Efficiency with Equilibrium Marginal Product Dispersion and Firm Selection*

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Abstract

Disparities in revenue productivity for narrowly defined industries is ubiquitous in firm-level data. Whereas often times such a heterogeneity is a symptom of factor misallocation, it is also possible that part of it is induced by firms' optimal decisions under technology and information constraints. To date, there is limited understanding of the implications of the observed revenue product heterogeneity for efficiency in frameworks where both sources for dispersion coexist. This paper fills the gap. Market distortions that generate inefficient factor accumulation may feed back into the equilibrium distribution of revenue productivity, and hence, empirical measures of allocative efficiency. Understanding such interaction is key for policy design. In this paper, I focus on the study of market distortions generated by firms' market power and I generate endogenous revenue product dispersion through either heterogeneous market power, non-convex production technologies, or information frictions. I characterize the market decentralization of efficient outcomes via policies that do not require firm-level information. Most importantly, I show that the welfare gains for a wide class of models when implementing these optimal policies follows a common pattern: welfare gains are proportional to the change in average revenue product and the number of operating units in the market.

Keywords: Revenue product dispersion, efficiency, optimal policy, welfare gains.

[**JEL Codes:** E32,L11,E23].

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1 Introduction

Disparities in revenue productivity for narrowly defined industries is ubiquitous in firm-level data.[1] Often times, those differences are interpreted as a symptom of distortions in factors' allocation. However, such a dispersion can also be consistent with features of the technologies that firms operate (including investment uncertainty and adjustment costs) and with optimal factor allocation.[2] To date, there is limited understanding of the implications of the observed revenue product heterogeneity for efficiency in frameworks where both sources for dispersion coexist. Hence, there is scarce guidance for the design of policies that aim at improving productivity in an industry and more broadly, efficiency.

In general, the level of dispersion in revenue productivity is determined endogenously by market and firm level distortions and the technologies that firm operate. I focus on the study of market distortions generated by firms' market power. The literature that studies efficiency in markets with imperfect competition is extensive, including markets where firms are heterogeneous in productivity. Typically, inefficiencies in those markets arise because (a) firms that should (not) be in the market are selected out (in); or (b) because firms operating in the market do not produce at their efficient level; or (c) because aggregate factor supply (capital, firms) is inefficient. The second source of inefficiency has the potential of generating revenue product dispersion. However, unless there is a source of endogenous revenue product dispersion (such as disparate market power across firms), allocations are typically constrained efficient. When revenue product dispersion is generated endogenously, the inefficient factor accumulation induced by market power may feed back into the equilibrium distribution of revenue productivity. Hence, distortions on aggregate factors' supply may impact empirical measures of allocative efficiency across firms. Understanding such interaction is key for policy design.

A few questions arise: How does market power shape the degree of revenue productivity dispersion observed in the data? What are the conditions such that market allocations that display revenue productivity dispersion are constrained efficient? Can simple policies (that do not target firm characteristics directly) improve welfare? What is the size of the welfare

[1] For cross country evidence refer to Asker et al. (2011). For evidence for Korea, refer to Midrigan and Xu (2009). Also, Hsieh and Klenow (2009) provide evidence for the US, India and China. For evidence in Latin America, see Buso et al. (2013).

[2] Economies with adjustment costs have been analyzed in Asker et al. (2011) and Midrigan and Xu (2009); Caunedo (2013) study minimum capacities; and David et al. (2016) study imperfect information.

gains associated to them?

The first contribution of the paper is to characterize conditions under which market distortions are reflected into static firm-level measures of distortions such as revenue product dispersion. This characterization is particularly relevant for optimal policy design. Heterogeneity in revenue product across firms can be reduced by lifting market-wide distortions. Hence, policy instruments that do not target particular firms can still achieve endogenous improvements in factor allocations. I call these type of policies, simple policies. If the policy maker is uncertain as of the sources of revenue productivity dispersion, the design of policy is problematic. Even when knowing the source of distortions, policy implementation often requires large amount of information about each production unit, and a rich set of instruments. Studying conditions under which efficient allocations can be decentralized with simple policies, is the second contribution of the paper.

Third and most important, the paper provides a characterization of the welfare gains associated to the implementation of such a class of policies. I show that welfare gains are proportional to the change in average revenue product, and the measure of active firms in the market. These two statistics can be easily computed in most firm-level micro datasets. Hence, when assessing the impact of a simple policy welfare gains can be computed directly from the data before and after the policy intervention. Market deregulation, lower trade tariffs or subsidies to the cost of capital are some of the policies whose welfare gains can be accounted for by the formula provided in the paper.

I first focus on economies where the demand system is one with constant elasticity of substitution across goods. Later, I extend the analysis to economies where the price elasticity depends on the number of firms producing each particular good. Hence, markups are heterogeneous across different goods.

To start, I focus in the plain vanilla model of misallocation, as in Restuccia and Rogerson (2008). Distortions are a set of subsidies and taxes on firms inputs and output. When markets are competitive, prices equal the shadow value of each commodity/firm for the planner, and feasibility constraints are satisfied. This result is robust to allowing firms to invest in productivity or capacity, i.e. innovation decisions with associated sunk costs, or export decision in models of trade (as in Melitz (2003)). If competition is imperfect and the demand system features constant elasticity of substitution, factor allocation across firms is constrained efficient but the aggregate factor supply (including the equilibrium number of incumbent firms) is distorted. If the elasticity of substitution is not constant, markups are heterogeneous across firms, which induces revenue product dispersion. In the specification studied in the paper, such an elasticity depends on the number of operating firms in the market. Hence, distortions in the equilibrium number of active firms feedbacks into markups and through them, the distribution of revenue products.

If we allow for a source of endogenous revenue product dispersion, such as non-convex technologies or adjustment costs, allocations are still constrained efficient under perfect competition, but in general they are not when firms have market power. Even when firms charge a common markup over marginal cost, distortions in the aggregate factor supply, will feed back into the observed dispersion in revenue products. Consider the case of capital adjustment costs that induce sS investment strategies. Which firms are in the inaction region at any point in time will depends on the equilibrium cost of capital. In general equilibrium, the latter is determined by the aggregate factor supply, which in this market is distorted. When the set of constrained firms shifts, so does the distribution of revenue product.

For each environment studied in the paper, I characterize a simple policy that yields efficient outcomes in the market allocation. Those policies are characterized by a tax/subsidy on the return to capital and on entry costs. If firms are allowed to upgrade technologies by incurring sunk costs, the optimal policy typically targets those upgrade costs too. When the revenue product distribution is endogenous, the size of the policy instrument is proportional to average TFPR in the economy.

Finally, although the paper studies a broad class of models and can accommodate different predictions for equilibrium quantities and prices, welfare gains from the implementation of simple policies follows a common pattern. In particular, the change in welfare is proportional to the change in average TFPR and the change in the measure of active firms in the market. The factors of proportionality depend on the investment to output ratio before the policy implementation, the degree of aggregate factor returns and the elasticity of substitution across goods.[3]

Literature Review. Non-degenerate firm size distribution requires some curvature in the firm's profit function. Such a curvature can be achieved through decreasing returns to factors in an environment with perfect competition (Hopenhayn (1992)), or through im-

[3]In an environment in which preferences are defined over a set of goods, rather than over a final aggregator, the aggregate factor returns are analogous to the curvature of the utility function for the composite of goods.

perfect competition (Asplund and Nocke (2006), Melitz (2003)). Under perfect competition, allocations are constrained efficient whenever marginal revenue product dispersion is induced by characteristics of the technology that firms operate (i.e. adjustment costs) or by the information structure of the economy (i.e. uncertainty, learning). This paper shows that this is in general not the case under imperfect competition.

The study of optimal policy in economies with heterogeneous firms is not new. It has been done in models of international trade under oligopolistic competition in prices and quantities (Eaton and Grossman (1986)). The implications of firm heterogeneity has also been studied in Dhingra and Morrow (2016). The contribution of this paper relative to theirs is twofold. First, the characterization of the feedback between market distortions, and the distribution of revenue products which is typically used to argue for allocative inefficiency. Second, the characterization of the welfare gains from implementation of simple policies (which include opening to trade as modelled in their paper) for a wide class of models that are typically used in the literature. In particular, I show that welfare gains can be computed from statistics that are readily available from firm-level micro data: average revenue productivity and the number of active production units.

For a model of industry dynamic without capital accumulation, Lee and Mukoyama (2008) study the impact of alternative policies on labor regulations. However, their policies are ad hoc in the sense that there is no notion of efficiency associated to them. They consider i.i.d. policies and policies correlated with the productivity of the firms. Guner et al. (2008) study policies that target the size of the establishment, which in turn is correlated with idiosyncratic productivity, and find a substantial role in shaping aggregate productivity. This paper contributes to the literature by disentangling the key factors that determine allocation efficiency. It highlights the link between aggregate supply of factors, such as capital accumulation decisions, labor supply and entrepreneurial activity (active firms) with equilibrium static allocation of factors.

2 Environment

I use the demand structure proposed by Benassy (1996), which extends the Dixit-Stiglitz demand structure to disentangle the elasticity of substitution across goods from the concept of "taste of variety" (Dixit and Stiglitz (1977)).

I consider economies where the only input of production is capital, and where allocating capital to firms is costly, i.e. for each k_t units used in production, τk_t of capital need to be allocated to such production unit. Goods are characterized by productivity-cost pairs, (a, τ) which evolve exogenously according to a law of motion, Γ. Productivity is hicks neutral and the cost of capital allocation is specific to the production of such good (a distortion from now on). Each good $y(a, \tau)$ is produced by $M(a, \tau)$ firms, with a technology

$$y(a,\tau) = (\sum_{i}^{M(a,\tau)} \frac{(a^{\frac{1}{\alpha}} k)^{\rho}}{M(a,\tau)})^{\frac{1}{\rho}}$$

where $\frac{1-\rho}{\rho}$ is the "taste for variety" parameter. Each firm operates a linear technology in capital, ak. Firms exit the market at a exogenous rate δ and entrepreneurs can enter the market at cost f in units of capital. When an entrepreneur enters the market he gets a productivity-distortion pair assigned exogenously according to $G(a, \tau)$[4]. Aggregate output is

$$Y_t = (\sum_{a,\tau} M(a,\tau) y(a,\tau)^{\alpha})^{\frac{\gamma}{\alpha}},$$

where $\frac{1}{1-\alpha}$ is the elasticity of substitution across goods, and γ indexes aggregate decreasing returns.[5]

The choice of technologies is not arbitrary. For $M(a, \tau) < \infty$, the markup of monopolistically competitive firms will be different than the "taste for variety" parameter $\frac{1-\rho}{\rho}$ and production scales will be inefficient (as in Benassy (1996)). However as $M(a, \tau) \to \infty$, the economy approximates a standard economy as studied in Dixit and Stiglitz (1977), where production scales are efficient. Still, incentives to entry and capital accumulation are distorted in a market allocation with imperfect competittion.

Consumers derive utility from consumption of the final good and preferences, $U(C_t)$, follow standard regularity conditions. Consumers accumulate capital and own shares to the firms operating in the market.

[4]When production technologies have only one input, distortions that affect their marginal product (cost in the decentralized allocation) and distortions that affect firm productivity are indistinguishable from each other. In the paper, we will endogenize marginal product dispersion in inputs, and hence we choose the first convention for this example.

[5]We could have alternatively set up a demand system where these are preferences of homogeneous consumers for a set of differentiated goods $y(a, \tau)$.

3 Efficient allocation

The social planner in this economy solves

$$\max_{k_t, M_t^e, M_t, K_{t+1}} E_t \sum_{t=1}^{\infty} \beta^{t-1} U(C_t)$$

subject to

$$C_t \leq (\sum_{a,\tau} M(a,\tau) y(a,\tau)^{\alpha})^{\frac{\gamma}{\alpha}} + (1-\widehat{\delta})K_t - K_{t+1}$$

$$y(a,\tau) = (\sum_{i}^{M(a,\tau)} \frac{(a^{\frac{1}{\alpha}} k)^{\rho}}{M(a,\tau)})^{\frac{1}{\rho}}$$

$$\sum_{a,\tau} M_t(a,\tau)\tau k_t + f M_t^e \leq K_t \qquad (r_t U'(C_t))$$

$$M_t(a,\tau) = M_t^e G(a,\tau) + (1-\delta) \sum_{a',\tau'} M_{t-1}(a',\tau')\Gamma \qquad (\lambda(a,\tau)U'(C_t))$$

$$M_0(a,\tau) = G(a,\tau) \quad K_1 \text{ given}$$

$$(a,\tau) = \Gamma(a',\tau') \text{ i.i.d. for each pair } (a',\tau')$$

In equilibrium the allocation of capital and its shadow value satisfy

$$k_t = (K_t - f M_t^e) \frac{\frac{a}{\tau}^{\frac{1}{1-\alpha}}}{\sum_{a,\tau} (\frac{a}{\tau^{\alpha}})^{\frac{1}{1-\alpha}} M_t(a,\tau)}$$

$$r_t = \gamma (K_t - f M_t^e)^{\gamma-1} (\sum_{a,\tau} (\frac{a}{\tau^{\alpha}})^{\frac{1}{1-\alpha}} M_t(a,\tau))^{\gamma \frac{1-\alpha}{\alpha}}$$

With the optimal capital allocation we can compute aggregate output

$$Y_t = (K_t - f M_t^e)^{\gamma} (\sum_{a,\tau} (\frac{a}{\tau^{\alpha}})^{\frac{1}{1-\alpha}} M_t(a,\tau))^{\gamma \frac{1-\alpha}{\alpha}}$$

Finally, the optimal measure of active firms in the market is determined by the free entry condition

$$\sum_{a,\tau} \lambda_t(a,\tau)G(a,\tau) = fr_t$$

where $\lambda_t(a,\tau)$ corresponds to the shadow value of a firm (a,τ). In a stationary equilibrium, the value of the firm is

$$\lambda(a,\tau) = \frac{\gamma\frac{1-\alpha}{\alpha}}{1-\beta(1-\delta)} Y_t \frac{(\frac{a}{\tau^\alpha})^{\frac{1}{1-\alpha}}}{\sum_{a,\tau}(\frac{a}{\tau^\alpha})^{\frac{1}{1-\alpha}} M_t(a,\tau)}$$

because idiosyncratic shocks are *i.i.d.*

Proposition 1 *The efficient allocation in a stationary equilibrium is characterized by the aggregate capital supply, the equilibrium measure of firms in the market and the capital allocation across production units following equations (O1)-(O3).*

From the free-entry condition and equilibrium interest rate,

$$f = \frac{\gamma\frac{1-\alpha}{\alpha}}{1-\beta(1-\delta)} (\frac{K_t - fM_t^e}{M_t}) \frac{\sum_{a,\tau}(\frac{a}{\tau^\alpha})^{\frac{1}{1-\alpha}} G_t(a,\tau)}{\sum_{a,\tau}(\frac{a}{\tau^\alpha})^{\frac{1}{1-\alpha}} v_t(a,\tau)} \tag{O1}$$

where $v_t(a,\tau)$ is the distribution of firms associated to Γ, and $M_t^e = \delta M_t$.

From the euler equation,

$$1 = \beta[(1-\widehat{\delta}) + \gamma(\frac{K_t - fM_t^e}{M_t})^{\gamma-1} M_t^{\frac{\gamma}{\alpha}-1} (\sum_{a,\tau}(\frac{a}{\tau^\alpha})^{\frac{1}{1-\alpha}} v_t(a,\tau))^{\gamma\frac{1-\alpha}{\alpha}}] \tag{O2}$$

and from the optimal capital demands

$$\frac{k_j}{k_i} = (\frac{\frac{a_j}{\tau_j}}{\frac{a_i}{\tau_i}})^{\frac{1}{1-\alpha}} \tag{O3}$$

Proof. See Appendix ■ Notice that $\frac{K_t - fM_t^e}{M_t}$ corresponds to the average size of a firm in terms of their capital intake. Also, given that the planner takes the cost of capital allocation to each firm as given, the optimal factor allocation across firms follows the relative productivity of each unit adjusted by this cost.

4 Decentralized Allocations

Assume there is a final good producer that maximizes profits

$$\max_{y(a,\tau)}(\sum_{a,\tau} M(a,\tau)y(a,\tau)^{\alpha})^{\frac{\gamma}{\alpha}} - \sum_{a,\tau} p(a,\tau)M(a,\tau)y(a,\tau)$$

by choosing intermediate inputs of each type (a,τ), $y(a,\tau)$.

For each pair (a,τ) there is a representative firm that maximizes profits by choosing intermediates indexed by i. A natural way to think about (a,τ) is a sector, populated by $M(a,\tau)$ firms all of which produce a homogeneous goods which is used in production by the representative firm with technology $y(a,\tau)$.

The problem of the firm is

$$V(a,\tau) = \max_{yi} p(a,\tau)y(a,\tau) - \sum_{i=1}^{M(a,\tau)} y_i p_i + \beta E_t V(a',\tau')$$

subject to

$$y(a,\tau) = (\sum_i \frac{y_i^{\rho}}{M(a,\tau)})^{\frac{1}{\rho}}$$

Each producer chooses capital to maximize the value of the firm according to

$$\max_k p_i a^{\frac{1}{\alpha}} k_t - r_t \tau k_t$$

4.1 Competitive Markets

Definition 1 (CE) *A competitive equilibrium is an allocation and an invariant distribution of firms types, $v(a,\tau)$, such that given prices, $\{p_i, p(a,\tau), r\}$, the law of motion for idiosyncratic shocks, Γ, and the entry distribution $G(a,\tau)$, i) households maximize utility, ii) firms maximize profits; iii) the feasibility conditions in goods and capital are satisfied; iv) and the following free entry condition holds*

$$fr = E_{\Gamma}(V(a,\tau))$$

The definition of (CE) is standard. To ease the exposition I have already imposed rational expectations on the law of motion, $v(a, \tau)$.

Proposition 2 *The competitive equilibrium is (constrained) efficient*

The proof of this statement accounts for showing that in a competitive allocation conditions O1-O3 plus the feasibility constraint of the economy are satisfies. Condition O1 corresponds to the free entry condition in the market; condition O2 is the household euler equation and O3 can be derived from the optimality conditions for intermediate goods producers. Feasibility constraints are trivially satisfied.

4.2 Imperfect Competition

In this section I assume firms compete imperfectly both across and within sectors (a, τ). Each representative firm of a sector faces a residual demand as derived from the final good firm problem. The elasticity of substitution of goods across "sectors" (a, τ) is $\frac{1}{1-\alpha}$. Each intermediate good producer faces a residual demand as derived from the representative in each sector, (a, τ) with an elasticity of substitution that may or may not depend on the number of active firms in the sector (to be characterized below), $\eta(a)$.

Representative firm in a sector The problem of the firm is

$$\max_{yi} p(a, \tau)y(a, \tau) - \sum_{i=1}^{M(a,\tau)} y_i p_i$$

subject to

$$y(a, \tau) = M(a, \tau)^{\frac{1}{\alpha}} (\sum_i \frac{y_i^\rho}{M(a, \tau)})^{\frac{1}{\rho}}$$

$$\gamma Y_t^{1-\frac{\alpha}{\gamma}} y(a, \tau)^{\alpha-1} = p(a, \tau)$$

where the last constraint is the residual demand for good (a, τ).

Assumption I: Intermediate goods are substitutes in production, $\alpha \in (0, 1)$.

Firm producing intermediate good i The problem of firm i in sector (a,τ) is

$$V_i(a,\tau) = \max_k p_i a^{\frac{1}{\alpha}} k_t - r_t \tau k_t + \beta E_t V_i(a',\tau')$$

subject to

$$p_i = p(a,\tau)(\frac{y(a,\tau)}{y_i})^{1-\rho} M(a,\tau)^{\frac{\rho}{\alpha}-1}$$

where this last constraint corresponds to the inverse demand function for good i as derived from the problem of the intermediate good producer in sector (a,τ).

Note that the demand elasticity for each firm i depends on the measure of firms operating within that sector. The price index in each sector is

$$p(a,\tau) = M(a,\tau)^{\frac{1}{\rho}-\frac{1}{\alpha}} (\sum_{i=1}^{M(a,\tau)} p_i^{\frac{\rho}{\rho-1}})^{\frac{\rho-1}{\rho}}$$

Let $\eta(a) = \frac{\partial y_i}{\partial p_i}\frac{p_i}{y_i}$ the price elasticity for intermediate good i

$$\eta(a,\tau) = \frac{1}{\rho-1} + (\frac{1}{\alpha-1} - \frac{1}{\rho-1})\frac{1}{M(a,\tau)}$$

$$\lim_{M(a,\tau)\to\infty} -\eta(a,\tau) = \frac{1}{1-\rho}$$

Assumption II: Intermediate goods within a sector are substitutes, $\rho \in (0,1)$.

Assumption III: Goods within a sector are more substitutes that goods across sectors, $\alpha < \rho$.

Given the elasticity of demand for any arbitrary measure $M(a,\tau)$ it is possible to characterize the markup as

$$p_t \frac{\partial y}{\partial k}(\frac{1+\eta(a)}{\eta(a))}) = \tau r$$

$$\varepsilon(a,\tau) = \frac{\eta(a,\tau)}{1+\eta(a,\tau)}$$

where $\lim_{M(a,\tau)\to\infty} \varepsilon(a,\tau) = \frac{1}{\rho}$ and the demand elasticity falls with the number of active firms in the sector $\frac{\partial \varepsilon(a,\tau)}{\partial M(a,\tau)} < 0$. Hence, as the measure of firms within a sector increases, markups drop.

4.2.1 Constant markups

If goods are perfect substitutes within a sector, $\rho = 1$, markups are constant and equal to $\varepsilon(a, \tau) = \frac{1}{\alpha}$. The economy resembles one with monopolistic competition across sectors, (a, τ) and a demand system with constant elasticity of substitution.

The allocation of capital across firms in different sectors is constrained efficient as

$$\frac{k_j}{k_i} = \left(\frac{\frac{a_j}{\tau_j}}{\frac{a_i}{\tau_i}}\right)^{\frac{1}{1-\alpha}}$$

which is identical to condition O3.However, as long as the aggregate supply of capital is elastic and the entry margin is endogenous, imperfect competition will distort those margins.

Proposition 3 *If $\rho = 1$, the efficient allocation can be decentralized as a market allocation under monopolistic competition, where the return (cost) of capital for the households (firms) is $\widehat{r}_t = r_t^m \theta_k$ with $\theta_k = \frac{1}{\alpha}$.*

Hence, the optimal policy entails, increasing the cost of capital for firms. In this case, the entry condition is undistorted.

Proposition 4 *If $\rho = 1$, and entry costs are denominated in final goods, the efficient allocation can be decentralized as a market allocation under monopolistic competition, where the return (cost) of capital for the households (firms) is $\widehat{r}_t = r_t \theta_k$ with $\theta_k = \frac{1}{\alpha}$, and the entry cost is $\widehat{f} = f\theta_f$ where $\theta_f = \alpha = \frac{1}{\theta_k}$.*

In this case the optimal policy entails increasing the cost of capital for firms and lowering entry costs, as both incentives for capital accumulation and entry of firms are distorted. The decentralized allocation is such that in steady state $K^{mc} < K$ and $M^{mc} < M$.

Finally, it is worth defining revenue TFP in the context of the model, as well as its mean and dispersion in the industry. By definition, revenue TFP is

$$TFPR(a, \tau) = \frac{\pi(a, \tau)}{k} = \frac{(1-\alpha)}{\alpha} \tau r^m$$

where r^m is the return to capital in the decentralized allocation. Hence, $TFPR(a, \tau)$ is proportional to the demand elasticity. Mean and dispersion in TFPR are given by

$$TFPR = \frac{(1-\alpha)}{\alpha} r^m \qquad \text{and} \quad \sigma_{TFPR} = \sigma_\tau$$

(where the average distortion was normalized to 1).

4.2.2 Heterogeneous markups

For $\rho \in (0,1)$, markups within each "sector" (a,τ) depend on the measure of firms operating. In general the relative allocation of capital across sectors is distorted since

$$\frac{k_j}{k_i} = \left(\frac{\frac{a_j}{\tau_j \varepsilon_j}}{\frac{a_i}{\tau_i \varepsilon_i}}\right)^{\frac{1}{1-\alpha}}$$

and $\varepsilon_j \neq 1$ for any finite measure of firms $M(a,\tau)$. Hence, as long as the efficient measure of firms is bounded relative factor allocations will be distorted. Markups decrease with the measure of firms in a given sector. If the invariant measure of firms is such as there are very few very productive firms, their markups will be relatively high, and capital will be allocated away from them, relative to the efficient allocation.

As before, entry and capital accumulation are inefficient in the market allocation. In this case however, it is not possible to restore efficiency by targeting aggregates only. It s possible to restore efficiency if the distribution of exogenous distortions is degenerate at τ.

Proposition 5 *When the distribution of exogenous distortions is degenerate (at τ), the optimal policy is a tax on the return to capital*

$$\widehat{r}_t = r_t \theta_k^e \qquad \textit{with} \quad \theta_k^e = \frac{\left(\psi_v\left(\frac{a^{\frac{\alpha}{\rho}}}{(\tau\varepsilon)^{\alpha}}\right)\right)^{\frac{\gamma}{\alpha}-1}\left(\psi_v\left(\frac{a^{\frac{\alpha}{\rho}}}{\tau\varepsilon}\right)\right)^{1-\gamma}}{\left(\psi_v(a/\tau^{\alpha})\right)^{\frac{\gamma}{\alpha}-1}\left(\psi_v(a/\tau)\right)^{1-\gamma}}$$

and a subsidy on the cost of entry

$$\widehat{f} = f\theta_f \quad \textit{where} \quad \theta_f = \frac{1}{1-\rho}\frac{\psi_v(a/\tau)}{\psi_v\left(\frac{a^{\frac{\alpha}{\rho}}}{\tau\varepsilon}\right)}\frac{\psi_G\left((1-\varepsilon)^{1-\alpha}\left(\frac{a^{\frac{\alpha}{\rho}}}{(\tau\varepsilon)^{\alpha}}\right)\right)}{\left(\psi_G\left(\frac{a^{\frac{\alpha}{\rho}}}{(\tau)^{\alpha}}\right)\right)}$$

Hence, the size of the tax on capital depends on the stationary distribution of firms through their impact on endogenous markups. When there are infinitely many firms in the market, both these policies approximate the ones described under perfect substitution.

Corollary 1 *As the measure of firms operating in each sector increases, the optimal policy converges to the one with constant markups,* $\lim_{M(a,\tau)\to\infty}\theta_k^e = \theta_k$ *and* $\lim_{M(a,\tau)\to\infty}\theta_f^e = \theta_f = 0$

In this economy, we can compute firm-level TFPR as

$$TFPR(a,\tau) = \frac{\pi(a,\tau)}{k} = (1-\varepsilon(a))\gamma Y^{1-\frac{\alpha}{\gamma}} a^{\frac{1}{\rho}} k^{\alpha-1}$$

Dispersion in TFPR is a non-linear function of the inverse of the markups

$$\sigma_{TFPR} = \sigma_{1-\varepsilon(a)}$$

while average TFPR is a non-linear function of the whole distribution of markups,

$$TFPR \simeq (\psi_v(\frac{a^{\frac{\alpha}{\rho}}}{(\tau\varepsilon)^{\alpha}}))^{\frac{\gamma}{\alpha}-1}(\psi_v(\frac{a^{\frac{\alpha}{\rho}}}{\tau\varepsilon}))^{1-\gamma}$$

Corollary 2 *The optimal tax on capital θ_k^e is proportional to average TFPR.*

5 Welfare

In the previous sections, we characterized policies that implement the constrained efficient allocation. But, what are the welfare gains associated to them? In other words, how costly are the distortions in the decentralized allocation in terms of their consumption equivalent?

It is possible to characterize welfare gains for a category of economies that share the following characteristics:

Assumption (A1): The demand structure for intermediate inputs is CES

Assumption (A2): Factors are elastically supplied.

Assumption (A3): Logarithm utility.

Assumption (A4): If $\rho \neq 1$,consider symmetric allocations within sectors.

Let s be the savings rate in the economy $s = \frac{\delta K}{Y}$, $TFPR$ is average $TFPR(a,\tau)$ and M_t is the measure of firms. Revenue TFP at the firm level is characterized by

$$TFPR(a,\tau) = \frac{\pi_t(a,\tau)}{k_t}$$

Hence, aggregate TFPR is

$$TFPR = \sum_{a,\tau} M_t(a,\tau) TFPR(a,\tau)$$

Let $\widehat{x} = \frac{x^{opt}}{x^{mkt}}$ where x^{opt} is the value of variable x in the stationary allocation under the optimal policy, and x^{mkt} is its value before the policy is implemented.

Proposition 6 *Welfare gains when implementing optimal simple policies in economies that satisfy (A1)-(A4) equal*

$$\widehat{W}_t = \frac{1}{(1-s)}\widehat{TFPR}_t - \psi\widehat{M}_t$$

where $\psi = 1$ if entry cost are denominated in factors, and $\psi = (\frac{1}{\gamma} - \frac{1-\alpha}{\alpha})$ if entry cost are denominated in goods, and $s = \frac{\delta k}{y}$.

This is the main result of the paper. In equilibrium, under assumption (A1)-(A4) aggregate output movements are a convex combination of movements in TFPR and the supply of factors, in this case, capital.

Assumption (A3) guaranties that savings rate is constant, while (A1) assures that through the free entry condition, changes in aggregate factor supply are proportional to the aggregate measure of firms. When entry costs are denominated in factors prices, capital and the measure of firms adjust one to one to satisfy the free entry condition. When entry costs are denominated in final goods, they adjust proportionally to each other. The factor of proportionality depends on the degree of decreasing returns in aggregate production, and the elasticity of substitution across intermediate goods.

6 Extensions

6.1 A model of non-convex production technologies

Suppose that firms face non-convex production technologies. In particular, I focus on technologies that display minimum capacities (as in *Caunedo, 2014)* and assume away any distortion (the results follow through if describing and economy with adjustment costs).

Centralized allocation The planner problem is analogous to that in section 3, extended to consider minimum capacity constrains in the allocation of capital.

$$\max_{k_t, M_t^e, M_t, K_{t+1}} \sum_{t=0}^{\infty} \beta^{t-1} (\sum_{a,\tau} M_t(a) a k_t^{\alpha})^{\frac{\gamma}{\alpha}} + (1-\widehat{\delta})K_t - K_{t+1}$$

subject to

$$\sum_a M_t(a)k_t + fM_t^e \leq K_t \qquad (r_t)$$

$$M_t(a) = M_t^e G(a) + (1-\delta)\sum_{a'} M_{t-1}(a')\Gamma(a|a') \qquad M_0(a) = G(a) \quad K_1 \text{ given} \qquad (\lambda(a))$$

$$k_t(a) \geq \underline{k} \qquad (\mu(a))$$

Define, $\widetilde{K}_t = K_t - fM_t^e - \underline{k}M_t^c$ where $M_t^c = \sum_{a\leq a_c} M_t(a)$ is the measure of constrained firms. The shadow value of capital satisfies

$$r_t^c = \alpha\gamma \frac{Y_t^{1-\frac{\alpha}{\gamma}}}{(\widetilde{K}_t)^{1-\alpha}} (\sum_{unc} (a)^{\frac{1}{1-\alpha}} M_t(a))^{1-\alpha}$$

and the efficient capital allocation satisfies

$$k_t = \max\{\widetilde{K}_t \frac{a^{\frac{1}{1-\alpha}}}{\sum_{unc} M_t(a)a^{\frac{1}{1-\alpha}}}, \underline{k}\}$$

The value of a firm for the planner satisfies

$$\lambda_t(a) = \gamma\frac{1-\alpha}{\alpha} Y_t^{1-\frac{\alpha}{\gamma}} ak_t^{\alpha} - \mu_t(a)\underline{k}_t + \beta\ (1-\delta)\sum_{a\prime} \lambda_{t+1}(a')\Gamma(a'|a)$$

If $\Gamma(a'|a)$ is i.i.d. then $E(\lambda_{t+1}(a')) = \lambda_t(a)$ and the value of the firm reduces to

$$\lambda_t(a) = \frac{\gamma\frac{1-\alpha}{\alpha} Y_t^{1-\frac{\alpha}{\gamma}} ak_t^{\alpha} - \mu_t(a)\underline{k}_t}{1-\beta\ (1-\delta)}$$

Aggregate output in the economy is given by

$$Y_t = (\sum_a M_t(a)ak_t^{\alpha})^{\frac{\gamma}{\alpha}} = \widetilde{K}_t^{\gamma}((\sum_{unc} M_t(a)a^{\frac{1}{1-\alpha}})^{1-\alpha} + \frac{\underline{k}^{\alpha}}{\widetilde{K}_t^{\alpha}} \sum_c M_t(a)a)^{\frac{\gamma}{\alpha}}$$

With these conditions we can fully characterize the efficient allocation,

Proposition 7 *The efficient allocation in a stationary equilibrium is characterized by the aggregate capital supply, the equilibrium measure of firms in the market and the capital allocation across production units following equations (MC1)-(MC3).*

From the free-entry condition and equilibrium interest rate,

$$\sum_a \frac{\gamma^{\frac{1-\alpha}{\alpha}} Y_t^{1-\frac{\alpha}{\gamma}} a k_t^{\alpha} - \mu_t(a) \underline{k}_t}{1 - \beta\ (1-\delta)} G(a) = f r_t^c \tag{MC1}$$

where $v_t(a)$ is the distribution of firms associated to Γ, and $M_t^e = \delta M_t$.

From the euler equation,

$$1 = \beta[(1-\widehat{\delta}) + \gamma \frac{Y_t^{1-\frac{\alpha}{\gamma}}}{(\widetilde{K}_t)^{1-\alpha}} (\sum_{unc} (a)^{\frac{1}{1-\alpha}} M_t(a))^{1-\alpha}] \tag{MC2}$$

and from the optimal capital demands

$$\frac{k_j}{k_i} = (\frac{a_j}{a_i})^{\frac{1}{1-\alpha}} \text{ for two unconstrained firms} \tag{MC3}$$

$$\frac{k_j}{k_i} = \frac{\widetilde{K}_t}{\underline{k}} \frac{a_j^{\frac{1}{1-\alpha}}}{\sum_{unc} M_t(a) a^{\frac{1}{1-\alpha}}} \text{ if firm } j \text{ is unconstrained and } i \text{ constrained}$$

An important statistic to consider in characterizing the allocation is the productivity of the marginal constrained firm, which is

$$a_c = (\frac{\underline{k}_t}{K_t - f M_t^e - \underline{k} \sum_{a \le a_c} M_t(a)})^{1-\alpha} (\sum_{a > a_c} M_t(a) a^{\frac{1}{1-\alpha}})^{1-\alpha}$$

Hence, the higher the capital allocated to unconstrained firms the lower the productivity threshold for a constrained firm. The higher the minimum capacity, the higher the threshold for constrained firms. The threshold also increases in the average productivity of unconstrained firms in the market. Notice that distortions in the measure of active firms in the market and the aggregate capital supply will have a first order impact on the productivity of the marginal firm, and therefore on the revenue product distribution.

Decentralized allocation, constant markups Consider the market structure of section 4.2.1 where goods are perfect substitutes within a sector, $\rho = 1$, and markups are constant and equal to $\varepsilon(a, \tau) = \frac{1}{\alpha}$.

The allocation of capital across firms in different sectors may not be constrained efficient as the productivity of the marginal firm in the market allocation will in general differ from the efficient one, $a_c^m \neq a_c$.Capital demands are still characterized as in the efficient allocation by

$$k_j = \widetilde{K}_t \frac{a_j^{\frac{1}{1-\alpha}}}{\sum_{unc} M_t(a) a^{\frac{1}{1-\alpha}}}$$

but the measure of unconstrained firms in the market, and the capital allocated to them need not coincide.

Proposition 8 *If $\rho = 1$, the efficient allocation can be decentralized as a market allocation under monopolistic competition, where the return (cost) of capital for the households (firms) is $\widehat{r}_t = r_t \theta_k$ with $\theta_k = \frac{1}{\alpha}$.*

Hence, the optimal policy entails, increasing the cost of capital for firms. In this case, the entry condition is undistorted.

Proposition 9 *If $\rho = 1$, and entry costs are denominated in final goods, the efficient allocation can be decentralized as a market allocation under monopolistic competition, where the return (cost) of capital for the households (firms) is $\widehat{r}_t = r_t \theta_k$ with $\theta_k = \frac{1}{\alpha}$, and the entry cost is $\widehat{f} = f \theta_f$ where $\theta_f = \alpha = \frac{1}{\theta_k}$.*

Notice that in either case, the optimal policy is identical to the one provided with exogenous distortions. The reason is that the source of inefficiency in the market is the same. Firm's market power distort the optimal capital accumulation and the number of active firms in the market. By restoring efficiency in both margins, we make sure that the productivity of the marginal firm that is constrained by the minimum capital capacity is the efficient one. Hence, the distribution of revenue productivity is constrained efficient.

6.2 A model with incomplete information

UNDER CONSTRUCTION

7 Conclusion

TBC

References

J. Asker, A. Collard-Wexler, and J. De Loecker. Dynamic inputs and resource (mis)allocation. *CEPR Discussion Papers*, 8469, 2011.

Marcus Asplund and Volker Nocke. Firm Turnover in Imperfectly Competitive Markets - super-1. *Review of Economic Studies*, 73(2):295–327, 2006. URL `https://ideas.repec.org/a/oup/restud/v73y2006i2p295-327.html`.

Jean-Pascal Benassy. Monopolistic competition, increasing returns to specialization and output persistence. *Economics Letters*, 52(2):187–191, August 1996.

M. Buso, L. Madrigal, and C. Pages. Productivity and resource misallocation in latin america. *The B.E. Journal of Macroeconomics*, 13(1):30, 2013.

J. Caunedo. Industry dynamics, investment and business cycles. *Working Paper*, 2013.

Joel M. David, Hugo A. Hopenhayn, and Venky Venkateswaran. Information, Misallocation, and Aggregate Productivity. *The Quarterly Journal of Economics*, 131(2):943–1005, 2016. URL `https://ideas.repec.org/a/oup/qjecon/v131y2016i2p943-1005..html`.

Swati Dhingra and John Morrow. Monopolistic Competition and Optimum Product Diversity Under Firm Heterogeneity. Technical report, 2016.

Avinash K Dixit and Joseph E Stiglitz. Monopolistic Competition and Optimum Product Diversity. *American Economic Review*, 67(3):297–308, June 1977. URL `https://ideas.repec.org/a/aea/aecrev/v67y1977i3p297-308.html`.

J. Eaton and G. Grossman. Optimal trade and industrial policy under oligopoly. *The Quarterly Journal of Economics*, 101(2):383–406, 1986.

N. Guner, G. Ventura, and Y. Xu. Macroeconomic implications of size-dependent policies. *Review of Economic Dynamics*, 11(4):721–744, 2008.

H. Hopenhayn. Entry, exit and firm dynamics in long run equilibrium. *Econometrica*, 60: 1127–50, 1992.

C. Hsieh and Peter Klenow. Misallocation and manufacturing tfp in china adn india. *Quarterly Journal of Economics*, CXXIV, 2009.

Y. Lee and T. Mukoyama. Entry, exit, and plant-level dynamics over the business cycle. *Center for Economic Studies, US Census Bureau*, 08-17, 2008.

M. Melitz. The impact of trade on intra-industry reallocations and aggregate industry productivity. *Econometrica*, 71(6):1695–1725, 2003.

V. Midrigan and D. Xu. Accounting for plant-level misallocation. *Society of Economic Dynamics, Meeting Papers*, 2009.

D. Restuccia and R. Rogerson. Policy distortions and aggregate productivity with heterogeneous plants. *Review of Economic Dynamics*, 11:707–720, 2008.

A Proofs

INCOMPLETE

Proof. (*Proposition 1*) The result follows from the optimality conditions of the planner's problem. Optimality conditions are standard, and read

$$\gamma Y_t^{1-\frac{\alpha}{\gamma}} y(a,\tau)^{\alpha-\rho} a^{\frac{\rho}{\alpha}} k_t^{\rho-1} = r_t \tau \qquad (k_t)$$

$$1 = \frac{U'(C_{t+1})}{U'(C_t)} \beta[(1-\widehat{\delta}) + r_{t+1}] \qquad (K_t+1)$$

$$\sum_{a,\tau} \lambda_t(a,\tau) G(a,\tau) = f r_t \qquad (M_t^e)$$

$$\lambda_t(a,\tau) = \frac{\gamma}{\alpha} Y_t^{1-\frac{\alpha}{\gamma}} y(a,\tau)^{\alpha} - r_t \tau k_t + \beta(1-\delta) \sum_{a',\tau'} \lambda_{t+1}(a',\tau')\Gamma \qquad (M_t(a,\tau))$$

The first one equalizes the marginal product capital in any production unit to its marginal cost. The second condition is the standard Euler equation under linear utility. The third condition equalizes the expected value of a firm $E[\lambda_t(a,\tau)]$ to the entry cost, fr_t. The fourth equation dictates that the value of a firm is the expected discounted value of the net output that it generates. The per period value of a firm of type (a,τ) for the social planner is $\gamma \frac{1-\alpha}{\alpha} Y_t^{1-\frac{\alpha}{\gamma}} a k_t^{\alpha}$.

The free entry condition follows from the optimality condition with respect to the measure of firms. The euler equation from the optimality condition for aggregate capital, using the equilibrium shadow value of capital. Capital allocation at the firm level follow optimal capital demands. ■

Proof. (*Proposition 6*) Use the feasibility condition in goods for the planner evaluated at a stationary equilibrium (so that investment equals δK). Compute equilibrium output as function of the optimal allocation of capital across firms. Define average TFPR as in 4.2.1 and the results follows. ■

Proof. (*Proposition 8*) There are two optimality conditions that are modified in the planner's problem relative to the benchmark in section 3

$$\gamma Y_t^{1-\frac{\alpha}{\gamma}} a k_t^{\alpha-1} = r_t - \mu_t(a) \qquad (k_t)$$

with $\mu_t(a) = 0$ if $k_t(a) > \underline{k}$,and

$$\lambda_t(a) = \frac{\gamma}{\alpha} Y_t^{1-\frac{\alpha}{\gamma}} a k_t^\alpha - r_t k_t + \beta\ (1-\delta) \sum_{a\prime} \lambda_{t+1}(a')\Gamma(a'|a) \qquad (M_t(a))$$

Using equilibrium aggregate output, we can compute the shadow value of capital

$$r_t^c = \gamma \frac{Y_t^{1-\frac{\alpha}{\gamma}}}{(\widetilde{K}_t)^{1-\alpha}} (\sum_{unc} (a)^{\frac{1}{1-\alpha}} M_t(a))^{1-\alpha}$$

$$r_t^c = \frac{\gamma}{\widetilde{K}_t^{1-\gamma}} (1 + \frac{\underline{k}^\alpha}{\widetilde{K}_t^\alpha} \frac{\sum_c M_t(a) a}{(\sum_{unc} M_t(a) a^{\frac{1}{1-\alpha}})^{1-\alpha}})^{\frac{\gamma}{\alpha}-1} (\sum_{unc} M_t(a) a^{\frac{1}{1-\alpha}})^{(\frac{\gamma}{\alpha}-1)(1-\alpha)} (\sum_{unc} (a)^{\frac{1}{1-\alpha}} M_t(a))^{1-\alpha}$$

$$r_t^c = \frac{\gamma}{\widetilde{K}_t^{1-\gamma}} (1 + \frac{\underline{k}^\alpha}{\widetilde{K}_t^\alpha} \frac{\sum_c M_t(a) a}{(\sum_{unc} M_t(a) a^{\frac{1}{1-\alpha}})^{1-\alpha}})^{\frac{\gamma}{\alpha}-1} (\sum_{unc} M_t(a) a^{\frac{1}{1-\alpha}})^{\frac{\gamma}{\alpha}(1-\alpha)}$$

The marginal constrained firm has an operating value that makes it indifferent between being constrained or not. Given that $\Gamma(a'|a)$ is independent of whether the firm is constrained or not this period, the marginal constrained firm satisfies

$$\frac{1-\alpha}{\alpha} \gamma Y_t^{1-\frac{\alpha}{\gamma}} a_c \underline{k}_t^\alpha = \gamma \frac{1-\alpha}{\alpha} Y_t^{1-\frac{\alpha}{\gamma}} a_c \frac{a_c^{\frac{\alpha}{1-\alpha}}}{(\sum_{unc} M_t(a) a^{\frac{1}{1-\alpha}})^\alpha} \widetilde{K}_t^\alpha$$

$$\gamma \frac{1-\alpha}{\alpha} Y_t^{1-\frac{\alpha}{\gamma}} a_c (\underline{k}_t^\alpha - \widetilde{K}_t^\alpha \frac{a_c^{\frac{\alpha}{1-\alpha}}}{(\sum_{unc} M_t(a) a^{\frac{1}{1-\alpha}})^\alpha}) = 0$$

$$a_c = (\frac{\underline{k}_t}{\widetilde{K}_t})^{1-\alpha} (\sum_{unc} M_t(a) a^{\frac{1}{1-\alpha}})^{1-\alpha}$$

Finally, the free entry condition reads

$$\sum_a \frac{\gamma \frac{1-\alpha}{\alpha} Y_t^{1-\frac{\alpha}{\gamma}} a k_t^\alpha - \mu_t(a) \underline{k}_t}{1-\beta\ (1-\delta)} G(a) = f r_t^c$$

$$\sum_{a \le a_c} \frac{\gamma \frac{1-\alpha}{\alpha} Y_t^{1-\frac{\alpha}{\gamma}} a \underline{k}_t^\alpha}{1-\beta\ (1-\delta)} G(a) + \sum_{a > a_c} \frac{\gamma \frac{1-\alpha}{\alpha} Y_t^{1-\frac{\alpha}{\gamma}} a k_t^\alpha}{1-\beta\ (1-\delta)} G(a) = f \gamma \frac{Y_t^{1-\frac{\alpha}{\gamma}}}{(\widetilde{K}_t)^{1-\alpha}} (\sum_{unc} (a)^{\frac{1}{1-\alpha}} M_t(a))^{1-\alpha}$$

$$f = \frac{1-\alpha}{\alpha} \frac{1}{1-\beta\ (1-\delta)} \frac{\widetilde{K}_t}{\sum_{a>a_c} (a)^{\frac{1}{1-\alpha}} M_t(a))}$$
$$(\sum_{a \le a_c} a (\frac{\underline{k}_t \sum_{a>a_c} M_t(a) a^{\frac{1}{1-\alpha}}}{\widetilde{K}_t})^\alpha G(a) + \sum_{a>a_c} a^{\frac{1}{1-\alpha}} G(a))$$

$$f=\frac{1-\alpha}{\alpha}\frac{1}{1-\beta\ (1-\delta)}\frac{\widetilde{K}_t}{\sum_{a>a_c}(a)^{\frac{1}{1-\alpha}}M_t(a))}$$
$$(\sum_{a\leq a_c}a(\frac{\underline{k}_t\sum_{a>a_c}v_t(a)a^{\frac{1}{1-\alpha}}}{\frac{\widetilde{K}_t}{M_t}})^{\alpha}G(a)+\sum_{a>a_c}a^{\frac{1}{1-\alpha}}G(a))$$

and $\frac{\widetilde{K}_t}{M_t}=\frac{K_t}{M_t}-f\delta-\underline{k}\frac{M_t^c}{M_t}$.

Decentralized allocation

In the decentralized allocation under monopolistic competition, the first order conditions for firms are

$$\alpha\gamma Y_t^{1-\frac{\alpha}{\gamma}}ak_t^{\alpha-1}=r_t^{mc}-\mu_t^{mc}(a)$$

where $\mu_t^{mc}(a)=0$ if $k_{it}>\underline{k}$. The equilibrium interest rate should satisfy

$$r_t^c=\alpha\frac{\gamma}{\widetilde{K}_t^{1-\gamma}}(1+\frac{\underline{k}^{\alpha}}{\widetilde{K}_t^{\alpha}}\frac{\sum_c M_t(a)a}{(\sum_{unc}M_t(a)a^{\frac{1}{1-\alpha}})^{1-\alpha}})^{\frac{\gamma}{\alpha}-1}(\sum_{unc}M_t(a)a^{\frac{1}{1-\alpha}})^{\frac{\gamma}{\alpha}(1-\alpha)}$$

and capital demands are $k_t=\max\{\widetilde{K}_t\frac{a^{\frac{1}{1-\alpha}}}{\sum_{unc}M_t(a)a^{\frac{1}{1-\alpha}}},\underline{k}\}$. The marginal constrained firm has an operating value that makes it indifferent between being constrained or not. Given that $\Gamma(a'|a)$ is independent of whether the firm is constrained or not this period, the marginal constrained firm satisfies

$$(1-\alpha)\gamma Y_t^{1-\frac{\alpha}{\gamma}}a_c^{mc}\underline{k}_t^{\alpha}=(1-\alpha)\gamma Y_t^{1-\frac{\alpha}{\gamma}}a_c\frac{a_c^{\frac{\alpha}{1-\alpha}}}{(\sum_{unc}M_t(a)a^{\frac{1}{1-\alpha}})^{\alpha}}\widetilde{K}_t^{\alpha}$$
$$(1-\alpha)\gamma Y_t^{1-\frac{\alpha}{\gamma}}a_c^{mc}(\underline{k}_t^{\alpha}-(\widetilde{K}_t^{mc})^{\alpha}\frac{a_c^{\frac{\alpha}{1-\alpha}}}{(\sum_{unc}M_t(a)a^{\frac{1}{1-\alpha}})^{\alpha}})=0$$
$$a_c^{mc}=(\frac{\underline{k}_t}{K_t-fM_t^e-\underline{k}\sum_{a\leq a_c}M_t(a)})^{1-\alpha}(\sum_{a>a_c}M_t(a)a^{\frac{1}{1-\alpha}})^{1-\alpha}$$

Finally the free entry condition is as before

$$f=\frac{1-\alpha}{\alpha}\frac{1}{1-\beta\ (1-\delta)}\frac{\widetilde{K}_t}{\sum_{a>a_c}(a)^{\frac{1}{1-\alpha}}M_t(a))}$$
$$(\sum_{a\leq a_c}a(\frac{\underline{k}_t\sum_{a>a_c}v_t(a)a^{\frac{1}{1-\alpha}}}{\frac{\widetilde{K}_t}{M_t}})^{\alpha}G(a)+\sum_{a>a_c}a^{\frac{1}{1-\alpha}}G(a))$$

Hence, the market distortions are identical as in the case in which revenue product dispersion is generated exogenously. The optimal policy is therefore the same as before. ■