Financial Frictions, Volatility, and Skewness *

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Abstract

A number of recent papers use the interaction of firm idiosyncratic volatility shocks with firm financial frictions to explain business cycle fluctuations. I argue that a key parameter for these models is the cost of default, as it has a quantitatively first-order effect on the magnitude of the decline in employment and other aggregates in response to idiosyncratic volatility shocks. I use firm-level panel data and a structural model of financial frictions and volatility shocks to assess the role of volatility shocks and the cost of default on firm and aggregate employment over the business cycle. I find that when the cost of default is calibrated to the range of estimates coming from the corporate finance literature, the model reproduces key cross-sectional moments of equity volatility, bond spreads, and employment growth. However, this calibration implies aggregate employment losses driven by shocks to firm idiosyncratic volatility are modest. I propose two additional shocks calibrated using firm-level panel data which could amplify the decline in employment in the context of such a model. First, the decline in employment is amplified when the increase in firm idiosyncratic risk is modeled not only as a positive second moment shock but also a negative third moment shock. Second, a plausible increase in the cost of default over the business cycle can interact with volatility shocks to dramatically reduce aggregate employment.

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1 Introduction

The countercyclical behavior of the cross-sectional dispersion of economic variables has been well documented by economists, and convincing arguments have been made that this reflects the underlying volatility of firm-level idiosyncratic shocks.[1] A recent strand of literature has generated aggregate fluctuations by using the interaction of volatility shocks with firm-level financial market frictions to distort firm-level employment or investment decisions.[2] Several of these papers find that idiosyncratic volatility shocks, operating through financial frictions, are important for explaining business cycle dynamics.[3] Financial frictions affect not only firm investment but also firm employment decisions if labor markets are not frictionless; several recent papers document that firm employment does in fact respond to changes in financial constraints.[4]

In this paper, I use firm-level panel data, together with a structural model, to assess how firm employment responds to volatility shocks in the presence of financial frictions. I use cross-sectional patterns in the data directly related to the response of firm employment and credit spreads to volatility shocks in order to discipline parameters key to the magnitude of this response. One parameter which crucially affects the response of firm employment to volatility shocks is the cost of default. This is true in a large number of models, including papers using the financial accelerator framework of Bernanke, Gertler, and Gilchrist (1999) (hereafter BGG).[5] If default is costless, corresponding to the assumptions of Modigliani and Miller (1958), then the heightened default risk caused by an increase in volatility does not affect firm decisions. As the cost of default increases, the magnitude of its effect on firm employment increases as well. I argue that the cost of default is important not only for the impact of volatility shocks on firm employment over the business cycle, but also for the strength of many cross-sectional relationships relating credit spreads, default rates, and firm employment. I use firm-level panel data to identify shocks to the cost of default and to the skewness of firm idiosyncratic risk, and evaluate their impact on firm decisions and aggregates over the business cycle.

Many of the models in this literature would generate cross-sectional implications if

[1] Recent papers include Eisfeldt and Rampini (2006) and Bloom (2009). Additionally, Herskovic et al. (2014) document that idiosyncratic equity volatility obeys a strong factor structure and spikes during recessions.

[2] See Christiano, Motto, and Rostagno (2013), Arellano, Bai, and Kehoe (2012), and Gilchrist, Sim, and Zakrajsek (2014).

[3] For instance, Christiano et al. (2013) find that changing volatility is the most important shock over the business cycle. Similarly, Arellano, Bai, and Kehoe (2012) find that most of the decline in output and employment during the recent recession can be explained by idiosyncratic volatility shocks.

[4] See Benmelech, Bergman, and Seru (2015) and Chodorow-Reich (2014).

[5] In a BGG framework, the monitoring cost can be interpreted as the cost of default.

firms were to receive heterogeneous shocks to the level of idiosyncratic volatility.[6] These cross-sectional implications arise from the same channels that lead to aggregate fluctuations over the business cycle. Therefore, I argue that the cross-section can be used to calibrate parameters which affect business cycle dynamics.

The specific structural model I use to assess the role of volatility shocks and financial frictions on firm employment is based on the model of Arellano, Bai, and Kehoe (2012) (hereafter, ABK). Firms face idiosyncratic shocks which have a persistent effect on the profitability of the firm, and the stochastic volatility of these shocks are time varying.[7] The main mechanism at play in the model is the following: An increase in the firm's idiosyncratic volatility increases its probability of default. This, in turn, increases credit spreads and decreases firm employment. The decline in employment occurs because firms must choose their level of employment before realizing their idiosyncratic shocks, receiving proceeds from production, and paying the wage bill. Thus, choosing higher employment is risky and increases the probability of default, because the heightened wage bill increases the firm's liabilities the following period (therefore, the realized level of the idiosyncratic shock needed to pay off both debt and the higher wage bill increases). This effect of firm employment on default risk can be thought of as increasing the *operating leverage* of the firm, as the additional risk of a higher wage bill due in the future increases the fixed cost due similar to how it is increased by debt (financial leverage). A key property of this mechanism is that it is operational at the firm level. Therefore, an increase in the idiosyncratic volatility faced by a firm should increase its credit spread and reduce employment, even if the increase in volatility only affects that firm in isolation. Thus, this mechanism generates testable cross-sectional implications.

I base my analysis on the model of ABK as it is tractable and has meaningful heterogeneity in firm productivity and leverage. Additionally, their model focuses on the effect of volatility shocks and financial frictions on labor, rather than investment, and is able to generate a large decline in employment with volatility shocks calibrated for the 2007-2009 recession while matching important business cycle facts.[8] I expand this model by adding heterogeneity in volatility and generalizing the parameterization of the default cost.[9] I

[6]These include papers using the financial accelerator framework following Bernanke, Gertler, and Gilchrist (1999), notably Christiano, Motto, and Rostagno (2013).

[7]These shocks are motivated as *demand shocks*, but they are isomorphic to productivity shocks.

[8]ABK are able to generate a large decline in employment with volatility shocks consistent with observed sales growth dispersion during the 2007-2009 recession. Their model is able to do so without declining aggregate labor productivity.

[9]There is also heterogeneity in innovations to firm productivity, leverage, and volatility in my model. This heterogeneity is important because structural models of default risk and credit spreads, such as Merton (1974), use firm leverage and volatility as the key explanatory variables.

use this rich heterogeneity to generate cross-sectional implications. The same approach should work for an array of other models, notably those using the BGG framework and volatility shocks (such as Christiano, Motto, and Rostagno (2013)). The main mechanism in these models is effectively the same: in order to lower the risk of costly default firms/entrepreneurs reduce the scale of their operations. Increases to the likelihood or cost of default amplifies this reduction in scale.[10]

I use firm-level panel data from U.S. public firms on credit spreads, equity prices, and accounting statements to test the presence and magnitude of cross-sectional patterns predicted by the model. I document that there is considerable cross-sectional variation in innovations to firm asset volatility, even during non-recession years.[11] The variation in innovations to volatility in the cross-section is significant relative to the average increase in asset volatility during the crisis. I show that innovations to asset volatility are associated with significant increases in credit spreads and are predictive of decreases in employment. These results are robust to a large number of controls, year effects, and substituting sales for employment.[12] This validates the key qualitative implications of the model, and stresses that the association of volatility shocks with credit spreads and employment is present not only in aggregate time-series, but also in explaining patterns between firms in panel data.

I calibrate the model with moments relating the probability of default and credits spreads, and using the joint distribution of innovations to employment, credit spreads, and firm-level volatility measures. I show that the slope of the relationship between the probability of default and credit spreads depends primarily on the cost of default; credit spreads are a function of both the probability of default and recovery rates conditional on default, which depends crucially on the costs of default. When I set the cost of default to 100%, as in ABK where defaulting firms exit and lose all firm value, the relationship between the probability of default and credit spreads in the model is very steep. Similarly, if default is costless, the slope of this relationship is very flat. The slope implied by historical default probabilities and credit spreads by ratings class (for speculative grade

[10]BGG style models do not have meaningful heterogeneity in firms/entrepreneurs who are exposed to default, they are ex-ante identical. This allows the distribution of these agents to be summarized by the total wealth they hold. To implement my approach on such a model, meaningful heterogeneity would have to be added to generate cross-sectional implications.

[11]I consider innovations, as opposed to the level, of most of the measures I examine. This is because firm capital structure, given sufficient time, can adjust in response to differences in the level of volatility. Additionally, looking at innovations in volatility is the natural approach given the desire to examine the response to volatility *shocks*.

[12]There are some known measurement issues associated with firm employment in Compustat (see Bloom (2009)). I use sales, which is measured more accurately, to confirm that this measurement error is not significantly distorting my results.

firms) is much different than either of these two extremes. The slopes corresponding to estimates of the cost of default from the corporate finance literature, ranging from $8.4 - 30\%$ of firm value lost, generate a relationship much more in line with the data.[13]

With such default costs, my calibrated model generates a modest decline in employment in response to common shocks to firm idiosyncratic volatility. Feeding in volatility shocks corresponding to the 2007-2009 recession generates at most a 2.5% decline in employment under the upper bound calibration. This is much lower than that implied by the calibration of ABK, where all firm value is lost upon default. This suggests that idiosyncratic volatility shocks, by themselves, have trouble explaining much of the employment dynamics in the recession of 2007-2009 through this mechanism.

However, there is empirical evidence of other important changes during the great recession that affect employment in this model. First, Bloom, Guvenen, and Salgado (2015) document substantial evidence of a large negative skewness shock to the growth rates of U.S. public firms. This suggests that something may be lost by modeling the change in firm idiosyncratic risk only as a second moment shock, as skewness affects the relative amounts of left and right tail risk. Second, recovery rates, the fraction of debt obligations (principal and accrued interest) debtholders receive upon firm default, are procyclical and fell significantly during the 2007-2009 recession, which may be indicative of an increase in the cost of default.[14] I show that the cross-sectional relationship between default probabilities and credit spreads among U.S. public firms changes in a way consistent with a heightened cost of default during this recession. Specifically, I estimate that it became around 2.5 times steeper, corresponding to a significant increase in the cost of default.[15] Feeding in either of these shocks together with volatility shocks can amplify the decline in employment. If firm upside and downside volatility are parameterized separately, differing shocks to these volatilities consistent with micro data from the 2007-2009 recession (using the measures of upside and downside dispersion in Bloom, Guvenen, and Salgado (2015)) amplify the fall in employment by around 1% of employment. Shocks to the cost of default, when interacted with volatility shocks, can generate an enormous decline in employment. Simultaneous shocks to volatility and default cost, calibrated to micro data, explain the majority of employment losses during the 2007-2009 recession. These losses

[13]See Davydenko, Strebulaev, and Zhao (2012) (which includes a summary of estimates in the literature), Kaplan and Andrade (1998), and Hennessy and Whited (2007).

[14]See Altman (2006) and Moody's (2015).

[15]For the purpose of the model, I consider the cost of default to be exogenous. One could microfound such a shock through disruptions in financial markets which reduce the liquidation value of firm assets, see Shleifer and Vishny (1992) and Shleifer and Vishny (2011). Additionally, Gilchrist et al. (2014) consider shocks to the liquidation value of capital, which can make default more expensive as some capital is liquidated upon default in their model.

are substantially larger than if the cost of default were permanently high (calibrated to the maximum cost of default realized during the 2007-2009 in my parameterization). This suggests the interaction of these two shocks is key in explaining large employment losses.

Related Literature There are a number of papers which use volatility shocks to drive business cycle dynamics. Several of these papers use the increased volatility of firm-level shocks interacted with adjustment costs in capital or labor to generate business cycle fluctuations; see, for example Bloom (2009), Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2014), and Bachmann and Bayer (2013). Schaal (2015) uses the interaction of volatility shocks with labor market frictions in a search and matching model to generate significant declines in employment. The papers in this literature closest to mine are those which study the interplay between volatility shocks and financial frictions, notably Arellano et al. (2012), Christiano et al. (2013), and Gilchrist et al. (2014). I focus on volatility shocks operating through financial frictions for two reasons. First, there is evidence suggesting that this interaction may be able to deliver larger declines in output and employment than the interaction of volatility shocks with other frictions.[16] Second, the interaction of volatility shocks with financial frictions leads to clear cross-sectional implications which can be compared to available firm-level data.

My main contributions to this literature are as follows: First, I use cross-sectional patterns to discipline key parameters that control the impact of volatility shocks on aggregates over the business cycle. While some papers in this literature use the cross-section to parameterize volatility shocks, my paper focuses on other key relationships to parametrize the cost of default as well. Second, my paper is novel in the shocks I consider, notably the role of changing skewness of firm idiosyncratic shocks and its impact for aggregates in this setting. There is a strand of the finance literature which discusses the asset pricing implications of idiosyncratic skewness; see, for example Amaya et al. (2015), Feunou et al. (2015), Kraus and Litzenberger (1976), and Barberis and Huang (2008). However, these changes have been under-explored in macroeconomics — idiosyncratic risk shocks have typically been modeled only as volatility shocks.[17] Bloom, Guvenen, and Salgado (2015) provide convincing evidence in firm-level data of significant changes in idiosyncratic skew-

[16] Gilchrist et al. (2014) find, in a model with volatility shocks interacting with both adjustment costs and financial frictions, that the quantitatively important channel is through financial frictions. Bachmann and Bayer (2013) also call into question the quantitative relevance of adjustment costs interacting with volatility shocks.

[17] There are some papers in macroeconomics related to possibly time-varying skewness, but they differ substantially from my approach. A notable example is Orlik and Veldkamp (2014), who provide a micro-foundation for time-varying uncertainty about aggregate shocks by having agents update beliefs about the skewness of aggregate shocks.

ness over the business cycle, and my paper investigates its implications in a business cycle model.

My paper also contributes to this body of literature through my analysis of shocks to the cost of default. My paper is novel in that I stress the interaction of this default cost shock with volatility shocks and show that this interaction leads to amplification effects. While Gilchrist, Sim, and Zakrajsek (2014) do consider both idiosyncratic volatility and capital liquidity (related to the cost of default) shocks, my paper differs in that it stresses the interaction between the two happening simultaneously.[18] Additionally, I use the change in a key cross-sectional relationship to help parameterize the magnitude of the shock to the cost of default.

My paper is also related to a strand of literature which aims to disentangle the effects of financial and volatility (or uncertainty) shocks. The challenge faced by this literature is that the predictions of the two shocks are quite similar for the time series. Caldara, Fuentes-Albero, Gilchrist, and Zakrajsek (2014) state that "distinguishing between these two types of shocks... is difficult because increases in uncertainty are frequently associated with a widening of credit spreads, an indication of a tightening in financial conditions" and use a penalty function approach to assess the individual impact of these two shocks on the economy. Stock and Watson (2012) use a dynamic factor model and conclude that uncertainty and financial shocks were the primary drivers of the recent recession, although they note the high correlation between the shocks and question whether they are distinct. Christiano, Motto, and Rostagno (2013) use time-series data and a structural model following BGG with volatility and a number of other shocks to estimate the key drivers of business cycle dynamics.[19] My paper's contribution relative to this literature is the use of firm-level panel data to pin down shocks and their effects. For instance, while it is difficult to separately identify shocks to the cost of default as opposed to idiosyncratic volatility using only time series data on macroeconomic and financial aggregates, there are some rather stark cross-sectional implications of these shocks. Namely, in the framework I consider, the slope of the relationship between credit spreads and probability of default is very sensitive to the cost of default but not to volatility. This is because the effect of volatility on credit spreads occurs primarily through its effect on the probability of default.

Finally, my paper is related to a number of studies which aim to quantify the cost

[18] Additionally, a large literature looks at the effect of a variety of financial shocks, including the cost of default or liquidation, on the macroeconomy.

[19] They do not consider shocks to the cost of default/monitoring; indeed it would be difficult to separately identify them in a BGG-style model. They do, however, estimate the constant level of default/monitoring; their estimate of 20% is consistent with estimates from the finance literature.

of default. The main challenge lies not in estimating the direct costs (such as legal bills) but rather the indirect costs (for example, the loss of key employees/customers due to greater risk of unemployment/discontinued support). This is a difficult task due to potential selection effects and the anticipation of default by markets. Andrade and Kaplan (1998) is the most widely used of these studies, which looks at a sample of highly leveraged transactions which became distressed and finds costs of default from 10-23%. Other studies include Davydenko, Strebulaev, and Zhao (2012), which take advantage of the fact that default is only partially anticipated to estimate cost of default from the valuation of firm equity and debt before and after default. They produce estimates larger than previously found, finding that default destroy around 30% of firm value while debt renegotiations destroy around 15%. On the low end of estimates, Hennessy and Whited (2007) use structural estimation to find costs of default as low as 8.4% for large firms. My results are consistent with the range of estimates in this literature. My paper shows that this range of estimates not only corresponds to patterns in the pricing of debt (as others have), but is also consistent with firm dynamics, namely the magnitude of the decline in employment or sales predicted by innovations to credit spreads.

Road Map The rest of the paper follows as such: Section 2 introduces a simplified version of the model to introduce the key mechanism along with the strategy by which cross-sectional relationships are used to discipline model parameters. Section 3 describes the data I use and establishes facts about the cross-sectional relationships of innovations to asset volatility, credit spreads, and firm input decisions. Section 4 describes the model of firm volatility and financing frictions and characterizes an equilibrium. Section 5 describes the parameterization of the model and the resulting implications for both the cross-section and the business cycle. Section 6 documents the implications of negative idiosyncratic skewness shocks and shocks to the cost of default. Section 7 concludes.

2 Simple Model

I begin with a simplified version of the model to illustrate the mechanism of interest, through which firm-level volatility shocks can generate business cycle dynamics. Specifically, I show how changes in the distribution of a firm idiosyncratic shock can, in the presence of financial frictions, lead to fluctuations in the level of employment and credit spreads. The simple model will demonstrate how the costliness of default plays a key role in determining how large of an impact fluctuations in firm-level risk have on firm employment. Also, the simple model generates cross-sectional implications, which can be compared to the data to discipline the theory.

The primary mechanism in this model is based on employment *operating leverage*, where firms' labor decisions affect their probabilities of default. Firms make their employment decision before realizing a firm-level demand shock, affecting the profits generated by a given number of workers. They then receive revenues from production (which depend on the shock) less the wage bill (which does not depend on the shock) and any debt outstanding. If a firm chooses a greater amount of labor, it takes on additional risk due to greater operating leverage (a higher fixed cost next period) in exchange for higher expected profits. A higher labor choice hurts firm net revenues if the realized shock is low, and thus can increase the probability of default. Default is costly, so firms have an incentive to reduce their labor demand in order to reduce their default risk.

This section also demonstrates a few key points of the paper. First, I show that the cost firms face upon default has an enormous impact on the magnitude of the distortion to firm labor, as well as the magnitude of labor losses due to a volatility shock. Second, I show that since the mechanism is operational in partial equilibrium, the cross-sectional implications of the model can be used to discipline the parameterization. For example, the slope of the relationship between the probability of default and credit spreads is effectively pinned down by the cost of default and can be used to discipline parameter choices controlling the magnitude of the cost.

The simple model not only provides intuition about the mechanism of interest; in the following subsections I use it to demonstrate the key points made in this paper. In subsection 2.1, I introduce the simple model and outline how labor is distorted due to the presence of financial frictions (there are incomplete markets, as equity holders can only raise revenues with a one period state-uncontingent debt which can lead to costly default) and idiosyncratic shocks. In subsection 2.2, I characterize the distortion to labor and show how the cost of default and the distribution of idiosyncratic shocks affect the distortion. Namely, I show that the distortion is roughly proportional in the cost of default and the marginal probability of default (which naturally depends on the distribution of idiosyncratic shocks). In subsection 2.3, I show the cross-sectional implications of this mechanism and argue that they can be used for calibrating key parameters, such as the cost of default, to ensure the model is consistent with the data.

2.1 Model Setup and Characterization

Consider a two-period model of operating leverage and financial frictions. In the first period, firms issue uncontingent debt and choose the amount of labor input l to hire.

Equity holders receive cash flows from debt issuance in that first period, with the debt priced as the expected discounted present value of cash flows to debtholders. In the second period, a firm-level demand shock which augments revenues, z, is realized. If the firm does not default, equity holders receive revenues from production, zl^{α}, less the cost of labor, wl, and the amount of debt due, b. They also receive a continuation value V. If the firm defaults, equity holders receive nothing while debt holders receive net revenues from production, $zl^{\alpha}-wl$, as well as a portion $1-c$ of the continuation value V. The parameter c represents how costly default is. Thus, if $c=1$, then all remaining firm value above and beyond current operating profit is lost upon default. If $c=0$, then default is costless. Firms default if $zl^{\alpha}-wl-b+V<0$, that is if the value to equity holders in the second period (net cash flows from production less the debt payment plus the continuation value) are less than zero.[20] The default threshold can be characterized by the level of the shock z below which a firm defaults, denoted as $\overline{z}(l,b)$.

The revenues the firm receives for issuing debt are:

$$bq(l,b) = \underbrace{(1-F(\overline{z}(l,b)))}_{\text{prob. no default}}\beta b + \underbrace{F(\overline{z}(l,b))}_{\text{prob. default}}\beta\left(\underbrace{\mathbb{E}[z|z<\overline{z}(l,b)]\,l^{\alpha}-wl}_{\text{exp. operating profit}} + \underbrace{(1-c)}_{\text{default recovery}} \times V\right), \quad (1)$$

where $F(z)$ is the cumulative density function of the shock z. The optimization problem governing the labor demand of equity holders can be expressed as follows:

$$\max_{l}\ \underbrace{bq(l,b)}_{\text{debt issuance}} + \underbrace{(1-F(\overline{z}(l,b)))}_{\text{prob. no default}}\beta\left(\underbrace{\mathbb{E}[z|z>\overline{z}(l,b)]\,l^{\alpha}-wl}_{\text{exp. operating profit}} - \underbrace{b}_{\textit{debt due}} + \underbrace{V}_{\text{cont. val}}\right). \quad (2)$$

Plugging (1) into (2) yields:

$$\max_{l}\ \beta\left(\underbrace{\mathbb{E}[z]\,l^{\alpha}-wl}_{\text{exp. operating profit}} + \underbrace{V}_{\text{cont. val}} - \underbrace{F(\overline{z}(l,b))}_{\text{prob. default}} \times \underbrace{cV}_{\text{default cost}}\right). \quad (3)$$

The first order condition of (3) with respect to firm employment is the following:

[20] In the full model, default occurs if net cash flows from production less debt due are low enough such that revenues raised through debt issuance are insufficient to cover the firm's obligations. This is an endogenous threshold solved in an infinite-horizon model, so I use this simple default threshold for illustration here.

$$l^* = \left(\frac{\alpha E[z]}{w + V \times c \times f(\overline{z}(l^*, b)) \frac{\partial \overline{z}(l^*,b)}{\partial l^*}} \right)^{\frac{1}{1-\alpha}}. \quad (4)$$

As a contrast, if default is costless (corresponding to the assumptions in Modigliani and Miller (1958)), the first order condition reduces to the following:[21]

$$l^*_{MM} = \left(\frac{\alpha E[z]}{w} \right)^{\frac{1}{1-\alpha}} \quad (5)$$

The key difference between these two cases is the term $Vcf(\overline{z}(l^*, b)) \frac{\partial \overline{z}(l^*,b)}{\partial l^*}$ in the denominator. In this expression, Vc represents the cost of default, while $f(\overline{z}(l^*, b)) \frac{\partial \overline{z}(l^*,b)}{\partial l^*}$ represents the derivative of the probability of default with respect to firm's labor demand. This second term can be split into $f(\overline{z}(l^*, b))$, which represents the probability density function of the shock z at the default threshold, and $\frac{\partial \overline{z}(l^*,b)}{\partial l^*}$, which represents the effect of the firms labor decision on the default threshold. While V, c, and $f(\overline{z}(l^*, b))$ are non-negative by definition, $\frac{\partial \overline{z}(l^*,b)}{\partial l^*}$ depends on the parametrization as well as firm debt and labor. The following proposition formalizes the conditions under which this derivative is positive and its effect on firm employment:

Proposition 1. *The following are equivalent:*

(i) $wl^* \geq \alpha(wl^* + b - V)$

(ii) $\frac{\partial \overline{z}(l^*,b)}{\partial l^*} \geq 0$

(iii) $Vcf(\overline{z}(l^*, b)) \frac{\partial \overline{z}(l^*,b)}{\partial l^*} \geq 0$ *if* $V, c, f(\overline{z}(l^*, b)) > 0$

(iv) $l^* \leq l^*_{MM}$ *if* $V, c, f(\overline{z}(l^*, b)) > 0$

Proof. See Appendix A.

The parameter restriction $wl \geq \alpha(wl + b - V)$ can be understood as requiring that the wage bill is at least as large as a fraction α of the fixed cost less continuation value. To violative this condition, firms need to have sufficiently high levels of debt relative to their operating costs. Intuitively, if the wage bill is a small part of the fixed cost, a high labor choice increases the potential revenues from production but does little to the total

[21](5) is also the first order condition which would arise if I relaxed market incompleteness and allowed the debt payments to be conditional on the realized shock z.

fixed cost due. If firms have sufficiently high debt, the firm will default anyway for low realizations of z, and a higher labor choice helps the firm generate enough revenues for higher realizations of z to avoid default in some cases. However, firms with sufficient levels of debt to violate the condition in Proposition 1 are rare, both in the full model and the data. For most firms, the prospect of risky default distorts firm employment decisions downwards.

2.2 Distortion to Firm Employment

In this subsection, I demonstrate how changes to the distribution of z, such as volatility or skewness shocks, can decrease employment. Further, I discuss why the magnitude of the change in employment in response to these distributional changes depends crucially on the cost of default, c. Finally, I discuss why changing the cost of default in itself can reduce firm labor demand and why the combined effect of shocks to volatility and the cost of default can interact to generate large declines in employment.

In this simple model, the distribution of z affects only the term $f\left(\overline{z}\left(l^{*}, b\right)\right)$ in (4).[22] The following proposition demonstrates that if a threshold $\overline{z}$ is sufficiently below (or above) the mean, $f\left(\overline{z}\right)$ is increasing in volatility:

Proposition 2. *Let $f_1(z)$ and $f_2(z)$ denote normal probability density functions with identical means μ and variances σ_1 and σ_2, respectively.*

(i) If $|\mu-\overline{z}|>\sigma_1$, then $\frac{\partial f_1}{\partial \sigma_1}\left(\overline{z}\right)>0$

(ii) If $|\mu-\overline{z}|>\sigma_2$ and $\sigma_2>\sigma_1$, then $f_2\left(\overline{z}\right)>f_1\left(\overline{z}\right)$

Proof. See Appendix A.

Figure 1 illustrates how changing the distribution of the shock z by increasing the variance can raise the probability density function of the shock at a given default threshold. Naturally, other changes to the distribution also change the probability density function and this term. Notably, a decrease in skewness of the distribution following the empirical findings of Bloom, Guvenen, and Salgado (2015) leads to substantially more dispersion of outcomes in the left tail than the right; this higher variance of the left tail can significantly raise the probability density function at the default threshold for many firms.

The cost of default, c, crucially affects the magnitude of the impact of volatility shocks (and other changes to the distribution of z) on firm employment and output. First, note

[22] In the full model, where the continuation value and default threshold are endogenous, the distribution of z does affect other components in the problem. Quantitatively, however, the first-order effect still comes through $f\left(\overline{z}\left(l^{*}, b\right)\right)$.

that c multiplicatively enters the term which leads to the distortion from the case without the possibility of default in (4). In a world consistent with Modigliani and Miller (1958), $c = 0$ and changes to the distribution of z have no impact on employment at all, as long as they do not affect the expected value of z. I can illustrate this by finding an expression for the distortion from the no-default case. (4) and (5) can be re-arranged to express the distortion from the case without default as the following:

$$\log\left(l^*\right) - \log\left(l^*_{MM}\right) = -\frac{1}{1-\alpha}\log\left(1 + \frac{cVf\left(\overline{z}\left(l^*, b\right)\right)\frac{\overline{z}\left(l^*,b\right)}{\partial l^*}}{w}\right).$$

As the marginal cost of labor demand is primarily driven by wages, $\frac{cVf\left(\overline{z}\left(l^*,b\right)\right)\frac{\partial \overline{z}\left(l^*,b\right)}{\partial l^*}}{w}$ will be relatively small. Thus the following is a good approximation:

$$\frac{l^* - l^*_{MM}}{l^*_{MM}} \approx -\frac{c}{1-\alpha}\frac{Vf\left(\overline{z}\left(l^*, b\right)\right)\frac{\partial \overline{z}\left(l^*,b\right)}{\partial l^*}}{w}. \tag{6}$$

If the right hand side terms in (6) do not change very much in the firm's labor decision, then the percent deviation in labor choice from the costless default (Modigliani-Miller) case is roughly proportional in both how costly default is, c, as well as the probability density function at the default threshold, $f\left(\overline{z}\left(l^*, b\right)\right)$. This suggests that the level of c will roughly proportionally affect the magnitude of the impact of shocks to the distribution, which change $f\left(\overline{z}\left(l^*, b\right)\right)$, on employment. Also note that changing the level of c, all else fixed, will also affect firm employment – thus shocks to the cost of default can affect employment on their own.

This can be numerically demonstrated in this simple model, which can be done with a simple parameterization. I normalize the wage w to 1, and parameterize z as a lognormal distribution with the mean chosen so that $\mathbb{E}\left[z\right] = 1$. I set $V = 1.81$ to hit the median ratio of market capitalization to sales, and set $\alpha = .6091$, the degree of decreasing returns to scale in ABK.[23] I allow c, the volatility of z, and the debt outstanding b to vary. The ranges of volatility and leverage are chosen to be reasonable given the distribution of firms sales growth and leverage. If default leads to firm death ($c = 1$), as in ABK, the induced distortions are much larger than if the higher estimates from the corporate finance literature ($c = 0.3$) are used. In this simple model, this will increase the magnitude of distortions by almost 70%. Figure 2 details the impact of a doubling in volatility of

[23]This number accounts for both the decreasing returns to scale in their intermediate good production function and the extent of decreasing returns to scale inherent in the technology through which intermediate goods are aggregated into output.

employment for a range of values of c.

I can also demonstrate that there are significant interaction effects between shocks to the cost of default and volatility (or other distributional) shocks. For instance, if c and $f(\overline{z}(l^*, b))$ were each hit with s_1 and s_2 percent positive shocks, then (6) implies that they individually would amplify the distortion in labor demand from the no-default case by roughly s_1 and s_2 percent, respectively (thus employment fall by roughly $s_1 s_0$ and $s_2 s_0$ percent, where s_0 is the percent distortion of labor demand before the shocks). If they arrived together they would amplify the distortion in labor demand by $s_1 + s_2 + s_1 s_2$ percent (and thus labor demand falls by $s_0(s_1 + s_2 + s_1 s_2)$ percent). The calibrated shocks to the equivalents of these objects during the recent recession are quite large; the increase in idiosyncratic volatility can lead to a more than doubling of $f(\overline{z}(l^*, b))$, and I also find that c increases substantially. Therefore, the additional term due to the interaction, $s_1 s_2$, may be quite large.[24] Figure 3 details the implied employment response to a doubling of volatility and a doubling of the cost of default (from $c = .15$ to $.3$), demonstrating that the interaction term is significant and potentially large.

2.3 Implications for the Joint Distribution of Credit Spreads, Leverage, and Observable Volatility

The above theory has several implications for the cross-sectional behavior of firms that can be used to discipline its application over the business cycle. A key property of the mechanism detailed above is that it is operational in partial equilibrium. In other words, a rise in firm business risk (changing the distribution $f(z)$) will distort a firm's behavior even if it happens only to that single firm alone. This implies that there is a link between the cross-sectional implications of this mechanism and its business cycle implications. Therefore, cross-sectional patterns, even outside of business cycle episodes, can be used to calibrate the model. This is useful, as many significant changes to conditions over the business cycle (such as changes to financial or product market conditions) are omitted from the model that likely affect observables over the business cycle and make calibration with data during recessions more challenging.

One of the key parameters for governing the strength of this operating leverage mechanism distorting firm labor is the costliness of default, c. In this section I demonstrate a key observable implication of this parameter: the derivative of credit spreads with respect to the probability of default is increasing in c. I argue that values of this parameter which

[24]The reason why the two shocks do not separate if looking at log changes in (4) is that w is quite large relative to $cVf(\overline{z}(l^*, b))\frac{\partial \overline{z}(l^*,b)}{\partial l^*}$. If w were small, then the approximation in (6) would be poor and the interaction effects would be negligible.

correspond to the range of values found in the corporate finance literature (generally between 8.4-30% of firm value) generate more reasonable dynamics than the assumption of 100% of firm value lost.

I also show that the model has implications for the joint distribution of equity volatility, credit spreads, and employment. Namely, innovations to equity volatility should move together with credit spreads and predict employment declines.

2.3.1 Probability of Default and Credit Spreads

The probability of default, $F(\overline{z}(l,b))$, is the cumulative probability of receiving a shock below the default threshold implied by firm decisions. The credit spread faced by a firm can be written as $\frac{b}{bq(l,b)} - \frac{1}{\beta}$. This can be expanded to find:

$$cs(b,l) = \frac{b}{\beta\left(b - (b + wl - V(1-c))F(\overline{z}(l,b))\right) + \beta\left(\int_0^{\overline{z}(l,b)} z dz\right) l^{\alpha}} - \frac{1}{\beta}. \tag{7}$$

Note that the credit spread is increasing in $F(\overline{z}(l,b))$, as it decreases the denominator in the expression of $cs(b,l)$ above. The two are positively correlated, as greater leverage or volatility of z both lead to an increased probability of default, and therefore a lower probability of full repayment of debt and thus higher credit spreads. The strength of the relationship between the two, however, depends crucially on c. When c is high, default is very costly and recoveries conditional on default are low. Thus, as the probability of default increases, credit spreads increase substantially. Conversely, if c is low, debtholders are able to recover most of the firm's value (and more of the debt face value) upon default, and thus credit spreads rise less in the probability of default. Assuming that l and b remain fixed, I can characterize the slope of credit spreads relative to default probability as the following:

$$\frac{\partial\, cs}{\partial\, P(def)} = \frac{b\beta(b + wl - V) + bVc}{\left(\beta\left(b - (b + wl - V(1-c))F\left(wl^{1-\alpha} + (b-d)l^{-\alpha}\right)\right) + \beta\left(\int_0^{wl^{1-\alpha}+(b-d)l^{-\alpha}} z dz\right) l^{\alpha}\right)^2}.$$

In the above equation, increasing the cost of default, c, increases the slope for reasonable parameterizations, as increasing the term in the numerator dominates the term in the denominator. In practice (allowing l, b, and the volatility of z to vary), in the simple model the relationship between the probability of default and credit spreads is approximately linear for reasonable probabilities of default.

Figure 4 shows how credit spreads vary with the probability of default (due to heterogeneity in both leverage and asset volatility) for a range of values for c.[25] Note that the slope of credit spreads with respect to default probability is increasing in c — it is near zero for $c = 0$, and in excess of one for $c = 1$. Also plotted on the figure are credit spreads and one-year realized default probabilities for speculative grade firms by rating class.[26] The teal lines (c=0.084,0.3) represent the range of estimates for the cost of default from the corporate finance literature. Note that, as expected, the relationship in the data lies between these two lines.

What this plot suggests is that the assumption that default leads to firm death ($c = 1$) leads to much too strong of a relationship between default probabilities and credit spreads compared to the data. On the other hand, using c in a range of values found in the corporate finance literature, where default is much less costly, leads to a relationship much more in line with the data. As the choice of c is very important for the magnitude of the distortion to employment, this has significant implications for the effect of volatility shocks over the business cycle.

Correcting for Risk Premia I also account for the potential bias of risk premia on this relationship. Credit spreads are often decomposed into two parts: compensation for default losses and the risk premium. The simple model corresponds only to the first part. Cross-sectional patterns in risk premia may affect this cross-sectional relationship and potentially bias calibration strategies reliant on it. A number of recent studies have estimated cross-sectional patterns in risk premia by computing excess bond returns, namely Choi and Kim (2015) and Chordia, Goyal, Nozawa, Subrahmanyam, and Tong (2015). Their results indicate that while firms with a low probability of default indeed do have a greater *proportion* of their credit spreads explained by risk premia, the *level* of the risk premium is increasing as firms face a higher risk of default. Both of these studies find that excess bond returns are decreasing in firm credit ratings (a worse credit rating implies higher risk premia). Chordia et al. (2015) find that mean excess bond returns are decreasing in distance to default, a common proxy for default risk. This implies that firms with lower distance-to-default (and thus higher default probabilities) have higher risk premia.

These results suggest that the model-implied relationship between the probability of

[25] I follow the parameterization outlined above for V, α, and $f(z)$. Volatility and debt outstanding are both allowed to vary — they are the reasons firms have differential credit spreads and default probabilities.

[26] I consider only speculative grade firms because investment grade firms default incredibly rarely within one year, and when they do it is almost always during a recession. Speculative grade firms, on the other hand, have sufficient observed short-term default rates outside of recessions to allow for meaningful comparison across rating classes.

default and credit spreads would be steeper if we were to perfectly account for cross-sectional patterns in risk premia. This will reduce model-implied estimates of c. For the sake of tractability, I perform this correction using the data. I compute the slope of the risk premium with respect to default probability using estimates by rating class of excess bond returns from Choi and Kim (2015) and historical default probabilities from Standard and Poor's. I find that the slope of the annualized risk premium with respect to the annual probability of default is 0.14.[27] This can be interpreted as the following: C/CCC rated firms, with an annual default rate of roughly $20-25\%$, should have annual risk premia of around 3% greater than firms with no probability of default. Figure 5 demonstrates the relationship between default probability and credit spreads less risk premia. The range of estimates discussed earlier, $c \in [.084, .3]$, still bounds the relationship in the data from below and above. With this correction, the lower end of this range corresponds better to the data, whereas without it the upper end of this range matches the data better.

2.3.2 Innovations to Asset Volatility, Credit Spreads, and Employment

The model also has implications for how innovations to the level of firm fundamental idiosyncratic risk (the volatility of z) affects asset volatility, credit spreads, and firm's employment decisions. In the model, an increase in the volatility of z increases the volatility of the value of the firm's assets and the probability of default. This raises credit spreads and decreases firm employment decisions.[28] Therefore the model has two clear predictions: First, innovations in measures of firm volatility should co-move with credit spreads on the firm's debt. Second, innovations in measures of firm volatility should predict (negative) innovations in firm employment.

Figure 6 shows the implied credit spreads and labor choice generated in the simple model, conditional on the amount of debt due, for varying levels of volatility of z. The simple model is parameterized as before. Note that credit spreads are increasing in volatility, but are higher if the debt due is greater. The firm's labor choice is decreasing in volatility, and depends on the indebtedness of the firm. Figure 2 shows the implied impact of volatility doubling on firm employment in the simple model for varying values of c. It is clear that higher c leads to a greater decline in firm employment in response to volatility shocks. Note that this effect should exist in both the cross-section and over the business cycle.

[27] I compute the slope using default probabilities and excess bonds returns for AAA and C/CCC rated firms.

[28] In (4) and (7), this manifests itself by raising $f\left(\overline{z}\left(l^{*}, b\right)\right)$ and $F\left(\overline{z}\left(l^{*}, b\right)\right)$.

Implications for Financial Market Measures of Volatility As the volatility of firm fundamentals is difficult to measure at the firm level (due to the low frequency of observations), it is common to use financial market measures of volatility, particularly measures of realized or expected equity volatility.[29] The draw of z, reflecting stochastic firm fundamentals or demand, affects the market value of claims on the firm. Therefore, the volatility of z should be reflected in the volatility of equity returns. I can easily compute the model-implied equity volatility in this two period case introduced above, or a measure of instantaneous volatility implied by a Merton model related to this two-period case. Both expressions are functions of not only the volatility of z, but also capital structure considerations. If a firm is highly leveraged, a small volatility of z can imply larger volatility of equity returns because small movements in z can have large percent changes in the value of assets less liabilities. The presence of default itself also affects the volatility of equity returns. Appendix A outlines the computation of equity volatility in this simple model.

Relationship Between Equity Volatility, Credit Spreads, Employment Figure 7 shows the relationship between equity volatility, credit spreads, and employment in the simple model.[30] This is quite similar to the relationship with theoretical volatility — the model predicts that equity volatility will co-vary with credit spreads and predict employment losses.

This is a key qualitative prediction of the model I test in the data. As the relationship between equity volatility and the volatility of z relies crucially on assumptions about the variance of the continuation value and its correlation with z, I do not use the magnitude of this relationship to calibrate the model.[31] Instead, I rely on the relationship between the probability of default and credit spreads, as well as the relationship between innovations to credit spreads and innovations to year-ahead employment.

3 Empirical Evidence

In this section, I describe the dataset and the measures of volatility I derive from asset prices, and document key aspects of the cross-sectional relationship between volatility,

[29] For instance, see Gilchrist et al. (2014).

[30] The relationship here is plotted for equity volatility from a Merton model, however, the relationship is similar if I use the full model or measures of the volatility of the value of the firm as a whole.

[31] There is likely significant variation in equity returns driven by factors other than near-term shocks. This could be addressed in the model by making V stochastic (which it is in the full model), but its correlation with z would affect how much equity volatility responds do changes in the volatility of z and thus these cross-sectional relationships.

credit spreads, and firm labor decisions. The main findings of this section are the following: Cross-sectional evidence suggests that an increase in measures of firm idiosyncratic risk are significantly correlated with changes in credit spreads, and are predictive of changes in firm employment. This is consistent with the main qualitative predictions of the model for the cross-section. I also document how predictive innovations in credit spreads are of employment, as this relationship can be compared to the (full) model.

3.1 Data and Measurement

I use data on public firm equity prices, financial statements, analyst sales forecasts, and credit spreads from 1984-2013. Equity price data is drawn from the Center for Research in Security Prices (CRSP), accounting data from Compustat, and data on credit spreads from the Lehman-Warga (1984-1998) and Merill Lynch (1997-2013) databases.

3.1.1 Measuring Volatility

A key empirical challenge is constructing a panel dataset of measures of the time-varying idiosyncratic risk at the firm-level. The primary measure I consider is a measure of idiosyncratic equity volatility following Gilchrist, Sim, and Zakrajsek (2014), though I do consider alternative measures for robustness.

Measuring Volatility from Asset Returns Asset prices, in principle, should summarize all of the publicly available information relevant for a firm's future prospects. This corresponds closely to theory — a stochastic volatility shock is an innovation in the distribution of non-forecastable shocks affecting future firm cash flows. I thus follow the literature and use a measure of the idiosyncratic component of daily return volatility as one of my measures of volatility.

I use a standard linear factor model to remove the portion of excess returns attributable to aggregate factors:

$$R_{it_d} - r^f_{t_d} = \alpha_i + \beta_i' f_{it_d} + u_{it_d}. \tag{8}$$

$R_{it_d} - r^f_{t_d}$ is the daily excess equity return on stock i in trading day d of year t. f_{it_d} corresponds to a vector of aggregate factors — in this case a four-factor model consisting of the Fama and French (1992) factors along with a momentum measure. I use u_{it_d} to denote the variation in excess returns beyond the four aggregate factors, for reasons one can consider idiosyncratic. The residual of running an OLS regression on (8), $\hat{u}_{it_d}$, is the idiosyncratic component of excess returns, and greater variation in this corresponds to higher levels of idiosyncratic volatility. The standard deviation of this component can thus be computed over an annual frequency as:

$$\sigma_{it}^{EI} = \sqrt{\frac{250}{D_t}\sum_{d=1}^{D_t}\left(\hat{u}_{it_d} - \bar{u}_{it}\right)^2}. \tag{9}$$

$\bar{u}_{it}$ denotes the average of $\hat{u}_{it_d}$ over the year, and D_t the number of trading days. Further details of this measure can be found in Gilchrist, Sim, and Zakrajsek (2014).

As measures of equity volatility are affected by leverage, I also compute a measure of asset volatility from returns. I follow a procedure consistent with Bharath and Shumway (2008) and Gilchrist and Zakrajsek (2012) in measuring firm asset volatility, and further derive a measure of idiosyncratic asset volatility, detailed in Appendix B.

Measure of Volatility from Sales Growth To allay concerns about variation in financial market returns representing factors other than changes in firm fundamentals or expectations of them, I construct measures of sales growth volatility over controls as a measure of fundamental volatility. This data is available at no more than a quarterly frequency, so I use a GARCH process to estimate the volatility of this process. I consider only firms with at least 30 quarters of consecutive sales data. I first remove all forecastable variation in firm growth rates by running a regression of firm sales growth on lagged firm fundamentals (investment rate, size, profitability) with time-industry and firm-season fixed effects.[32] The residuals of this regression represent the variation in firm sales growth beyond those explained by predictive factors, and also help eliminate any seasonal trends, including those which exist only for given industries or firms. I then run a GARCH(1,1) procedure to estimate a volatility series from the time series of residuals for each firm. I test each firm's residual series individually for ARCH effects, and keep only those which reject the null (support the presence of ARCH effects). This results in a panel dataset of quarterly firm-level volatility.

3.1.2 Other Firm Measures

Firm measures of leverage, profitability, and Tobin's Q are all computed from Compustat annual data. I define the book value of debt as the sum of short term debt outstanding and $\frac{1}{2}$ of long term debt outstanding (following Gilchrist and Zakrajsek (2012)). Leverage

[32]By time-industry fixed effects I mean a dummy for every 1 digit sic code for every year-quarter. By firm-season fixed effects I mean a fixed effects factor for every firm in a given season/quarter (i.e. Microsoft quarter 1).

is computed as the ratio of the book value of debt and the sum of the value of equity and the book value of debt. Profitability is defined as the ratio of operating profits to the book value of assets (from the balance sheet). Tobin's Q is defined as the ratio of the sum of the value of equity and the book value of debt over the book value of assets (from the balance sheet). I compute credit spreads from the databases following the procedure outlined in Gilchrist et al. (2014). I take employment from Compustat as well, and allay concerns about measurement error by replicating my results with sales.

3.2 Empirical Results

3.2.1 Cross-section of Innovations to Volatility

There is a large dispersion of innovations (year-over-year percent changes) to firm-level measures of volatility, even during non-recession years. The magnitude of this dispersion is significant compared to the size of the average innovation in the equivalent measures of volatility during the great recession of 2007-2009. Figures 8 and 9 show histograms of the frequency of innovations to measures of volatility, measured as year-over-year percent changes, during non-recession years. Overlaid on the histogram is the average year-over-year percent change that occurs during the rise in volatility during the great recession.[33] The dispersion in innovations to my measures of volatility outside of recessions is substantial. In terms of the volatility of assets, the mean increase in volatility in 2007/2008 represents approximately two standard deviations of the non-recession distribution. Thus, while the mean innovation to volatility during the crisis is clearly large, the cross-sectional variation during non-recession years is comparably large enough that cross-sectional relationships are empirically relevant for understanding the effect of volatility shocks on firm decisions.

3.2.2 Relationship to Credit Spreads

I now look at the response of credit spreads to innovations to financial market measures of volatility. If greater volatility in asset prices reflects increases in volatility about future firm prospects, it will increase the probability of default and raise credit spreads.

In the data, credit spreads increase significantly in response to firm level innovations in volatility, suggesting that the financial market measure of volatility reflects economically significant changes in volatility. Figure 10 shows this relationship of year-over-year inno-

[33]The change from 2007-2008 is used because it is by far the greatest average annual increase in volatility that occurs in the time series.

vations of credit spreads against idiosyncratic equity volatility, for non-recession years.[34] The observations are binned by the x-axis since there are thousands of observations. Figure 11 shows the entire scatter in a heatmap and a line of best fit confirming the positive correlation. Table 1 shows that this positive relationship is robust to controlling for firm characteristics which are known determinants of credit spreads, specifically innovations in leverage, profitability, and equity values. This is consistent with the findings of Gilchrist, Sim, and Zakrajsek (2013), who find that an increase in idiosyncratic volatility is associated with a significant widening of credit spreads. They also find that increases in idiosyncratic volatility also has statistically significant effects on capital expenditures. I therefore assess the impact of idiosyncratic volatility on employment, my corresponding variable of interest.

3.2.3 Relationship with Firm Employment

I evaluate the magnitude of a decrease in firm employment predicted by increases in the level of idiosyncratic volatility facing the firm. I find that the relationship is negative and robust, though the magnitude of the effect is modest. Figure 12 shows the relationship between the year-ahead year-over-year percent change in firm employment against year-over-year change in idiosyncratic equity volatility, for non-recession years. Observations are binned by idiosyncratic equity volatility as there are thousands of observations. Figure 13 shows the entire scatter in a heatmap, which confirms the trend. This relationship remains negative and significant (though the magnitude is relatively modest, with a doubling in volatility implying a 2-3% decrease in employment), even after controlling for innovations in firm characteristics, such as profitability, leverage, and Tobin's Q, or year/industry fixed effects. Table 2 shows the results of the regression:

$$\Delta\, emp_t = \beta_0 + \beta_1 \Delta \sigma^{AI}_{it-1} + \gamma\, X_{i,t-1} \tag{10}$$

where $X_{i,t-1}$ represents the controls mentioned above. This illustrates the robustness of the magnitude and the sign of the decrease in employment associated with an increase in firm idiosyncratic risk. To alleviate concerns about measurement error in Compustat employment, Figure 14 shows that the relationship between innovations to equity volatility and year-ahead sales growth is quite similar in both direction and magnitude.

[34] The relationship between idiosyncratic asset volatility and credit spreads is very similar.

3.2.4 Relation to Model

The relationships documented above agree with the qualitative predictions of the model, but comparing them quantitatively is difficult. The true volatility of firm processes are not observed in the data, and the financial market proxies from realized returns are difficult to convincingly characterize in the model.[35] Therefore, I turn to a relationship which measures the extent to which default risk affects employment: how predictive innovations in credit spreads are to changes in employment. Figure 15 documents this relationship for non-recession years, binning firms by credit spreads; Figure 16 displays the entire scatter. There is a significant and robust (to standard controls) negative relationship between innovations to credit spreads and employment.[36] This is consistent with the work of Benmelech, Bergman, and Seru (2015), who document that financial constraints play a role in reducing firm (and aggregate) employment.

4 Model

To evaluate the role of volatility shocks in driving aggregate dynamics, I use a dynamic model with financial frictions and variation in the level of volatility at the firm level. The model is a generalized version of ABK.[37]

My model differs from ABK in two primary ways. First, I allow for the extent of default costs to vary, controlled by a parameter c. Second, I add heterogeneity in both the level and changes in firm idiosyncratic volatility. Specifically, I let innovations to the volatility of firm specific demand z, vary across firms. This additional heterogeneity allows me to generate cross-sectional implications which can reproduce patterns in the data and be used for calibration.

The model has a continuum of final good firms, intermediate good firms, financial intermediaries, and households. Intermediate good firms produce differentiated goods using labor which are aggregated into the final good. Final good firms are competitive and produce the final good from intermediate goods. The production technology of final good firms is subject to idiosyncratic shocks augmenting the usage of individual intermediate goods. These shocks need not be technological: as in ABK they can be interpreted

[35] For instance, the variance of realized equity returns may be affected by liquidity considerations or other financial market changes not captured in the model.

[36] Figure 17 documents that this relationship is similar when sales growth is used instead of employment growth.

[37] ABK have, as of October 2015, presented an updated version of their paper, which include some changes to the model. The now use a different agency problem to motivate firms to hold debt and consider the risk-free interest rate to be exogenous. However, the mechanism through which volatility shocks affect employment and the main business cycle results are qualitatively unchanged.

as idiosyncratic *demand* shocks. The volatility of these demand shocks is stochastic. Competitive financial intermediaries lend to the intermediate good firms using standard defaultable debt contracts. Households own all firms and financial intermediaries and pay lump-sum taxes. They make decisions over consumption and leisure, which they both value.

The timing of the model is as follows. First, households make their labor supply decision and firms make their labor demand decisions, and the wage rate is set to clear the labor market. Then, shocks to demand and volatility are realized. Then intermediate good firms produce with the labor choice they made the previous period and the demand shock they received and sell the goods to final good firms, pay workers, choose whether to repay the financial intermediaries, and choose labor for the next period as well as issue additional debt. The final good firms buy the intermediate goods and aggregate them into the final good, which is either consumed or used by entering firms to pay the cost of entry. Potential entrants decide whether to pay the entry cost (in units of the final good) and enter. Households receive all of the net cash flows from intermediaries and firms, and consume the final good.

4.1 Physical Environment

Intermediate good firms produce differentiated goods with technology $y_{it} = l_{it}^{\alpha}$, where l_{it} indexes the labor stock of firm i at time t. The price that they receive for these goods, as well as how important these goods are for the final good, depend on an idiosyncratic demand shock z_{it}. z_{it} follows a Markov process with transition function $\pi_z\left(z_{it}|z_{t-1,i},\sigma_{it}\right)$ and depends on the lagged value of z_{it} as well as the level of stochastic volatility the firm faces, σ_{it}. I do not assume σ_{it} must be equal for all firms — it is a function of an idiosyncratic component, σ_{it}^{id}, and an aggregate component, σ_t^{ag}. Both of these follow Markov processes, with transition functions $\pi_{\sigma^{id}}\left(\sigma_{it}^{id}|\sigma_{t-1,i}^{id}\right)$ and $\pi_{\sigma^{ag}}\left(\sigma_t^{ag}|\sigma_{t-1}^{ag}\right)$.

I denote the idiosyncratic state of a firm as $x_{it} = \left\{z_{it}, l_{it}, b_{it}, \sigma_{it}^{id}\right\}$, where l_{it} is the employment of the firm and b_{it} its borrowing, and the aggregate state of a firm as $S_t = \{\sigma_t^{ag}, S_{bt}\}$, where S_{bt} is the beginning-of-period aggregate state prior to the realization of shocks. The beginning-of-period aggregate state, $S_{bt} = \left\{\sigma_{t-1}^{ag}, \Upsilon_t, B_t\right\}$, is summarized by the aggregate level of volatility before the shock, σ_{t-1}^{ag} , the distribution of firms $\Upsilon_t(x_t)$, and the household's wealth, B_t.

Final good firms buy the intermediate goods and produce the final good with technology:

$$Y_t = \left(\int z(x_{it}) y(x_{it})^{\frac{\gamma-1}{\gamma}} d\Upsilon_t(x_{it}) \right)^{\frac{\gamma}{\gamma-1}} \tag{11}$$

where $z(x_{it})$ and $y_t(x_{it})$ denote the demand shock and intermediate good production of a firm with state x_{it}.

4.2 Final Good Firm's Problem

The final good is used for consumption or to pay the entry cost. Competitive final good firms choose their purchase of intermediate goods to maximize profits, $Y_t - \int \times p_t(x_{it}) y_t(x_{it}) d\Upsilon_t(x_{it})$, subject to (11). This yields the relative price of the intermediate firm good x_{it} relative to the aggregate price index:

$$p_t(x_{it}) = z(x_{it}) \left(\frac{Y_t}{y_t(x_{it})} \right)^{\frac{1}{\gamma}}. \tag{12}$$

4.3 Intermediate Good Firm's Problem

Intermediate good firms make labor, debt issuance, dividend, and entry decisions. They are restricted from paying non-negative dividends, and if they would be compelled to do so they default and any cash flows or remaining firm value goes to the financial intermediaries. It is useful to separately consider the problems of incumbents and entrants.

4.3.1 Incumbent Firms

Incumbents enter the period with a stock of labor and debt, l_{it} and b_{it}, and receive their new demand shock and idiosyncratic component of firm-level volatility, z_{it} and σ_{it}^{id}. They choose new amounts of labor and debt issuance, l'_{it} and b'_{it}, as well as dividends d_{it}. The dividend for an incumbent firm can be written as:

$$d_t = p_t(x_{it}) l_{it}^{\alpha} - W(S_{bt}) l_{it} - b_{it} + b_{t+1,i} Q\left(z_{it}, \sigma_{it}^{id}, l_{t+1,i}, b_{t+1,i}, S_t\right) \tag{13}$$

where $W(S_{bt})$ is the wage. The cash flow from debt issuance depends on the firm idiosyncratic shocks, z_{it}, σ_{it}^{id}, as well as firm decisions, $l_{t+1,i}, b_{t+1,i}$, and the aggregate state, S_t. The price of state-uncontingent debt is $Q\left(z_{it}, \sigma_{it}^{id}, l_{t+1,i}, b_{t+1,i}, S_t\right)$.

Firms seek to maximize the discounted present value of dividends, modified by a Jensen effect parameter κ. This is motivated by ABK, referring to Jensen (1986), as the following: Managers have incentives to spend built up cash by the firm in ways that benefit themselves at the expense of shareholders, and thus shareholders change management incentives to pay out dividends rather than retain them. The value function of intermediate good firms can thus be expressed as the following:

$$V(x_{it}, S_t) = \underset{l_{t+1,i}, b_{t+1,i}}{max} \begin{cases} \kappa d_t + (1-\kappa)\, E_t\left[P_{S_t,S_{t+1}} V(x_{t+1,i}, S_{t+1}) | x_{it}, S_t, l_{t+1,i}, b_{t+1,i}\right] & d_t \geq 0 \\ 0 & d_t < 0 \end{cases}$$

such that (13) holds. Here, $P_{S_t,S_{t+1}}$ denotes the stochastic discount factor implied by the consumer's problem. If firms cannot pay a non-negative dividend, they default and are seized by creditors, and the equity holders walk away with no cash.

4.3.2 Entrants

Firms can enter by paying an entry cost ξ in terms of the final good. This cost is paid by household in exchange for equity (this is the only equity issuance allowed in the model), and they enter with no debt and idiosyncratic states x_e and σ_e^{id}. Thus the free entry equation can be expressed, modified by the Jensen effect parameter, as the following:

$$0 = \underset{l_{t+1,e}}{max} - \kappa\xi + (1-\kappa)\, E_t\left[P_{S_t,S_{t+1}} V(x_{t+1,i}, S_{t+1}) | S_t, l_{t+1,e}\right].$$

4.3.3 Distribution of firms

The distribution of firms, $\Upsilon_t(x)$, depends on firm leverage, employment, default, and entry decisions. The law of motion for it can be characterized as the sum of several components:

First, incumbent firms which do not default:

$$\Upsilon_t^{ND}(z', l', b', \sigma'^{id}) = \left(1 - brup(z', l', b', \sigma^{id\prime}, S_t)\right) \sum_x P\left(z', \sigma'^{id} | x, S_{t-1}\right) \mathbb{1}_{\{l'=l^*(x,S_{t-1}), b'=b^*(x,S_{t-1})\}} \Upsilon_{t-1}(x),$$

where $brup(z', l', b', \sigma^{id\prime}, S_t)$ is a dummy variable which is equal to 1 if a firm defaults conditional on realized shocks $z', \sigma^{id\prime}$, choices l', b' and aggregate state S_t.

Second, incumbent firms which do default (but survive default with probability $1-c$):

$$\Upsilon_t^{D}(z', l', 0, \sigma'^{id}) = (1-c)\left(brup(z', l', b', \sigma^{id\prime}, S_t)\right) \sum_x P\left(z', \sigma'^{id} | x, S_{t-1}\right) \mathbb{1}_{\{l'=l^*(x,S_{t-1}), b'=b^*(x,S_{t-1})\}} \Upsilon_{t-1}(x).$$

Finally, entering firms:

$$\Upsilon_t^e(z', l', 0, {\sigma'}^{id}) = e_t P\left(z', {\sigma'}^{id} | x_e, S_{t-1}\right) \mathbb{1}_{\{l'=l_e^*(S_t)\}}.$$

Using these components, the law of motion for the distribution of firms can be written as the following:

$$\Upsilon_t(x) = \Upsilon_t^{ND}(x) + \Upsilon_t^{D}(x) + \Upsilon_t^{e}(x)$$

4.4 Financial Intermediation

Financial intermediaries lend non-contingent debt to the firms and borrow state-contingent debt from the households. They are perfectly competitive and thus the price of outstanding debt $Q\left(z_{it}, \sigma_{it}^{id}, l_{t+1,i}, b_{t+1,i}, S_t\right)$ is pinned down by the following zero profit condition:

$$\begin{aligned} Q\left(z_{it}, \sigma_{it}^{id}, l_{t+1,i}, b_{t+1,i}, S_t\right) = \; & E\Bigg[P_{S_t,S_{t+1}}\Bigg(1 - brup\left(x_{t+1,i}, S_{t+1}\right) \\ + \quad & (1-\kappa)\left(brup\left(x_{t+1,i}, S_{t+1}\right)\right) rr_{t+1}\left(x_{t+1}, S_{t+1}, b_{t+1,i}\right)\Bigg) \,|\, z_{it}, \sigma_{it}^{id}, l_{t+1,i}, b_{t+1,i}, S_t\Bigg]. \end{aligned}$$

I assume here that if firms default, the resulting cash flows to debtholders are discounted at the discount rate of equity holders, $(1-\kappa)\beta$. This is micro-founded by specifying that the remaining value of the firm is contracted to be sold to equity holders (or the managers of these firms) in case of default. If I did not make this assumption, then firms would have a perverse incentive to default, as cash flows conditional on defaulting would be discounted at a lower rate than cash flows conditional on surviving.

The recovery rate on debt in the event of default is denoted $rr_{t+1}\left(x_{t+1}, S_{t+1}, b_{t+1,i}\right)$ and is defined as follows:

$$rr_{t+1}\left(x_{t+1}, S_{t+1}, b_{t+1,i}\right) = max\left\{\frac{p_{t+1}(x_{t+1,i}) l_{t+1,i}^{\alpha} - W(S_{bt+1}) l_{t+1,i} + (1-c) V_{it}^{C}\left(x_{t+1,i}, S_t\right)}{b_{t+1,i}}, 0\right\},$$

where $V_{it}^{C}\left(x_{t+1,i}, S_t\right)$ is the continuation value of the unlevered firm after being seized by debtholders, and c is the probability that the firm is destroyed in default. The continuation value of the unlevered firm is expressed as the following:

$$V_{it}^{C}\left(x_{t+1,i}, S_t\right) = \underset{l_{t+2,i}}{max} E\left[P_{S_{t+1}S_{t+2}} V\left(x_{t+2,i}, S_{t+2}\right) | x_{t+1,i}, S_{t+1}, l_{t+2,i}, b_{t+2,i} = 0\right].$$

4.5 Households

Households have preferences over consumption of the final good and leisure of the form $U(C_t, L_t)$. They own (and receive) all dividends paid by firms. They also buy state-contingent securities from the financial intermediaries, effectively lending them cash they then extend as credit to intermediate good firms. They discount the future with discount factor β. The problem of households can thus be written as the following maximization problem:

$$V^H(S_{bt}) = \underset{L_t}{max} E\left[\underset{C_t, B_t(S_{t+1})}{max} U(C_t, L_t) + \beta V^H(S_{bt+1})\right]$$

such that the budget constraint of the household is satisfied:

$$C_t + \sum_{S_{t+1}} B_t(S_{t+1}) P(S_{t+1}|S_{b,t}) = W_t L_t + B_{t-1}(S_t) + \sum d_{it} - \xi M_t(S_t),$$

where M_t is the mass of entry.

4.6 Equilibrium

The equilibrium in this model is a sequence of allocations $\{C(S_t), L(S_{bt}), Y(S_{bt})\}$, distribution of firms Υ_t, firm decision functions $\{l(x_t, S_{bt}), b(x_t, S_{bt}), brup(x_t, S_{bt})\}$ and prices $\{W(S_{bt}), Q(l', b', x_t, S_{bt}), P(S_{bt})\}$ such that the intermediate good firm's problem is satisfied, the household's problem is satisfied, financial intermediaries earn zero economic profit, final good firms minimize costs, the labor and final good markets clear, the law of motion for the distribution of firms is satisfied, and the free entry condition holds.

5 Quantitative Results

This section outlines the parameterization of the model and discusses the solution method. It then documents the cross-sectional implications of the parameterized model and the business cycle implications of a volatility shock.

5.1 Parameterization

I parameterize the utility function of be of the form $U\left(C,L\right)=\frac{C^{1-\varsigma}}{1-\varsigma}-\frac{L^{1+\nu}}{1+\nu}$, following ABK. I parameterize the process for innovations in z to be the following: $\log(\frac{z_{t+1}}{z_t})=\mu_z+\sigma_{it}\epsilon_{zit}$, where $\epsilon_{zit}\sim N(0,1)$ and $\mu_z=-\frac{\sigma_{it}^2}{2}$. Therefore, $E[z]$ does not change in σ_{it}. The process for $log(\sigma_{it}^{id})$ is set to be an AR(1) process with mean, μ_σ, variance, ϕ_σ, and persistence, ρ_σ. The firm parameters which are chosen are $c,\kappa,\beta,\mu_{\sigma^{id}},\rho_{\sigma^{id}},\phi_{\sigma^{id}}$. I allow for a range of parameterizations for c, allowing for both $c=1$ (default very costly) as in ABK, as well as setting c to the range of values found in the corporate finance literature ($c\in(.084,0.3)$). The remaining parameters are calibrated in the $c=0.3$ case, which is on the high end of estimates from the corporate finance literature. Table 3 summarizes the parameterization of the remaining parameters in the model. κ is chosen to hit the degree of financial leverage help by non-financial firms in Compustat. I compute the sum of a measure of total liabilities and the sum of market capitalization for all firms for which I have both data series and I compute the ratio of the sum of liabilities to the sum of market capitalizations across firms. μ_σ, ϕ_σ, and ρ_σ are chosen to hit the behavior of firm idiosyncratic asset volatility, including the mean, persistence, volatility, and how much of the time series variation is represented by the common component of idiosyncratic equity volatility.[38] I parameterize aggregate shocks to firm volatility to match dispersion in firm sales growth over the business cycle.

5.1.1 Measuring Equity Volatility in the Model

The parameter σ_{it} measures the amount of idiosyncratic volatility of the firm-specific fundamental shock z. However, it does not have a corresponding observable in the data. Thus, I compute a measure of idiosyncratic asset volatility to correspond to the measure I compute from the data. To this end, I compute the expected discounted present value of all cash flows from the firm (both those going to debt and equity holders) for a firm with a given set of state variables. Given the transition probabilities for firm fundamental z and for firm volatility σ_{it}^{id}, I compute the theoretical expected return for the value of firm's assets as well as the standard deviation of returns for the firm's assets. This volatility differs from the measure in the data in that it does not have measurement error and only accounts for changes in business risk or leverage, and not for other variation in asset prices (due to liquidity or other financial market factors). I proxy for these factors by adding noise to the measurement of variance. The process for σ_{it} and for the noise affecting

[38] I find that a common component is responsible for about 25% of within-firm variation in idiosyncratic equity volatility.

observation of asset volatility is chosen to hit the cross-sectional variation in percent changes in sales growth, the cross-sectional variation in idiosyncratic asset returns, and the persistence and volatility of idiosyncratic firm asset volatility.

5.2 Solution Method

The solution method and parameterization I use differs from that used in ABK in a few ways. First, I use very fine grid of productivity shocks, as the quantitative implications of structural models of default can be very sensitive to the grid points. My parameterization for z, which assumes a random walk instead of an AR1, allows me to solve the model for a very fine grid, as I can represent all firm decisions and state variables relative to firm productivity z.[39] The cross-sectional implications of my model are generated from the solved steady state. Business cycle implications are generated in the following way: The economy is first assumed to be in steady state, and then a sequence of aggregate shocks are fed in, including volatility shocks calibrated to the micro data for the recent recession. Then firm decision rules are computed via backward value iteration, from which the resulting path of aggregate employment is recovered.

5.3 Cross-Sectional Relationships

To analyze the cross-sectional implications of the model, I compute the steady state of the model with idiosyncratic shocks to firm fundamental volatility z. Figure 18 shows that the relationship between default rates and credit spreads are entirely inconsistent for the first parameterization ($c = 1$), but generate reasonable results for $c \in [0.084, 0.3]$ (I correct for risk premia in the data using the procedure outlined in section 2.3.1). This confirms the results from the simple model. However, it is possible that while default destroys less than 30% of firm value, firms may make decisions as if default were much more costly. For instance, if managers face unemployment in case of default, they have an additional incentive to avoid default. To address this, I consider the relationship between innovations to credit spreads and innovations to year-ahead employment. In the context of the model, this is a measure of how the risk of default (as reflected by credit spreads) affects firm employment choices. If firms made employment decisions as if default were more costly due to higher default costs borne by managers, this relationship should reflect the distortion caused by managerial incentives and correspond to a higher cost of default. Figure 19 shows the scatter plot of this relationship in the model in the $c = 0.3$ case.

[39] Additionally, the literature cited by ABK indicate the demand shocks which are represented by z are quite persistent.

There is a negative relationship between innovations to credit spreads and employment, but it is not perfectly correlated as firms vary in their productivity, volatility, and leverage, and receive shocks to both productivity and volatility. Figure 20 shows the lines of best fit summarizing this relationship in the data and in the model, for a range of values of c. Once again, the relationship in the data lies somewhere between the slopes implied by $c = .084$ and $c = 0.3$. This suggests that firm employment decisions, not just debt pricing, in my model is consistent with firm level data for $c \in [0.084, 0.3]$. If the cost of default were instead parameterized to be $c = 1$, the relationship between these two variables would be much stronger in the model than in firm-level data.

5.4 Business Cycle Implications

As demonstrated in the simple model, the parameterization of c has large effects on the impact of volatility shocks on employment. Figure 21 illustrates how the aggregate employment of firms responds to the sequence of volatility shocks implied by the dispersion in firm sales growth seen during the recent 2007-2009 recession. If one follows ABK and assumes default leads to the loss of all firm value ($c = 1$), the model generates large declines in output ($> 6\%$), explaining a great deal of the decline in employment per capita over the recession. However, if c is set to the upper bound of estimates from the corporate finance literature, the decline is much more modest, representing at most about a 2.5% decrease. Further reducing c lowers the impact of volatility shocks even more. In the lower bound calibration ($c = .084$), employment only declines slightly before over-correcting. In comparison to these plots, employment per capita fell by around 10%, and the employment of Compustat firms (who did not enter/exit during the recession) fell by just over 5%. Therefore, after calibrating c to match cross-sectional patterns, volatility shocks alone are not sufficient to explain the decline in employment.

6 Other Shocks

However, there are other changes which occurred over the business cycle which may affect the decline in employment. I focus on two which are consistent with firm-level data and have a meaningful effect on firms in my model: First, a decline in the skewness of innovations to firm idiosyncratic shocks z; second, a worsening of the cost of default, c.

6.1 Skewness Shock

Figure 24 shows the distribution of year-over-year sales growth during and before the 2007-2009 recession. It is clear that during the recession, firms grew less on average and that the dispersion in growth rates was greater, but the distribution seems to also have become slightly more asymmetric. This becomes much more apparent when looking at measures of outlier-robust dispersion and skewness following Bloom, Guvenen, and Salgado (2015). Figure 23 plots three lines: the difference between the 90th and 10th percentile of sales growth, the difference between the 50th and 10th percentile, and the difference between the 90th and 50th percentile. It is apparent that there was a marked increase in the dispersion of sales growth during the recent crisis, as the difference between the 90th and 10th percentile of sales growth widened significantly. However, this was essentially all driven by downside dispersion, the difference between the 50th and 10th percentile, rather than upside dispersion. Bloom, Guvenen, and Salgado (2015) document that this marked decline in skewness is a robust result, and that in general the skewness of firm growth rates in compustat is procyclical. In my model with only volatility shocks, the model underestimates the increase in downside dispersion and overestimates the increase in upside dispersion.

I parameterize a negative skewness shock by assuming that ϵ_{zit} follows a binormal distribution (about the median)[40]. The probability density function of this distribution is the following:

$$f(\epsilon) = \begin{cases} \frac{1}{\sigma_1\sqrt{2\pi}} e^{-\frac{1}{2}\frac{(\epsilon - m)^2}{2\sigma_1^2}} & x \leq m \\ \frac{1}{\sigma_2\sqrt{2\pi}} e^{-\frac{1}{2}\frac{(\epsilon - m)^2}{2\sigma_2^2}} & x > m \end{cases}$$

I can thus parameterize the standard deviations for the upper and lower tails separately, and specify them to hit the numbers documented in Bloom, Guvenen, and Salgado (2015) for upside (90th percentile less 50th percentile) and downside (50th percentile less 10th percentile) of the dispersion of sales growth.

As default is typically a left-tail event, shocks to the skewness of the distribution can increase marginal default probabilities and reduce firm employment further. Figure 24 shows the results of an increase in idiosyncratic risk that increased volatility and decreased skewness. This shows that skewness shocks can amplify the decline in employment, but its magnitude is modest (around 1 percentage additional decline in employment).

[40]The binormal distribution has been used to parameterize skewness in financial modeling, see Feunou, Jahan-Parvar, and Tédongap (2014), for example.

6.2 Changing Cost of Default

There is considerable evidence consistent with an increased cost of default during recessions. A number of papers document that recovery rates are procyclical (thus losses to debtholders, conditional on default, are countercyclical) and that there is a negative correlation between the default rate and recovery rates conditional on default.[41] This negative relationship is only partially explained by the state of the aggregate economy; this has been interpreted as consistent with the "fire-sale" effect of Shleifer and Vishny (1992).[42] Figure 25 plots the decline in average recovery rates reported by Moody's during the 2007-2009 recession. Additionally, I argue that a key cross-sectional relationship, the slope of credit spreads vs. the probability of default, changed during the recent crisis in a way consistent with an increased cost of default. As historical default probabilities are highly variable for individual years, I have to use another measure of default risk. I follow Gilchrist and Zakrajsek (2012) and use the probability of default implied by firm micro-data and a Merton "distance-to-default" model.[43] Figure 26 displays this relationship both before and during the great recession. There are two clear changes: first, credit spreads increase substantially given their implied default probability; second, the slope between credit spreads and default probability increases. The level effect, indicating that credit spreads rise more than can be explained by the extent of default risk, is consistent with the finding in Gilchrist and Zakrajsek (2012) that the *excess bond premium*, a measure of credit spreads in excess of what can be explained by default risk, increased substantially in the 2007-2009 recession. The increase in the slope is consistent with a higher cost of default in my model. The slope increases substantially, more than doubling.

As the cost of default is a first-order driver of the incentive for firms to reduce employment in the face of risk, a shock to the cost of default can reduce employment. I parameterize the shock to the bankruptcy cost c to hit the relative changes in recovery rates during the 2007-2009 recession. Figure 27 documents the result of a volatility shock as well as a shock to the level of the default cost.[44] By itself, a shock to the cost of default does relatively little, reducing employment by around 1.5%. However, when fed in together with idiosyncratic volatility shocks, employment declines markedly, generating a roughly 7% decline in employment. The large losses are due to the interaction of the

[41] Altman (2006) and Altman, Brady, Resti, and Sironi (2005) document key facts and provide a detailed overview of the literature. These results are consistent with Moody's data and analysis on recovery rates, see Cantor and Varma (2004) and Moody's (2015).

[42] See Acharya, Bharath, and Srinivasan (2007).

[43] The probability of default is a function of the market value of the firm, its liabilities, and the volatility of equity, all taken from firm micro-data.

[44] The cost of default here is 30% before the shock, and increases to just over 50% of firm value during the crisis.

two shocks, not just higher default costs. If c is parametrized as constant and equal to the maximum value of the shock to the cost of default realized in the great recession, the decline in employment is only about half as much (the line labeled "Volatility shock, high constant cost of default" illustrates this in Figure 27). The intuition here is similar to the simple model: volatility shocks and shocks to the cost of default interact, as together they raise the incentive for firms to reduce employment in order to avoid losses upon default more than they do separately. In addition, both firm risk-taking and the distortion of firm employment from the costless default case *before the shock* depend substantially on the level of c. If c is high prior to the sequence of shocks, firms choose lower employment and take less risk in terms of leverage and operating leverage than if c was low. Therefore the fact that c is low prior to the sequence of shocks helps amplify the magnitude of the decline in employment.

Figure 28 shows the decline in employment implied by the interaction of these shocks for varying mean levels of the cost of default, $c = .084, 0.3$. The shocks are parameterized to hit the *relative* decline in recovery rates; therefore, the maximum level of c parameterized is lower if the cost of default is lower ($c = .084$) before the shock. If I instead fed in the same sequence of shocks to the cost of default, the decline in employment would be even greater if the cost of default was lower prior to the recession. The decline in employment by the interaction of these shocks still depends on the typical magnitude of the cost of default. However, given a cost of default, the decline in employment is much larger in response to the combination of these shocks than to volatility shocks alone.

7 Conclusion

In this paper I investigate the role of idiosyncratic volatility shocks interacting with financial frictions to drive a decline in employment over the business cycle. I argue that the cost of default has a first-order effect on the size of this decline, and use cross-sectional implications of the model to calibrate the cost of default. The assumption of very high default costs leads to counterfactual implications and are in conflict with an established literature in corporate finance. Parameterizing the cost of default to lower levels, consistent with both the cross-sectional implications and the range of estimates from the literature, greatly reduces the impact of this mechanism over the business cycle. I investigate two additional shocks which are consistent with the micro-data for the 2007-2009 recession — a negative idiosyncratic skewness shock and a shock to the cost of default. I find that modeling the risk shock as not only as a volatility shock, but also as a negative skewness shock can amplify employment losses, but not sufficiently to explain the decline in aggregate employment. Shocks to the cost of default, on their own, only have a modest effect on employment. However, when shocks to the cost of default and volatility are fed in together, they can interact to generate a large ($> 7\%$) decline in employment. This suggests that the factors leading to a change in cost of default are key to understanding the effects of higher idiosyncratic volatility during the great recession.

Appendix

A Simple Model

Proof of Proposition 1

The default threshold $\overline{z}\left(l,b\right)$ is defined as follows:

$$\overline{z}\left(l,b\right)l^{\alpha}-wl-b+V=0.$$

This can be rearranged:

$$\overline{z}\left(l,b\right) = wl^{1-\alpha}+bl^{-\alpha}-Vl^{-\alpha}.$$

Taking the derivative with respect to labor yields:

$$\frac{\partial\overline{z}\left(l,b\right)}{\partial l} = \left(1-\alpha\right)wl^{-\alpha}-\alpha\,bl^{-1-\alpha}+\alpha\,Vl^{-1-\alpha}.$$

This can be rearranged to find:

$$\frac{\partial\overline{z}\left(l,b\right)}{\partial l} = l^{-1-\alpha}\left(wl-\alpha\left(wl+b-V\right)\right).$$

It immediately follows that (i) and (ii) are equivalent.

The positivity of parameters V,c, and probability density function f immediately imply that (ii) and (iii) are equivalent.

(4) immediately implies that (iii) and (iv) are equivalent.

Proof of Proposition 2

The normal pdf has the form $f(z)=\frac{1}{\sigma\sqrt{2\pi}}e^{\frac{(z-\mu)^2}{2\sigma^2}}$. Thus the derivative of this with respect to σ is:

$$\frac{\partial\,f(z)}{\partial\sigma}=-\frac{1}{\sigma^2\sqrt{2\pi}}e^{\frac{(z-\mu)^2}{2\sigma^2}}+\frac{\left(z-\mu\right)^2}{\sigma^3}\frac{1}{\sigma\sqrt{2\pi}}e^{\frac{(z-\mu)^2}{2\sigma^2}}.$$

This can be simplified as:

$$\frac{\partial\,f(z)}{\partial\sigma}=\frac{1}{\sigma^4\sqrt{2\pi}}e^{\frac{(z-\mu)^2}{2\sigma^2}}\left(\left(z-\mu\right)^2-\sigma^2\right).$$

Which implies that $\frac{\partial f(z)}{\partial \sigma} > 0$ if $|\mu - z| > \sigma$, and thus (i) immediately follows. (ii) follows from the fact that $|\mu - z| > \sigma_2$ and $\sigma_2 > \sigma_1$ then $f_2(z) = f_1(z) + \int_{\sigma_1}^{\sigma_2} \frac{\partial f(z)}{\partial \sigma} d\sigma > f_1(z)$.

Derivation of Equity Volatility in the Simple Model

Two Period Case Real equity returns can be computed as follows:

$$\frac{V_{E,2} + div}{\beta V_{E,1}},$$

where

$$V_{E,2} = \beta \begin{cases} 0 & z < \overline{z}(l,b) \\ zl^{\alpha} - wl - b + V & z \geq \overline{z}(l,b) \end{cases}$$

is the value of equity after z is realized,

$$V_{E,1} = \beta \left(E[z] l^{\alpha} - wl + V - F(\overline{z}(l,b)) cV \right)$$

is the value of equity before debt issuance, and

$$div = \beta \left(b(1 - F(\overline{z}(l,b))) + \int_0^{\overline{z}(l,b)} (zl^{\alpha} - wl) \, dF(z) + F(\overline{z}(l,b))(1-c)V \right)$$

is the income from debt issuance paid to equity holders. These can be combined to compute the volatility of equity returns in closed form. The expression is omitted here for simplicity, but it is a function of the volatility of z, the amount of leverage, the incidence of default, and the relative size of V to the operating profits. Shocks to the volatility of z will have a first-order effect on the size of equity volatility.

Merton Model The role of the volatility of z on equity volatility can most easily be demonstrated using a Merton model equivalent of this two-period model. The equivalent of "asset value" in the Merton model is $A = zl^{\alpha}$; this means that if z follows geometric Brownian motion, so does A_t. The equivalent of liabilities are $B = wl + b - V$; the continuation value has to be subtracted here because it does not move proportional in z. Default occurs, as in a Merton model, if $A < B$. The standard solution of a Merton model then implies that equity volatility, σ_E, can be expressed as the following:

$$\sigma_E = \sigma_z \frac{z_0 l^\alpha \Phi\left(\frac{log\left(\frac{z(l^\alpha)}{b+wl-V}\right)+\left(r+\frac{\sigma_z^2}{2}\right)(T-t)}{\sigma_z\sqrt{T-t}}\right)}{\left(1-F\left(\overline{z}\left(l,b\right)\right)\right)\left(E\left[z_T|z_T>\overline{z}\left(l,b\right)\right]l^\alpha - wl - b + V\right)}$$

Here it is clear that changes to σ_z will have a large effect on σ_E, though the magnitude of the response also depends on the effect on capital structure considerations.

B Dataset Construction

Data Sources

Daily equity prices are extracted from the Center for Research in Security Prices (CRSP). Data from firm accounting statements, at either the annual or quarterly frequency are taken from Compustat. I combine data on credit spreads from the Lehman-Warga and Merill Lynch databases. The data on credit spreads is limited to the time frame of 1984-2012, thus my empirical analysis is limited to those years.

Computing Asset Volatility

As measures of equity volatility are affected by leverage, I also compute a measure of asset volatility from returns. I follow a procedure consistent with Bharath and Shumway (2008) and Gilchrist and Zakrajsek (2012) in measuring total firm value V_{it}^A and firm asset volatility, σ_{it}^A, whose procedures are all in the spirit of Merton (1974). V_A is the value of assets, V_{it}^B is the value of debt, μ_{it}^A is the mean rate of asset growth, and σ_{it}^A is asset volatility. I recover V_{it}^A and σ_{it}^A from the data closely following the procedure outlined by Gilchrist and Zakrajsek (2012). For each firm, I linearly interpolate the quarterly value of debt from Compustat to a daily frequency. I use daily data on the market value of equity; call this V_{it}^E. I guess a value of asset volatility, $\sigma_{it}^A = \sigma_{it}^E \frac{V_{it}^B}{V_{it}^E + V_{it}^B}$, where the standard deviation of equity is calculated as the square root of the annualized moving average of squared returns for a firm. To recover the value of assets and the volatility of assets, I follow the procedure outlined in Merton (1974).

Given a guess of σ_{it}^A, I then use the equation

$$V_{it}^E(t) = V_{it}^A(t)\Phi(d_1) - e^{-r(T-t)} * V_{it}^B\Phi(d_2),$$

where $d_1 = \frac{log\left(\frac{V_{it}^A}{V_{it}^B}\right)+\left(r+\frac{1}{2}\left(\sigma_{it}^A\right)^2\right)T}{\sigma_{it}^A\sqrt{T}}$ and $d_2 = d_1 - \sigma_{it}^A\sqrt{T}$, to recover the value of assets. I define r to be the one-year Treasury-constant maturity, which is taken from the Federal Reserve's H.15 report. After converging on V_{it}^A for the given σ_{it}^A, I recompute σ_A from the implied V_{it}^A using the same methodology I use to compute σ_{it}^E. I ultimately converge on σ_{it}^A through a slow-updating procedure.

This is a measure of asset volatility, but not the idiosyncratic component. I use the theory of Merton above to derive a measure of idiosyncratic asset volatility from idiosyncratic equity volatility and asset volatility. Assuming that equity prices follow geometric brownian motion and are a function of asset prices and debt, Ito's lemma allows us to express them as the following: [45]

$$\sigma_{it}^A = \sigma_{it}^E \frac{V_{it}^E}{V_{it}^A}\frac{\partial V_{it}^A}{\partial V_{it}^E}. \tag{14}$$

Rearranging (14) and splitting asset and equity volatility into aggregate and idiosyncratic components yields:

$$\left(\sigma_{it}^{AM}\right)^2 + \left(\sigma_{it}^{AI}\right)^2 = \left(\sigma_{it}^{EM}\right)^2\left(\frac{V_{it}^E}{V_{it}^A}\frac{\partial V_{it}^A}{\partial V_{it}^E}\right)^2 + \left(\sigma_{it}^{EI}\right)^2\left(\frac{V_{it}^E}{V_{it}^A}\frac{\partial V_{it}^A}{\partial V_{it}^E}\right)^2,$$

where σ_{it}^{AI} is idiosyncratic asset volatility of firm i and time t, σ_{it}^{AM} is the firm's aggregate (market) asset volatility, and $\left(\sigma_{it}^A\right)^2 = \left(\sigma_{it}^{AM}\right)^2 + \left(\sigma_{it}^{AI}\right)^2$ (and similarly for equity volatility). Assuming that the idiosyncratic components in this equation correspond to each other and plugging in (14) yields idiosyncratic asset volatility as the product of idiosyncratic equity volatility and a de-leveraging factor:

$$\sigma_{it}^{AI} = \sigma_{it}^{EI}\frac{\sigma_{it}^A}{\sigma_{it}^E}. \tag{15}$$

[45] This is equivalent to equation 6 in Gilchrist and Zakrajsek (2012)

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Table 1: Regression of innovations in credit spreads on innovations in volatility

	(1) $\%\Delta cs_t$	(2) $\%\Delta cs_t$	(3) $\%\Delta cs_t$	(4) $\%\Delta cs_t$
$\%\Delta\sigma_{Ai,t}$	0.249***	0.031**		
$\%\Delta\sigma_{sale,t}$			0.187***	-0.023
$\%\Delta lvg_t$		-0.024		-0.072
$\%\Delta\Pi_t$		-0.404***		-0.329***
$\%\Delta V_{E,t}$		-0.369***		-0.375***
Year Effects	No	Yes	No	Yes
Bond Characteristic Controls	No	Yes	No	Yes
Bond Rating Effects	No	Yes	No	Yes
N	15474	14700	6865	6631
r2	0.025	0.438	0.002	0.407

* Data is an unbalanced panel for 1984-2012, excluding NBER recession years. Percent changes are computed as $\frac{x_{t+1}-x_t}{\frac{1}{2}(x_{t+1}+x_t)}$. Bond characteristics considered include the duration, coupon, and the face value of debt. $\sigma_{Ai,t}$ is idiosyncratic asset volatility, $\sigma_{sale,t}$ is a measure of volatility estimated from a GARCH process on firm sales growth, lvg_t is market leverage, Π_t is profitability (operating profits to book value of assets), and $V_{E,t}$ denotes the market capitalization of the firm.

Table 2: Regression of innovations in employment on innovations in volatility

	(1) $\%\Delta l_{t+1}$	(2) $\%\Delta l_{t+1}$	(3) $\%\Delta l_{t+1}$	(4) $\%\Delta l_{t+1}$	(5) $\%\Delta l_{t+1}$	(6) $\%\Delta l_{t+1}$
$\%\Delta\sigma_{Ai,t}$	-0.024***	-0.021***	-0.028***			
$\%\Delta\sigma_{sale,t}$				-0.029	-0.028	-0.015
$\%\Delta\Pi_t$		0.033***	0.005		0.042*	0.007
$\%\Delta Q_{t-1}$		0.023***	0.025***		0.027***	0.029***
$\%\Delta sale_t$		0.173***	0.126***		0.193***	0.151***
$\%\Delta V_{E,t}$			0.087***			0.074***
$\%\Delta lvg_t$			0.026***			0.016
Year Dummies	No	Yes	Yes	No	Yes	Yes
N	37511	30567	29292	9797	8381	8100
r2	0.001	0.057	0.080	0.000	0.067	0.086

* Data is an unbalanced panel for 1984-2012, excluding NBER recession years. Percent changes are computed as $\frac{x_{t+1}-x_t}{\frac{1}{2}(x_{t+1}+x_t)}$. Bond characteristics considered include the duration, coupon, and the face value of debt. $\sigma_{Ai,t}$ is idiosyncratic asset volatility, $\sigma_{sale,t}$ is a measure of volatility estimated from a GARCH process on firm sales growth, lvg_t is market leverage, Π_t is profitability (operating profits to book value of assets),Q_t is Tobin's Q, and $V_{E,t}$ denotes the market capitalization of the firm.

Table 3: Baseline parameterization

Parameter	Meaning	Value	Notes
	Non-firm Parameters:		
ς	CES parameter in utility function	2	Taken from ABK
ν	Controls labor elasticity	0.5	Taken from ABK, implies labor elasticity of 2
γ	Elasticity of substitution	7.7	Taken from ABK, implies 15% markup
	Firm parameters:		
α	Returns to scale in labor	0.7	Taken from ABK
κ	Jensen effect parameter	0.05	Chosen to hit ratio of the sum of total liabilities to the sum of market capitalization
β	Discount rate	0.99	
	Firm-specific portion of idiosyncratic risk:		
$log(\mu_{\sigma^{id}})$	Mean	.09	Chosen to hit IQR of sales growth and proportion of idiosyncratic equity volatility not explained by common component
$\rho_{\sigma^{id}}$	Persistence	.92	From autocorrelation of firm idiosyncratic equity volatility
$\phi_{\sigma^{id}}$	Variance	.12	From volatility of innovations to firm idiosyncratic equity volatility

*All variables are at a quarterly frequency.

Figure 1: Effect of volatility shock on $f\left(\overline{z}\left(l^{*},b\right)\right)$

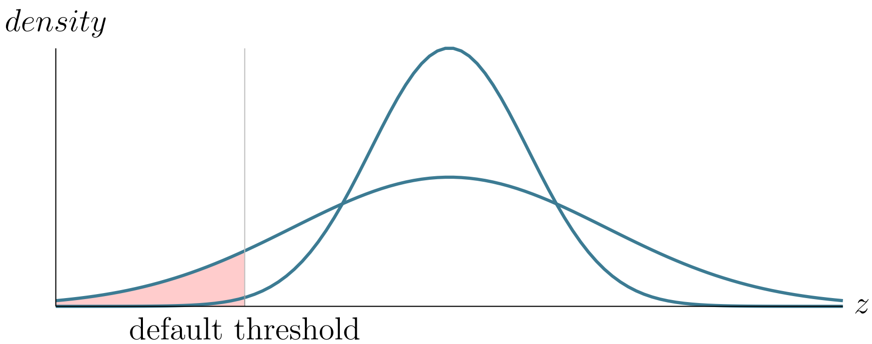
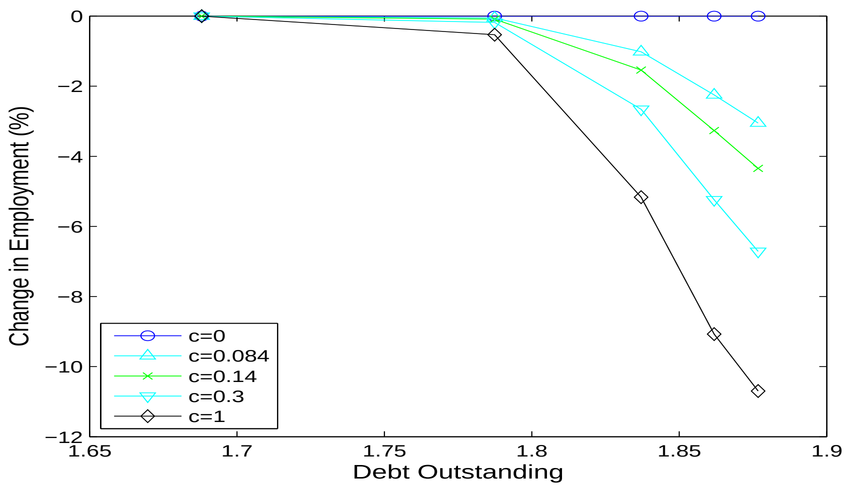
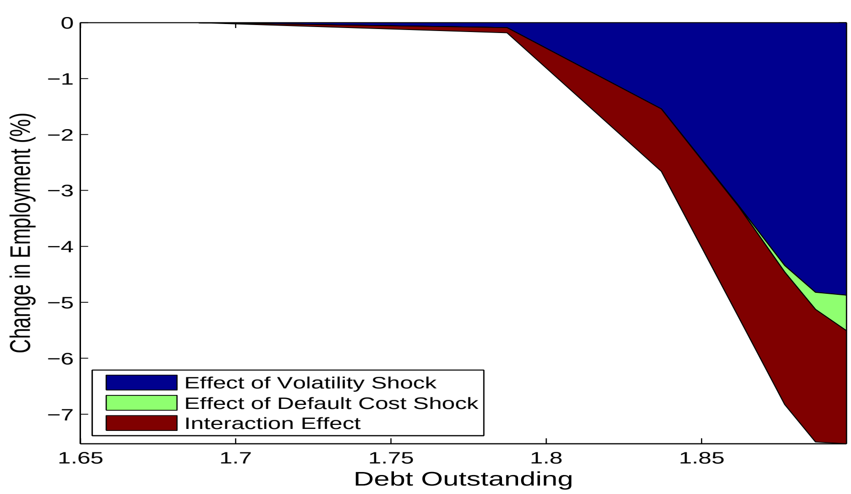


Figure 2: Volatility shock



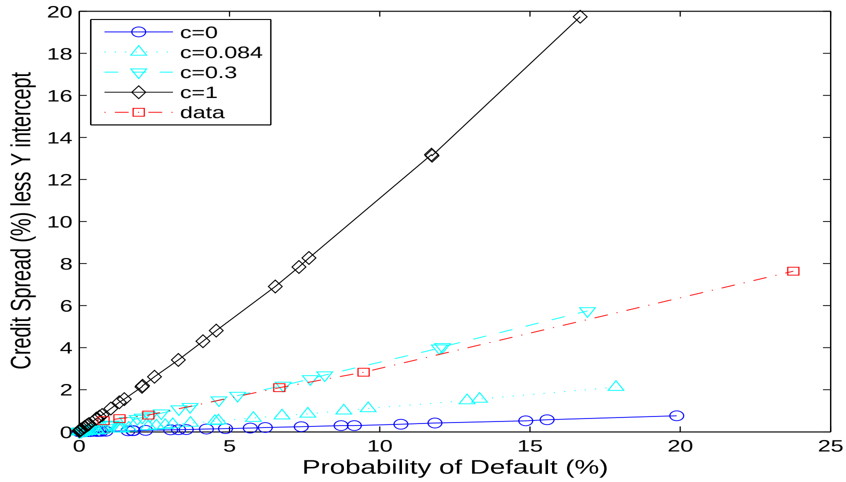
* Lines represent the implied effect of a doubling in the volatility of z on firm employment in the simple model, for a variety of values of c.

Figure 3: Volatility shock



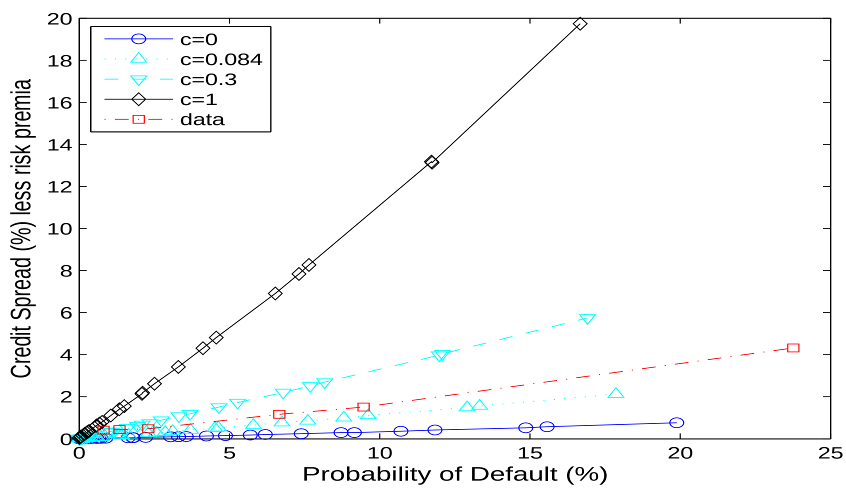
* Lines represent the implied effect of a doubling in the volatility of z or of a doubling in the cost of default on firm employment in the simple model, denoting the individual and interaction effects of the two shocks.

Figure 4: Credit spreads vs. probability of default



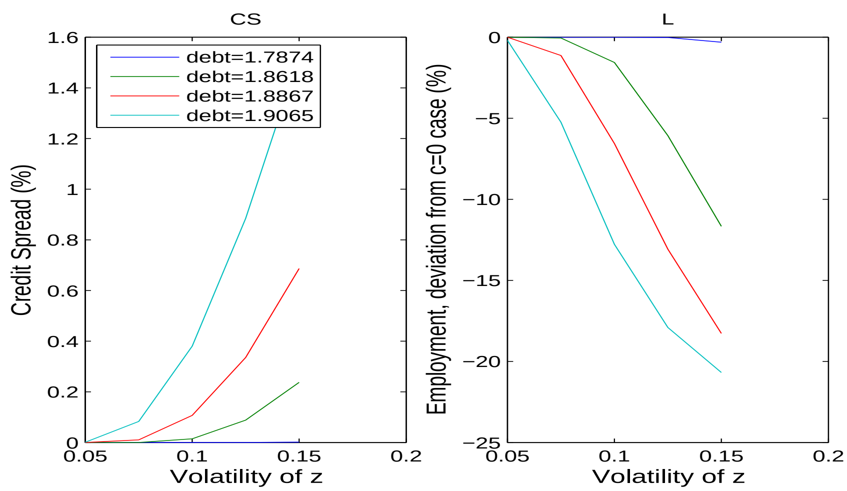
* The red dashed line reflects data from speculative grade firms, by rating class, for the time period 1984-2012 as reported by Standard and Poor's. The remaining lines are implied by the simple model, for varying values of c. All series have the y-intercept from their line of best fit subtracted from them so the slopes can be easily compared.

Figure 5: Credit spreads vs. probability of default, controlling for risk premia



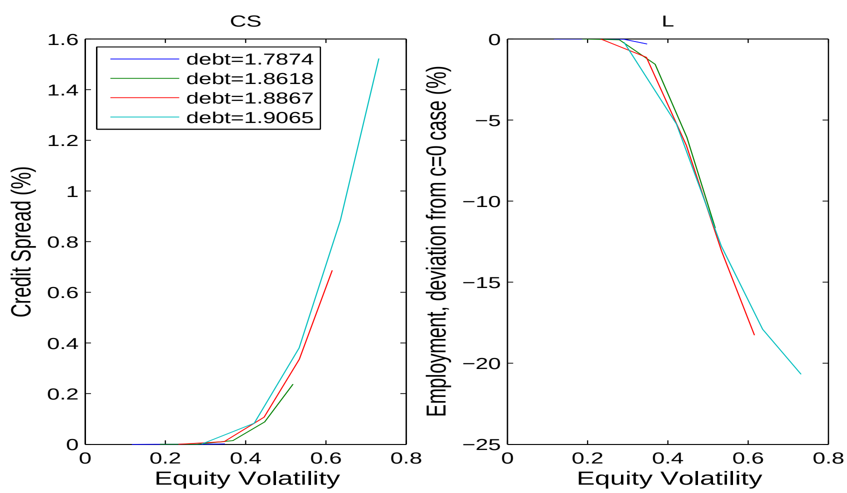
* The red dashed lines reflects data from speculative grade firms, by rating class, for the time period 1984-2012 as reported by Standard and Poor's, less a risk premia effect implied by estimates from Choi and Kim (2015) by ratings class. The remaining lines are implied by the simple model, for varying values of c. All series have the y-intercept from their line of best fit subtracted from them so the slopes can be easily compared.

Figure 6: Effect of volatility



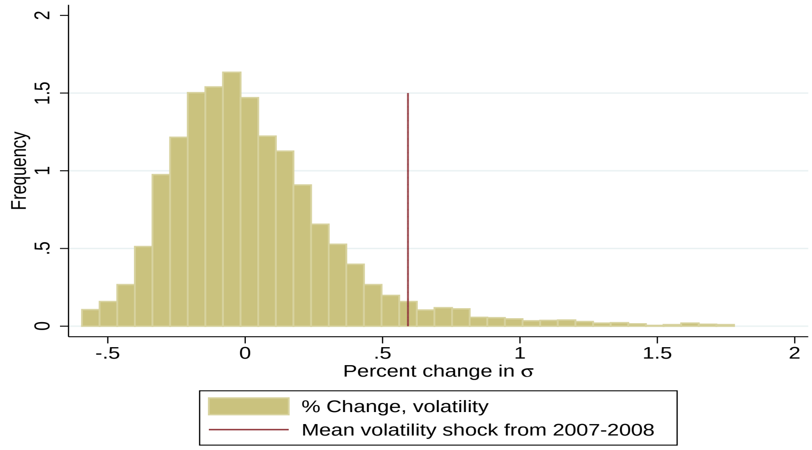
* These results are generated from the simple model, with $c = 0.3$.

Figure 7: Volatility shock



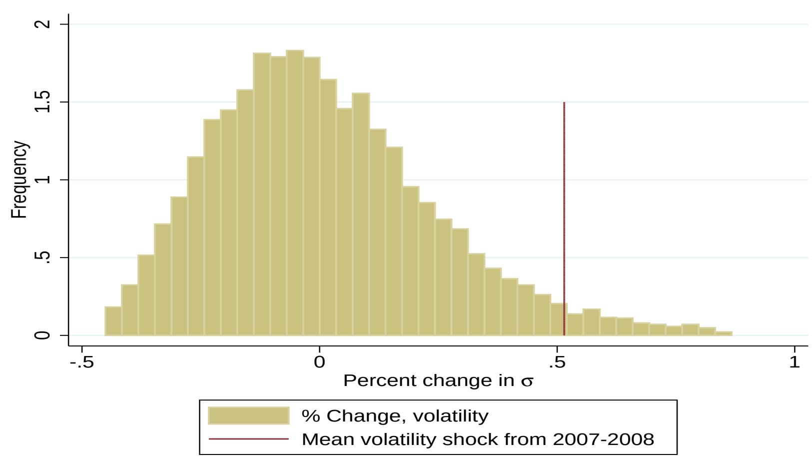
* These results are generated from the simple model, with $c = 0.3$.

Figure 8: Cross-section of shocks to asset volatility during non-recession years



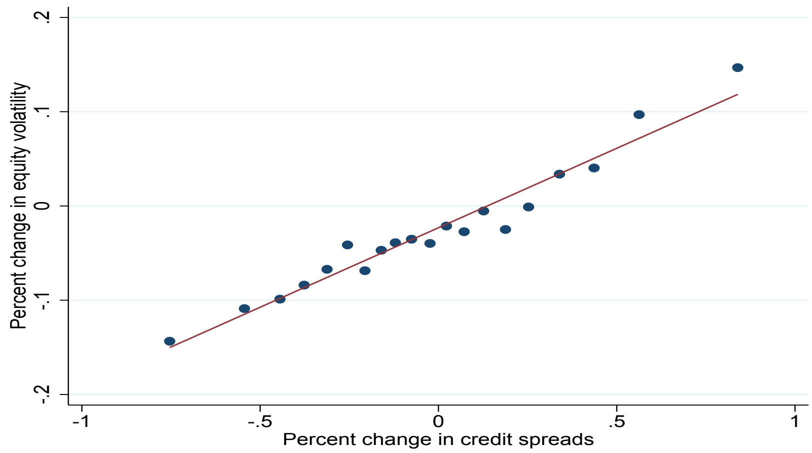
*Data is an unbalanced panel for 1984-2012, excluding NBER recession years. Percent changes are computed as $\frac{x_{t+1}-x_t}{\frac{1}{2}\left(x_{t+1}+x_t\right)}$.

Figure 9: Cross-section of shocks to idiosyncratic equity volatility during non-recession years



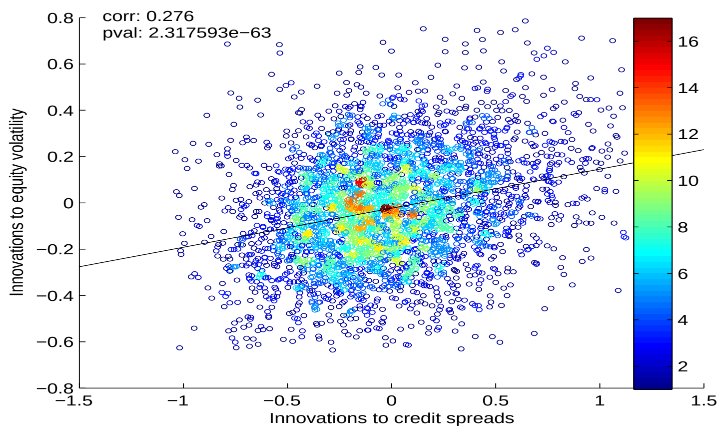
*Data is an unbalanced panel for 1984-2012, excluding NBER recession years. Percent changes are computed as $\frac{x_{t+1}-x_t}{\frac{1}{2}\left(x_{t+1}+x_t\right)}$.

Figure 10: Innovations in credit spreads vs. volatility



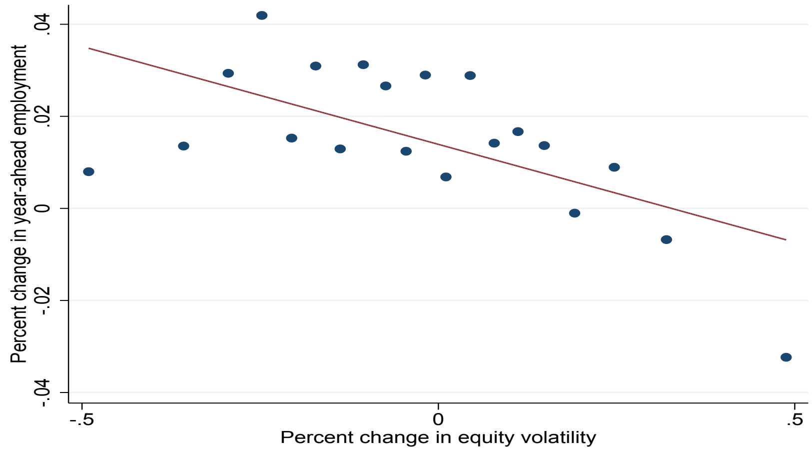
*Data is an unbalanced panel for 1984-2012, excluding NBER recession years. Percent changes are computed as $\frac{x_{t+1}-x_t}{\frac{1}{2}\left(x_{t+1}+x_t\right)}$. Observations are binned into 5-percentile bins by the x-axis, with a point in the scatter reflecting the mean of those observations in both the x and y axis variables.

Figure 11: Innovations in credit spreads vs. volatility



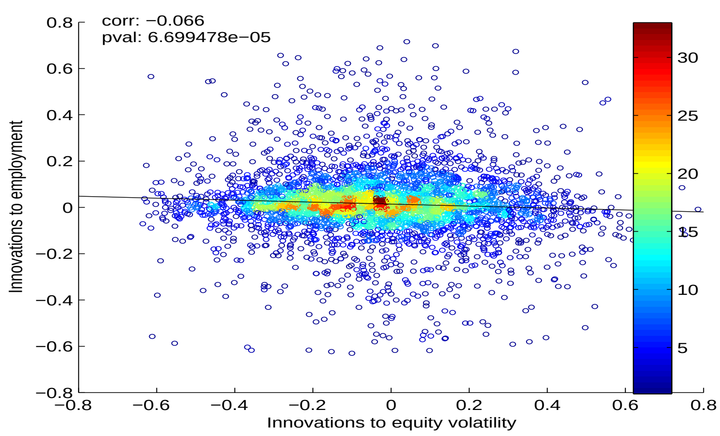
*Data is an unbalanced panel for 1984-2012, excluding NBER recession years. Percent changes are computed as $\frac{x_{t+1}-x_t}{\frac{1}{2}\left(x_{t+1}+x_t\right)}$. The color denotes the number of observations at a given point.

Figure 12: Innovations in firm employment vs. volatility



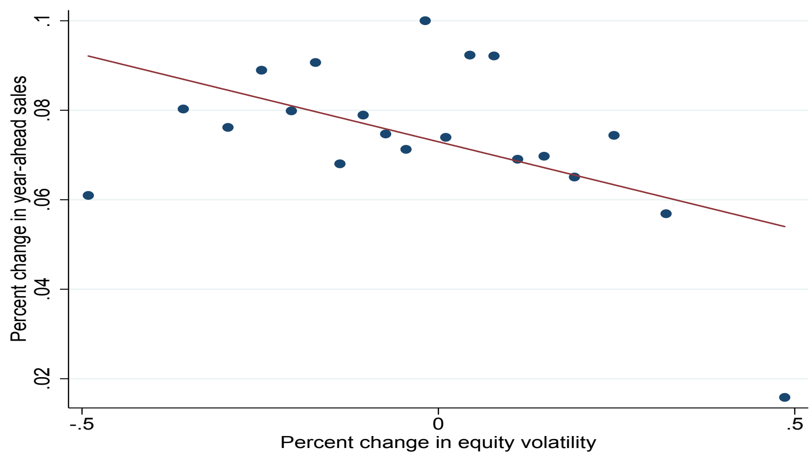
*Data is an unbalanced panel for 1984-2012, excluding NBER recession years. Percent changes are computed as $\frac{x_{t+1}-x_t}{\frac{1}{2}\left(x_{t+1}+x_t\right)}$. Observations are binned into 5-percentile bins by the x-axis, with a point in the scatter reflecting the mean of those observations in both the x and y axis variables.

Figure 13: Innovations in firm employment vs. volatility



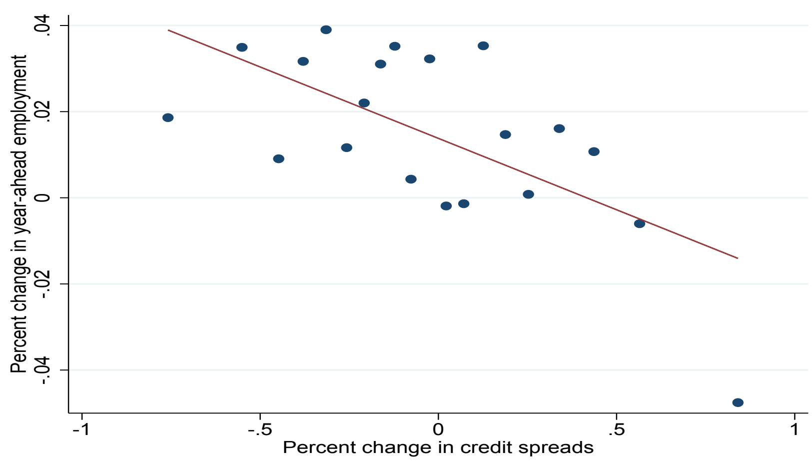
*Data is an unbalanced panel for 1984-2012, excluding NBER recession years. Percent changes are computed as $\frac{x_{t+1}-x_t}{\frac{1}{2}\left(x_{t+1}+x_t\right)}$. The color denotes the number of observations at a given point.

Figure 14: Innovations in firm sales vs. volatility



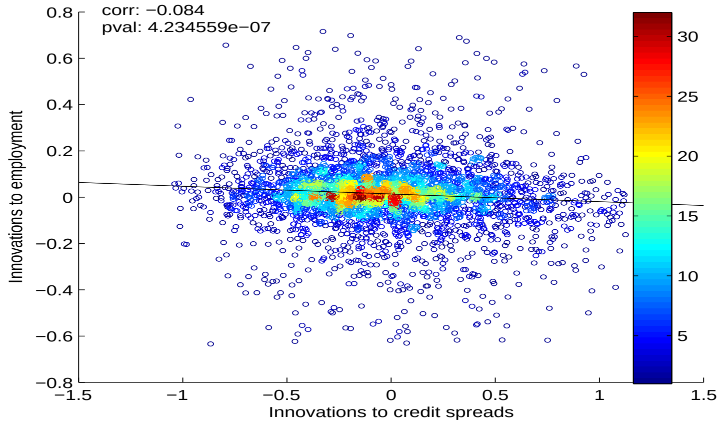
*Data is an unbalanced panel for 1984-2012, excluding NBER recession years. Percent changes are computed as $\frac{x_{t+1}-x_t}{\frac{1}{2}\left(x_{t+1}+x_t\right)}$. Observations are binned into 5-percentile bins by the x-axis, with a point in the scatter reflecting the mean of those observations in both the x and y axis variables.

Figure 15: Innovations in firm employment vs. credit spreads



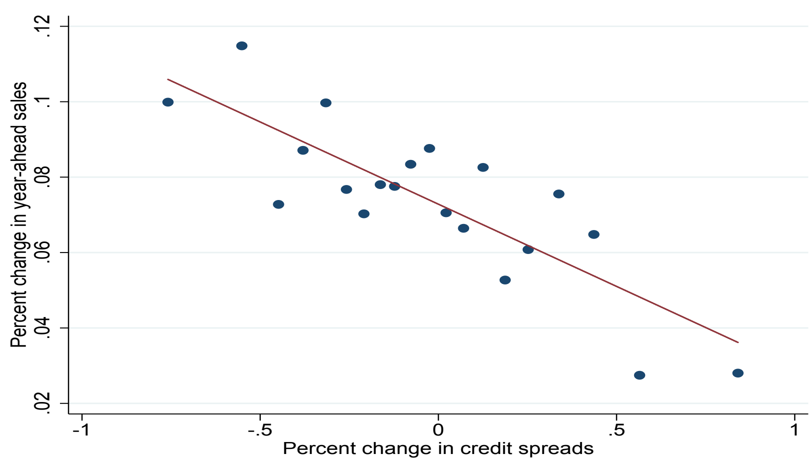
*Data is an unbalanced panel for 1984-2012, excluding NBER recession years. Percent changes are computed as $\frac{x_{t+1}-x_t}{\frac{1}{2}\left(x_{t+1}+x_t\right)}$. Observations are binned into 5-percentile bins by the x-axis, with a point in the scatter reflecting the mean of those observations in both the x and y axis variables.

Figure 16: Innovations in firm employment vs. credit spreads



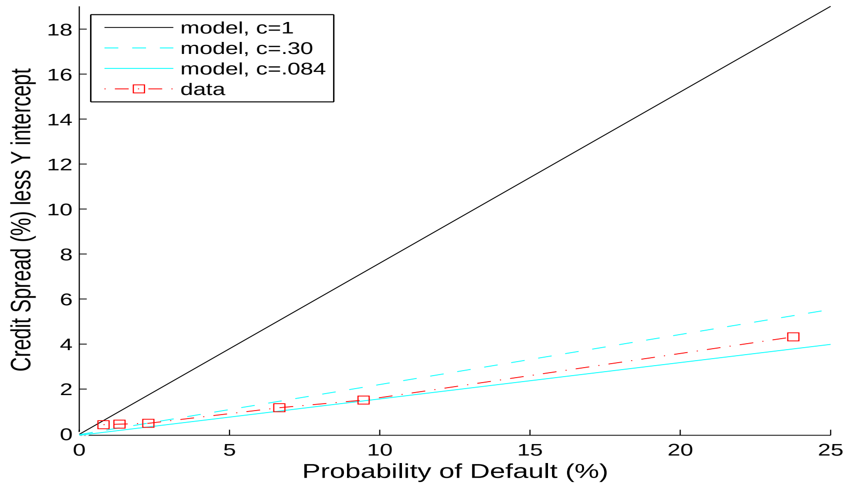
*Data is an unbalanced panel for 1984-2012, excluding NBER recession years. Percent changes are computed as $\frac{x_{t+1}-x_t}{\frac{1}{2}\left(x_{t+1}+x_t\right)}$. The color denotes the number of observations at a given point.

Figure 17: Innovations in firm sales vs. credit spreads



*Data is an unbalanced panel for 1984-2012, excluding NBER recession years. Percent changes are computed as $\frac{x_{t+1}-x_t}{\frac{1}{2}\left(x_{t+1}+x_t\right)}$. Observations are binned into 5-percentile bins by the x-axis, with a point in the scatter reflecting the mean of those observations in both the x and y axis variables.

Figure 18: Probability of default vs. credit spreads



* The red dashed line reflects data from speculative grade firms, by rating class, for the time period 1984-2012 as reported by Standard and Poor's. I adjust this data to account for risk premia, with the procedure outlined in section 2.3.1. The remaining lines are implied by the full model, for varying values of c. All series have the y-intercept from their line of best fit subtracted from them so the slopes can be easily compared.

Figure 19: Innovations to employment vs. credit spreads, model (c=.3)

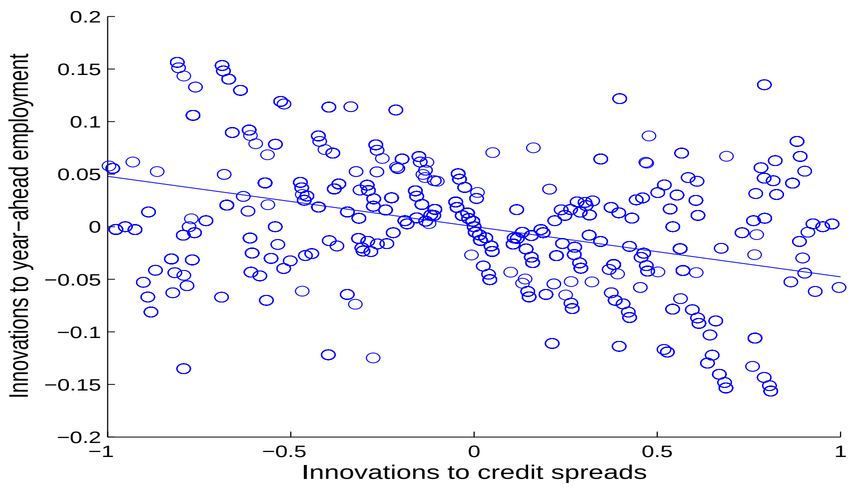
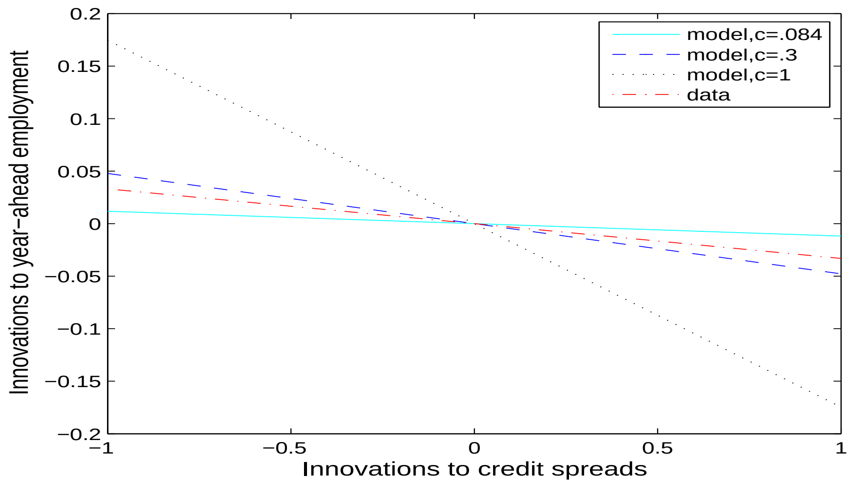
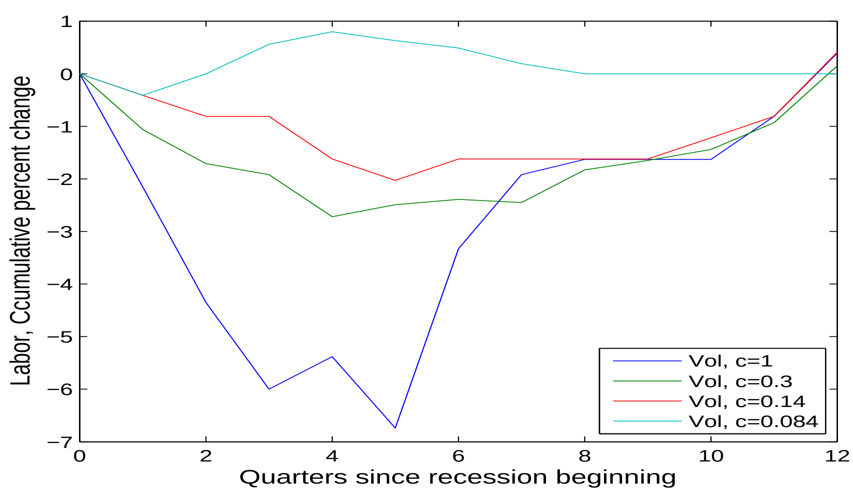


Figure 20: Innovations to employment vs. credit spreads



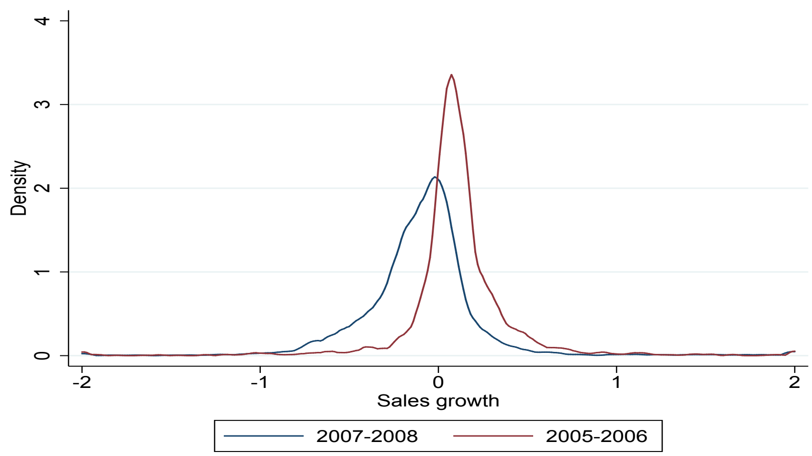
* The red line reflects the line of best fit from an unbalanced panel from 1984-2012, excluding NBER recession years. The remaining lines of best fit are computed using the full model, for varying values of c, in the steady-state.

Figure 21: Aggregate implications of a volatility shock



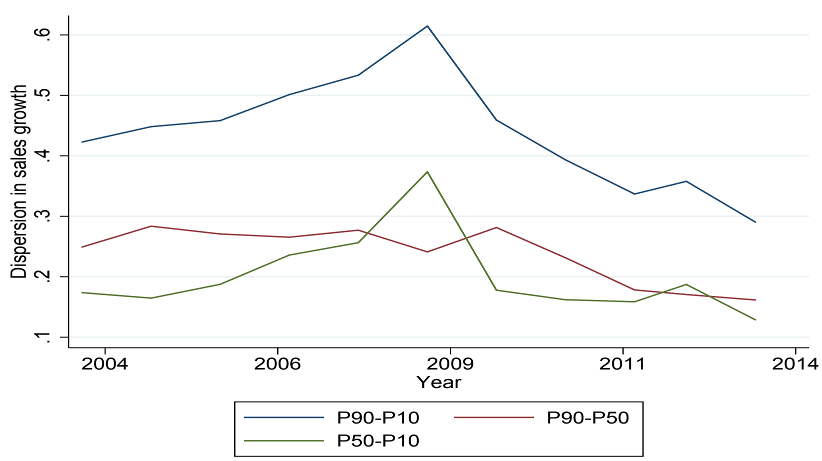
* The lines depict the response of employment, for a variety of calibrations of c, to a sequence of shocks corresponding to the IQR of sales growth during the 2007-2009 recession.

Figure 22: Distribution of firm sales growth



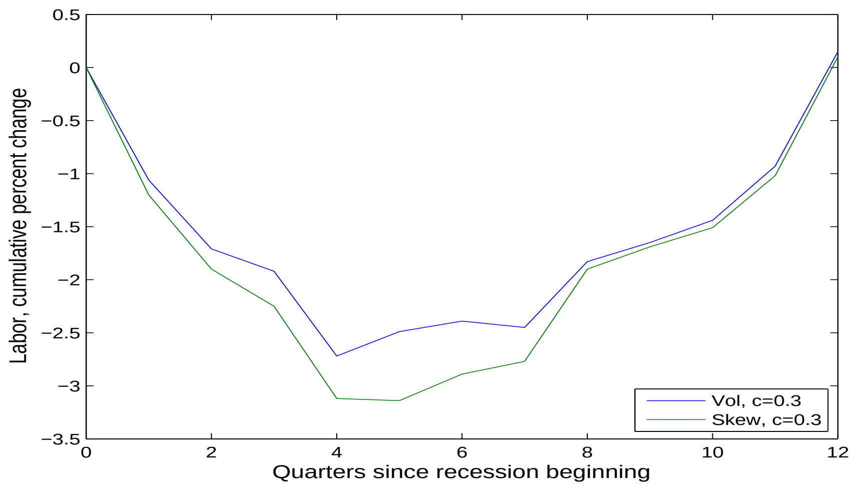
*Percent changes are computed as $\frac{x_{t+1}-x_t}{\frac{1}{2}(x_{t+1}+x_t)}$.

Figure 23: Measures of Dispersion, Upside and Downside



*Percent changes are computed as $\frac{x_{t+1}-x_t}{\frac{1}{2}(x_{t+1}+x_t)}$.

Figure 24: Aggregate implications of a volatility and skewness shock

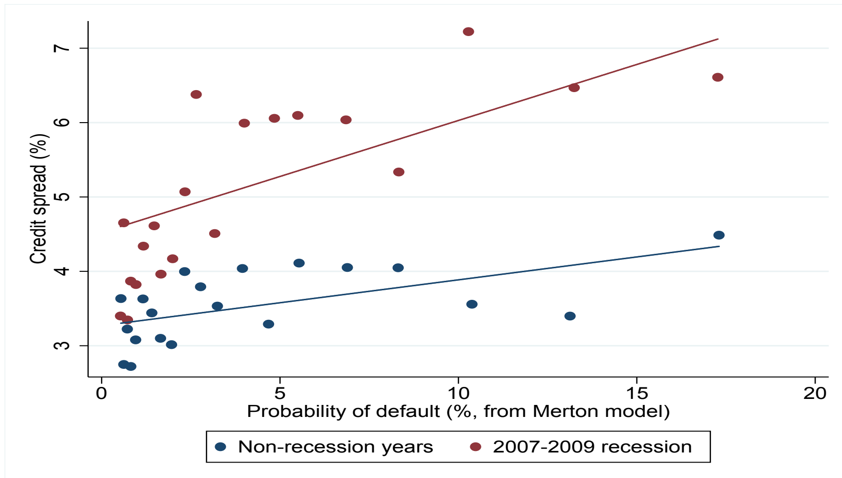


* The lines depict the response of employment to a sequence of shocks corresponding to the IQR of sales growth and the observed changes in the difference between upside and downside dispersion (documented by differences in percentiles of sales growth) during the 2007-2009 recession.

Figure 25: Procyclical Recovery Rate

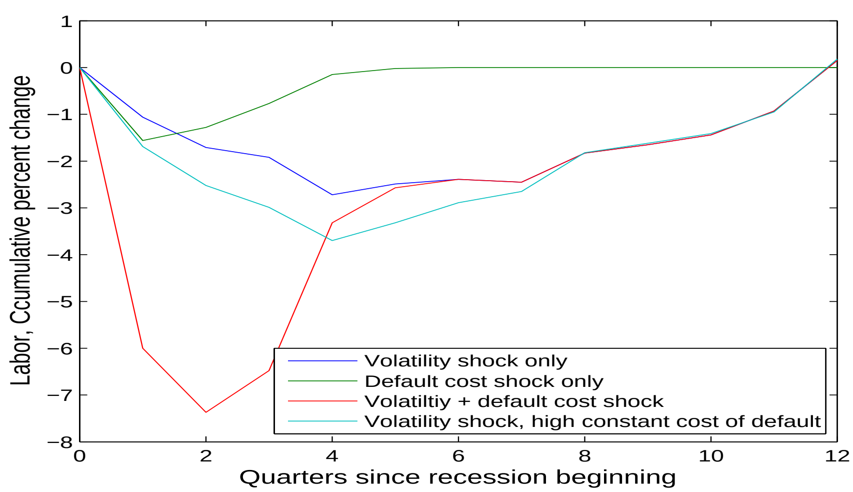
* The data is taken from the average recovery rate as reported by Moody's.

Figure 26: Probability of Default vs. Credit Spreads



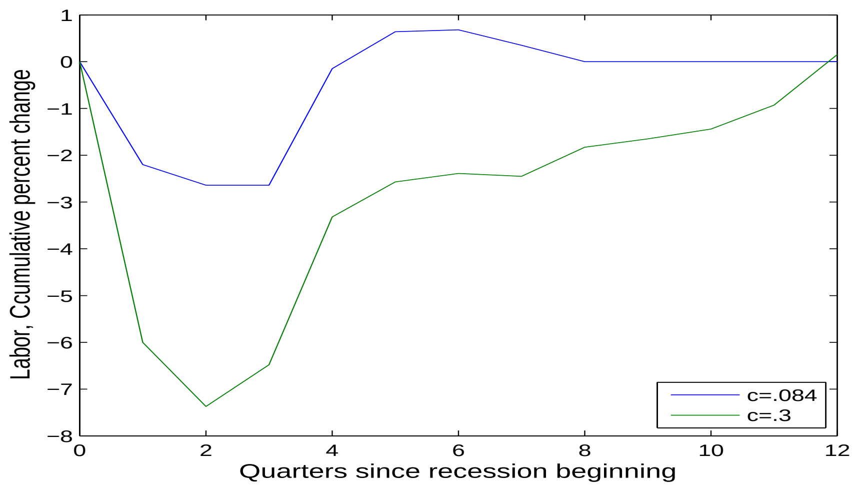
* The x-axis denotes the probability of default implied by a Merton model, computed from the market capitalization of a firm, its liabilities, and realized equity volatility.

Figure 27: Aggregate Implications of a volatility and default cost shock



* The lines depict the response of employment to a sequence of shocks corresponding to the IQR of sales growth and calibrated shocks to the cost of default during the 2007-2009 recession. The line denoted "Volatility shock, high constant cost of default" considers the impact of volatility shocks if the cost of default is set to the maximum value of it attained by the calibrated shock during this recession.

Figure 28: Aggregate Implications of a volatility and default cost shock, varying c



* The lines depict the response of employment, for a variety of calibrations of c, to a sequence of shocks corresponding to the IQR of sales growth and calibrated shocks to the cost of default during the 2007-2009 recession.