Taxation, Redistribution and Frictional Labor Supply*

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Abstract

We analyze the implications of ex ante dispersion in worker talents and a frictional labor market for the design of tax and benefit systems. Our model features on and off the job search, job ladders and equilibrium income and profit dispersion within talent markets. In a baseline setting with no talent dispersion, the optimal system consists of an unemployment benefit financed out of a simple lump sum tax on workers. The benefit is high enough to suppress worker income and firm profit dispersion, deter worker poaching and collapse job ladders. With talent dispersion, high benefit levels drive less talented workers out of the market and are prohibitively costly. Active talent markets are frictional. Taxes impact the dispersion of worker incomes and firm profits within these markets. These effects shape and modify conventional optimal tax formulas. *(JEL D31, H21, H24, J60, J62, J65)*

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1 Introduction

A worker's pay depends upon the surplus that she generates in production and the share of that surplus that she captures as income. She may be poor because she has low marketable talent or because she is matched with a firm that extracts most of the surplus she produces (or both). The opportunity to (re)match with a firm that pays more and extracts less creates job ladders for equally talented workers. We explore the optimal design of tax and benefit policies in the face of both ex ante dispersion in worker talent and the ex post risk of falling off or getting stuck on the job ladder. In our model, higher marginal taxes increase the pre-tax incomes firms must pay to keep workers at the bottom of job ladders active. This increase ripples up through job ladders as higher paying firms seek to maintain their size and successfully poach workers from lower paying ones. Firm profits are correspondingly squeezed and with them the revenues derived from profit taxation. The social value of higher marginal taxes in an affine tax setting turns on which workers benefit from the profit squeeze. On the one hand, an income tax induced profit squeeze in combination with profit taxation redistributes *within* talent markets from those at the top to those at the bottom of job ladders. This is desirable under standard redistributive social preferences. On the other hand, it redistributes *across* talent markets from lower to higher talents who both generate more surplus and give up more in profit. This is undesirable. Quantitative analysis suggests that the second is the dominant force and that frictional considerations create a motive for lower marginal taxes in affine settings. When the policy maker is free to select any non-linear tax function, the profit squeeze channel creates a motive for lower marginal tax rates across a range of incomes.

Our model combines a standard public finance framework featuring heterogeneous worker talent, an intensive labor supply margin and taxes with a Burdett and Mortensen (1998) type frictional labor market. As in standard public finance models, private variation in worker talent creates a motive for redistribution, that must be traded off against the distortion to the consumption-effort margin created by redistributive taxation. In addition, as in the Burdett-Mortensen model, workers engage in off and on the job search in a frictional labor market. They experience "market luck" unrelated to their talent: job destruction shocks that consign them temporarily to unemployment and randomly arriving opportunities to climb job ladders and find less extractive employers.[1] The policymaker is assumed to observe

[1]This combination of assumptions enriches the standard treatment of the labor market in public finance models and aligns with both recent microeconometric work that incorporates on-the-job

(and tag) the current employment status (working or not working) and the income of the worker, but nothing else.[2] She is, thus, constrained to select a benefit paid to those who are not working and a tax function applied to the incomes of those who are. The policymaker observes firm profit and in our benchmark version (optimally) taxes it at 100%.[3] In this setting, tax and benefit policy impacts workers' intensive effort and extensive participation margins. Moreover, it affects the equilibrium shape of the job ladders that emerge and, hence, the amount of ex post or career "luck" that workers face.

We derive and quantitatively evaluate optimal tax equations for such an environment. We focus first on the affine tax case and then generalize results to the nonlinear setting. In our frictional economy, some part of the surplus created by a worker is captured by the firm in the form of profit. Within each talent market there is a minimal income that firms must deliver to workers and a corresponding maximal profit-per-worker level that they can extract. Firms distribute themselves across talent markets and profit-per-worker offers so as to maximize total profit. Some firms make high profit-per-worker offers (low income offers) to workers. These firms fail to attract and retain many workers and, thus, remain small. They make a large markup on a small population of (bottom-of-the-job-ladder) workers. Other firms make low profit-per-worker offers (high income offers). They make equal profit to the first type of firm via low markups on many workers. An increase in the marginal tax rate at a given income reduces the effort and output produced by workers in a neighborhood of that income. In addition, it mechanically reduces the after-tax income of more talented workers earning higher incomes. In the absence of any adjustment in the benefit paid to non-working agents, the maximal profit that a firm can extract from these workers is reduced (and the minimal pre-tax income received by them correspondingly raised). This reduction ripples through the entire distribution of profits-per-worker within higher talent markets. Both tax revenues and worker utilities are impacted by this "profit channel" effect. On

search and macroeconomic work that stresses the role of on-the-job search in reconciling observed wage dispersion with the short duration of job search and unemployment. However, as we discuss, our qualitative insights extend to other frictional models that generate heterogeneity in both the surplus workers generate and the share of that surplus that they extract.

[2] We exclude dynamic tax functions that condition on histories. In our model income is related to both talent and progress up the job ladder. Our restrictions on policy imply that a policymaker cannot distinguish between two workers earning the same income one of whom is an unlucky talented worker who has not progressed far up the job ladder and the other a lucky talented worker who has.

[3] We assume that the social marginal value of public funds exceeds that of profits if left in private hands. Since the former can be redistributed as a lump sum transfer, it equals the average marginal utility of workers.

the revenue side, lower profits depress profit tax receipts, while raising labor incomes and labor income tax receipts. On the (direct) utility side, the increase in incomes and, hence, worker consumption partially offsets the adverse mechanical impact of higher taxes on worker welfare. Thus, the suppression of profits via higher income taxes has both costs (lower profit tax revenues) and benefits (higher pre-tax labor incomes). In the affine tax setting, the policymaker applies the same marginal tax to all working agents. A rise in this rate suppresses profits and raises incomes across all active workers. However, the costs and benefits of this are not distributed uniformly across the population. On the one hand, *within* each talent market, workers lower on the job ladder benefit more from the suppression. Since, under standard welfare criteria these poorer workers have higher social marginal welfare weights, this creates a motive for higher marginal taxation. On the other hand, workers in high talent markets generate more surplus and surrender more as profit. Thus, profit suppression redistributes across talent markets away from poorer low talents and towards richer high talents. Under standard welfare criteria this is a motive for lower marginal taxation. The impact of the profit channel on tax determination depends on the balance of these forces. In the nonlinear tax setting similar considerations play out. Optimal nonlinear tax formulas are extended to include terms that capture the costs and benefits of suppressing the profits collected from (and correspondingly raising the incomes received by) those on higher incomes.

To assess the quantitative implications of the profit channel for optimal policy, we calibrate our model to US income data. Under a baseline assumption on the ratio of the job finding to the job destruction rate (and, hence, degree of labor market frictions) taken from the literature and an approximation to current US tax policy, we back out the talent distribution implied by income data. We then compute optimal affine and optimal nonlinear tax rates. To explore the implications of abstracting from labor market frictions, we raise the job finding-job destruction ratio, recalibrate the talent distribution and recompute optimal policy. Our quantitative results imply that as the job finding-job destruction ratio increases above our baseline value (and the economy approaches a frictionless one), optimal affine tax rates rise. Thus, the profit channel (in combination with an optimal profit tax) works in the direction of lower marginal rates (with higher rates disproportionately benefiting more talented higher income earners who have more to gain from profit suppression). Optimal nonlinear tax schedules also exhibit lower marginal rates over wide range of middle and high incomes at our baseline job finding-job destruction rate values. This stems partly from the desire to avoid suppressing profits and profit revenues extracted from even higher earners.

The remainder of the paper proceeds as follows. After a brief literature review, Section 2 describes our baseline environment (with affine taxes). This section outlines how taxes impact competitive equilibria in our heterogeneous talent/frictional labor market setting. Sections 3 and 4 state and characterize the solution to the policymaker's (affine tax) problem in this setting. The latter also highlights two limiting cases: the non-frictional model and the homogenous talent model. Section 5 describes the policy implications of some extensions of our baseline environment. Section 6 presents quantitative results for the affine tax environment. Optimal nonlinear tax formulas are derived in Section 7 and quantitatively evaluated in Section 8. Section 9 concludes. Proofs and calibration details are provided in the appendices.

Literature The literature on optimal nonlinear income taxation originates with Mirrlees (1971). Saez (2001) recasts the optimal tax formulas of Mirrlees (1971) in terms of tax elasticities of income and attributes of the income distribution. These papers and most other contributions to the optimal nonlinear tax literature adopt a simple specification of the labor market in which there is a single market (and price) for effective labor and no market frictions. More recently, a new generation of normative public finance models has stressed fiscal spillovers and general equilibrium effects in richer labor market settings. Papers by Rothschild and Scheuer (2013) and Rothschild and Scheuer (2014) consider policymaking in a Roy model with multidimensional talent. Ales et al. (2015) analyze the implications of technical change for tax policy prescriptions in a related model. Ales and Sleet (2016) and Scheuer and Werning (2016b) use an assignment model to analyze the taxation of high income earners. Papers by Scheuer and Werning (2016a) and Sachs et al. (2016) provide unifying treatments across a variety of environments.

A smaller literature considers tax design in situations with search frictions. Boone and Bovenberg (2002) explore how taxes can be used to correct inefficiencies in settings with random (off-the-job) search, Nash bargaining between firms and workers and either free entry or a limited supply of firms. There is no on-the-job intensive effort margin, but unemployed workers exert costly effort in job search. Hungerbühler et al. (2006) augments the framework of Boone and Bovenberg (2002) with variation in talent (but omits search effort). In this setting, all workers with the same talent earn the same wage, but wages vary across talents. Optimal nonlinear taxation redistributes from more to less talented workers, but also deters workers from "bargaining aggressively". Thus, high marginal taxes distort the economy by encouraging too much job creation and output. Further variations on and extensions of these models are contained in Boone and Bovenberg (2006) and Lehmann et al. (2011). More recently Golosov et al. (2013) consider policy design in a model with directed search and no ex ante talent heterogeneity. Our paper advances this literature by analyzing optimal income taxation in an environment featuring both on and off the job search, ex ante talent heterogeneity and an intensive effort margin. Our focus on on-the-job search is partly motivated by the widespread adoption of the on-the-job search assumption in the structural job search literature.[4] It is also partly motivated by the observation of Hornstein et al. (2011) that large unemployment to work transitions and the relatively short duration of unemployment is inconsistent with very much frictional wage dispersion in models that only feature off-the-job search, whether random or directed, and have plausible implications for the value of non-market time and for worker discount factors.

2 Baseline Environment

Worker preferences and constraints The economy is inhabited by a unit mass of agent-workers and a unit mass of firms. In each period, a fraction ρ of the worker population survives and a fraction $1-\rho$ dies and is replaced by an equal number of new borns. Workers have preferences over stochastic lifetime consumption and effort allocations $\{c_t, y_t\}_{t=0}^{\infty}$ of the form:

$$(1-\beta)\sum_{t=0}^{\infty}\beta^t E[U(c_t - h(y_t))],$$

where U and h are, respectively, concave and strictly convex functions and both are increasing and twice differentiable. In addition, $h(0)$ is normalized to 0. The discount factor $\beta \leq \rho$ incorporates the worker's survival probability and her patience.[5]

In our baseline environment we abstract from private insurance or savings. Hence, the consumption of an employed worker earning x and facing tax function T is just $c = x - T[x]$. We also initially restrict attention to affine tax functions of the form: $T[x] = -L + \tau x$, with L a lump sum transfer given to all employed agents and τ the marginal income tax rate. Later we generalize the analysis to allow for nonlinear income tax functions. Unemployed agents exert no effort and receive a

[4]Rich structural job search models featuring on-the-job search are analyzed by Bontemps et al. (2000), Postel-Vinay and Robin (2002), Lise et al. (2016) amongst many others.

[5]We sometimes consider the limiting case $\beta = \rho = 1$. Worker payoffs are then identified with the aggregator $\lim_{T\to\infty}\frac{1}{T+1}\sum_{t=0}^{T}E[U(c_t - h(y_t))]$.

benefit b. Hence, for them $c = b$. It is convenient to call the argument $c - h(y)$ of U a worker's *current payoff* and to denote the current payoff of an employed worker earning income x and exerting effort y by:

$$\Psi^e(x, y) = x - T[x] - h(y).$$

Similarly, the current payoff of an unemployed worker is:

$$\Psi^u(b, 0) = b.$$

Letting $i_t \in \{e, u\}$ denote the employment status of a worker at t and substituting the expressions for Ψ^i into the worker's preferences gives the following worker objective over employment-income-effort allocations $\{i_t, x_t, y_t\}_{t=0}^{\infty}$:

$$(1 - \beta) \sum_{t=0}^{\infty} \beta^t E[U(\Psi^{i_t}(x_t, y_t))].$$

Worker employment, income and effort allocations We now discuss the determination of a worker's employment, income and effort allocation. Workers draw a permanent talent shock $\theta \in \Theta := [\underline{\theta}, \overline{\theta}]$ from a distribution K with density k at the beginning of their lives. Worker sell their effort in talent specific labor markets. Firms observe worker talent and sort themselves across these markets. As discussed below firms may be unable to earn non-negative profits in the lowest talent markets in which case these markets are inactive and the workers within them do not participate in the labor market.

Workers search for employment opportunities in active talent markets. Surviving workers whether unemployed or employed encounter new firms with probability λ. Firms post profit-per-worker amounts q. A firm posting q commits to taking (no more than) q from each worker with which it matches. Let $F[\cdot|\theta]$ denote the cross sectional distribution of profit-per-worker offers in market θ. A surviving worker draws a profit-per-worker offer less than q with probability $\lambda F[q|\theta]$. A worker drawing a firm offering q and accepting this offer then becomes the residual claimant on the surplus she creates. She selects an effort y, produces θy and takes the residual surplus $x = \theta y - q$ as pre-tax income so as to maximize her current payoff. Equivalently, she chooses pre-tax income x to maximize:

$$\Psi^e\left(x, \frac{x + q}{\theta}\right) := L + (1 - \tau)x - h\left(\frac{x + q}{\theta}\right).$$

Her optimal choice of income is $x(q,\theta) := \theta \frac{\partial h}{\partial y}^{-1}(\theta(1-\tau)) - q$ and her optimized current payoff is:

$$\Phi(q,\theta) := L - (1-\tau)q + \Gamma(\theta),$$

where $\Gamma(\theta) := (1-\tau)\{x(q,\theta)+q\} - h(\frac{x(q,\theta)+q}{\theta})$. Note that x, Φ and Γ all depend upon the policy parameters and, in particular, depend upon the marginal income tax rate τ, but for now we suppress this dependence in the notation. Note also that this formulation is dual to assuming that firms post utilities, assign joint surplus maximizing efforts to accepting workers and pay incomes that attain the accepted utilities after reimbursement for effort.[6] Since the probability of finding a new job does not depend upon the current one and since a worker's current payoff from a job is decreasing in the firm's profit-per-worker offer, the decision of an employed worker to accept a new profit-per-worker offer is a static one: she will accept if the offer is below her current one. A worker accepting such an offer will obtain a greater income and, in this sense, climb the within-talent market job ladder.

The job of a surviving worker is destroyed with probability δ. Such a worker joins new born θ workers and surviving workers who failed to find a job last period in the unemployment pool. Unemployed workers obtain utility b. Since they are no less (and no more) effective in finding jobs than employed workers, their decision to accept a job is a static one. They will do so if they encounter a firm making a profit-per-worker offer q below $\overline{q}(\theta)$, where:

$$\Phi(\overline{q}(\theta),\theta) = b. \tag{1}$$

Hence, $\overline{q}(\theta)$ gives the maximal profit offer in talent market θ. Inverting (1), it is given by:

$$\overline{q}(\theta) = -\frac{b-L}{1-\tau} + \frac{1}{1-\tau}\Gamma(\theta). \tag{2}$$

Let $\underline{q}(\theta)$ denote the minimal profit offer in talent market θ.

Firm problem Let $N(q|\theta)$ denote the number of workers attracted to a firm posting profit per worker offer q, where $N(\cdot|\theta)$ is a decreasing function. Firm profits are given by:

$$\pi(q|\theta) = N(q|\theta)q$$

[6] In Burdett and Mortensen (1998), workers do not supply effort and firms make simple income offers x. Their model could equally be formulated as one in which firms make profit-per-worker offers.

and firms simply select θ (the talent market in which they post) and q (the profit per worker amount they post) to maximize π. In our benchmark environment we simply assume that profits are taxed at 100% by the policymaker.[7] Profits in our model are pure rents to firm ownership. Provided that the social marginal value of a dollar of tax revenues distributed to all agents in the form of an addition to the lump sum employment transfer L and the unemployment benefit b exceeds the social marginal value of a dollar distributed to firm owners, it is optimal to tax this dollar at 100%. We assume that this is true for the first (and all subsequent) dollars potentially earned by firm owners.

Active and Inactive Talent Markets As noted, firms allocate themselves across θ markets and profit-per-worker offers so as to maximize their profits. No firm enters a market in which the maximal profit-per-worker that a worker will accept is negative. Hence, firms only enter productivity markets $\theta \in [\tilde{\theta}, \overline{\theta}]$, where:

$$\tilde{\theta} = \max\{\underline{\theta}, \hat{\theta}\} \quad \text{and} \quad \overline{q}(\hat{\theta}) = 0.$$

From (2), the threshold $\tilde{\theta}$ is given by:

$$\tilde{\theta} = \max\left\{\underline{\theta}, \Gamma^{-1}(\max\{b - L, 0\})\right\}. \tag{3}$$

We refer to talent markets $\theta \in [\tilde{\theta}, \overline{\theta}]$ as active and talent markets $\theta \in [\underline{\theta}, \tilde{\theta})$ as inactive.

Steady-state Equilibrium Distributions We focus on steady-state competitive equilibria. Recall that workers are born into unemployment and that workers with types $[\underline{\theta}, \tilde{\theta})$ never leave unemployment and are effectively out of the labor force. The inflow into unemployment in each active talent market consists of new borns and survivors who loose their jobs. The outflow consists of unemployed workers who die and those who survive and find jobs. In steady state these must be identical. Hence, the unemployment rate in each active talent market satisfies:

$$1 - \rho + \rho(1 - u)\delta = (1 - \rho)u + \rho\lambda u.$$

So that:

$$u = \frac{\tilde{\delta}}{\tilde{\lambda} + \tilde{\delta}},$$

[7]From the firm's point of view posting q is costless, thus they lose nothing by maximizing profits even under 100% taxation. Alternatively, 100% taxation may be regarded as a limiting case. For all profit tax rates less than 100% it is strictly profitable for firms to choose q to maximize $\pi(q|\theta)$.

where $\tilde{\lambda} = \rho\lambda$ and $\tilde{\delta} = 1 - \rho + \rho\delta$ denote the survival adjusted job finding and job destruction probabilities. The total steady-state mass of unemployed (inclusive of those out-of the labor force) is:

$$K[\tilde{\theta}] + u(1 - K[\tilde{\theta}]).$$

Recall that $F[\cdot|\theta]$ denotes the distribution of profit-per-worker offers made by firms entering the $\theta \geq \tilde{\theta}$ market and let $G[\cdot|\theta]$ denote the distribution of employed θ-type workers across such offers. In active talent markets, the inflow of workers into firms posting profit-per-worker amounts greater than q consists solely of surviving unemployed agents who encounter firms posting profit amounts greater than q, i.e. the inflow equals $\tilde{\lambda}(1 - F[q|\theta])u$. The outflow consists of workers previously matched with firms posting profit offers greater than q who either die, exogenously separate or encounter a better (lower) profit-per-worker offer firm. This outflow equals $\{\tilde{\delta} + \tilde{\lambda}F[q|\theta]\}(1 - G[q|\theta])(1 - u)$. Equating these terms, substituting for u and rearranging gives:

$$G[q|\theta] = \left(\frac{\tilde{\delta} + \tilde{\lambda}}{\tilde{\delta} + \tilde{\lambda}F[q|\theta])}\right) F[q|\theta]. \tag{4}$$

The "mass" of θ workers recruited by a firm offering q consists of those surviving workers who make contact with the firm and either are employed at firms offering $q' > q$ or are unemployed. It is given by:

$$R(q|\theta) = \tilde{\lambda}[(1 - u)(1 - G[q|\theta]) + u]\frac{k(\theta)}{m(\theta)},$$

where m is the density of firms across θ markets. Workers separate from such firms if they die, exogenously loose their job or receive a better offer. The rate at which they separate is, thus, $S(q|\theta) = \tilde{\delta} + \tilde{\lambda}F[q|\theta]$. Let $N(q|\theta)$ denote the number of θ workers per firm offering q. Equating steady-state recruitment and separations implies:

$$N(q|\theta) = \frac{\tilde{\lambda}\tilde{\delta}}{\{\tilde{\delta} + \tilde{\lambda}F[q|\theta]\}^2}\frac{k(\theta)}{m(\theta)}.$$

Hence, the total profit of a firm entering market θ and offering q is:

$$\pi(q|\theta) = q\frac{\tilde{\lambda}\tilde{\delta}}{\{\tilde{\delta} + \tilde{\lambda}F[q|\theta])\}^2}\frac{k(\theta)}{m(\theta)}. \tag{5}$$

In particular, the profit made by the firm posting the maximum profit-per-worker in this market $\overline{q}(\theta)$ is:

$$\pi(\overline{q}(\theta)|\theta) = \overline{q}(\theta)\frac{\tilde{\lambda}\tilde{\delta}}{\{\tilde{\delta}+\tilde{\lambda})\}^2}\frac{k(\theta)}{m(\theta)}. \tag{6}$$

Firms distribute themselves across θ markets and q offers so as to maximize and, hence, equate profits. This determines the density of firms across markets:

$$m(\theta) = \frac{\overline{q}(\theta)k(\theta)}{\int_{\tilde{\theta}}^{\overline{\theta}} \overline{q}(\theta')k(\theta')d\theta'} \tag{7}$$

and the distribution of profit offers within each market, for $q \in [\underline{q}(\theta), \overline{q}(\theta)]$,

$$F[q|\theta] = -\frac{\tilde{\delta}}{\tilde{\lambda}} + \frac{\tilde{\delta}+\tilde{\lambda}}{\tilde{\lambda}}\sqrt{\frac{q}{\overline{q}(\theta)}}. \tag{8}$$

Since $F(\underline{q}(\theta)) = 0$, (8) implies that the minimal proft-per-worker offer made in the θ talent market is $\underline{q}(\theta) = \left(\frac{\tilde{\delta}}{\tilde{\delta}+\tilde{\lambda}}\right)^2 \overline{q}(\theta)$. The corresponding density on $[\underline{q}(\theta), \overline{q}(\theta)]$ is $f(q|\theta) = \frac{\tilde{\delta}+\tilde{\lambda}}{2\tilde{\lambda}}\frac{1}{q}\sqrt{\frac{q}{\overline{q}(\theta)}}$. Finally, combining (4) and (8) gives the distribution of workers across offers:

$$G[q|\theta] = \frac{\tilde{\delta}+\tilde{\lambda}}{\tilde{\lambda}} - \frac{\tilde{\delta}}{\tilde{\lambda}}\sqrt{\frac{\overline{q}(\theta)}{q}}.$$

The corresponding density is $g(q|\theta) = \frac{\tilde{\delta}}{2\tilde{\lambda}}\sqrt{\frac{\overline{q}(\theta)}{q}}\frac{1}{q}$.

Policy Maker Budget Balance Equilibrium requires that the policymaker balances her budget and, in particular, that transfers to the employed L and to the unemployed b are funded by profit and income taxation. We assume that the government cannot borrow or lend. Using the previously derived expressions for the fraction of workers unemployed, steady state budget balance implies:

$$\begin{aligned}\left\{K[\tilde{\theta}] + (1-K[\tilde{\theta}])\frac{\tilde{\delta}}{\tilde{\lambda}+\tilde{\delta}}\right\}b + \left\{(1-K[\tilde{\theta}])\frac{\tilde{\lambda}}{\tilde{\lambda}+\tilde{\delta}}\right\}L \\ = \frac{\tilde{\lambda}}{\tilde{\lambda}+\tilde{\delta}}\int_{\tilde{\theta}}^{\overline{\theta}}\int_{\underline{q}(\theta)}^{\overline{q}(\theta)} \{q + \tau x(q,\theta)\}\, g(q|\theta)k(\theta)dqd\theta.\end{aligned} \tag{9}$$

The Impact of Taxes Before stating and analyzing the policymaker's problem in detail, we briefly describe the impact of taxes in this economy. First, as in standard non-frictional public finance models, τ affects a (participating) worker's intensive effort margin and, so, the total output produced by that worker. Second, the income tax rate τ raises the threshold talent $\tilde{\theta}$ and, hence, reduces the fraction of active talent markets. Consequently, it impacts the fraction of agents who participate in the labor market. Third, τ affects the distribution of profits-per-worker. Specifically, a higher tax rate reduces a worker's utility holding the profit extracted from that worker q fixed. Recall that $L + (1-\tau)x - h(\frac{x+q}{\theta})$ is an employed worker's current payoff. Lowest earning workers in an active talent market θ are indifferent between work and unemployment. They receive current payoffs b. Consequently, if τ rises and $b - L$ does not change, then to ensure such workers remain attached to the work force, the profit extracted from them, $\overline{q}(\theta)$, must fall. Firms previously making profit-per-worker offers equal or close to $\overline{q}(\theta)$ must reduce their offers. As they do so they limit the ability of other firms previously making lower profit-per-worker offers to poach workers. This reduces the size of such firms, lowers their total profits and modifies their tradeoff between firm size and profit-per-worker. They respond by reducing their profit-per-worker demands. In this way, the impact of the tax rise on high profit-per-worker firms ripples through the entire profit offer distribution in each talent market. In doing so it directly impacts profit tax revenues. Moreover, since a worker's income is the difference between her output and the profit extracted by firms, this effect also offsets the impact of reduced effort and output on worker income. Hence, it affects income tax revenues and has direct implications for worker welfare. This "profit channel" is the primary difference between the operation of tax policy in our frictional model and a more standard setting.

The combination of the second and third effects described above cause the support of the pre-tax distribution of incomes within a talent market to shift downwards and become more compressed. To see this explicitly define $x(s,\theta)$ to be the pre-tax income earned by a θ talent worker at the s-th quantile within her talent market specific income distribution. This income is given by:

$$x(s,\theta) := z(\theta) - B(s)\overline{q}(\theta), \tag{10}$$

where $B(s) = \left(\frac{\tilde{\delta}}{\tilde{\delta}+\tilde{\lambda}s)}\right)^2$ and $z(\theta) = \theta \frac{\partial h}{\partial y}^{-1}(\theta(1-\tau))$ denotes worker output. Clearly, $x(s,\theta) \leq z(\theta)$ since some output is captured by firm profit. Using the definition of $\overline{q}(\theta)$, a small increase in τ causes a change in $x(s,\theta)$ of:

$$\frac{\partial x}{\partial \tau}(s,\theta) = \frac{\partial z}{\partial \tau}(\theta) + B(s)\left\{\frac{z(\theta) - \overline{q}(\theta)}{1-\tau}\right\} > \frac{\partial z}{\partial \tau}(\theta). \tag{11}$$

This change is always greater than the change (reduction) in worker output $\frac{\partial z}{\partial \tau}$

because some of the burden of income taxation falls on firm profits via the profit channel described above. Indeed, an increase in the marginal tax rate τ can lead to an *increase* in the pre-tax income of some workers if labor supply is relatively inelastic.[8] In this case the reduction in worker output induced by the tax rise is small and is exceeded by the reduction in profit-per-worker needed to keep workers active, implying an increase in pre-tax worker income. In addition, holding θ fixed, $\frac{\partial x}{\partial \tau}(s,\theta)$ is smaller for higher values of s. In economic terms, those higher up the job ladder experience larger reductions (or smaller increases) in income in response to a tax rise, since the reduced amount of profit extracted from them absorbs less of the fall in worker output. Consequently, the income distribution within a given active talent market becomes more compressed as taxes rise. Figure 1 illustrates this compression on the within talent market income density for the case $h(y) = \frac{1}{2}y^2$.

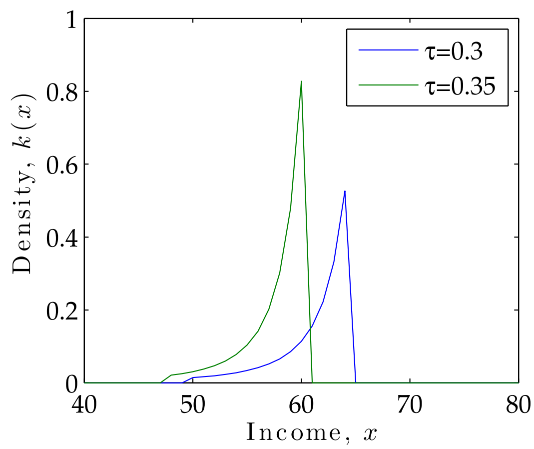


Figure 1: The blue line shows the income density within a given talent market for a value of $\tau = 0.3$. The green line shows the income density for a slightly higher tax rate of 0.35. The second density is shifted downwards and has a more compressed support and correspondingly higher values on this support. These densities are drawn for the case $h(y) = -\frac{1}{2}y^2$.

The overall impact of a small increase in τ on income tax revenues combines the mechanical gain from a higher tax rate holding pre-tax incomes fixed with the effect on pre-tax incomes from the three channels described above. Further, there is a loss of profit tax revenues stemming from the profit channel. A higher tax

[8]Precisely, using the worker's first order conditions: $\frac{\partial x}{\partial \tau}(s,\theta) = -\frac{\theta^2}{h''(z(\theta))} + B(s)\left\{\frac{z(\theta)-\bar{q}(\theta)}{1-\tau}\right\} = \frac{z(\theta)}{(1-\tau)}\left\{-\frac{h'(\frac{z(\theta)}{\theta})}{\frac{z(\theta)}{\theta}h''(\frac{z(\theta)}{\theta})} + B(s)\left\{1-\frac{\bar{q}(\theta)}{z(\theta)}\right\}\right\}$. The term $\frac{h'(\frac{z(\theta)}{\theta})}{\frac{z(\theta)}{\theta}h''(\frac{z(\theta)}{\theta})}$ is the worker's effort supply elasticity, while the term $B(s)\left\{1-\frac{\bar{q}(\theta)}{z(\theta)}\right\} \in [0,1]$. If the effort elasticity is small enough, then some workers will experience an increase in the pre-tax income in response to a tax rise.

rate also directly affects worker utility. Holding transfers b and L fixed, this effect combines the mechanical loss in after-tax income (holding pre-tax income fixed) with the impact of taxes on pre-tax income and effort. It is given by: $-x(s,\theta) + (1-\tau)\frac{\partial x}{\partial \tau}(s,\theta) + \mathcal{E}(\theta)z(\theta)$, where $\mathcal{E}(\theta) = \frac{h'(y(\theta))}{h''(y(\theta))y(\theta)}$ is the conventional wage elasticity of effort. In the standard setting, an envelope theorem implies that the second and third terms in the preceding expression sum to zero and, hence, that the tax impact on a worker's current payoff is given entirely by the mechanical effect. However, in the current frictional model, the impact of taxes on incomes is dampened by the absorption of some of the reduction in output by profit. Consequently, the second and third terms in the loss expression do not sum to zero and the overall utility loss is less than the mechanical loss.

3 The Affine Tax Policymaker Problem

Define $r := b - L$ to be the additional transfer paid to the unemployed over and above that paid to the employed and collect policy parameters into $P := (r, L, \tau)$. In the notation below we make dependence of variables on P explicit in the notation.

We assume that the policy maker maximizes the lifetime utility of each cohort of new born agents at the steady state. Hence, the policymaker's objective is:

$$\int_{\underline{\theta}}^{\overline{\theta}} (1-\beta) \sum_{t=0}^{\infty} \beta^t E[U(\Phi_t(P))|\theta; P]k(\theta)d\theta, \tag{12}$$

where $\Phi_t(P) = \Phi(q_t, \theta; P)$ for an employed θ talent agent with profit offer q_t at t (under policy parameter P) and $\Phi_t(P) = b$ for an unemployed agent at t. The expectation operator in (12) is that induced by F, $\tilde{\theta}$, $\underline{q}$ and $\overline{q}$ (all of which depend on P) and the parameters λ and δ.

Let $\{Q_t[\cdot|\theta, P]\}$ denote the induced probability distributions over a θ talent worker's employment outcomes in each period of her life given policy P.[9] Define the discount-adjusted probability measure over lifetime employment outcomes for a talent θ worker by:

$$\tilde{Q}[\cdot|\theta, P] = (1-\beta) \sum_{t=0}^{\infty} \beta^t Q_t[\cdot|\theta, P].$$

Let $\tilde{E}[\cdot|\theta, P]$ denote the expectation operator implied by this probability measure and let $\tilde{Q}[\cdot|P]$ and $\tilde{E}[\cdot|P]$ denote the corresponding unconditional probability mea-

[9] Specifically, a θ worker of is unemployed with probability 1 in period 0 of her life. If she survives into the next period she transitions to employment with a profit-per-worker offer of less than q with probability $\lambda F^*[q|\theta, P]$ and remains unemployed with probability $1-\lambda$ and so forth.

sure and expectation after integration over the talent distribution. If $\beta = \rho$ and survival risk is the worker's sole motive for discounting, then $\tilde{Q}[\cdot|\theta, P]$ equals the cross sectional distribution of θ talent workers across employment outcomes:

$$Q[\cdot|\theta, P] = (1-\rho)\sum_{t=0}^{\infty} \rho^t Q_t[\cdot|\theta, P],$$

with corresponding expectation $E[\cdot|\theta, P]$. More generally, if $\beta < \rho$, then it places less weight on outcomes attained (more frequently) later in life than the cross sectional distribution. The policymaker's problem may be conveniently stated as:

$$\sup_{P} \; \tilde{E}[U(\Phi(P))|P]. \tag{13}$$

subject to the budget constraint:

$$\begin{aligned} \mathcal{G} + \left\{K[\tilde{\theta}(P)] + (1 - K[\tilde{\theta}(P)])\frac{\tilde{\delta}}{\tilde{\lambda}+\tilde{\delta}}\right\}r \\ = -L + \frac{\tilde{\lambda}}{\tilde{\lambda}+\tilde{\delta}}\int_{\tilde{\theta}(P)}^{\overline{\theta}}\int_{\underline{q}(\theta;P)}^{\overline{q}(\theta;P)} \{q + \tau x(q,\theta)\}\, g(q|\theta;P)k(\theta)dqd\theta, \end{aligned} \tag{14}$$

where $\mathcal{G}$ denotes government spending, $\overline{q}(\theta; P)$ is given by (2), $\tilde{\theta}(P)$ by (3), $\underline{q}(\theta; P) = \left(\frac{\tilde{\delta}}{\tilde{\delta}+\tilde{\lambda}}\right)^2 \overline{q}(\theta; P)$ and $x(q, \theta; P) = \theta \frac{\partial h}{\partial y}^{-1}(\theta(1-\tau)) - q$.

4 Analysis of the Policymaker's Problem

This section describes optimal policy in the benchmark environment. Optimal values are denoted by * superscripts, e.g. P^* denotes the optimal policy triple, $E^*[\cdot] = E[\cdot|P^*]$ and so forth.

Choosing L Let μ^* denote the optimal multiplier on the government budget constraint. The policymaker's first order condition for L is simply $\mu^* = \tilde{E}^*[U'(\Phi^*)]$. Thus, the social shadow value of government funds is simply the discount-adjusted average worker marginal utility.

Choosing τ Proposition 1 derives an expression for τ^* in a form that is comparable to those found elsewhere in the literature. In particular, it incorporates additional terms that capture the role of the profit channel discussed previously in shaping policy. Public finance models that treat the labor market as being non-frictional omit this channel and the corresponding terms. As a precursor to the statement of the proposition, let $\tilde{\text{Cov}}$ denote a covariance under the discount-adjusted measure and let $\mathcal{E}_z^*$ denote the elasticity of a variable z with respect to $1-\tau$ at the optimal policy. We will refer to the latter as the "tax elasticity of z" for brevity.

Proposition 1. *The optimal marginal tax rate satisfies the condition:*

$$
\begin{aligned}
0 = &- \tilde{Cov}^*(U'(\Phi), x) - \tilde{E}^*[U'(\Phi)]\tau^* \frac{E^*[x]\mathcal{E}^*_{E[x]}}{1-\tau^*} \\
&+ \tilde{E}^*[U'(\Phi)]r^* \frac{\tilde{\theta}^*\mathcal{E}^*_{\tilde{\theta}}}{1-\tau^*}\frac{\tilde{\lambda}k(\tilde{\theta}^*)}{\tilde{\lambda}+\tilde{\delta}} \\
&+ \tilde{Cov}^*(U'(\Phi), \mathcal{E}_{\bar{q}}q) - \tilde{E}^*[U'(\Phi)]\tau^* \frac{\pi^*\mathcal{E}^*_{\pi}}{1-\tau^*} \\
&+ \tilde{E}^*[U'(\Phi)]\{E^*[x] - \tilde{E}^*[x]\} - \tilde{E}^*[U'(\Phi)]\{E^*[\mathcal{E}_{\bar{q}}q] - \tilde{E}^*[\mathcal{E}_{\bar{q}}q]\}. \qquad (15)
\end{aligned}
$$

Proof. See Appendix A. □

Proposition 1 has the following immediate corollary.

Corollary 1. *If $\beta = \rho$, the optimal marginal tax rate satisfies the condition:*

$$
\begin{aligned}
0 = &- Cov^*(U'(\Phi), x) - E^*[U'(\Phi^*)]\tau^* \frac{E^*[x]\mathcal{E}^*_{E[x]}}{1-\tau^*} \\
&+ E^*[U'(\Phi)]r^* \frac{\tilde{\theta}^*\mathcal{E}^*_{\tilde{\theta}}}{1-\tau^*}\frac{\tilde{\lambda}k(\tilde{\theta}^*)}{\tilde{\lambda}+\tilde{\delta}} \\
&+ Cov^*(U'(\Phi), \mathcal{E}_{\bar{q}}q) - E^*[U'(\Phi)]\tau^* \frac{\pi^*\mathcal{E}^*_{\pi}}{1-\tau^*}. \qquad (16)
\end{aligned}
$$

Interpreting Proposition 1. We first interpret the simpler equation (16) in Corollary 1 and then the more complicated expression (15) in Proposition 1. The first two terms in (16) are present in a standard (non-frictional) public finance setting with heterogeneous agents. Indeed in that setting (16) reduces to:

$$
-\text{Cov}^*(U'(\Phi), x) - E^*[U'(\Phi)]\frac{\tau^*}{1-\tau^*}E^*[x]\mathcal{E}^*_{E[x]} = 0. \qquad (17)
$$

The first term in either (16) or (17) gives the mechanical social benefit from collecting and redistributing income via a higher tax. It abstracts from behavioral and equilibrium responses. Specifically, a small increment in the tax rate of $\delta\tau$ levied on a (θ, q) worker depresses her income and consumption. This in turn depresses social welfare by: $-U'(\Phi^*)x^*(q,\theta)\delta\tau$.[10] Aggregating across workers gives a social cost of $-E^*[U'(\Phi)x]\delta\tau$. The increased tax rate mechanically raises tax revenues by $E^*[x]\delta\tau$. The use of these revenues to fund an increased lump sum transfer has a social benefit of $E^*[U'(\Phi)]E^*[x]\delta\tau$.[11] Combining these terms gives an expression proportional to the first covariance term in (15), i.e. proportional to $-E^*[U'(\Phi)x] + E^*[U'(\Phi)]E^*[x]$. This covariance is negative and thus describes a redistributive motive for higher marginal taxes.

The second term in either (16) or (17) gives additional adverse behavioral consequences of a small increase in the income tax rate for income tax revenues. In the standard model this stems from an intensive margin response: working agents reduce their effort in response to higher taxes, so reducing the (residual) surplus they claim as pre-tax income. This channel is present in the frictional model, but it is supplemented by two other effects. First, there is an extensive margin response as the less talented working agents drop out of the labor market in response to a higher tax rate. This effect reinforces the usual intensive margin one. Second, some of the burden of the income tax falls on firm profits. Specifically, and consistent with our earlier description, the increase in τ reduces the maximal profit offer needed to make a worker indifferent between working and unemployment and, hence, causes the distribution of profit offers within each θ market to shift leftwards and place more mass on lower profit offers. This raises and, hence, partially offsets the depressing effects of the intensive and extensive channels on worker income. In our frictional model all three of these effects are folded into the variable $\mathcal{E}^*_{E[x]}$ in (16). It corresponds to the (aggregate) *elasticity of taxable income*.

In addition to folding multiple channels into the elasticity of taxable income, the frictional model (with $\beta = \rho$) introduces three new terms into the standard expression (17). The first (given on the second line of (16)) describes the fiscal cost of the additional lump sum transfer $r = b - L$ paid to those who drop out of the labor market as a result of the tax rise. It supplements the social cost of lost income tax revenue at the extensive margin already incorporated into the second term of (16). The remaining terms capture other implications of the profit channel for the determination of the optimal income tax rate. First, by depressing profits, the higher tax rate raises worker incomes and, hence, consumption. At the same time it re-

[10]In the standard (non-frictional) setting, there are no further direct effects on the worker's welfare from the tax change: by an envelope theorem, the welfare loss from behavioral adjustments in income and consumption exactly match the benefit from reduced leisure. However, in our frictional setting there are additional behavioral and equilibrium effects which are accounted for by some of additional terms in (16) relative to (17).

[11]Where we use the fact that a unit of lump sum transfer has a social marginal benefit of $^*E[U'(\Phi)]$ when $\beta = \rho$ in our setting.

duces profit tax revenues and the lump sum transfer received by workers. The term $\text{Cov}^*(U'(\Phi), \mathcal{E}_{\bar{q}}q)$ captures the social benefit of this redistribution. It is analogous to the first term in (16) which described the social benefit of the (mechanical) income tax redistribution. In contrast to that term, this redistributive term enters (15) positively since the redistribution is away from rather than towards the lump sum transfer. In addition, its sign is more ambiguous. On the one hand, within each talent market, workers from whom more profit is extracted are lower on the job ladder and have lower current payoffs, Φ. This is a force for a negative correlation between current worker payoffs and profit. Since U' is decreasing in Φ, if this force predominated the final covariance term in (16) would be positive. In this case, socially desirable redistribution is achieved via the suppression of profits and this is a force for a higher marginal tax rate. On the other hand, if workers in high talent markets generate more surplus and surrender more, on average, as profit, suppressed profit may disproportionately benefit these higher utility workers. The covariance is then negative, reducing profit is undesirable and the profit channel works in favor of a lower marginal tax rate. A priori the sign of this term is indeterminate.[12] The last term in (16) captures the fact that the suppression of profits (and, hence, profit tax revenues) induced by higher τ exceeds the increase in aggregate worker consumption occurring as a result of this suppression. This is because some of the increase in income received by workers as a result of the profit fall is then paid in income tax (and the benefit of these revenues has already been incorporated into the second term in (16)).

Equation (15) in Proposition 1 generalizes (16) to the case in which $\beta < \rho$. When $\beta < \rho$, the discount-adjusted probability measure of workers across employment states no longer coincides with the actual distribution of workers across these states. The former weights employment states attained earlier in life more heavily. These include unemployment and higher profit-per-worker employment states lower on the job ladder that are more likely to be attained earlier in a career. The relevant covariance terms and the shadow value of resources in (15) are computed using the discount adjusted probability measure. However, total labor income $E^*[x]$ and the impact of the tax rate on total profit per worker offers $E^*[\mathcal{E}_{\bar{q}}q]$ are computed using the actual rather than the discount adjusted probability measures requiring the adjustments to the covariance terms contained in the second line of (15).

Connection to Saez Saez (2001) analyzes the optimal linear tax on incomes above a threshold. His focus is on the taxation of top incomes, but the analysis applies to

[12]Sufficient conditions for signing it are discussed in Appendix A.

affine taxation of all incomes. He organizes the first order condition for the optimal marginal tax rate as: $(1-\bar{g})M + \tau B = 0$, where M is the mechanical effect on income tax revenues of a small income tax rate change, B is the behavioral effect and $\bar{g}$ is the marginal social value of dollars taken from people earning incomes greater than the threshold above which the higher tax rate is applied. The term $(1-\bar{g})M$ is analogous to the first term in (15). It captures the social value of income tax revenues mechanically collected via the tax rise net of the cost to tax payers. Saez models all agents as being on the intensive margin. Hence, if the higher tax rate is applied to all incomes, B reduces to the average uncompensated elasticity of income with respect to the retention rate $1-\tau^*$ scaled by the ratio of income to the retention rate. Thus, B corresponds to the second term in (15).

4.1 Special Cases

We briefly describe two limiting cases. In the first, there is a frictional labor market, but no talent dispersion. In the second, there is talent dispersion, but $\tilde{\lambda}/\tilde{\delta}$ is arbitrarily large implying that there is no frictional labor market: in steady state all workers are "at the top of the job ladder" and collect all of the rents. To simplify, we assume $\beta = \rho$.

Frictional labor market without talent dispersion In the absence of talent dispersion (with the common θ normalized to 1), a first best allocation is attainable in which those in work supply the first best efficient level of effort 1 and agents are fully insured against job loss. The optimal tax system consists of a lump tax paid by those in work, a zero marginal tax rate and a (fully insuring) unemployment benefit. This tax system eliminates dispersion in job offers and the associated ex post luck.

More precisely, suppose that the unemployment benefit b is set to $\frac{\tilde{\lambda}}{\tilde{\lambda}+\tilde{\delta}}\{y^* - h(y^*)\}$, where y^* satisfies $1-h'(y^*)=0$. Marginal tax rates are zero and the lump sum tax is set to $L = -\frac{\tilde{\delta}}{\tilde{\lambda}+\tilde{\delta}}\{y^* - h(y^*)\}$. Thus a worker matched with a firm offering q selects effort equal to the first best level and earns a utility of $\frac{\tilde{\lambda}}{\tilde{\lambda}+\tilde{\delta}}\{y^* - h(y^*)\} - q$. It follows that firms can only attract workers if they make $q=0$ offers. Dispersion in such offers (and in firm profits) is endogenously eliminated by the tax system. Consequently, job ladders and frictional income dispersion cannot exist in the face of an optimally designed tax system absent private variation in talent. When talent variation *is* present, driving the unemployment benefit high enough to set all profit offers to zero in a given talent market involves closing markets for less talented workers. This is costly and is only done up to the threshold $\tilde{\theta}^*$. Dispersion in firm profit offers in active talent markets above the threshold and job ladders survive.

Dispersion in talent without a frictional labor market Suppose that the support of the productivity distribution h is not a singleton, but an interval $[\underline{\theta}, \overline{\theta}]$ and that $\tilde{\lambda}/\tilde{\delta}$ is arbitrarily large. In this case in steady state, there is no unemployment and all θ workers receive the best possible profit offers from firms: $q = 0$. Studying either the original problem (13) or its first order condition with respect to τ as $\tilde{\lambda}/\tilde{\delta} \to \infty$ reveals that in the limit the first order condition for τ reduces to (17).

5 Some Extensions

This section details two extensions to the benchmark. The first assumes that profits are only partially rather than fully taxable. The second introduces private saving. A third (more substantial) extension to nonlinear tax functions is considered in Section 7.

5.1 Partial taxation of profits

The preceding assumes that profits are fully taxable (and that the policymaker chooses to tax them at 100%). If profits are not fully taxable several issues are raised: Are workers shareholders? Can workers accumulate shares during their working lives? If workers are not shareholders, does the policymaker value the welfare of shareholders? We consider the simplest case in which workers are not shareholders and shareholders are not socially valued. However, we assume that profits can only be taxed at a fixed rate τ_p.[13] Given Corollary 1, straightforward algebra yields:

Proposition 2. *If profits are taxed at the rate τ_p and $\beta = \rho$, then the optimal marginal*

[13] This assumption of an upper bound on profit taxation can be motivated in various ways. First, the analysis may be interpreted as a partial reform of income taxation in the face of an institutional constraint on profit taxes. Second, profit taxation may distort firm creation.

tax rate satisfies the condition:

$$
\begin{aligned}
0 = &- Cov^*(U'(\Phi), x) - E^*[U'(\Phi)]\tau^* \frac{E^*[x]\mathcal{E}^*_{E[x]}}{1-\tau^*} \\
&+ E^*[U'(\Phi)]r^* \frac{\tilde{\theta}^* \mathcal{E}^*_{\tilde{\theta}}}{1-\tau^*} \frac{\tilde{\lambda} k(\tilde{\theta}^*)}{\tilde{\rho}+\tilde{\lambda}} \\
&+ Cov^*(U'(\Phi), \mathcal{E}_{\bar{q}} q) - E^*[U'(\Phi)] \frac{\tau^*}{1-\tau^*} \pi^* \mathcal{E}^*_{\pi} + E^*[U'(\Phi)] \frac{1-\tau_p}{1-\tau} \pi^* \mathcal{E}^*_{\pi}. \qquad (18)
\end{aligned}
$$

Proof. See Appendix A. □

The expression in (18) augments (16) with $E^*[U'(\Phi)]\frac{1-\tau_p}{1-\tau}\pi^*\mathcal{E}^*_{\pi}$. In the preceding section, higher income tax rates reduced profits and, hence, profit tax revenues, which was socially undesirable. When profit cannot be taxed, that cost is removed and the marginal benefit of income taxation increased. The additional term in (18) captures this effect.

5.2 Private Savings

Our benchmark environment omits private savings. Thus it does not permit older workers to self-insure against job loss through the accumulation of private claims. However, our tax formula (15) in Proposition 1 survives in more general environments that do incorporate private savings. For example, if workers are able to purchase annuities at an exogenously given price of r and income from annuities is untaxed, then as described in Appendix A, tax formula (15) continues to hold. However, the values of the covariances (and other terms) that enter the formula are modified by this extension.

6 Quantitative Analysis of Optimal Affine Taxes

This section explores quantitative implications of our theory for optimal affine tax determination.

Calibration Worker preferences are assumed to be of the form:

$$
U(c - h(y)) = \frac{1}{1-\sigma}\left(c - \frac{1}{1+\gamma} y^{1+\gamma}\right)^{1-\sigma}.
$$

The primitives of our model are then the preference parameters σ, γ and β, the survival probability ρ, the exogenous job separation and job finding rates δ and λ and the talent distribution. Our benchmark calibration sets $\sigma = 2$ and $\gamma = 1$. The latter implies a unit effort supply elasticity. We perform sensitivity analysis with respect to this parameter and consider economies with lower effort elasticities (and higher values of γ). The model's time period is set to be one month and the discount factor β and survival probability ρ are set to a common and limiting value of 1.[14] We choose δ and λ to be in line with the empirical job destruction and job finding rates reported in the literature. Specifically, Shimer (2012) computes monthly job finding rates in the neighborhood of 0.03 for the US in the post 1985 period. Thus, we set $\delta = 0.03$. Shimer (2012) reports monthly job finding rates for unemployed workers that fluctuate around 0.4 in the same period. Shimer (2012) does not report job finding rates for employed workers. Hornstein et al. (2011) calibrate a model with job-to-job transitions to US data and recover values for the job finding rates of employed workers of between 0.08 and 0.12. Our model does not incorporate distinct job finding rates for the employed and the unemployed. Consequently, we set λ equal to a weighted average of the value for the unemployed given in Shimer (2012) and the midpoint value for the employed in Hornstein et al. (2011): $\lambda = 0.12 \approx 0.4 \times 0.06 + 0.1 \times 0.94$.[15] Thus, our calibrated value for λ/δ is 4. As λ/δ increases, holding the other parameters fixed, labor market frictions are reduced and the model converges to a standard frictionless one. Thus, higher values of λ/δ reveal the implications of abstracting from labor market frictions and attributing all empirical income variation to talent dispersion (as is typically done in public finance). We consider such alternative values in our quantitative analysis. We set government spending $\mathcal{G}$ to equal 25% of equilibrium output.

Calibration of k The standard labor market model used in public finance assumes that differently talented workers supply labor in a frictionless market. The first order condition of these workers implies a one-to-one mapping between talent and income. This mapping can be inverted to allow the recovery of the underlying talent distribution from the empirically observed income distribution. In contrast our model does not imply a one-to-one mapping between talent and income. Rather the earnings density j is related to the talent density k via a mapping $j = \mathcal{T}(k; P)$,

[14] Hence, the actual and survival adjusted δ and $\tilde{\delta}$ and λ and $\tilde{\lambda}$ coincide.

[15] Allowing job finding rates to differ across employment and unemployment complicates the analysis by making the decision to accept a job dynamic: a worker must trade off a higher wage against the greater option value of unemployment. We leave this extension to future work.

where:

$$j(x) = \mathcal{T}(k;P)(x) := \int_{\underline{\theta}(x;P)}^{\overline{\theta}(x;P)} \left\{ \frac{\delta}{2\lambda} \left(\frac{\overline{q}(\theta;P)}{\tilde{q}(\theta,x;P)} \right)^{\frac{1}{2}} \frac{1}{\tilde{q}(\theta,x;P)} \right\} k(\theta) d\theta, \tag{19}$$

and $P = (T, b)$. Inversion of $\mathcal{T}(\cdot;P)$ gives an empirical counterpart to k on $[\tilde{\theta}(P), \overline{\theta}]$.

The data source for our calibration exercise is the March release of the 2016 Current Population Survey (CPS).[16] We first form an affine approximation to current US government tax policy, $T[x] = -L + \tau x$, by regressing total taxes paid on labor income. Our estimates (with standard deviations in parentheses) are:

$$T[x] = -\underset{(2.526)}{302.56} + \underset{(0.000361)}{0.336}\ x.$$

The smallest active talent $\tilde{\theta}(P)$ is then determined by: $(1-\hat{\tau}) - \frac{x_1^{\gamma}}{\tilde{\theta}(P)^{1+\gamma}} = 0$, where $\hat{\tau} = 0.336$, $\underline{x}$ is the smallest income in our sample and γ equals its calibrated value. In addition, once $\tilde{\theta}(P)$ is determined, b is pinned down by:

$$\hat{L} + (1-\hat{\tau})\underline{x} - \frac{1}{1+\gamma} \left(\frac{\underline{x}}{\tilde{\theta}(P)} \right)^{1+\gamma} = b, \tag{20}$$

where $\hat{L}$ equals our estimated value 302.56. In preceding sections b has been interpreted both as the value of inactivity to a worker and the direct cost of such inactivity to the policy maker. To better align the model with data, it is useful to distinguish these concepts. We (continue to) use b to label the per period value of inactivity to the worker, while denoting the transfer from the policymaker to the worker by b^U.[17] Given the values for T, $\tilde{\theta}$ and $\underline{x}$, an empirical counterpart for b can be recovered from the data using (20). A value for b^U is not needed to calibrate the talent distribution.[18] Given empirical values for the tax policy parameters and for b, empirical counterparts for the functions $\overline{q}(\cdot;P)$, $\tilde{q}(\cdot;P)$, $\underline{\theta}(\cdot;P)$ and $\overline{\theta}(\cdot;P)$ may be constructed using the relevant formulas from preceding sections. In addition, $\underline{x}(\cdot;P)$ and $\overline{x}(\cdot;P)$, the inverses of $\underline{\theta}(\cdot;P)$ and $\overline{\theta}(\cdot;P)$, may be obtained.

Let $\{x_m\}_{m=1}^{M+1}$ and $\{\theta_n\}_{n=1}^{N+1}$ denote fine grids of labor income and talent values and let $\mathbf{j} = \{j_m\}_{m=1}^{M} = \{J(x_m) - J(x_{m-1})\}_{m=1}^{M}$ and $\mathbf{k} = \{k_n\}_{n=1}^{N} = \{K(\theta_n) - K(x_{\theta-1})\}_{n=1}^{N}$ denote quantiles of the income and talent distributions. We recover estimates of the quantiles of the earnings distribution, $\mathbf{j}$ from our data set. Equation (19) implies

[16] Detailed description of the data and of our sample selection is give in Appendix C.

[17] The difference $b - b^U$ could be positive because the worker derives additional utility benefit from inactivity, has additional time to engage in home production, or receives transfers from other agents. Alternatively, it could be negative because there is a stigma attached to not working.

[18] However, $b - b^U$ is needed to evaluate the fiscal cost of delivering b to inactive workers.

the following approximate map between such $\mathbf{j}$ and $\mathbf{k}$ of the income distribution:

$$j_m \approx \sum_{n=1}^{N} \left\{ \int_{\underline{x}_{m-1}(\theta_n;P)}^{\overline{x}_m(\theta_n;P)} \left\{ \frac{\delta}{2\lambda} \left(\frac{\overline{q}(\theta_n)}{\tilde{q}(\theta_n,x)} \right)^{\frac{1}{2}} \frac{1}{\tilde{q}(\theta_n,x)} \right\} dx \right\} k_n,$$

where $\overline{x}_m(\theta;P) = \min\{x_m, \overline{x}(\theta;P)\}$, and $\underline{x}_m(\theta;P) = \max\{x_{m-1}, \underline{x}(\theta;P)\}$. Thus, we focus on the linear equations:

$$\mathbf{j} = \mathbf{T}\mathbf{k},$$

where $\mathbf{T}$ is an $M \times N$ matrix with (m,n)-th element:

$$\mathbf{T}(m,n) = \int_{\underline{x}_{m-1}(\theta_n;P)}^{\overline{x}_m(\theta_n;P)} \left\{ \frac{\delta}{2\lambda} \left(\frac{\overline{q}(\theta_n)}{\tilde{q}(\theta_n,x)} \right)^{\frac{1}{2}} \frac{1}{\tilde{q}(\theta_n,x)} \right\} dx.$$

We calibrate $\mathbf{k}$ by selecting a value $\hat{k}$ that minimizes the sum of squares:

$$\min_{\mathbf{k}} \sum_{m=1}^{M} (\mathbf{j}(m) - \mathbf{T}(m,\cdot) \cdot \mathbf{k})^2.$$

This procedure provides values for $\{\mathbf{k}_n\}_{\tilde{n}+1}^{N}$, where $\tilde{n}+1$ denotes the least talented working quantile. In our model, $\theta_{\tilde{n}+1}$ is determined by the closest grid point in excess of $\tilde{\theta}(P)$. The left tail of the distribution is effectively censored: we observe the mass, but not the conditional talent distribution of workers who do not work. Thus, the only restriction on $\{\mathbf{k}_n\}_{n=1}^{\tilde{n}}$ is that $\sum_{n=1}^{\tilde{n}} k_n$ equals the mass of non-working agents. We apply a kernel density smoother to the part of the distribution we observe and fit a Pareto right tail. We conduct sensitivity analysis with respect to the left tail of the talent distribution and consider various fitted left tails.

Quantitative Results Table 1 reports optimal affine tax results (and equilibrium aggregate profit) for our benchmark parameter values and two alternative and higher values for γ.[19] The optimal marginal tax rate ranges from 31.5% (at $\gamma = 1$) to 47.9% (at $\gamma = 2$). These results imply optimal marginal tax rates close to or somewhat greater than our estimated rate with higher rates at lower effort elasticity values.[20]

[19]For each alternative parameter value, the talent distribution is recalibrated.

[20]Although the optimal affine tax arrangement is combined with 100% taxation of profit. The OECD reports that the overall tax rates on dividend income in the US was 57.6 per cent in 2015. This value includes the marginal statutory corporate income tax rate on distributed profits and the sum of the maximum federal personal and average state income tax rate on dividends. Data taken from OECD Tax Database, Corporate and Capital Income Taxes: Table II.4. Historical values for

Table 1: Optimal Affine Tax Policy

Variable	$\mathcal{G} = 0.25$ GDP, $\lambda = 0.12$		
	$\gamma = 1$	$\gamma = 1.5$	$\gamma = 2$
τ^*	31.5	41.1	47.9
L^*	323	805	1200
b^*	741	1120	1540
π^*	338	444	552

Notes: L^*, b^* and π^* are monthly 2015 US \$ amounts. π^* is per capita monthly profit.

Table 2 reports results for a range of values for the job finding to job destruction ratio λ/δ.[21] As λ/δ rises, optimal affine tax rates show a marked increase. As

Table 2: Optimal Affine Tax Policy

Variable	$\mathcal{G} = 0.25$ GDP, $\gamma = 1$			
	$\frac{\lambda}{\delta} = 4$	$\frac{\lambda}{\delta} = 10$	$\frac{\lambda}{\delta} = 20$	$\frac{\lambda}{\delta} = 200$
τ^*	31.0	37.9	39.7	40.9
L^*	323	727	917	1028
b^*	720	1005	1108	1130
π^*	338	126	78	9

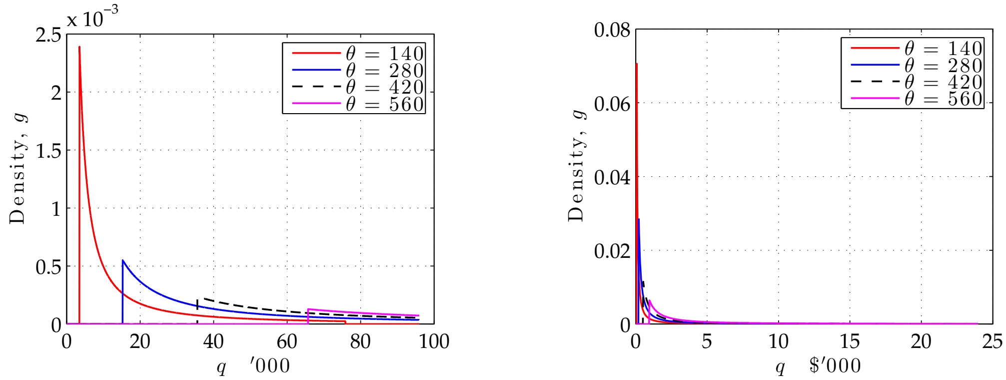
Notes: L^*, b^* and π^* are monthly 2015 US \$ amounts. π^* is per capita monthly profit.

noted previously, λ/δ controls the degree of labor market frictions: as it increases the economy converges to a frictionless one. This is illustrated in Figure 2 which shows conditional profit offer densities $g(\cdot|\theta)$ for different values of θ and two distinct λ/δ values. Figure 2(a) shows these densities for the benchmark value $\lambda/\delta = 4$ and Figure 2(b) shows them for a larger value. In the former case profit offers are dispersed over tens of thousands of dollars within mid and high level θ markets. In the latter case they concentrated over a few thousands of dollars. The smaller role of labor market frictions at higher values of λ/δ is also reflected in the lower profit amounts reported in Table 2. The results in Table 2 suggest that labor market frictions (in combination with optimal 100% taxation of pure profit) create a rationale

the US range from 51.7 per cent in 2003 to 88.7 in 1981. The average value for OECD countries in 2015 is 43.1 per cent.

[21] Again we recalibrate the talent density for different parameter, in this case λ/δ, values. Higher values of λ/δ imply that more income variation is attributed to talent variation and that the talent distribution has a fatter tail.

for lower rates of income taxation.



(a) $\frac{\lambda}{\delta} = 4$

(b) $\frac{\lambda}{\delta} = 20$

Figure 2: Conditional profit offer densities.

Table 3 provides perspective on this result. It reports the values of the various terms in (16) (with $\rho = 1$). These terms decompose the marginal benefits and costs of a small change in τ around the optimum. The terms in the last two rows of the table are associated with the profit channel. Recall that an increase in taxes suppresses profits and profit revenues, while raising labor incomes and labor income tax revenues. The covariance term in the third row captures the distributional benefit of slightly reducing profit tax revenues (that are redistributed lump sum) and raising workers' incomes in proportion to the profit extracted from them. The term in the fourth row adjusts this to take into account that workers will pay some of their higher income in income tax (the revenue benefits of which have already been accounted for in the second row terms). Both of these terms are negative indicating that they introduce an additional marginal cost of raising taxes and are force for lower taxes. However, as λ/δ rises these terms become small implying that as frictions become less important this force is diluted.

The negative sign of the covariance between marginal utility and $\mathcal{E}_{\overline{q}}q$ in the third row of Table 3 is, perhaps, surprising. It implies that an increase in tax rates that suppresses profits and profit tax revenues is undesirable on redistributional grounds. However, it is natural to suppose that poorer agents at the bottom of job ladders face more extractive employers and that a reduction in profits would benefit them, aligning with the policy makers distributional goals. Decomposition

Table 3: Optimal Affine Tax Policy

Variable	$\mathcal{G} = 0.25,\ \gamma = 1$			
	$\frac{\lambda}{\delta} = 4$	$\frac{\lambda}{\delta} = 10$	$\frac{\lambda}{\delta} = 20$	$\frac{\lambda}{\delta} = 200$
$-\text{Cov}^*(U', x)$	3.0×10^{-3}	1.8×10^{-3}	1.6×10^{-3}	1.5×10^{-3}
$-E^*[U']\frac{\mathcal{E}^*_{E[x]}\tau^*}{1-\tau^*}$	-1.7×10^{-3}	-1.6×10^{-3}	-1.5×10^{-3}	-1.5×10^{-3}
$\text{Cov}^*(U', \mathcal{E}_{\bar{q}}q)$	-3.3×10^{-4}	-3.8×10^{-5}	-1.7×10^{-5}	-1.7×10^{-6}
$-E^*[U']\frac{\mathcal{E}^*_{\pi}\tau^*\pi^*}{1-\tau^*}$	-2.5×10^{-4}	-5.1×10^{-5}	-2.4×10^{-5}	-2.4×10^{-6}

of this covariance term at the benchmark $\frac{\lambda}{\delta} = 4$ reveals:

$$\underbrace{\text{Cov}^*(U', \mathcal{E}_{\bar{q}}q)}_{-3.28\times 10^{-4}} = \underbrace{E^*[\text{Cov}^*(U', \mathcal{E}_{\bar{q}}q|\theta)]}_{1.00\times 10^{-4}} + \underbrace{\text{Cov}^*(E^*[U'|\theta], E^*[\mathcal{E}_{\bar{q}}q|\theta])}_{-4.28\times 10^{-4}}.$$

The first term captures the redistributional benefits of reducing profits within talent markets and the second the redistributional benefits of reducing profits across markets. The former is positive capturing the fact that the lowest paid in a talent market have more profit extracted from them. However, the second term is negative: more highly talented workers "pay" more in profit on average and suppression of profits redistributes from less to more talented workers. This second term dominates.

7 Nonlinear Taxation

In this section the environment from the previous sections is modified to permit the policymaker to select a twice continuously differentiable non-linear tax function $T : \mathbb{R}_+ \to \mathbb{R}$ applied to the earnings of working agents. Preliminary concepts and notation is introduced first. Throughout $\beta = \rho = 1$ (so that $\tilde{\delta} = \delta$ and $\tilde{\lambda} = \lambda$) and $\bar{\theta} = \infty$ is assumed.

Tax perturbations and elasticities First note that a working agent's first order condition in the nonlinear tax setting is:

$$1 - T'[x(q, \theta; T)] = \frac{1}{\theta} h' \left(\frac{x(q, \theta; T) + q}{\theta} \right), \tag{21}$$

where $x(q,\theta;T)$ is the worker's optimal income choice at tax function T. Consider a worker facing a perturbed tax function $\tilde{T}[x] = T[x] + \kappa\Omega(x) = T[x] + \kappa \int_0^x \omega(x')dx'$, where ω is a smooth function. In this case, the worker's first order condition is:

$$(1 - T'[x(q,\theta;T+\kappa\Omega)] - \kappa\omega(x(q,\theta;T+\kappa\Omega))) = \frac{1}{\theta}h'\left(\frac{x(q,\theta;T+\kappa\Omega)+q}{\theta}\right), \tag{22}$$

Assuming that $x(q,\theta;\cdot)$ is locally differentiable in κ around 0 (i.e. that the conditions of the implicit function theorem hold), differentiating (22) with respect to κ, rearranging and taking the limit as κ goes to 0 gives:

$$-\left.\frac{\partial x(q,\theta;T+\kappa\Omega)}{\partial\kappa}\right|_{\kappa=0} = \frac{x(q,\theta;T)\omega(x(q,\theta;T))}{1-T'[x(q,\theta;T)]}\mathscr{E}(q,\theta;T), \tag{23}$$

where $\mathscr{E}(q,\theta;T)$ is the elasticity of income with respect to the tax perturbation:

$$\mathscr{E}(q,\theta;T) := \frac{1}{\frac{x(q,\theta;T)}{x(q,\theta;T)+q}\frac{\frac{x(q,\theta;T)+q}{\theta}h''\left(\frac{x(q,\theta;T)+q}{\theta}\right)}{h'\left(\frac{x(q,\theta;T)+q}{\theta}\right)} + \frac{T''[x(q,\theta;T)]x(q,\theta;T)}{1-T'[x(q,\theta;T)]}}.$$

Note that this differs from $\mathcal{E}(q,\theta;T) = \frac{\theta}{x(q,\theta;T)}\frac{h'(x(q,\theta;T)/\theta)}{h''(x(q,\theta;T)/\theta)}$, the conventional wage elasticity of effort (or tax elasticity of income), because workers do not receive all of the surplus they create and because taxes are non-linear.[22]

The income distribution The workers' income choice problems imply that all workers earning income x must share the same marginal rate of substitution between income and consumption:

$$\frac{1}{\theta}h'\left(\frac{x+\tilde{q}(\theta,x;T)}{\theta}\right) = 1 - T'[x],$$

where $\tilde{q}(\theta,x;T)$ gives the profit extracted from a worker of type θ, earning x under tax system T. Thus, $\tilde{q}(\cdot,\cdot;T)$ defines a family of iso-income curves in (q,θ) space satisfying:

$$\tilde{q}(\theta,x;T) = \theta(h')^{-1}(\{1-T'[x]\}\theta) - x. \tag{24}$$

The distribution of firms and workers across (q,θ) is determined analogously to the affine tax case using the steady state labor flow and firm profit maximization conditions. In particular, let $\underline{q}(\theta;T)$ and $\overline{q}(\theta;T)$ denote the minimal and maximal profit-per-worker offers made to type θ workers given tax function T. As in the affine

[22] Non-linearity of taxes also ensures that worker effort choice depends on q as well as θ. See (21).

tax case, $\overline{q}(\theta;T)$ is the maximal profit per worker such that θ-types are indifferent between accepting a job and remaining unemployed:

$$x(\overline{q}(\theta;T),\theta;T) - T[x(\overline{q}(\theta;T),\theta;T)] - h\left(\frac{x(\overline{q}(\theta;T),\theta;T)+\overline{q}(\theta;T)}{\theta}\right) = b. \tag{25}$$

Further, $\underline{q}(\theta;T) = \frac{\delta}{\delta+\lambda}\overline{q}(\theta;T)$. For $\tilde{q}(\theta,x;T)$ to be consistent with firm profit maximization, it must be that $\tilde{q}(\theta,x;T) \in [\underline{q}(\theta;T),\overline{q}(\theta;T)]$. The following lemma further characterizes $\tilde{q}(\cdot,x;T)$.

Lemma 1. *$\tilde{q}(\cdot,x;T)$ is increasing in θ between two endpoints $\underline{\theta}(x;T)$ and $\overline{\theta}(x,T)$.*

Proof. See Appendix B. □

The upper end point $\overline{\theta}(x;T)$ in Lemma 1 is determined by the talent such that the iso-income curve $\tilde{q}(\cdot,x;T)$ reaches the upper bound on profits $\overline{q}(\cdot;T)$, i.e. $\tilde{q}(\overline{\theta}(x;T),x;T) = \overline{q}(\overline{\theta}(x;T);T)$. Combining formulas (24) and (25) gives:

$$\overline{\theta}(x;T) = \frac{1}{1-T'[x]}h'(h^{-1}\{x-T[x]-b\}).$$

Let $\tilde{\theta}(T)$ denote the threshold talent such that all firms make zero profits per worker on any working agent: $\overline{q}(\tilde{\theta}(T);T) = 0$. The lower end point $\underline{\theta}(x;T)$ in Lemma 1 is such that the iso-income curve $\tilde{q}(\cdot,x;T)$ reaches the lower bound on profits $\underline{q}(\cdot;T)$, i.e. $\tilde{q}(\underline{\theta}(x;T),x;T) = \underline{q}(\underline{\theta}(x;T);T) = (\frac{\delta}{\delta+\lambda})^2\overline{q}(\underline{\theta}(x,T);T)$. In addition, $\tilde{\theta}(T)$ satisfies:

$$x - T[x] - h\left(\frac{x}{\tilde{\theta}(T)}\right) = b \quad \text{and} \quad x = h'\{1-T'[x]\}\tilde{\theta}(T).$$

The following figure illustrates the situation. Talents below the threshold $\tilde{\theta}(T)$ do not work. A fraction $\frac{\lambda}{\lambda+\delta}$ of talents θ above this threshold do work and receive profit-per worker offers between $\underline{q}(\theta;T)$ and $\overline{q}(\theta;T)$. Workers earning income x have talents between $\underline{\theta}(x;T)$ and $\overline{\theta}(x;T)$ and profit-per-worker offers along the iso-income schedule $\tilde{q}(\cdot,x;T)$.

Let $J(\cdot;T)$ denote the income distribution given tax function T. The counter cumulative $1-J[\cdot;T]$ equals the mass of employed workers between the $\overline{q}(\theta;T)$ and $\underline{q}(\theta;T)$ curves and to the right of the $\tilde{q}(x,\theta;T)$ curve. This is given by:

$$1-J[x;T] := \int_{\underline{\theta}(x;T)}^{\overline{\theta}(x;T)}\int_{\underline{q}(\theta;T)}^{\tilde{q}(\theta,x;T)} g(q|\theta;T)dqk(\theta)d\theta + \int_{\overline{\theta}(x;T)}^{\infty}\int_{\underline{q}(\theta;T)}^{\overline{q}(\theta;T)} g(q|\theta;T)dqk(\theta)d\theta.$$

Let $j(\cdot;T)$ denote the corresponding income density.

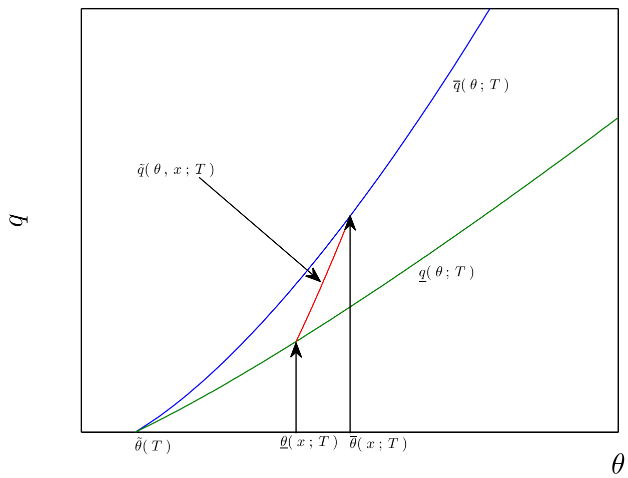


Figure 3: Talent, profit-per-worker and income combinations in equilibrium.

Policymaker's Problem Given these preliminaries, we turn to the policymaker's nonlinear tax policy problem. The policy problem (with $\beta = \rho = 1$) is:

$$\sup_{T,b} \left(\frac{\delta}{\delta+\lambda} + \frac{\lambda}{\delta+\lambda}K[\tilde{\theta}(T)]\right)U(b) \\ + \frac{\lambda}{\delta+\lambda}\int_{\tilde{\theta}(T)}^{\infty}\int_{\underline{q}(\theta;T)}^{\overline{q}(\theta;T)} U\left(\Phi(q,\theta;T)\right) g(q|\theta;T)dqk(\theta)d\theta. \tag{26}$$

subject to:

$$\left(\frac{\delta}{\delta+\lambda} + \frac{\lambda}{\delta+\lambda}K[\tilde{\theta}(T)]\right)b = \frac{\lambda}{\lambda+\delta}\int_{\tilde{\theta}(T)}^{\infty}\int_{\underline{q}(\theta;T)}^{\overline{q}(\theta;T)} \{\tilde{q} + T[x(q,\theta;T)]\}\, g(q|\theta;T)dqk(\theta)d\theta,$$

where:

$$\Phi(q,\theta;T) = x(q,\theta;T) - T[x(q,\theta;T)] - h\left(\frac{x(q,\theta;T)+q}{\theta}\right).$$

We characterize the solution to (26) by analyzing perturbations of a tax function around an optimum. These perturbations are rigorously defined and analyzed in Appendix B. Theorem 1 describes the implications of a small increase in the marginal tax rate "at" an income x_0. In this sense it generalizes Proposition 1. We again use the * superscript to denote evaluations at an optimal tax function T^*, e.g. $\underline{\theta}^*(x_0) = \underline{\theta}(x_0, T^*)$, $\overline{\theta}^*(x_0) = \overline{\theta}(x_0, T^*)$ and so forth.

Theorem 1. *Let* $\hat{q}^*(x_0,\theta) = \tilde{q}^*(x_0,\theta)$ *if* $\theta \in [\underline{\theta}^*(x_0), \overline{\theta}^*(x_0)]$ *and* $\hat{q}^*(x_0,\theta) = \overline{q}^*(x_0,\theta)$ *if* $\theta \in (\overline{\theta}(x_0,T^*),\infty)$ *and let* $\overline{\mathscr{E}}^*(x_0)$ *denote the average value of the elasticity* $\mathscr{E}^*(q,\theta)$

amongst those workers earning x_0. For x_0 such that $\underline{\theta}^*(x_0) > \tilde{\theta}^*$, the optimal nonlinear tax function satisfies the following condition:

$$
\begin{aligned}
&- \int_{\underline{\theta}^*(x_0)}^{\infty} \int_{\underline{q}^*(\theta)}^{\hat{q}^*(\theta,x_0)} U'\left(\Phi^*(q,\theta)\right) g^*(q|\theta)k(\theta)dqd\theta \\
&\quad + \int_{\overline{\theta}^*(x_0)}^{\infty} \frac{\mathcal{E}^*_{\overline{q}}(\theta)}{1-T^{*\prime}[x_0]} \int_{\underline{q}^*(\theta)}^{\overline{q}^*(\theta)} U'\left(\Phi^*(q,\theta)\right)(1-T^{*\prime}[x^*(q,\theta)])qg^*(q|\theta)dqk(\theta)d\theta \\
&\qquad + E^*[U'(\Phi^*)]\Bigg\{ (1-J^*[x_0]) - \frac{x_0 T^{*\prime}[x_0]}{1-T^{*\prime}[x_0]}\overline{\mathscr{E}^*}(x_0)j^*(x_0) \\
&\qquad\quad + \int_{\overline{\theta}^*(x_0)}^{\infty} \frac{\mathcal{E}^*_{\overline{q}}(\theta)}{1-T^{*\prime}[x_0]} \int_{\underline{q}^*(\theta)}^{\overline{q}^*(\theta)} T^{*\prime}[x^*(q,\theta)] \left\{ \frac{x^*(q,\theta)+q}{x^*(q,\theta)} \frac{\mathscr{E}^*(q,\theta)}{\mathcal{E}^*(q,\theta)} \right\} qg^*(q|\theta)dqk(\theta)d\theta \\
&\qquad\qquad - \int_{\overline{\theta}^*(x_0)}^{\infty} \frac{\mathcal{E}^*_{\overline{q}}(\theta)}{1-T^{*\prime}[x_0]} \int_{\underline{q}^*(\theta)}^{\overline{q}^*(\theta)} qg^*(q|\theta)dqk(\theta)d\theta \Bigg\} = 0.
\end{aligned}
\tag{27}
$$

Proof. See Appendix B. □

Interpreting Theorem 1 In the standard Mirrlees setting, a local tax perturbation in the marginal tax rate around the optimum at an income x_0 has three effects. First, it leads to the mechanical collection of extra (socially valued) tax revenues from all workers earning more than x_0. Second, it directly reduces the utility of those who earn more than x_0 and, thus, pay more tax. Third it induces a small intensive margin behavioral response from workers earning x_0. This lowers income tax revenues, but, since workers are optimizing before and after the tax change, this behavioral response has no additional direct impact on their welfare. Optimal policy trades off these effects. All of them are present in our model, but they are supplemented with additional impacts on the distribution of incomes that stem from the profit maximizing behavior of firms in a frictional labor market setting. In addition, the tax perturbation impacts firm profits and profit tax revenues.[23]

Impact on Revenues The tax perturbation causes each worker earning more than x_0 to pay an extra dollar in tax. This leads to a mechanical increase in tax revenues equal to (after normalization by the fraction of agents working) $1 - J^*(x_0)$. In addition, θ-talent workers earning x_0 slightly reduce their effort and, hence,

[23]These additional effects parallel those in the affine case. However, the terms in (27) are slightly reorganized relative to the optimal affine tax equation (16). For a rearrangement of (27) that parallels (16) see Appendix C.

incomes by:

$$\frac{x_0}{1 - T^{*\prime}[x_0]} \mathscr{E}^*(\tilde{q}^*(x_0, \theta), \theta), \tag{28}$$

where $\mathscr{E}^* = \mathscr{E}(\cdot; T^*)$ is defined as in (23). This leads to a reduction in tax revenues collected from these workers of $\frac{x_0 T^{*\prime}[x_0]}{1-T^{*\prime}[x_0]} \overline{\mathscr{E}}^*(x_0) j^*(x_0)$, where $\overline{\mathscr{E}}^*(x_0)$ is the average elasticity of income with respect to taxes amongst x_0 income earners.

The tax perturbation also has an impact on the profit-per-worker offers received by workers earning x_0 and above, and, hence, the distribution of workers earning such incomes. This has further implications for revenue collection. Specifically, the lowest paid workers with talents $\theta > \overline{\theta}^*(x_0)$ earn more than x_0 and, so, pay more tax (on the infra-marginal x_0-th dollar that they earn). Consequently, the maximal profit consistent with such a worker accepting a job, $\overline{q}^*(\theta)$, must be reduced. This is illustrated in Figure 4 by a downward pivot of $\overline{q}(\theta)$ at talents above $\overline{\theta}^*(x_0)$. The distribution of $\theta > \overline{\theta}^*(x_0)$ workers across profit-per-worker offers is shifted downwards and the minimal profit extracted from such workers $\underline{q}^*(\cdot)$ shifts downwards as well. On the one hand, this distributional shift increases worker incomes and raises income tax revenues. On the other, it depresses profits and profit tax revenues. These effects are captured by the terms in the fourth and fifth lines of (27). The shadow value of tax revenues remains $E^*[U'(\Phi)]$ and the terms on the third, fourth and fifth lines of (27).

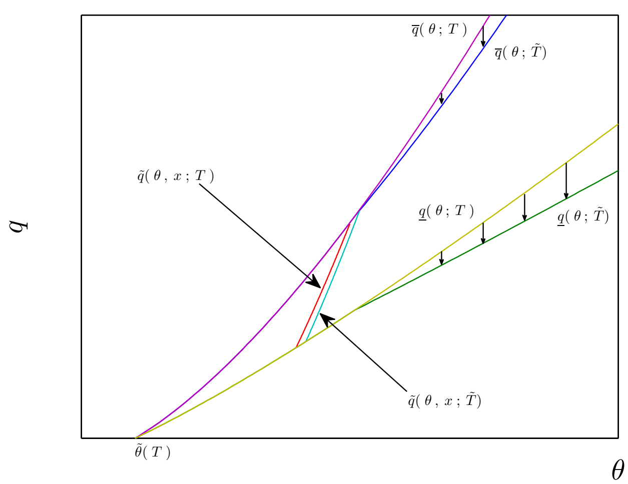


Figure 4: Impact of a Tax Perturbation at income x. The rise in taxes paid by workers earning more than x causes the maximal and minimal profit curves to pivot downwards.

Direct impact on Social Welfare Workers earning more than x_0 pay additional taxes and this reduces their consumption and utility. The welfare cost of this is given by the first term in (27). In the standard Mirrlees model, this is the only direct welfare effect of taxation. However, in our setting there is an additional effect. As described previously, profit extracted from workers with talents greater than or equal to $\overline{\theta}^*(x_0)$ falls. The income and, hence, utility of these workers rises and this is captured by the term on the second line of (27).

8 Quantitative Analysis of Optimal NonLinear Taxes

This section reports quantitative optimal nonlinear tax results. The benchmark parameter values and calibration procedure described in Section 6 is used. Figure 5 plots optimal tax rates for the baseline λ/δ value of 4 and a larger value of 20. Consistent with the affine case, optimal tax rates increase with λ/δ. Hence, an analysis that abstracted from labor market frictions and attributed all or most income variation to talent variation would prescribe higher marginal tax rates. To

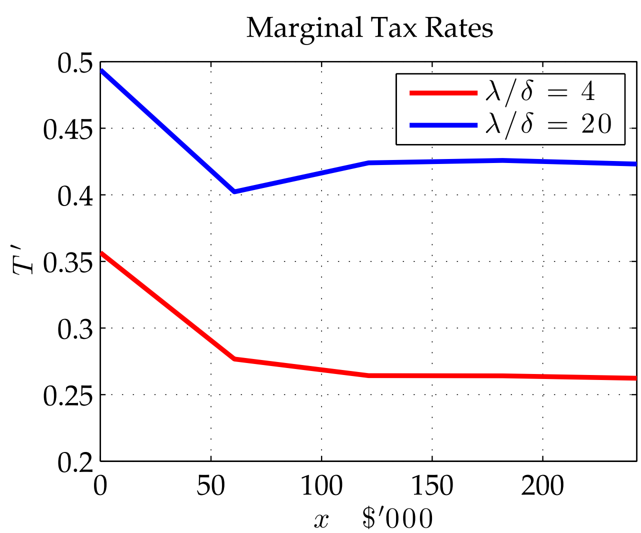


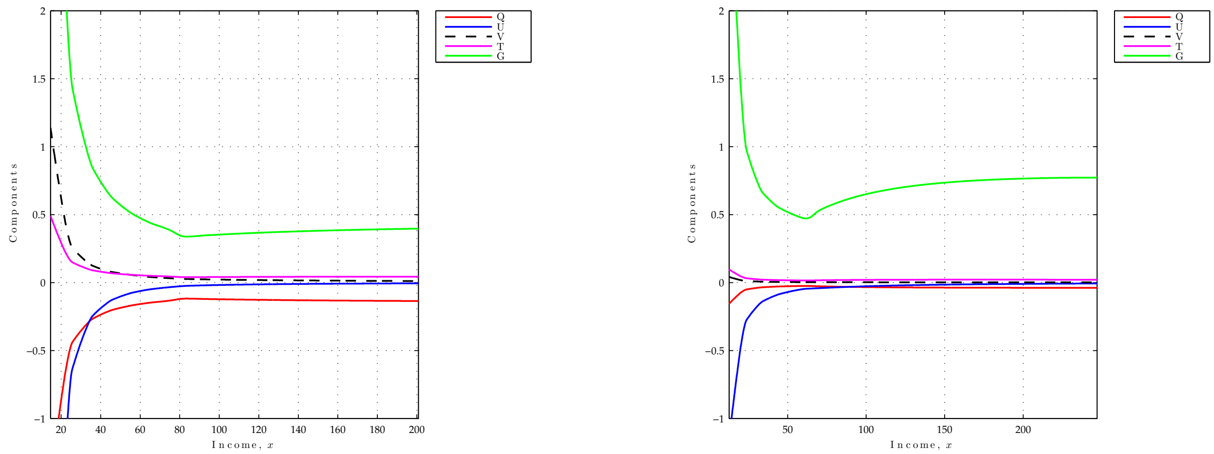
Figure 5: Marginal tax rates as function of income x. Plotted for the baseline value of $\lambda = 0.12$ and $\lambda = 1$.

better understand the forces behind this result, it is convenient use (27) to decompose marginal taxes into component "wedges". As a first step, (27) may be rearranged as:

$$
\begin{aligned}
\frac{T^{*\prime}[x_0]}{1-T^{*\prime}[x_0]} = & \underbrace{\frac{1}{\overline{\mathscr{E}^*}(x_0)} \frac{1-J^*[x_0]}{x_0 j^*(x_0)}}_{G} \\
& \underbrace{- \frac{1}{L} \int_{\underline{\theta}^*(x_0)}^{\infty} \int_{\underline{q}^*(\theta)}^{\hat{q}^*(\theta,x_0)} U'\left(\Phi^*(q,\theta)\right) g^*(q|\theta) k(\theta) dq d\theta}_{U} \\
& + \underbrace{\frac{1}{L} \int_{\overline{\theta}^*(x_0)}^{\infty} \frac{\mathcal{E}^*_{\overline{q}}(\theta)}{1-T^{*\prime}[x_0]} \int_{\underline{q}^*(\theta)}^{\overline{q}^*(\theta)} U'\left(\Phi^*(q,\theta)\right)(1-T^{*\prime}[x^*(q,\theta)]) q g^*(q|\theta) dq k(\theta) d\theta}_{T} \\
& + \underbrace{\frac{1}{M} \int_{\overline{\theta}^*(x_0)}^{\infty} \frac{\mathcal{E}^*_{\overline{q}}(\theta)}{1-T^{*\prime}[x_0]} \int_{\underline{q}^*(\theta)}^{\overline{q}^*(\theta)} T^{*\prime}[x^*(q,\theta)] \mathscr{R}(q,\theta) q g^*(q|\theta) dq k(\theta) d\theta}_{V} \\
& \underbrace{- \frac{1}{M} \int_{\overline{\theta}^*(x_0)}^{\infty} \frac{\mathcal{E}^*_{\overline{q}}(\theta)}{1-T^{*\prime}[x_0]} \int_{\underline{q}^*(\theta)}^{\overline{q}^*(\theta)} q g^*(q|\theta) dq k(\theta) d\theta.}_{Q} \qquad (29)
\end{aligned}
$$

where: $L := \overline{\mathscr{E}^*}(x_0) x_0 j^*(x_0) E^*[U'(\Phi^*)]$, $M := \overline{\mathscr{E}^*}(x_0) x_0 j^*(x_0)$ and $\mathscr{R} := \frac{x^*(q,\theta)+q}{x^*(q,\theta)} \frac{\mathscr{E}^*(q,\theta)}{\mathcal{E}^*(q,\theta)}$. The first left hand side term (labelled G) is a conventional term that appears in optimal tax equations derived under the assumption of frictionless labor markets (e.g. Saez (2001)). It equals (the reciprocal of) the product of the income Pareto coefficient and the elasticity of income with respect to a local marginal tax perturbation at x_0. The second term (labelled U) also appears in the frictionless setting and captures the social cost of redistributing away from those earning more than x_0. The remaining terms (labelled, respectively, T, V and Q) capture the impact of the redistribution away from profits and towards incomes on those earning more than x_0, the income tax revenues collected from those earning more than x_0 and profit tax revenues.

Our benchmark calibration is shown in Figure 6(a). The green curve shows the conventional term G. This inherits the shape of the (optimal) Pareto coefficient reciprocal, which initially falls and then stabilizes. The blue curve shows the other conventional term U, which is initially large (in absolute terms) and negative reflecting the higher social marginal cost of redistributing from a population that includes relatively low earners. However, the term rapidly decays to zero, reflecting the low social marginal cost of redistributing from high income earners (under the assumed social criterion). In a conventional model only these terms would be present and at



(a) $\frac{\lambda}{\delta} = 4$ (b) $\frac{\lambda}{\delta} = 20$

Figure 6: Optimal Tax Wedges. The wedge terms Q, U, V, T and G in the figure correspond to the terms in equation (29).

high incomes the former would shape the optimal marginal tax function. The additional terms introduced by the model with frictions correspond to the dashed black V, pink T and red Q curves. These imply that the social marginal costs and benefits associated with redistributing from profits to incomes are nontrivial at low incomes and that the marginal loss of profit revenues associated with higher income tax rates remains a significant inhibiting factor even at higher incomes. In contrast Figure 6(b) shows that, when $\frac{\lambda}{\delta}$ is assumed equal to 20 and labor market frictions are correspondingly assumed small, these additional terms become negligible (and optimal tax rates are larger).

Higher values for λ/δ value also imply higher values for the term G. As illustrated in Figure 7, this stems largely from an elevated reciprocal Pareto coefficient of income $\frac{1-J^*(x_0)}{x_0 j^*(x_0)}$. When $\frac{\lambda}{\delta} = 4$ the adoption of lower marginal tax rates (relative to empirical ones and in order to sustain profit tax revenues) combined with greater transfers to non-working agents shifts raises worker incomes. However, this rightward shift is greater for low to mid incomes causing mass to pile up at the higher incomes shown in Figure 7. This raises the income density relative to the counter cumulative over this range so reinforcing the adoption of lower marginal tax rates at these incomes.

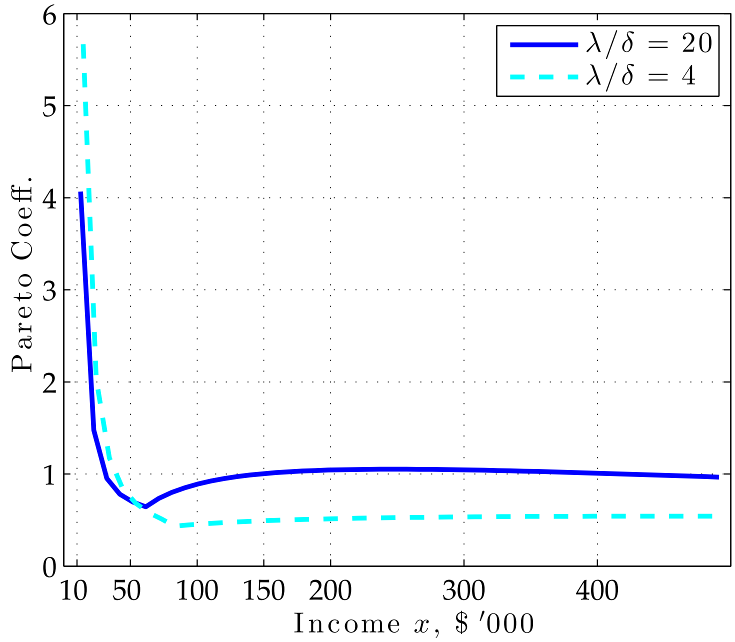


Figure 7: Reciprocal Pareto Coefficients.

9 Conclusion

A worker's pay depends upon her marketable talent and the extractiveness of her employer. Variation in the latter creates job ladders as workers search on the job for better firms offering higher wages. We analyze the implications of this for tax design in a model that blends standard public finance features with a frictional labor market. In doing so we highlight a novel "profit channel". Higher marginal income tax rates squeeze firm profits and, hence, raise the share of the surplus captured by workers. Through this channel they lower profits and profit tax revenues on the one hand and raise worker incomes and income tax revenues on the other (relative to a standard public finance model that omits this channel). Such profit squeezing is socially desirable and a motive for higher income tax rates if profits are disproportionately extracted from low income workers (or, more generally, from those with higher marginal social welfare weights). But is undesirable if profits and profit tax revenues are largely taken from more highly talented workers earning higher incomes. Theory highlights the tradeoffs, but is ambiguous on the overall implications of this channel for policy. Our quantitative analysis suggests that it works in the direction of lower marginal tax rates over a large range of incomes in the nonlinear setting and a lower overall tax rate in the affine setting.

We have explored the role of the profit channel in a particular (well known) frictional setting. But it is more general and would emerge in other frictional models in which higher income taxes (in combination with a given or policy determined outside option for workers) squeeze firm profits and different amounts of profits are collected from different workers. The latter might occur because workers are heterogeneous with respect to mobility or replaceability or because firms are heterogeneous with respect to their bargaining ability. We anticipate that our qualitative findings would be robust to these alternative settings, but leave exploration of this to future work.

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A Proofs

Proof of Proposition 1

Proof. Let $W^*(\theta) = W(P^*;\theta)$ denote the payoff to a new born θ talent at the optimal policy P^*. If $\theta \in [\underline{\theta}, \tilde{\theta}^*)$, then τ has no impact on $W^*(\theta)$. For $\theta \in [\tilde{\theta}^*, \overline{\theta}]$ and recalling that new borns are initially unemployed:

$$W^*(\theta) = U(b) + \beta\lambda \int_{\underline{q}^*(\theta)}^{\overline{q}^*(\theta)} V^*(q|\theta) f^*(q|\theta) dq + \beta(1-\lambda) W^*(\theta),$$

where $V^*(q|\theta)$ denotes the continuation payoff to an employed θ talent at a firm making profit offer q and f^* is the profit offer density both evaluated at the optimal policy. Differentiating with respect to τ gives:

$$\begin{aligned}\frac{\partial W^*}{\partial \tau}(\theta) = \beta\lambda \int_{\underline{q}^*(\theta)}^{\overline{q}^*(\theta)} \frac{\partial V^*}{\partial \tau}(q|\theta) f^*(q|\theta) dq + \beta\lambda \int_{\underline{q}^*(\theta)}^{\overline{q}^*(\theta)} V^*(q|\theta) \frac{\partial f^*}{\partial \tau}(q|\theta) dq \\ + V^*(q^*|\theta) f^*(q^*|\theta) \frac{\partial \overline{q}^*}{\partial \tau}(\theta) - V^*(q|\theta) f^*(q|\theta) \frac{\partial \underline{q}^*}{\partial \tau}(\theta) + \beta(1-\lambda) \frac{\partial W^*}{\partial \tau}(\theta). \quad \text{(A.1)}\end{aligned}$$

Using the definition of f^*,

$$\int_{\underline{q}^*(\theta)}^{\overline{q}^*(\theta)} V^*(q|\theta) \frac{\partial f^*}{\partial \tau}(q|\theta) dq = \frac{1}{2} \frac{\mathcal{E}^*_{\overline{q}}(\theta)}{1-\tau^*} \int_{\underline{q}^*(\theta)}^{\overline{q}^*(\theta)} V^*(q|\theta) f^*(q|\theta) dq, \quad \text{(A.2)}$$

where $\mathcal{E}^*_{\overline{q}}$ is the elasticity of $\overline{q}^*(\theta)$ with respect to $1-\tau$ evaluated at the optimum. Integrating by parts:

$$\begin{aligned}\int_{\underline{q}^*(\theta)}^{\overline{q}^*(\theta)} V^*(q|\theta) f^*(q|\theta) dq &= -\int_{\underline{q}^*(\theta)}^{\overline{q}^*(\theta)} \frac{\partial V^*(q|\theta) f^*(q|\theta)}{\partial q} q dq \\ &\quad + V^*(\overline{q}^*(\theta)) f(\overline{q}^*(\theta)) \overline{q}^*(\theta) - V^*(\underline{q}^*(\theta)) f(\underline{q}^*(\theta)) \underline{q}^*(\theta) \\ &= -\int_{\underline{q}^*(\theta)}^{\overline{q}^*(\theta)} \frac{\partial V^*}{\partial q}(q|\theta) f^*(q|\theta) q dq - \int_{\underline{q}^*(\theta)}^{\overline{q}^*(\theta)} V^*(q|\theta) \frac{\partial f^*}{\partial q}(q|\theta) q dq \\ &\quad + V^*(\overline{q}^*(\theta)) f(\overline{q}^*(\theta)) \overline{q}^*(\theta) - V^*(\underline{q}^*(\theta)) f(\underline{q}^*(\theta)) \underline{q}^*(\theta)\end{aligned}$$

Using $\frac{\partial f(q|\theta)}{\partial q} = -\frac{1}{2q} f(q|\theta)$, we then obtain:

$$\begin{aligned}\frac{1}{2} \int_{\underline{q}^*(\theta)}^{\overline{q}^*(\theta)} V^*(q|\theta) f^*(q|\theta) dq = -\int_{\underline{q}^*(\theta)}^{\overline{q}^*(\theta)} \frac{\partial V^*}{\partial q}(q|\theta) f^*(q|\theta) q dq \\ + V^*(\overline{q}^*(\theta)) f(\overline{q}^*(\theta)) \overline{q}^*(\theta) - V^*(\underline{q}^*(\theta)) f(\underline{q}^*(\theta)) \underline{q}^*(\theta)\end{aligned}$$

(A.3)

Combining (A.1) to (A.3) gives:

$$\frac{\partial W^*}{\partial \tau}(\theta) = \beta\lambda \int_{\underline{q}^*(\theta)}^{\overline{q}^*(\theta)} \frac{\partial V^*}{\partial \tau}(q|\theta) f^*(q|\theta) dq + \beta\lambda \frac{\mathcal{E}^*_{\overline{q}}(\theta)}{1-\tau^*} \int_{\underline{q}^*(\theta)}^{\overline{q}^*(\theta)} \frac{\partial V^*}{\partial q}(q|\theta) q f^*(q|\theta) dq$$
$$+ \beta(1-\lambda)\frac{\partial W^*}{\partial \tau}(\theta). \tag{A.4}$$

Next note that:

$$V^*(q|\theta) = U(\Phi^*(q,\theta)) + \beta\lambda \int_{\underline{q}^*(\theta)}^{q} V^*(q'|\theta) f^*(q'|\theta) dq' + \beta(1-\lambda F^*(q|\theta) - \delta) V^*(q|\theta) + \beta\delta W^*(\theta).$$

Differentiating $V^*(q|\theta)$ with respect to q gives:

$$\frac{\partial V^*}{\partial q}(q|\theta) = -(1-\tau^*)U'(\Phi^*(q,\theta)) + \beta(1-\lambda F^*(q|\theta) - \delta)\frac{\partial V^*}{\partial q}(q|\theta). \tag{A.5}$$

Differentiating $V^*(q|\theta)$ with respect to τ (and proceeding along similar lines to the calculation of $\frac{\partial W^*}{\partial \tau}(\theta)$) gives:

$$\frac{\partial V^*}{\partial \tau}(q|\theta) = - x^*(q,\theta)U'(\Phi^*(q,\theta)) + \beta\lambda \int_{\underline{q}}^{q} \frac{\partial V^*}{\partial \tau}(q'|\theta) f^*(q'|\theta) dq'$$
$$+ \beta(1-\lambda F^*[q|\theta] - \delta)\frac{\partial V^*}{\partial \tau}(q|\theta) - \beta\lambda \frac{\mathcal{E}^*_{\overline{q}}(\theta)}{1-\tau^*} \int_{\underline{q}}^{q} \frac{\partial V}{\partial q}(q'|\theta) q' f^*(q'|\theta) dq' + \beta\delta \frac{\partial W^*}{\partial \tau}(\theta). \tag{A.6}$$

In period 0 a new born worker is unemployed with probability 1. In period 1, the worker remains unemployed with probability $1-\lambda$ and transitions to employment with a profit-offer of less than q with probability $\lambda F^*[q|\theta]$. In period 2, the worker is unemployed with probability $(1-\lambda)^2 + \lambda\delta$ and so forth. Let $Q^*_t[\cdot|\theta]$ denote the probability distribution over employment outcomes of a θ talent worker at age t at the optimum and let:

$$\tilde{Q}^*[\cdot|\theta] = (1-\beta)\sum_{t=0}^{\infty} \beta^t Q^*_t[\cdot|\theta].$$

If $\beta = \rho$ and survival risk is the worker's sole motive for discounting, then $\tilde{Q}^*[\cdot]$ gives the cross sectional distribution of θ talent workers across employment outcomes. More generally, if $\beta < \rho$, then it places less weight on outcomes attained (more frequently) later in life than the cross sectional distribution. Let $\tilde{E}^*[\cdot|\theta]$ denote the expectation under $\tilde{Q}^*[\cdot|\theta]$. Combining (A.4) to (A.6) and iterating gives:

$$(1-\beta)\frac{\partial W^*}{\partial \tau}(\theta) = -\tilde{E}^*[xU'(\Phi)|\theta] + \mathcal{E}^*_{\overline{q}}(\theta)\tilde{E}^*[U'(\Phi)q|\theta]. \tag{A.7}$$

And taking the expectation over θ gives the direct marginal impact of a tax raise on the welfare of new born workers:

$$(1-\beta)\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial W^*}{\partial \tau}(\theta)k(\theta)d\theta = -\tilde{E}^*[xU'(\Phi)] + \tilde{E}^*[\mathcal{E}_{\overline{q}}U'(\Phi)q].$$

We turn next to the budget constraint. The shadow value of an extra unit of government funds is $(1-\beta)\tilde{E}^*[U'(\Phi)|\theta]$. The direct (mechanical) increase in revenue from a small marginal tax rate increase is:

$$E^*[x] = \frac{\rho\lambda}{1-\rho(1-\lambda-\delta)}\int_{\tilde{\theta}^*}^{\overline{\theta}}\int_{\underline{q}^*(\theta)}^{\overline{q}^*(\theta)} x^*(q,\theta)g^*(q|\theta)k(\theta)dqd\theta,$$

where $E^*[\cdot]$ denotes the cross sectional average labor income at the optimum. The intensive margin labor supply response is:

$$-\frac{\tau^*}{1-\tau^*}E[x^*]\mathcal{E}^*_{E[x]} = \tau^*\frac{\partial}{\partial \tau}\left\{\frac{\rho\lambda}{1-\rho(1-\lambda-\delta)}\int_{\tilde{\theta}}^{\overline{\theta}}\int_{\underline{q}(\theta)}^{\overline{q}(\theta)} x(q,\theta)g(q|\theta)k(\theta)dqd\theta\right\}\Bigg|_{\tau=\tau^*},$$

where $\mathcal{E}^*_{E[x]}$ denotes the elasticity of $E^*[x]$ with respect to $1-\tau$ at the optimum. In addition, the tax rate rise also impacts the extensive margin and since the optimal $r^* = b^* - L^*$ need not equal 0, this has the following marginal impact on revenues:

$$r^*\frac{\tilde{\theta}^*}{1-\tau}\mathcal{E}^*_{\tilde{\theta}}\frac{\rho\lambda}{1-\rho(1-\lambda-\delta)}k(\tilde{\theta}^*),$$

where $\mathcal{E}^*_{\tilde{\theta}}$ is the elasticity of $\tilde{\theta}$ with respect to $1-\tau$ at the optimum. Finally, the marginal tax rate affects profits and, hence, profit tax revenues:

$$\begin{aligned}\frac{\partial \pi^*}{\partial \tau} =& \tau^*\frac{\partial \pi^*}{\partial \tau} + (1-\tau^*)\frac{\partial}{\partial \tau}\int_{\tilde{\theta}^*}^{\overline{\theta}}\int_{\underline{q}^*(\theta)}^{\overline{q}^*(\theta)} qg^*(q|\theta)k(\theta)dqd\theta \\ =& -\frac{\tau^*}{1-\tau^*}\mathcal{E}^*_{\pi}\pi^* - \int_{\tilde{\theta}^*}^{\overline{\theta}}\mathcal{E}^*_{\overline{q}}(\theta)\int_{\underline{q}^*(\theta)}^{\overline{q}^*(\theta)} qg^*(q|\theta)dqk(\theta)d\theta = -\frac{\tau^*}{1-\tau^*}\pi^*\mathcal{E}^*_{\pi} - E^*[\mathcal{E}_{\overline{q}}q],\end{aligned}$$

where the derivation of the second equality is similar to the derivation for (A.4) (but with q replacing $V^*(q,\theta)$) and $\mathcal{E}^*_{\pi}$ is the elasticity of aggregate profit π with respect to $1-\tau$ at the optimum. Combining terms gives the overall first order condition for τ:

$$\begin{aligned}0 =& -\tilde{E}^*[xU'(\Phi)] + \tilde{E}^*[\mathcal{E}_{\overline{q}}U'(\Phi^*)q] \\ &+ \tilde{E}^*[U'(\Phi)]\left\{E^*[x] - \tau^*\frac{E^*[x]\mathcal{E}^*_{E[x]}}{1-\tau^*} + r^*\frac{\tilde{\theta}^*\mathcal{E}^*_{\tilde{\theta}}}{1-\tau^*}\frac{\rho\lambda k(\tilde{\theta}^*)}{1-\rho(1-\lambda-\delta)} - E^*[\mathcal{E}_{\overline{q}}q] - \tau^*\frac{\pi^*\mathcal{E}^*_{\pi}}{1-\tau^*}\right\}.\end{aligned} \tag{A.8}$$

Equivalently, using the definitions of $\tilde{\delta}$ and $\tilde{\delta}$:

$$\begin{aligned}0 =& - \tilde{\mathrm{Cov}}^*(U'(\Phi), x) + \tilde{\mathrm{Cov}}^*(U'(\Phi), \mathcal{E}_{\bar{q}} q) \\ &+ \tilde{E}[U'(\Phi^*)]\{E[x] - \tilde{E}[x]\} - \tilde{E}[U'(\Phi^*)]\{E[\mathcal{E}^*_{\bar{q}} q] - \tilde{E}[\mathcal{E}^*_{\bar{q}} q]\} \\ &+ \tilde{E}^*[U'(\Phi)] \left\{ -\tau^* \frac{E^*[x]\mathcal{E}^*_{E[x]}}{1-\tau^*} + r^* \frac{\tilde{\theta}^* \mathcal{E}^*_{\tilde{\theta}}}{1-\tau^*} \frac{\tilde{\lambda} k(\tilde{\theta}^*)}{\tilde{\lambda} + \tilde{\delta}} - \tau^* \frac{\pi^* \mathcal{E}^*_\pi}{1-\tau^*} \right\}. \end{aligned} \tag{A.9}$$

□

Proof of Proposition 2

Proof. In the proof of Proposition 1, the impact of the tax perturbation on profits and, hence, profit tax revenues was shown to be:

$$\frac{\partial \pi^*}{\partial \tau} = - E^*[\mathcal{E}_{\bar{q}} q] - \frac{\tau^*}{1-\tau^*} \pi^* \mathcal{E}^*_\pi.$$

In the current setting with partial taxation of profits, this becomes:

$$\tau_p \frac{\partial \pi^*}{\partial \tau} = - E^*[\mathcal{E}_{\bar{q}} q] - \frac{\tau^*}{1-\tau^*} \pi^* \mathcal{E}^*_\pi + (1-\tau_p) E^*[\mathcal{E}_{\bar{q}} q] + (1-\tau_p) \frac{\tau^*}{1-\tau^*} \pi^* \mathcal{E}^*_\pi.$$

The last two terms can be regrouped as:

$$\frac{1-\tau_p}{1-\tau^*} \pi^* \mathcal{E}^*_\pi.$$

The inclusion of this additional term leads to the first order condition in Proposition 2.

□

Extension to include private savings Formula (A.9) was derived under the assumption that workers cannot save. The argument may be extended to accommodate saving. Assume that workers can purchase annuities at an exogenously given price. Further assume that income from annuities is untaxed. By accumulating annuities older workers can self insure against job loss. Now, worker value functions W^* and V^* depend on wealth as well as employment status. However, by an envelope theorem, savings adds no additional terms to the expression for $\frac{\partial W^*}{\partial \tau}(\theta)$, which following the argument leading to (A.7) in the proof of Proposition 1 becomes:

$$(1-\beta) \frac{\partial W^*}{\partial \tau}(\theta) = -\tilde{E}^*[xU'(D+\Phi)|\theta] + \mathcal{E}^*_{\bar{q}}(\theta) \tilde{E}^*[U'(D+\Phi)q|\theta], \tag{A.10}$$

where D is consumption in excess of after-tax labor income consumed in each employment-wealth state and $\tilde{Q}^*[\cdot|\theta]$ is modified to be the (discount-adjusted) distribution over employment status *and* wealth and $\tilde{E}^*[\cdot|\theta]$ is the corresponding expectation. The other times entering the optimal tax first order condition are unaltered by the inclusion of savings. The optimality condition (A.9) becomes:

$$
\begin{aligned}
0 = &- \tilde{\mathrm{Cov}}^*(U'(D+\Phi), x) + \tilde{\mathrm{Cov}}^*(U'(D+\Phi), \mathcal{E}_{\bar{q}} q) \\
&+ \tilde{E}^*[U'(D+\Phi)]\{E^*[x] - \tilde{E}^*[x]\} - \tilde{E}^*[U'(D+\Phi)]\{E^*[\mathcal{E}_{\bar{q}} q] - \tilde{E}^*[\mathcal{E}_{\bar{q}} q]\} \\
&+ \tilde{E}^*[U'(D+\Phi)] \left\{ -\tau^* \frac{E^*[x]\mathcal{E}^*_{E[x]}}{1-\tau^*} + r^* \frac{\tilde{\theta}^* \mathcal{E}^*_{\tilde{\theta}}}{1-\tau^*} \frac{\rho\lambda k(\tilde{\theta}^*)}{1-\rho(1-\lambda-\delta)} - \tau^* \frac{\pi^* \mathcal{E}^*_{\pi}}{1-\tau^*} \right\}.
\end{aligned}
\tag{A.11}
$$

This expression is essentially identical to (15). The only modification is that marginal utilities of consumption depend on D in addition to Φ.

Signing the redistributive term $\tilde{\mathrm{Cov}}^*(U'(\Phi), \mathcal{E}_{\bar{q}} q)$ Whether the redistributive term $\tilde{\mathrm{Cov}}^*(U'(\Phi), \mathcal{E}_{\bar{q}} q)$ is positive or negative depends upon whether the reductions in profit-per-worker (and increases in pre-tax worker incomes) induced by a tax change fall mainly on poorer agents at the bottom of job ladders from whom more profit is extracted relative to others in their talent market or richer, more talented who generate more surplus. We explore this more formally below at a possibly non-optimal policy P.

Recall that: $x(q,\theta) + q = \theta \frac{\partial h}{\partial y}^{-1}(\theta(1-\tau))$ and

$$
\bar{q}(\theta) = -\frac{b-L}{1-\tau} + \frac{1}{1-\tau}\Gamma(\theta) = -\frac{b-L}{1-\tau} + x(q,\theta) + q - \frac{1}{1-\tau} h\left(\frac{x(q,\theta)+q}{\theta}\right). \tag{A.12}
$$

Assume that $b \geq L$, then $\bar{q}(\theta) < x(q,\theta) + q$ and:

$$
\mathcal{E}_{\bar{q}}(\theta) = \frac{1-\tau}{\bar{q}(\theta)} \frac{\partial \bar{q}}{\partial(1-\tau)}(\theta) = -1 + \frac{x+q}{\bar{q}(\theta)} = -1 + \frac{\theta \frac{\partial h}{\partial y}^{-1}(\theta(1-\tau))}{\bar{q}(\theta)} > 0.
$$

After some algebra, one can show that:

$$
\frac{\partial \mathcal{E}_{\bar{q}}}{\partial \theta}(\theta) = \frac{x+q}{\theta \bar{q}(\theta)} \left[\mathcal{E}(\theta) + 1 - \frac{x+q}{\bar{q}(\theta)} \right].
$$

Further with $b \geq L$, $\frac{x+q}{\bar{q}(\theta)} \geq \frac{1}{1 - \frac{h(\frac{x+q}{\theta})}{\frac{x+q}{\theta} h'(\frac{x+q}{\theta})}}$. Hence, if $h(y) = \frac{1}{1+\gamma} y^{1+\gamma}$, $\gamma > 1$, then $\frac{x+q}{\bar{q}(\theta)} > \mathcal{E} + 1$ and we conclude that $\frac{\partial \mathcal{E}_{\bar{q}}}{\partial \theta}(\theta) \leq 0$. Thus, under plausible assumptions on preferences, the highest and, hence, the distribution of profit-per-worker offers is less sensitive to changes in tax rates in higher talent markets.

We have by the law of total covariance:

$$
\mathrm{Cov}(U', \mathcal{E}_{\bar{q}} q) = E[\mathrm{Cov}(U', \mathcal{E}_{\bar{q}} q | \theta)] + \mathrm{Cov}(E[U'|\theta], E[\mathcal{E}_{\bar{q}} q | \theta]).
$$

Now,

$$\mathrm{Cov}(U', \mathcal{E}_{\bar{q}} q|\theta) = \mathcal{E}_{\bar{q}}(\theta) \int_{\underline{q}(\theta)}^{\overline{q}(\theta)} U'(\Phi(q,\theta;P)) q g(q|\theta;P) dq$$
$$- \mathcal{E}_{\bar{q}}(\theta) \int_{\underline{q}(\theta)}^{\overline{q}(\theta)} U'(\Phi(q,\theta;P)) g(q|\theta;P) dq \int_{\underline{q}(\theta)}^{\overline{q}(\theta)} q g(q|\theta;P) dq.$$

Since $\Phi(q,\theta;P)$ is decreasing in q and U' is decreasing in Φ, we conclude that $\mathrm{Cov}(U', \mathcal{E}_{\bar{q}} q|\theta)$ is positive, i.e. *within* each talent market those lowest on the job ladder with the highest marginal utility benefit the most from a tax induced reduction in profit per worker offer. Since each $\mathrm{Cov}(U', \mathcal{E}_{\bar{q}} q|\theta) > 0$, it follows that $E[\mathrm{Cov}(U', \mathcal{E}_{\bar{q}} q|\theta)] > 0$.

We now verify that $E[U'|\theta]$ is decreasing in θ. To see this let $v(q|\theta) = \Phi(q,\theta;P)$ and change variables to write $E[U'|\theta]$ as:

$$-\int_{b}^{\overline{v}(\theta)} U'(v) g\left(\frac{q(v|\theta)}{q(b|\theta)}\right) \frac{dq}{dv} dv.$$

We have that: $-\frac{dq}{dv} = h'\left(\frac{x(v|\theta)+q(v|\theta)}{\theta}\right)\frac{1}{\theta} = 1-\tau$. Also, $\frac{\partial q}{\partial \theta}(v,\theta) = \frac{x(v|\theta)+q(v|\theta)}{\theta}$. Let $K(v,\theta) := \frac{q(b|\theta)^{\frac{1}{2}}}{q(v|\theta)^{\frac{3}{2}}}$. It follows that: $\frac{\theta}{K(v,\theta)}\frac{\partial K}{\partial \theta}(v,\theta) = -\frac{3}{2}\frac{x(v|\theta)+q(v|\theta)}{q(v|\theta)} + \frac{1}{2}\frac{x(b|\theta)+q(b|\theta)}{q(b|\theta)} = -\frac{x(v|\theta)}{q(v|\theta)} + \frac{x(b|\theta)}{q(b|\theta)} - 1$. But by the worker's first order condition: $1-\tau = \frac{1}{\theta}h'\left(\frac{x+q}{\theta}\right)$. Hence, for a given θ, $x+q$ equals a fixed number and $\frac{x}{q}$ increases as v rises and q falls. It follows that $\frac{\theta}{K(v,\theta)}\frac{\partial K}{\partial \theta}(v,\theta) < 0$. Hence, at higher θ values, the distribution over v first order stochastically dominates the distribution over lower values. But U' is falling in v and so we conclude that $E[U'|\theta]$ is falling in θ.

Next consider $E[\mathcal{E}_{\bar{q}} q|\theta]$. Using the formula for $\mathcal{E}_{\bar{q}}$ (and recalling that this variable does not depend on q), the formula for $\overline{q}(\theta)$ and ignoring unimportant constants:

$$\mathcal{E}_{\bar{q}}(\theta) E[q|\theta] \propto \mathcal{E}_{\bar{q}}(\theta) \int_{\underline{q}(\theta)}^{\overline{q}(\theta)} \left(\frac{\overline{q}(\theta)}{q}\right)^{\frac{1}{2}} dq \propto \left(-1 + \frac{x(q,\theta)+q}{\overline{q}(\theta)}\right)\overline{q}(\theta)$$
$$= \frac{b-L}{1-\tau} + \frac{1}{1-\tau} h\left(h'^{-1}(\theta(1-\tau))\right).$$

This term is falling in θ and so $\mathrm{Cov}(E[U'|\theta], E[\mathcal{E}_{\bar{q}} q|\theta]) < 0$. This captures the fact that *even though* profits are proportionately less sensitive to the income tax rate in higher talent markets, since average profits collected per worker are greater in higher talent markets, workers in these markets benefit more from the suppression of profits and on average these workers earn higher incomes and have higher consumption and utility. These calculations highlight the basic tension: the profit channel benefits workers at the bottom of the job ladder (who tend to be poorer, suggesting that the profit channel enhances the motive for redistributive taxation) *and* more highly talented workers (who tend to be richer suggesting that the profit channel dampens the motive for redistributive taxation).

We briefly indicate properties that are needed for the covariance $\tilde{\text{Cov}}(U'(\Phi), \mathcal{E}_{\bar{q}}q)$ to be negative (and, hence, a motive for lower taxes). First, assume that at the optimum $b = L$ and that $h(y) = \frac{1}{2}y^2$. Under these conditions $\mathcal{E}_{\bar{q}} = 1$, the profit-per-worker distribution of higher talent workers is no less sensitive that that of lower ones and $\tilde{\text{Cov}}(U'(\Phi^*), \mathcal{E}_{\bar{q}}q) = \tilde{\text{Cov}}(U'(\Phi), q)$. To assess the sign of this term, first change the order of integration and compute:

$$E[U'q] = \int_{\underline{q}}^{\bar{q}} \int_{\underline{\theta}(q)}^{\bar{\theta}(q)} U'(\Phi(q, \theta; P))qg(q|\theta; P)dq,$$

where $\underline{\theta}(q)$ and $\bar{\theta}(q)$ are defined implicitly by $q = \bar{q}(\underline{\theta}(q))$ and $q = \underline{q}(\bar{\theta}(q))$. Next consider:

$$\begin{aligned} E[U'|q] &= \int_{\underline{\theta}(q)}^{\bar{\theta}(q)} U'(\Phi(q, \theta; P)) \frac{\frac{\delta}{2\lambda}\left(\frac{\bar{q}(\theta)}{q}\right)^{\frac{1}{2}} \frac{1}{q}k(\theta)}{\int_{\underline{\theta}(q)}^{\bar{\theta}(q)} \frac{\delta}{2\lambda}\left(\frac{\bar{q}(\theta')}{q}\right)^{\frac{1}{2}} \frac{1}{q}k(\theta')d\theta'} d\theta \\ &= \int_{\underline{\theta}(q)}^{\bar{\theta}(q)} U'(\Phi(q, \theta; P)) \frac{\bar{q}(\theta)^{\frac{1}{2}}k(\theta)}{\int_{\underline{\theta}(q)}^{\bar{\theta}(q)} \bar{q}(\theta')^{\frac{1}{2}}k(\theta')d\theta'} d\theta. \end{aligned}$$

Next change variables in the preceding integration. Define $v = \Phi(q, \theta; P)$, so that (given $h(y) = \frac{1}{2}y^2$) $dv = \theta(1-\tau)^2 d\theta$ and $\theta(v; q) = \sqrt{\frac{2}{1-\tau}\left\{\frac{v-L}{1-\tau} + q\right\}}$, then:

$$E[U'|q] = \int_b^{\frac{\lambda}{\delta}q(1-\tau)+b} \psi'(v) \frac{\left[\frac{v-b}{1-\tau} + q\right]^{\frac{1}{2}} \frac{1}{(1-\tau)^2} \frac{k(\theta(v;q))}{\left[\frac{2}{1-\tau}\left\{\frac{v-L}{1-\tau}+q\right\}\right]^{\frac{1}{2}}}}{\int_b^{\frac{\lambda}{\delta}q(1-\tau)+b} \left[\frac{v'-b}{1-\tau} + q\right]^{\frac{1}{2}} \frac{1}{(1-\tau)^2} \frac{k(\theta(v';q))}{\left[\frac{2}{1-\tau}\left\{\frac{v'-L}{1-\tau}+q\right\}\right]^{\frac{1}{2}}} dv'} dv.$$

This can be simplified to give:

$$E[U'|q] = \int_b^{\frac{\lambda}{\delta}q(1-\tau)+b} U'(v) \frac{\left[\frac{v-b}{1-\tau} + q\right]^{\frac{1}{2}} \frac{k(\theta(v;q))}{\left[\frac{v-L}{1-\tau}+q\right]^{\frac{1}{2}}}}{\int_b^{\frac{\lambda}{\delta}q(1-\tau)+b} \left[\frac{v'-b}{1-\tau} + q\right]^{\frac{1}{2}} \frac{k(\theta(v';q))}{\left[\frac{v'-L}{1-\tau}+q\right]^{\frac{1}{2}}} dv'} dv.$$

Consider the term:

$$\left[\frac{v-b}{1-\tau} + q\right]^{\frac{1}{2}} \frac{k(\theta(v;q))}{\left[\frac{v-L}{1-\tau} + q\right]^{\frac{1}{2}}}. \tag{A.13}$$

Let $\gamma(q) = \left[\left\{\frac{2}{1-\tau}\right\}\frac{v-b}{1-\tau} + q\right]^{\frac{1}{2}}$. Using the definitions of $\theta(v;q)$ and $\gamma(v;q)$, the derivative of (A.13) with respect to q is:

$$\frac{\gamma(v;q)k(\theta(v;q))}{\theta(v;q)}\left[\frac{1}{\gamma(v;q)}\frac{\partial\gamma}{\partial q}(v;q)-\frac{1}{\theta(v;q)}\frac{\partial\theta}{\partial q}(v;q)+\left(\frac{\theta k'(\theta(v;q))}{k(\theta(v;q))}\right)\frac{1}{\theta(v;q)}\frac{\partial\theta}{\partial q}(v;q)\right].$$

If $b = L$, this reduces to:

$$k(\theta(v;q))\left[\left(\frac{\theta k'(\theta(v;q))}{k(\theta(v;q))}\right)\frac{1}{\theta(v;q)}\frac{\partial\theta}{\partial q}(v;q)\right],$$

which is negative if $k' < 0$ (e.g. if k is a Pareto distribution). Thus, in this case, higher values of q place more mass on higher values of v. But these higher v are associated with lower values for U'. Hence, U' and q are negatively correlated. We conclude that in this case $-E[U']E[q] + E[U'q] < 0$.

B Nonlinear Tax Analysis

We first prove Lemma 1 from the main text. For convenience we restate the lemma.

Lemma 1 : *$\tilde{q}(\cdot, x; T)$ is increasing in θ between two endpoints $\underline{\theta}(x;T)$ and $\overline{\theta}(x,T)$.*

Proof. Recall the definition of $\tilde{q}$: $\tilde{q}(\theta, x; T) = \theta(h')^{-1}(\{1 - T'[x]\}\theta) - x$. Hence:

$$\frac{\partial}{\partial\theta}\tilde{q}(\theta,x;T) = (h')^{-1}(\{1-T'[x]\}\theta) + \frac{\theta\{1-T'[x]\}}{h''(\{1-T'[x]\}\theta)} > 0.$$

$\square$

In the remainder of this appendix, we characterize the optimal nonlinear tax. For convenience, we restate the policymaker's problem:

$$\sup_{T,b}\ \left(\frac{\delta}{\delta+\lambda}+\frac{\lambda}{\delta+\lambda}K[\tilde{\theta}(T)]\right)U(b)+\frac{\lambda}{\delta+\lambda}\int_{\tilde{\theta}(T)}^{\infty}\int_{\underline{q}(\theta;T)}^{\overline{q}(\theta;T)}U\left(\Phi(q,\theta;T)\right)g(q|\theta;T)dqk(\theta)d\theta. \tag{B.1}$$

subject to:

$$\mathcal{G}+\left(\frac{\delta}{\delta+\lambda}+\frac{\lambda}{\delta+\lambda}K[\tilde{\theta}(T)]\right)b=\frac{\lambda}{\lambda+\delta}\int_{\tilde{\theta}(T)}^{\infty}\int_{\underline{q}(\theta;T)}^{\overline{q}(\theta;T)}\{\tilde{q}+T[x(q,\theta;T)]\}\,g(q|\theta;T)dqk(\theta)d\theta,$$

where:

$$\Phi(q,\theta;T)=x(q,\theta;T)-T[x(q,\theta;T)]-h\left(\frac{x(q,\theta;T)+q}{\theta}\right).$$

It is convenient to let $r := b + T[0]$, $\tilde{T}[x] = T[x] - T[0]$ and $P = (r, T[0], \tilde{T})$ and to re-express this problem as:

$$\sup_P \left(\frac{\delta}{\delta+\lambda} + \frac{\lambda}{\delta+\lambda} K[\tilde{\theta}(P)]\right) U(r - T[0]) \\ + \frac{\lambda}{\delta+\lambda} \int_{\tilde{\theta}(P)}^{\infty} \int_{\underline{q}(\theta;P)}^{\overline{q}(\theta;P)} U\left(\Phi(q,\theta;P) - T[0]\right) g(q|\theta;P) dq k(\theta) d\theta. \tag{B.2}$$

subject to:

$$\mathcal{G} + \left(\frac{\delta}{\delta+\lambda} + \frac{\lambda}{\delta+\lambda} K[\tilde{\theta}(P)]\right)(r - T[0]) \\ = \frac{\lambda}{\lambda+\delta} \int_{\tilde{\theta}(P)}^{\infty} \int_{\underline{q}(\theta;P)}^{\overline{q}(\theta;P)} \left\{\tilde{q} + \tilde{T}[x(q,\theta;P)] + T[0]\right\} g(q|\theta;P) dq k(\theta) d\theta,$$

where:

$$\Phi(q,\theta;P) = x(q,\theta;P) - \tilde{T}[x(q,\theta;T)] - h\left(\frac{x(q,\theta;P)+q}{\theta}\right).$$

First order condition for $T[0]$. Let μ^* denote the optimal multiplier on the policymaker's budget constraint. Differentiating the policymaker's Lagrangian with respect to $T[0]$ (holding r and $\tilde{T}$ fixed at their optimal values) gives the first order condition:

$$E^*[U'] = \mu^*,$$

where $E^*[U']$ denotes the optimal average marginal utility.

First order condition for r. We next derive the first order conditions for r. Recall that r is the premium received by unemployed workers over the tax paid by workers who work, but earn nothing, i.e. $r = b - T[0]$. As precursors to the first order condition, we give formulas for the variables $\frac{\partial \overline{q}}{\partial r}(\theta;P)$ and $\frac{\partial \tilde{\theta}}{\partial r}(\theta;P)$ which appear in the condition. First, recall that the variable $\overline{q}$ satisfies:

$$x(\overline{q}(\theta;P),\theta;P) - \tilde{T}[x(\overline{q}(\theta;P),\theta;P)] - h\left(\frac{x(\overline{q}(\theta;P),\theta;P) + \overline{q}(\theta;P)}{\theta}\right) = r.$$

Hence,

$$\frac{\partial \overline{q}}{\partial r}(\theta;P) = -\frac{1}{\frac{1}{\theta} h'\left(\frac{x(\overline{q}(\theta;P),\theta;P)+\overline{q}(\theta;P)}{\theta}\right)}. \tag{B.3}$$

In addition, the variable $\tilde{\theta}(P)$ satisfies:

$$x(0,\tilde{\theta}(P);P) - \tilde{T}[x(0,\tilde{\theta}(P);P)] - h\left(\frac{x(0,\tilde{\theta}(P);P)}{\tilde{\theta}(P)}\right) = r.$$

And so,

$$\frac{\partial \tilde{\theta}}{\partial r}(\theta; P) = \frac{1}{\frac{x(0,\tilde{\theta}(P);P)}{\tilde{\theta}(P)^2} h'\left(\frac{x(0,\tilde{\theta}(P);P)}{\tilde{\theta}(P)}\right)}. \tag{B.4}$$

Denoting optimal quantities with * superscripts, the first order condition for r is:

$$\begin{aligned}
&\Big(\frac{\delta}{\delta+\lambda} + \frac{\lambda}{\delta+\lambda} K[\tilde{\theta}^*]\Big) U'(b^*) \\
&\quad + \frac{\lambda}{\lambda+\delta} \int_{\tilde{\theta}^*}^{\infty} \frac{1}{\overline{q}^*(\theta)} \frac{\partial \overline{q}^*}{\partial r}(\theta) \int_{\underline{q}^*(\theta)}^{\overline{q}^*(\theta)} U'\left(\Phi^*(q,\theta)\right)(1 - T^{*\prime}[x^*(q,\theta)]) q g^*(q|\theta) dq k(\theta) d\theta \\
&\quad + \frac{\lambda}{\lambda+\delta} E^*[U'(\Phi^*)] \Bigg\{ - k[\tilde{\theta}^*] \Big(T^*[x^*(0,\tilde{\theta}^*)] + b^*\Big) \frac{\partial \tilde{\theta}^*}{\partial r} \\
&\qquad + \int_{\tilde{\theta}^*}^{\infty} \frac{1}{\overline{q}^*(\theta)} \frac{\partial \overline{q}^*}{\partial r}(\theta) \int_{\underline{q}^*(\theta)}^{\overline{q}^*(\theta)} T^{*\prime}[x^*(q,\theta)] \left\{ \frac{x^*(q,\theta)}{x^*(q,\theta)+q} \frac{\mathscr{E}^*(q,\theta)}{\mathcal{E}^*(q,\theta)} \right\} q g^*(q|\theta) dq k(\theta) d\theta \\
&\qquad - \int_{\tilde{\theta}^*}^{\infty} \frac{1}{\overline{q}^*(\theta)} \frac{\partial \overline{q}^*}{\partial r}(\theta) \int_{\underline{q}^*(\theta)}^{\overline{q}^*(\theta)} q g^*(q|\theta) dq k(\theta) d\theta \Bigg\} = 0.
\end{aligned} \tag{B.5}$$

The first term in (B.5) gives the direct utility of benefit of an increase in r (and, hence, b) to an unemployed worker at the optimum. Although higher benefits raise $\tilde{\theta}^*$ (see (B.4)) and, hence, the fraction of workers who are unemployed, this has no direct effect on worker utility as the worker of talent $\tilde{\theta}^*$ is indifferent between employment and unemployment at the optimum. However, a higher value of r reduces $\overline{q}^*$ (see (B.3)) and shifts the distribution of profit-per-worker offers. In so doing it raises the incomes and utilities of those workers who work. This is captured by the second term in (B.5). The remaining terms capture the impact on (the shadow value of) the policymaker's budget of perturbations in r around the optimum. The first of these terms (on the third line) gives the adverse effect of raising $\tilde{\theta}^*$ and increasing unemployment. This effect combines the loss of tax revenues and the increased benefits paid. The next term on the fourth line gives the increased income tax revenues stemming from the higher incomes induced by the downward shift in the profit-per-worker distribution. The final term gives the loss in profit tax revenues from the same effect.

First order condition for T'. Finally, we consider the optimal determination of $\tilde{T}$. Specifically, we derive the first order condition stated in Theorem 1 for T' or, equivalently, for $\tilde{T}'$. To this end, let $\hat{T}[x] = T^*[x] + \kappa \Omega_\varepsilon^{x_0}(x)$ denote a tax perturbation around an optimal tax function T, where κ is a real number and $\Omega_\varepsilon^{x_0}(x) = \int_0^x \omega_\varepsilon^{x_0}(x')dx'$ is a function perturbing the tax amount at each income. Thus, the functions $\{\omega_\varepsilon^{x_0}\}$ perturb the marginal tax rate at each income. They will be constructed to allow us in a limiting sense to perturb the marginal tax rate "at" the single income level $x_0 > 0$. Each function $\omega_\varepsilon^{x_0}$ is assumed to be twice continuously differentiable and the family of perturbation functions $\omega_\varepsilon^{x_0}$ is assumed to satisfy for any sequence $\varepsilon_n \downarrow 0$, (i) $\lim_{n\to\infty} \int_0^\infty \omega_{\varepsilon_n}^{x_0} = 1$ and (ii) $\lim_{n\to\infty} \omega_{\varepsilon_n}^{x_0} \overset{\text{weak}}{\to}$

δ_{x_0}, where δ_{x_0} denotes the Dirac delta function and $\overset{\text{weak}}{\to}$ denotes weak convergence. Such a family satisfies for any continuous function l and sequence $\varepsilon_n \downarrow 0$, $\lim_{n\to\infty} \int_0^\infty \omega_{\varepsilon_n}^{x_0}(x)l(x)dx = l(x_0)$. In addition, note that for any $x < x_0$, $\lim_{n\to\infty} \Omega_{\varepsilon_n}^{x_0}(x) = 0$ and for $x > x_0$, $\lim_{n\to\infty} \Omega_{\varepsilon_n}^{x_0}(x) = 1$. As a concrete example consider the family $\{\omega_\varepsilon^{x_0}\}_\varepsilon$, where:

$$\omega_\varepsilon^{x_0}(x) = \begin{cases} 0 & x \le x_0 - \varepsilon - \varepsilon^2 \\ \frac{1}{2\varepsilon} \exp\left(- \left\{ \frac{x_0 - \varepsilon - x}{x - x_0 + \varepsilon + \varepsilon^2} \right\}^2 \right) & x \in [x_0 - \varepsilon - \varepsilon^2, x_0 - \varepsilon] \\ \frac{1}{2\varepsilon} & x \in [x_0 - \varepsilon, x_0 + \varepsilon] \\ \frac{1}{2\varepsilon} \exp\left(- \left\{ \frac{x - x_0 - \varepsilon}{x_0 + \varepsilon + \varepsilon^2 - x} \right\}^2 \right) & x \in [x_0 + \varepsilon, x_0 + \varepsilon + \varepsilon^2] \\ 0 & x \ge x_0 + \varepsilon + \varepsilon^2. \end{cases}$$

Suppose that l is a continuous function. Then, using the mean value theorem with $x' \in [x_0 - \varepsilon, x_0 + \varepsilon]$,

$$\begin{aligned} \int \omega_\varepsilon^{x_0}(x)l(x)dx &= \int_{x_0-\varepsilon-\varepsilon^2}^{x_0-\varepsilon} \omega_\varepsilon^{x_0}(x)l(x)dx + \int_{x_0+\varepsilon}^{x_0+\varepsilon+\varepsilon^2} \omega_\varepsilon^{x_0}(x)l(x)dx + \frac{1}{2\varepsilon}\int_{x_0-\varepsilon}^{x_0+\varepsilon} l(x)dx \\ &\le \frac{1}{2\varepsilon}\int_{x_0-\varepsilon-\varepsilon^2}^{x_0-\varepsilon} |l(x)|dx + \frac{1}{2\varepsilon}\int_{x_0+\varepsilon}^{x_0+\varepsilon+\varepsilon^2} |l(x)|dx + \frac{1}{2\varepsilon}\int_{x_0-\varepsilon}^{x_0+\varepsilon} l(x)dx \\ &\le \frac{\varepsilon}{2} \sup_{x\in[x_0-\varepsilon-\varepsilon^2, x_0-\varepsilon]} |l(x)| + \frac{\varepsilon}{2} \sup_{x\in[x_0+\varepsilon, x_0+\varepsilon+\varepsilon^2]} |l(x)| + l(x'). \end{aligned}$$

Given the continuity of l as $\varepsilon \to 0$ this bound converges to $l(x_0)$. Similarly, a lower bound can be computed that converges to $l(x_0)$. We conclude that for any continuous function l, $\lim_{\varepsilon\downarrow} \int \omega_\varepsilon^{x_0}(x)l(x)dx = l(x_0)$.

We first consider incomes x_0 that are above the minimum income paid (to an active worker) (and, hence, above the income paid to the threshold type $\tilde{\theta}^*$), i.e. that are greater than $x^*(0, \tilde{\theta}^*)$.

Perturbing the objective For a small $\kappa > 0$ and $\Omega_\varepsilon^{x_0}$ in the family of perturbation functions described above, the perturbed objective for (26) is:

$$\begin{aligned} &\Big(\frac{\delta}{\delta+\lambda} + \frac{\lambda}{\delta+\lambda} K[\tilde{\theta}(T^* + \kappa\Omega_\varepsilon^{x_0})]\Big) U(b) \\ &\quad + \frac{\lambda}{\delta+\lambda} \int_{\tilde{\theta}(T^*+\kappa\Omega_\varepsilon^{x_0})}^{\infty} \int_{\underline{q}(\theta;T^*+\kappa\Omega_\varepsilon^{x_0})}^{\overline{q}(\theta;T^*+\kappa\Omega_\varepsilon^{x_0})} U\left(\Phi(q,\theta;T^*+\kappa\Omega_\varepsilon^{x_0})\right) g(q|\theta;T^*+\kappa\Omega_\varepsilon^{x_0})k(\theta)dqd\theta. \end{aligned} \tag{B.6}$$

Our first goal is to differentiate this perturbed objective with respect to κ and evaluate at $\kappa = 0$. We will call this derivative the direct impact of the tax perturbation (on social welfare). We start by evaluating the impact of the tax perturbation on the utility of workers at each (q, θ), deferring for the moment the impact of the perturbation on the distribution of workers across (q,θ). First note that by the definition of Φ:

$$\Phi(q,\theta;T^*+\kappa\Omega_\varepsilon^{x_0}) = x(q,\theta;T^*+\kappa\Omega_\varepsilon^{x_0}) - T^*[x(q,\theta;T^*+\kappa\Omega_\varepsilon^{x_0})] - \kappa\Omega_\varepsilon^{x_0}[x(q,\theta;T^*+\kappa\Omega_\varepsilon^{x_0})] - h\left(\frac{x(q,\theta;T^*+\kappa\Omega_\varepsilon^{x_0})+q}{\theta}\right),$$

where $x(q,\theta;T^*+\kappa\Omega_\varepsilon^{x_0})$ is the worker's optimal income choice at $(q,\theta;T^*+\kappa\Omega_\varepsilon^{x_0})$. Hence:

$$\frac{\partial\Phi}{\partial\kappa}(q,\theta;T^*+\kappa\Omega_\varepsilon^{x_0})\bigg|_{\kappa=0} = -\Omega_\varepsilon^{x_0}[x^*(q,\theta)]$$

And so:

$$\begin{aligned}\lim_{\varepsilon\downarrow 0}\frac{\lambda}{\delta+\lambda}\frac{\partial}{\partial\kappa}\int_{\tilde\theta^*}^{\infty}\int_{\underline{q}^*(\theta)}^{\overline{q}^*(\theta)} U\left(\Phi(q,\theta;T^*+\kappa\Omega_\varepsilon^{x_0})\right)g^*(q|\theta)k(\theta)dqd\theta\bigg|_{\kappa=0} \qquad \text{(B.7)}\\ = -\frac{\lambda}{\delta+\lambda}\int_{\underline{\theta}^*(x_0)}^{\infty}\int_{\underline{q}^*(\theta)}^{\hat q^*(\theta,x_0)} U'\left(\Phi^*(q,\theta)\right)g^*(q|\theta)k(\theta)dqd\theta,\end{aligned}$$

where for $\theta\in[\underline{\theta}^*(x_0),\overline{\theta}^*(x_0)]$, $\hat q^*(\theta,x_0) = \min\{\tilde q^*(\theta,x_0),\overline{q}^*(\theta)\}$ and for $\theta > \overline{\theta}^*(x_0)$, $\hat q^*(\theta,x_0) = \overline{q}^*(\theta)$. Next consider the impact of the tax perturbation on the distribution of profit offers made and, hence, the utility of workers:

$$\begin{aligned}&\frac{\lambda}{\delta+\lambda}\frac{\partial}{\partial\kappa}\int_{\tilde\theta^*}^{\infty}\int_{\underline{q}(\theta;T^*+\kappa\Omega_\varepsilon^{x_0})}^{\overline{q}(\theta;T^*+\kappa\Omega_\varepsilon^{x_0})} U\left(\Phi^*(q,\theta)\right)g(q|\theta;T^*+\kappa\Omega_\varepsilon^{x_0})k(\theta)dqd\theta\bigg|_{\kappa=0}\\ &= -\frac{\lambda}{\delta+\lambda}\int_{\underline{\theta}^*(x_0)}^{\infty} U\left(\Phi^*(\underline{q}^*(\theta),\theta)\right)\frac{\partial\underline{q}^*}{\partial\kappa}(\theta)g^*(\underline{q}^*(\theta)|\theta)k(\theta)d\theta\\ &\quad+\frac{\lambda}{\delta+\lambda}\int_{\underline{\theta}^*(x_0)}^{\infty} U\left(\Phi^*(\overline{q}^*(\theta),\theta)\right)\frac{\partial\overline{q}^*}{\partial\kappa}(\theta)g^*(\overline{q}^*(\theta)|\theta)k(\theta)d\theta\\ &\quad+\frac{\lambda}{\delta+\lambda}\int_{\underline{\theta}^*(x_0)}^{\infty}\int_{\underline{q}^*(\theta)}^{\overline{q}^*(\theta)} U\left(\Phi^*(q,\theta)\right)\frac{\partial g^*}{\partial\kappa}(q|\theta)k(\theta)dqd\theta. \qquad \text{(B.8)}\end{aligned}$$

Define $\mathcal{E}_{\overline{q}}^*(\theta) := -\frac{1-T^{*\prime}[x_0]}{\overline{q}^*(\theta)}\frac{\partial\overline{q}^*}{\partial\kappa}(\theta)$. Note that $\overline{q}(\theta;T^*+\kappa\Omega_\varepsilon^{x_0})$ is defined implicitly by:

$$\begin{aligned}&x(\overline{q}(\theta;T^*+\kappa\Omega_\varepsilon^{x_0}),\theta;T^*+\kappa\Omega_\varepsilon^{x_0}) - T^*[x(\overline{q}(\theta;T^*+\kappa\Omega_\varepsilon^{x_0}),\theta;T^*+\kappa\Omega_\varepsilon^{x_0})]\\ &-\kappa\Omega_\varepsilon^{x_0}[x(\overline{q}(\theta;T^*+\kappa\Omega_\varepsilon^{x_0}),\theta;T^*+\kappa\Omega_\varepsilon^{x_0})] - h\left(\frac{x(\overline{q}(\theta;T+\kappa\Omega_\varepsilon^{x_0}),\theta;T^*+\kappa\Omega_\varepsilon^{x_0})+\overline{q}(\theta;T+\kappa\Omega_\varepsilon^{x_0})}{\theta}\right) = b.\end{aligned}$$

Differentiating with respect to κ and setting κ equal to 0 gives:

$$\frac{\partial\overline{q}^*}{\partial\kappa}(\theta) := \frac{\partial\overline{q}}{\partial\kappa}(\theta;T^*+\kappa\Omega_\varepsilon^{x_0})\bigg|_{\kappa=0} = -\frac{\theta\Omega_\varepsilon^{x_0}[x^*(\overline{q}^*(\theta),\theta)]}{h'\left(\frac{x^*(\overline{q}^*(\theta),\theta)+\overline{q}^*(\theta)}{\theta}\right)}.$$

Thus, in the limit as $\varepsilon \downarrow 0$, $\mathcal{E}^*_{\overline{q}}(\theta)$ is zero for θ less than $\overline{\theta}^*(x_0)$.

The definitions of $\mathcal{E}^*_{\overline{q}}(\theta)$ and $\underline{q}^*$ imply that the term in the second line of (B.8) can be written as:

$$\begin{aligned}
&-\frac{\lambda}{\delta+\lambda}\int_{\underline{\theta}^*(x_0)}^{\infty} U\left(\Phi^*(\underline{q}^*(\theta),\theta)\right)\frac{\partial \underline{q}^*}{\partial \kappa}(\theta) g^*(\underline{q}^*(\theta)|\theta)k(\theta)d\theta \\
&\qquad\qquad = \frac{\lambda}{\delta+\lambda}\int_{\underline{\theta}^*(x_0)}^{\infty} U\left(\Phi^*(\underline{q}^*(\theta),\theta)\right)\frac{\mathcal{E}^*_{\overline{q}}(\theta)}{1-T^{*\prime}[x_0]}\underline{q}^*(\theta) g^*(\underline{q}^*(\theta)|\theta)k(\theta)d\theta. \qquad \text{(B.9)}
\end{aligned}$$

Similarly, the term in the third line of (B.8) can be written as:

$$\begin{aligned}
&\frac{\lambda}{\delta+\lambda}\int_{\underline{\theta}^*(x_0)}^{\infty} U\left(\Phi^*(\overline{q}^*(\theta),\theta)\right)\frac{\partial \overline{q}^*}{\partial \kappa}(\theta) g^*(\overline{q}^*(\theta)|\theta)k(\theta)d\theta \\
&\qquad\qquad = -\frac{\lambda}{\delta+\lambda}\int_{\underline{\theta}^*(x_0)}^{\infty} U\left(\Phi^*(\overline{q}^*(\theta),\theta)\right)\frac{\mathcal{E}^*_{\overline{q}}(\theta)}{1-T^{*\prime}[x_0]}\overline{q}^*(\theta) g^*(\overline{q}^*(\theta)|\theta)k(\theta)d\theta. \qquad \text{(B.10)}
\end{aligned}$$

Next recall that:

$$g(q|\theta;T^*+\kappa\Omega^{x_0}_{\varepsilon}) = \frac{\delta}{2\lambda}\left(\frac{\overline{q}(\theta;T^*+\kappa\Omega^{x_0}_{\varepsilon})}{q}\right)^{\frac{1}{2}}\frac{1}{q}.$$

And, hence,

$$\frac{\partial g^*}{\partial \kappa}(q|\theta) := \frac{\partial g}{\partial \kappa}(q|\theta;T^*+\kappa\Omega^{x_0}_{\varepsilon})\bigg|_{\kappa=0} = -\frac{1}{2}g^*(q|\theta)\frac{\mathcal{E}^*_{\overline{q}}(\theta)}{1-T^{*\prime}[x_0]}.$$

Thus, the final term in (B.8) satisfies:

$$\begin{aligned}
&\frac{\lambda}{\delta+\lambda}\int_{\tilde{\theta}^*}^{\infty}\int_{\underline{q}^*(\theta)}^{\overline{q}^*(\theta)} U\left(\Phi^*(q,\theta)\right)\frac{\partial g^*}{\partial \kappa}(q|\theta)k(\theta)dq d\theta \\
&\qquad = -\frac{1}{2}\frac{\lambda}{\delta+\lambda}\int_{\tilde{\theta}^*}^{\infty}\frac{\mathcal{E}^*_{\overline{q}}(\theta)}{1-T^{*\prime}[x_0]}\left\{\int_{\underline{q}^*(\theta)}^{\overline{q}^*(\theta)} U\left(\Phi^*(q,\theta)\right)g^*(q|\theta)dq\right\}k(\theta)d\theta. \qquad \text{(B.11)}
\end{aligned}$$

Integrating the inner integral in (B.11) by parts:

$$\begin{aligned}
&\int_{\underline{q}^*(\theta)}^{\overline{q}^*(\theta)} U\left(\Phi^*(q,\theta)\right) g^*(q|\theta) dq \\
&\quad = U\left(\Phi^*(q,\theta)\right) g^*(q|\theta) q \Big]_{\underline{q}^*(\theta)}^{\overline{q}^*(\theta)} - \int_{\underline{q}^*(\theta)}^{\overline{q}^*(\theta)} \frac{\partial}{\partial q} \left\{ U\left(\Phi^*(q,\theta)\right) g^*(q|\theta) \right\} q dq \\
&\quad = U\left(\Phi^*(q,\theta)\right) g^*(q|\theta) q \Big]_{\underline{q}^*(\theta)}^{\overline{q}^*(\theta)} + \int_{\underline{q}^*(\theta)}^{\overline{q}^*(\theta)} U'\left(\Phi^*(q,\theta)\right) (1 - T^{*\prime}[x^*(q,\theta)]) q g^*(q|\theta) dq \\
&\qquad\qquad - \int_{\underline{q}^*(\theta)}^{\overline{q}^*(\theta)} U\left(\Phi^*(q,\theta)\right) \frac{\partial g^*}{\partial q}(q|\theta) dq \\
&\quad = U\left(\Phi^*(q,\theta)\right) g^*(q|\theta) q \Big]_{\underline{q}^*(\theta)}^{\overline{q}^*(\theta)} + \int_{\underline{q}^*(\theta)}^{\overline{q}^*(\theta)} U'\left(\Phi^*(q,\theta)\right) (1 - T^{*\prime}[x^*(q,\theta)]) q g^*(q|\theta) dq \\
&\qquad\qquad + \frac{3}{2} \int_{\underline{q}^*(\theta)}^{\overline{q}^*(\theta)} U\left(\Phi^*(q,\theta)\right) g^*(q|\theta) dq.
\end{aligned}$$

Consequently,

$$\begin{aligned}
\frac{1}{2} \int_{\underline{q}^*(\theta)}^{\overline{q}^*(\theta)} U\left(\Phi^*(q,\theta)\right) g^*(q|\theta) dq = &-U\left(\Phi^*(q,\theta)\right) g^*(q|\theta) q \Big]_{\underline{q}^*(\theta)}^{\overline{q}^*(\theta)} \\
&- \int_{\underline{q}^*(\theta)}^{\overline{q}^*(\theta)} U'\left(\Phi^*(q,\theta)\right) (1 - T^{*\prime}[x^*(q,\theta)]) q g^*(q|\theta) dq. \qquad \text{(B.12)}
\end{aligned}$$

Combining (B.8) to (B.12) gives:

$$\begin{aligned}
&\frac{\lambda}{\delta+\lambda} \frac{\partial}{\partial \kappa} \int_{\tilde{\theta}^*}^{\infty} \int_{\underline{q}(\theta; T^* + \kappa \Omega_\varepsilon^{x_0})}^{\overline{q}(\theta; T^* + \kappa \Omega_\varepsilon^{x_0})} U\left(\Phi^*(q,\theta)\right) g(q|\theta; T^* + \kappa \Omega_\varepsilon^{x_0}) k(\theta) dq d\theta \Bigg|_{\kappa=0} \\
&\quad = \frac{\lambda}{\delta+\lambda} \int_{\tilde{\theta}^*(x_0)}^{\infty} \frac{\mathcal{E}_{\overline{q}}^*(\theta)}{1 - T^{*\prime}[x_0]} \int_{\underline{q}^*(\theta)}^{\overline{q}^*(\theta)} U'\left(\Phi^*(q,\theta)\right) (1 - T^{*\prime}[x^*(q,\theta)]) q g^*(q|\theta) dq k(\theta) d\theta.
\end{aligned}$$
(B.13)

Putting everything together:

$$\lim_{\varepsilon\downarrow 0}\frac{\lambda}{\delta+\lambda}\frac{\partial}{\partial\kappa}\int_{\tilde{\theta}^*}^{\infty}\int_{\underline{q}(\theta;T^*+\kappa\Omega_\varepsilon^{x_0})}^{\overline{q}(\theta;T^*+\kappa\Omega_\varepsilon^{x_0})} U\left(\Phi(q,\theta;T^*+\kappa\Omega_\varepsilon^{x_0})\right) g(q|\theta;T^*+\kappa\Omega_\varepsilon^{x_0})k(\theta)dqd\theta\Bigg|_{\kappa=0}$$

$$=\frac{\lambda}{\delta+\lambda}\int_{\underline{\theta}^*(x_0)}^{\infty}\int_{\underline{q}^*(\theta)}^{\hat{q}^*(\theta,x_0)} U'\left(\Phi^*(q,\theta)\right) g^*(q|\theta)k(\theta)dqd\theta$$

$$+\frac{\lambda}{\delta+\lambda}\int_{\tilde{\theta}^*(x_0)}^{\infty}\frac{\mathcal{E}^*_{\overline{q}}(\theta)}{1-T^{*\prime}[x_0]}\int_{\underline{q}^*(\theta)}^{\overline{q}^*(\theta)} U'\left(\Phi^*(q,\theta)\right)(1-T^{*\prime}[x^*(q,\theta)])qg^*(q|\theta)dqk(\theta)d\theta \tag{B.14}$$

Perturbing Tax Revenues We next consider the impact of the perturbation on tax revenues. Perturbed tax revenues are given by:

$$\frac{\lambda}{\delta+\lambda}\int_{\tilde{\theta}^*}^{\infty}\int_{\underline{q}(\theta;T^*+\kappa\Omega_\varepsilon^{x_0})}^{\overline{q}(\theta;T^*+\kappa\Omega_\varepsilon^{x_0})}\{T^*[x(q,\theta;T^*+\kappa\Omega_\varepsilon^{x_0})]+\kappa\Omega_\varepsilon^{x_0}[x(q,\theta;T^*+\kappa\Omega_\varepsilon^{x_0})]\}g(q|\theta;T^*+\kappa\Omega_\varepsilon^{x_0})dqk(\theta)d\theta$$

$$+\frac{\lambda}{\delta+\lambda}\int_{\tilde{\theta}(T^*+\kappa\Omega_\varepsilon^{x_0})}^{\infty}\int_{\underline{q}(\theta;T^*+\kappa\Omega_\varepsilon^{x_0})}^{\overline{q}(\theta;T^*+\kappa\Omega_\varepsilon^{x_0})} qg(q|\theta;T^*+\kappa\Omega_\varepsilon^{x_0})dqk(\theta)d\theta.$$

From (23):

$$-\frac{\partial x(q,\theta;T^*+\kappa\Omega_\varepsilon^{x_0})}{\partial\kappa}\Bigg|_{\kappa=0}=\frac{x^*(q,\theta)\omega_\varepsilon^{x_0}(x^*(q,\theta))}{1-T^{*\prime}[x^*(q,\theta)]}\mathscr{E}^*(q,\theta). \tag{B.15}$$

And so:

$$\lim_{\varepsilon\downarrow 0}-\frac{\partial x(q,\theta;T^*+\kappa\Omega_\varepsilon^{x_0})}{\partial\kappa}\Bigg|_{\kappa=0}=\begin{cases}\frac{x_0}{1-T^{*\prime}[x_0]}\mathscr{E}^*(q,\theta) & \text{if } x^*(q,\theta)=x_0\\ 0 & \text{otherwise}\end{cases}. \tag{B.16}$$

Define:

$$\overline{\mathscr{E}}(x_0;T)=\int_{\underline{\theta}(x_0;T)}^{\overline{\theta}(x_0;T)}\mathscr{E}(\tilde{q}(x_0,\theta;T),\theta;T)\frac{g(\tilde{q}(x_0,\theta;T)|\theta;T)k(\theta)}{\int_{\underline{\theta}(x_0;T)}^{\overline{\theta}(x_0;T)} g(\tilde{q}(x_0,\theta;T)|\theta;T)k(\theta)}d\theta$$

to be the average tax elasticity amongst workers earning x_0 at tax function T and let $J[\cdot;T]$ and $j(\cdot;T)$ denote the induced labor income distribution and density. Note that:

$$1-J[x_0;T]=\lim_{\varepsilon\downarrow 0}\int_{\underline{\theta}(T)}^{\infty}\int_{\underline{q}(\theta;T)}^{\overline{q}(\theta;T)}\Omega_\varepsilon^{x_0}(x(q,\theta;T))g(q|\theta;T)k(\theta)dqd\theta. \tag{B.17}$$

Proceeding analogously to the derivation of (B.14),

$$\lim_{\varepsilon\downarrow 0}\frac{\partial}{\partial\kappa}\int_{\tilde{\theta}(T^*)}^{\infty}\int_{\underline{q}(\theta;T^*+\kappa\Omega_\varepsilon^{x_0})}^{\overline{q}(\theta;T^*+\kappa\Omega_\varepsilon^{x_0})}\{T^*[x(q,\theta;T^*+\kappa\Omega_\varepsilon^{x_0})]+\kappa\Omega_\varepsilon^{x_0}(x(q,\theta;T^*+\kappa\Omega_\varepsilon^{x_0}))\}g(q|\theta;T^*+\kappa\Omega_\varepsilon^{x_0})k(\theta)dqd\theta\Big|_{\kappa=0}$$
$$=(1-J[x_0;T^*])-\frac{x_0T^{*\prime}[x_0]}{1-T^{*\prime}[x_0]}\overline{\mathscr{E}}(x_0;^*T)j(x_0;T^*)$$
$$+\int_{\overline{\theta}(x_0;T^*)}^{\infty}\frac{\mathcal{E}_{\overline{q}}(\theta;T^*)}{1-T^{*\prime}[x_0]}\int_{\underline{q}(\theta;T^*)}^{\overline{q}(\theta;T^*)}T^{*\prime}[x(q,\theta;T^*)]\left\{\frac{x(q,\theta;T^*)}{x(q,\theta;T^*)+q}\frac{\mathscr{E}(q,\theta;T^*)}{\mathcal{E}(q,\theta;T^*)}\right\}qg(q|\theta;T^*)dqk(\theta)d\theta. \tag{B.18}$$

Finally, turning to profit taxes:

$$\lim_{\varepsilon\downarrow 0}\frac{\partial}{\partial\kappa}\int_{\tilde{\theta}(T^*)}^{\infty}\int_{\underline{q}(\theta;T^*+\kappa\Omega_\varepsilon^{x_0})}^{\overline{q}(\theta;T^*+\kappa\Omega_\varepsilon^{x_0})}qg(q|\theta;T^*+\kappa\Omega_\varepsilon^{x_0})k(\theta)dqd\theta\Big|_{\kappa=0}$$
$$=-\frac{\lambda}{\delta+\lambda}\int_{\overline{\theta}(x_0;T^*)}^{\infty}\frac{\mathcal{E}_{\overline{q}}(\theta;T^*)}{1-T^{*\prime}[x_0]}\int_{\underline{q}(\theta;T^*)}^{\overline{q}(\theta;T^*)}qg(q|\theta;T^*)dqk(\theta)d\theta. \tag{B.19}$$

The optimal resource multiplier $\mu^*=E^*[U'(\Phi)]$ and so combining everything, the first order condition for the marginal tax rate $T'[x_0]$ is:

$$-\int_{\underline{\theta}(x_0;T^*)}^{\infty}\int_{\underline{q}(\theta;T^*)}^{\hat{q}(\theta,x_0;T^*)}U'\left(\Phi(q,\theta;T^*)\right)g(q|\theta;T^*)k(\theta)dqd\theta$$
$$+\int_{\overline{\theta}(x_0;T^*)}^{\infty}\frac{\mathcal{E}_{\overline{q}}(\theta;T^*)}{1-T^{*\prime}[x_0]}\int_{\underline{q}(\theta;T^*)}^{\overline{q}(\theta;T^*)}U'\left(\Phi(q,\theta;T^*)\right)(1-T^{*\prime}[x(q,\theta;T^*)])qg(q|\theta;T^*)dqk(\theta)d\theta$$
$$+E^*[U'(\Phi)]\Bigg\{(1-J[x_0;T^*])-\frac{x_0T^{*\prime}[x_0]}{1-T^{*\prime}[x_0]}\overline{\mathscr{E}}(x_0;T^*)j(x_0;T^*)$$
$$+\int_{\overline{\theta}(x_0;T^*)}^{\infty}\frac{\mathcal{E}_{\overline{q}}(\theta;T^*)}{1-T^{*\prime}[x_0]}\int_{\underline{q}(\theta;T^*)}^{\overline{q}(\theta;T^*)}T^{*\prime}[x(q,\theta;T^*)]\left\{\frac{x(q,\theta;T^*)}{x(q,\theta;T^*)+q}\frac{\mathscr{E}(q,\theta;T^*)}{\mathcal{E}(q,\theta;T^*)}\right\}qg(q|\theta;T^*)dqk(\theta)d\theta$$
$$-\int_{\overline{\theta}(x_0;T^*)}^{\infty}\frac{\mathcal{E}_{\overline{q}}(\theta;T^*)}{1-T^{*\prime}[x_0]}\int_{\underline{q}(\theta;T^*)}^{\overline{q}(\theta;T^*)}qg(q|\theta;T^*)dqk(\theta)d\theta\Bigg\}=0. \tag{B.20}$$

This equation is readily reorganized to more closely resemble the affine optimal equation (16):

$$
\begin{aligned}
&-\left\{\int_{\underline{\theta}(x_0;T^*)}^{\infty}\int_{\underline{q}(\theta;T^*)}^{\hat{q}(\theta,x_0;T^*)} U'\left(\Phi(q,\theta;T^*)\right) g(q|\theta;T^*)k(\theta)dqd\theta - E^*[U'(\Phi)](1-J[x_0;T^*])\right\} \\
&\quad + E^*[U'(\Phi)]\Bigg\{-\frac{x_0 T^{*\prime}[x_0]}{1-T^{*\prime}[x_0]}\overline{\mathscr{E}}(x_0;T^*)j(x_0;T^*) \\
&\qquad\qquad + \int_{\overline{\theta}(x_0;T^*)}^{\infty}\frac{\mathcal{E}_{\overline{q}}(\theta;T^*)}{1-T^{*\prime}[x_0]}\int_{\underline{q}(\theta;T^*)}^{\overline{q}(\theta;T^*)} T^{*\prime}[x(q,\theta;T^*)]\mathscr{R}(q,\theta;T^*)qg(q|\theta;T^*)dqk(\theta)d\theta\Bigg\} \\
&\quad + \Bigg\{\int_{\overline{\theta}(x_0;T^*)}^{\infty}\frac{\mathcal{E}_{\overline{q}}(\theta;T^*)}{1-T^{*\prime}[x_0]}\int_{\underline{q}(\theta;T^*)}^{\overline{q}(\theta;T^*)} U'\left(\Phi(q,\theta;T^*)\right)\left(1-T^{*\prime}[x(q,\theta;T^*)]\right)qg(q|\theta;T^*)dqk(\theta)d\theta \\
&\qquad\qquad -E^*[U'(\Phi)]\int_{\overline{\theta}(x_0;T^*)}^{\infty}\frac{\mathcal{E}_{\overline{q}}(\theta;T^*)}{1-T^{*\prime}[x_0]}\int_{\underline{q}(\theta;T^*)}^{\overline{q}(\theta;T^*)}(1-T^{*\prime}[x(q,\theta)])qg(q|\theta;T^*)dqk(\theta)d\theta\Bigg\} \\
&\quad - E^*[U'(\Phi)]\left\{\int_{\overline{\theta}(x_0;T^*)}^{\infty}\frac{\mathcal{E}_{\overline{q}}(\theta;T^*)}{1-T^{*\prime}[x_0]}\int_{\underline{q}(\theta;T^*)}^{\overline{q}(\theta;T^*)} T^{*\prime}[x(q,\theta;T^*)]qg(q|\theta;T^*)dqk(\theta)d\theta\right\} = 0,
\end{aligned}
\tag{B.21}
$$

where $\mathscr{R}(q,\theta;T^*) := \frac{x(q,\theta;T^*)}{x(q,\theta;T^*)+q}\frac{\mathscr{E}(q,\theta;T^*)}{\mathcal{E}(q,\theta;T^*)}$. The first line contains the analogue of the covariance between marginal utility and labor earnings from the affine case. The second and third lines contain the analogue of the second term form the affine case: it gives the impact on labor earnings and, hence, revenues from the tax perturbation. This term includes the usual intensive margin income response and the profit channel income response. The fourth and fifth lines give the analogue of the covariance between marginal utility and the profit adjustment. As in the affine case, it captures redistributive consequences of the profit channel. The final line adjusts for the fact that workers only receive the after-tax profit adjustment.

We next consider incomes x_0 that are below the minimum income paid (to an active worker) (and, hence, above the income paid to the threshold type $\tilde{\theta}(T^*)$), i.e. that are below $x(0,\tilde{\theta}(T^*);T^*)$.

First note that:

$$
\left.\frac{\partial\tilde{\theta}}{\partial\kappa}\right|_{\kappa=0} = \frac{\Omega_{\varepsilon}^{x_0}(x(0,\tilde{\theta}(T);T))}{h'\left(\frac{x(0,\tilde{\theta}(T);T)}{\tilde{\theta}(T)}\right)\frac{x(0,\tilde{\theta}(T);T)}{\tilde{\theta}(T)^2}}.
$$

Thus, perturbations to the tax function at incomes below $x(0,\tilde{\theta}(T);T)$ perturb $\tilde{\theta}(T)$ (by raising the level of taxes at $x(0,\tilde{\theta}(T);T)$). This does not impact the social objective since $\tilde{\theta}(T)$ workers are indifferent between working and not working. However, it does reduce revenues by:

$$-\frac{\lambda}{\delta+\lambda}k[\tilde{\theta}(T)]\Big(T[x(0,\tilde{\theta}(T);T)]+b\Big)\left.\frac{\partial\tilde{\theta}}{\partial\kappa}\right|_{\kappa=0}.$$

Other impacts are computed along the same lines as for incomes $x_0 > x(0,\tilde{\theta}(T);T)$. The overall effect evaluated at the optimum is:

$$\begin{aligned}
&-\int_{\tilde{\theta}(T^*)}^{\infty}\int_{\underline{q}(\theta;T^*)}^{\overline{q}(\theta;T^*)} U'\left(\Phi(q,\theta;T^*)\right) g(q|\theta;T^*)k(\theta)dqd\theta \\
&\quad+\int_{\tilde{\theta}(T^*)}^{\infty}\frac{\mathcal{E}^*_{\overline{q}}(\theta;T^*)}{1-T^{*\prime}[x_0]}\int_{\underline{q}(\theta;T^*)}^{\overline{q}(\theta;T^*)} U'\left(\Phi(q,\theta;T^*)\right)(1-T^{*\prime}[x(q,\theta;T^*)])qg(q|\theta;T^*)dqk(\theta)d\theta \\
&\quad\quad+E^*[U'(\Phi)]\Bigg\{(1-J[x(0,\tilde{\theta}(T^*);T^*);T^*])-k[\tilde{\theta}(T^*)]\frac{T^*[x(0,\tilde{\theta}(T^*);T^*)]+b}{h'\left(\frac{x(0,\tilde{\theta}(T^*);T^*)}{\tilde{\theta}(T^*)}\right)\frac{x(0,\tilde{\theta}(T^*);T^*)}{\tilde{\theta}(T^*)^2}} \\
&\quad\quad\quad+\int_{\tilde{\theta}(T^*)}^{\infty}\frac{\mathcal{E}_{\overline{q}}(\theta;T^*)}{1-T^{*\prime}[x_0]}\int_{\underline{q}(\theta;T^*)}^{\overline{q}(\theta;T^*)} T'[x(q,\theta;T^*)]\left\{\frac{x(q,\theta;T^*)}{x(q,\theta;T^*)+q}\frac{\mathscr{E}(q,\theta;T^*)}{\mathcal{E}(q,\theta;T^*)}\right\}qg(q|\theta;T^*)dqk(\theta)d\theta \\
&\quad\quad\quad\quad-\int_{\tilde{\theta}(T^*)}^{\infty}\frac{\mathcal{E}_{\overline{q}}(\theta;T^*)}{1-T^{*\prime}[x_0]}\int_{\underline{q}(\theta;T^*)}^{\overline{q}(\theta;T^*)} qg(q|\theta;T^*)dqk(\theta)d\theta\Bigg\}=0. \qquad \text{(B.22)}
\end{aligned}$$

C Calibration Details

Data Our main data source is the Current Population Survey (CPS) administered by the US Census Bureau and the US Bureau of Labor Statistics. We focus on the March release of the 2016 survey which provides information for the calendar year 2015 and use March supplement sample weights to produce our estimates. The total number of observations in the raw sample is 185,487. We focus on population of working age individuals and so we drop people who are younger than 25 or older than 65 years of age. This reduces the sample size to 97,168.

Our main variable of interest is labor income. We measure this as the sum of wage and salary income earned during the last calendar year. We compute total hours worked from data on average hours worked per week and total number of weeks worked. Following Heathcote, Perri and Violante (2010), we drop those who work a very small number of (less than 100) hours and those who earned very small annual amounts (less than $250). We also drop those giving inconsistent answers to questions about labor income and hours worked (i.e. those who claim they received labor income, but did not work or vice versa). We calculate the hourly wage rate for the remaining working agents in our sample by dividing total labor income earned by total hours worked, and drop individuals whose hourly wage rate is below half of the federal minimum wage.

In addition to working agents, our sample contains agents who have not worked at all during the last calendar year. These agents are asked their reasons for not working. In our model agents who do not work have very low talent. To align non-workers in the data with those in the model, we retain those giving disability, sickness or inability to find work despite searching as reasons for not working, while dropping those giving taking care of home or family, going to school, or retirement as reasons. We are left with 68,749 working agents and 8,120 people who did not work during 2015. We identify the latter group of people with those below the active talent threshold $\tilde{\theta}$ implied by current US policy (and our assumed preference and labor market transition prameters)

Policy Approximation To form an approximation to $\mathcal{T}(\cdot; P)$, an approximation of US policy is needed. We form an affine approximation to government tax policy P by regressing total taxes paid on income. We obtain the estimates (with standard deviations in parentheses):

$$T[x] = \underset{(2.526)}{302.56} + \underset{(0.000361)}{0.336}\ x.$$

[Put plot of taxes and incomes with regression line here]

D Numerical Algorithm for Affine Tax Analysis

Derivations used in the Algorithm For a given policy $P \equiv (b, L, \tau)$, workers in active talent market $\theta \geq \tilde{\theta}$ solve:

$$\max_x L + (1-\tau)x - \frac{(\frac{x+q}{\theta})^{1+\gamma}}{1+\gamma}.$$

This implies worker income and surplus policy functions:

$$x(q,\theta) = [(1-\tau)\theta]^{1/\gamma}\theta - q$$

and

$$y(\theta) \equiv x(q,\theta) + q = [(1-\tau)\theta]^{1/\gamma}\theta.$$

A talent market θ is active if a worker receiving a profit offer $q = 0$ can obtain a utility of at least b. In active talent markets, the maximal profit offer $\overline{q}(\theta)$ that can be made (and accepted with positive probability) is:

$$L + (1-\tau)\{y(\theta) - \overline{q}(\theta)\} - \frac{1}{1+\gamma}\left(\frac{y(\theta)}{\tilde{\theta}}\right)^{1+\gamma} = b.$$

The activity threshold $\tilde{\theta}$ is such that $\bar{q}(\tilde{\theta}) = 0$. Hence, extending $\bar{q}$ from $[\tilde{\theta}, \bar{\theta}]$ to $\underline{\theta}, \bar{\theta}]$, we have:

$$\bar{q}(\theta) = -\frac{b-L}{1-\tau} + v(\theta) < 0, \quad \text{for } \theta < \tilde{\theta}, \tag{D.1}$$
$$\bar{q}(\tilde{\theta}) = -\frac{b-L}{1-\tau} + v(\tilde{\theta}) = 0,$$
$$\bar{q}(\theta) = -\frac{b-L}{1-\tau} + v(\theta) > 0, \quad \text{for } \theta > \tilde{\theta},$$

where $v(\theta) = y(\theta) - \frac{(\frac{y(\theta)}{\theta})^{1+\gamma}}{(1+\gamma)(1-\tau)}$. Substituting v into and rearranging the policymaker's budget constraint gives L as a function of b and τ:

$$L = \frac{1}{A}\Big[-Bb + \frac{\tilde{\lambda}}{\tilde{\delta}+\tilde{\lambda}}\tau \int_{\tilde{\theta}}^{\bar{\theta}} y(\theta)k(\theta)d\theta + \Big(\frac{\tilde{\delta}\tilde{\lambda}}{\tilde{\delta}+\tilde{\lambda}}\Big)^2(1-\tau)(1-K(\tilde{\theta}))\int_{\tilde{\theta}}^{\bar{\theta}} v(\theta)k(\theta)d\theta\Big] \tag{D.2}$$

where the constants A and B satisfy:

$$A = \Big[(1-K(\tilde{\theta}))\frac{\tilde{\lambda}}{\tilde{\delta}+\tilde{\lambda}} - (1-K(\tilde{\theta}))^2\Big(\frac{\tilde{\delta}\tilde{\lambda}}{\tilde{\delta}+\tilde{\lambda}}\Big)^2\Big]$$
$$B = \Big[K(\tilde{\theta}) + (1-K(\tilde{\theta}))\frac{\tilde{\delta}}{\tilde{\delta}+\tilde{\lambda}} + (1-K(\tilde{\theta}))^2\Big(\frac{\tilde{\delta}\tilde{\lambda}}{\tilde{\delta}+\tilde{\lambda}}\Big)^2\Big],$$

with $\tilde{\delta} := 1-\rho(1-\delta)$ and $\tilde{\lambda} = \rho\lambda$.

Algorithm The algorithm combines an outer grid search over b and τ values and an inner step in which an equilibrium and the corresponding social welfare is computed at b and τ points on the grid. In successive runs progressively finer grids are applied to smaller neighborhoods of b and τ around the calculated optimum. The inner equilibrium calculation uses a grid of θ values. It has the following sub-steps.

Substep 0. For a given b and τ, set $\tilde{\theta} = \theta_1$. Set the substep counter$k = 1$ and proceed to the next substep.

Substep k. Using (D.1), check if $\bar{q}(\theta_k) > 0$. If so, then set $\tilde{\theta} = \theta_1$, compute L using (D.2), evaluate $\underline{q}(\theta)$ and $g(\cdot|\theta)$ for all $\theta \geq \tilde{\theta}$ using the formulas in the paper, compute social welfare and end the inner step. If not, set $k = k+1$ and repeat sub step k.

E Numerical Algorithm for Nonlinear Tax Analysis

Recall the policy problem:

$$\sup_{T,b} \left(\frac{\delta}{\delta+\lambda} + \frac{\lambda}{\delta+\lambda}K[\tilde{\theta}(T)]\right)U(b) + \frac{\lambda}{\delta+\lambda}\int_{\tilde{\theta}(T)}^{\infty}\int_{\underline{q}(\theta;T)}^{\overline{q}(\theta;T)} U\left(\Phi(q,\theta;T)\right) g(q|\theta;T)dqk(\theta)d\theta. \tag{E.1}$$

subject to:

$$\mathcal{G} + \left(\frac{\delta}{\delta+\lambda} + \frac{\lambda}{\delta+\lambda}K[\tilde{\theta}(T)]\right)b = \frac{\lambda}{\lambda+\delta}\int_{\tilde{\theta}(T)}^{\infty}\int_{\underline{q}(\theta;T)}^{\overline{q}(\theta;T)} \{\tilde{q} + T[x(q,\theta;T)]\}\, g(q|\theta;T)dqk(\theta)d\theta,$$

Numerical Procedure

Step 0 Set parameters of the problem. These should include a large maximal income $\overline{x}$ and a large domain $\mathcal{D} := [0,\overline{q}] \times [\underline{\theta},\overline{\theta}]$. Select an initial premium paid to the unemployed $r_1 = b_1 + T_1[0]$. Fix a tax amount $T_1[0]$ and derivative $T_1' : [0,\overline{x}] \to \mathbb{R}_+$. Hence, for $x \in [0,\overline{x}]$, $T_1[x] - T[0] = \int_0^x T_1'[x']dx$. Proceed to Step 1.

Step $n > 0$

Substep a Approximate x_n on $\mathcal{D}$. At each $(q,\theta) \in \mathcal{D}$, set:

$$x_n(q,\theta) = \theta h'^{-1}\left(\theta\{1 - T_n'[x_n(q,\theta)]\}\right) - q.$$

Substep b Find $\tilde{\theta}_n$, where:

$$x_n(0,\tilde{\theta}_n) - \{T_n[x_n(0,\tilde{\theta}_n)] - T_n[0]\} - h\left(\frac{x_n(0,\tilde{\theta}_n)}{\tilde{\theta}_n}\right) = r_n. \tag{E.2}$$

Substep c On a domain $[\tilde{\theta}_n,\overline{\theta}]$, set $\overline{q}_n$:

$$x_n(\overline{q}_n(\theta),\theta) - h\left(\frac{x_n(\overline{q}_n(\theta),\theta) + \overline{q}_n(\theta)}{\theta}\right) = r_n. \tag{E.3}$$

and $\underline{q}_n(\theta) = \left(\frac{\delta}{\delta+\lambda}\right)^2 \overline{q}_n(\theta)$.

Substep e Define various functions used to construct integrals:

- $g_n(q|\theta) = \frac{1-\rho(1-\delta)}{2\rho\lambda}\sqrt{\frac{\overline{q}_n(\theta)}{q}}\frac{1}{q}$

- $\tilde{q}_n(\theta,x) = \theta(h')^{-1}(\{1-T_n'[x]\}\theta) - x$.
- $\underline{\theta}(x)$, where: $\tilde{q}_n(\underline{\theta}_n(x),x) = \underline{q}_n(\underline{\theta}_n(x)) = (\frac{\delta}{\delta+\lambda})^2\overline{q}_n(\underline{\theta}_n(x)))$.
- $\overline{\theta}_n(x) = \min\left(\overline{\theta}, \frac{1}{1-T_n'[x]}h'(h^{-1}\{x-T_n[x]-b\})\right)$.
- $\hat{q}_n(\theta,x) = \tilde{q}_n(\theta,x)$ if $\theta \in [\underline{\theta}(x), \overline{\theta}(x)]$ and $\hat{q}_n(\theta,x) = \overline{q}_n(\theta)$ if $\theta \in [\overline{\theta}(x), \overline{\theta}]$.
- $\Phi_n(q,\theta) = x_n(q,\theta) - T_n[x_n(q,\theta)] - h\left(\frac{x_n(q,\theta)+q}{\theta}\right)$.
- $\mathscr{E}_n(q,\theta) = \dfrac{1}{\frac{x_n(q,\theta)}{x_n(q,\theta)+q}\frac{\frac{x_n(q,\theta)+q}{\theta}h''\left(\frac{x_n(q,\theta)+q}{\theta}\right)}{h'\left(\frac{x_n(q,\theta)+q}{\theta}\right)} + \frac{T_n''[x_n(q,\theta)]x_n(q,\theta)}{1-T_n'[x_n(q,\theta)]}}$.
- $\mathcal{E}_n(q,\theta) = \frac{\theta}{x_n(q,\theta)}\frac{h'(x_n(q,\theta)/\theta)}{h''(x_n(q,\theta)/\theta)}$.
- $\frac{\mathcal{E}_{\overline{q},n}(\theta)}{1-T_n'[x_0]} = \frac{\theta}{h'\left(\frac{x_n(\overline{q}_n(\theta),\theta)+\overline{q}_n(\theta)}{\theta}\right)}\frac{1}{\overline{q}_n(\theta)}$, for $\theta \geq \overline{\theta}_n(x)$.

Substep f Update the marginal tax function $T_{n+1}'[\hat{x}]$ at each $\hat{x}$ on a grid of incomes from $[\tilde{x}_{n+1}, \overline{x}]$ according to:

$$\begin{aligned}
\frac{x_0 T_{n+1}'[\hat{x}]}{1-T_{n+1}'[\hat{x}]}\overline{\mathscr{E}_n}(\hat{x})j_n(\hat{x}) &= -\frac{1}{E[U'(\Phi_n)]}\int_{\underline{\theta}_n(\hat{x})}^{\overline{\theta}}\int_{\underline{q}_n(\theta)}^{\hat{q}_n(\theta,\hat{x})} U'\left(\Phi_n(q,\theta)\right) g_n(q|\theta)k(\theta)dqd\theta \\
&\quad + \frac{1}{E[U'(\Phi_n)]}\int_{\overline{\theta}_n(\hat{x})}^{\overline{\theta}}\frac{\mathcal{E}_{\overline{q},n}(\theta)}{1-T_n'[\hat{x}]}\int_{\underline{q}_n(\theta)}^{\overline{q}_n(\theta)} U'\left(\Phi_n(q,\theta)\right)(1-T_n'[x_n(q,\theta)])qg_n(q|\theta)dqk(\theta)d\theta \\
&\quad + (1-J_n[x_0]) - \int_{\overline{\theta}_n(x_0)}^{\overline{\theta}}\frac{\mathcal{E}_{\overline{q},n}(\theta)}{1-T_n'[x_0]}\int_{\underline{q}_n(\theta)}^{\overline{q}_n(\theta)} qg_n(q|\theta)dqk(\theta)d\theta \\
&\quad + \int_{\overline{\theta}_n(x_0)}^{\overline{\theta}}\frac{\mathcal{E}_{\overline{q},n}(\theta)}{1-T_n'[x_0]}\int_{\underline{q}_n(\theta)}^{\overline{q}_n(\theta)} T_n'[x_n(q,\theta)]\left\{\frac{x_n(q,\theta)}{x_n(q,\theta)+q}\frac{\mathscr{E}_n(q,\theta)}{\mathcal{E}_n(q,\theta)}\right\} qg_n(q|\theta)dqk(\theta)d\theta,
\end{aligned}$$

where:

$$1 - J_n[x] := \int_{\underline{\theta}_n(x)}^{\overline{\theta}_n(x)}\int_{\underline{q}_n(\theta)}^{\tilde{q}_n(\theta,x)} g_n(q|\theta)dqk(\theta)d\theta + \int_{\overline{\theta}_n(x)}^{\overline{\theta}}\int_{\underline{q}_n(\theta)}^{\overline{q}_n(\theta)} g_n(q|\theta)dqk(\theta)d\theta$$

and $j_n(\cdot)$ is the corresponding income density and:

$$\overline{\mathscr{E}}_n(x_0)j_n(x_0) = \int_{\underline{\theta}_n(x_0)}^{\overline{\theta}_n(x_0)} \mathscr{E}_n(\tilde{q}_n(x_0,\theta),\theta)g_n(\tilde{q}_n(x_0,\theta)|\theta)k(\theta)d\theta.$$

Substep g Rearrange (B.22) and use this to update the premium r_{n+1} according to:

$$
\begin{aligned}
k[\tilde{\theta}_n]&\frac{r_{n+1}-T_n[\tilde{x}_n]+T_n[0]}{h'\left(\frac{\tilde{x}_n}{\tilde{\theta}_n}\right)\frac{\tilde{x}_n}{\tilde{\theta}_n^2}}\\
=&-\frac{1}{E_n[U'(\Phi_n)]}\int_{\tilde{\theta}_n}^{\infty}\int_{\underline{q}_n(\theta)}^{\overline{q}_n(\theta)}U'\left(\Phi_n(q,\theta)\right)g_n(q|\theta)k(\theta)dqd\theta\\
&+\frac{1}{E_n[U'(\Phi_n)]}\int_{\tilde{\theta}_n}^{\infty}\frac{\mathcal{E}_{\overline{q},n}(\theta)}{1-T_n'[\tilde{x}_n]}\int_{\underline{q}_n(\theta)}^{\overline{q}_n(\theta)}U'\left(\Phi_n(q,\theta)\right)(1-T_n'[x_n(q,\theta)])qg_n(q|\theta)dqk(\theta)d\theta\\
&+(1-J_n[\tilde{x}_n])+\int_{\tilde{\theta}_n}^{\infty}\frac{\mathcal{E}_{\overline{q},n}(\theta)}{1-T_n'[\tilde{x}_n]}\int_{\underline{q}_n(\theta)}^{\overline{q}_n(\theta)}T_n'[x(q,\theta)]\left\{\frac{x_n(q,\theta)}{x_n(q,\theta)+q}\frac{\mathscr{E}_n(q,\theta)}{\mathcal{E}_n(q,\theta)}\right\}qg_n(q|\theta)dqk(\theta)d\theta\\
&-\int_{\tilde{\theta}_n}^{\infty}\frac{\mathcal{E}_{\overline{q},n}(\theta)}{1-T_n'[\tilde{x}_n]}\int_{\underline{q}_n(\theta)}^{\overline{q}_n(\theta)}qg_n(q|\theta)dqk(\theta)d\theta.
\end{aligned}
$$

Substep h Update the zero income tax amount to $T_{n+1}[0]$ using the budget constraint according to:

$$
\Big(\frac{\delta}{\delta+\lambda}+\frac{\lambda}{\delta+\lambda}K[\tilde{\theta}_n]\Big)r_{n+1}=\frac{\lambda}{\lambda+\delta}\int_{\tilde{\theta}_n}^{\infty}\int_{\underline{q}_n(\theta)}^{\overline{q}_n(\theta)}\left\{q+T_n[x_n(q,\theta)]-T_n[0]\right\}g_n(q|\theta)dqk(\theta)d\theta+T_{n+1}[0],
$$

Substep l Calculate *test* $:=\sup_x|T_{n+1}(x)-T_n(x)|$. If test $<\varepsilon$ finish. Otherwise, set $n=n+1$ and return to Step n.

Appendix References

Heathcote, J., F. Perri, G. Violante (2010). Unequal we stand: An empirical analysis of economic inequality on the United States, 1967-2006. *Review of Economic Dynamics 13*(1), 15–51.