The Consequences of An Aging Chinese Miracle*

Michael Dotsey Wenli Li Fang Yang†

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Abstract

China is aging rapidly stemming from declines in fertility rates and increases in life expectancy. It is projected that the fraction of Chinese population ages 65 and above will exceed that of the U.S. by mid 2030.

Keywords:

JEL Classifications:

1 Introduction

China is ageing rapidly (Figures 1 and 2), according to the United Nations' Population and Vital Statistics Reports. Chinese fertility rate, measured as the average number of children of women between the age of 15 and 49, has been falling sharply. It was 5.5 in 1970 compared to 2.5 in the U.S. By the early 1990s, the fertility rate in China fell below that in U.S., 1.7 versus 2.0. Although Chinese fertility rate is expected to recover somewhat, it is projected to remain at less than 2.0, and below that of the U.S. at least until 2050.[1] Population growth rates, as a result, decline consistently between 1970 and 2010 for both countries and the trends are projected to continue till 2050. The decline in population growth is much more severe in China than in the U.S. The demographic effects of falling utility have been reinforced by improvements in old age mortality. In 1970, the Chinese life expectancy was 60 years of age. This number rose to 75 years of age in 2010 and is expected to reach 80 years of age by 2050, less than 5 years shorter than the U.S. life expectancy (Figure 2). The combined effect of declining fertility rates and increasing life expectancy is that population over the age 60, a measure of old-age dependency ratio, is increasing dramatically in China and is expected to exceed that of the U.S. in 2030 reaching a level of over 30 percent by 2050.

*The views expressed are those of the authors and do not necessarily reflect those of the Federal Reserve Bank of Philadelphia or the Federal Reserve System.

†Michael Dotsey and Wenli Li: Research Department, Federal Reserve Bank of Philadelphia, Ten Independence Mall, Philadelphia, PA 19106 (Email: michael.dotsey@phil.frb.org; wenli.li@phil.frb.org). Fang Yang: Department of Economics, Louisiana State University, Business Education Complex, Room 2300, Baton Rouge, LA 70803-6306 (Email: fyang@lsu.edu).

[1]The Chinese government recently announced the two-child policy which may ease the falling trend in fertility rates.

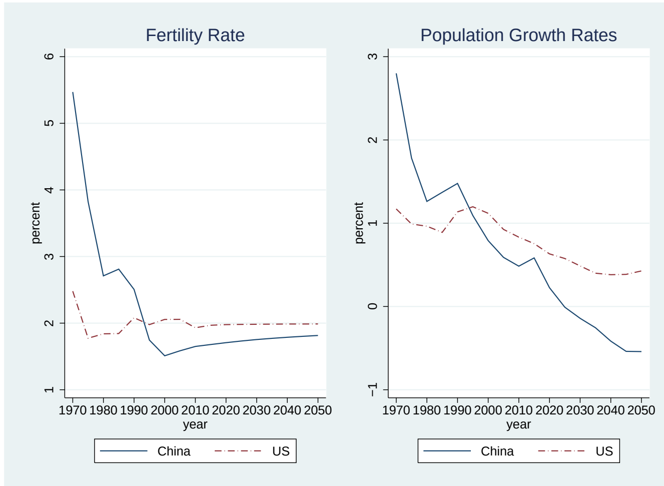

Figure 1. Fertility Rates and Population Growth Rates: China versus U.S.

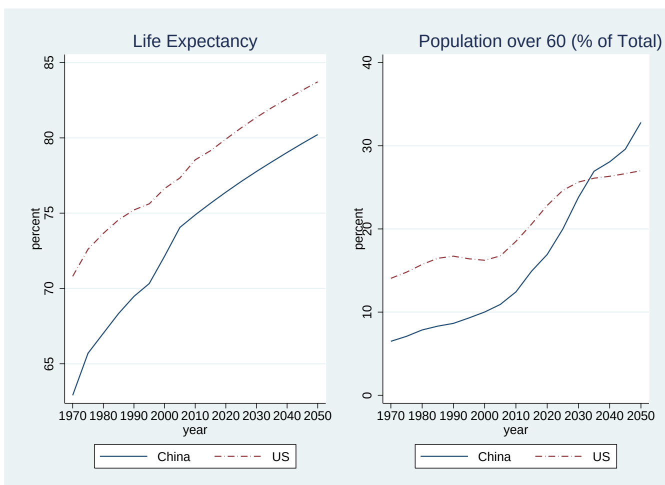

Figure 2. Life Expectancy and Old Age Dependency Rates: China versus U.S.

Despite the fast ageing Chinese population, the Chinese economy has been growing robustly. While the per capital GDP in China relative to that in the U.S. remained at a low of about 2 percent between 1970 and 2000, after 2000, the Chinese economy took off. During the next ten years, the relative ratio climbed up to over 12 percent and is expected to reach 23 percent by 2050 (Figure 3). Part of the high growth is driven by the high savings rate in China as evidenced in Figure 4 which far outpaces that of the U.S.

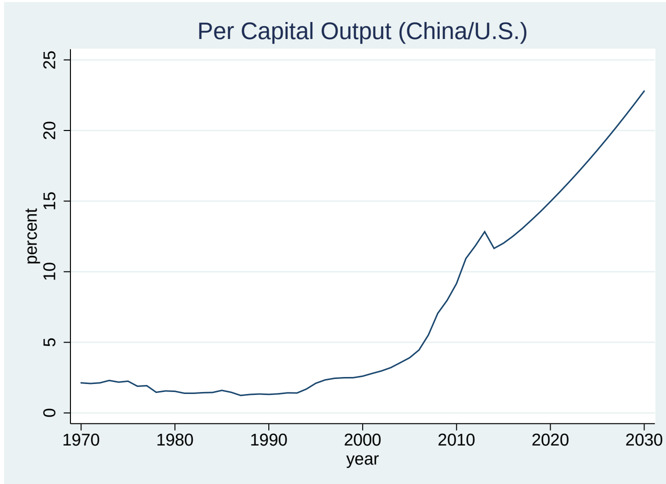

Figure 3. Relative per Capita GDP: China/U.S.

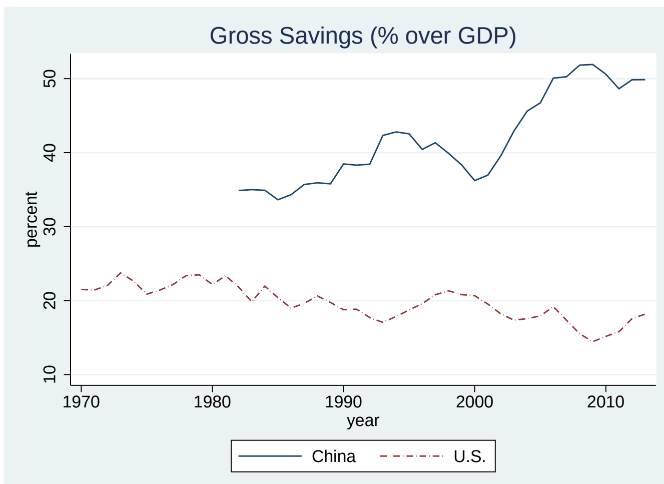

Figure 4. Gross Savings Rate: China versus U.S. (gross savings are calculated as gross national income less total consumption plus net transfer, source: World Bank)

In this paper, we study the consequences of the demographic changes in China and the U.S. on the two nation's output growth, capital accumulation, labor supply, and welfare in various economic environments including closed economies, economies with perfect capital mobility and no trade barriers, and economies with imperfect capital mobility and /or trader barriers. Our focus on only two of the largest economies in the world, China and U.S., allow us to more realistically capture each economy and make use of micro data that exhibits unique and interesting patterns such as having savings rate in China. More important, we endogenize the growth in both economies along the lines of Parente and Prescott (1994) where firms spend resources to improve their total factor productivity. This together with the introduction of trade and trade barriers are two unique features that differentiate our paper from the large literature on demographics and world economy. In particular, Attanasio, Kitao, and Violante (2006) evaluates the impact of demographic changes on emerging economies' factor prices, savings rate, output, and welfare. Attanasio, Bonfatti, Kitao, and Weber (2015) study similar questions of Latin America and Caribbean countries. Krueger and Ludwig (2006) study the impact of demographic changes of developed countries including the U.S., and the rest of the OECD countries.

The rest of the paper is organized as follows. In Section 2, we describe our model. In Section 3 we discuss how we estimate and/or calibrate the model's parameters in the closed economy. In Section 4, we present the results from a two country model with perfect capital mobility and no trade barriers.

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2 The Model

We model the two countries similarly and omit the superscript that denotes each country in the discussion.

2.1 The Firm

Following Parente and Prescott (1994), we assume that the production function of each country takes the following form,

$$Y_t = A_t L_t^{\theta_l} K_{t-1}^{\theta_k},\ 0 \leq \theta_n, \theta_k \leq 1, \tag{1}$$

where A_t denotes its technology level at t, L_t denotes the labor input, and K_{t-1} denotes the capital input. The representative firm can advance its technology between time t and $t+1$ by making an investment X_{At} at t.

$$X_{At} = \pi \int_{A_t}^{A_{t+1}} (\frac{\Gamma}{W_t})^{\alpha} d\Gamma, \tag{2}$$

where π is the parameter that indexes the size of barriers to technology adoption and W_t indicates the technology frontier and it grows at the constant rate of γ,

$$W_t = W_0(1+\gamma)^t. \tag{3}$$

Under the assumption that Γ is normally distributed, integration of (2) gives

$$(\alpha+1)X_{At} = \pi \frac{A_{t+1}^{\alpha+1} - A_t^{\alpha+1}}{W_0^{\alpha}(1+\gamma)^{\alpha t}}. \tag{4}$$

It follows immediately then,

Lemma 1. The amount of required investment X_{At} increases as the level of the adopted technology A_{t+1} increases, with these increases occurring at an increasing rate. It decreases as the level of the firm's current technology A_t increases, with these decreases occurring at a decreasing rate.

Define

$$Z_{t-1} = \frac{A_t^{\alpha+1}}{W_0^{\alpha}(1+\gamma)^{\alpha(t-1)}(1+\alpha)}, \tag{5}$$

$$X_{zt} = X_{At}, \tag{6}$$

$$\theta_z = \frac{1}{1+\alpha}, \tag{7}$$

we have

$$Z_t = \frac{1}{(1+\gamma)^{\frac{1-\theta_z}{\theta_z}}} Z_{t-1} + \frac{1}{\pi} X_{zt}. \tag{8}$$

In other words,

$$A_t = [W_0^\alpha (1+\gamma)^{\alpha(t-1)}(1+\alpha)]^{\frac{1}{\alpha+1}} Z_{t-1}^{\frac{1}{\alpha+1}}. \tag{9}$$

Equation (1) then becomes

$$Y_t = [W_0^\alpha (1+\gamma)^{\alpha(t-1)}(1+\alpha)]^{\frac{1}{\alpha+1}} Z_{t-1}^{\frac{1}{\alpha+1}} L_t^{\theta_l} K_{t-1}^{\theta_k}, \tag{10}$$

we impose $\frac{1}{\alpha+1} + \theta_l + \theta_k = 1$, and define

$$\mu = [\frac{W_0^\alpha (1+\alpha)}{1+\gamma}]^{\frac{1}{\alpha+1}}, \tag{11}$$

the production function can be rewritten as

$$Y_t = \mu (1+\gamma)^{\frac{\alpha t}{\alpha+1}} Z_{t-1}^{\theta_z} L_t^{\theta_l} K_{t-1}^{\theta_k}.$$

The law of motion for the capital is

$$K_t = (1-\delta_k) K_{t-1} + X_{kt}.$$

Firm's Problem The representative firm chooses labor, physical capital, and investment in technology catch up to maximize the dividend stream paid to its owners. The dividends at each period are given by,

$$V_{ft} = Y_t - \omega_t L_t - (r_{kt} + \delta^k) K_{t-1} - X_{zt},$$

where ω_t is the wage rate and r_{kt} is the rate of return to physical capital at time t. Let r_{zt} denote the implicit rental rate of return to technology capital. The firm maximizes its life time after-tax dividend given by

$$V_{f0} = \sum_{t=0}^{\infty} (1-\tau) p_t V_{ft},$$

where p_t is the price of output at date t. Assuming that labor and capital markets are competitive, we then have

$$\begin{aligned} \omega_t &= \theta_n \frac{Y_t}{L_t}, \\ r_{kt} + \delta^k &= \theta_k \frac{Y_t}{K_{t-1}}, \\ r_{zt} &\equiv \theta_z \frac{Y_t}{Z_{t-1}} = \pi [\frac{p_{t-1}}{p_t} - \frac{1}{(1+\gamma)^\alpha}]. \end{aligned}$$

Let S_{t-1}^f denote the number of equity shares at time t-1, and we assume firm issues new shares to keep up with the population growth, that is, we set $S_t^f = N_t$, and let q_t denote the price of the equity after payment at time t. We have

$$q_t S_t^f = \frac{1}{p_t} \sum_{j=t+1}^{\infty} (1-\tau) p_j V_{fj}.$$

2.2 The Household

Consider a continuum of infinitely lived households of measure 1. The households own the physical capital as well as the firm. In addition, each household has one unit endowment of time which is allocated between labor and leisure. The number of person in each household at time t is N_t. Households have preferences over consumption C_t and leisure $(N_t - L_t)$ over time given by

$$\sum_{t=0}^{\infty} \beta^t N_t \frac{\{(\frac{C_t}{N_t})^{\phi}[\zeta_t(1-L_t/N_t)]^{1-\phi}\}^{1-\sigma}}{1-\sigma}, \tag{12}$$

where β is the discount rate, ϕ is the weight on consumption in the period utility function, and σ captures risk aversion. The term ζ_t denotes the preference for leisure that may vary over time, and N_t is the population size at time t.

The Household's Problem Households are the equity holders of the firm. The problem of the household is to maximize (12), subject to its budget constraint as follows

$$C_t+A_t+q_tS_t \leq \omega_tL_t+(1+r_t)A_{t-1}+v_{ft}S_{t-1}\frac{S_t^f}{S_{t-1}^f}+q_tS_{t-1}\frac{S_t^f}{S_{t-1}^f}+V_{gt}-\tau(\omega_tL_t+r_tA_{t-1}+v_{ft}S_{t-1}\frac{S_t^f}{S_{t-1}^f}), \tag{13}$$

where $v_{ft} = \frac{V_{ft}}{S_t^f}$ denotes the dividend per share, V_{gt} denotes lump-sum government transfer, and τ is the income tax rate. The term $S_{t-1}\frac{S_t^f}{S_{t-1}^f}$ captures the renormalization of the equity shares as firms issue new shares each period and v_{ft} and q_t denote dividend and equity price per time t share.

Let us write the budget constraint as follows:

$$\sum_{t=0}^{\infty} p_t[C_t+A_t+q_tS_t] \leq \sum_{t=0}^{\infty} p_t[\omega_tL_t+(1+r_t)A_{t-1}+v_{ft}S_{t-1}\frac{S_t^f}{S_{t-1}^f}+q_tS_{t-1}\frac{S_t^f}{S_{t-1}^f}+V_{gt}-\tau(\omega_tL_t+r_tA_{t-1}+v_{ft}S_{t-1}\frac{S_t^f}{S_{t-1}^f})], \tag{14}$$

let $s_{t-1} \equiv \frac{S_{t-1}}{S_{t-1}^f}$ denote the fraction of equity shares owned by the household, then the budget constraint can be rewritten as

$$\sum_{t=0}^{\infty} p_t[C_t+A_t+q_tS_t] \leq \sum_{t=0}^{\infty} p_t[\omega_tL_t+(1+r_t)A_{t-1}+v_{ft}s_{t-1}S_t^f+q_ts_{t-1}S_t^f+V_{gt}-\tau(\omega_tL_t+r_tA_{t-1}+v_{ft}s_{t-1}S_t^f)], \tag{15}$$

2.3 The Government

The government chooses $\{G_t, \tau_t, V_{gt}\}$, where G_t is the government expenditure per household, to balance its period budget constraint

$$G_t + V_{gt} = \tau(\omega_tL_t + r_tA_{t-1} + v_{ft}S_{t-1}).$$

2.4 Equilibrium

The competitive equilibrium for the economy consists of prices $\{p_t, \omega_t, r_{kt}\}_{t=0}^{\infty}$ and allocations $\{K_0, Z_0, K_0, S_0\}$ and $\{Y_t, L_t, K_t, Z_t, X_{zt}, V_{ft}, C_t, L_t, A_t, S_t\}_{t=1}^{\infty}$ such that

1. Household maximize their utilities;
2. The representative firm maximizes its dividends; and
3. Markets clear, for $t = 0, 1, 2, ...$, we have
   (a) Goods market: $C_t + G_t + NX_t = Y_t - X_{zt} - X_{kt}$, $X_{kt} = K_t - (1-\delta^k)K_{t-1}$, where NX_t denotes net export.
   (b) Labor market: $N_t l_t = L_t$,
   (c) Physical capital market: $A_{t-1}+B_{t-1} = K_{t-1}$, B_{t-1} denotes borrowing from foreigners.
   (d) Equity share market: $S_t = S_t^f$.

2.5 Equilibrium conditions

Given the aggregate technology

$$Y_t = \mu(1+\gamma)^{\frac{\alpha t}{\alpha+1}} Z_{t-1}^{\theta_z} L_t^{\theta_l} K_{t-1}^{\theta_k}.$$

We define per capita total output as follows

$$y_t \equiv \frac{Y_t}{N_t} = \mu(1+\gamma)^{\frac{\alpha t}{\alpha+1}} z_{t-1}^{\theta_z} l_t^{\theta_l} k_{t-1}^{\theta_k},$$

where $z_{t-1} \equiv \frac{Z_{t-1}}{N_t}$, $l_t \equiv \frac{L_t}{N_t}$, $k_{t-1} \equiv \frac{K_{t-1}}{N_t}$. The per capita output growth rate is therefore

$$g_y \equiv \frac{y_{t+1}}{y_t} = (1+\gamma)^{\frac{\alpha}{\alpha+1}} (\frac{z_t}{z_{t-1}})^{\theta_z} (\frac{l_{t+1}}{l_t})^{\theta_l} (\frac{k_t}{k_{t-1}})^{\theta_k}.$$

At balanced growth path, per capita z_t and k_t all grow at rate g_y. It then follows

$$g_y = \{(1+\gamma)^{\frac{\alpha}{\alpha+1}}\}^{\frac{1}{\theta_l}}.$$

Additionally, variables $\{c_t, x_{kt}, \omega_t, g_t, v_{gt}, v_{ft}\}$ all grow at the same rate g_y.

To detrend all the economic variables, we define $\widetilde{y}_t \equiv \frac{y_t}{(g_y)^t}, \widetilde{z}_{t-1} \equiv \frac{z_{t-1}}{(g_y)^t}, \widetilde{k}_{t-1} \equiv \frac{k_{t-1}}{(g_y)^t}$. And

$$\begin{aligned}
\widetilde{y}_t &= \frac{\mu(1+\gamma)^{\frac{\alpha t}{\alpha+1}}}{(g_y)^{t\theta_l}} \widetilde{z}_{t-1}^{\theta_z} \widetilde{k}_{t-1}^{\theta_k} l_t^{\theta_l} \\
&= \mu \widetilde{z}_{t-1}^{\theta_z} \widetilde{k}_{t-1}^{\theta_k} l_t^{\theta_l}.
\end{aligned}$$

Households' utility function can be rewritten as

$$\begin{aligned}
&\sum_{t=0}^{\infty} \beta^t N_t \frac{\{(\frac{C_t}{N_t})^{\phi}[\zeta_t(1-L_t/N_t)]^{1-\phi}\}^{1-\sigma}}{1-\sigma} \\
&= \sum_{t=0}^{\infty} N_t \beta^t \frac{\{(c_t)^{\phi}[\zeta_t(1-l_t)]^{1-\phi}\}^{1-\sigma}}{1-\sigma} \\
&= \sum_{t=0}^{\infty} N_t (\beta {g_y}^{\phi(1-\sigma)})^t \frac{\{\widetilde{c}_t{}^{\phi}[\zeta_t(1-l_t)]^{1-\phi}\}^{1-\sigma}}{1-\sigma},
\end{aligned}$$

where $c_t \equiv \frac{C_t}{N_t}, \widetilde{c}_t \equiv \frac{c_t}{(g_y)^t}$.

Furthermore, dividend per share is

$$\begin{aligned}
v_{ft} &\equiv \frac{V_{ft}}{S_t^f} = \frac{V_{ft}}{N_t} \\
&= \frac{Y_t - \omega_t L_t - (r_{kt} + \delta^k) K_{t-1} - X_{zt}}{N_t} \\
&= y_t - \omega_t l_t - (r_{kt} + \delta^k) k_{t-1} - x_{zt},
\end{aligned}$$

and

$$\begin{aligned}
\widetilde{v}_{ft} &\equiv \frac{v_{ft}}{(g_y)^t} \\
&= \widetilde{y}_t - \widetilde{\omega}_t l_t - (r_{kt} + \delta^k) \widetilde{k}_{t-1} - \widetilde{x}_{zt}
\end{aligned}$$

where $\widetilde{\omega}_t \equiv \frac{\omega_t}{(g_y)^t}$. Furthermore,

$$\begin{aligned}
q_t S_t^f &= \frac{1}{p_t} \sum_{j=t+1}^{\infty} (1-\tau) p_j V_{fj} \\
&= \frac{1}{p_t} \sum_{j=t+1}^{\infty} (1-\tau) p_j S_j^f v_{fj} \\
&= \frac{1}{p_t} \sum_{j=t+1}^{\infty} (1-\tau) p_j S_j^f (g_y)^j \widetilde{v}_{fj}.
\end{aligned}$$

Therefore, on the balanced growth path

$$q_t = \frac{N_0}{N_t p_t} \sum_{j=t+1}^{\infty} (1-\tau) p_j (g_y g_n)^j \widetilde{v}_{fj}.$$

Furthermore,

$$\begin{aligned}
\frac{q_{t+1} S_t}{q_t S_{t-1}} &= \frac{q_{t+1}}{q_t} g_n = \frac{p_t}{p_{t+1}} \frac{\sum_{j=t+2}^{\infty} (1-\tau) p_j S_j (g_y)^j \widetilde{v}_{fj}}{\sum_{j=t+1}^{\infty} (1-\tau) p_j S_j (g_y)^j \widetilde{v}_{fj}} \\
&= \frac{p_t}{p_{t+1}} \frac{\sum_{j=t+2}^{\infty} p_j N_0 (g_n g_y)^j \widetilde{v}_{fj}}{\sum_{j=t+1}^{\infty} p_j N_0 (g_n g_y)^j \widetilde{v}_{fj}} = \frac{1}{\widetilde{\beta}} \widetilde{\beta} g_n g_y = g_n g_y \\
\frac{q_{t+1}}{q_t} &= g_n g_y \frac{S_{t-1}}{S_t} = g_y.
\end{aligned}$$

Define $\widetilde{q}_t \equiv \frac{q_t}{(g_y)^t}$, $s_t \equiv \frac{S_t}{N_t}$, the household's budget constraint can be rewritten as

$$C_t + A_t + q_t S_t \le \omega_t L_t + (1+r_t)A_{t-1} + v_{ft}s_{t-1}S_t^f + q_t s_{t-1} + v_{gt}S_t^f - \tau(\omega_t L_t + r_t A_{t-1} + v_{ft}s_{t-1}S_t^f),$$

divide both sides by population N_t, given $S_t^f = N_t$, we have

$$c_t + \frac{N_{t+1}}{N_t}a_t + q_t s_t \le \omega_t l_t + (1+r_t)a_{t-1} + v_{ft}s_{t-1} + q_t s_{t-1} + v_{gt} - \tau(\omega_t l_t + r_t a_{t-1} + v_{ft}s_{t-1}).$$

Detrend the budget constraint by dividing both sides again by $(g_y)^t$, we have

$$\widetilde{c}_t + g_y\frac{N_{t+1}}{N_t}\widetilde{a}_t + \widetilde{q}_t s_t \le \widetilde{\omega}_t l_t + (1+r_t)\widetilde{a}_{t-1} + \widetilde{v}_{ft}s_{t-1} + \widetilde{q}_t s_{t-1} + \widetilde{v}_{gt} - \tau(\widetilde{\omega}_t l_t + r_t\widetilde{a}_{t-1} + \widetilde{v}_{ft}s_{t-1}).$$

Equilibrium Characterization in detrended terms From the household's problem (FOC on C_t), we have

$$\begin{aligned}
p_t\lambda &= \beta^t N_t \frac{(\frac{C_t}{N_t})^{\phi(1-\sigma)-1}[\zeta_t(1-L_t/N_t)]^{(1-\phi)(1-\sigma)}}{N_t^{\phi(1-\sigma)}}\phi \\
&= \beta^t(\frac{C_t}{N_t})^{\phi(1-\sigma)-1}[\zeta_t(1-L_t/N_t)]^{(1-\phi)\{^{1-\sigma}\}}\phi \\
&= \beta^t(\widetilde{c}_t(g_y)^t)^{\phi(1-\sigma)-1}[\zeta_t(1-l_t)]^{(1-\phi)\{^{1-\sigma}\}}\phi \\
&= \beta^t(g_y^{\phi(1-\sigma)-1})^t(\widetilde{c}_t)^{\phi(1-\sigma)-1}[\zeta_t(1-l_t)]^{(1-\phi)\{^{1-\sigma}\}}\phi. \qquad (16)
\end{aligned}$$

From firms' problem we have

$$\widetilde{y}_t = \mu\widetilde{z}_{t-1}^{\theta_z}\widetilde{k}_{t-1}^{\theta_k}l_t^{\theta_l}.$$

From the firm's problem, we obtain (firm maximizing dividend by choosing $\widetilde{z}_t$).

$$\begin{aligned}
r_{zt+1} &= \pi[\frac{p_t}{p_{t+1}} - \frac{1}{(1+\gamma)^{\frac{1-\theta_z}{\theta_z}}}] \\
&= \pi\{[1+(1-\tau)r_{t+1}] - \frac{1}{(1+\gamma)^{\alpha}}\}.
\end{aligned}$$

Additionally, we have

$$\begin{aligned}
\widetilde{\omega}_t &= \theta_k\widetilde{y}_t, \\
r_{kt} + \delta^k &= \theta_k\frac{\widetilde{y}_t}{\widetilde{k}_{t-1}}, \\
r_{zt} &\equiv \theta_z\frac{\widetilde{y}_t}{\widetilde{z}_{t-1}}.
\end{aligned}$$

Households' maximization problem can be rewritten as

$$\max\sum_{t=0}^{\infty} N_t(\beta g_y{}^{\phi(1-\sigma)})^t\frac{\{\widetilde{c}_t{}^{\phi}[\zeta_t(1-l_t)]^{1-\phi}\}^{1-\sigma}}{1-\sigma}, \qquad (17)$$

$$\widetilde{c}_t+g_y\frac{N_{t+1}}{N_t}\widetilde{a}_t+\widetilde{q}_ts_t\leq\widetilde{\omega}_tl_t+(1+r_t)\widetilde{a}_{t-1}+\widetilde{v}_{ft}s_{t-1}+\widetilde{q}_ts_{t-1}+\widetilde{v}_{gt}-\tau(\widetilde{\omega}_tl_t+r_t\widetilde{a}_{t-1}+\widetilde{v}_{ft}s_{t-1}) \quad (18)$$

First order conditions with respect to $\{\widetilde{c}_t,\widetilde{a}_t,s_t,l_t\}$ are

$$\lambda_t = N_t\phi(\beta {g_y}^{\phi(1-\sigma)})^t\{\widetilde{c}_t^{\phi(1-\sigma)-1}[\zeta_t(1-l_t)]^{(1-\phi)(1-\sigma)}\} \quad (19)$$

$$1+(1-\tau)r_{t+1} = \frac{\lambda_t}{\lambda_{t+1}}\frac{N_{t+1}}{N_t}g_y \quad (20)$$

$$\widetilde{q}_{t+1}+(1-\tau)\widetilde{v}_{ft+1} = \widetilde{q}_t\frac{\lambda_t}{\lambda_{t+1}} \quad (21)$$

$$(1-\tau)\omega_t = \frac{1-\phi}{\phi}\frac{c_t}{(1-l_t)}, \quad (22)$$

and from the non-arbitrage condition, we get

$$r_t = r_{kt}.$$

From equation (16) we can show that $p_t=\lambda_t/(N_t{g_y}^t)$,

$$\frac{p_t}{p_{t+1}} = \frac{\lambda_t}{\lambda_{t+1}}\frac{N_{t+1}}{N_t}g_y = 1+(1-\tau)r_{t+1}$$

From the household's problem, we have

$$\frac{\lambda_{t+1}}{\lambda_t} = \frac{N_{t+1}\beta {g_y}^{\phi(1-\sigma)}\{\widetilde{c}_{t+1}^{\phi(1-\sigma)-1}[\zeta_{t+1}(1-l_{t+1})]^{(1-\phi)(1-\sigma)}\}}{N_t\{\widetilde{c}_t^{\phi(1-\sigma)-1}[\zeta_t(1-l_t)]^{(1-\phi)(1-\sigma)}\}}$$

$$\frac{1}{1+(1-\tau)r_{t+1}} = \beta {g_y}^{\phi(1-\sigma)-1}(\frac{\widetilde{c}_{t+1}}{\widetilde{c}_t})^{(\phi-1-\sigma\phi)}(\frac{1-l_{t+1}}{1-l_t})^{(1-\sigma)(1-\phi)}(\frac{\zeta_{t+1}}{\zeta_t})^{(1-\sigma)(1-\phi)},$$

$$\widetilde{q}_{t+1}+(1-\tau)\widetilde{v}_{ft+1} = \frac{\widetilde{q}_t(1+(1-\tau)r_{t+1})}{g_y}\frac{N_t}{N_{t+1}} = \frac{\widetilde{q}_t}{g_y}\frac{p_t}{p_{t+1}}\frac{N_t}{N_{t+1}}$$

$$(1-\tau)\widetilde{\omega}_t = \frac{1-\phi}{\phi}\frac{\widetilde{c}_t}{(1-l_t)}.$$

From the goods market clearing condition, we have,

$$\widetilde{c}_t+\widetilde{nx}_t+\widetilde{g}_t = \widetilde{y}_t-\widetilde{x}_{zt}-\widetilde{x}_{kt}$$

Define $g_{nt}=\frac{N_{t+1}}{N_t}$. By definition, we also have

$$g_yg_{nt}\widetilde{z}_t = \frac{1}{(1+\gamma)^{\frac{1-\theta_z}{\theta_z}}}\widetilde{z}_{t-1}+\frac{1}{\pi}\widetilde{x}_{zt}$$

$$\widetilde{v}_{ft} = \widetilde{y}_t-\widetilde{\omega}_tl_t-(r_{kt}+\delta^k)\widetilde{k}_{t-1}-\widetilde{x}_{zt}.$$

Now government budget constraint

$$\widetilde{g}_t+\widetilde{v}_{gt} = \tau(\widetilde{\omega}_tl_t+r_t\widetilde{a}_{t-1}+s_{t-1}\widetilde{v}_{ft}).$$

Put all the equations together and simplify, we have

$$\theta_z \frac{\widetilde{y}_{t+1}}{\widetilde{z}_t} = \pi\{[1 + (1-\tau)(\theta_k \frac{\widetilde{y}_{t+1}}{\widetilde{k}_t} - \delta_k)] - \frac{1}{(1+\gamma)^{\alpha}}\}, \tag{23}$$

$$\frac{1}{1 + (\theta_k \frac{\widetilde{y}_{t+1}}{\widetilde{k}_t} - \delta_k)(1-\tau)} = \beta {g_y}^{\phi(1-\sigma)-1} (\frac{\widetilde{c}_{t+1}}{\widetilde{c}_t})^{(\phi-1-\sigma\phi)} (\frac{1-l_{t+1}}{1-l_t})^{(1-\sigma)(1-\phi)} (\frac{\zeta_{t+1}}{\zeta_t})^{(1-\sigma)(1-\phi)}, \tag{24}$$

$$(1-\tau)\theta_n \frac{\widetilde{y}_t}{l_t} = \frac{1-\phi}{\phi} \frac{\widetilde{c}_t}{(1-l_t)}, \tag{25}$$

$$\widetilde{y}_t = \mu \widetilde{z}_{t-1}^{\theta_z} \widetilde{k}_{t-1}^{\theta_k} l_t^{\theta_l}, \tag{26}$$

$$g_{nt} g_y \widetilde{z}_t = \frac{1}{(1+\gamma)^{\frac{1-\theta_z}{\theta_z}}} \widetilde{z}_{t-1} + \frac{1}{\pi} \widetilde{x}_{zt}, \tag{27}$$

$$g_{nt} g_y \widetilde{k}_t = (1-\delta_k) \widetilde{k}_{t-1} + \widetilde{x}_{kt}, \tag{28}$$

$$\widetilde{c}_t + \widetilde{g}_t + \widetilde{nx}_t = \widetilde{y}_t - \widetilde{x}_{zt} - \widetilde{x}_{kt}, \tag{29}$$

$$\widetilde{g}_t = \tau(\widetilde{\omega}_t l_t + r_t \widetilde{a}_{t-1} + s_{t-1} \widetilde{v}_{ft}), \tag{30}$$

$$= \tau[\widetilde{\omega}_t l_t + r_t \widetilde{a}_{t-1} + s_{t-1}(\widetilde{y}_t - \widetilde{\omega}_t l_t - (r_{kt} + \delta^k)\widetilde{k}_{t-1} - \widetilde{x}_{zt})]$$

$$= \tau[\widetilde{\omega}_t l_t + r_t \widetilde{a}_{t-1} + \widetilde{y}_t - \widetilde{\omega}_t l_t - (r_{kt} + \delta^k)\widetilde{k}_{t-1} - \widetilde{x}_{zt}] \tag{31}$$

$$= \tau[r_t \widetilde{a}_{t-1} + \widetilde{y}_t - (r_{kt} + \delta^k)\widetilde{k}_{t-1} - \widetilde{x}_{zt}], \tag{32}$$

$$\widetilde{a}_{t-1} + \widetilde{b}_{t-1} = \widetilde{k}_{t-1} \tag{33}$$

3 Closed Economy

In the closed economy, we have $\widetilde{nx}_t = 0, \widetilde{b}_{t-1} = 0$, given that we assume representative agent. it follows immediately $s_{t-1} = 1$. The government's budget constraint becomes

$$\widetilde{g}_t = \tau(\widetilde{y}_t - \delta^k \widetilde{k}_{t-1} - \widetilde{x}_{zt})$$

Other equilibrium variables:

$$p_t = \beta^t (g_y^{\phi(1-\sigma)-1})^t (\widetilde{c}_t)^{\phi(1-\sigma)-1} [\zeta_t(1-l_t)]^{(1-\phi)\{^{1-\sigma}\}} \phi$$

$$\widetilde{v}_{ft} = \widetilde{y}_t - \widetilde{\omega}_t l_t - (r_{kt} + \delta^k)\widetilde{k}_{t-1} - \widetilde{x}_{zt}$$

$$q_t = \frac{1}{p_t S_t^f} \sum_{j=t+1}^{\infty} (1-\tau) p_j S_j^f v_{fj}$$

$$\widetilde{a}_{t-1} = \widetilde{k}_{t-1}; s_t = 1;$$

3.1 Steady state

Given the tax rate τ, we need to solve 7 unknowns, $\{g^*, l^*, n^*, \widetilde{x}_z^*, y^*, z^*, k^*\}$

$$\theta_z \frac{y^*}{z^*} = \pi\{[1 + (1-\tau)(\theta_k \frac{y^*}{k^*} - \delta_k)] - \frac{1}{(1+\gamma)^\alpha}\} \tag{34}$$

$$\frac{1}{[1 + (\theta_k \frac{y^*}{k^*} - \delta_k)(1-\tau)]\beta} = g_y{}^{\phi(1-\sigma)-1} \tag{35}$$

$$(1-\tau)\theta_n \frac{y^*}{l^*} = \frac{1-\phi}{\phi}\frac{c^*}{(1-l^*)}, \tag{36}$$

$$y^* = \mu z^{*\theta_z} k^{*\theta_k} l^{*\theta_l} \tag{37}$$

$$g_n g_y z^* = \frac{1}{(1+\gamma)^{\frac{1-\theta_z}{\theta_z}}} z^* + \frac{1}{\pi} x_z^* \tag{38}$$

$$g_n g_y k^* = (1-\delta_k)k^* + x_k^*, \tag{39}$$

$$c^* + g^* = y^* - x_z^* - x_k^* \tag{40}$$

$$g^* = \tau[y^* - x_z^* - \delta^k k^*]. \tag{41}$$

3.2 Calibration

Table 1 presents the calibrated parameters for the U.S. and Chinese economies in the closed economy case. The two countries are calibrated to have the same parameters except for three shocks to population growth, barrier to technology adoption, and preference fore leisure. The parameters for which the two countries share the same values are calibrated to the U.S. economy. For the other variables, we set the U.S. population to grow at an annual rate of 1 percent and normalize the barrier to their technology adoption and preference to leisure to 1. Figure 5 charts the exogenous path of technology barrier and population growth we give to the Chinese economy and Figure 6 charts the exogenous path of the preference shock. Note that the technology barrier has dropped drastically between 1970 to 2000. This series is chosen to match the relative GDP growth rate during the same period (Figure 7). By comparison, the population growth rate dropped much more gradually. The gradual increase in preference for leisure is to capture the increased longevity in China.

Table 1. Calibration

parameter		U.S.	China
θ_k	physical capital income share	0.2	
θ_l	labor income share	0.4	
θ_z	firm technology capital share	0.4	
w_0	initial technology frontier	1	
γ	growth rate of technology frontier	0.0085	
τ	income tax rate	0.24	
σ	parameter of risk aversion	2	
ϕ	weight on consumption in utility	0.4379	
δ_k	physical capital depreciation rate	0.06	
β	discount rate	0.95	
g_n	population growth rate	1.012	see figure 5
ζ	leisure preference shock	1.3	see figure 6
π	barrier to technology adoption	1	see figure 5

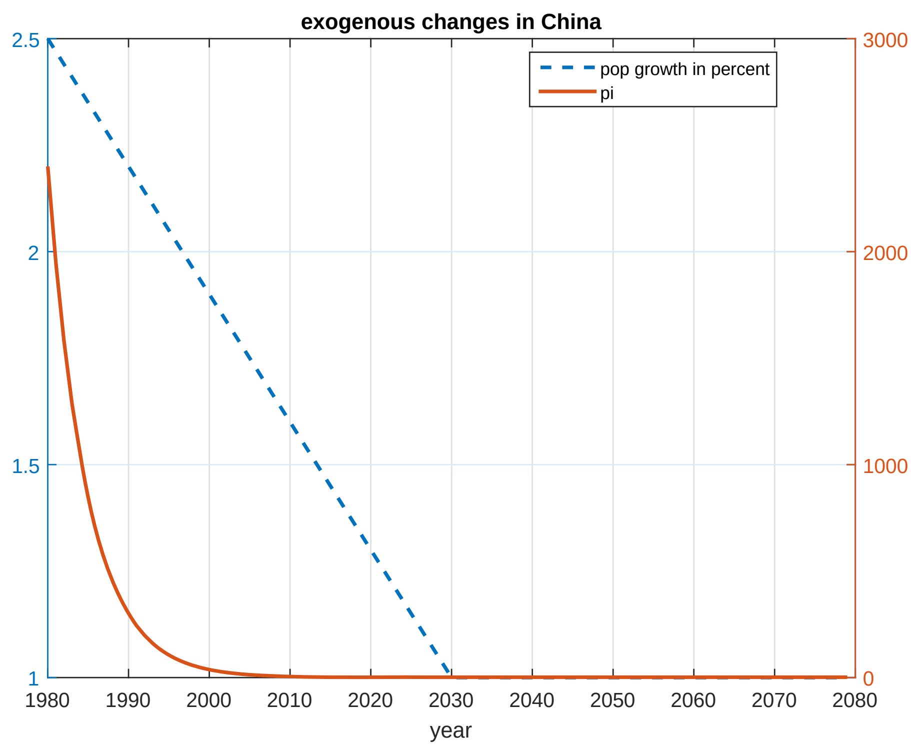


Figure 5. Exogenous Processes: Population Growth Rates and Barrier to Technology Adoption

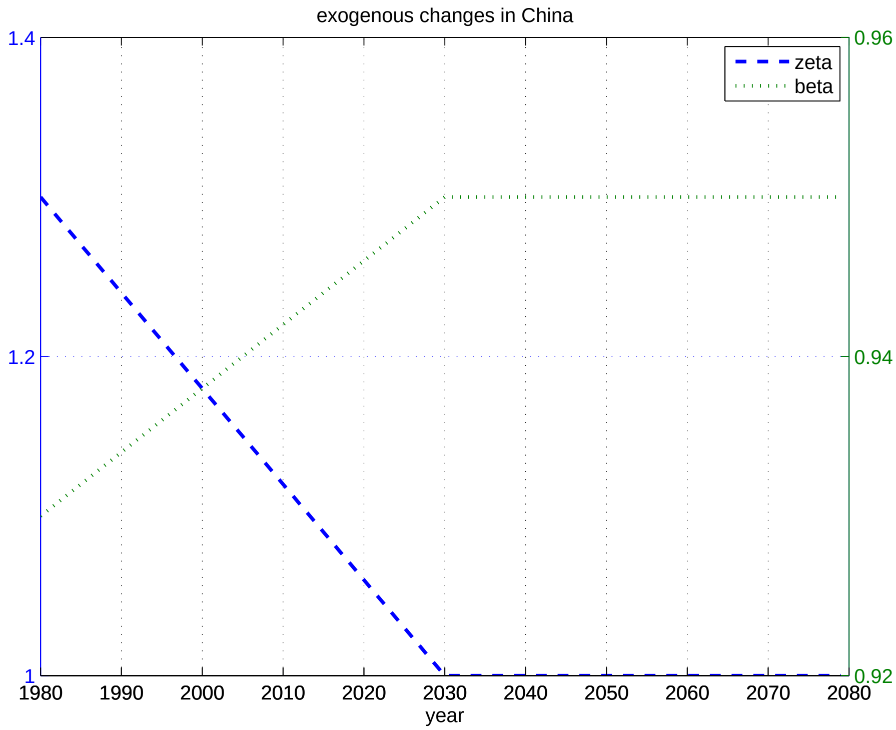


Figure 6. Exogenous Processes: Preference Shock

3.3 Results

Table 2 presents our initial and final steady states for both economies. While the U.S. economy is on a balanced growth path, the Chinese economy starts at one initial steady state which we calibrate to 1980 and then with changes in the technology adoption barrier, population growth rate, and preference for leisure, transitions into a new steady state where the barrier becomes 3, while the population growth rate slows down to the same level as the U.S. economy and the preference for leisure increases to 1.3, the same as that in the U.S. economy.

Table 2. Steady States in Closed Economies

		U.S. Steady State	China Initial Steady State	New Steady State
K/Y	physical capital output ratio	2.448	2.448	2.448
C/Y	consumption output ratio	0.499	0.441	0.499
X_k/Y	physical capital investment rate	0.214	0.251	0.14
Z/Y	technology output ratio	1.788	0.0004	0.596
X_z/Y	technological capital investment rate	0.108	0.136	0.108
G/Y	gov. consumption output ratio	0.179	0.172	0.179
l	labor supply	0.322	0.349	0.322
$\widetilde{\omega}_t$	wage rate	2.916	0.042	1.458
r_{kt}	return to K	0.163	0.163	0.163
r_{zt}	return to Z	0.112	548.1	0.447
g_y	per capita output growth rate	0.0171	0.0171	0.017
ζ	leisure preference shock	1.3	1.0	1.3
g_n	population growth rate	0.010	0.025	0.010

Figure 7 presents the relative per capita GDP growth rate of the two economies along the transition. Our calibration replicates the catch up of the Chinese economy to the U.S. economy reasonably well. Figure 8 presents total output, capital, consumption, and labor supply of the Chinese economy during the transition path and Figure 9 presents wages, interest rate, and the implicit return to technological capital during the transition. When the technology adoption barrier first declines, labor input increases significantly to take advantage of the higher productivity. By period 40, however, labor supply begins to decline while physical capital investment catches up. The technological capital input also increases as lower technological adoption barrier makes it more profitable to invest in technology adoption. As a result, interest rate goes up significantly and reaches a peak of about 30 percent by period 40. By contrast, wages increase gradually over time with the take off occurring at around period 25. The return to technological capital investment is very high initially but falls almost immediately. By period 20, the return to technological capital investment approaches the steady value of 0.447.

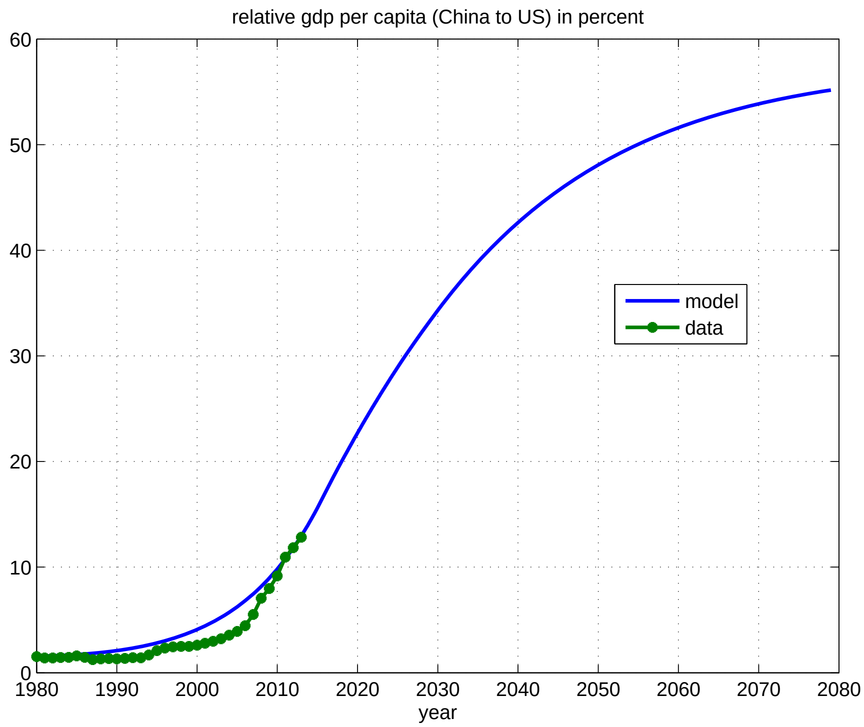


Figure 7. Relative per Capita GDP (China/U.S.): the Baseline

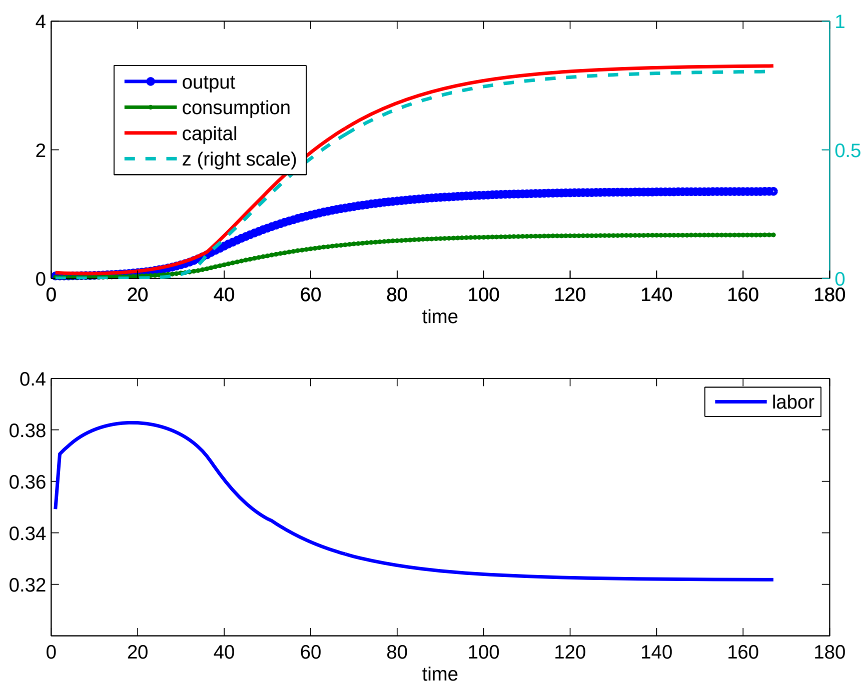


Figure 8. Transition Dynamics of Quantity: the Baseline

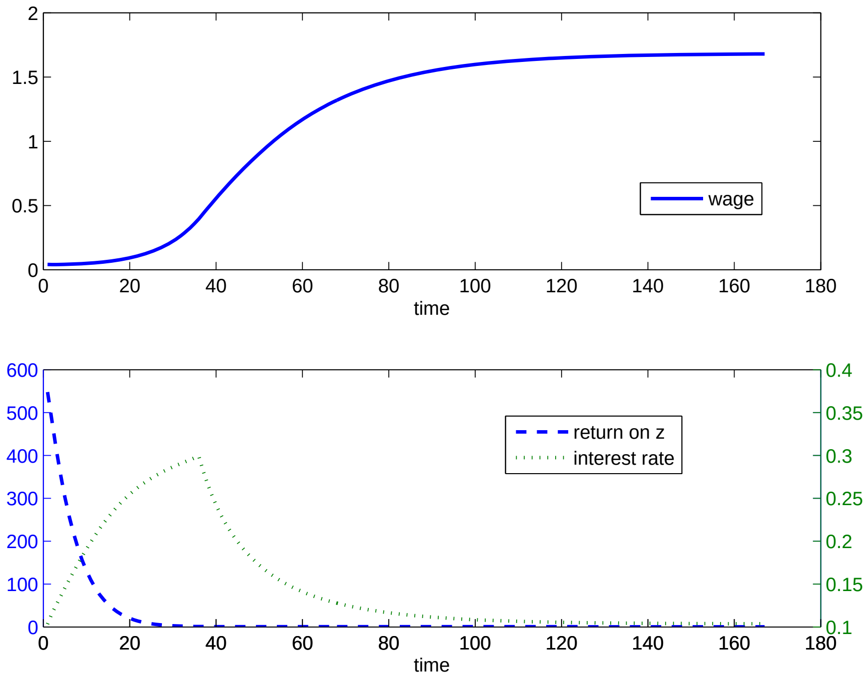


Figure 9. Transition Dynamics of Prices: the Baseline

3.4 Decomposition

In this subsection, we decompose the effects of the three shocks, namely population growth, barriers to technology adoption, and preference to leisure, and investigate how they each contribute to the economic transition and new steady state. Table 3 presents final steady states for each case. The change of preference parameter ζ does not affect the steady state at all.

Table 3. Steady States Comparison in Closed Economies

		China			
		all shocks	g_n only	π only	ζ only
K/Y	physical capital output ratio	2.448	2.448	2.448	2.448
C/Y	consumption output ratio	0.499	0.499	0.441	0.441
X_k/Y	physical capital investment rate	0.214	0.251	0.214	0.251
Z/Y	technology output ratio	0.596	0.0004	0.596	0.0004
X_z/Y	technological capital investment rate	0.108	0.108	0.136	0.136
G/Y	gov. consumption output ratio	0.179	0.179	0.172	0.172
l	labor supply	0.322	0.322	0.349	0.349
$\widetilde{\omega}_t$	wage rate	1.458	0.042	1.458	0.042
r_{kt}	return to K	0.163	0.163	0.163	0.163
r_{zt}	return to Z	0.336	548.1	0.336	548.1
g_y	per capita output growth rate	0.017	0.017	0.017	0.017
ζ	leisure preference shock	1.3	1.0	1.0	1.3
g_n	population growth rate	0.010	0.01	0.025	0.025
π	barrier to technology adoption	3	4900	3	4900

Figures 10, 11, and 12 chart the scenario where we keep only the population growth shock**.** Chart 9 indicates that without the dramatic decline in the barriers to technology adoption, the relative per capita GDP of the Chinese economy to that of the U.S. economy hardly moves compared to the benchmark. This is more evident in Figure 11. Investments in technology adoption remain much smaller than in the benchmark. With declining population growth, both capital input and labor supply drop over time. As a result, total output and consumption also decline somewhat. For prices, the return to technology adoption drops in the first period but then moves up before declines again at period 40. This is in strong contrast to the benchmark case where the return to technology adoption is much higher because of the decline in barrier and it declines monotonically over time. The wage rate, on the other hand, jumps up in the first period, then moves down gradually. It goes up again after period 40. The interest rate to physical capital investment increases for the first 40 periods and then falls slightly afterwards. In the first period, wage rate jumps and return to Z and K drops because K and Z are predetermined before the shock and a reduction in g_n increases $\widetilde{z}_t$ and $\widetilde{k}_t$.

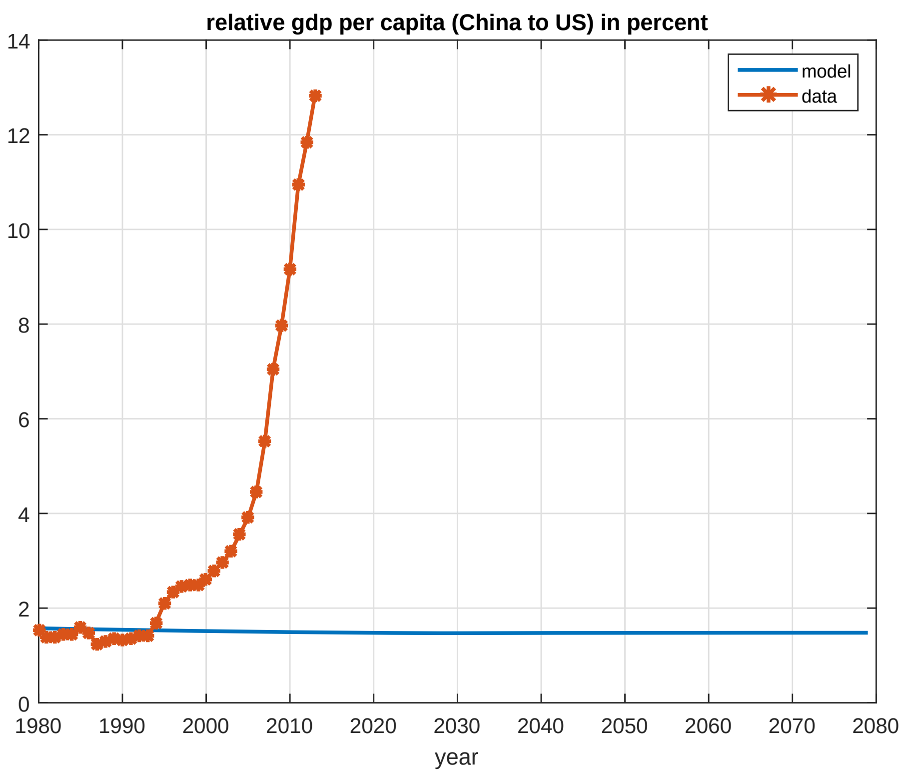


Figure 10. Relative per Capita GDP (China/U.S.): Population Growth Rate Changes Only

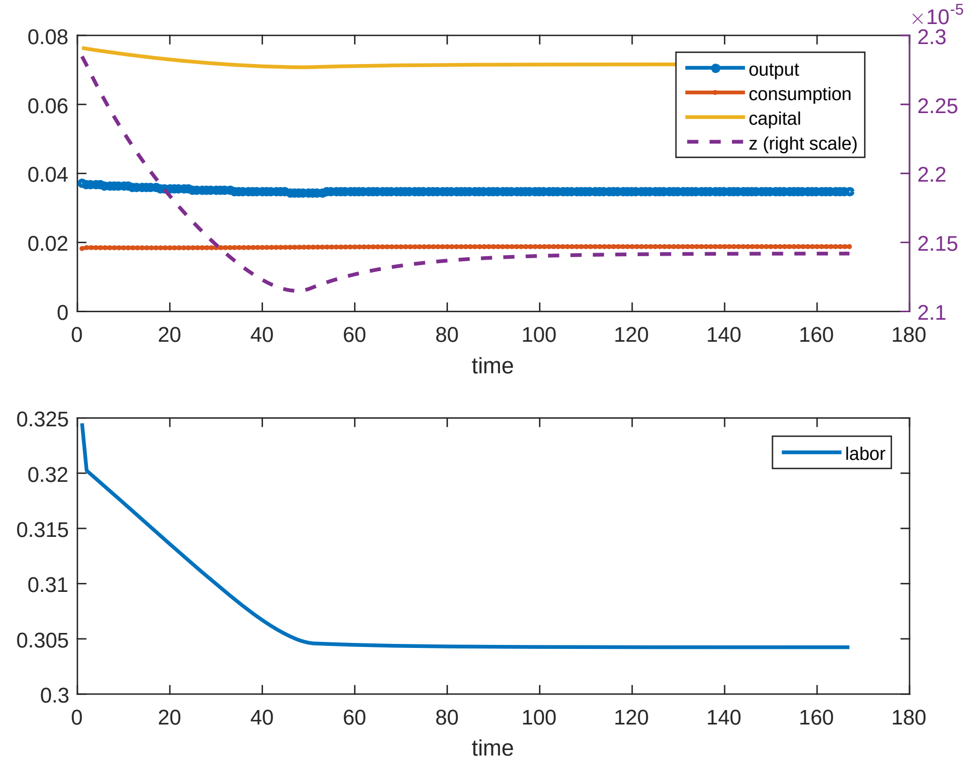


Figure 11. Transition Dynamics of Quantity: Population Growth Rate Changes Only

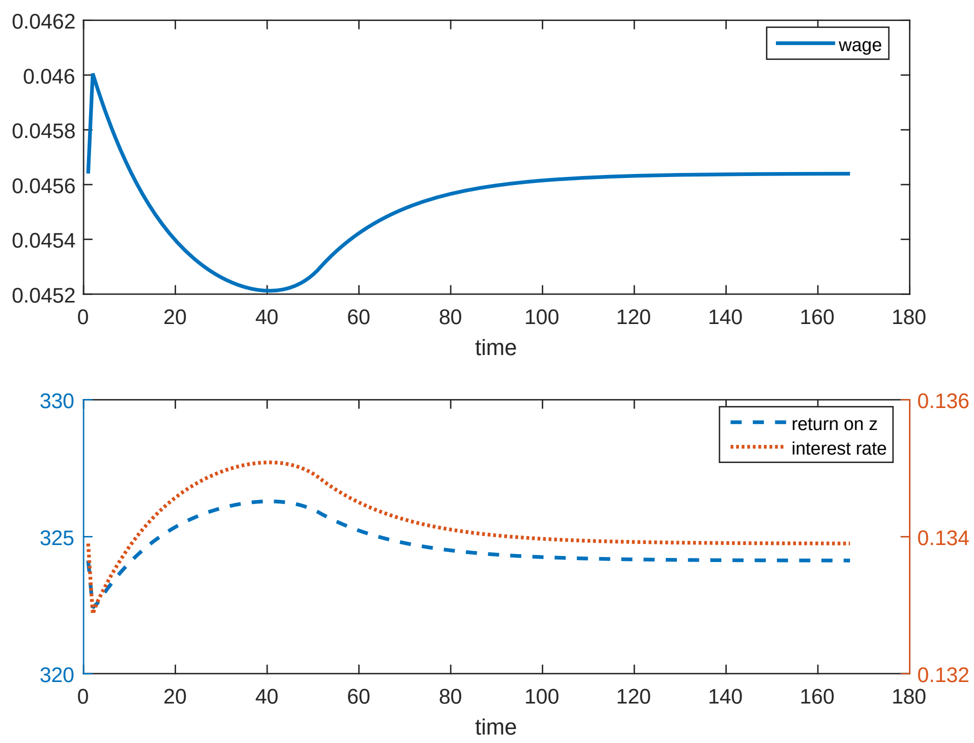


Figure 12. Transition Dynamics of Prices: Population Growth Rate Changes Only

Figures 13, 14, and 15 describe the scenario when we keep only the barrier to technology adoption shock. The economic evolution resembles very much that in the benchmark with the exception that the transition to the new steady state is slightly faster than in the benchmark economy. This is particularly true with labor supply, it jumps up immediately and reaches its peak at period 20 instead of period 40.

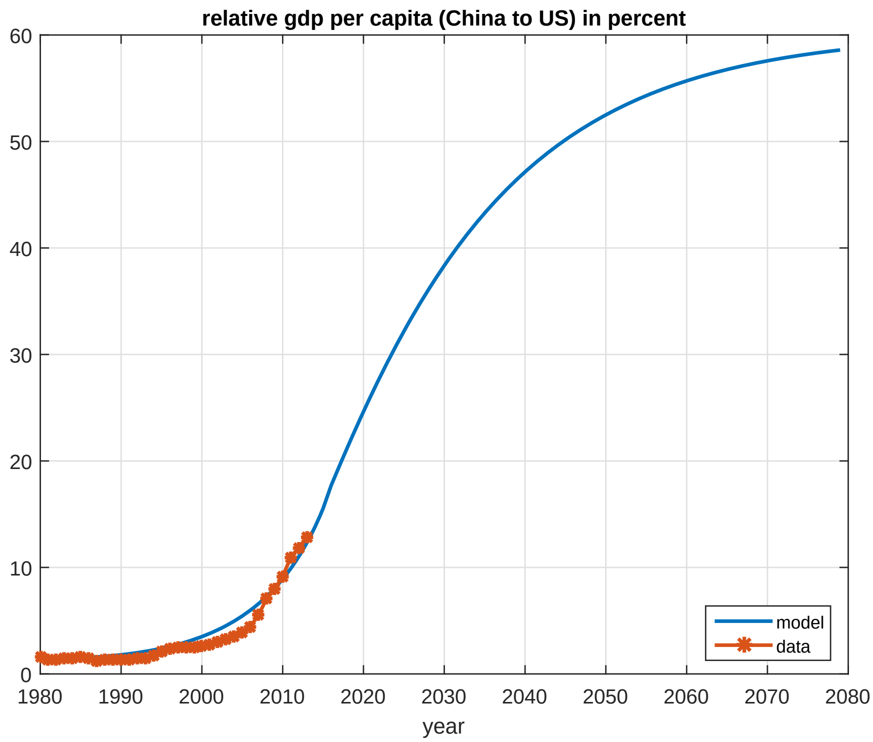


Figure 13. Relative per Capita GDP (China/U.S.): Barriers to Technology Adoption Shock Only

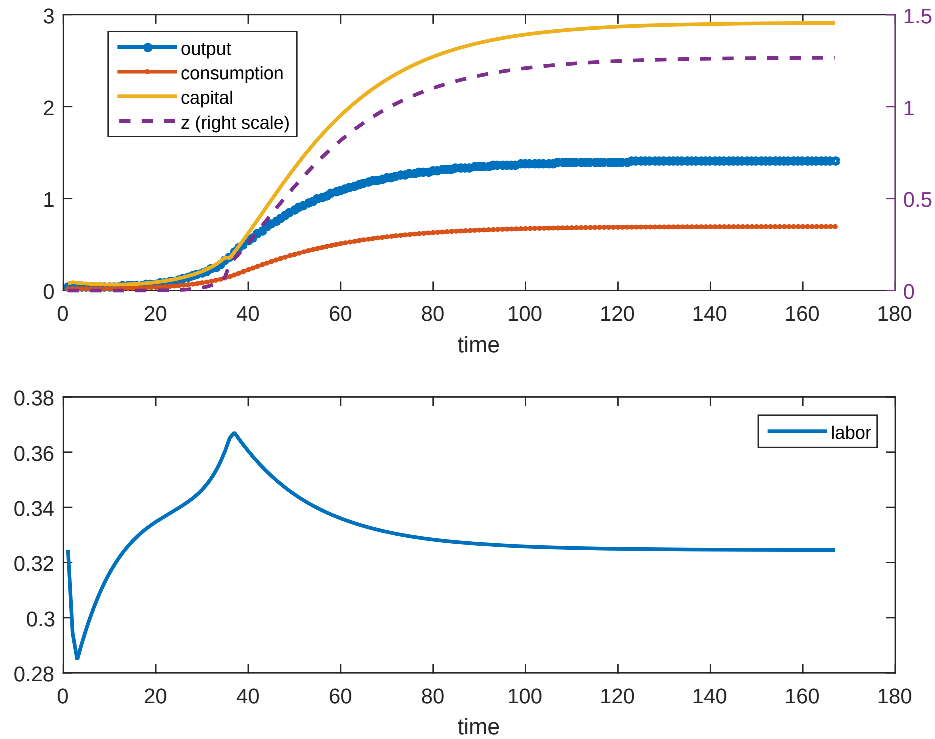
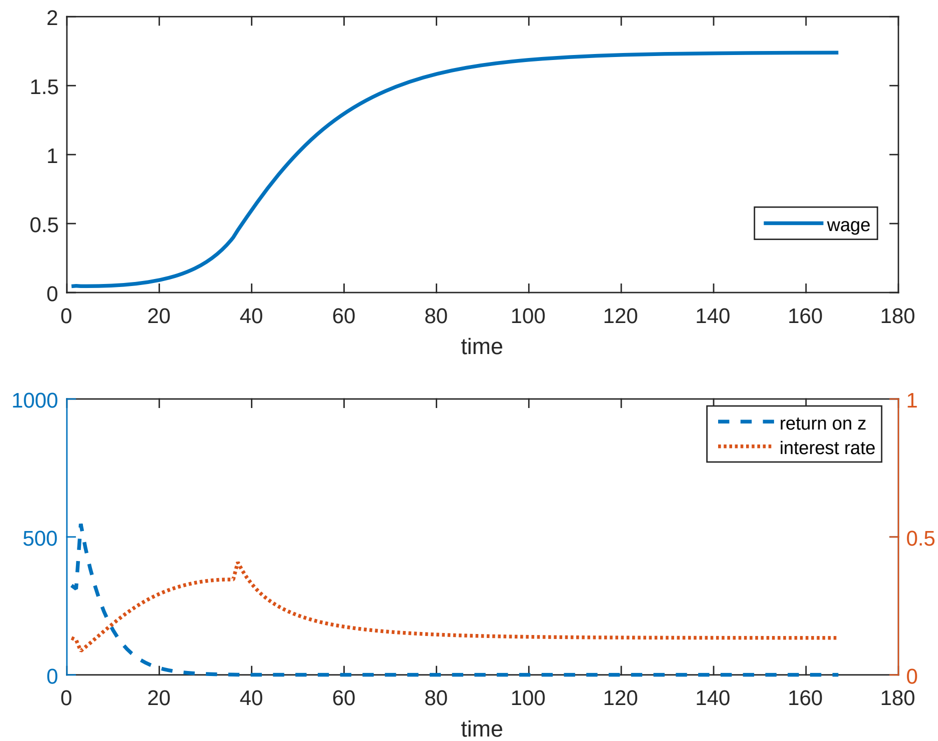


Figure 14. Transition Dynamics of Quantities: Barriers to Technology Adoption Shock Only



Figue 15. Transition Dynamics of Prices: Barriers to Technology Adoption Shock Only

Finally, we study the case where we keep only the preference shock to leisure as summarized in Figures 16 to 18. In our framework, an increase in the preference shock from 1.0 to 1.3 actually amounts to people living longer, resulting a lower discount rate during the transition. Again, without the changes in barrier to technology adoption, the Chinese per capita GDP does not catch up with that of the U.S. as the data indicate. Investments in technology adoption is very small despite the fact that the return increases initially. Labor supply increases initially before the preference shock kicks in and then flattens out. As a result, wage initially declines before rising again. Return to physical capital investment follows the return to technology adoption but at lower

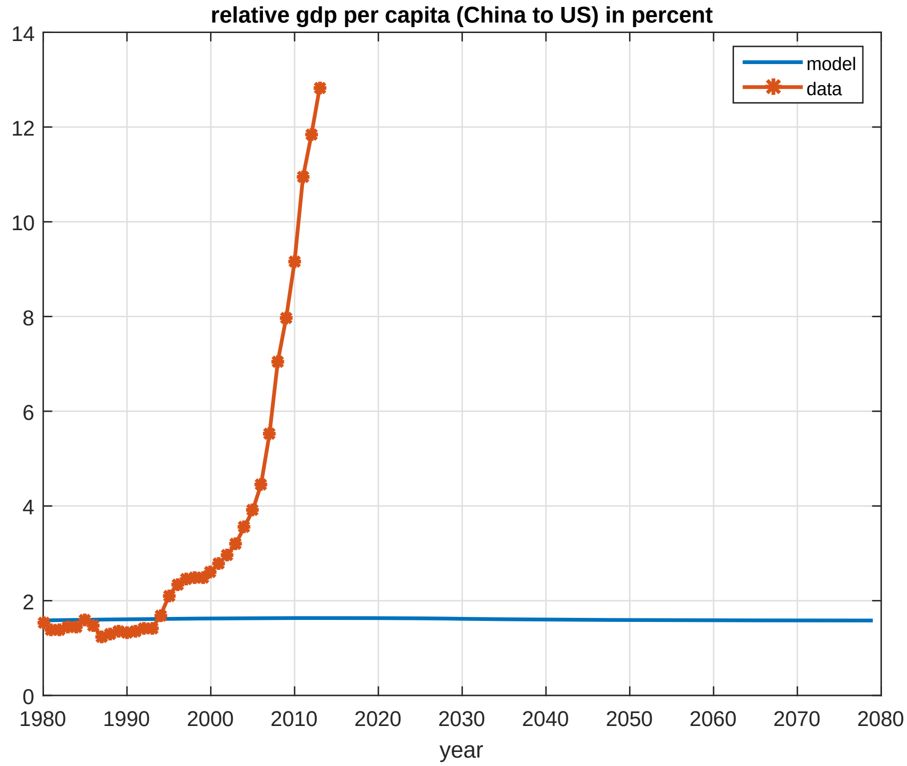


Figure 16. Relative per Capita GDP (China/U.S.): Preference Shock to Leisure Only

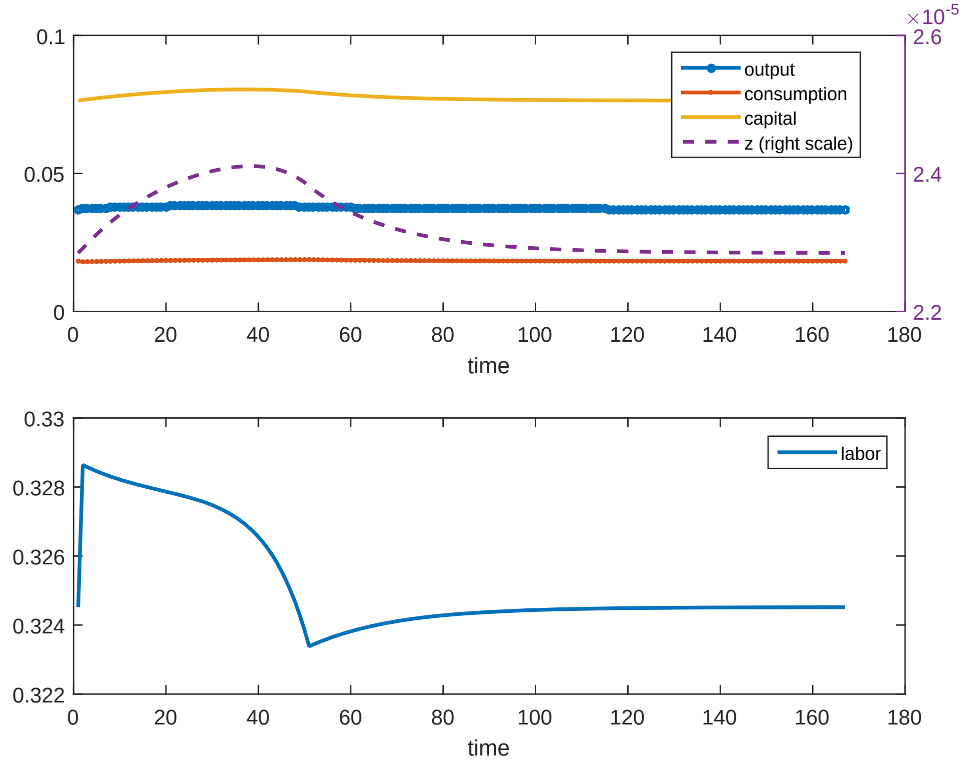


Figure 17: Transition Dynamics of Quantity: Preference Shock to Leisure Only

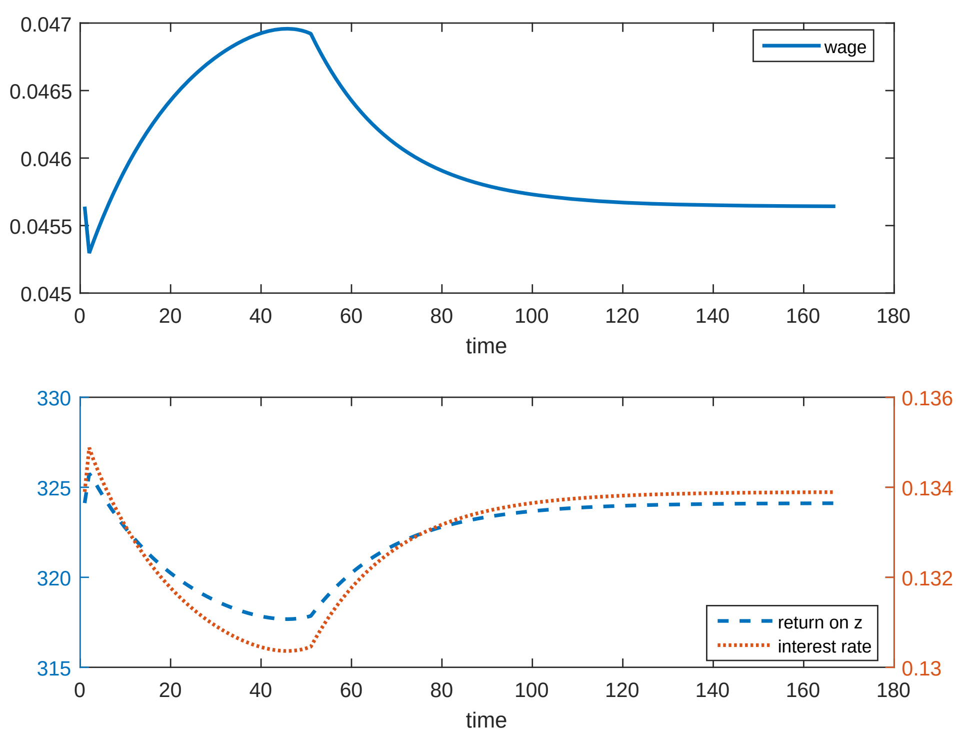


Figure 18: Transition Dynamics of Prices: Preference to Leisure Shock Only

4 Two country model

to be continued

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