Credit Regimes and the Seeds of Crisis*

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Abstract

This paper develops a theory of housing boom-bust cycles driven by endogenous shifts in lending standards. The key friction is asymmetric information about default risk, implying that the economy occasionally and endogenously switches between two credit regimes. These regimes differ by whether or not lenders use income verification to screen the marginal homebuyer. A switch from the "screening" regime to the "pooling" regime leads to rapid home price appreciation, a collapse in down-payment requirements, and a reduction in income documentation — consistent with the shift in lending standards during the US housing boom. The episode ends in a foreclosure crisis once fundamentals revert and the screening regime returns. The theory predicts patterns of debt accumulation and mortgage spreads consistent with existing micro evidence.

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Beginning in 2003, the US mortgage market underwent an unprecedented pivot in lending standards — between 2003 and 2004, the share of non-prime mortgage originations increased from 10% to 28%, peaking at 39% in 2006. Simultaneously, the labor market began to recover from the recession of 2001 and the housing market boomed. Employment growth jumped from 0% in 2003 to just below 2% in 2004, and growth in national home price indices doubled from 6% to 12%. This boom ended in the housing crisis of 2006-2007, the financial crisis of 2007-2008, and, ultimately, the Great Recession. A large and growing literature points to the initial change in mortgage markets as central to understanding this period. To assess policies meant to protect against similar episodes, economists need a theory explaining this shift in lending standards.

This paper develops a theory of rational non-prime lending and housing boom-bust cycles. In addition to matching existing microeconomic evidence, the theory allows us to understand the role of changing lending standards in fueling a housing boom, and how rising home prices influence the allocation of credit across different borrowers.

The theory is built on rational and competitive behavior by lenders who face asymmetric information about borrower income risk. Borrowers are either safe, having stable income, or risky, with uncertain income. Lenders can only observe borrower outcomes such as their credit history and employment status, and based on this limited information they offer mortgage contracts to individual borrowers.

The market is endogenously split into a pooling segment and a screening segment. Lenders pool together borrowers who appear unlikely to be risky, and offer them a low interest rate. Safe borrowers in this segment have little gain from revealing their type. In contrast, the screening segment consists of those borrowers who appear likely to be risky. Safe borrowers differentiate themselves by either making a high downpayment or undergoing income verification, and benefit from a low interest rate after revealing their type. In-equilibrium, risky borrowers in the screening segment have their type revealed and can only borrow at a high interest rate.

The pooling segment can be interpreted as the market for Alt-A mortgages — where loans require high credit scores but do not require income verification. In contrast, the screening segment corresponds to prime and subprime mortgage markets. Prime mortgages typically have low interest rates but require income documentation or a sizable downpayment, while subprime mortgage have high interest rates and may have either limited income documentation or a low downpayment. Together, the pooling segment and the subprime portion of the screening segment represent non-prime mortgages. By endogenizing the share of the overall mortgage market where pooling occurs, the theory can speak to the rise of non-prime lending during the mid 2000's.

In doing so, we also get a theory of housing boom-bust cycles driven by shifts in lending standards. Due to how segmentation in the mortgage market changes with economic fundamentals and home prices, the economy may operate in two different credit regimes — a "pooling regime" and a "screening regime." Here, the regime corresponds to whether the marginal home owner is offered a pooling contract or is screened out of the prime market.

When the marginal homeowner is screened, they only have access to a subprime mortgage with a high interest rate. Given this elevated interest rate, they have a low home valuation. Since the marginal home buyer must be indifferent between owning and renting, home prices must be low in the screening regime.

However, if risky borrowers become less likely to default — say during an economic expansion — then safe borrowers have less reason to differentiate themselves and the pooling segment of the mortgage market expands. This expansion triggers the pooling regime once lenders begin to offer a pooling contract to the marginal home buyer.

During this regime change, interest rates discretely fall, home prices rapidly rise, and lenders substitute away from using income verification toward using downpayment requirements to ration credit. These results come from joint mortgage and housing market clearing, and how the nature of rationing in the mortgage market depends on home prices. When a fall in income risk increases the extent of pooling, it generates an excess demand for housing. Risky borrowers gain access to low interest rate loans, which attracts former renters into the housing market. They begin bidding up home prices. For the housing market to clear, some of these former renters must be rationed back out of the mortgage market as home prices rise.

The market clearing mechanism comes from how incentive constraints link downpayment requirements to home prices. Rising home prices make the outside option of renting more attractive, relaxing incentive constraints that determine downpayment requirements. While lenders use downpayment-based screening to ration credit, the screening segment of the mortgage market expands when rising home prices relax downpayment requirements. This expansion in screening rations risky borrowers out of the market and dampens housing demand.

However, if lenders do not use downpayment-based screening, then the housing market cannot clear through this mechanism. For the housing market to clear, lenders must first substitute away from income-verification based screening. They will do so only if they can offer a small enough downpayment to attract safe borrowers, which requires a large enough relaxation of incentive constraints. As a result, housing market clearing requires a large appreciation of home prices to reduce downpayment requirements and incentivize a switch away from income verification. As a result, joint equilibrium between housing and mortgage markets implies a rapid home price appreciation, a relaxation of downpayment requirements, and an abandonment of income verification. This result provides an explanation for the sudden fall in income documentation during the housing boom (Jiang et al. (2014)).

The credit regime captures strong non-linear feedback between housing and mortgage markets and leads to housing boom-bust cycles[1]. If the underlying fundamentals of the economy are mean reverting, then a boom triggered by falling income risk must eventually end. When the economy returns to the screening regime, credit tightens for risky borrowers, demand for housing falls, and

[1]The mechanism in this theory operates through the terms of mortgage contracts and lets us match the shift in lending standards during this period. Boom-bust cycles may arise through other mechanisms such as adaptive or extrapolative expectations and speculative and social dynamics — see Lo (2004), Barberis et al. (2015), DeFusco et al. (2017), and Burnside et al. (2016).

home values collapse. As home prices fall, borrowers go underwater on their homes, which generates a foreclosure crisis. Note that both safe and risky borrowers go into default, which lines up with the microeconomic evidence on foreclosures during the housing crisis (Ferreira and Gyourko (2015), Adelino et al. (2016)). In this theory, a foreclosure crisis is the ultimate unwinding of a credit-fueled housing boom.

Non-linearity is central to this result. The regime summarizes a discontinuity in equilibrium outcomes, which makes standard perturbation solution methods inappropriate. To solve the infinite horizon economy, I apply the regime-switching perturbation method in Lind (2015). This solution method leverages the speed of perturbation and absorbs non-linearities into an endogenous regime variable. As a result, despite having a highly non-linear mechanism and many state variables, we can study equilibrium dynamics driven by endogenous credit regimes.

By highlighting the importance of state-dependent amplification in driving boom-bust cycles[2], this paper builds on and complements the results of Gorton and Ordoñez (2012) and Gorton and Ordoñez (2016). In these papers, financial crises arise in short-term collateralized debt markets when market participants have an ex-post incentive to secretly produce information about collateral (mortgage-backed security) quality. Similarly, my model is built on asymmetric information and generates highly non-linear dynamics[3]. It differs in that I study the joint determination of mortgage contracts and home prices, and the crisis does not come from an ex-post incentive problem. Rather, the key mechanism comes from general equilibrium feedback between the mortgage market and the housing market. Since the foreclosure crisis led investors to worry about the quality of mortgage-backed securities, this paper complements the contributions of Gorton and Ordoñez (2012) and Gorton and Ordoñez (2016). It endogenizes the housing crisis that precipitated the financial crisis.

To diagnose the crisis, economists have turned to credit bureau data to ask whether or not there was reallocation of credit towards risky borrowers during the boom. Different studies use different measures of borrower riskiness and find seemingly conflicting results. Mian and Sufi (2015) use a measure of *historical* riskiness based on fixed groups of borrowers. They find that borrowers with low FICO scores in 1997 had a larger increase in debt than borrowers with high FICO scores in 1997. In contrast, Albanesi et al. (2016) use a measure of *observable* riskiness. They examine the distribution of debt across borrowers by FICO score two years before origination and show that debt by FICO score grew more for high FICO score borrowers.

The theory proposed by this paper can simultaneously match the evidence from both papers. The difference in their results highlights the importance of distinguishing between fundamental borrower types (risky versus safe) and lender information about borrower types. To the extent that types are persistent, I argue that the Mian and Sufi (2015) approach is more likely to measure reallocation across borrower types. In my model, the average loan-to-value ratio of safe borrowers

[2]For recent work on non-linear dynamics that generate limit cycles see Beaudry et al. (2015).

[3]Guerrieri and Uhlig (2016) also examine how adverse selection in credit markets influences the housing market. Their model generates housing cycles through switching between multiple equilibria. In contrast, the theory in this paper focuses on a unique stable equilibrium and generates boom-bust cycles in response to shocks to fundamentals through a non-linear feedback mechanism between housing and mortgage markets.

falls during the boom, while risky borrower loan-to-value ratios are stable — matching the evidence on loan-to-value ratios in Mian and Sufi (2015). Given this difference in loan-to-value ratios, safe borrowers accumulate less debt than risky borrowers during a housing boom.

However, these results reverse if we condition on lender beliefs about borrower types rather than fundamental types. Those safe borrowers who appear risky to lenders end up using high-downpayment mortgages to signal their type during the boom. Those safe borrowers who are obviously safe do not signal their type and have stable loan-to-value ratios in parallel to risky borrowers. As a result, loan-to-values are more stable among borrowers who seem safe from the perspective of lenders at the time of origination. Therefore, conditioning on lender information reverses the pattern of debt accumulation. This predicted reversal is consistent with the evidence of Albanesi et al. (2016), who study debt growth conditional on lender information close to the time of origination. By matching both sets of results, the theory may help explain why these authors get seemingly conflicting results from the same data — the measure of riskiness that each study uses captures a distinct theoretical concept. Patterns of debt accumulation can differ significantly depending on whether or not we condition on the information available to lenders at the time of mortgage origination.

The theory also provides an explanation for the mortgage rate conundrum of Justiniano et al. (2016). These authors document that from mid-2003 to 2006 mortgage rates were low relative to their historical relationship with US treasury yields. In the summer of 2003 the yield curve began to rise following the Federal Reserve's June 2003 announcement that it was ending its easing cycle. Following similar past annoucements, mortgage rates rose with the yield curve. This time, they did not.

The theory explains this puzzle through a switch from the screening regime to the pooling regime. The interest rates of risky borrowers fall when they get pooled with safe borrowers — pooling dilutes their default risk. The interest rates of safe borrowers also fall on average. Those safe borrowers who switch to high-downpayment based screening have reduced loan-to-value ratios. With a larger equity cushion, they are less likely to default reducing the risk premium on their loan. The switch away from income verification toward downpayment-based rationing implies that mortgage rates fall for safe borrowers on average. As a result, when a fall in income risk triggers a shift in the credit regime, mortgage rates fall relative to the risk free rate for all borrowers. The endogenous response of monetary policy to improving labor market conditions accounts for the rise in the yield curve, and an endogenous shift in the credit regime accounts for the low level of mortgages rates relative to treasury yields.

This paper also speaks to the macroeconomic literature on the housing boom by endogenizing the shocks that explain the boom-bust cycle. Previous research points to credit supply shocks as an explanation of the housing boom (for instance, Mian and Sufi (2009), Justiniano et al. (2015a), and Mian and Sufi (2016)). The model can explain these shocks through switches in the credit regime. Even a small change in income risk can trigger a regime change, generating a large shift away from high documentation lending and a rapid appreciation in home prices. When this occurs, credit rationing decreases and it is as-if a credit supply shock hit the economy.

Kaplan et al. (2016) replicate the boom-bust episode in a quantitative heterogenous agent macroeconomic model calibrated to cross-sectional facts. They find that three shocks are necessary to match the time series data: an income shock, a loan-to-value shock, and a home price expectation shock[4]. The theory I propose explains the loan-to-value ratio shock through the endogenous increase in loan-to-value ratios that occurs when rising home prices relax incentive constraints. The shock to home price expectations is explained through the shift in the credit regime. From the perspective of a traditional asset pricing condition for homes, a discrete switch in the contract faced by the marginal home owner appears as a wedge shock. With rational expectations and some persistence in the credit regime, it generates an endogenous anticipation of high home prices while lenders continue to offer pooling contracts to the marginal home buyer. The mechanism developed in this paper may enable economists to reduce the number of shocks that are required to explain the housing boom and bust.

Because the mechanism generates endogenous boom-bust cycles by amplifying underlying business cycle fundamentals, this theory points to possible non-prime lending booms during future macroeconomic expansions — indeed, the re-emergence of subprime lending in both mortgages and car loans has made headlines recently[5]. This prediction is consistent with the historical evidence, which suggests that credit-driven housing booms and busts are rare but systemic events. For instance, Jordà et al. (2015) show that, across countries, an expansion of credit during a housing boom predicts a subsequent bust. The theory proposed in this paper formalizes this perspective.

More broadly, this model provides a framework for studying credit rationing in business cycle analysis. One lesson is that the price of assets that serve as collateral can change rapidly and even discontinuously in fundamentals when there is adverse selection in credit markets[6]. This strong form of amplification is distinct from workhorse models of collateralized debt used when studying business cycles — for example, Kiyotaki and Moore (1997) and Bernanke et al. (1999). This theory provides a new approach to modeling asset price booms and financial frictions in business cycle analysis.

The paper is organized as follows. Section 1 introduces a partial equilibrium model of endogenous segmentation in the mortgage market. Section 2 examines joint mortgage market and housing market equilibrium and shows how small changes to income risk can sometimes generate big changes in mortgage contracts and home prices. In Section 3, I extend the model to examine equilibrium dynamics. I show that the accumulation of small shocks to income risk can trigger a housing boom which then leads to a housing crisis. Section 4 shows that the theory can simultaneously match the seemingly conflicting results in the empirical literature on debt reallocation and also accounts for

[4]Similarly, Gelain et al. (2015) find that shocks to lending standards can explain the boom-bust episode in a representative agent framework.

[5]For example, see "Subprime Bonds Are Back With Different Name Seven Years After U.S. Crisis" Bloomberg News, January 27, 2015.

[6]See Bigio (2015) for a model where adverse selection plays a key role in market for liquidity and business cycles.

the mortgage rate conundrum. Finally, Section 5 concludes.

1 Endogenous Segmentation in the Mortgage Market

This section develops a model of the mortgage market where borrowers are endogenously segmented by the type of equilibrium that prevails — either a screening, separating, or pooling equilibrium. This segmentation arises from how lenders tailor mortgage offers to individual borrowers based on their observable characteristics.

I first setup the economic environment, and then I examine equilibrium segmentation. Note that for this partial equilibrium analysis home prices are exogenous. In Section 2 I endogenize home prices.

1.1 Setup

The economy consists of a unit mass of borrowers and at least two lenders. There are two periods, $t = 0, 1$, with mortgage contracts signed at $t = 0$ before uncertainty is resolved at $t = 1$.

Each borrower $i \in [0, 1]$ has the same linear preferences:

$$u(C_{i,0}, C_{i,1}) = C_{i,0} + \beta \mathbb{E} C_{i,1}$$

where $C_{i,0}$ denotes period 0 consumption of numeraire, $C_{i,1}$ denotes period 1 consumption, and $\beta > 0$. Borrowers come in two types, safe and risky. The variable τ_i indicates whether or not individual i is risky.

$$\tau_i = \begin{cases} 0 & \text{if } i \text{ is safe} \\ 1 & \text{if } i \text{ is risky} \end{cases}$$

Safe borrowers have income of $Y_i = Y$ for sure at $t = 1$, while risky borrowers have income of $Y_i = Y$ with chance $1 - \phi$ and $Y_i = 0$ with chance ϕ. The parameter

$$\phi \equiv \mathbb{P}[Y_i = 0 \mid \tau_i = 1]$$

measures the *income risk* of risky borrowers. Since income risk falls in expansions and increases in recessions[7], ϕ captures the state of the business cycle.

Lenders are risk neutral and maximize expected profits given an opportunity cost of funds of $1 + r < 1/\beta$. Because lenders are patient relative to borrowers, there are potential gains from trade.

Several frictions limit the scope for trade. First, limited liability and limited commitment implies that borrowers may default either by choice or if they have insufficient income to cover their debts. Second, lack of ex-post income verification makes state-contingent contracting impossible. Third, there is asymmetric information between borrowers and lenders about borrower types. Fourth, lender screening behavior is publicly unverifiable.

[7]See Busch et al. (2016).

Lenders have access to an income verification technology. This technology requires borrowers to supply income documentation, requires lenders to pay a cost, and perfectly reveals a borrower's type. Denote the lender's fixed cost of income verification by κ. I also assume that provide income documentation imposes some infinitesimal application cost on borrowers, which will keep risky borrowers from applying when they anticipate being rejected after their type is revealed.

If a lender does not verify a borrower's income, then they must form a belief based on freely available information (for example, a borrower's credit history and employment status). Let X_i be a vector of all variables that a lender can costlessly observe related to individual i's type. Denote the density of X_i conditional on borrower type by $f_X(x \mid \tau_i = \tau)$.

Lenders care about an individual borrower's likelihood of being the risky type. By Bayes' law, the share of risky borrowers among borrowers with $X_i = x$ is

$$\mathbb{P}[\tau_i = 1 \mid X_i = x] = \frac{f_X(x \mid \tau_i = 1)\mathbb{P}[\tau_i = 1]}{f_X(x \mid \tau_i = 1)\mathbb{P}[\tau_i = 1] + f_X(x \mid \tau_i = 0)\mathbb{P}[\tau_i = 0]} \equiv \rho(x)$$

The function ρ maps borrower observables to lender beliefs about borrower types. Instead of working directly with X_i, it will be convenient to work with the following *risk index*:

$$\rho_i \equiv \rho(X_i).$$

This index is the proportion of risky borrowers among borrowers with observables of X_i. Individual borrowers with a large value of ρ_i are perceived by lenders as likely to be the risky type.

Given that there are no state-contingent contracts, mortgage contracts specify period 0 and period 1 transfers. Since lender screening behavior is publicly unverifiable, mortgage contracts cannot specify the type of screening technology that lenders use, but can specify whether or not lenders require borrowers to provide income documentation. A *mortgage contract* then consists of transfers and an income documentation requirement.

Definition 1 A *mortgage contract* is a triple, (T, L, Doc), where

1. T is a period 0 transfer from the lender to the borrower.

2. L is a period 1 transfer from the borrower to the lender.

3. $Doc = 0$ if income documentation is not required, and $Doc = 1$ if it is required.

Lenders will have an ex-post incentive to verifying borrower income when documentation is provided. I will refer to contracts with $Doc = 0$ as *low-documentation contracts* and contracts with $Doc = 1$ as *high-documentation contracts*.

Each borrower lives in a single unit of housing which they either own or rent. The purchase price of a house at $t = 0$ is P units of numeraire and the rental rate is R. The ex-post value of individual i's home (at $t = 1$) is idiosyncratic:

$$\text{Period 1 home value} = Z_i \overset{iid}{\sim} F.$$

Z_i is strictly positive, independent of borrower income, and continuously distributed with positive support and density F'.

Let $h(z) \equiv F'(z)/(1 - F(z))$ be the hazard rate of Z_i. I assume that the hazard rate satisfies the following regularity condition.

Assumption 1 (Monotone Scaled Hazard Rate) *$zh(z)$ is strictly increasing and continuous in z.*

This condition is a weak regularity condition that is common in the literature[8] and is satisfied by many standard distributions[9].

Due to limited liability and limited commitment, unsecured borrowing is impossible. However, a foreclosure technology exists which enables lenders to recover contracted debts through the liquidation of a borrower's home following default at the start of $t = 1$.

Foreclosure is costly[10]. When a lender initiates foreclosure, this reduces the sale value of the home to $(1 - \lambda)Z_i$. The parameter λ captures losses from foreclosure[11]. The foreclosure technology then transfers funds from the home sale to the lender to cover any debt outstanding against the home. Given debt of L, the payoffs from foreclosure are

$$\text{Borrower Foreclosure Payoff} = \max\{(1 - \lambda)Z_i - L, 0\}$$
$$\text{Lender Foreclosure Payoff} = \min\{L, (1 - \lambda)Z_i\}.$$

Figure 1 depicts these payoffs. This technology provides an ex-post incentive for borrowers to re-pay contracted debts, allowing homes to serve as collateral for borrowing.

I make the following assumption to rule out contracts which imply certain default.

Assumption 2 (Certain Default is Inefficient) *The scaled hazard rate gets sufficiently large:*

$$\lim_{z \to \infty} zh(z) > \frac{1 - (1 + r)\beta}{(1 - \phi)\lambda}.$$

This condition implies that arbitrarily large loans — which all borrowers will default on for sure — are inefficient. Borrowers will strategically default when underwater on their mortgage and as loan size increases the probability of default increases. This condition states that at large loan sizes, the likelihood of strategic default increases too much to be worth increasing any borrowers loan size. Together with Assumption 1, this condition ensures that equilibrium contract offers are (almost surely) finite and unique.

8 Bernanke et al. (1999) assume the same condition on the idiosyncratic return on capital.

9 For example, any distribution with a monotone hazard rate, like the log-normal distribution, satisfies this condition.

10 As a result, there are possible gains from ex-post renegotiation. I assume that there are sufficiently large renegotiation costs that rule out ex-post renegotiation.

11 For a discussion of foreclosure costs see Hayre and Saraf (2008). For empirical estimates of losses due to forced sales of homes see Campbell et al. (2011).

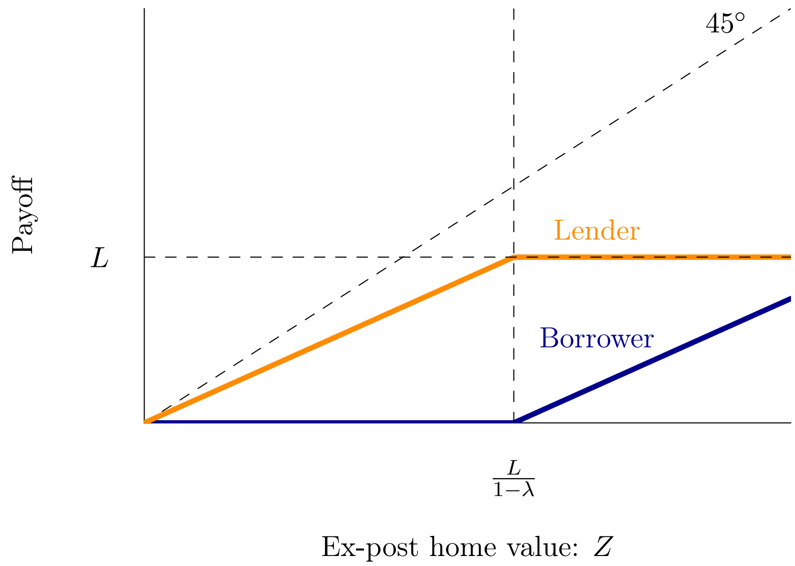


Figure 1: Foreclosure payoffs given debt level of L.

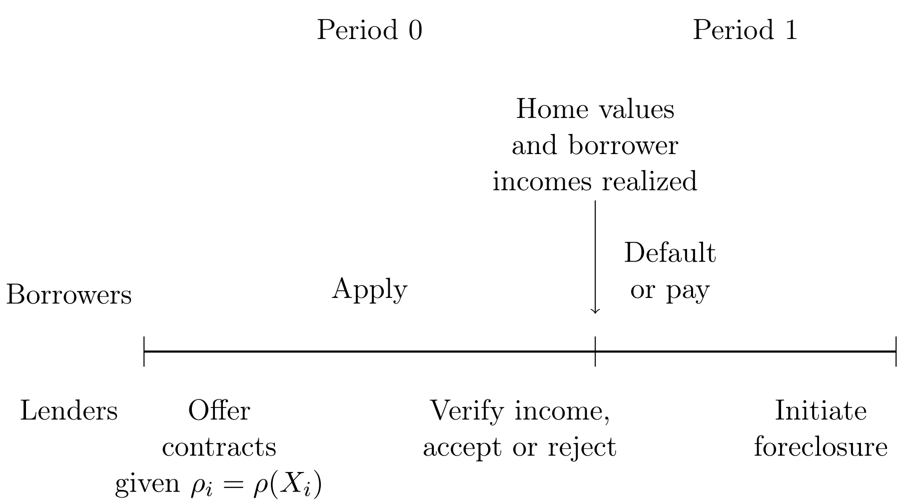


Figure 2: Within-period timing.

Timing within the two periods is as follows (see Figure 2). During period 1, lenders offer mortgages to individual borrowers. Then, borrowers apply to an individual lender. Finally, lenders perform income verification, and accept or reject applicants. At the end of $t = 0$, transfers occur and borrowers consume numeraire, which is not storable. Between period 0 and period 1, borrowers learn the value of their home and whether or not they have income. They then choose to make their mortgage payment, or not, and lenders initiate foreclosure. Finally, homes are liquidated and borrowers consume numeraire at the end of $t = 1$.

1.2 Equilibrium

Models of asymmetric information typically have a large number of equilibria. I focus on the unique efficient equilibrium. It is stable in the spirit of Kohlberg and Mertens (1986) and satisfies a number of selection criteria used in the literature on signaling games[12]. This selection is similar to the approach taken by Dell'Ariccia and Marquez (2006), who examine how equilibrium in a credit market changes as the composition of types in the market varies[13].

Up to the identity of the lenders who make contract offers and the application behavior of borrowers when indifferent, there is a unique stable Nash equilibrium among borrowers with $\rho_i = \rho$ for almost all $\rho \in [0, 1]$.

Proposition 1 (Existence and Uniqueness) *Let Assumption 1 and Assumption 2 hold. Suppose that risky borrowers do not apply for screening contracts whenever they anticipate rejection. Then for almost every $\rho \in [0, 1]$, there exists a unique stable Nash equilibrium up to deletion of non-traded mortgage offers and the identity of the lender making each offer. In this equilibrium lenders make zero profits and safe borrowers only use contracts that maximize their value.*

Proof. See Appendix A. ■

This proposition states that equilibrium is unique up to an equilibrium selection with respect to the application behavior of borrowers. The assumption that risky borrowers do not apply for high-documentation contracts can be formalized by a small cost to supply income documentation. This selects the equilibrium where risky borrowers do not apply for high-documentation contracts.

In Appendix A I provide proofs and characterize equilibrium. The following sub-sections summarize equilibrium outcomes.

1.2.1 Period 1: Foreclosure, and Default

Since homes have strictly positive value, initiating foreclosure following a borrower default is a strictly dominant strategy. Borrowers then anticipate that foreclosure will always occur following any default.

Borrowers prefer to default if the foreclosure value of $\max\{(1-\lambda)Z_i - L, 0\}$ is greater than their equity of $Z_i - L$. This condition holds if any only if they have negative equity: $Z_i < L$. Underwater borrowers strategically default on their mortgage.

Even if they are willing to repay, they may be unable to pay. Borrowers are unable to make any payment which is larger than their potential income of Y. For simplicity, I assume that Y is large enough that repayment is feasible whenever income is available[14]. This assumption means that the

[12] See also Cho and Kreps (1987), Banks and Sobel (1987), Cho and Sobel (1990), and Ramey (1996)

[13] Also see Wilson (1977), Stiglitz and Weiss (1981), Bester (1985), and Hellwig (1987).

[14] More generally, if a lender made a loan with $Y < L$, the loan will be defaulted on with probability 1. In the presence of foreclosure costs, these loans are inefficient and will not be made. Lenders will then impose a debt-to-income constraint: $L \leq Y$.

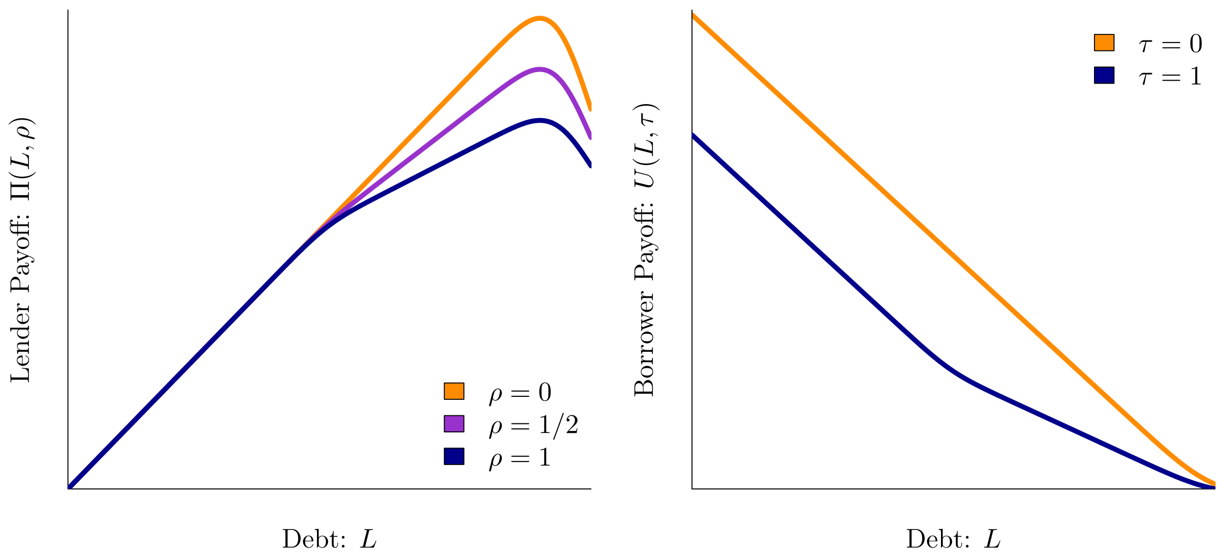


Figure 3: Anticipated contract payoffs varying ρ and τ when $\phi = 1/2$ and Z is log-normal.

model highlights the role of income risk in influencing mortgage contract terms and abstracts from the level of borrower income[15].

Given debt of L and ex-post home value of $Z_i = z$, the ex-post default rate among borrowers with risk index of ρ is

$$D(L, \rho\phi, z) = \mathbf{1}\{z < L\} + \mathbf{1}\{z \geq L\}\rho\phi. \tag{1}$$

If the borrower's home is underwater, then they default for sure. Otherwise, they only default if their income is unavailable, which occurs with chance of 0 for safe borrowers and chance of ϕ for risky borrowers. Among borrowers with $\rho_i = \rho$, a fraction ρ are risky, and a fraction ϕ of these risky borrowers are forced into default.

1.2.2 Period 0: Anticipated Contract Payoffs

Lenders anticipate defaults when originating mortgages. Let $\Pi(L, \rho\phi)$ denote a lender's expected payoff on a loan of size L to a borrower with risk index of ρ given income risk of ϕ:

$$\begin{aligned}\Pi(L, \rho\phi) &= \mathbb{E}\left[D(L, \rho\phi, Z_i)\min\{(1-\lambda)Z_i, L\} + (1 - D(L, \rho\phi, Z_i))L\right] \\ &= L - \int_0^\infty D(L, \rho\phi, z)\max\{L - (1-\lambda)z, 0\}F'(z)\mathrm{d}z \end{aligned} \tag{2}$$

If default occurs, the lender gets their foreclosure payoff, and if the household doesn't default then the lender receives the face value of the debt. The second line expresses the payoff as the loan's face value net of anticipated losses from foreclosure. The left panel of Figure 3 plots Π for $\rho = 0, 1/2, 1$

[15]See Greenwald (2016) for a model in which the level of borrower income and debt-to-income constraints are key determinants of monetary transmission and macroeconomic dynamics.

when $\phi = 1/2$ and Z_i is log-normal.

Let $U(L, \tau\phi)$ denote the expected payoffs of a borrower with type τ given income risk of ϕ:

$$U(L, \tau\phi) = \mathbb{E}\left[D(L, \tau\phi, Z_i) \max\{(1-\lambda)Z_i - L, 0\} + (1 - D(L, \tau\phi, Z_i))(Z_i - L)\right]$$
$$= \bar{z} - L + \int_0^\infty D(L, \tau\phi, z) \max\{-\lambda z, L - z\} F'(z) \mathrm{d}z. \tag{3}$$

where $\bar{z} = \int_0^\infty zF'(z)\mathrm{d}z$. If the borrower defaults, they get the payoff from foreclosure. If they do not default, they keep their home equity. The second line expresses this valuation in terms of expected home equity plus a term capturing the net benefit to the household of forced and strategic defaults. Borrowers benefit from default whenever they are underwater. Otherwise, default is costly. As a result, safe borrowers always benefit from their default option, while risky borrowers occasionally default on positive equity. The right panel of Figure 3 plots U for $\tau = 0, 1$ when $\phi = 1/2$ and Z_i is log-normal.

The following lemma establishes a single crossing condition which ensures that safe borrowers can signal their type by making a large down payment.

Lemma 1 (Single Crossing Condition) *For any debt level L such that $\mathbb{P}[(1-\lambda)Z_i < L < Z_i] > 0$, risky borrowers value a marginal increase in debt more than safe borrowers. In particular:*

$$\frac{\partial}{\partial L}\left[U(L, \phi) - U(L, 0)\right] = \phi\left[F\left(\frac{L}{1-\lambda}\right) - F(L)\right].$$

Proof. See Appendix A. ∎

Although risky borrowers value mortgages less than safe borrowers (U is decreasing in its second argument), they value debt more *on the margin* than safe households. This occurs because of the difference in default behavior between the two types. Safe borrowers only default strategically. They never face foreclosure costs because they only default when they have negative equity, passing the costs onto their lender. Risky borrowers also always default on negative equity, but also default on positive equity with chance ϕ. Whenever $\mathbb{P}[(1-\lambda)Z_i < L < Z_i] > 0$, there is a chance that a risky borrower will default on positive equity but the cost of foreclosure destroys that equity by reducing the value of the home. In this case, risky borrowers do not internalize the cost of holding more debt, but safe borrowers do internalize this cost. As a result, risky borrowers value increases in debt more than safe borrowers.

1.2.3 Period 1: Income Verification, Rejections, and Applications

Lenders will reject any application which they anticipate a loss on. For instance, a high-documentation contract might be profitable only if the lender rejects all risky applicants. Due to zero profits, lenders will have an ex-post incentive to verify income and reject risky borrowers[16].

[16]Note that there exist unstable equilibria where lenders randomly reject additional borrowers. These equilibria arise because lenders make zero profits ex-post and therefore are indifferent between accepting and

Anticipating lender income verification and rejection choices, borrowers will apply to whichever contract gives them the best payoff among all offers that are profitable for the lender. Given an infinitesimal application cost, risky borrowers will not apply for high-documentation contracts since they anticipate rejection.

1.2.4 Period 1: Offers

When making offers, lenders observe individual observables of X_i and infer that individual i is risky with chance $\rho_i = \rho(X_i)$. They then make an offer based on this belief.

To characterize equilibrium contracts, it is useful to first consider the full information case where lenders can observe borrower types. With full information, the safe borrower's most preferred contract solves the following program.

$$\begin{aligned} V^{safe} = \max_{T,L} \quad & T + \beta U(L,0) \\ \text{s.t.} \quad & -T + \frac{1}{1+r}\Pi(L,0) \geq 0 \qquad \text{(PC)} \end{aligned}$$

The constraint PC is the lender's zero profit (participation) constraint. Competition drives lender profits to zero so that this constraint binds. We can solve the problem by substituting in the participation constraint, finding the optimal loan size, and then backing out the initial transfer consistent with zero profits. Denote the loan size for this safe contract by L^{safe}.

The full information contract for risky borrowers gives them value of

$$V^{risky} = \max_{L} \quad \frac{1}{1+r}\Pi(L,\phi) + \beta U(L,\phi).$$

Here, I have used the zero profit condition for lenders to eliminate the period 0 transfer from the program.

Given these full information contracts, safe borrowers get more value from being a homeowner than risky borrowers.

Remark 1 *Safe borrowers value their full information contract more than risky borrowers value their full information contract:*

$$V^{safe} > V^{risky}.$$

Proof. Define $V(x) = \max_L \frac{1}{1+r}\Pi(L,x) + \beta U(L,x)$. We have $V(0) = V^{safe}$ and $V(\phi) = V^{risky}$. By the envelope theorem $V' < 0$ because Π and U are both decreasing in their second arguments. Therefore, $V(0) > V(\phi)$ and $V^{safe} > V^{risky}$. ■

Relative to risky borrowers, safe borrowers are natural homeowners. Even for relatively high home values, safe borrowers will prefer to purchase a house. In contrast, risky borrowers will only

rejecting applications of borrowers that remain after rejecting borrowers who are unprofitable to keep in the pool. However, these equilibria are unstable because borrowers have a strong incentive to accept slightly worse terms in order to give lenders a small positive profit and avoid these arbitrary rejections.

be willing to use their full information contract when home values are relatively low.

Now consider a safe borrower's most preferred low-documentation contract which signals their type through a high downpayment — the competitive high-downpayment contract. The outside option for a risky borrower is to either apply for their full information contract, or to rent. The value of their full information contract net of the home price is $-P+V^{risky}$, and the value of renting is $-R$. Risky borrowers prefer their outside option to a low-documentation contract $(T, L, 0)$ whenever the following incentive compatibility constraint holds.

$$T - P + \beta U(L, \phi) \leq \max\{-R, -P + V^{risky}\}. \tag{IC}$$

The competitive high-downpayment contract then maximizes value for safe borrowers subject to this incentive constraint.

$$\begin{aligned} V^{down}(P-R) = \max_{L} \quad & \frac{1}{1+r}\Pi(L,0) + \beta U(L,0) \\ \text{s.t.} \quad & \frac{1}{1+r}\Pi(L,0) + \beta U(L,\phi) \leq \max\{P-R, V^{risky}\} \qquad \text{(IC)} \end{aligned}$$

Here, I used the zero profit condition to eliminate T, and simplified the IC constraint by adding P to both sides. I call the quantity $P - R$ the *net home price*. It captures how the relative cost of owning versus renting influences this contract.

Note that the value of the high-downpayment contract does not depend on borrower observables — in a separating equilibrium, risky borrowers and safe borrowers reveal their types and so a lender's initial information is irrelevant. However, this value may depend on the net home price, $P - R$. Changes in home prices relative to rental rates change the incentive for risky borrowers to enter the mortgage market. When home prices are high, risky borrowers find owning less attractive, and it takes less of a down payment for safe borrowers to signal their type (high $P - R$ relaxes the IC constraint). As a result, the value of separation for safe borrowers will vary with the net home price. Once we consider joint housing and mortgage market equilibrium in Section 2, this link will lead to general equilibrium feedback and amplification of shocks to income risk.

When is the IC constraint slack? When risky borrowers do not wish to use the safe borrower's full information contract. If a risky borrower were to gain access to this contract, they'd get value of

$$V^* = \frac{1}{1+r}\Pi(L^{safe}, 0) + \beta U(L^{safe}, \phi).$$

Whenever the net home price is above this threshold, risky borrowers prefer to rent over using the loan intended for safe borrowers. As a result, the IC constraint will not bind if $P - R \geq V^*$. In this case, there is no issue of adverse selection. Safe borrowers use their full information contract and become homeowners, risky borrowers become renters, and types are revealed in equilibrium.

But for lower home prices — when $P - R < V^*$ — risky borrowers will enter the market and introduce adverse selection (the IC constraint will bind). In this case, safe borrowers may prefer the allocation under income-verification based screening or under pooling.

Consider income verification. Since only safe borrowers apply, the value of a high-documentation contract is constant across safe borrowers:

$$V^{doc} = \max_{L} \quad \frac{1}{1+r}\Pi(L,0) + \beta U(L,0) - \kappa = V^{safe} - \kappa.$$

They get the value of their full information contract net of the income verification cost. Note that, like with downpayment-based screening, the value of screening through income verification is constant across borrowers. Income verification removes all risky borrowers from the pool of funded loans. The chance that a risky borrower has no income is irrelevant for the terms a safe borrower receives on a high-documentation contract.

We can see that the value of the two methods of screening are the same across market segments. Define the overall value of screening as

$$V^{screen}(P-R) = \max\{V^{down}(P-R), V^{doc}\}.$$

Lenders will use whichever method of screening is most preferred by safe borrowers. If they didn't another lender could offer a contract using the alternative form of screening and capture the market. The implication is that whether or not the value of screening changes with the net home price depends on whether or not lenders are actively using income verification or high downpayment requirements to ration credit. When they use income verification, the value of screening does not depend on home prices through the incentive constraint channel. However at high enough home prices, lenders will substitute away from income verification and toward downpayment requirements, making the incentive constraint channel active.

Finally, consider pooling. In a pooling equilibrium, lenders offer a single contract and accept all applicants. Due to zero profits, the value of the mortgage to safe borrowers depends on how many risky borrowers there are in the pool. In the group of borrowers with risk index of ρ, a fraction ρ are risky and a fraction $1-\rho$ are safe. The value of the contract to safe borrowers will depend on how many of these risky borrowers apply for the contract.

In equilibrium, risky borrowers will always apply whenever they are offered a contract that leads to pooling. This outcome comes from the fact that whenever risky borrowers are willing to opt out of a pooling contract, lenders have an incentive to offer the competitive separating contract. Intuitively, a contract which fails to attract risky borrowers away from renting effectively acts like a separating contract. Any contract other than the separating contract is relatively inefficient at inducing separation. If a lender offered such a contract, another lender would introduce the separating contract and make a profit.

Because of this result, we can focus on a pooling equilibrium in which all risky borrowers apply. In this case, the lender zero profit condition implies a transfer of $T = \frac{1}{1+r}\Pi(L, \rho\phi)$. A safe borrower with risk-index of ρ then prefers a low-documentation contract that solves

$$V^{pool}(\rho) = \max_{L} \quad \frac{1}{1+r}\Pi(L, \rho\phi) + \beta U(L,0).$$

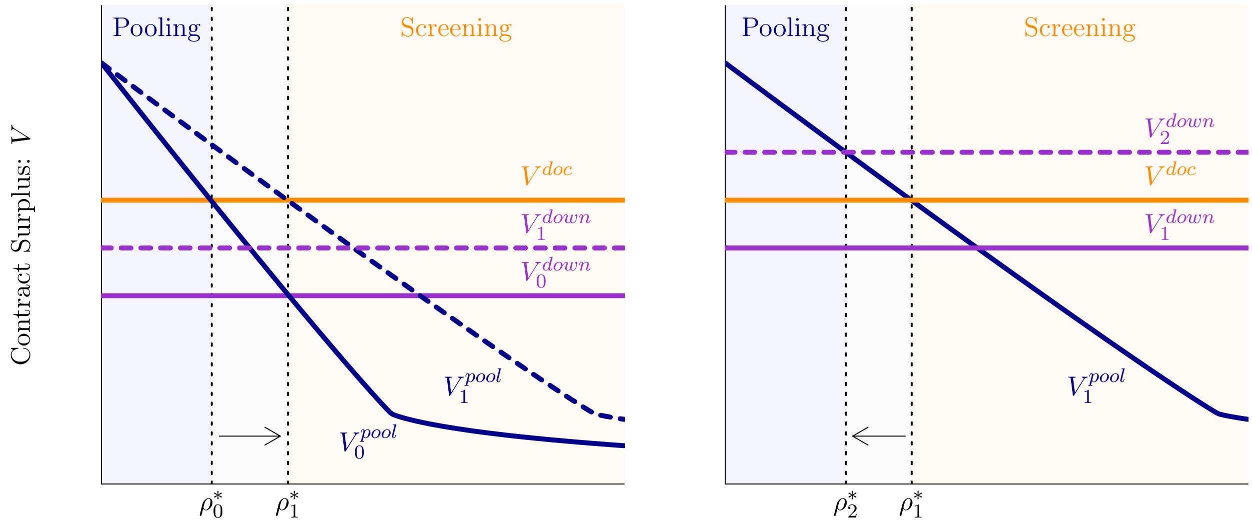


Figure 4: Comparative statics of market segmentation: fall in ϕ (left) followed by increase in $P-R$ (right).

Note that the value of pooling across ρ-values will change as income risk varies. In particular, a fall in income risk will increase the value of pooling across all borrowers:

$$\frac{\partial}{\partial\phi}V^{pool}(\rho) = \frac{1}{1+r}\Pi_2(L^{pool}(\rho), \rho\phi)\rho = -\rho\int_{L^{pool}(\rho)}^{\frac{L^{pool}(\rho)}{1-\lambda}}(L^{pool}(\rho) - (1-\lambda)z)F'(z)\mathrm{d}z < 0.$$

Intuitively, safe borrowers face a higher interest rate when they are pooled with risky borrowers, but as risky borrowers become less likely to default, loan terms improve. As a result, the pooling value function for safe borrowers will increase when the income risk of risky borrowers falls.

In equilibrium lenders will make offers that maximize value for safe borrowers. We can determine the pattern of segmentation by comparing the pooling and screening value functions. The following proposition characterizes the pattern of market segmentation.

Proposition 2 (Equilibrium Segmentation) *Let Assumption 1 and Assumption 2 hold. When $P-R \geq V^*$ there is no segmentation. Lenders offer the full information safe contract to all borrowers. Safe borrowers apply and are accepted to this contract. Risky borrowers do not apply and instead rent their home. When $P-R < V^*$, the market is divided into a pooling segment and a screening segment. Let ρ^* satisfy $V^{screen}(P-R) = V^{pool}(\rho^*)$. Borrowers with $\rho < \rho^*$ are in a pooling equilibrium and those with $\rho > \rho^*$ are in a screening equilibrium. Lenders use downpayment requirements to screen borrowers when $V^{doc} < V^{down}(P-R)$ and use income verification when $V^{doc} > V^{down}(P-R)$.*

Proof. See Appendix A. ■

Lenders offer contracts that maximize value for safe borrowers, as otherwise another a competing lender could attract away safe borrowers. As a result, lenders offer those borrowers with $\rho_i = \rho$ whichever contract gives the most value to safe borrowers and the pattern of equilibrium segmentation corresponds to the upper envelope of the pooling, documentation, and downpayment value functions.

Note that the separating segment consists of the borrowers who are likely to be risky from the perspective of lenders. This result comes from the fact that the value of screening is constant across ρ-values, while the value of pooling is decreasing in ρ. In high risk parts of the market, safe borrowers face high interest rates when using pooling contracts, and find it worthwhile to differentiate themselves by either undergoing income verification or making a large downpayment.

There are two key comparative statics for the pattern of market segmentation which lead to the key general equilibrium results in Section 2. First, as income risk in the economy falls (increases), the pooling segment expands (contracts). The boundary ρ-value between the pooling and screening segments, ρ^*, quantifies the size of the pooling segment. Formally, if $P - R < V^*$ so that $\rho^* > 0$ then the pooling segment is decreasing in income risk:

$$\frac{\partial \rho^*}{\partial \phi} < 0.$$

Intuitively, as risky borrowers become more likely to default safe borrowers are less willing to be pooled and a smaller fraction of the market is in a pooling equilibrium.

Second, so long as lenders use downpayment requirement to ration credit, then the pooling segment is decreasing in the net home price. If lenders instead use income verification, then the size of the pooling segment is constant in $P - R$. Formally, if $P - R < V^*$ and $V^{down}(P - R) > V^{doc}$ then the pooling segment is decreasing in the net home price:

$$\frac{\partial \rho^*}{\partial (P - R)} < 0,$$

while if $V^{down}(P - R) < V^{doc}$ then it is constant:

$$\frac{\partial \rho^*}{\partial (P - R)} = 0.$$

Figure 4 depicts these comparative statics. The left diagram shows a fall in income risk which expands the pooling segment. The right diagram then shows an increase in the net home price that offsets the expansion by reducing the size of the pooling segment. Note that a smaller increase in the net home price — which didn't shift the value of downpayment-based screening above the value of income-verification-based screening — would not reduce the size of the pooling segment.

Note that the homeownership rate in the economy is the number of borrowers who gain access to credit and purchase a home. Since credit access varies with the net home price through the incentive constraint channel, the comparative static for the net home price implies a housing demand curve and the shape of this housing demand curve will depend on the nature of screening — whether lenders use downpayment requirements or income verification to ration credit.

Also, for a given home price, changes in the mortgage market due to variation in income risk will change credit access to risky borrowers and influence homeownership. For example, if income risk falls, then a portion of the market that previously was in a screening equilibrium shifts to pooling, and this expands credit access to risky borrowers. Before, they were rationed out of the market through screening, but after the fall in income risk, they have access to pooling contracts and will become homeowners.

Together, these two results suggest an important potential interaction between mortgage markets and housing markets. An analysis of joint housing and mortgage market equilibrium is necessary to fully characterize how the share of non-prime loans varies with economic fundamentals.

2 Joint Mortgage and Housing Market Equilibrium

This section examines joint housing market and mortgage market equilibrium. I show that general equilibrium feedback between mortgage and housing markets leads to a strong form of state-dependent amplification. In particular, small shocks to income risk can sometimes lead to sudden and large changes in equilibrium outcomes. We can interpret this result as coming from endogenous switching between two distinct credit regimes: a screening regime where the marginal home buyer is credit rationed, and a pooling regime where they instead are pooled with safe borrowers. During a transition from the screening regime to the pooling regime home prices boom. Simultaneously lenders stop verifying borrower incomes and rapidly relax down-payment requirements.

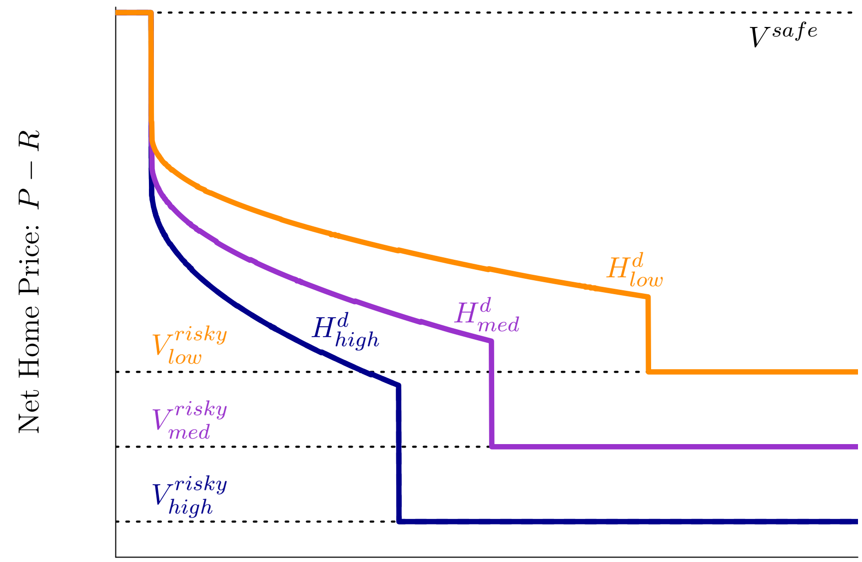
2.1 Housing Demand

Housing demand equals the number of borrowers in the economy who choose to enter the mortgage market and become homeowners. We can trace out a housing demand curve by considering how equilibrium in the mortgage market changes as home prices vary.

First, consider when home prices are sufficiently high so that all risky borrowers prefer to rent — when $P - R > V^*$. In this case, lenders are able to offer safe borrowers their full information contract without worrying about risky borrowers applying. So long as the home price is not too high, safe borrowers will become homeowners. They are indifferent between owning and renting when $P - R = V^{safe}$. They prefer to rent when $P - R > V^{safe}$ and prefer to own when $P - R < V^{safe}$.

Once $P - R \leq V^*$, risky borrowers begin to enter the mortgage market. Their entry leads to adverse selection and the introduction of rationing in the mortgage market. By Proposition 2, the market is split into a pooling segment, where risky borrowers gain access to credit, and a screening segment, where they are rationed out of the prime market.

First consider when the home price is high enough that high-documentation contracts are not introduced. By Proposition 2, this outcome occurs when $V^{doc} < V^{down}(P-R)$. In this case, lenders use downpayment requirements to ration credit in the screening segment of the market. Over this range of the net home price, the size of the pooling segment is increases as the net home price falls. Therefore, more risky borrowers become homeowners as the net home price falls and we have a



Homeownership Rate: H

Figure 5: Housing demand as income risk varies. The orange corresponds to low income risk, purple to medium income risk, and blue to high income risk.

downward sloping housing demand curve. Note that the marginal home buyer is a risky borrower who has access to a pooling contract. I refer to this case as the pooling regime.

Once home prices are low enough, the market enters the alternative screening regime. This occurs once lenders introduce income verification requirements — once $V^{doc} > V^{down}(P - R)$. Now, further reductions in the net home price no longer increase credit access to risky borrowers and the size of the pooling segment does not change with the net home price.

As a result, the homeownership rate becomes inelastic. Expansion in homeownership requires more risky borrowers to become homeowners, but the introduction of income verification rations the marginal borrower out of the market. Additional risky borrowers will become homeowners only if they find it worthwhile to use their full information contract, which occurs once $P - R \leq V^{risky}$.

Figure 5 shows this housing demand curve as income risk varies. Housing demand is 0 when $P - R$ is above V^{safe}. Once $P - R < V^{safe}$ all safe borrowers become homeowners. Then, if $P - R < V^*$, the mortgage market enters the pooling regime and housing demand increases as $P - R$ falls. For a sufficiently low home price, the mortgage market enters the screening regime, and housing demand becomes is inelastic. Once $P - R = V^{risky}$, risky borrowers are willing to use their full information contract to buy a home, and if $P - R < V^{risky}$ then all risky borrowers become homeowners. The shape of the housing demand curve summarizes how adverse selection and changes in credit rationing impact homeownership.

This demand curve shifts with income risk. A fall in income risk increases V^{risky} (because risky borrowers value homes more when they are less likely to default), and also makes pooling relatively attractive for safe borrowers. The expansion in the pooling segment of the mortgage market increases credit access to risky borrowers, which shifts the inelastic portion of the housing demand curve rightward.

2.2 General Equilibrium

I now consider joint housing market and mortgage market equilibrium. For simplicity, I focus on the case of a fixed stock of housing. The intuition developed here will carry forward once I consider dynamics and the accumulation of housing in Section 3.

With an inelastic supply of housing, if a fall in income risk expands the pooling segment of the mortgage market enough (if the inelastic portion of housing demand shifts far enough to the right), then it will trigger a rapid increase in home prices, a full shift away from screening contracts (a switch to the pooling regime), and a rapid relaxation of down-payment requirements.

Let H^s denote the stock of homes. As income risk falls, housing demand increases, as in Figure 5. If the level of housing demand attributable to risky borrowers gaining access to pooling contracts remains below H^s, then for the housing market to clear some risky borrowers must be willing to use their full information contract and become homeowners. Their participation in the market requires that $P - R \leq V^{risky}$. All risky borrowers would want to become homeowners if $P - R < V^{risky}$ and the market would not clear. We must have $P - R = V^{risky}$. The marginal homeowner is a risky borrower using a high interest rate loan who is indifferent between owning and renting. Note that in this case, lenders must be using income verification to ration credit access, so the screening regime prevails.

Figure 6 depicts this outcome. The top panel corresponds to the mortgage market (as in Figure 4), depicting the pattern of segmentation across risk indices. The bottom panel corresponds to the housing market, with a demand curve as in Figure 5 and an inelastic housing supply.

Now consider a fall in income risk and suppose that the inelastic portion of the housing demand curve surpasses H^s. This increase beyond H^s represents risky borrowers who previously rented gaining access to pooling loans. Because these risky borrowers are pooled with safe borrowers, they face a relatively low interest rate, get more value from the pooling contract than they get from their full information contract, and therefore strictly prefer to enter the housing market. Given the fixed housing stock, this increase in demand cannot be met. Since all market participants value homeownership more than the prevailing home price of $P - R = V^{risky}$, borrowers begin to bid up home prices.

As home prices rise, homeownership becomes less attractive relative to renting, which reduces downpayment requirements (the IC constraint relaxes). Figure 7 depicts this outcome. The net home price rises from $P_0 - R$ to $P_1 - R$, relaxing the IC constraint, and reducing downpayment requirements. This improvement in loan terms increases the value of the downpayment contract for safe borrowers from V_0^{down} to V_1^{down}, and shifts the mortgage market toward using downpayment

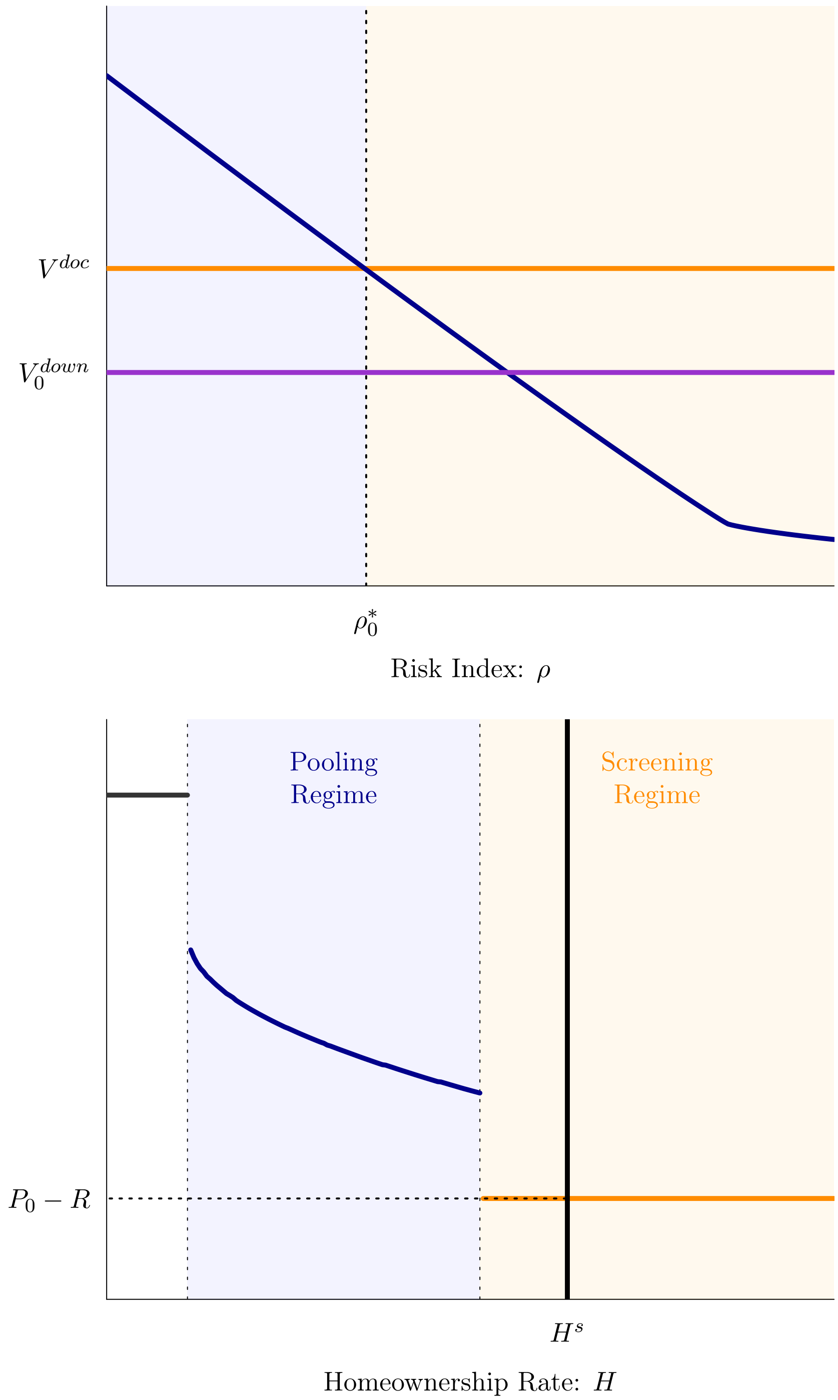


Figure 6: Joint equilibrium in the mortgage market (top) and housing market (bottom) when the screening regime occurs (high income risk).

requirements to ration credit instead of using income verification.

It is through the switch to downpayment-based credit rationing and the incentive constraint channel that we get housing market clearing. Home prices must increase enough to generate a

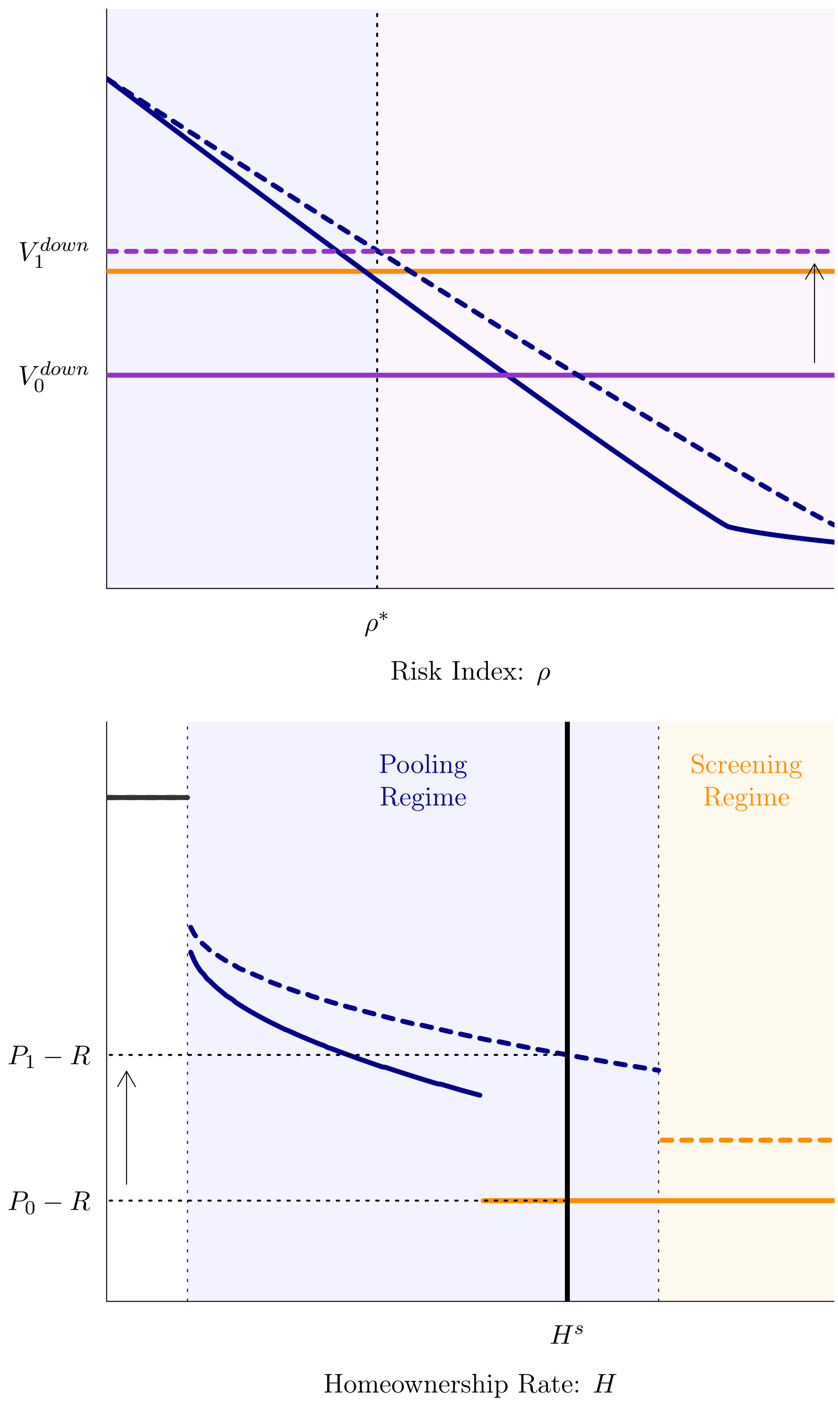


Figure 7: Joint equilibrium in mortgage market (top) and housing market (bottom) during a shift from the screening regime to the pooling regime (fall in income risk).

shift away from income-verification based screening toward downpayment-based screening. With the incentive constraint channel active, further increases in the net home price will reduce the size of the pooling segment, reducing credit access to risky borrowers, and dampening housing demand.

Before the shock to income risk, the marginal home buyer was a risky borrower who was screened out of the market and was indifferent between owning and renting. Now, that same marginal home buyer (because the homeownership rate has not changed) instead has access to a pooling contract. The economy has switched from the original screening regime to the pooling regime.

Note that outcomes change discontinuously during the regime change. Joint housing and mortgage market equilibrium implies that a small fall in income risk can generate a sudden collapse in income documentation requirements, a rapid appreciation in home prices, and a relaxation of down-payment requirements.

We can interpret the rise in the home price as coming from a switch in the indifference condition pricing homes. In the screening regime, risky borrowers had to be indifferent between owning and renting so that they would be willing to purchase homes. But in the pooling regime, risky borrowers gain access to low interest rate loans and now strictly prefer to become homeowners. Instead of through their indifference condition, homes are now priced implicitly through the indifference condition of those safe borrowers who are on the margin between using a pooling contract and a high-downpayment contract contract. It is their indifference between contract types which determines the extent of credit access to risky borrowers and the level of housing demand. In effect, homes are priced implicitly through the IC constraint.

This result has two interpretations which tie to previous results in the literature on the origins of the housing boom. Justiniano et al. (2015b) show that shocks to an exogenous limit on credit supply can generate a housing boom and bust. In this model, the screening regime captures such a credit supply constraint, and the shift to the pooling regime is an endogenous relaxation of this constraint. While income risk is high, adverse selection in the mortgage market leads to credit rationing through income documentation requirements. When the credit regime switches from screening to pooling, it is as-if a credit limit ceases to bind.

The results can also be interpreted as endogenizing the shock to loan-to-value ratios that Kaplan et al. (2016) find are necessary to explain the boom and bust. This interpretation comes from how the regime shift changes the role of the IC constraint. This constraint can be interpreted as a loan-to-value constraint because it specifies (given prevailing home prices), how large of a down payment a safe borrower must make to signal their type. When home prices rise, this constraint relaxes, allowing safe borrowers who use high-downpayment contracts to increase their leverage as home prices appreciate. This mechanism generates a shift in loan-to-value ratios as-if there were a shock to a loan-to-value constraint.

Note that the mechanism works both to generate booms and busts. Starting from the pooling regime, an increase in income risk can lead to a tightening of lending standards and a collapse in home prices. This result implies that mean reversion in income risk can explain a bust following a housing boom. The next section extends the model to a dynamic setting and formalizes this intuition.

3 Housing Boom-Bust Cycles

To examine the dynamics of home prices, I extend the model to an infinite horizon setting and introduce simple house production and rental unit sectors. The resulting rational expectations equilibrium is characterized by housing cycles due to shifts in the credit regime. These cycles are driven by small underlying shocks to borrower income risk which get amplified through the non-linear general equilibrium mechanism from Section 2.

The non-linear mechanism at the heart of the model leads to both a housing boom and housing crisis following a shock that lowers income risk and then smoothly reverts. The resulting dynamics appear as-if a large positive credit supply shock hits the economy and then a large negative credit supply shock follows up to generate a housing crisis. However, instead of the dynamics arising from a series of negatively correlated shocks, the theory generates the entire episode through the non-linear amplification mechanism arising from joint housing and mortgage market clearing. As income risk falls, the economy reaches the threshold triggering the pooling regime and an endogenous credit-boom occurs. Then, as income risk reverts to steady state, the economy passes back over regime-change threshold and reverts to the screening regime. Credit contracts and a housing crisis occurs. In this model, a housing crisis is the ultimate unwinding of the initial credit-fueled boom.

3.1 Dynamic Extension

Time is indexed by $t \in \{0, 1, \dots\}$. I re-interpret period 0 of the previous static model as the end of each time period t, and period 1 as the beginning of $t + 1$. All notation carries forward, but with a time subscript t appended.

For instance, I denote income risk by ϕ_t, which I assume evolves as a second order autoregressive process in logs:

$$\ln \phi_t = (1 - \rho_1^\phi - \rho_2^\phi)\mu^\phi + \rho_1^\phi \ln \phi_{t-1} + \rho_2^\phi \ln \phi_{t-2} + \sigma^\phi \varepsilon_t^\phi.$$

This setup generates momentum, which will lead to persistent rather than transitory shifts in the credit regime. I view this specification for income risk as a reduced form for a model of the labor market in which individuals have a higher risk of losing income during an economic downturn[17]. For expositional simplicity, I take income risk as exogenous. This exogeneity assumption shuts down potential general equilibrium feedback from home prices to the state of the business cycle which may be important for understanding the housing boom and bust (Mian and Sufi (2011) Berger et al. (2015)). However, abstracting from this additional source of amplification helps to clarify the mechanism introduced in this paper.

I assume that the resale value of a home purchased in $t-1$ is proportional to the equilibrium price at time t, so that $P_t Z_{it}$ is the value of individual i's home[18]. The value of the home in foreclosure

[17] Appendix C presents a model of labor market search where fundamental shocks to firm productivity generate fluctuations in employment risk and therefore generate fluctuations in income risk.

[18] This assumption can be formalized by a setup where homeowners have idiosyncratic home maintenance and investment opportunities. The amount of effective housing that they own after maintenance and investment is Z_{it}.

is then $P_t(1-\lambda)Z_{it}$.

Given this proportionality, anticipated contract payoffs in $t+1$ for loan of size L_{it} to borrower i at time t are

$$\mathbb{E}_t\Pi\left(\frac{L_{it}}{P_{t+1}}, \rho_{it}\phi_{t+1}\right)P_{t+1}, \quad \text{and} \quad \mathbb{E}_tU\left(\frac{L_{it}}{P_{t+1}}, \tau_i\phi_{t+1}\right)P_{t+1}.$$

Ex-post payoffs are homogenous of degree one in the realized home price and the face value of debt, and so we can re-use the functions Π and U after normalizing by the next period home price.

I assume that borrower observables take three values: $\rho_{it} \in \{0, \rho_L, \rho_H\}$ where $0 < \rho_L < \rho_H < 1$. Borrower with $\rho_{it} = 0$ are *known safe borrowers*. In this segment of the market, there is no problem of adverse selection. All borrowers will receive the full information contract intended for safe borrowers and so they face neither an income documentation requirement or a high down payment requirement. We can interpret this segment of the market as representing the traditional Alt-A segment of the mortgage market — low documentation loans to borrowers with very high credit scores.

In contrast, adverse selection bites in the other two portions of of the market. Borrowers with $\rho_{it} = \rho_L$ are *low risk borrowers* while those with $\rho_{it} = \rho_H$ are *high risk borrowers*. These two portions of the market both represent borrowers with less-than-perfect credit. A portion ρ_L of low risk borrowers are risky and a portion ρ_H of high risk borrowers are risky. Let n_L denote the fraction of the population with $\rho_{it} = \rho_L$ and n_H denote the fraction with $\rho_{it} = \rho_H$. The remaining $1 - n_L - n_H$ borrowers are known safe borrowers with $\rho_{it} = 1$.

Because I no longer assume a continuous distribution for ρ_{it}, equilibrium now requires that lenders occasionally use mixed strategies. In particular, in the parameterization I will consider, it is the middle segment of the market which will determine the prevailing credit regime. The segment of known safe borrowers always receives their full information contract and the high risk segment will always be in a separating equilibrium. The middle segment will either be in a screening equilibrium where all risky borrowers have their type revealed, or in a pooling equilibrium where lenders randomly mix between offering pooling contracts and high-downpayment contracts. Those risky borrowers who are offered the pooling contract will gain access to credit, while those risky borrowers who are offered the high-downpayment contract will opt to rent their home.

The homeownership rate in the pooling regime equals the number of risky borrowers who gain access to pooling contracts (since they strictly prefer pooling), plus the number of safe borrowers (who all become homeowners). Housing market clearing then pins down the fraction of risky borrowers in segment $\rho_{it} = \rho_L$ who are offered pooling contracts. Safe borrowers in this group are happy to receive a random offer so long as they are indifferent between pooling and separating contracts — which is precisely the indifference condition pricing housing in the pooling regime.

Let H_t denote the homeownership rate and let μ_t denote the probability that a lender offers the pooling contract in the pooling regime. Housing market clearing requires that

$$H_t = \underbrace{(1 - n_L - n_H)}_{\text{Known Safe}} + \underbrace{n_L(1-\rho_L)}_{\text{Safe Among Low Risk}} + \underbrace{n_L\rho_L}_{\text{Risky Among Low Risk}} \mu_t + \underbrace{n_H(1-\rho_H)}_{\text{Safe Among High Risk}}$$

The equilibrium homeownership rate must equal the demand for housing. All safe borrowers own a home — captured by the first, second, and fourth terms on the right hand side. The third term is the number of risky borrowers who become homeowners. These borrowers only come from the low risk segment because the high risk segment is in a screening equilibrium. A fraction μ_t of risky borrowers in the low risk segment get offered a pooling contract.

Solving for μ_t gives an expression for the chance that a low risk borrower is offered a pooling contract.

$$\mu_t = \frac{H_t - (1 - n_L\rho_L - n_H\rho_H)}{n_L\rho_L}.$$

The numerator is the portion of homeowners who are risky borrowers. The number of safe borrowers in the whole population is $1-n_L\rho_L-n_H\rho_H$ and any housing that is not purchased by a safe borrower must be purchased by a risky borrower. The denominator is the portion of borrowers in the low risk segment who are risky.

I refer to the portion of overall mortgages which are either intended for risky borrowers or are require neither income documentation or a high downpayment as the *non-prime share*. It is this quantity which we observed suddenly increase between 2003 and 2004, and then collapse in the housing crisis. In the model it corresponds to known safe borrowers, low risk borrowers who are offered a pooling contract, and risky borrowers using their full information contract. This quantity is

$$\text{Non-Prime Share}_t = \begin{cases} \frac{(1-n_L-n_H)+H_t-(1-\rho_L n_L-\rho_H n_H)}{H_t} & \text{if } S_t = \text{Screening} \\ \frac{(1-n_L-n_H)+n_L\mu_t}{H_t} & \text{if } S_t = \text{Pooling} \end{cases}$$

In the screening regime, non-prime lending consists of mortgages to known safe borrowers and mortgages to risky borrowers. In the pooling regime, it consists of mortgages to known safe borrowers and pooling contracts in the low risk segment.

The prevailing regime depends on the value of alternative contracts to safe borrowers in the low risk segment. The high-documentation contract has a loan size satisfying

$$0 = \frac{1}{1+r}\mathbb{E}_t\Pi_1\left(\frac{L_t^{doc}}{P_{t+1}}, 0\right) + \beta\mathbb{E}_t U_1\left(\frac{L_t^{doc}}{P_{t+1}}, 0\right).$$

The value of this contract to a safe borrower in the low risk segment is

$$V_t^{doc} = \frac{1}{1+r}\mathbb{E}_t\Pi\left(\frac{L_t^{doc}}{P_{t+1}}, 0\right)P_{t+1} + \beta\mathbb{E}_t U\left(\frac{L_t^{doc}}{P_{t+1}}, 0\right)P_{t+1} - \kappa.$$

The pooling contract in the low risk segment has a loan size satisfying

$$0 = \frac{1}{1+r}\mathbb{E}_t\Pi_1\left(\frac{L_t^{pool}}{P_{t+1}}, \rho_L\phi_{t+1}\right) + \beta\mathbb{E}_t U_1\left(\frac{L_t^{pool}}{P_{t+1}}, 0\right).$$

Safe borrowers get value of

$$V_t^{pool} = \frac{1}{1+r}\mathbb{E}_t\Pi\left(\frac{L_t^{pool}}{P_{t+1}}, \rho_L\phi_{t+1}\right)P_{t+1} + \beta\mathbb{E}_t U\left(\frac{L_t^{pool}}{P_{t+1}}, 0\right)P_{t+1}$$

from this contract.

The prevailing credit regime is determined by these two values. Let S_t indicate the credit regime:

$$S_t = \begin{cases} \text{Screening} & \text{if } V_t^{doc} \geq V_t^{pool} \\ \text{Pooling} & \text{if } V_t^{doc} < V_t^{pool}. \end{cases}$$

If $V_t^{doc} \geq V_t^{pool}$ then safe borrowers in the low risk segment prefer screening, and otherwise they prefer pooling.

In the screening regime, risky borrowers are indifferent between their full information contract and renting. The first order condition determining the loan size of their full information contract, L_t^{risky}, is

$$0 = \frac{1}{1+r}\mathbb{E}_t\Pi_1\left(\frac{L_t^{risky}}{P_{t+1}}, \phi_{t+1}\right) + \beta\mathbb{E}_t U_1\left(\frac{L_t^{risky}}{P_{t+1}}, \phi_{t+1}\right).$$

They get value of

$$V_t^{risky} = \frac{1}{1+r}\mathbb{E}_t\Pi\left(\frac{L_t^{risky}}{P_{t+1}}, \phi_{t+1}\right)P_{t+1} + \beta\mathbb{E}_t U\left(\frac{L_t^{risky}}{P_{t+1}}, \phi_{t+1}\right)P_{t+1}$$

from this contract. The pricing condition in the screening regime is then

$$P_t = R_t + V_t^{risky} \quad \text{if} \quad S_t = \text{Screening}.$$

In the pooling regime, safe borrower indifference between pooling and high-downpayment contracts pins down the home price. The binding IC constraint implies that the high-downpayment contract's loan size must satisfy

$$\frac{1}{1+r}\mathbb{E}_t\Pi\left(\frac{L_t^{down}}{P_{t+1}}, 0\right)P_{t+1} + \beta\mathbb{E}_t U\left(\frac{L_t^{down}}{P_{t+1}}, \phi_{t+1}\right)P_{t+1} = P_t - R_t$$

giving safe borrowers value of

$$V_t^{down} = \frac{1}{1+r}\mathbb{E}_t\Pi\left(\frac{L_t^{down}}{P_{t+1}}, 0\right)P_{t+1} + \beta\mathbb{E}_t U\left(\frac{L_t^{down}}{P_{t+1}}, 0\right)P_{t+1}.$$

Their indifference condition (implicitly) pricing the home is then

$$V_t^{pool} = V_t^{down} \quad \text{if} \quad S_t = \text{Pooling}.$$

The last contract we need is the full information safe contract. It has loan size satisfying

$$0 = \frac{1}{1+r}\mathbb{E}_t\Pi_1\left(\frac{L_t^{safe}}{P_{t+1}}, 0\right) + \beta\mathbb{E}_t U_1\left(\frac{L_t^{safe}}{P_{t+1}}, 0\right).$$

Safe borrowers get value of

$$V_t^{safe} = \frac{1}{1+r}\mathbb{E}_t\Pi\left(\frac{L_t^{safe}}{P_{t+1}}, 0\right)P_{t+1} + \beta\mathbb{E}_t U\left(\frac{L_t^{safe}}{P_{t+1}}, 0\right)P_{t+1}.$$

from this contract.

Given these contracts, we can calculate the ex-post depreciation of the housing stock due to foreclosure losses. On a contract with loan size of L, the ex-post loan-to-value ratio is L/P_t and the ex-post default rate among borrowers with type τ is $D(L/P_t, \tau\phi_t, z)$. As a result, the average home quality after foreclosure losses given loan size of L and type τ is

$$\Gamma(L/P_t, \tau\phi_t) = \int_0^\infty \left[1 - \lambda D\left(L/P_t, \tau\phi_t, z\right)\right] zF'(z)\mathrm{d}z$$

We can then calculate the stock of housing after idiosyncratic value shocks and foreclosure losses. This value depends on the allocation of contracts from the prior period, and therefore depends on the prior credit regime. Let $\tilde{H}_t$ denote this remaining stock of housing.

If the screening regime prevailed in the previous period, then this quantity satisfies

$$\begin{aligned}\tilde{H}_t = {} & \Gamma\left(\frac{L_{t-1}^{safe}}{P_t}, 0\right)(1 - n_L - n_H) + \Gamma\left(\frac{L_{t-1}^{doc}}{P_t}, 0\right)\left[n_L(1-\rho_L) + n_H(1-\rho_H)\right] \\ & + \Gamma\left(\frac{L_{t-1}^{risky}}{P_t}, \phi_t\right)(H_{t-1} - (1 - n_L\rho_L - n_H\rho_H) \quad \text{if} \quad S_{t-1} = 1\end{aligned}$$

Known safe borrowers used the safe contract. The remaining safe borrowers used the high-documentation contract. All remaining housing demand must come from risky borrowers who opted to use the risky contract.

On the other hand, if the pooling regime prevailed in the prior period, then the remaining stock of housing is

$$\begin{aligned}\tilde{H}_t = {} & \Gamma\left(\frac{L_{t-1}^{safe}}{P_t}, 0\right)(1 - n_L - n_H) + \Gamma\left(\frac{L_{t-1}^{pool}}{P_t}, 0\right)n_L(1-\rho_L)\mu_{t-1} \\ & + \Gamma\left(\frac{L_{t-1}^{down}}{P_t}, 0\right)n_L(1-\rho_L)(1-\mu_{t-1}) + \Gamma\left(\frac{L_{t-1}^{pool}}{P_t}, \phi_t\right)n_L\rho_L\mu_{t-1} \\ & + \Gamma\left(\frac{L_{t-1}^{down}}{P_t}, 0\right)n_H(1-\rho_H) \quad \text{if} \quad S_{t-1} = 2.\end{aligned}$$

This expression differs from the previous because risky borrowers no longer use the risky contract, safe borrowers in the low risk segment use both the pooling and high-downpayment contracts, safe borrowers in the high risk segment use high-downpayment contracts, and risky borrowers in the low risk segment use the pooling contract.

3.2 Rental and Housing Supply

I assume that absentee landlords supply rental units using apartment buildings. Apartment buildings are produced from numeraire using a constant returns to scale technology. Given the cost of funds of r faced by landlords, the rental rate is equal to the user cost of apartments:

$$R_t = \Xi - \frac{1-\delta_A}{1+r}\Xi$$

where Ξ is the landlord's conversion rate between apartments and numeraire and δ_A is the depreciation rate on apartments. Under this assumption, the supply of rental units is perfectly elastic. This setup simplifies the analysis and lets the model match the stability of rental rates over the housing boom. Although a stark simplification, the main results go through if landlords can produce rental services by combining housing and apartments — see Appendix B for an example extension.

Housing units are produced by a competitive housing sector. New homes are built using a Cobb-Douglas production technology in land and numeraire. In each period, a quantity $\ell > 0$ of new land is auctioned to home builders by the government. The supply of new housing then solves:

$$\max_{H_t^{new}, I_t^H} \quad P_t H_t^{new} - I_t^H$$
$$\text{s.t.} \quad H_t^{new} \leq \ell^\alpha \left(\frac{I_t^H}{1-\alpha}\right)^{1-\alpha}$$

where I_t^H denotes investment in new housing. Since land is purchased from the government and the production of housing has constant returns to scale, developers make zero profits and the government receives land rents. The implied stock of new housing is

$$H_t^{new} = P_t^{\frac{1-\alpha}{\alpha}} \ell.$$

The quantity $(1-\alpha)/\alpha$ is the elasticity of new home construction to the home price.

The supply of housing is then the amount of housing from the prior period after depreciation and foreclosure costs, plus new housing. This quantity is

$$H_t = \tilde{H}_t + H_t^{new}$$

Note that both $\tilde{H}_t$ and H_t^{new} are increasing in the home price so housing supply is upward sloping. It is no longer inelastic, as in Section 2.

Table 1: Baseline parameterization (quarterly)

Parameter	Value	Target/Source
λ	0.27	Campbell et al. (2011)
δ_o	.0038	1.5% ann. depreciation (Piazzesi and Schneider (2016))
σ_z	.07	.14 ann. s.d. of home prices (Piazzesi and Schneider (2016))
β	.95	Iacoviello (2005)
r	$\ln(1/.99)$	Iacoviello (2005)
R	1	normalize rental rate to 1
ℓ	3.52×10^{-5}	65% s.s. homeownership rate
$\frac{1-\alpha}{\alpha}$	1.5	median supply elasticity in Saiz (2010)
$\exp(\mu^{\phi})$	11.1%	6% s.s. spread on risky contract
κ	0.0119	0.75% s.s. spread on high-doc contract in segment L
ρ_L	8.35%	0.75% s.s. spread on pooling contract in segment L
ρ_H	47.4%	0.36% foreclosure rate in steady state (Corbae and Quintin (2013))
n_L, n_H	29.3%, 70.1%	1% s.s. share pooling and 1% s.s. share risky borrowers
ρ_1^{ϕ}	1.6	
ρ_2^{ϕ}	$-.63$	
σ^{ϕ}	.05	

3.3 Numerical Example: Parameterization

I parameterize the model to benchmark values from the macroeconomic literature on housing. For the new parameters, I target historical mortgage rates and spreads, as well as the historical foreclosure rate. For the exogenous process for income risk, I choose parameter values that generate momentum. The qualitative results only require that income risk exhibits hump-shaped impulse responses. Table 1 summarizes the model parameterization.

Mortgage contract payoffs depend on the foreclosure loss parameter, λ, and the distribution of Z_{it}. I choose $\lambda = .27$ to match the estimate of the sale discount on a foreclosed homes from Campbell et al. (2011). I assume that Z_{it} has a log-normal distribution with mean equal to $1 - \delta_o$ and with standard deviation parameter of σ_z. The parameter δ_o can be interpreted as the depreciation rate on owner-occupied housing. I choose this depreciation rate and the standard deviation parameter based on Piazzesi and Schneider (2016). Specifically, I choose δ_o to lie on the lower bound of the range that they suggest, and use the standard deviation parameter to match idiosyncratic volatility in home prices.

In order to be consistent with the literature, I set the discount factor for borrowers to match the discount factor for impatient households in Iacoviello (2005). Similarly, I choose the lender cost of funds parameter, r, equal to the discount rate for patient households in Iacoviello (2005).

I parameterize land supply to match homeownership — given the steady state home price, we can use ℓ to match a 65% homeownership rate — the average homeownership rate in the three decades prior to 2003. I choose the expenditure share on land, α, to imply an housing supply elasticity, $\frac{1-\alpha}{\alpha}$, equal to the median of 1.5 in Saiz (2010).

In steady state, the full information contract for risky borrowers depends crucially on the income risk parameter μ^ϕ. Since this contract represents a high interest rate loan used by borrowers who are known to have elevated default risk, I choose this parameter to imply a 6% spread of the contract's interest rate above the cost of funds for lenders. This choice implies a steady state mortgage rate of about 10% faced by risky borrowers in the subprime market, which is in the typical range for interest rates faced by subprime borrowers prior to the housing boom.

Next, I choose values for ρ_L and κ to imply that both the pooling contract in the low risk segment and high-documentation contracts have a 0.75% spread over the lender's cost of funds. In the model, when these two contracts have the same interest rate, safe borrowers in the low risk segment will prefer the high-documentation contract because it gives them their optimal loan size. However, keeping the spread on the two contracts close to each other ensures that there is some chance that shocks to income risk can trigger a switch to the pooling regime. Since the high-documentation contract interest rate depends on both parameters, I first choose ρ_L to imply a 0.75% spread on the pooling contract, and then choose κ (given ρ_L) to get a 0.75% spread on the screening contract. This parameterization implies that the mortgage rate for the high-documentation contract is about 4.5%, in line with levels of prime mortgage rates (after accounting for inflation) during the early 2000's (see Justiniano et al. (2016)).

Finally, we can jointly pin down the share of risky borrowers in the high risk segment, the size of the low risk segment, and the size of the high risky segment to target the share of pooling contracts, the share of contracts going to borrowers who are revealed to be risky (subprime mortgages), and the steady state foreclosure rate. The former two targets I choose as 1% to reflect that both forms of non-prime lending were rare prior to the housing boom. For the later moment, I match a 0.36% foreclosure rate following Corbae and Quintin (2013).

Finally, I choose parameter values for the exogenous income risk process to imply that income risk has some momentum. The exact parameter values are not important for the qualitative results. A hump-shaped response is necessary to get persistent changes in the credit regime from slightly different paths for income risk. I choose the standard deviation parameter σ^ϕ to be just large enough so that the solution algorithm (see Appendix **??**) converges to an equilibrium where credit regime changes occasionally arise. If this parameter is too small, then equilibrium remains in a small enough neighborhood of the steady state that the algorithm converges to the usual local linear approximation coming from traditional perturbation methods. For a sufficiently large standard deviation, the algorithm will instead converge to a regime-switching linear approximation, capturing the non-linear equilibrium dynamics arising in this model.

3.4 Numerical Example: Simulations

Figure 8 shows equilibrium outcomes from two simulations of the model. In the first simulation (dashed lines), I start the economy at its stochastic steady state[19], and then shock the economy for

[19]The stochastic steady state is the long-run outcome in the absence of shocks and is not necessarily equal to the deterministic steady state.

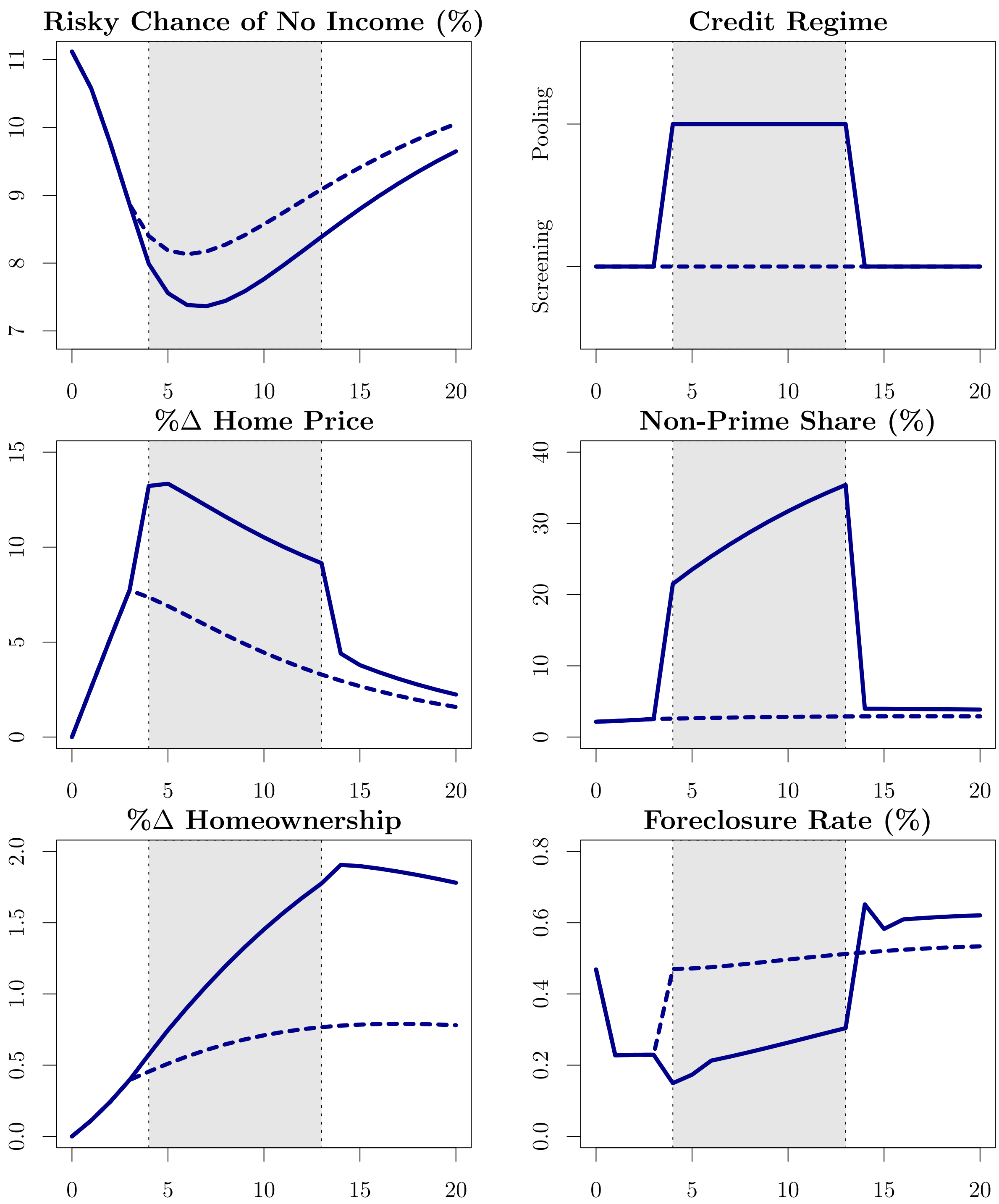


Figure 8: Response to 3 (dashed) and 4 (solid) -1/2 standard deviation shocks to income risk. Shaded region indicates pooling regime in second simulation.

three periods with a series of -0.5 standard deviation income risk shocks. These shocks push the economy to the brink of regime change, but do not quite trigger the pooling regime. In the second simulation, I perform the same exercise, but add a fourth -0.5 standard deviation shock, which pushes the economy past the tipping point and triggers the pooling regime. The difference between the dashed and solid impulse responses lets us visualize the effect of the regime change.

The top left panel shows the implied paths for income risk in the two scenarios. Both have a smooth hump-shaped response to the shocks. The responses are identical for the first three periods, but then diverge in the fourth period. For both simulations, income risk steadily and smoothly reverts to steady state after peaking.

These two slightly different paths for income risk lead to very different equilibrium outcomes. The top right panel shows the equilibrium credit regime. In the three-shock simulation, the economy stays in the screening regime throughout and exhibits a smooth impulse response. In the four-shock simulation, the additional shock in period 4 triggers the pooling regime which persists until period 14.

The regime change leads to a sudden and rapid shift in equilibrium outcomes. Home prices rapidly appreciate (middle left panel), the share of non-prime lending in the economy jumps up and begins to grow (middle right panel), the growth rate of homeownership increases (bottom left panel), and the foreclosure rate slightly falls and then begins to grow (bottom right panel).

As income risk reverts smoothly toward steady state, the economy eventually reaches the regime-change threshold again. The resulting shift back to the screening regime leads to a credit contraction (collapse in non-prime share) and a sudden collapse in home prices. The collapse in home prices pushes homeowners underwater and generates a foreclosure crisis — which appears as a spike in the foreclosure rate in period 14.

Again, note that no shock hits the economy to generate the bust — the only driving force underlying these responses is the smoothly varying path of income risk. From the accumulated effect of shocks in the early periods of the simulation, we get a rich boom-bust pattern through the model's state-dependent amplification mechanism. Although it appears as-if a series of highly correlated shocks hit the economy in periods 4 and then again in period 14, the true underlying income risk shocks all occurred within the first 4 periods of the simulation. The turning points of the cycle can be entirely attributed to the propagation and amplification of these initial shocks.

It took a number of these initial shocks to push the economy to the tipping point — specifically four -0.5 standard deviation shocks — so this housing boom-bust episode is a relatively rare event. The conditions that trigger the housing cycle develop steadily through the cumulative effect of small shocks to the aggregate state of the economy. This result formalizes the idea that developing conditions in an economy may lead up to a boom. This model provides micro-foundations to study how evolving economic conditions might signal an imminent credit boom.

This theory provides a framework for modeling credit-driven housing booms which end in housing crises. The predictions of the model line up with the historical evidence of Jordà et al. (2015), who show that a simultaneous boom in credit and home prices predicts a subsequent bust. This theory proposes adverse selection in the mortgage market and general equilibrium between housing and the mortgage market as an explanation for this type of boom-bust pattern. Because of the non-linear mechanism underlying this result, the model does not rely on shocks to drive the turning points in the cycle. Rather, small shocks accumulate into the state of the economy to trigger the boom, then mean reversion in fundamentals triggers the bust.

4 Discussion of Empirical Evidence

This section examines the theory's predictions for the evolution of debt across borrowers, and for mortgage rates. I argue that the model matches existing empirical evidence from individual-level mortgage outcomes during the housing boom and bust.

4.1 The Allocation of Debt During the Boom

The microeconomic literature has recently turned to credit bureau data to examine how mortgage markets allocated debt during the housing boom. Mian and Sufi (2015) examine the growth in debt of individuals borrowers, binned according to their FICO score in 1997. They find that debt grew more for borrowers with low FICO scores in 1997 than for borrowers with high FICO scores in 1997 and show that the difference across the credit score distribution comes from the dynamics of loan-to-value ratios. High FICO score borrowers reduces their loan-to-value ratios during the boom while low FICO score borrowers did not. In contrast, Albanesi et al. (2016) look at the allocation of debt within each year by FICO score. They find that the percent difference in debt between low FICO score borrowers in 2005 and low FICO score borrowers in 2000 was smaller than the percent difference in debt between high FICO score borrowers in 2005 and high FICO score borrowers in 2000. Based on different conditional moments calculated from similar data, these authors come to different conclusions[20].

I argue that the theory presented in this paper may help to reconcile these results. When comparing the theory to this empirical evidence, it is crucial to distinguish between borrower time-invariant characteristics (captured by τ_i), and borrower observable characteristics relevant for credit decisions (captured by ρ_i). A switch from the screening regime to the pooling regime leads to a change in how lenders ration credit. They substitute down-payment based rationing for income-verification based rationing. The implication is that safe borrowers in the prime market will reduce their loan-to-value ratios after the regime change. In contrast, risky borrower loan-to-value ratios are relatively stable since they find high-downpayment loans undesirable. As a result, risky borrowers have debt which moves in proportion to home prices while safe borrower debt moves less than one-for-one with home prices. This pattern of loan-to-value ratios over time by borrower type implies that debt grows more for risky borrowers during the boom.

However, fundamental types are not directly measurable. The variable, ρ_i, measures lender initial beliefs about the riskiness of borrowers before they make mortgage offers — which is likely to be correlated with a borrowers credit score close to the time of mortgage origination. Since the mortgage market is segmented according to this variable, the pattern of debt growth will vary by

[20] Albanesi et al. (2016) replicate the result of Mian and Sufi (2015) so the difference is unlikely due to a discrepancy in their samples.

ρ_i. Average debt growth in each market segment depends on both the composition of funded loans (related to the share of risky borrowers in the segment) and the terms of those loans. In low risk parts of the market, lenders pool borrowers which leads to a relatively stable loan-to-value ratio regardless of the credit regime. In contrast, lenders screen in the high risk segment, and loan-to-value ratios will change depending on whether or not they use downpayment requirements or income verification to ration credit. The implication is that loan-to-value ratios will fall in the screening segment during a switch to the pooling regime because lenders begin to rely on downpayment requirements to separate out borrowers. As a result, debt growth will be larger for borrowers that are *observably low risk* (low ρ_i) because they have more stable loan-to-value ratios.

As a result, the distinction between fundamental types and lender beliefs about fundamental types can reverse the pattern of debt growth predicted by the theory. We must be careful when mapping the theory to microeconomic evidence. Whether a given measure of borrower riskiness is more related to borrower types or lender information about types changes how it relates to the theory.

To the extent that the measure used by Mian and Sufi (2015) captures time-invariant dimensions of borrower riskiness, I argue that the theory is consistent with their evidence. They track fixed groups of individuals based on their FICO scores in 1997 and report how debt changed for each group during the boom. If borrower fundamental types are persistent, we would expect the share of risky borrowers to be larger among those borrowers with low FICO scores in 1997 relative to those borrowers with high FICO scores in 1997. As a result, I argue that their measure of borrower riskiness relates to τ_i in the theory. Under this interpretation, the model's predictions for debt growth and loan-to-value ratios are consistent with the evidence of Mian and Sufi (2015).

Simultaneously, I argue that the model also matches the evidence of Albanesi et al. (2016) since their measure of riskiness is specifically designed to reflect lender information close to the time of mortgage origination. They measure borrower riskiness based on observables close to the time of mortgage origination. This measure of riskiness related to the risk index, ρ_i. Along this dimension, the model predicts that borrowers that appeared risky at the time or mortgage origination had less debt growth during the boom — consistent with their findings.

Figure 9 shows the model's predictions for debt growth by borrower fundamental types (τ_i) and by borrower observables (ρ_i). I show the cumulative change in debt in the two simulations considered in Section 3. In the first simulation (dashed lines) three -1/2 standard deviation shocks hit the economy. In the second (solid lines), an additional fourth shock hits the economy and triggers the pooling regime.

In the three-shock simulation, debt grows symmetrically across fundamental borrower types and across borrower observables. In the absence of a change in the credit regime, lenders use income verification to screen risky borrowers which leads to stable loan-to-value ratios within individual borrowers. As a result, debt moves in proportion to home prices and the pattern of debt accumulation reflects the response of home prices to the fall in income risk. There are no differential patterns of debt growth.

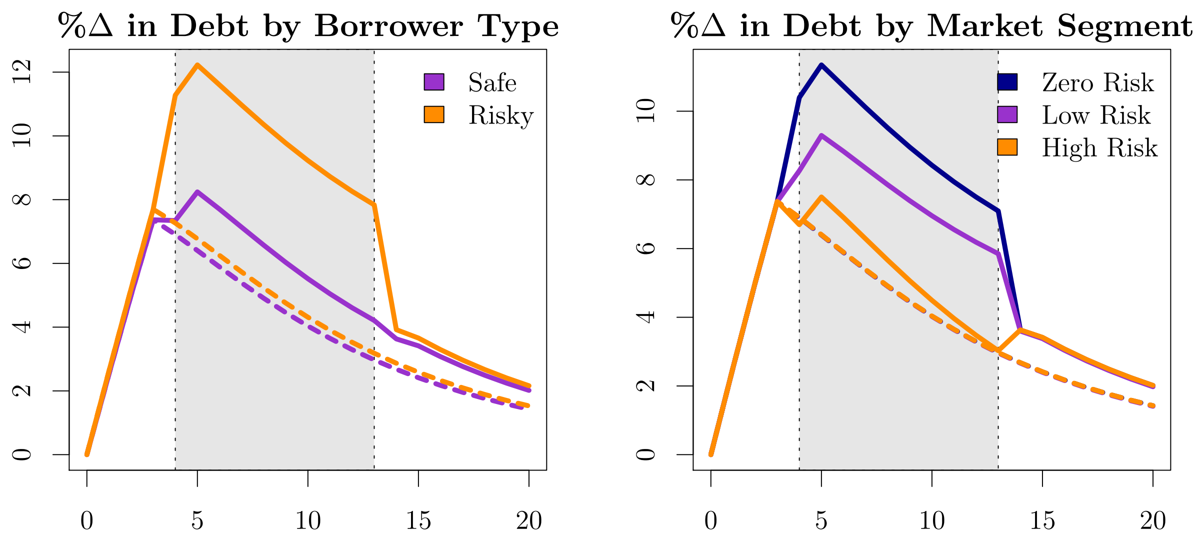


Figure 9: Debt growth by fundamental borrower types (left) and borrower observables at origination (right). Response to 3 (dashed) and 4 (solid) -1/2 standard deviation income risk shocks. Shaded region indicates pooling regime in second simulation.

In contrast, we do get differences in debt growth by borrower type and borrower observables when a fourth shock triggers the pooling regime. First consider the right panel — debt growth by borrower observables (ρ_i). In the zero-risk segment all borrowers are known to be safe and are offered pooling contracts in both regimes. They have relatively stable loan-to-value ratios and so their cumulative debt growth (blue line) moves one-for-one with home prices. On the other extreme, the high risk segment of the market consists entirely of safe borrowers who switch from income-verification based screening to downpayment based screening. These borrowers see a fall in their loan-to-value ratios because of this shift in the nature of credit rationing. As a result, their cumulative debt growth (orange line) moves less than one-for-one with the home price during the boom. In the low risk segment of the market, safe borrowers are indifferent between pooling and making a large downpayment. Lenders mix between offering the two types of contracts. As a result, a portion of borrowers switch from income-verification based screening to pooling and a portion switches from income verification based screening to downpayment-based screening. As a mixture of the two kinds of contracts, their response for debt accumulation is between the extremes of the zero risk and the high risk segments. This pattern — with more debt growth among observably safe borrowers — is qualitatively consistent with the findings of Albanesi et al. (2016) if we assume that their measure of borrower riskiness captures ρ_i in the theory.

Moving to the left graph, we get a reverse pattern that I argue is consistent with the evidence of Mian and Sufi (2015). The debt growth of risky borrowers (orange line) follows the path of home prices because they have relatively stable loan-to-value ratios. In the screening regime they use their full information contract and in the pooling regime they use a pooling contract. Neither require a particularly large downpayment, and so their debt moves in proportion to the home price. Safe borrowers use a mixture of contracts because they get loans from multiple segments of the overall market. However, because a large portion of safe borrowers in the prime market switch from income-verification based screening to downpayment based screening, there is an a fall in the average loan-to-value ratio among safe borrowers. The shift in the nature of credit rationing implies that safe borrower debt moves less than one-for-one with home prices. In contrast, risky borrowers use their full information contract in the screening regime and switch to using pooling contracts in the pooling regime. Neither has a large downpayment requirement, and so loan-to-value does not change very much for risky borrowers. Their pattern for debt growth reflects the movement of home prices. As a result, risky borrower debt grows more than safe borrower debt during the boom. These patterns for debt growth and loan-to-value ratios is consistent with Mian and Sufi (2015) if we assume that their measure of borrower riskiness proxies for borrower fundamental types. This assumption is reasonable so long as borrower fundamental types are relatively persistent.

Note that the above discussion abstracts from possible compositional effects from more risky borrowers entering the market. In the theory, the homeownership rate is relatively constant over the simulation (increasing by two percentage points). If home prices didn't rise, then more risky borrowers would enter the market and drive up homeownership rate. But, in general equilibrium, rising home prices lead to relaxed incentive constraints, and a shift to downpayment based credit rationing. The rationing effect of higher home prices keeps risky borrowers from driving up the homeownership rate and ensures that the housing market clears. Consistent with this result, Foote et al. (2016) show that, indeed, the boom was a bad time to become a new homeowner.

In summary, I argue that the theory is consistent with both the evidence in Mian and Sufi (2015) and the evidence in Albanesi et al. (2016). Rather than being conflicting, I argue that these studies complement each other by giving two distinct sets of moments which any theory of the boom ought to match. By following fixed individuals over time, Mian and Sufi (2015) let us think about reallocation of debt by persistent borrower characteristics. By examining how debt was allocated in each year across borrower observables, Albanesi et al. (2016) show how the market responded to available information. Each approach captures a distinct theoretical concept — borrower fundamental types versus observable characteristics. The theory offered by this paper is consistent with these moments, and highlights the importance of distinguishing between borrower types and borrower observables when interpreting the available microeconomic evidence.

4.2 The Mortgage Rate Conundrum

Justiniano et al. (2016) examine individual level loan data and document a puzzle — during the boom, mortgage rates were low relative to the level we would expect given prevailing treasury yields. In particular, they show that after composition adjusting the sample of loans, average mortgage rates fell relative to treasury yields by about 50 basis points during the later half of 2003, and this persisted for the duration of the housing boom.

Figures 10 and 11 show a stylized replication of their findings using aggregate data. Figure 10 shows the average spread on 15 year and 30 year fixed rate mortgages over the ten year treasury yield from January of 2000 to January of 2008. Between the middle of 2003 and the end of 2005,

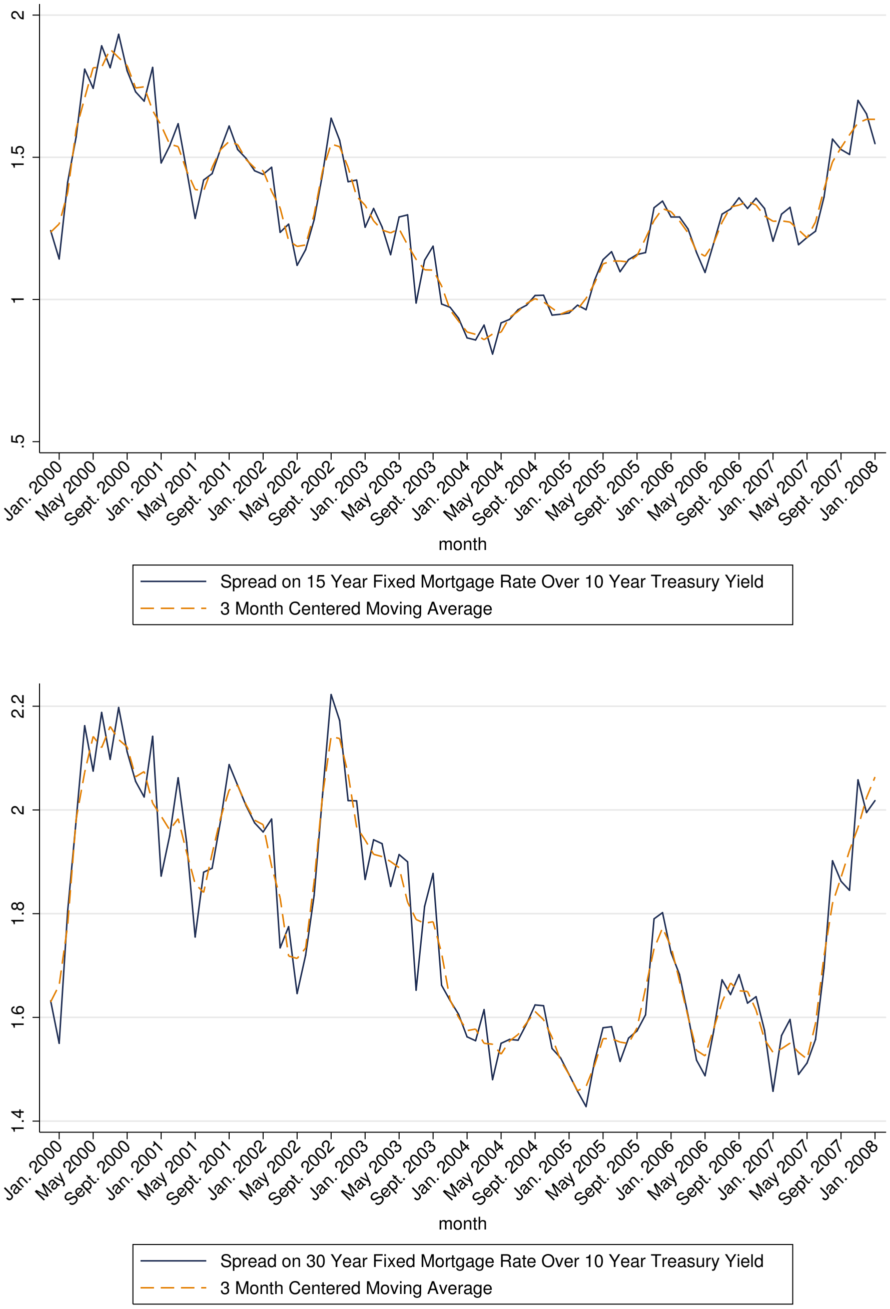
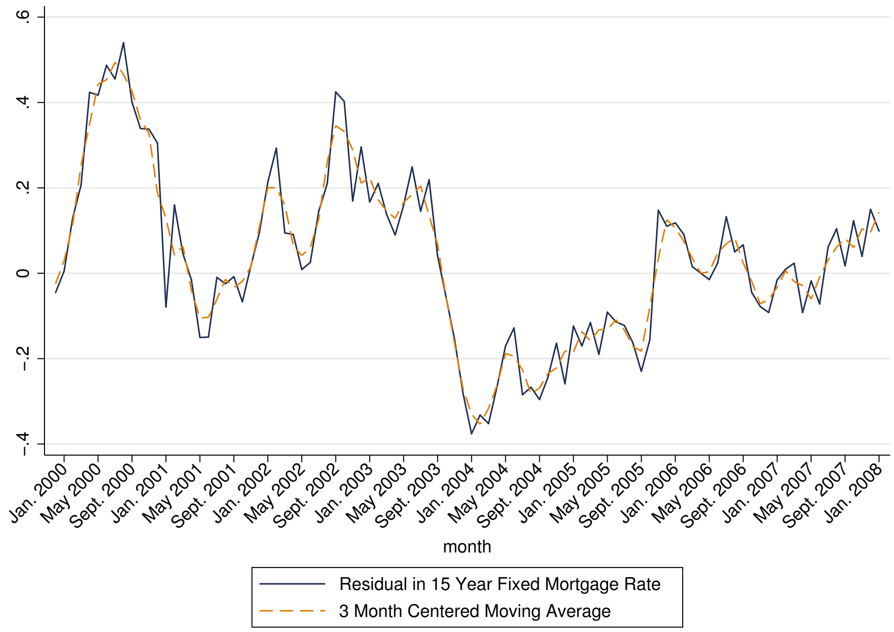


Figure 10: Average spread on fixed rate mortgages over ten year treasuries

mortgage rate spreads were low relative to the overall sample.

We should expect these spreads to move endogenously with various aggregate factors. I follow Justiniano et al. (2016) and calculate a measure of residual variation in mortgage rates that is unexplained by the historical relationship of mortgage rates to the yield curve and macroeconomic factors.



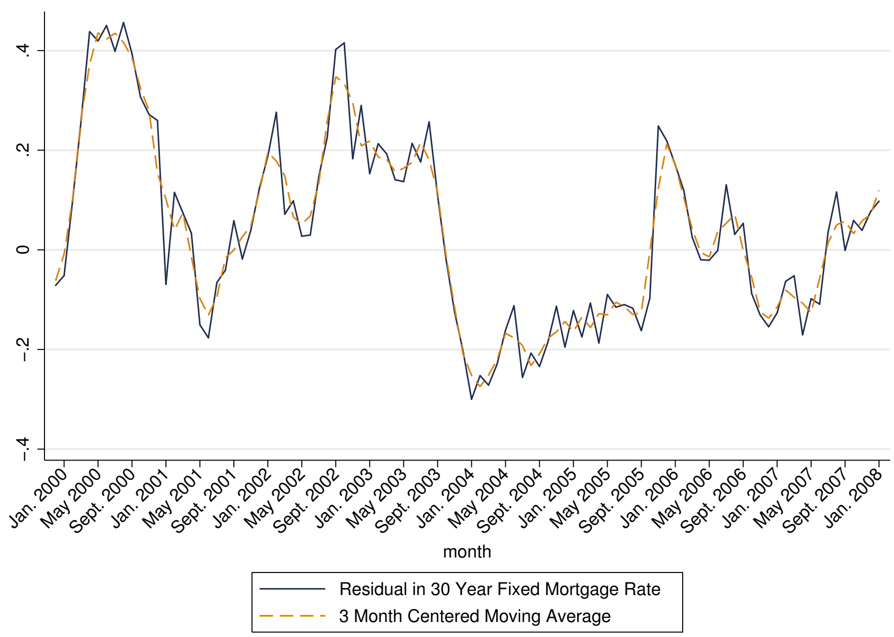


Figure 11: Residual variation in fixed rate mortgages (OLS residuals from (4)).

Specifically, on a sample from January of 1990 to January of 2008 I run the regression

$$i_t = \alpha_m + \beta' X_t + u_t \tag{4}$$

where i_t is either the average interest rate on 15 or 30 year fixed rate mortgages, α_m is a dummy for the month in the year, and X_t is a vector containing the contemporaneous and lagged 1, 5 ,and

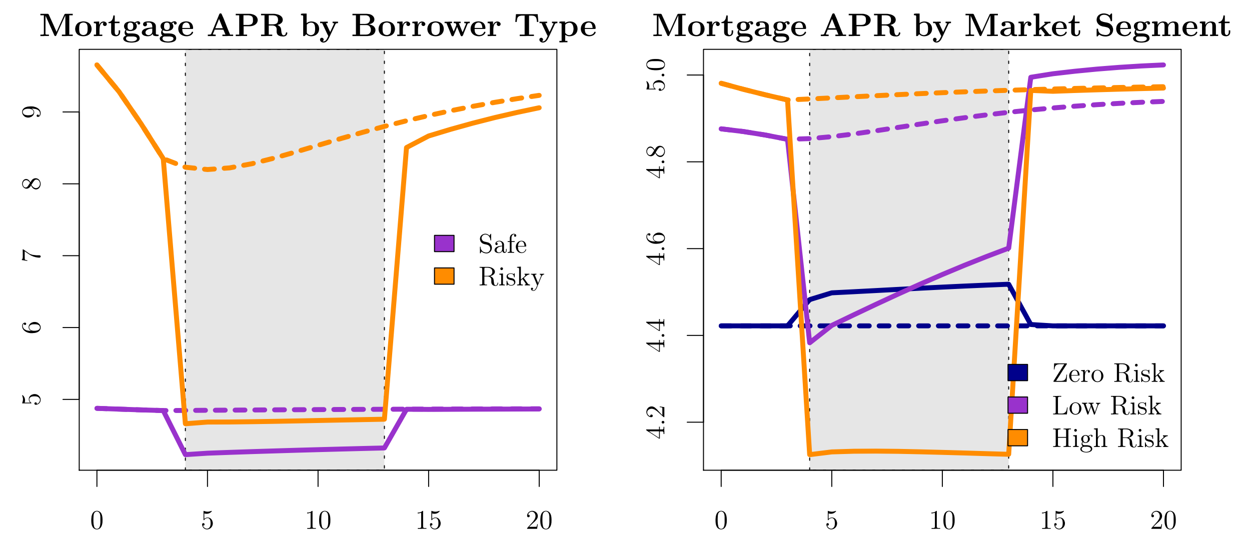


Figure 12: Interest rates by fundamental borrower type (left) and borrower observables at origination (right). Response to 3 (dashed) and 4 (solid) -1/2 standard deviation income risk shocks. Shaded region indicates pooling regime in second simulation.

10 year treasury yields, the lagged unemployment rate, lagged growth in industrial production, and lagged inflation. The residuals from this regression represent movements in mortgage rates that are not explained by historical co-movement with these aggregate factors. Figure 11 shows these residuals from January of 2000 to January of 2008.

The residual variation in mortgage rates moves closely together for both series. Moreover, the pattern follows precisely the results of Justiniano et al. (2016). In the later half of 2003, both series show a sudden and rapid fall in mortgage rates relative to what we'd expect given historical co-movements with aggregate factors. This effect persists during the housing boom, reverting to zero around 2005-2006.

The theory offered by this paper can explain the mortgage rate conundrum documented by Justiniano et al. (2016) as coming from a compression of risk spreads on mortgages, and as well as a reduction in screening costs that get priced into contracts. In particular, it explains how a sudden fall in mortgage rates relative to aggregate factors can occur — through a shift from a screening regime to a pooling regime in the mortgage market.

Figure 12 shows the paths for mortgage rates in a simulation where three -1/2 standard deviation shock to income risk hit the economy and a simulation where four -1/2 standard deviation shocks hit the economy. It shows responses of mortgage rates disaggregated by borrower type and by borrower observables at origination. In the first simulation the economy remains in the screening regime throughout, while the economy enters and exits the pooling regime in the second simulation. The grey shaded region shows when the pooling regime occurs.

A switch to the pooling leads to a collapse in mortgage rates, particularly for risky borrowers and borrowers in the low risk and high risky market segments. This result is consistent with the findings of Justiniano et al. (2016) — particularly their finding that mortgage rates fell more for subprime loans.

Why do mortgage rates fall? Consider a risky borrower who gains access to a pooling contract. Initially, they used their full information contract which fully priced in their elevated default risk. When they gain access to a pooling contract, this default risk gets diluted through pooling with safe borrowers. As a result, they see a fall in the risk premium on their mortgage due to an implicit subsidy from safe borrowers.

For safe borrowers who switch from high-documentation to high-downpayment contracts, the fall in the interest rate comes from reduced screening costs for lenders. When screening, lenders sink resources into producing information about individual borrowers and the screening contract prices these costs into the interest rate. When a safe borrower moves from a high-documentation contract to a high-downpayment contract, these cost are no longer paid by lenders. A high down payment additionally reduces risk of strategic default. As a result, these contracts have relatively low interest rates and the shift implies a fall in mortgage spreads.

The non-linear mechanism in this theory, where small shocks to income risk can trigger large changes in equilibrium outcomes, provides an explanation for the mortgage rate conundrum as an endogenous outcome driven by macroeconomic fundamentals. As emphasized by Justiniano et al. (2016), the timing of the relative fall in mortgage rates coincided with the end of the Federal Reserve's easing cycle. Specifically, in the summer of 2003, the Fed signaled that, due to improving conditions in labor markets, they would no longer continue to lower their policy rate. At this point, long-term yields began to rise in anticipation of a higher future policy rate, yet mortgage rates didn't rise with treasury yields. I argue that we can understand the mortgage rate puzzle as coming from the simultaneous endogenous response of both the mortgage market and policy makers to improvements in the macroeconomy.

Justiniano et al. (2016) also show that the size of the mortgage origination industry didn't follow its historical pattern of shrinking at the end of a Fed easing cycle. Similar to past business cycles, there was a mortgage refinance boom as monetary policy lowered interest rates after the recession of 2001. During this period, employment in the mortgage industry grew steadily. But this time it continued to grow when the Fed signaled higher future interest rates, likely due to the boom in non-prime lending that began at the end of 2003. The theory of this paper explains the non-prime lending boom as the rational profit maximizing response of lenders to falling income risk, and therefore can rationalize the continued expansion of the mortgage industry at the end of the Fed's policy cycle.

The expansion in non-prime lending explains both low mortgage rates relative to the yield curve, and the continued expansion of the mortgage industry. This pattern was at odds with the historical pattern for precisely the same reasons that the emergence of non-prime lending was at odds with historical patterns. This narrative explains the mortgage rate conundrum by explaining the simultaneity of the non-prime lending boom, the housing boom, rising yields, and improving labor market conditions.

5 Conclusion

In this paper, I develop a theory of mortgage credit which provides an explanation for the rise of non-prime lending during the early 2000's, the associated housing boom, and the subsequent housing crisis. This theory is built on the assumption that competitive lenders face asymmetric information about borrower income risk during mortgage origination, which leads to endogenous segmentation of the mortgage market. As a result, the model makes rich predictions for the allocation of credit and mortgage rates across borrowers. I show that it can match and may to help reconcile the existing microeconomic evidence on debt allocation during the boom, as well as explain the mortgage rate conundrum. The dynamics in the theory rely solely on variation in income risk over the business cycle, and the model generates boom-bust cycles through the endogenous and state-dependent amplification of these fundamentals. This theory allows us to explain housing boom-bust episodes without appealing to either new or large shocks.

The key non-linear mechanism operates through general equilibrium feedback between housing and mortgage markets. During an expansion, income risk falls, which increases the share of borrowers offered pooling mortgage contracts. If this expansion in credit access attracts former renters into the housing market, then housing demand increases. This increase in demand triggers a regime change — lenders stop offering mortgages with high documentation requirements, loan-to-value ratios suddenly increase, and home prices rapidly appreciate. As the cycle peaks and income risk begins to revert, there is a reversal in the credit regime. Credit access falls, home prices collapse, and, as a result, there is a foreclosure crisis as borrowers go underwater on their homes. Through the non-linear mechanism implied by joint housing and mortgage market clearing, we can explain housing boom-bust cycles resulting from small shocks to economic fundamentals.

The dynamic model has a structure that can be embedded into standard business cycle models. An important next step is to introduce the mechanism into a production economy that includes frictions in financial intermediation. Feedback from home prices to consumption and output may amplify housing boom-bust episodes and influence business cycle dynamics generally. Also, because the housing crisis precipitated a financial panic in markets that relied on mortgage-backed securities as collateral, frictions in the intermediation of funds between lenders and savers may be particularly important for understanding recessions which follow after housing crises. This theory provides a novel mechanism for understanding the origins of housing crises and provides micro-foundations for studying policies intended to reduce the likelihood of future crises and recessions.

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A Equilibrium Characterization and Proofs

Lemma 2 (Sub-game Perfect Period 1 Expected Payoffs) *Consider expected payoffs from the sub-game beginning in period 1. Suppose that an individual borrower i has type τ_i and has been accepted into a contract with transfer of T_i and loan size of $L_i > 0$. Then the expected payoff from the contract for the borrower is $T_i + \beta U(L_i, \tau_i \phi)$ and the expected payoff to the lender is $-T_i + \frac{1}{1+r}\Pi(L_i, \rho_i \phi)$ where ρ_i is the lender's subjective belief that i is the risky type.*

Proof. The result follows from backward induction in period 1. At the end of period 1, lenders choose whether or not to initiate foreclosure following a borrower default. Their payoff from initiating foreclosure is $-T_i + \frac{1}{1+r}\min\{L_i, (1-\lambda)Z_i\}$. If they do not initiate foreclosure, their payoff is $-T_i$. As a result, since $\mathbb{P}[Z_i > 0] > 0$, they have a strictly dominant strategy to initiate foreclosure.

Borrowers anticipate that foreclosure will occur following any default. If $Y_i = 0$, then they cannot make their payment of L_i, and must default. If they have income and do not default, they get $-T_i + \beta(Y_i + Z_i - L_i)$. If they default they get $-T_i + \beta(Y_i + \max\{(1-\lambda)Z_i - L_i, 0\})$. The gain from default is $\beta \max\{-\lambda Z_i, L_i - Z_i\}$. The gain is strictly positive when $L_i > Z_i$, is zero when $L_i = Z_i$, and is strictly negative when $L_i < Z_i$. Since Z_i is continuously distributed, the borrower has a strictly dominant strategy with probability 1.

Therefore, the payoffs from this sub-game are

$$\begin{aligned} &\mathbf{1}\{Z_i < L_i \text{ or } Y_i = 0\}\left[T_i + \beta(Y_i + \max\{(1-\lambda)Z_i - L_i, 0\})\right] \\ &\qquad + \mathbf{1}\{Z_i \geq L_i \text{ and } Y_i = Y\}\left[Y_i + Z_i - L_i\right] \end{aligned}$$

for the borrower, and

$$\begin{aligned} &\mathbf{1}\{Z_i < L_i \text{ or } Y_i = 0\}\left[-T_i + \frac{1}{1+r}\min\{L_i, (1-\lambda)Z_i\}\right] \\ &\qquad + \mathbf{1}\{Z_i \geq L_i \text{ and } Y_i = Y\}\left[-T_i + \frac{1}{1+r}L_i\right] \end{aligned}$$

for the lender. Given type τ_i and the lender's belief about i's type of ρ_i, the result follows from calculating expected values of these payoffs. ■

These payoff functions are the continuation payoffs for the sub-game following the borrower's application decision and the lender's screening and accept/reject decision. For any finite set of contracts that are offered to an individual borrower, this sub-game is a finite signaling game. The message space of the borrower (sender) is the set of contracts and the action space of the lender (receiver) is the choice of screening (if the contract applied to is a screening contract) and the choice of whether to accepting or reject the contract. Note that because this sub-game is a standard signaling game, there can be many equilibria.

However, competition among lenders over contract offers will select among these possible continuation outcomes, and I can show that, up to the application behavior of risky borrowers when offered screening contracts, the sub-game perfect equilibrium of the overall game is unique, despite there existing many equilibria of the sub-game that begins with the borrower application decision.

To do so, I define competitive contracts that may emerge as equilibrium offers, and show that the contracts satisfying these definitions are unique.

First, I calculate the derivatives of Π and U in L. We have

$$\begin{aligned}\Pi(L,\phi) &= L - \int_0^\infty D(L,\rho\phi,z)\max\{L-(1-\lambda)z,0\}F'(z)\mathrm{d}z \\ &= L - \int_0^L [L-(1-\lambda)z]F'(z)\mathrm{d}z - \phi\int_L^{\frac{L}{1-\lambda}}[L-(1-\lambda)z]F'(z)\mathrm{d}z\end{aligned}$$

and so, by Leibniz's rule,

$$\begin{aligned}\Pi_1(L,\phi) &= 1-\lambda LF'(L) - F(L) - \phi\left[-\lambda LF'(L) + F\left(\frac{L}{1-\lambda}\right) - F(L)\right] \\ &= 1-(1-\phi)(\lambda LF'(L)+F(L)) - \phi F\left(\frac{L}{1-\lambda}\right).\end{aligned}$$

Similarly,

$$\begin{aligned}U(L,\phi) &= \bar{z} - L + \int_0^\infty D(L,\tau\phi,z)\max\{-\lambda z, L-z\}F'(z)\mathrm{d}z \\ &= \bar{z} - L + \int_0^L (L-z)F'(z)\mathrm{d}z + \phi\int_L^{\frac{L}{1-\lambda}}(L-z)F'(z)\mathrm{d}z + \phi\int_{\frac{L}{1-\lambda}}^\infty(-\lambda z)F'(z)\mathrm{d}z\end{aligned}$$

and

$$\begin{aligned}U_1(L,\phi) &= -1 + F(L) + \phi\frac{1}{1-\lambda}\left(L - \frac{L}{1-\lambda}\right)F'\left(\frac{L}{1-\lambda}\right) \\ &\qquad + \phi\left[F\left(\frac{L}{1-\lambda}\right) - F(L)\right] + \phi\frac{\lambda}{1-\lambda}\frac{L}{1-\lambda}F'\left(\frac{L}{1-\lambda}\right) \\ &= -1 + (1-\phi)F(L) + \phi F\left(\frac{L}{1-\lambda}\right).\end{aligned}$$

From this result, we can prove Lemma 1.

Proof of Lemma 1. We have

$$\frac{\partial}{\partial L}\left[U(L,\phi) - U(L,0)\right] = U_1(L,\phi) - U_1(L,0) = \phi\left[F\left(\frac{L}{1-\lambda}\right) - F(L)\right].$$

Since $\phi > 0$, this quantity is strictly positive if and only if $\mathbb{P}[L < Z < L/(1-\lambda)] = \mathbb{P}[(1-\lambda)Z < L < Z] > 0$. ■

The following lemma establishes an intermediate result for proving the uniqueness of equilibrium contracts.

Lemma 3 *Let Assumption 1 and Assumption 2 hold. Fix some $\rho \in [0,1]$ and $\tau \in [0,\rho]$. Then the* *program*

$$\begin{aligned} \max_{T,L} \quad & T + \beta U(L, \tau\phi) \\ s.t. \quad & T \leq \frac{1}{1+r}\Pi(L, \rho\phi) \end{aligned} \tag{PC}$$

has a unique and finite solution.

Proof. Since the objective is strictly increasing in T, the lender's zero profit (participation) constraint must bind, pinning down T for any given L. The first order necessary condition for the loan size is then

$$\begin{aligned} 0 &= \frac{1}{1+r}\Pi_1(L, \rho\phi) + \beta U_1(L, \tau\phi) \\ &= \frac{1}{1+r}\left[1 - (1-\rho\phi)(\lambda L F'(L) + F(L)) - \rho\phi F\left(\frac{L}{1-\lambda}\right)\right] \\ &\qquad\qquad + \beta\left[-1 + (1-\tau\phi)F(L) + \tau\phi F\left(\frac{L}{1-\lambda}\right)\right] \end{aligned}$$

Re-arrange to get

$$\begin{aligned} (1-(1+r)\beta)(1-F(L)) &= (1-\rho\phi)\lambda L F'(L) + (\rho - (1+r)\beta\tau)\phi\left[F\left(\frac{L}{1-\lambda}\right) - F(L)\right] \\ &= (1-\rho\phi)\lambda L F'(L) + (\rho - (1+r)\beta\tau)\phi \int_1^{\frac{1}{1-\lambda}} L F'(Lx)\mathrm{d}x \end{aligned}$$

For any L such that $F(L/(1-\lambda)) = 0$, this condition cannot hold because the left-hand-side is strictly positive, while the right hand side must equal zero. For any L such that $F(L) = 1$, the condition will hold — this case corresponds to when all borrowers default for sure on the loan. To see if such a large loan is optimal, we need to compare to the case where $F(L) < 1$.

In this case, we can re-write in terms of the hazard function $h(z) = F'(z)/(1-F(z))$.

$$\begin{aligned} 1-(1+r)\beta &= (1-\rho\phi)\lambda L\frac{F'(L)}{1-F(L)} + (\rho-(1+r)\beta\tau)\phi\int_1^{\frac{1}{1-\lambda}} L\frac{F'(Lx)}{1-F(L)}\mathrm{d}x \\ &= (1-\rho\phi)\lambda L h(L) + (\rho-(1+r)\beta\tau)\phi\int_1^{\frac{1}{1-\lambda}} Lh(Lx)\frac{1-F(Lx)}{1-F(L)}\mathrm{d}x \end{aligned} \tag{5}$$

Note that because $zh(z)$ is strictly increasing in z

$$\begin{aligned} \frac{\partial}{\partial L}\ln\frac{1-F(Lx)}{1-F(L)} &= \frac{xF'(Lx)}{1-F(Lx)} - \frac{F'(L)}{1-F(L)} = \frac{1}{L}Lxh(Lx) - \frac{F'(L)}{1-F(L)} \\ &> \frac{1}{L}h(L) - \frac{F'(L)}{1-F(L)} = \frac{1}{L}\frac{LF'(L)}{1-F(L)} - \frac{F'(L)}{1-F(L)} = 0 \end{aligned}$$

which implies $Lh(Lx)\frac{1-F(Lx)}{1-F(L)}$ is increasing and continuous in L for each x. As a result, the right-hand side of Equation (5) is strictly increasing and continuous in L. Since the left-hand side is constant in L, there can be at most one L solving this equation.

The right-hand side is equal to 0 at $L = 0$. The limit of the left-hand side of Equation (5) as $L \to \infty$ is

$$\begin{aligned}
\text{Limit} \equiv & \lim_{L\to\infty} (1-\rho\phi)\lambda Lh(L) + (\rho - (1+r)\beta\tau)\phi \int_1^{\frac{1}{1-\lambda}} Lh(Lx)\frac{1-F(Lx)}{1-F(L)}\mathrm{d}x \\
&= (1-\rho\phi)\lambda \lim_{L\to\infty} Lh(L) + (\rho - (1+r)\beta\tau)\phi \int_1^{\frac{1}{1-\lambda}} \lim_{L\to\infty} Lh(Lx)\frac{1-F(Lx)}{1-F(L)}\mathrm{d}x \\
&= (1-\rho\phi)\lambda \lim_{L\to\infty} Lh(L) + (\rho - (1+r)\beta\tau)\phi \int_1^{\frac{1}{1-\lambda}} \frac{1}{x} \lim_{L\to\infty} Lh(L)\mathrm{d}x \\
&= (1-\rho\phi)\lambda \lim_{L\to\infty} Lh(L) + (\rho - (1+r)\beta\tau)\phi \ln\frac{1}{1-\lambda} \lim_{L\to\infty} Lh(L) \\
&= \left[(1-\rho\phi)\lambda + (\rho - (1+r)\beta\tau)\phi \ln\frac{1}{1-\lambda}\right] \lim_{L\to\infty} Lh(L)
\end{aligned}$$

We then have

$$\begin{aligned}
\text{Limit} &= \left[(1-\rho\phi)\lambda + (\rho - (1+r)\beta\tau)\phi \ln\frac{1}{1-\lambda}\right] \lim_{L\to\infty} Lh(L) \\
&> \left[(1-\rho\phi)\lambda + (\rho - \tau)\phi \ln\frac{1}{1-\lambda}\right] \lim_{L\to\infty} Lh(L) \\
&> (1-\rho\phi)\lambda \lim_{L\to\infty} Lh(L) \\
&> (1-\phi)\lambda \lim_{L\to\infty} Lh(L) > 1-(1+r)\beta
\end{aligned}$$

where the final inequality follows from Assumption 2. Since the last expression is equal to the left-hand side of Equation (5), a solution to Equation (5) exists. Finally, since the objective is decreasing in L as $L \to \infty$, we can rule out the case that $F(L) = 1$. Therefore, the program has a unique and finite solution. ■

We can apply this result to establish the uniqueness of contracts satisfying various requirements. For example, consider full information contracts:

Definition 2 (Competitive Full Information Contracts) A *competitive safe contract* is a non-screening contract $(T, L, 0)$ whose transfer and loan size solve

$$\begin{aligned}
V^{safe} = \max_{T,L} \quad & T + \beta U(L, 0) \\
\text{s.t.} \quad & T \leq \frac{1}{1+r}\Pi(L, 0) \qquad \text{(PC)}
\end{aligned}$$

Similarly, a *competitive risky contract* is a non-screening contract $(T, L, 0)$ whose transfer and loan size solve

$$
\begin{aligned}
V^{risky} = \max_{T,L} \quad & T + \beta U(L,\phi) \\
\text{s.t.} \quad & T \leq \frac{1}{1+r}\Pi(L,\phi) && \text{(PC)}
\end{aligned}
$$

Corollary 1 (Unique Competitive Full Information Contracts) *Let Assumption 1 and Assumption 2 hold. Then each full information contract is unique. Denote the competitive safe contract by $(T^{safe}, L^{safe}, 0)$ and the competitive risky contract by $(T^{risky}, L^{risky}, 0)$.*

Proof. The result follows from Lemma 3 for the case where $\rho = 0$ and $\tau = 0$, and the case where $\rho = 1$ and $\tau = 1$. ■

Similarly, there are unique competitive pooling contracts across risk indices.

Definition 3 (Competitive Pooling Contract) A *competitive pooling contract* among borrowers with risk index of ρ is a non-screening contract $(T, L, 0)$ whose transfer and loan size solve

$$
\begin{aligned}
V^{pool}(\rho) = \max_{T,L} \quad & T + \beta U(L,0) \\
\text{s.t.} \quad & T \leq \frac{1}{1+r}\Pi(L,\rho\phi) && \text{(PC)}
\end{aligned}
$$

Corollary 2 (Unique Competitive Pooling Contract) *Let Assumption 1 and Assumption 2 hold. Then there is a unique competitive pooling contract for each $\rho \in [0,1]$. Denote its transfer by $T^{pool}(\rho)$ and its loan size by $L^{pool}(\rho)$. Moreover, $T^{pool}(0) = T^{safe}$, $L^{pool}(0) = L^{safe}$, and $V^{pool}(\rho)$ is strictly decreasing, continuous, and convex in ρ.*

Proof. For each ρ, there is a unique competitive pooling contract due to Lemma 3 for the case where $\tau = 0$. The pooling contract for $\rho = 0$ is identical to the full information competitive safe contract because both contracts correspond to the case when $\rho = 0$ and $\tau = 0$. Finally, note that

$$
V^{pool}(\rho) = \max_{L} \quad \frac{1}{1+r}\Pi(L,\rho\phi) + \beta U(L,0)
$$

and Π is affine in its second argument. As a result, by the envelope theorem $V^{pool}(\rho)$ is continuous and differentiable in ρ with

$$
\frac{\partial}{\partial \rho} V^{pool}(\rho) = \frac{1}{1+r}\Pi_2(L^{pool}(\rho), \rho\phi)\phi = -\phi \int_{L^{pool}(\rho)}^{\frac{L^{pool}(\rho)}{1-\lambda}} (L^{pool}(\rho) - (1-\lambda)z) F'(z) \mathrm{d}z < 0.
$$

Finally, as the maximum of a collection of affine functions, $V^{pool}(\rho)$ is convex. Specifically, for any

$x \in [0,1]$ and $\rho, \rho' \in [0,1]$ we have

$$\begin{aligned}
xV^{pool}(\rho') + (1-x)V^{pool}(\rho) &= x \max_L \left\{ \frac{1}{1+r}\Pi(L, \rho'\phi) + \beta U(L,0) \right\} \\
&\qquad + (1-x) \max_L \left\{ \frac{1}{1+r}\Pi(L, \rho\phi) + \beta U(L,0) \right\} \\
&\geq \max_L \quad x\frac{1}{1+r}\Pi(L, \rho'\phi) + (1-x)\frac{1}{1+r}\Pi(L, \rho\phi) + \beta U(L,0) \\
&= \max_L \quad \frac{1}{1+r}\Pi(L, (x\rho' + (1-x)\rho)\phi) + \beta U(L,0) \\
&= V^{pool}(x\rho' + (1-x)\rho).
\end{aligned}$$

■

The competitive screening contracts for each screening technology are also unique.

Definition 4 (Competitive High-Documentation Contract) A *competitive high-documentation contract* is a high-documentation contract $(T, L, 1)$ whose transfer and loan size solve

$$\begin{aligned}
V^{doc} = \max_{T,L} &\quad T + \beta U(L,0) \\
\text{s.t.} &\quad T \leq \frac{1}{1+r}\Pi(L,0) - \kappa_m
\end{aligned} \tag{PC}$$

Corollary 3 (Unique Competitive Screening Contract) *Let Assumption 1 and Assumption 2 hold. Then there is a unique competitive high-documentation contract. Denote this contracts by* $(T^{doc}, T^{doc}, 1)$*. These quantities are*

$$T^{doc} = T^{safe} - \kappa, \quad \textit{and} \quad L^{doc}) = L^{safe}.$$

Proof. Fix $\rho \in [0,1]$. Let $\tilde{T} = T + \kappa$. The program is equivalent to solving

$$\begin{aligned}
\max_{\tilde{T},L} &\quad \tilde{T} + \beta U(L,0) \\
\text{s.t.} &\quad \tilde{T} \leq \frac{1}{1+r}\Pi(L,0)
\end{aligned} \tag{PC}$$

We can therefore apply Lemma 3 for the case where $\rho = 0$ and $\tau = 0$, implying that the unique solution to this program must have $\tilde{T} = T^{safe}$ and $L = L^{safe}$. Therefore, $T = \tilde{T} - \kappa = T^{safe} - \kappa$.
■

Definition 5 (Competitive High-Downpayment Contract) Given net home price of $P - R$, a *competitive high-downpayment contract* is a low-documentation contract whose transfer and loan size solve the following program

$$
\begin{aligned}
V^{down}(P-R) = \max_{T,L} \quad & T + \beta U(L,0) \\
\text{s.t.} \quad & T + \beta U(L,\phi) \leq \max\{P-R, V^{risky}\} && \text{(IC)} \\
& T \leq \frac{1}{1+r}\Pi(L,0) && \text{(PC)}
\end{aligned}
$$

Lemma 4 (Unique Competitive Separating Contracts) *Let Assumption 1 and Assumption 2 hold. Then there is a unique high-downpayment contract. Denote its transfer by $T^{down}(P-R)$ and its loan size by $L^{down}(P-R)$.*

Proof. The PC constraint must bind at any solution. Suppose that the IC constraint is slack. Then L must solve

$$
\max_{L} \quad \frac{1}{1+r}\Pi(L,0) + \beta U(L,0)
$$

this program has a unique solution by Lemma 3 for the case where $\rho = 0$ and $\tau = 0$. Denote it by L^{safe}. Define $V^* = \frac{1}{1+r}\Pi(L^{safe},0) + \beta U(L^{safe},\phi)$. We can see that if $V^* < P-R$, then we must indeed have a slack IC constraint in the original program, and if $V^* = P-R$, then L^{safe} remains feasible in the original program. In either case L^{safe} must solve the original program. Otherwise, when $V^* > P-R$, L^{safe} is not in the feasible set of the original program. In this case, the IC constraint must bind and the program reduces to

$$
\max_{L \in \{L | \frac{1}{1+r}\Pi(L,0) + \beta U(L,\phi) = P-R\}} \quad \max\{P-R, V^{risky}\} + \beta U(L,0) - \beta U(L,\phi)
$$

Due to Lemma 1, the objective of this program is strictly decreasing in L. The solution must be the smallest value of L such that the IC constraint binds. Note that

$$
\begin{aligned}
\frac{1}{1+r}\Pi_1(L,0) + \beta U_1(L,\phi) &= \frac{1}{1+r}\left[1 - \lambda L F'(L) - F(L)\right] \\
&\quad + \beta\left[-1 + (1-\phi)F(L) + \phi F\left(\frac{L}{1-\lambda}\right)\right] \\
&> \frac{1}{1+r}\left[1 - \lambda L F'(L) - F(L)\right] + \beta\left[-1 + F(L)\right] \\
&= \frac{1}{1+r}\Pi_1(L,0) + \beta U_1(L,0)
\end{aligned}
$$

and we have $\frac{1}{1+r}\Pi_1(L,0) + \beta U_1(L,\phi) > 0$ for $L < L^{safe}$. Since $\frac{1}{1+r}\Pi(0,0) + \beta U(0,\phi) < \max\{P-R, V^{risky}\}$ and $\frac{1}{1+r}\Pi(L^{safe},0) + \beta U(L^{safe},\phi) > \max\{P-R, V^{risky}\}$, there is then a unique solution for $L < L^{safe}$. Therefore, there is a unique solution to the original program. Denote this solution by $L^{down}(P-R)$ and define $T^{down}(P-R) = \frac{1}{1+r}\Pi(L^{down}(P-R),0)$. ■

We can now use these definitions and results to simultaneously prove Proposition 1 and Proposition 2. The proof proceeds by showing that only these competitive contracts can be offered in any equilibrium, and any equilibrium where lenders randomly reject borrowers must be unstable.

Proof of Proposition 1 and Proposition 2.

Let Assumption 1 and Assumption 2 hold. Consider when $P - R > V^{safe}$. Since V^{safe} is an upper bound on both safe borrower and risky borrower valuations for any contract which gives lenders non-negative profits, all borrowers have a strictly dominant strategy to become renters for any set of contract offers. As a result, the set of contracts offered is indeterminate, and all borrowers become renters.

Now suppose that $P - R \leq V^{safe}$. I first show that there is a unique strictly dominant competitive contract for almost all risk indices, and characterize the sets of ρ-values where each competitive contract strictly dominates. Then, I show that equilibrium is unique for ρ-values in each of these sets.

Consider whether or not the IC constraint for the competitive high-downpayment contract binds.

1. Suppose that $V^* \leq P - R < V^{safe}$. In this case the IC constraint for the competitive separating contract doesn't bind and so $T^{down}(P - R) = T^{safe}$, $L^{down}(P - R) = L^{safe}$, and $V^{down}(P - R) = V^{safe}$. Since $V^{safe} > V^{pool}(\rho)$ for all $\rho \in (0, 1]$ and $V^{safe} > V^{doc}$, the competitive high-downpayment contract strictly dominates both the competitive pooling and competitive high-documentation contracts for almost all borrowers.

2. Suppose that $P - R < V^*$. Now, the IC constraint binds and so $V^{down}(P - R) < V^{safe}$. Let $V^{screen}(P - R) = \max\{V^{down}(P - R), V^{doc}\} \leq V^{safe}$. Since $V^{pool}(0) = V^{safe}$, we may have a $\rho > 0$ such that $V^{pool}(\rho) = V^{screen}(P - R)$. If $V^{pool}(1) < V^{screen}(P - R)$ then there is a unique solution for $\rho \in (0, 1)$. Otherwise, we have $V^{pool}(\rho) > V^{screen}(P - R)$ for $\rho \in [0, 1)$. Take ρ^* to be the solution in the former case, and otherwise take $\rho^* = 1$. We then have that the competitive pooling contract strictly dominates for $\rho \in [0, \rho^*)$. For $\rho \in (\rho^*, 1]$ either the competitive high-downpayment contract or the competitive high-documentation contract will dominate depending on whether $V^{down}(P - R) > V^{doc}$, $V^{down}(P - R) = V^{doc}$, or $V^{down}(P - R) < V^{doc}$.

These results establish that for almost all ρ-values from 0 to 1, one of the competitive contracts strictly dominates. I next show that within each region where a specific type of competitive contract dominates, that contract must be offered in equilibrium.

First consider ρ such that the competitive high-downpayment contract dominates. Consider the sub-game beginning with a borrower's decision to apply for contracts. Let C_i denote the set of contract offers made to individual i. Suppose C_i does not include the competitive high-downpayment contract. I will show that lenders can make a profitable deviation by introducing a contract of the form $(T^{down}(P - R) - \epsilon, T^{down}(P - R), 0)$ for $\epsilon > 0$ sufficiently small along side the full information contract for risky borrowers.

Let $\epsilon > 0$ and consider equilibrium in the sub-game starting with borrower application choices when the contract offer set is $\{(T^{down}(P - R) - \epsilon, T^{down}(P - R), 0), (T^{risky}, L^{risky}, 0)\} \cup C_i$. There exist two types of equilibrium. In one type, enough risky borrowers apply for the new contract that it becomes unprofitable, and lenders reject all applications. I show that these equilibria are unstable. In the second type of equilibrium, risky borrowers never apply for the new contract, and lenders make a strictly positive profit on the new contract. This equilibrium is unique and stable. Selecting this stable equilibrium to the sub-game, we conclude that lenders can make a profitable deviation by introducing the new contract.

Let θ denote the anticipated rejection rate when i applies to the new contract. If $\tau_i = 1$, then their expected value from applying to contract $(T^{down}(P-R)-\epsilon, T^{down}(P-R), 0)$ is

$$(1-\theta)(T^{down}(P-R)-\epsilon+\beta U(T^{down}(P-R),\phi))+\theta(P-R) = (1-\theta)(\max\{V^{risky}, P-R\}-\epsilon)+\theta(P-R)$$

The left hand side is strictly less than $\max\{V^{risky}, P-R\}$ whenever $\theta < 1$ or $\theta = 1$ and $V^{risky} > P-R$. In either case, we have that applying for contract $(T^{down}(P-R)-\epsilon, T^{down}(P-R), 0)$ is strictly dominated. If $\theta = 1$ and $P-R \geq V^{risky}$, then risky borrowers may apply. However, any equilibrium with $\theta = 1$ is unstable because a small reduction of θ will imply that applying is strictly dominated. We can therefore focus on the case where $\theta < 1$.

In this case, risky borrowers will never apply, and any applicants must be safe borrowers. Lenders then have expected profit of

$$-(T^{down}(P-R)-\epsilon)+\frac{1}{1+r}\Pi(T^{down}(P-R),0) = \epsilon > 0$$

on any accepted application. Since they have a strictly positive profit, they must accept all applications so that $\theta = 0$. Anticipating this acceptance, borrowers with $\tau_i = 0$ get value from applying of

$$T^{down}(P-R)-\epsilon+\beta U(T^{down}(P-R),0)) = V^{down}(P-R)-\epsilon$$

Since $V^{down}(P-R)$ is strictly larger than $V^{pool}(\rho)$ and V^{doc}, and both these values represent the best case for pooling and high-documentation contracts, by taking ϵ sufficiently small, we can conclude that the new contract strictly dominates any contract in C_i. As a result, we can conclude that after introducing the new contract, there is a unique stable equilibrium in the following sub-game and in this new equilibrium, risky borrowers do apply for the new contract while all safe borrowers do apply for the new contract, all applications are accepted, and lenders make a strictly positive expected profit. Given this profitable deviation, we can conclude that whenever $P-R \leq V^{safe}$, and ρ is in the region where the competitive high-downpayment contract dominates, the competitive separating contract must be offered.

Now, suppose that the competitive high-downpayment contract is in C_i. Zero profits requires that no risky borrowers apply for the contract, but also implies that lenders are indifferent between accepting and rejecting applicants. This indifference might lead to multiple equilibria because lenders may randomly reject applications. Suppose that they reject a fraction $\theta > 0$. I show that another lender could introduce a contract of the form $(T^{down}(P-R)-\epsilon, L^{down}(P-R), 0)$, capture the market for safe borrowers, and make a strictly positive profit. Safe borrowers get expected value from the competitive high-downpayment contract of

$$(1-\theta)V^{down}(P-R)+\theta(P-R)<V^{safe}$$

We can then choose an $\epsilon>0$ such that $\epsilon<V^{safe}-[(1-\theta)V^{down}(P-R)+\theta(P-R)]$. Given that risky borrowers previously didn't use the high-downpayment contract, the new contract must be strictly dominated from their perspective since it has value reduced by ϵ. As a result, in the equilibrium following the introduction of the new contract, lenders must believe that only safe borrowers apply. They then get expected profit of

$$-(T^{down}(P-R)-\epsilon)+\frac{1}{1+r}\Pi(L^{down}(P-R),0)=\epsilon>0$$

and so safe borrowers anticipate that they will be accepted if they apply for the new contract. Although they get a lower transfer, they no longer face a chance of rejection and strictly prefer the new contract because

$$V^{down}(P-R)-\epsilon>(1-\theta)V^{down}(P-R)+\theta(P-R).$$

Therefore, all safe borrowers apply for the new contract, get accepted with probability 1, and lenders make a strictly positive profit. Therefore, we can conclude that in any equilibrium, lenders must accept all applicants for the competitive high-downpayment contract (so that $\theta=0$).

Using the same line of reasoning for high-documentation and pooling contracts, we get that for almost all ρ-values and in any stable equilibrium, the strictly dominating competitive contract must be offered and used by all safe borrowers. Risky borrowers use either pooling contracts or their full-information contract. Since the identity of the lender offering specific contracts is irrelevant, this stable equilibrium is unique up to the identity of which lender makes each contract offer. It is also unique up to the deletion of un-traded contracts.

■

B Extension: Conversion of Housing to Rental Units

To capture conversion of housing into rental units, I extend the absentee landlord's production of rental services to incorporate both houses as well as apartment buildings. I assume that the production of rental services uses both housing and apartments. The production function is

$$S_t^R=\mu A_{t-1}^{\gamma}(H_t^R)^{1-\gamma}.$$

where S_t^R denotes rental services, A_{t-1} denotes the stock of apartments from the previous period, and H_t^R denotes the current stock of homes owned by landlords. These stocks evolve as

$$H_t^R=(1-\delta_H)H_{t-1}^R+I_t^R,\quad \text{and} \quad A_t=(1-\delta_A)A_{t-1}+I_t^A$$

where I_t^A is investment in new apartments. This specification implies that landlords produce apartment buildings one-for-one from numeraire, which normalizes the units of the stock of apartments.

Given an opportunity cost of funds of $1+r$, the landlord's first order condition for investment in housing is

$$P_t = R_t(1-\gamma)\mu\left(\frac{A_{t-1}}{H_t^R}\right)^\gamma + \frac{1-\delta_H}{1+r}\mathbb{E}_t P_{t+1}.$$

This condition states that the converted-house-to-apartment ratio is an increasing function of the rental rate relative to the user cost of housing:

$$H_t^R = A_{t-1}\left((1-\gamma)\mu\frac{R_t}{P_t - \frac{1}{1+r}\mathbb{E}_t P_{t+1}}\right)^{\frac{1}{\gamma}}$$

Their first order condition for investment in apartments is

$$r + \delta_A = \mathbb{E}_t R_{t+1}\gamma\mu\left(\frac{H_{t+1}^R}{A_t}\right)^{1-\gamma}$$

where δ_A is the depreciation rate on apartment buildings. This condition states that landlords equate the expected marginal revenue product of apartment investment to their cost of funds plus depreciation.

C Relating Income Risk to Labor Market Fundamentals

This appendix presents a simple extension of the baseline model in 3 in which borrower income risk comes from employment risk due to search in the labor market.

There is a large family of workers who are either employed or unemployed in each period. This family provides imperfect income insurance to its members. When an individual worker is unemployed, the family will help them to cover their debt payments with chance $1-\bar{\phi}$. As a result, an individual worker who is unemployed will be forced into default with chance $\bar{\phi}$. When they are employed, they are never forced to default on their mortgage.

Employed workers separate from their current employer with chance s at the end of each period. Individual workers learn whether or not they will keep their job between t and $t+1$ prior to signing mortgage contracts at time t. Lenders cannot observe their separation, and so there is asymmetric information between workers and lenders about individual worker's labor search status.

Both recently separated workers and currently unemployed workers search for a new jobs between periods, while workers who retain their job between periods do not need to search for work. The former have some risk of having no income, while the later have income for sure. As a result, we can interpret the group of workers searching for work as the risky borrowers in the baseline model, and the group with stable employment as the safe borrowers.

I assume that current employment status is observable to lenders. As a result, the population is split into a group of potentially risky borrowers — those who were employed today and might have separated from their current employer — and a group of known risky borrowers — those who didn't have work this period and therefore must be searching for work. Among borrowers with current employment, a fraction s are searching for work, and so we have $\rho_{it} = s$ among workers with current employment. The fraction searching among workers without current employment is 1, so $\rho_{it} = 1$. This models maps precisely into the setup in section 3 when we set $\rho = s$, and let the proportion of potentially risky borrowers in baseline model, n, be time varying and equal to the employment rate in this model, N_t.

The ex-post chance of having no income among workers searching for work is

$$\phi_{t+1} = \bar{\phi}\mathbb{P}_{t+1}[E_{i,t+1} = 0 \mid \text{Search}_{it}] = \bar{\phi}(1 - f_{t+1})$$

where f_{t+1} is the ex-post job-finding rate. Borrowers who do not need to search for work have income for sure.

I assume that competitive firms post job vacancies and are randomly matched to workers. Define θ_t as labor market tightness

$$\theta_t = \frac{V_t}{1 - (1 - s)N_{t-1}}$$

where V_t is total vacancies posted at time t and N_{t-1} is the employment rate in the prior period.

Let M denote the matching function, mapping total vacancy postings and total number of searching workers to a total number of matched workers to vacancies. Assume it is homogenous of degree one, so we can write the job-finding rate and hiring rate by

$$f_t = M(\theta_t, 1), \quad \text{and} \quad h_t = M(1, 1/\theta_t).$$

Assume that posting a vacancy costs a firm κ_v units of numeraire. Firms operate a constant returns to scale production function in labor. Let A_t denote the marginal product of labor in this technology, and let W_t denote the wage paid by firms. Then the job creation condition for firms is

$$\frac{\kappa_v}{h_t} = A_t - W_t + \frac{1 - s}{1 + r}\mathbb{E}_t \frac{\kappa_v}{h_{t+1}}$$

where I assume that firms face the same exogenous opportunity cost of funds as lenders. The quantity κ_v/h_t represents the effective cost of adding a worker to the labor force.

Following Haefke et al. (2013) and Michaillat (2012), I assume that wages are rigid and linked to the marginal product of labor as

$$W_t = A_t^\gamma$$

where $\gamma < 1$. As a result, we can solve for the current hiring rate as a function of expectations of future productivity

$$\frac{\kappa_v}{h_t} = \sum_{j=0}^{\infty} (1 - s)^j \left(\frac{1}{1 + r}\right)^j (A_{t+j} - A_{t+j}^\gamma)$$

Since hiring rates pin down labor market tightness and therefore job finding rates, we get that the job finding rate depends on expectations of the path of productivity. In turn, the income risk of workers who are searching for work depends on the ex-post job-finding rate. Since productivity and the opportunity cost of funds are exogenous, the job finding rate and income risk are both effectively exogenous.

D Sketch of Solution Algorithm

The model of Section 3 is a infinite horizon rational expectations model with endogenous regime-switching. In particular, the distribution of the credit regime, conditional on the state of the economy, is an equilibrium object. I apply a solution algorithm based on the approach of Lind (2015) to jointly solve for this conditional regime distribution and the associated regime-conditional policy functions.

In general the approach applies to models whose equilibrium conditions can be expressed as

$$\begin{aligned} 0 &= \mathbb{E}_t f_{S_{t+1},S_t,S_{t-1}}(Y_{t+1}, Y_t, X_{t+1}, X_t) \\ X_{t+1} &= h_{S_t,S_{t-1}}(X_t, Y_t) + \Sigma\varepsilon_{t+1} \\ S_t &= \sigma_{S_{t-1}}(Y_t, X_t) \end{aligned}$$

where Y_t is an endogenous vector of control variables, X_t is an endogenous vector of state variables, S_t is an endogenous regime variable taking values in $\{1, \ldots, N_S\}$, ε_t is an exogenous vector of *iid* shocks, and Σ is a matrix mapping shocks into future states. The function f captures the forward looking equilibrium condition of the model, the function h captures the endogenous evolution of the state variables, and the function σ assigns a regime to a particular equilibrium outcome at time t. Note that Y_t and S_t are simultaneously determined because S_t and expectations of S_{t+1} can influence Y_t through the forward looking equilibrium conditions and realizations of Y_t are linked to S_t through the regime assignment equation.

I focus on a minimum-state variable equilibrium concept that accounts for this simultaneity between the regime and contemporaneous outcomes.

Definition 6 A *regime-switching equilibrium* is a collection of measurable functions $\{g_{s,s_-}(x)\}_{s=1}^{N_S}$ and a conditional regime distribution $Q_{s',s}(x)$ (measurable in x) such that

1. (**Rational Expectations**) For each (s, s_-) and x

$$0 = \int \sum_{s_+=1}^{N_S} f_{s_+,s,s_-}(g_{s_+,s}(h_{s,s_-}(x, g_{s,s_-}(x)) + \Sigma\varepsilon), g_{s,s_-}(x), h_{s,s_-}(x, g_{s,s_-}(x)) + \Sigma\varepsilon, x) \\ \times Q_{s_+,s}(h_{s,s_-}(x, g_{s,s_-}(x)) + \Sigma\varepsilon) f_\varepsilon(\varepsilon)\mathrm{d}\varepsilon$$

2. (**Regime Consistency**) For each (s, s_-) and x, if $Q_{s,s_-}(x) > 0$ then $s = \sigma(g_{s,s_-}(x), x)$.

3. (**Fundamental Volatility**) For each (s, s_-) and x, $Q_{s,s_-}(x) \in \{0, 1\}$.

The first condition requires that the policy functions solve the model given rational expectations. Implicitly, this definition requires that out-of-equilibrium outcomes (in regimes that never occur given $S_{t-1} = s_-$ and $X_t = x$) also solve the model equations, though a more general definition need not make this particular equilibrium selection. In this context, the selection is inconsequential. The second condition requires that regimes can only occur if outcomes in that regime are consistent with the regime assignment equation. The third condition requires that all randomness in outcomes can be attributed to variation in the state variables (say due to fundamental shocks) — a requirement which rules out sunspots.

The solution algorithm uses backwards induction and perturbation based approximation to approximate such an equilibrium. At iteration n, let $g^{(n-1)}_{s,s_-}(x)$ and $Q^{(n-1)}_{s,s_-}(x)$ denote the policy functions and regime distribution from the previous iteration. Consider finding an updated policy function, $g^{(n)}_{s,s_-}$, solving

$$0 = \int \sum_{s_+=1}^{N_S} f_{s_+,s,s_-}(g^{(n-1)}_{s_+,s}(h_{s,s_-}(x, g^{(n)}_{s,s_-}(x)) + \Sigma\varepsilon), g^{(n)}_{s,s_-}(x), h_{s,s_-}(x, g^{(n)}_{s,s_-}(x)) + \Sigma\varepsilon, x) \\ \times Q^{(n-1)}_{s_+,s}(h_{s,s_-}(x, g^{(n)}_{s,s_-}(x)) + \Sigma\varepsilon) f_\varepsilon(\varepsilon)\mathrm{d}\varepsilon.$$

Introduce a perturbation parameter of $\eta \in [0,1]$ as

$$0 = \int \sum_{s_+=1}^{N_S} f_{s_+,s,s_-}(g^{(n-1)}_{s_+,s}(h_{s,s_-}(x, g^{(n)}_{s,s_-}(x,\eta), 1) + \eta\Sigma\varepsilon + (1-\eta)\bar{v}^{(n-1)}_{s_+,s,s_-}), \\ g^{(n)}_{s,s_-}(x,\eta), h_{s,s_-}(x, g^{(n)}_{s,s_-}(x,\eta)) + \eta\Sigma\varepsilon + (1-\eta)\bar{v}^{(n-1)}_{s_+,s,s_-}, x) \\ \times Q^{(n-1)}_{s_+,s}(h_{s,s_-}(x, g^{(n)}_{s,s_-}(x)) + \Sigma\varepsilon) f_\varepsilon(\varepsilon)\mathrm{d}\varepsilon.$$

where $\bar{v}^{(n-1)}_{s_+,s,s_-}$ is a regime-transition conditional approximation node.

We can approximate the policy function $g^{(n)}_{s,s_-}$ in two steps. First, consider the case with $\eta = 0$, and solve for a nodes $\bar{y}^{(n)}_{s,s_-}$ such that the equation is satisfied for a given point in the state space of $x = \bar{x}^{(n)}_{s,s_-}$. Then, since the model is solved for $y = \bar{y}^{(n)}_{s,s_-}$ for $x = \bar{x}^{(n)}_{s,s_-}$ and $\eta = 0$, we can apply the implicit function theorem to get a first order approximation of $g^{(n)}_{s,s_-}$.

Next we can update the conditional distribution of the regime to $Q^{(n)}$ such that it is consistent with the assignment of regimes implied by the approximation to $g^{(n)}_{s,s_-}$ and the regime function σ_{s,s_-}. I do so by keeping track of conditional regime transitions on a grid of ε-values (transitions to s_+ conditional on $S_t = s$ and $S_{t-1} = s_-$). Also, I update the shock approximation nodes, $\bar{v}^{(n)}_{s_+,s,s_-}$, to equal the expected value of $\Sigma\varepsilon'$ condition on the regime transition. Finally, I allow the approximation nodes $\bar{x}^{(n)}_{s,s_-}$ to slowly adjust based on the average predicted outcomes for the state at each step of the algorithm. I iterate on these approximation and update steps until the algorithm converges.