

Skills, occupations, and the allocation of talent over the business cycle

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Abstract

Business cycles have heterogeneous effects in the labor market. Workers with different characteristics and skills are not equally affected by the cyclical swings in the demand for labor of different occupations. This paper studies the employment and occupational decisions of workers with heterogeneous skill portfolios and how business cycle conditions affect the patterns of sorting of workers into occupations, the accumulation of skills, and earnings. For this I develop and estimate a dynamic general equilibrium Roy (1951) model with aggregate shocks and propose a new method to characterize the solution recursively. The estimation shows that workers' comparative advantage strongly influence their occupational choices, but changes in business cycle conditions affect the sorting of workers into occupations and can have long lasting effect in the accumulation of skills. The model is able to capture the direction and cyclical patterns of occupational switching. I compute measures of the missallocation of talent and labor productivity losses from recessions.

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1 Introduction

Business cycles have heterogeneous effects in the labor market. The employment of younger and less educated workers fluctuates more than the employment of older and highly educated workers. Similarly, employment in production and construction occupations typically experience a sharper contraction in recessions, far more pronounced than that for professionals and clerical occupations. This rises the questions, how do workers with different characteristics and skills adapt to the asymmetric cyclical swings in the labor demand? To what extent business cycles affect the allocation of talent and labor productivity?

To answer these questions, I develop and estimate a dynamic [Roy \(1951\)](#) model of the labor market where workers have a multidimensional set of skills. As in [Roy \(1951\)](#), differences in skill endowments determine workers' comparative advantage in different occupations. In the model, occupational choice is influenced by workers' comparative advantage, non-pecuniary costs of switching occupations, and economic conditions in the different labor markets. The labor demand for different occupations changes with the business cycle, thus affecting wages and workers' employment and occupational choices.

The economic literature has long recognized that worker's skills and human capital may evolve over time and depend on past occupation and employment choices. For example, earnings may increase on-the-job, as workers accumulate sector or occupation specific experience. They may also decrease in periods of non-employment as these erode worker's human capital.¹ In this way, while business cycles may have temporary effects on the demand for labor, they may have long-lasting effects on worker's skills, productivity and future employment.

The estimation and solution of a dynamic [Roy \(1951\)](#) model presents several challenges. First, workers' underlying skill-portfolio and its dynamic transitions are unobserved and must be estimated from data on workers' occupational choices and wages. I follow the intuition in [Yamaguchi \(2012\)](#) and model skills as a hidden Markov process which I estimate using a large panel of U.S. workers over the period 2000 to 2008. The data comes from the Survey of Income and Program Participation, and I use the 2001 and 2004 panels. However, my estimation procedure is different from [Yamaguchi \(2012\)](#) as I use the Expectation Maximization (EM) algorithm and follow closely [Arcidiacono and Miller \(2011\)](#), in particular, their two-stage estimator. The use of the EM algorithm together with the conditional choice probabilities (CCPs) greatly simplifies the computational burden of the estimation and allows for more flexible structural assumptions.

Second, solving for equilibrium in an economy with many segmented labor markets in the presence of aggregate shocks raises an important technical difficulty. When workers

¹See for example, [Altonji and Shakotko \(1987\)](#), [Topel \(1991\)](#), [Dustmann and Meghir \(2005\)](#) and [Kambourov and Manovskii \(2009\)](#), who estimate returns to experience and tenure. Similarly, [Pissarides \(1992\)](#), [Ljungqvist and Sargent \(1998\)](#) and [Huckfeldt \(2016\)](#) study how worker's skills and human capital are affected by job loss and unemployment.

decide on whether to work and where to reallocate, they must be able to make rational predictions about the evolution of wages in the different labor markets. These wages, in turn, depend on the labor supply and reallocation decisions of all workers in the economy. Put simply, individual decisions depend on the distribution of workers in all sectors of the economy, and with aggregate shocks, this distribution is not time invariant. In quantitative applications involving a non-trivial number of labor markets, using moments to summarize this distribution, as in [Krusell and Smith \(1998\)](#), is not useful, since this would require a large number of moments. In this paper I follow a large literature on dynamic discrete choice models with random utility to characterize the worker's problem and propose a new procedure to solve for the recursive equilibrium with aggregate uncertainty. There are two key steps. First, using idiosyncratic extreme-value shocks in the worker's problem, I can aggregate discrete individual work and mobility choices into the smooth optimization problem of a market-representative agent. Second, I use perturbation methods. These methods can accommodate a very large number of state variables, which in this model consists of all aggregate shocks and the distribution of workers across all the different labor markets.

This paper is closely related to [Lee and Wolpin \(2006\)](#), who develop and estimate a dynamic general equilibrium Roy model to study the large increase in employment in the U.S. service sector since 1950. I relate to this work in two ways. First, I study the dynamics of worker's reallocation across U.S. labor markets, but while they study the long run dynamics, here I focus on business cycle aspects. I too find that while employment in different industries and occupations experience large fluctuations, wage differentials do not, a result that is consistent with [McLaughlin and Bils \(2001\)](#). Second, at a technical level, [Lee and Wolpin \(2006\)](#) solve their model using [Krusell and Smith \(1998\)](#) approximate rationality method, while I propose a different procedure to solve general equilibrium dynamic discrete choice models with aggregate uncertainty.

[Poletaev and Robinson \(2008\)](#) use the Dictionary of Occupational Titles, which nowadays has morphed into O*Net, to characterize the skill portfolio of jobs which they use to measure the distance between tasks. They find that worker's wages are negatively affected when they switch to a job with a very different skill content. My model shares this feature but I also show how occupational choice is affected by the skill content of different occupations and how it varies with the business cycle. [Yamaguchi \(2012\)](#), [Lindenlaub et al. \(2012\)](#) and [Lise and Postel-Vinay \(2015\)](#) also use O*Net to measure workers' multidimensional skills and study the complementarity between worker's skills and occupations and how it shapes the patterns of occupational choice.

In this work, I relate to a large literature started by [Lucas and Prescott \(1974\)](#) on reallocation and unemployment in competitive labor markets, better known as island models. Essentially all this literature analyzes stationary environments where aggregate shocks are ruled out and little is known about cyclical properties of island models. To the best of my knowledge, [Veracierto \(2008\)](#) is the sole exception. By assuming complete markets, using

employment lotteries, and introducing random search, he is able to write down the problem as that of a representative household or a social planner, who instructs workers to move out of islands.² In addition, in [Veracierto \(2008\)](#), as in most island models, labor market flows and unemployment duration are not defined. The reason is that, in equilibrium, a positive measure of workers are indifferent between being employed or not. This implies that we can interchange the employment status of many agents without altering equilibrium. However, by doing that, key labor market variables like transitions between employment status and unemployment duration are not uniquely pinned down as they depend on individual labor histories. I introduce aggregate shocks into an island model, but I do not assume complete markets or employment lotteries. Moreover, in my model workers are heterogeneous and labor market histories are pinned down from micro-foundations.

My model also relates to a recent literature that studies how workers' specific skills affect their employment and career decisions during the business cycle. [Carrillo-Tudela and Visschers \(2013\)](#) develop a model of occupational mobility and reallocation over the business cycle, which shares features of neoclassical island models and search and matching labor models. Similarly, [Huckfeldt \(2016\)](#) studies the interplay between recessions and different types of occupations to explain the large earnings cost of job loss during recessions. These models have a block recursive structure ([Menzio and Shi, 2011](#)), which makes the problem very tractable. However, they both assume that all occupations are perfect substitutes in production/consumption, as workers in the different occupations produce the same final consumption good. A different assumption, for example a Cobb-Douglas or CES production function that aggregates the output of the different occupations, breaks the block recursive structure. In my model I have nested CES production and consumption aggregators, and I propose how to solve for the recursive equilibrium using a different approach. [Moscarini \(2001\)](#) develops a Roy model with search-and-matching frictions to study the occupational choice of workers with multidimensional skills over the business cycle. In this work I abstract from search-and-matching frictions, but am able to enrich the set of occupations, skills and workers characteristics to capture other features of the data.

This paper is organized as follows. Section 2 presents the dynamic general equilibrium Roy model with aggregate uncertainty. Section 3 discusses some functional form assumptions, calibration and estimation. Section 4 describes the solution method, focusing largely on the intuition and leaving other details in the Appendix. Section 5 presents features of the model, like the distribution of skills, their dynamic evolution, and so on, in a stationary environment where aggregate shocks are muted. Section 6 show how occupational choice, wages and the distribution of skills are affected by business cycle conditions in different labor markets. Section 7 concludes.

²This reduces to a stochastic nonlinear control problem which he solves using a linear-quadratic approximation. [Judd \(1996\)](#) discusses the limitations and potential problems that may arise in using linear-quadratic approximations.

2 A model of occupational choice, skills and labor market dynamics

Time is discrete and infinite. There is a finite number of industries in the economy. In each industry $i = 1, \dots, I$, competitive firms produce a different good or service. Production exhibits constant returns to scale and uses labor from J occupations and intermediate inputs from all industries. Firms face no fixed costs of production nor any entry or exit barriers. I assume the production function in each industry is:

$$q_{i,t} = A_{i,t} \left[\psi_i \left(\sum_{j=1}^J \psi_{ji} L_{ji,t}^{(\eta-1)/\eta} \right)^{\frac{\eta}{(\eta-1)}} \right]^{\alpha_{li}} \prod_{h=1}^I \varphi_{hi,t}^{\alpha_{hi}} \quad (1)$$

where $q_{i,t}$ is total output of good i at time t , $\varphi_{hi,t}$ is the amount of good h that is used as intermediate in sector i and $L_{ji,t}$ is the amount of efficiency units of labor from occupation $j = 1, \dots, J$ hired by the firm in sector i . α_{li} and α_{hi} are positive parameters. The assumption of constant returns to scale implies that $\alpha_{li} + \sum_{h=1}^I \alpha_{hi} = 1$. ψ_i and ψ_{ji} are positive parameters related to the share of labor from occupation j in value added of industry i , and η is the elasticity of substitution between different occupations in production. Total factor productivity, A_i , is random, and may be correlated over time and across sectors.

There is a measure one of workers which can be in one of two employment states: employed or non-employed. Each worker can be employed in only one of J occupations. Employed workers obtain labor income which they use to consume final goods. The non-employed obtain utility in terms of home production or a consumption equivalent value of leisure.

Workers are heterogeneous in several dimensions. They differ in terms of age, education, their occupation and employment status last period (or last occupation if non-employed), and skills. Workers' skills are described by a vector in which different elements denote the endowment of different types of skills (i.e., manual, cognitive, and so on). The combination of all these characteristics defines a worker type denoted by τ . The total efficiency units of labor of a worker with type τ employed in occupation j is $x_{j\tau}$. In this way, workers can have different productivity in different occupations as in [Roy \(1951\)](#). Workers' type evolve over time according to the transition function $\pi(\tau'|j\tau)$, which depend on workers' skills, occupation and employment status. This transition can accommodate important features discussed in the literature like skill upgrading and accumulation of occupational human capital while employed, skill depreciation if non-employed, skills mismatch and so on.

Workers use their income to consume a Cobb-Douglas basket of final goods from all industries. The expenditure shares of good i in this basket is equal to γ_i . I use the "ideal" price index stemming from this Cobb-Douglas consumption basket as the *numeraire* of the economy to which all other prices are normalized. Future utility is discounted with factor

β .

Workers face two decisions each period, whether to work or not, and whether to switch occupations. These decisions are affected by economic conditions, costs from switching occupations and idiosyncratic shocks. I denote the shocks affecting the occupational decision by vector ϵ of size J , where each element is the preference for occupation j . Similarly, I denote by ν the idiosyncratic shock to home production affecting the work/non-work decision. Moreover, non-employed workers are able to return to work with probability $(1 - \delta)$. In this way, δ captures, in reduced-form, additional frictions that may be present in the labor market and are not explicitly modeled here.

The timing of the model is as follows. All aggregate shocks are realized and observed by all agents in the economy. Workers start the period with skills τ and attached to some occupation, which is the result of past career choices. With probability $(1 - \delta)$, non-employed workers have the option to work in the current period. Workers observe ν and decide to work or not. After this first decision, those working observe ϵ and decide their occupation.³ Next, production and consumption take place. Finally, workers' type evolves, and death and aging shocks are realized.

Firms and workers are price takers in the goods and labor markets. The wage per efficient unit of labor in each occupation, $w_{j,t}$, and the price for industry goods, $p_{i,t}$, are set by perfect competition.

2.1 Firm's problem

As already mentioned, production is constant returns to scale and there are no fixed costs nor any entry or exit barriers for producers. Therefore, the total number of competitive producers in an industry is not pinned-down and, due to symmetry, it suffices to analyze the problem of the "representative firm" in an industry. In every period the firm solves the following static problem:

$$\max_{\{L_{ji,t}\}_{j=1}^J, \{\varphi_{hi,t}\}_{h=1}^I} p_{i,t} A_{i,t} \left[\psi_i \left(\sum_{j=1}^J \psi_{ji} L_{ji,t}^{(\eta-1)/\eta} \right)^{\frac{\eta}{(\eta-1)}} \right]^{\alpha_{li}} \prod_{h=1}^I \varphi_{hi,t}^{(1-\alpha_{hi})} - \sum_{j=1}^J w_{j,t} L_{ji,t} - \sum_{h=1}^I p_{h,t} \varphi_{hi,t}$$

³I assume a sequential choice (sequential logit) for exposition purposes only. The model would be virtually identical if the decision were simultaneous but with a slightly different error structure (i.e. Generalized Extreme Value and nested logit).

The optimal, minimum-cost demands for labor and intermediate goods conditional on output level $q_{i,t}$ are:

$$L_{ji,t} = \alpha_{li} p_{i,t} q_{i,t} \left(\frac{\psi_{ji}}{p_{j,t}} \right)^\eta \frac{1}{\sum_{j=1}^J \psi_{ji}^\eta p_{j,t}^{1-\eta}} \quad (2)$$

$$\varphi(ki, t) = \alpha_{hi} \frac{p_{i,t} q_{i,t}}{p_{h,t}} \quad (3)$$

where I simplified the previous expressions by assuming that the price of good i equals its marginal cost of production, a condition that holds in equilibrium due to free entry and no fixed costs of production.

2.2 Worker's problem

Since reallocation is costly, the worker's problem is dynamic. In particular, my assumptions allow me to specify the problem recursively as a dynamic discrete choice model with random utility. The problem of a worker of type τ that was employed in the previous period in labor market j can be written recursively as,

$$V^E(j, \tau, \nu, \Upsilon) = \max \left\{ \mathbb{E}_\epsilon \left[\max_{j'} \left\{ \frac{w(j', \Upsilon) x_{j'\tau}}{P(\Upsilon)} - \chi_{jj'\tau} + \mathbb{E}_{\nu', \tau', \Upsilon'} \left[\beta V^E(j', \tau', \nu', \Upsilon') \mid j, \tau, \Upsilon \right] + \sigma \epsilon_j \right\} \right], \right. \\ \left. b_\tau + \kappa \nu + \mathbb{E}_{\nu', \tau', \Upsilon'} \left[\beta V^N(j, \tau', \nu', \Upsilon') \mid j, \tau, \Upsilon \right] \right\} \quad (4)$$

where $V^E(j, \tau, \nu, \Upsilon)$ and $V^N(j, \tau, \nu, \Upsilon)$ are the value of working and not working, respectively, for workers of type τ with past occupation j and idiosyncratic shock ν . The variable Υ summarizes the aggregate state of the economy and contains the TFP shocks and the distribution of workers by type, occupations and employment status. Parameters σ and κ scale the variance of the idiosyncratic shocks.

According to equation (4), a worker will chose to work in the current period if the value from working, the first term in the maximum, is higher than the value of not working. The idiosyncratic shock ν , introduces heterogeneity in the work/non-work decision. Focusing on the value of working, the first part is the period utility from consumption of market goods, where $P(\Upsilon) = \prod_{i=1}^I \left(\frac{p(i, \Upsilon)}{\gamma_i} \right)^{\gamma_i}$ is the ideal price index of the Cobb-Douglas consumption basket. Parameter $\chi_{jj'\tau}$ is the cost of switching occupation from j to j' , which may vary with workers' characteristics. The last part is the continuation value. The reallocation decision is captured by the choice of j' . Workers choose the best labor market to work and this decision is influenced by economic conditions in the different occupations, switching costs and idiosyncratic preferences ϵ_j .⁴ These preference shocks will create heterogeneity across otherwise-identical workers.⁵ Given that at the time of the first decision

⁴Note that I deviate from the classic [Lucas and Prescott \(1974\)](#) model of segmented labor markets by introducing additional sources of worker heterogeneity within a labor market. Later I discuss how this assumptions are useful to reconcile the model with the data and to solve the model numerically.

⁵In real life workers change occupations for many reasons which are not captured by purely economic

the preference shocks are not observed, the expectation is taken with respect of these shocks.

The value of non-employment consists on the flow payoff in terms of home production, b_τ , which may depend on worker's characteristics, and the continuation value. b can be interpreted as the common component of home production across workers and ν the idiosyncratic component.

The value for a worker that was non-employed last period is:

$$V^N(j, \tau, \nu, \Upsilon) = \delta \{b_\tau + \kappa \nu + \mathbb{E}_{\nu', \tau', \Upsilon'} [\beta V^N(j, \tau', \nu', \Upsilon') | j, \tau, \Upsilon]\} + (1 - \delta) V^E(j, \tau, \nu, \Upsilon)$$

Equation (5) shows that a non-employed will have the option to work with probability $(1 - \delta)$. In this case, the worker faces the same choices as a worker that was employed last period, that is, will decide whether to work or not, and her occupation.

It is worth stressing that workers search over all occupations, including their past one. From the point of view of a single worker, search over occupations is directed, as she chooses her most preferred occupation taking into account her comparative advantage, economic conditions and the realization of the ϵ shocks. In this way, occupations are an inspection good. However, from an *ex-ante* perspective, search can be interpreted as semi-directed, as ϵ introduce randomness across workers, leading to different mobility patterns. Parameter σ affects how random or directed search is.

2.3 Aggregation and Equilibrium

I assume that each element in ϵ is distributed iid, mean-zero Type I extreme value and that ν is distributed iid, mean-zero logistic. These are widely used and studied distributions in the literature of dynamic discrete choice models (Rust, 1987, 1994).⁶ I define $V^E(j, \tau, \Upsilon) = \mathbb{E}_\nu [V^E(j, \tau, \nu, \Upsilon)]$ and $V^N(j, \tau, \Upsilon) = \mathbb{E}_\nu [V^N(j, \tau, \nu, \Upsilon)]$, which are the expected employment and non-employment lifetime utilities, respectively, of a worker in occupation j and type τ which has not yet observed the realization of the shock ν .

Proposition 1. *Assume each element in ϵ is distributed iid, mean-zero, Type I Extreme Value, and ν is distributed iid, mean-zero logistic, then*

$$V^E(j, \tau, \Upsilon) = \kappa \log \left(\exp \left[\frac{\sigma}{\kappa} \log \left(\sum_{j'} e^{\frac{w(j', \Upsilon) x_{j' \tau}}{P(\Upsilon)} - \chi_{jj' \tau} + \beta \mathbb{E}[V^E(j', \tau', \Upsilon') | j, \tau, \Upsilon]} \right) \right] + \exp \left[\frac{1}{\kappa} \mathbb{E} [b_\tau + \beta V^N(j, \tau', \Upsilon') | j, \tau, \Upsilon] \right] \right) \quad (5)$$

$$V^N(j, \tau, \Upsilon) = \delta (b_\tau + \mathbb{E} [\beta V^N(j, \tau', \Upsilon') | j, \tau, \Upsilon]) + (1 - \delta) V^E(j, \tau, \Upsilon) \quad (6)$$

differences. For example, a worker needs to move to a different city within her firm, which may trigger an occupational reallocation for the spouse.

⁶The main advantage of the extreme value and logistic distributions, which lead to their popularity, is the closed-form solution for the expectation of the maximum of a set of random variables and for the probability of a particular choice being maximal.

In addition, let $\mu^E(j, j', \tau, \Upsilon)$ be the proportion of workers with last occupation j and skills τ that decide to work in occupation j' , and $\mu^N(j, \tau, \Upsilon)$ those that decide not to work. Then,

$$\mu^E(j, j', \tau, \Upsilon) = \frac{\exp \left[\frac{\sigma}{\kappa} \log \left(\sum_{j'} e^{\frac{w(j', \Upsilon) x_{j' \tau}}{P(\Upsilon)} - \chi_{jj' \tau} + \beta \mathbb{E} [V^E(j', \tau', \Upsilon') \mid j, \tau, \Upsilon] / \sigma} \right) \right]}{\exp [V^E(j, \tau, \Upsilon) / \kappa]} \times \frac{e^{\frac{w(j', \Upsilon) x_{j' \tau}}{P(\Upsilon)} - \chi_{jj' \tau} + \beta \mathbb{E} [V^E(j', \tau', \Upsilon') \mid j, \tau, \Upsilon] / \sigma}}{\sum_{\ell} e^{\frac{w(\ell, \Upsilon) x_{\ell \tau}}{P(\Upsilon)} - \chi_{j\ell \tau} + \beta \mathbb{E} [V^E(\ell, \tau', \Upsilon') \mid j, \tau, \Upsilon] / \sigma}} \quad (7)$$

$$\mu^N(j, \tau, \Upsilon) = \frac{\exp [b_{\tau} + \beta \mathbb{E} [V^N(j, \tau', \Upsilon') \mid j, \tau, \Upsilon] / \kappa]}{\exp [V^E(j, \tau, \Upsilon) / \kappa]} \quad (8)$$

Finally, denote by $\Phi^E(j, \tau)$ and $\Phi^N(j, \tau)$ the mass of workers at the beginning of the period with skills τ , last occupation j , and past employment status E or N , respectively, then the total supply of efficiency units of labor in occupation j is

$$L^s(j, \Upsilon) = \sum_{\tau} x_{j\tau} \left[\sum_{j'} \mu^E(j', j, \tau, \Upsilon) (\Phi^E(j', \tau, \Upsilon) + (1 - \delta) \Phi^N(j', \tau, \Upsilon)) \right] \quad (9)$$

Proof. In Appendix A □

Proposition 1 is an aggregation result. All the heterogeneity induced by the shocks ν and ϵ can be integrated out. $V^E(j, \tau, \Upsilon)$ and $V^N(j, \tau, \Upsilon)$ have a structural interpretation as the utilities of the "representative agents" with last occupation j , type τ , by employment status. Contrary to other aggregation results in macroeconomics which lead to a representative agent for the whole economy (for example by using employment lotteries as in Hansen (1985) and Rogerson (1988)), here there are many representative agents, one per occupation, skill level and employment status. Moreover, as workers move out of some occupations and into others, the mass of workers that each of these agents represent evolves endogenously in response to the changing economic conditions.

Note that $\mu^E(j, j, \tau, \Upsilon)$ is the proportion of workers that decide to work in the same occupation they did before. In other words, although these workers have the option to switch occupations, they find optimal, given their comparative advantage, switching costs and economic conditions, to stay in their current occupation.

Definition. A *Recursive Competitive Equilibrium* in this economy is a set of functions for: prices $p(i, \Upsilon)$, wages $w(j, \Upsilon)$, labor supply $L^s(j, \Upsilon)$ and demand $L^d(i, \Upsilon)$, production of goods $q(i, \Upsilon)$, demand for intermediate inputs φ_{hi} and demand for final consumption goods $c(ji, \Upsilon)$, for all industries $i, h = \{1 \dots I\}$ and occupations $j = \{1 \dots J\}$; policy functions on employment and mobility $\mu^E(j, j', \tau, \Upsilon)$, $\mu^N(j, \tau, \Upsilon)$, value functions for the previously employed and nonemployed,

$V^E(j, \tau, \Upsilon)$ and $V^N(j, \tau, \Upsilon)$, a distribution of workers $\Phi^E(j, \tau, \Upsilon)$ and $\Phi^N(j, \tau, \Upsilon)$, for all occupations $j, j' = \{1 \dots J\}$, skills τ and past employment status, and an operator G over the aggregate state Υ such that:

- Given prices and wages, $V^E(j, \tau, \Upsilon)$ and $V^N(j, \tau, \Upsilon)$, are the maximum lifetime utilities of occupation j , skills τ 's representative agent (or ex-ante utility of the worker), optimal labor supply decisions in occupation j aggregate to $L^s(j, \Upsilon)$, optimal work and mobility decisions aggregate to $\mu^E(j, j', \tau, \Upsilon)$, $\mu^N(j, \tau, \Upsilon)$. The optimal demands for good i from workers in occupation j aggregate to $c(ji, \Upsilon)$.
- Given prices and wages, firms maximize profits. $L^d(ji, \Upsilon)$ and $\varphi(ki, \Upsilon)$ are the optimal (cost minimizing) demand for labor and intermediate goods conditional on production level $q(i, \Upsilon)$.
- Firms make zero profits in all markets at all times.
- All Labor and goods markets clear.
- $\Upsilon' = G(\Upsilon)$ and G is consistent with the employment and mobility decisions.

It is important to note that with a finite number of sectors and occupations, TFP shocks will have aggregate effects in the economy. The equilibrium is not stationary and the distribution of workers over sectors is not time invariant. These shocks affect the incentives to move as workers trade-off macroeconomic conditions in different labor markets versus their own comparative advantage for each occupation and the costs of moving.

It is convenient to define here the solution to a version of the model with no aggregate uncertainty. A stationary equilibrium in this model requires that TFP shocks are equal to their expected value at all times. In this case, the distribution of workers over occupations is time-invariant and wages are constant. Nonetheless, workers transit along a stationary distribution by effect of their idiosyncratic type and preference shocks, generating gross flows. Gross job creation and destruction will be positive but exactly offset each other at all times in this stationary equilibrium, leading to an invariant distribution of employment and non-employment across labor markets. Similarly in a stationary economy, gross mobility across occupations is positive but, on net, will cancel.⁷ The characteristics of the stationary version in this model are different from that of [Lucas and Prescott \(1974\)](#) model. This is a direct consequence of the finite number of industries and occupations (islands).

3 Parametrization

The model has a very large number of parameters. To pin-down their values I use a combination of calibration and estimation. Exogenously calibrated parameters include most of

⁷[Coen-Pirani \(2010\)](#) is also able to generate gross flows that are different from net flows in an island model by introducing a binary taste shock.

the production coefficients, consumption shares, together with parameters related to aging and death shocks and the discount factor β . In addition, the reemployment opportunity δ will be chosen such that the model is able to match the average level of non-employment to the data. Most parameters on utility, mobility costs and skill transitions are estimated assuming an stationary environment. Conditional on these values, the elasticity of substitution across occupations and the variance of the idiosyncratic shocks are estimated using information on time series fluctuations on aggregate variables. In addition, I impose some structure on workers' productivity. In particular, I assume that $x_{j\tau}$ is related to workers' characteristics and occupations' characteristics in the following way:

$$\begin{aligned} \log(x_{j\tau}) = & a_0 + a_e \mathbb{I}(\text{edu} - \text{gender} = e) + a_g \mathbb{I}(\text{age} = g) + a_j \mathbb{I}(\text{occ} = j) \\ & + a_{m1} \text{man} + a_{m2} y_{mj} \text{man} + a_{c1} \text{cog} + a_{c2} y_{cj} \text{cog} \end{aligned} \quad (10)$$

where $\mathbb{I}$ is the indicator function. I assume that the observed components of τ are gender, age and education, while the unobservables are the levels of manual (*man*) and cognitive (*cog*) skills of workers. y_{mj} and y_{cj} are the levels of manual and cognitive skill requirements for occupation j , respectively, which are estimated from O*Net as described in the Appendix. This functional form implies that labor productivity depends on the absolute level of skill endowments of workers, given by the values of coefficients a_{m1} and a_{c1} , and also on how useful these skills are for a particular occupation j , according to coefficients a_{m2} and a_{c2} . If a_{m2} and a_{c2} are positive, workers' skills and occupations are complements in production, increasing workers' labor productivity.⁸

In the empirical analysis I use three education levels, leading to six education-gender categories and two age groups, young and old. Young workers are those with ages between 25 and 35 and old workers have ages between 35 and 55.⁹ Finally, I model unobserved heterogeneity as a finite mixture (Heckman and Singer, 1984). For this I discretize skills into three levels of manual and three levels of cognitive skills.

Before describing the details of the calibration and estimation, I discuss some normalizations.

3.1 Normalizations

Some absolute levels in the model are not pinned down. First, there is monetary neutrality in the model and the level of nominal prices is not defined. I normalize the price level of the aggregate consumption basket to be one at all times and states (the *numeraire*). Second, production exhibits constant return to scale and the demand for goods is homothetic. In a stationary equilibrium this implies that, for example, doubling all workers' productivities

⁸It would be possible to adopt a more flexible specification for productivity but this would increase the set of parameters to be estimated.

⁹As is usual in the literature, I chose these age ranges in an effort to abstract from education and retirement decisions.

will lead to an economy that is twice as big, but otherwise is identical. A normalization on the level of the aggregate economy is needed. For this I normalize aggregate demand, $\sum_j L(j)w(j)$, to be equal to one in the stationary equilibrium.

Given data on earnings, the wage per efficient units of labor and the total amount of units of labor cannot be identified separately in the stationary equilibrium. To see this, note that if we double the wage in occupation j and cut the productivity of workers in occupation j by half, nothing changes as workers will have the same earnings in that occupation and thus, no incentives to change their decision. For this, I will work with the levels of earnings of workers ($w(j)x_{j\tau}$) in the stationary equilibrium.¹⁰ Similarly, firms will spend the same share of production in labor from that occupation.

In addition, the units of skills are not identified separately from the coefficients in (10). Thus, I normalize skills to be between 0 and 1.

Finally, note that in a stationary equilibrium we can characterize the demand for labor given parameters on production and expenditure shares. To see this, note that in equilibrium, the value of production (supply) for each good i must equal the value of its final demand and its intermediate demand. Using the normalization on the aggregate final demand, we have

$$(\mathbf{p} \circ \mathbf{q}) = \boldsymbol{\gamma} + \boldsymbol{\alpha}(\mathbf{p} \circ \mathbf{q}) \quad (11)$$

where $\circ$ denotes the element by element (or Hadamard) product, $\boldsymbol{\gamma}$ is the vector of parameters α_i (final demand shares) and $\boldsymbol{\alpha}$ is the matrix with parameters α_{hi} (the share of intermediate inputs in production). Thus, to the extent that the matrix $(\mathbf{I} - \boldsymbol{\alpha})$ is invertible, we can pin-down the value of production in each industry.

Next, using equation (2) and under the calibrated share form for the CES production (see the Appendix for a discussion), we have

$$(\mathbf{w} \circ \mathbf{L}) = \boldsymbol{\psi} [\hat{\boldsymbol{\alpha}} \circ (\mathbf{p} \circ \mathbf{q})] = \boldsymbol{\psi} [\hat{\boldsymbol{\alpha}} \circ ((\mathbf{I} - \boldsymbol{\alpha})^{-1} \boldsymbol{\gamma})] \quad (12)$$

where $\hat{\boldsymbol{\alpha}}$ is the vector of parameters α_{li} (the shares of value added in production) and $\boldsymbol{\psi}$ is the matrix with the shares of occupation j in the value added of industry i .

This equation states that the value of the demand for labor in the different occupations in the stationary equilibrium is fully pinned down by parameters (and the normalization of the aggregate demand).

3.2 Estimation

Most of the parameters on utility, mobility costs and and skill transitions are estimated assuming a stationary environment. This estimation takes the calibrated values for the dis-

¹⁰Outside of the stationary equilibrium, fluctuations in earnings are driven exclusively by fluctuations in wages, which will move earnings proportionately. Exogenous TFP shocks and the structure of the model will determine how these wages fluctuate. Alternatively I could have assumed $w(j) = 1$ for all j in the stationary equilibrium.

count factor, aging and death probabilities as given, as well as values for parameters κ and σ which will be estimated in a different way to be described next. The sample consists of N individuals for which I observe their sequences of employment status, d_{it}^E , occupation, d_{it}^j , and earnings, ω_{it} , for $t = 1, \dots, T$, and also their age and education.¹¹ Details on the data source and sample selection are contained in the Appendix.

To simplify notation, let τ capture only the unobserved component of the state variables of workers. That is, the notation abstracts from age and education, but these are observable and taken into account in the estimation. In addition, note that for individuals not working, $d_{it}^j = 1$ for $j = j_{it-1}$ and 0 otherwise. The estimation uses the Expectation Maximization (EM) algorithm and follows closely [Arcidiacono and Miller \(2011\)](#), in particular, their two-stage estimator.

Let the likelihood of observing $(d_{it}^E, d_{it}^j, \omega_{it})$ given the individual's past employment status (E_{it-1}) and occupation (j_{it-1}) be,

$$\begin{aligned}\mathcal{L}(d_{it}^E = 1, d_{it}^j, \omega_{it} | \tau, E_{it-1} = 1, j_{it-1}; \theta, \pi, \mu) &= \prod_{j=1}^J \left[\ell_{jt}^{\omega_i}(\tau; \theta, \pi, \mu) \ell_{jt}^E(j_{t-1}, \tau; \theta, \pi, \mu) \right]^{d_{it}^j} \\ \mathcal{L}(d_{it}^E = 0, d_{it}^j, \omega_{it} | \tau, E_{it-1} = 1, j_{it-1}; \theta, \pi, \mu) &= \prod_{j=1}^J \left[\ell_{jt}^N(j_{t-1}, \tau; \theta, \pi, \mu) \right]^{d_{it}^j} \\ \mathcal{L}(d_{it}^E = 1, d_{it}^j, \omega_{it} | \tau, E_{it-1} = 0, j_{it-1}; \theta, \pi, \mu) &= \prod_{j=1}^J \left[\ell_{jt}^{\omega_i}(\tau; \theta, \pi, \mu) \ell_{jt}^E(j_{t-1}, \tau; \theta, \pi, \mu) \right]^{d_{it}^j} (1 - \delta) \\ \mathcal{L}(d_{it}^E = 0, d_{it}^j, \omega_{it} | \tau, E_{it-1} = 0, j_{it-1}; \theta, \pi, \mu) &= \prod_{j=1}^J \left[\ell_{jt}^N(j_{t-1}, \tau; \theta, \pi, \mu) \right]^{d_{it}^j} (1 - \delta) \delta\end{aligned}$$

where $\ell_{jt}^{\omega_i}$ is the likelihood of the observed earnings, ℓ_{jt}^E is the likelihood of working in occupation j and ℓ_{jt}^N is the likelihood of being non-employed with last occupation j . In constructing the likelihood for earnings, I assume that observed log-earnings are equal to (10) plus a normally distributed shock with mean zero and constant variance to be estimated.

Let $\pi(\tau_t | j_{it-1}, \tau_{t-1}, E_{it-1}) = \bar{\pi}(\tau_t | j_{it-1}, \tau_{t-1})^{E_{it-1}} \underline{\pi}(\tau_t | j_{it-1}, \tau_{t-1})^{(1-E_{it-1})}$, then the likelihood of the observed sequences for an individual is,

$$\begin{aligned}\mathcal{L}(d_i^E, d_i^j, \omega_i | \theta, \pi, \mu) &= \sum_{\tau_1} \sum_{\tau_2} \dots \sum_{\tau_T} \pi_1(\tau_1 | E_{i0}, j_{i0}) \mathcal{L}(d_{i1}^E, d_{i1}^j, \omega_{i1} | \tau, E_{i0}, j_{i0}; \theta, \pi, \mu) \times \\ &\quad \prod_{t=2}^T \mathcal{L}(d_{it}^E, d_{it}^j, \omega_{it} | \tau, E_{it-1}, j_{it-1}; \theta, \pi, \mu) \pi(\tau_t | j_{it-1}, \tau_{t-1}, E_{it-1})\end{aligned}\quad (13)$$

where $\pi_1(\tau_1, E_{i0}, j_{i0})$ is the probability that an individual i has state τ_1 , past employment E and occupation j the first period of the sample. In addition, let $L_{it}(d_i^E, d_i^j, \omega_i, \tau_{it} = \tau | \theta, \pi, \mu)$

¹¹In the data, T is between 36 to 48 months. Because of this, I abstract in estimation from death and aging.

be the joint likelihood of $\tau_{it} = \tau$ and the observed sequences (d_i^E, d_i^j, ω_i) for individual i :

$$\begin{aligned} \mathcal{L}_{it}(d_i^E, d_i^j, \omega_i, \tau_{it} = \tau | \theta, \pi, \mu) &= \sum_{\tau_1} \sum_{\tau_2} \dots \sum_{\tau_{t-1}} \sum_{\tau_{t+1}} \dots \sum_{\tau_T} \pi_1(\tau_1 | E_{i0}, j_{i0}) \mathcal{L}(d_{i1}^E, d_{i1}^j, \omega_{i1} | \tau_1, E_{i0}, j_{i0}; \theta, \pi, \mu) \times \\ &\quad \prod_{t'=2}^{t-1} \mathcal{L}(d_{it'}^E, d_{it'}^j, \omega_{it'} | \tau_{t'}, E_{it'-1}, j_{it'-1}; \theta, \pi, \mu) \pi(\tau_{t'} | j_{it'-1}, \tau_{t'-1}, E_{it'-1}) \times \\ &\quad \mathcal{L}(d_{it}^E, d_{it}^j, \omega_{it} | \tau, E_{it-1}, j_{it-1}; \theta, \pi, \mu) \pi(\tau | j_{it-1}, \tau_{t-1}, E_{it-1}) \times \\ &\quad \mathcal{L}(d_{it+1}^E, d_{it+1}^j, \omega_{it+1} | \tau_{t+1}, E_{it}, j_{it}; \theta, \pi, \mu) \pi(\tau_{t+1} | j_{it}, \tau, E_{it}) \times \\ &\quad \prod_{t'=t+2}^T \mathcal{L}(d_{it'}^E, d_{it'}^j, \omega_{it'} | \tau_{t'}, E_{it'-1}, j_{it'-1}; \theta, \pi, \mu) \pi(\tau_{t'} | j_{it'-1}, \tau_{t'-1}, E_{it'-1}) \quad (14) \end{aligned}$$

The log-likelihood of the sample is,

$$\sum_{i=1}^N \ln \mathcal{L}(d_i^E, d_i^j | \theta) \quad (15)$$

Estimation is done in two stages using the estimator proposed by [Arcidiacono and Miller \(2011\)](#), where the use of the EM algorithm together with the conditional choice probabilities (CCPs) greatly simplify the computational burden of the estimation.¹² The Appendix contains a thorough description on the estimation algorithm. Here I provide some intuitions.

[Arcidiacono and Miller \(2011\)](#), using ideas from [Hotz and Miller \(1993\)](#) and [Arcidiacono and Jones \(2003\)](#), propose a two stage estimator where in the first stage, the EM algorithm is combined with a CCP estimator. In short, the EM algorithm assigns a probability that an individual i has unobserved state τ at time t , denoted by $q_{i\tau t}$. Then treats τ as an observed variable and weights observations using $q_{i\tau t}$ to construct the likelihood. Moreover, CCPs (which in the model are μ^E and μ^N) are also constructed using $q_{i\tau t}$. In this first stage I estimate the values for the parameters in (10), the skill transitions, π , and the CCPs, μ . Given these estimated values I can obtain the stationary distributions of workers across occupations, types and employment status, and characterize the labor supply by occupation in a stationary equilibrium using,

$$\begin{aligned} \Phi^E(j, \tau) &= \sum_{j'} \sum_{\tau'} (1 - \phi) \bar{\pi}(\tau | j\tau') \mu^E(j', j, \tau') (\Phi^E(j', \tau') + (1 - \delta) \Phi^N(j', \tau')) + \phi \Phi_1^E(j, \tau) \\ \Phi^N(j, \tau) &= \sum_{\tau'} (1 - \phi) \underline{\pi}(\tau | j\tau') [\mu^N(j, \tau') (\Phi^E(j, \tau') + (1 - \delta) \Phi^N(j, \tau')) + \delta \Phi^N(j, \tau')] + \phi \Phi_1^N(j, \tau) \\ L(j) &= \sum_{\tau} x_{j\tau} \sum_{j'} \mu^E(j', j, \tau) (\Phi^E(j', \tau) + (1 - \delta) \Phi^N(j', \tau)) \end{aligned}$$

where Φ_1^E and Φ_1^N are the distribution of new young workers that enter the economy every

¹²Some other recent application of this estimator are [Scott \(2016\)](#) and [Traiberman \(2016\)](#).

period and ϕ is the death probability, which varies with age. It is important to note that the values for κ , σ , or η are not needed in this first stage, nor is it required to solve other elements of the model for this estimation.

In the second stage, the values for home production, b , and mobility costs χ are estimated conditional on values for κ , σ . The structure of the model, and in particular, its *stochastic terminal action*, greatly simplify this estimation which can be done non-parametrically. In particular, the Appendix shows that using the CCPs we can obtain model consistent estimates for the value function using the following expressions,

$$\begin{aligned} V^N(j, \tau) &= b_\tau + \beta \mathbb{E} [V^N(j, \tau') | j, \tau] - \kappa(1 - \delta) \log(\mu^N(j, \tau)) \\ V^E(j, \tau) &= b_\tau + \beta \mathbb{E} [V^N(j, \tau') | j, \tau] - \kappa \log(\mu^N(j, \tau)) \end{aligned}$$

which can, in turn, be replaced into the expressions for the work and mobility decisions in (7) and (8). The intuition is that parameters b and χ are estimated to bring the moments on mobility and work decisions in the model close to those “observed” in the data (CCPs).

The last step in estimation is for parameters κ , σ and η . I estimate these parameters by indirect inference, using time series moments on employment for the different occupations and occupational switches. For this I generate TFP shocks for the different industries from some estimated distribution (see the Appendix for details) and compute moments on employment and occupational switching. I iterate on parameters to minimize the distance between the model and the data. While this is effectively an outside loop in estimation, the computational burden is not big since these parameters do not enter the first stage estimation. Thus only the second stage has to be repeated, meaning that new set of values for b and χ have to be estimated for each new evaluation of κ , σ and η .

4 Numerical Solution

Solving and simulating general equilibrium dynamic discrete choice models with aggregate uncertainty is challenging. There is no difficulty in writing down the model, determining what the state variables are, or defining equilibrium. However, in equilibrium, prices and wages depend on the distribution of workers over all markets. Therefore, individual work and occupational decisions depend on this distribution. At the same time, this distribution is just an aggregate of workers’ decisions. [Lee and Wolpin \(2006\)](#) introduce a “forecasting” rule that workers use to predict prices and wages. For this they follow the approximate rationality method of [Krusell and Smith \(1998\)](#), and the parameters of the forecasting rule are calibrated to minimize agent’s forecasting errors. In this section I depart from [Lee and Wolpin \(2006\)](#) and propose a new approach to solve for the model’s recursive dynamic equilibrium and decision rules in a model with a large number segmented labor markets.

The model does not have a closed form solution and in order to solve it I need to rely on

numerical techniques. The state variables of the model are the exogenous TFP shocks and the distribution of workers across occupations, types and employment status. As I mentioned before, I denote the set of state variables by $\Upsilon = \left\{ \{A_i\}_{i=1}^I, \{\Phi^E(j, \tau), \Phi^N(j, \tau)\}_{j=1, \tau=1}^{J, T} \right\}$. The solution to the recursive competitive equilibrium is a set of *functions* for prices $p(i, \Upsilon)$, wages $w(j, \Upsilon)$, labor supply $L(j, \Upsilon)$, production of goods $q(i, \Upsilon)$, policy functions on employment and mobility $\mu^E(j, j', \tau, \Upsilon)$, $\mu^N(j, \tau, \Upsilon)$, value functions for the previously employed and nonemployed, $V^E(j, \tau, \Upsilon)$ and $V^N(j, \tau, \Upsilon)$, and the next period distribution of workers $\Phi^E(j, \tau, \Upsilon)$ and $\Phi^N(j, \tau, \Upsilon)$ that solves the system of equilibrium conditions for all possible values of the state variables.¹³

The high dimensionality of Υ and the fact that it contains all continuous variables makes the problem difficult to solve using standard numerical methods. Even for a small number of occupations the curse of dimensionality has a strong bite. Here I overcome this difficulty using perturbation methods. These methods are used extensively in economics, mainly in macroeconomic models with a representative agent, and can easily accommodate a very large number of state variables.¹⁴ In the model developed here there are several representative agents, one per occupation (j), type (τ), and employment status, and their relative importance (measure) changes over time as the distribution of workers changes with economic conditions. The idea of using perturbation in problems with a distribution of agents and aggregate shocks goes back to [Campbell \(1998\)](#) where he used it in analyzing entry and exit of firms over the business cycle. More recently [Reiter \(2009\)](#) used perturbation in a Bewley-Hugget-Aiyagari model with aggregate shocks. In addition, I will be perturbing the Value Function which is the key element in the extensive margin decision on whether to work and where. Perturbation of the Value function is discussed in [Judd \(1998\)](#) but has seldom been applied in economic problems. A recent exception is [Caldara et al. \(2011\)](#) where they use it to solve a problem with recursive preferences. [Kline \(2008\)](#) uses perturbation methods to solve a model with 3 sectors.

In order to use perturbation methods all the functions that characterize the equilibrium must be differentiable, as the technique relies on Taylor's and the Implicit Function theorems. In dynamic discrete choice models, preference shocks are a way to reconcile the model and the data. The reason is that in the data, workers that are otherwise identical not always make the same choices. Thus preference shocks ν and ϵ introduce unobserved individual heterogeneity bringing the model closer to the data. However, shocks ν and ϵ are also convenient for computational reasons. Specifically, they make the individual discrete problem smooth in the aggregate. To see this note that if all workers have identical preferences over discrete choices, then both at the individual and aggregate level, the optimal decisions and labor supply would not be smooth (differentiable) functions. Figure 1 shows

¹³This is a subset of all variables in the model. Once a solution for these has been found, the solution for the rest can be found immediately. The Appendix lists the set of equilibrium conditions that these variables must satisfy, which is just a collection and rearrangement of many of the equations derived in Section 3.

¹⁴For a complete exposition on perturbation theory in economics see [Judd \(1998\)](#).

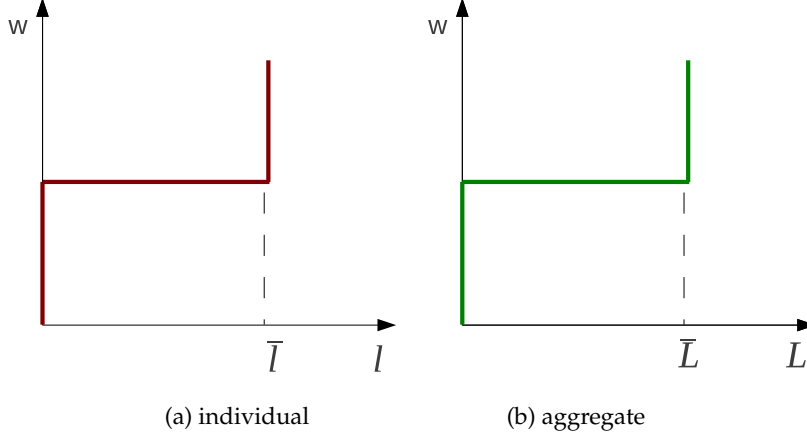


Figure 1: Labor supply with homogeneous workers in the sector

visually the labor supply for the homogeneous preferences case, in which workers have the same reservation wage (panel a), which translates to a non-smooth labor supply in the aggregate (panel b). In the model I develop here, an individual worker also has preferences of perfect substitutes on the work decision and occupation decision, but the introduction of the idiosyncratic preference shocks generates smooth functions for the (aggregate) representative agent. This can easily be seen in Figure 2, where in panel (a) I plot the labor supply of three possible individual workers, each with a different reservation wage as a result of different idiosyncratic shocks. For the occupation as a whole, the aggregate labor supply is smooth (panel b) as there is a continuum of workers, each with a different reservation wage. While in principle this aggregation can be achieved with any distributional assumption (provided the distribution is continuous and with adequate support) the use of the Extreme Value distribution substantially simplifies the problem by delivering almost closed-form expressions.¹⁵

Another key element in the solution is to characterize the law of motion for the aggregate state Υ . For the TFP shocks is trivial as these variables move exogenously. However, for the evolution of the distribution of workers over sectors and skill types, is more difficult. Given that this distribution has a discrete support, the solution will be a function that defines how the mass of workers at each sector-skill combination evolves with changes in the aggregate state.

To build some intuitions on how perturbation techniques work with a distribution, in

¹⁵Note that in the original [Lucas and Prescott \(1974\)](#) model, workers have homogeneous preferences over the employment decision in a labor market, leading to non-smooth decision rules. The reader can see the shape of the solution in the traditional island model in [Alvarez and Veracierto \(2000\)](#), where it is clear that the Value Function and the labor supply present sharp kinks and areas where the derivative is zero, violating the conditions for the Taylor and Implicit Function Theorems. Note also that the use of employment lotteries do not help overcome the problem. In fact, [Alvarez and Veracierto \(2000\)](#) assume complete markets with employment lotteries.

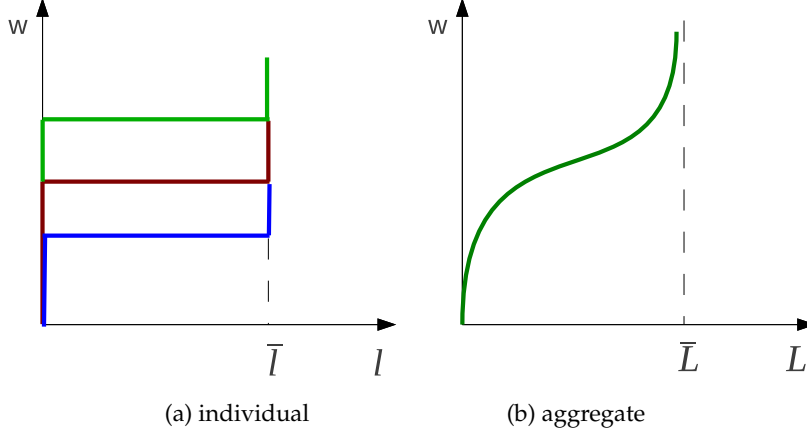


Figure 2: Labor supply with heterogeneous workers in the sector

Figure 3 I plot a possible distribution of workers over five occupations. As the number of occupations and types τ are discrete, the distribution is discrete and can be represented by a histogram where each bar denotes the fraction (measure) of workers on each j - τ pair. In this example, the light-colored bars represent the initial distribution and the dark-colored bars represent the distribution after a shock hits the economy. An unknown function F determines how this mass moves over time as the aggregate state changes, i.e. $\Phi'^E(j, \tau, \Upsilon) = F_{j\tau}(\Upsilon)$. Perturbation methods approximate with a Taylor expansion this function. In this way, an unknown functional expression is approximated by a polynomial on the variables in Υ . For example with a first order approximation we have,

$$\begin{aligned} \Phi'^E(j, \tau, \Upsilon) \approx & a_{0,j\tau} + a_{1,j\tau} A_1 + \dots + a_{I,j\tau} A_I + \dots + a_{I+1,j\tau} \Phi^E(1, 1) + \dots + \\ & + a_{I+JT,j\tau} \Phi^E(J, T) + a_{I+JT+1,j\tau} \Phi^N(1, 1) + \dots + a_{I+2JT,j\tau} \Phi^N(J, T) \end{aligned}$$

The rest of the variables in the model can be approximated in a similar way. The coefficients of this polynomial are functions of the deep parameters of the model and their value can be found numerically using methods that are now standard in economics. For example, for a first order approximation one can use [Blanchard and Kahn \(1980\)](#) or [Sims \(2002\)](#) and for a second order approximation [Gomme and Klein \(2011\)](#). The intuition is that the value of the coefficients is such that, as aggregate uncertainty is reduced to zero, the solution is closer to the stationary equilibrium of the economy. Moreover, the coefficients ensure that after the economy is hit by a shock, all endogenous control variables assume values that are consistent with a unique saddle-path equilibrium, provided such equilibrium exists, which can be verified numerically. For models where the actual decision rules are linear, the method discussed here will find the exact solution. Further technical details on the numerical solution are described in the Appendix and a complete exposition can be found in [Judd \(1998\)](#).

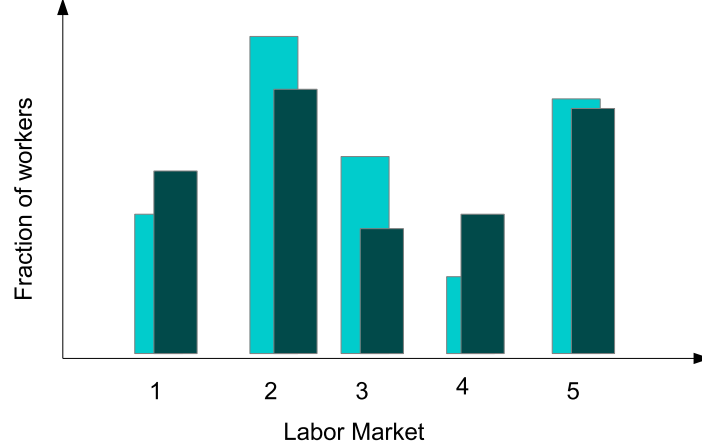


Figure 3: Perturbation of the distribution

It is worth highlighting that the proposed solution method differs from [Lee and Wolpin \(2006\)](#), where to overcome the curse of dimensionality, the approximate forecasting rule typically contains a small set of variables. In the method I present here there is no need to propose a rule for prices that agents follow. Agents in the model are fully rational and understand the economy completely. Both workers' decisions and prices are mutually consistent and the solution of the model delivers functions for prices and wages that depend on all the state variables of the model, namely all aggregate shocks and the full distribution of workers over labor markets.¹⁶

5 Estimation results, occupational choice and skills in the stationary equilibrium

In this section I present the estimation results and describe the features of the model assuming a stationary equilibrium. This “long-run” view will be a clean benchmark to which compare how workers' occupations and skills are affected by business cycle conditions.

5.1 skills and productivity

Table 1 shows the estimated coefficients from equation (10), which describe how workers' characteristics and skills affect their productivity (efficient units of labor) in the different occupations. The estimated coefficients show that the age and education premiums are much lower than in a simple Mincer regression that does not account for unobserved skills. The intuition is simple, age and education are correlated with skills since skills accumulate with time and more educated workers tend to have higher skills. Regarding the returns to skills, the table shows that there are strong complementarities between worker's skills

¹⁶Numerical errors may nonetheless arise if the order of the Taylor expansion is not sufficient to capture the curvature in the decision rules.

and occupational skill requirements, where the estimated skill requirements are shown in Figure 4. For example, workers with high cognitive skills can get 200 log points higher earnings in managerial occupations than a similar worker with low cognitive skills, all else equal. However, the estimation shows important differences between manual and cognitive skills. On the one hand, the estimated complementarity between workers' manual skills and occupation requirements is not as strong as that for cognitive skills. This implies that, comparative advantage will not have such a strong pull on that dimension. On the other hand, the returns to the absolute levels of skills show an opposite pattern, with the returns to manual skills being higher than for cognitive skills.

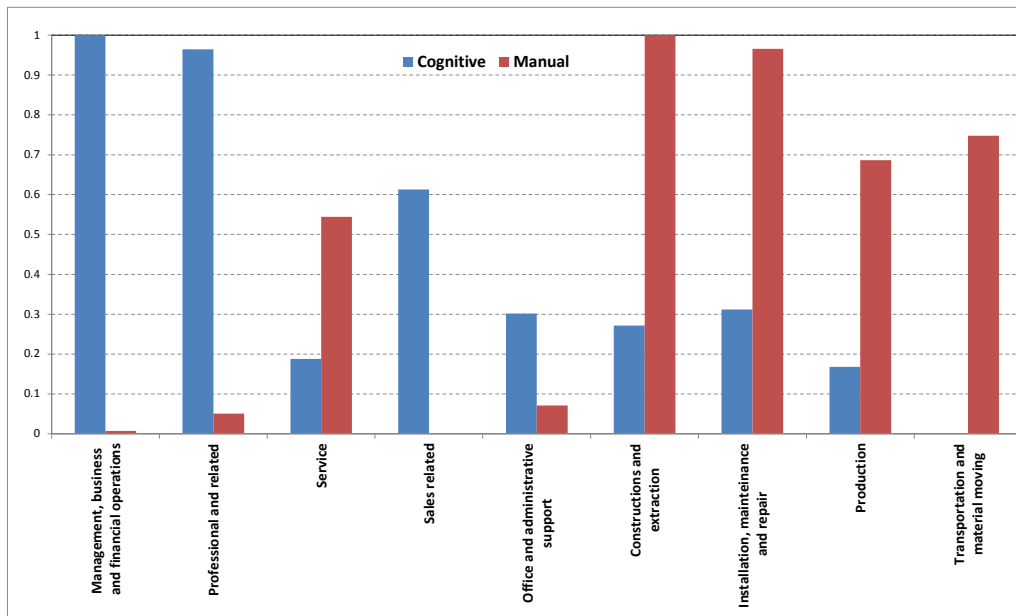
Table 1: Productivity parameters and skill complementarities

	log earnings
demographics	
old	0.05
female HS	0.12
female College+	0.20
male HS dropout	0.19
male HS	0.40
male College+	0.61
skills	
manual	1.27
manual x occ requirement	0.46
cognitive	0.70
cognitive x occ requirement	1.30
Occupation dummy	Yes
Variance of residual	0.04

5.2 The distribution of skills

How are cognitive and manual skills distributed in the population? How do people with different skills sort into occupations? These two questions are answered here. Figure 5 shows the distribution of cognitive and manual skills in the population (top panel), for the non-employed (bottom-left panel) and for the employed (bottom-right panel). A few important results emerge. First, people with low skill endowments have a comparative advantage for non-market activities (home production), and thus represent a large share of non-employment. Second, among the employed, there is a mild negative correlation in skills. That is, workers with high manual skills tend to have low cognitive skill and vice versa.

Figure 4: Occupation skill requirements



Skill requirements estimated by principal component analysis using data from ONet 19.0.

Figure 5: Estimated distribution of skills in the population

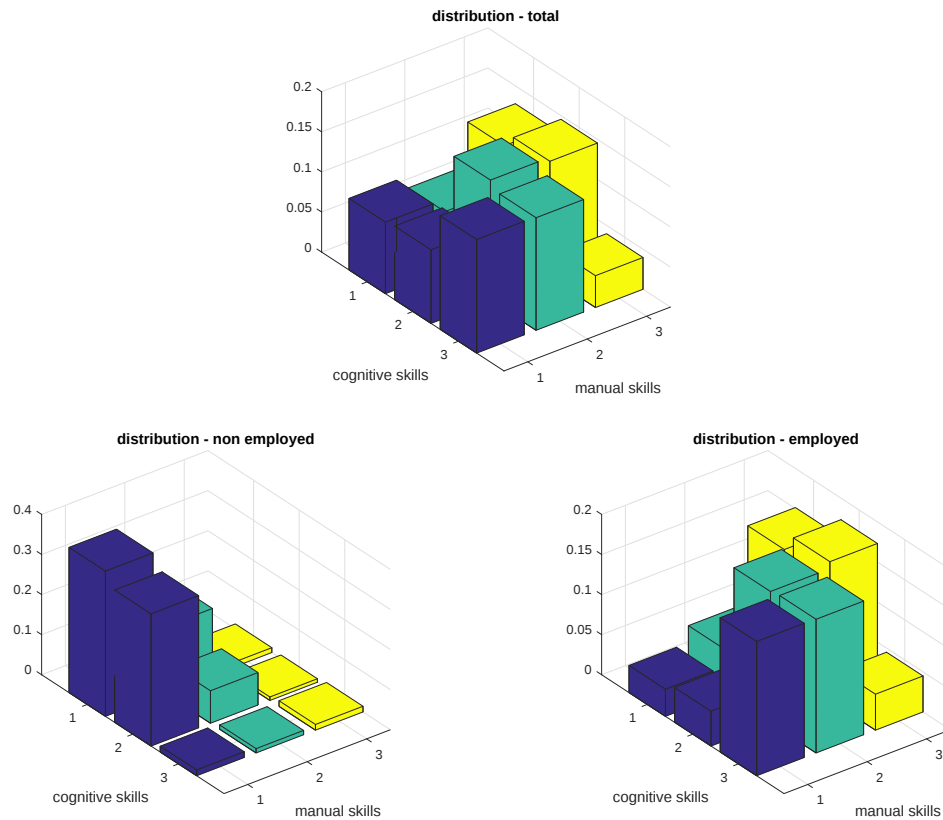


Figure 6: Relative density of skills by education groups

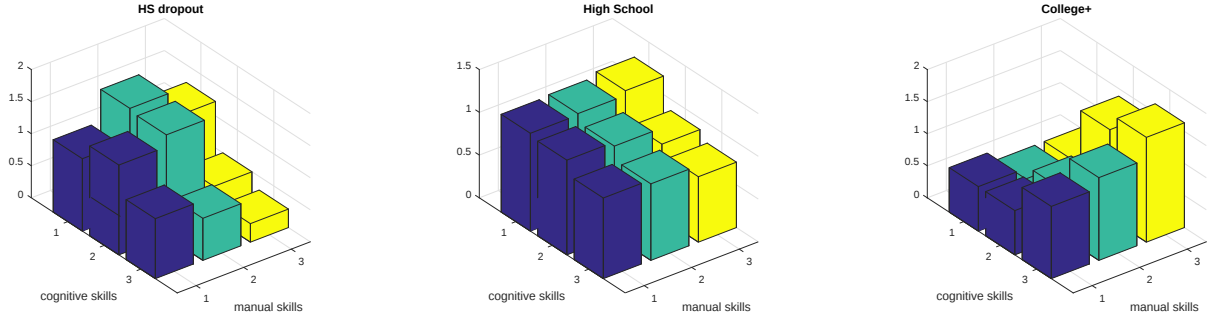


Figure 6 presents a different cut of the data. It shows the ratio of the distribution of employed workers with a particular education level (high school dropout, high school and college) to the distribution of employed workers. If a bar is above one, it means that there is a higher than average density of those skills in that education group. In this way, the graph shows the distribution of skills of a certain group, controlling for differences in the size of the different groups. The estimates show that high school dropouts tend to have low cognitive skills and somewhat higher manual skills. On the other hand, the college educated have a much larger than average concentration of workers with high cognitive skills, with the high school group lying in between.

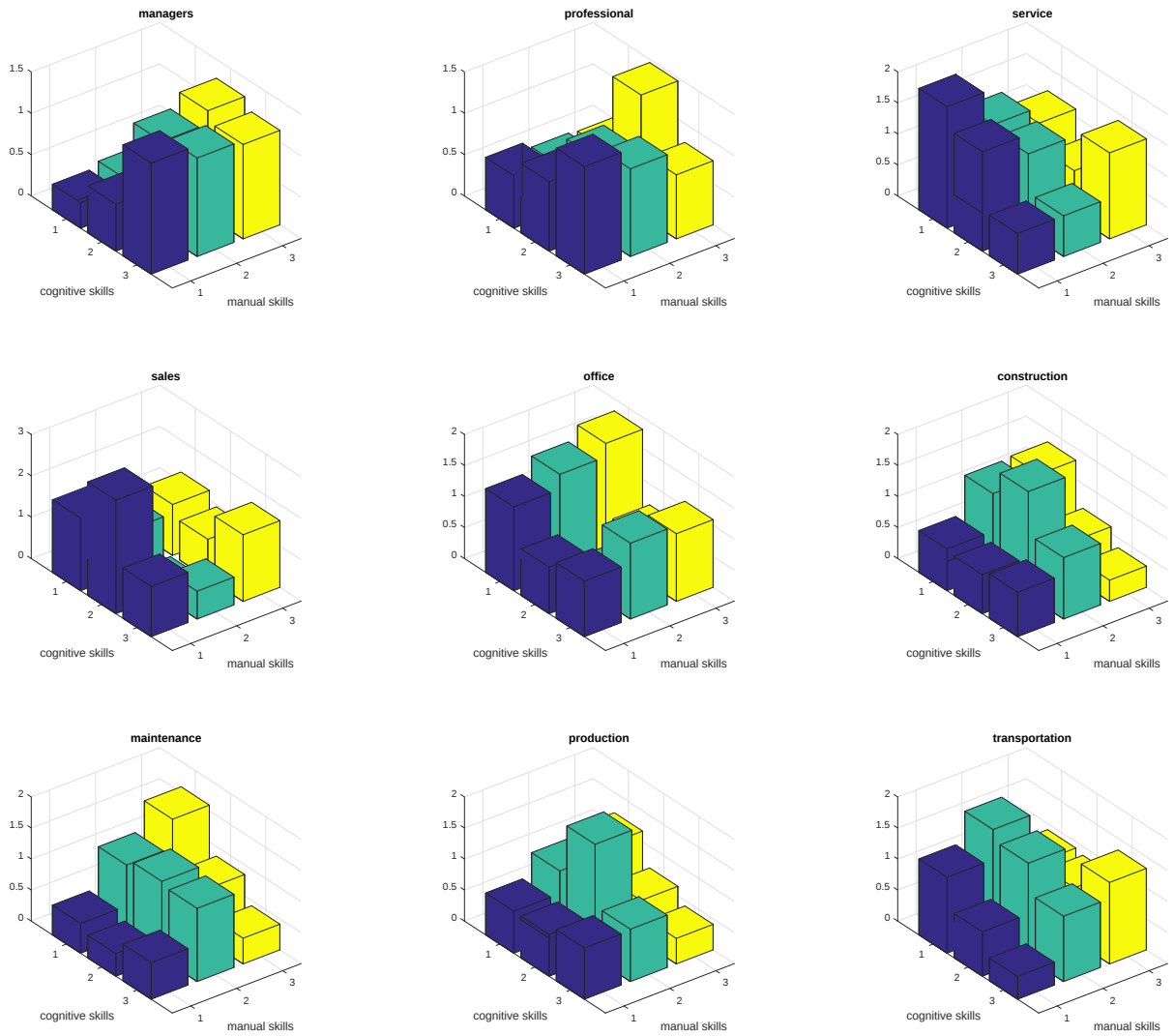
In the model, education is an invariant characteristic of workers, thus, this figure is not meant to reflect any sorting pattern. Nonetheless, it serves to check if the estimation is picking reasonable characteristics of the data.

In terms of sorting, Figure 7 shows the distribution of skills of employed workers in the different occupations relative to the aggregate distribution of employment. Managers have a larger than average concentration of high skills, both cognitive and manual. Professional occupations have a high concentration of cognitive skills. On the other hand, construction, production and maintenance occupations tend to have workers with high manual skills and low cognitive skills. This aspect of the estimation shows that workers sort into occupations according to their comparative advantage as cognitive skills are better compensated in managerial and professional occupations, while manual skills are better compensated in construction and production occupations.

5.3 Skill dynamics

TBC

Figure 7: Relative density of skills by occupation



6 Business cycles, skills and the allocation of talent

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7 Conclusions

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Appendix

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