

Velocity in the Long Run: Money and Structural Transformation¹

Antonio Mele
University of Surrey

Radoslaw Stefanski
University of St. Andrews and University of Oxford (OxCarre)

July 28, 2016

Abstract

Monetary velocity declines as economies grow. We argue that this is due to the process of structural transformation - the shift of workers from agricultural to non-agricultural production associated with rising income. A calibrated, two-sector model of structural transformation with monetary and non-monetary trade accurately generates the long run monetary velocity of the US between 1869 and 2013 as well as the velocity of a panel of 92 countries between 1980 and 2010. Three lessons arise from our analysis: 1) Developments in *agriculture*, rather than non-agriculture, are key in driving monetary velocity; 2) Inflationary policies are disproportionately more costly in richer than in poorer countries; and 3) Nominal prices and inflation rates are not ‘always and everywhere a monetary phenomenon’: the composition of output influences money demand and hence the secular trends of price levels.

JEL codes: O1; O4; E4; E5; N1

Keywords: structural transformation, monetary shares, velocity, agricultural productivity, non-monetary exchange

¹ We would like to thank Martin Ellison, Alexander Berentsen, Fernando Martin, Domenico Ferraro, Berthold Herrendorf, Joe Kaboski, Marti Mestieri, Rachel Ngai, Edward C. Prescott, Todd Schoellman, Akos Valentyni, Gustavo Ventura, Xiaodong Zhu, and all other participants of the Structural Transformation Workshop at Arizona State University as well as seminar participants at the University of Oxford and the University of St Andrews for helpful comments and suggestions. All errors are our own.

Corresponding author: Radoslaw (Radek) Stefanski, School of Economics and Finance - Castlecliffe, The Scores - University of St Andrews, Fife KY16 9AR, United Kingdom. **Email address:** radek.stefanski@gmail.com; rls7@st-andrews.ac.uk

1 Introduction

How does a country's long-run money demand change with its economic development? An extensive literature² finds that for broad enough measures of the money stock and over long periods of time, increases in income per capita tend to be associated with increases in the money-to-GDP ratio or, equivalently, a falling monetary velocity.³ The possible sources of this stylized fact have been widely debated and include institutional changes, financial innovations, improvements in communication and information-gathering technologies as well as changes in the composition of output.⁴ Perhaps surprisingly, this research has been almost entirely empirical in nature, which has made it challenging to quantify the role played by individual channels.⁵ In this paper we address this gap by constructing and calibrating a model of long-run monetary demand, and we use the model to quantify the role played by a single mechanism potentially driving the income-velocity relationship: *structural transformation*. This process, also known as industrialization, is the change in the composition of an economy's employment and output, from agricultural towards non-agricultural goods, associated with economic growth. Whilst structural transformation is certainly known to influence money demand (see for example Jonung (1983), Friedman (1959) or Chandavarkar (1977)), no theoretical models of the process exist, and the quantitative importance of this channel is unclear. We use a calibrated model to show how structural transformation influences money demand over the development process and that quantitatively this channel is the main driver of within and across country differences in long-run monetary velocity.

The suggested mechanism is simple. Agriculture - especially traditional agriculture - is largely a non-monetary sector. As is argued by Chandavarkar (1977), this is true for at least three reasons. First, agricultural workers are often compensated in kind, either through share-cropping or other informal credit arrangements. Second, households in poorer countries tend to home-produce their agricultural consumption. Third, the agricultural setting is conducive to barter transactions.⁶

² See for example Wicksell (1936), Warburton (1949), Friedman (1959), Bordo and Jonung (1981, 1987), Jonung (1983) or Ireland (1991).

³ Income velocity of money, V , is defined as the ratio of gross domestic product, Y (measured in current prices, P) to the nominal money stock, M : $V \equiv \frac{P \times Y}{M}$. This implies that the inverse of velocity is simply the money-share: $V^{-1} = \frac{M}{P \times Y}$. Velocity is thus a convenient measure of how money demand compares to income, with lower money demand relative to income translating into higher velocity and viceversa. This also means that a rising money share is equivalent to a falling velocity.

⁴ Wicksell (1936), for example, argues that this trend stems from an increase in the prominence of organized markets, a shift from home to market production as well as the increase in both worker specialization and the complexity of goods. Friedman and Schwartz (1963) suggest that real money balances are luxury goods and hence have an income elasticity larger than unity. Bordo and Jonung (1987, 1990) emphasize institutional changes and financial innovations as systematically influencing velocity over long periods, whilst Townsend (1987) and Goodfriend (1991) highlight that improvements in communications and information-gathering technologies can contribute to a falling velocity. Finally, Friedman (1959), Chandavarkar (1977) and Jonung (1983), assert that the composition of an economy can be important in driving money demand.

⁵ One notable exception is Ireland (1994), who constructs a theoretical model of changes in the composition of monetary aggregates. Our paper instead focuses on the changes in the total share of money-stock-to-GDP and abstracts from compositional effects.

⁶ Since most individuals wish to consume as varied a bundle of food as possible, most household-producers will wish to consume the agricultural goods of other household-producers in addition to their own. Thus, double coincidence of wants is easy to achieve when traders meet and thus the probability of barter is likely to be relatively high.

The non-agriculture sector, however, tends to be far more complex and hence is more likely to need money to enable exchange.⁷ As an economy grows, its structure will change from one dominated by a (predominantly non-monetary) agricultural sector towards one dominated by a (predominantly monetary) non-agricultural sector. This changing composition of the economy results in a rising demand for money, a rising money-to-GDP ratio and consequently a falling monetary velocity.

To capture the above mechanisms, we build a simple model with two sectors: agriculture, which produces goods traded without money, and non-agriculture, in which an endogenous share of goods is exchanged using money. The demand for money in the non-agricultural sector is introduced through a cash-in-advance constraint on consumption goods following Cole and Kocherlakota (1998). Building on a long-tradition of models exemplified by Herrendorf et al. (2014), Buera and Kaboski (2012), Gollin et al. (2002), Restuccia et al. (2008), Yang and Zhu (2013) and Stefanski (2014a,b), structural transformation is generated by non-homothetic preferences: consumers have a subsistence level of agricultural consumption. Intuitively, households need to eat a minimum quantity of food to survive. In poor countries where agricultural productivity tends to be low, more workers need to be employed in the agricultural sector in order to produce enough food to satisfy subsistence needs. A poor economy is thus dominated by non-monetary agriculture and most transactions take place without money. As agricultural productivity increases, less workers are needed to satisfy subsistence consumption, prompting workers to migrate into the non-agricultural sector and thus increasing that sector's share in total employment and output. Given that a part of non-agricultural goods are traded with money, the shift in the composition of the economy towards the non-agricultural sector will result in an increase in monetary transactions, an increase in the money-to-GDP ratio and hence in a lower velocity.

The model is calibrated to match the 1869-2013 patterns of US growth, structural transformation and monetary supply. Our simple framework replicates several features of the long run data including agricultural labor shares, GDP per worker, sectoral prices, nominal interest rates and aggregate inflation. Most importantly, the model reproduces the evolution of the US long run money-to-GDP ratio over 140 years, capturing nearly 91% of the variation in the data. A similarly calibrated one-sector model fails to replicate observed money-share, as it is unable to match the observed price dynamics - and in particular the so-called 'Great-Deflation' of the late 19th century. The model also accurately predicts the variability in velocity *across* countries. Keeping preference parameters of the US, we recalibrate the model to a panel of 92 countries between 1980 and 2010. The model accurately captures cross-country differences in incomes, employment shares, interest rates and inflation rates. It also does well in reproducing velocities and does exceptionally well in replicating the income-velocity relationship.⁸

⁷ Crucially, there are more varieties of goods in the non-agricultural sector. Thus, the probability that an economics professor - for example - can barter his output for the services of a car mechanic is far lower than the probability that two farmers producing different types of vegetables may wish to trade. For an in-depth discussion on this transactions role of money see Ostroy and Starr (1990).

⁸ In particular, a one percent increase in GDP per worker is found to increase the money-to-GDP ratio by 0.14 percentage points in both the data and the model.

The multi-sector framework, and in particular the reallocation of workers from agriculture to non-agriculture, is the key driver of our results. To illustrate this point, we perform a number of exercises that isolate individual channels affecting monetary velocity, like variation in monetary growth rates, productivity growth rates and productivity levels. We also extend the baseline model to allow for heterogenous endowments of capital across countries.⁹ Quantitatively, we find that differences in agricultural productivity levels are the key source of cross-country variation in velocity since they influence the size of the non-agricultural, predominantly-monetary sector. A one-sector version of our model without structural transformation fails entirely in capturing across and within country variation in monetary velocity.

Finally, we examine how the costs of suboptimal monetary policies vary with income. Inflation is more costly in richer countries than in poorer countries - where the monetary part of the economy is smaller and therefore distortions from the inflation tax are less damaging. For example, a hyperinflation of approximately 400% a year in a poor country like Zimbabwe (where GDP per worker is 2% of that of the US) will have negligible welfare costs. By contrast, the same hyperinflation in a country like Argentina (30% of US GDP per worker) will have very large welfare costs. Argentinean incomes would have to rise by approximately 25% to deliver the same expected flow utility as without the hyperinflation. The low cost of inflation in poorer countries may help explain why poorer countries tend to have much higher inflation rates than richer countries.

There are three main lessons from our work. First, monetary velocity is not constant over the development process but falls with income. Most of the existing explanations of this observation are rooted in the non-agricultural sector - and focus on factors such as financial innovation or the expansion of the banking sector. The surprising finding of this paper is that the evolution of monetary velocity is driven almost exclusively by developments in the *agricultural* sector. It is the variation in agricultural productivity that influences the size of the non-agricultural sector and in turn influences monetary demand and hence velocity. Second, since velocity depends systematically on the composition of output - a country's price levels and inflation rates may not 'always and everywhere be a monetary phenomenon' (Friedman and Schwartz, 1963). The price level in an economy, P , is defined as $P \equiv (M/Y)V$ where M is the money stock, Y is output and V is velocity. Two countries, with identical money stocks and identical output *levels* - but with different output *compositions* - will have entirely different velocities, V , and hence different price levels. The message to researchers from these findings is that a one sector model cannot be successfully used to understand the long run dynamics of monetary velocity and hence the evolution of price levels or inflation rates. These findings should also be of interest to policymakers in developing countries who may have overlooked the importance of the agricultural sector in their monetary policy decisions. Finally, the third lesson of this paper is that the cost of inefficient monetary policy varies with income and is higher in richer countries than in poorer countries. An inflation tax offers a relatively

⁹ Capital is largely a credit good and so a variation in capital stock levels can influence the demand for money and hence velocity.

cheap source of income to poor-country governments, and its distortive effects are relatively small in economies that are dominated by large, non-monetary, agricultural sectors. This may help explain why we observe persistently higher inflation in poorer countries than in richer countries - despite recommendations of strong anti-inflationary policies by international financial institutions such as the International Monetary Fund (IMF).¹⁰

In the next section we document the main facts regarding structural transformation, monetary velocity and the extent of non-monetary production in agriculture. In section 3, we construct and solve our simple baseline model. In section 4 we calibrate the model to the experience of the United States and in section 5 we show the results for the US. Section 6 carries out the cross-country analysis by examining how the model performs in international, cross-country data and by running a number of counterfactuals to quantify the importance of the different mechanisms of the model. Section 7 examines the different costs of inflation in rich and poor countries. Finally, section 8 offers some concluding remarks on the importance of our findings.

2 Facts

In this section we present three stylized facts. First, there is a positive relationship between the money to GDP ratio (or the inverse of monetary velocity) and income per capita. Second, the proportion of workers employed in agriculture tends to fall as income rises. Finally, non-monetary activities are most prevalent in countries with large agricultural sectors. These facts taken together suggest that as countries grow richer, they tend to use more money relative to the size of their economy, and that this process is potentially linked with the decline of a predominantly non-monetary sector - agriculture.¹¹

Money Shares We are interested in the pattern of the ratio of the stock of money to nominal GDP, over time and across countries.¹² It is therefore crucial to define exactly what we mean by the stock of money. Throughout the paper all classifications of monetary data follow the definitions set out by the IMF in the International Financial Statistics (IFS). The IFS classifies money into ever broadening bands from M0 to M3.¹³ We are not directly interested in the narrowest definition of the monetary stock such as M0 or M1: as argued by Ireland (1994), economies undergo a change in

¹⁰ Weisbrot et al. (2009), for example, examine existing IMF loan agreements with 41 developing countries in 2009. They find that the IMF took a stance on monetary policy in 25 of these countries and in 22 of those 25 countries it recommended a contractionary or anti-inflationary monetary policy.

¹¹ See Appendix A for details of sources and construction of all data.

¹² From now on, unless otherwise stated, we will only refer to money share relative to the GDP instead of referring to (the inverse of) velocity. This saves repetition and - in our eyes - the money share has a cleaner and more intuitive interpretation than velocity.

¹³ The narrowest definition of money stock, M0, is the value of currency and deposits in the central bank. The next classification, M1, includes all of M0 as well as transferable deposits and electronic currency. Next, M2, includes M1 but also measures time and savings deposits, foreign currency transferable deposits, certificates of deposit and securities repurchase agreements. Finally, M3, includes M2 as well as travelers checks, foreign currency time deposits, commercial paper and shares of mutual/market funds.

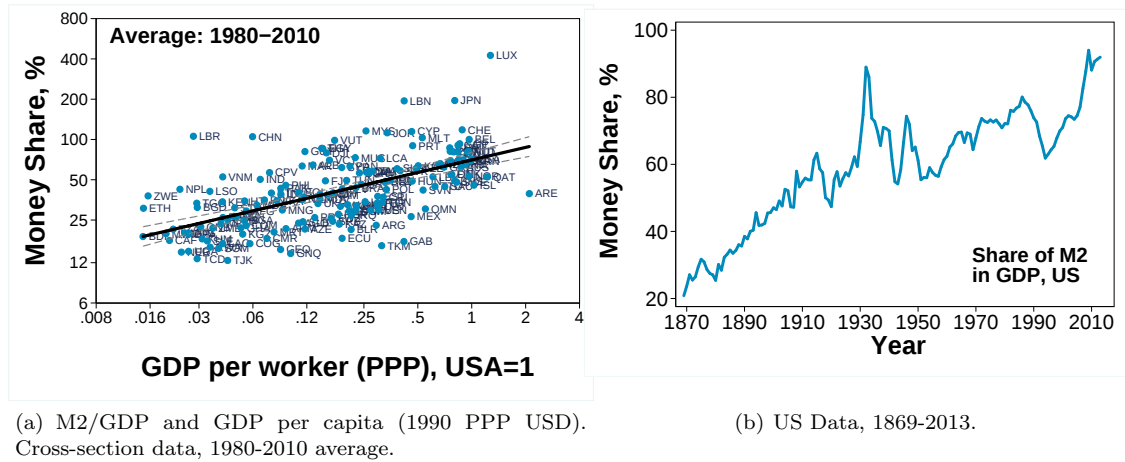


Figure 1: M2/GDP and income per capita. Share of money (M2) in GDP with income and over time. (Source: WDI, PWT, Anderson (2006))

monetary technologies as they develop, and they therefore use less currency and more sophisticated forms of exchange such as electronic currency or time deposits. These technological changes and the subsequent changes in the *composition* of money are not the focus of this paper. Instead we are interested in how the *quantity* of money evolves with income. This suggests that we should focus on the wider definitions of money. The data for M3 however is relatively scarce - both across countries and for longer periods of time. Consequently, we choose our measure of money stock to be M2.¹⁴

For each country we calculate the average money share (M2 divided by nominal GDP) and the average GDP per worker over the 1980-2010 period in order to focus on the long run relationship between these two variables.¹⁵ Figure 1(a) then plots the average money share versus GDP per worker (relative to the US average GDP) for the resulting cross-section of 166 countries. A strikingly positive relationship emerges: richer countries tend to have a higher money share (or a lower velocity) than poorer countries. This fact is statistically significant, as apparent from the 5% error bands on the regression line. Quantitatively, a 1% increase in GDP per worker is associated with a 0.16 percentage point increase in the money share. A similar fact can be observed in long run time series data. Figure 1(b) shows the share of M2 money stock relative to nominal GDP in the United States between 1869 and 2014. There is a clear rising trend in the money stock to GDP ratio. Since GDP per worker in the US roughly grew at a constant rate over the period, there is also a positive relationship between money share and income per worker in the US time-series data.¹⁶ Finally, notice that this is not an isolated case. Bordo et al. (1993) have documented similar historical patterns in long run data for Canada, the United Kingdom, Norway and Sweden.

¹⁴ See Dorich (2009) and McCallum and Nelson (2010) for a discussion of which monetary aggregate better captures the liquidity services of money.

¹⁵ Data on M2 as well as data on nominal GDP in local currency units comes from the IMF (2015) and is conveniently collected in the World Bank's WDI database. Data on GDP per worker expressed in constant (1990) purchasing power parity terms comes from the Penn World Tables (PWT) version 7.1 (Heston et al., 2012).

¹⁶ The semi-elasticity of the money share with respect to GDP per worker in the US over the period is 0.19.

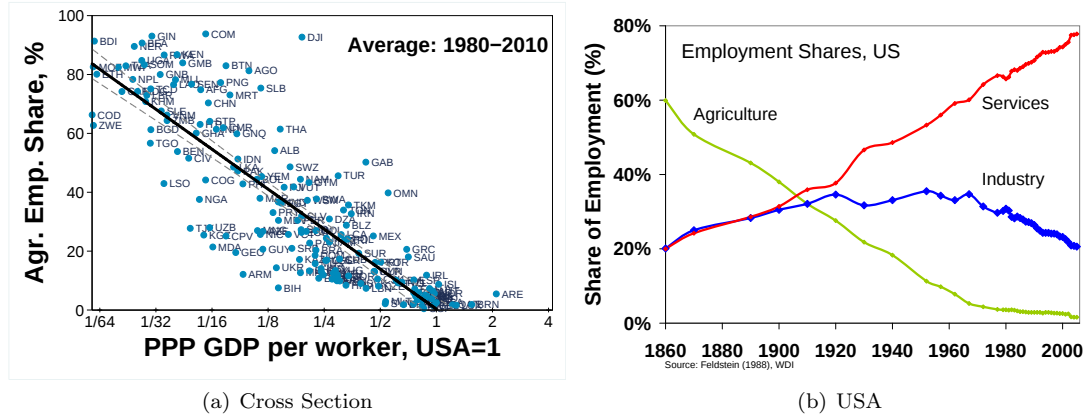


Figure 2: Share of employment in agriculture with GDP per worker and over time. (Source: FAOSTAT, Urquhart (1984) and WDI (2016).)

Structural Transformation It is a widely documented fact that agricultural employment shares fall with rising income in a process referred to as structural transformation.¹⁷ Figure 2(a) documents this pattern for a cross-section of 173 countries by plotting the average agricultural employment shares over the 1980-2010 period versus the average (1990, PPP) GDP per worker of each country.¹⁸

A similar pattern is visible within countries over time as their income per capita increases. Figure 2(b), for example, depicts falling agricultural employment shares in the US between 1860 and 2004 from Urquhart (1984) and WDI (2016). This process will prove to be the key element driving the rising monetary shares depicted in Figure 1.

The non-monetary economy Finally, we present evidence on the extent of non-monetary or subsistence activities, and their relationship with the agricultural sector. Blades (1975) carries out an analysis for the OECD using survey data for 48 developing countries over the 1970-1975 period, documenting the proportion of GDP that can be classified as a non-monetary or subsistence activity. His conclusions were quite strong: “Agriculture is obviously the main item in non-monetary production, accounting often for over 80 percent of the total.” Figure 3 plots the share of non-monetary value added in GDP from Blades (1975) versus agricultural employment share in 1980.¹⁹ Notice the strong positive relationship: agricultural economies tend to have a far greater proportion of their value added in non-monetary sectors than more industrialized countries. As we argue in the remainder of the paper, this relationship will be crucial in explaining cross-country and within country differences in long run monetary velocity.

¹⁷ See for example, Maddison (1982), Echevarria (1997) or Duarte and Restuccia (2010).

¹⁸ The employment share data is obtained from FAOSTAT (2012).

¹⁹ We use the 1980 agricultural employment share, since earlier data is not easily available for all countries.

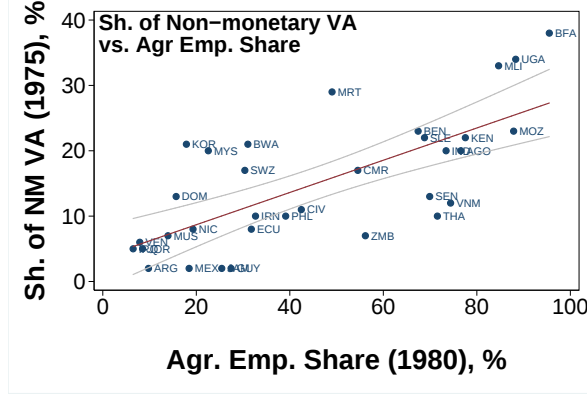


Figure 3: Share of non-monetary value added, relative to the agricultural employment share. (Source: WDI, Blades (1975))

3 The Baseline Model

In this section we build a very simple cash-in-advance model of structural transformation guided by the three facts presented in the previous section: money shares that rise with income, agricultural employment shares that decline with income, and the dominance of non-monetary transactions in predominantly agricultural economies. Our model is a version of Cole and Kocherlakota (1998) but with two sectors: a completely non-monetary sector (agriculture) and a partially monetary sector (non-agriculture). In this baseline version, we assume that production technologies are linear in labor. In Appendix B.2, we extend the model to include capital accumulation in the non-agricultural sector, obtaining very similar results. In order to generate structural transformation, we impose non-homothetic preferences over agricultural goods. This assumption, in conjunction with rising agricultural productivity, generates a shift of labor from agriculture to non-agriculture. A calibrated version of the model will be used to quantify the role that structural transformation plays in driving across- and within- country differences in monetary shares.

Households The representative household maximizes welfare subject to its budget constraint, a cash in advance constraint, a non-negativity constraint on money holdings, and a non-Ponzi condition:

$$\begin{aligned}
 & \max_{a_t, c_{m,t}, c_{n,t}, b_{t+1}, m_{t+1}} \sum_{t=0}^{\infty} \beta^t (\alpha \log(a_t - \bar{a}) + (1 - \alpha - \gamma) \log(c_{m,t}) + \gamma \log(c_{n,t})) \\
 & \text{s.t. } p_{a,t}a_t + p_{c,t}(c_{m,t} + c_{n,t}) + b_{t+1} + m_{t+1} \leq w_t + (1 + r_t)b_t + m_t + T_t \\
 & p_{c,t}c_{m,t} \leq m_t \\
 & m_t \geq 0 \text{ and } b_t \geq -\bar{B}
 \end{aligned} \tag{1}$$

The household owns a unit of labor which it sells on the market for a wage w_t . In addition, the household comes into the period with money holdings m_t , bond holdings b_t (which it sells at a price

$1 + r_t$) and it receives a helicopter transfer of money from the government of T_t . Given this income, the household purchases agricultural goods a_t (at the price $p_{a,t}$) as well as non-agricultural goods $c_{m,t}$ and $c_{n,t}$ (at the price $p_{c,t}$). It also purchases bonds b_{t+1} that promise to pay out $1 + r_{t+1}$ dollars next period, and chooses its (non-negative) stock of money, m_{t+1} , for the next period. We assume that there are two kinds of non-agricultural goods: those that can be bought without money (c_t^n) and those that must be bought with money ($c_{m,t}$). This establishes a role for money: the household needs to put aside a part of its income, m_{t+1} , each period in order to be able to buy monetary goods in the following period. To capture this idea, we impose the cash-in-advance (CIA) constraint (25) on $c_{m,t}$ goods. Only cash held from the previous period can be used to purchase monetary goods in the current period and cash transfers from government can only be used in the subsequent period. Finally, we impose a lower bound, $-\bar{B}$, on bond holdings to avoid Ponzi schemes.

Households have non-homothetic preferences for agricultural goods a_t . In other words, there exists a subsistence level of agriculture, $\bar{a}$, that must be consumed every period. Intuitively, households need to obtain a minimum quantity of food or calories in order to survive. A crucial assumption of our model is that agriculture is a non-cash good. The intuition here is that agriculture - especially traditional agriculture - is often characterized by compensation in kind or subsistence production. Farmers - especially in poorer countries - often work for landlords as sharecroppers and get paid in kind for their efforts, or produce much of their food at home for their own consumption (Blades, 1975). Furthermore, barter is far more likely in agriculture than in non-agriculture, as most people wish to eat a varied basket of foods. Thus, when two agricultural producers (or households) meet to trade, double coincidence of wants for agricultural products is relatively likely. As such, little to no cash is needed to obtain agricultural goods in poorer countries. Of course, the above assumption is a simplification and in richer countries, cash will be far more readily used in the agricultural sector. However, in richer countries, the agricultural sector will also tend to be very small - and hence the influence of agricultural money demand on the overall money demand will also be very small. Therefore, as a first approximation, the assumption that all agricultural goods are traded without money is a reasonable one.

Money use in non-agriculture, however, will be important, since a double coincidence of wants in that sector is far less likely. Due to the massive variety of goods produced in non-agriculture, the probability that two randomly matching producers or consumers (say an economist and a car mechanic) would want each other's services will be far smaller. As such, money becomes necessary to overcome this mismatch problem. Of course, some goods in non-agriculture are nonetheless still traded without cash using credit arrangements or payments in kind (e.g. employer-provided cars, housing, health insurance etc.). Hence, following Chari et al. (1996b), in our model a fixed proportion γ of non-agricultural goods do not require cash.²⁰

²⁰ In Appendix B.2, we follow Cole and Kocherlakota (1998) by adding capital to the model, and show that if we treat investment as a non-monetary, credit good we can go a long way in endogenizing the cash-credit split of the non-agricultural sector.

Firms There are two representative firms: agricultural (a) and non-agricultural (c). Each firm $s = a, c$, hires labor, $L_{s,t}$, and produces output, $Y_{s,t}$, using a simple linear technology that combines labor with exogenous, sector-specific total factor productivity, $B_{s,t}$. The profit maximization problems are:

$$\max_{L_{s,t}} p_{s,t} Y_{s,t} - w_t L_{s,t} \quad \text{s.t.} \quad Y_{s,t} = B_{s,t} L_{s,t}. \quad (2)$$

Output of non-agricultural firms, $Y_{c,t}$, can be sold as both a monetary and a non-monetary good, whereas all agricultural output is assumed to be non-monetary.

Money Supply The government is assumed to have a so-called helicopter monetary policy:

$$M_{t+1} = T_{t+1} + M_t. \quad (3)$$

Market Clearing Finally, in each period, markets clear in a standard fashion.

$$\begin{aligned} a_t &= Y_{a,t}, \quad c_{m,t} + c_{n,t} = Y_{c,t} \\ m_t &= M_t, \quad b_t = 0 \\ L_{a,t} + L_{c,t} &= 1. \end{aligned} \quad (4)$$

Competitive Equilibrium For a given monetary policy, $\{T_t\}_{t=0}^{\infty}$, a competitive equilibrium in this economy is a sequence of prices, $\{p_{a,t}, p_{c,t}, w_t, r_t\}$, and quantities, $\{a_t, c_{m,t}, c_{n,t}, b_t, m_t, L_{a,t}, L_{c,t}\}_{t=0}^{\infty}$, such that (1) given prices and monetary policy, households and firms solve their optimization problem, (2) the government budget constraint is satisfied and (3) markets clear.

Solution The first order conditions for the household are given by:

$$a_t: \frac{\alpha \beta^t}{a_t - \bar{a}} = \lambda_t p_{a,t}; \quad c_{m,t}: \frac{(1 - \alpha - \gamma) \beta^t}{c_{m,t}} = p_{c,t}(\lambda_t + \mu_t); \quad c_{n,t}: \frac{\gamma \beta^t}{c_{n,t}} = p_{c,t} \lambda_t \quad (5)$$

$$b_{t+1}: \lambda_t = (1 + r_{t+1}) \lambda_{t+1}; \quad m_{t+1}: \lambda_t = \lambda_{t+1} + \mu_{t+1} \quad (6)$$

$$\text{CIA: } \mu_t(p_{c,t} c_{m,t} - m_t) = 0 \quad \text{and} \quad \mu_t \geq 0 \quad (7)$$

In the above, λ_t and μ_t are multipliers on the budget and CIA constraints respectively. The firms' first-order conditions are:

$$L_{a,t}: p_{a,t} B_{a,t} = w_t \quad \text{and} \quad L_{c,t}: p_{c,t} B_{c,t} = w_t. \quad (8)$$

The market clearing conditions are given by the equations in (4). Finally, the transversality conditions for the above problem are:

$$\liminf_{t \rightarrow \infty} \lambda_t (b_{t+1} + \bar{B}) = 0 \quad \text{and} \quad \liminf_{t \rightarrow \infty} \lambda_t m_{t+1} = 0. \quad (9)$$

Following Cole and Kocherlakota (1998), we solve the model by imposing the following assumption on interest rates:

Assumption 3.1. *Assume that interest rates are always positive, i.e. $r_t > 0$.*

The consequence of the above assumption is that the CIA constraint always binds since there is a positive opportunity cost of holding money.²¹ We impose this assumption for two reasons. First and most importantly, despite recent negative nominal interest rates, this paper's focus is the very long run, and long-run interest rates have tended to be positive. Second, imposing this assumption allows us to find a simple, analytic and unique solution.

Given a sequence of government policies and Assumption 3.1 (so that the CIA constraint holds with equality), equations (5)-(8), in addition to the market clearing conditions, characterize the competitive equilibrium. In particular, sectoral employment is given by:

$$L_{a,t} = \frac{\alpha\tau_t + (1 - \alpha - \gamma(1 - \tau_t)) \frac{\bar{a}}{B_{a,t}}}{1 - (\alpha + \gamma)(1 - \tau_t)} \quad \text{and} \quad L_{c,t} = 1 - L_{a,t}, \quad (10)$$

where $\tau_t \equiv \frac{1}{\beta} \frac{M_{t+1}}{M_t}$. Non-agricultural output is divided between cash and non-cash goods:

$$c_{n,t} = (1 - \phi_t)Y_{c,t} \quad \text{and} \quad c_{m,t} = \phi_t Y_{c,t}, \quad (11)$$

where $\phi_t = \frac{1 - \alpha - \gamma}{(1 - \alpha - \gamma) + \gamma\tau_t}$. Finally, sectoral prices are

$$p_{a,t} = \frac{M_t}{\phi_t B_{a,t} L_{c,t}} \quad \text{and} \quad p_{c,t} = \frac{M_t}{\phi_t B_{c,t} L_{c,t}} \quad (12)$$

and the nominal interest rate and wage rate are

$$r_t = \tau_t - 1 \quad \text{and} \quad w_t = \frac{M_t}{\phi_t L_{c,t}}. \quad (13)$$

The impact of non-homothetic preferences on agricultural employment can be seen in equation (10). When productivity in agriculture, $B_{a,t}$, is low, a large fraction of the labor force must work in agriculture in order to produce enough food for subsistence. As productivity in agriculture increases the subsistence level can be met with a smaller fraction of the population working in the agricultural sector resulting in a shift of workers from agriculture to non-agriculture. In other words, from equation (10), $\partial L_{a,t} / \partial B_{a,t} < 0$.

Next, notice that the composition of the economy affects sectoral prices. Since sectoral employment shares depend on agricultural productivity, from equations (10) and (12), we can write $\partial p_{s,t} / \partial B_{a,t} < 0$, $s = a, c$. In other words, *ceteris paribus*, prices of agricultural and non-agricultural goods decline with rising agricultural productivity and hence with structural transformation. Thus it is not only *monetary* factors that drive nominal prices, as has been suggested by the literature

²¹ To see this divide the equations in (6) by each other to obtain the expression for interest rates, $r_{t+1} = \frac{\mu_{t+1}}{\lambda_{t+1}}$. Thus, the nominal interest rate is positive if and only if money yields liquidity services ($\mu_{t+1} > 0$). In particular, if the nominal interest rate is positive, the CIA constraint is binding.

(Friedman and Schwartz, 1963), but *real* factors also play a crucial role. Rising agricultural productivity changes the composition of the economy, and hence influences the nominal price of goods. This feature of the model is particularly important at the early stages of industrialization. In section 5, we show that this process was key in generating the so-called “Great Deflation” in the late 19th century United States.

Finally, notice that monetary policy has an impact both on sectoral employment and the consumption of monetary goods. By choosing the transfers, the government directly controls the evolution of the money stock in the economy, M_t , and hence the variable τ_t , which determines how costly it is to hold cash from one period to another. From equation (13), the higher the τ_t , the higher the nominal interest rate, and hence the greater the opportunity cost of holding cash. Thus, higher τ_t results in workers shifting away from cash dominated sectors, $\partial L_{c,t}/\partial \tau_t < 0$ and consumers consuming less cash goods, $\partial c_{m,t}/\partial \tau_t < 0$. This first channel highlights a new cost of monetary policy in this two-sector CIA model - higher inflation taxes can reverse or delay structural transformation. Interestingly, the impact of monetary policy on the size of the agricultural sector is itself dependent on the level of agricultural productivity, given that $\frac{\partial(\partial L_{a,t}/\partial \tau_t)}{\partial B_{a,t}} > 0$. In other words, an inflation tax in a country with higher agricultural productivity will have a greater distortionary effect than in a country with lower productivity. Countries with high agricultural productivity have larger non-agricultural and hence monetary sectors. The inflation tax will hence impact a larger fraction of those economies. This suggests that the same policies will have different effects in rich and poor countries. We quantify these effects in section 7, where we look at the welfare cost of inflation.

Optimality The distortion in this environment arises - as in the standard CIA model - from the lag between households being paid their wage income and their ability to buy non-agricultural cash goods with that income (Cole and Kocherlakota, 1998). In particular, households can only use last period’s money holdings to purchase current period non-agricultural cash goods. This forces households to hold a low-yield asset (money) instead of a higher yield asset (bonds) in order to have money holdings to purchase cash goods in the future. Thus, as long as nominal interest rates are positive (i.e., the CIA constraint binds), the economy will not reach the first best. If, however, nominal interest rates were set to zero, then households would be indifferent between being paid today or being paid in the future (and indeed between holding money and a bond), and the distortion associated with the trading arrangement would be eliminated.²² Hence, since the CIA binds if and only if $r_t > 0$, and, as we showed above, $r_t = \tau_t - 1$, we need $\tau_t \rightarrow 1$ to eliminate the distortion. In other words, we need to implement the Friedman rule, i.e. we must have $\frac{M_{t+1}}{M_t} \rightarrow \beta$.²³

²² More specifically, money does not expand the production possibility frontier. As such, the Pareto optimal allocations can be found by solving the corresponding social planner’s problem without money. It is then easy to show that the decentralized problem and the social planner’s problem are identical when nominal interest rates are zero.

²³ A word of caution is needed here. Whilst it is true that as $\tau_t \rightarrow 1$, the equilibrium allocations approach the Pareto optimal allocations, directly setting $\tau_t = 1$ in equations (10)-(13) violates Assumption 3.1. It is relatively easy to show that it is nonetheless true that the allocations and prices implied by the above equations when $\tau_t = 1$ are also a competitive equilibrium. However, as is argued by Cole and Kocherlakota (1998), an equilibrium such as this (i.e. one where $r_t = 0$) can be achieved with a large set of monetary policies - including but not restricted to the policy

Velocity The share of monetary stock relative to the nominal GDP (or the *inverse* of monetary velocity) can be written as:

$$\begin{aligned} V_t^{-1} &= \frac{M_t}{p_{a,t}a_t + p_{c,t}C_t} = \frac{\phi_t p_{c,t}C_t}{p_{a,t}a_t + p_{c,t}C_t} \\ &= \phi_t L_{c,t}. \end{aligned} \quad (14)$$

The first equality follows by definition. The second equality follows from the assumption that the CIA constraint binds, and from equation (11), which splits non-agricultural consumption into its monetary and non-monetary components. The final equality follows from equations (10)-(13).

There are two channels driving the money share. First, the term ϕ_t determines what proportion of non-agriculture is bought with cash. This variable itself is influenced by the preference parameter, γ , which captures (in a reduced form) the non-monetary activity in the non-agricultural sector, as well as by τ_t . Since $\partial\phi_t/\partial\tau_t < 0$, a higher inflation tax results in households wanting to purchase fewer cash goods, which in turn lowers money demand and the money share. Second, the money share crucially and positively depends on $L_{c,t}$ - the share of employment in the non-agricultural sector. As a greater proportion of workers shifts to a largely cash sector, the share of money in the economy rises - and the velocity falls.

From (14), two facts of interest emerge. First, since higher agricultural productivity results in greater non-agricultural employment, it also implies a higher money share. In other words, $\frac{\partial V_t^{-1}}{\partial B_t^a} > 0$. Second, a higher inflation tax means it is more costly to hold cash, which leads to a lower employment share in non-agriculture (as workers move away from a cash to a non-cash sector) and a lower ϕ_t (as consumers want to consume less cash goods). Together this means that the money share decreases in response to a higher τ_t so that $\frac{\partial V_t^{-1}}{\partial \tau_t} < 0$.

These two facts highlight that both monetary policy and agricultural productivity can influence a country's monetary demand and hence monetary shares. We quantify the role that each of these channels play in explaining within and across country variation in monetary shares, by calibrating the model in the next section.

4 US Calibration

We calibrate the model to the experience of the United States between 1869 and 2012.²⁴ Following Solow (1956), we measure the growth of technological residuals with production functions. In particular, measured productivity in the agricultural ($B_{a,t}$) and non-agricultural sectors ($B_{c,t}$) is:

$$B_{a,t} = \frac{Y_{a,t}}{L_{a,t}} \text{ and } B_{c,t} = \frac{Y_{c,t}}{L_{c,t}}. \quad (15)$$

$M_{t+1}/M_t = \beta$. Thus, whilst the limit is indeed an equilibrium, Pareto optimal and can be implemented with a policy $\tau_t = 1$, it is not necessarily unique as other monetary policies could also sustain zero nominal interest rates.

²⁴ See Appendix A for details of sources and construction of all data.

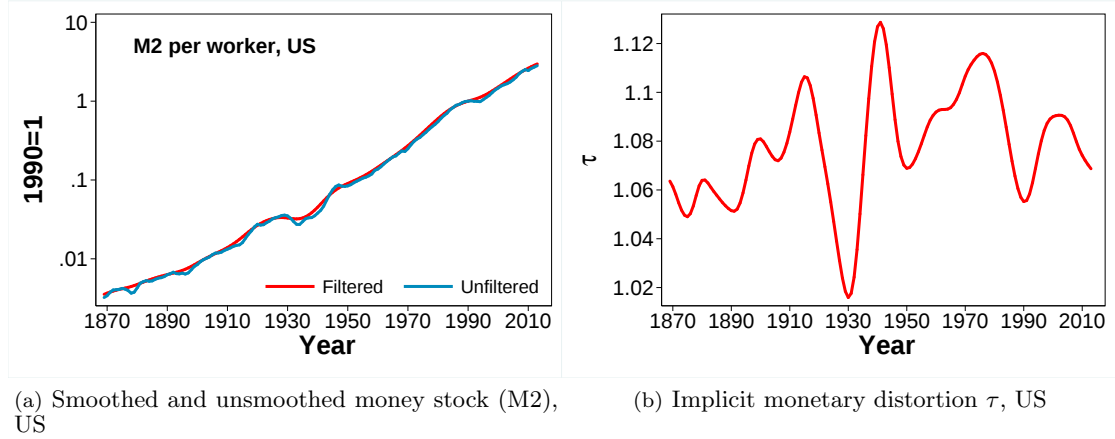


Figure 4: The evolution of the money stock in the US.

In Appendix A we explain how we construct constant price sectoral value added ($Y_{a,t}$ and $Y_{c,t}$), total labor force and agricultural employment data for the US between 1869 and 2013. We then calculate sequences of sectoral labor productivity in the data for each sector using equation (15). Next, we smooth these resulting sequences using a Hodrick-Prescott filter with smoothing parameter 100, and calculate the annualized growth rate of the smoothed labor productivity in each sector between 1869 and 2013.²⁵ We find that annualized growth rate of labor productivity for the period in agriculture was 2.657%, and in non-agriculture 1.276%. We normalize $B_{a,1869} = B_{c,1869} = 1$ and assume that productivity in each sector grows at the corresponding annualized average. Letting $g_a \equiv 1 + 0.02657$ and $g_c \equiv 1 + 0.01276$, we define sectoral productivity in our model as:

$$B_{a,t} = g_a^{t-1869} \text{ and } B_{c,t} = g_c^{t-1869}. \quad (16)$$

We take the sequence of money stock per worker, $\{M_t\}_{t=1869}^{2014}$, directly from the data as an exogenous input. We follow Anderson (2006) in the construction of the long run money stock. Since the labor force is assumed constant in the model, we divide our sequence of money by the size of the labor force in the US and smooth the resulting sequence using the HP filter with smoothing parameter 100. The resulting money stock per worker is presented in Figure 4.

We then turn to the discount factor, β . Recall that the nominal interest rate, r_t , satisfies $r_t = \tau_t - 1$, where $\tau_t = \frac{1}{\beta} \frac{M_{t+1}}{M_t}$. Thus, $\beta = \frac{1}{1+r_t} \frac{M_{t+1}}{M_t}$. Taking the nominal interest rate from Officer and Williamson (2016b), and combining it with monetary growth rates in the US, we calculate the average of the right hand side of this equation between 1980 and 2012 to obtain a value of $\beta = 0.972$.²⁶ Given β and the sequence of M_t we calculate the series of monetary wedges $\{\tau_t\}_{t=1869}^{2013}$ using the formula $\tau_t \equiv \frac{1}{\beta} \frac{M_{t+1}}{M_t}$.

Finally, we calibrate the variables γ, α and $\bar{a}$. The calibration of γ follows Chari et al. (1996a).

²⁵ Here the annualized growth of a sequence x_t between periods T and $T + N$ is given by $\left(\frac{x_{T+N}}{x_T}\right)^{\frac{1}{N}} - 1$.

²⁶ The average nominal interest rate and monetary growth rate for 1980-2012 is 7.95% and 4.97% respectively.

Parameter	Values	Target
$B_{a,1869}, B_{c,1869}$	1	Normalization.
$g_a - 1$	0.02657	Annualized growth rate of (HP-Smoothed) agricultural labor productivity in US, 1869-2012. See Appendix for detailed sources.
$\{B_{a,t}\}_{t=1869}^{2013}$	$B_{a,t} = g_a^t$	Constant, exogenous sectoral productivity growth.
$g_c - 1$	0.01276	Annualized growth rate of (HP-Smoothed) non-agricultural labor productivity in US, 1869-2012. See Appendix for detailed sources.
$\{B_{c,t}\}_{t=1869}^{2013}$	$B_{c,t} = g_c^t$	Constant, exogenous sectoral productivity growth.
$\{M_t\}_{t=1869}^{2014}$	$\{\cdot\}$	M2 money stock from IMF (2015), Rasche (1990), Friedman and Schwartz (1963) and Carter et al., eds (2006).
β	0.972	Money growth rate and nominal interest rate in 1980.
$\{\tau_t\}_{t=1869}^{2013}$	$\tau_t = \frac{1}{\beta} \frac{M_{t+1}}{M_t}$	Constructed from above.
γ	0.2339	Money share in non-agricultural value added, 1980.
α	0.00003626	Employment share in agriculture in 1980.
$\bar{a}$	0.641	Employment share in agriculture in 2012.

Table 1: Calibrated parameters

In particular, this parameter determines the importance of cash versus non-cash goods in the non-agricultural sector. In our model, the share of cash goods in non-agriculture, call it s_t^M , is given by:

$$s_t^M \equiv \frac{p_{C,t} c_{m,t}}{p_{C,t}(c_{m,t} + c_{n,t})} = \phi_t = \frac{1}{1 + \frac{\gamma}{1-\alpha-\gamma} \tau_t}. \quad (17)$$

Solving for γ , we obtain $\gamma = \frac{1-\alpha}{1 + \frac{s_t^M}{1-s_t^M} \tau_t}$. Taking the average of the right hand side between 1980 and 2012, we obtain $\gamma = 0.23(1-\alpha)$.²⁷ Notice that α determines the long run employment share in agriculture, whilst $\bar{a}$ determines the initial employment in agriculture (given the normalization $B_{a,1869} = 1$). We take the agricultural employment share in 1980 and 2012 directly from the data (3.5% and 1.5% respectively). We then plug these values into equation (10) and obtain two additional equations. Together these two equations with the equation for γ yield the following parameter values: $\bar{a} = 0.641$, $\alpha = 0.00003626$ and $\gamma = 0.2339$. All the parameters from the above calibration are summarized in Table 1. We can then use equations (10)-(13) to obtain solutions to the model which we review below.

5 US Results

Despite its simplicity, the model does well in capturing the evolution of a number of features of the US economy. Figure 5 shows the results for employment shares and GDP per worker over nearly one and a half centuries. Recall that the model was calibrated to match employment shares in agriculture in 1980 and 2010 - and yet, we capture the process of structural transformation over the entire period. This process, coupled with the exogenous evolution of sectoral productivities, also yields an accurate picture of the evolution of GDP per worker. Between 1869 and 2013 the model

²⁷ The average s_t^M between 1980 and 2012 in the data is approximately 0.75, whilst the average τ_t over the same period is 1.09.

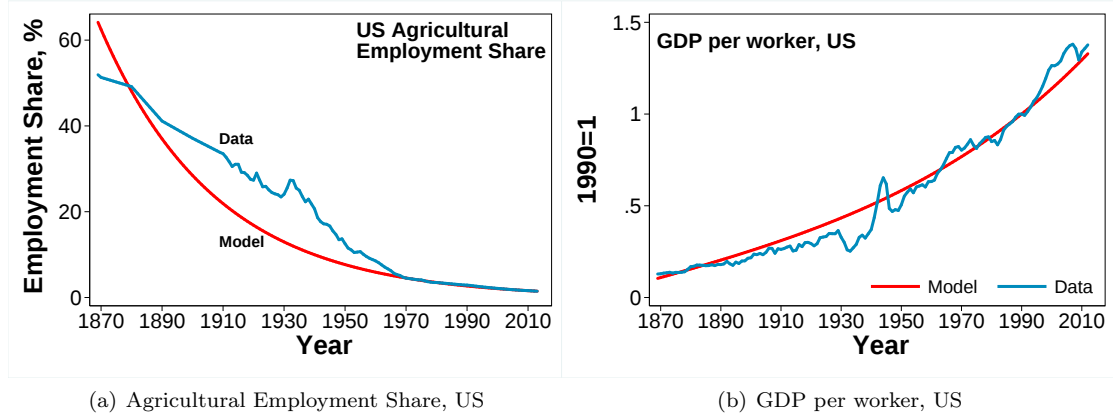


Figure 5: Simulations and data for US prices, 1869-2012.

predicts an average annualized labor productivity growth rate of 1.8%, versus 1.7% in the data.

Importantly, the model also succeeds in capturing the evolution of nominal and relative prices. Figures 6(a) and 6(b) show sectoral price indices (in dollar terms) in the data and the model. Whilst the model predicts a larger increase in prices of agricultural and non-agricultural goods at the end of the period, the fit in general is excellent. This is emphasized in Figure 6(c), which shows that the model captures the decline in the relative price of agricultural to non-agricultural goods (driven largely by the higher productivity growth in the agricultural sector). Figures 6(d)-6(h) then show the evolution of nominal wages,²⁸ consumer prices,²⁹ HP-filtered and unfiltered aggregate inflation rates³⁰ and nominal interest rates³¹ in the model and the data. Whilst we do not capture the short term volatility, the fit of the trend and the changes in the price level is excellent. Interestingly, our model captures the deflationary period in the late 19th century. This turns out to be a key feature of our model and we discuss this in more detail in the subsequent paragraphs. As for interest rates, recall that β was calibrated to match the interest rates between 1980 and 2012 - and so the model does relatively well over this period. In the earlier periods, the model implies higher interest rates than observed (in line with the equity premium puzzle exhibited by many of these types of models), however the correlation between the data and the model remains positive.

Finally, Figure 7 shows the striking results for money share. This very simple model generates virtually the entire increase in the money share of the United States economy over the 140-odd year period. The correlation between the model and the data is 0.91, the R^2 in a regression of the data on the model is 0.83, the model captures 74% of the standard deviation in the data and 54% of the

²⁸ Wage data come from Officer and Williamson (2016b) and capture the money wage of unskilled labor. See <https://www.measuringworth.com/datasets/uswage/source.php> for more details.

²⁹ Data comes from Officer and Williamson (2016a). Notice that this more aggregated data is available for the entire period.

³⁰ Here the inflation rates are calculated from the consumer price index. Very similar results emerge when the GDP deflator is used instead. The smoothing coefficient in the HP filter is set to 100.

³¹ The interest rate between 1869 and 1959 refers to long-term US bond yields and comes from Carter et al., eds (2006). Data after 1959 comes from the WDI (2016) and is the sum of the real interest rates and the GDP deflator.

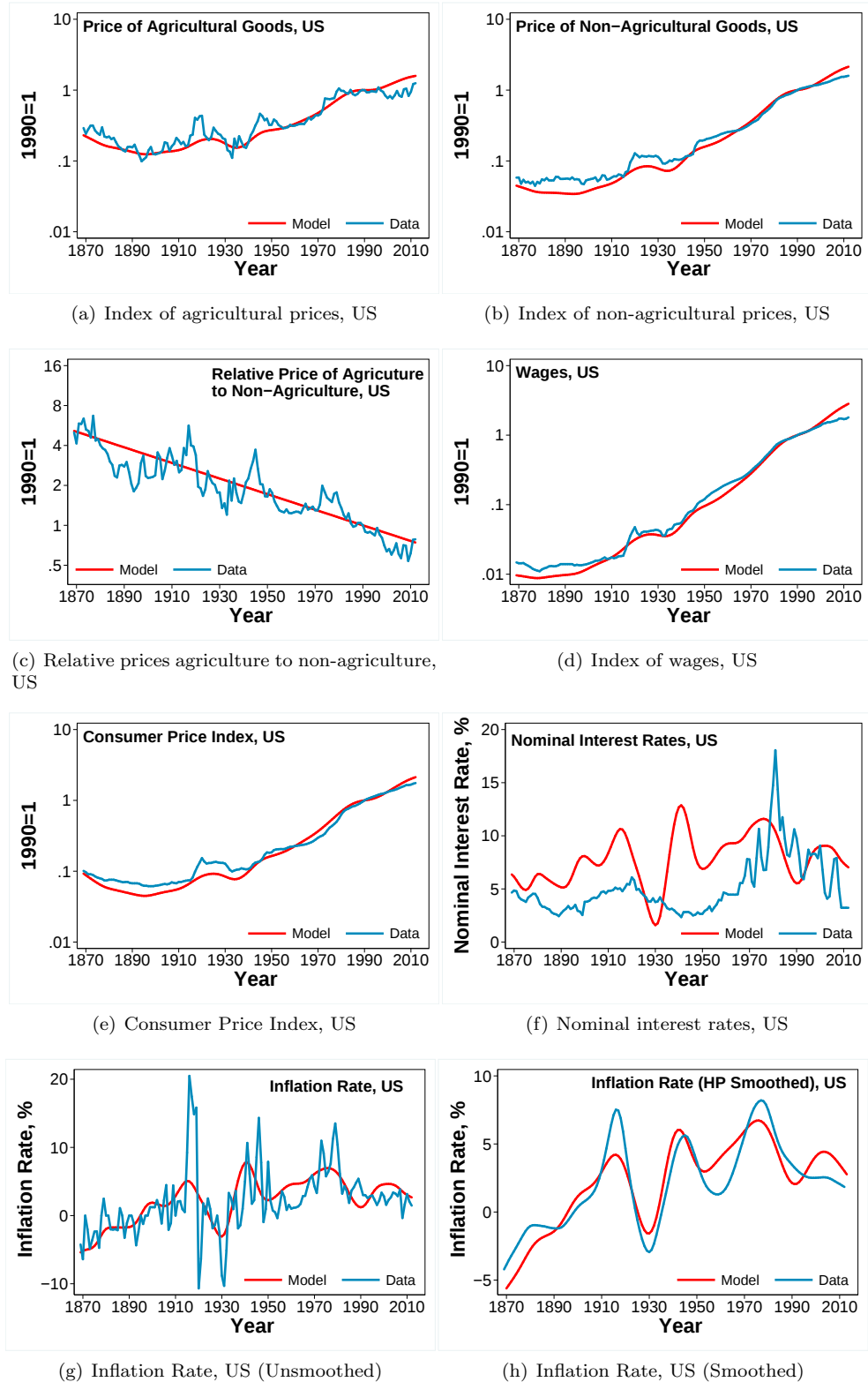


Figure 6: Simulations and data for US prices, 1869-2012.

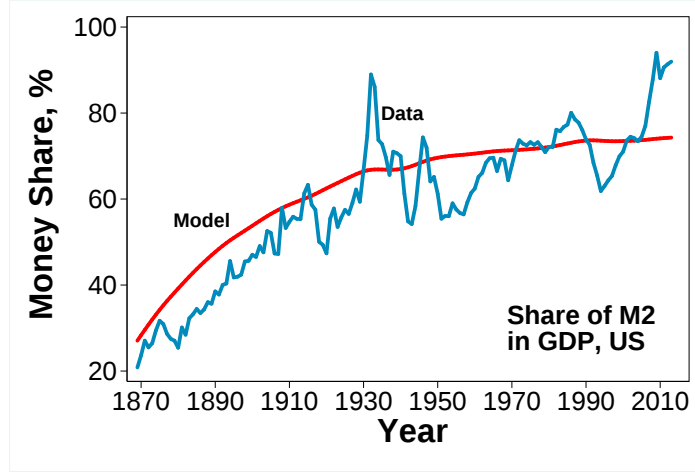


Figure 7: Money (M2) Share in current price GDP.

variance. In other words, whilst the model does not capture short term fluctuations in the money share, its fit of the long-run trend is exceptional. Importantly, although the model was calibrated to match non-agricultural money shares post-1980 (via the γ parameter), the evolution of the money share over time emerges entirely from the mechanism of the model. Since the model takes money supply as given, and is calibrated to match constant price GDP, the success of the model arises from its ability to match the evolution of nominal aggregate prices.

US Counterfactual To highlight the mechanism driving the above result, let us consider how an (effectively) one-sector version of the model would perform in replicating monetary shares. Assume that productivity in the agricultural sector in 1869 is set equal to the agricultural productivity of the US in 2010, and re-calibrate the non-agricultural productivity so as to maintain the same (period-by-period) GDP-per-worker as in the baseline model - whilst keeping all other parameters identical to the calibration in Table 1. Given the new, high agricultural productivity, equation (10) suggests that the employment share in agriculture will be close to α - and hence very small. Thus, this counterfactual effectively replicates a one-sector version of the baseline model. Figure 8(a) shows that monetary shares in the one-sector model are almost constant over the entire period. Variations in monetary growth rates generate some limited fluctuations, but these are not large or systematic enough to generate the observed changes in money shares. Why does the one-sector model fail? By definition, money share is $\frac{M_t}{P_t \times Y_t}$ - where M_t is the money stock, P_t is the aggregate price level and Y_t is GDP. In both the baseline and the counterfactual, we effectively take M_t and Y_t directly from the data. Thus the baseline and the one-sector models differ in their ability to reproduce the evolution of aggregate prices P_t over time. Figure 8(b) shows the evolution of the price index in the baseline, the data and the one-sector counterfactual. The one-sector model fails to capture the evolution of aggregate prices in the early periods. In particular, from the equations in (12), an increase in the employment share of the non-agricultural sector in the baseline model decreases sectoral prices $p_{a,t}$

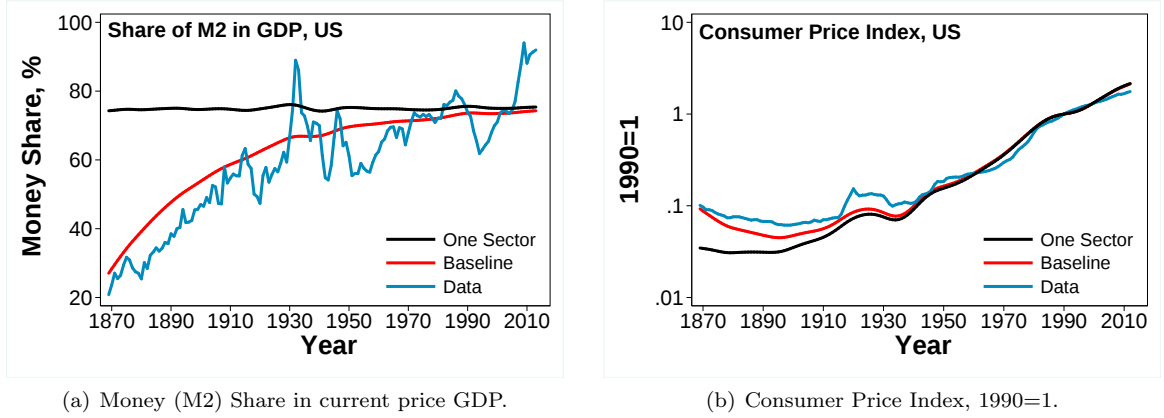


Figure 8: Results for US when we consider an (effectively) one-sector version of the model, 1869-2012.

and $p_{c,t}$. This downward pressure on sectoral prices is stronger when $L_{c,t}$ is small, i.e. at early stages of the structural transformation process. This mechanism therefore helps the baseline model to closely match the observed deflation in the late 19th century, a task that the one-sector model cannot accomplish. Historically, there has been an energetic debate among economists about the sources of this so-called ‘Great-Deflation’. One perspective, held by Tooke and Newmarch (1857), Wells (1891), Landes (1969), Rostow (1948, 1978) and Lewis (1978), highlights the importance of ‘real’ factors behind falling prices - such as the process of industrialization or globalization. The competing view - taken by Friedman and Schwartz (1963) or Bordo and Schwartz (1980, 1981) - argues that it was inadequate growth of money supply relative to real output that drove observed prices at the end of the 19th century (Reti, 1998). Our model gives support to the proponents of the first explanation - by emphasizing the important role that structural transformation played in influencing monetary velocity and prices. Thus the lesson to be drawn from our results is that the multi-sector framework is essential in capturing the historical long-run evolution of monetary shares and prices in the US.

6 Cross-Country Analysis

In this section we test the model’s ability to explain contemporary cross-country monetary shares, and we quantify the importance of structural transformation as a driver of the observed results. This is done in order to provide further external validity to the model and to examine its performance using modern data.

Cross Country Calibration We focus on an international panel of 92 countries for 1980-2010.³² We assume that each country is a closed economy with the same structural parameters of the US,

³² We consider all countries for which all necessary data is available. Unless otherwise stated all data sources and construction methodology is presented in Appendix A.

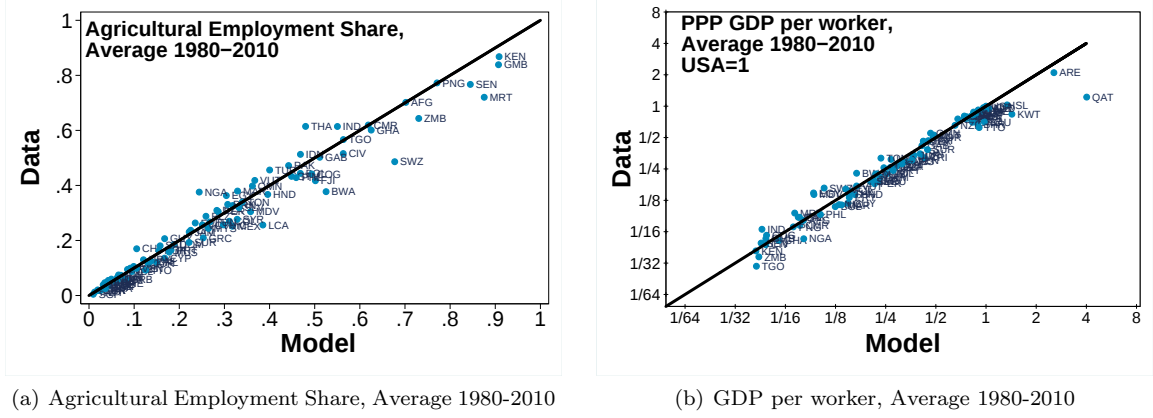


Figure 9: Simulations and data for cross-section of the average for 1980-2010. Line depicts 45 degrees.

with the exception of sectoral productivity and monetary policy. Money stock per worker in each country and each year is taken directly from the data. We assume that labor productivity in sector s and in country i grows at a constant rate, $g_s^i - 1$, and is given by:

$$B_{s,t}^i = B_s^i \times (g_s^i)^{t-1980}. \quad (18)$$

We then choose g_s^i to match observed average sectoral labor productivity growth rates in each country between 1980-2010, and pin down sectoral productivity levels, B_s^i , by following the approach of Duarte and Restuccia (2010). B_a^i is chosen to match the agricultural employment share in country i in 1980, whilst B_c^i is chosen to match the ratio of GDP per worker in country i to that of the USA in 1980. Productivity levels are inferred using the model rather than directly from the data because, as explained by Duarte and Restuccia (2010), internationally comparable sectoral price levels are not as readily available and perhaps not as accurate as aggregate price levels - especially in the non-agricultural sector, which tends to be dominated by services.

Cross-Country Results We will focus on average values over the 1980-2010 period for each country, in order to emphasize the long-run cross-sectional fit of the model. Figure 9 compares the output and employment shares in the model to those in the data, whilst Figure 10 shows the results for inflation rates³³ and nominal interest rates.³⁴ Despite some variance, countries lie close to the 45-degree line and the model does well on all fronts. Next, we examine the model's performance in predicting money shares. Figure 11(a) shows the ratio of money to current price GDP in the data and in the model. Here too the model performs relatively well - although there is more variation than before, and the model is unable to capture money shares greater than 100%, as observed in several European countries and Japan. Nonetheless, notice from Figure 11(b) that the simple model performs exceptionally well in predicting the (semi-)elasticities of money-shares

³³ The inflation data is constructed using the Consumer Price Index from WDI (2016). Very similar results hold when using the GDP deflator.

³⁴ These are constructed as the sum of the real interest rates and the CPI deflator using data from WDI (2016).

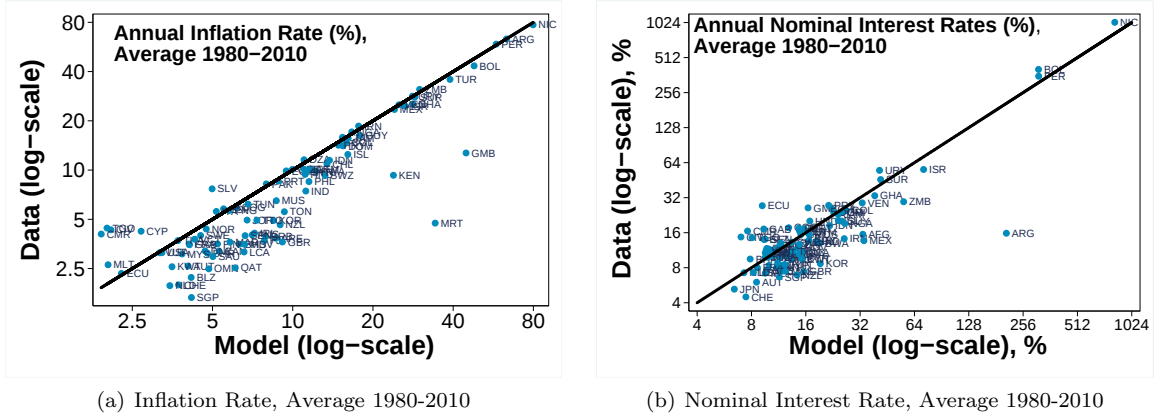


Figure 10: Simulations and data for cross-section of the average 1980-2010, annual inflation rates and nominal interest rates. Line depicts 45 degrees.

with respect to GDP per worker, which are found to be 0.14 in both the data and the model.³⁵ The simple model thus successfully captures the cross-sectional variation in monetary shares and the long-run, cross-sectional trend between monetary shares and income.

Cross Country variation in velocity We now turn to understanding which specific mechanism or mechanisms drive the success of the model. Recall that countries are assumed to differ in their monetary policies M_t^i (and hence τ_t^i) and in their sectoral productivities, $B_{s,t}^i = B_s^i \times (g_s^i)^{t-1980}$. The following two counterfactuals quantify the extent to which variations in each of these factors contributes to explaining cross-country differences in velocities as well as in the velocity-income relationship.

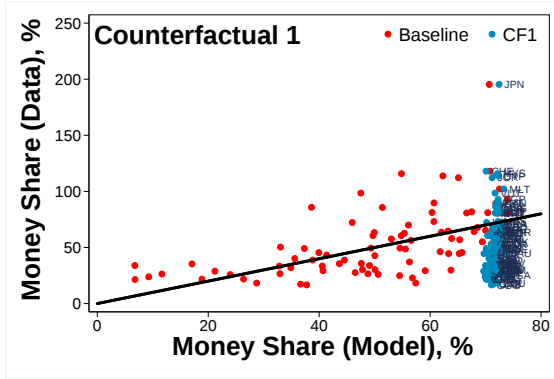
Counterfactual 1 In the first counterfactual, we assume that each country follows the US monetary policy, i.e. $\tau_t^i = \tau_t^{US}$. We also assume that all countries have the same agricultural productivity as the US in 1980, so that $B_a^i = B_a^{US}$. We re-calibrate non-agricultural productivity in 1980, B_c^i , to match each country's observed GDP per worker in 1980. Countries thus differ only in sectoral productivity growth rates and non-agricultural productivity levels. Since every country now has a high agricultural productivity (as the US), the agricultural sector will be vanishingly small in every country. As such, this experiment is akin to examining how well a one-sector model under US monetary policy in each country would match the data. Figure 12(a) compares money-shares in the counterfactual and the data, whilst Figure 12(b) plots money-shares versus the (log) GDP per worker in the counterfactual and the data. Predicted monetary shares are almost constant. Thus, an (effectively) one-sector version of the model with no variation in monetary policy cannot replicate the data. The reasons for this are clear from equation (14): $(V_t^i)^{-1} = \phi_t^i L_{c,t}^i$. First, non-agricultural

³⁵ In other words a doubling of GDP per worker in a country is associated with a 14 percentage point increase in the money share. The reason that this is different from the 0.16 elasticity found in Figure 1 is that we now have a smaller sample of countries.

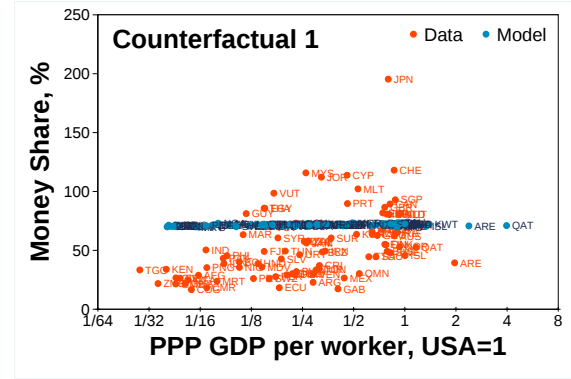
Figure 11: Money Share in model and data as well as the money share versus GDP (log-scale) in Model and Data, Average 1980-2010.

Counterfactual 2 In the second counterfactual we keep agricultural and non-agricultural productivity exactly as in Counterfactual 1, but we allow each country to have its own monetary policy taken directly from the data. Thus, this experiment examines whether differences in monetary policies in an (effectively) one sector model are capable of explaining cross-country differences in money shares and the velocity-income relationship. The results are shown in Figures 12(c) and 12(d). Whilst there is some additional variation coming from differences in monetary policies across countries - especially in some high inflation countries like Argentina or Peru - monetary shares do *not* change significantly with respect to the previous counterfactual. Thus, whilst differences in monetary growth rates do technically influence money shares across countries, the message from this experiment is that, *quantitatively*, velocity and the velocity-income relationship is overwhelmingly *not* driven by differences in monetary policies across countries.

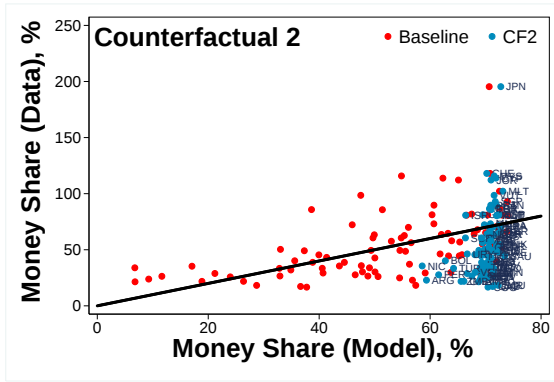
Baseline Finally, maintaining all the assumptions from Counterfactual 2, we also allow countries to differ in their agricultural productivity levels - and hence in the size of their agricultural sectors. In other words, we revert to the baseline economy where countries differ in their monetary policies as well as in agricultural and non-agricultural productivity growth rates and levels. The results



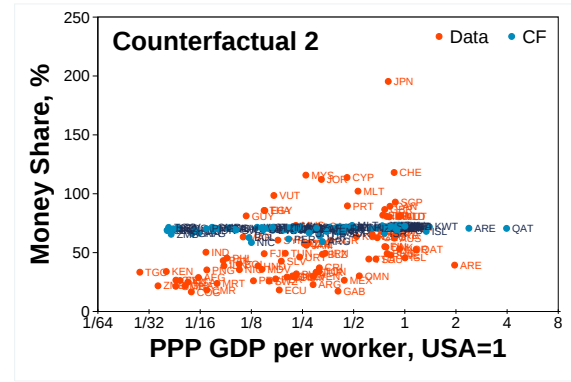
(a) Counterfactual 1 (US agr. productivity and monetary policy): Money shares in data and model.



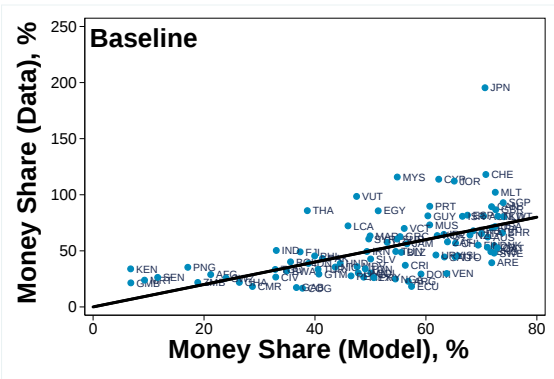
(b) Counterfactual 1 (US agr. productivity and monetary policy): Money shares versus GDP per capita.



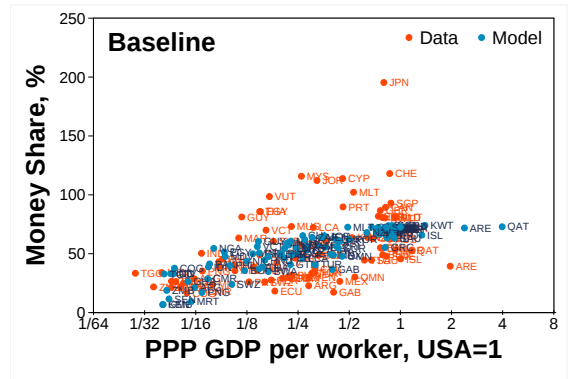
(c) Counterfactual 2 (US agr. productivity): Money shares in data and model.



(d) Counterfactual 2 (US agr. productivity): Money shares versus GDP per capita.



(e) Baseline Model: Money shares in data and model.



(f) Baseline Model: Money shares versus GDP per capita.

Figure 12: Money shares in the baseline model and the counterfactuals (average for 1980-2010). Drawn versus corresponding data and (log of) GDP per worker.

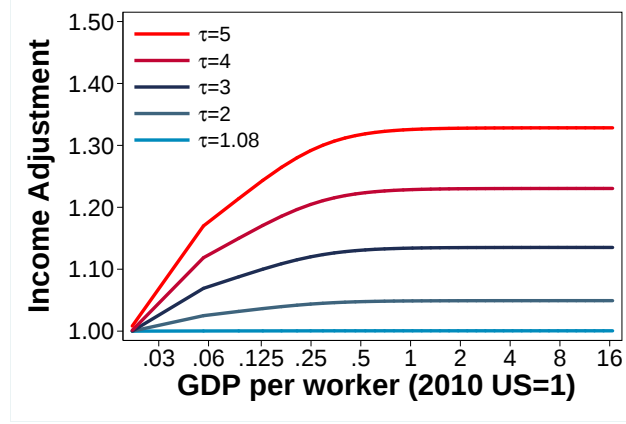


Figure 13: Income adjustment necessary to make an household indifferent between living in a world with sub-optimal monetary policy and a world with optimal monetary policy at different income levels.

are striking and shown in Figures 12(e) and 12(f). As was argued before, the model does well in capturing the cross-country variation in money-shares and very well in capturing the relationship between money-shares and income. We can therefore conclude that most of the variation in money shares in the model comes entirely from the multi-sector framework, and in particular from the cross-country differences in agricultural productivity levels, which generate agricultural sectors of different sizes.³⁶ Since the model captures practically the entire trend in cross-country velocity-income variation, this suggests - perhaps somewhat surprisingly - that structural transformation, and the decline of the agricultural sector in particular, is a key driver of the long run monetary velocity.³⁷

7 Costs of Inflation

In this section we examine the welfare costs of inflation in rich and poor countries. In particular, we calculate a compensating variation measure of welfare that determines how much higher a household's income would have to be in order to compensate for permanently higher inflation. We proceed as follows. The lifetime indirect utility of a household is given by:

$$V(\{w_t, p_{a,t}, p_{c,t}, \tau_t\}_{t=T}^{\infty}) \equiv \sum_{t=T}^{\infty} \beta^t u(a(w_t, p_{a,t}, \tau_t), c_n(w_t, p_{a,t}, p_{c,t}, \tau_t), c_m(w_t, p_{a,t}, p_{c,t}, \tau_t)). \quad (19)$$

In equation (19), $a(\cdot)$, $c_n(\cdot)$ and $c_m(\cdot)$ are standard demand functions for agricultural goods as well as monetary and non-monetary non-agricultural goods.³⁸ Suppose that $V(\cdot; \lambda)$ denotes an household's

³⁶ A more detailed analysis of the results is shown in Table 2 in Appendix B.1.

³⁷ Of course, there are some striking outliers. Countries like Cyprus, Switzerland or Japan have monetary shares far above their predicted values. A limitation of our model is that it cannot generate money shares larger than 1. However, the focus here is on broad cross country trends, rather than the specifics of individual countries - and in this respect the model does very well. A model with capital, discussed later, can come closer to generating the observed patterns - whilst a model with intermediate goods would presumably do even better. We leave the latter for future research.

³⁸ In particular, from the household's first order conditions and its budget constraint we can show that: $a(w_t, p_{a,t}, \tau_t) =$

lifetime indirect welfare when income, w_t , is multiplied by a factor λ in each time period:

$$V(\{w_t, p_{a,t}, p_{c,t}, \tau_t\}_{t=T}^\infty; \lambda) \equiv \sum_{t=T}^\infty \beta^t u(a_t(\lambda w_t, p_{a,t}, \tau_t), c_{n,t}(\lambda w_t, p_{a,t}, p_{c,t}, \tau_t), c_{m,t}(\lambda w_t, p_{a,t}, p_{c,t}, \tau_t)). \quad (20)$$

We wish to know by what factor, λ , must income be multiplied to make a household indifferent between living in a world with some sub-optimal monetary policy, $\{\hat{\tau}_t\}_{t=0}^\infty$, and a world with optimal monetary policy. The answer to this question satisfies the following equation:

$$V(\{w_t, p_{a,t}, p_{c,t}, 1\}_{t=T}^\infty) = V(\{w_t, p_{a,t}, p_{c,t}, \hat{\tau}_t\}_{t=T}^\infty; \lambda), \quad (21)$$

where prices are equilibrium prices for the optimal economy (i.e. one with $\tau_t = 1$) from equations (12) and (13).³⁹

We calculate such λ for a hypothetical economy that is identical in all respects to the US economy at different stages of its development, with the exception of its inflation tax.⁴⁰ In particular, we set different values of this tax, ranging from $\tau_t = 1.08$, (the average rate in the US) up to $\tau = 5$. The results are shown in Figure 13. A higher inflation tax results in higher welfare costs - independent of a country's level of income. However the cost of a sub-optimal policy will be *lower* in poorer countries. The reason for this is that most of the output in poorer countries is concentrated in (non-monetary) agriculture. An inflation tax on the small monetary sector will thus have relatively little effect. In richer countries, where more of the output is produced in (monetary) non-agriculture, inflation taxes can have larger welfare effects. For example, in a country with GDP per worker equal to the US GDP in 2010, a monetary policy of $\tau_t = 5$ will require an increase in income of approximately 32% to make an household indifferent to a world with optimal monetary policy. However, the same monetary policy in a country that has approximately 3% of the US's GDP in 2010 will only require an increase in income of 5%. Thus, the same inflationary policies have very different costs in rich and poor countries. This suggests why we see higher inflation in poorer countries: these countries have a lower welfare cost of implementing inflationary policies.

An alternative approach for highlighting this result is to find the maximum inflation tax, $\bar{\tau}$, in each country that makes the country's welfare cost of inflationary policy, λ , identical to the welfare cost of the US's inflationary policy, λ^{US} . This inflation tax satisfies the following equation:

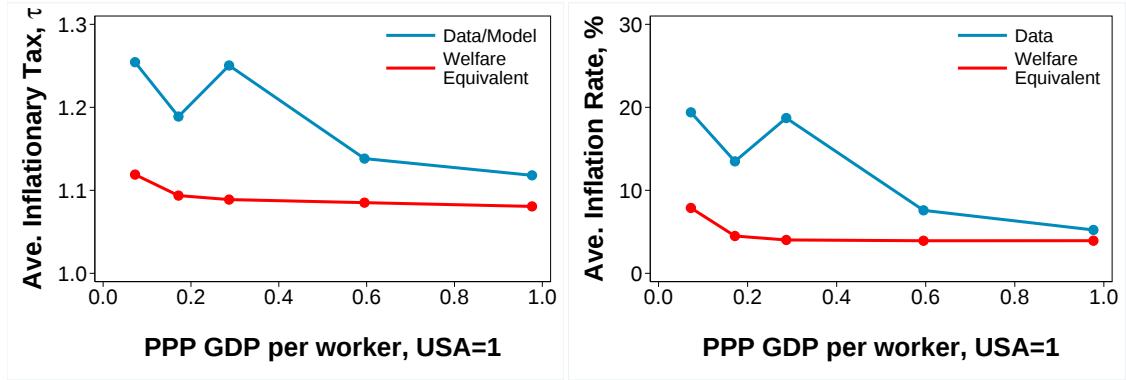
$$V(\{w_t, p_{a,t}, p_{c,t}, 1\}_{t=T}^\infty) = V(\{w_t, p_{a,t}, p_{c,t}, \bar{\tau}\}_{t=T}^\infty; \lambda^{US}). \quad (22)$$

The red line in Figure 14(a) shows this average welfare-equivalent $\bar{\tau}$ in each income quintile of the sample. In the lowest income quintile $\bar{\tau}^{q5} = 1.12$, whilst in the highest $\bar{\tau}^{q5} = 1.08$. The red line in Figure 14(b) then shows how each of these inflation taxes translate into observed average, annual

$\bar{a} + \frac{\alpha(w_t - p_{a,t}\bar{a})\tau_t}{p_{a,t}(1-\alpha-\gamma+(\alpha+\gamma)\tau_t)}, c_n(w_t, p_{a,t}, \tau_t) = \frac{\gamma(w_t - p_{a,t}\bar{a})\tau_t}{p_{c,t}(1-\alpha-\gamma+(\alpha+\gamma)\tau_t)}$ and $c_m(w_t, p_{a,t}, \tau_t) = \frac{(1-\alpha-\gamma)(w_t - p_{a,t}\bar{a})}{p_{c,t}(1-\alpha-\gamma+(\alpha+\gamma)\tau_t)}$.

³⁹ It does not matter whether we choose prices from the optimal or the sub-optimal economy, as the demand functions $a(\cdot)$, $c_n(\cdot)$ and $c_m(\cdot)$ depend only on relative prices and these are entirely independent of τ_t .

⁴⁰ Thus, for this exercise only, we assume each country follows the same growth path as the US. Poorer countries can thus be simply thought of as the US at some point in the past.



(a) Observed and welfare equivalent measures of τ in each income quintile. (b) Observed and welfare equivalent measures of average, annual inflation rates in each income quintile.

Figure 14: Observed and welfare equivalent measures of inflation taxes and rates across income quintiles.

inflation rates. Welfare-equivalent inflation rates in the lowest and highest income quintiles are 7.9% and 3.9% a year, respectively. Thus the poorest countries can have inflation rates that are twice as high compared to rich countries, and yet have the same welfare costs as rich, low-inflation countries. The blue lines in Figure 14 show the corresponding inflationary taxes and the average, annual inflation rates in the data. Notice from Figure 14(b) that the observed inflation rate in the richest countries is 3.7 times higher than the inflation rate in the poorest countries. Thus, whilst inflation rates vary enormously between rich and poor countries, the welfare costs of these policies do not vary nearly as much.

8 Conclusions

We put forward a new theory of money demand in the long run, where velocity depends on a country's GDP level. The main drivers are structural transformation - i.e. the reallocation of labor from agriculture to non-agriculture associated with development - and an endogenous share of monetary transactions in different sectors. Despite its stark simplicity, our theory is very successful at matching both within- and across-country changes in monetary velocity and the velocity-income relationship. We can replicate - almost perfectly - the long run monetary velocity in the United States between 1869 and 2013 and in a large cross-sectional data set of 92 countries.

There are three lessons to be learned from our work. First, our findings suggest that the evolution of monetary velocity is driven almost exclusively by developments in the *agricultural* rather than the non-agricultural sector. Second, we show that the costs of bad monetary policy are disproportionately higher in richer than in poorer countries. Finally, we demonstrate that, since velocity depends systematically on the composition of output, a country's price levels and inflation rates may not 'always and everywhere be a monetary phenomenon' (Friedman and Schwartz, 1963). These lessons

should be of utmost interest to central bankers, especially in developing countries, who may downplay the role of the agricultural sector in their policy decisions. Moreover, our findings may help explain why we observe persistently higher inflation rates in poorer countries than in richer countries - despite strong anti-inflationary policies recommended by international institutions. Finally, our results are also important to researchers, as they highlight that one-sector models cannot be successfully used to understand the long run dynamics of monetary velocity and nominal price levels. Our work on the evolution of long-run monetary velocity thus offers key insights into understanding the secular trends in sectoral and aggregate price levels, with strong implications for policy and future research.

A Data Appendix

A.1 US Data

GDP (PPP) and Labor Force Data We use data from Penn World Tables version 7.1. (see Heston et al. (2012)) to construct annual time series of PPP-adjusted GDP in constant 1990 prices, PPP-adjusted GDP per worker in 1990 constant prices, and total employment between 1950-2010. For each country, we construct total employment L using the variables Population (POP), PPP Converted GDP Chain per worker at 2005 constant prices (RGDPWOK), and PPP Converted GDP Per Capita (Chain Series) at 2005 constant prices (RGDPCH) as $L = (RGDPCH \times POP)/RGDPWOK$. We then construct PPP-adjusted GDP in constant 2005 prices using the variables “PPP Converted GDP Per Capita (Laspeyres), derived from growth rates of c, g, i, at 2005 constant prices (RGDPL)” and “Population (POP)” as $GDPK = RGDPL \times POP$. We then re-base the 2005 data to 1990 prices.

We extend the labor force and GDP data calculated above back in time for the period 1869-1950 using growth rates from Maddison (2007). In particular, we calculate the pre-1950 growth rates for Maddison’s GDP measure (in 1990 Geary-Khamis dollars) and use it to extend the GDP measure calculate above. We also calculate the population growth rates from Maddison’s data and assume that the growth rate of the labor force pre-1950 is the same as the growth rate of the population (which is true if the labor force to population ratio stays constant over time). We then use these population growth rates to extend the labor force data calculated above back in time. Finally, we extend the series for GDP and labor force forward in time between 2011 and 2014 using growth rates for labor force and constant price PPP GDP from the WDI (2016). The series for PPP-adjusted GDP per worker in constant prices is then computed as $y = GDP/L$.

Agricultural and Non-Agricultural Employment We construct contemporary (1980-2014) agricultural employment share using data from the FAOSTAT (2012) by taking the ratio of economically active population (labor force) in agriculture to total economically active population (labor force). For the US, we then combine this with the agricultural employment share from Alvarez-Cuadrado and Poschke (2011) for 1869-1980. Missing data is interpolated. We then obtain total employment in agriculture by multiplying the agricultural employment shares calculated above by the labor force data found in the previous paragraph. Non-agricultural employment is then the difference between total employment and agricultural employment. These are the $L_{a,t}$ and $L_{c,t}$ referred to in the main body of the text.

Labor Productivity Growth Rates To calculate sectoral labor productivity growth rates for our cross-sectional experiment we use constant price sectoral value added data in local currency units from the UN (2008), sectoral employment shares from FAOSTAT (2012) and labor force data from Heston et al. (2012). We multiply the employment shares by the labor force data to calculate

sectoral employment and then use this data to calculate sectoral labor productivities for agriculture and non-agriculture in each country between 1980 and 2010. Next, we HP-filter these series (using a smoothing parameter of 100), and for each country, i , and sector $s = a, c$ we calculate an annualized average sectoral labor productivity growth rate, $g_s^i - 1$.

Sectoral Value Added, Constant price For the period 1970-2014 we construct constant price (1990) value added data for agriculture and total value added from UN (2008). We then extend this data for the US backwards using sectoral growth rates. First, we obtain constant price agricultural and total value added data for the period 1947-1970 from the 10-sector database constructed by Timmer et al. (2014) and use this data to extend our value added measures back to 1947. Then, we extend these concatenated data back to 1869 using the Historical National Accounts database constructed by the Timmer et al. (2014). This database provides historical constant price indices of GNP for agriculture and the total economy. Missing values are interpolated. Notice that - like Duarte and Restuccia (2010) - we do *not* directly use the resulting series in our model. Instead, we use the above constant (1990) price agricultural and total value added data to calculate constant price shares of agricultural value added in total value added. Then, in order to remain consistent with our aggregate GDP data calculated above, we multiply these constant price shares by the aggregate PPP GDP data calculated above. This gives us a consistent estimate of 1990 value added of agriculture. Subtracting this estimate from the GDP PPP gives us an estimate of non-agricultural value added. These are the $Y_{a,t}$ and $Y_{c,t}$ referred to in the main body of the text.

US Sectoral prices Sectoral prices between 1929 and 2013 for the United States are obtained by dividing sectoral value added in current prices (obtained from the BEA and (Timmer et al., 2014)) by constant price sectoral value added (calculated above). Obtaining pre-1929 prices is more complicated. In particular, as far as we know, no dependable series of data on sectoral prices exists. As such, we use nominal wheat prices obtained from Table Cc205-266 in Carter et al., eds (2006) to extend the agricultural price index back in time between 1869 and 1929. Then, using data on constant- and current-price aggregate GDP, constant price sectoral value added and the agricultural price index calculated above, we can infer a non-agricultural price index as well. In particular, multiplying the constant-price agricultural value added by the agricultural price index, gives us an estimate of current price agricultural value added, $p_{a,t}Y_{a,t}$. We can then subtract this from current price GDP, to obtain an estimate of current price non-agricultural value added, $p_{c,t}Y_{c,t}$. Then, taking the ratio of current price non-agricultural value added to constant price non-agricultural value added we obtain an estimate of the price index of the non-agricultural sector.

Money We follow Anderson (2006) in the construction of the long run money stock series for the US. The source of the data for the years 1959-2014 are lines 34 and 35 in the International Financial

Statistics (IMF, 2015).⁴¹ For the year 1948-1958 we use data constructed by Rasche (1990) and available online on the Historical Statistics of the United States website (Carter et al., eds, 2006). We extend this for 1869-1947 using data from Friedman and Schwartz (1963) also reported in the Historical Statistics of the United States.⁴²

Aggregate Capital We follow Kuralbayeva and Stefanski (2013) and Caselli (2005) in constructing capital and make use of the perpetual inventory method. Capital is accumulated according to:

$$K_{t+1} = (1 - \delta)K_t + I_t, \quad (23)$$

where I_t is investment and δ is the depreciation rate. We measure I_t from the PWT 7.1 as real aggregate investment in PPP terms.⁴³ As is standard, we compute the initial capital stock K_0 as $I_0/(g + \delta)$, where I_0 is the value of the above investment series in the first period that it is available, and g is the average geometric growth rate for the investment series in the first twenty years the data is available. Finally, we set the depreciation rate, δ , to 0.0496 to match the depreciation rates in the US.⁴⁴ The results are not very sensitive to choices in either δ , g or initial capital stock. The above process gives us sequences of capital stocks, K_t , derived from PWT data.

We extend our US capital data back in time using Piketty (2014) who provides data on the capital-output ratio in the US between 1770 and 2010 (measured in current-period prices). We consider only the reproducible part of his capital measure by subtracting the value of land from his measure of ‘national capital’. The resulting capital series corresponds much more closely to our modern measure of capital (Jones, 2015). We use the implied growth rates in the capital-output ratio from Piketty (2014) to extend our capital-output ratio data backwards in time and maintain comparability. We find that the implied (reproducible) capital-output ratio in the US in 1869 (in current-period prices) was approximately 1.97. This is of course is a very crude estimate and should be approached cautiously - nonetheless this is the best we can do over this long time horizon.

TFP growth rates In the version of the model with capital, we use total factor productivity growth rates of the non-agricultural sector directly from the data. To calculate these, we assign all capital in the economy to the non-agricultural sector. Then, taking $Y_{c,t}$, $L_{c,t}$ and K_t from the above, and using the country-specific capital share, ν , the TFP in non-agriculture in a country is simply

⁴¹ Notice, that for 1980-2014 this coincides with the WDI data used earlier.

⁴² Notice that Friedman and Schwartz (1963) have slightly different definitions of monetary aggregates to those currently used. According to Anderson (2006), Friedman and Schwartz’s ‘M4 resembles, in many respects, the Federal Reserve’s current M2’ and ‘(h)ence, from an economic viewpoint, Friedman and Schwartz’s M4 and the currently published Federal Reserve M2 aggregates are more similar than first appearances might suggest’. As such, for the 1898-1947 period - when it is available - we use Friedman and Schwartz’s measure of M4. For the period 1869-1897 only the M3 measure is available. As such, we use the growth rate of M3, to extend the data back in time.

⁴³ So that $I_t \equiv RGDPL \cdot POP \cdot KI$, where $RGDPL$ is real income per capita obtained with the Laspeyres method, POP is the population and KI is the investment share in real income. In the above we have re-based $RGDPL$ into 1990 dollars.

⁴⁴ The value of δ is chosen so that the average ratio of depreciation to GDP using the constructed capital stock series matches the average ratio of depreciation to GDP in the data over the corresponding period. The OECDs Annual National Accounts report depreciation in the data as “Consumption of Fixed Capital.”

(a) Standard Deviation of Money Shares				(b) Money-Share Income Semi-Elasticities			
	Standard Deviation	Relative to Baseline	Relative to Data		Money-share Semi-elasticity	Relative to Baseline	Relative to Data
Data	0.292	-	1.00	Data	0.139	-	1.00
Baseline	0.174	1.00	0.60	Baseline	0.141	1.00	1.02
CF1	0.009	0.05	0.03	CF1	0.003	0.02	0.02
CF2	0.027	0.15	0.09	CF2	0.007	0.05	0.05

Table 2: Decomposition of results.

calculated as a Solow residual: $Y_{c,t}/(K_t^\nu L_{c,t}^{1-\nu})$. Calculating the annualized average growth rate of this term between 1980 and 2010, gives us the TFP growth rate used in the model.

B Theoretical Appendix

B.1 Detailed Baseline Results

In this section we give more details about the baseline model result. In particular, we decompose the money share standard deviation to identify which factor is more important, and we do the same with the money share income semi-elasticities. Our results are summarized in Table 2.

Table 2(a) decomposes the contribution of each counterfactual to the money share's standard deviation. In the data, the standard deviation is 0.292, while it is 0.174 in our baseline model. The baseline thus captures 60% of the standard deviation of monetary shares in the data. The second column of 2(a) then shows the contribution of each counterfactual to the baseline model's results. Notice that the (effectively) one-sector model with country-specific monetary growth can only explain 15% of the deviation of the baseline model. This means that 85% of the deviation comes from the two-sector framework alone. Table 2(b) shows that both in the data and the model the money share-income (semi-)elasticity is approximately 0.14. Again, it is the two-sector framework - rather than variation in capital stock - that overwhelmingly contributes to the success of the model along this dimension.

B.2 Introducing Capital

In this section we show that our results remain unchanged when we allow for capital accumulation in the non-agricultural sector. In particular we now assume that the production of non-agricultural goods now takes labor and capital, K_t , as an input.

Household's problem The representative households's problem is now given by:

$$\begin{aligned}
 & \max_{a_t, c_{m,t}, c_{n,t}, b_{t+1}, m_{t+1}, K_{t+1}} \sum_{t=0}^{\infty} \beta^t (\alpha \log(a_t - \bar{a}) + (1 - \alpha - \gamma) \log(c_{m,t}) + \gamma \log(c_{n,t})) \\
 & \text{s.t. } p_{a,t} a_t + p_{c,t} (c_{m,t} + c_{n,t} + x_t) + b_{t+1} + m_{t+1} \leq w_t + r_{k,t} K_t + (1 + r_t) b_t + m_t + T_t
 \end{aligned}$$

$$K_{t+1} = x_t + (1 - \delta)K_t \quad (24)$$

$$p_{c,t}c_{m,t} \leq m_t \quad (25)$$

$$K_t, m_t \geq 0 \text{ and } b_t \geq -\bar{B}$$

The problem is very similar to the baseline model. The household owns a unit of labor and a stock of capital K_t which it rents out on the market for a wage w_t and a rental rate $r_{k,t}$ respectively. In addition, the household enters period t with money holdings m_t , bond holdings b_t , and a helicopter transfer of money from the government, T_t . With these resources, it can buy agricultural goods a_t (at the price $p_{a,t}$), cash- and non-cash non-agricultural goods (respectively, $c_{m,t}$ and $c_{n,t}$, at the price $p_{c,t}$), investment goods x_t (at the price $p_{c,t}$), bonds b_{t+1} for a gross return of $1 + r_{t+1}$ dollars next period, and it can decide to hold a (non-negative) stock of money m_{t+1} for the next period.

Firms There are representative agricultural and non-agricultural firms. The agricultural firm's problem is identical to that in the baseline: the firm hires labor, $L_{a,t}$, and produces output, $Y_{a,t}$, using a simple linear technology that combines labor with exogenous, sector-specific total factor productivity, $B_{a,t}$. Its profit maximization problem is given as before by equation (2). Non-agricultural firms however, are now assumed to hire both labor, $L_{c,t}$, and to rent capital, K_t^f , from households. With these inputs they produce output, $Y_{c,t}$, using a Cobb-Douglas technology that combines labor and capital with exogenous, sector-specific total factor productivity, $B_{c,t}$. Its profit maximization problem is therefore given by:

$$\max_{K_t^f, L_{c,t}} p_{c,t}Y_{c,t} - w_t L_{c,t} - r_{k,t} K_t^f \quad \text{s.t. } Y_{c,t} = B_{c,t}(K_t^f)^\nu (L_{c,t})^{1-\nu} \quad (26)$$

Notice that output of non-agricultural firms, $Y_{c,t}$, can be sold as both a monetary and non-monetary consumption good as well as a non-monetary investment good. As in the baseline model, agricultural output is assumed to be a non-monetary, consumption good.

Money Supply The government is assumed to have a helicopter monetary policy as before:

$$M_{t+1} = T_{t+1} + M_t. \quad (27)$$

Market Clearing Finally, market clearing is standard.

$$\begin{aligned} a_t &= Y_{a,t}, \quad c_{m,t} + c_{n,t} + x_t = Y_{c,t} \\ m_t &= M_t, \quad b_t = 0 \\ L_{a,t} + L_{c,t} &= 1, \quad K_t = K_t^f. \end{aligned} \quad (28)$$

Solution We follow the same solution strategy as in the baseline model. In particular, we assume that nominal interest rates are always positive and we use the first order conditions of the household and firms problems, in combination with the market clearing conditions and the evolution of money by government to obtain three equations that (together with transversality conditions for bonds, capital and money) pin down the equilibrium solutions of the problem.

The first equation defines a split of non-agricultural consumption between monetary and non-monetary goods. Defining $C_t \equiv c_{m,t} + c_{n,t} = Y_{c,t} - (K_{t+1} - (1 - \delta)K_t)$ we can write:

$$c_{n,t} = (1 - \phi_t)C_t \quad \text{and} \quad c_{m,t} = \phi_t C_t, \quad (29)$$

where $\phi_t = \frac{1-\alpha-\gamma}{(1-\alpha-\gamma)+\gamma\tau_t}$. Second, given initial capital endowment, K_0 , the path of capital is pinned down by a transversality condition and the following Euler Equation:

$$\frac{\tau_{t+1}}{\tau_t} \frac{\phi_{t+1}}{\phi_t} \frac{C_{t+1}}{C_t} = \beta (\nu B_{t+1}^c (K_{t+1})^{\nu-1} (L_{t+1}^c)^{1-\nu} + 1 - \delta). \quad (30)$$

Finally, employment in the non-agricultural sector, $L_{c,t}$, is determined by the following:

$$\frac{\alpha\tau_t}{(1 - L_{c,t}) - \frac{\bar{a}}{B_{a,t}}} = \frac{(1 - \nu)(1 - \alpha - \gamma)B_{c,t}K_t^\nu (L_{c,t})^{-\nu}}{\phi_t C_t}. \quad (31)$$

We solve the model following the strategy of Echevarria (1997). We assume that after some point in time, T , the variables M_t , $B_{a,t}$ and $B_{c,t}$ grow at constant rates ($g_m - 1$, $g_a - 1 > 0$ and $g_c - 1$ respectively) and hence $\tau_t \rightarrow \bar{\tau}$ and $\phi_t \rightarrow \bar{\phi}$ converge to constants. Given these assumptions, the role of the non-homotheticity disappears in the limit as $\lim_{t \rightarrow \infty} \frac{\bar{a}}{B_{a,t}} = 0$ in equation (31). The model thus converges asymptotically to a balanced growth path where capital, investment and non-agricultural consumption grow at the rate $g_c^{\frac{1}{1-\nu}}$, agricultural consumption grows at a rate g_a and employment in agriculture and non-agriculture are constant. Consequently, we can re-write the model in terms of variables that are stationary in the long run. In particular, defining $\tilde{k}_t \equiv K_t/B_{c,t}^{\frac{1}{1-\nu}}$ and $\tilde{c}_t \equiv C_t/B_{c,t}^{\frac{1}{1-\nu}}$ equations (30) and (31) become:

$$\left(\frac{B_{c,t+1}}{B_{c,t}} \right)^{\frac{1}{1-\nu}} \frac{c_{t+1}}{c_t} \frac{\tau_{t+1}}{\tau_t} \frac{\phi_{t+1}}{\phi_t} = \beta \left(\nu \tilde{k}_{t+1}^{\nu-1} (L_{t+1}^c)^{1-\nu} + 1 - \delta \right) \quad \text{and} \quad (32)$$

$$\frac{\alpha\tau_t\phi_t}{(1 - L_{c,t}) - \frac{\bar{a}}{B_{a,t}}} = \frac{(1 - \nu)(1 - \alpha - \gamma)\tilde{k}_t^\nu (L_{c,t})^{-\nu}}{(1 - \delta)\tilde{k}_t - \left(\frac{B_{c,t+1}}{B_{c,t}} \right)^{\frac{1}{1-\nu}} \tilde{k}_{t+1} + \tilde{k}_t^\nu (L_{c,t})^{1-\nu}} \quad (33)$$

Since $\lim_{t \rightarrow \infty} \frac{\bar{a}}{B_{a,t}} = 0$, using the above it is easy to show that $\tilde{k}_t \rightarrow \tilde{k}^*$ and $L_t^c \rightarrow L_c^*$, where:

$$L_c^* = \frac{(1 - \nu) \left(g_c^{\frac{1}{1-\nu}} - \beta(1 - \delta) \right)}{\left(g_c^{\frac{1}{1-\nu}} - \beta(1 - \delta) \right) + \frac{\alpha}{1-\alpha-\gamma} \bar{\tau} \bar{\phi} \left(g_c^{\frac{1}{1-\nu}} (1 - \beta\nu) - \beta(1 - \delta)(1 - \nu) \right)}, \quad \text{and} \quad (34)$$

$$\tilde{k}^* = \beta^{-\frac{1}{1-\nu}} (g_c^{\frac{1}{1-\nu}} - \beta(1 - \delta))^{\frac{1}{1-\nu}} \nu^{\frac{1}{1-\nu}} L_c^*. \quad (35)$$

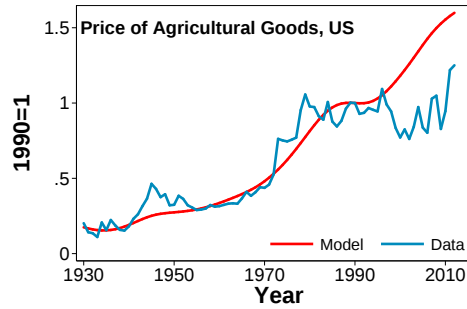
Parameter	Values	Target
$B_{a,1869}, B_{c,1869}$	1	Normalization
$g_a - 1$	0.02657	Annualized growth rate of (HP-Smoothed) agricultural labor productivity in US, 1869-2012. See Appendix for detailed sources.
$\{B_{a,t}\}_{t=1869}^{2013}$	$B_{a,t} = g_a^t$	Constant, exogenous sectoral productivity growth.
$g_c - 1$	0.00984	Annualized growth rate of (HP-Smoothed) non-agricultural labor productivity in US, 1869-2012. See Appendix for detailed sources.
$\{B_{c,t}\}_{t=1869}^{2013}$	$B_{c,t} = g_c^t$	Constant, exogenous sectoral productivity growth.
$\{M_t\}_{t=1867}^{2014}$	$\{\cdot\}$	M2 money stock from IMF (2015), Rasche (1990), Friedman and Schwartz (1963) and Carter et al., eds (2006).
β	0.97	Money growth rate and nominal interest rate in 1980.
$\{\tau_t\}_{t=1867}^{2013}$	$\tau_t = \frac{1}{\beta} \frac{M_{t+1}}{M_t}$	Constructed from above.
γ	0.1	Money share in non-agricultural value added, 1980.
α	0.00003626	Employment share in agriculture in 1980.
$\bar{a}$	0.64	Employment share in agriculture in 2012.
δ	0.0493	Ratio of consumption of fixed capital relative to US GDP (1980-2012).
ν	0.23	Caselli and Feyrer (2007)

Table 3: Calibrated parameters

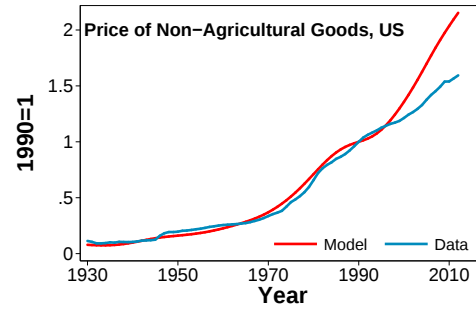
We then use standard numerical methods to solve for the sequences $\tilde{k}_t$ and $L_{c,t}$ that converge to L_c^* and $\tilde{k}^*$ using equations (32) and (33). Given these, we can obtain solutions for K_t and the other non-detrended variables.

US Calibration and Results We follow the same calibration strategy used for the model without capital. Table 3 summarizes the parameters' values.

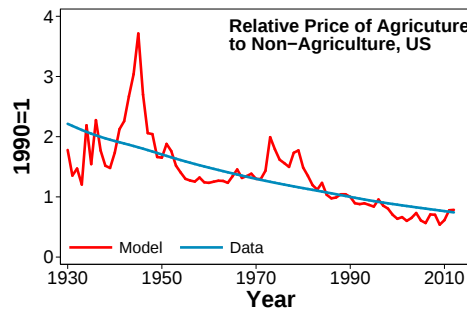
The only additional parameters are δ , ν and initial capital stock. We choose δ to match the average ratio of consumption of fixed capital relative to GDP in the US between 1980 and 2012 from the BEA. We choose a country specific ν directly from Caselli and Feyrer (2007) who provide estimates of reproducible capital shares for 54 countries. They start by taking aggregate capital shares using data from Gollin (2002) and Bernanke and Gurkaynak (2001). They then make use of the World bank's "Where is the Wealth of Nations?: Measuring Capital for the 21st Century" database (WB, 2005) to adjust these shares by excluding non-reproducible capital. We take these capital shares as our ν 's. For countries not included in their data set, we assign the average capital shares value of the 54 countries, which is 0.19. We assume that capital shares are constant in each country over time. Another crucial aspect of this calibration is the choice of initial capital in 1869. We use Piketty (2014)'s data to determine this value, by choosing the initial capital stock in the model to replicate his (reproducible-)capital-output ratio in the US in 1869 (details on our calculation are in Appendix A). Although this is most likely an imprecise measure of capital, it is probably the best we can do given the lack of historical data. Figure 15 illustrates that our model with capital does a good job in fitting the long-run data for US, especially nominal variables. In Figure 16 we can see that our model predicts money shares trends quite accurately. We slightly overpredict money shares in the earlier years, however we perfectly catch the trend. This is quite



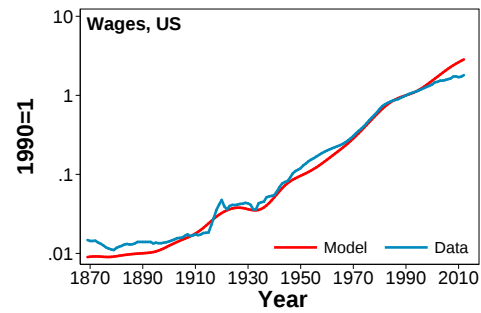
(a) Index of agricultural prices, US



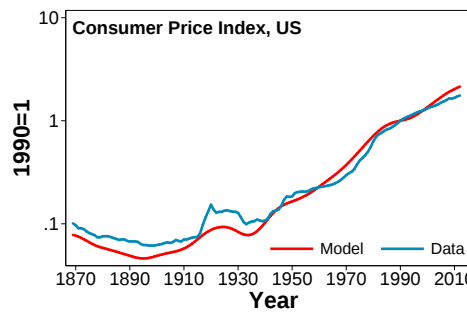
(b) Index of non-agricultural prices, US



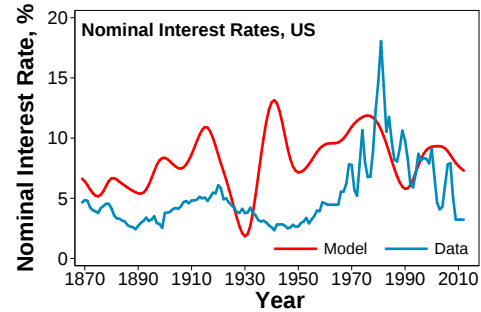
(c) Relative prices agriculture to non-agriculture, US



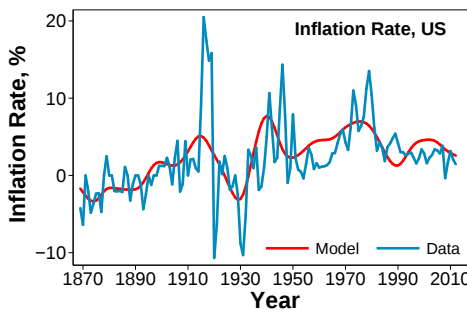
(d) Index of wages, US



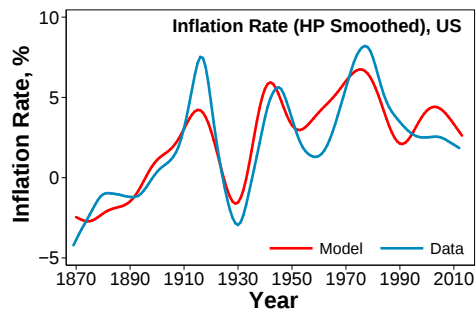
(e) Consumer Price Index, US



(f) Nominal interest rates, US



(g) Inflation Rate, US (Unsmoothed)



(h) Inflation Rate, US (Smoothed)

Figure 15: Simulations and data for US prices, 1869-2012.

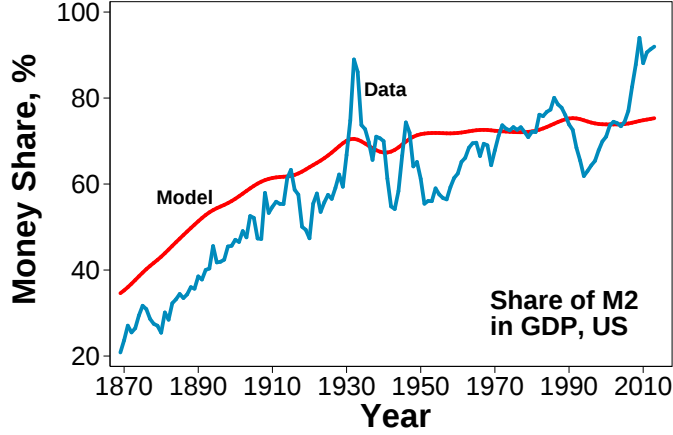


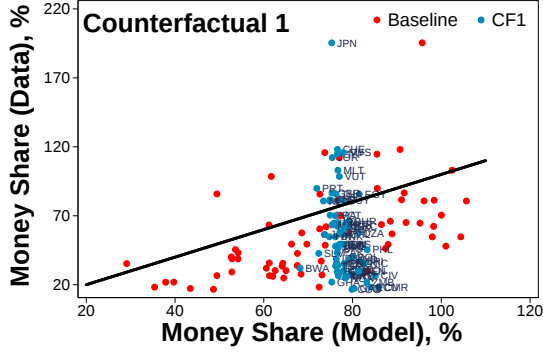
Figure 16: Money (M2) Share in current price GDP.

remarkable also in light of caveats regarding the measure of capital used.

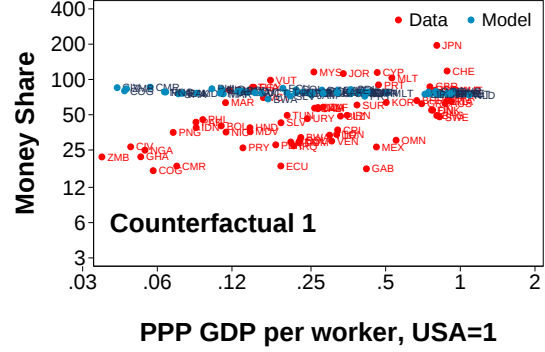
Counterfactuals In order to disentangle the mechanisms at work, we perform a different set of counterfactuals. In the same spirit of section 6, we assume that countries have the same structural parameters of the US economy. For more details on how we construct the international series for sectoral productivities, capital and monetary aggregates, see Appendix A.

Counterfactual 1 The aim of the first counterfactual is to isolate the effect of different sectoral growth rates across countries. We assume that each country has the same monetary policy as the US. We then choose $B_a^i = B_a^{US}$ for each country and choose B_c^i to match the ratio of GDP per worker in country i to that of the US in 1980. Furthermore, we choose a country's initial capital-output ratio in 1980 to its balanced growth path level. Countries thus vary only in their country-specific, sectoral productivity growth rates. Given that agricultural productivity is very high, the agricultural sector is small, and therefore this experiment is akin to having a one-sector model where all countries have balanced-growth-path capital levels and the same monetary policy, while varying only in sectoral growth rates. Figure 17(a) compares the results of our model in the counterfactual 1 with the data, while Figure 17(b) shows money shares predicted by the model as a function of the GDP per worker. As expected, and analogously to the model without capital, predicted money shares are about constant.

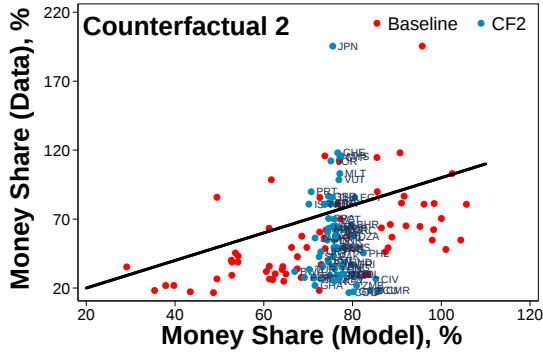
Counterfactual 2 In this counterfactual, we make the same assumptions as in the counterfactual 1, but we let countries have their own monetary policy. We therefore aim to see if differences in monetary policy can help explaining the variation in money shares across countries. In other words, in this counterfactual, our model is akin to a one-sector model in which all countries start from balanced-growth-path capital level, and they differ in sectoral growth rates and money growth



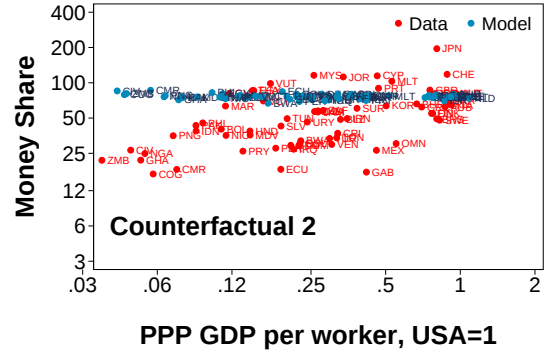
(a) Counterfactual 1 (US agr. productivity and monetary policy, steady state capital level): Money shares in data and model.



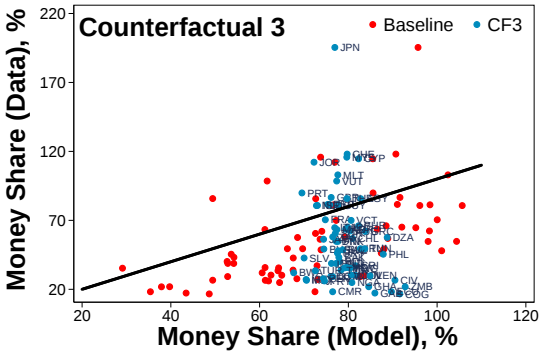
(b) Counterfactual 1 (US agr. productivity and monetary policy, steady state capital level): Money shares versus GDP per capita.



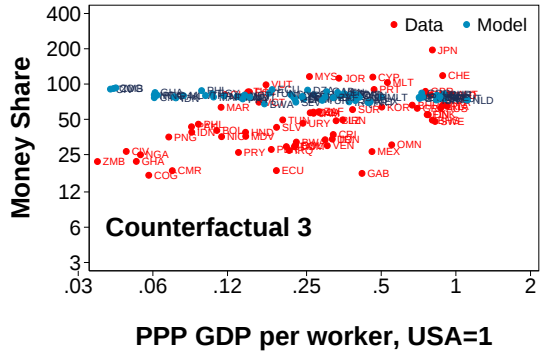
(c) Counterfactual 2 (US agr. productivity, steady state capital level): Money shares in data and model.



(d) Counterfactual 2 (US agr. productivity, steady state capital level): Money shares versus GDP per capita.

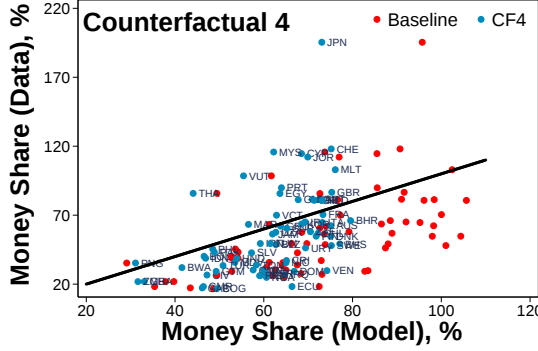


(e) Counterfactual 3 (US agr. productivity, country-specific capital level): Money shares in data and model.

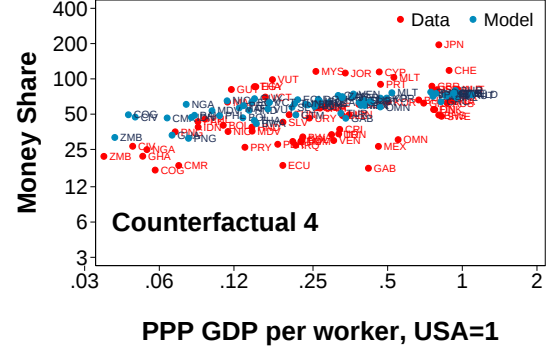


(f) Counterfactual 3 (US agr. productivity, country-specific capital level): Money shares versus GDP per capita.

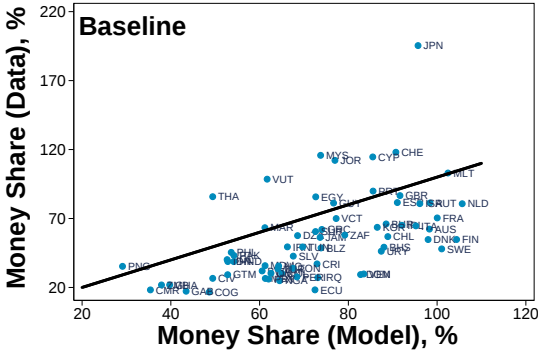
Figure 17: Money shares in the the baseline model and the counterfactuals (average for 1980-2010). Drawn versus corresponding data and (log of) GDP per worker.



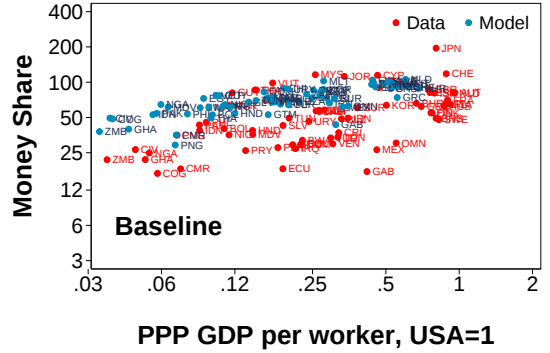
(a) Counterfactual 4 (Country-specific agr. productivity, steady state capital level): Money shares in data and model.



(b) Counterfactual 4 (Country-specific agr. productivity, steady state capital level): Money shares versus GDP per capita.



(c) Baseline Model: Money shares in data and model.



(d) Baseline Model: Money shares versus GDP per capita.

Figure 18: Money shares in the the baseline model and the counterfactuals (average for 1980-2010). Drawn versus corresponding data and (log of) GDP per worker.

rates. From Figures 17(c) and 17(d) it is evident that predicted money shares remain approximately constant. Hence, differences in monetary policies have only a small effect on money shares.

Counterfactual 3 In this exercise, we test if variation in initial capital endowments drives cross-country differences in money shares. The calibration is therefore the same as for counterfactual 2, but we assume country-specific initial levels of capital, taken directly from the data. Therefore, our exercise is akin to a one-sector model in which countries differ in sectoral growth rates, money growth rates and initial capital endowments. In this case, Figures 17(e) and 17(f) show that the model helps in explaining differences for rich countries, but it does a poor job for less developed countries.

Counterfactual 4 In our last counterfactual, we aim to isolate the effect of the two-sector structure of the economy. We therefore modify counterfactual 2, by re-calibrating B_c^i to match the ratio

(a) Standard Deviation of Money Shares				(b) Money-Share Income Semi-Elasticities			
	Standard Deviation	Relative to Baseline	Relative to Data		Money-share Semi-elasticity	Relative to Baseline	Relative to Data
Data	0.314	-	1.00	Data	0.168	-	1.00
Baseline	0.186	1.00	0.59	Baseline	0.182	1.00	1.08
CF1	0.031	0.17	0.10	CF1	-0.013	-0.07	-0.08
CF2	0.032	0.17	0.10	CF2	-0.011	-0.06	-0.07
CF3	0.052	0.28	0.16	CF3	-0.022	-0.12	-0.13
CF4	0.117	0.63	0.37	CF4	0.106	0.58	0.63

Table 4: Decomposition of results with capital.

of GDP per worker in each country relative to the US and by calibrating B_a^i for each country to match agricultural labor shares in 1980. Therefore, this counterfactual is akin to the baseline two-sector model, but with all countries starting with their balanced-growth-path level of capital. From Figures 18(a) and 18(b) we observe that the two sector structure does a very good job, especially for the developing countries, while it has a more difficult time in matching money-shares in richer countries. However, the differences with the baseline model (in Figures 18(c)-18(d)) are relatively small.

Baseline and summary Our fully-fledged model with capital is strikingly good at predicting money shares across countries and the money share-income relationship. The crucial lesson is that variation in agricultural productivity levels is still the dominant driver, although of course capital accumulation now also plays a role. Table 4(a) decomposes the money share's standard deviation into the contribution of each counterfactual. In the data, the standard deviation is 0.314, while it is 0.186 in our baseline model. The baseline thus captures 59% of the standard deviation of the data. The second column of 4(a) then shows the contribution of each counterfactual to the baseline model's results. By comparing counterfactuals 3 and 4 with counterfactual 2, we can observe the role played by capital variation and the two-sector framework respectively. Adding capital heterogeneity helped explain an additional 11(=28-17) percentage points of the deviation, whilst adding the two-sector framework helped explain an additional 46(-63-17) percentage points of the deviation. Thus, the two sector framework is crucial to obtaining the baseline result. Finally, Table 4(b) shows that, in the data, the money share-income (semi-)elasticity is approximately 0.168, whilst the baseline model predicts a slightly higher 0.182. Again, it is the two-sector framework - rather than variation in capital stock - that overwhelmingly contributes to the success of the model along this dimension.

References

- Alvarez-Cuadrado, Francisco and Markus Poschke**, “Structural Change Out of Agriculture: Labor Push versus Labor Pull,” *American Economic Journal: Macroeconomics*, 2011, 3 (3), 127–58.
- Anderson, Richard G.**, “Monetary Aggregates (CJ),” in Susan B. Carter, Scott Sigmund Gartner, Michael R. Haines, Alan L. Olmstead, Richard Sutch, and Gavin Wright, eds., *Historical Statistics of the United States, Earliest Times to the Present: Millennial Edition*, Cambridge University Press, 2006.
- Bernanke, B.S. and R. Gurdikaynak**, “Is Growth Exogenous? Taking Mankiw, Romer and Weil Seriously,” *NBER Macroeconomic Annual*, 2001.
- Blades, Derek W.**, “Subsistence activities in the national accounts of developing countries with special reference to Latin America,” *Review of Income and Wealth*, 1975, 21 (4), 391–410.
- Bordo, Michael D. and Anna J. Schwartz**, “Money and Prices in the Nineteenth Century: An Old Debate Rejoined,” *The Journal of Economics History*, March 1980, 40 (1), 61–67.
- and – , “Money and Prices in the 19th Century: Was Thomas Tooke Right?,” *Explorations in Economic History*, 1981, 18, 97–121.
- and **Lars Jonung**, “The Long Run Behavior of the Income Velocity of Money in Five Advanced Countries: 1870-1975: An Institutional Approach,” *Economic inquiry*, Jan 01 1981, 19 (1), 96. Last updated - 2013-02-23.
- and – , *The Long-Run Behavior of the Velocity of Circulation: The International Evidence*, Cambridge University Press, 1987.
- and – , “The long-run behavior of velocity: The institutional approach revisited,” *Journal of Policy Modeling*, 1990, 12 (2), 165–197.
- , – , and **Pierre Siklos**, “The Common Development of Institutional Change as Measured by Income Velocity: A Century of Evidence from Industrialized Countries,” *NBER Working Paper No. 4379*, 1993.
- Buera, Francisco J. and Joseph P. Kaboski**, “Scale and the Origins of Structural Change,” *Journal of Economic Theory*, March 2012, 147 (2), 684–712.
- Carter, Susan B., Scott Sigmund Gartner, Michael R. Haines, Alan L. Olmstead, Richard Sutch, and Gavin Wright**, eds, *Historical Statistics of the United States, Earliest Times to the Present: Millennial Edition*, Cambridge University Press, 2006.

- Caselli, Francesco**, “Accounting for Cross-country income Differences,” in Philippe Aghion and Steven N. Durlauf, eds., *Handbook of Economic Growth, Volume 1A.*, Elsevier B.V., 2005, pp. 680–738.
- **and James Feyrer**, “The Marginal Product of Capital,” *Quarterly Journal of Economics*, 2007, 122 (2).
- Chandavarkar, Anand G.**, “Monetization of Developing Economies,” *Staff Papers (International Monetary Fund)*, 1977, 24 (3), 665–721.
- Chari, V.V., Larry E. Jones, and Rodolfo E. Manuelli**, “Inflation, growth, and financial intermediation,” *Proceedings*, 1996, (May), 41–58.
- **, Lawrence J. Christiano, and Patrick J. Kehoe**, “Optimality of the Friedman rule in economies with distorting taxes,” *Journal of Monetary Economics*, 1996, 37, 203–223.
- Cole, Harold L. and Narayana Kocherlakota**, “Zero Nominal Interest Rates: Why Theyre Good and How to Get Them,” *Federal Reserve Bank of Minneapolis Quarterly Review*, 1998, 22(2), 210.
- Dorich, Jose**, “Resurrecting the Role of Real Money Balance Effects,” Staff Working Papers 09-24, Bank of Canada 2009.
- Duarte, M. and D. Restuccia**, “The Role of the Structural Transformation in Aggregate Productivity,” *The Quarterly Journal of Economics*, 2010, 125(1), 129–173.
- Echevarria, C.**, “Changes in Sectoral Composition Associated with Economic Growth,” *International Economic Review*, 1997, 38.
- FAOSTAT**, *Food and Agriculture Organization of the United Nations Database* 2012.
- Friedman, Milton**, “The Demand for Money: Some Theoretical and Empirical Results,” *The American Economic Review, Papers and Proceedings*, 1959, Vol. 49, 525–527.
- **and Anna J. Schwartz**, *A Monetary History of the United States, 1867-1960*, Princeton University Press, 1963.
- Gollin, Douglas**, “Getting Income Shares Right,” *Journal of Political Economy*, April 2002, 110 (2), 458–474.
- **, Stephen Parente, and Richard Rogerson**, “The Role of Agriculture in Development,” *American Economic Review*, 2002, 92(2), 160–164.
- Goodfriend, Marvin**, “Money, Credit, Banking, and Payments System Policy,” *Federal Reserve Bank of Richmond Economic Review*, January/February 1991, 77, 7–23.

- Herrendorf, Berthold, Richard Rogerson, and Akos Valentinyi**, “Growth and Structural Transformation,” in Philippe Aghion and Steven N. Durlauf, eds., *Handbook of Economic Growth*, North Holland, 2014, chapter 6, p. 855.
- Heston, Alan, Robert Summers, and Bettina Aten**, *Penn World Table Version 7.1* Center for International Comparisons of Production, Income and Prices at the University of Pennsylvania 2012.
- IMF**, “IMF International Financial Statistics, 1948-2015,” UK Data Service 2015.
- Ireland, P.**, “Money and Growth: An Alternative Approach,” *American Economic Review*, March 1994, *84* (1), 47–65.
- Ireland, Peter N.**, “Financial Evolution and the Long-Run Behavior of Velocity: New Evidence from U.S. Regional Data,” *FRB Richmond Economic Review*, November/December 1991, *77* (6), 16–26.
- Jones, Charles I.**, “Pareto and Piketty: The Macroeconomics of Top Income and Wealth Inequality,” *Journal of Economic Perspectives*, 2015, *29* (1), 29–46.
- Jonung, L.**, “Monetization and the Behavior of Velocity in Sweden, 1871-1913,” *Explorations in Economic History*, 1983, *20*, 418–439.
- Kuralbayeva, Karlygash and Radoslaw Stefanski**, “Windfalls, Structural Transformation and Specialization,” *Journal of International Economics*, 2013, *90*, 273–301.
- Landes, David S.**, *The Unbound Prometheus: Technological Change and Industrial Development in Western Europe from 1750 to the Present*, Cambridge University Press, 1969.
- Lewis, W. A.**, *Growth and Fluctuations 1870-1913.*, George Allen and Unwin, 1978.
- Maddison, A.**, *Phases of Capitalist Development*, Oxford: Oxford University Press, 1982.
- Maddison, Angus**, “Historical Statistics for the World Economy: 1-2003 AD,” 2007.
- McCallum, Bennett T. and Edward Nelson**, “Chapter 3 - Money and Inflation: Some Critical Issues?,” in Benjamin M. Friedman and Michael Woodford, eds., *Handbook of Monetary Economics*, Vol. 3 of *Handbook of Monetary Economics*, Elsevier, 2010, pp. 97 – 153.
- Officer, Lawrence H. and Samuel H. Williamson**, “The Annual Consumer Price Index for the United States, 1774-2014,” MeasuringWorth 2016.
- **and** —, “Annual Wages in the United States, 1774-Present,” <http://www.measuringworth.com/uswages/> 2016.

- Ostroy, Joseph M. and Ross M. Starr**, “Chapter 1 The transactions role of money,” in “Handbook of Monetary Economics,” Vol. 1, Elsevier, 1990, pp. 3 – 62.
- Piketty, Thomas**, *Capital in the Twenty-first Century*, Harvard University Press, 2014.
- Rasche, Robert H.**, “Demand Functions for Measures of U. S. Money and Debt,” *Financial Sectors in Open Economies: Empirical Analysis and Policy Issues*, 1990.
- Restuccia, Diego, Dennis Tao Yang, and Xiaodong Zhu**, “Agriculture and Aggregate Productivity: A Quantitative Cross-Country Analysis,” *Journal of Monetary Economics*, 2008, 55(2), 234–250.
- Reti, Steven P.**, *Silver and gold : the political economy of International Monetary Conferences, 1867-1892*, Greenwood Press, 1998.
- Rostow, W. W.**, *British Economy of the Nineteenth Century*, Calrendon Press, 1948.
- , *The World Economy: History and Prospects*, University of Texas Press, 1978.
- Solow, Robert M.**, “A Contribution to the Theory of Economic Growth,” *The Quarterly Journal of Economics*, 1956, 70, 65–94.
- Stefanski, Radoslaw**, “Dirty Little Secrets: Inferring Fossil-Fuel Subsidies from Patterns in Emission Intensities,” *OxCarre Working Papers*, 2014, 134.
- , “Structural Transformation and the Oil Price,” *Review of Economic Dynamics*, 2014, 17 (3), 458–504.
- Timmer, M.P., G.J. de Vries, and K. de Vries**, “Patterns of Structural Change in Developing Countries,” *GGDC research memorandum 149*, 2014.
- Tooke, T. and W. Newmarch**, *A History of Prices*, Vol. 6, Longman, Brown, Green, 1857.
- Townsend, Robert M.**, “Economic Organization with Limited Communication,” *American Economic Review*, December 1987, 77, 954–970.
- UN**, *National Accounts Statistics database* 2008.
- Urquhart, Michael**, “The employment shift to services: Where did it come from?,” *Monthly Labor Review*, 1984, 107, 15–22.
- Warburton, Clarck**, “The Secular Trend in Monetary Ve,” *Quarterly Journal of Economics*, February 1949, 63.
- WB**, *Where Is the Wealth of Nations?*, The World Bank, 2005. doi:10.1596/978-0-8213-6354-6.
- WDI**, “World Development Indicators,” 2016.

Weisbrot, Mark, Rebecca Ray, Jake Johnston, Jose Antonio Cordero, and Juan Antonio Montecino, “IMF-Supported Macroeconomic Policies and the World Recession: A Look at Forty-One Borrowing Countries,” *CEPR Reports*, October 2009.

Wells, David A., *Recent Economic Changes and Their Effect on Production and Distribution of Wealth and Well-Being of Society*, D. Appleton and Co., 1891.

Wicksell, Knut, *Interest And Prices*, Macmillan And Company Limited, 1936.

Yang, Dennis Tao and Xiaodong Zhu, “Modernization of Agriculture and Long Term Growth,” *Journal of Monetary Economics*, 2013, 60 (3).