Turbulence and Unemployment in Matching Models

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Abstract

Ljungqvist and Sargent (2007) show that increases in turbulence, in the sense of worse skill transition probabilities for workers who suffer involuntary layoffs, generate higher unemployment in a welfare state. den Haan, Haefke and Ramey (2005) challenge this finding and argue that if turbulence also exposes voluntary quits to a tiny risk of skill loss, then higher turbulence leads to a reduction in unemployment. In this paper we explore the source of these disparate results within the two adopted matching models. We find that the latter authors' assumption of additional exposure of the high-skilled unemployed to skill losses following unsuccessful job market encounters, together with the parameterization of their model, implies small incentives for labor mobility in tranquil times. Hence, any small cost to mobility would cause voluntary separations to shut down, e.g., tiny government mandated layoff costs would have counterfactually large effects of suppressing unemployment. Once the additional exposure to turbulence is dismissed and the parameterization is adjusted to account for the unemployment dynamics in data, the positive relationship between turbulence and unemployment reemerges.

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1 Introduction

Ljungqvist and Sargent (1998) investigate the causes of high and persistent unemployment in European welfare states after 1980, and propose the interaction of economic turbulence—immediate skill loss after a layoff—and a generous welfare state, as a force that can explain the unemployment dynamics. They find that in a pure search model, increased turbulence in the economic environment within welfare states is the cause of persistently higher unemployment. The welfare state adversely affects the response to economic shocks, making unemployment more persistent, and the response to changes in the economic environment (turbulence) which explains the higher unemployment rate at steady state. The key force is that turbulence decreases unemployed workers' willingness to accept new jobs as their unemployment benefits are linked to past jobs where their skills were high. In a follow-up paper, Ljungqvist and Sargent (2007) [LS from now on] show that the turbulence story is robust to introducing a matching framework.

In contrast to the LS's rationale for decreased willingness of unemployed workers to accept new jobs, den Haan, Haefke and Ramey (2005) [dHHR from now on] focus on the response by employed workers. Since displacement induces costly skill obsolescence in a turbulent environment, workers ought to be much more reluctant to part with their existing jobs, instead offering wage concessions to avoid layoffs, i.e. as the speed of skill obsolescence rises, workers become more reluctant to separate, and job destruction falls. Indeed, this intuition became commonly-held among economic observers. Under this argument, an increase in turbulence should have induced a reduction in the rate of job destruction, exerting downward pressure on unemployment and working against the job rejection channel highlighted by LS. In this paper, we show the robustness of the LS result to the enhancement of job retention incentives.

A puzzle The main finding of dHHR is that "if turbulence has only a tiny effect on the skills of workers experiencing endogenous separation, then the results of Ljungqvist and Sargent (1998, 2004a) are reversed, and higher turbulence leads to a *reduction* in unemployment".

Specifically,

"allowing for a skill loss probability following endogenous separation that is only 3% of the probability following exogenous separation eliminates the positive turbulence-unemployment relationship. Increasing this proportion to 5% gives rise to a strong negative relationship between turbulence and unemployment." (dHHR, p. 1362)

It is surprising that so small probabilities can have such large effects on equilibrium outcomes. In contrast, when we explored skill loss at endogenous separations in the model of Ljungqvist and Sargent (2007), we found small effects. Endogenous turbulence has to be more than 40% of exogenous turbulence before it can suppress unemployment relative to tranquil economic times (when turbulence is zero). This is a puzzling difference, and our goal is to understand these disparate outcomes. Therefore, we first describe four important differences between the models of dHHR and LS.

Four key differences The den Haan, Haefke and Ramey (2005) and Ljungqvist and Sargent (2007) frameworks differ in four dimensions:

1. **Exogenous vs. endogenous labor market tightness.** While dHHR assume an exogenous matching probability (i.e. the measures of both firms and workers are assumed to be exogenously equal to unity which in turn delivers a constant value of market tightness), LS adopt the standard assumption of free entry of firms and a zero-profit condition for posting vacancies.

2. **Timing of the consummation of skill upgrade.** dHHR assume that a worker who receives a positive skill shock at work must remain with the same firm for one period in order to consummate the higher skill level. Moreover, to guarantee that such continued relationships are privately optimal, it is assumed that a positive human capital shock is accompanied with a new productivity draw where the lower support is given by the endogenous reservation wage of a high-skilled worker in an ongoing employment relationship, where the rationale for that assumption is to limit the number of worker categories that must be considered. In contrast, LS assume that skill upgrades are immediately consummated, and to limit the number of worker categories in the analysis, it is assumed that upon skill upgrades workers immediately become entitled to high unemployment benefits (even if matches break up at skill upgrades).[1]

3. **Productivity distributions.** dHHR assume uniform productivity distributions with narrow support, while LS postulate truncated normal distributions with wider support.

4. **Skill loss because of job market encounters.** dHHR assume that after an encounter between a firm and an unemployed worker which does not result in a match being formed, the worker faces the same risk of losing human capital as if she is experiencing an endogenous separation from an ongoing employment relationship.

[1] As emphasized by LS, their alternative assumption enables them to discard dHHR's simplifying, but debatable, assumption that a skill upgrade is accompanied with a new productivity draw where the lower bound on possible draws is the reservation productivity of an ongoing match with a high-skilled worker. In our model, the distributions from which productivities are drawn do not change with endogenous reservation productivities.

Solution to the puzzle Our cleanest resolution to the dHHR puzzle is to start with the LS model as the basis for our comparison and make one perturbation at a time. We discuss the four potential candidates above: (i) exogenous versus endogenous market tightness, (ii) timing of the consummation of skill upgrade, (iii) productivity distributions, and (iv) turbulence affecting job market encounters.

Regarding the first difference—the type of labor market matching—we find that with endogenous matching, there is a dramatic decline in market tightness in response to turbulence as the "invisible hand" struggles to restore the profitability of firms; this force cannot be accommodated in the exogenous matching framework and thus the profitability of vacancies plummets in response to turbulence. Nevertheless, this force is not strong enough and unemployment dynamics are not significantly altered when we move from endogenous to exogenous tightness.

Regarding the second difference—the timing of consummation of skills—the alternative assumptions affect workers' bargaining power vis-à-vis firms. The assumptions of dHHR (2001, 2005) of delayed consummation effectively erode the bargaining position of high-skilled workers. When studying the employment effects of layoffs costs, Ljungqvist (2002) found that different assumptions about relative bargaining positions could have large impact on equilibrium outcomes. Indeed, under delayed consummation, wages of upgraded workers become negative (firms are extracting rents from the workers). Still, this force does not happen to be strong enough to alter other equilibrium outcomes.

Regarding the third difference—the productivity distribution—the range of the distributions is of overriding importance. The dHHR parameterization is associated with a much less robust incentive for high-skilled workers to transit between jobs, because of the uniform productivity distribution with narrow support that lacks a left-tail with really bad productivity outcomes (as compared to LS's truncated normal distribution with wider support).

Finally, regarding the fourth difference above—turbulence affecting high-skilled unemployed—dHHR seem to have slipped this additional exposure to endogenous turbulence into their framework as a simplification that allows to reduce the number of workers to keep track off and it looks entirely unintended. However, this exposure radically changes the incentives for labor mobility, as high-skilled workers hold onto their jobs given the risk that they face skill obsolescence now also affects them in case they are unemployed.

In summary, the last two perturbations, combined, hold the resolution to the dHHR puzzle. We find that the latter authors' assumption of also exposing the high-skilled unemployed to skill losses following unsuccessful job market encounters, together with the parameterization of their model, implies small incentives for labor mobility in tranquil times. Hence, any small cost to mobility would cause voluntary separations to shut down. Once the additional exposure to turbulence is dismissed and the parameterization is adjusted to account for the unemployment dynamics in data, the positive relationship between turbulence and unemployment reemerges.

Layoff costs How come that tiny risk of skill loss at voluntary separations—endogenous turbulence—does not overturn LS's positive relationship between turbulence and unemployment, but that extending those tiny risks of skill losses to encounters between vacancies and the unemployed, has those dramatic effects? As long as the average number of encounters needed to form a match is pretty small, one would expect that those tiny risks of skill loss would not overturn LS's relationship.

To further analyze the forces at play and strengthen our point regarding labor mobility, we conduct a layoff cost analysis in the dHHR model, in which a small layoff cost will close down unemployment and illustrate the fickleness of the dHHR parameterization of productivity distributions, e.g., tiny government mandated layoff costs would have counterfactually large effects of suppressing unemployment. We find that mobility of high-skilled employed shuts down with a labor tax equivalent to 70% of their quarterly wage, which is indeed a small number. We confirm find that the dHHR parameterization implies minuscule incentives for labor mobility in tranquil times. A tiny government mandated layoff cost (or any small cost to mobility, such as endogenous turbulence) has counterfactually large effects of suppressing unemployment by shutting down voluntary separations.

Organization Section 2 develops a matching framework with endogenous and exogenous turbulence. It builds on the models of LS and dHHR. Section 3 documents the puzzle. Section 4 dissects the analysis and finds that dHHR's framework and parametrization implies minuscule incentives for labor mobility in tranquil times and hence, any small cost to mobility would cause voluntary separations to shut down. Section 5 conducts the layoff tax analysis and Section 6 performs a cost-benefit analysis of labor mobility for high-skilled workers to shed more light on the puzzle. The Appendix presents our analysis perturbing the dHHR model and come to the same conclusion.

2 A matching framework with turbulence

We develop a modified version of the matching model in LS that includes both exogenous turbulence—worse skill transition probabilities for workers who suffer involuntary layoffs—, as well as endogenous turbulence—worse skill transition probabilities for workers who suffer voluntary quits. The model also considers differences in the rates at which the high-skilled employed and the high-skilled unemployed suffer skill obsolescence, either after voluntary quits or mere job market encounters.

2.1 Environment

Workers Consider a unit mass population consisting of workers, which are either employed or unemployed, and retirees. Workers differ in two dimensions: their current skill level i, which can be either low (l) or high (h), and their skill level during their last employment spell j, which in turn determines their entitlement to unemployment benefits. Workers experience accumulation or deterioration of skills, conditional on employment status and instances of exogenous and endogenous job terminations. We denote variables with two indices (i, j), the first for current skill and the second for benefit entitlement.

Workers are risk neutral and only value consumption. Their preferences are ordered according to

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t c_t \tag{1}$$

where they discount future utilities with $\beta \equiv \hat{\beta}(1 - \rho^r)$, which includes the subjective time discount factor $\hat{\beta} \in (0, 1)$ and the constant probability of retirement ρ^r. In order to keep the mass of workers constant, it is assumed that a retired worker exits the labor force permanently and is substituted by a newborn worker. All newborn workers enter the labor force with low skills.

Firms and employment relationships Firms post vacancies at a cost μ. We think of a job opportunity as a productivity draw z from a stationary distribution $v_i(z)$, which is indexed by the worker's skill level i. It is assumed that the high-skilled distribution first-order stochastically dominates the low-skilled distribution: $v_h(z) \leq v_l(z)$. An employment relationship produces output z per period.

Sources of uncertainty Besides retirement, there are six additional sources of uncertainty. There are three types of productivity shocks (upgrad, switch and termination) and three types of turbulence shocks (exogenous, endogenous for employed, and endogenous for unemployed).

(i) During employment, skills can be accumulated. Conditional on no exogenous job termination, a worker's skills can get upgraded from low to high with probability γ^u.

(ii) Idiosyncratic shocks within a worker-firm match affect employed workers productivities in the job. With probability γ^s, a new productivity is drawn from a distribution that depends on the current skill level. As a consequence of the new draw, a relationship may continue or may be endogenously terminated (a voluntary quit).

(iii) An employed worker faces a probability ρ^x of having her job terminated for exogenous reasons (a layoff).

(iv) Upon an exogenous job termination, a high skilled worker is subject to a potential risk of losing her skills with probability $\gamma^{d,x}$. This risk is called exogenous turbulence.

(v) Upon an endogenous job termination, high skilled workers are subject to the risk of losing their skills with probability γ^d. This risk is called endogenous turbulence. Following dHHR, we parametrize endogenous turbulence as a fraction ϵ of exogenous turbulence, and we vary it from zero—only exogenous turbulence— to one—both types of turbulence are equal—as follows: $\gamma^d = \epsilon\gamma^{d,x}$.

(vi) During unemployment, high-skilled workers that reject an offer after a job market encounter with a firm, are subject to the risk of losing their skills with probability γ^e.

In the DHRR framework, high-skilled unemployed workers are equal to the high-skilled employed workers as they face the same risk of losing skills from endogenous turbulence, i.e. $\gamma^d = \gamma^e$. The assumption used in dHHR to reduce the number of worker types, as in this case, both workers can be collapsed into one category in terms of their choices (reservation productivities) and their outside values. In the LS framework, endogenous turbulence only affects currently employed workers, i.e. $\gamma^e = 0$.

Timing We make the following assumption regarding the timing of turbulence shocks. In the beginning of the period, exogenous separations from jobs happen and exogenous turbulence $\gamma^{d,x}$ hits. These workers become part of the high-skilled unemployed, and will sit in the matching function one period. Then employed workers upgrade their skills and get new productivity draws, while unemployed workers have market encounters and get productivity draws. If they reject the draws, they are subject to endogenous turbulence (at their particular rates) that happens at the end of the period. The high-skilled employed and unemployed workers that are hit with endogenous turbulence become part of the low-skilled unemployed at the beginning of next period, joining the workers that were hit with exogenous turbulence.

Labor market and matching technology Let u_{ij} be the number of unemployed workers with current skill i and benefit entitlement j, then the total number of unemployed workers is $u = \sum_{i,j} u_{ij}$. Let v be the endogenous number of vacancies posted by firms. We assume a single matching function for all unemployed workers and vacancies. Matches are created according to an increasing, concave and linearly homogenous matching function $M(v,u)$ that takes as inputs the aggregate unemployment u and endogenously posted vacancies v. Let the ratio $\theta \equiv \frac{v}{\theta}$ denote labor market tightness. Then the probability that a worker with skill i and benefit entitlement j is matched with a vacancy, which is independent of (i,j), is an increasing function of market tightness

$$\lambda^w(\theta) \equiv \frac{M(v,u)}{u} = M\left(\frac{v}{u}, 1\right) = m(\theta)$$

and the probability that a firm meets a worker with skill i and benefit entitlement j, which does depend on (i,j) is a decreasing function of market tightness

$$\lambda^f_{ij}(\theta) \equiv \frac{M(v,u)}{v} \frac{u_{ij}}{u} = \frac{m(\theta)}{\theta} \frac{u_{ij}}{u}$$

Assuming that the matching technology is Cobb-Douglas, $M(v,u) = Av^\alpha u^{1-\alpha}$, we get that:

$$\lambda^w(\theta) = A\theta^{1-\alpha}, \qquad \lambda^f_{ij}(\theta) = A\theta^{-\alpha}\frac{u_{ij}}{u} \tag{2}$$

2.2 Match surplus, outside values and continuation values

Match surplus When an unemployed worker with skill i and benefit entitlement j meets a firm with a vacancy, the firm-worker pair draws productivity z from the stationary distribution $v_i(z)$. The firm and the worker will stay together and produce if the match is successful, which happens if the surplus is positive.

The joint surplus for a new $s^o_{ij}(z)$ or continuing $s_{ij}(z)$ relationship is given by the after-tax productivity $(1-\tau)z$ plus the future joint continuation value $g_i(z)$, minus the outside options of the match: the worker's future value from entering the unemployment pool in the current period ω_{ij} when he receives an unemployment benefit of b_j, and the firm's future value from entering the vacancy pool in the current period ω^f. For simplicity, and given that in the LS framework the outside value of the firm is always zero by free entry, we define the joint outside option ω_{ij} to be equal to the sum of the worker's outside option ω^w_{ij} (excluding benefits) and the firm's outside option ω^f, i.e. $\omega_{ij} \equiv \omega^w_{ij} + \omega^f$.

Then the joint surplus for a new or continuing relationship with a low-skilled worker with benefits j, is given by[2]

$$s_{lj}(z) = (1-\tau)z + g_l(z) - b_j - \omega_{lj}, \qquad j = l, h \tag{3}$$

To compute the surplus for relationships with high-skilled workers, we must differentiate between new and continuing relationships, because of the different probabilities of facing endogenous turbulence risk. The surplus for a new relationship with a high-skilled worker who is currently a high-skilled unemployed, denoted by s^o_{hh}, considers outside options that are affected by turbulence upon labor market encounters at rate γ^e as follows:

$$s^o_{hh}(z) = (1-\tau)z + g_h(z) - b_h - (1-\gamma^e)\omega_{hh} - \gamma^e\omega_{lh} \tag{4}$$

In contrast, the surplus for a continuing relationship with a high-skilled worker, or a new relationship with a low-skilled worker that receives a skill upgrade that is immediately consummated, is affected by turbulence at rate γ^d

$$s_{hh}(z) = (1-\tau)z + g_h(z) - b_h - (1-\gamma^d)\omega_{hh} - \gamma^d\omega_{lh} \tag{5}$$

Productivity thresholds The worker and firm split the match surplus through Nash bargaining and outside values as threat points. Since they both want a positive surplus, there is mutual agreement whether to form a match or not. The reservation productivity $\underline{z}_{ij}$ is the lowest productivity that makes a match profitable and satisfies

$$s_{ij}(\underline{z}_{ij}) = 0 \tag{6}$$

for new or continuing matches, respectively. Given $\underline{z}_{ij}$, let ν_{ij} denote the rejection probability, which is given by all productivities below the threshold

$$\nu_{ij} = \int_{-\infty}^{\underline{z}_{ij}} dv_i(y) \tag{7}$$

Also denote with E_{ij} the expected continuation value (including outside options) conditional on acceptance

$$E_{ij} = \int_{\underline{z}_{ij}}^{\infty} [(1-\tau)y + g_j(y)]dv_i(y) \tag{8}$$

[2]The equality between surpluses for new and continuing relationships holds without in the absence of layoff taxes. In Section 5 we introduce layoff taxes and the expressions for the match surplus are no longer equal for employed and unemployed low-skilled workers.

Match continuation values Consider a match between a firm and a worker with benefit entitlement j with productivity z. Let $g^w_j(z)$ be the continuation value of the match for the worker and $g^f(z)$ be the continuation of the match for the firm, both including their outside options. Finally let $g_i(z) \equiv g^w_j(z) + g^f(z)$ be the joint continuation of such match—including outside options.

The joint continuation value—including outside options—of a match with a low-skilled worker with current productivity z, denoted by $g_l(z)$, is given by:

$$\begin{aligned}
&\text{(Exogenous separation)} & g_l(z) &= \beta\rho^x(b_l + \omega_{ll}) \\
&\text{(No changes)} & &+ \beta(1-\rho^x)(1-\gamma^u)(1-\gamma^s)[(1-\tau)z + g_l(z)] \\
&\text{(Productivity switch)} & &+ \beta(1-\rho^x)(1-\gamma^u)\gamma^s[E_{ll} + \nu_{ll}(b_l + \omega_{ll})] \\
&\text{(Human capital upgrade)} & &+ \beta(1-\rho^x)\gamma^u[E_{hh} + \nu_{hh}(b_h + \underbrace{(1-\gamma^d)\omega_{hh} + \gamma^d\omega_{lh}}_{\text{endogenous turbulence}})]
\end{aligned} \tag{9}$$

where $E_{ij} = \int_{\underline{z}_{ij}}^{\infty}[(1-\tau)y + g_j(y)]dv_i(y)$ is the expected value following a new draw from productivity distribution $v_i(y)$ (either from upgrades or switches), conditional on acceptance. Since we assume that skill status is upgraded immediately (unlike dHHR), low skilled workers face the risk of endogenous turbulence when their human capital is upgraded and they reject the new productivity draw that comes with it (last line).

The joint continuation value—including outside options—of a match with a high-skilled worker with current productivity z, denoted by $g_h(z)$, is given by:

$$\begin{aligned}
&\text{(Exogenous separation)} & g_h(z) &= \beta\rho^x[\underbrace{\gamma^{d,x}(b_h + \omega_{lh}) + (1-\gamma^{d,x})(b_h + \omega_{hh})}_{\text{exogenous turbulence}}] \\
&\text{(No changes)} & &+ \beta(1-\rho^x)(1-\gamma^s)[(1-\tau)z + g_h(z)] \\
&\text{(Productivity switch)} & &+ \beta(1-\rho^x)\gamma^s[E_{hh} + \nu_{hh}(b_h + \underbrace{(1-\gamma^d)\omega_{hh} + \gamma^d\omega_{lh}}_{\text{endogenous turbulence}})
\end{aligned} \tag{10}$$

Endogenous turbulence affects the continuation value after a switch is rejected (last line).

Value of unemployment (outside value for employed workers) A worker that enters the unemployment pool obtains benefits b_j according to the skill level at its last job—benefits are determined by a replacement rate ϕ on the average labor income within the worker's skill category when last employed—and a continuation value of ω^w_{ij}.

A low-skilled worker with benefit entitlement j has an outside option with value of $b_j + \omega^w_{lj}$, where

$$\omega^w_{lj} = \underbrace{\beta\lambda^w(\theta)\int_{\underline{z}_{lj}}^{\infty}\pi s^o_{ij}(y)dv_l(y)}_{\text{match + accept}} + \underbrace{\beta(b_j + \omega^w_{lj})}_{\text{match + reject or no match}} \tag{11}$$

A high-skilled worker has an outside option wit value $b_h + \omega^w_{hh}$, where

$$\begin{aligned}\omega^w_{hh} &= \underbrace{\beta\lambda^w(\theta)\int_{\underline{z}_{hh}}^{\infty}\pi s^o_{hh}(y)dv_h(y)}_{\text{match + accept}} + \underbrace{\beta\lambda^w(\theta)(b_h + (1-\gamma^e)\omega^w_{hh} + \gamma^e\omega^w_{lh})}_{\text{match + reject (possibility of skill loss)}} \\ &+ \underbrace{\beta(1-\lambda^w(\theta))(b_h + \omega^w_{hh})}_{\text{no match}}\end{aligned} \tag{12}$$

Value of a vacancy (firms' outside value Let ω^f be a firm's future value of entering the vacancy pool, which is given by:

$$\omega^f = -\mu + \sum_{(i,j)}\lambda^f_{ij}(\theta)\beta\int_{\underline{z}_{ij}}^{\infty}(1-\pi)s^o_{ij}(y)dv_i(y) + \beta\omega^f \tag{13}$$

where μ is the cost of opening the vacancy, $\lambda^f_{ij}(\theta)$ is the probability of filling the vacancy with an unemployed worker with type (i,j) and $s^o_{ij}(y)$ is the match specific surplus from a new relationship, of which the firm obtains a fraction $(1-\pi)$. Because of free entry in the LS framework, in equilibrium firms expect to break even when posting a vacancy and the value of a firm ω^f is always equal to zero. In the dHHR, this value might defer from zero, and in some cases, can even become negative.

Zero profit condition and equilibrium market tightness Free entry by firms ensures that expected profits from posting a vacancy are equal to zero, i.e. $\omega^f = 0$. This implies in equation (13) that

$$\mu = \beta(1-\pi)\sum_{(i,j)}\lambda^f_{ij}(\theta)\int_{\underline{z}_{ij}}^{\infty}s^o_{ij}(y)dv_i(y)$$

The previous condition pins down the equilibrium value of market tightness. Substituting the vacancy filling rate $\lambda^f_{ij} = A\theta^{-\alpha}\frac{u_{ij}}{u}$ from (2) and rearranging we obtain an expression for market tightness:

$$\theta = \left(\frac{A\beta(1-\pi)\bar{s}}{\mu}\right)^{1/\alpha} \tag{14}$$

where $\bar{s} \equiv \sum_{i,j}\left[\frac{u_{ij}}{u}\int_{\underline{z}_{ij}}^{\infty}s^o_{ij}(y)dv_i(y)\right]$ is the expected surplus, which takes into account the different types of workers that make up the unemployment mass.

2.3 Wages, benefits, and taxes

Wage determination Wages are determined through Nash bargaining, with π and $1-\pi$ the bargaining weights of workers and firms, respectively. Since there are no layoff costs in our baseline model, there is only wage function that applies for both an initial round of negotiations between a newly matched firm and worker and for renegotiations in an ongoing match. In Section 5 we introduce layoff taxes and a two-tier wage structure that takes into account the different threat points that a firm has in a new negotiations versus a renegotiation.

The wage of a type (i,j) worker with current productivity z, denoted by $p_{ij}(z)$ solves the maximization problem:

$$\max_{p_{ij}(z)} \quad \left((1-\tau)z - p_{ij}(z) + g^f(z) - \omega^f\right)^{1-\pi}\left(p_{ij}(z) + g_i^w(z) - b_j - \omega_{ij}^w\right)^{\pi}$$

where $g^f(z)$ and $g_i^w(z)$ are the continuation values for firm and worker respectively (the joint continuation value defined above is the sum $g_i \equiv g^f + g_i^w$) and ω_{ij}^w and ω^f the outside options, respectively. The solution yields the following wage function:

$$p_{ij}(z) = \pi s_{ij}(z) + b_j + \omega_{ij} - g_i^w(z) \tag{15}$$

where $g_i^w(z)$ is the worker's future value from continuing the relationship. For low-skilled workers, it is equal to

$$\begin{aligned} g_l^w(z) &= \beta(1-\rho^x)\left\{(1-\gamma^u)\left[(1-\gamma^s)\pi s_{ll}(z) + \gamma^s\int_{\underline{z}_{ll}} \pi s_{ll}(y)dv_l(y)\right] + \gamma^u\int_{\underline{z}_{hh}} \pi s_{hh}(y)dv_h(y)\right\} \\ &+ \beta\rho^x(b_l + \omega_{ll}^w) \end{aligned} \tag{16}$$

and for high-skilled workers

$$g_h^w(z) = \beta(1-\rho^x)\left[(1-\gamma^s)\pi s_{hh}(z) + \gamma^s\int_{\underline{z}_{hh}} \pi s_{hh}(y)dv_h(y)\right] + \beta\rho^x(b_l + \omega_{ll}^w) \tag{17}$$

Benefits and Income Taxes For a (i,j) worker, income taxes are a constant fraction τ of the average productivity of her skill group. Let $e_i \equiv \sum_j e_{ij}$ denote the total number of workers in skill level i, then the average productivity $\bar{z}_i$ of skill group i is

$$\bar{z}_i = \sum_j \frac{e_{ij}}{e_i} \int_{\underline{z}_{ij}}^{\infty} y \, \frac{dv_i(y)}{1 - v_i(\underline{z}_{ij})} \tag{18}$$

Therefore, average taxes by skill level are $t_i = \tau \bar{z}_i$, and total government revenue

$$t_l e_l + t_h e_h = \tau(\bar{z}_l e_l + \bar{z}_h e_h) \tag{19}$$

Unemployment benefits are a constant fraction ϕ of the average wage of her entitlement group. Let $e_j \equiv \sum_i e_{ij}$ denote the total number of workers in entitlement level j, then the average wage of entitlement group j is

$$\bar{p}_j = \sum_i \frac{e_{ij}}{e_j} \int_{\underline{z}_{ij}}^{\infty} p_{ij}(y) \, \frac{dv_i(y)}{1 - v_i(\underline{z}_{ij})} \tag{20}$$

Thus, average benefits by entitlement level are $b_j = \phi \bar{p}_j$, and total government expenditure

$$b_l u_{ll} + b_h(u_{lh} + u_{hh}) = \phi(\bar{p}_j u_{ll} + \bar{p}_h(u_{lh} + u_{hh})) \tag{21}$$

Government budget The government satisfies its budget constraint period by period. It adjusts its tax rate in order to equalizing the total value of expenditure to its tax revenues:

$$\underbrace{\phi(\bar{p}_l u_{ll} + \bar{p}_h(u_{lh} + u_{hh}))}_{\text{gov. expenditure}} = \underbrace{\tau(\bar{z}_l e_l + \bar{z}_h e_h)}_{\text{gov. revenues}} \tag{22}$$

2.4 Worker flows

Workers flow between employment and unemployment status, skills and benefit entitlement. Within all the flows, we are interested in highlighting those where turbulence has an effect: the low-skilled unemployed with high benefits u_{lh} and the high-skilled employed e_{hh}. See the Appendix for more details on the flow expressions for other groups of workers.

The first group is the low-skilled unemployed with high benefits u_{lh}. They are at the core of the LS mechanism that generates larger unemployment duration as turbulence increases. Flows out of the u_{lh} group occur with retirement and successful job market encounters that end with an offer. Flows into this group occur in the following instances. Exogenous turbulence affects exclusively high-skilled workers e_{hh} that get laid off; with probability $\gamma^{d,x}$, they become part of the low-skilled workers with high benefit entitlement u_{lh}. Endogenous turbulence for the employed kicks-in with probability γ^d when a high-skilled worker e_{hh} rejects a productivity switch (i.e. she quits), and when a low-skilled worker (e_{ll} or e_{lh}) gets a human capital upgrade and reject the new offer (recall that benefits are upgraded immediately); finally, endogenous turbulence for high-skilled unemployed u_{ll} activates with probability γ^e after an unsuccessful job market encounter (we denote with ν^o_{hh} the rejection rate of high-skilled unemployed workers):

$$\begin{aligned}\Delta u_{lh} &= -\rho^r u_{lh} - (1-\rho^r)\lambda(\theta)(1-\nu_{lh})u_{lh} \\ &+ (1-\rho^r)\left\{\underbrace{\rho^x\gamma^{d,x}e_{hh}}_{\text{exogenous turbulence}} + \underbrace{(1-\rho^x)\gamma^d\nu_{hh}[\gamma^s e_{hh}+\gamma^u(e_{ll}+e_{lh})]}_{\text{end. turb. employed}} + \underbrace{\lambda(\theta)\gamma^e\nu^o_{hh}u_{hh}}_{\text{end. turb. unemployed}}\right\}\end{aligned} \tag{23}$$

As the mirror from the u_{lh} flows, turbulence also affects the flows in and our u_{hh}.

$$\begin{aligned}\Delta u_{hh} &= -\rho^r u_{hh} - (1-\rho^r)\lambda(\theta)[\gamma^e\nu^o_{hh} + (1-\nu^o_{hh})]u_{hh} \\ &+ (1-\rho^r)\left\{\rho^x(1-\gamma^{d,x})e_{hh} + (1-\rho^x)(1-\gamma^d)\nu_{hh}[\gamma^s e_{hh}+\gamma^u(e_{ll}+e_{lh})]\right\}\end{aligned} \tag{24}$$

The second group of key importance in the puzzle is the high-skilled employed e_{hh}. They are at the core of the dHHR mechanism that reduces unemployment with turbulence by reducing endogenous separations. High-skilled workers have very strong incentives to hold on to their jobs when they face a positive probability of skill loss, as well as high-skilled unemployed more willing to accept jobs. These forces are captured in lower rejection rates ν_{hh} and ν^o_{hh}, which in turn increase the flow into (or decrease the flow out of) high-skilled employed.

$$\Delta e_{hh} = -\rho^r e_{hh} + (1-\rho^r)\lambda(\theta)(1-\nu^o_{hh})u_{hh} \tag{25}$$

$$+ (1-\rho^r)\left\{(1-\rho^x)\gamma^u(1-\nu_{hh})(e_{ll}+e_{lh}) - (\rho^x + (1-\rho^x)\gamma^s\nu_{hh})e_{hh}\right\} \tag{26}$$

2.5 Calibration

We consider the semi-quarterly calibration of LS with an adjustment to the matching efficiency and vacancy creation cost that delivers a market tightness of $\theta = 1$ when turbulence is zero.[3] In contrast to the dHHR article, the parameterization constitutes a serious attempt to calibrate to trans-Atlantic employment observations.

[3] Under the original LS calibration $(A, \mu) = (0.45, 0.5)$, no layoff taxes and and no turbulence, the equilibrium market tightness is equal to $\theta = 0.9618$. We adjust the calibration in order to deliver a market tightness of 1, the same job finding rate for workers as in old calibration and a vacancy filling rate equal to worker job finding rate. Let $(\hat{A}, \hat{\mu})$ be a new calibration given by: $\hat{A} = \kappa^{1-\alpha}A, \quad \hat{\mu} = \kappa\mu$. By setting κ equal to the market tightness under the old calibration $\kappa = \theta = 0.9618$, the new calibration achieves the desired outcomes.

Table 1: Model Calibration

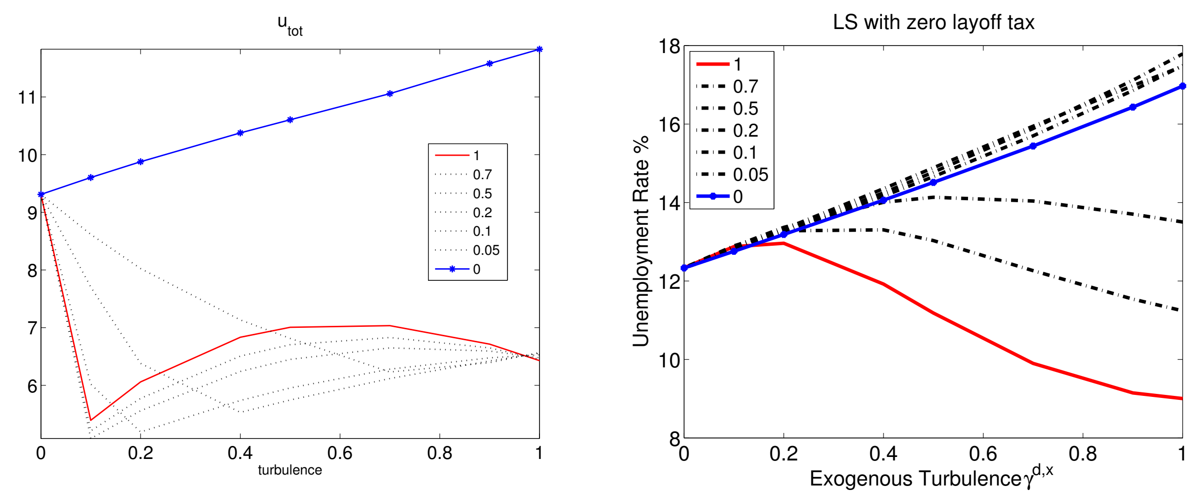
Parameter	**Definition**	**dHHR**	**LS**
Frequency		quarterly	semi-quarterly
Preferences			
$\hat{\beta}$	discount factor	0.995	0.99425
ρ^r	retirement probability	0.005	0.0031
$\beta = \hat{\beta}(1-\rho^r)$	adjusted discount	0.990	0.991
Risk sources			
ρ^x	exogenous break probability	0.01	0.005
γ^s	switch probability	0.1	0.05
γ^u	upgrade probability	0.025	0.0125
$\gamma^{d,x}$	exogenous turbulence	$\in [0,1]$	$\in [0,1]$
γ^d	endogenous turbulence	$[0, \gamma^{d,x}]$	$[0, \gamma^{d,x}]$
γ^e	endogenous turbulence	$[0, \gamma^d]$	$[0, \gamma^d]$
Labor market institutions			
π	worker bargaining	0.5	0.5
ϕ	replacement rate	0.7	0.7
Endogenous labor matching			
A	matching efficiency		0.441
α	elasticity of job finding		0.5
μ	flow cost of a vacancy		0.481
Productivity distributions			
$\mathbb{E}[z_l]$	low-skilled mean	1	1
$\mathbb{E}[z_h]$	high-skilled mean	2	2
$supp[z_l]$	low-skilled support	[0.5, 1.5]	[-1, 3]
$supp[z_h]$	high-skilled support	[1.5, 2.5]	[0, 4]
$std[z_l]$	low-skilled dispersion	$\frac{1}{\sqrt{12}}$	1
$std[z_h]$	high-skilled dispersion	$\frac{1}{\sqrt{12}}$	1

3 A Puzzle

Turbulence in dHHR05 We first document the dHHR puzzle: if voluntary quits are exposed to a tiny risk of skill loss, then higher turbulence leads to a reduction in unemployment. As a first step, we reproduce the results in dHHR with a minor but tractable change to the original dHHR model that does not affect our fundamental characterization of the puzzle but allow us to simplify our subsequent decomposition the puzzle. We let new entrants in the labor market be eligible for unemployment benefits equivalent to those of low skilled workers instead of zero benefits as in the original setup. This modification keeps down the number of classes of agents but has hardly any effect on aggregate outcomes.[4]

The left panel in Figure 1a shows the effect of turbulence, both exogenous and endogenous, on the unemployment rate for the modified dHHR model. As stated in their paper, the amount of endogenous turbulence needed in the dHHR framework to revert the unemployment-turbulence relationship is small. When endogenous turbulence is equal to 5% of exogenous turbulence ($\epsilon = 0.05$), unemployment falls as turbulence increases.

Figure 1: Turbulence and unemployment in dHHR (left) and LS (right) frameworks.



(a) Modified dHHR framework

(b) LS framework without layoff tax

Both figures use dHHR original calibration. The x-axis shows exogenous turbulence. Each line has a different endogenous turbulence, represented as a fraction ϵ of exogenous turbulence, where $\epsilon \in \{0, 0.05, 0.1, 0.2, 0.5, 0.7, 1\}$.

[4]In dHHR, new borns are entitled to benefits b_e which are set to zero. In LS, entrant benefits are assumed to be equal to b_l and there is no need to keep track of new workers, their productivity thresholds and rejection rates. We adjust worker flows and the government budget constraint to reflect this change.

Turbulence in LS07 Next, we introduce endogenous turbulence at voluntary quits in the LS model. Figure 1b shows the effect of turbulence, both exogenous and endogenous, on the unemployment rate for the calibration for the welfare state economy in LS, which sets layoff taxes to zero to make it compatible with the dHHR framework.[5]. We observe that endogenous turbulence needs to be very high, about 50% of exogenous turbulence, before the aggregate unemployment rate starts varying negatively with turbulence, and in such case, this negative effect only occurs at low levels of exogenous turbulence.

4 Key differences

Our strategy to uncover the resolution to the puzzle is to start with the LS model and make one perturbation at a time:

1. exogenous vs. endogenous matching
2. timing of consummation of skills
3. productivity distributions (functional form and support)
4. turbulence during job market encounters

In the Appendix, we start from the dHHR model and work through the perturbations in reverse.

4.1 First candidate: Differences in labor market tightness

The first candidate concerns differences in the matching framework. In LS, market tightness is endogenous and determined by the relative number of vacancies posted by firms and unemployed workers. In contrast, dHHR assumes a fixed and equal mass of workers and firms such that tightness always equal to unity.

From endogenous to exogenous matching Under exogenous matching, the mass of workers and firms is the same which implies $\theta = 1$ and matching rates $\lambda^w = \lambda^f = A$ for any level of turbulence. This means that the value of the firm with exogenous matching is:

$$\omega^f = \frac{1}{1-\beta}\left[-\mu + A\beta(1-\pi)\bar{s}(\gamma^{d,x})\right], \qquad \bar{s}(\gamma^{d,x}) \equiv \sum_{i,j}\left[\frac{u_{ij}}{u}\int_{\underline{z}_{ij}}^{\infty} s^o_{ij}(y)dv_i(y)\right]$$

[5] The quadrature method was not correctly implemented in that paper, however the distributions are still proper. Correcting first for the quadrature method and recalibrate the model yields the same results qualitatively.

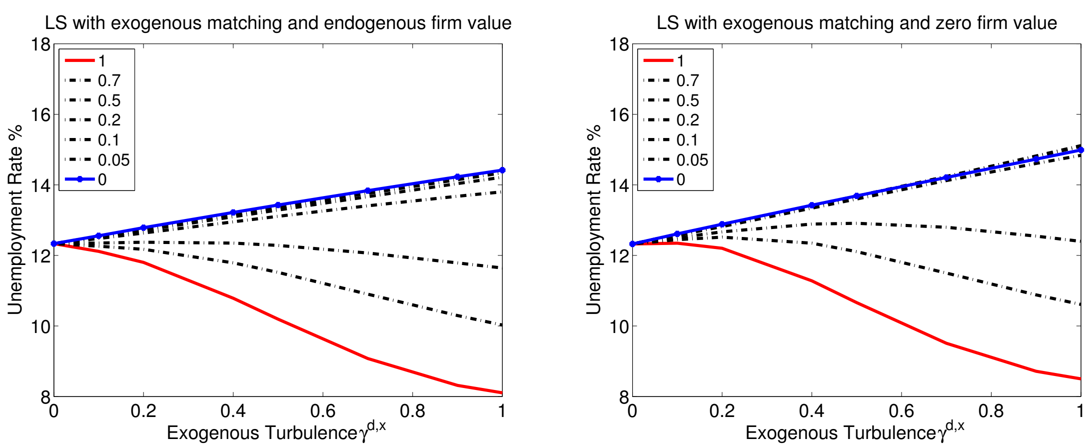
where $\bar{s}(\gamma^{d,x})$ is the expected surplus from a match for a level of turbulence $\gamma^{d,x}$. Fixing the vacancy creation cost to $\mu = A\beta(1-\pi)\bar{s}(0)$, we have that exogenous matching also features $\omega^f = 0$ for zero turbulence. For other levels of exogenous turbulence $\gamma^{d,x} \neq 0$, we will consider two alternatives:

1. Allow endogenous determination of firms' value ω^f, which changes with turbulence. Since turbulence reduces surpluses, ω^f will become negative as we increase turbulence. Given that in some cases firms would incur in losses, this experiment could be thought of a *subsidy to firm's profits.*

2. Fixed firms' outside value to zero ($\omega^f = 0$). This experiment could be thought of as a *subsidy to vacancy creation* so that firms break even when maintaining a constant market tightness (or levying a tax if the outside value turns positive for some initial range of turbulence). The value of the subsidy (tax) is given by: $1 - subsidy(\gamma^{d,x}) = \frac{A\beta(1-\pi)\bar{s}(\gamma^{d,x})}{\mu}$.

Results Figure 2 shows the unemployment response to turbulence for endogenous and exogenous firm value. Both scenarios show the same unemployment response to turbulence. Now, comparing either panel of Figure 2, with exogenous matching, with Figure 1b, with endogenous matching, we do not observe any differences. Thus we eliminate the type of matching as a potential candidate in explaining the puzzle.

Figure 2: Turbulence and unemployment in the LS model with exogenous matching.



(a) Endogenous firm value (b) Fixed firm value at $\omega^f = 0$

4.2 Second candidate: Differences in timing of consummation of skill upgrades

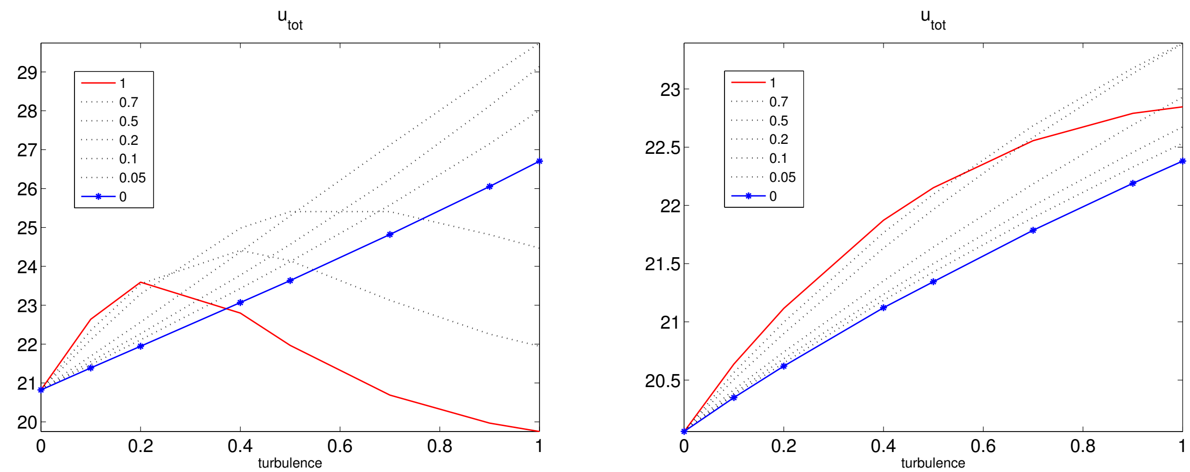
dHHR assume that a worker who received a positive skill shock at work must remain with the same firm for one period in order to consummate the higher skill level. Moreover, in order to guarantee that such continued relationships are privately optimal, it is assumed that a positive human capital shock is accompanied with a new productivity draw where the lower support is given by the endogenous reservation wage of a high-skilled worker in an ongoing employment relationship. The rationale for that assumption is to reduce the number of worker categories to be considered.

In contrast, the LS framework assumes that the skill upgrades are immediately consummated and it is assumed that upon skill upgrades workers immediately become entitled to high unemployment benefits (even if matches break up at skill upgrades). This assumption allows to limit the number of worker categories to be considered. Furthermore, it keeps the productivity distribution's support exogenous.

From immediate to delayed consummation of benefits Moving from immediate to delayed consummation of benefits generates new worker types and affects the flow equations between workers. In particular, it has consequential effects for wages. Even though wage determination under Nash bargaining is not relevant for the equilibrium determination, they are a useful tool to shed light on the mechanisms that affect the turbulence-unemployment relationship.

Results Figure 3 show the unemployment response to turbulence for the two cases. The left panel is our original framework. The right panels shows that the delayed timing actually works agains the dHHR story, increasing the correlation between unemployment with turbulence.

Figure 3: Turbulence and unemployment in LS for different timings of consummation of benefits



(a) Immediate benefit upgrade

(b) Delayed benefit upgrade

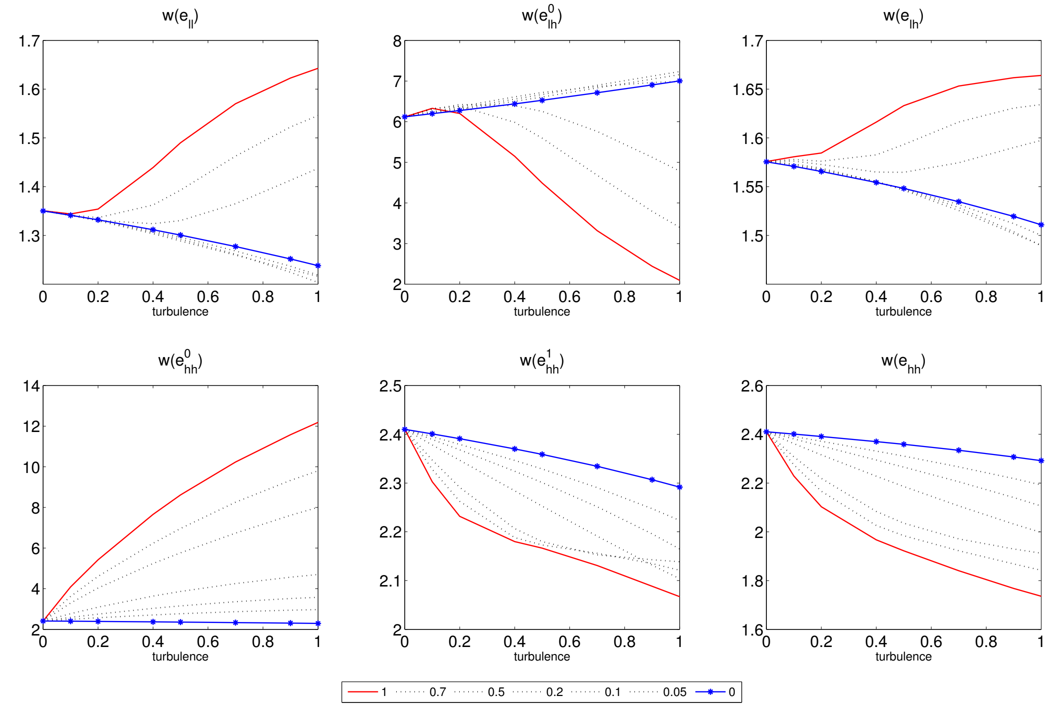
A "ransom" interpretation Why does delayed benefit upgrade à la dHHR actually work against them when we perturb the LS model, i.e., the turbulence story is strengthened under the timing assumption of dHHR, relative to LS's assumption of immediate benefit upgrade? As we noted under the assumption of delayed upgrade, the reservation productivities of unemployed workers with low skill/high benefit as well as those with high skill/high benefit monotonically increase with endogenous turbulence. Therefore, those unemployed workers' reluctance to accept new jobs makes unemployment to robustly increase with turbulence.

Firms under dHHR's timing assumption are able to "rip off" workers whenever they transition from low to high skill at work, because the consummation of that higher skill is conditional upon a worker remaining with his current employer for at least one more period, in which the firm can "steal" parts of the worker's capital gain as reflected in a negative Nash-bargained wage during that period of skill consummation.

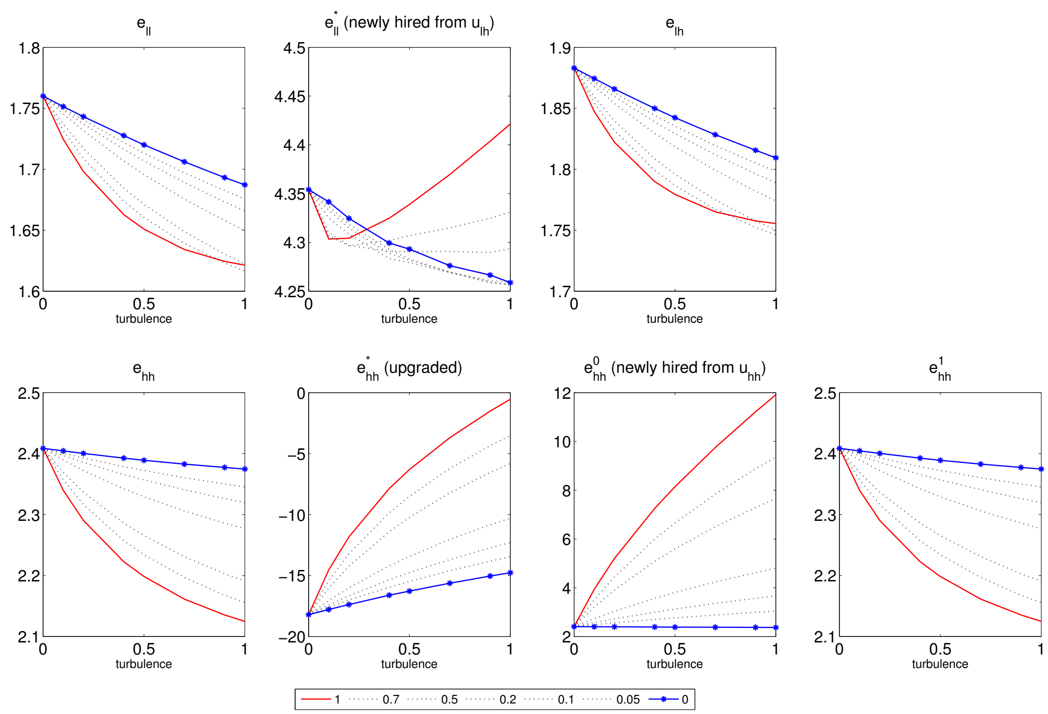
So returning to a currently unemployed worker with low-skills and high-benefits who fully anticipates that a future employer will indeed steal the capital gain of any future skill upgrade, the return to accepting a job is diminished and it looks more attractive to remain unemployed and collect high benefits. A similar argument, although not as strong, applies to a currently unemployed worker with high skills and high benefit, for whom the prospect of living on high benefits becomes more advantageous and will only be given up for a new job if the productivity is higher (as compared to LS?s timing assumption under which the capital gain of future skill dynamics are shielded from theft by employers).

We compare wages across immediate and delayed upgrade of benefits.

Figure 4: Wages and turbulence for different timings of consummation of benefits



(a) Wages for immediate upgrade

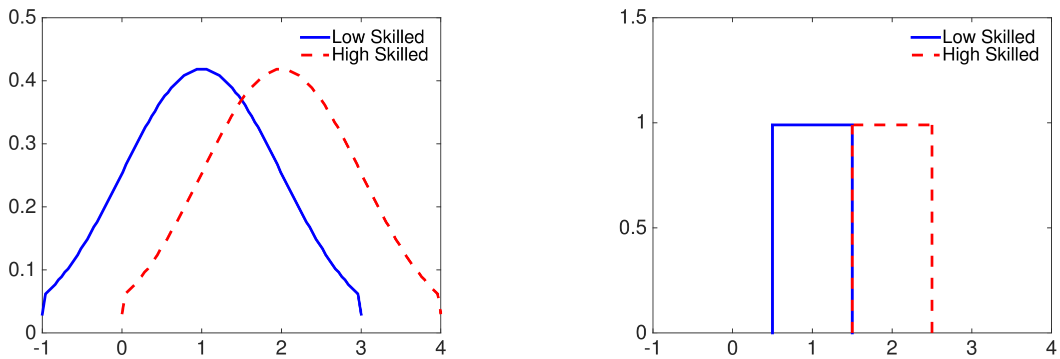


(b) Wages for delayed upgrade

4.3 Third candidate: Productivity distributions

dHHR assume uniform productivity distributions with small support while LS postulate truncated normal distributions with wide support. We explore the differences both in the shapes of the distributions as in the support. The original LS model assumes productivities are drawn from truncated normals $z_l \sim \mathcal{N}(1,1)$ for low skilled workers over the support $[-1,3]$ and $z_h \sim \mathcal{N}(2,1)$ for high skilled workers over the support $[0,4]$. There is an overlap over the range $[0,3]$ which represents 75% each support. In contrast, dHHR assumes uniform distributions $z_l \sim \mathcal{U}([0.5,1.5])$ and $z_h \sim \mathcal{U}([1.5,2.5])$ which implies no overlap. In both frameworks, the means are 1 for low skilled and 2 for high skilled; the dispersion is 1 in the Normal case and $1/\sqrt{12}$ in the Uniform case.

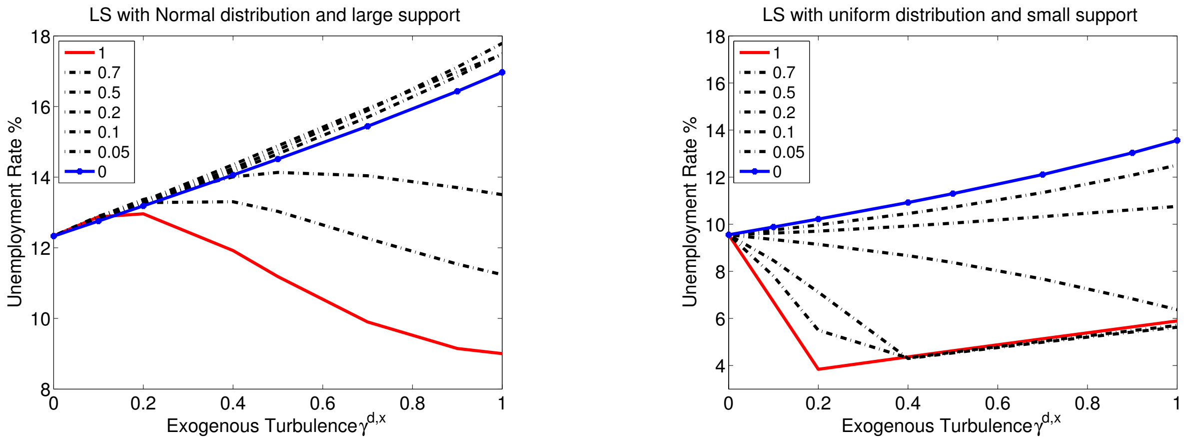
Figure 5: Two alternative productivity distributions



(a) Normal with wide support (b) Uniform with narrow support

Results Figure 6 shows the unemployment response to turbulence for the two alternative productivity distributions. The left panel is the original LS framework. The right panels shows that under the uniform distribution with narrow support, the dHHR force kicks in. Productivity draws on the job create reasons for worker reallocation in order to reproduce observed inflow rates into unemployment, while all prospects for long-run earnings growth are tied to human capital accumulation through learning-by-doing that takes place regardless of firm and current productivity. The implied value of mobility under the narrow support is very low.

Figure 6: Turbulence and unemployment in LS model with wide support



(a) Normal with wide support

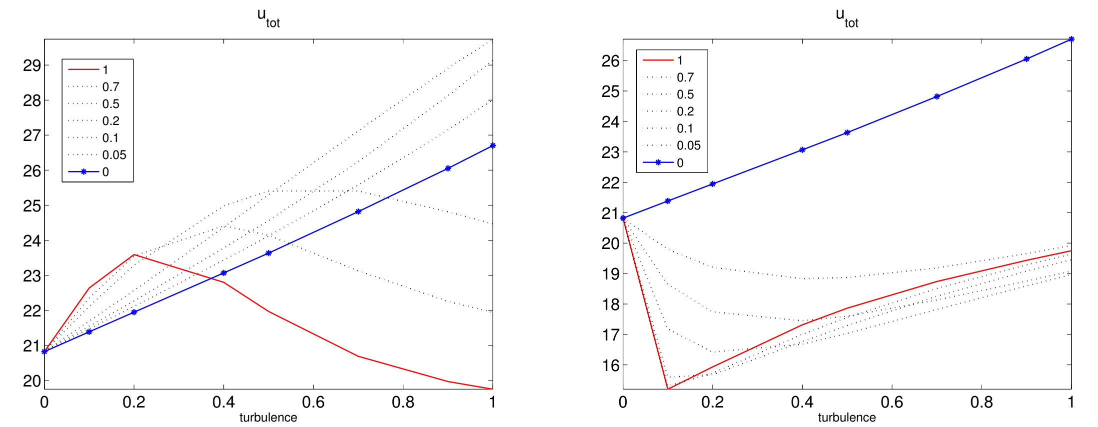
(b) Uniform with narrow support

4.4 Fourth candidate: Turbulence affecting job market encounters

The following results show different outcomes for two cases: $\gamma^e = 0$, which means that job market encounters do not pose any risk for unemployed workers with high skills; and $\gamma^e = \gamma^d$, which means that high-skilled unemployed workers that reject a job offer face the same risk of skill loss that an employed worker with high skills.

Results Figure 7 plots the unemployment-turbulence relationship for the two extreme cases, $\gamma^e = 0$ and $\gamma^e = \gamma^d$. We find that adding the exposure of the unemployed to turbulence through job market encounters has a sizeable effect on unemployment. Therefore, the "simplifying" assumption by dHHR is actually not innocuous. An unemployed high-skilled worker faces additional significant risk of skill loss after each period of unemployment. Using the metric of equilibrium outcomes at zero turbulence in the LS analysis, an unemployed worker has a match probability of around 0.5 at a semi-quarterly frequency and a rejection rate of around 0.5 (implying an average unemployment duration of 4 model periods, i.e., two quarters). Thus, an unemployed high-skilled worker faces a probability 0.5*0.5=0.25 of being exposed to endogenous turbulence after each period of unemployment. And this skill-loss risk is in addition to the exposure to endogenous turbulence at the separation itself. No wonder that the curves fall off sharply in the second figure above.

Figure 7: Turbulence and unemployment for different assumptions about job market encounters



(a) Unemployment rate assuming $\gamma^e = 0$ (b) Unemployment rate assuming $\gamma^e = \gamma^d$

4.5 Solution: Productivity distribution plus turbulence risk for unemployed

As the final resolution of the puzzle, starting from the LS framework, we add turbulence risk for the unemployed $\gamma^e = \gamma^d$ and then compare the unemployment-turbulence relationship under the two alternative productivity distributions. Under the productivity distribution with narrow support, which has a small implied value of labor mobility in tranquil times, turbulence and unemployment are negatively related as in dHHR.

Figure 8: Turbulence and unemployment in the dHHR model

(a) Uniform with narrow support and $\gamma^e = 0$ (b) Uniform with narrow support and $\gamma^e = \gamma^d$

Compare Figures 8a and 8b. Both cases assume Uniform distribution with narrow support. The difference lies in that the first one assumes no turbulence risk for unemployed $\gamma^e = 0$ whereas the second assumes symmetric turbulence risk for unemployed $\gamma^e = \gamma^d$. We see that the solution to the puzzles lies in the interaction between Candidates 3 and 4, which together determine a very low value of labor mobility in tranquil times.

5 Introducing layoff taxes

We show the fickleness of the dHHR calibration by introducing layoff taxes into both frameworks. We introduce a fixed layoff tax Ω that affects currently employed workers of all skill levels. The layoff tax applies to every endogenous and exogenous separation, except for retirement, and firms are fully liable of its payment. This introduces changes in the surpluses and the wage calculations. When computing wages we will assume that once a worker is hired, firms are the only ones liable for the layoff cost. This generates a two-tier wage system a la Mortensen and Pissarides (1999)[6].

It has been shown in other setups that the timing of the payment of the layoff tax is innocuous. The conjecture is that the wage profile is not important in this setup either, but given that the benefits are computed according to the average wages, the wage profile *might* have an effect on equilibrium outcomes.

Layoff taxes in matching models Ljungqvist (2002) studies the relationship between layoff taxes and unemployment across various frameworks. Regardless of the bargaining protocol, an increase in layoff taxes generates lower reservation productivity and less layoffs, which reduces output per worker; less layoffs but larger total layoff costs as fraction of GNP are increased; and a reduction in the probability of finding a job. This effect is extremely large in matching models with firm-liable tax.

The effect of layoff taxes on unemployment depends crucially on how the layoff tax enters the bargaining protocol between firms and workers. If layoff costs do not affect the relative share of the match surplus: slower reallocation of labor and increase in job tenure *decrease* unemployment rate. On the contrary, in matching models where the taxes affect bargaining and affect the relative share of the match surplus, layoff taxes increase unemployment. The main force is the zero profit condition for vacancy creation in the second. Firms surplus is eroded by the layoff tax and in order to finance vacancy and layoff costs, there is an *increase* in unemployment that increases the vacancy filling rate and lowers the bargaining position of workers.

[6]The risk neutral firm and worker would be indifferent between adhering to this two-tier wage system or one in which workers receive a fraction π of the match surplus s_{ij} in every period (which would have the worker paying a share π of any future layoff tax). As emphasized by Ljungqvist (2002), the wage profile, not the allocation, is affected by the two-tier wage system. Optimal reservation productivities remain the same. Under the two-tier wage system, a newly hired worker in effect posts a bond that equals his share of the future layoff tax

5.1 Layoff taxes in LS

The introduction of layoff taxes Ω in LS introduces a wedge between the joint surplus of a continuing match and a new match. However, this wedge only affects the reservation productivities for ongoing matches that most take into account the tax:

$$s_{ij}(\underline{z}_{ij}) = -\Omega$$

To implement the layoff tax, we set the firm's threat point to $-\Omega$ in future Nash Bargaining protocols. This introduces a two-tier wage scheme for new entrants and incumbent workers: $p^o_{ij}(z)$ vs. $p_{ij}(z)$.

Two-tier wage scheme Considering layoff costs, the wage $p_{ij}(z)$ solves the following problem:

$$\max_{p_{ij}(z)} \quad \left((1-\tau)z - p_{ij}(z) + g^f(z) - \omega^f + \Omega \mathbb{1}_{\{current\}}\right)^{1-\pi} \left(p + g^w_j(z) - b_j - \omega^w_{ij}\right)^{\pi}$$

where g^f and g^w are the future values for firm and worker respectively ($g = g^f + g^w$) and ω^w and $\omega^f - \mathbb{1}_{\{current\}}\Omega$ their future outside values. The indicator function is equal to 1 for all workers that are currently employed and 0 for newly employed workers. This problem has the solution:

$$p_{ij}(z) \quad = \quad \pi\left[s_{ij}(z) + \Omega \mathbb{1}_{\{current\}}\right] + b_j + \omega^w_{ij} - g^w_i(z)$$

where the future value of the worker is given by[7]

$$g^w_i(z) = \pi g_i(z) - \beta\pi(b_j + \omega^{w\prime}_{ij} + \omega^{f\prime} - \Omega) + \beta(b_j + \omega^{w\prime}_{ij})$$

Government revenue With the introduction layoff tax, the government's revenue now includes also the layoff costs collected:

$$revenue = (1-\tau)\bar{p} + s_{tot}\,\Omega$$

[7]The definition of ω^f does not include the layoff cost (in the endogenous matching framework this value is equal to zero and in the exogenous matching this value is equal to the firm's capital value). This is helpful for programming purposes. Also note that in the expression for the future value of a worker $g^w_i(z)$, the outside options considered are future outside options (two periods ahead) and therefore the layoff cost is always present (does not matter if the bargaining is with a new or current match) because it is the future outside value of a firm that has hired.

where $\bar{p}$ are the weighted average wages and s_{tot} are total separations (exogenous and endogenous) except retirement. To compute the income tax rate, we set revenues equal to benefits:

$$(1-\tau)\bar{p} + s_{tot}\,\Omega = benefits \quad \Longrightarrow \quad \tau = 1 - \frac{benefits - s_{tot}\,\Omega}{\bar{p}}$$

If layoff costs cover all the benefits ($s_{tot}\Omega \geq benefits$) then there is no need for income tax (we do not consider subsidies) and we set $\tau = 0$.

Results Figure 9 shows the unemployment rate, rejection rates and labor flows as a function of layoff costs $\Omega \in [0, 40]$ in our framework. The unemployment rate for all types of workers falls as the layoff cost increases. High-skilled employed workers are particularly affected by the layoff cost as their rejection rates fall dramatically. However, the mobility of these workers shuts down after the layoff cost has reached 30, which is equivalent to $XX\%$ of their yearly salary, an implausible number. Lastly, the separation rate falls from 3% to 1%.

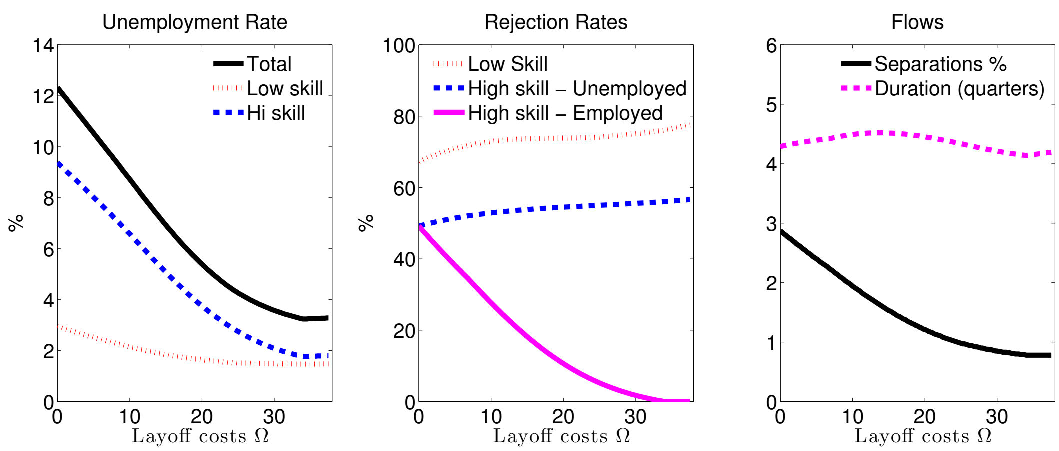


Figure 9: Unemployment and layoff taxes LS framework.The x-axis shows layoff taxes.

5.2 Layoff taxes in dHHR

We introduce a layoff cost Ω in the modified dHHR framework with the two inconsequential differences: (i) The new born workers receive the same benefits as low skilled workers (instead of zero) and (ii) the benefit status is updated immediately (instead of requiring one period to update). We use the exact same dHHR quarterly calibration with uniform distribution and small support.

Figure 10 shows the effect of increasing layoff costs on equilibrium outcomes for the dHHR framework. Mobility of high-skilled employed shuts down at $\Omega = 0.92$, which is equivalent to 70% of their quarterly wage. We confirm find that the dHHR parameterization implies minuscule incentives for labor mobility in tranquil times. A tiny government mandated layoff cost (or any small cost to mobility, such as endogenous turbulence) has counterfactually large effects of suppressing unemployment by shutting down voluntary separations.

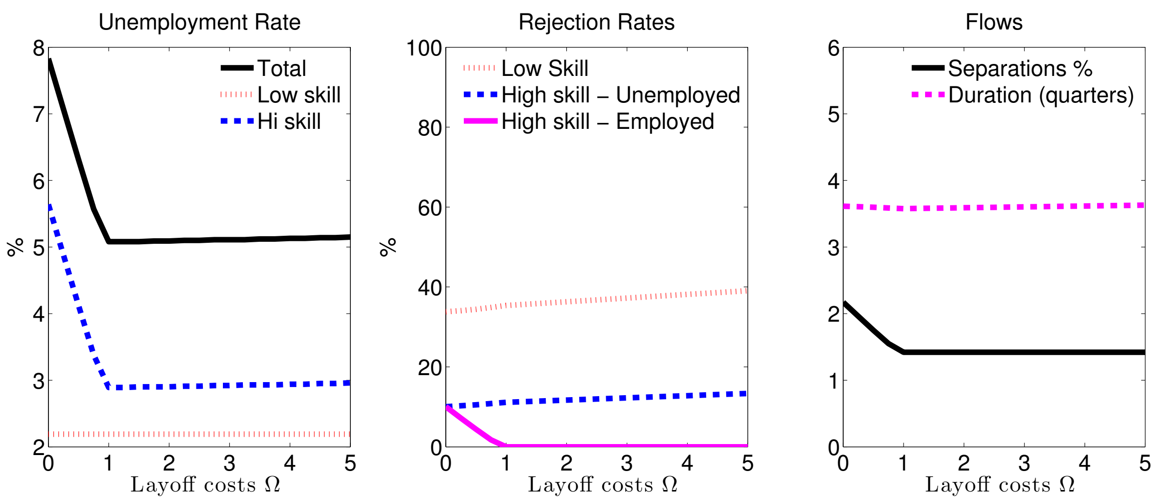


Figure 10: Unemployment and layoff taxes in dHHR framework.The x-axis shows layoff taxes.

6 Cost-Benefit Analysis of Labor Mobility

As an additional way to shed light on the puzzle resolution, we perform a cost-benefit analysis regarding incentives for labor mobility. Recall that the joint (for firm and worker) continuation value of a relationship with a worker with current productivity z and skills j, $g_i(z)$ can be approximated linearly

$$g_i(z) = \alpha_j + \beta_j z$$

The slope coefficient β_j on productivity represents the extra value that is to be had from accepting the present productivity that is expected to last for some given duration (that depends on the probability of drawing a new productivity at work γ^s, the probability of upgrade γ^u as well as the probability for being exogenously laid off ρ^x). Turbulence does not affect the slope directly, only through its effect on the tax rate.

The intercept represents the capital value of a worker of a particular skill, not including the described extra value associated with the present productivity draw. Turbulence does affect the intercept α_h. In the LS framework, it affects the continuation value after a rejected productivity switch; in the dHHR framework, there is an additional effect that amplifies the costs. Since unemployed workers also face turbulence risk in their setup, the outside option of high-skilled workers is also reduced. See Appendix C for expressions for α and β and details on the accuracy of the approximation.

We use this approximation to compute benefits and costs for a high-skilled worker to quit her job in search for better opportunities. We then measure labor mobility as the mass of workers that engage in this behavior within a given level of turbulence.

Costs of labor mobility For a high skilled worker, the cost of quitting and search for a new job is the risk to lose skills which affects the capital value of a worker, which is incorporated on the intercept α_j. Therefore, the cost is measured as the difference between the intercepts of the g functions for high and low workers *times* the level of endogenous turbulence:

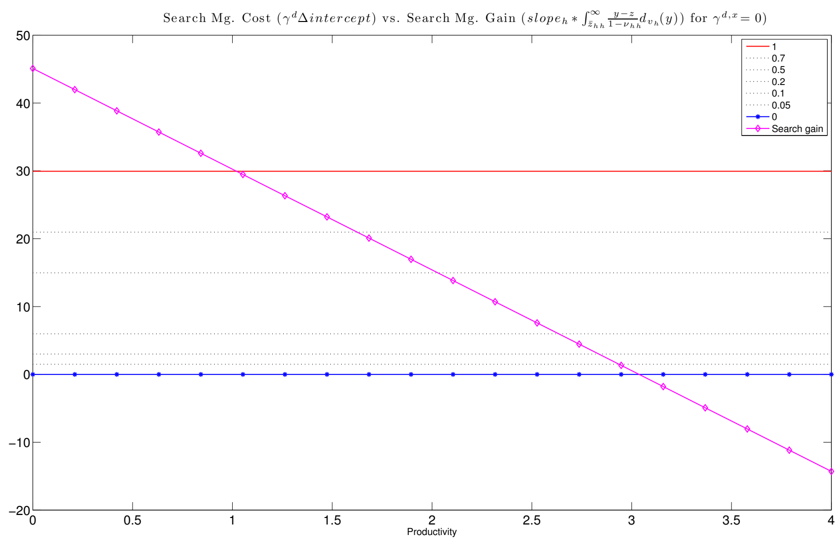
$$costs = \gamma^d(\alpha_h - \alpha_l)$$

Benefits of labor mobility For a high skilled worker, the benefit of quitting and search for a new job is the possibility to draw a better productivity that will last certain amount of time as measured by the slope of g. Therefore, the gain is measured as the expected productivity draw (conditional on acceptance) above the current level of productivity z *times* the slope of g_h:

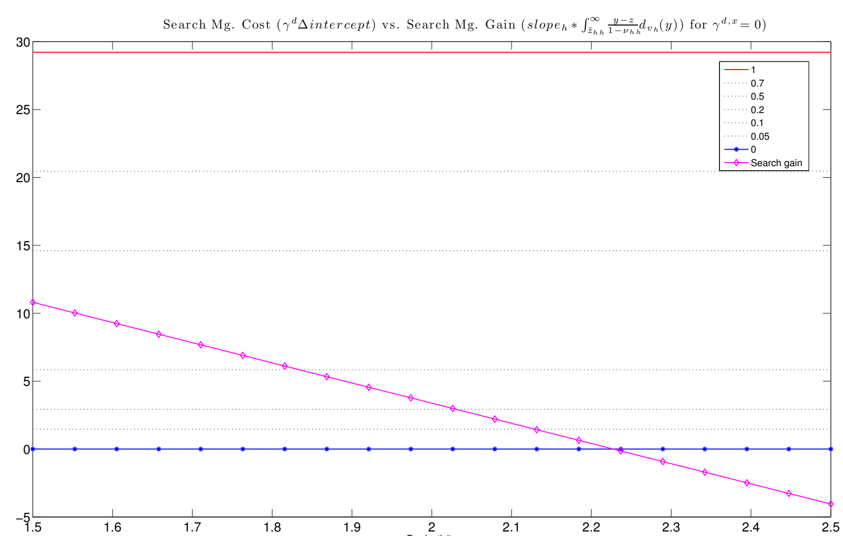
$$benefit(z) = \beta_h \left[\int_{\bar{z}_{hh}} \frac{y - z}{1 - \nu_{hh}} dv_h(y) \right] = \beta_h \left(\mathbb{E}[y | y \geq \bar{z}_{hh}] - z \right)$$

Figures plot the cost and benefit of labor mobility for a high-skilled worker as a function of current productivity z for different levels of endogenous turbulence γ^d. We freeze the equilibrium outcomes at $\gamma^{d,x} = \gamma^d = 0$, which implies that $\alpha_h, \alpha_l, \beta_h, \mathbb{E}[y|y \geq \bar{z}_{hh}]$ are fixed. The downward sloping line measures the benefit of quitting, which decreases as current productivity increases. The horizontal lines measure the costs of quitting the job, which is increasing in turbulence γ^d. The intersection of the benefit and the cost defines the productivity level z^* such that quitting is optimal for all $z < z^*$.

Figure 11: Marginal cost and benefit of labor mobility for a high skilled worker.



(a) Normal with wide support $\mathcal{N}[0, 4]$



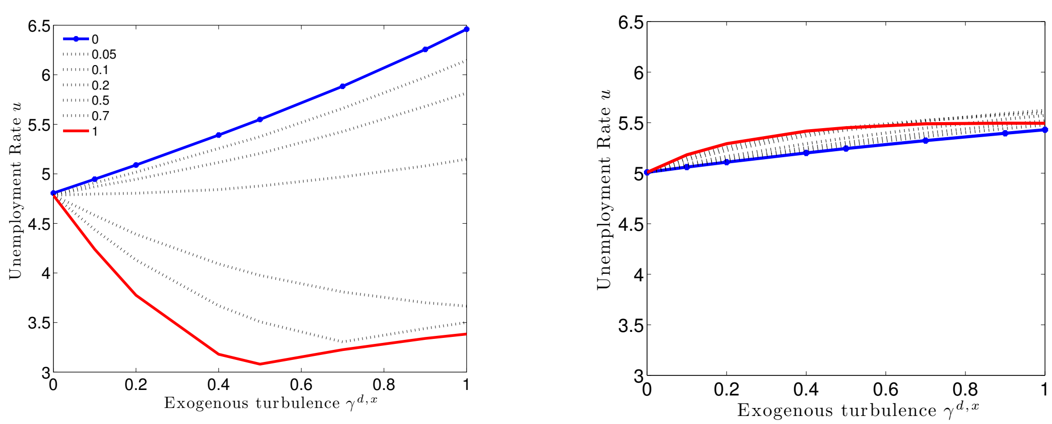
(b) Uniform with narrow support $\mathcal{U}[1.5, 2.5]$

Figure 11a focuses on the LS framework. Observe that for all levels of endogenous turbulence, z^* falls within the support of the distribution. Even for the highest level of endogenous turbulence, there is a fraction of workers that engage in quitting in the hope to get a better productivity draw. There are large incentives for labor mobility. Figure 11b focuses on the dHHR framework. In this case, high-skilled workers completely stop quitting when endogenous turbulence reaches 35% of exogenous turbulence. Also note that the cost seems to increase non-linearly, i.e. the horizontal lines move up faster as turbulence increases, compared to the LS case. The reason is the additional cost imposed to high-skilled workers in the dHHR setup by affecting their outside option as described above.

7 Welfare State vs. Laissez-Faire

In this section we compare the effect of endogenous turbulence in unemployment rates for the welfare state and laissez-faire economy. The following figures are generated within the original LS framework.[8]. For the welfare state parameters, we use a replacement rate of $\phi = 0.7$ and a layoff cost of $\Omega = 35$. This choice yields a welfare state unemployment rate of 4.8% at zero turbulence. Endogenous turbulence only affects currently employed workers and it is parametrized as a fraction ϵ of exogenous turbulence.

Figure 12: Turbulence and unemployment for different institutional arrangements



(a) Welfare state $(\phi, \Omega) = (0.7, 35)$

(b) Laissez-faire $(\phi, \Omega) = (0, 0)$

As expected, the effects of turbulence in the laissez-faire economy are miniscule. The institutions in the welfare state—the layoff tax and the large replacement rate—amplify the effects of turbulence, both exogenous and endogenous.

[8]Two modifications: the quadrature method is corrected and the matching efficiency parameter is re-calibrated to $A = 2.7$ such that the unemployment rate in the laissez-faire economy equals $u = 5\%$

8 Conclusion

We have shown that the negative turbulence-unemployment relationship is robust to introducing the risk of losing human capital for workers that quit. Once the parameterization is adjusted to account for the unemployment dynamics in data, the positive relationship between turbulence and unemployment reemerges.

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A Starting from dHHR framework

We now work in reverse the perturbations, starting from the dHHR framework. Recall that their original framework features exogenous labor market tightness, delayed consummation of skill upgrade, a uniform productivity distribution with narrow support, and assume that high-skilled unemployed workers face the turbulence risk after a job market encounter.

A.1 First candidate: Differences in labor market tightness.

In the dHHR framework, the value of the firm does not take into account the vacancy creation cost μ and thus its value is always non-negative (Eq. 11 in dHHR): $w^f = A\beta(1-\pi)\bar{s}(\gamma^d) + \beta w^f$, where $\bar{s}(\gamma^d) \equiv \sum_{i,j}\left[\frac{u_{ij}}{u}\int_{\underline{z}_{ij}}^{\infty} s^o_{ij}(y)dv_i(y)\right]$ is the expected surplus from a match. In order to implement the endogenous matching framework in which firms post vacancies, we first change the value of the firm to include a vacancy creation cost.

$$w^f = -\mu + A\beta(1-\pi)\bar{s}(\gamma^d) + \beta w^f$$

Secondly, we calibrate μ such that under exogenous matching, the value of the firm is equal to zero at zero turbulence, i.e. $w^f = 0$ if $\gamma^d = 0$, which yields $\mu = A\beta(1-\pi)\bar{s}(0) = 0.617$. We then fix that value of $\mu = 0.617$ as we introduce endogenous matching. In this way, the value of the firm is zero under both matching assumptions at zero turbulence. Under endogenous matching and free entry, market tightness is given by the following expression:

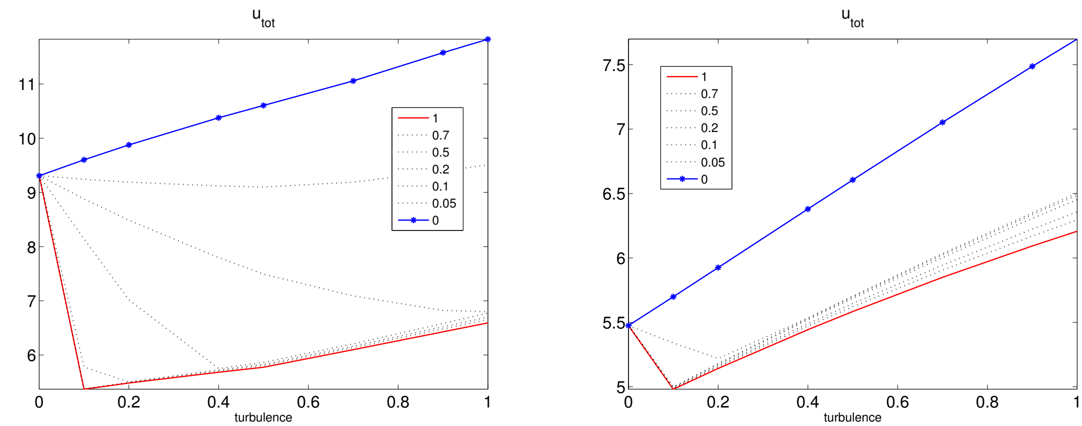
$$\theta = \left(\frac{A}{\mu}\beta(1-\pi)\bar{s}(\gamma^d)\right)^{1/\alpha} = \left(\frac{\bar{s}(\gamma^d)}{\bar{s}(0)}\right)^{1/\alpha}$$

Since $\mu = A\beta(1-\pi)\bar{s}(0)$, we have that at zero turbulence $\gamma^d = 0$ the matching outcomes are equal in both cases, namely $\theta = 1$, $\lambda^w = A$ and $\lambda^f = A\frac{u_{ij}}{u}$.

Figure 13 shows the unemployment-turbulence relationship for the dHHR framework under the two alternative matching frameworks. In both cases, a tiny exposure to endogenous turbulence reduces unemployment significantly. Even more, we find that the effect is stronger under endogenous matching.

Note that the unemployment rate differs at zero turbulence across the two matching frameworks. The reason is that with endogenous matching the value of the firm is equal to zero, whereas in the exogenous matching the value of the firm is large and positive, reducing the surplus, changing productivity thresholds, rejection rates, and other endogenous objects. We conclude, as before, that the type of matching is irrelevant for the disparate outcomes across frameworks.

Figure 13: Turbulence and unemployment in the dHHR model with alternative matching



(a) Unemployment rate with exogenous matching

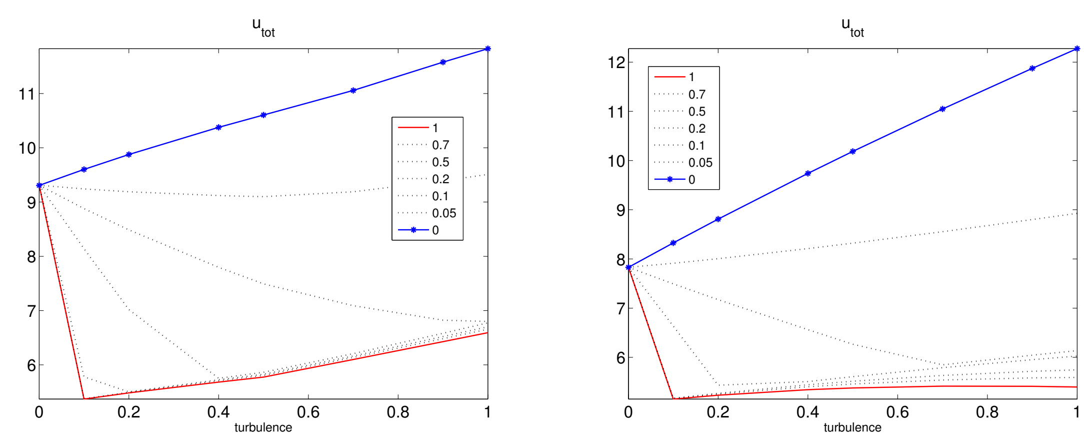
(b) Unemployment rate with endogenous matching

A.2 Second candidate: Differences in timing of consummation of skill upgrades

dHHR assume that after a skill upgrade, one period has to pass before workers are entitled to high benefits. In this section we introduce immediate benefit upgrade as in LS. This change reduces the number of workers that need to be tracked.

Figure 14 shows that there is no big difference between the timing in the dHHR. Recall the ransom interpretation in Section 4.2, that showed how wages became negative for recently upgraded high-skilled workers as the firms extract the surplus. Show that this is not the case here.

Figure 14: Turbulence and unemployment in the dHHR model with alternative consummation of skills



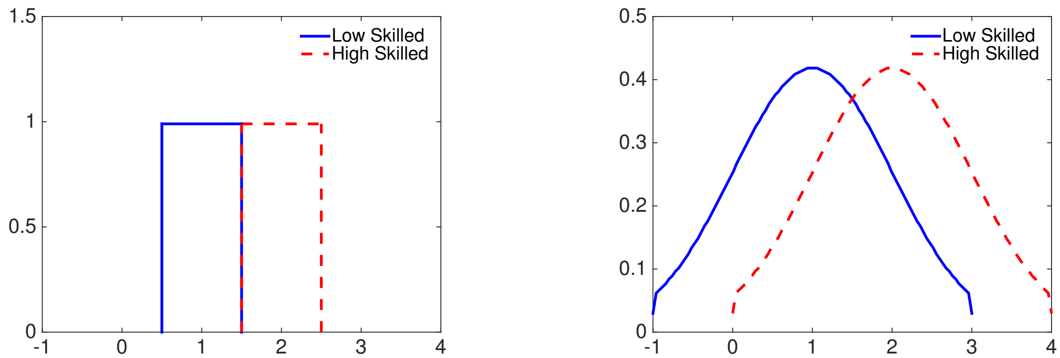
(a) Delayed consummation of benefits

(b) Immediate consummation of benefits

A.3 Third candidate: Productivity distributions

dHHR assumes uniform distributions with narrow support $z_l \sim \mathcal{U}([0.5, 1.5])$ and $z_h \sim \mathcal{U}([1.5, 2.5])$. In contrast, LS assume productivities are drawn from truncated normals $z_l \sim \mathcal{N}(1, 1)$ for low skilled workers over the support $[-1, 3]$ and $z_h \sim \mathcal{N}(2, 1)$ for high skilled workers over the support $[0, 4]$. In both frameworks, the means are 1 for low skilled and 2 for high skilled; the dispersion is 1 in the Normal case and $1/\sqrt{12}$ in the Uniform case.

Figure 15: Two alternative productivity distributions

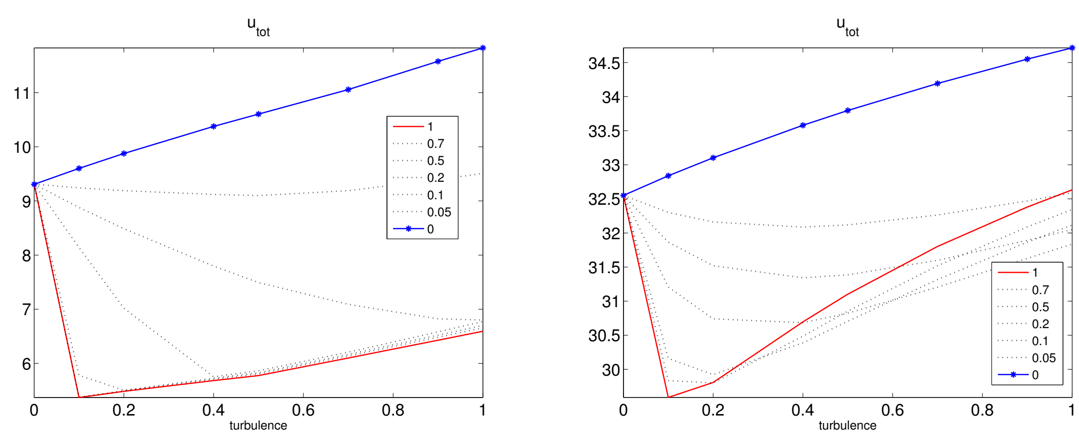


(a) Uniform with narrow support and no overlap.

(b) Normal with wide support and overlap.

Figure 16 shows the results for the original calibration of the productivity distribution and the alternative with wider support. We observe that...

Figure 16: Turbulence and unemployment in the dHHR model with alternative productivity distributions



(a) Uniform distribution with narrow support

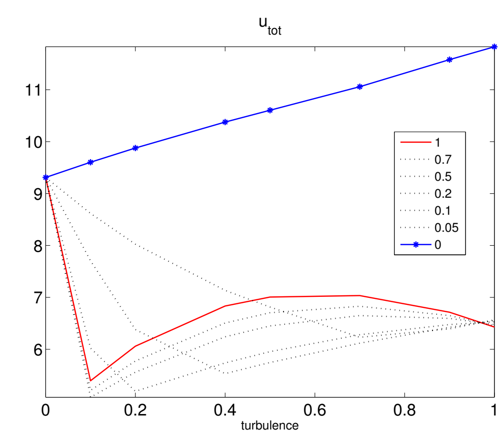
(b) Normal distribution with wide support

A.4 Fourth candidate: Turbulence affecting job market encounters.

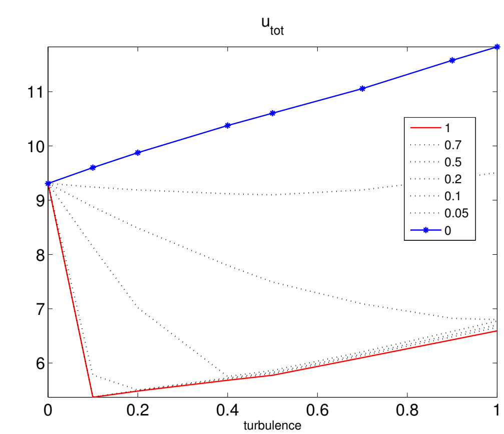
dHHR assume that after an encounter between a firm and an unemployed worker which does not result in a match being formed, the worker faces the same risk of losing human capital as if she is experiencing an endogenous separation from an ongoing employment relationship. They claim that this assumption is only for simplification as it allows to reduce the number of workers to keep track off. However, as we have shown above, this change is rather substantive and crucial for the explanation of the puzzle. This assumption works on their favor. We explore the alternative in which mere encounters of firms and workers in the labor market would not expose the latter ones the risk of losing skills.

Figure 17 presents results for the original dHHR framework with $\gamma^e = \gamma^d$ and our modified version with $\gamma^e = 0$. It can be observed that this additional risk does not change unemployment in a meaningful way. Clearly, dHHR unintended inclusion of additional exposure to endogenous turbulence by assuming that mere encounters between vacancies and unemployed workers are associated with the risk of losing human capital, will dampen rewards to labor mobility of high-skilled workers. But coupled with the compressed productivity distributions in dHHR, there is not much added harm from such a suppression. In contrast, the significant reasons for labor mobility coming from LS?s choice of productivity distributions, are indeed severely impacted and suppressed by that auxiliary assumption of dHHR, as shown in Figure 7.

Figure 17: Turbulence and unemployment in the dHHR model and turbulence for the unemployed



(a) Symmetric turbulence risk for unemployed $\gamma^e = \gamma^d$ (Original dHHR)

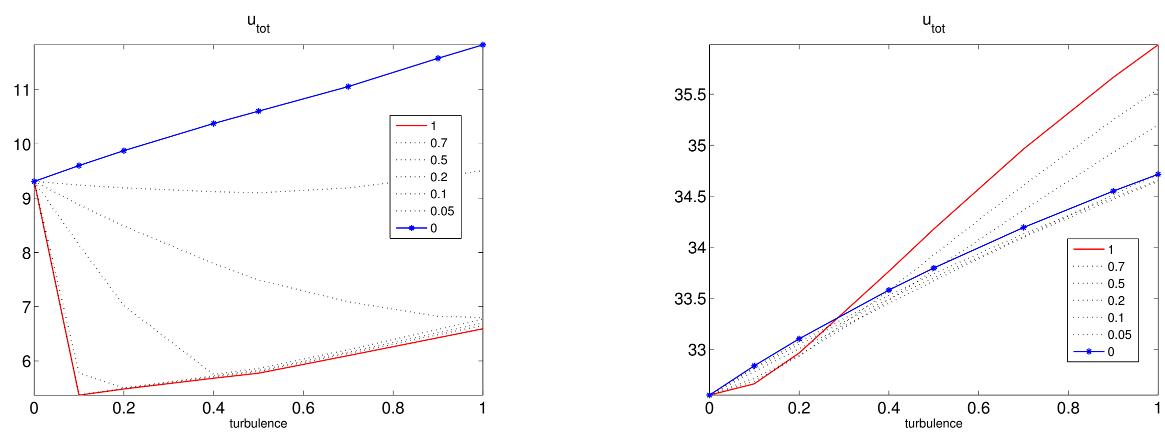


(b) No turbulence risk for unemployed $\gamma^e = 0$ (Our modified version of dHHR)

A.5 Solution: Productivity distribution plus turbulence risk for unemployed

As the final resolution of the puzzle, starting from the dHHR framework, we eliminate the turbulence risk for the unemployed $\gamma^e = 0$ and then compare the unemployment-turbulence relationship under the two alternative productivity distributions. Under the productivity distribution with wide support, which has a large implied value of labor mobility in tranquil times, turbulence and unemployment are positive related. Note that the unemployment rate is more than three times higher under the new calibration.

Figure 18: Turbulence and unemployment in the dHHR model



(a) Uniform with narrow support and $\gamma^e = 0$ (b) Normal with wide support and $\gamma^e = 0$

Compare Figures 16b and 18b. Both cases assume a Normal distribution with wide support. The difference lies in that the first assumes symmetric turbulence risk for unemployed $\gamma^e = \gamma^d$ whereas the second does not. We see that the solution to the puzzles lies in the interaction between Candidates 3 and 4, which together determine the value of labor mobility in tranquil times.

B General algorithm structure

The algorithm approximates continuation values of joint surpluses $g_i(z)$ using a **quadratic** polynomial on a productivity grid: $\hat{g}_i(z) = c_i^1 + c_i^2 z + c_i^3 z^2 = [1\ z\ z^2]c_i$

1. Guess initial values for:

   - b_j: benefits
   - ω_{ij}^w: outside value of workers after benefits
   - ω^f: firms' outside option (in LS, we guess an initial tightness θ?)
   - τ: tax rate
   - c_k: coefficients for approximation
   - u_{ij}, e_{ij}: masses of unemployed and wokers

2. Compute cutoffs $\underline{z}_{ij}$ such that $s_{ij}(\underline{z}_{ij}) = 0$ using a **linear** approximation for g (ignore the quadratic term).

$$
\begin{aligned}
s_{ij}(\underline{z}_{ij}) &= 0 \\
(1-\tau)\underline{z}_{ij} + \hat{g}_i(\underline{z}_{ij}) - b_j - \omega_{ij} &= 0 \\
(1-\tau)\underline{z}_{ij} + c_j^1 + c_j^2 \underline{z}_{ij} - b_j - \omega_{ij} &= 0 \\
\underline{z}_{ij} &= \frac{b_j + \omega_{ij}^w - c_j^1}{(1-\tau+c_j^2)}
\end{aligned}
$$

   where $\tau = 0$ if $b_j + \omega_{ij}^w - c_j^1 < 0$.

   - When cutoff is out of the grid ($\underline{z}_{ij} < z_{min}$ or $\underline{z}_{ij} > z_{max}$), then evaluate $\hat{g}(z_{min})$ or $\hat{g}(z_{max})$ respectively and solve as:

$$
\underline{z}_{ij} = \frac{b_j + \omega_{ij}^w - c_j^1 - c_j^2 z_{min}}{(1-\tau)}, \qquad or \qquad \underline{z}_{ij} = \frac{b_j + \omega_{ij}^w - c_j^1 - c_j^2 z_{max}}{(1-\tau)}
$$

     where $\tau = 0$ if $b_j + \omega_{ij}^w - c_j^1 - c_j^2 z_{min} < 0$.

3. Given cutoffs $\underline{z}_{ij}$, compute joint future values $g_i(z)$ using formulas.

$$\begin{aligned}
g_l(z) &= \beta\rho^x(b_l+\omega_{ll})+\beta(1-\rho^x)(1-\gamma^u)(1-\gamma^s)((1-\tau)z+g_l(z)) \\
&+ \beta(1-\rho^x)(1-\gamma^u)\gamma^s[\int_{\underline{z}_{ll}}^{\infty}[(1-\tau)y+g_l(y)]dv_l(y)+\nu_{ll}(b_l+\omega_{ll})] \\
&+ \beta(1-\rho^x)\gamma^u\left[\int_{\underline{z}_{hh}}^{\infty}[(1-\tau)y+g_h(y)]dv_h(y)+\nu_{hh}(b_h+(1-\gamma^d)\omega_{hh}+\gamma^d\omega_{lh})\right] \\
g_h(z) &= \beta\rho^x[\gamma^{d,x}(b_h+\omega_{lh})+(1-\gamma^{d,x})(b_h+\omega_{hh})]+\beta(1-\rho^x)(1-\gamma^s)[(1-\tau)z+g_h(z)] \\
&+ \beta(1-\rho^x)\gamma^s\left[\int_{\underline{z}_{hh}}^{\infty}[(1-\tau)y+g_h(y)]dv_h(y)+\nu_{hh}(b_h+(1-\gamma^d)\omega_{hh}+\gamma^d\omega_{lh})\right]
\end{aligned}$$

3.1 Compute rejection probabilities:

$$\nu_{ij}=\int_{-\infty}^{\underline{z}_{ij}}dv_i(y)$$

3.2 Compute expected continuation values without outside options:

$$E_{ij}=\int_{\underline{z}_{ij}}^{\infty}[(1-\tau)y+\hat{g}_j(y)]dv_i(y)$$

3.3 The expectation of joint future values (usually written in terms of joint surpluses s_{ij}):

$$\int_{\underline{z}_{ij}}^{\infty}s_{ij}(y)dv_i(y)=E_{ij}-[b_j+\omega^w_{ij}+\omega^f](1-\nu_{ij})$$

4. Update coefficients c_i by running the regression over a small grid of z (ngrid=41)

$$g_i(z)=[1\ z\ z^2]c_i'$$

5. In LS: given cutoffs and old measures of unemployed compute $\theta, \lambda^f(\theta), \lambda^w(\theta)$ using

$$\theta=\left(\frac{A}{\mu}\beta(1-\pi)\sum_{i,j}\left[\frac{u_{ij}}{u}\int_{\underline{z}_{ij}}^{\infty}s^o_{ij}(y)dv_i(y)\right]\right)^{1/(1-\alpha)}$$

6. In dHHR: given cutoffs and old measures of unemployed, update firms' outside option

$$\omega^f=\frac{\beta}{1-\beta}(1-\pi)\lambda\sum_{i,j}\left[\frac{u_{ij}}{u}\int_{\underline{z}_{ij}}^{\infty}s^o_{ij}(y)d\nu i(y)\right]$$

7. Update future continuation values ω_{ij}^{w} for workers

8. Compute masses of workers in u_{ij} and e_{ij} using flow equations and separation rates

9. Given masses of workers, compute average wages

$$\bar{p}_{ij} = \sum_{i,j} e_{ij} \int_{\underline{z}_{ij}}^{\infty} p_{ij}(y) dv_i(y)$$

where the conditional wages are given by Nash Bargaining

$$p_{ij}(z) = \pi s_{ij}(z) + b_i + \omega_{ij}^{w} - g_j^{w}(z)$$

With layoff taxes, we take into account the two-tier wage schedule that distinguishes between wages of new or continuing workers.

10. Update benefits b as a fraction ϕ of average wages

11. Adjust tax rate τ to balance budget

$$b_l u_{ll} + b_h(u_{lh} + u_{hh}) = \tau(e_l \hat{z}_l + e_h \hat{z}_h)$$

12. Check convergence of a set of moments

C More details on g and its approximation

The joint continuation value of a relationship with a low skilled worker is given by[9]:

$$
\begin{aligned}
g_l(z) &= \underbrace{\left[\frac{\beta(1-\rho^x)(1-\gamma^u)(1-\gamma^s)}{1-\beta(1-\rho^x)(1-\gamma^u)(1-\gamma^s)}\right](1-\tau)\, z}_{\alpha_l \;=\; slope_l} \\
&+ \frac{\beta(1-\rho^x)\left[(1-\gamma^u)\gamma^s(E_{ll}+\nu_{ll}(b_l+w_{ll})+\gamma^u(E_{hh}+\nu_{hh}(b_h+(1-\gamma^d)w_{hh}+\gamma^d w_{lh}))\right]}{1-\beta(1-\rho^x)(1-\gamma^u)(1-\gamma^s)} \\
&+ \frac{\beta\rho^x(b_l+w_{ll})}{1-\beta(1-\rho^x)(1-\gamma^u)(1-\gamma^s)}
\end{aligned}
$$

where $E_{ll} = \int_{\underline{z}_{ll}}^{\infty}(1-\tau)y + g_l(y)dv_l(y)$ and $E_{hh} = \int_{\underline{z}_{hh}}^{\infty}(1-\tau)y + g_h(y)dv_h(y)$.

The joint continuation value of a relationship with a high skilled worker is given by:

$$
\begin{aligned}
g_h(z) &= \underbrace{\left[\frac{\beta(1-\rho^x)(1-\gamma^s)}{1-\beta(1-\rho^x)(1-\gamma^s)}\right](1-\tau)\, z}_{\alpha_h \;=\; slope_h} \\
&+ \frac{\beta(1-\rho^x)\gamma^s\left[E_{hh}+\nu_{hh}(b_h+(1-\gamma^d)w_{hh}+\gamma^d w_{lh})\right]}{1-\beta(1-\rho^x)(1-\gamma^s)} \\
&+ \frac{\beta\rho^x(b_h+(1-\gamma^{d,x})w_{hh}+\gamma^{d,x}w_{lh})}{1-\beta(1-\rho^x)(1-\gamma^s)}
\end{aligned}
$$

Some observations:

1. The slope coefficient on productivity represents the extra value that is to be had from accepting the present productivity that is expected to last for some given duration (that depends on the probability of drawing a new productivity at work γ^s, the probability of upgrade γ^u as well as the probability for being exogenously laid off ρ^x).

2. In the calibration used, the difference in slopes between skills is only given by the upgrade probability: the slope for high skilled is equal to the slope of low skilled setting $\gamma^u = 0$. Since $\frac{\partial slope_l}{\partial \gamma^u} < 0$, then $\forall\ \gamma^u > 0$ we have $slope_h > slope_l$.

3. Turbulence does not affect the slope directly, only through its effect on the tax rate.

4. The intercept represents the capital value of a worker of a particular skill, not including the described extra value associated with the present productivity draw.

[9] $\beta = \hat{\beta}(1-\rho^r)$ includes the retirement probability.