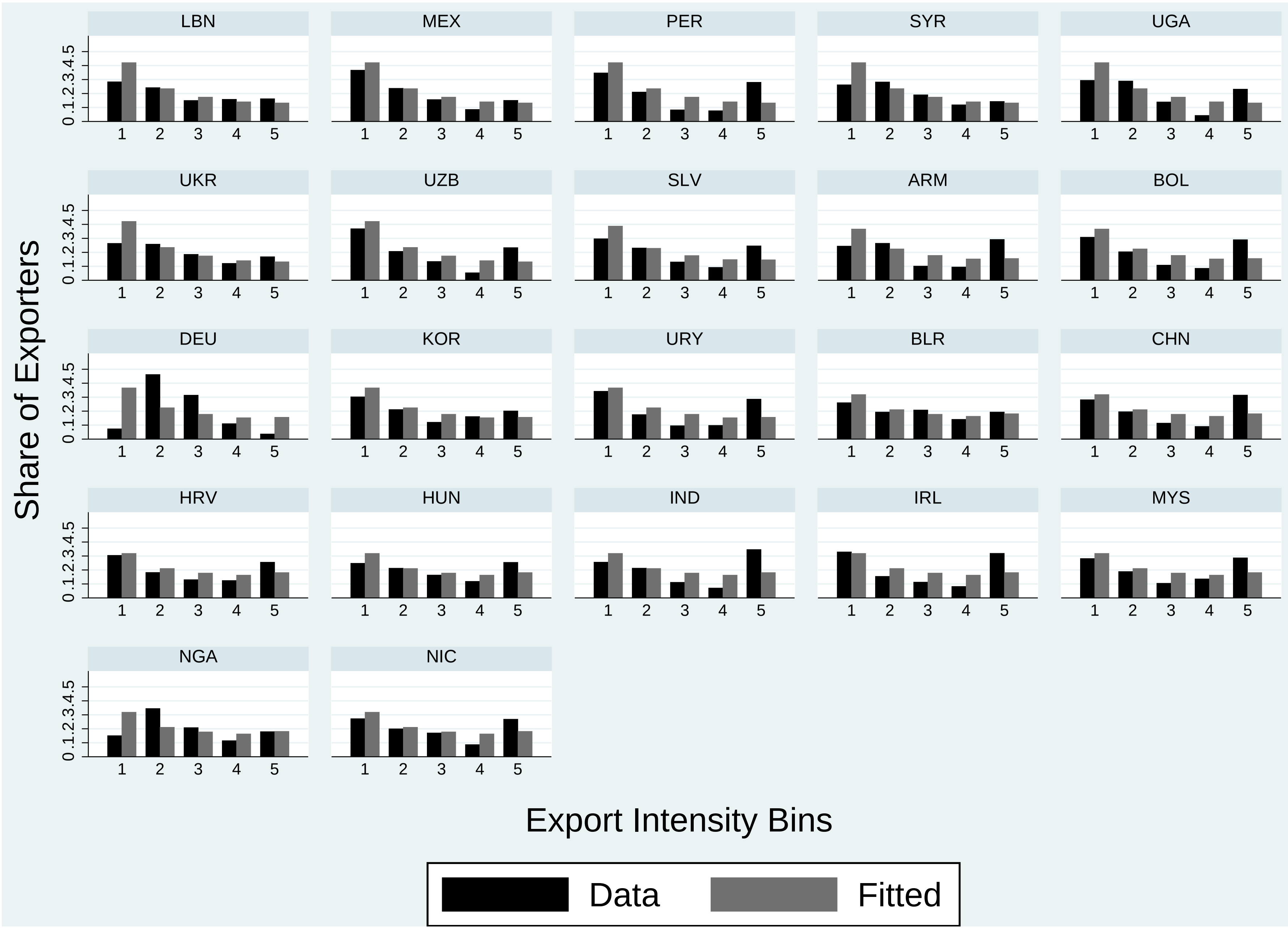
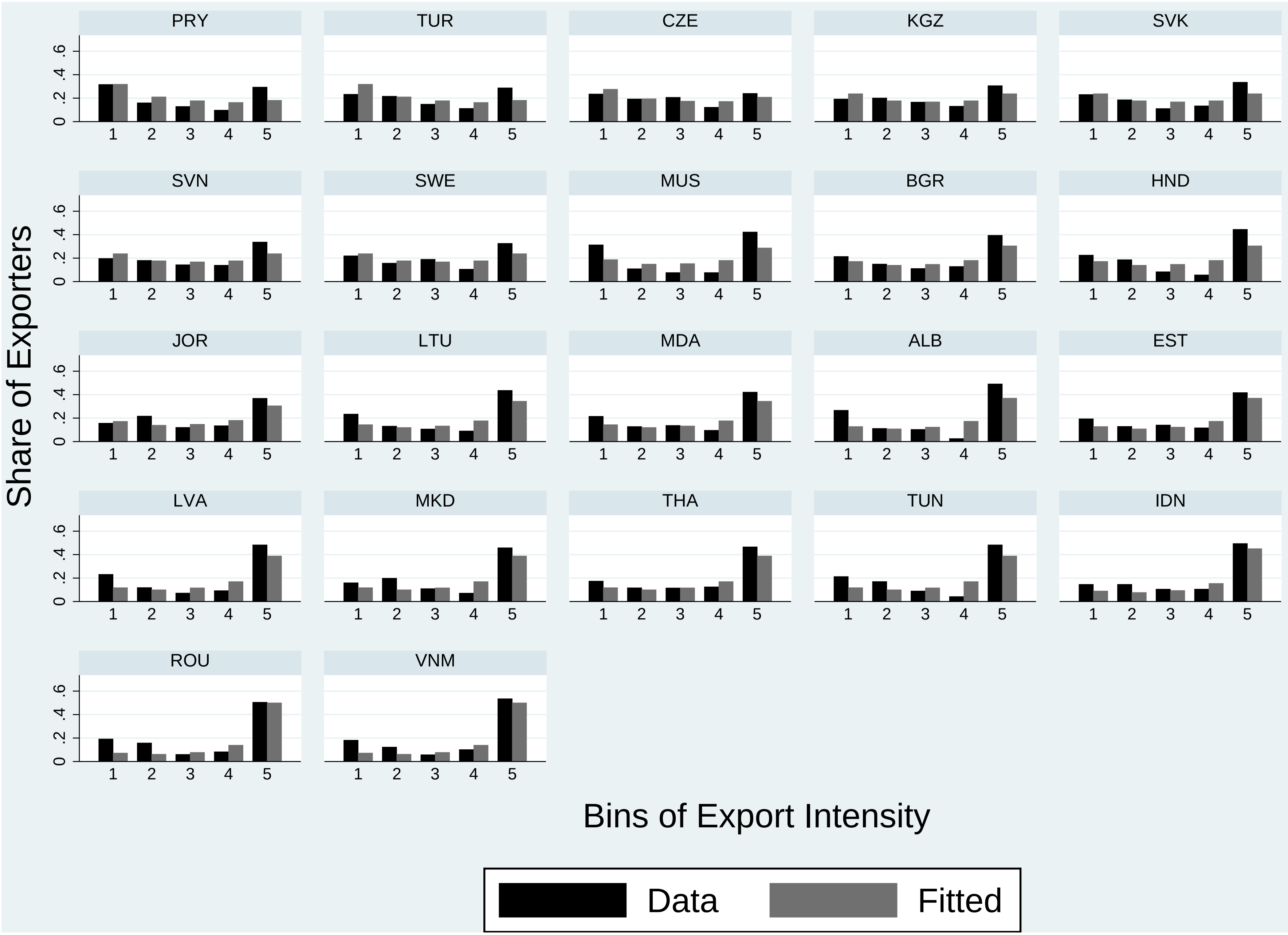


Figure 6: Model Fit Export Intensity Distribution, Continued



6 Country size and Bimodality

With $\alpha > 0$, all countries should exhibit a bimodal export-intensity distribution. While this may be the case theoretically, this may not be necessarily the case from an observational (statistical) point of view. In fact, bimodal distributions have probability distribution functions that exhibit more than one local maxima; however, from a statistical perspective, there is not a clear cut of the size of these peaks that determines whether an empirical distribution is bimodal or not. Hartigan and Hartigan (1985) propose a statistical test of the degree of departure from unimodality of the empirical conditional density function. The 'dip' test is based on the fact that the cdf of a unimodal distribution must have a single inflection point.[7]

Using $\alpha = 0.893$, Figure 7 plots the Hartigan's dip test statistic calculated from 10,000 simulated export intensity draws for different relative market sizes, which in turn determine the median export intensity (recall Corollary 1). For $S_d/S_d = 1$, the median export-intensity is 1/2 (Panel A) and the dip-test indicates an extreme bi-modal export-intensity distribution (Panel B). For $S_d/S_d = 0.429$ and $S_d/S_d = 2.333$, the median export-intensity is respectively 0.3 and 0.7, and the dip-tests indicate that such differences in country sizes are associated with a non-significant dip-test. From a statistical point of view, these distributions can not longer be differentiated from a unimodal distribution.

The dip test values follow an inverted-U relationship with respect to the median export-intensity of distributions. In other world, a Fréchet shape parameter is below 1 can encompass both the unimodal case in which most exporters sell the majority of their output either at home or abroad, as in the first and second panels of Figure 1, but also a situation featuring twin peaks. The latter is more likely to be observed in countries for which the domestic and foreign market sizes are relatively similar. Going back to the data, Figure 8 relates each country's median export-intensity with the value of the dip-test. An inverted-U relationship also emerges from the data. Countries with a relatively high or low median export-intensity

[7] See Henderson et al. (2008) for applications of this test to evaluate the multimodality of the cross-country distribution of economic outcomes.

are also the one associated with non significant Dip-tests.

Figure 7: Relative Foreign Market Size and Bimodality of the Export Intensity Distribution (using $\alpha = 0.893$)

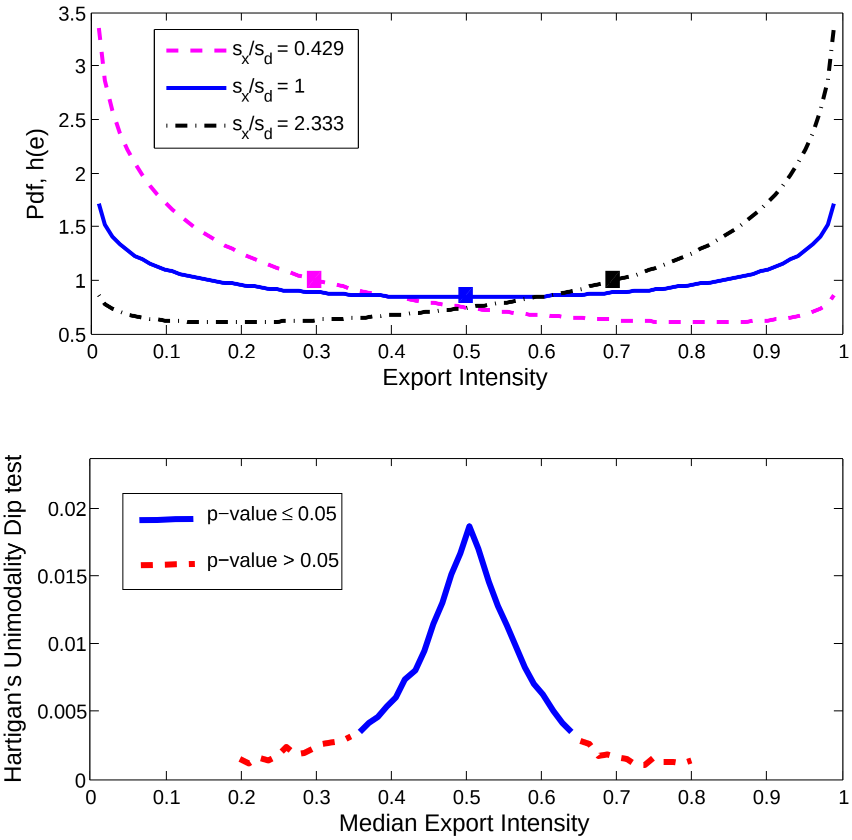
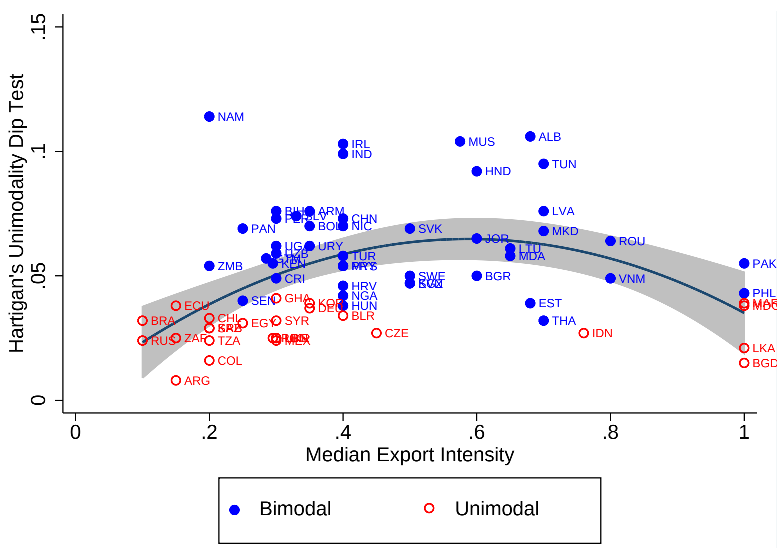


Figure 8: Prevalence of Bimodality of the Export Intensity Distribution across Countries



7 Other Distributions

Lognormal distributions Let us assume that z_d and z_x are distributed following a lognormal distribution with mean $\mu = 0$ and variance σ_d^2 and σ_x^2 respectively. Then $Z_d \equiv A_d z_d$ and $Z_x \equiv \tau^{1-\sigma} A_x z_x$ are themselves lognormally distributed with shape parameter σ_d^2 and σ_x^2, and scale parameter $s_d = A_d$ and $s_x = \tau^{1-\sigma} A_x$ respectively.

$$Zd \sim \ln \mathcal{N}(\ln(s_d), \sigma_d^2) \quad \text{with} \quad s_d = A_d \tag{12}$$

$$Zx \sim \ln \mathcal{N}(\ln(s_x), \sigma_x^2) \quad \text{with} \quad s_x = \tau^{1-\sigma} A_x \tag{13}$$

The ratio of two lognormal distribution is also a lognormal distribution.[8]

$$Zd/Zx \sim \ln \mathcal{N}(\ln(Sd/Sx), \sigma_d^2 + \sigma_x^2) \tag{14}$$

$Zd/Zx + 1$ follows a shifted lognomal distribution and the export intensity distribution ($E = Zx/(Zd+Zx) = 1/(Zx/Zd+1)$) follows the inverse of a shifter lognormal distribution. The pdf, cdf and moment generator of such distributions are unknown. Nevertheless, as quantiles of a random variable are invariant under monotonic transformation, shifting the distribution by one simply add one to each quantile, and reversing it, reverse the ordering except for the median. Hence, we can defined the quantile (or inverse) function. Solving $p = F(x)$ for x in terms of p, $F^{-1}(p) = exp[\mu + \sigma \Phi^{-1}(p)], p \in (0, 1)$. Φ is obtained from the special distribution calculator.

Proposition 3. *Assume that demand shifters Z_i, $\; i \in \{d, x\}$ follow a lognormal distribution with variance σ_i^2 and scale parameter $s_d = A_d$ and $s_x = \tau^{1-\sigma} A_x$, then the quantile function of the export-intensity distribution is given by:*

$$Q = \frac{1}{e^{ln(Sd/Sx) + \sqrt{\sigma_d^2 + \sigma_x^2}\Phi^{-1}(1-p)} + 1}, p \in (0, 1) \tag{15}$$

[8]This come from the fact that $XY \sim \ln \mathcal{N}(\mu_X + \mu_Y, \sigma_X^2 + \sigma_Y^2)$ and $1/X \sim \ln \mathcal{N}(-\mu_X, \sigma_X^2)$ for X and Y being two independent lognormal distributed functions.

Using the well known properties of the lognormal distribution, the median of Zd/Zx is given by: median $= e^{\ln(Sd/Sx)}$, where $\ln(Sd/Sx)$ are the mean of the underlying normal distribution of Zd/Zx. Then,

$$\text{median} = 1/[e^{\ln(Sd/Sx)} + 1] = 1/(Sd/Sx + 1) \tag{16}$$

Interestingly, the median value of the inverse of a shifter lognormal distribution is independent to the shape/variance parameters. A similar relationship between the median and the scale parameters will also arise assuming Fréchet distributions. However, the relationship is more general in the case of the lognormal distribution, as the Fréchet distributions require identical shape parameters across markets.

The use of the lognromal distribution is also interesting as all its moment are well defined for any shape parameters. While

8 Fitting Domestic and Export Sales Distributions

A different way to characterize the distribution of export intensity is by fitting directly the distribution of domestic and export sales for each country in our sample. To do so, we assume that sales in each market are distributed according to a Generalized Extreme Value (GEV), which includes Fréchet as a special case, or a lognormal distribution. For each country-destination pair we estimate a vector of two parameters using Maximum Likelihood: the shape and scale parameters for the GEV case and the mean and variance of the underlying normal distribution in the lognormal case.

The cdf of the GEV distribution is given by:

$$G(z) = \exp\left\{ -\left[1 + \xi \left(\frac{z}{s} \right) \right]^{-\frac{1}{\xi}} \right\},$$

where ξ is the shape parameter and s is the scale parameter. The GEV encompasses the Gumbel, Fréchet and Weibull distributions depending of the value of the shape parameter. More precisely, when $\xi = 1/\alpha > 0$, we obtain the Fréchet distribution, with α being its shape parameter as defined previously.

Figure 9a plots the estimated α_d and α_x, for the distribution of domestic and export sales in each country. Three key messages emerge from it. First, the fact that all estimated αs are positive, indicates that the distribution of sales are better fitted by the Fréchet distribution compared to the other two GEV sub-families. Second, while Figure 9 reveals substantial dispersion in the shape parameter across countries, it is remarkable that the shape parameters for domestic and export sales within countries are very similar. Given the large proportion of values closely tight to the 45 degree line, assuming $\alpha_h = \alpha_x$ seem to be harmless assumption. Third, almost all estimated shape parameters are below 1, with the exception of Sweden and Bosnia and Herzegovina for domestic sales and Bosnia and Herzegovina, Morocco and Bandladesh for export sales. In all the other cases, $\alpha < 1$ indicates that the distribution of sales are sufficiently heavy-tailed to generate bimodality in the export intensity distribution.

Figure 9: Shape Parameters Obtained by Fitting Domestic and Export Sales

(a) Generalized Extreme Value

(b) Log-normal distribution

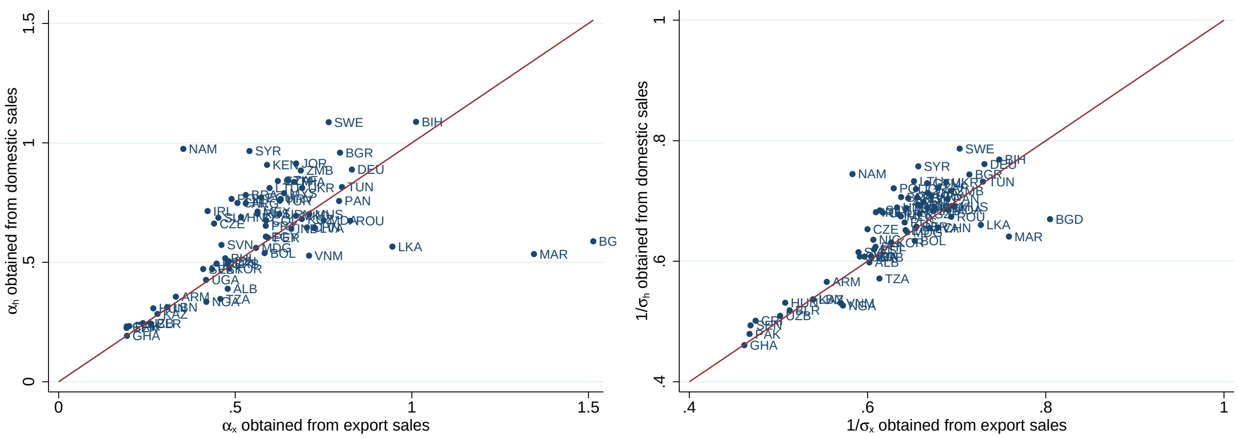


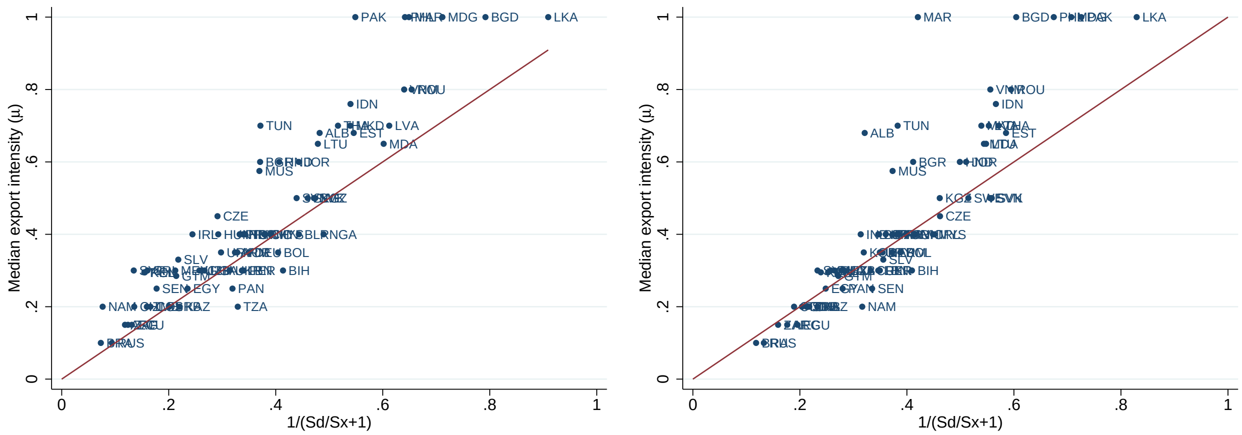
Figure 9b depicts the estimated the inverse of the standard deviation of domestic and export sales when we assume that these follow a lognormal distribution. For all countries the estimated values of $1/\sigma_d$ and $1/\sigma_x$ are below one, a level at which simulations show that the resulting export intensity distribution exhibits bimodality.

Our estimated scale parameters (the underlying mean of in the lognormal case) reveal a very tight one-to-one relationship between a country's median export intensity and the ratio $\frac{s_x}{s_d+s_x}$ — the relationship we used to recover the relative market size in the case of Fréchet distributed shifters with the same shape parameter across markets and countries (see Figures 10a and 10b). Thus, by relying on country-firm level sales data, we show that the assumptions that allowed us to estimate the distribution of export intensity in closed form are not unduly restrictive.

Figure 10: Median Export Intensity and the Ratio of Scale Parameters $\frac{s_x}{s_x+s_d}$

(a) Generalized Extreme Value

(b) Log-normal distribution



8.1 Productivity Vs demand shifters

In this section, we want to look at both the domestic and export sale observed in the data as well as the one can predict using the shape and scale parameters obtain from fitting the sales data assuming lognormal distributions for both domestic and export sales are as follow:

$$\text{Domestic Sales} = \Theta z_d Sd \quad (17)$$

$$\text{Export Sales} = \Theta z_x Sx \quad (18)$$

With Θ being the productivity of the firm, which affects the sales of the firm in all markets. Z_d is the demand shock, which is destination specific. If Θ and z_d are both distributed as independent Log-normal distribution, then the domestic sales will also be following a Log-normal distribution. More precisely, if $\Theta \sim \ln \mathcal{N}(0, \sigma_\theta^2)$ and $z_h \sim \ln \mathcal{N}(0, \sigma_h^2)$ then:

$$\text{Domestic Sales} \sim \ln \mathcal{N}(\ln(Sd_j), \sigma_\theta^2 + \sigma_d^2) \quad (19)$$

To make sales data comparable across countries, we re-scale both the domestic and export sales by dividing them by the their median (To be verified!).

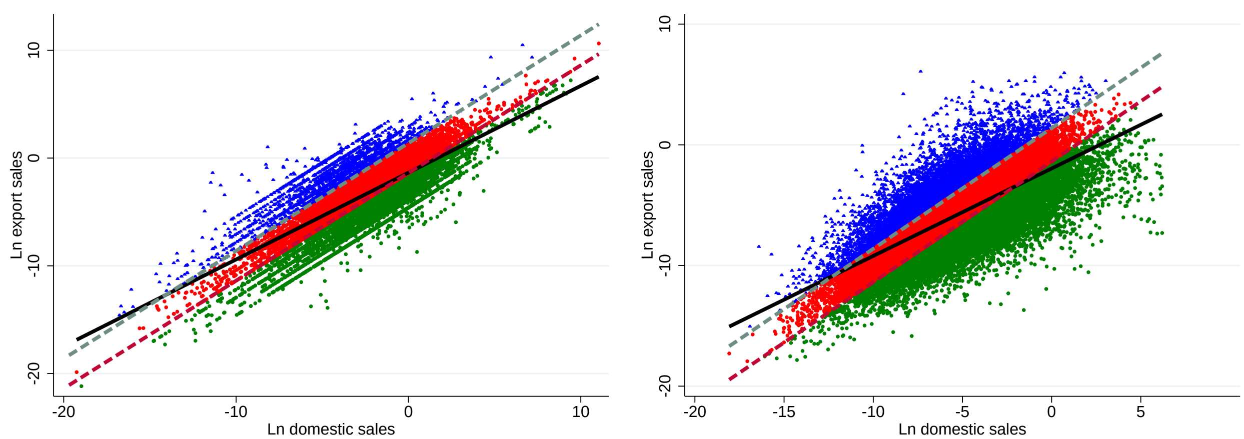
$$\text{Domestic Sales} \sim \ln \mathcal{N}(0, \sigma_\theta^2 + \sigma_d^2) \tag{20}$$

Interestingly, when fitting the sales data, the scale parameters Sd and Sx are well 'structurally' estimated, however the total variance attributed to z_d and z_x may be overestimated, as the country specific variance may be confounded with the productivity variance σ_θ^2.

Figure 11 shows a large heterogeneity in the sales among firms. Notably, there is a large heterogeneity in the relative export compared to domestic sales. In Panel (a), the two dash lines are associated with two constant export-intensity respectively at 80 percent and 20 percent. A large fraction of firms have an export-intensity either below 20 percent of higher than 80 percent. In addition, there is a strong correlation between domestic and export sales in the data (correlation=0.755). This is expected from a simple Melitz (2003) model, where the a firm's sales generated in each market is directly proportional to it productivity. The bold line fit the data, and will be discussed latter.

Figure 11: Correlation between domestic and export sales

(a) Data (b) Simulated



It is not simple to distinguish from the sales what is due to Θ or to z_d or z_x. However, when fitting the export-intensity distribution, we did not faced this problem as Θ does not impact a firm's export-intensity. Hence we can use the predicated $\alpha = 0.867$, to generate freshet demand shocks. First. we generate these two demand shocks by fixing the α, which allow us to generate the scale parameters, Sd and Sx, estimated for these distributions. (Note: Here I am not sure that the Sd and Sx are well estimated (as they are not estimated in a structural way), as we don't consider the existence of the Θ which affect the sales).

Second, we use these two demand shocks that we both multiply by a Θ which we assume follow a log-normal distribution $\mathcal{N}(0, \sigma_\theta^2)$ where σ_θ^2 will be 'calibrated' such as to obtain a similar correlation to the one observed in the data. With $\sigma_\theta^2 = 5.5$, we obtain a correlation of 0.755 between the log of the domestic sales and the log of the export, identical to the one observed in the data.

(Note that from Panel (a) and (b), the bold line indicates the correlation fit, which is less steep than the two dash-lines. This indicates that larger sales in the domestic market, implies a smaller export-intensity which is at odd with the common view. e.g. Arkolakis. Second, the slope of the fitting line is also very similar in the case of the simulated data. No idea why!)

9 Export-intensity distribution and trade liberalization

We now investigate the relationship between country size, external market potential and existence of twin peaks in the export intensity distribution in a general equilibrium environment. We use the 'small economy' variant of the Melitz (2003) model proposed by Demidova and Rodríguez-Clare (2009, 2013) as our benchmark to which we introduce Fréchet-distributed, firm-destination-specific demand shifters. Two key assumptions distinguish the small economy case from a standard two-country set up with heterogenous firms: (i) Home firms' decisions do not affect the national income, price index and mass of firms in the rest of the world, and (ii) the foreign demand for Home goods exported at a price p can be expressed as $q = A_x p^{-\sigma}$, where A_x is a constant not affected by changes at Home. The latter assumption implies that just as in the model presented in Section 2, firm-level export demand shifters is distributed Fréchet with shape parameter α and scale parameter $s_x = A_x \tau^{1-\sigma}$, while the scale parameter driving domestic demand, $s_d = wLP_h^{\sigma-1}$, is now endogenous.

After briefly sketching the model, we illustrate how changes in the cost of exporting and importing affect the size of the twin peaks.

Preferences and Demand. Home is populated by L individuals who supply one unit labor inelastically. The representative consumer has CES preferences:

$$\mathcal{U} = \left(\int_{\omega \in \Omega_d} [z_d(\omega)]^{\frac{1}{\sigma}} [q_d(\omega)]^{\frac{\sigma-1}{\sigma}} d\omega + \int_{\omega \in \Omega_m} [z_m(\omega)]^{\frac{1}{\sigma}} [q_m(\omega)]^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}, \tag{21}$$

where $q_d(\omega)$ and $q_m(\omega)$ denote consumption of domestically-produced and imported varieties, Ω_d and Ω_m represent respectively the set of available domestic and imported varieties and $\sigma > 1$ is the elasticity of substitution. The terms $z_d(\omega)$ and $z_m(\omega)$ are variety-destination-specific demand shifters. Home's expenditure in domestic and imported varieties is given by:

$$r_d(\omega) = (wL)P^{\sigma-1}z_d(\omega)p_d(\omega)^{1-\sigma} \quad \text{and} \quad r_m(\omega) = (wL)P^{\sigma-1}z_m(\omega)p_m(\omega)^{1-\sigma}, \tag{22}$$

where,

$$P = \left[\int_{\omega\in\Omega_d} z_d(\omega)p_d(\omega)^{1-\sigma}d\omega + \int_{\omega\in\Omega_m} z_m(\omega)p_m(\omega)^{1-\sigma}d\omega\right]^{1/1-\sigma},$$

is Home's ideal price index.

Production. A measure M_e of monopolistically competitive firms pay a fixed cost wf_e (where w denotes the wage at Home) to enter the market and draw their idiosyncratic productivity φ from a Pareto distribution $G_\varphi(\varphi) = 1-\varphi^{-\beta}$. After observing its productivity, but before knowing the realizations of domestic and export demand shifters, a firm chooses whether to operate or not and what markets to serve (i.e. only produce for the domestic market, or sell both at home and abroad). Domestic production involves a fixed cost wf_d, while firms that choose to export need to pay an additional cost wf_x. As noted above, expenditure in Home variety ω from the rest of the world, $r_x(\omega)$ is given by:

$$r_x(\omega) = z_x(\omega)A_x p_x(\omega)^{1-\sigma}, \tag{23}$$

where the term A_x incorporates both the national income and the price index of the rest of the world. Decisions made by Home firms do not affect A_x.

We assume that Home firms draw z_d and z_x from the same Fréchet distribution with shape parameter $\alpha > 0$. Furthermore, we assume that $z_m(\omega) = 1$ for all $\omega \in \Omega_m$. This assumption is not unduly restrictive, since we only need to keep track of the aggregate volume of imports for the trade balance condition to be satisfied; assuming that import demand shifters are distributed Fréchet does not affect our results.

Firms choose their mode of production $k \in \{D, X, \varnothing\}$, where D denotes domestic production, X, exporting and $\varnothing$ indicates a firm that does not produce at all, by solving the following problem:

$$\max_{D,X,\varnothing} \Big\{ E[\pi_d] - wf_d, E[\pi_d] + E[\pi_x] - w(f_d + f_x), 0 \Big\}. \tag{24}$$

In solving the problem (24), we assume that demand shifters do not affect entry and exit or the extensive margin of exports. Furthermore, as di Giovanni and Levchenko (2012), we assume that the marginal firm is small enough so that it ignores the impact of its demand realizations on Home's aggregate expenditure and price index. This assumption allows us to characterize firms' choice of production mode by means of a set of non-stochastic productivity cutoffs, φ_k^* defined implicitly by $E[\pi_k(\varphi_k^*)] = 0$.

Expected profits in each market are,

$$E[\pi_d] = \kappa(wL)P^{\sigma-1}\left(\frac{\varphi}{w}\right)^{\sigma-1} E[z_d], \tag{25}$$

$$E[\pi_x] = \kappa A_x \left(\frac{\varphi}{\tau w}\right)^{\sigma-1} E[z_x], \tag{26}$$

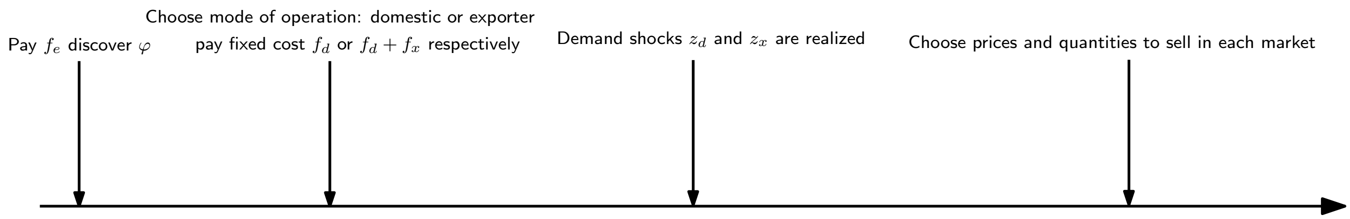
where $\kappa \equiv (\sigma-1)^{\sigma-1}\sigma^{-\sigma}$, and $\tau \geqslant 1$ is the iceberg transport cost involved in shipping goods from Home to the rest of the world. In deriving (25) and (26), we have made use of the fact that firms' optimal prices are $p_d = \frac{\sigma}{\sigma-1}\frac{w}{\varphi}$ and $p_x = \tau p_d$, and are therefore independent of the realization of demand shifters.[9]

Moreover, note that since variable profits are always positive in both markets, once a firm has incurred the fixed cost necessary to export, it always finds it optimal to sell some of its output domestically and abroad, regardless of the realization of demand shifters. Figure 12 illustrates the timeline of events faced by firms at Home.

Rest of the World. The measure M_e^* of monopolistically competitive firms in the rest

[9] Note that if $z \sim$ Fréchet(α), then $E[z] \to \infty$ when $\alpha < 1$. To deal with this issue, we solve the model by discretizing the distribution of demand shifters using a numerical quadrature rule (see Miranda and Fackler, 2004), which ensures that the demand shifters have finite moments. We make use of the quantile-based moments proposed by Moors (1988) to gauge the accuracy of our discretized approximation.

Figure 12: Timeline of Production



of the world is exogenous. We assume that that firms in the rest of the world draw their productivity from the same Pareto distribution, G_φ, as in Home. Exporters from the rest of the world pay the same fixed cost of exporting as Home firms but can potentially face a different iceberg transport cost of shipping goods to Home, which we denote τ^* — this implies that the measure of imported varieties available for consumption at Home, Ω_m, is an endogenous object in the model. Export supply from the rest of the world adjusts to changes in Home so that there is balanced trade between Home and the rest of the world.

Closing the Model. Choosing the mass of firms in the rest of the world, M_e^*, as the numeraire, equilibrium in the model is characterized by a vector of three endogenous variables, (M, P, w), the mass of active firms, the price index and wage prevailing at Home, such that expected profits exactly cover entry costs, the labor market clears, and there is balanced trade between Home and the rest of the world.

Demidova and Rodríguez-Clare (2013) show that equilibrium in the small economy version of the Melitz (2003) model can be easily represented using two curves, i.e. the upward-sloping competitiveness curve which is implicitly defined by the relationship between Home's wage w and the productivity cutoff to start exporting, φ_x^*, and the downward-sloping trade balance curve, which governs the trade flows equilibrium between Home and the rest of the world. As a result Demidova and Rodríguez-Clare (2009, 2013) show that $s_x/s_d = [\tau^{1-\sigma}A_x]/[(wL)P^{\sigma-1}]$ will increase following a decrease in import transport cost or in export transport cost.

Reduction in Export Costs. We first investigate a reduction in the iceberg cost of serving foreign markets, τ. This policy change increases the attractiveness of exporting for Home-based firms. Everything else equal, improved export prospects lower the productivity cutoff of exporting; the expansion of exporters increases labor demand and drives up wages, thereby inducing exit of low productivity firms. The price index is affected by two opposing forces: the increase in wages and the reduction in the mass of operating firms result in higher individual prices, whereas the increase in import competition necessary to have balanced trade, induces downward pressure on the price index. However, the former effect dominates and the price index increases, albeit by less than the wage, thus increasing welfare. Since both wages and the price index increase following the reduction in export costs, the effective domestic market size $s_d = (wL)P^{\sigma-1}$ increases.

Reduction in Import Costs. When the variable cost of importing goods from the rest of the world falls, there is no direct effect on the export profitability of Home firms and therefore the 'competitiveness' curve stays constant. This also implies that the effective export demand shifter does not change. As Home imports surge, however, Home needs to increase its exports to maintain balanced trade — this is achieved through a reduction of wages, which also drives down the price index.

We now explore how changes in export and import market access affect the size of the twin peaks observed in a country's export intensity distribution. From Proposition 2 that an increase in the relative size of the export market faced by Home firms increases the share of high-intensity exporters in detriment of low-intensity ones. Since the effective size of the export market remains exogenous under the small economy assumption, the question is how does a given policy change affect the endogenous domestic market size, and in turn, how does this translate in changes in the shape of export intensity distribution.

We define the 'height' of the left and right peak of the distribution as the share of exporters with export intensity below κ and above $1 - \kappa$ respectively; more formally, $\underline{\text{PEAK}} = H(\kappa)$

and $\overline{\text{PEAK}} = 1 - H(1-\kappa)$. Therefore, we can define the elasticity of each peak with respect to the relative effective market size s_x/s_d as follows:

$$\varepsilon_{\underline{\text{PEAK}},(s_x/s_d)} \equiv \frac{d \ln \underline{\text{PEAK}}}{d \ln(s_x/s_d)} = -\frac{\alpha[((1-\kappa)/\kappa)(s_x/s_d)]^\alpha}{1 + [((1-\kappa)/\kappa)(s_x/s_d)]^\alpha} < 0, \tag{27}$$

$$\varepsilon_{\overline{\text{PEAK}},(s_x/s_d)} \equiv \frac{d \ln \overline{\text{PEAK}}}{d \ln(s_x/s_d)} = \frac{\alpha}{1 + [(\kappa/(1-\kappa))(s_x/s_d)]^\alpha} > 0. \tag{28}$$

As Proposition 2 shows, an increase in the relative size of the export market increases the share of exporters in the left peak, while at the same time increasing the share of high-intensity exporters. The magnitude of the elasticity is decreasing in s_x/s_d, hence, everything else equal, we should expect a given policy change to have a smaller effect on the tails of the export intensity distribution in countries of small size.

Thus, following a unilateral import trade liberalization, the share of high-intensity exporters increases.

10 Export-intensity distribution and volatility

The implication of heavy-tailed sale distributions has numerous implications notably in order to understand the microeconomic origins of aggregate fluctuations (Gabaix, 2011; Acemoglu et al., 2015) and the role played by trade openness (di Giovanni et al., 2011; di Giovanni and Levchenko, 2012), particularly in small developing countries. In this section, we argue that the 'twin peaks' feature of export-intensity distribution, that heavy-tailed sale distribution generate, have some implication on its own.

To provide a simple example of the implication of different shape of the export intensity distribution, let us consider the revenue function described in equation (1) augmented with a time-varying i.i.d shocks ϵ_{it}.

$$r_{it} = \epsilon_{it} z_i A_i p_i^{1-\sigma}, \tag{29}$$

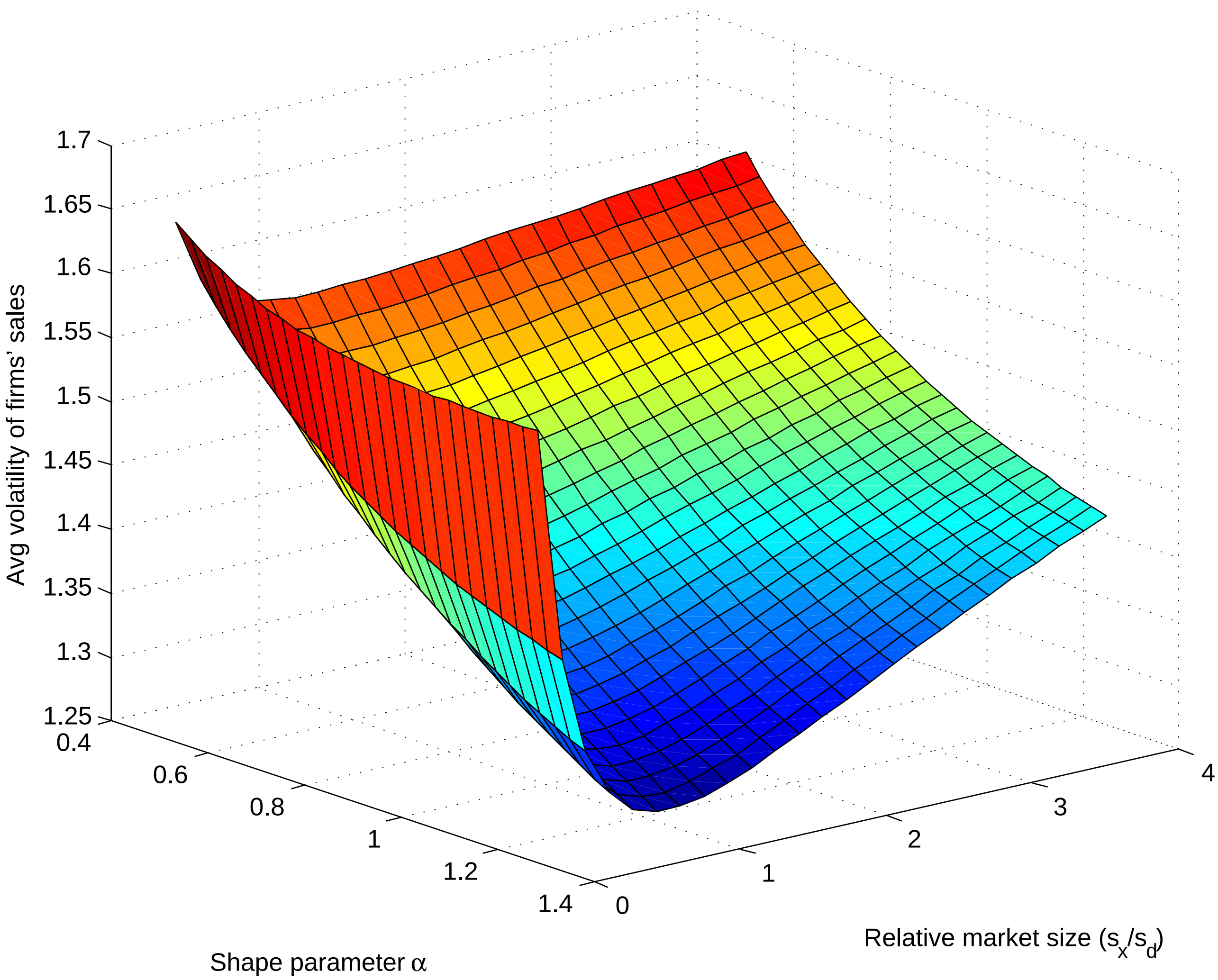
We assume that ϵ_{it} is drawn independently in each period from a time invariant country-specific distribution with mean μ_i and variance σ_i. For simplicity, there is no autocorrelation in the ϵ_{it}, and no correlation between shocks in the two countries. Assuming no inventories, a firm's action at time t does not influence its problem in other periods. The firm therefore maximizes its per-period profits. Assuming that μ_i, the mean of the time-varing shocks, is equal to one then the demand and foreign demand shifter parameters z_d and z_x previously defined then correspond to the long run demand parameters of a firm in a country. We can then defined $E(Z_d, Z_x) = Z_x/(Z_d + Z_d)$ as the long-term export-intensity of a firm.

We now turn to the predictions of the model for the volatility of sales at the firm level. [Need to include the calculus here].

$$\text{Firm Volatility} = \left(1 - E(z_d, z_x)\right)^2 \sigma_d^2 + \left(E(z_d, z_x)\right)^2 \sigma_x^2 \tag{30}$$

If $\sigma_d^2 = \sigma_x^2$, it is easy to see that the firm-level volatility will be minimized for an long-term export-intensity $E(z_d, z_x) = 1/2$. This firm is less volatile due to the diversification effect across markets. At the other extreme, a firm exporting only a small fraction of its output or almost all its output abroad would be more volatile. One can see how a 'twin peaks' export-intensity distribution would be associate with more volatility on aggregate. We can then calculate the aggregate volatility of a country. Similarly to di Giovanni & Levchenko (2012), we find that the average firm-level volatility is higher in smaller countries. In addition, Graph 13 show that the shape parameter α impacts the aggregate volatility at the country level.

Figure 13: Why does the export intensity distribution matter?



11 Conclusions

In this paper we have provided solid evidence demonstrating the remarkable extent of heterogeneity in the distribution of export intensity, the share of total sales accounted for by exports, across a large range of countries. We showed that a distribution characterized by a majority of exporters selling most of their output domestically together with only a few high-intensity exporters is not as predominant as previously thought. We show that in several countries the majority of exporters sell almost all their output abroad and also that there are countries in which the export intensity is characterized by twin peaks — a large number of exporters on both the left and right tails of the distribution.

We establish that the importance of low and high-intensity exporters is strongly driven by the external market size relative to the domestic market. The share of low-intensity exporters increases with the size of the domestic market; conversely, high-intensity exporters tend to be more prevalent in small countries or in locations that are geographically close to large foreign markets.

From a theoretical perspective, we show that when we augment a standard model of trade with constant demand elasticity with firm-destination-specific demand shifters which are distributed Fréchet with the same shape parameter across the domestic and export markets, we can characterize the distribution of export intensity for exporters in a tractable and parsimonious closed form, which can easily reproduce the potential bi-modality observed in the data. Furthermore, we show that the structural parameters governing demand shifters are very easily estimated using readily available public data on manufacturing firm-level export intensity. Our parametrization explains 89% of the cross-country variation in the export distribution across a wide range of countries.

Finally, we embed our simple partial equilibrium model of exporting into a small economy version of the workhorse model of trade with heterogeneous firms in order to investigate how unilateral changes in export and import costs result in changes in a country's export intensity distribution. We show that both forms of trade liberalization lead to reallocation of exporters from the left to the right tail of the distribution, with an import liberalization producing a slightly larger quantitative effect on the size of the right tail of the distribution.

References

Acemoglu, D., A. Ozdaglar, and A. Tahbaz-Salehi (2015): "Microeconomic Origins of Macroeconomic Tail Risks," Working Paper 20865, National Bureau of Economic Research.

Arkolakis, C. (2010): "Market Penetration Costs and the New Consumers Margin in International Trade," *Journal of Political Economy*, 118, 1151 – 1199.

Bernard, A. B., J. Eaton, J. B. Jensen, and S. Kortum (2003): "Plants and Productivity in International Trade," *American Economic Review*, 93, 1268–1290.

Bernard, A. B., J. B. Jensen, S. J. Redding, and P. K. Schott (2007): "Firms in International Trade," *Journal of Economic Perspectives*, 21, 105–130.

Brooks, E. (2006): "Why Don't Firms Export More? Product Quality and Colombian Plants," *Journal of Development Economics*, 80, 160–178.

Defever, F. and A. Riaño (2014): "Protectionism Through Exporting: Subsidies with Export Share Requirements in China," Manuscript, University of Nottingham.

Demidova, S. and A. Rodríguez-Clare (2009): "Trade policy under firm-level heterogeneity in a small economy," *Journal of International Economics*, 78, 100–112.

——— (2013): "The simple analytics of the Melitz model in a small economy," *Journal of International Economics*, 90, 266–272.

di Giovanni, J. and A. A. Levchenko (2012): "Country Size, International Trade, and Aggregate Fluctuations in Granular Economies," *Journal of Political Economy*, 120, 1083 – 1132.

di Giovanni, J., A. A. Levchenko, and R. Rancière (2011): "Power laws in firm size and openness to trade: Measurement and implications," *Journal of International Economics*, 85, 42–52.

Eaton, J., S. Kortum, and F. Kramarz (2011): "An Anatomy of International Trade: Evidence From French Firms," *Econometrica*, 79, 1453–1498.

Gabaix, X. (2011): "The Granular Origins of Aggregate Fluctuations," *Econometrica*, 79, 733–772.

Hartigan, J. A. and P. M. Hartigan (1985): "The Dip Test of Unimodality," *The Annals of Statistics*, 13, 70–84.

Henderson, D. J., C. F. Parmeter, and R. R. Russell (2008): "Modes, weighted modes, and calibrated modes: evidence of clustering using modality tests," *Journal of Applied Econometrics*, 23, 607–638.

Lu, D. (2010): "Exceptional Exporter Performance? Evidence from Chinese Manufacturing Firms," Manuscript, University of Chicago.

Melitz, M. J. (2003): "The Impact of Trade on Intra-Industry Reallocations and Aggregate Productivity," *Econometrica*, 71, 1695–1725.

Melitz, M. J. and S. J. Redding (2014): "Heterogeneous Firms and Trade," in *Handbook of International Economics*, ed. by K. R. Elhanan Helpman and G. Gopinath, Elsevier, vol. 4 of *Handbook of International Economics*, 1 – 54.

Miranda, M. J. and P. L. Fackler (2004): *Applied Computational Economics and Finance*, The MIT Press.

Moors, J. J. A. (1988): "A Quantile Alternative to Kurtosis," *Journal of the Royal Statistical Society. Series D (The Statistician)*, 37, 25–32.

Nadarajah, S. and S. Kotz (2006): "On the Ratio of Fréchet Random Variables," *Quantity & Quality*, 40, 861–868.

Twin Peaks
Appendix: Theory

A Comparison with other models

Krugman (1980)
Helpman and Krugman (1985)
Melitz (2003)
Eaton and Kortum (2002)
Arkolakis
Eaton, Kortum and Kramartz (2011)
Melitz and Ottaviano (2007)

To be citet
THE DYNAMICS OF COMPARATIVE ADVANTAGE Gordon H. Hanson Nelson Lind Marc-Andreas Muendler Working Paper 21753

B Proofs

Proof of Proposition 1. As noted above, $Z_i \sim \text{Fréchet}(\alpha, s_i)$ for $i \in \{d, x\}$, where $\alpha > 0$ is the shape parameter and $s_i > 0$ is the scale parameter.

$$\begin{aligned}
H(e) \equiv \text{Prob}[E \leqslant e] &= \text{Prob}\left[\frac{Z_x}{Z_d + Z_x} \leqslant e\right], \\
&= \text{Prob}\left[\frac{1-e}{e} \leqslant \frac{Z_d}{Z_x}\right], \\
&= 1 - \text{Prob}\left[Z \leqslant \frac{1-e}{e}\right], \\
&= 1 - F\left(\frac{1-e}{e}\right), \\
&= 1 - \frac{\left[\left(\frac{s_x}{s_d}\right)\frac{1-e}{e}\right]^{\alpha}}{1 + \left[\left(\frac{s_x}{s_d}\right)\frac{1-e}{e}\right]^{\alpha}}, \\
&= \frac{1}{1 + \left[\left(\frac{s_x}{s_d}\right)\frac{1-e}{e}\right]^{\alpha}}.
\end{aligned}$$

Where we have used the fact that $Z \equiv Z_d/Z_x$ and the definition of $F(z)$ given in (6). Taking the first derivative of $H(e)$ with respect to e yields (8).

Proof of Proposition 2. Recall that the pdf of export intensity is:

$$h(e) = \frac{\alpha(s_d s_x)^\alpha e^{\alpha-1}(1-e)^{\alpha-1}}{\left[(s_d e)^\alpha + (s_x(1-e))^\alpha\right]^2}.$$

The distribution of export intensity has support $[0,1]$. Since $\alpha > 0$, it follows that the denominator of $h(e)$ becomes $s_x^{2\alpha}$ when $e \to 0$ and $\lim_{e\to 0} e^{\alpha-1}(1-e)^{\alpha-1} \to +\infty$. Likewise, when $e \to 1$, the denominator of $h(e)$ becomes $s_d^{2\alpha}$, and $\lim_{e\to 1} e^{\alpha-1}(1-e)^{\alpha-1} \to +\infty$. Since $h(e)$ is strictly convex when $\alpha \in (0,1)$, it follows that $\text{Mode}(h(e)) = \sup_{e\in[0,1]} h(e) \in \{0,1\}$.

To prove that the major mode occurs at 0 when $s_d > s_x$, we need to show that $H(\kappa) > 1 - H(\kappa)$, for an arbitrary $\kappa \in (0,1)$. In other words, that there are more exporters with export intensity between 0 and κ than there are exporters with intensity between κ and 1.

$$\begin{aligned}
\frac{1}{1+\left(\frac{s_x}{s_d}\frac{1-\kappa}{\kappa}\right)^\alpha} &> 1 - \frac{1}{1+\left(\frac{s_x}{s_d}\frac{1-(1-\kappa)}{1-\kappa}\right)^\alpha} \\
1+\left(\frac{s_x}{s_d}\frac{\kappa}{1-\kappa}\right)^\alpha &> \left[1+\left(\frac{s_x}{s_d}\frac{1-\kappa}{\kappa}\right)^\alpha\right]\left(\frac{s_x}{s_d}\frac{\kappa}{1-\kappa}\right)^\alpha \\
s_d^{2\alpha}(1-\kappa)^\alpha + (s_d s_x)^\alpha \kappa^\alpha &> s_x^{2\alpha}(1-\kappa)^\alpha + (s_d s_x)^\alpha\kappa^\alpha \\
\left(\frac{s_d}{s_x}\right)^{2\alpha} &> 1.
\end{aligned}$$

When $s_x > s_d$ the major mode occurs at 1, and when $s_d = s_x$, we have that $H(\kappa) = H(1-\kappa)$, which means that the two modes are symmetric.

C Proofs

First, let us provide the first and second derivative of the pdf.

$$m = \frac{Sx}{Sd}\frac{1-e}{e} \tag{31}$$

$$(pdf)' = \frac{\alpha(\alpha(m)^\alpha(2e-\alpha-1) + (m)^{2\alpha}(2e+\alpha-1))}{e^2(-1+e)^2(1+(m)^\alpha)^3} \tag{32}$$

We can find the root value, using the envelop theorem.

$$(pdf)'' = \tag{33}$$

Second, we show that the second derivative of the pdf is always positive for $\alpha < 1$, such that we can be sure that the pdf is strictly convex.

Third, we provide the unique root value for the first derivative when $\alpha > 1$, and show that the second derivative is negative for this value. This demonstrate that the distribution is unimodal, with the root value being the mode of the distribution.

show the value of the mode

D Properties of Demand Shifters and Export Intensity Distributions

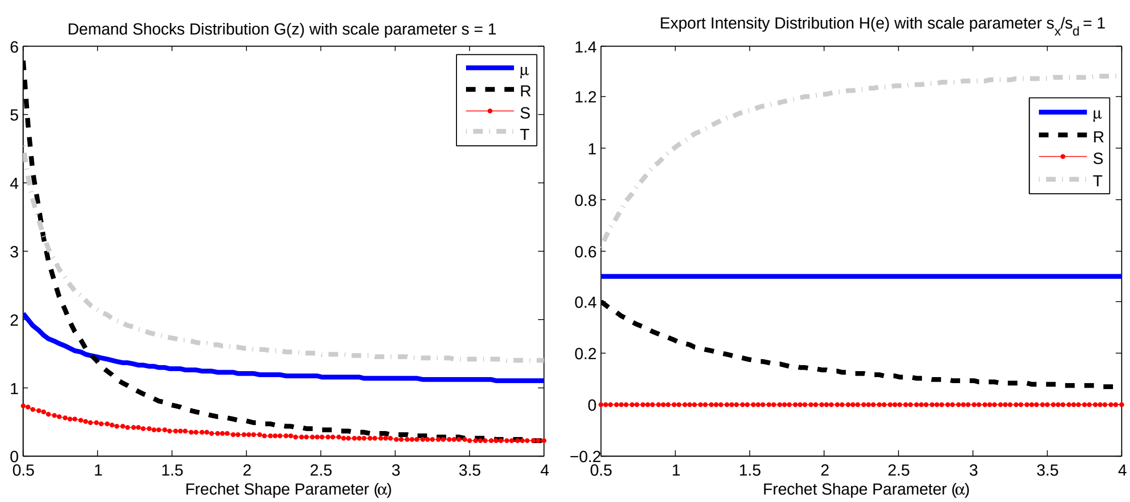
Moors (1988) provides quantile-based alternatives for the first four moments of a given probability distribution, i.e. its mean, variance, skewness and kurtosis. The corresponding alternatives are (i) the median (μ), (ii) the half inter-quantile range (R), (iii) the quantile coefficient of skewness (S) and (iv) the quantile coefficient of kurtosis (T). These are defined precisely below:

$$\begin{aligned}
\mu &= E_4, \\
R &= \frac{E_6 - E_2}{2}, \\
S &= \frac{E_6 - 2E_4 + E_2}{E_6 - E_2}, \\
T &= \frac{(E_7 - E_5) + (E_3 - E_1)}{E_6 - E_2},
\end{aligned}$$

where E_i denotes the i-th octile of a continuous random variable X with cdf F, which is implicitly defined by $F(E_i) = i/8, \quad i = 1, \ldots, 7$.

These quantile-based measures of summary statistics have three key desirable properties: (i) they exist even for distributions for which no moments exist, such as the Fréchet distribution with shape parameter $\alpha < 1$, (ii) they are not influenced by the tails of the distribution, and (iii) their calculation is quite simple, provided that the quantile function (i.e. the inverse of the cdf) is well defined. Figure D.1 shows how the summary statistics of the distribution of demand shifters and export intensity vary as a function of the shape parameter α of the Fréchet-distributed demand shifters.

Figure D.1: Quantile-based Summary Statistics for Demand Shifters and Export Intensity Distributions



E Simulated Export Intensity Distributions

In this section we provide Monte Carlo simulations based on different primitive distributions for the demand shifters that also generate twin peaks in the export intensity distribution. In all cases, we simulate a sample of a 1,000 firms 500 times. We also assume that firms draw both their destination-specific demand shifters from the same distribution.

Here we present five examples of export intensity distributions featuring twin peaks, which are depicted in Figure E.1. Demand shifters are distributed:

(i) Weibull, with shape and scale parameters, $a = b = 0.25$,

(ii) Log-Normal, with underlying mean $\mu = -0.5\sigma^2$ and variance $\sigma^2 = 4$,

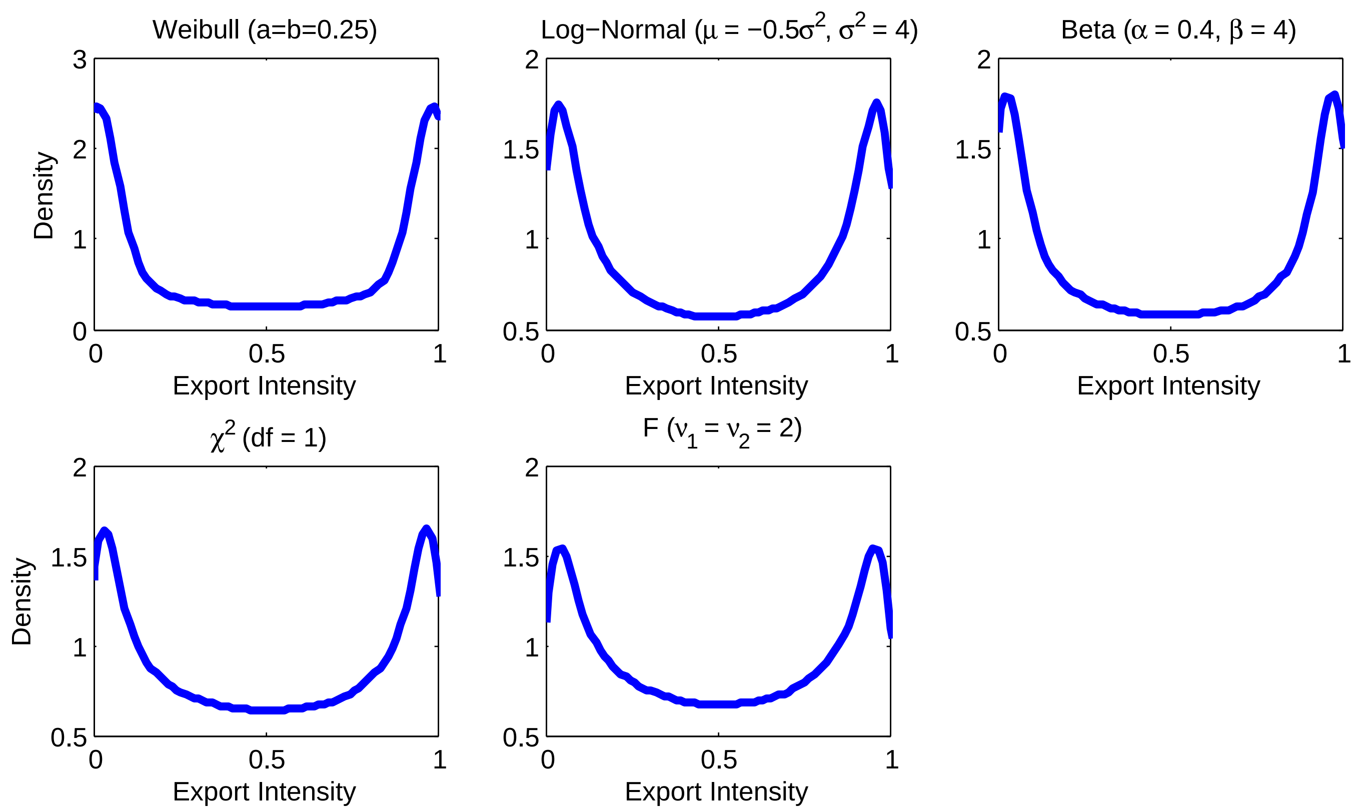
(iii) Beta, with shape parameters, $\alpha = 0.4$ and $\beta = 4$,

(iv) Chi-squared, with 1 degree of freedom,

(v) F, with 2 degrees of freedom both in the numerator and denominator.

All the simulated export intensity distributions reject the dip test of unimodality of Hartigan and Hartigan (1985) at the 1% significance level.

Figure E.1: Examples of Export Intensity Distributions given Different Underlying Demand Shifter Distributions



Twin Peaks
Appendix: Data

F World Bank Enterprise Survey

List of countries: Our sample consists of the following 71 countries drawn from the World Bank Enterprise Surveys for the years 2000-2014: Albania, Argentina, Armenia, Bangladesh, Belarus, Bolivia, Bosnia and Herzegovina, Brazil, Bulgaria, Chile, China, Colombia, Costa Rica, Croatia, Czech Republic, Ecuador, Egypt, El Salvador, Estonia, FYR Macedonia, Germany, Ghana, Guatemala, Honduras, Hungary, India, Indonesia, Ireland, Jordan, Kazakhstan, Kenya, Korea Republic, Kyrgyz Republic, Latvia, Lebanon, Lithuania, Madagascar, Malaysia, Mauritius, Mexico, Moldova, Morocco, Namibia, Nicaragua, Nigeria, Pakistan, Panama, Paraguay, Peru, Philippines, Poland, Romania, Russia, Senegal, Serbia, Slovak Republic, Slovenia, South Africa, Sri Lanka, Sweden, Syrian Arab Republic, Tanzania, Thailand, Tunisia, Turkey, Uganda, Ukraine, Uruguay, Uzbekistan, Vietnam and Zambia.

G World Bank Enterprise Survey

Comparison with other manufacturing surveys: Hongary
China
Ireland
Germany

H Data collection on Export-Share Requirements

A key output delivered by our paper is the first comprehensive cross-country dataset on the use of subsidies with Export Share Requirements (ESR) and their characteristics for the period 2000-2014. We collect information on 4 types of subsidies with ESR, namely policy measures that are: (T1) only available to multinational firms; (T2) in place in demarcated geographic areas such as FTZ; (T3) provided to firms engaged in processing trade not included in T2 and (T4) available to all firms irrespective of location and ownership status. Three criteria need to be satisfied for a policy to be classified as a subsidy with ESR: (C1) it needs to provide a fiscal incentive; (C2) should be available to manufacturing firms and (C3) the incentive has to be conditioned on an explicitly stated ESR.

Data collection follows a systematic procedure consisting of 4 steps. Each step (with the exception of S4) is carried out for each of the 71 developing countries of our sample and for each year in our period of analysis. (S1) We collect information from the Investment Climate Statements (ICS) published by the US Department of State. These reports are produced annually by US embassies and consulates, and provide country-specific information and assessments on laws and measures affecting business decision-making in foreign markets; they are publicly available for approximately 200 countries for the period 2000-2014. Information on subsidies with ESR is available in the sections Performance Requirements and Incentives and Foreign Trade Zones/Free Trade Zones in each statement. (S2) Information from S1 will be complemented with Trade Policy Review reports from the WTO; these feature sections on Export Procedures and Requirements and Export Support and Promotion, which can also indicate if a country is utilising measures satisfying the criteria stated above. Communications of the WTOs Committee on Subsidies and Countervailing Measures, which are also publicly available, will be searched for complaints related to subsidies with ESR. (S3) The data collected in S1 and S2 will be cross-checked and complemented by examining countries trade and investment laws and regulations, which are available in online legal information websites such as Lexis-Nexis and Bloomberg Law. (S4) Finally, countries with missing information but for which there is strong suspicion of use of subsidies with ESR are classified in a separate category (e.g. if criteria C1-C2 are satisfied but not C3).

The cases of China and Morocco are presented to illustrate the data collection procedure: a) China: S1: ICS report fiscal advantages available to multinational firms and in FTZ. ESR are mentioned, but no explicit threshold values are stated (C3 is not satisfied). S2: subsidy notifications mandated by Chinas Transitional Review Mechanism reveal the existence of a 70% ESR associated with tax concessions available to foreign-owned firms in 2004; Chinas 2010 TPR shows that these were phased out in 2008 with a transition period of 5 years. S3: Several fiscal incentives available to firms exporting more than 70% and also to firms exporting all their output are identified in Chinese laws and regulations.

b) Morocco: S1: ICS does not report any subsidies with ESR. S2: The 2003 TPR for Morocco reports a 100% ESR in place in the Tangier Free Trade Zone. Communications at the Committee on Subsidies and Countervailing Measures show that it remains in place in 2004. S3: The law and regulations governing FTZ in Morocco do not indicate the use of any ESR subsidies. Fiscal incentives are available at Tangier FTZ but are not explicitly associated with an export requirement (C3 is not satisfied). This would be a case in which steps S4 would be employed and the country classified in a separate category.

List of countries with ESR: Albania, Argentina, Armenia, Bolivia, Bulgaria, Chile, Colombia, Croatia, Czech Republic, Ecuador, Estonia, Macedonia, Germany, Hungary, Jordan, Kazakhstan, Kenya, Kyrgyz Republic, Latvia, Lebanon, Lithuania, Mauritius, Mexico, Panama, Poland, Russia, Serbia, Slovak Republic, Slovenia, South Africa, South Korea, Sweden, Syria, Ukraine, Uruguay.