Firm Dynamics with Subjective Beliefs
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Jose Maria Barrero*

February 12, 2017

Abstract

A key question regarding firms' performance is how their perceptions and beliefs about the future impact their behavior. I use novel data on US firms' subjective beliefs to study whether they exhibit overoptimism (i.e. their expectations exceed rational forecasts) and overconfidence (i.e. they underestimate the variance across potential outcomes) in their perceptions of future sales growth. I document that US firms are overoptimistic, overestimating future sales growth by 2 to 5 percentage points, and also overconfident, underestimating the uncertainty about future sales by about 70 percent. I then study the quantitative implications of distorted subjective beliefs in a model of firm dynamics with investment subject to rich adjustment costs, and calibrate the model to match the degree of overconfidence and overoptimism observed in the data. Overoptimistic firms in the model expect better conditions in the future, leading them to have lower exit rates and lower profitability on average; by contrast, overconfident firms perceive less uncertainty about the future, leading them to exit quickly in the face of bad shocks and to grow disproportionately large in the face of good ones. Preliminary quantitative results also suggest that overconfidence rather than overoptimism is the more significant of the two distortions in terms of its effects on firm dynamics.

*Stanford University, barreroj@stanford.edu. I thank Nick Bloom, Monika Piazzesi, and Chis Tonetti for their invaluable guidance and advice with this project. I also thank Steve Davis and various seminar participants at Stanford's Macro Lunch Workshop and in Nick Bloom's reading group for helpful comments. I'm also grateful with Michael Bryan, Brent Meyer, and Nicholas Parker at the Federal Reserve Bank of Atlanta for their help with the data from the Decision Maker Survey. Financial support from the Ewing Marion Kauffman Foundation via the Kauffman Dissertation Fellowship is gratefully acknowledged. All errors are my own.
This research was partially funded by the Ewing Marion Kauffman Foundation. The contents of this paper are solely the responsibility of Jose Maria Barrero.

1 Introduction

Economists have long recognized that firms make dynamic decisions in the face of risk and uncertainty about the future. The standard assumption in both empirical and theoretical work has supposed that firms, and in particular their managers, forecast future economic conditions correctly; that they have rational expectations and decide to invest, hire, conduct R&D, and even start up or liquidate their firms based on these correctly specified beliefs. Much of the literature has, therefore, refrained from investigating individual firms' and mangers' beliefs about the future and the potential impact of distortions to those beliefs. After all, rational expectations mean that firms' beliefs are consistent with their environment and with the dynamics of observable data.

Some empirical work in economics and finance has recently begun to consider the long-established view in psychology and behavioral economics that individuals' beliefs are often biased, in particular when thinking about chief cxecutives making high-stakes decisions for their firms. Malmendier and Tate (2008, 2005) and Malmendier et al. (2007), for instance, identify overconfident and overoptimistic CEOs as those who maintain significant exposure to their own firm's risk in their personal portfolio, documenting that these managers make investment and financing decisions, as well as mergers and acquisitions differently from other firms. Similarly, Ben-David et al. (2013) use a decade-long survey of CFO beliefs about future S&P 500 returns to conclude that top managers grossly underestimate stock market volatility. They also find that more biased CFOs choose bolder investment strategies and are more inclined to use debt over equity financing. With the same survey data Gennaioli et al. (2016) find that managers' reported expectations about future earnings explain much of the variation in planned and actual investment, and are more informative than typical proxies for investment oppotunities like Tobin's q.

Despite these recent advances, it remains the case that economists have not thoroughly investigated the nature of firms' subjective beliefs and their impact on firm decisions and performance. To a large extent, this knowledge gap has been due to lack of data on firm beliefs about their own future prospects as well as subsequent outcomes. Bachmann and Elstner (2015), for example, make a bold attempt at studying optimism among German firms using a long panel dataset. But they lack straightforward quantitative evidence and instead rely heavily on assumptions to combine qualitative data on firms' expectations with limited quantitative data on their production. The lack of strong empirical evidence also also hindered theoretical and quantitative work on subjective beliefs by restricting researchers' ability to discipline and test their models against data. Fuster et al. (2010), for example, posit and analyze a model of firms with biased subjective expectations, but fail to justify the way they model these biased expectations, much less to quantify them due to lack of data.

In this paper I aim to provide solid empirical evidence about the the nature of US firms' subjective beliefs, in particular asking whether firms seem to be overoptimistic (overestimating the mean) and/or overconfident (underestimating the variance) about future sales growth, or whether they appear to have accurate beliefs about the future[1]. I use a novel "Decision Maker Survey" run out of the Federal Reserve Bank of Atlanta in consultation with researchers at Stanford University and the University of Chicago, in which respondents periodically provide subjective probability distributions about future firm-specific outcomes like sales growth, employment, and capital expenditures. The dataset is longitudinal, so it also contains information about past and future realized performance of its respondent firms. On several dimensions this is the ideal dataset for studying firm-specific subjective beliefs about their own outcomes, although it has some limitations (such as being a short panel). The fact that it contains quantitative evidence about beliefs and elicits probability distributions (rather than point estimates) from its respondents allow me to avoid making many of the heroic assumptions made by Bachmann and Elstner (2015) and also lets me study both overoptimism and overconfidence.

My empirical analysis reveals that US firms on average exhibit overoptimism and overconfidence in their beliefs about future sales growth. Namely, respondents' sales growth expectations typically exceed realizations by 3 to 10 percentage points, and are 2 to 8 percentage points larger than the average growth rate across firms of similar size or within their broad sector, both of which are evidence of overoptimism. In turn, firms' subjective uncertainty (perceived standard deviation) over future sales growth grossly underestimates benchmark measures of the objective sales growth dispersion, by up to 70 or 80 percent, meaning that they are severely overconfident.

Armed with these empirical estimates of overoptimism and overconfidence among firms in the US, I develop a model of firm-level investment with free entry and exit in which a fraction of firms have distorted subjective beliefs about future fundamentals. The model features a rich specification of convex and non-convex capital adjustment costs, investment dynamics are consistent with the empirical evidence for lumpy investment behavior[2]; additionally rich

[1] I use separate terms to refer to distortions about the level of expectations over future outcomes (overoptimism), and distortions to perceived uncertainty about the future (overconfidence). I believe this terminology is broadly consistent with previous practice in behavioral finance, and with the notions that I intend to convey, although different terms have been used in the literature. Malmendier and Tate (2005, 2008) and Malmendier et al. (2007) use "overconfidence" to refer to behavior that is consistent with a combination of overoptimism and overconfidence as I define them. Ben-David et al. (2013) similary term "overconfidence" to include both kinds of distortions, using "miscalibration" or "overprecision" to refer to my notion of overconfidence, and the "above-average effect" to refer to what I overoptimism. Bachmann and Elstner (2015) focus on "optimism" in a manner that is consistent with me.

[2] See, for example, Cooper and Haltiwanger (2006), Khan and Thomas (2008), and Winberry (2015)

adjustment costs imply that firm decisions crucially depend on their perceived uncertainty about the future, as well as their expectations. This incorporation of rich adjustment costs contrasts with previous attempts at embedding subjective beliefs in structural models, with Bachmann and Elstner (2015) considering frictionless input adjustments and Fuster et al. (2010) using purely convex (i.e. quadratic) adjustment costs.

I calibrate the model to quantitatively match the observed degree of overoptimism and overconfidence in the Decision Maker Survey as well as standard moments in firm and investment behavior. To investigate the impact of each of the two beliefs distortions, I consider specifications that focus on each distortion at a time, but also study a full specification with firms that are simultaneously overoptimistic and overconfident. These exercises reveal that overoptimistic firms in the model exit at lower rates, are on average less productive and profitable, and smaller than unbiased firms. Intuitively, these dynamics reflect the fact that overoptimistic firms expect tomorrow's business conditions to improve upon today's, but those improvements never arrive and therefore they remain small and unprofitable. Overconfident firms, by contrast, are more certain about the future than an unbiased firm would be, so they perceive the real option of waiting before investing, disinvesting, or liquidating the firm to be less valuable. As a result, these firms exit with higher probability when hit by adverse shocks, and by contrast grow disproportionately large in the presence of positive shocks. Their reduced uncertainty leads them to be less cautions as they adjust their capital stock in reaction to shocks. In the full specification where biased firms are simultaneously overconfident and overoptimistic, it appears that overconfidence is the more salient distortion. Biased firms in this specification behave almost like the biased firms the specification with only overconfidence, both from a quantitative and qualitative standpoint.

Remarkably, the model delivers predictions that are supported by the empirical evidence in the Decision Maker Survey, but not targeted in the calibration. In the data, the degree of overconfidence increases monotonically with firm size, which is also true in the model because overconfident firms exit quickly in the face of adverse shocks and invest aggresively in response to positive shocks. On the other hand, there is no robust evidence of a statistical relationship between firm size and the degree of overoptimism in the Decision Maker Survey, consistent to some degree with the model's prediction that overoptimistic firms are prevalent among the smallest firms (unprofitable firms refusing to exit) but also among the largest (productive firms that overinvest in expectation of even better conditions tomorrow).

Overoptimism and overconfidence have significant implications for shareholders. Conditioning on a firm's current business conditions (productivity) and existing capital, its shareholders receive the greatest net present value from the firm's cash flows under a manager with rational expectations. But the distribution of firms across capital-holdings and business conditions is endogenous to the presence and severity of overconfidence and overoptimism. This is because distorted firms follow different investment policies and also make entry and exit decisions on their beliefs about future fundamentals. Thus, I quantitatively show that an economy calibrated to exhibit overconfidence, or both overconfidence and overoptimism, has a corporate sector that is about 50 percent more valuable than an economy with only unbiased firms. This counterintuitive result is a consequence of unproductive overconfident firms exiting at high rates, and productive ones growing disproportionately large and increasing the value of the firm. However, average firm value in an economy with overoptimistic firms would be 7 percent lower than in an economy with only biased firms.

It is still true that shareholders would be made better off in all specifications of the model if the managers running biased firms were replaced by rational ones. These potential gains range from 2 percent of average firm value in the specification with only overoptimistic firms, to 8 and 10 percent in economies with overconfidence, and overconfidence and overoptimism respectively. Furthermore, I compute the ex-ante value of a potential start-up conditional on its type (unbiased or biased), but before knowing the initial state of its business conditions or revenue productivity. In all specifications, an unbiased firm delivers a greater net present flow of dividends to its shareholders than a firm with distorted beliefs. Moreover, start-ups that are known to be overoptimistic, or overoptimistic and overconfident, but whose initial productivity is unknown deliver negative net present value to their shareholders.

My results in this paper shed new perspectives about firm dynamics in the real world. My insights about the prevalence and impact of overoptimism, for example, are reminiscent of the struggles of Theranos, Inc., a biotechnology firm from Silicon Valley that has struggled to roll-out its products despite having promising prospects. On the other hand, it is easy to think of Steve Jobs as a talented and lucky, although overconfident, individual responsible for building Apple into a corporate behemoth, much like the overconfident firms in my model.

Ultimately, this paper is an initial but serious attempt at understanding firm dynamics from an empirical and quantitative standpoint in the presence of distorted beliefs, leaving room for much future work on the subject. Many of the model's predictions remain to be tested with richer data, for example the Census Bureau's Longitudinal Research Database. The paper also abstracts completely from financial frictions, which may well interact with overconfidence and overoptimism and help explain empirical facts about how businesses raise funds via equity and debt. An additional unanswered question concerns why businesses end up hiring overoptimistic and overconfident managers, given how potentially detrimental they can be to shareholder value.

1.1 Related Literature

This paper contributes to the literature in behavioral corporate finance that has sought to identify business executives' impact on their organizations, especially when those executives seem to exhibit overconfidence and overoptimism. These include seminal contributions by Bertrand and Schoar (2003) on the impact of CEOs on firm performance, and above-mentioned work by Malmendier and Tate (2005); Malmendier et al. (2007); Malmendier and Tate (2008) and Ben-David et al. (2013) on overconfidence and overoptimism in organizations and their impact on corporate actions. However, I believe this is the first paper to use rich quantitative panel data on firms' beliefs about their own real outcomes to measure the degree of overconfidence and overoptimism in the corporate sector. After all Malmendier and Tate (2005); Malmendier et al. (2007); Malmendier and Tate (2008) rely on CEOs' personal portfolio decisions, and Ben-David et al. (2013) measure overconfidence using CFOs' beliefs about stock market returns. Bachmann and Elstner (2015) undertake an empirical exercise that is very similar in spirit to mine, but they only have qualitative data on expectations and also lack hard data about sales realizations, which makes them rely on strong assumptions to even measure optimism.

Some previous work has considered the economic impact of subjective beliefs (and, particulary, belief distortions) in the context of a model. Jurado (2016) notably considers the impact of advance information and belief distortions in the context of a full-scale DSGE framework, uncovering that distorted beliefs have significant explanatory power over fluctuations in consumption and stock prices. Fuster et al. (2010), like me, focus on the impact of beliefs in a model with firm making investment decisions subject to some adjustment costs. However, they focus on firms who are mistaken about the degree of mean-reversion in fundamentals, leading them, for instance, to be overoptimistic in good times and overly pessimistic in bad times. Furthermore, Fuster et al. (2010) do not use empirical evidence to discipline their quantitative evidence. Bachmann and Elstner (2015) calibrate a model with firm heterogeneity in which some firms are optimistic and pessimistic and match the extent of these biases to their empirical estimates, as I do, but they abstract from all adjustment frictions. To the best of my knowledge, this paper is the first to embed empirical estimates of both overconfidence and overoptimism in a firm dynamics model with a rich set of convex and non-convex adjustment costs, so it is simultaneously faithful to well-established empirical evidence on firm- and plant-level dynamics and to my estimates of subjective belief distortions.

There is an emerging literature in macroeconomics attempting to consider behavioral, or seemingly behavioral distortions and their impact on aggregate dynamics. Gabaix (2014, 2016), for example, microfounds the origin of behavioral distortions via a theory of "sparse maximization", which is capable of generating agents with mistaken beliefs about the dynamics of stochastic processes they are subject to. Terry (2016) considers one particular distortion, whereby firm managers seem to have short-termist incentives compelling them to meet quarterly earnings forcasts and distorting innovative activity and long-run growth. My paper contributes to this literature by adding empirical and quantitative evidence about overconfidence and overoptimism on firm dynamics in particular.

More broadly, my paper relates to work on firm dynamics and investment under uncertainty and adjustment costs. This includes contributions by Bernanke (1983), Abel and Eberly (1997), Pindyck (1988), as well as Cooper and Haltiwanger (2006) and Bloom (2009) on adjustment costs and, in particular, irreversible investments. Other important contributions using computational models, often considering general equilibrium and lumpy investment behavior include Khan and Thomas (2008), Bloom et al. (2014), Clementi and Palazzo (2013), and Winberry (2015). Finally, this paper also draws on the broad empirical literature about firms in the US, including the work by Davis et al. (1996) on job creation and destruction, Davis et al. (2007) on volatility and dispersion of business growth rates, Foster et al. (2008) on productivity, profitability, and reallocation, and also by Haltiwanger (2011) on productivity and firm dynamics, among others. In a similar sense, I contribute to the large literature investigating the misallocation of factors across firms in an economy, including the seminal contribution by Hsieh and Klenow (2009) and more recent attempts at uncovering the sources behind this misallocation. The latter category includes work by Asker et al. (2014) on dynamic frictions to input choices, and also papers by David et al. (2016) and David and Venkateswaran (2017) on imperfect information (that is, the fact that firms make decisions under risk and uncertainty about the future). Both of these lines of research are closely related to the contributions of my paper.

The rest of this paper is structured as follows: Section 2 introduces the Decision Maker Survey data and contains the main empirical analysis about the magnitude of overconfidence and overoptimism as observed empirically; Section 3 describes the model of firm dynamics and investment with potentially biased beliefs; Section 4 discusses how I solve and calibrate the model, and how I map biased beliefs in the model to the data; Section 5 shows the quantitative results and Section 6 performs some robustness checks on the quantitative results; Section 7 concludes.

2 Firms' Subjective Beliefs: Empirical Evidence

2.1 The Decision Maker Survey

My main source of data on firm subjective beliefs is the Decision Maker Survey (DMS), run out of the Federal Reserve Bank of Atlanta by Michael Bryan, Brent Myer, and Nicholas Parker in consultation with Nicholas Bloom of Stanford University and Steven J. Davis of the University of Chicago. The DMS is a novel longitudinal dataset surveying high-level firm managers, typically Chief Executive or Chief Financial Officers, on a monthly basis regarding their subjective beliefs about own-firm outcomes over the next year. Each month respondents are asked to answer questions about a subset of the following topics: sales, employment, capital expenditures, costs, profit margins and prices, with particular topics rotating month to month so that respondents have a chance to answer questions about most if not all modules once per quarter.

The Decision Maker Survey differs from other well-known data sources with subjective beliefs, firstly, in that respondents are firm insiders answering quantitative questions about their own firm's prospects. This is in contrast, for example, with the Philadelphia Federal Reserve Bank's Survey of Professional Forecasters (SPF) and data from the Institutional Brokers' Estimate System (I/B/E/S), as the former surveys professionals about the overall US economy and the latter asks outsiders about the earnings of publicly-listed firms.

The Decision Maker Survey is also particular because it elicits subjective probability distributions from its respondents. This feature is shared with the SPF, except that respondents to the DMS are not asked to assign probabilities to predefined grid of outcomes, instead providing their own forecast estimates and associated probabilities. For example, when answering questions about sales, respondents are asked to provide up to five scenarios for sales growth over the next four quarters, corresponding to a lowest, low, middle, high, and highest scenario, and then assign a probability to each of the scenarios. Respondents are specifically reminded that the total probability should add up to 100 percent. This design makes the Decision Maker Survey apt for studying businesses' subjective uncertainty (i.e. the standard deviation of their subjective distributions) about future outcomes, as well as subjective expectations.

Respondents to the Decision Maker Survey come from all sectors of the US economy, as shown in Figure 1. They are also prodominantly smaller, potentially entrepreneurial firms, with the mean and median number of employees at 273 and 130, respectively. This is in contrast with the publicly-traded firms that appear in I/B/E/S and even with the well-known Duke CFO Survey, whose respondents are probably roughly similar to S&P 1500 type firms (Ben-David et al. 2013).

The quality of responses in the Decision Maker Survey is remarkably high. Over 90 percent of all responses to questions about sales, employment or capital expenditures include probabilities that add up to 100 percent. In nearly all cases the outcome scenarios are monotonic (lowest is under low, which is under middle, etc.), and similarly almost no responses have 100 percent of the probability mass in a single bin. Summary statistics on these and other quality metrics are provided in Table 1.

For the purposes of this paper, I focus my attention on firms' subjective beliefs about future sales, and in particular on measuring the degree of overoptimism (how much firms' expectations exceed realizations and sample averages) and overconfidence (how much do firms underestimate the uncertainty or standard deviation of the future) over sales growth. However, similar empirical analysis is possible for, say, capital expenditures, or employment. Figure 3 displays screenshots of the questions about sales beliefs included in the in the Decision Maker Survey. All participants are first asked to provide the total dollar value of sales in the current quarter. Prior to September 2016, participants were then asked for their five scenarios for quarterly sales looking a year ahead, and their associated probabilities. Since September 2016, respondents have provided five scenarios for sales growth over the next four quarters, again with associated probabilities, as shown in Figure 3. A similar set of questions is asked in the modules relating to employment and capital expenditures.

Readers should refer to Appendix A for details on how the Decision Maker Survey data are selected and cleaned for analysis.

2.2 Optimism in the DMS

2.2.1 Constructing subjective expectations

Unlike many datasets with information about subjective beliefs, the Decision Maker Survey does not ask respondents for point estimates (best guesses) of future sales or sales growth. Instead, respondents provide subjective probability distributions over a set of plausible scenarios.

It is straighforward to calculate a firm's subjective expectation for sales growth over the next four quarters by combining the firm's forecasts for sales growth for the different scenarios, g_{it+4}, with the probabilities assigned to each, $\tilde{p}_{it+4}$:

$$\tilde{\mathbf{E}}_t[g_{t+4}] \quad = \quad \sum_{i=1}^{5} \tilde{p}_{it+4} g_{it+4}$$

Here, i indexes each of the firm's proposed scenarios. The notation $\tilde{\mathbf{E}}_t[\cdot]$ denotes the subjective expectations operator from the standpoint of quarter t, which works just like the regular expectations operator, but uses the firm's subjective probability distribution.

Prior to September 2016 the Decision Maker Survey did not ask for sales growth forecasts directly, instead soliciting estimates of the dollar value of quarterly sales four quarter ahead. I therefore construct growth forecasts for these responses using by combining the sales forecast information with respondents' reported dollar value of current sales. Following Davis et al. (1996), I measure these growth forecasts as the proposed change in sales over the four quarters, divided by the average between the initial and final sales figures. If s_t denotes the current level of sales reported and s_{it+4} denotes a respondent's prediction sales for quarter $t+4$ in scenario i, that means that the proposed growth rate for scenario i over four quarters is $g_{it+4} = (s_{it+4} - s_t)/(\frac{1}{2}(s_{it+4} + s_{it}))$. Since September 2016 the survey has asked for sales growth estimates g_{it+4} directly, allowing me to skip this step and compute the subjective expectations directly.

2.2.2 Measuring optimism

A straightforward measure of the sales optimism of firm j on date t is the difference between its subjective expected growth rate and a measure $\bar{g}_j$ of the objective expectation the firm should have

$$\text{Optimism}_{jt} = \tilde{\mathbf{E}}_t^j[g_{jt+4}] - \bar{g}_j.$$

The question is how to measure $\bar{g}_j$? I proceed using two approaches.

In the first approach I exploit the longitudinal nature of the Decision Maker Survey and take $\bar{g}_j$ to be a firm's own realized sales growth as observed a year after it made its predictions. To the extent that firms have incorporated all of the information available to them in constructing their subjective beliefs, the difference between expected and realized growth should only reflect unforeseeable disturbances. Furthermore, to the extent that shocks are independently drawn across firms, they should wash out on average across the population and any systematic deviations in measured optimism across firms will capture firms' tendency to over- or underestimate future sales growth depending on the sign of the deviation. To control for unforeseeable common shocks that may look like over- or underoptimism I construct versions of my optimism variable that strip out variation attributable to date and sector effects and compare these against the "raw" measure of optimism.

The first approach to measuring optimism effectively constructs the firm's own forecast error, which is an intuitive way of thinking about optimism but results in a relatively small sample, since the Decision Maker Survey is still a short panel (data begins in late 2014), and ex-post realizations of sales growth are only available for firms that fail to drop out of the survey over the course of a year[3]. To mitigate issues with these sample sizes, in the second approach I take $\bar{g}_j$ to be an empirical estimate of the average growth rate of the firms in my sample. The advantage in this second approach is that I can measure the average realized growth rate $\bar{g}_j$ for a class of firms and use it as a benchmark for all firms with sales expectations, even if I do not observe a subsequent realization of growth for some of them.

Because firms of different sizes or sectors may be subject to different growth processes and shocks, I measure $\bar{g}_j$ alternatively within sector and within quintile of the sales distribution. I also have the choice of calculating these average growth rates using recall information, i.e. based on firms' reported sales growth over the past four quarters[4], or to use actual growth rates calculated from "current sales" reports given a year apart. While recall data might be less accurate, it results in larger samples for calculating average sales growth rates. Again, in order to construct a realized growth rate, it is necessary for a firm to remain in the panel for several months. The number of realized growth rate observations should increase with time, which should benefit both this approach and the "forecast error" version above, but since the dataset is still fairly young the small sample size is still potentially an issue.

Comparing firms' subjective expectations to observed averages in this second apprach is a useful way of measuring optimism for the same reasons that it is useful to consider forecast errors. To the extent that the average growth rate for a class of firms $\bar{g}_j$ reflects the underlying objective expectation for sales growth, any systematic deviations between expectations and this benchmark should reflect over- or underoptimism. Individual firms may have some advance information about their own growth rate relative to the benchmark that may lead optimism to deviate from zero. But as long as this is advance information about idiosyncratic rather than common shocks, it should wash out on average. To control for common shocks to advance information, I also strip out variation attributable to date and sector effects and compare these "purged" variables with their "raw" counterparts in the next section.

While constructing any of my measures of optimism, I use the approach in Davis et al. (1996) whenever it is necessary to compute sales growth rates from dollar values of sales

[3]Bachmann and Elstner (2015) similarly measure optimism based on systematic deviations in forecast errors, but they have access to a much longer panel of data. My advantage over their analysis is that they only have qualitative data on firms expectations, and similarly lack reliable quantitative observations on realized output or sales. This means that they rely on a set of strong assumptions to construct firms' forecast errors, even if their samples are much bigger.

[4]Prior to September 2016 firms were asked for their current dollar value of sales, and the value of their sales four quarters prior to then, which can be used to construct recall growth rates. Since September 2016, the recall question asks firms directly for an estimate of rate of sales growth over the past year.

provided a year apart. That is, I compute realized growth rates as the change divided by the average between the initial and final value.

2.2.3 Optimism results

Figures 4 and 5 show the empirical distributions of measured optimism, respectively for the "forecast error" approach and taking the benchmark growth rate to be the mean growth rate in a firm's one-digit sector. The figures also show the distribution with and without controlling for date and sector effects, and include a vertical line at zero for reference to where optimism should be naturally centered in a population of firms with rational expectations over future growth. In all cases there are firms that expect their future growth to be above and below the benchmark, be that their ex-post realized growth, or the average growth within their sector or size quintile (the latter not shown). However, in all cases it is visually clear that the mass above zero is larger; that is, there is a bigger proportion of firms that overestimate their future growth relative to the benchmark, indicating that firms in the Decision Maker Survey tend to be overoptimistic.

I explore the empirical evidence on overoptimism quantitatively in Table 2. I compute the mean and median optimism and their standard errors, respectively, via regression and quantile regression on a constant, using alternative measures of optimism as the dependent variable. I consider both raw versions of the optimism variables and others that have been stripped of variation attributable to date and sector, or date-by-sector effects.

Although the results vary somewhat quantitatively depending on the benchmark growth rate, the estimates in Table 2 document that the firms in the Decision Maker Survey are significantly overoptimistic. The median firm's expectations for annual growth exceed the benchmark growth rate by 2 to 7 percentage points across the various specifications, with the mean firm exceeding them by 5 to 10 percent depending on the specification. These are large magnitudes, with some firms clearly predicting that they are going to grow by significant amounts when ex-post they shrink, or their peers are on average experiencing much more modest growth. However, the results are not unexpected in light of ample evidence of overconfidence and overoptimism in the literature, for example as documented by Malmendier and Tate (2005, 2008) and Malmendier et al. (2007). In fact, my estimates of optimism are in line with those in Bachmann and Elstner (2015), where it is documented that among firms that systematically overestimate their future production, the magnitude of the error is about ranges from about 3 to 6 percentage points.

2.3 Overconfidence in the DMS

2.3.1 Constructing subjective uncertainty

Just as a measure of subjective growth expectations is the key ingredient for measuring optimism, for overconfidence it is necessary to obtain the firm's subjective uncertainty over future growth, i.e. the standard deviation over one-year growth outcomes. Using the subjective growth rate scenarios g_i obtained as in Section 2.2.1 along with subjective probabilities $\tilde{p}_{it+4}$, this subjective uncertainty can be calculated as:

$$\widetilde{\mathbf{SD}}_t[g_{t+4}] \;=\; \sqrt{\sum_{i=1}^{5} \tilde{p}_{it+4}(g_{it+4} - \tilde{\mathbf{E}}_t[g_{t+4}])^2}, \qquad \text{where} \quad \tilde{\mathbf{E}}_t[g_{t+4}] = \sum_{i=1}^{5} \tilde{p}_{it+4} g_{it+4}.$$

2.3.2 Measuring overconfidence

I define overconfidence as the ratio between a firm j's subjective uncertainty about growth and an objective measure of what that uncertainty should be on date t:

$$\text{Overconfidence}_{jt} \;=\; \frac{\widetilde{\mathbf{SD}}_t^j[g_{jt+4}]}{\bar{\sigma}_j}.$$

Using a ratio to measure overconfidence is useful because both the numerator and denominator are positive numbers, so the measure is easily interpretable keeping in mind that firms with rational expectations should exhibit overconfidence of 1 under this measure. Note that greater overconfidence is associated with lower values of this raw overconfidence variable.

For my analysis I identify $\bar{\sigma}_j$ with an estimate of the cross sectional dispersion (i.e. standard deviation) of sales growth across firms. Ideally, this would be an estimate of firm-specific sales growth volatility, rather than cross-sectional dispersion, but the fact that the Decision Maker Survey is a short panel poses limitations for estimating within-firm volatility. However, in simple stochastic processes like random walks or the first order autoregressive process that I use in the model in Section 3 volatility and dispersion are both identical to the standard deviation of innovations to the process, and in U.S. firm-level data they tend to follow each other closely, for example in Davis et al. (2007).

One might be concerned that similar firms are subject to correlated shocks, so that my measurements of $\bar{\sigma}_j$ are biased. To mitigate it potential issue I proceed as with optimism, by alternatively measuring the benchmark uncertainty/standard deviation of growth $\bar{\sigma}_j$ within sector or size category. I argue that it is more likely for such confounding shocks to impact firms in the same sector, than across sectors, for a size category. Also, to the extent that there are correlated shocks, these should bias my empirical measure of uncertainty downwards, increasing my overconfidence variable and therefore biasing me against finding firms to be overconfident.

As with optimism, I alternatively use growth rates calculated off of recall or realizations data to compute sales dispersion across firms. For each of the possible ways of constructing the overconfidence variable I also build a version that removes date and sector effects to control for common shocks to subjective uncertainty.

2.3.3 Overconfidence results

As with optimism, I plot the distribution of overconfidence in Figures 6 and 7, alternatively with and without controlling for sector and date effects, and respectively taking the objective measure of uncertainty to be the dispersion of sales growth rates within one-digit sector and within quintile of the sales distribution. Each of these histograms includes a vertical line at unity, where overconfidence would be for a rational expectations firm that knows the objective distribution of its future sales growth. The empirical evidence shows that firms on-the-whole have subjective uncertainties about sales that are significantly below the objective dispersion of sales growth rates observed in the population. Moreover, all of these distributions have a long right tail, with very few firms reporting overconfidence above unity, or .5 for that matter.

The apparently extreme results from Figure 6 are corroborrated with a closer look at the central tendency measures for my overconfidence variables in Table 3. As with optimism, I report the mean and median overconfidence computed from regression and quantile regression, respectively, on a constant along with standard errors. Using recall-based growth rates to calculate the benchmark dispersion in sales growth rates, within sector or within sales quintile, the median firm has an overconfidence of about 0.2, or 0.3 to 0.35 after controlling for sector and date effects. The mean overconfidence is somewhat robust between 0.3 and 0.35 when using recall growth rates to measure sales growth dispersion. This suggests that the average firm underestimates the variance of its future sales growth by something like 65 to 70 percent. Using sales growth realizations these statistics are even more pronounced, with the median closer to 0.1 or 0.2 using raw and common-shock-adjusted measures, respectively, and the mean close to 0.2. These more extreme figures arise because realized growth rates are computed using fewer observations than recall are available with recall, which leads the benchmark standard deviation to be more sensitive to outliers with extreme growth experiences[5]. In the calibration and estimation of the model I thus rely on the results

[5]Indeed, the measured dispersions are about two or three times as large when using realizations rather

based on recall growth rates.

3 A Model of Firm Dynamics with Subjective Beliefs

I develop a model of entrepreneurial firms subject to idiosyncratic risk with free entry and exit. The firms make a periodic, forward-looking investment decision subject to capital adjustment costs, but their subjective beliefs about future fundamentals may be biased relative to the objective driving process. It is natural to think that there are managers running the firms in my model, and that it is these managers who may have distorted beliefs, but this distinction is largely irrelevant for my analysis. In what follows, I therefore refer interchangeably to "biased firms" and "firms whose managers are biased".

3.1 Technology and Environment

There is a continuum of firms, each having access to a decreasing returns to scale revenue production function $R(z_t, K_t) = z_t K_t^\alpha$ in quarter t. K_t is the amount of capital currently owned by the firm, while z_t is the current realization of a Hicks-neutral, idiosyncratic stochastic process augmenting the firms' capacity to raise revenue. This setup arises, for example, with an underlying Cobb-Douglas production function employing capital and labor, the latter optimized out statically, and with each firm facing a constant constant elasticity demand curve.

Fluctuations in z_t are meant to capture shocks to business conditions as a whole, incorporating movements in total-factor-productivity as well as demand. Business conditions evolve following an autoregressive process in logs:

$$\log(z_{t+1}) \quad = \quad \rho \log(z_t) + \sigma\varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim \mathcal{N}(0,1) \tag{1}$$

The firm's capital holdings evolve according to a standard law of motion, with a fraction δ of the current stock K_t depreciating after production occurs so that the amount available for production tomorrow equals the undepreciated stock plus any investment made today:

$$K_{t+1} \quad = \quad K_t(1-\delta) + I_t$$

Investment is subject to a rich set of convex and non-convex adjustments costs. The firm incurrs a fixed disruption cost F as a fraction of quarterly sales if it adjusts its capital stock above and beyond the effects of depreciation (i.e. if its target capital stock K_{t+1} falls outside

than recall growth rates in the data.

of $[K_t(1-\delta), K_t]$). Investments are also partially irreversible, so capital is sold at a fraction $1-\gamma$ of its purchase price p_k. Finally, investment is also subject to standard quadratic adjustment costs. The full specification of the adjustment costs in terms of the current and target capital stock is, therefore:

$$AC(K_t, K_{t+1}) = \left\{ \begin{array}{c} F \cdot R(z_t, K_t) \cdot \mathbf{1}(K_{t+1} \notin [K_t(1-\delta), K_t]) \\ +p_k\gamma(K_{t+1} - K_t(1-\delta)) \cdot \mathbf{1}(K_{t+1} < K_t(1-\delta)) \\ +\lambda \left(\frac{K_{t+1}-(1-\delta)K_t}{K_t}\right)^2 K_t \end{array} \right\} \tag{2}$$

This specification follows the evidence in the literature in favor of convex and non-convex adjustment costs, including Bloom (2009) andCooper and Haltiwanger (2006), who each estimate a similarly rich set of adjustment costs and finds strong evidence for all of them, particularly for "sharp" adjustment costs like fixed disruption costs and irreversibilities. For my particular context, adjustment costs are an important feature of the model because they help generate realistic investment dynamics, and, significantly, ensure that the firm's investment, entry, and exit decisions are sensitive to the firm's perception of the risks and uncertainties it faces. In particular, overconfident firms will underestimate the degree of of uncertainty in the future and perceive the real option of waiting before investing to be less valuable than unbiased firms will.

Finally, the firm incurrs overhead costs c every period during which it is active and raises revenue.

My baseline setup abstracts from financial frictions, so that in any particular period firms are free to run as large deficits as they wish, financing current investment with retained earnings in the future. This is as if the firm was backed by a deep-pocketed venture capitalist willing to inject cash into the firm whenever necessary.

3.2 Firms' Subjective Beliefs

Recall that the firms in the model are subject to idiosyncratic risk in the state of business conditions, which evolve according to Equation 1. However, firm managers have the following subjective beliefs about the evolution of the driving process:

$$\log(z_{t+1}) = \tilde{\mu} + \rho \log(z_t) + \tilde{\sigma}\varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim \mathcal{N}(0,1) \tag{3}$$

The parameters $\tilde{\mu}$ and $\tilde{\sigma}$, respectively, distort managers' subjective conditional expectations and subjective uncertainty about future business conditions. If $\tilde{\mu} > 0$, the manager is overoptimistic about the firms future prospects, since the subjective expectation for tomorrow's business conditions is higher than what is warranted by the objective driving process. The ratio between the subjective and objective conditional standard deviations over future business conditions, $\tilde{\sigma}/\sigma$, determines whether managers are over- or underconfident (in a statistical sense) about the future. If this ratio is less than unity, managers are less uncertain about the nature of future conditions than they should be, so they are overconfident in their ability to forecast the future and underestimate the probability of extreme (good or bad) outcomes.

All of my quantitative specifications will consider an economy with two types of firms: (1) unbiased firms whose subjective beliefs coincide with the objective stochastic process for business conditions, and (2) biased firms for whom either $\tilde{\mu} \neq 0$ or $\tilde{\sigma} \neq \sigma$, or both. In what follows I will denote firm type with the symbol θ, using $\theta = 1$ to refer to unbiased firms and $\theta = 2$ for biased firms.

The explicit specification for firms' subjective beliefs is the main innovation in my model, and is meant to capture the empirical findings about firm managers' beliefs about their future prospects, as detailed in Section 2. I will be taking as given the existence of firms run by managers with subjective beliefs and remaining agnostic as to why managers may develop belief distortions, or why they are kept as managers in the first place. Recent contributions by Gabaix (2014, 2016) have developed theory that micro-founds and justifies behavioral decision-making as the product of "sparse maximization" under which agents optimally choose to pay more or less attention to certain parameters and features of their environment. This theory is capable of generating agents whose beliefs about underlying stochastic processes are mistaken in ways that are similar to how the agents in my model are mistaken about future fundamentals.

3.3 Ownership and Control of the Firm

The standard assumption about firm behavior is that managers and shareholders have aligned interests, and therefore that managers aim to (and in fact do) maximize shareholder value. In my baseline setup, I still assume that managers attempt to maximize the net present value of profits, but because they may have beliefs that are at odds with the objective risks faced by the firm, managers' actions may in fact destroy shareholder value. At this point in the paper, I assume that shareholders do not or cannot replace overconfident or overoptimistic managers for correctly calibrated ones. This may be the result of either of the following empirically plausible scenarios:

- The agents in the economy who are willing to serve as managers are precisely those who tend to display overconfidence and overoptimism. Thus, shareholders are limited to a pool of predominantly biased individuals when choosing who to manage their firms and cannot ex-ante tell a manager's type before starting their business. In the results section I will show that firms run by biased managers will typically still generate positive value, and potentially even more value *ex-ante* than firms run by unbiased managers.

- The firms are small entrepreneurial projects owned and managed by the same (potentially biased) entity, which is unwilling or unable to sell the business to an unbiased entity, say, due to large transactions costs.

To summarize, I will assume that firms will be run by managers attempting to maximize their *subjective* valuation of the business.

3.4 Incumbent Firms' Problem

In quarter t, an incumbent firm observes its current business conditions z_t, produces output resulting in revenue $R(z_t, K_t)$ and pays for overhead operating costs c. The firm's manager then makes an investment decision, choosing the capital stock that will be available for production in quarter $t+1$. Investments entail paying for acquired capital at a price p_k in terms of the output good (or re-selling capital the firm no longer needs), and may lead the firm to pay some adjustment costs as specified by the function $AC(K_t, K_{t+1})$ detailed in equation 2. The firm may choose to exit by divesting all its assets, that is by choosing $K_{t+1} = 0$, paying the associated adjustment costs $AC(K_t, 0)$, and foregoing all future revenues and overhead costs.

The manager's objective is to maximize the (risk-neutral) net present value of cash flows provided by the firm, discounted at rate $1+r$, with beliefs about future cash flows depending on the managers' perception of the evolution of future business conditions z_{t+1} under equation 3. Formally, a manager of type θ[6] solves the optimal investment problem represented by the following Bellman equation:

$$
\begin{aligned}
\tilde{V}(z_t, K_t; \theta) \quad &= \quad \max_{K_{t+1} \geq 0} \left[\begin{array}{c} z_t K_t^{\alpha} - p_k(K_{t+1} - (1-\delta)K_t) - AC(K_t, K_{t+1}) - c \\ + \frac{1}{1+r} \tilde{\mathbf{E}}_t^{\theta}[\tilde{V}(z_{t+1}, K_{t+1}; \theta)] \end{array} \right] \\
&\quad s.t. \\
\tilde{V}(z_{t+1}, 0; \theta) \quad &= \quad 0, \quad \forall z_{t+1}, \theta
\end{aligned} \tag{4}
$$

[6] Recall that I consider model economies with two types of agents, using $\theta = 1$ to refer to unbiased managers and $\theta = 2$ to refer to biased managers.

Here the operator $\tilde{\mathbf{E}}_t^\theta[\cdot]$ computes the conditional expectation over realizations of z_{t+1} using all information available on date t, and under the subjective beliefs process for firms of type θ. Therefore $\tilde{V}(\cdot;\theta)$ denotes the *subjective* value of the business from the perspective of a manager of type θ.

3.5 Entering Firms' Problem

Every quarter there is a mass M of potential entrants. These entrants pay a sunk cost c_e to draw an initial shock z_0, which is distributed Pareto(ζ) to ensure heterogeneous entry. In the baseline case, I assume that upon drawing the shock, potential entrants are "born" unbiased with probability p, and biased with probability $1-p$. Upon seeing the initial state of business conditions, z_0, newborn managers may choose to make an initial capital investment K_1, subject to quadratic adjustment costs if K_1 exceeds some value K_{min}, or may decide not to enter ($K_0 = 0$) and forego all future cash flows. The entrant's initial investment problem, depending on its type, θ, is therefore:

$$\tilde{V}^E(z_0;\theta) = \max\left\{0, \max_{K_1>0}\left[\begin{array}{c} -p_k K_1 + \lambda_e\left(\frac{K_1-(1-\delta)K_{min}}{K_{min}}\right)^2 K_{min}\cdot\mathbf{1}(K_1>K_{min}) \\ +\frac{1}{1+r}\tilde{\mathbf{E}}_0^\theta[V(z_1,K_1;\theta)] \end{array}\right]\right\} \quad (5)$$

3.6 Objective Firm Value

I use the notation $V(z_t, K_t;\theta)$ (without the tilde superscript) to denote the *objective* value of a business with current business conditions z_t and capital stock K_t, taking as given that the firm is run by a manager of type θ. Define $K^*(z_t, K_t;\theta)$ to be the optimal policy of a manager of type θ solving the incumbent's problem in 4. Then, $V(\cdot;\theta)$ represents the net present value of cash flows delivered by policy $K^*(\cdot;\theta)$ under correct beliefs about the evolution of z_t:

$$\begin{aligned} V(z_t, K_t;\theta) &= z_t K_t^\alpha - p_k(K^*(z_t,K_t,\theta)-(1-\delta)K_t) - AC(K_t, K^*(z_t,K_t,\theta)) - c \\ &\quad + \frac{1}{1+r}\mathbf{E}_t[V(z_{t+1}, K^*(z_t,K_t;\theta);\theta)] \end{aligned}$$

Now $\mathbf{E}_t(\cdot)$ represents the conditional expectation based on the objectively correct beliefs about future business conditions. Note that $V(\cdot;\theta)$ will in general differ from $\tilde{V}(\cdot;\theta)$, the manager's subjective valuation of the business, except for firms whose managers are unbiased.

Similarly, I use $V^E(z_0;\theta)$ to denote the objective value of a new business business run by a manager of type θ, whose initial capital investment policy is $K_1^*(z_0;\theta)$:

$$V^E(z_0;\theta) = \begin{cases} 0 & \text{if} \quad K_1^*(z_0;\theta)=0 \\ \left[\begin{array}{c} -p_k K_1^*(z_0;\theta) \\ +\lambda\left(\frac{K_1^*(z_0;\theta)-(1-\delta)K_{min}}{K_{min}}\right)^2 K_{min}\cdot \mathbf{1}(K_1^*(z_0;\theta)>K_{min}) \\ +\frac{1}{1+r}\mathbf{E}_0[V(z_1,K_1^*(z_0;\theta);\theta)] \end{array}\right] & \text{if} \quad K_1^*(z_0;\theta)>0 \end{cases}$$

again, with $\mathbf{E}_0(\cdot)$ denoting the unbiased conditional expectation.

Because the model will not include an active equity market, objective business values will not play a large role in the dynamics of the model. However, these objective valuations are important for assessing the implications of subjective beliefs for shareholders. The setup in this subsection can easily be generalized to consider firm valuations from the perspective of outsiders whose beliefs also differ from the firm manager's, but are not necessarily correctly specified according to the objective driving process either.

3.7 Stationary Distribution of Firms

The objective driving process in equation 1, together with type-specific optimal investment policies arising from incumbents' problem, $K^*(z_t,K_t;\theta)$, define a transition operator $G(z_t,z_{t+1},K_t,K_{t+1},\theta)$ which gives the density of incumbent firms of type θ currently with state (z_t,K_t) that transition to state (z_{t+1},K_{t+1}) in one quarter.

Let $\phi(z_{0,}K_1,\theta)$ denote the measure of potential entrants of type θ whose initial draw for the state of business conditions is z_0 and choose to make an initial investment in K_1 units of capital, following their optimal policy $K_1^*(z_0,\theta)$. Note that if $K_1=0$, the manager chooses not to enter after observing z_0. In the baseline setup with two types of firms, $p=\int_{Z\times K}\phi(z,k,1)\mathrm{d}z\mathrm{d}k$ is the (exogenous) probability that a firm will be born unbiased ($\theta=1$), and $1-p=\int_{Z\times K}\phi(z,k,2)\mathrm{d}z\mathrm{d}k$ analogously is the probability that a firm is born biased ($\theta=2$).

The model generates a stationary distribution of firms $\lambda(z,k,\theta)$ across the state of business conditions, capital-holdings, and types at the beginning of each quarter before any investment decisions take place, but after the previous quarter's exiting firms have left and last quarter's potential entrants have made their initial investment decisions.

Formally, $\lambda(\cdot)$ satisfies the following properties (with some abuse of notation):

1. $\lambda: Z\times\{k>0\}\times\Theta\rightarrow[0,1]$ and $\int_{Z\times\{k>0\}\times\Theta}\lambda(z,k,\theta)\mathrm{d}z\mathrm{d}k\mathrm{d}\theta{=}1$

2. The distribution of firms across the state space is invariant across quarters, after considering: (1) endogenous capital choices and exogenous fluctuations in business conditions of incumbents , summarized by transition function $G(\cdot)$; and (2) birth of potential entrants across types and business conditions, leading to endogenous entry decisions as summarized by $\phi(\cdot)$:

$$\lambda(z',k',\theta) \;=\; \int_{Z\times\{k>0\}} \lambda(z,k,\theta)\cdot G(z,z',k,k',\theta)\mathrm{d}z\mathrm{d}k + M\phi(z',k',\theta)$$

3. The mass of potential entrants, M, is pinned down so that the mass of firms that ultimately decide to make a positive initial investment ($K_1 > 0$) is equal to the share of incumbents who exit:

$$M\int_{Z\times\{k>0\}\times\Theta} \phi(z,k,\theta)\mathrm{d}z\mathrm{d}k\mathrm{d}\theta = \int_{Z\times\{k>0\}\times\Theta} \lambda(z,k,\theta)\cdot G(z,z',k,0,\theta)\mathrm{d}z\mathrm{d}k\mathrm{d}\theta.$$

4. Free entry pins down the sunk costs of drawing an initial z_0 such that c_e equals the *ex-ante* expected value of an enterprise that may turn out to be run by either an unbiased or a biased manager: $c_e = \int_{Z\times\Theta} V^E(z;\theta)\cdot\psi(z,\theta)\mathrm{d}z\mathrm{d}\theta$, where $\psi(z,\theta)$ is the (exogenous) joint distribution of potential entrants across initial business conditions and types, and $V^E(z,\theta)$ is the objective value of a potential business drawing a manager of type θ and initial business conditions $z_0 = z$.

Effectively, this paper considers a model economy that is at its aggregate steady-state, although individual firms in the economy are subject to idiosyncratic risk. Therefore, aggregates and population moments are stable across time, but individual firms are always receiving shocks, growing, shrinking, entering, and exiting.

4 Model solution and calibration

I solve the model in Section 3 for a series of specifications that include two types of firms: (1) unbiased firms run by managers whose beliefs about future business conditions coincide with the objective stochastic process in 1, and (2) biased firms, whose subjective belief process implies they are either overoptimistic ($\tilde{\mu} > 0$), overconfident ($0 < \tilde{\sigma} < \sigma$), or both. As in the above section, I use the notation $\theta = 1$ and $\theta = 2$ to refer to unbiased and biased firms, respectively, for each specification.

4.1 Solving the Model with Subjective Beliefs

Solving and simulating economic models in which agents' beliefs are not consistent with the objective stochastic fluctuations in the economy imposes relatively few constraints over standard rational-expectations modeling. As explained in great detail in Jurado (2016), the model is consistent if agents' subjective beliefs agree with the objective process on set of outcomes may occur with positive probability, potentially disagreeing on what that positive probability is. Since both the subjective and objective processes in the model in equations 1 and 3 have infinite support and receive Gaussian shocks, the model in Section 3 satisfies this requirement[7].

Firms' optimal policy functions are the product of optimization under the assumption that the next quarter's business conditions will evolve from current conditions following the potentially biased subjective process. Thus, solving for the optimal investment policies of incumbents and entrants from problems 4 and 5 amounts to solving a standard dynamic programming problem. See Appendix B for details regarding this computation.

The dynamics of the model depend both on the firm-level policies $K^*(\cdot;\theta)$ and $K_1^*(\cdot;\theta)$ which depend on the type θ, and on the actual, exogenous evolution of business conditions z_t. Therefore, in solving for the stationary distribution of firms by capital holdings K, business conditions z, and types θ , I consider the combination of deterministic, endogenous capital choices and stochastic evolution of business conditions according to the true process in equation 1. The Markovian structure of the model makes it convenient to compute the stationary distribution $\lambda(z,k,\theta)$ by non-stochastic simulation, largely following the procedure outlined by Young (2010). See the simulation appendix B for details on this procedure.

Although firms are born unbiased or biased with exogenous probabilities p and $1-p$ which will be calibrated, the share of firms belonging to each type in the stationary distribution is an endogenous result depending on this distribution of types at birth and on the propensity of unbiased versus biased firms to exit. Different parameterizations of the subjective beliefs processes will therefore lead to different shares of biased and unbiased firms among the whole the population.

4.2 Calibration

4.2.1 Technological parameters

Most of the parameters governing the physical technology are set externally to conventional values from the literature. The elasticity of revenues with respect to capital is set to 0.625,

[7] Formally, this requires the subjective conditional variance $\tilde{\sigma}$ to be strictly greater than zero, although it could be arbitratily small.

consistent with constant returns to scale in the physical production function involving capital and labor (the latter optimized out statically and therefore abstracted from in the model), and 25% markups. The price of capital in terms of revenue p_k is set to 1.016 following Khan and Thomas (2008), and the annual interest rate (the rate at which managers discount future cash flows) is set to 5 percent. Depreciation is currently set to 15 percent on an annual basis, and is meant to account for both physical depreciation and (unmodelled) growth in the long-run level of revenue-generating capacity.

The parameters of the objective stochastic process for $\log(z_t)$ are set to standard values from the literature that estimates the dynamics and volatility of idiosyncratic components of revenue-based total-factor productivity (TFPR), including Haltiwanger (2011) andCastro et al. (2015). TFPR is the closest analogue to fundamentals z_t in the model, and furthermore it represents revenue-raising capability conditional on observed input choices and costs. Therefore, existing estimates of the idiosyncratic fluctuations in TFPR do not depend strongly on the co-movement of revenues and investment, which would be worrisome as the latter would be directly affected by overoptimistic and overconfident beliefs about future fundamentals. I thus proceed by calibrating z_t to have an autocorrelation ρ of 0.95 on a quarterly basis and the standard deviation of innovations, σ, to 0.2, in line with the findings in the above papers and Foster et al. (2008).

The tail parameter for the Pareto distribution from which potential entrants draw their initial business conditions z_0 is 3, broadly in line with values used by Clementi and Palazzo (2013) and Begenau and Salomao (2014).

The adjustment costs parameters broadly follow from consensus estimates in the literature, including Cooper and Haltiwanger (2006) and Bloom (2009). However, as existing estimates are typically inferred structurally from the comovement of sales and investment activity, I also attempt to match some well-known moments in the literature, focusing in particular on the behavior of unbiased firms. The fixed disruption cost of adjusting the capital stock, F, amounts to 1 percent of quarterly revenues, which is in line with Bloom (2009) and prevents firms from making small adjustments. The resale loss from disposing of installed capital, γ is 17.5 percent, which is consistent with a rate of inaction (i.e. the fraction of firms for which the absolute value of the quarterly investment rate under 0.25 percent) of about 8 percent among unbiased firms. The quadratic adjustment costs parameter λ is set to 0.2, which is within the accepted range in the literature, and coupled with overhead costs of 52.5 leads to an annual exit rate of 10 percent for incumbents. Initial investments K_1 are subject to quadratic adjustment costs with parameter λ_e set to 0.006 and apply to the portion of initial investment K_1 exceeding 100 units of capital. Jointly, these two parameter choices result in unbiased entrants being about 60 percent as large as the average incumbent.

In Section 6 I explore how the results change for different parameterizations of the capital adjustment costs.

4.2.2 Subjective beliefs parameters: calibration strategy

The following parameters remain to be calibrated:

1. The probability that a firm is born unbiased, p
2. Biased firms' degree of overoptimism over future business conditions, $\tilde{\mu}$
3. Biased firms' degree of overconfidence over future business conditions, $\tilde{\sigma}$

I choose these three parameters to jointly match data moments about overconfidence and/or overoptimism in the Decision Maker Survey, as well as an overall exit rate of 10 percent of all firms in the stationary distribution. The data moments about subjective beliefs distortions are obviously informative about their respective parameters, but the exit rate is as well because overoptimistic firms tend to refuse to exit even in bad times expecting better conditions in the near future; overconfident firms, by contrast, tend to exit at higher rates because their perceived lower uncertainty about future fundamentals lowers the option value of waiting relative to liquitating the firm's assets. Therefore, using the exit rate as an extra target allows me to calibrate the subjective belief parameters and the share of firms born unbiased, p, jointly.

I consider three separate specifications:

1. A specification in which the biased firms are overoptimistic and not overconfident. Here, $\tilde{\sigma} = \sigma$, but $\tilde{\mu}$ and p are calibrated to match the median overoptimism about sales growth rates in the Decision Maker Survey and an percent annual exit rate of 10 percent. I choose the median overoptimism because I believe the results about mean optimism in Section 2 are biased upward by extreme values. Median overoptimism is smaller but still statistically and economically significant.

2. A specification in which biased firms are overconfident but not overoptimistic. In this case $\tilde{\mu} = 0$ for all firms. $p$and $\tilde{\sigma}$ are chosen to match the mean overconfidence about sales growth in the Decision Maker Survey, and, again, a target overall exit rate of 10 percent over four quarters. I choose the mean overconfidence because it is less extreme than the median.

3. A specification in which biased firms are both overconfident and overoptimistic, targeting the two moments about beliefs distortions and the exit rate simultaneously.

Working with calibrations that address only one distortion at a time is useful for understanding the impact of that distortion in isolation. However, the full specification does justice to the fact that I document the presence of both overoptimism and overconfidence among firms in the Decision Maker Survey, and can indicate which of the two is more crucial to firm behavior.

4.2.3 Measuring optimism and overconfidence in the model

In order to use the overall degree of overoptimism and overconfidence as target moments in the calibration of the model requires a formal definition of these variables within the model. I define sales overconfidence and overoptimism in the model analogously to how I do it for the Decision Maker Survey data in Section 2.

An incumbent firm of type θ, with current business conditions z_t and capital-holdings K_t has a subjective distribution over its potential sales growth over the next four quarters, which depends on its subjective beliefs process and the firm's policy function $K^*(\cdot;\theta)$. Given the firm's current state (z_t, K_t), the firm's optimism is the difference between the subjective and objective expectations for four-quarter sales growth, taking as given the firm's investment policy:

$$\text{Optimism}(z_t, K_t, \theta) \quad = \quad \tilde{\mathbf{E}}_t^{\theta}\left[\frac{z_{t+4}K_{t+4}^{\alpha}}{z_t K_t^{\alpha}} - 1\right] - \mathbf{E}_t\left[\frac{z_{t+4}K_{t+4}^{\alpha}}{z_t K_t^{\alpha}} - 1\right].$$

I combine this definition of optimism with the stationary distribution of firms across the state-type space to obtain the economy-wide median optimism.

In turn, overconfidence is measured as the ratio between a firm's uncertainty (i.e. standard deviation) about this four-quarter growth rate and the true standard deviation implied by the objective stochastic process, once again the firm's policy function as given:

$$\text{Overconfidence}(z_t, K_{t,}\theta) \quad = \quad \frac{\tilde{\mathbf{SD}}_t^{\theta}\left[\frac{z_{t+4}K_{t+4}^{\alpha}}{z_t K_t^{\alpha}} - 1\right]}{\mathbf{SD}_t\left[\frac{z_{t+4}K_{t+4}^{\alpha}}{z_t K_t^{\alpha}} - 1\right]}$$

As with optimism, the model definition of overconfidence can then be combined with the stationary distribution $\lambda(\cdot)$ to obtain the average overconfidence in the economy. Note that for unbiased firms ($\theta = 1$), optimism and overconfidence are respectively equal to zero and one throughout the state space.

A similar procedure is used to obtain the average annual exit rate in the economy. Given a firm's current state (z_t, K_t), it is possible to compute the probability that the firm will choose to liquidate and exit over the next four quarters, based on a combination of the objective dynamics of the driving process z_t, and the firm's type-specific policy function $K^*(\cdot;\theta)$. The overall exit rate of the economy is then obtained by aggregating these exit probabilities by state (z_t, K_t) using the stationary distribution of firms in the economy.

4.3 Calibration Procedure

Formally the calibration attempts to minimize the squared distance between the vector of data moments G^D and the corresponding model-implied moments $G^M(\psi)$ by choosing the appropriate vector of parameters $\psi = (p, \tilde{\mu}, \tilde{\sigma})'$. Namely, the calibration looks for a global minimum of the objective function $(G^D - G^M(\psi))'W(G^D - G^M(\psi))$, where W is a positive definite weighting matrix, over the space of parameters. Note that in the overoptimism- and overconfidence-only specifications $\tilde{\sigma}$ and $\tilde{\mu}$ are respectively fixed to $\sigma$and 0. $G^M(\psi)$ is in all cases computed directly from the stationary distribution of the model, with the latter obtained from a non-stochastic simulation procedure in the spirit of Young (2010). Therefore, I do not rely on Monte Carlo simulations of data from the model. See Simulation Appendix B for details.

Tables 4 and 5 contain preliminary results from the calibration procedure for each of the specifications. In all cases, a minority of all potential entrants are born unbiased. The largest proportion of unbiased potential entrants is the optimism-only specification, with about 40 percent; by contrast, in both the overconfidence-only and combined overconfidence and overoptimism specifications the share of unbiased potential entrants is well under 10 percent. This speaks to some of the main quantitative results, whereby overconfident firms tend to remain in the population despite being hit by negative shocks because they keep expecting conditions to improve in the near future. Therefore, the model creates a large population of overoptimistic firms from a relatively low share share of all potential births. The opposite is true for overconfident firms, which tend to exit at higher rates because they perceive a lower option value to waiting before making an investment or liquidation decision. To sustain a large population of significantly overconfident firms, the model needs almost all entrants to be overconfident.

The parameters governing overoptimism and overconfidence in the beliefs process, namely $\tilde{\mu}$ and $\tilde{\sigma}$, indicate that biased firms are somewhat more biased than the "average" firm in the economy. This is natural, given that there will be biased and unbiased firms simultaneously in the model economy and so the model requires the biased types to be somewhat more extreme than average for the combination of unbiased and biased firms to match the average degree of distortion as targeted.

5 Results

In this section I present the main quantitative results from the model of firm dynamics in Section 3. I begin by presenting the results from the specifications that introduce overoptimism and overconfidence in isolation to develop economic intuition and a sense of the quantitative magnitude of the effects of each distortion as calibrated in isolation. Then I move on to consider results from the specification with firms that are both overconfident and overoptimistic.

5.1 Effects of Overoptimism and Overconfidence in Isolation

Each of optimism and overconfidence induces a distinct set of distortions to firm dynamics, so I begin by studying the specifications of the model which include unbiased firms and biased firms that are either overoptimistic or overconfident, but not both. Since firm type is a fixed characteristic, the aggregate stationary distribution for both of these specifications is effectively a mixture of two stationary distributions: one for the population of unbiased firms, and a second one for the overoptimistic or overconfident population depending on the specification. Obviously, the population of unbiased firms behaves identically across specifications for the subjective beliefs process because they are assumed to hold rational expectations regardless of how the biased population's beliefs are specified.

Table 6 shows some key moments pertaining to the dynamic behavior of the unbiased population of firms in column (1), in contrast with the overoptimistic and overconfident populations in their respective calibrations, displayed in columns (2) and (3). Intuitively, overoptimistic firms perpetually believe tomorrow's business conditions to be better than the objective expectation would dictate: $\tilde{\mathbf{E}}[\log(z_{t+1})] > \mathbf{E}[\log(z_{t+1})]$. Thus, overoptimistic firms choose to enter the market with business conditions that are about 30 percent lower than the unbiased population's, their overall exit rate is 10 percent lower as firms choose to remain despite receiving bad shocks, and ultimately have lower productivity (9 percent) and profits (37 percent) relative to the unbiased firms in the economy. This means that they also end up being about 10 percent smaller than their unbiased counterparts.

For the overconfident population, the key mechanism is related to real options effects. Recall that overconfident firms underestimate the standard deviation of future business conditions: $\widetilde{\mathbf{SD}}[\log(z_{t+1})] < \mathbf{SD}[\log(z_{t+1})]$. Since investment is subject to a rich set of convex and non-convex adjustment costs, overconfident firms perceive the real option of waiting before making an investment, distinvestment, or liquidation decision to be less valuable. Therefore, these firms make active decisions more often and more aggressively. They choose to liquidate the firm and exit the market more often, as they are reluctant to wait for more information about future conditions, leading to a 40 percent larger exit rate than the unbiased population. Similarly, they enter the market with 16 percent higher productivity because they don't particularly value the option of entering and waiting for a good shock to arrive. And when they do get lucky in the face of a series of positive shocks, overconfident firms are happy to make large investments since they have fewer concerns about having to adjust that investment in the future once more shocks are realized. This combination of low entry and high exit rates, along with large reactions to positive shocks means that overconfident firms that do survive are on average larger than the typical unbiased firm by 56 percent, more 9 percent more productive, and 74 percent more profitable[8].

An interesting moment to look at for different types of firms is the dispersion across firms' marginal products, often regarded as a key statistic about the degree of misallocation of factors across firms. Under frictionless conditions, all firms should equate their marginal product to the (common) cost of capital so dispersion should be zero. Empirically, this is usually not the case, due to frictions preventing adequate reallocation of capital and labor to where they are most productive. All of the specifications in my model generate significant dispersions in marginal products, largely as a consequence of the rich adjustment costs I incorporate in the model, in accordance with the findings of Asker et al. (2014). Indeed, I uncover only small deviations (on the order of about 5 percent) in the dispersion of marginal products for unbiased and biased firms in my model. Among overoptimistic firms, the marginal product of capital is about 4 percent larger than for the unbiased population, whereas for overconfident firms it is about 6.5 percent smaller. Qualitatively, these findings are intuitive to the extent that adjustment costs are, in practice, a smaller friction for the population of overconfident firms, so they overcome the main source of misallocation in the model. By contrast, when overoptimistic firms hold on to their capital and refuse to exit (or overaccumulate capital) this increases the degree of misallocation above that generated by the adjustment frictions. Through the lens of this model, however, belief distortions do not seem to be large contributors to the overall degree of misallocation in the economy, at least when misallocation is measured as the dispersion of marginal products.

The mechanisms that characterize the distortions induced by overoptimism and overconfidence on firm behavior are evident in Figures 8 and 9, which respectively show the distribution of the populations of unbiased, overoptimistic, and overconfident firms across business conditions z and output. Because of free entry and exit, there are few unbiased

[8]Similarly, looking at the endogenous share of unbiased versus biased firms in the stationary distribution, shown in Table7, it is obvious that overoptimistic firms account for a larger share of all firms in the stationary distribution than among all potential entrants, and the opposite is true for overconfident firms in the second specification. Each of these is a result of the distortions to entry and exit induced by the subjective beliefs biases.

firms with low productivity (business conditions) because unlucky firms opt to exit, leaving more productive firms in the market. However, among overoptimistic firms there is a larger proportion of unproductive firms, because unlucky firms are refusing to leave the market as they continue to expect conditions to improve in the future. The opposite is true when looking at the population of overconfident firms, who exit the market more quickly than their unbiased cousins when hit by negative shocks, leaving only the most productive firms to constitute the majority of all firms.

Similarly, when looking at the distribution of firms according to output, overoptimistic firms tend to form a disproportionate share of smaller firms, because they are reluctant to exit despite becoming unproductive and small relative to the rest of the distribution. But there is also a subpopulation of overoptimistic firms that is larger than the largest unbiased firms. These are presumably the lucky overoptimists who respond more strongly to good shocks in expectation of more of them to come in the near future. Overconfident firms, by contrast, are scarce among the smallest firms by output, but constitute an increasing share of the population at higher firm sizes. This result is due both to overconfident firms' higher propensity to exit in the face of adverse shocks, and their more aggressive responses to positive shocks.

5.2 Overoptimism and Overconfidence Together

The full specification with biased firms that are simultaneously overconfident and overoptimistic exhibits a some of the patterns attributed to each of the distortions, while also suggesting that overconfidence is quantitatively more significant. The latter fact is already somewhat evident looking at the calibration results for the full specification in Table 5. Matching the three target moments (exit rate, mean overconfidence, and median overoptimism) in the joint overoptimism-overconfidence specification is achieved largely by adding a degree of optimism to the overconfident calibration, with relatively small changes in the share of firms born unbiased, p, and the degree of overconfidence $\tilde{\sigma}$.

The moments of the overconfident and overoptimistic population in column (4) of Table 6 tell a similar story. All of the moments pertaining to the overconfident and overoptimistic population of firms in the full specification lie between the corresponding moments of the unbiased population and those of the purely overconfident population, and overall close to the latter. Note than for the purely overoptimistic calibration, the moments relating to biased firms go in the opposite direction. So one interpretation for the full specification's results is that overconfidence is a stronger distortion for firm behavior, but its effects are slightly curtailed by the presence of overoptimism. For example, biased and unbiased entrants and incumbents are equally productive in the full specification, as overconfidence and overoptimism push in favor of more and less productive entrants, respectively, and ultimately cancelling each other out.

The same flavor of results is confirmed by looking at the stationary distribution firms by business conditions and by output, in Figures 10 and 11m which closely resemble those of the overconfidence only specification. In particular, there are fewer biased firms with bad business conditions and more that are highly productive, relative to the unbiased population. And there are correspondingly few small biased firms in this specifications, with many monstrously large firms, just as when the biased population is only overconfident.

This specification of the model is remarkably able to match the shape of the distributions of overconfidence and overoptimism in the data, despite making no attempt to match these in the calibration. Compare the model's implied distributions for overconfidence and overoptimism (defined as in section 4.2.3 and calculated for the entire population of firms) in Figure 12 against the corresponding distributions in the Decision Maker Survey, seen in Figures 4 to 7. In both the model and the data, overoptimism looks at first glance roughly symmetric around zero, but with more of the mass to the right of zero so that the average and median firm is overoptimistic about the future. In both the model and the data the distribution of overconfidence seems to be right skewed, with most of the mass between zero and 0.4 so that most firms are severely overconfident, but a long right tail. In the model there is a significant share of firms with overconfidence of 1 (i.e. the unbiased population), whereas in the data there is a fraction of the population that is only mildly overconfident (i.e. overconfidence just under unity) and another fraction that is even underconfident (greater than unity). The similarity between the model and the data on these distribution shapes, which were not directly targeted, suggests that the mechanisms at play in the model are supported by the evidence in the data.

5.2.1 Does overconfidence or overoptimism explain the data better? A formal decomposition exercise

Under construction.

5.3 Firm Size, Overconfidence and Overoptimism: Model vs. Data

Some of the starkest predictions of the model are about the prevalence of biased firms across the firm size distribution. Purely overoptimistic firms tend to cluster among the smallest and the largest firms, as unproductive optimists shrink but fail to exit, while productive ones overinvest in expectation of even better future conditions. Purely overconfident firms, in turn, constitute an increasing share of the population when looking at larger firms, as unproductive ones exit the market and productive ones grow disproportionately large. These findings are evident in Figure 13, for the simple specifications with overconfident or overoptimistic biased firms. In the joint specification, both overconfidence and overoptimism are increasing in firm size because overconfidence is the stronger force in the joint specification, as discussed in Section 5.2.

A natural question to ask at this point is whether there are any empirical relationships between the degree of overoptimism or overconfidence and firm size? I explore this question graphically in Figure 14, in which I show locally weighted regressions of overconfidence and overoptimism against firm size, measured as the log of the mean number of employees, for firms in the Decision Maker Survey. For this particular measure of optimism, there seems to be a slightly negative relationship, but as seen in Table 8 this result is not robust to using different measures of optimism. For overconfidence, by contrast, there is strong and robust evidence that larger firms underestimate future uncertainty to larger degrees, which is consistent across several of my measures of overconfidence as seen in Table 9.

The fact that the model can reproduce this stark feature of the data without having targeted it suggests that the quantitative results from the model are the result of mechanisms that are also at play in the data, effectively serving as "overidentifying restrictions" for the model. Furthermore, the absence of a robust relationship between optimism and size in the Decision Maker Survey data gives some credence to the overall result that overconfidence is the more significant distortion in the full specification with overconfident and overoptimistic biased firms.

5.4 Implications of Belief Distortions on Shareholders

So far, I have considered the implications of belief distortions for firm behavior and dynamics without considering how they impact welfare, in particular how they impact the firms' shareholders. Recall that firms in the model act to maximize the net present value of their profits under their own subjective expectations about the future (see Section 3). However, the firm's own subjective valuation $\tilde{V}(\cdot;\theta)$ in general differs from the objective net present value of cash flows $V(\cdot;\theta)$, coinciding only for firms that are unbiased (θ =1). It is also straightforward to show that conditional on state (z_t, K_t), unbiased firms in fact attain the maximum net present value of profits, while distorted firms, whether they are overoptimistic, overconfident, or both, cannot possibly generate more value: i.e. $V(z_t, K_t; \theta = 2) \leq V(z_t, K_t; \theta = 1)$.

It is tempting to conclude immediately that an economy in which firms are not distorted generates more value for shareholders than one including distorted firms. This reasoning ignores the fact that the distribution of firms across the $Z \times K$ state space is endogenous to the type and degree of distorted beliefs they might have. If the firm distribution with and without distortions is similar, then an economy with distorted firms will tend to generate less value for shareholders. But this might not be the case if, for example, distorted firms' productivity (business conditions, z) and capital-holdings are significantly larger, leading them to be more valuable than smaller, less productive unbiased firms. Both of these things are true for the specifications with overconfident and overconfident-overoptimistic firms, so it is ultimately a quantitative statement whether changes in the endogenous distribution of firms across the state space can overturn losses in firm value due to overconfidence.

I explore the impact of belief distortions on realized shareholder value in the first column of Table 10. For each of the three specifications of the model I compute the aggregate objective value of all firms in the economy, as well as the hypothetical aggregate value in an economy populated by only unbiased firms and display the ratio between the two. The findings show that in specifications where firms are overconfident, the value of the corporate sector is higher than in an economy with only unbiased firms. Aggregate value is almost 60 percent higher when overconfidence is the only distortion and 55 percent when firms are both overconfident and overoptimistic. Firms that are overconfident are more productive and also larger (recall Table 6 and Figures 8 and 9), thus the realized value is higher. This result carries over to the combined specification, consistent with the findings above that overconfidence is the more salient distortion. Overoptimism, on the other hand, takes a toll on the aggregate firm value in the economy because it increases the proportion of small, unproductive firms in the economy, and it also encourages firms to devote resources to investment in "bright futures" that never come to fruition. These dynamics ultimately mean that aggregate value in the specification with overoptimism is 7 percent smaller than it would be for the benchmark unbiased economy.

I consider a second exercise asking how much firm value could be gained by eliminating belief distortions, holding fixed the (initial) distribution of firms across the state space in each specification. In other words, this considers a world in which it is possible to take the current stationary distribution of firms and replace all of the biased managers with unbiased ones. What would be the increase in aggregate value from making that change today? The answers for each specification are displayed in column (2) of Table 10, ranging from 2 percent for the economy with only overoptimism to 8 to 10 percent, respectively in the overconfidence only and combined specifications. On some level these results are relatively counterintuitive, because previously it seemed like there were greater negative consequences from overoptimism, namely in terms of reduced productivity and profitability. But ultimately, as suggested in figure 9, the distribution of firms across the state space in the specification with overoptimism is more similar than when there is overconfidence. There are also fewer biased firms in the stationary distribution with only overoptimism. The large gains from removing overconfidence arise from shareholders gaining access to a lot of the capital that is tied up in the monstrously large firms created by lucky overconfident managers, on one hand, and to the profits lost due to the premature exit of unlucky ones.

The final question about shareholders' welfare under biased versus unbiased firms is whether it objectively makes sense for biased potential entrants to be allowed to enter the economy based only on type, i.e. before knowing their initial draw for business conditions z_0?. There are several forces at play here, since overconfident and overoptimistic entrants, respectively, tend to be more and less productive than the average unbiased entrant. They also tend to be larger than unbiased firms with the same business conditions at entry, meaning that larger initial investments need to be made. Looking at columns (3) and (4) of Table 10, the overconfidence only specification is the sole scenario for which a biased firm with unknown initial business conditions generates positive net present value. As it stands, this means that if an unbiased venture capitalist were to know the type of a manager (but not the firm's productivity), they would be better off not backing overoptimistic or overoptimistic-overconfident managers than by staying out of the market. It would be worth-while for them to back a firm run by an overconfident manager relative to staying out, but would still be better off choosing an unbiased manager.

The analyses in this section are a first step to understanding the welfare implications of overconfidence and overoptimism. But they are also a first look at the question of why there are overoptimistic and overconfident managers running firms in the real world. I intend to look at these issues more closely in further work.

6 Robustness

6.1 Different Adjustment Costs Specification

Under construction.

6.2 Financial Frictions

Under construction.

7 Conclusion

This paper empirically investigates the nature of US firms' beliefs about future sales growth, revealing that most firms are overoptimistic, systematically overestimating growth by 2 to 5 percentage points and underestimating the uncertainty (standard deviation) of future sales by up to 70 percent. This analysis is possible thanks to novel data from a Decision Makers Survey which collects information on businesses' subjective probability distributions over future outcomes on a monthly basis, and also tracks realized outcomes over time.

I use the empirical results about US businesses' beliefs over future sales growth to investigate the impact of overconfidence and overoptimism in a quantitative model of firm-level investment and free entry and exit. Calibrating the model to match the observed degree of belief distortions in the Decision Maker Survey data, I find that overoptimistic firms tend to be unproductive, small, and reluctant to exit the market as they keep expecting improvements in business conditions that fail to materialize. Overoptimistic firms, on the other hand, are quick to make investment, entry, and liquidation decisions because they are unduly certain about future outcomes, and therefore dither less than rational firms would in the face of risk. When the model includes firms that are simultaneously overconfident and overoptimistic, and is calibrated to match the severity both distortions, preliminary results indicate that overconfidence is the more significant distortion to firm behavior.

The results presented in this paper are a first empirical look at firms' subjective beliefs about their own future outcomes, and the quantitative impact of distortions to those beliefs for firm dynamics. As a first look, the paper also points to a number of questions to address in future work. Why do firms hire managers who are overconfident and overoptimistic? What are the quantitative implications of overconfidence and overoptimism on firms' financing choices? Moreover, how do belief distortions in the coporate sector impact the business cycle? The baseline analyses and methods used here will serve as useful starting points to further push the frontiers of economic knowledge.

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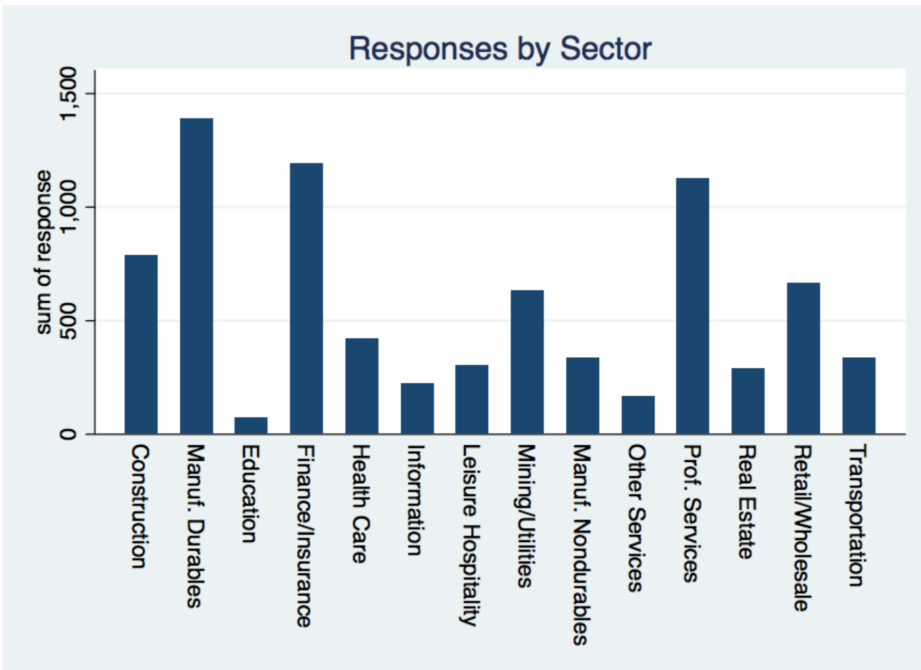
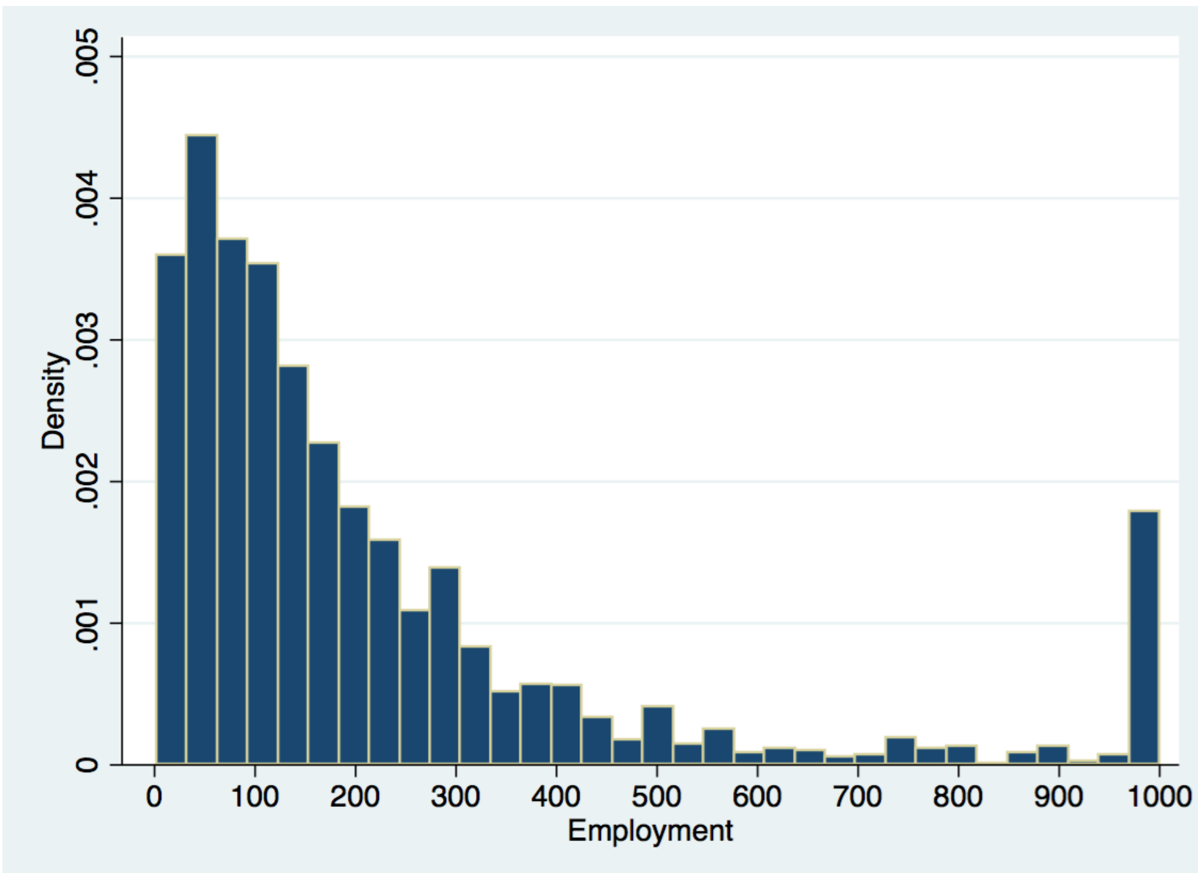
Figure 1: DMS Responses by Sector



Notes: Sectoral affiliation of firms in the Decision Maker Survey. An observation is a firm-month response.

Figure 2: Firms in the DMS by Employment



Notes: Current employment of respondents to the Decision Maker Survey, censored at 1000 employees. An observation is a firm-month response, with firms answering questions about employment once or twice per quarter.

Table 1: Quality of Responses in the DMS

Criterion	(1) **Sales**	(2) **Employment**	(3) **Capex**
Probabilities add to 100%	0.930	0.968	0.923
Scenarios monotonic (Lowest<=Low, etc.)	0.976	0.990	0.969
No scenario has 100% probability	0.992	0.982	0.986
Not all scenarios equal	0.928	0.946	0.915
All	0.897	0.928	0.872
N	2,252	2,324	2,274

Notes: This table shows the fraction of firm-months in the Decision Maker Survey with responses for the indicated module (sales, employment or capital expenditures) that satisfy one or more of the quality criteria for subjective beliefs responses. Observations that fail to report a current value for sales, employment, capital expenditures and profit margins, and which didn't respond to questions about unit costs and average prices are treated as-nonresponses. The quality criteria are whether the subjective probabilities add up to 100%; whether predictions are monotonic (lowest lower than low, etc.); whether no scenario among lowest/low/middle/high/higherst is given 100% likelihood; and whether at least two scenarios have different predictions.

Figure 3: Decision Maker Survey Questions about Sales

Decision Maker Panel

Stanford | CHICAGO BOOTH | FEDERAL RESERVE BANK of ATLANTA

For the <u>current</u> quarter, what would you estimate the total dollar value of your **SALES REVENUE** will be?

$ ______

Looking <u>back</u>, over the last 12 months, what was your approximate percentage **SALES REVENUE** GROWTH rate?

______ %

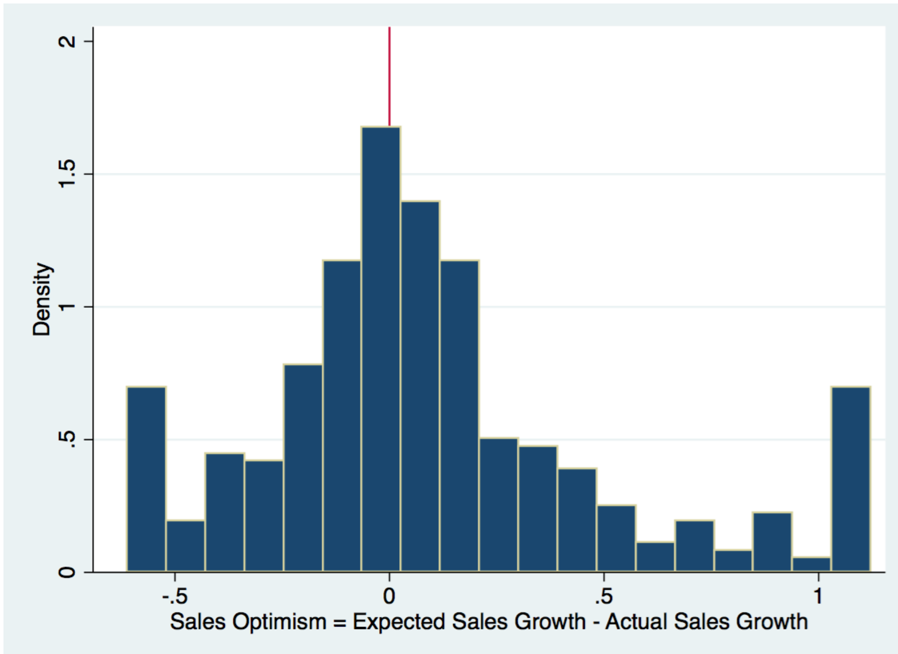
Looking <u>ahead</u>, from now to four quarters from now, what approximate percentage **SALES REVENUE** growth rate would you assign to each of the following scenarios?

The LOWEST percentage sales revenue growth rate would be about:	1	%
A LOW percentage sales revenue growth rate would be about:	2	%
A MIDDLE percentage sales revenue growth rate would be about:	3	%
A HIGH percentage sales revenue growth rate would be about:	4	%
The HIGHEST percentage sales revenue growth rate would be about:	5	%

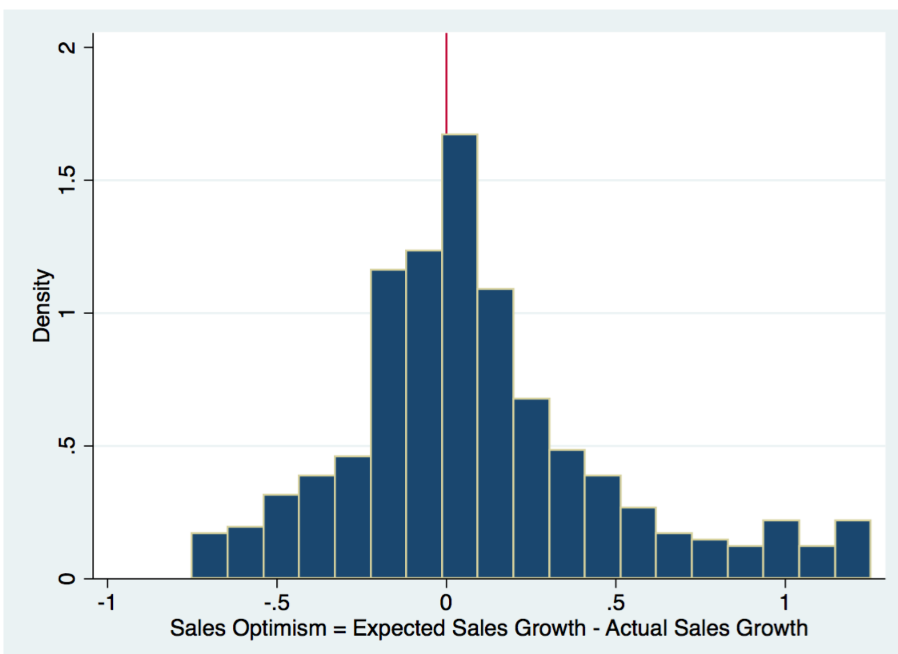
Please assign a percentage likelihood to the **SALES REVENUE** growth rates you entered. (Values should sum to 100%)

LOWEST: The likelihood of realizing a **1%** sales revenue growth rate would be:	20	%
LOW: The likelihood of realizing a **2%** sales revenue growth rate would be:	20	%
MIDDLE: The likelihood of realizing a **3%** sales revenue growth rate would be:	20	%
HIGH: The likelihood of realizing a **4%** sales revenue growth rate would be:	20	%
HIGHEST: The likelihood of realizing a **5%** sales revenue growth rate would be:	20	%
Total	100	%

Figure 4: Distribution of Optimism (Forecast Error)



(a) Raw

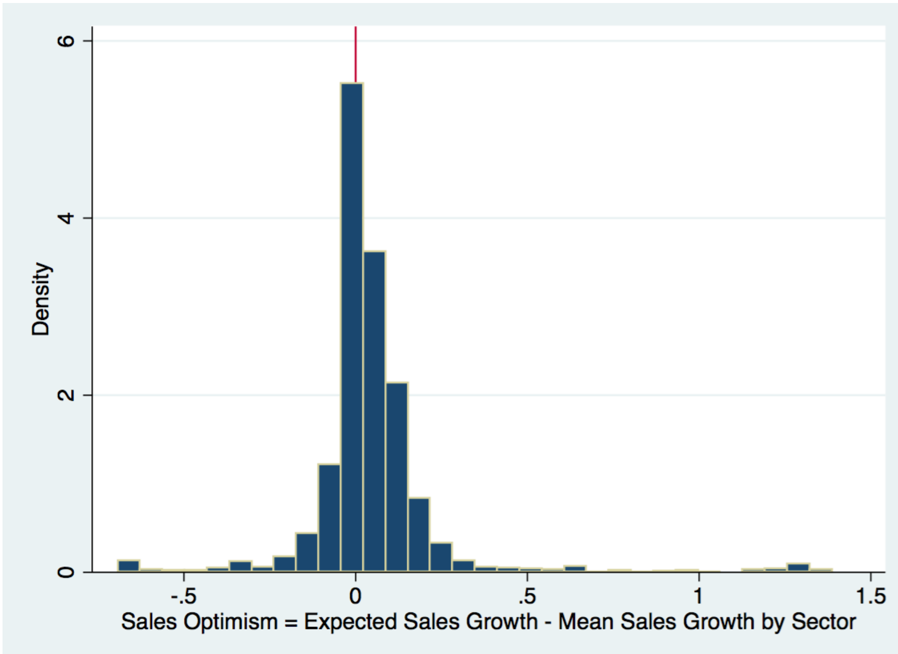


(b) Adjusted for Date, Sector Effects

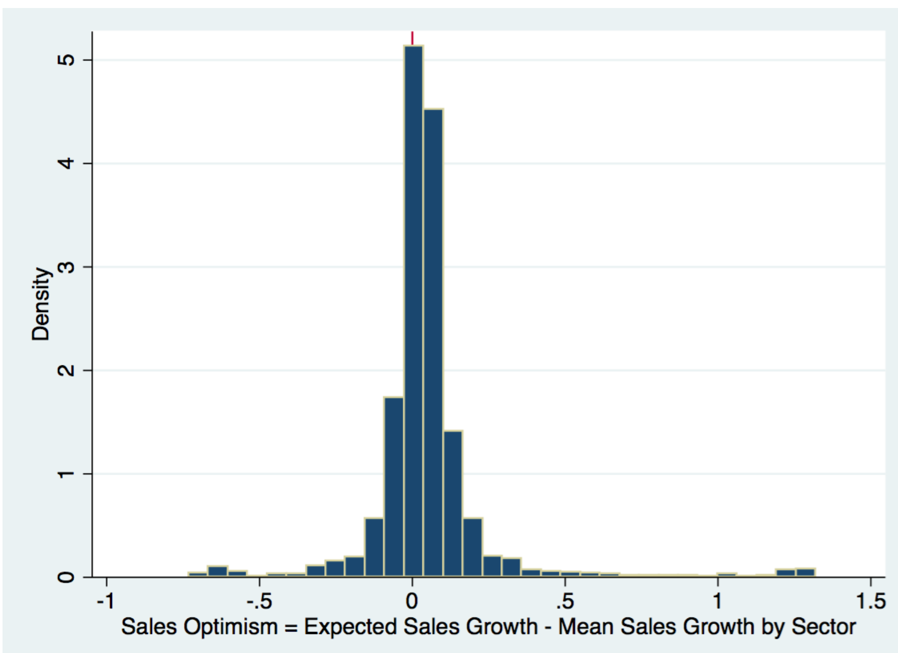
Notes: The distribution of optimism among respondents of the Decision Maker Survey. Optimism here is measured as the difference in percentage points between a firm's expected sales growth rate for the next 12 months and their actual realized growth rate. Panel 5ashows the raw distribution of this measure of optimism, and panel 5b shows the distribution after removing variation attributed to date and sector effects. N = 392.

Figure 5: Distribution of Optimism Over Mean Sectoral Growth

(a) Raw



(b) Adjusted for Date, Sector Effects



Notes: The distribution of optimism among respondents of the Decision Maker Survey. Optimism here is measured as the difference in percentage points between a firm's expected sales growth rate for the next 12 months and average sales growth within their 1-digit sector. Panel 5a shows the raw distribution of this measure of optimism, and panel 5b shows the distribution after removing variation attributed to date and sector effects. N = 1956.

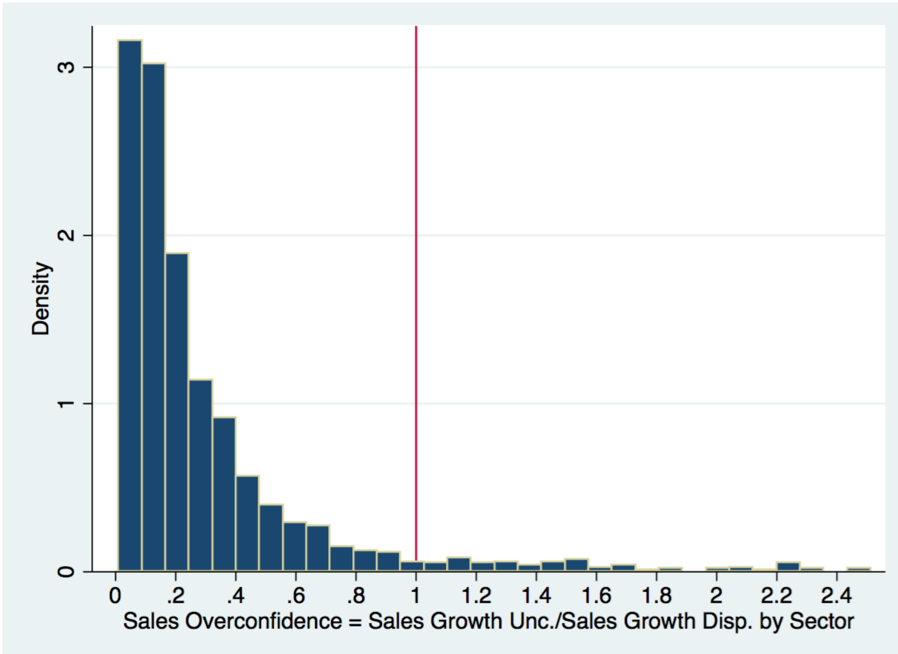
Table 2: Optimism in the DMS

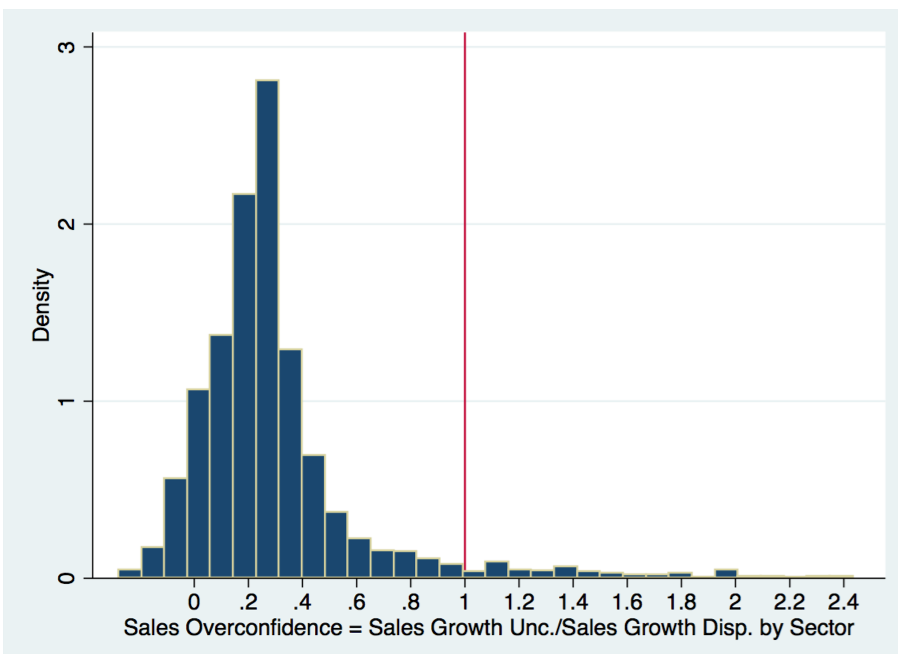
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Sales Optimism = Expected Growth - Benchmark Growth							
Benchmark Growth	Recall, w/in sector		Recall, w/in sales quintile		Realized, w/in sector		Realized, firm-specific	
Mean	0.048***	0.048***	0.048***	0.048***	0.083***	0.083***	0.097***	0.097***
	(0.005)	(0.005)	(0.005)	(0.005)	(0.006)	(0.005)	(0.023)	(0.021)
Median	0.021***	0.034***	0.024***	0.033***	0.070***	0.070***	0.031*	0.047***
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.016)	(0.016)
Sector, Date FE		Y		Y		Y		Y
Observations	1,956	1,956	1,956	1,956	1,932	1,932	392	392
Firms	740	740	740	740	728	728	168	

Notes: This table shows the mean and median sales optimism for each of the specified benchmark growth rates. Robust standard errors in parentheses, clustered by firm for the mean. Means are the product of OLS regression, and medians from quantile regression. Data is from the DMS, with optimism measured as the difference between subjective expected growth and a benchmark growth rate, defined as the mean DHS growth based on recall or realizations data within the stated group (columns 1-6), or as the firm's realized growth rate in columns (7) and (8). In columns (2), (4), (6) and (8), sector and date effects are first removed from the sample, and then the mean and median are estimated by regression and quantile regression on a constant. Sectors correspond to NAICS-1. *** p<0.01, ** p<0.05, * p<0.1

Figure 6: Distribution of Overconfidence, within-sector Sales Growth Dispersion

(a) Raw



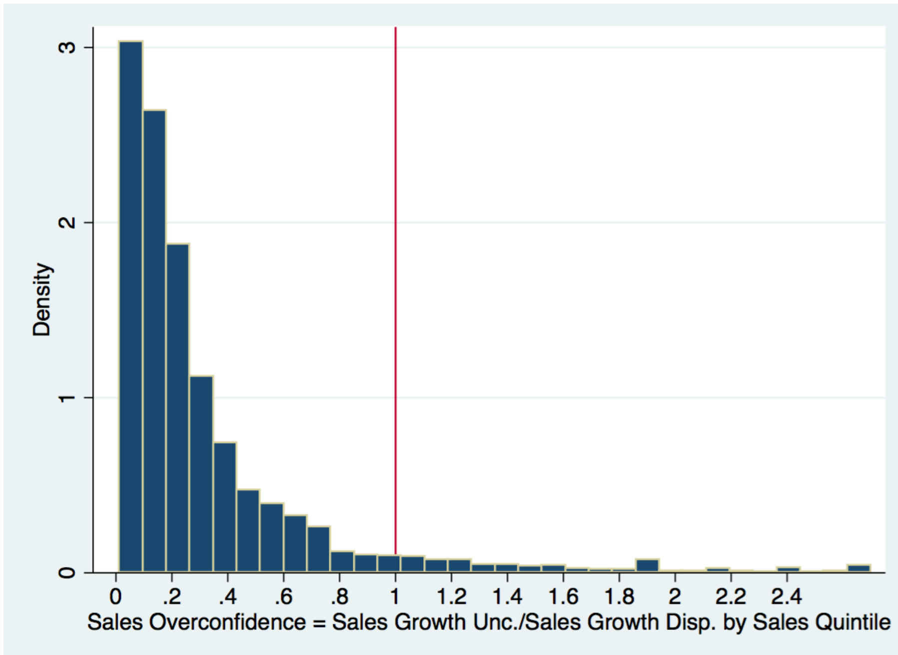


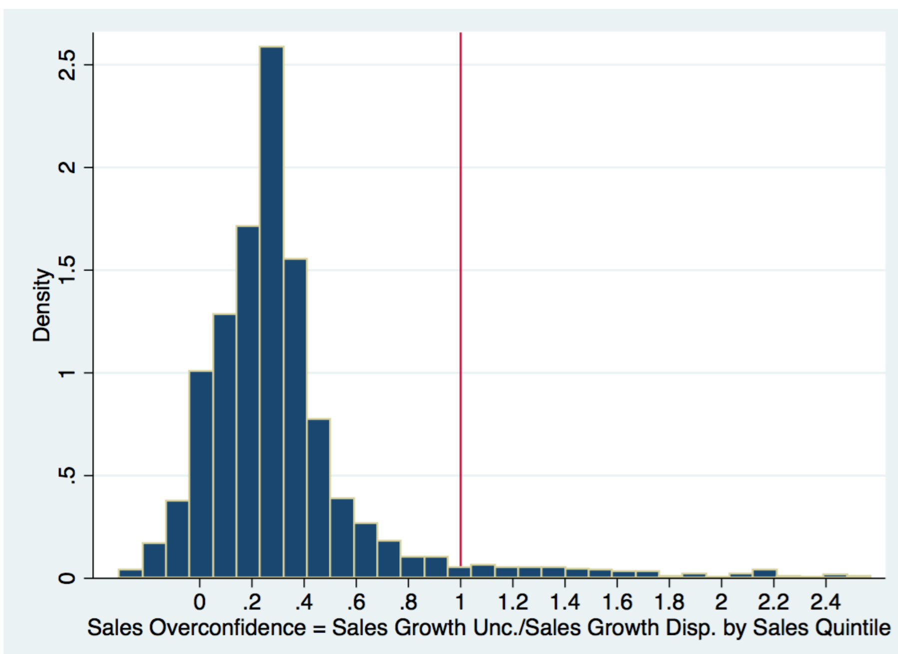
(b) Adjusted for Sector, Date Effects

Notes: Distribution of overconfidence among respondents of the Decision Maker Survey. Overconfidence here is measured as the ratio between a firm's subjective uncertainty about sales growth over the next 12 months and the cross-sectional dispersion of sales growth among firms within their 1-digit sector. Panel 6ashows the raw distribution of this measure of optimism, and panel 7b shows the distribution after nemoving variation attributed to date and sector effects. N = 1954.

Figure 7: Distribution of Overconfidence, within Sales Quintile Sales Growth Dispersion

(a) Raw





(b) Adjusted for Sector, Date Effects

Notes: Distribution of overconfidence among respondents of the Decision Maker Survey. Overconfidence here is measured as the ratio between a firm's subjective uncertainty about sales growth over the next 12 months and the cross-sectional dispersion of sales growth among firms within the same sales quintile. Panel 7ashows the raw distribution of this measure of optimism, and panel 8b shows the distribution after nemoving variation attributed to date and sector effects. N = 1954.

Table 3: Overconfidence in the DMS

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Sales Overconfidence = Subjective Uncertainty (SD) / Objective SD							
Benchmark SD	Recall, w/in sector		Recall, w/in sales quintile		Realized, w/in sector		Realized, w/in sales quintile	
Mean	0.298***	0.298***	0.323***	0.323***	0.204***	0.204***	0.169***	0.169***
	(0.011)	(0.010)	(0.012)	(0.011)	(0.009)	(0.008)	(0.007)	(0.006)
Median	0.180***	0.295***	0.197***	0.354***	0.131***	0.224***	0.124***	0.196***
	(0.005)	(0.040)	(0.006)	(0.054)	(0.004)	(0.040)	(0.004)	(0.037)
Sector, Date FE		Y		Y		Y		Y
Observations	1,954	1,954	1,954	1,954	1,930	1,930	1,941	1,941
Firms	739	739	739	739	727	727	738	738

Notes: This table shows the mean and median sales overconfidence for several specified benchmark dispersions in growth rates. Robust standard errors in parentheses, clustered by firm for the mean. Means are the product of OLS regression, and medians from quantile regression. Data is from the DMS, with overconfidence measured as the ratio between subjective uncertainty and a benchmark dispersion measure for sales growth rates, defined as the cross sectional standard deviation of DHS growth based on recall or realizations data within the stated group. Sectors correspond to NAICS-1. *** p<0.01, ** p<0.05, * p<0.1

Table 4: Calibration

Specification	Target Moment	Model	Target
Overoptimism	Annual Exit Rate	0.093	0.100
	Median Sales Optimism	0.030	0.030
Overconfidence	Annual Exit Rate	0.135	0.100
	Mean Sales Overconfidence	0.295	0.300
Overconfidence and Overoptimism	Annual Exit Rate	0.132	0.100
	Median Sales Optimism	0.020	0.030
	Mean Sales Overconfidence	0.300	0.300

Notes: Model-implied moments and their data counterparts that are targeted for each specification. Model moments are computed from the stationary distribution of the model obtained by non-stochastic simulation. The target annual exit rate data is from Business Dynamics Statistics from the US Bureau of Census. Optimism and overconfidence targets come from the from the Decision Maker Survey.

Table 5: Calibration Results

Specification	Parameter		
	p	$\tilde{\mu}$	$\tilde{\sigma}$
Overoptimism	0.277	0.008	$\mathbf{1}\sigma$
Overconfidence	0.040	**0**	0.187σ
Overconfidence and Overoptimism	0.083	0.018	0.196σ

Notes: Calibration results from minimizing the squared distance between model and data moments for each specification. Values in bold are fixed in that specification and thus not part of the minimum distance calibration.

Table 6: Features of the Unbiased, Overoptimistic, and Overconfident Populations

	(1)	(2)	(3)	(4)
Moment	**Unbiased**	**Overoptimistic**	**Overconfident**	**Overoptimistic & Overconfident**
Annual exit rate	0.10	0.09	0.14	0.13
Mean z of entrants	1.84	1.32	2.14	1.84
Mean z of incumbents	1.55	1.41	1.69	1.61
Mean profits	57.35	36.10	99.75	83.61
Mean output	165.83	149.37	259.83	255.99
$SD[\log(MRPK)]$	0.338	0.351	0.316	0.320

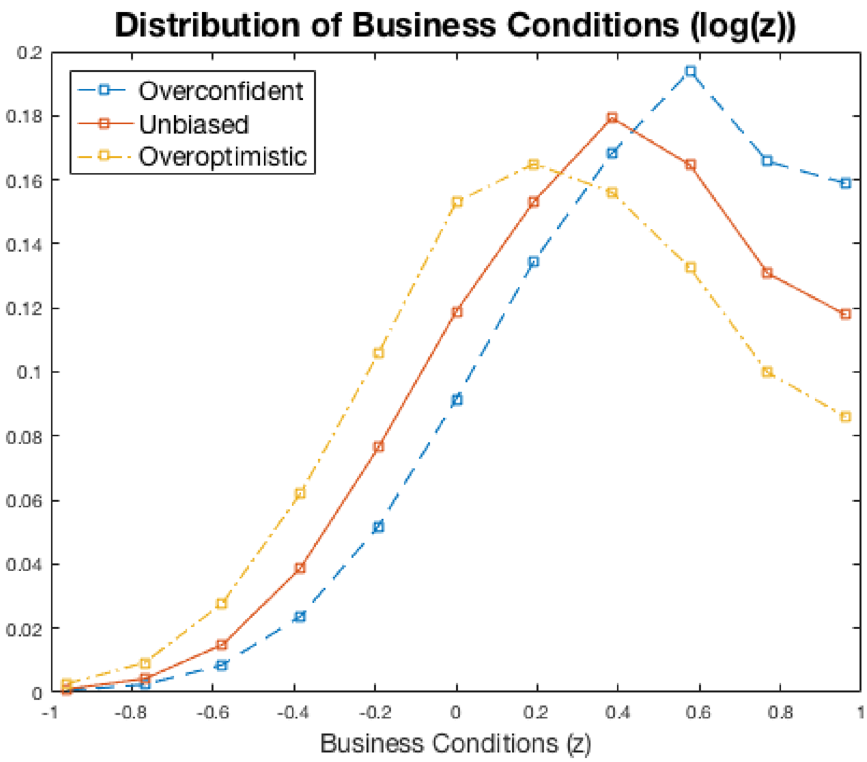
Notes: This table displays moments pertaining to each of the unbiased, and biased populations of firms across the several specifications in the model. Column (1) shows moments for in the unbiased population, which behaves identically across all specifications. Columns (2) and (3) respectively show moments for the biased population in the two specifications of the model that include unbiased firms and biased firms that are either overconfident or overoptimistic but not both. Column (4) reports moments about the behavior of firms in the full specification, in which the biased firms are both ooveroptimistic and overconfident.

Table 7: Shares of Biased and Unbiased Firms by Specification

		Share in Stationary Distribution (Endogenous)	
Specification	**Unbiased Share of Potential Entrants, p**	**Unbiased**	**Biased**
Overoptimism	0.277	0.258	0.742
Overconfidence	0.040	0.053	0.947
Overconfidence & Overoptimism	0.083	0.108	0.892

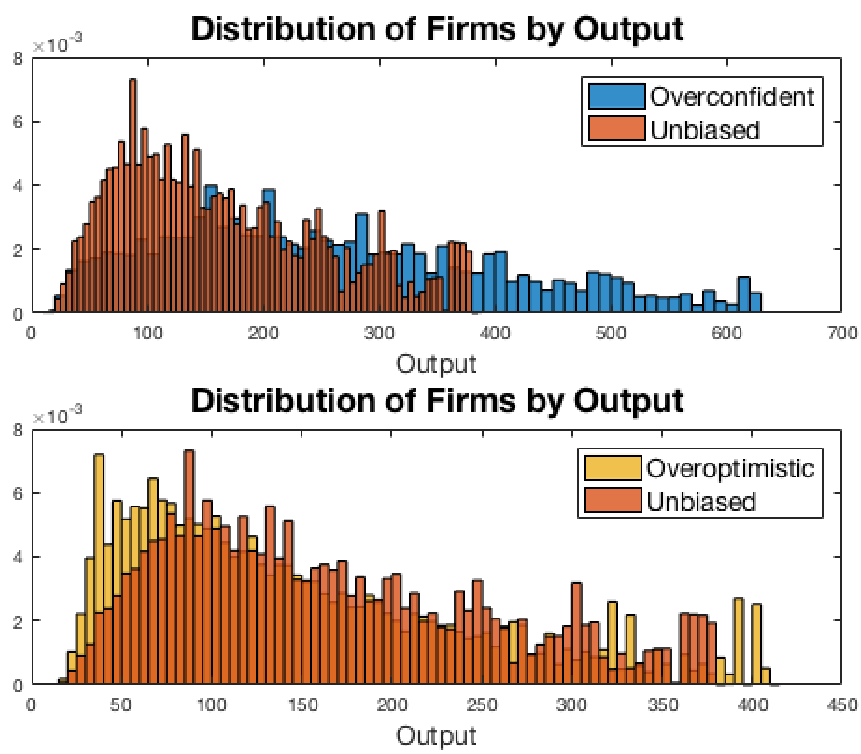
Notes: This table shows for each of the specifications for which the model in Section 3 is calibrated, the unbiased share of all potential entrants, which corresponds to calibration parameter p. It also shows the shares of unbiased and biased firms in the endogenous stationary distribution of firms across the state space. The latter are calculated directly off the stationary distribution, which is computed using non-stochastic simulation.

Figure 8: Distribution of Firms by Business Conditions z: Basic Specifications



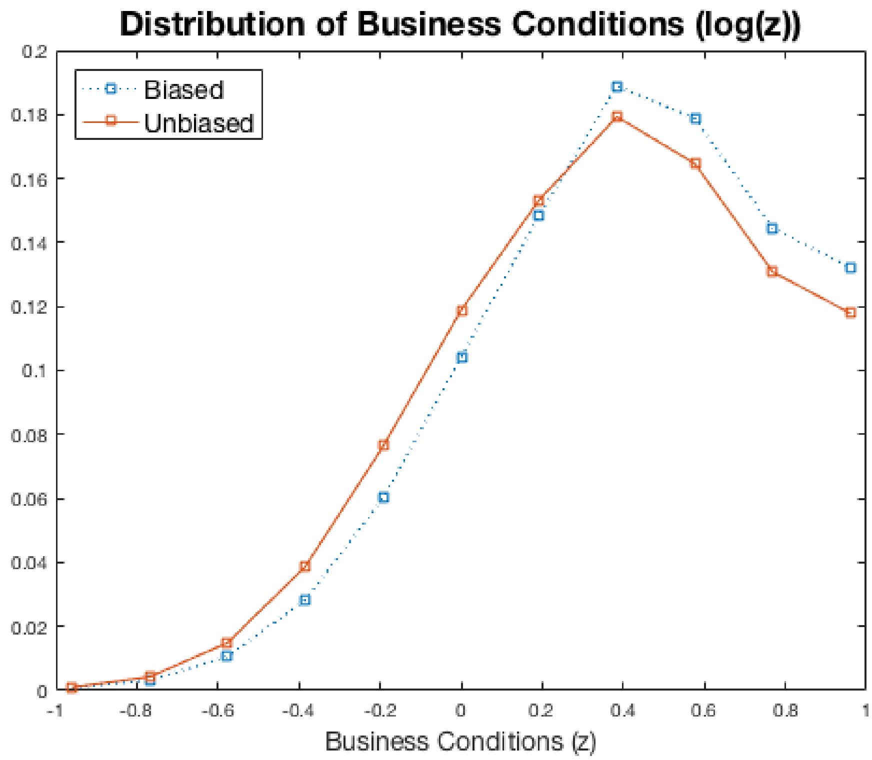
Notes: This figure plots the distribution of firms by business conditions for the specifications in which biased firms are either overconfident or overoptimistic. The unbiased population is identical in both specifications. The plots labeled "overconfident" and "overoptimistic" show the distribution for the population of biased firms in the relevant specification. Calculated from the stationary distribution for each specification.

Figure 9: Distribution of Firms by Output (Revenue): Simple Specifications



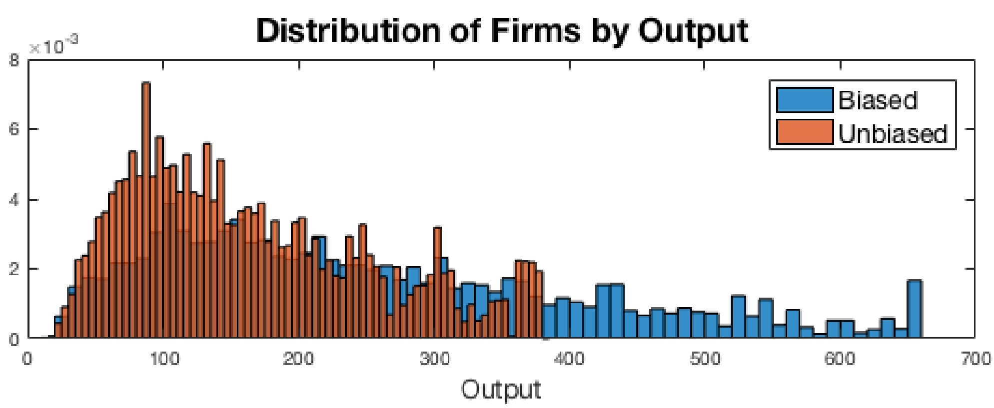
Notes: This figure plots the distribution of biased and unbiased firms by output in the overconfidence-only and overoptimism-only specifications of the model. Histograms constructed by making 100,000 random draws from the stationary distribution for each specification.

Figure 10: Distribution of Firms by Business Conditions z: Full Specification



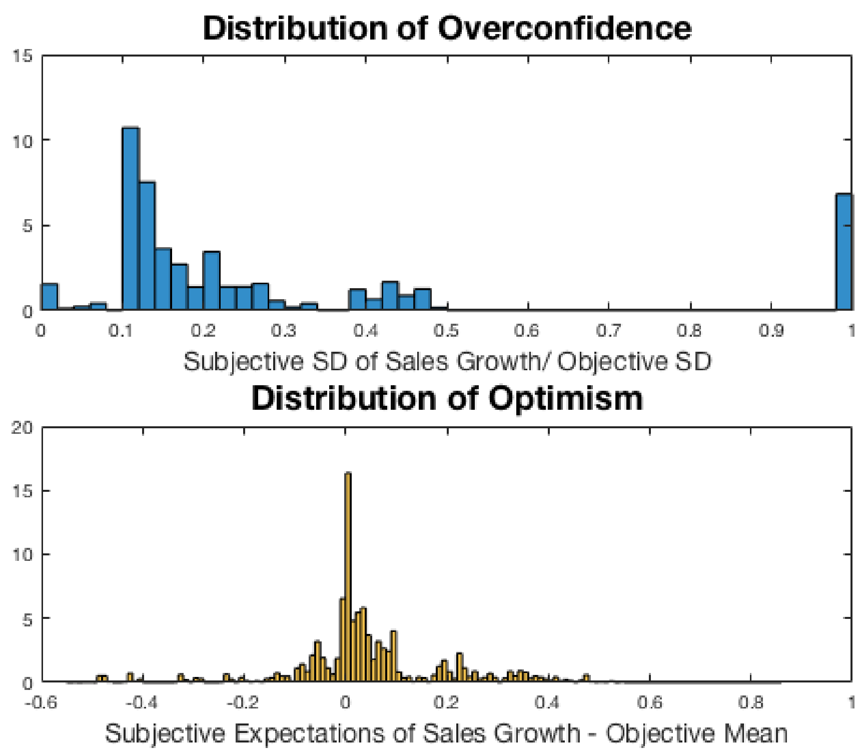
Notes: This figure plots the distribution of unbiased and biased firms by business conditions in the full specification of the model with biased firms that are both overconfident and overoptimistic. Calculated from the stationary distribution implied by the full specification.

Figure 11: Distribution of Firms by Output (Revenue): Full Specification



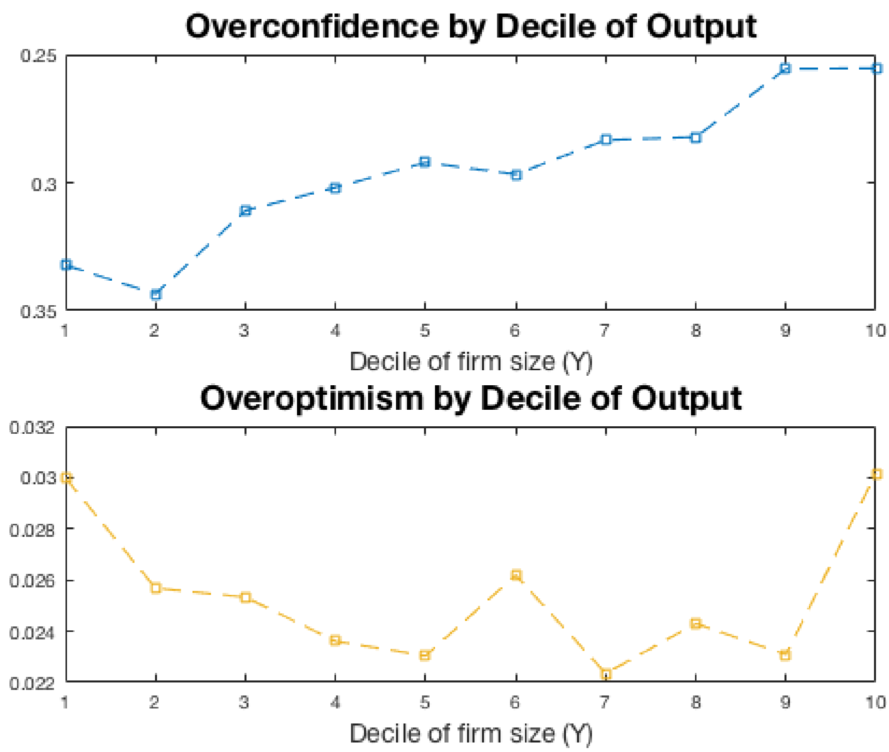
Notes: This figure plots the distribution of biased and unbiased firms by output in the full specification with biased firms that are both overoptimistic and overconfident. Histograms constructed by making 100,000 random draws fromt he stationary distribution for each specification.

Figure 12: Distribution of Overoptimism and Overconfidence in the Full Specification



Notes: This figure shows the distribution of overconfidence (above) and overoptimism (below) in the specification for which the biased population of firms is both overconfident and overoptimistic. The figure is constructed by making 100,000 draws from the stationary distribution and plotting the resulting histogram.

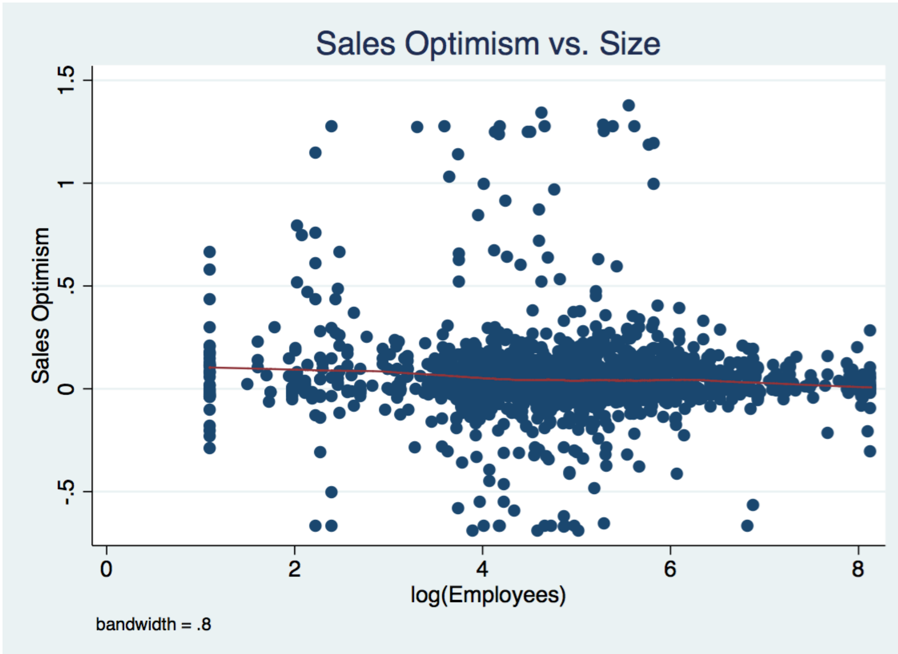
Figure 13: Belief Distortions and Firm Size in the Model



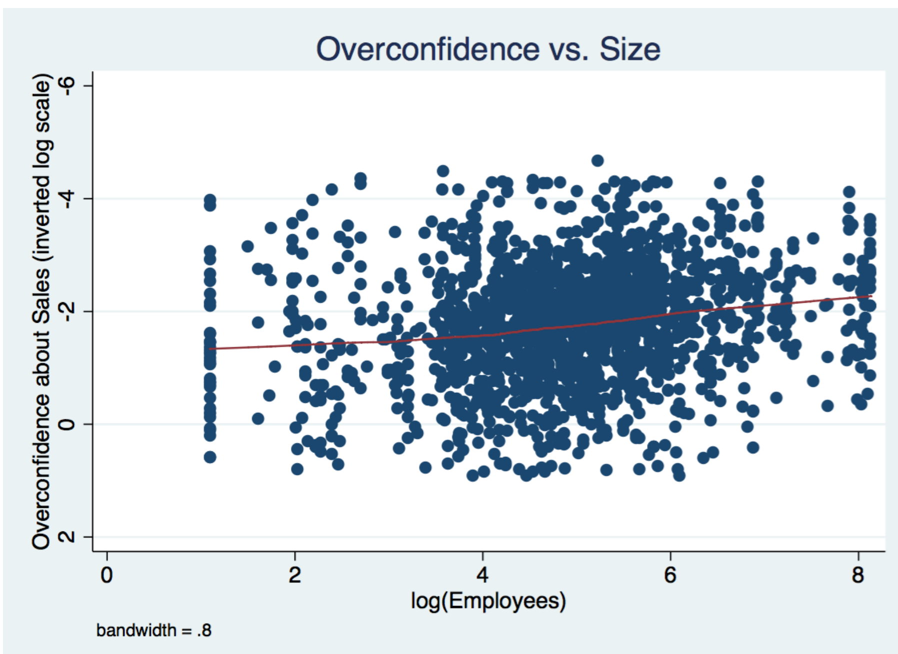
Notes: This figure plots the degree of optimism and overconfidence by decile of output in the specifications of the model which respectively include only overoptimistic or only overconfident biased firms. In each decile, the degree of overconfidence (overoptimism) is computed as the weighted average between the mean (median) overconfidence in the economy and one (zero), where the weights are the shares of biased and unbiased firms, respectively within that particular decile of the output distribution.

Figure 14: Belief Distortions and Firm Size in the DMS

(a) Overoptimism



(b) Overconfidence



Notes: These figures plot measures of overconfidence and overoptimism in the Decision Maker Survey against firm size, along with the corresponding locally-weighted regressions. Firm size is measured as the log of mean employment at the firm level. Overoptimism is measured as the difference between a firm's expected sales one-year sales growth and the mean sales growth within their sales quintile. Overconfidence is measured (in logs) as the ratio between a firm's subjective uncertainty about one-year sales growth and the dispersion of sales growth rates among firms in the same sales quintile, and plotted on an inverted scale. (More overconfidence means a smaller ratio between subjective and objective uncertainty).

Table 8: Overoptimism vs. Firm Size

Dependent Variable	**Sales Optimism = Subjective Sales Growth Expectations - Benchmark Growth**									
Benchmark Growth	Realized, w/in Sector		Recall, w/in Sales Quintile		Realized, w/in Sector		Realized, w/in Sales Quintile		Realized, firm ex-post	
log(Employment)	-0.010**	-0.007*	-0.022***	-0.022***	-0.004	-0.007*	0.047***	0.052***	0.014	0.013
	(0.004)	(0.004)	(0.004)	(0.004)	(0.005)	(0.004)	(0.004)	(0.005)	(0.019)	(0.021)
Sector Fixed Effects	N	Y	N	Y	N	Y	N	Y	N	Y
Date Fixed Effects	N	Y	N	Y	N	Y	N	Y	N	Y
Observations	1,876	1,876	1,876	1,876	1,856	1,856	1,863	1,863	392	392
R-squared	0.005	0.066	0.021	0.044	0.001	0.178	0.085	0.136	0.002	0.099
Firms	669	669	669	669	661	661	668	668	168	168

Notes: This table regresses different measures of optimism on the log of mean employment reported by the firm. Each observation is a firm-month, with firms responding questions about sales and employment once or twice per quarter. Optimism is measured as the difference between the subjective expectation of one-year sales growth and a benchmark sales growth measure. In columns 1 to 8 this benchmark is the mean sales growth within a sector or sales quintile, based on recall or realizations data as indicated, while in columns 9 and 10 it is actual ex-post sales growth as reported by the firm. Robust standard errors, clustered by firm in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.1$

Table 9: Overconfidence vs. Firm Size

Dependent Variable	**log(Sales Overconfidence) = -log(Subjective SD of Sales Growth/ Benchmark SD)**							
Benchmark SD	Recall, w/in Sector		Recall, w/in Sales Quintile		Realized, w/in Sector		Realized, w/in Sales Quintile	
log(Employment)	0.154***	0.162***	0.145***	0.125***	0.185***	0.162***	0.072***	0.047*
	(0.027)	(0.026)	(0.027)	(0.026)	(0.028)	(0.026)	(0.027)	(0.026)
Sector Fixed Effects	N	Y	N	Y	N	Y	N	Y
Date Fixed Effects	N	Y	N	Y	N	Y	N	Y
Observations	1,875	1,875	1,875	1,875	1,855	1,855	1,862	1,862
R-squared	0.038	0.347	0.033	0.358	0.052	0.379	0.008	0.350
Firms	669	669	669	669	661	661	668	668

Notes: This table regresses different measures of overconfidence on the log of mean employment reported by the firm. Each observation is a firm-month, with firms responding questions about sales and employment once or twice per quarter. Overconfidence is measured as the ratio between the subjective standard deviation (uncertainty) of one-year sales growth and a benchmark standard deviation for sales growth. This benchmark is the standard deviation of sales growth within a sector or sales quintile, based on recall or realizations data as indicated. Robust standard errors, clustered by firm in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.1$

Table 10: Belief Distortions and Shareholder Value

Specification	(1) **Relative Firm Value**	(2) **Potential Value Gains**	(3) **Ex-Ante Value of Biased Entrant**	(4) **Ex-Ante Value of Unbiased Entrant**
Overoptimism	0.93	1.02	-20.13	285.30
Overconfidence	1.59	1.08	112.35	285.30
Overconfidence and Overoptimism	1.55	1.10	-77.65	285.30

Notes: Column (1) of this table shows how much aggregate shareholder value is generated by each specification of the model as a fraction of the value generated by an economy with only unbiased agents. Column (2) displays the value that could be generated in each specification by replacing biased managers with unbiased managers and then letting the economy transition to the stationary distribution with only unbiased agents, as a fraction of the aggregate value generated by the economy with both biased and unbiased managers. Columns (3) and (4) show the ex-ante expected value of a potential entrant conditional on type, but before knowing its initial business conditions z_0.

A Data appendix

Under construction.

B Simulation appendix

Under construction.