

INTERBANK NETWORK DISRUPTIONS AND THE REAL ECONOMY*

Dasha Safonova[†]
University of Notre Dame

JOB MARKET PAPER

December 13, 2016

[\[Click here for the most recent version\]](#)

Abstract

Shocks to the structure of the interbank lending network can have important macroeconomic repercussions. This paper examines the impact of the dynamic structure of the interbank lending network on interest rates and investment in the nonfinancial sector. By incorporating a network of bank relationships into a general equilibrium model with monetary policy, I show that the aggregate interest rate increases in response to a shock that destroys a large fraction of bank relationships and decreases in response to a shock that destroys a small fraction of relationships. Moreover, the shape of the interbank network matters for these dynamics: the interest rate is least responsive to the network disruptions if the interbank network is scale-free. Additionally, the amplification and propagation of the network shocks depend on the corridor of the policy rates set by the central bank. In particular, as the difference between the discount rate and the excess reserve rate decreases, the effect of a network disruption on interest rates becomes less significant but more persistent, which in turn leads to a smaller but more prolonged effect on the real sector.

Keywords: banks; interbank network; liquidity; monetary policy

JEL classification: D85, E44, E52, G21

*I am extremely grateful to Timothy Fuerst, Eric Sims, and Rüdiger Bachmann for their guidance and advice during this project. I also thank Huberto Ennis, Gary Richardson, Ned Prescott, Joseph Kaboski, Christiane Baumeister, Matthew Knowles, and Jing Wang for their valuable comments and suggestions. Finally, I thank all participants at seminars at the FRB Richmond and University of Notre Dame for questions, comments, and insights that helped to develop and improve this paper.

[†]E-mail: dsafonov@nd.edu; tel.: +1 (757) 641 9851

1 Introduction

Interbank lending markets enhance the efficiency of the banking sector by allowing banks to resolve temporary imbalances and to smoothly supply credit to the nonfinancial sector, which, in turn, promotes economic growth. Traditionally, interbank lending markets are assumed to be frictionless in the context of macroeconomic analysis. However, the distress in the financial sector observed in recent years calls the validity of this assumption into question. Furthermore, empirical studies of interbank markets document that banks do not typically utilize interbank markets to their full capacity, providing evidence for barriers that prevent banks from trading with each other.¹ Such barriers can arise as a result of asymmetric information, costly coordination, geographical/time constraints, or other frictions.² Because of these frictions, not all banks may have the same chance of finding a trading partner in the interbank lending market.

This paper is the first to consider bank-specific trading opportunities within a dynamic general equilibrium model suited for monetary policy analysis. Even though this friction has been previously investigated in a static and/or partial equilibrium setting (for example, [Allen and Gale \(2000\)](#) and [Freixas et al. \(1998\)](#)), the existing models that analyze the banking sector within a dynamic general equilibrium environment abstract from bank-specific conditions in the interbank loan market, limiting our understanding of how changes in these conditions may affect the real economy. Given that central banks target the interest rate on short-term interbank loans, one crucial aspect is how this transmission depends on monetary policy in

¹For empirical evidence that documents the sparse nature of interbank market participation see: [Soramäki et al. \(2007\)](#) and [Bech and Atalay \(2010\)](#) for evidence from the federal funds market; [Iori et al. \(2008\)](#) for Italian interbank market; [Boss et al. \(2004\)](#) for Austrian interbank market; [Inaoka et al. \(2004\)](#) for Japanese interbank market; [Bräuning et al. \(2012\)](#) and [Craig and von Peter \(2014\)](#) for German interbank network; and [Vila et al. \(2010\)](#) for unsecured overnight market in the United Kingdom.

²For example, [Soramäki et al. \(2007\)](#) find that participation in the interbank market fell following the attacks of September 11, 2001, due to the decrease in coordination which was, most likely, a result of operational problems. [Calomiris and Carlson \(2016\)](#) show that, during the National Banking era, banks in areas with more manufacturing firms maintained more network connections. Finally, a vast literature covers a surge in expected counterparty risk following the failure of Lehman Brothers. Some examples are [Nier et al. \(2007\)](#), [Gupta et al. \(2013\)](#), [Blasques et al. \(2016\)](#), [Beltran et al. \(2015\)](#).

an economy that features *both* heterogeneous and time-varying trading opportunities in the interbank lending market.

Because banks face liquidity shocks, which arise due to desynchronized revenues and outlays, they rely on the interbank loan market at times of cash shortages. Even though loans from the central bank are also available, they are typically more costly than interbank loans; therefore, banks attempt to borrow in the interbank market before relying on last-resort loans from the monetary authority. Depending on the distribution of interbank trading opportunities, some banks may be more successful in getting interbank loans than others, which gives rise to heterogeneous liquidity funding costs. A bank with a higher expected cost of financing liquidity does two things *ex ante*: first, it charges a higher interest rate on loans supplied to the real sector; and second, it chooses to hold a greater share of its assets in cash. All else equal, both actions result in a decline of bank's lending to the real economy. What happens in general equilibrium, however, depends on how the bank's trading opportunities compare to the trading opportunities of other banks at a given point in time. In this paper, I provide a framework that qualifies these general equilibrium effects.

More specifically, I incorporate a dynamic network of bank relationships into the [Bianchi and Bigio \(2014\)](#) framework. I define a relationship as a potential bilateral trading opportunity in the interbank loan market. While [Bianchi and Bigio \(2014\)](#) bring insights from the liquidity management literature into a dynamic macro model and propose a novel mechanism for monetary policy transmission, they implicitly assume that all banks are connected to each other (i.e. the interbank network is complete). By relaxing this assumption, I can address the following unanswered questions. First, how do various network disruptions affect interest rates on loans to the nonfinancial sector, and, consequently, aggregate investment? Second, how does a particular shape of the interbank network matter for these dynamics? Third, how does the amplification and propagation of network shocks depend on monetary policy?

I consider two scenarios for network disruption shocks: an interbank market freeze, in

which all of the network connections are destroyed, and a partial network destruction, in which only a fraction of the network links ceases at the time of the shock. For both scenarios, I study two general cases for the steady-state interbank network: a complete network where all banks are connected to each other and an incomplete network in which the total number of links (connections) is below the maximum possible number of connections. I further consider three different sub-cases of an incomplete network. The first sub-case is the random network, in which banks have an equal probability of being connected to any other bank in the network. The second sub-case is the circle network, where banks have relationships with a given number of closest neighbors. Finally, I examine the scale-free network sub-case, in which only a small fraction of banks has many connections and the rest of banks have little to no connections.

In the first experiment, the economy starts with a complete interbank network and is subjected to an interbank market freeze. This shock captures a reduced level of trust in a world with asymmetric information about the counterparty risk. Following the shock, the expected cost of funding liquidity increases, resulting in an increase in the aggregate interest rate and a decrease in the aggregate investment in the real sector. However, as banks can always borrow funds from the central bank at the discount window rate, the response of the interest rate is limited by the spread between the discount window rate and the excess reserve rate (the interest rate corridor).³ For example, if the width of the corridor is 4 percent on an annual basis, the aggregate interest rate increases by 40 basis points following the interbank market freeze. Given that the model does not feature firm or bank default, the effect on the aggregate investment is modest. In the baseline calibration, aggregate loan supply decreases by 0.25 percent on impact with the maximum decrease of 0.45 percent three periods after the shock.⁴

In the second experiment, the economy starts with a complete interbank network and is subjected to a partial network destruction. The responses of the aggregate interest rate

³This result was first discussed in Proposition 5 in [Bianchi and Bigio \(2014\)](#).

⁴These values are in line with [Bianchi and Bigio \(2014\)](#).

and the aggregate investment depend largely on the size of the shock. As the fraction of connections destroyed at the time of the shock decreases from 100 percent (as in the interbank market freeze scenario), the response of the aggregate rate becomes less positive and switches to negative when the fraction of destroyed connections reaches a threshold value. In the baseline calibration, the threshold value is approximately 50 percent. That is, when more than a half of the links are terminated, the aggregate interest rate increases and the total investment decreases, and when less than a half of the connections are destroyed, the interest rate *decreases* and investment *increases*. This result highlights that the responses of interest rates and investment are nonlinear in the size of the network disruption, implying that smaller shocks could be important to study separately from interbank market freezes.

Network shocks also have significant cross-sectional implications for the distribution of bank equity: both the total and partial destruction shocks lead to a persistent increase in the equity variance. These distributional changes are observed because of the gradual nature of the interbank network recovery. In particular, some banks re-establish their connections sooner than others, gaining a competitive advantage in lending and accumulating equity at a faster rate. On the other hand, some banks do not regain their connections for a long time, which prevents them from lowering interest rates and increasing their supply of loans to the real sector. This, in turn, stagnates their equity growth for multiple periods. As a result, the standard deviation of bank equity remains large for multiple periods after the shock, even when the aggregate equity reverts to its pre-shock level. This result suggests that interbank network shocks are one potential source of observed differences in bank equity.

In the third experiment, I compare the responses of variables to an interbank market freeze in the complete network to those in the incomplete network cases (random, circle, scale-free). I find that both the mean and the variance of the interest rates on loans to the nonfinancial sector are highest in the scale-free network. However, the aggregate loan rate is most responsive to the network destruction shock in the cases of random and circle networks. The distribution of bank's equity is most affected (relatively to the initial distribution) when

the interbank network is scale-free.

To highlight the role of monetary policy in the transmission of network disruptions onto the real economy, I present an experiment in which I vary the width of the interest rate corridor (the difference between the discount window rate and excess reserve rate). In the complete network case, the aggregate interest rate is less sensitive to the network destruction shock when the corridor of policy rates is narrow. However, the narrower the corridor, the longer the shock affects the banks' equity. The amplification result holds for other network shapes. The differences in propagation of the shock caused by the corridor adjustment are network-specific.

The structure of the paper is as follows. Section 2 introduces the background for interbank markets and networks. Section 3 presents the model. Section 4 discusses the alternatives for the shape of the steady-state interbank network. Section 5 describes the choices for the model parameters. Section 6 presents the results. Section 7 provides policy implications. Section 8 concludes.

2 Background

2.1 Banking sector

2.1.1 Why does interbank lending exist?

Commercial bank assets can be categorized into two categories: liquid assets (low return) and illiquid assets (high return). Liquid assets of a typical bank include bank's reserves (vault cash and bank's deposits at its account with the central bank), short-term securities, and repurchase agreements.⁵ The main categories of illiquid assets in the banking sector are commercial and industrial loans and real estate loans, more generally loans to the non-financial sector. In this paper, I categorize all assets that can be converted to cash *within a*

⁵Repurchase agreement is a contract in which the seller of a security agrees to repurchase it from the buyer at an agreed price.

period as liquid, and all assets that mature at some future time as illiquid. For example, if one period is a day, a loan that matures tomorrow is considered illiquid today.

The main category on the liability side of a commercial bank is customer deposits. A large fraction of these deposits is demand deposits. The key characteristic of demand deposits is that customers can withdraw them or make additional deposits at any point during a period without a penalty. I further refer to both deposit withdrawals and additional deposits as withdrawals, with a negative withdrawal being a deposit. When a customer makes a withdrawal, the bank has to draw on its cash assets to meet this sudden demand. Every period, banks choose the portfolio shares of liquid and illiquid assets, taking in account that they may experience some withdrawals after the portfolio decision has been made.

Depending on the sign and size of deposit withdrawals, a bank can end up in three possible situations. Figure 1 illustrates these cases. If, after the portfolio decision, a bank experiences a negative withdrawal (deposit), the amount of liquid assets increases and the size of bank's portfolio expands. This situation is referred to as having excess reserves. If a bank experiences a positive withdrawal *and* the size of withdrawal is smaller than the amount of liquid assets, the resulting balance of liquid assets decreases and the size of bank's portfolio contracts. Finally, if the bank experiences a positive withdrawal *and* the amount of withdrawal is greater than the amount of liquid assets on hand, the bank has a deficit of liquid assets. This situation is referred to as having a reserve deficit. To avoid a default, a bank will borrow the amount of deficit from an available source. Banks can always take a last-resort loan from the central bank. Alternatively, they may seek a loan at a more beneficial interest rate from a bank with excess reserves. This constitutes the demand for liquidity. Why would a bank with excess reserves want to lend to a bank with a reserve deficit? Because otherwise, it earns no (or lower) interest on the excess reserves. This constitutes the supply of liquidity. A market for interbank lending arises.

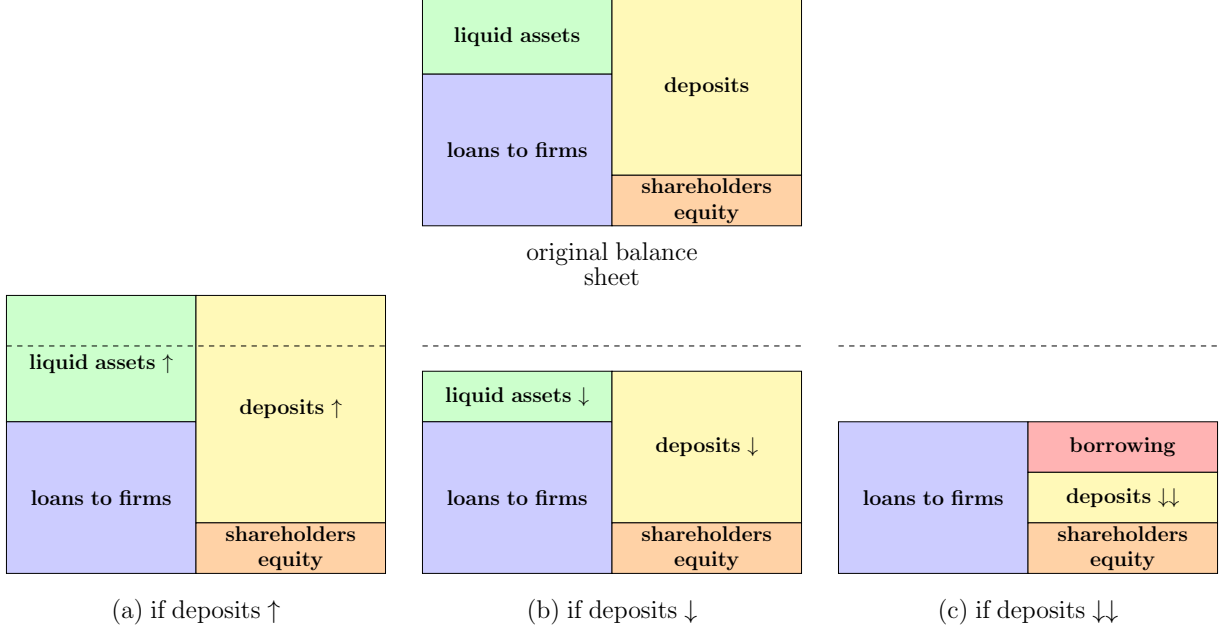


Figure 1. Bank's balance sheet and stochastic deposits/withdrawals. Dashed line in the three bottom panels indicates the original size of the bank's balance sheet.

2.1.2 Why do banks build relationships?

A typical feature of an interbank lending market is that it is an over-the-counter (OTC) market, which implies that, if a bank wants to trade, it has to find a partner first. A key characteristic of the demand for liquidity is that the bank with a reserve deficit has to find the lender during the same period it experiences the deposit withdrawals that lead to the deficit. Given the OTC nature, this might be difficult to do. Additionally, banks are subject to stochastic withdrawals every period, which implies that, in the absence of some longer-term corresponding relationship with another bank(s), they start their search for trading partners from zero every period. By creating a network of “friends” banks can reduce this search cost. Therefore, it is beneficial for banks to build long-term relationships with each other in order to reduce the cost of finding a trading partner.

In the context of this paper, I think of a relationship between two banks as being on each other's contact list. A relationship, however, is not a contract and does not obligate banks to trade with each other. It merely implies that a bank with a deficit/excess can contact another

bank to check whether it has an excess or a deficit of reserves. The list of all relationships constitutes the interbank network. Note that this is not the typical definition: an interbank network is usually defined as a set of *observed* interbank trades. This distinction matters for the sequence of the events within a period.

2.2 Network theory

Graphs. Network theory is the study of graphs that represent a relationship between discrete objects. A graph is an ordered pair $G = (V, E)$ where V is a set of nodes (vertices) and E is a set of edges (connections, links) which are two-element subsets of V . Graphs can be *directed* and *undirected*. An undirected graph is a graph in which edges have no orientation. A graph is *simple* if it does not contain multiple edges or self-loops are disallowed. Maximum number of edges in a graph is $L_{max} = N(N - 1)/2$. Network is called sparse if $L \ll L_{max}$.

I will consider simple undirected graphs in the context of this paper. Figure 2 shows an example of an undirected simple graph with five vertices and five edges. The set of nodes is $V = \{A, B, C, D, E\}$ and the set of edges is $E = \{\{A, B\}, \{A, E\}, \{B, C\}, \{C, E\}, \{E, D\}\}$. A graph can be represented with the *adjacency matrix*, which is a square $|V| \times |V|$ matrix

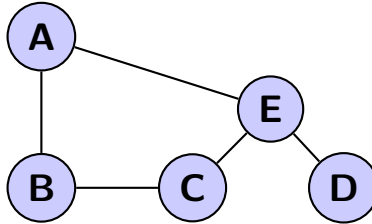


Figure 2. Simple undirected graph.

with non-zero elements for all the edges. An *unweighted* adjacency matrix has elements that are either 0 or 1. The adjacency matrix of an undirected graph is symmetric. The adjacency

matrix for the graph in Figure 2 is:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

Network types. One convenient measure of a network is a *node degree*, which is equal to the number of nodes a node is connected to. Networks are classified by their degree distributions. The *degree distribution*, p_k , is the probability distribution of node degrees over the whole network. If the degree distribution is binomial, the network is a *random network*. A concept of a random network was introduced by Erdős and Rényi (1959). Random networks observe a small world property – the distance between two randomly chosen nodes in a network is short. However, random networks do not observe high clustering, which is what most real-world networks observe.⁶ A network model introduced by Watts and Strogatz (1998) is an extension of the random network model that addresses the coexistence of high clustering and the small world property. It fails to explain, however, why high-degree nodes have a smaller clustering coefficient than low-degree nodes. If degree distribution follows a power law, a network is called a *scale-free network*. Scale-free networks were first discussed by Barabási and Albert (1999). Scale-free networks are commonly observed in the real world. An important property of the scale-free networks is the existence of “hubs” – nodes with an extremely high degree. Removing a hub from a network may turn the network into one with a lot of disconnected nodes.

⁶See Barabási (2016).

3 The model

3.1 Environment

Time is discrete, indexed by t , and has an infinite horizon. The economy consists of aggregate household, aggregate firm, a large number of banks, and a central bank. Household is risk-neutral, i.e. it is indifferent between consumption and bank deposits. The household supplies labor to the aggregate firm. Labor is the only factor of production. The firm is subject to the working-capital constraint, therefore it needs to borrow from the banking sector every period. The firm views loans from different banks as imperfect substitutes.

Banks take deposits from the household, provide loans to the non-financial firm, and hold cash assets (reserves) in their deposit accounts at the central bank. Demand deposits are the only type of liability.

3.2 Banking sector

3.2.1 Bank's Portfolio

In a given period, banks choose to allocate available funds towards two types of assets: liquid and illiquid. Liquid assets (cash assets or reserves) are held in banks' accounts at the central bank and are used to settle transactions. Illiquid assets cannot be used during a given period. Loans to nonfinancial firms constitute bank's illiquid assets. Banks take deposits from the aggregate household to fund the new loan issuances and new purchases of liquid assets.

In every period before portfolio decisions are made, banks pay out a constant fraction of equity, θ , as dividends. The bank makes portfolio decisions subject to the following balance sheet:

Assets		Liabilities and Equity	
Loans	$\tilde{B}_i$	Deposits	$\tilde{D}_i$
Cash assets	$\tilde{C}_i$	Equity	$(1 - \theta) E_i$

where $\tilde{B}_i$, $\tilde{C}_i$, and $\tilde{D}_i$ are the choice variables and E_i is the state variable.

Deposits. Deposits earn an interest rate of R^d , which is the same for all banks and is paid at the beginning of the next period. Deposits are perfectly elastic; however, they are bounded by the capital requirement κ :

$$\tilde{D}_i \leq \kappa (1 - \theta) E_i \quad (1)$$

The capital requirement is a policy instrument that imposes an upper bound on the debt-to-equity ratio. Capital requirements prevent taking on excess leverage by financial institutions.

A key feature of the model is that the deposits are callable on demand. At any point after the portfolio decisions have been made, $\tilde{D}_i$ may increase or decrease by a random amount. In particular, a bank experiences a withdrawal $\omega_i \tilde{D}_i$ where ω_i is a stochastic fraction drawn from a commonly known distribution $F(\cdot)$:

$$\begin{aligned} \omega_i &\in (-\infty, 1], \quad \omega_i \sim F(\cdot) \\ \text{s.t.} \quad &\sum_i \omega_i \tilde{D}_i = 0 \end{aligned} \quad (2)$$

I assume that deposits remain within the banking system, that is, withdrawal shocks simply reshuffle deposits, and do not constitute bank runs.

Loans. The flow of new loan issuances is denoted by I_i . When a bank provides a loan, it creates a deposit account for the borrower and deposits the loan amount to this account. The borrower promises to repay the loan in the beginning of the next period. The gross

interest rate on a loan is R_i^b . Loans are never defaulted on. Once a loan is made, it becomes illiquid, i.e. cannot be sold, for the rest of the period.

Cash assets. Cash assets are the primary means to settle transactions. They are kept in the bank's account at the central bank (reserve account). This account resembles a checking account: when a bank needs to make a payment, it draws its reserve account for the amount of the payment; when a bank receives funds, these funds are deposited to the reserve account. When banks experiences a withdrawal, the balance of the reserve account decreases by the amount of the withdrawal, $\omega_i \tilde{D}_i$. At the end of each period, banks must have a non-negative balance in their accounts with the central bank:

$$\tilde{C}_i - \omega_i \tilde{D}_i \geq 0 \quad (3)$$

Depending on the realization of ω_i , a bank can end up with a shortage or excess of cash. If a bank has a shortage, it must borrow the amount of the deficit. Banks can borrow from each other or from the central bank. A central bank loan is always available; therefore, banks will never default. The interest on the central bank loan is called the discount window rate and is denoted by r^{DW} . If a bank has an excess of cash after the withdrawal shock realization, it can either hold it in its deposit account with the central bank until the next period or loan it to another bank. Holding the excess cash in the reserve account earns r^{ER} , which is called the excess reserve rate.

3.2.2 Interbank Market

Banks can enter the interbank market for two reasons: first, banks with a shortage of cash seek to find a loan at a rate lower than r^{DW} ; second, banks with an excess seek to earn a rate higher than r^{ER} . I assume that an interbank loan cannot exceed the amount of the reserve deficit. Thus, the only purpose of the interbank market in this model is redistribution of reserves.

Bargaining problem. A bank with an excess places lending orders and a bank with a deficit places borrowing orders. Following [Atkeson et al. \(2012\)](#) and [Bianchi and Bigio \(2014\)](#), I assume that a bank places a continuum of orders of infinitesimal size (further referred to as \$1), and trades occur on dollar-per-dollar basis. The interest rate at which a trade occurs, r^{FF} , is determined by the bilateral Nash bargaining.

Problem 1 *The bargaining problem between a lending order and a borrowing order:*

$$\max_{r^{FF}} (r^{DW} - r^{FF})^\xi (r^{FF} - r^{ER})^{1-\xi}$$

where ξ is the bargaining power of the borrowing order.

The first-order condition implies that the interbank loan rate is a convex combination of the policy rates:

$$r^{FF} = \xi r^{ER} + (1 - \xi) r^{DW} \quad (4)$$

Note that for any Nash bargaining parameter, $r^{ER} \leq r^{FF} \leq r^{DW}$. Therefore, banks with deficit of cash will always try to borrow in the interbank market before borrowing from the central bank. Similarly, banks with excess reserves will always try to lend in the interbank market.

I assume that all orders on either side of the market have the same bargaining power, which implies that the interbank lending rate is identical for all matched orders. The probabilities of finding a matching order, however, vary across banks, depending on the bank's position in the interbank network.

Interbank network. An important feature of the interbank market is that, in order to trade, banks have to search for a trading partner. This gives rise to an interbank network. I define the interbank network as in [Lenzu and Tedeschi \(2012\)](#). Banks enter bilateral potential trading agreements (PTAs). These agreements constitute a promise to engage in trade when one of the banks has excess cash and another has a deficit. Note that if the two banks with a PTA end up on the same side of the market (both lenders or both borrowers), PTA is not

useful.

Definition 1 *The interbank network is an undirected graph (N, G) where $N = [1, \dots, n]$ is the set of nodes (banks) and G is the $n \times n$ symmetric adjacency matrix with elements $G_{ij} \in \{0, 1\}$ which represent a relationship between banks i and j :*

$$G_{ij} = G_{ji} = \begin{cases} 1 & \text{if there exists a PTA between } i \text{ and } j \\ 0 & \text{if there exists no PTA between } i \text{ and } j \end{cases}$$

where $G_{ii} = 0 \forall i$, i.e. a bank cannot be connected to itself.

The adjacency matrix G is an exogenous process and is known in the beginning of t . If every bank has a PTA with every other bank in the network, then the interbank network is “complete”; if there are no PTAs, the interbank network is “empty”; if there are some PTA, the network is “incomplete.” When the network is incomplete, banks have different probabilities of matching in the interbank market.

Interbank Matching. I define a vector of bank deficits:

$$\tilde{X}_i = \omega_i \tilde{D}_i - \tilde{C}_i \tag{5}$$

where a negative value of $\tilde{X}_i$ implies that a bank has a surplus.

Consider a bank i with a deficit of reserves. The mass of lending orders available to i is the sum of surpluses of i ’s connections:

$$\Upsilon_i^+ = \sum_j I(\tilde{X}_j < 0) G_{ij} \tilde{X}_j$$

where $I(\cdot)$ is the indicator function. The mass of borrowing orders that the lending orders

of i 's neighbors' can potentially match with is:

$$\Upsilon_i^- = \sum_k I(\tilde{X}_k \geq 0) \min[K_k, 1] \tilde{X}_k, \quad K_{1 \times N} = \sum_j G_{ij} G_j \quad (6)$$

I assume that borrowing and lending orders are paired at random. If $\Upsilon_i^+ < \Upsilon_i^-$, then there is more borrowing orders than lending orders in i 's neighborhood, and some of the borrowing orders are not matched. If $\Upsilon_i^+ \geq \Upsilon_i^-$, all borrowing orders are matched with certainty.⁷ The probability that a borrowing order from the bank i meets a lending orders is defined as:

$$p_i^B = \min \left[1, \frac{\Upsilon_i^+}{\Upsilon_i^-} \right] \quad (7)$$

The borrowing orders that are not matched in the interbank market are directed to the central bank lending facility. The average cost of i 's borrowing is:

$$\chi_i^B = p_i^B \cdot r^{FF} + (1 - p_i^B) \cdot r^{DW} \quad (8)$$

Similarly, if i is a lending bank, the fraction of its lending orders that finds a match in the interbank market is:

$$p_i^L = \min \left[1, \frac{\Gamma_i^-}{\Gamma_i^+} \right]$$

where Γ_i^- is the mass of borrowing orders from i 's neighbors and Γ_i^+ is the mass of lending orders available to i 's neighbors with borrowing orders. Refer to Appendix A for the derivation of masses Γ_i^- and Γ_i^+ . The average return on a unit of i 's excess reserves is:

$$\chi_i^L = p_i^L \cdot r^{FF} + (1 - p_i^L) \cdot r^{ER} \quad (9)$$

Note that, in general, $\Upsilon_i^+ \neq \Gamma_i^+$ and $\Upsilon_i^- \neq \Gamma_i^-$, meaning that deposit withdrawals cannot be perfectly insured against. This is because banks can only trade with banks they are

⁷This type of matching was used [Bech and Monnet \(2014\)](#), and [Bianchi and Bigio \(2014\)](#)

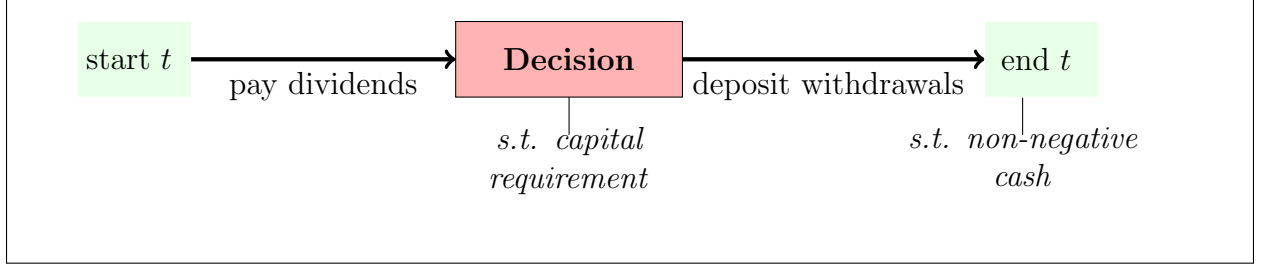


Figure 3. Sequence of events within a time period.

connected to. As a result, the total amount of lending in the interbank market is weakly less than the total amount of reserve deficit in the banking sector. Intuitively, in such setup it is more burdensome for a bank to end up with a reserve deficit, compare to the case where all banks are connected to each other.

From the prospective of an individual bank, χ_i^L and χ_i^B are general equilibrium objects, and individual banks cannot influence these values, even though these values are bank-specific.

3.2.3 Timing and Laws of Motion

Figure 3 displays the sequence of events within a period. Bank enters a period t with reserves, C_i , loans, B_i , and deposits, D_i . Bank's equity in the beginning of the period is:

$$E_i = B_i + C_i - D_i \quad (10)$$

Bank pays out a constant fraction θ of this equity as dividends:

$$DIV_i = \theta E_i \quad (11)$$

Decision. After the old loans have been repayed, the bank chooses new loan issuances, I_i , and new deposits, Δ_i . When a loan is made, the amount I_i is deposited to the customer's account. Thus, deposits increase when new loans are made. When the bank attracts Δ_i new

deposits, it deposits Δ_i in its reserve account. Thus, cash assets increase when new deposits are acquired.

The laws of motion for loans, cash assets, and deposits are:

$$\tilde{B}_i = I_i \quad (12)$$

$$\tilde{C}_i = C_i + \Delta_i - \theta E_i \quad (13)$$

$$\tilde{D}_i = D_i + I_i + \Delta_i \quad (14)$$

Substituting equations (12)-(13) into (14) yields:

$$(1 - \theta)E_i = \tilde{B}_i + \tilde{C}_i - \tilde{D}_i \quad (15)$$

The left-hand side is the funds that the bank allocates between loans, reserves, and deposits and it is known.

Post-decision. Withdrawal shocks happen, decreasing bank's deposits by $\omega_i \tilde{D}_i$:

$$D'_i = \tilde{D}_i - \omega_i \tilde{D}_i \quad (16)$$

Recall that a balance in the bank's account with the central bank must be non-negative before the end of the period (reserve requirement). I define the amount bank needs to be able to meet the reserve requirement as a reserve deficit, which I denote by $\tilde{X}_i$:

$$\tilde{X}_i = \omega_i \tilde{D}_i - \tilde{C}_i \quad (17)$$

When $\tilde{X}_i$ is positive, the bank has reserve deficit, and when $\tilde{X}_i$ is negative, the bank has a reserve surplus. The bank will attempt to borrow/lend the amount of deficit/surplus on the interbank market. The expected cost of borrowing is χ_i^B , and the expected return on

lending is χ_i^L . The unit return/cost on a reserve deficit can be defined as:

$$R_i^x = \begin{cases} \chi_i^L & \text{if } \tilde{X}_i \leq 0 \\ \chi_i^B & \text{if } \tilde{X}_i > 0 \end{cases} \quad (18)$$

This cost will be paid in the beginning of the next period. Once the bank proceeds with the loan, the level of cash assets changes to:

$$C'_i = \tilde{C}_i - \omega_i \tilde{D}_i + \tilde{X}_i \quad (19)$$

In the beginning of the next period, bank's equity is:

$$E'_i = C'_i + R_i^b \tilde{B}_i - R^d D'_i - R_i^x \tilde{X}_i$$

Using equations (16), (65), and (19), the value of equity in the beginning of the next period can be rewritten in terms of this period's variables:

$$E'_i = R_i^b \tilde{B}_i - R^d \tilde{D}_i + R_i^x \tilde{C}_i - \omega_i (R_i^x - R^d) \tilde{D}_i \quad (20)$$

3.2.4 Bank's Problem

Banks maximize the expected lifetime wealth subject to the capital requirement, dividend rule, and the balance sheet constraint. The aggregate state is summarized in vector $Z = \{r^{DW}; r^{ER}; F(\omega_i); G_t\}$, which includes policy rates, distribution of withdrawal shocks, and the network matrix. Banks are more impatient than the household, and their discount factor is augmented household discount factor, $\beta\zeta$.

Problem 2 Banks solve the following maximization problem:

$$\begin{aligned}
V(E_i, Z) &= \max_{\tilde{D}_i, \tilde{B}_i, \tilde{C}_i} DIV_i + \beta \zeta \mathbb{E}[V(E'_i, Z')] \\
s.t. \quad & (1 - \theta) E_i = \tilde{B}_i + \tilde{C}_i - \tilde{D}_i \\
& E'_i = R_i^b \tilde{B}_i - R^d \tilde{D}_i + R_i^x \tilde{C}_i - \omega_i (R_i^x - R^d) \tilde{D}_i \\
& \tilde{D}_i \leq \kappa (1 - \theta) E_i \\
& DIV_i = \theta E_i \\
& \tilde{B}_i, \tilde{C}_i, \tilde{D}_i \geq 0
\end{aligned}$$

E_i is known at the time of decision. Thus, solving for the loans, reserves, and deposits as fractions of after-dividend equity is equivalent to solving the original problem. I define:

$$\begin{bmatrix} \tilde{d}_i & \tilde{b}_i & \tilde{c}_i & \Omega_i \end{bmatrix} \equiv \begin{bmatrix} \frac{\tilde{D}_i}{(1 - \theta) E_i} & \frac{\tilde{B}_i}{(1 - \theta) E_i} & \frac{\tilde{C}_i}{(1 - \theta) E_i} & \frac{E'_i}{(1 - \theta) E_i} \end{bmatrix} \quad (21)$$

where Ω_i is the return on bank's portfolio. By Proposition 3 (Separation) in [Bianchi and Bigio \(2014\)](#), the bank's value is:

$$V(E_i, Z) = v(Z) \cdot (1 - \theta) E_i \quad (22)$$

where $v(Z)$ solves:

$$\begin{aligned}
v(Z) &= \theta + \beta \zeta (1 - \theta) \mathbb{E} v(Z'|Z) \max_{\tilde{d}_i, \tilde{b}_i, \tilde{c}_i} \mathbb{E}_\omega \Omega_i \\
s.t. \quad & 1 = \tilde{b}_i + \tilde{c}_i - \tilde{d}_i \\
& \Omega_i = R_i^b \tilde{b}_i - R^d \tilde{d}_i + R_i^x \tilde{c}_i - \omega_i (R_i^x - R^d) \tilde{d}_i \\
& \tilde{d}_i \leq \kappa \\
& \tilde{b}_i, \tilde{c}_i, \tilde{d}_i \geq 0
\end{aligned} \quad (23)$$

This setup features no dividend decision, the original problem can be reduced to a static portfolio maximization problem where bank chooses the shares of after-dividend equity it allocates to loans, reserves, and deposits. Substituting the budget constraint into the objective allows to write the problem as follows:

Problem 3 *The portfolio maximization problem of a bank:*

$$\begin{aligned} \max_{\tilde{c}_i, \tilde{d}_i} \quad & R_i^b - R_i^b \tilde{c}_i + (R_i^b - R^d) \tilde{d}_i + \mathbb{E}_{\omega_i} \left[R_i^x \tilde{c}_i - \omega_i (R_i^x - R^d) \tilde{d}_i \right] \\ \text{s.t.} \quad & \tilde{d}_i \leq \kappa \\ & \tilde{c}_i, \tilde{d}_i \geq 0 \end{aligned}$$

A non-standard feature of this problem is that R_i^x has a discontinuity at the point where the bank's reserve deficit is zero, i.e. $\omega_i \tilde{d}_i - \tilde{c}_i = 0$. This occurs when the value of the withdrawal shock is $\omega_i = \frac{\tilde{c}_i}{\tilde{d}_i}$. Since $\omega_i \leq 1$, then it must be that $\frac{\tilde{c}_i}{\tilde{d}_i} \leq 1$, which rules out $\hat{d}_i = 0$ in equilibrium. If the realized shock is below $\frac{\tilde{c}_i}{\tilde{d}_i}$, then the bank has excess reserves, which can be lent out at χ_i^L . If the realized shock is above $\frac{\tilde{c}_i}{\tilde{d}_i}$, then the bank has a reserve deficit and has to get a loan at χ_i^B .

The first order conditions are:

$$R_i^b = \chi_i^L F \left[\frac{\tilde{c}_i}{\tilde{d}_i} \right] + \chi_i^B \left(1 - F \left[\frac{\tilde{c}_i}{\tilde{d}_i} \right] \right) \quad (24)$$

$$R_i^b - R^d = \chi_i^L \int_{-\infty}^{\frac{\tilde{c}_i}{\tilde{d}_i}} \omega_i f(\omega_i) d\omega_i + \chi_i^B \int_{\frac{\tilde{c}_i}{\tilde{d}_i}}^1 \omega_i f(\omega_i) d\omega_i + \mu_i \quad (25)$$

$$(26)$$

where $F(\cdot)$ is the CDF of withdrawal shocks and μ_i is the multiplier on the capital requirement constraint. For the rest of the paper I consider the case of binding capital requirement.⁸

Refer to Appendix B for the details of derivation.

⁸Due to the linearity of bank's objective, when the capital requirement is non-binding, transition to steady state is instant (see Bianchi and Bigio (2014)).

The left-hand side of equation (24) is the opportunity cost of holding cash assets over loans and the right-hand side is the expected benefit of holding an additional unit of cash asset. The left-hand side of equation (25) is the arbitrage obtained by lending and the right-hand side is the expected increase in liquidity cost due to issuing an additional unit of deposits.

Once the optimal portfolio shares are found, the optimal levels of loans, cash, and deposits can be calculated as follows:

$$\begin{bmatrix} \tilde{D}_i^* & \tilde{B}_i^* & \tilde{C}_i^* \end{bmatrix} = (1 - \theta) E_i \cdot \begin{bmatrix} \kappa & \tilde{b}_i^* & \tilde{c}_i^* \end{bmatrix} \quad (27)$$

The expected return on the optimal portfolio is:

$$\mathbb{E}_\omega \Omega_i^* = R_i^b + \kappa \mu_i \quad (28)$$

Bank's value at the optimal portfolio:

$$v(Z) = \theta + \beta \zeta (1 - \theta) \mathbb{E} v(Z'|Z) \mathbb{E}_\omega \Omega_i^* \quad (29)$$

The value of dividend payout ratio, θ , will be calibrated such that the right-hand side of above equation is a contraction mapping operator.

3.3 Real sector

The aggregate household obtains utility from consumption, C_t , and disutility from labor, H_t . The household can save by providing deposits to the banking sector, D_t^A . Deposits receive a constant interest rate of R^d , which is paid at the beginning of the next period.

Problem 4 *The household solves the following maximization problem:*

$$\begin{aligned} \max_{C_t, H_t, D_t^A} \quad & \sum_{t=0}^{\infty} \beta^t \left[C_t - \frac{H_t^{1+\nu}}{1+\nu} \right] \\ \text{s.t.} \quad & D_t^A + C_t = W_t H_t + R^d D_{t-1}^A + \Pi_t + T_t \end{aligned}$$

where W_t is the real wage rate, Π_t is the firm's profit, T_t is the tax transfer, and ν is the inverse of the Frisch elasticity. The labor supply curve is:

$$H_t = W_t^{\frac{1}{\nu}} \quad (30)$$

which implies that the household's total wage income is $W_t^{\frac{\nu+1}{\nu}}$. If $R^d = \frac{1}{\beta}$, the household is indifferent between consumption and saving, and:

$$C_t \in [0, Y_t], \quad D_t^A = Y_t - C_t, \quad R^d = \frac{1}{\beta} \quad (31)$$

where Y_t is the output of an aggregate firm.

An aggregate profit-maximizing firm uses household's labor to produce output according to the following production function:

$$Y_t = A_t H_t^{1-\alpha} \quad (32)$$

where A_t is a technology index, and $1 - \alpha$ is the labor share. The firm has to pay workers before the output is realized, therefore, to cover the wage bill, it borrows the total amount I_t^A from the banking sector:

$$W_t H_t = I_t^A \quad (33)$$

I_t^A is collected via the CES technology:

$$I_t^A = \left[\sum_i \lambda_i^{\frac{1}{\epsilon}} I_{it}^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}} \quad (34)$$

where I_{it} is borrowing from bank i , λ_i is the bank i 's share, and ϵ is the elasticity of substitution between loans from different banks. The firm promises to repay the loan principal and accrued interest in the beginning of the next period. The total repayment to the banking sector is then $\sum_i R_i^b I_{it}$. The firm never defaults on loans.

Problem 5 *The aggregate firm solves the following maximization problem:*

$$\begin{aligned} \max_{I_t^A, I_{it}, H_t} \quad & \sum_{t=0}^{\infty} \beta^t \left[A H_t^{1-\alpha} - W_t H_t + I_t^A - \sum_i R_{it-1}^b I_{it-1} \right] \\ \text{s.t.} \quad & W_t H_t = I_t^A \\ & I_t^A = \left[\sum_i \lambda_i^{\frac{1}{\epsilon}} I_{it}^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}} \end{aligned}$$

Taking the first-order conditions and imposing labor market clearing implies the demand curve for a loan from a bank i :

$$R_i^b = \frac{(1-\alpha) A_t}{\beta} [I_t^A]^{\frac{1}{\epsilon} - \frac{\nu+\alpha}{\nu+1}} \left[\frac{I_{it}}{\lambda_i} \right]^{-\frac{1}{\epsilon}} \quad (35)$$

The aggregate repayment of loans to the banking sector is:

$$\sum_i R_i^b I_{it} = \frac{(1-\alpha) A_t}{\beta} [I_t^A]^{\frac{1-\alpha}{\nu+1}} = \frac{1-\alpha}{\beta} Y_t$$

and the firm's profit is:

$$\Pi_t = A_t H_t^{1-\alpha} - \sum_i R_{it-1}^b I_{it-1} = Y_t - \frac{1-\alpha}{\beta} Y_{t-1}$$

Refer to Appendix C for the details of derivations.

3.4 Central Bank

The central bank starts a period with M_t^0 reserves and D_t^{CB} banking sector deposits. It issues new reserves, Δ_t^{CB} , receives interest on discount window loans, pays interest on excess reserves, and makes transfers T_t to the household. The laws of motion for deposits and reserves:

$$D_{t+1}^{CB} = D_t^{CB} + \Delta_t^{CB} + r_t^{DW} X_t^- - r_t^{ER} X_t^+ - T_t \quad (36)$$

$$M_{t+1}^0 = M_t^0 + \Delta_t^{CB} \quad (37)$$

where X_t^- is the loans to the banking sector and X_t^+ is the aggregate excess reserves held at the central bank. Combing the laws of motion gives the next period budget constraint:

$$M_{t+1}^0 - M_t^0 = D_{t+1}^{CB} - D_t^{CB} - r_t^{DW} X_t^- + r_t^{ER} X_t^+ + T_t \quad (38)$$

Central bank's policy rates satisfy:

$$r_t^{DW} \geq r_t^{ER}$$

The difference between the two rates constitutes the interest rate corridor. The central bank chooses r_t^{DW} and r_t^{ER} to target a particular level of the interbank loan interest rate, r_t^{FF} .

3.5 Exogenous processes

3.5.1 Interbank Network Shock

I assume that the interbank network is static before the shock. The steady state network is characterized by a degree distribution p_k^* . I consider different options for this distribution, which will be discussed in a later section. The current steady-state network is a particular realization of the network implied by p_k^* . This network is denoted by (N, K^*) where N is set

of nodes (banks) and K^* is a set of edges that are unordered pairs of distinct nodes. The number of connection in (N, K^*) is:

$$L^* = \frac{1}{2} \sum_{i=1}^N k_i^*, \quad L^* \leq L^{max} = N(N-1)/2 \quad (39)$$

where k_i^* is the degree (number of connections) for a bank i .

At the time of a shock, a fraction of edges in K^* is destroyed. Before discussing exactly *how* the network transitions back to its steady state, it is important to understand *what* will the network be at the time it reaches the steady state again.

Degree-preserving rewiring (randomization) algorithm. I use the algorithm proposed in [Maslov and Sneppen \(2002\)](#) that generates a network in which all nodes have the same degrees as in the original (null) network, but the network's wiring had been randomized. First, a random edge (i, j) is selected from the original network, (N, K^*) . Next, a second random edge (u, v) is selected such that $i \neq u$, $j \neq v$, and edges (i, v) and (j, u) do not already exist in the network. Then, edges (i, j) and (u, v) are removed from the network and edges (i, v) and (j, u) are added. This process is repeated until each link in (N, K^*) is rewired at least once. Figure 4 demonstrates an example for a single iteration of this algorithm. The degree distribution of the resulting network, (N, K') , is:

$$p'_k = p_k^* \quad (40)$$

Transition to the steady state. At the time of a shock, a fraction $\zeta \in [0, 1]$ of edges is removed. In the baseline model, the subset of K^* that is removed is chosen at random. The number of remaining connections is $(1 - \zeta) L^*$. Next period, a number of new connections emerge. This number is determined by a fraction $s \in [0, 1]$ that corresponds to the speed with which the network is rebuilt. The new network (N, K_{t+1}) is characterized by an edge

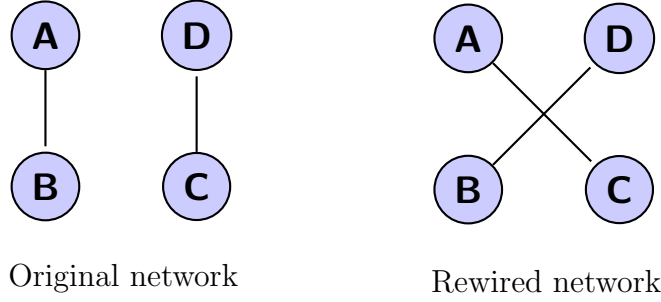


Figure 4. An iteration of a degree-preserving randomization algorithm for a four-bank network.

set K_{t+1} and a number of links L_{t+1} :

$$L_{t+1} = (1 - \zeta) L^* + s\zeta L^* \quad (41)$$

A list with a number $s\zeta L^*$ of new edges is generated according to the degree-preserving randomization process. At $t + 2$, if number of existing connection is equal to the original number of links, no new connections are created. Otherwise, a number $s(1 - s)\zeta L^*$ of links is created:

$$L_{t+2} = \begin{cases} L_{t+1} + s(1 - s)\zeta L^* & \text{if } L_{t+1} < L^* \\ L^* & \text{if } L_{t+1} = L^* \end{cases}$$

A general formula for the number of links at j periods after the shock:

$$L_{t+j} = \begin{cases} L_{t+j-1} + s \cdot \sum_{n=0}^j (-s)^n \cdot \zeta L^* & \text{if } L_{t+j-1} < L^* \\ L^* & \text{if } L_{t+j-1} = L^* \end{cases} \quad (42)$$

3.5.2 Withdrawal Shocks

I assume that withdrawal shocks ω_i are drawn from logistic distribution with mean $\bar{\omega}$ and standard deviation σ :

$$F^*(\omega) = \frac{1}{1 + e^{\frac{\bar{\omega} - \omega}{\sigma}}}$$

To account for the fact that ω_i is a fraction that takes a maximum value of 1, I truncate the distribution at 1 from the right:

$$F(\omega) = \frac{F^*(\omega)}{F^*(1)}$$

I assume that $F(\omega)$ is time-invariant.

3.6 Market clearing

Deposit Market. Supply of deposits must equal demand for deposits:

$$D_t^A = \sum_i \tilde{D}_{it} = \kappa(1 - \theta) \sum_i E_{it} \quad (43)$$

Money market. Cash assets held by banks during the decision stage must equal the supply of money:

$$\sum_i \tilde{C}_{it} = M_t^0 \quad (44)$$

The total amounts borrowed and lent to the central bank respectively are:

$$X_t^- = \sum_i I(X_{it} > 0) \cdot (1 - p_{it}^B) X_{it} \quad (45)$$

$$X_t^+ = \sum_i (1 - I(X_{it} > 0)) \cdot (1 - p_{it}^L) X_{it} \quad (46)$$

Loan Market. Supply of loans from bank i equals to the demand for loans from bank i :

$$\tilde{B}_i = I_i \quad (47)$$

Appendix D shows the full list of equilibrium conditions.

4 Degree distribution of the interbank network

I consider two general cases for the interbank network – complete interbank network and incomplete interbank network. For the incomplete case, I consider three sub-cases – random interbank network, circle interbank network, and scale-free interbank network. I discuss each case below and summarize them in Table 1.

4.1 Complete network

All banks are connected to each other. The total number of connections and the degree of each bank take their maximum possible values,

$$L^{CN} = L^{\max} = N(N-1)/2 \quad (48)$$

$$k_i^{CN} = k^{\max} = N-1 \quad (49)$$

respectively. The probabilities of matching in the interbank market for borrowing and lending orders are identical for all banks and equal to:

$$p^B = \min[1, \Psi] \quad \text{and} \quad p^L = \min\left[1, \frac{1}{\Psi}\right] \quad (50)$$

where

$$\Psi = \frac{F\left[\frac{\bar{c}}{\kappa}\right] \int_{-\infty}^{\frac{\bar{c}}{\kappa}} (\omega\kappa - \bar{c}) f(\omega) d\omega}{\left(1 - F\left[\frac{\bar{c}}{\kappa}\right]\right) \int_{\frac{\bar{c}}{\kappa}}^1 (\omega\kappa - \bar{c}) f(\omega) d\omega} \quad (51)$$

Table 1. Summary of cases for interbank network topology

Network	Degree distribution	L	$\bar{k}$	$\bar{c}$
Complete	Dirac delta centered at $N-1$	$N(N-1)/2$	$N-1$	$(N-1)/N$
Circle	Dirac delta centered at k^*	$Nk^*/2$	k^*	$3/4(k^*-2)/(k^*-1)$
Random	binomial with $p = k^*/(N-1)$	$Nk^*/2$	k^*	k^*/N
Scale-free	power law with $\gamma=3$, $m = k^*/2$	$Nk^*/2$	k^*	$k^*(\ln N)^2/(16N)$

is the mass of lending orders relative to borrowing orders. In this case, all banks choose the same reserve ratio and loan rate. Only the realized return on a unit of reserve surplus, R^x , and not individual withdrawal shocks, affect evolution of aggregate equity:

$$E'_A = (1 - \theta)E_A \left(R^b \tilde{b} + R^x \tilde{c} - R^d \kappa \right) \quad (52)$$

When the interbank network is complete, there is a perfect insurance against the liquidity (withdrawal) shocks and the model reduces to [Bianchi and Bigio \(2014\)](#).

4.2 Incomplete network

Not all banks are connected to each other, i.e. the number of connections is below network's capacity:

$$L^{IN} = \frac{\bar{k}N}{2} < L^{\max} \quad (53)$$

where $\bar{k}$ is the average degree of a bank. Individual banks, however, may differ in their degree, implying that probabilities of matching in the interbank market for borrowing and lending orders may also be different. Specifically, the probabilities in (50) are augmented by a network-dependent factor ψ_i^N :

$$p^B = \min \left[1, \psi_i^N \Psi \right] \quad \text{and} \quad p^L = \min \left[1, \psi_i^N \frac{1}{\Psi} \right] \quad (54)$$

where

$$\psi_i^N = \frac{\sum_j G_{ij} E_j}{\sum_k \min[K_k, 1] E_k}, \quad K = \sum_j G_{ij} G_j \quad (55)$$

In this case, the cost of reserve deficit, R_i^x , will be bank-specific, resulting in a distribution of loan rates. Moreover, the evolution of aggregate equity will depend on individual banks' deposit withdrawals, i.e. there will be no perfect insurance against the liquidity shocks.

Below, I describe three different cases for the incomplete network. For comparison purposes, I set the average degree, $\bar{k}$, to be a specific value k^* , such that the total number of

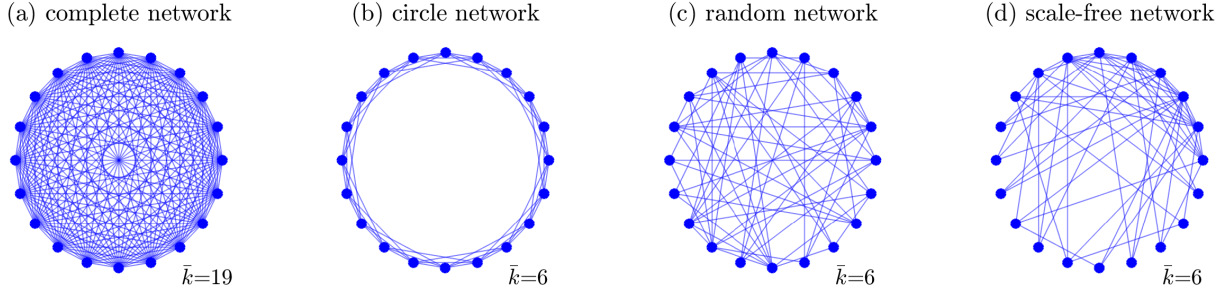


Figure 5. Interbank network examples. The figure displays 4 possible topologies for a network of $N = 20$ banks. Each node represents a bank, and each line represents an existing connection between two banks. Panel (a) displays a network with a maximum number of connections, which is equal to 190. Panels (b)-(d) show different network topologies for an incomplete network with 60 connections. The average degree is displayed below each panel.

connections and the average degree is the same in each case. Panels (b)-(c) in Figure 5 display network topologies for $k^* = 6$ in a network of 20 banks.

Circle network. Each bank is connected to exactly k^* neighbors and the total number of links is $Nk^*/2$. Degree distribution of a circle network is a Dirac delta function centered at k^* . The clustering coefficient is:

$$c^{circle} = \frac{3(k^* - 2)}{4(k^* - 1)} \quad (56)$$

Random network (Erdős-Rényi network). The degree distribution of a random network is the binomial distribution. The probability that a bank is connected to k banks is:

$$p_k = \binom{N-1}{k} p^k (1-p)^{N-1-k} \quad (57)$$

The average degree of a random network is $\bar{k} = p(N-1)$ and the clustering coefficient is:

$$c^{random} = \frac{\bar{k}}{N} \quad (58)$$

Scale-free network. Degree distribution follows a power law:

$$p_k = ak^{-\gamma}, \quad 2 < \gamma \leq 3, \quad a = 2m^2 \quad (59)$$

where a is a normalization constant and m is a scaling parameter.⁹ By the result in [Klemm and Eguiluz \(2002\)](#), the average degree and the clustering coefficient respectively are:

$$\bar{k} = 2m \quad (60)$$

$$c^{scale-free} = \frac{m (\ln N)^2}{8N} \quad (61)$$

5 Parameter choices

5.1 Parameter values

The household's discount factor is set to 1, which implies that the interest rate on deposits is 0. I assume a constant CES share parameter for all banks, $\lambda = 1/N$. I set the annual interest rate paid on excess reserves to $r^{ER} = 0$, and the annual discount window rate to $r^{DW} = 2.5\%$. I use the bargaining power parameter of $\xi = 0.5$, such that the target for the interbank loan rate is $r^{FF} = 1.25\%$. I set $\kappa = 10$, such that the required capital ratio is 10%. I set the average steady state reserve ratio to 1%.

5.2 Calibration of dividend payout ratio

Given capital requirement, κ , and the target reserve ratio, RR_{ss} , I calculate the steady-state portfolio share of cash assets, $\tilde{c}_{ss}$, and loans, $\tilde{b}_{ss}$. I then calculate the steady-state ratio Ψ_{ss} , interbank matching probabilities, and implied values of χ_{ss}^L and χ_{ss}^B for the complete network scenario. The average steady-state loan rate (in a complete network):

$$R_{ss}^b = (\chi_{ss}^L - \chi_{ss}^B) F \left[\frac{\tilde{c}_{ss}}{\kappa} \right] + \chi_{ss}^B \quad (62)$$

⁹Follows from a Barabási-Albert model. Given an initial network with m_0 nodes, m new nodes is added to each node at each step until the network reaches a size N . Typically, $m_0 = m$. See [Barabási and Albert \(1999\)](#) for more details.

Table 2. Parameter values

Parameter	Definition	Value
β	1	household's discount factor
α	0	capital input share
$1/\nu$	2.5	Frisch elasticity of labor supply
A_{ss}	1	steady-state firm productivity
ϵ	5	elasticity of substitution between loans
λ	$1/N$	bank's share in loan aggregator
ξ	0.5	bargaining power of a borrower in the interbank market
$\bar{\omega}$	0	mean of withdrawal shocks
σ	0.1	standard deviation of withdrawal shocks
r^{ER}	0	annual interest rate on excess reserves
r^{DW}	0.025	annual discount window rate
r^{FF}	0.0125	annual target interbank loan rate
N	1000	number of banks
ζ	1	fraction of links destroyed at the time of the shock
s	0.1	fraction of destroyed connections rebuilt each period after the shock
k^*	$0.6N$	average degree of a bank in incomplete network
κ	10	capital requirement to match 10% capital ratio
RR_{ss}	0.01	steady-state reserve ratio

and evolution of equity:

$$E' = (1 - \theta)E \left(R_{ss}^b \tilde{b}_{ss} - R^d \kappa + R^x \tilde{c}_{ss} \right)$$

Taking expectation over ω and using the fact that in equilibrium $R_{ss}^b = \mathbb{E}_\omega [R^x]$:

$$\mathbb{E}_\omega [E'] = (1 - \theta)E \left((1 + \kappa)R_{ss}^b - \kappa R^d \right)$$

A value of θ that makes the expected equity growth equal 1 is:

$$\theta^* = 1 - \frac{1}{(1 + \kappa)R_{ss}^b - \kappa R^d} \quad (63)$$

The loan demand equation (35) implies the steady-state level of expected average equity

for the complete network case:

$$\mathbb{E}_\omega [E_{ss}^{complete}] = \frac{1}{N(1 - \theta^*)(1 + \kappa - \tilde{c}_{ss})} \left[\frac{\beta R_{ss}^b}{(1 - \alpha) A_{ss}} \right]^{-\frac{\nu+1}{\nu+\alpha}} \quad (64)$$

For the incomplete network environments, I find the pair $(\theta^*, \mathbb{E}_\omega [E_{ss}])$ numerically for a given target reserve ratio.

6 Quantitative exercises

6.1 Distribution of interest rates and equity

Table 3 and Figure 6 display properties of loan rates and equity distributions for different steady-state topologies of the interbank network.

When the network is incomplete, the distribution of loan rate largely depends on the type of the incomplete network. Scale-free network has the highest average loan rate, but also the highest variability of both loan rates and equity.

6.2 Aggregate dynamics

6.2.1 100% destruction of complete network

First, I consider a removal of all interbank connections in a complete network, i.e. interbank network becoming empty. Figure 7 displays impulse responses of *average* variables. When banks cannot trade with each other, they will always use discount window loans if they have

Table 3. Steady-state equity and interest rate on loans to nonfinancial firms

	loan rate		equity	
	mean, %	st. dev, %	mean	st. dev, %
Complete	1.1036	0.0001	0.0001	0.0012
Circle	1.0340	0.0004	0.0001	0.0012
Random	1.0378	0.0055	0.0001	0.0012
Scale-free	1.1096	0.0396	0.0001	0.0015

Rows 3-5 display results for a network with average connectivity of 60%.

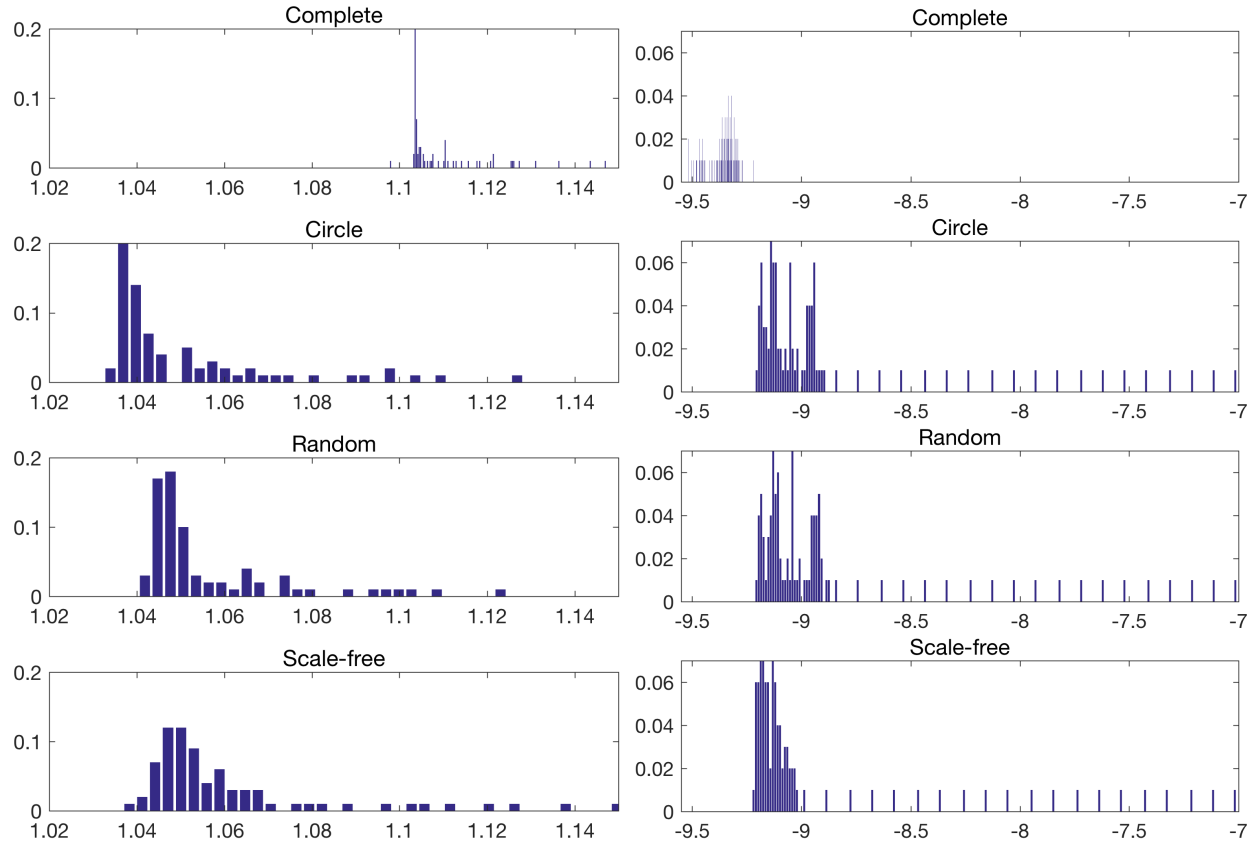


Figure 6. Steady-state distribution of interest rates and equity. The figure displays 4 possible cases of interbank network with 1000 banks. The left panel shows the distribution of interest rates. Interest rates are expressed in percent. The right panel displays the distribution of bank's equity on a log scale. Vertical axes are probabilities.

a reserve deficit. The cost of reserve deficit is at its highest – the discount window rate, and the return on excess cash is at its lowest – the central bank's interest rate paid on excess reserves. Thus, banks experience the highest liquidity cost, and, as a result, hold more cash and make less loans. In equilibrium, firm's output contracts and the household decreases its deposits to the banking system, which results in a further decrease in lending. Although banks initially hold more cash, the drop in deposits results in overall reduction of bank's portfolio size, decreasing both lending activity and holdings of cash assets.

Interestingly, as the average interest rate on loans to nonfinancial firms reduces after its initial hike, the variability of interest rates does not decrease for multiple periods after the shock. Figure 8 displays snapshots of interest rate and equity distributions during particular

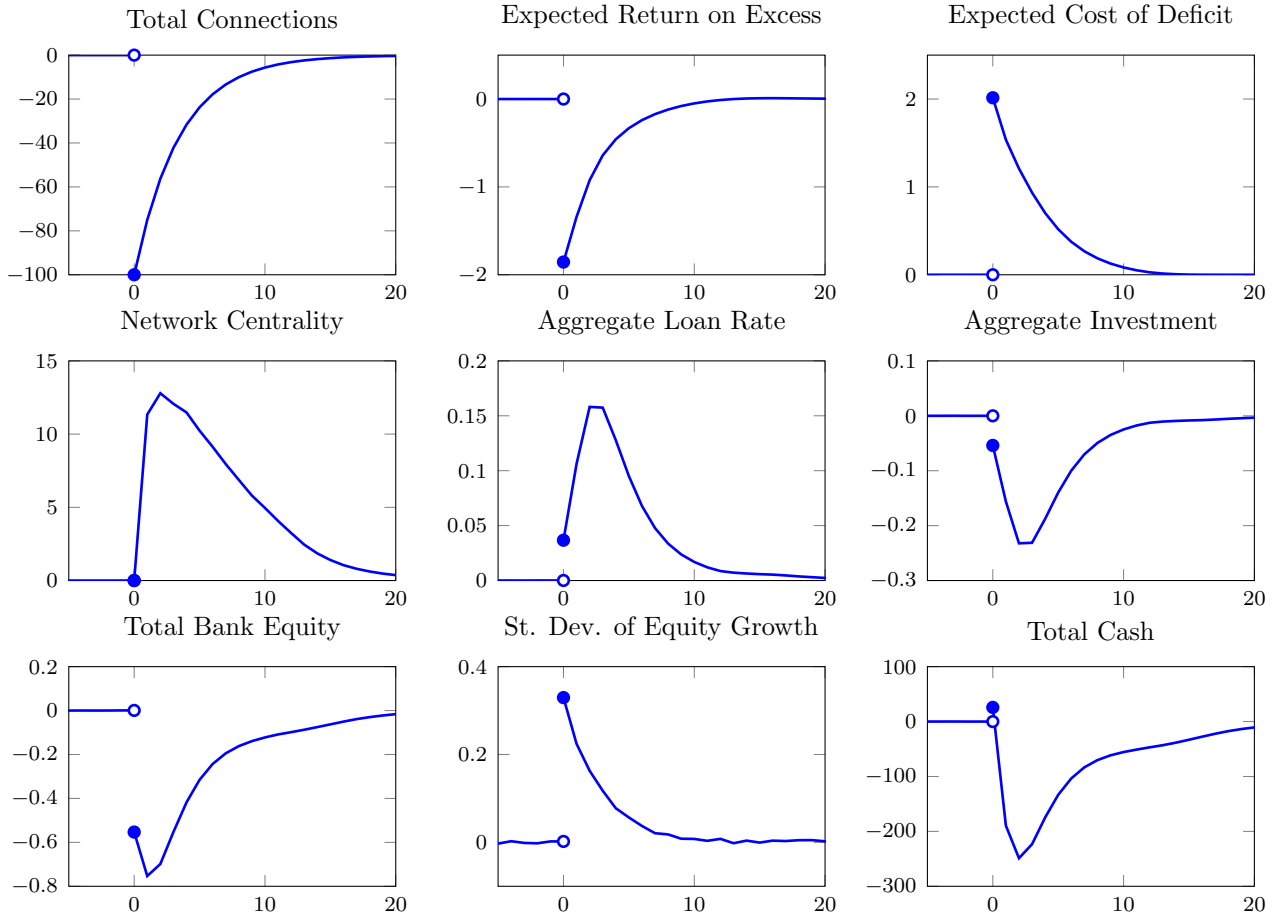


Figure 7. Complete network destruction shock (100 simulations). The figure displays impulse responses to removal of 100% links for a network with 100 banks. Vertical axes are percent deviations from steady state for level variables and deviations from steady state in percentage points for interest rates and standard deviations. Standard deviations σ are in relative terms.

periods after the shock. At 30 periods after the shock, the distribution of interest rates is wider than it is at 10th period post shock. Although the distribution of interest rates is initially (at 50th period after the shock) thinner, equity distribution widens over time.

The distributional changes in Figure 8 are observed because of “uneven” recovery of the interbank network. In particular, because only some banks are lucky to have some of their connections back sooner rather than later, they get a competitive advantage over others and accumulate equity faster. Even though other banks initially reconnect, they are at permanent disadvantage because other banks got to have higher level of equity sooner. Thus, there is a

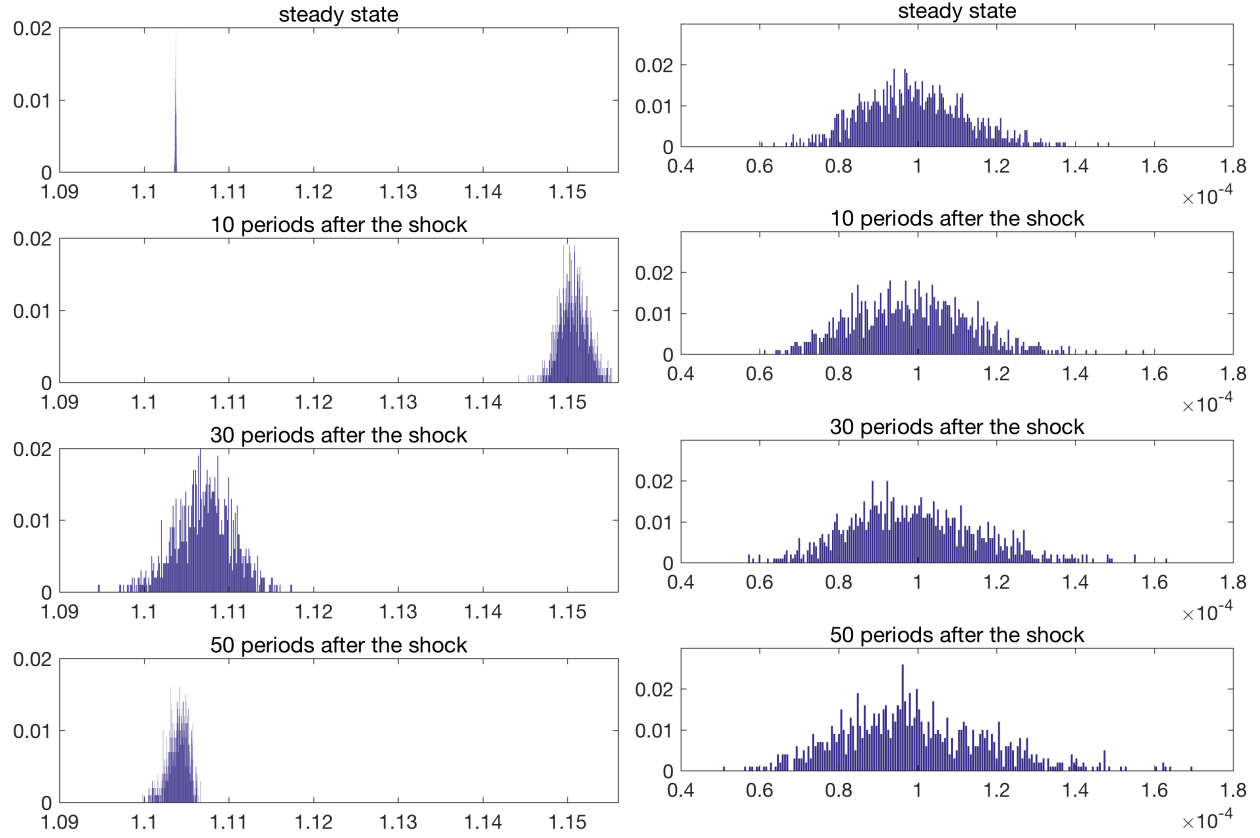


Figure 8. Distribution of interest rates and equity after the network destruction shock. The figure displays distributions during particular times after the shock. The left panel shows the distribution of interest rates. Interest rates are expressed in percent. The right panel displays the distribution of bank's equity. Vertical axes are probabilities.

new, wider, steady-state distribution of equity, which corresponds to the permanently higher value of $\sigma(E)$ in Figure 7. Even if the interbank network is thought of as being complete at most times, it is important to study network destruction shocks, as they could provide an endogenous mechanism for generating large variance in bank equity.

Result 1 *A temporary network destruction shock leads to a persistent change in banks' equity distribution.*

6.2.2 Partial destruction of a complete network

Figure 9 displays responses of variables to a partial network destruction shock. When less than a 100% of connections are removed, the number of connections per banks varies less

after the shock ($\sigma(k)$ is smaller). As a results, the average interest rate and average equity return to their original levels faster. However, the dispersion of equity increases at the same rate as in the case of 100% link removal.

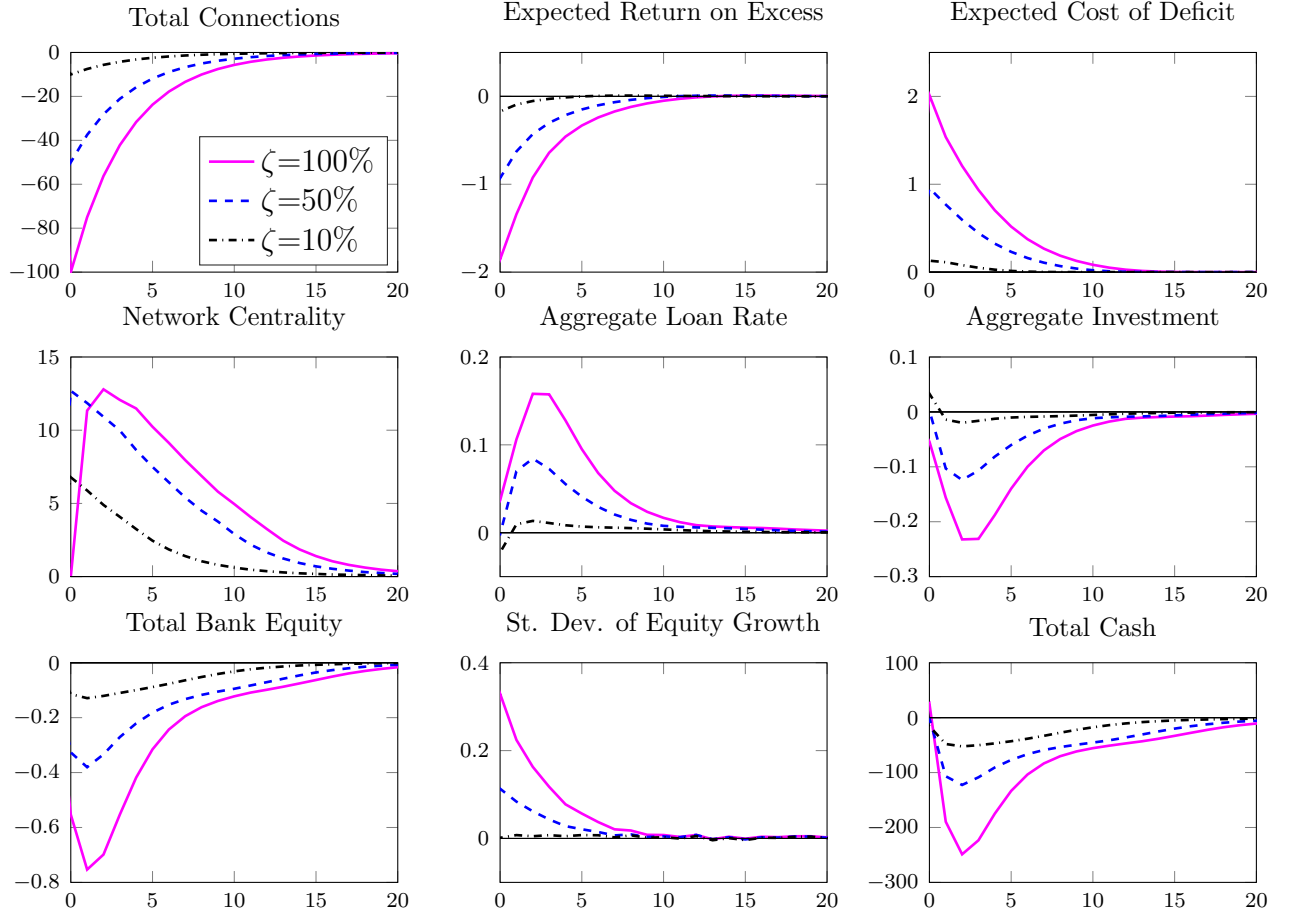


Figure 9. Partial destruction of complete network (100 simulations). The figure displays impulse responses to removal of different percentage of links , ζ , in a network with 100 banks. Vertical axes are percent deviations from steady state for level variables and deviations from steady state in percentage points for interest rates and standard deviations. Standard deviations σ are in relative terms.

Another results is that there exists some threshold value, ζ^* , such that when less than ζ^* of connections are destroyed, average interest rate decreases on impact, as opposed to increasing in the case when more that ζ^* of links are removed. This is because assets allocation decisions are different between the two regimes. In particular, on average, banks increase their non-financial loan asset holdings and decrease their cash asset holdings on

impact if $\zeta < \zeta^*$, and do the opposite if $\zeta \geq \zeta^*$. In both cases, however, equity decreases, resulting in drop of portfolio size one period after the shock.

Result 2 *There exists some threshold value for the fraction of interbank connections destroyed at the time of the shock, ζ^* , such that when $\zeta < \zeta^*$, average interest rate decreases at the time of the shock, as opposed to increasing in the case when $\zeta \geq \zeta^*$.*

6.2.3 Destruction shock for different network topologies

Next, I compare how the responses of variables to the 100% network destruction shock in the complete network compare to the responses in the circle, random, and scale-free networks. Figure 10 displays the results. The variables behave differently, depending on whether we start with a complete or incomplete network. If the network is incomplete, average equity does not experience such a large drop as it does in the case of complete network. Consequently, bank's portfolio size does not decrease, and banks increase their cash holdings relatively to steady state. The loan rate increases by most in the circle and random networks.

Figure 11 displays the cumulative probabilities of banks' degree and equity before the shock, at the time of the shock, and 50 periods after the shock. Dispersion of equity increases at a faster rate for incomplete networks. As a result, we observe a much wider distribution of equity 50 periods after the shock. This effect is most pronounced when the interbank network is a scale-free one.

Result 3 *The distribution of bank's equity is most affected (relatively to the initial distribution) by the network destruction shock when the degree distribution of interbank network follows the power law (i.e. the network is scale-free).*

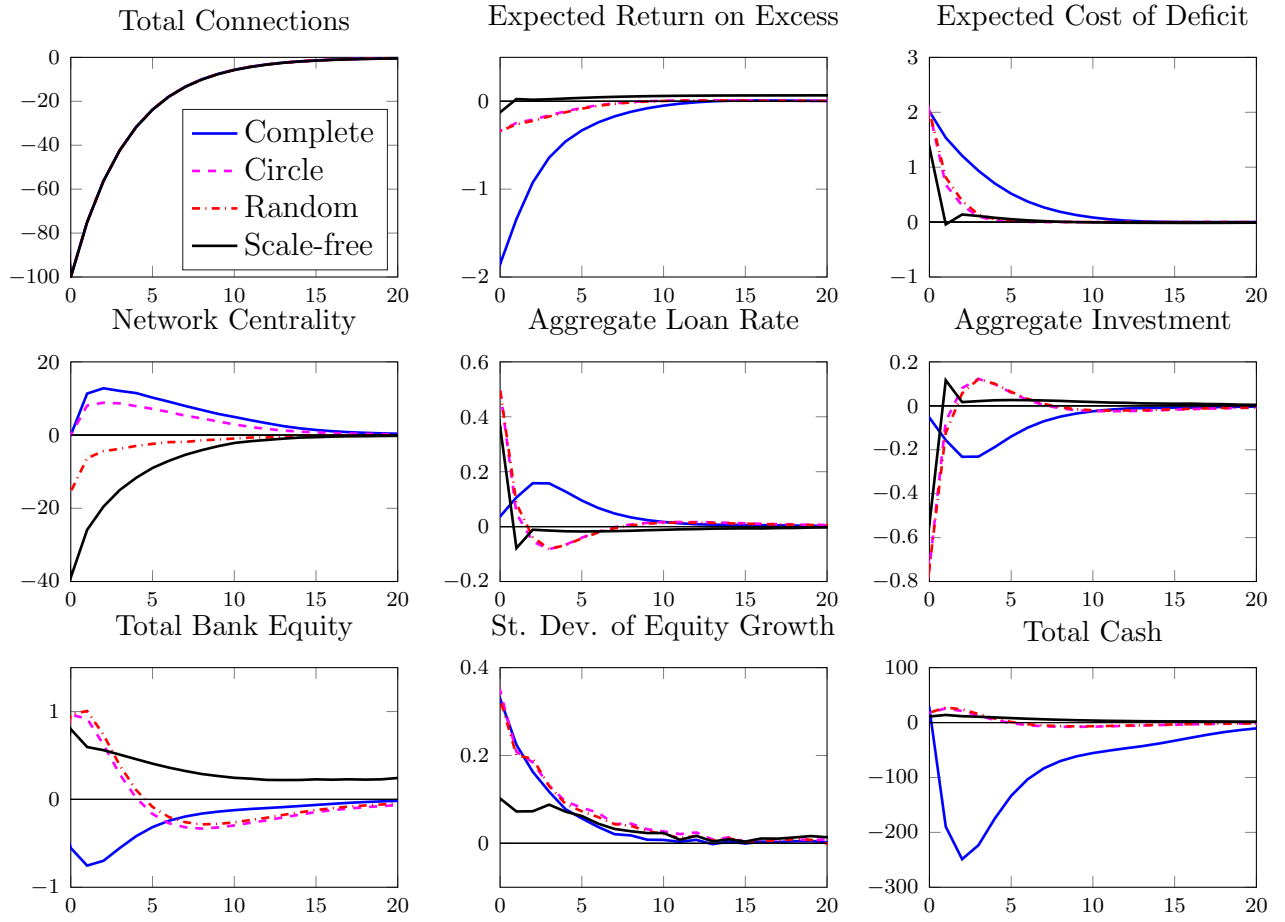


Figure 10. Destruction of different-topology networks (100 simulations). The figure displays impulse responses to removal of all links for networks with different topologies. Vertical axes are percent deviations from steady state for level variables and deviations from steady state in percentage points for interest rates and standard deviations.

7 Policy implications

7.1 Interest rate corridor

I next consider interbank network destruction shock for different central bank policy rates.

Recall that the central bank targets the interest rate on interbank loans, which is equal to

$$r^{FF} = \xi r^{ER} + (1 - \xi) r^{DW}.$$

I conduct an experiment where the interest rate corridor, $r^{DW} - r^{ER}$,

is changed such that r^{FF} is kept constant for all cases of the corridor. Figure 12 displays the

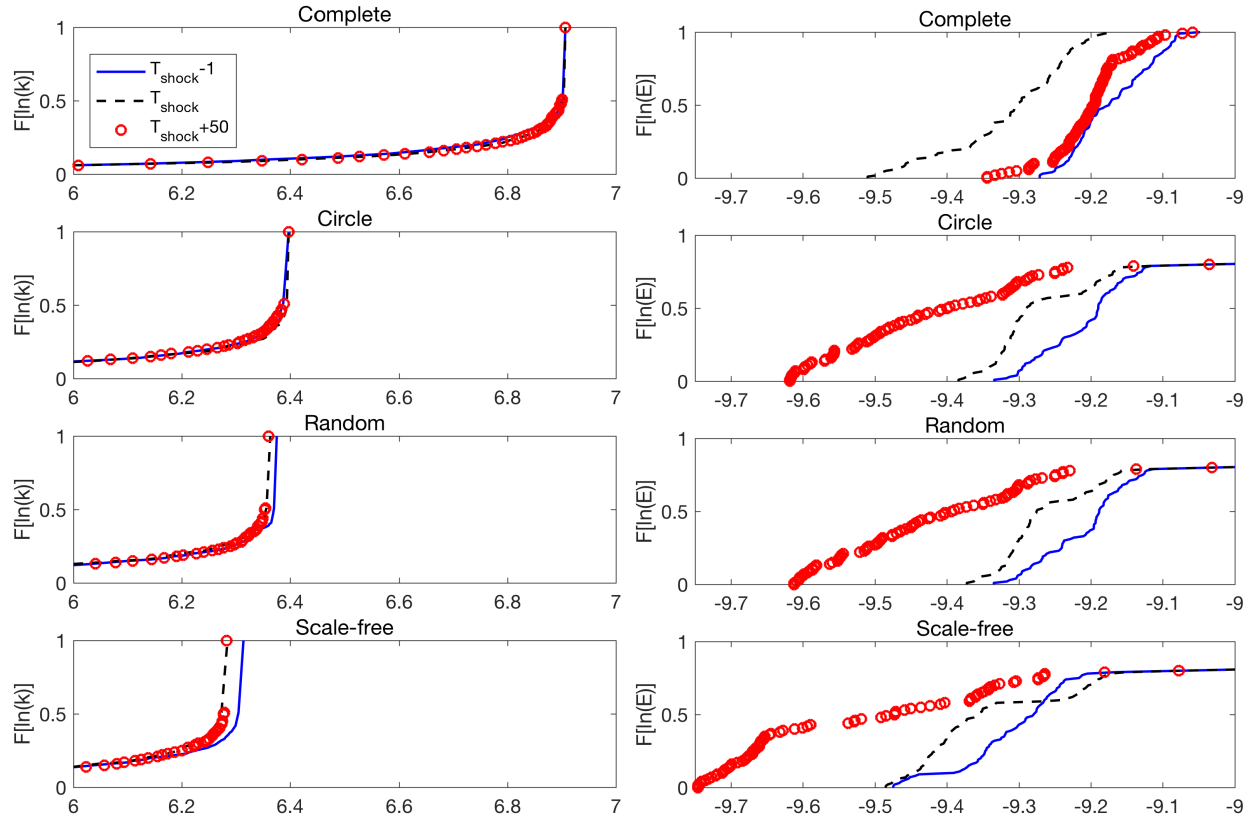


Figure 11. Degree distribution and equity dispersion. The left panel displays cumulative distribution of $\ln(\text{degree})$ and the right panel displays cumulative distribution of $\ln(\text{equity})$ during particular times. Horizontal axes are in logs. For illustrative purposes, axes are truncated.

responses of variables to 100% network destruction shock for the complete interbank network. When the corridor is at 2.5% (relatively wide), banks' equity reduces by approximately 0.9 percent more at the time of the shock than it does in a narrow corridor case. However, the shock propagates (in the response of equity) for a longer time when the interest rate corridor is narrow.

Result 4 *Interest rate is less responsive to the network destruction shock when the corridor of policy rates is narrow. However, the narrower the corridor, the longer the shock propagates on banks' equity.*

I next check if Result 5 is robust for different shapes of the interbank network. Figures 13-15 display the results.

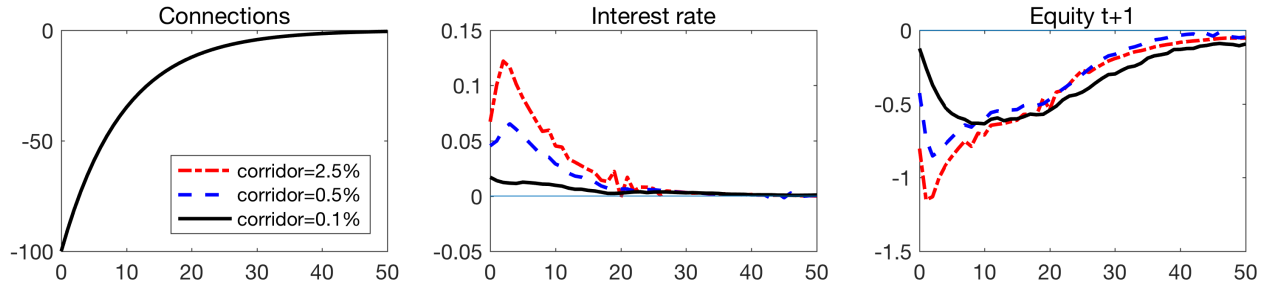


Figure 12. Interest rate corridor and network destruction shock (complete network). The figure displays impulse responses to removal of all links in the complete network for different interest rate corridors. Vertical axes are percent deviations from steady state for level variables and deviations from steady state in percentage points for interest rates and standard deviations.

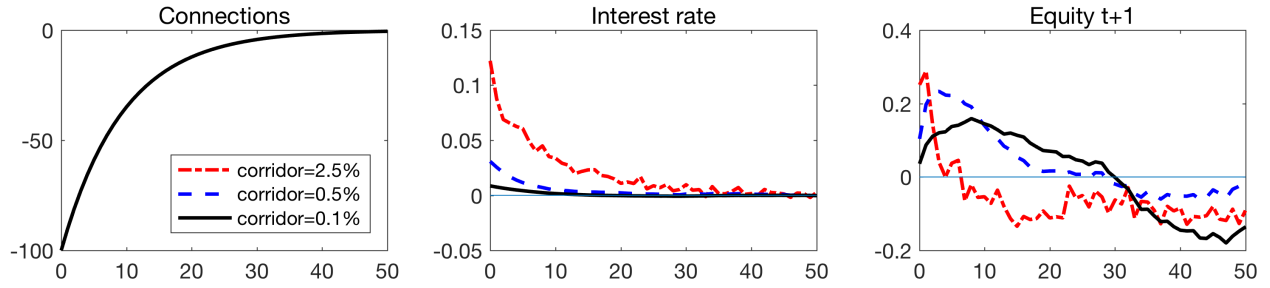


Figure 13. Interest rate corridor and network destruction shock (circle network). The figure displays impulse responses to removal of all links in the complete network for different interest rate corridors. Vertical axes are percent deviations from steady state for level variables and deviations from steady state in percentage points for interest rates and standard deviations.

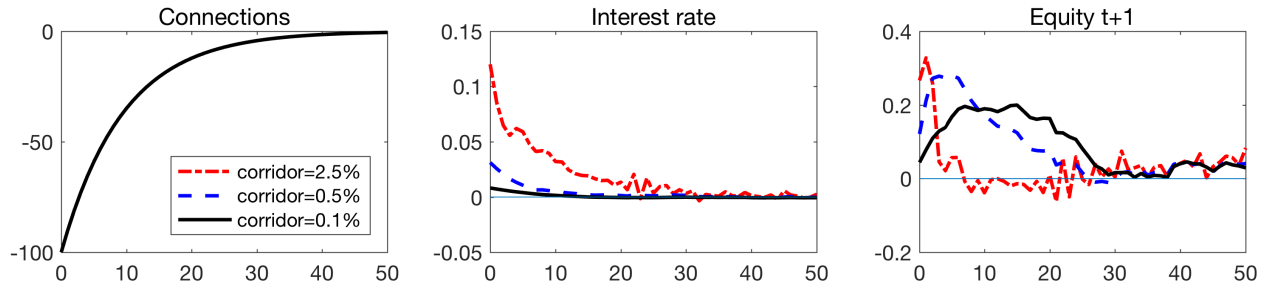


Figure 14. Interest rate corridor and network destruction shock (random network). The figure displays impulse responses to removal of all links in the complete network for different interest rate corridors. Vertical axes are percent deviations from steady state for level variables and deviations from steady state in percentage points for interest rates and standard deviations.

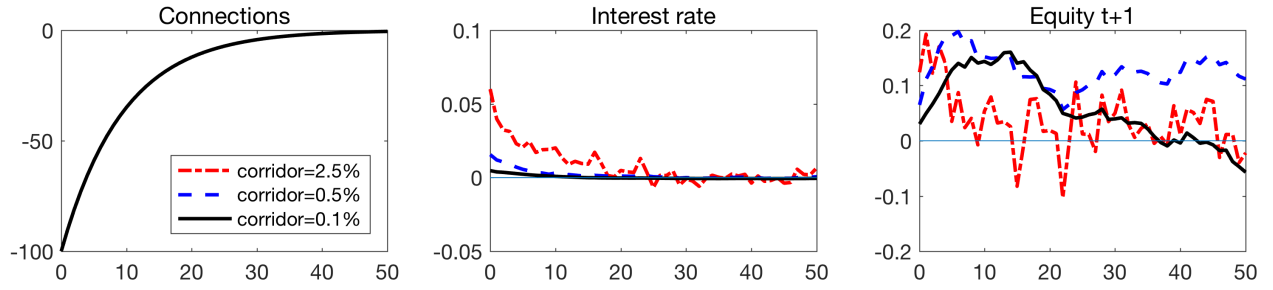


Figure 15. Interest rate corridor and network destruction shock (scale-free network). The figure displays impulse responses to removal of all links in the complete network for different interest rate corridors. Vertical axes are percent deviations from steady state for level variables and deviations from steady state in percentage points for interest rates and standard deviations.

8 Conclusion

This paper presents a dynamic macro model with an agent-based banking sector where banks are interconnected with each other via the interbank network. The quantitative exercises show that the structure and dynamics of the interbank network are essential for our understanding how the financial sector influences the real economy. In particular, the response of the economy to an interbank market freeze is qualitatively different from the response to a smaller interbank network disruption, which implies potentially different strategies for monetary policy. Depending on the central bank's policy, the shocks to the interbank network may matter more or less for the aggregate investment. This presents a potential tradeoff for monetary policy. Ongoing work includes further investigation of how monetary policy may be able to mitigate the distress in the financial sector. One other extension that is a subject of the ongoing research is embedding an endogenous mechanism of interbank network formation into the model.

References

Franklin Allen and Douglas Gale. Financial contagion. *Journal of Political Economy*, 108(1):1–33, February 2000. URL <http://ideas.repec.org/a/ucp/jpolec/>

[v108y2001i1p1-33.html](#).

Andrew G Atkeson, Andrea L Eisfeldt, and Pierre-Olivier Weill. The market for otc credit derivatives. In *UCLA Working Paper*. 2012.

A.L. Barabási. *Network Science*. Cambridge University Press, 2016. ISBN 9781107076266. URL <https://books.google.com/books?id=iLtGDQAAQBAJ>.

Albert-László Barabási and Réka Albert. Emergence of scaling in random networks. *science*, 286(5439):509–512, 1999.

Morten Bech and Cyril Monnet. Sorting in the interbank money market. Technical report, Citeseer, 2014.

Morten L Bech and Enghin Atalay. The topology of the federal funds market. *Physica A: Statistical Mechanics and its Applications*, 389(22):5223–5246, 2010.

Daniel O Beltran, Valentin Bolotnyy, and Elizabeth Klee. Un-networking: The evolution of networks in the federal funds market. 2015.

Javier Bianchi and Saki Bigio. Banks, liquidity management and monetary policy. NBER Working Papers 20490, National Bureau of Economic Research, Inc, September 2014. URL <http://ideas.repec.org/p/nbr/nberwo/20490.html>.

Francisco Blasques, Falk Brauning, and Iman Van Lelyveld. A dynamic network model of the unsecured interbank lending market. Working Papers 16-3, Federal Reserve Bank of Boston, April 2016. URL <https://ideas.repec.org/p/fip/fedbwp/16-3.html>.

Frédéric Boissay, Fabrice Collard, and Frank Smets. Booms and systemic banking crises. 2013.

Michael Boss, Helmut Elsinger, Martin Summer, and Stefan Thurner 4. Network topology of the interbank market. *Quantitative Finance*, 4(6):677–684, 2004.

- Falk Bräuning, Falko Fecht, et al. *Relationship lending in the interbank market and the price of liquidity*. Deutsche Bundesbank, 2012.
- Charles W Calomiris and Mark A Carlson. Interbank networks in the national banking era: their purpose and their role in the panic of 1893. 2016.
- Ben Craig and Goetz von Peter. Interbank tiering and money center banks. *Journal of Financial Intermediation*, 23(3):322–347, 2014. URL <http://ideas.repec.org/a/eee/jfinin/v23y2014i3p322-347.html>.
- Paul Erdős and Alfréd Rényi. On random graphs, i. *Publicationes Mathematicae (Debrecen)*, 6:290–297, 1959.
- Xavier Freixas, Bruno Parigi, and Jean Charles Rochet. Systemic risk, interbank relations and liquidity provision by the central bank. Economics Working Papers 440, Department of Economics and Business, Universitat Pompeu Fabra, October 1998. URL <http://ideas.repec.org/p/upf/upfgen/440.html>.
- Agam Gupta, Molly M King, James Magdanz, Regina Martinez, Matteo Smerlak, and Brady Stoll. Critical connectivity in banking networks. *SFI CSSS. SFI CSSS report*, 2013.
- Hajime Inaoka, Takuto Ninomiya, Ken Taniguchi, Tokiko Shimizu, Hideki Takayasu, et al. Fractal network derived from banking transaction—an analysis of network structures formed by financial institutions. 2004.
- Giulia Iori, Giulia De Masi, Ovidiu Vasile Precup, Giampaolo Gabbi, and Guido Caldarelli. A network analysis of the italian overnight money market. *Journal of Economic Dynamics and Control*, 32(1):259–278, 2008.
- Konstantin Klemm and Victor M Eguiluz. Growing scale-free networks with small-world behavior. *Physical Review E*, 65(5):057102, 2002.

- Simone Lenzu and Gabriele Tedeschi. Systemic risk on different interbank network topologies. Working Papers 375, Universita' Politecnica delle Marche (I), Dipartimento di Scienze Economiche e Sociali, April 2012. URL <http://ideas.repec.org/p/anc/wpaper/375.html>.
- Sergei Maslov and Kim Sneppen. Specificity and stability in topology of protein networks. *Science*, 296(5569):910–913, 2002. ISSN 0036-8075. doi: 10.1126/science.1065103. URL <http://science.sciencemag.org/content/296/5569/910>.
- Erlend Nier, Jing Yang, Tanju Yorulmazer, and Amadeo Alentorn. Network models and financial stability. *Journal of Economic Dynamics and Control*, 31(6):2033–2060, 2007.
- Kimmo Soramäki, Morten L Bech, Jeffrey Arnold, Robert J Glass, and Walter E Beyeler. The topology of interbank payment flows. *Physica A: Statistical Mechanics and its Applications*, 379(1):317–333, 2007.
- Anne Vila, Kimmo Soramaki, and Peter Zimmerman. The sterling unsecured loan market during 2006-08: insights from network theory. 2010.
- Duncan J. Watts and Steven H. Strogatz. Collective dynamics of /‘small-world/’ networks. *Nature*, 393(6684):440–442, 06 1998. URL <http://dx.doi.org/10.1038/30918>.

Appendix A Matching Probabilities

Banks have to make portfolio decisions before the deposit shocks are realized. I define a vector of bank deficits:

$$\tilde{X}_i = \omega_i \tilde{D}_i - \tilde{C}_i \quad (65)$$

where a negative value implies that a bank has a surplus.

Let ω_i^* be the value of deposit withdrawal that makes deficit $\tilde{X}_i$ equal 0:

$$\omega_i^* = \frac{\tilde{C}_i}{\tilde{D}_i} = \frac{\tilde{c}_i}{\tilde{d}_i}$$

Then if $\omega_i \leq \omega_i^*$, there is a surplus $\tilde{X}_i \leq 0$ and i is a lender, and if $\omega_i > \omega_i^*$, there is a deficit $\tilde{X}_i > 0$ and i is a borrower. Recall

$$p_i^B(G) = \min \left[1, \frac{\Upsilon_i^+(G_i)}{\Upsilon_i^-(G)} \right]$$

where $K = \sum_j G_{jk} \cdot \mathbb{1}_{\{G_{ij}=1\}}$.

$$\begin{aligned} \Upsilon_i^+ &= \sum_j G_{ij} \cdot F(\omega_j \leq \omega_j^*) \mathbb{E} [\tilde{X}_j(\omega_j) \mid \omega_j \leq \omega_j^*] \\ &= \sum_j G_{ij} \cdot F(\omega_j \leq \omega_j^*) \int_{-\infty}^{\omega_j^*} (\omega \tilde{D}_j - \tilde{C}_j) f(\omega) d\omega \\ &= \sum_j G_{ij} \cdot F(\omega_j \leq \omega_j^*) (1 - \theta) E_j \int_{-\infty}^{\omega_j^*} (\omega \tilde{d}_j - \tilde{c}_j) f(\omega) d\omega \\ &= (1 - \theta) \sum_j G_{ij} \cdot F(\omega_j \leq \omega_j^*) E_j \int_{-\infty}^{\omega_j^*} (\omega \tilde{d}_j - \tilde{c}_j) f(\omega) d\omega \end{aligned}$$

The integral $\int_{-\infty}^{\omega_j^*} (\omega \tilde{d}_j - \tilde{c}_j) f(\omega) d\omega$ is the same for all j , thus it can be taken out of the sum:

$$\begin{aligned} \Upsilon_i^+ &= (1 - \theta) F(\omega_j \leq \omega_j^*) \int_{-\infty}^{\omega_j^*} (\omega \tilde{d} - \tilde{c}) f(\omega) d\omega \cdot \sum_j G_{ij} \cdot E_j \\ &= (1 - \theta) F(\omega_j \leq \omega_j^*) \mathbb{E} [x(\omega) \mid \omega \leq \omega^*] \cdot \sum_j G_{ij} \cdot E_j \end{aligned}$$

Equivalently,

$$\begin{aligned} \Upsilon_i^- &= \sum_k \mathbb{1}_{\{K_k \geq 1\}} \cdot F(\omega_j > \omega_j^*) \mathbb{E} [\tilde{X}_k(\omega_k) \mid \omega_k > \omega_k^*] \\ &= (1 - \theta) \mathbb{E} [x(\omega) \mid \omega > \omega^*] \cdot F(\omega_j > \omega_j^*) \sum_k \mathbb{1}_{\{K_k \geq 1\}} E_k \end{aligned}$$

$$\begin{aligned}
p_i^B(G) &= \min \left[1, \frac{(1-\theta) F(\omega \leq \omega^*) \mathbb{E}[x(\omega) \mid \omega \leq \omega^*] \cdot \sum_j G_{ij} \cdot E_j}{(1-\theta) F(\omega > \omega^*) \mathbb{E}[x(\omega) \mid \omega > \omega^*] \cdot \sum_k \mathbb{1}_{\{K_k \geq 1\}} E_k} \right] \\
&= \min \left[1, \frac{F(\omega \leq \omega^*) \mathbb{E}[x(\omega) \mid \omega \leq \omega^*] \cdot \sum_j G_{ij} \cdot E_j}{F(\omega > \omega^*) \mathbb{E}[x(\omega) \mid \omega > \omega^*] \cdot \sum_k \mathbb{1}_{\{K_k \geq 1\}} \cdot E_k} \right] \\
p_i^B(G) &= \min \left[1, \frac{F(\omega \leq \omega^*) \mathbb{E}[x(\omega) \mid \omega \leq \omega^*]}{F(\omega > \omega^*) \mathbb{E}[x(\omega) \mid \omega > \omega^*]} \cdot \frac{\sum_j G_{ij} \cdot E_j}{\sum_k \mathbb{1}_{\{K_k \geq 1\}} \cdot E_k} \right] \tag{66}
\end{aligned}$$

where $K = \sum_j G_{jk} \cdot \mathbb{1}_{\{G_{ij}=1\}}$.

Similar procedure results in an expression for the probability of lending order matching with a borrowing order. Recall that:

$$p_i^L(G) = \min \left[1, \frac{\Gamma_i^-}{\Gamma_i^+} \right]$$

where Γ_i^- is the mass of reserve deficits for i 's neighbors and Γ_i^+ is the mass of lending orders available to i 's neighbors with borrowing orders.

$$p_i^L(G) = \min \left[1, \frac{F(\omega > \omega^*) \mathbb{E}[x(\omega) \mid \omega > \omega^*]}{F(\omega \leq \omega^*) \mathbb{E}[x(\omega) \mid \omega \leq \omega^*]} \cdot \frac{\sum_j G_{ij} \cdot E_j}{\sum_k \mathbb{1}_{\{K_k \geq 1\}} \cdot E_k} \right] \tag{67}$$

I separate the components of the probabilities into common and idiosyncratic:

$$\begin{aligned}
\text{common: } \psi &= \frac{F(\omega > \omega^*) \mathbb{E}[x(\omega) \mid \omega > \omega^*]}{F(\omega \leq \omega^*) \mathbb{E}[x(\omega) \mid \omega \leq \omega^*]} \\
\text{idiosyncratic: } \Psi_i(G_i) &= \frac{\sum_j G_{ij} \cdot E_j}{\sum_k \mathbb{1}_{\{K_k \geq 1\}} \cdot E_k}
\end{aligned}$$

Then,

$$p_i^B(G) = \min \left[1, \frac{1}{\psi} \Psi_i(G_i) \right] \tag{68}$$

$$p_i^L(G) = \min [1, \psi \Psi_i(G_i)] \tag{69}$$

Appendix B Generalized Bank's Problem

This section closely follows the derivation of the bank's problem in [Bianchi and Bigio \(2014\)](#). Banks maximize their expected lifetime utility:

$$\begin{aligned} \max_{\tilde{D}_i, \tilde{B}_i, \tilde{C}_i, DIV_i} \quad & \mathbb{E}_0 \sum_{t \geq 0} (\beta \zeta)^t \frac{DIV^{1-\gamma}}{1-\gamma} \\ \text{s.t.} \quad & E_i = \tilde{B}_i + \tilde{C}_i + DIV_i - \tilde{D}_i \end{aligned} \quad (70)$$

$$E'_i = R_i^b \tilde{B}_i - R^d \tilde{D}_i + R_i^x \tilde{C}_i - \omega_i (R_i^x - R^d) \tilde{D}_i \quad (71)$$

$$\tilde{D}_i \leq \kappa (\tilde{B}_i + \tilde{C}_i - \tilde{D}_i) \quad (72)$$

$$\tilde{B}_i, \tilde{C}_i, \tilde{D}_i \geq 0$$

I denote the aggregate state by Z and solve the above problem by the method of dynamic programming. The aggregate state is summarized in vector $Z = \{r^{DW}; r^{ER}; F(\omega); G\}$, which includes policy rates, distribution of withdrawal shocks, and the network matrix. Rewriting the problem:

$$V(E_i, Z) = \max_{\tilde{D}_i, \tilde{B}_i, \tilde{C}_i, DIV_i} \frac{DIV^{1-\gamma}}{1-\gamma} + \beta \zeta \mathbb{E}[V(E'_i, Z')] \quad (73)$$

$$\begin{aligned} \text{s.t.} \quad & E_i = \tilde{B}_i + \tilde{C}_i + DIV_i - \tilde{D}_i \\ & E'_i = R_i^b \tilde{B}_i - R^d \tilde{D}_i + R_i^x \tilde{C}_i - \omega_i (R_i^x - R^d) \tilde{D}_i \\ & \tilde{D}_i \leq \kappa (\tilde{B}_i + \tilde{C}_i - \tilde{D}_i) \\ & \tilde{B}_i, \tilde{C}_i, \tilde{D}_i \geq 0 \end{aligned}$$

B.1 Homogeneity

I define a fraction of equity that a bank allocates towards dividends as $div_i \equiv DIV_i/E_i$. The utility function can be written as:

$$U(DIV_i) = E_i^{1-\gamma} \cdot U(div_i)$$

I guess that the value function satisfies:

$$V(E_i, Z) = v(Z) E_i^{1-\gamma}$$

where $v(Z)$ is the slope of the value function. The value function (73) can be rewritten as:

$$V(E_i, Z) = E_i^{1-\gamma} \left[\max_{\tilde{D}_i, \tilde{B}_i, \tilde{C}_i, div_i} \frac{div_i^{1-\gamma}}{1-\gamma} + \beta \zeta \mathbb{E}_\omega \mathbb{E} v(Z'|Z) \left[\frac{E'_i}{E_i} \right]^{1-\gamma} \right]$$

Consider the budget constraint (70). Dividing it by E_i results in:

$$1 = \tilde{b}_i + \tilde{c}_i + div_i - \tilde{d}_i \quad (74)$$

where deposits, loans, and reserves are expressed as fractions of equity:

$$\begin{bmatrix} \tilde{d}_i & \tilde{b}_i & \tilde{c}_i \end{bmatrix} \equiv \begin{bmatrix} \tilde{D}_i & \tilde{B}_i & \tilde{C}_i \\ E_i & E_i & E_i \end{bmatrix} \quad (75)$$

The level of equity in the beginning of a period is non-negative, thus, dividing the capital requirement by E_i results in:

$$\tilde{d}_i \leq \kappa \left(\tilde{b}_i + \tilde{c}_i - \tilde{d}_i \right) \quad (76)$$

Consider the evolution of equity (71). All the terms on the right-hand side are linear in equity. Dividing the equation by E_i yields:

$$\frac{E'_i}{E_i} = R_i^b \tilde{b}_i - R^d \tilde{d}_i + R_i^x \tilde{c}_i - \omega_i (R_i^x - R^d) \tilde{d}_i \quad (77)$$

where $\frac{E'_i}{E_i}$ is equity growth between two consecutive periods, which is equal to the sum of the realized returns on loans and reserves net of the cost of deposits.

Problem 6 *The scale-invariant problem of a bank is:*

$$\begin{aligned} v(Z) &= \max_{\tilde{d}_i, \tilde{b}_i, \tilde{c}_i, \text{div}_i} \frac{\text{div}_i^{1-\gamma}}{1-\gamma} + \beta \zeta \mathbb{E}_\omega \mathbb{E} v(Z'|Z) \left[\frac{E'_i}{E_i} \right]^{1-\gamma} \\ \text{s.t.} \quad & 1 = \tilde{b}_i + \tilde{c}_i + \text{div}_i - \tilde{d}_i \\ & \frac{E'_i}{E_i} = R_i^b \tilde{b}_i - R^d \tilde{d}_i + R_i^x \tilde{c}_i - \omega_i (R_i^x - R^d) \tilde{d}_i \\ & \tilde{d}_i \leq \kappa \left(\tilde{b}_i + \tilde{c}_i - \tilde{d}_i \right) \\ & \tilde{b}_i, \tilde{c}_i, \tilde{d}_i \geq 0 \end{aligned} \quad (78)$$

Policy rules that solve the original problem are equivalent to the policy rules that solve Problem 6 multiplied by equity.

B.2 Portfolio Separation

Rewrite the budget constraint (74) as follows:

$$1 - \text{div}_i = \tilde{b}_i + \tilde{c}_i - \tilde{d}_i$$

The left-hand side constitutes the fraction of equity that is split between investment in assets with different returns. These can be thought of as portfolio shares of three assets: loans, reserves, and deposits. I define these shares as:

$$\hat{b}_i = \frac{\tilde{b}_i}{1 - \text{div}_i}, \quad \hat{c}_i = \frac{\tilde{c}_i}{1 - \text{div}_i}, \quad \hat{d}_i = \frac{\tilde{d}_i}{1 - \text{div}_i} \quad (79)$$

Using the definitions above, the budget constraint and the capital requirement can be rewritten as:

$$1 = \hat{b}_i + \hat{c}_i - \hat{d}_i \quad (80)$$

$$\hat{d}_i \leq \frac{\kappa}{1 + \kappa} (\hat{b}_i + \hat{c}_i) \quad (81)$$

respectively. Expressing the evolution of equity in terms of portfolio shares results in:

$$\frac{E'_i}{E_i} = (1 - \text{div}_i) \left[R_i^b \hat{b}_i - R^d \hat{d}_i + R_i^x \hat{c}_i - \omega_i (R_i^x - R^d) \hat{d}_i \right]$$

Substituting the budget constraint (80) into the above:

$$\frac{E'_i}{E_i} = (1 - \text{div}_i) \left[R_i^b + (R_i^x - R_i^b) \hat{c}_i + R_i^b \hat{d}_i - \omega_i (R_i^x - R^d) \hat{d}_i \right] \quad (82)$$

Substituting the budget constraint (80) into the capital requirement (81) yields:

$$\hat{d}_i \leq \kappa \quad (83)$$

Since div_i is known at $t + 1$, the value function (78) can be rewritten as:

$$v(Z) = \max_{\hat{d}_i, \hat{c}_i, \text{div}_i} \frac{\text{div}_i^{1-\gamma}}{1-\gamma} + \beta \zeta (1 - \text{div}_i)^{1-\gamma} \mathbb{E} v(Z'|Z) \mathbb{E}_\omega [R_i^E]^{1-\gamma}$$

where R_i^E is the realized return on bank's portfolio defined as:

$$R_i^E \equiv R_i^b \hat{b}_i - R^d \hat{d}_i + R_i^x \hat{c}_i - \omega_i (R_i^x - R^d) \hat{d}_i \quad (84)$$

Moreover, $\hat{c}_i$ and $\hat{d}_i$ enter only in the continuation value, thus, their optimal values can be found independently from optimal dividend. Solving problem 6 is equivalent to solving the following problem:

Problem 7 *The value function $v(\cdot)$ solves:*

$$\begin{aligned} v(Z) = \max_{\text{div}_i} & \frac{\text{div}_i^{1-\gamma}}{1-\gamma} + \beta \zeta (1 - \text{div}_i)^{1-\gamma} \mathbb{E} v(Z'|Z) \max_{\hat{d}_i, \hat{c}_i} \mathbb{E}_\omega [R_i^E]^{1-\gamma} \\ \text{s.t.} & R_i^E = R_i^b \hat{b}_i - R^d \hat{d}_i + R_i^x \hat{c}_i - \omega_i (R_i^x - R^d) \hat{d}_i \\ & \hat{d}_i \leq \kappa \\ & 0 \leq \hat{c}_i, \hat{d}_i \end{aligned}$$

Next I consider the portfolio maximization problem.

B.3 Portfolio Maximization Problem

$$\begin{aligned}
\max_{\hat{d}_i, \hat{c}_i} \quad & \mathbb{E}_\omega \left[R_i^b \hat{b}_i - R^d \hat{d}_i + R_i^x \hat{c}_i - \omega_i (R_i^x - R^d) \hat{d}_i \right]^{1-\gamma} \\
\text{s.t.} \quad & \hat{d}_i \leq \kappa \\
& 0 \leq \hat{c}_i, \hat{d}_i
\end{aligned} \tag{85}$$

A non-standard feature of this problem is that R_i^x has a discontinuity at the point where bank's reserve deficit is zero. This occurs when $\omega_i = \frac{\hat{c}_i}{\hat{d}_i}$. Since $\omega_i \leq 1$, then it must be that $\frac{\hat{c}_i}{\hat{d}_i} \leq 1$, which rules out $\hat{d}_i = 0$ in equilibrium. If the realized shock is below $\frac{\hat{c}_i}{\hat{d}_i}$, then the bank has excess reserves, which can be sold at χ_i^L . If the realized shock is above $\frac{\hat{c}_i}{\hat{d}_i}$, then the bank has a reserve deficit and has to buy reserves at χ_i^B . The portfolio problem can be rewritten as follows:

$$\begin{aligned}
\max_{\hat{d}_i, \hat{c}_i} \quad & \int_{-\infty}^{\frac{\hat{c}_i}{\hat{d}_i}} [R_i^E(\chi_i^L)]^{1-\gamma} f(\omega) d\omega + \int_{\frac{\hat{c}_i}{\hat{d}_i}}^1 [R_i^E(\chi_i^B)]^{1-\gamma} f(\omega) d\omega \\
\text{s.t.} \quad & \hat{d}_i \leq \kappa \\
& 0 \leq \hat{c}_i \\
\text{where} \quad & R_i^E(R_i^x) = R_i^b + (R_i^x - R_i^b) \hat{c}_i - (R^d - R_i^b + R_i^x \cdot \omega_i) \hat{d}_i
\end{aligned}$$

Rewriting the problem:

$$\begin{aligned}
\max_{\hat{d}_i, \hat{c}_i} \quad & \int_{-\infty}^{\frac{\hat{c}_i}{\hat{d}_i}} \left[R_i^b + (\chi_i^L - R_i^b) \hat{c}_i - (R^d - R_i^b + \omega_i \chi_i^L) \hat{d}_i \right]^{1-\gamma} f(\omega) d\omega \\
& + \int_{\frac{\hat{c}_i}{\hat{d}_i}}^1 \left[R_i^b + (\chi_i^B - R_i^b) \hat{c}_i - (R^d - R_i^b + \omega_i \chi_i^B) \hat{d}_i \right]^{1-\gamma} f(\omega) d\omega \\
& + \mu_i (\kappa - \hat{d}_i) + \lambda_{it}^1 \hat{c}_i + \lambda_{it}^2 \hat{d}_i
\end{aligned}$$

Differentiating w.r.t. $\hat{c}_i$:

$$\begin{aligned}
0 = & (1 - \gamma) (\chi_i^L - R_i^b) \int_{-\infty}^{\frac{\hat{c}_i}{\hat{d}_i}} \left[R_i^b + (\chi_i^L - R_i^b) \hat{c}_i - (R^d - R_i^b + \omega_i \chi_i^L) \hat{d}_i \right]^{-\gamma} f(\omega) d\omega \\
& + \frac{1}{\hat{d}_i} \left[R_i^b + (\chi_i^L - R_i^b) \hat{c}_i - \left(R^d - R_i^b + \frac{\hat{c}_i}{\hat{d}_i} \chi_i^L \right) \hat{d}_i \right]^{1-\gamma} \\
& + (1 - \gamma) (\chi_i^B - R_i^b) \int_{\frac{\hat{c}_i}{\hat{d}_i}}^1 \left[R_i^b + (\chi_i^B - R_i^b) \hat{c}_i - (R^d - R_i^b + \omega_i \chi_i^B) \hat{d}_i \right]^{-\gamma} f(\omega) d\omega \\
& - \frac{1}{\hat{d}_i} \left[R_i^b + (\chi_i^B - R_i^b) \hat{c}_i - \left(R^d - R_i^b + \frac{\hat{c}_i}{\hat{d}_i} \chi_i^B \right) \hat{d}_i \right]^{1-\gamma} + \lambda_{it}^1
\end{aligned}$$

Some terms cancel out:

$$\begin{aligned}
-\lambda_{it}^1 = & (1 - \gamma) (\chi_i^L - R_i^b) \int_{-\infty}^{\frac{\tilde{c}_i}{\hat{d}_i}} \left[R_i^b + (\chi_i^L - R_i^b) \hat{c}_i - (R^d - R_i^b + \omega_i \chi_i^L) \hat{d}_i \right]^{-\gamma} f(\omega) d\omega \\
& + \frac{1}{\hat{d}_i} \left[R_i^b - R_i^b \hat{c}_i - (R^d - R_i^b) \hat{d}_i \right]^{1-\gamma} - \frac{1}{\hat{d}_i} \left[R_i^b - R_i^b \hat{c}_i - (R^d - R_i^b) \hat{d}_i \right]^{1-\gamma} \\
& + (1 - \gamma) (\chi_i^B - R_i^b) \int_{\frac{\tilde{c}_i}{\hat{d}_i}}^1 \left[R_i^b + (\chi_i^B - R_i^b) \hat{c}_i - (R^d - R_i^b + \omega_i \chi_i^B) \hat{d}_i \right]^{-\gamma} f(\omega) d\omega
\end{aligned}$$

Divide by $1 - \gamma$:

$$\begin{aligned}
-\frac{\lambda_{it}^1}{1 - \gamma} = & (\chi_i^L - R_i^b) \int_{-\infty}^{\frac{\tilde{c}_i}{\hat{d}_i}} \left[R_i^b + (\chi_i^L - R_i^b) \hat{c}_i - (R^d - R_i^b + \omega_i \chi_i^L) \hat{d}_i \right]^{-\gamma} f(\omega) d\omega \\
& + (\chi_i^B - R_i^b) \int_{\frac{\tilde{c}_i}{\hat{d}_i}}^1 \left[R_i^b + (\chi_i^B - R_i^b) \hat{c}_i - (R^d - R_i^b + \omega_i \chi_i^B) \hat{d}_i \right]^{-\gamma} f(\omega) d\omega
\end{aligned}$$

Isolate R_i^b :

$$\begin{aligned}
-\frac{\lambda_{it}^1}{1 - \gamma} = & \int_{-\infty}^{\frac{\tilde{c}_i}{\hat{d}_i}} \chi_i^L \left[R_i^b + (\chi_i^L - R_i^b) \hat{c}_i - (R^d - R_i^b + \omega_i \chi_i^L) \hat{d}_i \right]^{-\gamma} f(\omega) d\omega \\
& - R_i^b \int_{-\infty}^{\frac{\tilde{c}_i}{\hat{d}_i}} \left[R_i^b + (\chi_i^L - R_i^b) \hat{c}_i - (R^d - R_i^b + \omega_i \chi_i^L) \hat{d}_i \right]^{-\gamma} f(\omega) d\omega \\
& + \int_{\frac{\tilde{c}_i}{\hat{d}_i}}^1 \chi_i^B \left[R_i^b + (\chi_i^B - R_i^b) \hat{c}_i - (R^d - R_i^b + \omega_i \chi_i^B) \hat{d}_i \right]^{-\gamma} f(\omega) d\omega \\
& - R_i^b \int_{\frac{\tilde{c}_i}{\hat{d}_i}}^1 \left[R_i^b + (\chi_i^B - R_i^b) \hat{c}_i - (R^d - R_i^b + \omega_i \chi_i^B) \hat{d}_i \right]^{-\gamma} f(\omega) d\omega
\end{aligned}$$

Using the definition $R_i^E \equiv R_i^b + (\chi_i^L - R_i^b) \hat{c}_i - (R^d - R_i^b + \omega_i \chi_i^L) \hat{d}_i$, the above equation can be rewritten as follows:

$$\mathbb{E}_\omega \left[R_i^x [R_i^E]^{-\gamma} \right] - R_i^b \mathbb{E}_\omega [R_i^E]^{-\gamma} + \frac{\lambda_{it}^1}{1 - \gamma} = 0$$

Applying formula for expectation of a product of two dependent variables:

$$\mathbb{E}_\omega R_i^x \cdot \mathbb{E}_\omega [R_i^E]^{-\gamma} + \text{COV} \left\{ R_i^x, [R_i^E]^{-\gamma} \right\} - R_i^b \mathbb{E}_\omega [R_i^E]^{-\gamma} + \frac{\lambda_{it}^1}{1 - \gamma} = 0$$

Dividing by $\mathbb{E}_\omega [R_i^E]^{-\gamma}$:

$$R_i^b = \mathbb{E}_\omega R_i^x + \frac{\text{COV} \left\{ R_i^x, [R_i^E]^{-\gamma} \right\}}{\mathbb{E}_\omega [R_i^E]^{-\gamma}} + \frac{\lambda_{it}^1}{(1 - \gamma) \mathbb{E}_\omega [R_i^E]^{-\gamma}}$$

where

$$\begin{aligned}
\mathbb{E}_\omega R_i^x &= \chi_i^L \int_{-\infty}^{\frac{\tilde{c}_i}{\tilde{d}_i}} f(\omega) d\omega + \chi_i^B \int_{\frac{\tilde{c}_i}{\tilde{d}_i}}^1 f(\omega) d\omega \\
&= \chi_i^L \cdot F\left[\frac{\tilde{c}_i}{\tilde{d}_i}\right] + \chi_i^B \left(1 - F\left[\frac{\tilde{c}_i}{\tilde{d}_i}\right]\right) \\
&= (\chi_i^L - \chi_i^B) F\left[\frac{\tilde{c}_i}{\tilde{d}_i}\right] + \chi_i^B
\end{aligned}$$

Differentiating the objective w.r.t. $\hat{d}_i$:

$$\begin{aligned}
& - (1 - \gamma) (R^d - R_i^b + \omega_i \chi_i^L) \int_{-\infty}^{\frac{\tilde{c}_i}{\tilde{d}_i}} \left[R_i^b + (\chi_i^L - R_i^b) \hat{c}_i - (R^d - R_i^b + \omega_i \chi_i^L) \hat{d}_i \right]^{-\gamma} f(\omega) d\omega \\
& - \frac{\hat{c}_i}{\hat{d}_i^2} \left[R_i^b + (\chi_i^L - R_i^b) \hat{c}_i - \left(R^d - R_i^b + \frac{\tilde{c}_i}{\tilde{d}_i} \chi_i^L \right) \hat{d}_i \right]^{1-\gamma} \\
& + \frac{\hat{c}_i}{\hat{d}_i^2} \left[R_i^b + (\chi_i^B - R_i^b) \hat{c}_i - \left(R^d - R_i^b + \frac{\tilde{c}_i}{\tilde{d}_i} \chi_i^B \right) \hat{d}_i \right]^{1-\gamma} \\
& - (1 - \gamma) (R^d - R_i^b + \omega_i \chi_i^B) \int_{\frac{\tilde{c}_i}{\tilde{d}_i}}^1 \left[R_i^b + (\chi_i^B - R_i^b) \hat{c}_i - (R^d - R_i^b + \omega_i \chi_i^B) \hat{d}_i \right]^{-\gamma} f(\omega) d\omega \\
& - \mu_i + \lambda_{it}^2 = 0
\end{aligned}$$

Equivalently:

$$\begin{aligned}
& - (R^d - R_i^b + \omega_i \chi_i^L) \int_{-\infty}^{\frac{\tilde{c}_i}{\tilde{d}_i}} \left[R_i^b + (\chi_i^L - R_i^b) \hat{c}_i - (R^d - R_i^b + \omega_i \chi_i^L) \hat{d}_i \right]^{-\gamma} f(\omega) d\omega \\
& - (R^d - R_i^b + \omega_i \chi_i^B) \int_{\frac{\tilde{c}_i}{\tilde{d}_i}}^1 \left[R_i^b + (\chi_i^B - R_i^b) \hat{c}_i - (R^d - R_i^b + \omega_i \chi_i^B) \hat{d}_i \right]^{-\gamma} f(\omega) d\omega \\
& - \frac{\mu_i}{1 - \gamma} + \frac{\lambda_{it}^2}{1 - \gamma} = 0
\end{aligned}$$

Rewriting in terms of R_i^E :

$$- (R^d - R_i^b) \mathbb{E}_\omega [R_i^E]^{-\gamma} - \mathbb{E}_\omega [\omega_i R_i^x [R_i^E]^{-\gamma}] + \frac{\lambda_{it}^2 - \mu_i}{1 - \gamma} = 0$$

Applying formula for expectation of a product of two dependent variables:

$$- (R^d - R_i^b) \mathbb{E}_\omega [R_i^E]^{-\gamma} - \mathbb{E}_\omega [\omega_i R_i^x] \mathbb{E}_\omega [R_i^E]^{-\gamma} - \mathbb{COV} \left\{ \omega_i R_i^x, [R_i^E]^{-\gamma} \right\} + \frac{\lambda_{it}^2 - \mu_i}{1 - \gamma} = 0$$

Dividing by $\mathbb{E}_\omega [R_i^E]^{-\gamma}$:

$$R^d - R_i^b = -\mathbb{E}_\omega [\omega_i R_i^x] - \frac{\mathbb{COV} \left\{ \omega_i R_i^x, [R_i^E]^{-\gamma} \right\}}{\mathbb{E}_\omega [R_i^E]^{-\gamma}} + \frac{\lambda_{it}^2 - \mu_i}{(1 - \gamma) \mathbb{E}_\omega [R_i^E]^{-\gamma}}$$

where

$$\mathbb{E}_\omega [\omega_i R_i^x] = \chi_i^L \int_{-\infty}^{\frac{\hat{c}_i}{\hat{d}_i}} \omega f(\omega) d\omega + \chi_i^B \int_{\frac{\hat{c}_i}{\hat{d}_i}}^1 \omega f(\omega) d\omega$$

The first order conditions for an interior solution imply:

$$R_i^b = \frac{\mathbb{E}_\omega [R_i^x \cdot [R_i^E]^{-\gamma}]}{\mathbb{E}_\omega [R_i^E]^{-\gamma}} \quad (86)$$

$$R_i^b - R^d = \frac{\mathbb{E}_\omega [R_i^x \cdot \omega_i \cdot [R_i^E]^{-\gamma}] + \frac{\mu_i}{1-\gamma}}{\mathbb{E}_\omega [R_i^E]^{-\gamma}} \quad (87)$$

where μ_i is the multiplier on the capital requirement constraint.

$$\begin{aligned} R_i^b &= \underbrace{\mathbb{E}_\omega R_i^x}_{\text{direct effect}} + \underbrace{\frac{\mathbb{COV} \left\{ R_i^x, [R_i^E]^{-\gamma} \right\}}{\mathbb{E}_\omega [R_i^E]^{-\gamma}}}_{\text{liquidity risk premium effect}} \\ R_i^b - R^d &\geq \underbrace{\mathbb{E}_\omega [R_i^x \cdot \omega_i]}_{\text{direct effect}} + \underbrace{\frac{\mathbb{COV} \left\{ R_i^x \cdot \omega_i, [R_i^E]^{-\gamma} \right\}}{\mathbb{E}_\omega [R_i^E]^{-\gamma}}}_{\text{liquidity risk premium effect}} \end{aligned}$$

where the latter holds with equality if the capital requirement is non-binding. The covariance terms are liquidity risk premia. For a risk-neutral bank ($\gamma = 0$) these terms disappear.

Once the optimal values for $\hat{c}_i$ and $\hat{d}_i$ are found, the expected value of $[R_i^E]^{1-\gamma}$ equals:

$$\Omega_{it}^* \equiv \mathbb{E}_\omega [R_i^E]^{1-\gamma} = F \left[\frac{\hat{c}_i^*}{\hat{d}_i^*} \right] [R_i^E (\chi_i^L)]^{1-\gamma} + \left(1 - F \left[\frac{\hat{c}_i^*}{\hat{d}_i^*} \right] \right) [R_i^E (\chi_i^B)]^{1-\gamma}$$

B.4 Dividends and Bank Value

The value function is linear in Ω_{it}^* :

$$v(Z) = \max_{div_i} \frac{div_i^{1-\gamma}}{1-\gamma} + \beta \zeta (1 - div_i)^{1-\gamma} \mathbb{E} v(Z'|Z) \Omega_{it}^*$$

Differentiating w.r.t. div_i :

$$\begin{aligned}
div_i^{-\gamma} &= \beta \zeta (1 - \gamma) \Omega_{it}^* (1 - div_i)^{-\gamma} \mathbb{E} v(Z'|Z) \\
\left(\frac{1 - div_i}{div_i} \right)^\gamma &= \beta \zeta (1 - \gamma) \mathbb{E} v(Z'|Z) \Omega_{it}^* \\
\frac{1}{div_i} &= 1 + [\beta \zeta (1 - \gamma) \mathbb{E} v(Z'|Z) \Omega_{it}^*]^\frac{1}{\gamma} \\
div_i &= \frac{1}{1 + [\beta \zeta (1 - \gamma) \mathbb{E} v(Z'|Z) \Omega_{it}^*]^\frac{1}{\gamma}}
\end{aligned}$$

Substituting back to the value function results in the following functional equation:

$$v(Z) = \frac{1}{1 - \gamma} \left[1 + [\beta \zeta (1 - \gamma) \mathbb{E} v(Z'|Z) \Omega_{it}^*]^\frac{1}{\gamma} \right]^\gamma \quad (88)$$

The right-hand side can be treated as a contraction mapping operator. Once the value function is solved, next period equity can be calculated:

$$E'_i = (1 - div_i) E_i R_i^E$$

This concludes the bank problem.

Appendix C Real Sector

C.1 Household

Household obtains utility from consumption, C_t , and disutility from labor, H_t . The household can save by providing deposits to the banking sector, D_t^A . Deposits receive a constant interest rate of R^d , which is paid at the beginning of the next period.

Problem 8 *The household solves the following maximization problem:*

$$\begin{aligned}
\max_{C_t, H_t, D_t^A} \quad & \sum_{t=0}^{\infty} \beta^t \left[C_t - \frac{H_t^{1+\nu}}{1+\nu} \right] \\
s.t. \quad & D_t^A + C_t = W_t H_t + R^d D_{t-1}^A + \Pi_t + T_t
\end{aligned}$$

where W_t is the real wage rate, Π_t is the firm's profit, T_t is the tax transfer, and ν is the inverse of the Frisch elasticity. The labor supply curve is:

$$H_t = W_t^{\frac{1}{\nu}} \quad (89)$$

which implies that the household's total wage income is $W_t^{\frac{\nu+1}{\nu}}$. If $R^d = \frac{1}{\beta}$, the household is indifferent between consumption and saving, and:

$$C_t \in [0, Y_t], \quad D_t^A = Y_t - C_t, \quad R^d = \frac{1}{\beta} \quad (90)$$

where Y_t is firm's output.

C.2 Firm

An aggregate profit-maximizing firm uses household's labor to produce output according to the following production function:

$$Y_t = A_t H_t^{1-\alpha} \quad (91)$$

where A_t is a technology index, and $1 - \alpha$ is the labor share. The firm has to pay workers before output is realized, therefore it borrows the total amount I_t^A from the banking sector to cover the wage bill:

$$W_t H_t = I_t^A \quad (92)$$

I_t^A is collected via the CES technology:

$$I_t^A = \left[\sum_i \lambda_i^{\frac{1}{\epsilon}} I_{it}^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}} \quad (93)$$

where I_{it} is borrowing from bank i , λ_i is the bank i 's share, and ϵ is the elasticity of substitution between loans from different banks. The firm promises to repay the loan principal and accrued interest in the beginning of the next period. The total repayment to the banking sector is then $\sum_i R_i^b I_{it}$. The firm never defaults on loans.

Problem 9 *The aggregate firm solves the following maximization problem:*

$$\begin{aligned} \max_{I_t^A, I_{it}, H_t} \quad & \sum_{t=0}^{\infty} \beta^t \left[A H_t^{1-\alpha} - W_t H_t + I_t^A - \sum_i R_{it-1}^b I_{it-1} \right] \\ \text{s.t.} \quad & W_t H_t = I_t^A \\ & I_t^A = \left[\sum_i \lambda_i^{\frac{1}{\epsilon}} I_{it}^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}} \end{aligned}$$

Substituting the constraints into the objective, the firm's problem can be written as an unconstrained maximization problem:

$$\max_{I_{it}} \sum_{t=0}^{\infty} \beta^t \left[\frac{A_t}{W_t^{1-\alpha}} \left[\sum_i \lambda_i^{\frac{1}{\epsilon}} I_{it}^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon(1-\alpha)}{\epsilon-1}} - \sum_i R_{it-1}^b I_{it-1} \right]$$

The first-order condition implies the demand curve for a loan from a bank i :

$$\begin{aligned}
R_i^b &= \frac{(1-\alpha) A_t}{\beta W_t^{1-\alpha}} \left[\sum_i \lambda_i^{\frac{1}{\epsilon}} I_{it}^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\frac{\epsilon(1-\alpha)}{\epsilon-1}-1}{\epsilon-1}} \lambda_i^{\frac{1}{\epsilon}} I_{it}^{-\frac{1}{\epsilon}} \\
&= \frac{(1-\alpha) A_t}{\beta W_t^{1-\alpha}} \left[\sum_i \lambda_i^{\frac{1}{\epsilon}} I_{it}^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\frac{\epsilon(1-\alpha)}{\epsilon-1}-\epsilon+1}{\epsilon-1}} \lambda_i^{\frac{1}{\epsilon}} I_{it}^{-\frac{1}{\epsilon}} \\
&= \frac{(1-\alpha) A_t}{\beta W_t^{1-\alpha}} \left[\sum_i \lambda_i^{\frac{1}{\epsilon}} I_{it}^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{1-\alpha\epsilon}{\epsilon-1}} \lambda_i^{\frac{1}{\epsilon}} I_{it}^{-\frac{1}{\epsilon}} \\
&= \frac{(1-\alpha) A_t}{\beta W_t^{1-\alpha}} \left[\sum_i \lambda_i^{\frac{1}{\epsilon}} I_{it}^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\frac{\epsilon}{\epsilon-1} \frac{1-\epsilon\alpha}{\epsilon}}{\epsilon-1}} \lambda_i^{\frac{1}{\epsilon}} I_{it}^{-\frac{1}{\epsilon}}
\end{aligned}$$

Equivalently,

$$R_i^b = \frac{(1-\alpha) A_t}{\beta W_t^{1-\alpha}} (I_t^A)^{\frac{1}{\epsilon}-\alpha} \lambda_i^{\frac{1}{\epsilon}} I_{it}^{-\frac{1}{\epsilon}} \quad (94)$$

C.3 Labor Market Clearing

Substituting the labor supply condition (89) into the working-capital constraint (92) gives the relationship between wages and total investment:

$$W_t^{\frac{1+\nu}{\nu}} = I_t^A$$

Substituting the above in the loan demand (94):

$$\begin{aligned}
R_i^b &= \frac{(1-\alpha) A_t}{\beta W_t^{1-\alpha}} (I_t^A)^{\frac{1}{\epsilon}-\alpha} \lambda_i^{\frac{1}{\epsilon}} I_{it}^{-\frac{1}{\epsilon}} \\
&= \frac{(1-\alpha) A_t}{\beta H^{\nu(1-\alpha)}} (I_t^A)^{\frac{1}{\epsilon}-\alpha} \lambda_i^{\frac{1}{\epsilon}} I_{it}^{-\frac{1}{\epsilon}} \\
&= \frac{(1-\alpha) A_t}{\beta (I_t^A)^{\frac{\nu(1-\alpha)}{\nu+1}}} (I_t^A)^{\frac{1}{\epsilon}-\alpha} \lambda_i^{\frac{1}{\epsilon}} I_{it}^{-\frac{1}{\epsilon}} \\
&= \frac{(1-\alpha)}{\beta} A_t [I_t^A]^{\frac{1}{\epsilon}-\alpha-\frac{\nu(1-\alpha)}{\nu+1}} \lambda_i^{\frac{1}{\epsilon}} I_{it}^{-\frac{1}{\epsilon}} \\
&= \frac{(1-\alpha) A_t}{\beta} [I_t^A]^{\frac{1}{\epsilon}-\frac{\nu+\alpha}{\nu+1}} \left[\frac{I_{it}}{\lambda_i} \right]^{-\frac{1}{\epsilon}}
\end{aligned}$$

Simplifying:

$$R_i^b = \frac{(1-\alpha) A_t}{\beta} [I_t^A]^{\frac{1}{\epsilon}-\frac{\nu+\alpha}{\nu+1}} \left[\frac{I_{it}}{\lambda_i} \right]^{-\frac{1}{\epsilon}} \quad (95)$$

$$\begin{aligned}
R_i^b &= \frac{(1-\alpha) A_t}{\beta} [I_t^A]^{\frac{1}{\epsilon} - \frac{\nu+\alpha}{\nu+1}} \lambda_i^{\frac{1}{\epsilon}} I_{it}^{-\frac{1}{\epsilon}} \\
R_i^b I_{it} &= \frac{(1-\alpha) A_t}{\beta} [I_t^A]^{\frac{1}{\epsilon} - \frac{\nu+\alpha}{\nu+1}} \lambda_i^{\frac{1}{\epsilon}} I_{it}^{-\frac{1}{\epsilon}} I_{it} \\
R_i^b I_{it} &= \frac{(1-\alpha) A_t}{\beta} [I_t^A]^{\frac{1}{\epsilon} - \frac{\nu+\alpha}{\nu+1}} \lambda_i^{\frac{1}{\epsilon}} I_{it}^{\frac{\epsilon-1}{\epsilon}}
\end{aligned}$$

Summing over i :

$$\begin{aligned}
\sum_i R_i^b I_{it} &= \frac{(1-\alpha) A_t}{\beta} [I_t^A]^{\frac{1}{\epsilon} - \frac{\nu+\alpha}{\nu+1}} \sum_i \lambda_i^{\frac{1}{\epsilon}} I_{it}^{\frac{\epsilon-1}{\epsilon}} \\
&= \frac{(1-\alpha) A_t}{\beta} [I_t^A]^{\frac{1}{\epsilon} - \frac{\nu+\alpha}{\nu+1}} \left[\left[\sum_i \lambda_i^{\frac{1}{\epsilon}} I_{it}^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}} \right]^{\frac{\epsilon-1}{\epsilon}} \\
&= \frac{(1-\alpha) A_t}{\beta} [I_t^A]^{\frac{1}{\epsilon} - \frac{\nu+\alpha}{\nu+1}} [I_t^A]^{\frac{\epsilon-1}{\epsilon}} \\
&= \frac{(1-\alpha) A_t}{\beta} [I_t^A]^{\frac{1}{\epsilon} + \frac{\epsilon-1}{\epsilon} - \frac{\nu+\alpha}{\nu+1}} \\
&= \frac{(1-\alpha) A_t}{\beta} [I_t^A]^{1 - \frac{\nu+\alpha}{\nu+1}} \\
&= \frac{(1-\alpha) A_t}{\beta} [I_t^A]^{\frac{\nu+1-\nu-\alpha}{\nu+1}} \\
&= \frac{(1-\alpha) A_t}{\beta} [I_t^A]^{\frac{1-\alpha}{\nu+1}} \\
&= \frac{1-\alpha}{\beta} Y_t
\end{aligned}$$

The firm's profit is then:

$$\begin{aligned}
\Pi_t &= \frac{A_t}{W_t^{1-\alpha}} \left[\sum_i \lambda_i^{\frac{1}{\epsilon}} I_{it}^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon(1-\alpha)}{\epsilon-1}} - \sum_i R_{iit-1}^b I_{it-1} \\
\Pi_t &= \frac{A_t}{W_t^{1-\alpha}} [I_t^A]^{1-\alpha} - \frac{(1-\alpha) A_{t-1}}{\beta} [I_{t-1}^A]^{\frac{1-\alpha}{\nu+1}} \\
\Pi_t &= A_t H_t^{1-\alpha} - \frac{(1-\alpha) A_{t-1}}{\beta} [I_{t-1}^A]^{\frac{1-\alpha}{\nu+1}} \\
\Pi_t &= A_t [I_t^A]^{\frac{1-\alpha}{1+\nu}} - \frac{(1-\alpha) A_{t-1}}{\beta} [I_{t-1}^A]^{\frac{1-\alpha}{\nu+1}} \\
\Pi_t &= Y_t - \frac{1-\alpha}{\beta} Y_{t-1}
\end{aligned}$$

The equilibrium household's budget constraint is:

$$\begin{aligned}
D_t^A + C_t &= W_t H_t + R^d D_{t-1}^A + \Pi_t + T_t \\
D_t^A + C_t &= I_t^A + R^d D_{t-1}^A + A_t [I_t^A]^{\frac{1-\alpha}{1+\nu}} - \frac{(1-\alpha) A_{t-1}}{\beta} [I_{t-1}^A]^{\frac{1-\alpha}{\nu+1}} + T_t \\
D_t^A + C_t &= I_t^A + \frac{1}{\beta} (Y_{t-1} - C_{t-1}) + A_t [I_t^A]^{\frac{1-\alpha}{1+\nu}} - \frac{(1-\alpha) A_{t-1}}{\beta} [I_{t-1}^A]^{\frac{1-\alpha}{\nu+1}} + T_t \\
D_t^A + C_t &= I_t^A + \frac{A_{t-1}}{\beta} [I_{t-1}^A]^{\frac{1-\alpha}{\nu+1}} - \frac{1}{\beta} C_{t-1} + A_t [I_t^A]^{\frac{1-\alpha}{1+\nu}} - \frac{(1-\alpha) A_{t-1}}{\beta} [I_{t-1}^A]^{\frac{1-\alpha}{\nu+1}} + T_t \\
D_t^A + C_t &= I_t^A + \frac{\alpha A_{t-1}}{\beta} [I_{t-1}^A]^{\frac{1-\alpha}{\nu+1}} + T_t - \frac{1}{\beta} C_{t-1} + A_t [I_t^A]^{\frac{1-\alpha}{1+\nu}} \\
D_t^A + C_t &= I_t^A + \frac{\alpha}{\beta} Y_{t-1} + T_t - \frac{1}{\beta} C_{t-1} + Y_t \\
Y_t &= I_t^A + \frac{\alpha}{\beta} Y_{t-1} + T_t - \frac{1}{\beta} C_{t-1} + Y_t \\
I_t^A + T_t &= \frac{1}{\beta} C_{t-1} - \frac{\alpha}{\beta} Y_{t-1} \\
I_t^A + T_t &= \frac{1}{\beta} (Y_{t-1} - D_{t-1}) - \frac{\alpha}{\beta} Y_{t-1} \\
I_t^A + T_t &= \frac{1}{\beta} ((1-\alpha) Y_{t-1} - D_{t-1})
\end{aligned}$$

Appendix D Equilibrium Conditions

$$\begin{aligned}
(1 - \theta) E_{it} &= \tilde{B}_{it} + \tilde{C}_{it} - \tilde{D}_{it} \\
\tilde{D}_{it} &\leq \kappa (1 - \theta) E_{it} \\
E_{it+1} &= R_{it}^b \tilde{B}_{it} - R^d \tilde{D}_{it} + R_{it}^x \tilde{L}_{it} - \omega_{it} (R_{it}^x - R^d) \tilde{D}_{it} \\
X_{it} &= \omega_{it} \tilde{D}_{it} - \tilde{L}_{it} \\
R_{it}^x &= \begin{cases} \chi_{it}^L = p_{it}^L r_t^{FF} + (1 - p_{it}^L) r_t^{ER} & \text{if } X_{it} \leq 0 \\ \chi_{it}^B = p_{it}^B r_t^{FF} + (1 - p_{it}^B) r_t^{DW} & \text{if } X_{it} > 0 \end{cases} \\
R_{it}^b &= \chi_{it}^L F \left[\frac{\tilde{C}_{it}}{\tilde{D}_{it}} \right] + \chi_{it}^B \left(1 - F \left[\frac{\tilde{C}_{it}}{\tilde{D}_{it}} \right] \right) \\
R_{it}^b - R^d &= \chi_{it}^L \int_{-\infty}^{\frac{\tilde{C}_{it}}{\tilde{D}_{it}}} \omega_{it} f(\omega_{it}) d\omega_{it} + \chi_{it}^B \int_{\frac{\tilde{C}_{it}}{\tilde{D}_{it}}}^1 \omega_i f(\omega_i) d\omega_i + \mu_{it}
\end{aligned}$$

$$\begin{aligned}
D_t^A + C_t &= W_t H_t + R^d D_{t-1}^A + \Pi_t + T_t \\
H_t &= W_t^{\frac{1}{\nu}} \\
D_t^A &= Y_t - C_t \\
R^d &= \frac{1}{\beta}
\end{aligned}$$

$$\begin{aligned}
Y_t &= A_t H_t^{1-\alpha} \\
I_t^A &= \left[\sum_i \lambda_i^{\frac{1}{\epsilon}} I_{it}^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}} \\
W_t H_t &= I_t^A \\
R_i^b &= \frac{(1 - \alpha) A_t}{\beta W_t^{1-\alpha}} (I_t^A)^{\frac{1}{\epsilon} - \alpha} \lambda_i^{\frac{1}{\epsilon}} I_{it}^{-\frac{1}{\epsilon}} \\
\Pi_t &= A_t H_t^{1-\alpha} - \sum_i R_{it-1}^b I_{it-1}
\end{aligned}$$

$$\begin{aligned}
M_{t+1}^0 - M_t^0 &= D_{t+1}^{CB} - D_t^{CB} - r_t^{DW} X_t^- + r_t^{ER} X_t^+ + T_t \\
X_t^- &= \sum_i \mathbb{1}_{\{X_{it} > 0\}} \cdot (1 - p_{it}^B) X_{it}, \quad X_t^+ = \sum_i (1 - \mathbb{1}_{\{X_{it} > 0\}}) \cdot (1 - p_{it}^L) X_{it}
\end{aligned}$$

$$\begin{aligned}
D_t^A &= \sum_i \tilde{D}_{it} \\
\tilde{B}_i &= I_i \\
\sum_i \tilde{C}_{it} &= D_t^{CB} = M_t^0
\end{aligned}$$