The drivers of life-cycle growth of manufacturing plants.*

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PRELIMINARY

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Abstract

We take advantage of rich microdata on Colombian manufacturing establishments to decompose growth over an establishment's life cycle into that attributable to fundamental sources of idiosyncratic growth–physical productivity, demand shocks (firm appeal), and input prices–and distortions that weaken the link between those fundamentals and actual growth. We rely on a nested CES structure for preferences over products by multiproduct businesses, and data on quantities and prices for individual products for each manufacturing establishment, to decompose profitability shocks into physical productivity and demand shocks. Pooling all ages, measured fundamentals explain around 67% of the variability of output relative to birth level, with the remaining 33% explained by distortions and other unobserved factors. Demand shocks and $TFPQ$ are equally important in the explained part, while input prices play a more minor role. Distortions explain more than 50% of growth up to age seven, but their contribution falls to less than 25% by around age 20. For the fraction explained by fundamentals, early life growth is explained by $TFPQ$ with demand and

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input prices playing a minor role. But demand is the crucial factor in long-run growth, with a contribution that surpases that of $TFPQ$ and unobserved factors by around age 15. In the 2000s compared to the 1980s, two decades separated by a wave of deep structural reforms, the contribution of $TFPQ$ to the variance in life cycle growth grows by around 10 p.p , with demand and input prices falling in importance. Interestingly, that of distortions remains basically constant.

Keywords: Life cycle of plants; post-entry growth; TFPQ; demand

JEL codes: O47; O14; O39

1 Introduction

The growing availability of detailed firm and establishment level data has allowed researchers to dig into the empirical micro foundations behind sluggish aggregate growth in many low- and middle-income economies. A recent strand of the literature has focused on how businesses grow over their life cycle, uncovering patterns that suggest that less developed economies are characterized by post-entry business growth slower than that observed in developed economies (Hsieh and Klenow, 2014; Buera and Fattal, 2014). Slow post-entry productivity growth—related to poor market selection or poor innovation– has been found to play a crucial role in explaining poor outcome growth in this context.

In trying to understand the reasons behind slow post-entry productivity growth, much of the focus has been on dimensions external to the business, such as institutions that discourage, or fail to encourage, healthy market selection and investment in productivity growth (e.g.Hsieh and Klenow, 2014). On the side of businesses, meanwhile, the focus has been on efforts conducive to improvements in technical efficiency. For instance, research on managerial practices that impact productivity has focused on production processes and employee management (e.g. Bloom and Van Reenen, 2007; Bloom et al. 2016).

But productivity in this context has been generally measured without distinguishing physical efficiency from quality or other demand-related factors (e.g. Hsieh and Klenow, 2014, p. 1056). Foster, Haltiwanger and Syverson (2016), in fact, have found that for US manufacturers in commodity-like sectors employment growth from birth to maturity closely tracks demand growth but not growth in physical productivity. The relative role of each of these dimensions in the determinants of growth as a business ages is thus an open question. Understanding these issues is critical for quantifying the differential barriers to post entry growth within and across countries. Policies designed to overcome such barriers likely depend on the nature of and relative importance of demand side vs. technical efficiency factors in accounting for which businesses succeed.

We decompose growth over an establishment's life cycle into that attributable to fundamental sources of growth–physical productivity, demand shocks, and input prices–, and idiosyncratic distortions that weaken the link between those fundamentals and actual growth. Rich microdata on all non-micro Colombian manufacturing establishments, where quantities and prices for individual products and inputs of establishments are observed, is taken advantage of for this purpose. We rely on a nested CES structure for preferences over products by multiproduct businesses (Hottman et al, 2016) to measure separately quality-adjusted output and prices at the establishment level, which, together with data on input use and total value of output, permits identifying physical productivity and demand shocks.

Our approach contrasts sharply with standard approaches that attempt to infer the dynamics of firm performance from patterns of measures of revenue productivity. It also contrasts with those aimed at disentangling idiosyncratic prices into marginal costs and markups (e.g. Hottman et al. 2016, De Loecker and Warszynski, 2012), to the extent that we focus directly on the fundamental determinants of each of these margins.

Hsieh and Klenow (2014) have pioneered the empirical exploration of growth over a manufacturing plant's life cycle, showing that average size growth over the life cycle of a manufacturing establishment varies considerably between the US, India and Mexico, with US plants growing much more dynamically than those in Mexico or India, and have shown that such differences could be attributed to variability in idiosycratic distortions across those economies. We aim to establish a series of facts related to those uncovered by Hsieh and Klenow: 1) The degree of dispersion in growth across plants in a sector, within the same economy. 2) The degree to which that dispersion in size growth is attributable to dispersion in the life cycle growth in specific fundamentals, in particular physical productivity, as compared to demand shock and input costs over different horizons, and also to idiosyncratic distortions due to which outcome growth may not respond to growth in fundamentals.

Establishment level data for the manufacturing sector in Colombia is uniquely rich. Since at least 1982, the Annual Manufacturing Survey has been recording information on all individual products produced by an establishment and all individual material inputs used by the establishment, besides information on input use and monetary value of production. Establishment level price indices can be constructed using this information. Moreover, the age of the establishment from the time of the start of its operations is reported in the survey (which includes all non-micro manufacturing establishments), and establishments can be followed longitudinally, some of them for over 30 years. The age indicator is not affected by restructuring or changes in ownership.

We take advantage of these unique data to assess the relative importance of different plant-level fundamentals as determinants of growth over the life cycle of businesses in Colombia's manufacturing industry, and in contrast to distortions that weaken the link between fundamentals and output (Restuccia and Rogerson, 2009; Hsieh and Klenow, 2009). On the side of fundamentals we can separate physical productivity ("$TFPQ$") from demand shocks and input prices at the plant level, an unusually rich set of measured fundamentals.[1]

Our analysis contributes to the growing literature on post-entry growth by expanding the set of plant-level determinants of growth to which attention is paid, from the traditional focus on physical productivity to a more comprehensive view that highlights the importance of input prices and demand, including building the business' client base, introducing new products, or increasing product quality in existing product lines. In this aspect, we build on the ideas recently proposed by Foster et al. (2016), but expand the empirical reach by enriching the characterization of fundamentals, and widening the sectoral scope to all manufacturing sectors. The latter requires an explicit treatment of the multi-product character of most manufacturing establishments to appropriately separate quantities and prices at the establishment level, for which we rely on a demand setup similar to that used by Hottman et al (2016), which allows us to construct quality-adjusted plant price indices.

Hottman et al (2016) decompose the variance of firm sales into the contributions of firm appeal, cost and markup. Beyond our different focus on life-cycle growth over the medium and long run for a single business, we also follow a different methodological approach where cost shocks and physical productivity measures are obtained from data on the plant's production process rather than infered through a structure that relies exclusively on data on prices and quantities. Interestingly, while physical productivity is frequently seen as particularly important for growth in the longer run, our results suggest that in fact demand becomes relatively more important to explain growth over longer horizons.

For an average manufacturing plant in our Colombian data, compared to its level at birth output has grown by a factor of 2.4 by age 5, almost four-fold by age 10, and ten-fold by age 25. Employment grows at a slower pace, with factors of growth relative to birth of around 1.5, 2, and 3 for ages 5, 10, and

[1] $TFPQ$ is defined here as tecnhical efficiency, as in Foster, Haltiwanger and Syverson (2008, 2016). This contrasts with the definition in Hsieh-Klenow (2009, 2014) where $TFPQ$ can be interpreted as either technical efficiency or quality.

25. Based on comparable data for the US and growth over cohorts, this pace of employment growth in Colombia is approximately half as fast as that in the US.

There is wide dispersion in the patterns of growth across firms, with average growth driven by a small fraction of rapidly growing businesses. We find that in the long run such dispersion is mostly explained by dispersion in fundamentals, rather than distortions and other unobserved factors, with $TFPQ$ and demand shocks both playing a crucial role. Pooling all ages, measured fundamentals explain around 67% of the variability of output relative to birth level, with the remaining 33% explained by distortions and other unobserved factors. Of the fraction explained by measured factors, input price growth explains 7 p.p, with demand shocks and $TFPQ$ being equally important in explaining the rest. Interestingly, distortions are particularly important to explain growth from birth to early ages, explaining more than 50% of the variance of growth up to age seven. But, they lose in importance for longer horizons, with their contribution falling to less than 25% by around age 20. For the fraction explained by fundamentals, variance in early life growth is explained by $TFPQ$ with demand and input prices playing a minor role. But demand is the crucial factor in accounting for the variation in long-run growth, with a contribution that surpases that of $TFPQ$ and unobserved factors by around age 15.

We also observe changes in the contribution of technology vs. demand and other factors over time in Colombia. The contribution of TFPQ to the variance in life cycle growth grows by around 10 p.p in the 2000s compared to the 1980s, while the contribution of demand and input prices falls. Interestingly, that of distortions remains basically constant. Many underlying factors probably changed between those two decades, but a crucial dimension is the implementation of wide market-oriented reforms in the 1990s.

The paper proceeds as follows. Section 2 presents our conceptual framework, defining each of the plant fundamentals that we characterize, and our approach to decompose growth into contributions of those fundamental sources. We then explain the data used in our empirical work, in section 3. Growth over establishments' life cycle in terms of output, employment and other outcomes, which is the object we aim at decomposing, is characterized in section 4). Section 5 explains the approach we use to measure fundamentals. Results for our growth decomposition are presented in section 6. Section 8 concludes.

2 Decomposing growth into fundamental sources

We start with a very simple model of firm optimal behavior given firm fundamentals, to derive the relationship that should be observed between size growth and growth in fundamentals as a firm ages. For consistency with the literature on business dynamics, we refer to a business as a "firm", even though the unit of observation for our empirical work is an establishment. The main fundamentals we consider are the productivity of the firm's productive process (often termed TFPQ in the literature) and a demand shock. The conceptual framework below makes clear what we mean by each of these, and the sense in which they are "fundamentals". Beyond measuring TFPQ and demand shocks, we observe unit prices for inputs, in particular material inputs and labor.

In the model, the firm chooses its size optimally given TFPQ, demand shocks and input prices. As a result, growth over its life cycle is driven by growth in each of them. This is the basis of our analysis.

Though for simplicity we take growth of fundamentals as exogenous in describing this conceptual framework, it is clear that the firm may make investments to modify both the physical productivity of its production process and dimensions of demand. The firm's efforts to strengthen demand may include investments in building its client base (Foster et al., 2014), and adding new products and/or improving the quality of its pre-existing product lines (Atkeson and Burstein, 2010; Acemoglu et al., 2014).

Taking fundamentals as given allows us to abstract from dynamic considerations that enter the decision of a firm to invest in improving future productivity and demand, rather than putting those resources towards current production. In turn, our focus on growth over lengthy periods of time, that even exceed decades in many of our observations, reduces the scope for adjustment costs to play a key role in firm growth for given sets of fundamentals. For instance, the time for employment and capital to fully adjust to shocks (e.g. productivity or demand shocks) is at most three years for U.S. manufacturing plants (Foster et al. 2016, based on a variety of estimates of adjustment costs). Taking advantage of these facts, we use a simple static model to frame our analysis of growth between birth and a future period t, where t is far away into the future. In the empirical sections, we further address questions about the potential role of adjustment costs by permitting sluggish response to fundamentals and by focusing on wider age bins.

2.1 Technology

Consider a firm indexed by f, that produces output Q_{ft} using a composite input X_{ft} to maximize its profits, with technology

$$Q_{ft} = A_{ft} X_{ft}^{\gamma} = a_{ft} A_t X_{ft}^{\gamma} \tag{1}$$

A_{ft} is the firm's physical total factor productivity, and γ the returns to scale parameter. In turn, $A_{ft} = a_{ft} A_t$ where A_t is an aggregate technology shock, and a_{ft} is an idiosyncratic component. We refer to a_{ft} as the firm's $TFPQ$. Equation (1) makes clear that $TFPQ$ captures the (idiosyncratic) physical efficiency of the productive process: how much physical product the firm expects to obtain from a unit of a basket of inputs, beyond that obtained by the average firm.

Some firms are multiproduct, and for them output Q_{ft} is a composite of individual products (see below). But though process efficiency is likely to vary across products in the firm, some aspects of it, such as production management and average worker ability for basic tasks, are common to different product lines within the firm. We focus on these firm-level components of efficiency, captured by a_{ft}, a crucial focus when trying to understand growth at the firm level.

2.2 Demand

As in Hottman et al. (2016), in the context of multiproduct firms we define firm output Q_{ft} as a CES composite of individual products $Q_{ft} = \left(\sum_{\Omega_t^f} d_{fjt} q_{fjt}^{\frac{\sigma_J - 1}{\sigma_J}} \right)^{\frac{\sigma_J}{\sigma_J - 1}}$, where q_{fjt} is period t purchases of good j produced by firm f and the weights d_{fjt} reflect consumers' relative preference for different goods within the basket offered by firm f, and Ω_t^f is the basket of goods produced by f in year t. In particular, consumers derive utility from a nested CES utility function, with a CES nest for firms and another for products within firms. Consumer's utility in period t is given by:

$$U\left(Q_{1t},...,Q_{Nt}\right) = \left(\sum_{f=1}^{N_{Ft}} d_{ft} Q_{ft}^{\frac{\sigma_F-1}{\sigma_F}}\right)^{\frac{\sigma_F}{\sigma_F-1}} \tag{2}$$

$$where\ Q_{ft} = \left(\sum_{\Omega_t^f} d_{fjt} q_{fjt}^{\frac{\sigma_J-1}{\sigma_J}}\right)^{\frac{\sigma_J}{\sigma_J-1}} \tag{3}$$

$$s.t. \quad \sum_{f=1}^{N_{Ft}} \sum_{\Omega_t^f} p_{fjt} q_{fjt} = E_t \tag{4}$$

where p_{fjt} is the price of q_{fjt}, and N_{Ft} is the number of firms in period t. We refer to d_{fjt} and d_{ft} as, respectively product (within firm) and firm "appeal", defined as in equations 2 and 3: the weight, in consumer preferences, of product fj in firm f's basket of products, and of firm f in the set of firms. Product appeal d_{fjt} captures the valuation of attributes specific to good fj relative to other goods produced by the firm, while firm appeal d_{ft} captures attributes that are common to all goods provided by firm f,, such as the firm's costumer service and average quality of firm f's products. Both firm and product appeal may vary over time. Parameters σ_F and σ_J capture, respectively, elasticities of substitution across firms and across goods produced by the same firm, where σ_J is assumed constant across firms within a sector.

Consumer optimization implies that the period t demand for product fj and the firm revenue are, respectively, given by

$$q_{fjt} = d_{ft}^{\sigma_F} d_{fjt}^{\sigma_J} \left(\frac{P_{ft}}{P_t}\right)^{-\sigma_F} \left(\frac{p_{fjt}}{P_{ft}}\right)^{-\sigma_J} \frac{E_t}{P_t} \tag{5}$$

$$and$$

$$R_{ft} = d_{ft}^{\sigma_F} P_{ft}^{1-\sigma_F} \frac{E_t}{P_t^{1-\sigma_F}} \tag{6}$$

where

$$P_{ft} = \left(\sum_{\Omega_t^f} d_{fjt}^{\sigma_J} p_{fjt}^{1-\sigma_J}\right)^{\frac{1}{(1-\sigma_J)}} \tag{7}$$

is the firm's exact price index, and $P_t = \left(\sum_{f=1}^{N_F} d_f^{\sigma_F} p_f^{1-\sigma_F}\right)^{\frac{1}{1-\sigma_F}}$ is an aggregate price index. Given the properties of CES demand, $Q_{ft} = \frac{R_{ft}}{P_{ft}}$ (which can be shown using these optimal demands and equation (2)).

Dividing (6) through by P_{ft}, and solving for P_{ft} we obtain demand equation (8), which we use in the empirics:

$$\frac{P_{ft}}{P_t} = D_{ft} Q_{ft}^{-\varepsilon} = D_t d_{ft} Q_{ft}^{-\varepsilon} \tag{8}$$

where $\varepsilon = \frac{1}{\sigma_F}$ is an inverse elasticity of demand, and D_{ft} is a demand shifter, with aggregate and idiosyncratic components $D_t = \frac{E_t}{P_t}$ and d_{ft}. A crucial insight on the measurement of firm appeal emerges from equation (8): d_{ft} is the price charged holding quantities constant, beyond aggregate effects. We refer to d_{ft} generically as the firm's (idiosyncratic) demand shock.

Inverting this equation and multiplying through by P_{ft} to obtain $R_{ft} = P_{ft}^{1-\frac{1}{\varepsilon}} D_{ft}^{\frac{1}{\varepsilon}}$ (where $R_{ft} = P_{ft}Q_{ft}$), one obtains the analogous interpertation of measured firm appeal (d_{ft}) used by Hottman et al (2016): it captures sales holding prices constant. This is akin to quality as defined by Khandelwal (2010), Hallak and Schott (2011), Fieler, Eslava and Xu (2016), and others. Foster et al (2016), in turn, interpret firm appeal as capturing the strength of the business' client base.

2.3 Equilibrium

The firm chooses its scale X_{ft} to maximize profits

$$\underset{X_{it}}{Max}\ \ (1-\tau_{ft}) P_{ft} Q_{ft} - C_{ft} X_{ft} = (1-\tau_{ft}) D_{ft} A_{ft}^{1-\varepsilon} X_{ft}^{\gamma(1-\varepsilon)} - C_{ft} X_{ft}$$

taking as given A_{ft}, D_{ft}, and unit costs of the composite input, C_{ft}. There may be idiosyncratic distortions τ_{ft}, that affect a firm's choice of size given all of these fundamentals (Restuccia and Rogerson, 2009; Hsieh an Klenow, 2009, 2014).[2] These distortions capture, for instance, adjustment

[2] Further below, we also considered factor-specific distortions that, for given choice of X_{it}, affect the relative choice of a given input with respect to others.

costs, product-specific tariffs, and size-dependent taxes. A full treatment of adjustment costs requires a fully dynamic approach beyond the reach of this paper. Instead, we take a reduced-form approach that recognizes adjustment costs that break the link between actual adjustment and the "desired adjustment" in an environment where the absence of such costs turns the dynamic problem into a series of static ones, as in our approach.[3] Profit maximization yields optimal input demand of

$$X_{ft} = \left(\frac{\gamma \left(1-\varepsilon\right)\left(1-\tau_{ft}\right) D_{ft} A_{ft}^{1-\varepsilon}}{C_{ft}} \right)^{\frac{1}{1-\gamma(1-\varepsilon)}} \tag{9}$$

Equation (9) is the starting point of our decompositions of life cycle growth.

Bear in mind that unit cost shocks also contain aggregate and idiosyncratic components: $C_{ft} = c_{ft} C_t$. In addition, measured $TFPQ$, demand shocks and cost shocks may deviate from a_{ft}, d_{ft} and c_{ft} due to measurement error, shocks realized by the firm after choosing X_{ft}, and other sources of noise. We denote noise in each of these three dimensions ($TFPQ$, demand and cost) by α_{ft}, δ_{ft} and ζ_{ft} respectively, and include these noise terms in the derivations below to later help in the interpretation of empirical results.

2.4 Life cycle growth

It follows from equation (9) that input growth over the life cycle of the firm, $\frac{X_{ft}}{X_{f0}}$ where 0 is the year of start of operations for plant f, can be attributed to growth in the different fundamentals:

$$\frac{X_{ft}}{X_{f0}} = \left(\frac{d_{ft}}{d_{f0}}\right)^{\kappa_1} \left(\frac{a_{ft}}{a_{f0}}\right)^{\kappa_2} \left(\frac{c_{ft}}{c_{f0}}\right)^{-\kappa_1} \kappa_t \kappa_{ft} \tag{10}$$

where $\frac{d_{ft}}{d_{f0}}$, $\frac{a_{ft}}{a_{f0}}$ and $\frac{c_{ft}}{c_{f0}}$ are, respectively, life cycle growth in idiosyncratic demand shocks, $TFPQ$ and input price shocks. Here, $\kappa_t = \left(\frac{D_t}{D_0}\right)^{\kappa_1} \left(\frac{A_t}{A_0}\right)^{\kappa_2} \left(\frac{C_t}{C_0}\right)^{-\kappa_1}$ captures growth between birth and age t in the aggregate components of fundamentals, and κ_{ft} captures residual variation from noise in fundamentals

[3] See, for instance, Caballero, Engel and Haltiwanger (1995, 1997), Eslava, Haltiwanger, Kugler, and Kugler (2010).

not observed by the firm at the time of choosing its scale in each period, $\kappa_{ft} = \frac{\delta_{ft}^{\kappa_1}\alpha_{ft}^{\kappa_2}\zeta_{ft}^{-\kappa_1}\left(1-\tau_{ft}\right)^{\kappa_1}}{\delta_{f0}^{\kappa_1}\alpha_{f0}^{\kappa_2}\zeta_{0t}^{-\kappa_1}(1-\tau_{0t})^{\kappa_1}}$. Notice that idiosyncratic distortions τ_{ft} decouple the choice of scale from fundamentals. The distortions that a firm faces may vary as it ages (that is, distortions may be considered age-specific), and thus also decouple life-cycle growth in output from the growth of fundamentals. This aspect is captured in our decomposition in the residual term κ_{ft}.

Parameters κ_1, and κ_2, with $\kappa_1 = \frac{1}{1-\gamma(1-\varepsilon)}$, $\kappa_2 = (1-\varepsilon)\,\kappa_1$, are constant across firms that face the same demand elasticity and same factor elasticities in production. Equation (10) decomposes growth in firm size into the contribution of firm level fundamentals, aggregate effects, and firm level unexpected shocks.

An analogous decomposition applies in terms of output, directly derived from $Q_{ft} = A_{ft}X_{ft}^{\gamma}$.Further assuming that $X_{ft} = K_{ft}^{\frac{\beta}{\gamma}}L_{ft}^{\frac{\alpha}{\gamma}}M_{ft}^{\frac{\phi}{\gamma}}$, moreover, we can decompose $\frac{c_{ft}}{c_{f0}}$ into the growth of specific dimensions of input prices, among which two are observed in the data: the price of material inputs, pm_{ft}, and average wage per worker, w_{ft}. There may also be distortions to the use of one input relative to other. Taking these aspects into account, the decomposition of life cycle output growth can be written (see Appendix B):

$$\frac{Q_{ft}}{Q_{f0}} = \left(\frac{d_{ft}}{d_{f0}}\right)^{\gamma\kappa_1}\left(\frac{a_{ft}}{a_{f0}}\right)^{1+\gamma\kappa_2}\left(\frac{pm_{ft}}{pm_{f0}}\right)^{-\phi\kappa_1}\left(\frac{w_{ft}}{w_{f0}}\right)^{-\beta\kappa_1}\chi_t\chi_{ft} \tag{11}$$

Equation (11) is our central object of interest. It decomposes life cycle growth in output into the contribution of different idiosyncratic fundamentals, as well as unobservables, including aggregate effects, distortions, and measurement error. In particular, we focus on four measured souces of fundamental idiosyncratic growth: demand shocks $\frac{d_{ft}}{d_{f0}}$, $TFPQ$ $\frac{a_{ft}}{a_{f0}}$, material input prices $\frac{pm_{ft}}{pm_{f0}}$, and wages $\frac{w_{ft}}{w_{f0}}$. Moreover, $\chi_t == \kappa_t^{\gamma}\left(\frac{A_t}{A_0}\right)$ captures aggregate growth, and $\chi_{ft} = \left(\frac{\left(1-\tau_{ft}\right)^{\kappa_1}\left(1+\tau_{ft}^{M}\right)^{-\phi\kappa_1}\left(1+\tau_{ft}^{L}\right)^{-\beta\kappa_1}\delta_{ft}^{\kappa_1}\alpha_{ft}^{1+\kappa_2}\zeta_{ft}^{-\kappa_1}r_{ft}^{\frac{-\alpha\kappa_1}{\gamma}}}{\left(1-\tau_{f0}\right)^{\kappa_1}\left(1+\tau_{f0}^{M}\right)^{-\phi\kappa_1}\left(1+\tau_{f0}^{L}\right)^{-\beta\kappa_1}\delta_{f0}^{\kappa_1}\alpha_{f0}^{1+\kappa_2}\zeta_{ft}^{-\kappa_1}r_{f0}^{\frac{-\alpha\kappa_1}{\gamma}}}\right)^{\gamma}$ captures residual variation from a number of sources, such as noise in fundamentals not observed by the firm at the time of choosing its scale in each period; growth in unobserved user cost of capital; and changes over the life cycle in distortions faced by the firm, both common across inputs and specific to the use of particular inputs. Here, τ_{ft}^{M} and τ_{ft}^{L} are idiosyncratic distortions to the use of materials and labor relative to capital, such as factor-specific adjustment costs, and subsidies/taxes to the use of one input. Notice that these distortions alter the link between the choice of scale and fundamentals a_{ft}, d_{ft}, pm_{ft} and w_{ft} (Restuccia and Rogerson, 2009; Hsieh and Klenow, 2009, 2014). In our decomposition, they are captured by the idiosyncratic residual term χ_{ft}. Equation 11 makes clear that growth over the life cycle also responds to changes over that cycle in the distortions faced by the firm. Age-dependent distortions are a clear example of such changes.[4]

In the rest of this document, we use rich data on Colombian manufacturing establishments to measure each of the fundamentals in the above decompositons, a_{ft}, d_{ft}, pm_{ft} and w_{ft}, and decompose outcome growth into that attributable to each of them, and subsequently to the distortions and other unobservables captured by the residuals.

Notice that dispersion in the growth of fundamentals relates to dispersion in the average product of inputs, as well as in $TFPR$, two concepts highlighted in Hsieh and Klenow's work. $TFPR$ has been defined by Foster et al (2008) as $TFPR_{ft} = P_{ft}A_{ft}$. As is apparent, in the absence of idiosyncratic distortons, dispersion in $TFPR$ is driven by that in $TFPQ$, as well as by dispersion in prices for given $TFPQ$, which in our framework closely corresponds to the concept of demand shock. In particular, assuming $\tau_{ft} = 0$, $TFPR_{ft} = \frac{C_{ft}}{\gamma(1-\varepsilon)}\left(\frac{\gamma(1-\varepsilon)D_{ft}A_{ft}^{1+\varepsilon\gamma}}{C_{ft}}\right)^{\frac{1-\gamma}{1-\gamma(1-\varepsilon)}}$.

It is clear from the last expression that the particular case of constant returns to scales, $\gamma = 1$, is one where dispersion in $TFPR$ arises only if there is dispersion in input costs. For any $\gamma \neq 1$, meanwhile, $TFPR$ dispersion is also driven by both $TFPQ$ and firm appeal dispersion. This is the case even in absense of distortions to both the process of accumulation of $TFPQ$ and demand, and to the optimal allocation of resources. As noted by Haltiwanger (2016), by allowing for $\gamma \neq 1$, the above derivation explicitly adds idiosyncratic $TFPQ$, demand and cost shocks to potential sources of $TFPR$ dispersion already identified in Hsieh and Klenow's original framework.[5]

[4]Some young firms may, for instance, have more dificulty in accessing financing, or face stronger adjustment costs than their older counterparts. Also, many startups enjoy benefits that older firms do not face. This is the case, as an example, of small young firms in Colombia who at times have been exempted from specific labor taxes.

[5]Furthermore, dispersion in the average product of inputs is not influenced by demand shock dispersion even under our general assumptions: $\frac{R_{ft}}{X_{ft}} = \frac{C_{it}}{\gamma(1-\varepsilon)}$. Notice that $TFPR$ and average product are equivalent only if $\gamma = 1$.

3 Data

3.1 Annual Manufacturing Survey

We use data from the Colombian Annual Manufacturing Survey (AMS) from 1982 to 2012. The survey, collected by the Colombian official statistical bureau DANE, covers all manufacturing establishments belonging to firms that own at least one plant with 10 or more employees, or those with production value exceeding a level close to US$100,000. The unit of observation in the survey is the establishment. An establishment is a specific physical location where production occurs.

Each establishment is assigned a unique ID that allows us to follow it over time. Since a plant's ID does not depend on an ID for the firm that owns the plant, it is not modified with changes in ownership, and such changes are not mistakenly identified as births and deaths. Plant IDs in the survey were modified in 1992 and 1993. We use the official correspondence that maps one into the other to follow establishments over that period.[6]

Surveyed establishments are asked to report their level of production and sales, as well as their use of employment and other inputs, their purchases of fixed assets, and the value of their payroll. We consruct a measure of plant-level wage per worker by dividing payoll into number of employees. Sector ID's are also reported, at the 3-digit level of the ISIC revision 2 classification.[7] Since 2004, respondents are also asked about their investments in innovation, with bi-annual frequency.

A unique feature of the AMS, crucial for our ability to decompose fundamental sources of growth, is that inputs and products are reported at a detailed level. Plants report separately each material input used and product

[6] Though there is supposedly a one-to-one mapping between the two correspondences, there seems to be some degree of mismatch, as suggested by higher exit rates in 1991 and 1992 compared to other years, as well as higher entry in 1993. DANE does report having undertaken efforts to improve actual coverage (compliance) in 1992, which may explain higher entry in 1993, but not higher exit in 1991 and 1992. Even for actual continuers that are incorrectly classified as entries or exits, however, our age variable is correct (see further below). That is, we may fail to properly identified entry into the survey and exit for a fraction of plants over 1992-1993, but this does not lead to mistaken age assignments in our calculations.

[7] The ISIC classification in the survey changed from revision 2 to revision 3 over our period of observation. The three-digit level of disaggregation of revision 2 is the level at which a reliable correspondence between the two classifications can be put together.

produced, at a level of disaggregation corresponding to seven digits of the ISIC classification (close to six-digits in the Harmonized System). For each of these individual inputs and products, plants report separately quantities and values used or produced, so that plant-specific unit prices can be computed for both individual inputs and individual outputs. We thus directly observe idiosyncratic input costs for individual materials. Furthermore, by taking advantage of product-plant-specific prices, we can produce firm-level price indices, and as a result generate measures of productivity based on physical output, and also estimate demand shocks. Details on how we go about these estimations are provided in section 5.

Importantly for this study, the plant's initial year of operation is also recorded–again, unaffected by changes in ownership–. We use that information to calculate an establishment's age in each year of our sample. Though we can only follow establishments from the time of entry into the survey, we can determine their correct age, and follow a subsample from birth. We denominate that subsample, composed of the establishments we observe from birth, as the *life cycle sample*. Based on the life cycle sample, we generate measurement adjustment factors that we then use to estimate life-cycle growth even for plants that we do not observe from birth (more on this in section 3.3).

With respect to studies that rely on data from economic censuses, one clear limitation of our approach is that we only observe a fraction of establishments from birth (about 30% of establishments in the sample), and that fraction is selected: it corresponds to establishments born at or beyond a given size. Moreover, we only observe establishments that satisfy exclusion criteria based on size, though those criteria cover all SMEs and large establishments, leaving out only microestablishments. And, while being formal is not a criterion for inclusion in the Manufacturing Survey, it is indeed likely that most informal establishments are micro, so our results under-represent informal firms. The crucial upside from these data, however, is that we can follow each establishment longitudinally, and do it at higher frequencies (annual, rather that inter-census). We also observe a census of SME and large manufacturing establishments, which are likely to account for a large fraction of any sustained growth actually observed. We attempt to deal with selection biases using a variety of approaches, from adjusting for expected accumulated life cycle growth at time of entry into the survey, to contrasting our findings for plants observed from birth to analogous figures for all of the other plants in the manufacturing survey.

Table 1: Descriptive Statistics

	Mean	Std. Dev.
Log output	10.573	1.717
Log revenue	11.765	1.556
Employment	52.935	110.184
Log capital	10.134	1.836
Log material expenditure	10.865	1.839
Log material	9.937	1.912
Log input prices (SV)	-0.296	0.809
Log output prices (SV)	-0.062	0.863
Log TPFQ	2.545	1.046
Log Demand shock	7.102	1.653
N (baseline sample)	172,734	
N (life cycle sample)	43,747	

Table 1 presents basic descriptive statistics for our sample. It is composed of over 170,000 observations, with numbers of plants per year fluctuating around 7,500. The average plant has just over 50 employees.

3.2 Plant-level prices

Our ability to separate $TFPQ$ from demand shocks–both defined as in section 2–depends crucially on being able to appropriately capture plant level prices. The exact plant level price index that turns into a quality-adjusted (more precisely, appeal-adjusted) deflator for plant output, $P_{ft} = \left(\sum_{\Omega_t^f} d_{fjt}^{\sigma_J} p_{fjt}^{1-\sigma_J}\right)^{\frac{1}{(1-\sigma_J)}}$, depends on unobservable σ and $\{d_{fjt}\}$. We follow here insights from a long and active literature on economically motivated price indices to construct appropriate price indices from observable information.[8] We describe in this section our approach to measure P_{ft}. The underlying derivations are included in Appendix A.

Denoting by $\Omega_{t,t-1}^f$ the set of goods produced by plant f in both period t and $t-1$, and $P_{ft}^{*} = \left(\sum_{\Omega_{t,t-1}^f} d_{fjt}^{\sigma_J} p_{fjt}^{1-\sigma_J}\right)^{\frac{1}{(1-\sigma_J)}}$, we take advantage of

[8] See Redding and Weinstein (2016), and references therein to Sato (1976), Vartia (1976), and Feenstra (2004), whose insights are key in our derivation.

Feenstra's (2004) insight, that

$$\frac{P_{ft}}{P_{ft-1}} = \left(\frac{\sum_{\Omega^f_{t,t-1}} s_{fjt}}{\sum_{\Omega^f_{t,t-1}} s_{fjt-1}} \right)^{\frac{1}{\sigma-1}} \frac{P^*_{ft}}{P^*_{ft-1}} \tag{12}$$

where $s_{fjt} = \frac{p_{fjt}q_{fjt}}{\sum_{\Omega^f_t} p_{fjt}q_{fjt}}$. Defining $s^*_{fjt} = \frac{p_{fjt}q_{fjt}}{\sum_{\Omega^f_{t,t-1}} p_{fjt}q_{fjt}}$, $s^*_{fjt-1,t} = \frac{p_{fjt-1}q_{fjt-1}}{\sum_{\Omega^f_{t,t-1}} p_{fjt-1}q_{fjt-1}}$, and $\omega_{ft,t-1} = \frac{\frac{(s^*_{fjt}-s^*_{fjt-1,t})}{\ln s^*_{fjt}-\ln s^*_{fjt-1,t}}}{\sum\limits_{\Omega_{t,t-1}} \left(\frac{(s^*_{fjt}-s^*_{fjt-1})}{\ln s^*_{fjt}-\ln s^*_{fjt-1,t}} \right)}$ (the "Sato-Vartia" weights), we furthermore rely on the assumption that $\prod\limits_{\Omega^f_{t,t-1}} d_{fjt}^{\omega_{fjt,t-1}} = \prod\limits_{\Omega^f_{t,t-1}} d_{fjt-1}^{\omega_{fjt,t-1}}$ to obtain (see Appendix A):

$$\frac{P^*_t}{P^*_{t-1}} = \prod_{\Omega^f_{t,t-1}} \left(\frac{p_{fjt}}{p_{fjt-1}} \right)^{\omega_{fjt,t-1}}$$

The assumption that $\frac{\widehat{d}^*_{ft}}{\widehat{d}^*_{ft-1}} = 1$, where $\widehat{d}^*_t = \prod\limits_{\Omega^f_{t,t-1}} d_{fjt}^{\omega_{fjt,t-1}}$ is weaker than the common assumption that individual product appeal is constant over time, $\frac{d_{fjt}}{d_{fjt-1}} = 1$. [9]

After obtaining plant-level price changes for the $\Omega^f_{t,t-1}$ basket of goods in each pair of consecutive years, price indices in levels are constructed recursively as $\ln P^*_{ft} = \ln P^*_{ft-1} + \ln \left(\frac{P^*_{ft}}{P^*_{ft-1}} \right)$. The price series for each plant is initialized at a given level P_{fB_f}, where B_f is the base year for plant f. We

[9] If $\frac{\widehat{d}^*_t}{\widehat{d}^*_{t-1}} \neq 1$, $\ln \left(\frac{P^*_t}{P^*_{t-1}} \right) = \sum\limits_{\Omega_{t,t-1}} \ln \left(\frac{p_{fjt}}{p_{fjt-1}} \right)^{\omega_{fjt,t-1}} - \frac{\sigma}{\sigma-1} \ln \left(\frac{\widehat{d}^*_{ft}}{\widehat{d}^*_{ft-1}} \right)$. The Sato-Vartia price index $\sum\limits_{\Omega_{t,t-1}} \ln \left(\frac{p_{fjt}}{p_{fjt-1}} \right)^{\omega_{fjt,t-1}}$ ignores the negative term $-\frac{\sigma}{\sigma-1} \ln \left(\frac{\widehat{d}^*_{ft}}{\widehat{d}^*_{ft-1}} \right)$. As Redding and Weinstein (2016) indicate, it is possible that product demand shocks $\frac{d_{fjt}}{d_{fjt-1}}$ are positively correlated with the weights $\omega_{fjt,t-1}$ so that $\frac{\widehat{d}^*_{ft}}{\widehat{d}^*_{ft-1}} > 1$. We keep the assumption that $\frac{\widehat{d}^*_t}{\widehat{d}^*_{t-1}} = 1$ in order to calculate $\ln \left(\frac{P^*_t}{P^*_{t-1}} \right)$ from observables, but acknowledge this potential upward bias in the Sato-Vartia price index that we use, which may induce a mistakenly low growth in quantities and therefore in $TFPQ$. This bias is likely to underestimate the contribution of $TFPQ$ to output growth relative to that of demand shocks.

construct the base price for plant f as: $\ln P_{fB_f} = \prod_{\Omega^f_{t,t-1}} \left(\frac{p_{fjB_f}}{\overline{p_{jB_f}}}\right)^{s_{fj}}$, where $\overline{p_{jB_f}}$ is the average price of product j in year B_f across plants, and year B_f is the first year in which plant f is present in the survey. Notice that this approach takes advantage of cross sectional variability across plants for any given product or input j. In the base year, $\left(\frac{P_{fjB_f}}{\overline{P_{jB_f}}}\right)$ will be normalized to one for the average producer of product j. For other plants, it will capture dispersion in price levels around that average.

We use output prices at the level of the plant to obtain a measure of the plant's output, by deflating the plant's revenue

$$Q_{ft} = \frac{R_{ft}}{P_{ft}} \tag{13}$$

In the baseline case, in which we use quality adjusted prices, we initially use the price indices based on the common $(t, t-1)$ common basket of goods, P^*. Since the Feenstra adjustment factor for changing baskets, $\left(\frac{\sum_{\Omega^f_{t,t-1}} s_{fjt}}{\sum_{\Omega^f_{t,t-1}} s_{fjt-1}}\right)^{\frac{1}{\sigma-1}}$, requires an estimate of the elasticiy of substitution which we lack initially, we consider it separately (see section 5).

We similarly obtain a measure of materials by deflating material expenditure by plant-level price indices for materials, P^M_{ft}. The index P^M_{ft} is constructed on the basis of information on individual prices and quantities of material inputs, using an analogous approach to that used to construct output prices. The underlying assumption is that M_{ft}, the index of materials quantities used, is a CES aggregate of individual inputs. P^M_{ft} is also one of the fundamentals we consider in our decomposition.

In an alternative approach against which we compare our baseline quality-adjusted prices, we examine the robustness of our results to using "statistical" price indices based on either constant baskets of goods, or on divisia approaches. These are discussed in section 7.

3.3 Life cycle growth

We focus on life cycle growth defined as $\frac{Q_{fa}}{Q_{f0}}$, where 0 is the year of start of operations for plant f and a is any age at which we observe the plant. We do not observe all plants from the actual time of their birth, though we do know what that actual year is. We thus proceed in the following way to estimate $\frac{Q_{ft}}{Q_{f0}}$:

Suppose B is the age of plant f when we first observe it in the survey. For variable Z ($Z = Y, L, TFPQ, etc$), we estimate size at age a relative to birth as:

$$\frac{Z_{fa}}{Z_{fB}} = \left(\frac{Z_{f,a}}{Z_{f,B}}\right)\overline{\left(\frac{Z_B}{Z_0}\right)}_{life\_cycle} \tag{14}$$

where the last term is an adjustment factor based on what we observe for the sample of plants that we do observe from birth, denominated the life cycle sample. That is, $\overline{\left(\frac{Z_B}{Z_0}\right)}_{life\_cycle}$ is relative Z at age B compared to birth averaged over all plants that we observe from birth. We conduct robustness analysis restricting the sample to that of plants observed from birth (the "life cycle sample"), for which we observe actual $\frac{Z_{fa}}{Z_{f0}}$.

If we just defined $\frac{Z_{fa}}{Z_{f0}} = \left(\frac{Z_{f,a}}{Z_{f,B}}\right)$ as our estimate of post entry growth for age a for plant f we would bias our estimate of actual growth up to a. Since at age $a = B$ the ratio $\left(\frac{Z_{f,a}}{Z_{f,B}}\right)$ is one, the presence of plants that we observe for the first time at age B biases our estimate of average post entry growth for age B towards one (which is, most frequently, downwards), with the size of this bias growing with the number of plants that appear for the first time in our sample at age B. Going to the alternative extreme of simply using $\left(\frac{Z_a}{Z_0}\right)_{life\_cycle}$ as an estimate of the true post-entry growth at age a would also be problematic. On the one hand, it reduces our numbers of observations to about a third of all of the plants that we observe, affecting the precision of our estimates. On the other hand, because the "life cycle" sample is a selected sample of the plants that are born sufficiently large to surpasss the inclusion threshold already at birth, these estimates are biased towards the post-entry growth of these selected samples, which may be faster or slower. By using equation (14) to estimate post-entry growth for plants that enter the sample after birth we expand our sample away from these selected plants.

4 Growth over the life cycle

We start by characterizing "outcome" growth over the life cycle of a manufacturing establishment (the left hand side of our growth decomposition). Our main outcome is output $Q_{ft} = \frac{R_{ft}}{P_{ft}}$. Because recent literature has focused on life cycle growth in terms of employment, we also describe employment growth for our sample for comparison with that literature.

4.1 Average life cycle growth

To characterize output growth for the average establishment in our sample, we estimate a full set of ϕ_{age} coefficients in equation:

$$\frac{Q_{ft}}{Q_{f0}} = \alpha_t + \alpha_s + \sum_{age=3}^{age=30+} \phi_{age} d_{age,f,t} + \varepsilon_{ft} \tag{15}$$

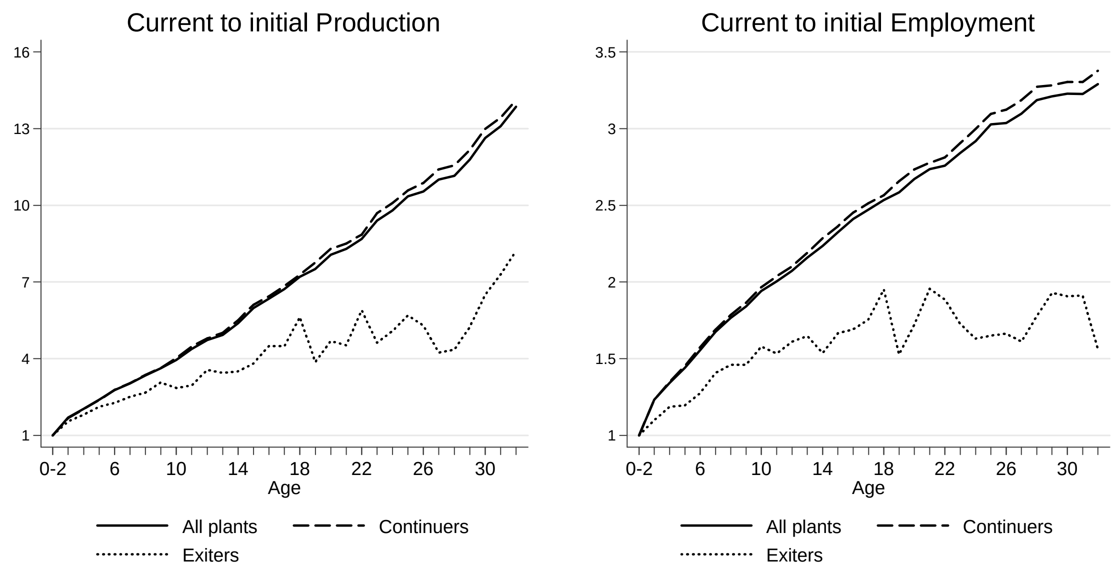
where $\frac{Q_{ft}}{Q_{f0}}$ is the ratio between plant $f's$ output level in year t and the level at plant's birth; $d_{age,f,t}$ is a dummy variable that takes the value of 1 if plant f is of age age in year t; and ε_{ft} is an estimation error. We control for (three-digit) sector effects and aggregate time effects. We define age as the difference between the current year, t, and the year when the plant began its operations, and define the plant's output level at birth Q_{f0} as the average output it reported in ages 0 to 2. By averaging over the first few years in operation we deal with measurement error coming, for instance, from partial-year reporting (if the plant, for instance, was in operation for only part of its initial year).

Since we have constructed deflators P^*_{ft} rather than P_{ft}, we actually estimate (15) using $\frac{R_{ft}}{P^*_{ft}}$ as dependent variable, and including $\lambda^Q_{ft} = \sum_B^t \ln \left(\frac{\sum_{\Omega^f_{l,l-1}} s_{fjt}}{\sum_{\Omega^f_{l,l-1}} s_{fjt-1}} \right)$, where B is the year in which we first observe plant f, as a regressor in the estimation. λ^Q_{ft} captures the ouput price index factor adjustment for changing baskets of products, and follows from equation (12) and the recursive construction of plant levels.

Figure 1, left panel, presents the coefficients associated with different ages in the estimated equation (15).[10] As in the rest of figures, we use a logarithmic scale. The average establishment in our sample grows by a factor of 2.4 in terms of production by age 5, almost four times from birth

[10]Estimation results for equation (15) are also shown in Appendix Table A1, though only up to age 12 to keep the table manageable.

Figure 1: Life Cycle Growth



Includes year and sector fixed effects

by age 10, and more than ten times by age 25.[11] The right panel presents analogous results for employment: $\frac{L_{ft}}{L_{0t}}$. By age 5 the average establishment has reached about 1.4 times its initial employment, by age 10 it has almost doubled, and 25 years after birth employment has grown three-fold.

Average growth over the life cycle is driven by both within-plant growth for surviving establishments and by exit. Figure 1 indicates that the average patterns described in the previous paragraph are driven by continuous plants (plants of age t that continue on to age $t+1$). Consistent with healthy market selection, plants that exit at age t (t plants that never reach age $t+1$) do exhibit less growth between birth and the age at which they exit than plants of the same age that go on to age $t+1$. However, the path of growth that we attempt to describe is mainly a continuers' story, as was the case in Hsieh-Klenow's (2014) investigation of life cycle growth in India, México and the US.

To give an idea of where these average patterns fit in the international

[11] More precisely, $\frac{Q_{fa}}{Q_{f0}} = 2.4$ when $a = 5$, $\frac{Q_{fa}}{Q_{f0}} = 3.95$ when $a = 10$, and $\frac{Q_{fa}}{Q_{f0}} = 10.35$ when $a = 25$. See Table $A1$.

spectrum, Figure 2 compares the cross sectional patterns of employment growth with US cross sectional patterns, calculating the two in an analogous manner, and including only manufacturing plants of 10 or more employees. US data is from the publicly available information in the Bureau of the Census' Business Dynamics Database, which shows average size for given age categories. The period is limited to 2002-2012, which is the time span for which we can assign age tags in the US data. The cross sectional version of life cycle growth, used for this graph, is calculated by dividing the average employment level of plants of a given age by the average size of plants at birth.[12] Results indicate that the growth speed of the average US establishment basically doubles that in Colombia for comparable manufacturing plants. For instance, in the US employment in the 16-20 age category more than doubles that of the 0-5 category, while for Colombia the analogous figure is 1.5 times. This is consistent with results in Hsieh and Klenow (2014) indicating that less developed economies are characterized by less dynamic post-entry growth.[13]

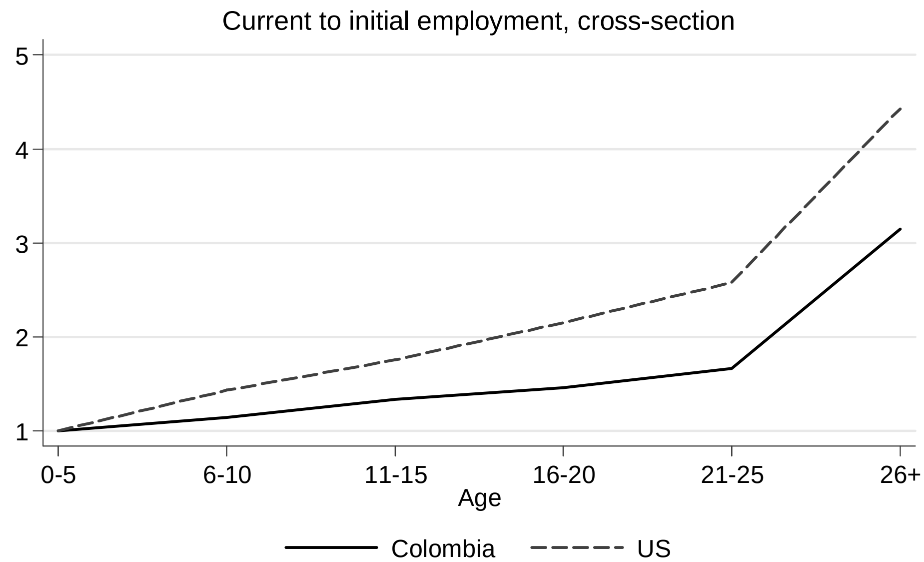
Hsieh and Klenow (2009) and Buera and Fattal (2014) attribute such cross-country differences to poor institutions in developing economies, that fail to encourage investments in productivity, as well as healthy market selection. Identifying the actual role that specific institutions play is an interesting area of future research. Within-country changes in institutions, either across businesses or over time (or both) offer a fruitful ground for such

[12]The estimated growth dynamics are considerably dampened in the cross sectional approach compared to the longitudinal one, which hints at the importance of being able to follow individual units longitudinally. Cross-sectional comparisons of cohorts, by contrast to our focus on $\frac{L_{ft}}{L_{f0}}$, implicitly give more weight to plants born larger, which results in the dampened cross-sectional dynamics for Colombia observed in Figure 2b compared to figure 2:

$$\frac{\overline{L_{age}}}{\overline{L_0}} = \frac{\sum_{i=1}^{N} L_{i,age}}{\sum_{i=1}^{N} L_{i,0}} = \frac{\sum_{i=1}^{N} \frac{L_{i,age}}{L_{i,0}} * L_{i,0}}{\sum_{i=1}^{N} L_{i,0}} = \sum_{i=1}^{N} \frac{L_{i,age}}{L_{i,0}} \frac{L_{i,0}}{\sum_{i=1}^{N} L_{i,0}}$$

[13]Though similar to Hsieh and Klenow's, our numbers for the US are not identical to theirs, even if we focused on the same year, because of several differences in the calculation. We use data from the Business Dynamics Statistics, which directly records the age of an establishment. It also records employment for establishments of all sizes. Meanwhile, Hsieh and Klenow impute age based on previous appearance in Census, and use imputed rather than observed employment for small businesses.

Figure 2: Employment over the life cycle of manufacturing plants
Colombian vs. the US, 2002-2012

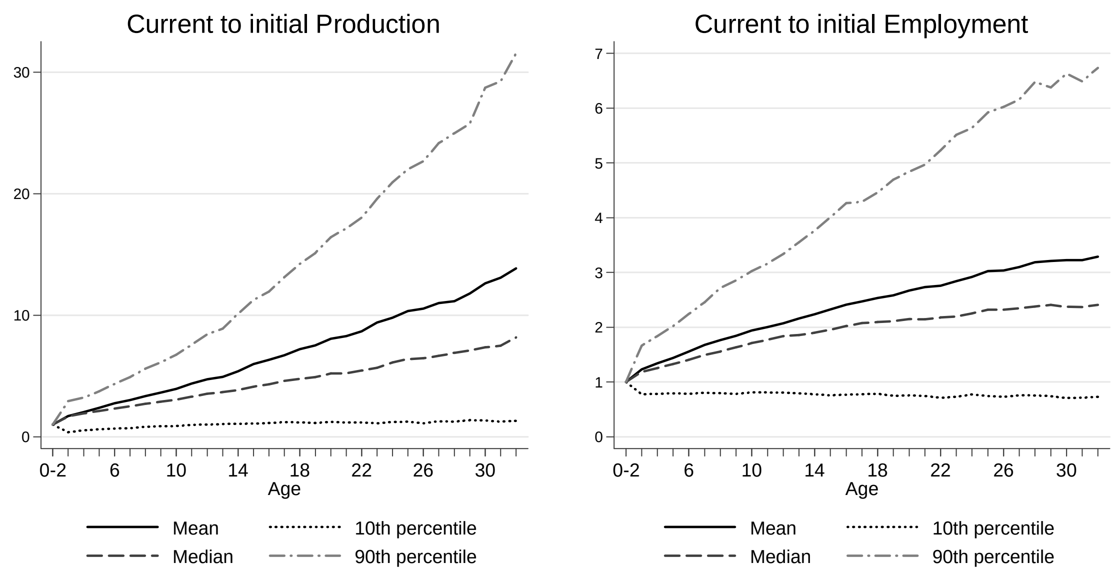


exploration, to the extent that they keep constant other factors potentially influencing business dynamics, from the macroeconomic environment to business culture. We undertake that exploration for Colombia, taking advantage of changes in import tariffs, in a separate paper.

The average growth dynamics described above, however, hide considerable heterogeneity. Figure 3 shows different moments of the distribution of life-cycle growth. For each age the figure depicts the respective moment of the distribution of $\frac{Q_{it}}{Q_{i0}}$ and $\frac{L_{it}}{L_{i0}}$. Some of those moments are also recorded in Table $A2$ in the appendix. Median growth falls under mean growth, highlighting the fact that it is a minority of fast-growing plants that drive mean growth. By age 5, while the average plant has multiplied its output at birth by a 2.4 factor, the plant in the 90th percentile has multiplied it by 3.75, the median plant by 2.1, and the plant in the 10th percentile has shrank to 60% of its original size. At age 10 the 90th percentile of life cycle similarly more more than doubles the median (6.78 rather than 3.1). Employment growth is also characterized by similarly wide dispersion.

Figure 4 further characterizes life cycle growht for other plant characteristics: revenue deflated by an industry level deflator, the capital stock,

Figure 3: Distribution of Life Cycle Growth



Includes year and sector fixed effects

purchases of material inputs, the share of non production worker and product scope. The capital stock and material inputs grow much faster than output and, especially, than employment (notice the different scales). This partly explains why output grows faster than employment: use of other factors is outgrowing that of labor inputs. At age 25 the real capital stock has multiplied by a factor of about 45 with respect to its level at birth for the average plant, and the use of material inputs is almost 20 times that of birth time. As noted, the corresponding figures for output and employment are close to 10 and 3.

Plants also seem to become more sophisticated as they age: both the share of non-production workers and the number of products grow over the life cycle. Ten years after starting operating, the average plant increases the number of 8-digit product categories in which it produces by about 30%. Skill composition also increases, at a slightly larger pace (1.6 times the birth level by age 10). Keep in mind, however, that the level of disaggregation of products in the Manufacturing Survey is insufficient to capture product introduction as captured, for instance, in bar code data (e.g. Hottman et al. 2016).

As with output and employment growth, there is wide dispersion and marked skewness in the patterns described by Figure 4. Mean growth overtakes median growth for all of the plant characteristics presented. At age 25, the 90th percentile of growth doubles the mean for all of the outcomes explored. Some plants also become less sophisticated as they age, as seen in a 10th percentile of life cycle growth below one in both the share of non production workers and product scope, even 25 years after birth.

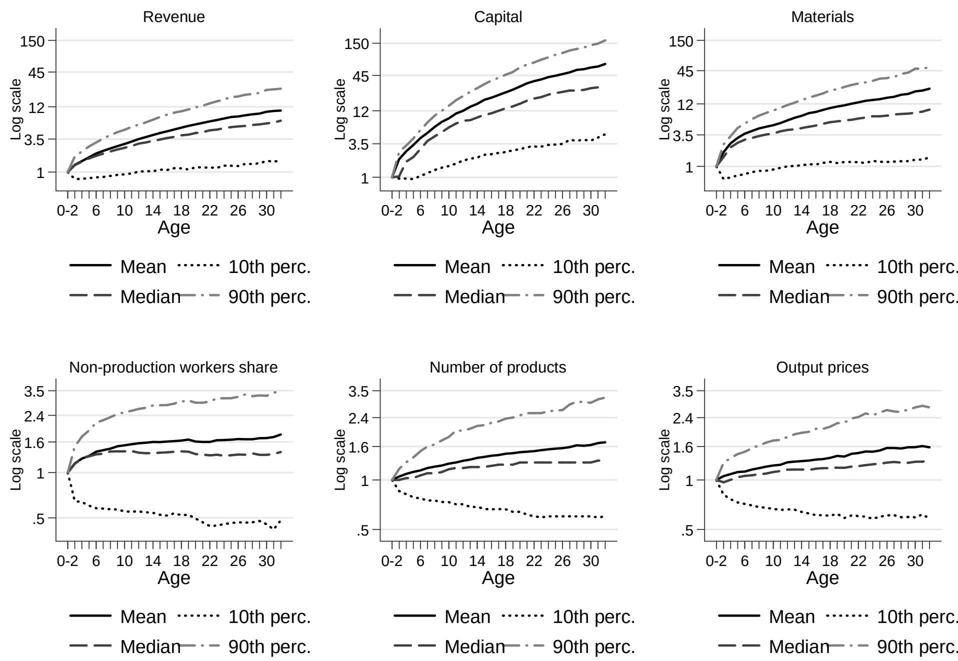
5 Estimation strategy

5.1 Decomposing firm level growth

This section explains our approach to estimating the fundamental dimensions into which we then decompose output and input growth. A key feature of our analysis is the availability prices and quantities sold at the product-firm level, constructed as explained in section 3.2. Taking advantage of this key feature, we proceed sequentially in the following way:

Figure 4: Distribution of growth: different plant characteristics

Current to inital



5.2 Physical productivity

With physical quantities of output (materials) constructed by deflating nominal output (material expenditures) by the plant-level output price index, and direct reports of the number of employees and the stock of physical capital, we estimate plant total $TFPQ$ as the (log) residual of production function (1) $Q_{ft} = A_{ft}X_{ft}^{\gamma}$,where $X_{ft} = K_{ft}^{\frac{\alpha}{\gamma}}L_{ft}^{\frac{\beta}{\gamma}}M_{ft}^{\frac{\phi}{\gamma}}$. A usual concern in the literature is that researchers observe values, rather than quantities, of output and inputs, so that the residual from estimating the (revenue) production function cannot be interpreted as A_{ft} (or $TFPQ$). The fact that we use theory-based plant-level prices to deflate output and materials deals with this concern. We estimate the log production function:

$$\ln Q_{ft} = \alpha \ln K_{ft} + \beta \ln L_{ft} + \phi \ln M_{ft} + \ln A_{ft} \tag{16}$$

$Q_{ft} = \left(\frac{R_{ft}}{P_{ft}}\right)$, but we have an estimate of P_{ft}^{*} rather than P_{ft}. Equation (12) and the recursive construction of plant level prices imply $\ln P_{ft} = \ln P_{ft}^{*} + \frac{1}{\sigma-1}\sum_{B}^{t} \ln \left(\frac{\sum_{\Omega_{l,l-1}^{f}} s_{fjt}}{\sum_{\Omega_{l,l-1}^{f}} s_{fjt-1}} \right)$, where B is the year in which plant f is initially observed in the survey. We rely on these facts and estimate the coefficients of 16 by estimating:

$$\ln \left(\frac{R_{ft}}{P_{ft}^{*}} \right) = \alpha \ln K_{ft} + \beta \ln L_{ft} + \phi \ln M_{ft} + \rho\lambda_{ft}^{Q} + \varphi\lambda_{ft}^{M} + \ln A_{ft} \tag{17}$$

where $\lambda_{ft}^{Q} = \sum_{B}^{t} \ln \left(\frac{\sum_{\Omega_{l,l-1}^{f}} s_{fjt}}{\sum_{\Omega_{l,l-1}^{f}} s_{fjt-1}} \right)$ captures the ouput price index factor adjustment for changing baskets of products. λ_{ft}^{M} is the analogous adjustment factor for the prices of materials, where $M_{ft} = \frac{\text{materials expenditure}}{P_{ft}^{*M}}$. See section 3.2 for details.

A remaining concern in estimating the production function is simultaneity bias: the fact that X is chosen as a function of the residual A_{ft}. We estimate the production function for each two-digit sector, using proxy methods. In particular, we follow the approach proposed by Ackerberg, Caves and Frazer (2015, ACF henceforth). Our control function includes lagged materials, current employment, current capital, plant input prices, λ^{Q}_{ft-1} and λ^{M}_{ft-1}.[14] Our inclusion of plant input prices follows De Loecker et al (2015), though in our case we take advantage of both output and input plant level prices as deflators.[15] We also plan to examine the robustness of our results to other methods for determining factor elasticities, including using a cost share approach. We plan to further examine robustness to using Ghandi, Navarro and Rivers (2013) approach to the estimation of production functions. We have already examined the distributions of $TFPQ$ estimates under alternative methods, finding high correlation between production function residuals from these different estimation approaches.

We obtain $\widehat{\ln A_{ft}}$ as a residual from this estimation. We then use $\widehat{A_{ft}} = \exp\left(\widehat{\ln A_{ft}}\right)$ as our estimate of total $TFPQ$. Table 2 presents the results of our estimation of the production function, carried at the two-digit level of ISIC revision 2. We estimate returns to scale in the range 0.9 to 1.1 , and a coefficient for labor that tends to double that of capital. In most sectors, the coefficient for materials is estimated to be above 0.5.

5.3 Demand shocks

Our (log) demand shock, $\ln D_{ft}$, is the residual from estimating the demand function (8)

$$\ln P_{ft} = \alpha - \varepsilon \ln Q_{ft} + \ln D_{ft} \tag{18}$$

Estimating (18) by OLS would yield a biased estimate of $\varepsilon = \frac{1}{\sigma}$, to the extent that $Q\left(= \frac{R_{ft}}{P^{*}_{ft}}\right)$ is correlated with the residual price for reasons

[14] Declaring labor, besides materials, as a free input, yields somewhat unplausible results for some sectors. Under that assumption, returns to scale are frequently (i.e. for several sectors and some periods of estimation) estimated to be increasing, and the coefficient for labor shoots up. Such implausible results support our prior that treating labor as a free input is not appropriate in the context in which we carry our estimation.

[15] De Loecker et al (2015), deflate output but not inputs using plant-level deflators. This induces biases that they address by including plant level prices in their control function. Though we do make use of plant-level materials prices, a bias may persist from the lack of access to plant-level capital deflators. Moreover, the inclusion of additional information in the control function helps deal with the concern that proxy methods based on a materials as single invertible proxy may fail to properly identify the coefficient for materials in a gross output production function (Ghandi et al, 2013). We therefore include our plant-level prices in the control function.

Table 2: Estimated factor and demand elasticties

Sector	β (L)	α (K)	ϕ (M)	Returns to scale	σ
Overall	0.372	0.173	0.508	1.053	1.403
31	0.296	0.139	0.614	1.049	2.066
32	0.265	0.120	0.609	0.994	1.650
33	0.360	0.154	0.471	0.985	1.321
34	0.629	0.273	0.198	1.101	1.318
35	0.437	0.211	0.464	1.112	1.332
36	0.449	0.187	0.438	1.074	1.613
37	0.437	0.223	0.461	1.121	1.348
38	0.368	0.109	0.585	1.061	1.166
39	0.330	0.180	0.503	1.013	1.582

This table reports estimates of the factor elasticities in the production function, and the demand elasticity in the demand function. The production function is estimated following ACF methods, with lagged materials, current employment, current capital, and plant input prices in the control function. The demand function is estimated using IV methods and TFPQ as an instrument for Q. Sectors are classified at the two digit level of the ISIC classification, revision 2.

beyond demand shocks. We thus estimate this demand function using IV methods. In particular, we use the log physical productivity shock $\ln A_{ft}$ as an instrument for the plant's output, as in Foster et al (2008, 2016) and Eslava et al (2013). In our case, however, we explicitly recognize the multi-product character of plants by using a theory-based price index that quality-adjusts (or "appeal"-adjusts) our measure of output. To the extent that our approach does appropriately deal with this adjustment, $TFPQ$ obtained as a residual from this adjusted Q measure should be stripped of apparent changes in productivity related to appeal changes. This adjustment is thus crucial for the exclusion restriction to hold when using $TFPQ$ as an instrument to estimate (18).

By using $\ln A_{ft}$ as an instrument we focus on pure demand: the variability that is orthogonal to supply side shocks. The estimate that we recover for ε_{it} is an unbiased estimate of the ability of the firm to charge a different price when observing a shock to its sold quantity that is unrelated to the efficiency of its production process. Orthogonality between demand and supply shocks is also the main identifying assumption in the estimation of elasticities of substitution in research that uses data on product prices and sales (Broda and Weinstein, 2006; Redding and Weinstein 2016; and Hottman et al. 2016).

Because we have estimated P^*_{ft} rather than P_{ft}, we actually estimate

$$\ln P^*_{ft} = \alpha - \varepsilon \ln Q_{ft} + \mu \lambda^Q_{ft} + \ln D_{ft} \tag{19}$$

rather than (18). We note that λ^Q_{ft} may be endogenous in this estimation. We currently work on modifying this estimation to deal with the potential endogeneity of λ^Q_{ft}. For the time being, we discuss further below the robustness of our results to using different price indices.

The last column of Table 2 reports obtained estimates of the elasticity of demand, $\frac{1}{\varepsilon} = \sigma$. We estimate elasticities between 1.2 and 2.1 for the different sectors. We obtain our demand shock as $\widehat{D_{ft}} = \exp\left(\widehat{\ln D_{ft}}\right)$, where the latter is the estimation residual from (18).

The lower panel of Table 1 presents basic descriptive statistics for our estimates of $\ln A_{ft}$ and $\ln D_{ft}$ on a log basis. As found by Eslava et al. (2013) for an earlier period, $TFPQ$ is negatively correlated with output prices, which is intuitive to the extent that more efficient production allows charging lower prices. Both $TFPQ$ and D are positively correlated with plant size (captured in the table by output). $TFPQ$ is highly correlated with $TFPR$. By construction, our estimate for $\log D_{ft}$ captures only the part of the price effect that is uncorrelated with $TFPQ$, so the correlation between $\ln TFPQ$ and $\ln D$ is zero.

6 Results: Decomposing growth into fundamental sources

To start, the distributions of the evolution over the life cycle of these plant fundamentals, measured by $\widehat{A_{ft}}$, $\widehat{D_{ft}}$ over the life cycle are displayed in Figure 5, which also shows the life cycle growth of output and input price indices. As we did before for employment, for a given variable Z each figure depicts statistics for $\frac{Z_{fa}}{Z_{f0}}$ for different ages a, on a log scale.

The average growth of demand shocks over the life cycle dominates that of $TFPQ$, as well as that of material input prices (notice the different scales). While, for the average plant, demand shocks grow more that three-fold over a thirty year period, compared to the level at birth, $TFPQ$ and unit prices of both input and output grow by less than 50% over the same horizon. The interplay between output prices (Figure 4) and demand shocks is particularly interesting: with growing output over the life cycle, downward sloping demand would imply that the plant would have to charge ever shrinking prices over its life cycle, unless the appeal of f to constumers changed over time. We do not observe such fall in output prices, signaling increasing ability of the firm to sell more at given prices. By construction, this is what the life cycle growth of the demand shock, $\widehat{D_t}$, captures.

There is also considerable dispersion in the growth of both TFPQ and demand shocks. The 90th-10th gap, however, is much wider (on a log basis) for demand shocks. Moreover, even the 10th percentile grows modestly in the case of the demand shocks, while $TFPQ$ falls markedly for the 10th percentile.

We now decompose the variance of $\frac{Q_{fa}}{Q_{f0}}$ into contributions associated with different fundamental sources, most notably $TFPQ$ and demand shocks (equation 11). We run a two stage procedure, similar to that in Hottman et al. (2016):

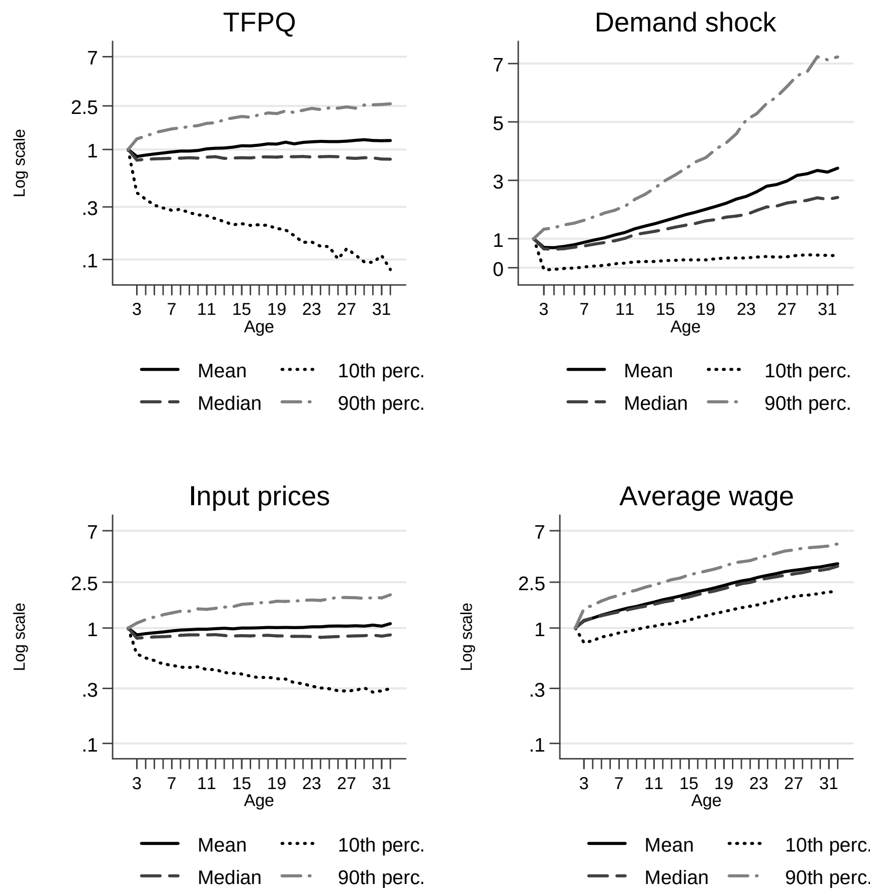
1. We estimate

$$\ln \frac{Q_{fa}}{Q_{f0}} = \beta_D \ln\left(\frac{D_{fa}}{D_{f0}}\right) + \beta_A \ln\left(\frac{A_{fa}}{A_{f0}}\right) + \beta_M \ln\left(\frac{pm_{fa}}{pm_{f0}}\right) + \beta_w \ln\left(\frac{w_{fa}}{w_{f0}}\right) + \upsilon_{fa}$$

where we include sector and time effects, so that $\upsilon_{at} = \upsilon_t + \upsilon_s + \upsilon'_{at}$. and coefficients β_D, β_D, β_M and β_w capture the contribution of idiosyncratic growth of demand shocks, TFPQ, material input prices and wages. As made clear by 11 $\widehat{\upsilon_{at}}$ captures distiortions, beyond aggregate effects and noise.

2. We then estimate the following equations:

Figure 5: Distribution of Fundamentals
Current to initial



Includes year and sector fixed effects

$$
\begin{aligned}
\beta_D \left(\frac{D_{fa}}{D_{f0}}\right) &= \rho_D \frac{Q_{fa}}{Q_{f0}} + \nu_{fa,D} \\
\beta_A \left(\frac{A_{fa}}{A_{f0}}\right) &= \rho_A \frac{Q_{fa}}{Q_{f0}} + \nu_{fa,A} \\
\beta_M \left(\frac{pm_{fa}}{pm_{f0}}\right) &= \rho_M \frac{Q_{fa}}{Q_{f0}} + \nu_{fa,M} \\
\beta_w \left(\frac{w_{fa}}{w_{f0}}\right) &= \rho_w \frac{Q_{fa}}{Q_{f0}} + \nu_{fa,w} \\
\widehat{\upsilon_{fa}} &= \rho_\upsilon \frac{Q_{fa}}{Q_{f0}} + \nu_{fa,\upsilon}
\end{aligned}
$$

by the properties of OLS, $\rho_D + \rho_A + \rho_M + \rho_w + \rho_\upsilon = 1$.

As with the estimation of demand and $TFPQ$ we deal with the fact that we are using P^*_{ft} rather than P_{ft} by including λ^Q_{ft} and λ^M_{ft} as additional factors in each stage of the decomposition.

The first bar in Figure 6 depicts the result of this decomposition, pooling across ages a, and reporting $(\rho_M + \rho_w)$ together to simplify the figure. We find that dispersion in life cycle growth is mostly explained by dispersion in fundamentals, rather than distortions and other unobserved factors. Measured fundamentals explain around 67% of the variability of output relative to birth level pooling across ages. The remaining 33% is explained by the error term χ_{ft} which, from equation (11), captures distortions and other unobserved factors, such as measurement error.[16]

Concentrating on the fraction explained by measured factors, input price growth explains 7 p.p, with the remaining variability accounted for by demand shocks and $TFPQ$, with each of them accounting for approimately 30 p.p. That is, demand shocks and $TFPQ$ are both equally important drivers of life cycle growth.

Both the contribution of fundamentals relative to distortions, and the relative contribution of different fundamentals within the former, vary markedly

[16] In particular, $\chi_{ft} = \left(\frac{(1-\tau_{ft})^{\kappa_1} \left(1+\tau^M_{ft}\right)^{-\phi\kappa_1} \left(1+\tau^L_{ft}\right)^{-\beta\kappa_1} \delta_{ft}^{\kappa_1} \alpha_{ft}^{1+\kappa_2} \zeta_{ft}^{-\kappa_1} r_{ft}^{\frac{-\alpha\kappa_1}{\gamma}}}{(1-\tau_{f0})^{\kappa_1} \left(1+\tau^M_{f0}\right)^{-\phi\kappa_1} \left(1+\tau^L_{f0}\right)^{-\beta\kappa_1} \delta_{f0}^{\kappa_1} \alpha_{f0}^{1+\kappa_2} \zeta_{ft}^{-\kappa_1} r_{f0}^{\frac{-\alpha\kappa_1}{\gamma}}} \right)^{\gamma}$. The aggregate growth term $\chi_t == \kappa_t^\gamma \left(\frac{A_t}{A_0}\right)$ is absorbed by time effects.

Figure 6: Production growth variance decomposition

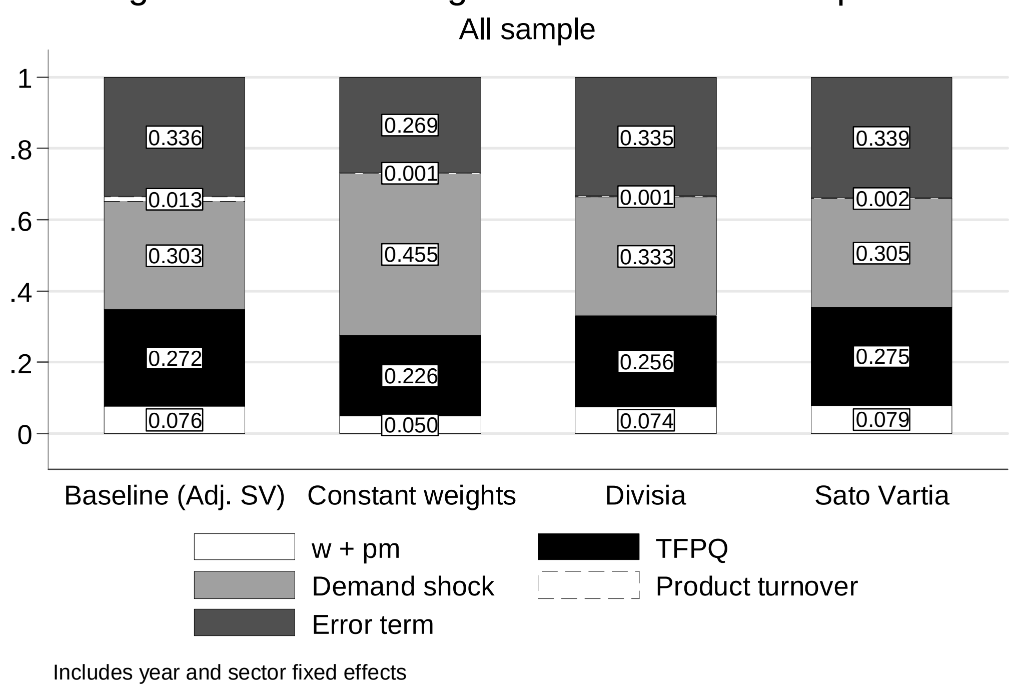
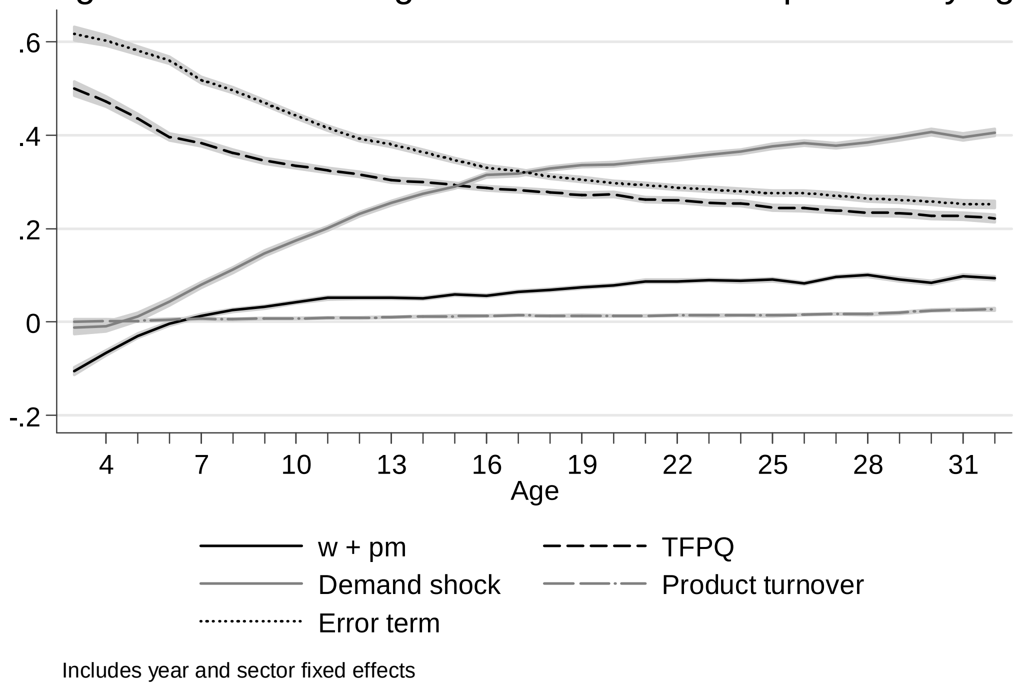


Figure 7: Production growth variance decomposition by age



over different time horizons from birth (Figure 7, with shades representing confidence intervals). Distortions (dotted line) are particularly important to explain early age growth, explaining more than 50% of the variance of growth up to age seven. But, they lose in importance for longer horizons, with their contribution falling to less than 25% by around age 20. For the fraction explained by fundamentals, variance in early life growth is fully explained by $TFPQ$. But the contribution of demand grows rapidly over the life cycle, growing to around 20% by age 10 and to close to 40% by age 20. Since the fraction explained by $TFPQ$ and distortions follows the opposite pattern, falling over the life-cycle, the importance of demand surpases that of $TFPQ$ and unobserved factors by around age 15.

As is probably not surprising, the growth of input prices (both for materials and employment) plays a more minor role than any of the other factors in explaining life cycle growth. Still, their contribution is 7% for the sample of pooled ages, growing from negative to almost 10% for the older ages.

This is a dimension not previously explored as a driver of post entry growth. For instance, Hsieh and Klenow (2014) assume that all plants face the same input prices, an assumption under which all marginal revenue dispersion is atributable to distortions. Our finding that the role of life cycle input price growth is non negligible suggests that, at least in the context of life cycle growth, dispersion in marginal products is not solely driven by distortions. In our data, it is the growth of wages, as opposed to material input prices, that drives most of this contribution.

Our findings in Figures 6 and 7 point to a minor importance of the λ_{ft}^{Q} and λ_{ft}^{M} adjustment factors for product and input turnover. This is probably a reflection that product turnover is less meaningful in the context of auto-reported eight digit ISIC categories than in studies where researchers have access to information on detailed product categories from, for instance, scan bar codes (e.g. Hottman et al, 2016).

Figure 8 directly depicts the different terms in decomposition (11) for different sections of the distribution of output growth, as well as actual output growth. The upper left panel shows the components of the decomposition for plant with average $\frac{Q_{fa}}{Q_{f0}}$. The upper right panel, meanwhile shows each of this components at age a for plants in the lowest decile of $\frac{Q_{fa}}{Q_{f0}}$. The two lower panels proceed similarly for the 45th to 55th percentile and the upper decile of $\frac{Q_{fa}}{Q_{f0}}$.

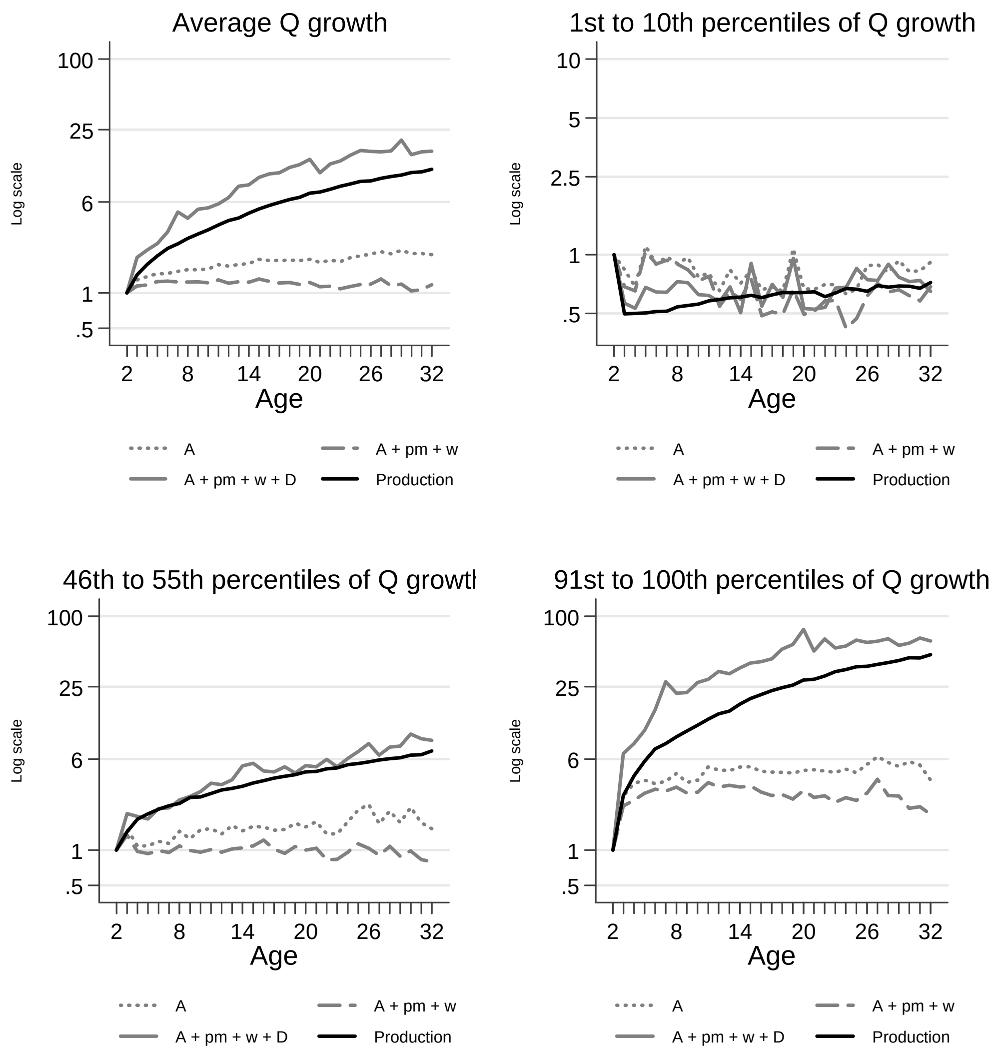
The black solid line represents actual outcome growth, $\frac{Q_{fa}}{Q_{f0}}$.The dotted grey line in each of the panels corresponds to $\left(\frac{a_{fa}}{a_{f0}}\right)^{1+\gamma\kappa 2}$, while the dashed line adds the (generally negative) contribution of input prices by depicting $\left(\frac{a_{fa}}{a_{f0}}\right)^{1+\gamma\kappa_1}\left(\frac{pm_{fa}}{pm_{f0}}\right)^{-\phi\kappa_1}\left(\frac{w_{ft}}{w_{f0}}\right)^{-\beta\kappa_1}$. The solid grey line further adds the contribution of demand shocks: $\left(\frac{d_{fa}}{d_{f0}}\right)^{\gamma\kappa_1}\left(\frac{a_{fa}}{a_{f0}}\right)^{1+\gamma\kappa_1}\left(\frac{pm_{fa}}{pm_{f0}}\right)^{-\phi\kappa_1}\left(\frac{w_{ft}}{w_{f0}}\right)^{-\beta\kappa_1}$. The difference between this solid grey line and the solid black line is the contribution of unmeasured factors, χ_{ft}. Aggregate effects, χ_t are soaked up, as are sector effects.

It is clear from Figure 8 that, as previously illustrated, both TFPQ and demand shocks play crucial roles. But the Figure also suggests that demand is particularly important to explain the dynamism of high growth plants.

The central finding that idiosyncratic demand shocks play a key role in the growth of output, at least as important as that of $TFPQ$ and especially for

Figure 8: Contribution of fundamentals to production growth

Current to initial



rapid growers, is in line with Foster et al's (2016) argument that consolidating a solid client basis is more central to post-entry business growth than physical efficiency gains, and their consistent results for selected US manufacturing industries. It also squares well with the findings in Hottman et al (2016) pointing at demand shocks as a major determinant of sales variability in the US.

7 Robustness

7.1 Alternative price indices

All of the results discussed so far use exact price indices calculated as explained in section 3.2. We examine now robustness of our results to two different alternative formulations of the price index. We first use a "statistical" approach based on Tornquist indices for a constant basket of goods or, alternatively, on the divisia price index that allows that basket to change and uses average t, $t-1$ expenditure shares (see section 3.2). In each of these approaches, $TFPQ$ and demand shocks are obtained from estimating production and demand functions 16 and 18, rather than their versions adjusted for product turnover, 17 and 19. Product turnover is only considered as a separate factor in the growth decompostion. For comparison, we also run a version of the decomposition where $TFPQ$ and demand shocks are obtained without product adjustments but output is deflated using P^*_{ft} (the Sato-Vartia prices).

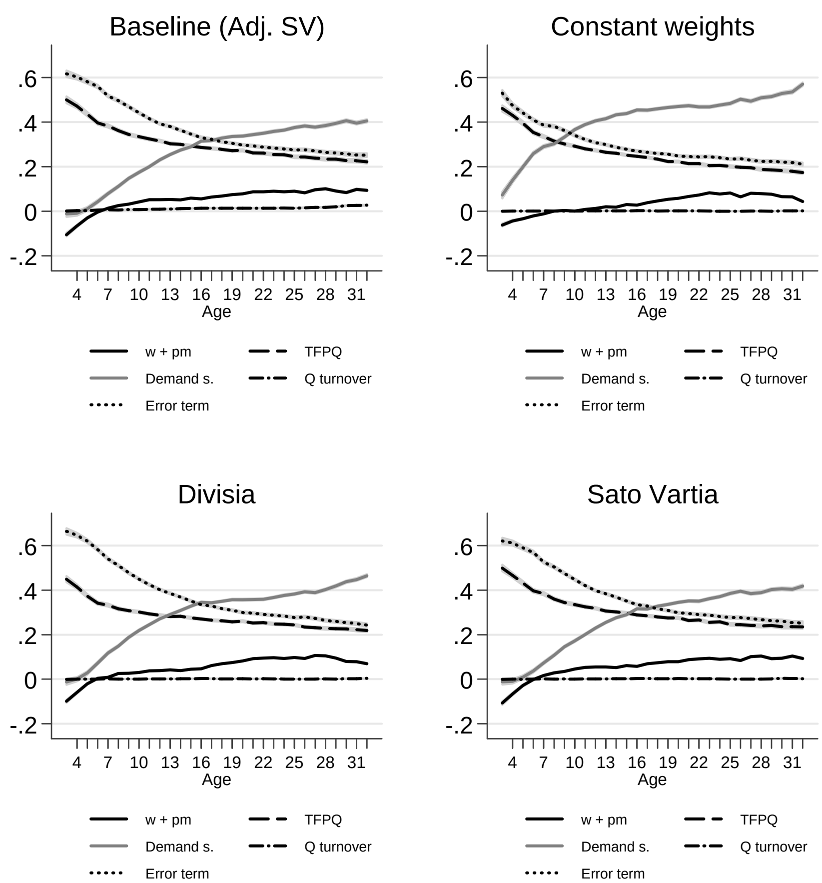
Our first alternative version of prices uses a basket of goods that is fixed over the life cycle, and constant weights for them. In particular, Tornqvist indices for the growth of prices of plant f at time t are constructed, as $\frac{P_{ft}}{P_{ft-1}} = \prod_{\Omega^f} \left(\frac{p_{fjt}}{p_{fjt-1}}\right)^{\overline{s}_{fj}}$, where Ω^f is a basket of all products produced (or materials used) by plant f at any point in which we observe f, and $\overline{s}_{fj}$ is the average share of j in that basket of products (or materials) plant f produces over the whole period. In this approach, the plant level index is initialized at $\ln P_{fA} = \sum_{\Omega^f} s_{fj} \left(\ln p_{fjA} - \ln \overline{p_{jA}}\right)$. If product j is not produced (or used as input) in years t or $t-1$ (or both), $\Delta \ln(P_{fjt})$ is inputed at the average growth of the price of that product (or input) for other plants within the sector. If no plant in the sector produces that good in t, then the average over all plants is used, independent of sector. Notice that this version of prices does not quality-adjust prices in any way, and does not take into account product turnover. Compared to this version, all versions allowing for evolving baskets of goods have the advantage of capturing evolving expenditure shares over time and therefore quality-adjusting prices, but the disadvantage of being more biased by errors from product coding and coarse aggregation, which more likely in our context than in that of prices from scan bar codes (Hottman et al 2016).

The second bar of Figure 6 presents results using this alternative approach, pooling observations across ages. Ignoring quality (more precisely, appeal) adjustment results in three important differences in the results using a fixed basket of goods over the life cycle, compared to our baseline, for the pooled sample (Figure 6, bars "constant weights" vs baseline). First, the error term is assigned a contribution 7 p.p. smaller in the version with a constant basket of goods, possibly reflecting the fact, mentioned above, that this version is less prone to measurement error from product coding error and aggregation. Second, the contribution of product turnover (to quality-unadjusted output) falls to about a tenth. The mechanical reason is that product turnover is mechanically confounded with $TFPQ$ shocks and demand shocks when prices are not adjusted for such turnover. Third, and most importantly, demand appears twice as important as $TFPQ$. That is, ignoring quality adjustments does lead to an overestimation of the role of physical efficiency in life cycle growth. Figure 9 shows that these three features are prevalent also when comparing the "constant weights" price version to our baseline for different time horizons (the upper left panel of Figure 9 simply reproduces Figure 7). Boh versions, however, displays the same patterns of increasing importance of demand and decreasing importance of $TFPQ$ and the error term.

The other versions of prices (divisia and Sato Vartia) progressively move towards our baseline. In the divisia version we consider the current basket at each period t, and use average $t, t-1$ shares of sales (materials expenditure) as weights for the respective products. Divisia prices do allow for changing baskets of goods over the life cycle, but the weights given to different products are not exact, in the sense of not capturing the exact shares derived from our demand theory. Sato-Vartia prices, in turn, ignore the product turnover adjustment (λ_{ft}^{Q} and λ_{ft}^{M}, which capture turnover over pairs of years) in the estimation of $TFPQ$ and demand shocks. Not surprisingly, then, the results of decompositions based on these alternative approaches stand between those

Figure 9: Production growth variance decomposition

Different price indices



Includes year and sector fixed effects

in our baseline and those in the constant weights version of prices. The divisia price index, by failing to quality-adjust prices in an exact manner, attributes more importance to demand relative to $TFPQ$ than our baseline. In turn, the Sato Vartia price index fails to assign a role to product turnover (also a demand factor).

7.2 Life cycle growth: 1980s vs 2000s

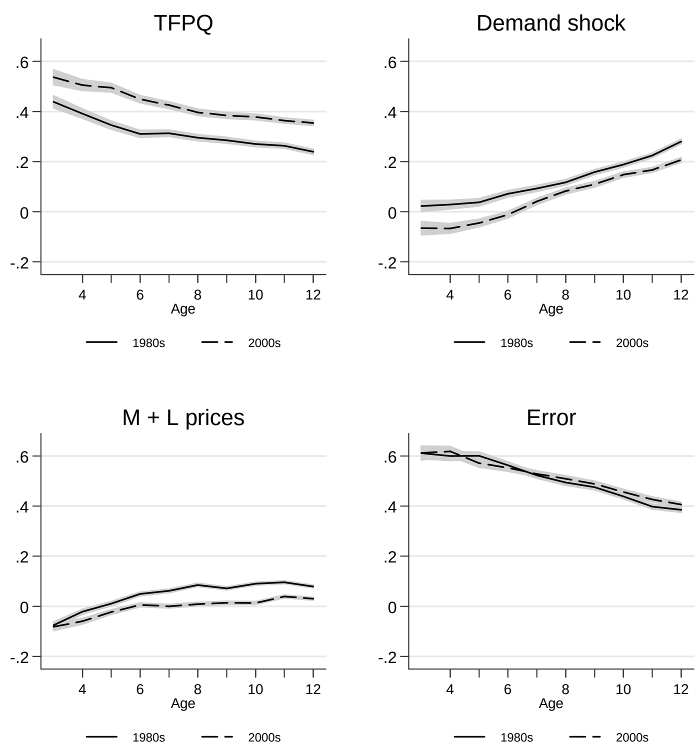
Hsieh and Klenow (2014) have explained cross country differences in patterns of life cycle growth from differences in institutions that may enhance or weaken the link between market fundamentals and growth, in turn affecting market selection and incentives to invest in improving those "fundamentals".

Colombia, as many other countries in Latin America and around the globe, undertook wide market-oriented reforms during the 1990s. These included unilateral trade openning, financial liberalization, and flexibilization of labor regulations. Eslava et al (2004, 2013, 2010) present evidence that these reforms did generate changes in business dynamics consistent with a reduction in the distortions to business incentives: allocative efficiency improved, the market selection mechanism was enhanced, capital and labor adjustment became more flexible (though in an apparently capital-biased way). We now ask whether there are noticeable differences in the contribution of our measured plant fundamentals to life cycle dynamics for Colombian manufacturing plants, and in turn whether life cycle growth seems affected by these differences.

Figure 10 depicts results of our decomposition of life cycle growth separating the 1980s (actually 1982-1992) and the 2000s (2002 to 2012). We leave out the years between 1993 and 2001 for two reasons: 1) Two the extent that the comparison provides meaningful information about changes related to the reforms, it is not clear whether firms born just as the reforms are being adopted behave in their first few years as pre-reform or post-reform firms. 2) Between 1997 and 2001, the country went through its deepest recession in 70 years. The 2002-2012 period displays macroeconomic conditions that are not as widely different from those in the first decade of our sample as is the case for the years between 1993 and 2001. Still, of course, market reforms are not the only conditions that changed between the 1980s and the 2000s. We do not claim that the differences between the two decades in life cycle growth and in the contribution of fundamentals to life cycle growth are to be attributed to the reforms of the 1990s. But those reforms are clearly an

Figure 10: Production growth variance decomposition

1980s vs. 2000s



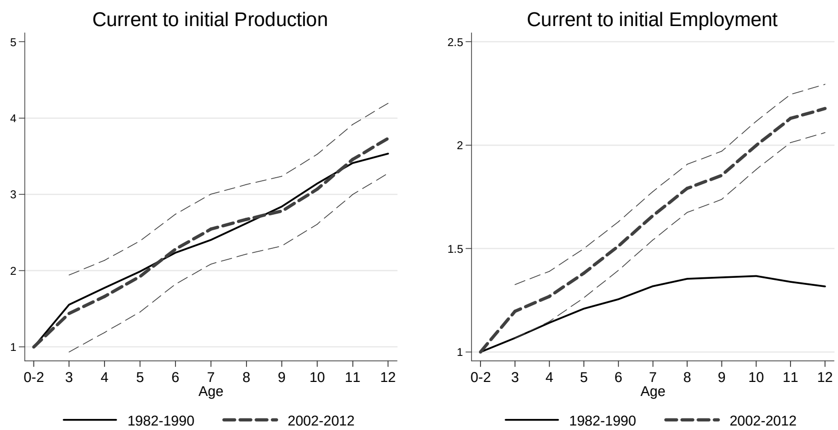
Includes year and sector fixed effects

important factor.

The contribution of $TFPQ$ to the life cycle output growth grows by almost 10% betwen the 2000s and the 1980s, for all horizons. This greater contribution of $TFPQ$ is basically offset by a composed decrease in that of demand shocks and input prices. As a result, the fraction of growth variability explained by the error terms remains roughly constant.

Figures 11 and 12 further depict output and employment life cycle growth in the 1980s vs the 2000s. For Figure 9, we pool the two samples, and run the regression

Figure 11: Life Cycle Growth: 1980s vs. 2000s



Includes year and sector fixed effects, 90% confidence interval for the difference pre vs. post

$$\frac{Q_{ft}}{Q_{f0}} = \alpha_t + \alpha_s + \sum_{age=1}^{age=10} \phi_{age} d_{age,f,t} + \sum_{age=1}^{age=10} \phi_{age,post} d_{age,f,t} * dpost_t + \varepsilon_{it} \quad (20)$$

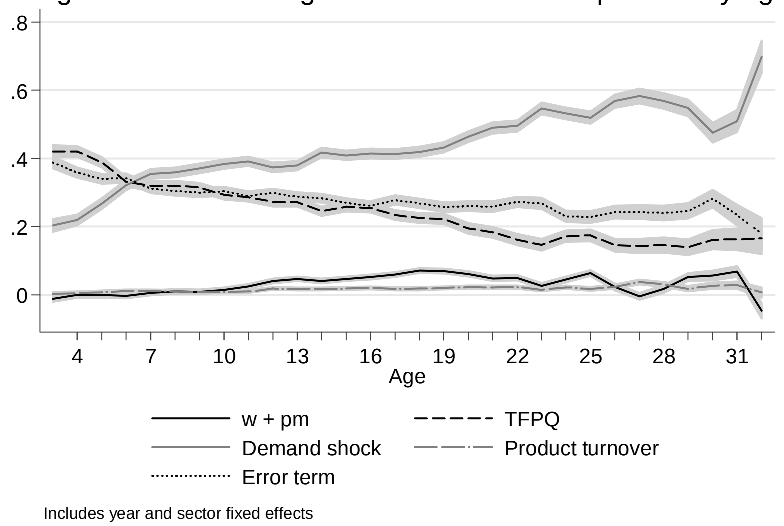
where $dpost_t = 1$ if $t \in [2002, 2012]$ and $dpost_t = 0$ otherwise.

The 2000s exhibit much more dynamic employment growth over the life cycle. The whole distribution of employment growth, not only its average, shifts to the right in the 2000s with respect to the 1980s. To the extent that market reforms in the 1990s are driving the contrasting patterns of life cycle employment growth between the two decades, these results are consistent with Hsieh and Klenow's (2014) finding of faster employment growth in the US as compared to India and Mexico, and their argument that poor market institutions drive less dynamic growth for the two latter. For output, however, significant difference between the two periods is observed.

7.3 Life cycle sample

Figure 12 reproduces the decomposition by ages of Figure 7 for the life cycle sample, composed of plants that we observe from birth. Those plants represent about a third of the overall sample (Table 1). The life cycle sample has the advantage of allowing a precise measurement of growth from birth,

Figure 7: Production growth variance decomposition by age



both in terms of outcomes and in terms of fundamentals. This comes at the cost of biasing the sample towars firms that are born larger. We do not know ex-ante what the direction of that bias is, neither for the patterns of outcome growth nor for the contribution of fundamentals to that growth.

The basic patterns we have found are robust of focusing on the life cycle sample: 1) the unexplained part of growth, which captures the role of distortions, is less important than the explained part, and becomes less important for longer horizons; 2) the role of both $TFPQ$ and demand is crucial, explaining most life cycle growth; 3) $TFPQ$ is particularly important for early life growth, but quickly becomes less important than demand, whose contribution grows markedly for longer horizons. Two important quantitative differences emerge with respect to the full sample, however: 1) the measured contribution of distortions falls (from 33.6% to 29.6% in the pooled sample); and 2) the weight of demand shocks relative to $TFPQ$ grows. In particular, while $TFPQ$ and demand are equally important in the baseline decomposition pooling ages, the contribution of demand shocks is 1.5 times that of $TFPQ$ in the life cycle sample pooling ages.

8 Conclusion

We take advantage of rich microdata on Colombian manufacturing establishments to decompose growth over an establishment's life cycle into that attributable to fundamental sources of growth–physical productivity, demand shocks, and input prices–and distortions that weaken the link between those fundamentals and actual growth. We rely on a nested CES structure for preferences over products by multiproduct businesses, and data on quantities and prices for individual products for each manufacturing establishment, to decompose profitability shocks into physical productivity and demand shocks. Pooling all ages, measured fundamentals explain around 67% of the variability of output relative to birth level, with the remaining 33% explained by distortions and other unobserved factors. Demand shocks and $TFPQ$ are equally important in the explained part, while input prices play a more minor role. Distortions explain more than 50% of growth up to age seven, but their contribution falls to less than 25% by around age 20. For the fraction explained by fundamentals, early life growth is explained by $TFPQ$ with demand and input prices playing a minor role. But demand is the crucial factor in long-run growth, with a contribution that surpases that of $TFPQ$ and unobserved factors by around age 15. In the 2000s compared to the 1980s, two decades separated by a wave of deep structural reforms, the contribution of $TFPQ$ to the variance in life cycle growth grows by around 10 p.p , with demand and input prices falling in importance. Interestingly, that of distortions remains basically constant.

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10 Appendix A: measuring exact price indices

The change in prices from one period to the next is, from 7:

$$\frac{P_{ft}}{P_{ft-1}} = \left(\frac{\sum_{\Omega_t^f} d_{fjt}^{\sigma} p_{fjt}^{1-\sigma}}{\sum_{\Omega_t^f} d_{fjt-1}^{\sigma} p_{fjt-1}^{1-\sigma}} \right)^{\frac{1}{1-\sigma}} \tag{21}$$

Defining as $\Omega_{t,t-1}^f$ the set of goods that is common to both periods, and multiplying both the numerator and the denominator by $\left(\sum_{\Omega_{t,t-1}^f} d_{fjt-1} p_{fjt-1}^{1-\sigma} * \sum_{\Omega_{t,t-1}^f} d_{fjt} p_{fjt}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$ we obtain:

$$\begin{aligned} \frac{P_{ft}}{P_{ft-1}} &= \left(\frac{\sum_{\Omega_t^f} d_{fjt}^{\sigma} p_{fjt}^{1-\sigma}}{\sum_{\Omega_{t,t-1}^f} d_{fjt}^{\sigma} p_{fjt}^{1-\sigma}} \frac{\sum_{\Omega_{t,t-1}^f} d_{fjt-1}^{\sigma} p_{fjt-1}^{1-\sigma}}{\sum_{\Omega_{t-1}^f} d_{fjt-1}^{\sigma} p_{fjt-1}^{1-\sigma}} \frac{\sum_{\Omega_{t,t-1}^f} d_{fjt}^{\sigma} p_{fjt}^{1-\sigma}}{\sum_{\Omega_{t,t-1}^f} d_{fjt-1}^{\sigma} p_{fjt-1}^{1-\sigma}} \right)^{\frac{1}{1-\sigma}} && (22) \\ &= \frac{\lambda_{ft-1,t}}{\lambda_{ft,t-1}} \left(\frac{\sum_{\Omega_{t,t-1}^f} d_{fjt}^{\sigma} p_{fjt}^{1-\sigma}}{\sum_{\Omega_{t,t-1}^f} d_{fjt-1}^{\sigma} p_{fjt-1}^{1-\sigma}} \right)^{\frac{1}{1-\sigma}} && (23) \end{aligned}$$

where $\lambda_{ft-1,t} = \left(\frac{\sum_{\Omega^f_{t,t-1}} d^{\sigma}_{fjt-1} p^{1-\sigma}_{fjt-1}}{\sum_{\Omega^f_{t-1}} d^{\sigma}_{fjt-1} p^{1-\sigma}_{fjt-1}} \right)^{\frac{1}{1-\sigma}}$ and $\lambda_{ft,t-1} = \left(\frac{\sum_{\Omega^f_{t,t-1}} d^{\sigma}_{fjt} p^{1-\sigma}_{fjt}}{\sum_{\Omega^f_{t}} d^{\sigma}_{fjt} p^{1-\sigma}_{fjt}} \right)^{\frac{1}{1-\sigma}}$.
Furthermore, since

$$s_{fjt} = \frac{p_{fjt} q_{fjt}}{R_{ft}} = \frac{p^{1-\sigma}_{fjt} \left(d^{\sigma}_{fjt} \right)}{P^{1-\sigma}_{fjt}} \tag{24}$$

notice from 24 that:

$$\lambda_{ft-1,t} = \left(\sum_{\Omega^f_{t,t-1}} \frac{d^{\sigma}_{fjt-1} p^{1-\sigma}_{fjt-1}}{\sum_{\Omega^f_{t-1}} d^{\sigma}_{fjt-1} p^{1-\sigma}_{fjt-1}} \right)^{\frac{1}{1-\sigma}} = \left(\sum_{\Omega^f_{t,t-1}} s_{fjt-1} \right)^{\frac{1}{1-\sigma}}$$

That is, $(\lambda_{ft-1,t})^{1-\sigma}$ is the share of period $t-1$ expenditures devoted to goods that are common to both periods. Similarly, $(\lambda_{ft,t-1})^{1-\sigma}$ is the share of period t expenditure devoted to goods common to both periods.

With this, the change in prices between the two periods can be written:

$$\frac{P_{ft}}{P_{ft-1}} = \left(\frac{\sum_{\Omega^f_{t,t-1}} s_{fjt}}{\sum_{\Omega^f_{t,t-1}} s_{fjt-1}} \right)^{\frac{1}{\sigma-1}} \frac{P^*_{ft}}{P^*_{ft,t-1}} \tag{25}$$

where $P^*_{ft} = \left(\sum_{\Omega^f_{t,t-1}} d^{\sigma}_{fjt} p^{1-\sigma}_{fjt} \right)^{\frac{1}{1-\sigma}}$ is a period t price index for the basket of goods common to t and $t-1$ for firm f, and $P^*_{ft} = \left(\sum_{\Omega^f_{t,t-1}} d^{\sigma}_{fjt-1} p^{1-\sigma}_{fjt-1} \right)^{\frac{1}{1-\sigma}}$ is a period $t-1$ price index for that same basket.

Now define the Sato-Vartia growth of prices as $\sum\limits_{\Omega_{t,t-1}} \ln \left(\frac{p_{fjt}}{p_{fjt-1}} \right)^{\omega_{fjt,t-1}}$ with $\omega_{fjt,t-1} = \frac{\frac{(s^*_{fjt} - s^*_{fjt-1})}{\ln s^*_{fjt} - \ln s^*_{fjt-1}}}{\sum\limits_{\Omega_{t,t-1}} \left(\frac{(s^*_{fjt} - s^*_{fjt-1})}{\ln s^*_{fjt} - \ln s^*_{fjt-1}} \right)}$, where s^*_{fjt} is the ratio of period t sales of product j for firm f to firm f's sales in period t of goods that belong to the basket $\Omega_{t,t-1}$.

The Marshalian demands 5, given by $q_{fjt} = d^{\sigma_F}_{ft} d^{\sigma_J}_{fjt} \left(\frac{P_{ft}}{P_t} \right)^{-\sigma_F} \left(\frac{p_{fjt}}{P_{ft}} \right)^{-\sigma_J} \frac{E_t}{P_t}$, imply

$$s^*_{fjt} = \frac{d_{ft}^{\sigma_F} d_{fjt}^{\sigma_J} \left(\frac{P_{ft}}{P_t}\right)^{-\sigma_F} \left(\frac{p_{fjt}}{P_{ft}}\right)^{-\sigma_J} \frac{E_t}{P_t}}{\sum\limits_{\Omega^f_{t,t-1}} d_{ft}^{\sigma_F} d_{fjt}^{\sigma_J} \left(\frac{P_{ft}}{P_t}\right)^{-\sigma_F} \left(\frac{p_{fjt}}{P_{ft}}\right)^{-\sigma_J} \frac{E_t}{P_t}} = \frac{d_{fjt}^{\sigma_J} p_{fjt}^{1-\sigma}}{\left(P^*_{ft}\right)^{1-\sigma}}$$

so that $\left(\frac{p_{fjt}}{p_{fjt-1}}\right) = \left(\frac{P^*_{ft}}{P^*_{ft-1}}\right)\left(\frac{s^*_{fjt}}{s^*_{fjt-1}}\right)^{\frac{1}{1-\sigma}}\left(\frac{d_{fjt}}{d_{fjt-1}}\right)^{-\frac{\sigma}{1-\sigma}}$. Given this and the facts that $\sum\limits_{\Omega^f_{t,t-1}} \omega_{fjt,t-1} = 1$ and $\sum\limits_{\Omega^f_{t,t-1}} s^*_{fjt,t-1} = 1$,

$$\begin{aligned}
&\sum_{\Omega^f_{t,t-1}} \ln\left(\frac{p_{fjt}}{p_{fjt-1}}\right)^{\omega_{fjt,t-1}} \\
= \;& \ln\left(\frac{P^*_{ft}}{P^*_{ft-1}}\right) + \frac{1}{(1-\sigma)} \sum_{\Omega^f_{t,t-1}} \frac{\frac{(s^*_{fjt}-s^*_{fjt-1})}{\ln s^*_{fjt}-\ln s^*_{fjt-1}}}{\sum\limits_{\Omega^f_{t,t-1}}\left(\frac{(s^*_{fjt}-s^*_{fjt-1})}{\ln s^*_{fjt}-\ln s^*_{fjt-1}}\right)} . \ln\left(\frac{s^*_{fjt}}{s^*_{fjt-1}}\right) \\
& + \frac{\sigma}{\sigma-1} \sum_{\Omega^f_{t,t-1}} \omega_{fjt,t-1} \ln\left(\frac{d_{fjt}}{d_{fjt-1}}\right) \\
= \;& \ln\left(\frac{P^*_{ft}}{P^*_{ft-1}}\right) + \frac{\sigma}{\sigma-1} \ln\left(\frac{\widehat{d}^*_{ft}}{\widehat{d}^*_{ft-1}}\right)
\end{aligned}$$

where $\widehat{d}^*_t = \prod\limits_{\Omega^f_{t,t-1}} d_{fjt}^{\omega_{fjt,t-1}}$ is a weighted geometric mean of individual product appeals for the constant basket of goods. Under the assumption that this geometric mean is invariant over time ($\frac{\widehat{d}^*_{ft}}{\widehat{d}^*_{ft-1}} = 1$), the common good price index $\left(\frac{P^*_{ft}}{P^*_{ft-1}}\right)$ can be calculated as in Sato-Vartia

$$\ln\left(\frac{P^*_{ft}}{P^*_{ft-1}}\right) = \sum_{\Omega_{t,t-1}} \ln\left(\frac{p_{fjt}}{p_{fjt-1}}\right)^{\omega_{fjt,t-1}}$$

This is the way in which we proceed. The assumption that $\frac{\widehat{d}^*_{ft}}{\widehat{d}^*_{ft-1}} = 1$ is weaker than the common assumption that $\frac{d_{fjt}}{d_{fjt-1}} = 1$. As Redding and Weinstein (2016) indicate, however, it is still likely that product demand shocks

$\frac{d_{fjt}}{d_{fjt-1}}$ are correlated with the weights $\omega_{fjt,t-1}$ so that $\frac{\widehat{d}^{*}_{ft}}{\widehat{d}^{*}_{ft-1}} > 1$. We keep the assumption that $\frac{\widehat{d}^{*}_{ft}}{\widehat{d}^{*}_{ft-1}} = 1$ in order to calculate $\ln\left(\frac{P^{*}_{ft}}{P^{*}_{ft-1}}\right)$ from observables, but acknowledge this potential upward bias in the Sato-Vartia price index that we use to approximate $\ln\left(\frac{P^{*}_{ft}}{P^{*}_{ft-1}}\right)$, which may induce a mistakenly high growth in quantities and therefore in $TFPQ$. This bias is thus likely to overestimate the contribution of $TFPQ$ to output growth relative to that of demand shocks.

11 Appendix B: firm problem with Cobb Douglas production function

Firm chooses X_t to solve:

$$\underset{\{X_{ft}\}}{Max} \quad \pi_{ft} = R_{ft} - C_{ft}X_{ft} = D_{ft}A_{ft}^{1-\varepsilon}X_{ft}^{\gamma(1-\varepsilon)} - C_{ft}X_{ft}$$

where $R_{ft} = P_{ft}Q_{ft}$.Optimal input demand is

$$X_{ft} = \left(\frac{\gamma\left(1-\varepsilon\right)D_{ft}A_{ft}^{1-\varepsilon}}{C_{ft}}\right)^{\frac{1}{1-\gamma(1-\varepsilon)}}$$

Suppose $X_{ft} = K_{ft}^{\frac{\alpha}{\gamma}}L_{ft}^{\frac{\beta}{\gamma}}M_{ft}^{\frac{\phi}{\gamma}}$ where K, L and M are, respectively, capital, labor and material inputs, and $\gamma = \alpha + \beta + \phi$. The firm's problem is

$$\underset{\{K_{ft},L_{ft},M_{ft}\}}{Max} \quad D_{ft}A_{ft}^{1-\varepsilon}K_{ft}^{\alpha(1-\varepsilon)}L_{ft}^{\beta(1-\varepsilon)}M_{ft}^{\phi(1-\varepsilon)} - C_{ft}\left(L_{ft}, K_{ft}, M_{ft}\right)$$

where $C_{ft}\left(L_{ft}, K_{ft}, M_{ft}\right) = w_{ft}L_{ft} + r_{ft}K_{ft} + pm_{ft}M_{ft}$. w_{ft} is the wage rate, r_{ft} is the unit cost of capital, and pm_{ft} is an index of material input prices. First order conditions for L_{ft}, K_{ft}, M_{ft}, yield the following optimal demands:

$$
\begin{aligned}
\frac{\beta R_{ft}}{L_{ft}} &= \frac{w_{ft}}{(1-\varepsilon)} \\
\frac{\beta K_{ft}}{\alpha L_{ft}} &= \frac{w_{ft}}{r_{ft}} \\
\frac{\phi L_{ft}}{\beta M_{ft}} &= \frac{pm_{ft}}{w_{ft}}
\end{aligned}
$$

Rewriting, and replacing the last two equations into the first one:

$$
\begin{aligned}
\frac{w_{ft}}{(1-\varepsilon)} &= \beta D_{ft} A_{ft}^{1-\varepsilon} \left(\frac{K_{ft}^{\alpha} L_{ft}^{\beta} M_{ft}^{\phi}}{L_{ft}^{\frac{1}{1-\varepsilon}}} \right)^{1-\varepsilon} \\
&= \beta D_{ft} A_{ft}^{1-\varepsilon} \left(\left(\frac{K_{ft}}{L_{ft}} \right)^{\alpha} \left(\frac{M_{ft}}{L_{ft}} \right)^{\phi} \frac{1}{L_{ft}^{\frac{1}{1-\varepsilon}-(\beta+\alpha+\phi)}} \right)^{1-\varepsilon} \\
L_{ft}^{\frac{1}{(1-\varepsilon)}-\gamma} &= \left(\frac{(1-\varepsilon)\, \beta D_{ft} A_{ft}^{1-\varepsilon}}{w_{ft}} \right)^{\frac{1}{(1-\varepsilon)}} \left(\frac{\alpha w_{ft}}{\beta r_{ft}} \right)^{\alpha} \left(\frac{\phi w_{ft}}{\beta pm_{ft}} \right)^{\phi} \\
L_{ft}^{\frac{1-\gamma(1-\varepsilon)}{(1-\varepsilon)}} &= \varpi_L \frac{D_{ft}^{\frac{1}{(1-\varepsilon)}} A_{ft} w_{ft}^{-\frac{1}{(1-\varepsilon)}+(\alpha+\phi)}}{r_{ft}^{\alpha} pm_{ft}^{\phi}}
\end{aligned}
$$

where ϖ_L is a function of parameters. Proceeding similarly for capital and materials, we obtain demands $M_{ft}^{\frac{1}{(1-\varepsilon)}-\gamma} = \varpi_M \frac{D_{ft}^{\frac{1}{(1-\varepsilon)}} A_{ft} pm_{ft}^{-\frac{1}{(1-\varepsilon)}+(\alpha+\beta)}}{r_{ft}^{\alpha} w_{ft}^{\beta}}$ and $K_{ft}^{\frac{1}{(1-\varepsilon)}-\gamma} = \varpi_K \frac{D_{ft}^{\frac{1}{(1-\varepsilon)}} A_{ft} r_{ft}^{-\frac{1}{(1-\varepsilon)}+(\beta+\phi)}}{w_{ft}^{\beta} pm_{ft}^{\phi}}$.

As a consequence:

$$
\begin{aligned}
X_{ft}^{\frac{1-\gamma(1-\varepsilon)}{(1-\varepsilon)}} &= \varpi_L^{\frac{\beta}{\gamma}} \varpi_K^{\frac{\alpha}{\gamma}} \varpi_M^{\frac{\phi}{\gamma}} D_{ft}^{\frac{1}{1-\varepsilon}} A_{ft} \\
&\quad * \left(\frac{pm_{ft}^{-\frac{\phi}{\gamma(1-\varepsilon)}+\frac{\phi}{\gamma}(\alpha+\beta)}}{r_{ft}^{\frac{\phi}{\gamma}\alpha} w_{ft}^{\frac{\phi}{\gamma}\beta}} \frac{r_{ft}^{-\frac{\alpha}{\gamma(1-\varepsilon)}+\frac{\alpha}{\gamma}(\beta+\phi)}}{w_{ft}^{\frac{\alpha}{\gamma}\beta} pm_{ft}^{\frac{\alpha}{\gamma}\phi}} \frac{w_{ft}^{-\frac{\beta}{\gamma(1-\varepsilon)}+\frac{\beta}{\gamma}(\alpha+\phi)}}{r_{ft}^{\frac{\beta}{\gamma}\alpha} pm_{ft}^{\frac{\beta}{\gamma}\phi}} \right) \\
&= \varpi_L^{\frac{\beta}{\gamma}} \varpi_K^{\frac{\alpha}{\gamma}} \varpi_M^{\frac{\phi}{\gamma}} \left(\frac{D_{ft}^{\frac{1}{1-\varepsilon}} A_{ft}}{w_{ft}^{\frac{\beta}{\gamma(1-\varepsilon)}} pm_{ft}^{\frac{\phi}{\gamma(1-\varepsilon)}} r_{ft}^{\frac{\alpha}{\gamma(1-\varepsilon)}}} \right)
\end{aligned}
$$

and

$$\frac{X_{ft}}{X_{f0}} = \left(\frac{d_{ft}}{d_{f0}}\right)^{\kappa_1}\left(\frac{a_{ft}}{a_{f0}}\right)^{\kappa_2}\left(\frac{pm_{ft}}{pm_{f0}}\right)^{-\frac{\phi}{\gamma}\kappa_1}\left(\frac{w_{ft}}{w_{f0}}\right)^{-\frac{\beta}{\gamma}\kappa_1}\kappa_t\widehat{\kappa}_{ft} \quad (26)$$

$$\frac{L_{ft}}{L_{f0}} = \left(\frac{d_{ft}}{d_{f0}}\right)^{\kappa_1}\left(\frac{a_{ft}}{a_{f0}}\right)^{\kappa_2}\left(\frac{pm_{ft}}{pm_{f0}}\right)^{-\phi\kappa_2}\left(\frac{w_{ft}}{w_{f0}}\right)_t^{-\kappa_1+(\alpha+\phi)\kappa_2}\xi_t\xi_{ft} \quad (27)$$

$$\frac{Q_{ft}}{Q_{f0}} = \left(\frac{d_{ft}}{d_{f0}}\right)^{\gamma\kappa_1}\left(\frac{a_{ft}}{a_{f0}}\right)^{1+\gamma\kappa_2}\left(\frac{pm_{ft}}{pm_{f0}}\right)^{-\phi\kappa_1}\left(\frac{w_{ft}}{w_{f0}}\right)^{-\beta\kappa_1}\chi_t\chi_{ft} \quad (28)$$

where $\kappa_1 = \frac{1}{1-\gamma(1-\varepsilon)}$; $\kappa_2 = (1-\varepsilon)\kappa_1$; $\kappa_t = \left(\frac{D_t}{D_0}\right)^{\kappa_1}\left(\frac{A_t}{A_0}\right)^{\kappa_2}\left(\frac{C_t}{C_0}\right)^{-\kappa_1}$ captures aggregate growth between birth and age t, and ;$\widehat{\kappa}_{ft} = \frac{\delta_{ft}^{\kappa_1}\alpha_{ft}^{\kappa_2}\zeta_{ft}^{-\kappa_1}r_{ft}^{\frac{-\alpha\kappa_1}{\gamma}}}{\delta_{f0}^{\kappa_1}\alpha_{f0}^{\kappa_2}\zeta_{ft}^{-\kappa_1}r_{f0}^{\frac{-\alpha\kappa_1}{\gamma}}}$ captures residual variation from noise in fundamentals not observed by the firm at the time of choosing its scale in each period, as well as from unobserved user cost of capital. ξ_t, ξ_{ft}, χ_t and χ_{ft} are analogous residuals for the specific cases of employment and output. In particular: $\chi_t == \kappa_t^{\gamma}\left(\frac{A_t}{A_0}\right)$ and $\chi_{ft} = \widehat{\kappa}_{ft}^{\gamma}\frac{\alpha_t}{\alpha_0}$.[17] Notice also that $C_{ft} = w_{ft}^{\frac{\beta}{\gamma}}pm_{ft}^{\frac{\phi}{\gamma}}r_{ft}^{\frac{\alpha}{\gamma}}$.

12 Appendix C: Appendix Tables

[17] $\xi_{ft} = \kappa_{ft}\left(\frac{r_{ft}}{r_{f0}}\right)^{-\alpha\frac{1-\gamma(1-\varepsilon)}{(1-\varepsilon)}}$