

Trade and Migration: A Quantitative Assessment

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Introduction

- ▶ Considerable advances on the quantification and understanding of the gains from economic integration
- ▶ Most of the focus has been on goods market
 - ▶ Direct consequence of observing reductions in trade costs (tariffs)
- ▶ Yet, most free trade agreements have an explicit commitment to liberalize factor markets, in particular labor markets
- ▶ In this study we use the EU 2003 accession, to study the gains from trade and migration
 - ▶ Exploit the variation in migration policy across countries and time to quantify the change in migration costs
 - ▶ Use a structural model of trade and migration to measure the gains from trade and migration

EU 2003 accession

- ▶ Agreement between member states of the European Union (EU) and New Member States (NMS)
- ▶ Integration in the goods market: zero tariffs starting in 2004
 - ▶ NMS countries resigned to previous FTA and joined EU FTA's
- ▶ Integration in the factor markets: Migration restrictions varied across countries (use LABour market REForm database)
 - ▶ Some EU countries granted access to citizens from NMS in 2004
 - ▶ For example, United Kingdom (UK)
 - ▶ Other countries followed a 2-3-2 years phase in period
 - ▶ For instance, Germany (DE)
 - ▶ Non discriminatory policy, except for Cyprus (CY)
 - ▶ All EU granted access starting 2004
- ▶ We exploit this variation in migration policies to estimate changes in migration costs

Preview of the model

- ▶ Ricardian trade model with multiple countries and trade costs
 - ▶ we introduce trade policy in the model
- ▶ Households with forward-looking migration decisions
 - ▶ migration decisions subject to time-varying migration costs, impacted by migration policy
 - ▶ Stock vs. flow of migrants: a NMS citizen residing in the UK has a different cost to migrate to DE relative to a EU citizen
- ▶ Guided by previous literature, the model accounts for congestion effects and scale effects
 - ▶ migration congests local fixed factors (land, infrastructure)
 - ▶ migrants also congest public goods, reducing households' utility
 - ▶ migration generates scale effects, i.e. positive effect on country's productivity
- ▶ Model builds on CDP (2015)

Mechanisms

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- ▶ Adding reduction in tariffs $\frac{w_i}{P_i} \downarrow\downarrow\uparrow\uparrow$

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- ▶ Adding scale effects $\frac{w_i}{P_i} \downarrow\downarrow\uparrow$
- ▶ Adding reduction in tariffs $\frac{w_i}{P_i} \downarrow\downarrow\uparrow\uparrow$
- ▶ With dynamics, welfare also includes the option value of migration

Results

- ▶ What are the welfare effects of the actual change in migration costs and the actual changes in tariffs as a consequence of the EU enlargement?
- ▶ Baseline model:
 - ▶ EU15's welfare increases 0.27%
 - ▶ NMS' welfare increases 4.70%
 - ▶ Europe's welfare increases 1.03%
- ▶ We quantify the welfare effects of the different margins in our model
 - ▶ Migration without changes to trade policy would have resulted in welfare losses for EU15

Model

A model of trade and migration

- ▶ Time is discrete, there is perfect foresight, and N locations
 - ▶ locations are indexed by i and j
 - ▶ nationality is indexed by n , for example NMS, EU
- ▶ The value of a n national in country i at time t is given by

$$v_{n,t}^i = \log(C_t^i) + \max_{\{j\}_{j=1}^N} \{ \beta E[v_{n,t+1}^j] - m_{n,t}^{ij} + \nu \epsilon_{n,t}^{ij} \}$$

- ▶ $\beta \in (0,1)$ discount factor
- ▶ $m_{n,t}^{ij}$ additive, migration cost from i to j conditional on n
- ▶ $\epsilon_{n,t}^{ij}$ are stochastic i.i.d idiosyncratic taste shocks
 - ▶ $\epsilon \sim$ Type-I Extreme Value distribution with zero mean
 - ▶ $\nu > 0$ is the dispersion of taste shocks

Household's problem - Dynamic migration decision

- ▶ Using properties of Type-I Extreme Value distributions one obtains:
- ▶ The expected (expectation over ϵ) lifetime utility of a worker at i

$$V_{n,t}^i = \log(C_t^i) + \nu \log \left[\sum_{j=1}^N \exp(\beta V_{n,t+1}^j - m_{n,t}^{ij})^{1/\nu} \right]$$

- ▶ Fraction of workers that reallocate from market i to j

$$\mu_{n,t}^{ij} = \frac{\exp(\beta V_{n,t+1}^j - m_{n,t}^{ij})^{1/\nu}}{\sum_{k=1}^N \exp(\beta V_{n,t+1}^k - m_{n,t}^{ik})^{1/\nu}}$$

- ▶ Evolution of the labor distribution of labor across markets

$$L_{n,t+1}^i = \sum_{j=1}^N \mu_{n,t}^{ji} L_{n,t}^j, \quad \text{for all } n$$

where $L_t^i = \sum_{n=1}^N L_{n,t}^i$

Household's problem

- ▶ Households supply a unit of labor inelastically
 - ▶ receive a competitive market wage w_t^i
- ▶ Consumption aggregator

$$C_t^i = \left(\frac{G^i}{L_t^i} \right)^{\alpha_i} \left((1 - \tau_{L,t}) \frac{w_t^i}{P_t^i} \right)^{1-\alpha_i},$$

where P_t^i is the local price index, and α_i is the fraction of public goods in total consumption

- ▶ $\frac{G^i}{L_t^i}$ is per capita provision of public goods => **congestion effect**
- ▶ Government finance spending with: tariff revenues, labor taxes ($\tau_{L,t}^i$), lump sum transfer from structure owners

Production - Static sub-problem

- ▶ Perfect competition, CRS technology, idiosyncratic productivity $z^i \sim \text{Fréchet}(1, \theta)$, deterministic sectoral regional TFP A_t^i (Eaton and Kortum 2002)

$$q_t^i(z^i) = z^i A_t^i (L_t^i)^{1-\gamma^i} (H^i)^{\gamma^i}$$

- ▶ $A_t^i = \phi L_t^i \Rightarrow$ scale effects, H^i fixed factor $\Rightarrow$ congestion effect
- ▶ Costly trade $\kappa_t^{ij} = (1 + \tau_t^{ij}) d_t^{ij}$
 - ▶ Ad-valorem tariffs $\tau_t^{ij} \geq 0$, with $\tau_t^{ii} = 0$
 - ▶ iceberg trade costs $d_t^{ij} \geq 1$, with $d_t^{ii} = 1$
- ▶ Trade shares, price index

$$\pi_t^{ij} = \frac{A_t^j (\kappa_t^{ij} \omega_t^j)^{-\theta}}{\sum_{k=1}^N A_t^k (\kappa_t^{ik} \omega_t^k)^{-\theta}}, \quad P_t^i = \xi \left(\sum_{j=1}^N A_t^j (\kappa_t^{ij} \omega_t^j)^{-\theta} \right)^{-\frac{1}{\theta}}$$

with $\omega_t^j = \zeta (w_t^j)^{1-\gamma^j} (r_t^j)^{\gamma^j}$

Production - Static sub-problem - Equilibrium conditions

- ▶ Government budget constraint

$$P_t^i G^i = T_t^i + \tau_{L,t}^i w_t^i L_t^i + R_t^i$$

- ▶ T_t^i are tariff revenues, and R_t^i are lump-sum transfers from the structure owners
- ▶ Goods market clearing

$$X_t^i = P_t^i G^i + (1 - \tau_{L,t}^i) w_t^i L_t^i + \iota^i \chi_t^i - R_t^i$$

with $\chi_t^i = \sum_{j=1}^N r_t^{ji} H^j$

- ▶ Labor market clearing

$$w_t^i L_t^i = (1 - \gamma^i) \sum_{j=1}^N \frac{\pi_t^{ji}}{(1 + \tau_t^{ji})} X_t^j$$

General equilibrium

- ▶ State of the economy = distribution of labor $L_t = \{L_{n,t}^i\}_{n=1,i=1}^{N,N}$
- ▶ Let $\Theta_t \equiv \left(\{m_{n,t}^{ij}\}, \{\kappa_t^{ij}\}, \{\tau_{n,t}^{ij}\}, \{A_t^i\} \right)_{n=1,i=1,j=1}^{N,N,N}$

Definition. Given (L_t, Θ_t) , a **temporary equilibrium** is a vector $w(L_t, \Theta_t)$ that satisfies the equilibrium conditions of the static sub-problem

Definition. Given (L_0, Θ_t) , a **sequential competitive equilibrium** of the model is a sequence $\{L_{n,t}, \mu_{n,t}, V_{n,t}, w(L_t, \Theta_t)\}_{t=0}^{\infty}$ that solves HH dynamic problem and the temporary equilibrium at each t

Solution Method

Counterfactuals

- ▶ We use the Dynamic Exact Hat Algebra of CDP
- ▶ Suppose we want to study changes in migration costs m_t^{ij} to a counterfactual value $m_t^{'ij}$
 - ▶ Let $\dot{x}_{t+1} = \frac{x_{t+1}}{x_t}$, and $\hat{x}_{t+1} = \dot{x}_{t+1}' / \dot{x}_{t+1}$
- ▶ Economy with actual change in fundamentals

$$\dot{\Theta}_t \equiv \left(\{\dot{m}_{n,t}^{ij}\}, \{\dot{\kappa}_t^{ij}\}, \{\dot{\tau}_{n,t}^{ij}\}, \{\dot{A}_t^i\} \right)_{n=1,i=1,j=1}^{N,N,N}$$

- ▶ Relative to an economy with all actual changes in fundamentals except for m_t^{ij} ,

$$\dot{\Theta}'_t \equiv \left(\{\dot{m}'_{n,t}^{ij}\}, \{\dot{\kappa}_t^{ij}\}, \{\dot{\tau}_{n,t}^{ij}\}, \{\dot{A}_t^i\} \right)_{n=1,i=1,j=1}^{N,N,N}$$

Solving for counterfactuals

Proposition. Given data (L_t, μ_t, π_t, X_t) , $(\nu, \theta, \beta, \alpha)$, and $\{\hat{\Theta}_t\}_{t=0}^{\infty}$ solving the model with the Dynamic Exact Hat Algebra does not require Θ_t , and solves

$$\hat{u}_{n,t+1}^i = (\hat{C}_{n,t+1}^i)^{1/\nu} \sum_{j=1}^N \mu_{n,t-1}'^{ij} \dot{\mu}_t^{jk} \left(\hat{m}_t^{ij} \right)^{-1/\nu} \left(\hat{u}_{t+1}^j \right)^{\beta/\nu}$$

$$\mu_{n,t}'^{ij} = \frac{\mu_{n,t-1}'^{ij} \dot{\mu}_t^{jj} \left(\hat{m}_t^{ij} \right)^{-1/\nu} \left(\hat{u}_{t+1}^j \right)^{\beta/\nu}}{\sum_{k=1}^N \mu_{n,t-1}'^{ik} \dot{\mu}_t^{jk} \left(\hat{m}_t^{ik} \right)^{-1/\nu} \left(\hat{u}_{t+1}^k \right)^{\beta/\nu}},$$

$$L_{n,t+1}'^i = \sum_{j=1}^N \mu_{n,t}'^{ji} L_{n,t}^j, \quad \text{for all } n$$

where $\hat{C}_{t+1}^i = \left(\hat{w}_t^i / \hat{P}_t^i \right)^{1-\alpha_i} \left(1 / \hat{L}_t^i \right)^{\alpha_i}$ solves the temporary equilibrium

Data and Estimation

Data

- ▶ To compute the model we need data on
 - ▶ Migration flows by nationality $\mu_{n,t}^{ij}$
 - ▶ Bilateral tariffs τ_t^{ij}
 - ▶ Trade and production data across countries π_t^{ij}, X_t^i
- ▶ estimates of
 - ▶ Migration cost elasticity ν
 - ▶ Changes in migration costs $m_{n,t}^{ij}$
- ▶ Next slides: data description, estimation of the migration elasticity, estimation of changes in migration costs

Data

- ▶ Migration flows - European labour force survey (EU-LFS)
 - ▶ Access to the confidential micro data
 - ▶ The basis for major European statistics, i.e. unemployment rate
- ▶ Construct migration flows based on information on past year residence
 - ▶ We keep only people between 15 and 65
- ▶ We can distinguish between “EU15 nationals”, “NMS nationals” and “others”
- ▶ Consistent data over 2002-2007 for
 - ▶ EU15: Austria, Belgium, Germany, Denmark, Spain, France, UK, Greece, Italy, Portugal
 - ▶ NMS: Cyprus, Czech Republic, Estonia, Hungary, Lithuania, Latvia, Poland

Data

- ▶ Bilateral tariff data - World integrated trade solutions (WITS)
 - ▶ We use effectively applied rate as tariff type
- ▶ Trade data and production data - World Input-Output database (WIOD)
 - ▶ Same sample of European countries plus a constructed Rest of the World
- ▶ α_i computed as final government consumption over total final consumption
- ▶ We estimate ν using the methodology in Artuç and McLaren (JIE 2016)
 - ▶ We use flows of EU nationals within the EU
 - ▶ We obtain $\nu = 2.1$

Changes in migration costs

Goal: evaluate the changes in migration costs due to EU accession (up to the elasticity ν)

- ▶ Main events:
 - ▶ UK opening to NMS8 countries in 2004
 - ▶ GR, ES, PT, & IT opening to NMS8 countries in 2006-7
 - ▶ NMS8 countries opening to each other in 2004
- ▶ Methodology: **Difference-in-difference** approach, guided by the model
- ▶ Key identifying assumption: trends in migration costs between treated countries would have been the same as those with (and between) control countries in the absence of the entry into the EU

Measuring changes in migration costs

- ▶ From the model, the log ratio of the number of n nationals migrating from country i to country j to the number of stayers in i is

$$\log \frac{\mu_{t,n}^{ij}}{\mu_{t,n}^{ii}} = -\frac{1}{\nu} \left(m_{t,n}^{ij} - m_{t,n}^{ii} \right) + \frac{\beta}{\nu} V_{t+1,n}^j - \frac{\beta}{\nu} V_{t+1,n}^i$$

- ▶ We identify the change, around the date of the EU accession, in $-\frac{1}{\nu} m_{t,n}^{ij}$...
 - ▶ Assuming $m_{t,n}^{ii}$ does not change
 - ▶ And controlling for $\frac{\beta}{\nu} V_{t+1,n}^j$ and $\frac{\beta}{\nu} V_{t+1,n}^i$ via destination-time-nationality and origin-time-nationality fixed effects, respectively

Changes in migration costs: NMS to UK

- Estimates of the change—before vs. after 2004—in $\left(-\frac{1}{\nu}\tau_{t,n}^{NMS8,UK}\right)$ for NMS8 and EU nationals

	NMS8 Nationals	EU Nationals
Change	4.19** (1.86)	0.37 (2.24)
Observations	275	222
R^2	0.86	0.21

Robust standard errors in parentheses

** $p < 0.05$, * $p < 0.1$

- We proceed in the same way to estimate all the changes in migration costs as a result of changes to migration policies over 2002-2007

Counterfactuals (preliminary)

Welfare effects from trade and migration policy

What are the welfare effects from the actual changes in migration and trade policy?

	Welfare effects (percent)				
EU15	0.03				
NMS	3.29				
Europe	0.59				
change to migration policy	✓				
congestion effects					
trade openness					
change to trade policy					
scale effects					

Welfare effects from trade and migration policy

What are the welfare effects from the actual changes in migration and trade policy?

	Welfare effects (percent)				
EU15	0.03	-0.12			
NMS	3.29	4.13			
Europe	0.59	0.60			
change to migration policy	✓	✓			
congestion effects		✓			
trade openness					
change to trade policy					
scale effects					

Welfare effects from trade and migration policy

What are the welfare effects from the actual changes in migration and trade policy?

	Welfare effects (percent)				
EU15	0.03	-0.12	-0.13		
NMS	3.29	4.13	4.18		
Europe	0.59	0.60	0.60		
change to migration policy	✓	✓	✓		
congestion effects		✓	✓		
trade openness			✓		
change to trade policy					
scale effects					

Welfare effects from trade and migration policy

What are the welfare effects from the actual changes in migration and trade policy?

	Welfare effects (percent)				
EU15	0.03	-0.12	-0.13	0.22	
NMS	3.29	4.13	4.18	4.98	
Europe	0.59	0.60	0.60	1.03	
change to migration policy	✓	✓	✓	✓	
congestion effects		✓	✓	✓	
trade openness			✓	✓	
change to trade policy				✓	
scale effects					

Welfare effects from trade and migration policy

What are the welfare effects from the actual changes in migration and trade policy?

	Welfare effects (percent)				
EU15	0.03	-0.12	-0.13	0.22	0.27
NMS	3.29	4.13	4.18	4.98	4.70
Europe	0.59	0.60	0.60	1.03	1.03
change to migration policy	✓	✓	✓	✓	✓
congestion effects		✓	✓	✓	✓
trade openness			✓	✓	✓
change to trade policy				✓	✓
scale effects					✓

Case study: UK

- ▶ What were the welfare effects of the EU enlargement for the UK?
 - ▶ UK's welfare increases 0.1 percent
- ▶ What would have been the costs/gains for the UK of going to trade and migration autarky with the rest of Europe?
 - ▶ UK's welfare would have declined by 0.9 percent

Conclusion

- ▶ Develop a dynamic model for trade and migration policy analysis
 - ▶ take into account role of tariff revenues, public goods, congestion effects, changes in migration costs, stock v/s flow of migrants
 - ▶ model guides the estimation of changes in migration costs as a consequence of changes in migration policy
- ▶ Compute the effects of actual changes in trade and migration policy as a consequence of the EU enlargement of 2004
 - ▶ preliminary results show welfare gains, heterogenous across countries
 - ▶ effects vary depending on the type of policy implemented
- ▶ Future work: more counterfactuals and results, more analysis on the mechanisms driving the results, different skills (if possible)

Thank you!

Equilibrium conditions

- Transition matrix (migration flows) $\{\mu_t^{ij}\}_{t=0}^T$, **Data**

$$\mu_{n,t}^{ij} = \frac{\exp(\beta V_{n,t+1}^j - m_{n,t}^{ij})^{1/\nu}}{\sum_{k=1}^N \exp(\beta V_{n,t+1}^k - m_{n,t}^{ik})^{1/\nu}}$$

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- Transition matrix at t , from Model given fundamentals m'_t

$$\mu_{n,t}^{'ij} = \frac{\exp(\beta V_{n,t+1}^{'j} - m_{n,t}^{'ij})^{1/\nu}}{\sum_{k=1}^N \exp(\beta V_{n,t+1}^{'k} - m_{n,t}^{'ik})^{1/\nu}}$$

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- Transition matrix at t , from Model given fundamentals m'_t

$$\mu_{n,t}^{'ij} = \frac{\exp(\beta V_{n,t+1}^{'j} - m_{n,t}^{'ij})^{1/\nu}}{\sum_{k=1}^N \exp(\beta V_{n,t+1}^{'k} - m_{n,t}^{'ik})^{1/\nu}}$$

- Take the differences at each t . Model relative to data

$$\mu_{n,t}^{'ij} = \frac{\mu_{n,t}^{ij} \exp(V_{n,t+1}^{'j} - V_{n,t+1}^j)^{\beta/\nu} \exp(m_{n,t+1}^{'j} - m_{n,t+1}^j)^{-1/\nu}}{\sum_{k=1}^N \mu_{n,t}^{ik} \exp(V_{n,t+1}^{'k} - V_{n,t+1}^k)^{\beta/\nu} \exp(m_{n,t+1}^{'k} - m_{n,t+1}^k)^{-1/\nu}}$$

Equilibrium in “hats”

- Denote by:

$$\hat{u}_t^i = \dot{u}_t^{'i} / \dot{u}_t^i$$

$$\hat{m}_t^{ij} = \exp(m_t^{'ij} - \textcolor{blue}{m}_t^{ij}) / \exp(m_{t-1}^{'ij} - \textcolor{blue}{m}_{t-1}^{ij})$$

$$\dot{\mu}_t^{ij} = \mu_t^{ij} / \mu_{t-1}^{ij}$$

- and generically

$$\hat{\Theta}_t = \hat{\Theta}_t^{'} / \dot{\Theta}_t$$

- Take the time difference to obtain

$$\mu_{n,t}^{'ij} = \frac{\mu_{n,t-1}^{'ij} \dot{\mu}_t^{ij} \left(\hat{m}_t^{ij} \right)^{-1/\nu} \left(\hat{u}_{t+1}^j \right)^{\beta/\nu}}{\sum_{k=1}^N \mu_{n,t-1}^{'ik} \dot{\mu}_t^{ik} \left(\hat{m}_t^{ik} \right)^{-1/\nu} \left(\hat{u}_{t+1}^k \right)^{\beta/\nu}}$$

Equilibrium in “hats”

- Analogously, trade shares

$$\dot{\pi}_t^{ij} = \dot{A}_t^j \left(\frac{\dot{\kappa}_t^{ij} \dot{\omega}_t^j}{\dot{P}_t^i} \right)^{-\theta}$$

$$\dot{\pi}_t^{'ij} = \dot{A}_t^j \left(\frac{\dot{\kappa}_t^{ij} \dot{\omega}_t^j}{\dot{P}_t^{'i}} \right)^{-\theta}$$

$$\hat{\pi}_t^{ij} = \left(\frac{\hat{\omega}_t^j}{\hat{P}_t^i} \right)^{-\theta}$$

- where

$$\hat{P}_t^i = \left(\sum_{j=1}^N \pi_{t-1}^{'ij} \dot{\pi}_t^{ij} (\hat{\omega}_t^j)^{-\theta} \right)^{-\frac{1}{\theta}}$$