Earnings Dynamics and Returns to Skills
(Preliminary draft. Please do not cite.)

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September 13, 2016

Abstract

In this paper, we study the role of returns to unobserved skills in the rising residual earnings inequality for the past few decades in the U.S. We identify and estimate a general model of earnings residuals that incorporates (i) changing returns to unobserved skills, (ii) changing distribution of unobserved skills, and (iii) changing volatility of earnings that is not related to skills. Using data from the PSID, we find that the returns to unobserved skills went down since the mid-1980s despite the steady increase in the residual inequality. Using a simple demand and supply framework, we show that both demand and supply factors contributed to the downward trend in the returns to skills over time.

1 Introduction

In the past few decades, the U.S. has experienced a sharp surge in labor income dispersion. Earnings differentials across workers with different observed characteristics such as education and age/experience all rose substantially, but residual earnings inequality (i.e. inequality conditional on observable characteristics) increased simultaneously. Although there is widespread agreement that the returns to observable skills have increased dramatically (e.g., Katz and Autor, 1999), there are different views about the factors underlying changes in residual earnings inequality. Changes in residual inequality have often been equated with changes in returns to unobserved skills (Juhn, Murphy, and Pierce, 1993) or changes in the distribution of unobserved skills (Lemieux, 2006) in the literature. However, a sizeable portion of the rise in residual inequality is attributable to short-term fluctuations in earnings (Gottschalk and Moffitt, 1994), which are unlikely to reflect changes related to skills. Distinguishing between these explanations is important for understanding the causes and welfare consequences of rising earnings inequality.

In this paper, we consider a general model of log earnings residuals that incorporates (i) changing returns to unobserved skills, (ii) changing distribution of unobserved skills, and (iii) changing volatility of earnings unrelated to skills. We begin by discussing identification when unobserved skills are subject to permanent shocks and the volatile (non-skill) component of earnings is subject to permanent and transitory shocks as in much of the literature on earnings dynamics. We show that, when the panel data cover long periods, the returns to unobserved skills for all periods are identified and the variances of unobserved skills as well as the volatile component of earnings are identified except for the last few periods. Intuitively, identification of the returns to unobserved skills derives from the fact that correlations in earnings residual changes far enough apart in time are due to unobserved skills and not permanent or transitory shocks unrelated to skills.

We estimate the returns to unobserved skills, the variances of unobserved skills, and the variances of the non-skill component of earnings using data on log annual/weekly earnings for men ages 30-59 in the Panel Study of Income Dynamics (PSID) separately by college attendance. We highlight five main findings. (i) The returns to unobserved skills were stable until the mid-1980s, and then fell afterwards. The decline of the return was more substantial for those who did not attend college ('high school workers'). (ii) The variance of unobserved skill rose throughout the sample period. (iii) The variance of the unobserved skill component (i.e., return to skill multiplied by unobserved skill) rose until the mid-1980s, decreased afterwards, and then rebounded slightly in the early 2000s only for those who attended college ('college workers'). (iv) The variance of permanent shocks increased since the mid-1980s. (v) The variance of transitory shocks increased sharply in the early 1980s among high school workers but increased only modestly in the long-run for both high school and college workers.

Our results demonstrate that accounting for changes in the distribution of unobserved skills and in the volatility of earnings is important in estimating the returns to unobserved skills. Time patterns of the returns to unobserved skills are quite different from those estimated in the existing studies (e.g., Juhn, Murphy, and Pierce, 1993; Moffitt and Gottschalk, 2012) that do not take into account these factors. The evolution of the variance of log earnings residuals after the mid-1980s cannot be easily explained by unobserved skills alone, as the decline in the returns to unobserved skills pushed down the share of skill-related component in total residual variance.

We interpret our empirical findings using a simple demand and supply framework and ask whether the changes in the returns to unobserved skills reflect changes in skill demand or supply. We present an assignment model of labor market that is similar to Sattinger (1979) where the returns to skills are determined by production technology and how workers with heterogeneous skills and jobs with heterogeneous productivity are combined in production. More skilled workers earn more partly because they work at more productive jobs. Skill distribution affects the returns to skills by changing the equilibrium assignment. An increase in the variance of skill reduces the returns to skills by reducing the productivity differential across jobs among workers different skill levels. We recover the demand and supply factors by combining our estimates—returns to skills and variance of skills—with the restrictions implied by the theoretical model. We find that both demand and supply factors played important roles in the decline of the returns to unobserved skills since the mid-1980s for both high school and college workers. The decrease in skill demand was a more important factor than supply changes for high school workers.

This paper proceeds as follows. In Section 2, we provide identification results for our model where the returns to unobserved skills, the variance of unobserved skills, and the variance of non-skill component of earnings change over time. Section 3 describes the PSID data used to estimate earnings dynamics for American men and reports our empirical findings. Section 4 interprets the time patterns of the estimated returns to unobserved skills in a demand and supply framework and Section 5 concludes.

2 Identifying the Returns to Unobserved Skills

In this section, we establish conditions under which the returns to unobserved skills, the variances of unobserved skills, and the variance of volatile component of earnings are identified.

Suppose that we observe log earnings residuals of a large number of individuals $i = 1, \ldots, N$ from period $t = \underline{t}$ to period $t = \bar{t}$. We consider log earnings residuals for individual i in period t of the form

$$w_{i,t} = \mu_t \theta_{i,t} + u_{i,t}, \tag{1}$$

where $\theta_{i,t}$ represents an unobserved skill, μ_t a return to unobserved skills, and $u_{i,t}$ idiosyncratic shocks. We assume that $\theta_{i,t}$ and $u_{i,t}$ are mean zero and uncorrelated with each other.

The earnings equation (1) reflects the classical idea of Friedman and Kuznets (1954) that one's earnings consists of a permanent component that is related to one's intrinsic skills and a transitory component that reflects a short-run variation in earnings that is not related to skills. Although the transitory component can be serially correlated, the correlation between transitory components that are far apart in time is likely to be small. Therefore, we begin with the assumption that there exists $k > 0$ such that $\mathrm{Cov}(u_t, u_{t'}) = 0$ for $|t' - t| \geq k$, which will be generalized later.[1] Then the "long" autocovariance of earnings residuals reflects only the skill-related component (Carroll, 1992; Moffitt and Gottschalk, 2012): for $|t' - t| \geq k$,

$$\mathrm{Cov}(w_t, w_{t'}) = \mu_t \mu_{t'} \mathrm{Cov}(\theta_t, \theta_{t'}). \tag{2}$$

Time-invariant Unobserved Skills When unobserved skills do not change over time, i.e., $\theta_{i,t} = \theta_i$, then $\mathrm{Cov}(\theta_t, \theta_{t'}) = \mathrm{Var}(\theta)$. In this case, the ratio of the covariances reveals the ratio of the returns to skills: for $|t' - t| > k$,

$$\frac{\mathrm{Cov}(w_t, w_{t'})}{\mathrm{Cov}(w_{t-1}, w_{t'})} = \frac{\mu_t \mu_{t'} \mathrm{Var}(\theta)}{\mu_{t-1} \mu_{t'} \mathrm{Var}(\theta)} = \frac{\mu_t}{\mu_{t-1}}. \tag{3}$$

Therefore, if we observe individual earnings for long periods, the returns to skills are identified up to a normalization that $\mu_{t^*} = 1$ for some t^*. Once the returns are identified, we can also recover the variance of unobserved skills $\mathrm{Var}(\theta) = \mathrm{Cov}(w_t, w_{t'})/(\mu_t \mu_{t'})$ as well as the variance of earnings shocks $\mathrm{Var}(u_t) = \mathrm{Var}(w_t) - \mathrm{Var}(\mu_t \theta)$.

Equation (3) can be interpreted in the framework of instrumental variable regression. We can obtain μ_t / μ_{t-1} by regressing $w_{i,t}$ on $w_{i,t-1}$ using $w_{i,t'}$ as an instrumental variable. To see why, first note that $w_{i,t}$ and $w_{i,t-1}$ are related as follows:

$$w_{i,t} = \mu_t \theta_i + u_{i,t} = \mu_t \left(\frac{w_{i,t-1} - u_{i,t-1}}{\mu_{t-1}} \right) + u_{i,t}. \tag{4}$$

Since $w_{i,t-1} = \mu_{t-1} \theta_i + u_{i,t-1}$ is a noisy measure of unobserved skill θ_i, it is correlated with the measurement error $u_{i,t-1}$ (and also correlated with $u_{i,t}$ if $u_{i,t}$ and $u_{i,t-1}$ are correlated) and running an OLS regression of $w_{i,t}$ on $w_{i,t-1}$

[1] Let x_t be a random variable and its realization for individual i be $x_{i,t}$. Denote its cross-sectional second moments by $\mathrm{Var}(x_t)$ and $\mathrm{Cov}(x_t, x_{t'})$.

would give a biased estimate of μ_t/μ_{t-1}. To address this endogeneity problem, an earnings residual of distant future or past $w_{i,t'}$ can be used as an instrumental variable since it is correlated with θ_i but uncorrelated with $u_{i,t-1}$ and $u_{i,t}$.

Time-varying Unobserved Skills In more general cases where unobserved skills change over time, let $\nu_{i,t} = \Delta\theta_{i,t} = \theta_{i,t} - \theta_{i,t-1}$ be the skill change between period $t-1$ and period t. Then Equation (4) becomes

$$w_{i,t} = \mu_t \left(\frac{w_{i,t-1} - u_{i,t-1}}{\mu_{t-1}} + \nu_{i,t} \right) + u_{i,t}.$$

As long as the skill changes are uncorrelated with past skills,[2] past earnings residuals are still valid instruments and μ_t for $t > \underline{t} + k$ is identified by Equation (3). However, future earnings residuals are not valid if the skill change has permanent effects on future skills. To see this, note that, for $t' > t$, $\theta_{t'} = \theta_t + \sum_{\tau=t+1}^{t'} \nu_\tau$ and $\mathrm{Cov}(\theta_t, \theta_{t'}) = \mathrm{Var}(\theta_t)$. Then, for $t' \geq t + k$,

$$\frac{\mathrm{Cov}(w_t, w_{t'})}{\mathrm{Cov}(w_{t-1}, w_{t'})} = \frac{\mu_t \mu_{t'} \mathrm{Var}(\theta_t)}{\mu_{t-1} \mu_{t'} \mathrm{Var}(\theta_{t-1})} = \frac{\mu_t}{\mu_{t-1}} \frac{\mathrm{Var}(\theta_{t-1}) + \mathrm{Var}(\nu_t)}{\mathrm{Var}(\theta_{t-1})}, \tag{5}$$

which is larger than μ_t/μ_{t-1} unless $\mathrm{Var}(\nu_t) = 0$. This poses an identification problem for μ_t in early periods where there are no past earnings to be used as instrumental variables. However, they can still be recovered by differencing out the bias generated by permanent skill changes. Let c_i be the period when individual i starts accumulating skills ('cohort') and suppose that there exist two cohorts c and c' such that $\mathrm{Var}(\theta_{t-1}|c) \neq \mathrm{Var}(\theta_{t-1}|c')$ and $\mathrm{Var}(\nu_t|c) = \mathrm{Var}(\nu_t|c')$. The second condition holds when skill shock variance depends only on time, or when there is a non-monotonic age trend in the variance of skill change.[3] In this case, the ratio of the difference of the long autocovariance between cohorts gives the ratio of the returns to skills for early periods: for $t' - t \geq k$,

$$\frac{\mathrm{Cov}(w_t, w_{t'}|c) - \mathrm{Cov}(w_t, w_{t'}|c')}{\mathrm{Cov}(w_{t-1}, w_{t'}|c) - \mathrm{Cov}(w_{t-1}, w_{t'}|c')} = \frac{\mu_t \mu_{t'} \left[\mathrm{Var}(\theta_{t-1}|c) - \mathrm{Var}(\theta_{t-1}|c') \right]}{\mu_{t-1} \mu_{t'} \left[\mathrm{Var}(\theta_{t-1}|c) - \mathrm{Var}(\theta_{t-1}|c') \right]} = \frac{\mu_t}{\mu_{t-1}}.$$

Then the variances of unobserved skills for all $t \leq \bar{t} - k$ are also identified from the covariance between current and future earnings residuals $\mathrm{Var}(\theta_t) = \mathrm{Cov}(w_t, w_{t'})/(\mu_t \mu_{t'})$ for $t' - t \geq k$. Although the return to unobserved skill is identified for all periods, the variance of unobserved skills is not identified for later periods near $\bar{t}$, because we cannot distinguish between unobserved skills and earnings shocks without future earnings residuals.

Permanent Non-skill Component of Earnings We have assumed that all permanent components of earnings are related to skills. However, there may be factors that are orthogonal to skills but also have permanent effects on earnings. For example, workers with identical skills may get paid differently because of the differences in non-pecuniary characteristics of jobs (Rosen, 1986). These differences are likely to be permanent if they reflect workers' preferences over jobs. Also permanent shocks to firm productivity can be translated into permanent earnings shocks

[2]This is the standard weak exogeneity assumption in dynamic models.

[3]For example, young (old) workers may experience larger skill changes than middle age workers due to training (health shocks). Baker and Solon (2003) and Blundell, Graber, and Mogstad (2015) find this U-shaped age profile in the variance of earnings shocks.

among similarly skilled workers when there are labor market frictions that prevent full insurance against firm level shocks (Guiso, Pistaferri, and Schivardi, 2005; Lamadon, 2014).

In this case, a similar identification strategy works under the modified assumption that $\mathrm{Cov}(u_t, \Delta u_{t'}|c) = 0$ for all (c,t,t') such that $t'-t \geq k$. With this assumption, the change in distant future earnings residual $\Delta w_{i,t'}$, instead of level $w_{i,t'}$, can be used as an instrumental variable in a regression of $w_{i,t}$ on $w_{i,t-1}$ unless the return to unobserved skill stays constant (i.e., $\Delta\mu_{t'} = 0$). To see this, note that $\Delta w_{i,t'}$ can be written as

$$\Delta w_{i,t'} = \Delta\mu_{t'}\theta_{i,t'} + \mu_{t'-1}\Delta\theta_{i,t'} + \Delta u_{i,t'}. \tag{6}$$

For $t' \geq t-k$, $\Delta w_{i,t'}$ is uncorrelated with $u_{i,t-1}$ and $u_{i,t}$, but it is correlated with $w_{i,t-1}$ through $\theta_{i,t'}$ if $\Delta\mu_{t'} \neq 0$. Therefore, the availability of an instrumental variable hinges on some variation in the returns to unobserved skills over time. In the extreme case where μ_t does not change at all, it is impossible to separately identify the permanent skill component and non-skill component. To simplify exposition, we impose the strong sufficient condition that $\Delta\mu_t \neq 0$ for all periods.

We summarize the above discussion in the following Proposition. The proofs for this and subsequent results are provided in Appendix A.

Proposition 1. *Suppose that (i) there exists $k > 0$ such that $\bar{t} - \underline{t} > 2k$ and $\mathrm{Cov}(u_t, \Delta u_{t'}|c) = 0$ for all (c,t,t') such that $t'-t \geq k$, (ii) $\Delta\mu_t \neq 0$ for all t, and (iii) $\theta_{i,t} = \theta_{i,t-1} + \nu_{i,t}$ where, for all t, $\mathrm{Cov}(\nu_t, \theta_{t-j}|c) = 0$ for all c and $j > 0$ and $\mathrm{Var}(\theta_{t-1}|c) \neq \mathrm{Var}(\theta_{t-1}|c')$ and $\mathrm{Var}(\nu_t|c) = \mathrm{Var}(\nu_t|c')$ for some $c \neq c'$. Then (i) μ_t is identified for all t, up to a normalization $\mu_{t^*} = 1$ for some t^*, and (ii) $\mathrm{Var}(\theta_t)$ and $\mathrm{Var}(u_t)$ are identified for all $t < \bar{t} - k$.*

3 Estimation

We estimate the returns to unobserved skills, the variance of unobserved skills, and the variance of earnings shocks using panel data from the Panel Study of Income Dynamics (PSID) on male annual/weekly earnings in the U.S. from 1970 to 2008.

3.1 Empirical Specification

Since the relative wages between those who did not attend college and those that did have diverged significantly during the sample periods (Katz and Murphy, 1992), we allow all parameters including the returns to unobserved skills to differ between the two groups, which we call 'sector'. Specifically, let s_i be an indicator variable for college workers for individual i. Then, the log earnings residual for individual i in year t is

$$w_{i,t} = \mu_t(s_i)\theta_{i,t} + u_{i,t}, \tag{7}$$

where $\mu_t(s)$ is the return to unobserved skills for an individual with $s_i = s$ and, as before, unobserved skill evolves according to

$$\theta_{i,t} = \theta_{i,t-1} + \nu_{i,t}. \tag{8}$$

We further assume that the non-skill earnings shock $u_{i,t}$ is the sum of a unit root process ('permanent component') and a lower order moving average process ('transitory component') as typically assumed in the literature:[4]

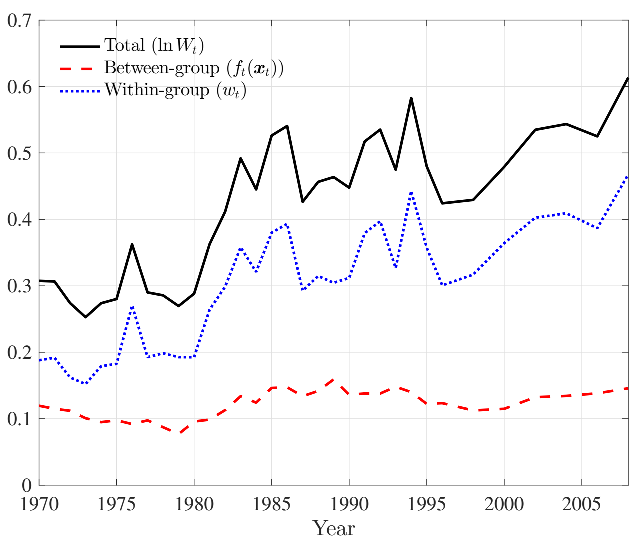
$$u_{i,t} = \kappa_{i,t} + \varepsilon_{i,t}, \tag{9}$$

$$\kappa_{i,t} = \kappa_{i,t-1} + \eta_{i,t}, \tag{10}$$

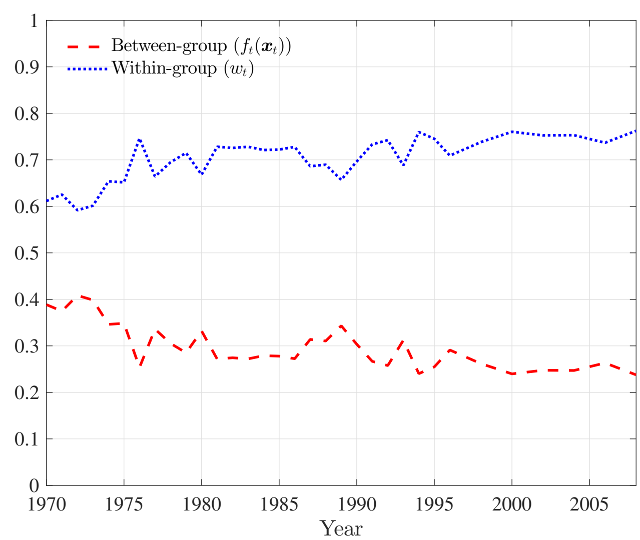
$$\varepsilon_{i,t} = \sum_{j=0}^{q} \beta_j(s_i)\xi_{i,t-j}, \tag{11}$$

where $\beta_0(s) = 1$. With this specification, the identifying assumption $\mathrm{Cov}(u_t, \Delta u_{t'}|s,c) = 0$ in Proposition 1 holds for $k = q+2$, so the returns to unobserved skills for all periods and the variances of unobserved skills for $t < \bar{t} - k = \bar{t} - q - 2$ are identified. Variances of permanent and transitory non-skill components for $t < \bar{t} - q - 2$ are also identified as follows: $\mathrm{Var}(\kappa_t|s) = \mathrm{Cov}(w_t, w_{t'}|s) - \mu_t(s)\mu_{t'}(s)\,\mathrm{Var}(\theta_t|s)$ for $t' \geq t+q+1$ and $\mathrm{Var}(\varepsilon_t|s) = \mathrm{Var}(w_t|s) - \mu_t(s)^2\,\mathrm{Var}(\theta_t|s) - \mathrm{Var}(\kappa_t|s)$.

3.2 PSID Data



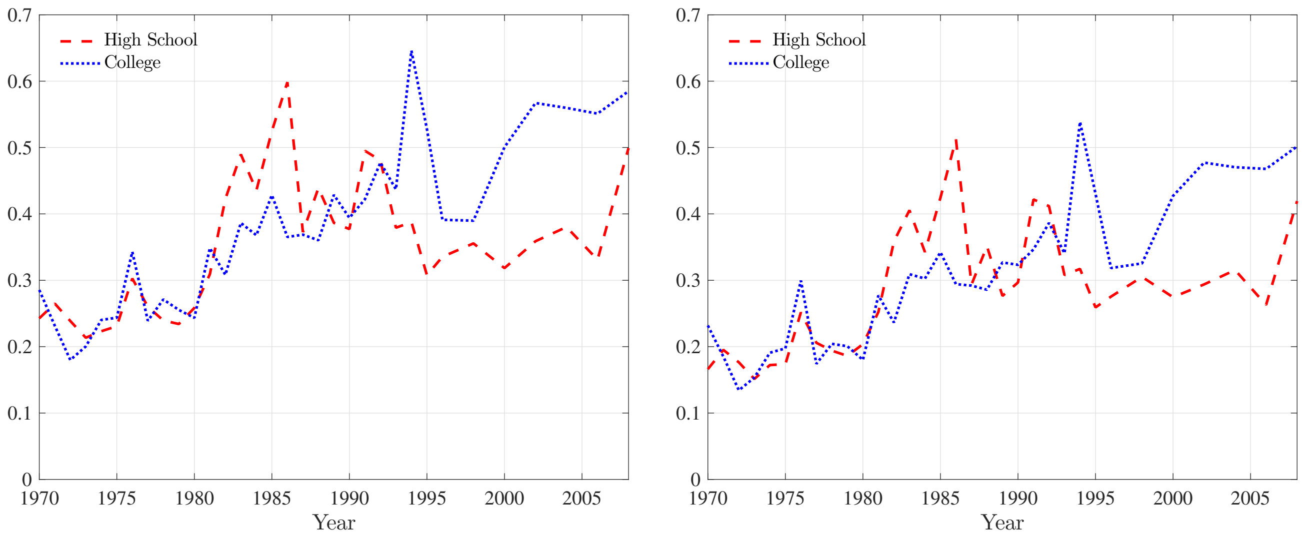
(a) Variance



(b) Share of Variance

Figure 1: Between- and Within-group Variance of Log Annual Earnings

[4]See, among others, Abowd and Card (1989); Blundell and Preston (1998); Meghir and Pistaferri (2004); Blundell, Pistaferri, and Preston (2008); Heathcote, Perri, and Violante (2010). See MaCurdy (2007) for a review.



(a) Total Variance (b) Within-group Variance

Figure 2: Variance of Log Annual Earnings by Sector

The PSID is a longitudinal survey of a representative sample of individuals and families in the U.S. beginning in 1968. The survey was conducted annually through 1997 and biennially since. We use data collected from 1971 through 2009. Since earnings and weeks of work were collected for the year prior to each survey, our analysis considers annual earnings from 1970 to 2008. (Key findings also hold for weekly earnings as shown in Appendix D.)

Our sample is restricted to male heads of households from the core (SRC) sample.[5] We use earnings from any year these men were ages 30-59, had positive wage and salary income, worked at least one week, and were not enrolled as a student. Our earnings measure reflects total wage and salary earnings (excluding farm and business income) and is denominated in 1996 dollars using the CPI-U-RS. We trim the top and bottom 1% of all earnings measures within year and sector by ten-year age cells. The resulting sample contains 3,194 men and 32,549 person-year observations – roughly ten observations for each individual.

Our sample is composed of 92% whites, 6% blacks and 1% hispanics with an average age of 42 years old. We create seven education categories based on current years of completed schooling: 1-5 years, 6-8 years, 9-11 years, 12 years, 13-15 years, 16 years, and 17 or more years. College workers are defined as those with more than 12 years of schooling. In our sample, 16% of respondents finished less than 12 years of schooling, 33% had exactly 12 years of completed schooling, 20% completed some college (13-15 years), 21% completed college (16 years), and 10% had more than 16 years of schooling.

Our analysis focuses on log annual earnings residuals after controlling for differences in educational attainment,

[5] We exclude those from any PSID oversamples (SEO, Latino) as well as those with non-zero individual weights. The earnings questions we use are asked only of household heads. We also restrict our sample to those who were heads of household and not students during the survey year of the observation of interest as well as two years earlier. Our sampling scheme is very similar to that of Moffitt and Gottschalk (2012), except that we do not include earnings measures before age 30.

race, and age. Let $W_{i,t}$ be earnings and $\boldsymbol{x}_{i,t}$ be a vector of observed characteristics for individual i in year t. Log earnings residual $w_{i,t}$ can be written as follows:

$$\ln W_{i,t} = f_t(\boldsymbol{x}_{i,t}) + w_{i,t}. \tag{12}$$

We run year- and sector-specific regressions of log earnings on age, race, and education indicators, along with interactions between race and education indicators and a third order polynomial in age.

Figure 1 shows total variance, between-group variance, and within-group variance (variance of residuals) of log annual earnings. The variance of log annual earnings increases sharply in the early 1980s and after the late 1990s. The evolution of the within-group variance closely mirrors the evolution of the total variance. The within-group variance explains a larger share of total variance than the between-group variance, and it also explains an increasing share of the total variance over time. Although the within- and between-group variance follow similar time patterns, they show a slight divergence in the mid- and late 1970s and the late 1990s.

The total and within-group variance of log annual earnings evolved quite differently across sectors, as shown in Figure 2. The rise in inequality in the early 1980s is stronger among high school workers, while the increasing inequality among college workers is the main driver of the surge in inequality in the early 2000s. Between the mid-1980s and mid-2000s, earnings inequality fell among high school workers and increased among college workers, which is consistent with the literature on 'polarization' in the U.S. labor market (Autor, Levy, and Murnane, 2003; Autor, Katz, and Kearney, 2008; Acemoglu and Autor, 2011; Autor and Dorn, 2012).

3.3 Minimum Distance Estimation

We use minimum distance estimation and the residual moments described above to estimate the model by choosing parameters that minimize the distance between the sample covariances and the theoretical covariances implied by the model. We assume that individuals enter labor market at age 20 with initial skill ψ_i and no other prior shocks, which implies that unobserved skill in year t for individual i born in year c_i ('birth cohort') can be written as

$$\theta_{i,t} = \psi_i + \sum_{\tau=c_i+20}^{t} \nu_{i,\tau}.$$

Variance of initial skill does not depend on birth cohort c_i and the variances of all shocks depend on only time and not on cohort or age. We also assume that the variances of all shocks prior to 1970 are the same as those in year 1970. We impose smooth time trend (cubic polynomials in year) in the variances of permanent skill and non-skill shocks, while the variances of transitory shocks are unrestricted. We choose a normalization that $\mu_{1985}(s) = 1$.

The parameters we estimate are, for each s, $\text{Var}(\psi|s)$, $\rho(s)$, $\{\beta_j(s)\}_{j=1}^{q}$, and parameters for $\text{Var}(\nu_t|s)$ and $\text{Var}(\eta_t|s)$, and for each (s,t), $\mu_t(s)$ (except for $t = 1985$) and $\text{Var}(\xi_t|s)$. For a given parameter vector $\boldsymbol{\Lambda}$, we can compute theoretical counterpart for $\text{Cov}(w_t, w_{t'}|s,c)$ implied by the model (7)-(11) and compare them with the sample covariances. Since some cohort (or, equivalently, age $a_t = t - c$) cells have few observations when calculating residual covariances, we partition the age set $A = \{30,\ldots,59\}$ into 10-year age groups A_1, A_2, and A_3, each corresponding to ages 30-39, 40-49, and 50-59, and aggregate within age groups.

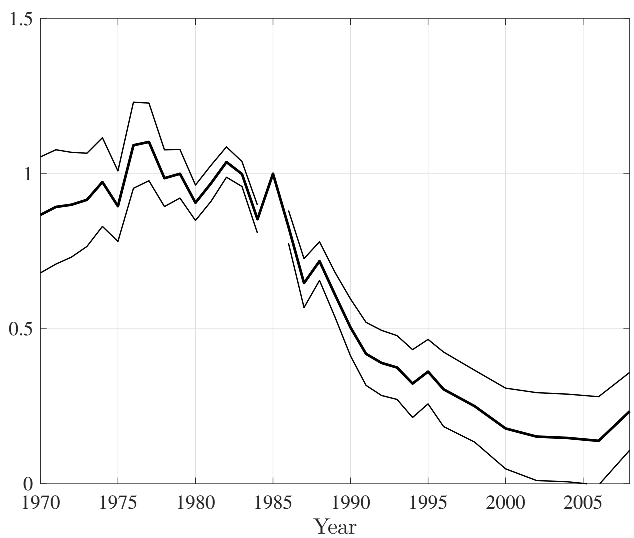
The minimum distance estimator $\hat{\mathbf{\Lambda}}$ solves

$$\min_{\mathbf{\Lambda}} \sum_{(s,j,t,t')\in\Gamma} \left\{ \widehat{\mathrm{Cov}}(w_t, w_{t'}|s, A_j) - \widetilde{\mathrm{Cov}}(w_t, w_{t'}|s, A_j, \mathbf{\Lambda}) \right\}^2,$$

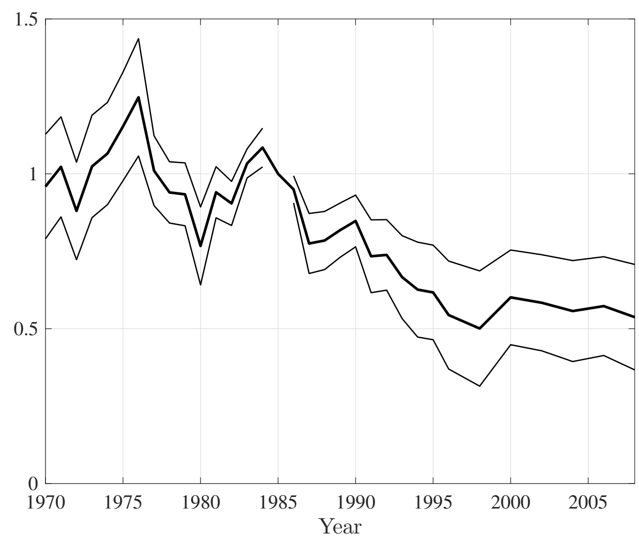
where $\Gamma = \{s, j, t, t' | 1970 \leq t \leq t' \leq 2008, s \in \{0,1\}, j \in \{1,2,3\}\}$, $\widehat{\mathrm{Cov}}(w_t, w_{t'}|s, A_j)$ is the sample covariance conditional on sector s and age group A_j in year t', and $\widetilde{\mathrm{Cov}}(w_t, w_{t'}|s, A_j, \mathbf{\Lambda})$ is the corresponding theoretical covariance.

3.4 Estimation Results

We discuss results for log annual earnings residuals with MA(3) transitory shocks in the text; however, conclusions are quite similar for log weekly earnings (Appendix D) and different specifications for the transitory component (Appendix C).



(a) High School



(b) College

Figure 3: Estimated $\mu_t(s)$ (thick lines) with 95% Confidence Interval (thin lines)

Estimated Returns Figure 3 reports the estimated returns and 95% confidence intervals over time. For high school workers, the return to unobserved skills increases in the early 1970s, levels off in the late 1970s and early 1980s, then falls about 80% between 1985 and 2000. It remains fairly constant thereafter. For college workers, the return to unobserved skills increases more sharply until the mid-1970s, decreases about 75% between 1976 and 1998, and stays constant afterwards. Overall, the returns to unobserved skills fell substantially after 1985 in both sectors despite the increasing residual inequality during the period. Although the returns for high school and college workers display similar patterns, the return for high school workers declines 30 percentage points more than that of college workers after 1985. Indeed, we can reject the hypothesis that the returns to unobserved skills are identical

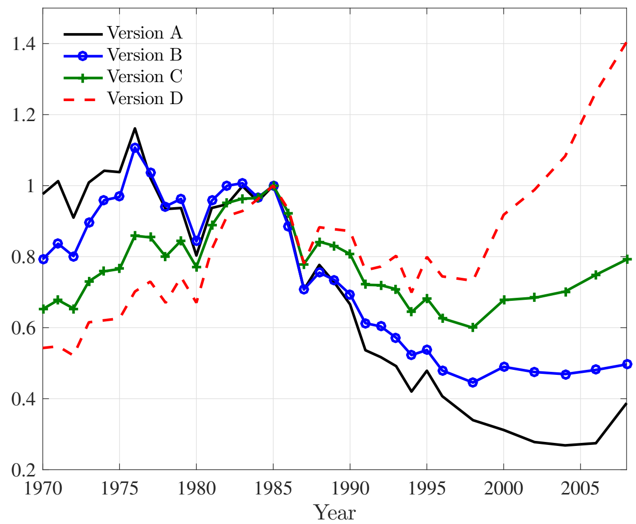


Figure 4: Estimated μ_t under Different Restrictions

across sectors at the 5% significance level.[6] The relatively larger decline in the returns to unobserved skills for high school workers after 1985 is in line with the falling relative wages of high school workers during that period (e.g., Autor, Katz, and Kearney, 2008). This suggests that the changing returns to unobserved skills can be an important determinant of the evolution of between-group inequality as well as within-group inequality.

Our finding that the returns to unobserved skills have fallen since the mid-1980s is consistent with the work of Castex and Dechter (2014), who showed that the returns to cognitive ability as measured by AFQT scores fell in the 2000s relative to the 1980s by comparing NLSY79 and NLSY97 cohorts. The decreasing returns to unobserved skills, however, is inconsistent with the conclusions reached in both the PSID- and CPS-based literatures. The CPS-based literature, which implicitly ignores earnings shocks and equates changes in the total variance log wages residuals with changes in the returns to unobserved skills, concludes that the returns to unobserved skills increased steadily after the early 1970s (Juhn, Murphy, and Pierce, 1993; Katz and Autor, 1999; Acemoglu, 2002).[7]

The PSID-based literature explores the relative importance of permanent and transitory shocks, typically ignoring variation in the returns of unobserved skills. A few exceptions are Haider (2001) and Moffitt and Gottschalk (2012); however, they reached different conclusions. Haider (2001) estimated that the returns to unobserved skills were stable in the 1970s and then increased throughout the 1980s. Moffitt and Gottschalk (2012)'s estimates suggest that the returns increased until the mid-1980s, stabilized, and then increased again in the mid-1990s.[8] We show that their very different conclusions are due to restrictions imposed in their models. Although assuming more persistent

[6] The Wald test is used to test the hypothesis $H_0 : \mu_t(0) = \mu_t(1)$ for all $t \neq 1985$.

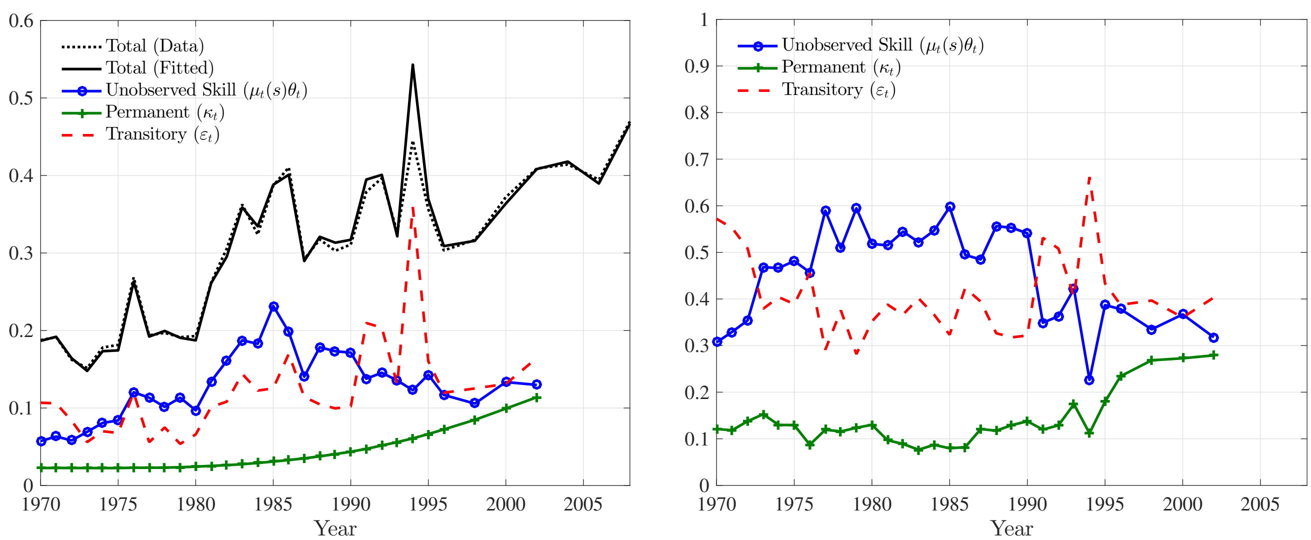
[7] One exception is Lemieux (2006), who finds that, after controlling for composition effects, the return to unobserved skills declined in the 1970s and 1990s. Autor, Katz, and Kearney (2008) show that the decline in the returns to unobserved skills between 1989 and 2005 is driven by decreasing returns among workers in the lower tail of the wage distribution. Lochner and Shin (2014) explore identification and estimation of the returns to unobserved skills allowing for differential changes in the returns across skill levels.

[8] There are other studies estimating models similar to Haider (2001) and Moffitt and Gottschalk (2012) using different panel data. Using the U.S. tax returns data from 1987 to 2009 (DeBacker et al., 2013) and Canadian tax returns data from 1976 to 1992 (Baker and Solon, 2003), they reach similar conclusions that the returns to unobserved skills increased during the sample period.

ARMA(1,1) transitory shocks, they assume that earnings processes are identical across sectors and also abstract from permanent skill and non-skill shocks with time-varying variances. Figure 4 shows how these restrictions affect the estimated returns.

In version A, we assume that transitory shocks follow an ARMA(1,1) process and the returns are identical across sectors.[9] The estimates show similar patterns to our baseline estimates, implying that different assumptions about the persistence of transitory shocks do not affect the results significantly. In version B, we further constrain all other parameters to be identical across sectors. Version C additionally imposes that there are no permanent non-skill shocks and version D further assumes that the variances of skill shocks are constant over time. As we move from version A to version D, the estimated return changes counter-clockwise, generating increasing trends before the mid-1980s and after the mid-1990s with a muted decline in the late 1980s and the early 1990s.

Version D is essentially identical to the models of Haider (2001) and Moffitt and Gottschalk (2012)[10] and the estimates are similar, suggesting that accounting for permanent skill and non-skill shocks with time-varying variances greatly affects the estimates of the returns to unobserved skills. In the absence of permanent shocks with time-varying variances, the model's ability to match increasing residual variance is limited, and all permanent increases in residual inequality is explained by increasing returns to skills.



(a) Variance Decomposition

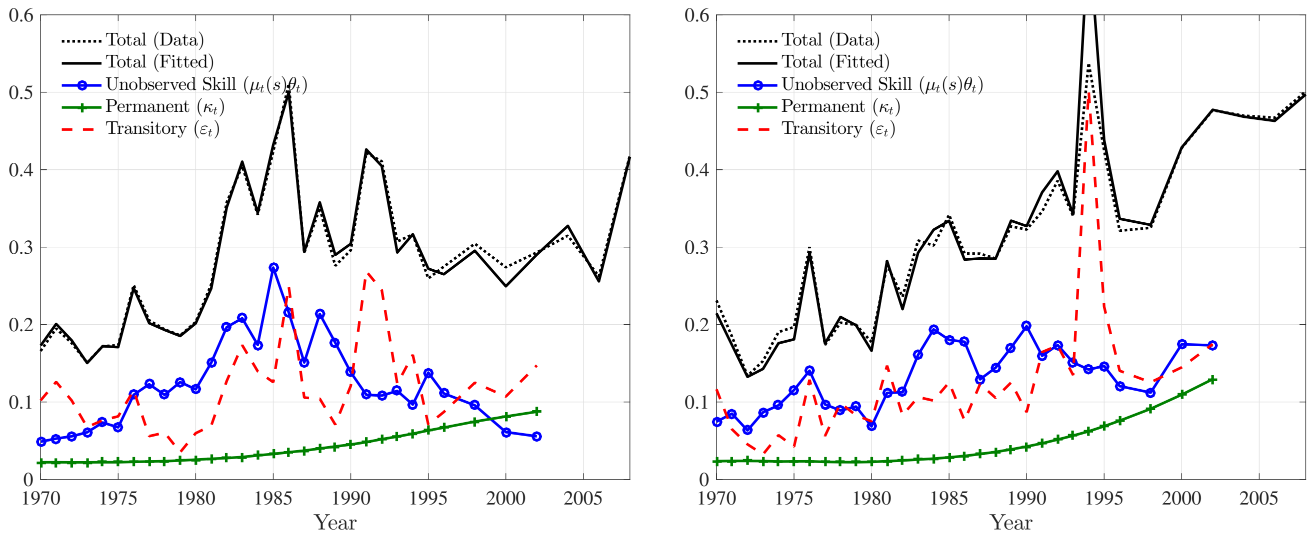
(b) Share of Each Component

Figure 5: Variance and Share of Each Component

Role of Unobserved Skills in Explaining Residual Variance The fact that estimated returns have evolved quite differently from the residual inequality implies that the role of unobserved skills in explaining residual inequality

[9]Transitory shocks that follow an ARMA(1,1) process fade out only asymptotically, so we need a different set of assumptions for identification, as shown in Appendix A.3.

[10]They also have heterogenous growth rate of unobserved skills that is correlated with initial unobserved skills. Including heterogenous income profiles to version D has negligible effects on the estimates.



(a) High School (b) College

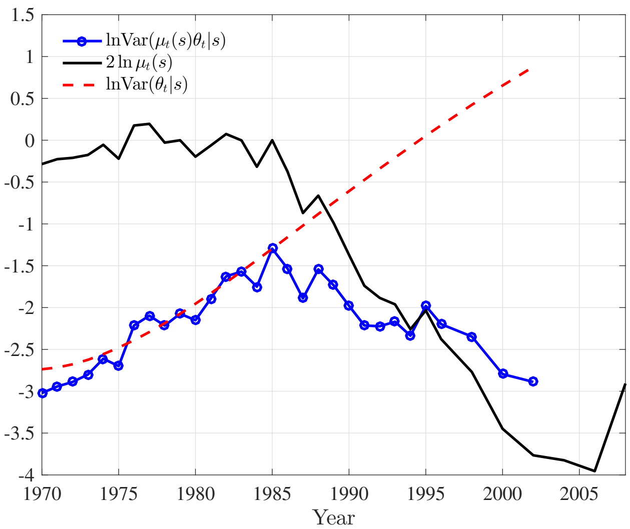
Figure 6: Variance Decomposition by Sector

has also changed. Figure 5 decomposes the residual variance of all sectors into its three components: unobserved skills ($\mu_t(s)\theta_t$), permanent (κ_t) and transitory (ε_t) components of non-skill earnings shocks.[11] Initially quite low, the variance of the unobserved skill component rises over the 1970s and early 1980s before falling after 1985. The variance of transitory component rises in the early 1980s and the variance of permanent non-skill component stays constant throughout the 1970s and then rises continuously afterwards. As a share of the total variance in log earnings residuals, the transitory component plays the largest role in the early 1970s, after which the unobserved skill component dominates. The unobserved skill component explains about 60% of the total residual variance at its peak in the mid-1980s and its share decreases afterwards. The permanent non-skill component explains an increasing share of the total residual variance after the mid-1980s. Between 1985 and 2002, the share of the permanent non-skill component increases from 10% to 30%.

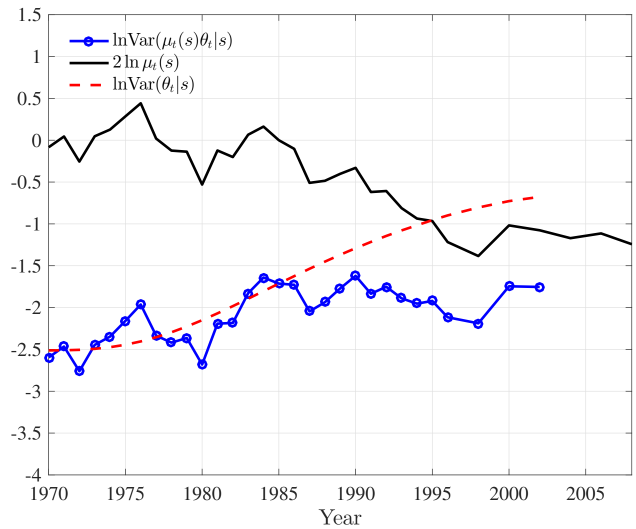
Figure 6 shows the decomposition of the residual variance by sector. The broad time patterns—the reversal of the importance of the unobserved skill component around 1985 and the increasing importance of permanent non-skill component afterwards—can also be found in each sector. However, the long-term fluctuation in the variance of unobserved skill component is more pronounced for high school workers: it quadruples between 1970 and 1985, then falls back to its original level by the early 2000s. For college workers, the variance of unobserved skill component decreases only modestly during the 1990s and then increases again in the early 2000s.

To better understand the evolution of the variance of unobserved skill component, note that the variance of residual earnings due to unobserved skills depends both on the returns to unobserved skills and the variance of

[11] As shown in Section 2, the variances of unobserved skills and non-skill earnings components are not identified for the last few years of our panel. Our figures report these variances trough 2002.



(a) High School



(b) College

Figure 7: Decomposing the Log Variance of Unobserved Skill Component

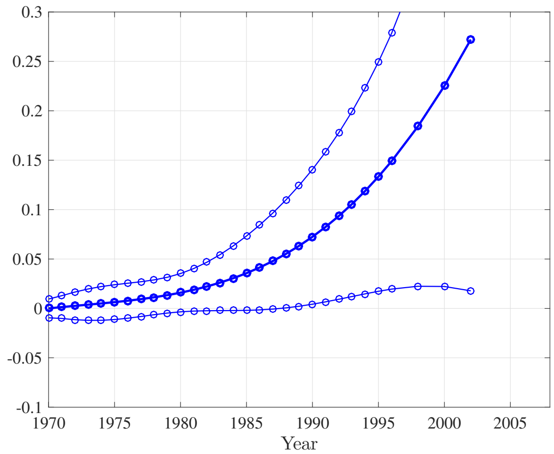
unobserved skills: $\text{Var}(\mu_t(s)\theta_t|s) = \mu_t(s)^2\,\text{Var}(\theta_t|s)$. By taking logs, we get the following additive decomposition:

$$\ln \text{Var}\left(\mu_t(s)\theta_t|s\right) = 2\ln\mu_t(s) + \ln\text{Var}(\theta_t|s).$$

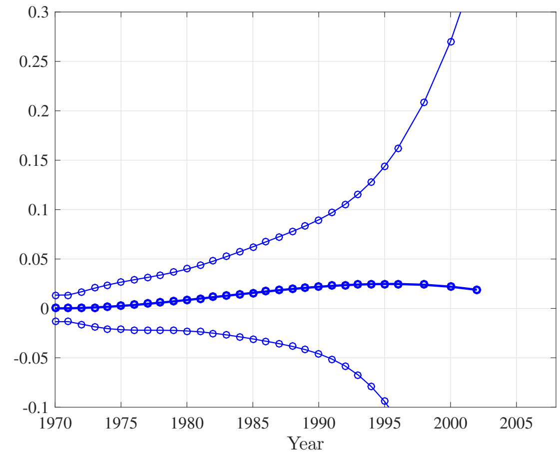
Figure 7 shows that the variance of unobserved skills increases continuously throughout our sample periods in both sectors, and that the increase is more dramatic for high school workers. Since the returns to unobserved skills stayed relatively constant until 1985, the rising variance of unobserved skills entirely explains the increasing variance of unobserved skill component before 1985 in both sectors. However, after 1985, the sharp decline in returns reduces the variance of unobserved skill component despite the sustained growth of the variance of unobserved skills.

Figure 8 shows that the variance of skill shocks increases substantially for high school workers after 1985 and the variance of permanent non-skill shocks for college workers increases more than high school workers. The variance of transitory shocks sharply increases for high school workers in the 1980s and early 1990s, and it has increased continuously for college workers since the mid-1970s, with a sharp spike in 1994.

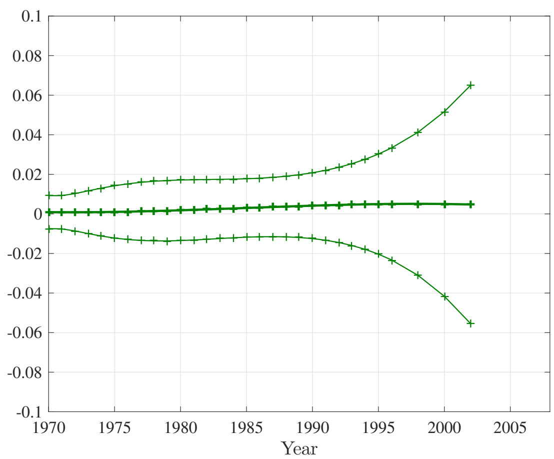
To summarize empirical findings, the returns to unobserved skills fell after 1985 and the decline was larger for high school workers than college workers. The variance of earnings residuals due to unobserved skills increased until 1985 and fell afterwards despite the sustained growth in the variance of unobserved skills. The reversal of residual inequality for high school workers around 1985 mainly reflects the change in the variance of unobserved skill component, although the transitory component non-skill shock also contributed. The rise in residual inequality since the mid-1990s is due to increasing variances of permanent and transitory non-skill shocks as well as increasing variance of unobserved skills among college workers.



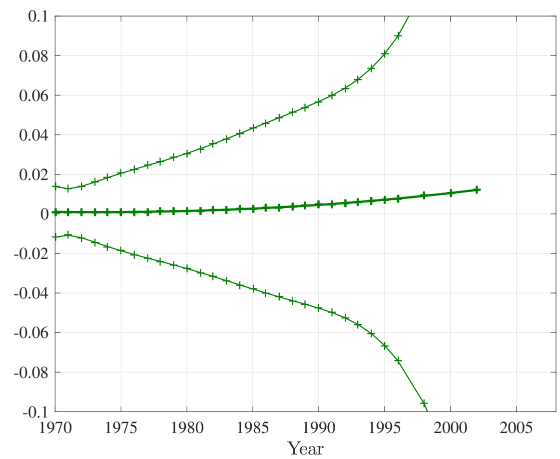
(a) Skill Shocks Var($\nu_t|s$) (High School)



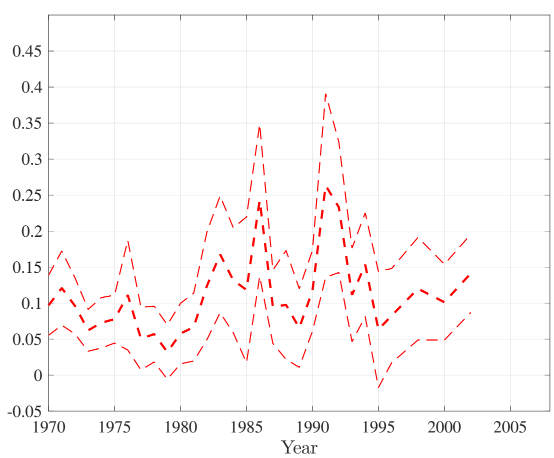
(b) Skill Shocks Var($\nu_t|s$) (College)



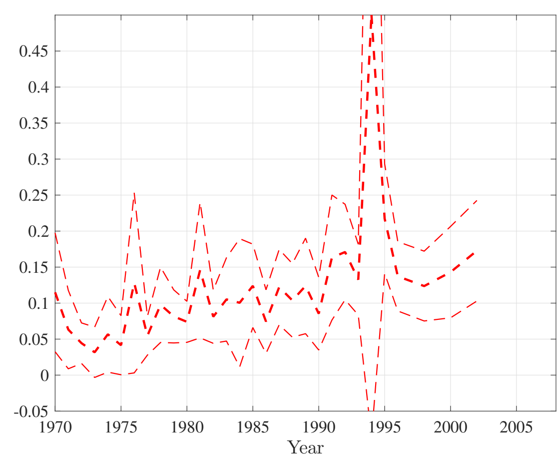
(c) Permanent Shocks Var($\eta_t|s$) (High School)



(d) Permanent Shocks Var($\eta_t|s$) (College)



(e) Transitory Shocks Var($\xi_t|s$) (High School)



(f) Transitory Shocks Var($\xi_t|s$) (College)

Figure 8: Variances of Shocks (thick lines) with 95% Confidence Interval (thin lines)

4 Interpreting the Returns to Skills in a Demand and Supply Framework

Our analysis shows that the changing returns to unobserved skills are important in understanding the evolution of residual inequality. The changes in the returns to skills documented in the previous section could reflect shifts in the demand for skills. However, the concurrent changes in the distribution of skills suggests that the supply of skills may also play a role. In this section, we assess the contributions of skill demand and supply in the evolution of returns to skills using a simple demand and supply framework based on an assignment model of Sattinger (1979).[12]

In the model, the return to skills is generated by differences in the productivity of skills, but is amplified by differences in the productivity of jobs for which skills are employed. Heterogeneous productivity across jobs can be thought of as the different amount of resources under each worker's control, such as the quantity of capital or hierarchy, or different vintages of technology embodied in capital. As in the models of 'span-of-control' (Lucas, 1978) and 'superstar' (Rosen, 1981), production technology determines how much differences in skills are magnified to differences in earnings. In contrast to these models, however, the demand for skills is not infinitely elastic because jobs are in fixed supply, which allows the supply of skills to also affect the return to skills through its effect on the equilibrium assignment of skills and jobs.

Our model is repeated static, where the distributions of skills and jobs exogenously change over time, abstracting from endogenous skill formation and job creation.[13] To focus on the role of skills and returns to skills, we also abstract from elastic labor supply and dynamics of non-skill component of earnings in this theoretical framework.[14]

4.1 Assignment Model of Labor Market

Endowment We consider an economy populated by a continuum of measure 1 of workers and jobs operating in different sectors. In each period t, the fraction of workers in sector s is the same as the fraction of jobs in that sector. In each sector, there exist workers endowed with heterogeneous skills $\Theta_t \geq 0$ distributed with cdf $F_t(\Theta_t|s)$ and jobs with heterogeneous productivity $z_t \geq 0$ distributed with cdf $G_t(z_t|s)$.

Technology In each sector, production takes place through one-to-one matching between workers and jobs. If a worker with skill Θ_t works at a job with productivity z_t in sector s, $y_t(s,\Theta_t,z_t) \geq 0$ units of output is produced. We assume that, for each s, $y_t(s,\cdot,\cdot)$ is twice continuously differentiable and strictly increasing, and strictly supermod-

[12]Assignment models have been a standard theory of income inequality in labor economics. See Sattinger (1993) for an early review. Recent theoretical and empirical studies of income inequality based on this framework include Terviö (2008); Gabaix and Landier (2008); Costinot and Vogel (2010); Lindenlaub (2014); Burstein, Morales, and Vogel (2015); Ales, Kurnaz, and Sleet (2015); Scheuer and Werning (2015).

[13]There is a literature explaining the trends in between- and within-group wage inequality in the framework of endogenous human capital accumulation with heterogeneous learning ability and imperfect substitution of human capital across groups (e.g., Heckman, Lochner, and Taber, 1998; Guvenen and Kuruscu, 2010). This literature does not take into account idiosyncratic earnings shocks that are not related to human capital, equating all changes in residual wage inequality with changes in the inequality of human capital quantity and time spent on training. Using a model that is similar to ours, Jovanovic (1998) studies the effects of technical changes on long-run inequality when the distributions of skills and jobs endogenously evolve over time in a theoretical framework.

[14]Violante (2002) develops a theory of how technical changes may affect transitory shocks. An acceleration of technological progress increases the productivity gaps across jobs, which is translated into more volatile earnings in a frictional labor market.

ular:

$$\frac{\partial y_t(s,\Theta_t,z_t)}{\partial\Theta_t\partial z_t}>0.$$

These assumptions imply that high skill workers are more productive (i.e., they have absolute advantages) than low skill workers at all jobs, but the productivity gap between high and low skill workers is larger at more productive jobs due to complementarity between skills and jobs. Therefore, the efficient assignment that maximizes aggregate output features positive assortative matching where more skilled workers work at more productive jobs (Becker, 1973).

Profit Maximization All markets are perfectly competitive. Workers in sector s with skill Θ_t earn $W_t(s,\Theta_t)$ and output in sector s is sold at price $p_t(s)$. Producers maximize profits, solving

$$\max_{\Theta_t}\Big\{p_t(s)y_t(s,\Theta_t,z_t)-W_t(s,\Theta_t)\Big\}. \tag{13}$$

Denote the solution by $\hat{\Theta}_t(s,z_t)$, which is monotone in z_t due to the supermodularity of $y_t(s,\cdot,\cdot)$. The necessary first order condition is

$$p_t(s)\frac{\partial y_t\big(s,\hat{\Theta}_t(s,z_t),z_t\big)}{\partial\Theta_t}=\frac{\partial W_t\big(s,\hat{\Theta}_t(s,z_t)\big)}{\partial\Theta_t}. \tag{14}$$

Market Clearing The labor market clears if the measure of producers demanding skills $\hat{\Theta}_t(s,z_t)$ or less, which equals $G_t(z_t|s)$, coincides with the measure of workers supplying skills $\hat{\Theta}_t(s,z_t)$ or less, which equals $F_t\big(\hat{\Theta}_t(s,z_t)|s\big)$:

$$F_t\big(\hat{\Theta}_t(s,z_t)|s\big)=G_t(z_t|s). \tag{15}$$

We also assume that both producers and workers have an option not to operate, in which case they earn zero. Therefore, (15) is the the market clearing condition provided that profits and earnings are nonnegative.

Equilibrium Earnings A competitive equilibrium in each sector consists of the assignment rule $\hat{\Theta}_t(s,z_t)$ and the earnings function $W_t(s,\Theta_t)$ that satisfy the first order condition for profit maximization (14) and the market clearing condition (15). Let $\hat{z}_t(s,\Theta_t)$ be the inverse of $\Theta_t=\hat{\Theta}_t(s,z_t)$ with respect to z_t. Then the equilibrium skill elasticity of earnings can be written as follows:[15]

$$\frac{\partial\ln W_t(s,\Theta_t)}{\partial\ln\Theta_t}=\frac{\partial\ln y_t\big(s,\Theta_t,\hat{z}_t(s,\Theta_t)\big)}{\partial\ln\Theta_t}\Bigg/\frac{W_t(s,\Theta_t)}{p_t(s)y_t\big(s,\Theta_t,\hat{z}_t(s,\Theta_t)\big)}. \tag{16}$$

The numerator is the partial skill elasticity (that is, holding productivity of job constant) of output at the equilibrium assignment and the denominator is the worker's share of revenue, or labor share. The following Proposition shows

[15] The same condition is derived by Costinot and Vogel (2010) for the case when workers take the whole revenue (i.e., labor share is 1) due to free entry of jobs. In their model, skill demand is still not infinitely elastic because outputs of different jobs are imperfect substitutes.

that, when the production function is Cobb-Douglas and skills and jobs are log-normally distributed, each of these terms is constant for all skill levels and the skill elasticity of earnings, which we call the 'return to skill' $\mu_t(s)$, can be derived in a simple closed form.

Proposition 2. *Suppose that (i)* $\ln y_t(s,\Theta_t,z_t) = \lambda_t(s)\ln\Theta_t + \gamma_t(s)\ln z_t$ *and (ii)* Θ_t *and* z_t *are log-normally distributed, conditional on s. Then the labor share in each sector is*

$$\frac{W_t(s,\Theta_t)}{p_t(s)y_t\big(s,\Theta_t,\hat{z}_t(s,\Theta_t)\big)} = \frac{\lambda_t(s)}{\lambda_t(s)+\gamma_t(s)\frac{\sigma(\ln z_t|s)}{\sigma(\ln\Theta_t|s)}} \tag{17}$$

and the return to skill in each sector is

$$\mu_t(s) \equiv \frac{\partial \ln W_t(s,\Theta_t)}{\partial \ln \Theta_t} = \lambda_t(s)+\gamma_t(s)\frac{\sigma(\ln z_t|s)}{\sigma(\ln\Theta_t|s)}, \tag{18}$$

where $\sigma(x|s)$ *is conditional standard deviation of x.*

More skilled workers are more productive at all jobs, which is captured by the partial skill elasticity of output $\lambda_t(s)$. But they also work for jobs with higher productivity, so part of the productivity increase comes from the productivity of jobs. Taking into account this sorting effect, the total skill elasticity of output is

$$\frac{\partial \ln y_t\big(s,\Theta_t,\hat{z}_t(s,\Theta_t)\big)}{\partial \ln\Theta_t} + \frac{\partial \ln y_t\big(s,\Theta_t,\hat{z}_t(s,\Theta_t)\big)}{\partial \ln z_t}\frac{\partial \ln \hat{z}_t(s,\Theta_t)}{\partial \ln\Theta_t} = \lambda_t(s)+\gamma_t(s)\frac{\partial \ln \hat{z}_t(s,\Theta_t)}{\partial \ln\Theta_t}. \tag{19}$$

With the log-normality assumption, the market clearing condition (15) implies

$$\frac{\partial \ln \hat{z}_t(s,\Theta_t)}{\partial \ln\Theta_t} = \frac{\sigma(\ln z_t|s)}{\sigma(\ln\Theta_t|s)}, \tag{20}$$

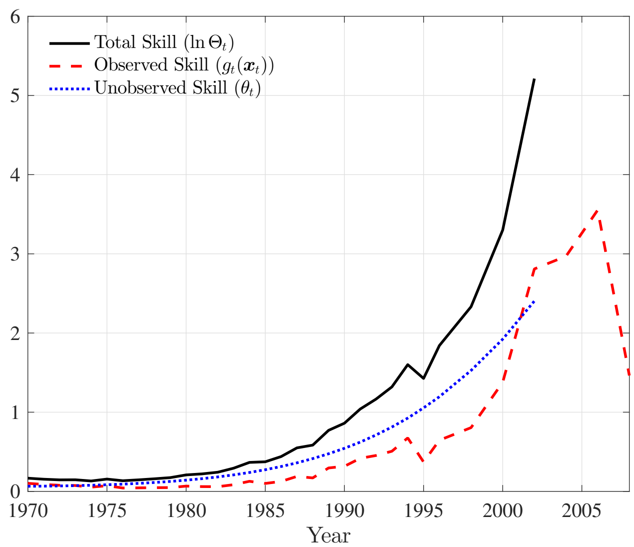
which shows that relative heterogeneity between workers and jobs is important for the equilibrium assignment. When workers are relatively homogeneous compared to jobs, a slightly more skilled worker is assigned to a much more productive job. This generates a large total skill elasticity of output, amplifying the innate productivity differences across workers. On the other hand, when jobs are relatively homogeneous compared to workers, the sorting effect is small, and the total skill elasticity of output mainly reflects the skill differences.

Equation (17) shows that the labor share is determined by the fraction of the worker's marginal contribution, partial skill elasticity of output, out of the sum of worker's and job's marginal contribution to output. By combining (16) and (17), we get (18), which equates the return to skill with the total skill elasticity of output.[16] The return to skill is determined by production technology $\lambda_t(s)$ and $\gamma_t(s)$, distribution of jobs $\sigma(\ln z_t|s)$, and distribution of skills $\sigma(\ln\Theta_t|s)$. An increase in the demand for high skills—an increase in $\lambda_t(s)$ and/or $\gamma_t(s)\sigma(\ln z_t|s)$—can be considered a skill-biased technical change in the sense that it increases the return to skills and widens residual earnings inequality. On the other hand, an increase in the dispersion of skills $\sigma(\ln\Theta_t|s)$ reflects a skill supply

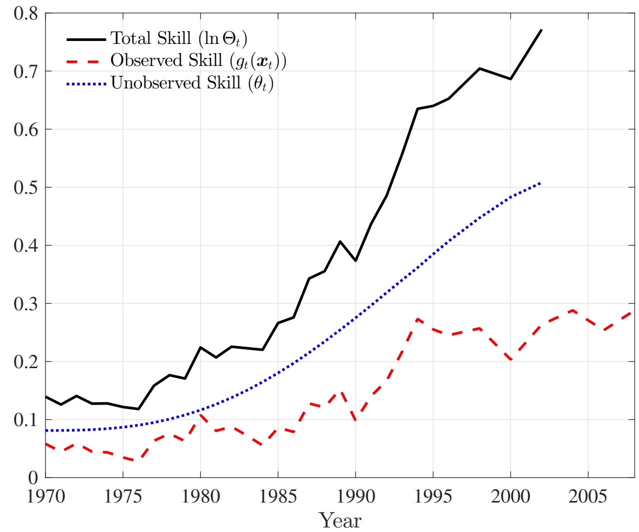
[16] To our knowledge, the formula (18) is novel. Gabaix and Landier (2008) derived a similar formula with Pareto distribution.

change that reduces the return to skills and residual earnings inequality.[17] In the next subsection, we discuss how we can decompose the changes in the returns to skills into the changes in each factor.

4.2 Recovering Supply and Demand Factors



(a) High School



(b) College

Figure 9: Variance of Observed and Unobserved Skills

Variance of Log Skill We assume that skills are correlated with the workers' observed characteristics but are not fully observed. Specifically, log skill of individual i in year t is

$$\ln\Theta_{i,t} = g_t(\boldsymbol{x}_{i,t}) + \theta_{i,t}.$$

From (18), the equilibrium earnings function can be written as $\ln W_t(s,\Theta_t) = \alpha_t(s) + \mu_t(s)\ln\Theta_t$ for some constant $\alpha_t(s)$. Assuming that individual log earnings $\ln W_{i,t}$ is the sum of a skill-related component that reflects the equilibrium earnings function and a non-skill component of earnings $u_{i,t}$,

$$\ln W_{i,t} = \alpha_t(s_i) + \mu_t(s_i)\ln\Theta_{i,t} + u_{i,t} = \underbrace{\alpha_t(s_i) + \mu_t(s_i)g(\boldsymbol{x}_{i,t})}_{\equiv f_t(\boldsymbol{x}_{i,t})} + \underbrace{\mu_t(s_i)\theta_{i,t} + u_{i,t}}_{=w_{i,t}}$$

gives the first stage regression equation (12). Although the time- and sector-specific constant $\alpha_t(s)$ is not separately identified from the location of the function $g_t(\boldsymbol{x}_t)$, the variance of log skill is identified by taking variance

[17] Note that all of these factors, in addition to output price $p_t(s)$ and the average skill and job productivity $\mathbb{E}[\ln\Theta_t|s]$ and $\mathbb{E}[\ln z_t|s]$ of each sector, also affect between-group inequality (i.e., average earnings differential across sectors). This raises a possibility that between- and within-group inequality are driven by a common factor, which may affect them differently.

conditional on s,

$$\mathrm{Var}(\ln \Theta_t|s) = \frac{\mathrm{Var}(\ln W_t|s) - \mathrm{Var}(u_t|s)}{\mu_t(s)^2}.$$

Therefore, the second stage estimates of $\mu_t(s)$ and $\mathrm{Var}(u_t|s)$ can be used to compute the variances of log skills in each period except for the last $k = q+2$ periods when $\mathrm{Var}(u_t|s)$ is not identified. Figure 9 shows that the variances of observed and unobserved skills increased over time, and the increase was much larger for high school workers.

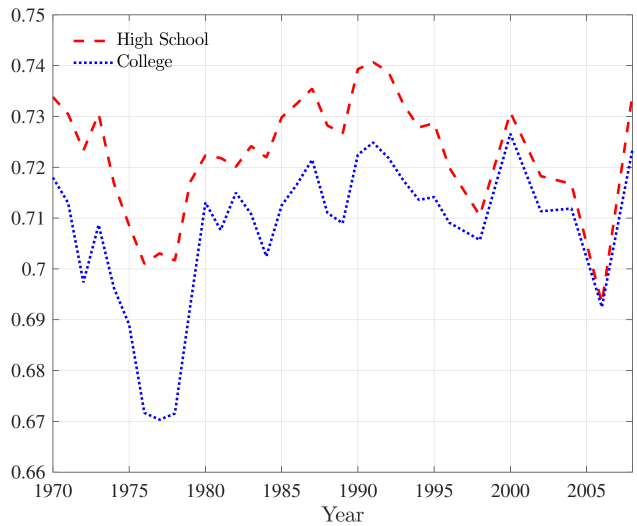
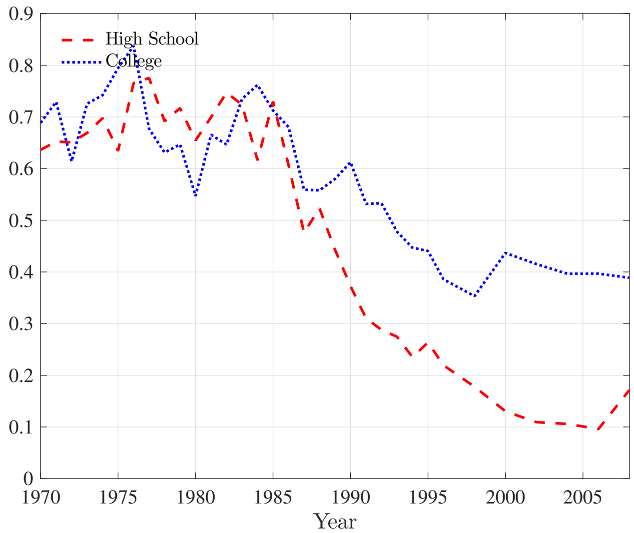


Figure 10: Average Labor Share

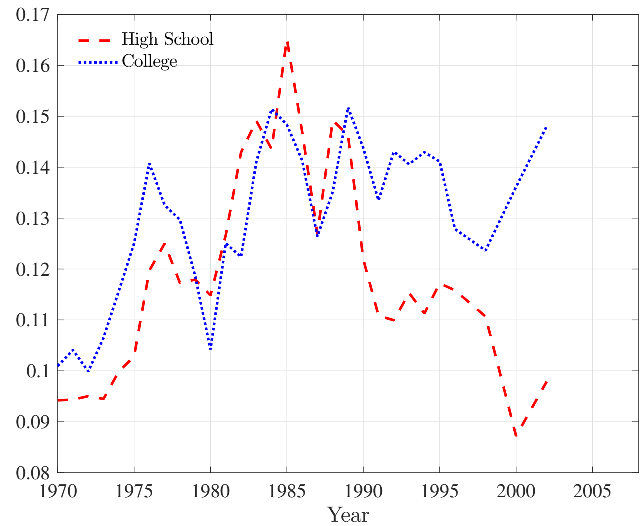
Production Function Parameters By combining (17) and (18), the labor share can be written as $\lambda_t(s)/\mu_t(s)$. Therefore, the output elasticity of skill $\lambda_t(s)$ can be identified from $\mu_t(s)$ and the labor share of each sector. Then we can also get $\gamma_t(s)\sigma(\ln z_t|s) = \sigma(\ln \Theta_t|s)(\mu_t(s) - \lambda_t(s))$, but we cannot separately identify $\gamma_t(s)$ and $\sigma(\ln z_t|s)$. Recovering the output elasticity from factor income share is a commonly used method since Cobb and Douglas (1928), but implementing it in this case is complicated by the need to know the sector-specific labor shares. The difficulty arises from the fact that data on value added by workers with different levels of education is not available.

To overcome this difficulty, we assume that the labor share in our model can be approximated by average industry-level labor share for workers in each sector. Suppose that there are $j = 1, \ldots, J$ industries with value added $V_{j,t}$ and labor compensation $L_{j,t}$ in year t. Let $E_{j,t}(s)$ be the number of workers in industry j, sector s, and year t, and let $E_t(s) = \sum_{j=1}^{J} E_{j,t}(s)$ be the total number of workers in all industries. Then the average labor share in sector s and year t is

$$\sum_{j=1}^{J} \frac{E_{j,t}(s)}{E_t(s)} \frac{L_{j,t}}{V_{j,t}}.$$



(a) $\lambda_t(s)$



(b) $\gamma_t(s)\sigma(\ln z_t|s)$

Figure 11: Production Function Parameters

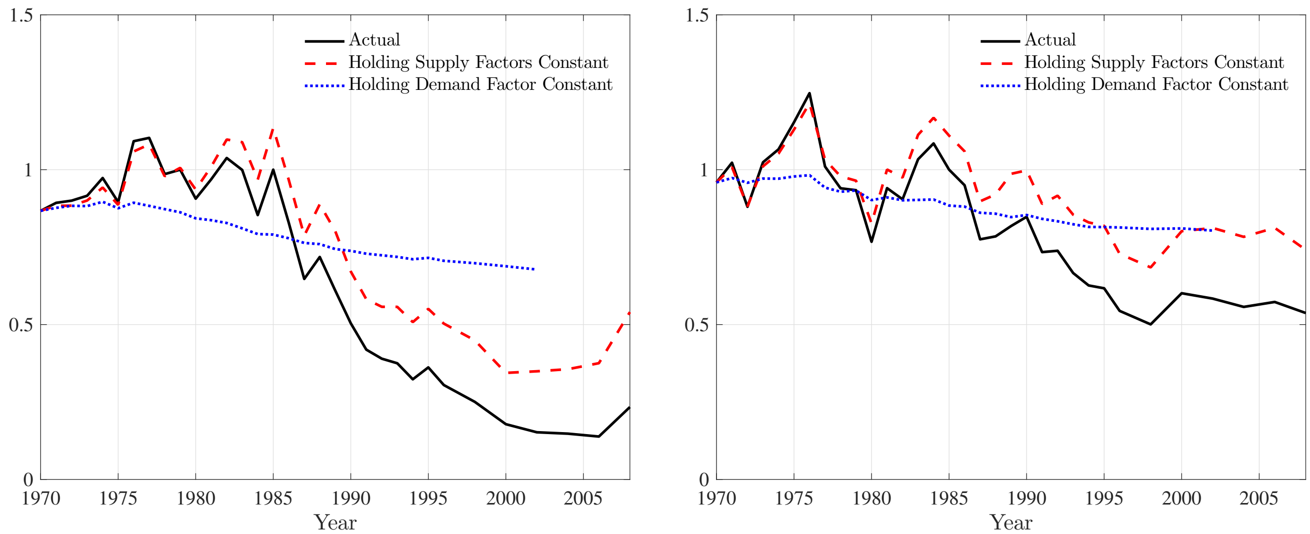
We use data on labor share and number of high school and college workers in the U.S. by 31 ISIC[18] industries from the World KLEMS dataset.[19] The number of workers is computed by counting male workers ages 25-64. Figure 10 shows the average labor share by sector. Overall, average labor shares in the two sectors tend to comove. The average labor share is lower for college workers until the late 1990s, but the gap is closed afterwards. The low average labor share for college workers means that college workers are more likely to work in industries with low labor share compared to high school workers. In our theoretical framework, the lower labor share in the college sector means that college workers enjoy a relatively larger productivity gain from jobs (i.e., $\lambda_t(s)$ is smaller compared to $\gamma_t(s)\sigma(\ln z_t|s)/\sigma(\ln\Theta_t|s)$) compared to high school workers.

Figure 11 shows the production function parameters calculated from the average labor share and the returns to skills. The time patterns of the partial skill elasticity of output $\lambda_t(s)$ are similar to those of the returns to skills $\mu_t(s)$: they are stable until 1985, decrease afterwards, and the decline is larger for high school workers. The productivity differential across jobs $\gamma_t(s)\sigma(\ln z_t|s)$, however, increases until the mid-1980s and is stable afterwards for college workers. For high school workers, $\gamma_t(s)\sigma(\ln z_t|s)$ decreases between the mid-1980s and 2000, showing divergence from that of college workers in the early and late 1990s.

4.3 Decomposing the Returns to Skills

We assess the contributions of each factor recovered in the previous subsection in the evolution of the returns to skills. Figure 12 shows the actual returns as well as counterfactual returns when demand factors ($\lambda_t(s)$ and $\gamma_t(s)\sigma(\ln z_t|s)$) and supply factor ($\sigma(\ln\Theta_t|s)$) are held constant at their values in 1970. Both demand and supply

[18]The International Standard Industrial Classification of All Economic Activities (ISIC) is a United Nations industry classification system.
[19]Available at http://www.worldklems.net. See Jorgenson, Ho, and Samuels (2012) for description.



(a) High School

(b) College

Figure 12: Actual and Counterfactual Returns ($\mu_t(s)$)

factors play important roles in the evolution of the returns to skills in the sense that each factor alone can explain the general decreasing trend of the returns to skills. Rising variance of observed and unobserved skills over time reduced the returns to skills by lowering productivity differentials across jobs among workers with different skills. For high school workers, the demand for skills modestly increased until 1985, which mainly reflects the change in the productivity of jobs ($\gamma_t(s)\sigma(\ln z_t|s)$) rather than the productivity of skills ($\lambda_t(s)$). The demand for skills for high school workers undergoes a steep decline between 1985 and 2000, explaining most of the decline in the returns to skills during that period, then slightly increases afterwards. For college workers, the decline of the demand for skills was not as drastic as that for high school workers, explaining about half of the decrease in the return to skills between 1970 and 2000.

5 Conclusion

In this paper, we show that the returns to unobserved skills and the variance of unobserved skills are separately identified from time-varying variances of non-skill earnings shocks. Using panel data from the PSID on male earnings in the U.S. from 1970-2008, we show that accounting for changes in the distributions of skills and the volatility of earnings is important for the estimation of the evolution of the returns to unobserved skills. The time patterns of the estimated returns to unobserved skills are different from those estimated in the literature. The returns to unobserved skills were stable in the 1970s and early 1980s, but they sharply decreased after 1985, and the decline was more sizable for high school workers than for college workers. Unobserved skills explain substantial and increasing share of the total variance of log annual earnings residual, but their importance declined after the mid-1980s despite the continuous increase in the variance of unobserved skills. Much of the increase in the residual variance after the late 1990s is explained by the rise in the variance of permanent non-skill component of earnings.

Using a simple demand and supply framework, we show that the decrease in the returns to unobserved skills after the mid-1980s is driven by both supply and demand factors. The decline in the demand for skills was more drastic for high school workers than for college workers. Our finding that the demand for skills decreased since the mid-1980s seem to reinforce the challenges (e.g., Card and DiNardo, 2002) faced by the skill-biased technical change (SBTC) hypothesis, which typically states that the steady increase in the demand for skills explains increasing inequality. Further work is required to understand why the skill demand declined during the 1990s, precisely when technological progress had accelerated (Cummins and Violante, 2002).

References

Abowd, John M. and David Card. 1989. "On the Covariance Structure of Earnings and Hours Changes." *Econometrica* 57 (2):411–445.

Acemoglu, Daron. 2002. "Technical Change, Inequality, and the Labor Market." *Journal of Economic Literature* 40 (1):7–72.

Acemoglu, Daron and David Autor. 2011. "Skills, Tasks and Technologies: Implications for Employment and Earnings." In *Handbook of Labor Economics*, vol. 4, edited by O. Ashenfelter and D. Card, chap. 12. Elsevier, 1043–1171.

Ales, Laurence, Musab Kurnaz, and Christopher Sleet. 2015. "Technical Change, Wage Inequality, and Taxes." *American Economic Review* 105 (10):3061–3101.

Autor, David and David Dorn. 2012. "The Growth of Low Skill Service Jobs and the Polarization of the U.S. Labor Market." IZA Discussion Papers 7068, Institute for the Study of Labor (IZA).

Autor, David, Frank Levy, and Richard Murnane. 2003. "The Skill Content of Recent Technological Change: An Empirical Exploration." *Quarterly Journal of Economics* 118 (4):1279–1333.

Autor, David H, Lawrence F Katz, and Melissa S Kearney. 2008. "Trends in U.S. Wage Inequality: Revising the Revisionists." *The Review of Economics and Statistics* 90 (2):300–323.

Baker, Michael and Gary Solon. 2003. "Earnings Dynamics and Inequality among Canadian Men, 1976–1992: Evidence from Longitudinal Income Tax Records." *Journal of Labor Economics* 21 (2):289–321.

Becker, Gary S. 1973. "A Theory of Marriage: Part I." *Journal of Political Economy* 81 (4):pp. 813–846.

Blundell, Richard, Michael Graber, and Magne Mogstad. 2015. "Labor Income Dynamics and the Insurance from Taxes, Transfers, and the Family." *Journal of Public Economics* 127:58 – 73.

Blundell, Richard, Luigi Pistaferri, and Ian Preston. 2008. "Consumption Inequality and Partial Insurance." *The American Economic Review* 98 (5):pp. 1887–1921.

Blundell, Richard and Ian Preston. 1998. "Consumption Inequality and Income Uncertainty." *The Quarterly Journal of Economics* 113 (2):603–640.

Burstein, Ariel, Eduardo Morales, and Jonathan Vogel. 2015. "Accounting for Changes in Between-Group Inequality." Working Paper 20855, National Bureau of Economic Research.

Card, David and John E. DiNardo. 2002. "Skill-Biased Technological Change and Rising Wage Inequality: Some Problems and Puzzles." *Journal of Labor Economics* 20 (4):733–783.

Carroll, Christopher D. 1992. "The Buffer-Stock Theory of Saving: Some Macroeconomic Evidence." *Brookings Papers on Economic Activity* 1992 (2):61–156.

Castex, Gonzalo and Evgenia Kogan Dechter. 2014. "The Changing Roles of Education and Ability in Wage Determination." *Journal of Labor Economics* 32 (4):685–710.

Cobb, Charles W and Paul H Douglas. 1928. "A Theory of Production." *The American Economic Review* 18 (1):139–165.

Costinot, Arnaud and Jonathan Vogel. 2010. "Matching and Inequality in the World Economy." *Journal of Political Economy* 118 (4):pp. 747–786.

Cummins, Jason G. and Giovanni L. Violante. 2002. "Investment-Specific Technical Change in the United States (1947–2000): Measurement and Macroeconomic Consequences." *Review of Economic Dynamics* 5 (2):243 – 284.

DeBacker, Jason, Bradley Heim, Vasia Panousi, Shanthi Ramnath, and Ivan Vidangos. 2013. "Rising inequality: transitory or persistent? New evidence from a panel of US tax returns." *Brookings Papers on Economic Activity* 2013 (1):67–142.

Friedman, Milton and Simon Kuznets. 1954. *Income from Independent Professional Practice*. National Bureau of Economic Research.

Gabaix, Xavier and Augustin Landier. 2008. "Why has CEO Pay Increased So Much?" *The Quarterly Journal of Economics* 123 (1):49–100.

Gottschalk, Peter and Robert Moffitt. 1994. "The Growth of Earnings Instability in the U.S. Labor Market." *Brookings Papers on Economic Activity* 25 (2):217–272.

Guiso, Luigi, Luigi Pistaferri, and Fabiano Schivardi. 2005. "Insurance within the Firm." *Journal of Political Economy* 113 (5):1054–1087.

Guvenen, Fatih and Burhanettin Kuruscu. 2010. "A Quantitative Analysis of the Evolution of the US Wage Distribution, 1970-2000." In *NBER Macroeconomics Annual 2009, Volume 24*. University of Chicago Press, 227–276.

Haider, Steven J. 2001. "Earnings Instability and Earnings Inequality of Males in the United States: 1967–1991." *Journal of Labor Economics* 19 (4):799–836.

Heathcote, Jonathan, Fabrizio Perri, and Giovanni L. Violante. 2010. "Unequal We Stand: An Empirical Analysis of Economic Inequality in the United States, 1967–2006." *Review of Economic Dynamics* 13 (1):15 – 51. Special issue: Cross-Sectional Facts for Macroeconomists.

Heckman, James J, Lance Lochner, and Christopher Taber. 1998. "Explaining Rising Wage Inequality: Explorations with a Dynamic General Equilibrium Model of Labor Earnings with Heterogeneous Agents." *Review of Economic Dynamics* 1 (1):1 – 58.

Jorgenson, Dale W, Mun S Ho, and Jon D Samuels. 2012. "A Prototype Industry-Level Production Account for the United States, 1947-2010." Second World KLEMS Conference, Harvard University, August 9, 2012.

Jovanovic, Boyan. 1998. "Vintage Capital and Inequality." *Review of Economic Dynamics* 1 (2):497 – 530.

Juhn, Chinhui, Kevin M. Murphy, and Brooks Pierce. 1993. "Wage Inequality and the Rise in Returns to Skill." *Journal of Political Economy* 101 (3):410–42.

Katz, Lawrence F. and David H. Autor. 1999. "Changes in the Wage Structure and Earnings Inequality." In *Handbook of Labor Economics*, *Handbook of Labor Economics*, vol. 3, edited by O. Ashenfelter and D. Card, chap. 26. Elsevier, 1463–1555.

Katz, Lawrence F. and Kevin M. Murphy. 1992. "Changes in Relative Wages, 1963–1987: Supply and Demand Factors." *The Quarterly Journal of Economics* 107 (1):35–78.

Lamadon, Thibaut. 2014. "Productivity Shocks, Dynamic Contracts and Income Uncertainty." *Working Paper* .

Lemieux, Thomas. 2006. "Increasing Residual Wage Inequality: Composition Effects, Noisy Data, or Rising Demand for Skill?" *The American Economic Review* 96 (3):461–498.

Lindenlaub, Ilse. 2014. "Sorting Multidimensional Types: Theory and Application." *Working Paper* .

Lochner, Lance and Youngki Shin. 2014. "Understanding Earnings Dynamics: Identifying and Estimating the Changing Roles of Unobserved Ability, Permanent and Transitory Shocks." Working Paper 20068, National Bureau of Economic Research.

Lucas, Robert E. 1978. "On the Size Distribution of Business Firms." *The Bell Journal of Economics* 9 (2):508–523.

MaCurdy, Thomas. 2007. "A Practitioner's Approach to Estimating Intertemporal Relationships Using Longitudinal Data: Lessons from Applications in Wage Dynamics." In *Handbook of Econometrics*, *Handbook of Econometrics*, vol. 6, Part A, edited by James J. Heckman and Edward E. Leamer, chap. 62. Elsevier, 4057 – 4167.

Meghir, Costas and Luigi Pistaferri. 2004. "Income Variance Dynamics and Heterogeneity." *Econometrica* 72 (1):1–32.

Moffitt, Robert A. and Peter Gottschalk. 2012. "Trends in the Transitory Variance of Male Earnings Methods and Evidence." *Journal of Human Resources* 47 (1):204–236.

Rosen, Sherwin. 1981. "The Economics of Superstars." *The American Economic Review* 71 (5):845–858.

———. 1986. "The Theory of Equalizing Differences." In *Handbook of Labor Economics*, *Handbook of Labor Economics*, vol. 1, chap. 12. Elsevier, 641 – 692.

Sattinger, Michael. 1979. "Differential Rents and the Distribution of Earnings." *Oxford Economic Papers* 31 (1):60–71.

———. 1993. "Assignment Models of the Distribution of Earnings." *Journal of Economic Literature* 31 (2):831–880.

Scheuer, Florian and Iván Werning. 2015. "The Taxation of Superstars." Working Paper 21323, National Bureau of Economic Research.

Terviö, Marko. 2008. "The Difference That CEOs Make: An Assignment Model Approach." *The American Economic Review* 98 (3):642–668.

Violante, Giovanni L. 2002. "Technological Acceleration, Skill Transferability, and the Rise in Residual Inequality." *The Quarterly Journal of Economics* 117 (1):297–338.

Appendix

A Proofs and Analytical Details

A.1 Proof of Proposition 1

First, note that, from (6), $\mathrm{Cov}(w_t, \Delta w_{t'}|c) = \mu_t \Delta\mu_{t'} \mathrm{Var}(\theta_t|c)$ for $t' - t \geq k$. For $t' - t \geq k$, the covariances conditional on cohort c are

$$\mathrm{Cov}(w_t, \Delta w_{t'}|c) = \mu_t \Delta\mu_{t'} \left[\mathrm{Var}(\theta_{t-1}|c) + \mathrm{Var}(\nu_t|c)\right], \tag{21}$$

$$\mathrm{Cov}(w_{t-1}, \Delta w_{t'}|c) = \mu_{t-1} \Delta\mu_{t'} \mathrm{Var}(\theta_{t-1}|c). \tag{22}$$

By taking a difference between cohort c and c' such that $\mathrm{Var}(\nu_t|c) = \mathrm{Var}(\nu_t|c')$, we get

$$\frac{\mathrm{Cov}(w_t, \Delta w_{t'}|c) - \mathrm{Cov}(w_t, \Delta w_{t'}|c')}{\mathrm{Cov}(w_{t-1}, \Delta w_{t'}|c) - \mathrm{Cov}(w_{t-1}, \Delta w_{t'}|c')} = \frac{\mu_t}{\mu_{t-1}}$$

In this way, μ_t is identified for all $t < \bar{t} - k$.

To identify μ_t for $t \geq \bar{t} - k$, note that, for $t' - t \geq k$,

$$\frac{\mathrm{Cov}(w_t, \Delta w_{t'+1}|c)}{\mathrm{Cov}(w_t, \Delta w_{t'}|c)} = \frac{\mu_t \Delta\mu_{t'+1} \mathrm{Var}(\theta_t|c)}{\mu_t \Delta\mu_{t'} \mathrm{Var}(\theta_t|c)} = \frac{\Delta\mu_{t'+1}}{\Delta\mu_{t'}}.$$

Therefore, $\mu_{\bar{t}-k}$ is identified using $\mathrm{Cov}(w_{\bar{t}-2k-1}, \Delta w_{\bar{t}-k-1}|c)$, $\mathrm{Cov}(w_{\bar{t}-2k-1}, \Delta w_{\bar{t}-k}|c)$, $\mu_{\bar{t}-k-2}$, and $\mu_{\bar{t}-k-1}$ provided that $\bar{t} - 2k - 1 \geq \underline{t}$. Similarly, μ_t for $t > \bar{t} - k$ are also identified.

Once we know μ_t, then $\mathrm{Var}(\theta_t)$ for all $t < \bar{t} - k$ can be identified from (22): for $t' - t \geq k$,

$$\mathrm{Var}(\theta_t) = \frac{\mathrm{Cov}(w_t, \Delta w_{t'})}{\mu_t \Delta\mu_{t'}}.$$

Finally, for all $t < \bar{t} - k$, $\mathrm{Var}(u_t) = \mathrm{Var}(w_t) - \mathrm{Var}(\mu_t \theta_t)$ is identified.

A.2 Proof of Proposition 2

With the assumption of log-normality, the market clearing condition (15) becomes

$$\frac{\ln \hat{\Theta}_t(s, z_t) - \mathbb{E}[\ln \Theta_t|s]}{\sigma(\ln \Theta_t|s)} = \frac{\ln z_t - \mathbb{E}[\ln z_t|s]}{\sigma(\ln z_t|s)},$$

which gives

$$\ln \hat{\Theta}_t(s, z_t) = \mathbb{E}[\ln \Theta_t|s] + \frac{\sigma(\ln \Theta_t|s)}{\sigma(\ln z_t|s)}(\ln z_t - \mathbb{E}[\ln z_t|s]).$$

Let $\hat{z}_t(s,\Theta_t)$ be the inverse of $\Theta_t = \hat{\Theta}_t(s,z_t)$ with respect to z_t, that is,

$$\ln \hat{z}_t(s,\Theta_t) = \mathbb{E}[\ln z_t|s] + \frac{\sigma(\ln z_t|s)}{\sigma(\ln \Theta_t|s)}(\ln \Theta_t - \mathbb{E}[\ln \Theta_t|s]).$$

Then the first order condition (14) becomes

$$\begin{aligned}\frac{\partial W_t(s,\Theta_t)}{\partial \Theta_t} =& p_t(s)\lambda_t(s)\Theta_t^{\lambda_t(s)-1}\hat{z}_t(s,\Theta_t)^{\gamma_t(s)}\\ =& p_t(s)\lambda_t(s)\exp\left(\gamma_t(s)\left\{\mathbb{E}[\ln z_t|s] - \frac{\sigma(\ln z_t|s)}{\sigma(\ln \Theta_t|s)}\mathbb{E}[\ln \Theta_t|s]\right\}\right)\Theta_t^{\mu_t(s)-1},\end{aligned}$$

where

$$\mu_t(s) = \lambda_t(s) + \gamma_t(s)\frac{\sigma(\ln z_t|s)}{\sigma(\ln \Theta_t|s)}.$$

Next, note that $W_t(s,0) = 0$ because $W_t(s,0) > 0$ would give negative profits for producers with $z_t = 0$. Then, by integrating the above equation, we get

$$\begin{aligned}W_t(s,\Theta_t) =& W_t(s,0) + \int_0^{\Theta_t}\frac{\partial W_t(s,\Theta_t')}{\partial \Theta_t}d\Theta_t'\\ =& p_t(s)\frac{\lambda_t(s)}{\mu_t(s)}\exp\left(\gamma_t(s)\left\{\mathbb{E}[\ln z_t|s] - \frac{\sigma(\ln z_t|s)}{\sigma(\ln \Theta_t|s)}\mathbb{E}[\ln \Theta_t|s]\right\}\right)\Theta_t^{\mu_t(s)},\end{aligned}$$

which gives (18).

Note that

$$\begin{aligned}p_t(s)y_t\left(s,\Theta_t,\hat{z}_t(s,\Theta_t)\right) =& p_t(s)\Theta_t^{\lambda_t(s)}\hat{z}_t(s,\Theta_t)^{\gamma_t(s)}\\ =& p_t(s)\exp\left(\gamma_t(s)\left\{\mathbb{E}[\ln z_t|s] - \frac{\sigma(\ln z_t|s)}{\sigma(\ln \Theta_t|s)}\mathbb{E}[\ln \Theta_t|s]\right\}\right)\Theta_t^{\mu_t(s)}\\ =& W_t(s,\Theta_t)\frac{\mu_t(s)}{\lambda_t(s)}.\end{aligned}$$

Therefore,

$$\frac{W_t(s,\Theta_t)}{p_t(s)y_t\left(s,\Theta_t,\hat{z}_t(s,\Theta_t)\right)} = \frac{\lambda_t(s)}{\mu_t(s)}.$$

A.3 Identification with ARMA(1,1) Transitory Shocks

Suppose that there exist (t,t',c) such that $\mathrm{Var}(\theta_t|c) = \mathrm{Var}(\theta_t|c+1)$ and $\mathrm{Var}(\theta_{t'}|c) = \mathrm{Var}(\theta_{t'}|c+1)$. Then the AR(1) coefficient ρ is identified. The assumption is satisfied if, for example, there is no cohort trend in initial skill and $\mathrm{Var}(\nu_t) = 0$ for some t. Once ρ is identified, the rest of the identification is simliar with that of the MA(q) process.

Identification of ρ Let c_i denote the year when individual i enters the labor market. Then

$$\begin{aligned}\varepsilon_{i,t} =& \xi_{i,t} + (\beta+\rho)\xi_{i,t-1} + \rho(\beta+\rho)\xi_{i,t-2} + \ldots, + \rho^{t-c_i-1}(\beta+\rho)\xi_{i,c_i},\\ \varepsilon_{i,t+k} =& \xi_{i,t+k} + (\beta+\rho)\xi_{i,t+k-1} + \rho(\beta+\rho)\xi_{i,t+k-2} + \ldots\\ &+ \rho^{k-1}(\beta+\rho)\xi_{i,t} + \rho^{k}(\beta+\rho)\xi_{i,t-1} + \ldots + \rho^{t+k-c_i-1}(\beta+\rho)\xi_{i,c_i}.\end{aligned}$$

Then the autocovariance of the transitory component is

$$\operatorname{Cov}(\varepsilon_t, \varepsilon_{t+k}|c) = \rho^{k-1}\Big\{(\beta+\rho)\operatorname{Var}(\xi_t) + \rho(\beta+\rho)^2\operatorname{Var}(\xi_{t-1}) + \ldots + \rho^{2(t-c)-1}(\beta+\rho)^2\operatorname{Var}(\xi_c)\Big\},$$

which gives the following autocovariance of earnings residual:

$$\begin{aligned}&\operatorname{Cov}(w_t, \Delta w_{t+k}|c)\\ =&\mu_t\Delta\mu_{t+k}\operatorname{Var}(\theta_t|c) + \operatorname{Cov}(\varepsilon_t, \varepsilon_{t+k}|c) - \operatorname{Cov}(\varepsilon_t, \varepsilon_{t+k-1}|c)\\ =&\mu_t\Delta\mu_{t+k}\operatorname{Var}(\theta_t|c) + \rho^{k-2}(\rho-1)\Big\{(\beta+\rho)\operatorname{Var}(\xi_t) + \rho(\beta+\rho)^2\operatorname{Var}(\xi_{t-1}) + \ldots + \rho^{2(t-c)-1}(\beta+\rho)^2\operatorname{Var}(\xi_c)\Big\}.\end{aligned}$$

By differencing across neighboring cohorts, we get

$$\begin{aligned}&\operatorname{Cov}(w_t, \Delta w_{t+k}|c) - \operatorname{Cov}(w_t, \Delta w_{t+k}|c+1)\\ =&\mu_t\Delta\mu_{t+k}\big[\operatorname{Var}(\theta_t|c) - \operatorname{Var}(\theta_t|c+1)\big] + \rho^{k-2+2(t-c)-1}(\rho-1)(\beta+\rho)^2\operatorname{Var}(\xi_c)\end{aligned}$$

Therefore, if there exists (t, t', c) such that $\operatorname{Var}(\theta_t|c) = \operatorname{Var}(\theta_t|c+1)$ and $\operatorname{Var}(\theta_{t'}|c) = \operatorname{Var}(\theta_{t'}|c+1)$, ρ is identified as follows:

$$\rho^{k-k'} = \frac{\operatorname{Cov}(w_t, \Delta w_{t+k}|c) - \operatorname{Cov}(w_t, \Delta w_{t+k}|c+1)}{\operatorname{Cov}(w_t, \Delta w_{t+k'}|c) - \operatorname{Cov}(w_t, \Delta w_{t+k'}|c+1)}.$$

Identification of μ_t Note that

$$\begin{aligned}\Delta w_{t+1} - \rho\Delta w_t =& [\Delta\mu_{t+1}\theta_{t+1} + \mu_t\nu_{t+1} + \eta_{t+1} + \Delta\varepsilon_{t+1}] - \rho[\Delta\mu_t\theta_t + \mu_{t-1}\nu_t + \eta_t + \Delta\varepsilon_t]\\ =& [\Delta\mu_{t+1}\theta_{t+1} + \mu_t\nu_{t+1} + \eta_{t+1}] - \rho[\Delta\mu_t\theta_t + \mu_{t-1}\nu_t + \eta_t] + \Delta\xi_{t+1} + \beta\Delta\xi_t.\end{aligned}$$

Then

$$\operatorname{Cov}(w_{t-2}, \Delta w_{t+1} - \rho\Delta w_t) = \mu_{t-2}(\Delta\mu_{t+1} - \rho\Delta\mu_t)\operatorname{Var}(\theta_{t-2}),$$

and the difference across cohorts is

$$\operatorname{Cov}(w_{t-2}, \Delta w_{t+1} - \rho\Delta w_t|c) - \operatorname{Cov}(w_{t-2}, \Delta w_{t+1} - \rho\Delta w_t|c') = \mu_{t-2}(\Delta\mu_{t+1} - \rho\Delta\mu_t)\Big\{\operatorname{Var}(\theta_{t-2}|c) - \operatorname{Var}(\theta_{t-2}|c')\Big\}.$$

Therefore, if $\text{Var}(\nu_{t-1}|c) = \text{Var}(\nu_{t-1}|c')$, then $\text{Var}(\theta_{t-2}|c) - \text{Var}(\theta_{t-2}|c') = \text{Var}(\theta_{t-3}|c) - \text{Var}(\theta_{t-3}|c')$ and

$$\frac{\text{Cov}(w_{t-2}, \Delta w_{t+1} - \rho \Delta w_t|c) - \text{Cov}(w_{t-2}, \Delta w_{t+1} - \rho \Delta w_t|c')}{\text{Cov}(w_{t-3}, \Delta w_{t+1} - \rho \Delta w_t|c) - \text{Cov}(w_{t-3}, \Delta w_{t+1} - \rho \Delta w_t|c')} = \frac{\mu_{t-2}}{\mu_{t-3}}.$$

B Calculating Standard Errors

Let $m = 1, \dots, M$ be the index of moments. Let $d_{i,m}$ be the indicator of whether individual i contributes to the m^{th} moment $\text{Cov}(w_t, w_{t'}|s, A_j)$. That is, both $w_{i,t}$ and $w_{i,t'}$ are non-missing and $s_i = s$ and $a_{i,t} \in A_j$. Also let $p_m(\boldsymbol{\Lambda}) = \widetilde{\text{Cov}}(w_t, w_{t'}|s, A_j, \boldsymbol{\Lambda})$. Then we can write

$$h_m(\boldsymbol{z}_i, \boldsymbol{\Lambda}) = d_{i,m}\big[w_{i,t} w_{i,t'} - p_m(\boldsymbol{\Lambda})\big],$$

where $\boldsymbol{z}_i$ includes $w_{i,t}$ $d_{i,m}$ for all t and m for individual i. Let $\boldsymbol{h}(\boldsymbol{z}, \boldsymbol{\Lambda}) = (h_1(\boldsymbol{z}, \boldsymbol{\Lambda}), \dots, h_M(\boldsymbol{z}, \boldsymbol{\Lambda}))$. Then the following moment condition holds for the true parameter $\boldsymbol{\Lambda}_0$:

$$\mathbb{E}[\boldsymbol{h}(\boldsymbol{z}, \boldsymbol{\Lambda}_0)] = 0.$$

The minimum distance estimator $\hat{\boldsymbol{\Lambda}}$ is equivalent to the GMM estimator that solves

$$\min_{\boldsymbol{\Lambda}} \left[\frac{1}{N}\sum_{i=1}^{N} \boldsymbol{h}(\boldsymbol{z}_i, \boldsymbol{\Lambda})\right]' \boldsymbol{W} \left[\frac{1}{N}\sum_{i=1}^{N} \boldsymbol{h}(\boldsymbol{z}_i, \boldsymbol{\Lambda})\right],$$

where $\boldsymbol{W} = \text{diag}[\frac{N^2}{N_1^2}, \dots, \frac{N^2}{N_M^2}]$ and $N_m = \sum_{i=1}^{N} d_{i,m}$.

The GMM estimator $\hat{\boldsymbol{\Lambda}}$ is asymptotically normal with a variance matrix $\boldsymbol{V} = (\boldsymbol{H}'\boldsymbol{W}\boldsymbol{H})^{-1}(\boldsymbol{H}'\boldsymbol{W}\Omega\boldsymbol{W}\boldsymbol{H})(\boldsymbol{H}'\boldsymbol{W}\boldsymbol{H})^{-1}$ where $\boldsymbol{H}$ is the Jacobian of the vector of moments, $\mathbb{E}[\partial \boldsymbol{h}(\boldsymbol{z}, \boldsymbol{\Lambda}_0)/\partial \boldsymbol{\Lambda}']$, and $\boldsymbol{\Omega} = \mathbb{E}[\boldsymbol{h}(\boldsymbol{z}, \boldsymbol{\Lambda}_0)\boldsymbol{h}(\boldsymbol{z}, \boldsymbol{\Lambda}_0)']$. Both expectations are replaced by sample averages and evaluated at the estimated parameter:

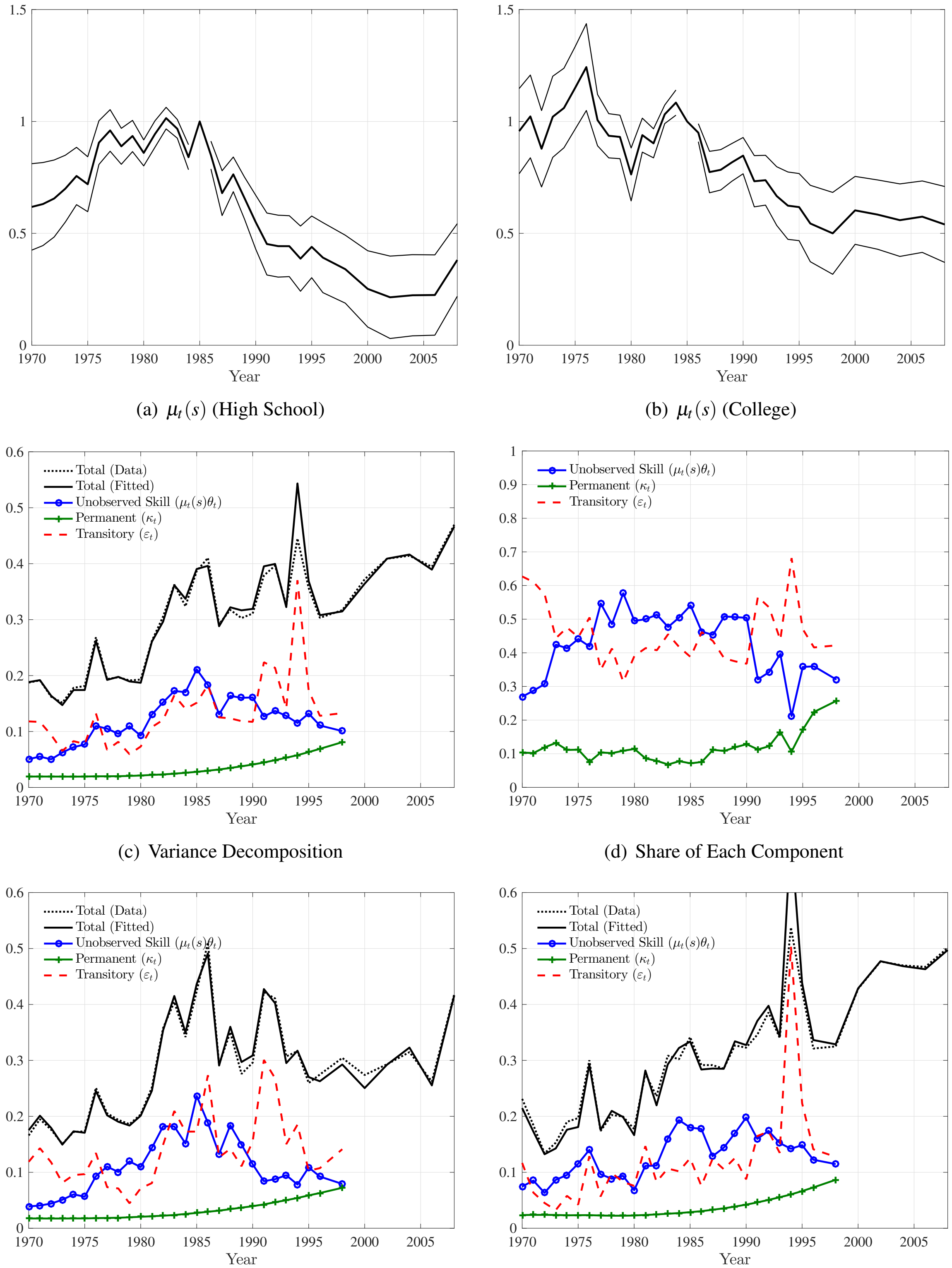
$$\begin{aligned}
\hat{\boldsymbol{H}} &= \frac{1}{N}\sum_{i=1}^{N} \frac{\partial \boldsymbol{h}(\boldsymbol{z}_i, \hat{\boldsymbol{\Lambda}})}{\partial \boldsymbol{\Lambda}'} = \boldsymbol{W}^{-\frac{1}{2}} \frac{\partial \boldsymbol{p}(\hat{\boldsymbol{\Lambda}})}{\partial \boldsymbol{\Lambda}'}, \\
\hat{\boldsymbol{\Omega}} &= \frac{1}{N}\sum_{i=1}^{N} \boldsymbol{h}(\boldsymbol{z}_i, \hat{\boldsymbol{\Lambda}})\boldsymbol{h}(\boldsymbol{z}_i, \hat{\boldsymbol{\Lambda}})',
\end{aligned}$$

where $\boldsymbol{W}^{-\frac{1}{2}} = \text{diag}[\frac{N_1}{N}, \dots, \frac{N_M}{N}]$.

We can test r linear parameter restrictions $H_0 : \boldsymbol{R}\boldsymbol{\Lambda} = 0$ using Wald test statistic:

$$\text{Wald} = N(\boldsymbol{R}\hat{\boldsymbol{\Lambda}})'(\boldsymbol{R}\hat{\boldsymbol{V}}\boldsymbol{R}')^{-1}\boldsymbol{R}\hat{\boldsymbol{\Lambda}} \xrightarrow{d} \chi^2_r.$$

C Additional Estimates for Log Annual Earnings



(a) $\mu_t(s)$ (High School)

(b) $\mu_t(s)$ (College)

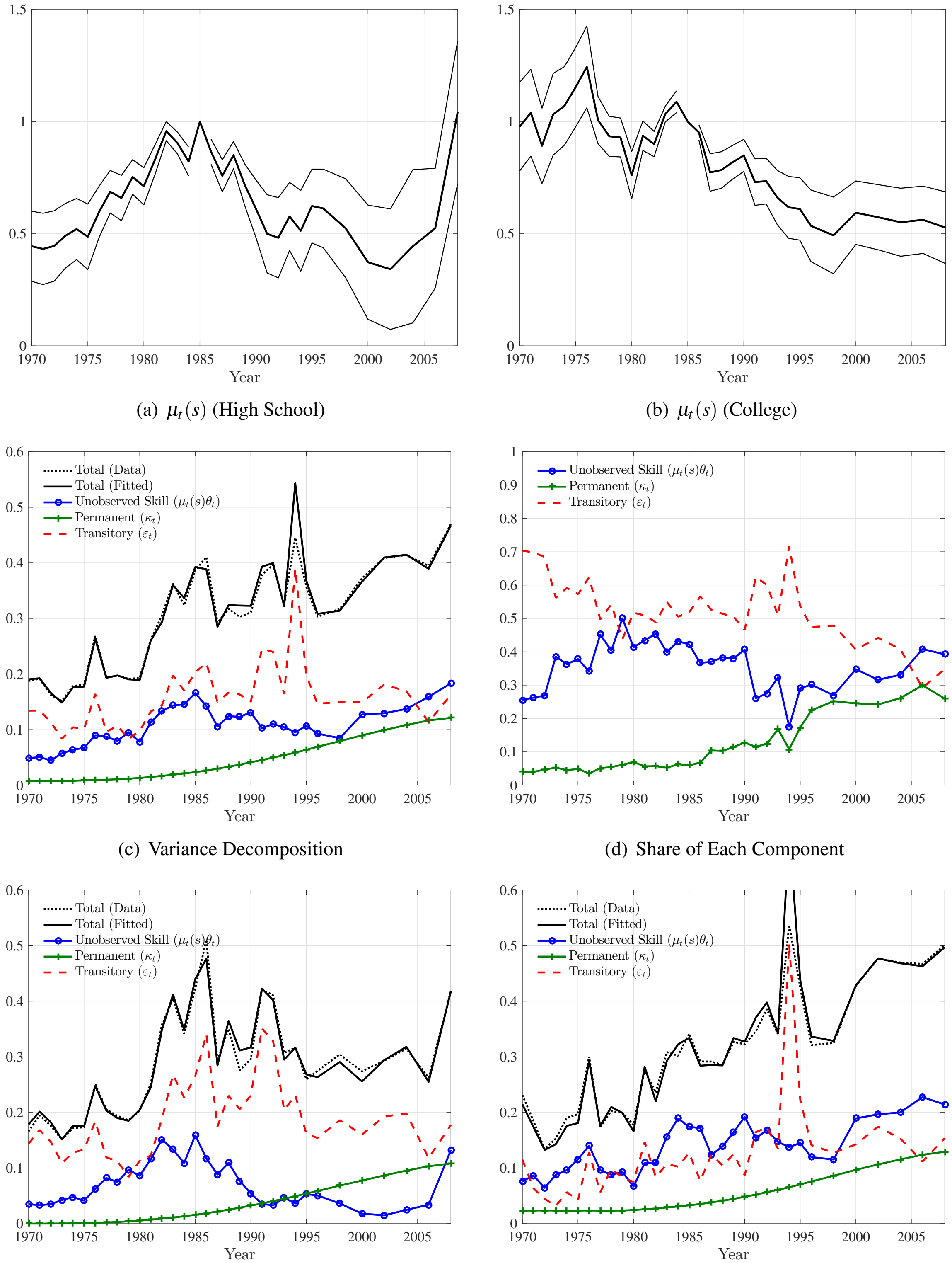
(c) Variance Decomposition

(d) Share of Each Component

(e) Variance Decomposition (High School)

(f) Variance Decomposition (College)

Figure 13: Estimates with MA(5) Transitory Shocks



(a) $\mu_t(s)$ (High School)

(b) $\mu_t(s)$ (College)

(c) Variance Decomposition

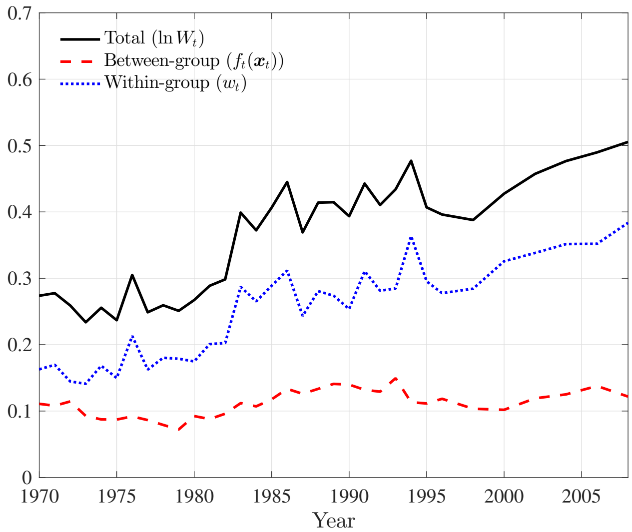
(d) Share of Each Component

(e) Variance Decomposition (High School)

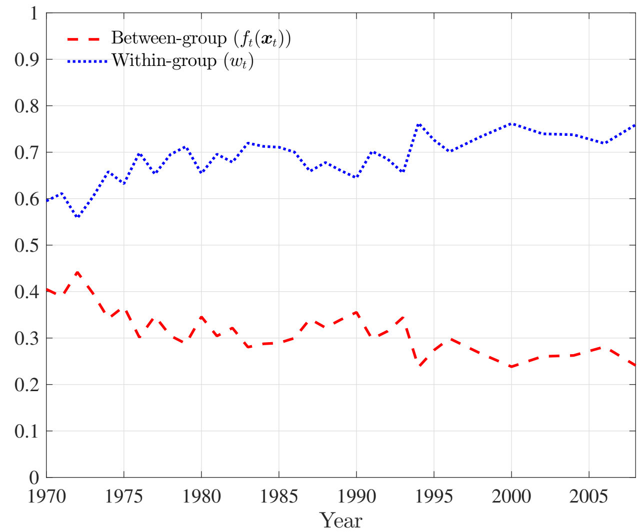
(f) Variance Decomposition (College)

Figure 14: Estimates with ARMA(1,1) Transitory Shocks

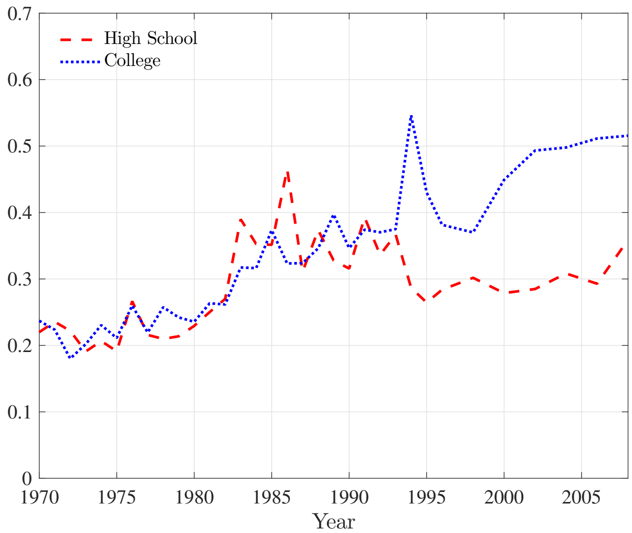
D Estimates for Log Weekly Earnings



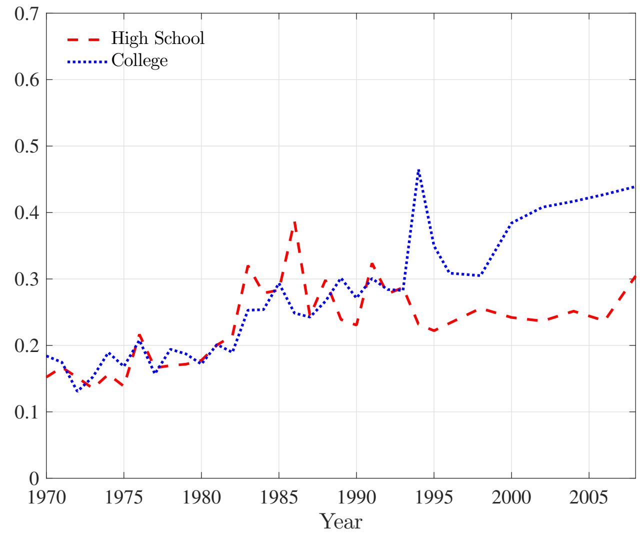
(a) Variance



(b) Share of Variance

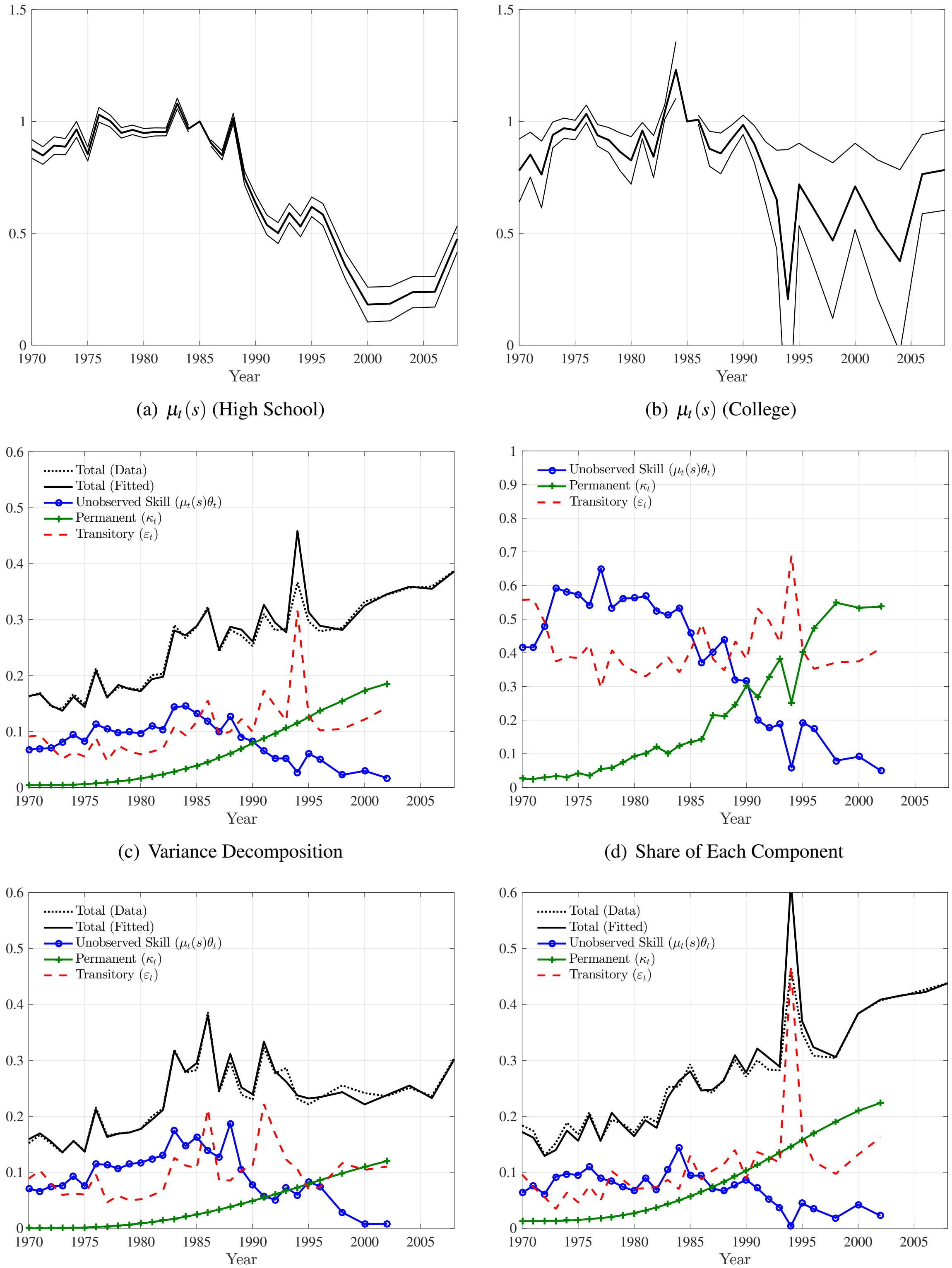


(c) Total Variance by Sector



(d) Within-group Variance by Sector

Figure 15: Total, Between-, and Within-group Variance of Log Weekly Earnings



(a) $\mu_t(s)$ (High School)

(b) $\mu_t(s)$ (College)

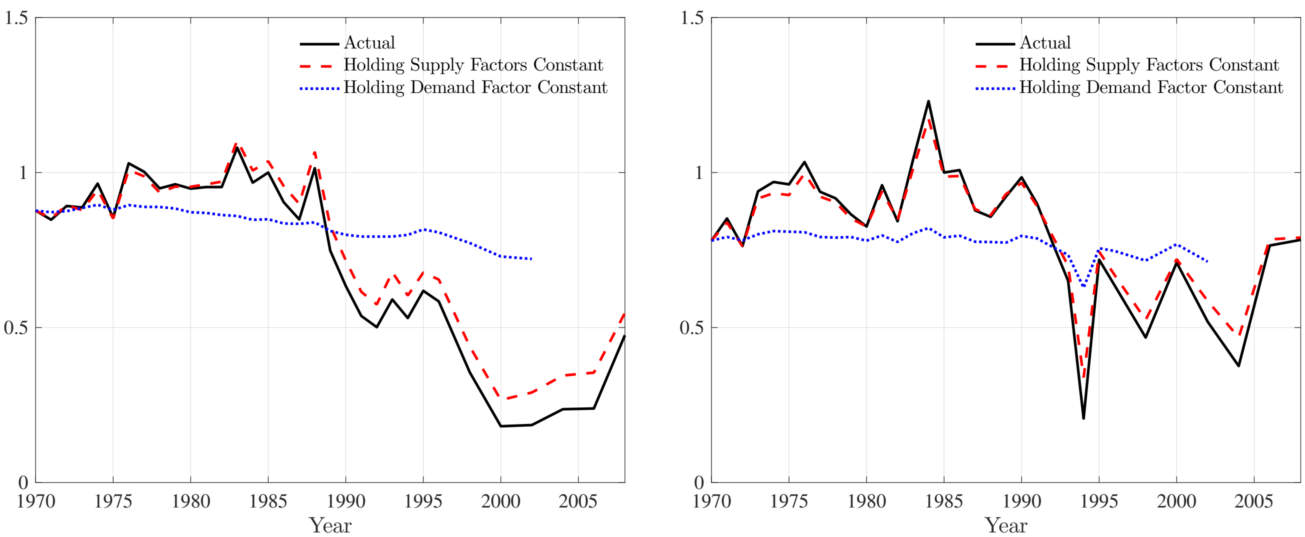
(c) Variance Decomposition

(d) Share of Each Component

(e) Variance Decomposition (High School)

(f) Variance Decomposition (College)

Figure 16: Estimates with Log Weekly Earnings and MA(3) Transitory Shocks



(a) High School (b) College

Figure 17: Actual and Counterfactual Returns ($\mu_t(s)$)