

# The Fiscal Multiplier<sup>\*</sup>

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## Abstract

This paper studies the size of the fiscal multiplier in a model with incomplete markets and rigid prices and wages. Allowing for incomplete markets instead of complete markets—the prevalent assumption in the literature—comes with two advantages. First, the incomplete markets model delivers a realistic distribution of the marginal propensity to consume across the population, whereas all households counterfactually behave according to the permanent income hypothesis if markets are complete. Second, in our model the response of prices, output, consumption and employment is uniquely determined by fiscal policy for any monetary policy including the zero-lower bound (ZLB) as opposed to most of the previous literature, where an infinite number of equilibria exists, leaving the researcher to arbitrarily pick one. Our preliminary findings indicate that the impact multiplier is quite large between 2 and 3 depending on whether tax or deficit financing is used and increase to values above 3 in a liquidity trap.

**Keywords:** Fiscal Multiplier, Incomplete Markets, Sticky Prices

**JEL codes:**

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<sup>\*</sup>Support from the National Science Foundation Grant Nos. SES-0922406 and SES-1357903 and FRIPRO Grant No. 250617 is gratefully acknowledged.

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# 1 Introduction

What is the impact of a government spending or a transfer stimulus on output? The idea behind this classic policy measure is quite simple. A government spending stimulus increases aggregate demand which in turn leads to higher labor demand and thus more employment and higher wages. This higher labor income then stimulates consumption, in particular of poor households, which again leads to more employment, higher labor income, more consumption and so on and so forth. The equilibrium impact of an initial government spending of 1\$ on output - the fiscal multiplier - is the sum of the initial increase in government spending and the induced private consumption response.

This simple idea is based on two essential elements/assumptions which ensure that the stimulus has a *direct* impact on output and employment and in addition has *indirect* multiplier effects on private consumption. The first element is that output is demand determined such that the increase in government spending stimulates aggregate demand. The underlying important assumption is that prices are rigid to avoid that firms adjust prices and not quantities as a response to more government demand. Firms then increase production to satisfy demand and this requires higher employment and higher wages, leading to higher household income. It is because of price rigidities that there is a role for demand and that one can talk about stimulating aggregate demand in a meaningful way. We name this demand and associated output stimulus through an increase in government spending the direct effect. The direct effect differs from the full equilibrium effect in that it keeps prices and taxes unchanged and most importantly does not take into account indirect multiplier effects which arise from higher private consumption leading to more labor demand, higher labor income and again more consumption and so on.

The second element/assumption ensures that these indirect multiplier effects on private consumption are significant. For this indirect channel to be quantitatively meaningful a significant deviation from the permanent income hypothesis is necessary. If this is the case, then a large fraction of higher income - the marginal propensity to consume (MPC) - is spent on consumption and not saved, whereas permanent income hypothesis households have a small MPC close to the real interest rate and thus save almost all of the additional income. The higher is the MPC and thus the additional consumption, the higher is the additional increase

of labor demand and of households labor income triggering another multiplier round of higher consumption etc.

A quantitatively meaningful assessment of a stimulus policy not only has to be based on these two elements - price rigidities and high MPC - but in addition these elements have to match observed households' and firms' behavior. This requires that a model first features the right amount of nominal rigidities such that the aggregate demand channel is as in the data. And, second, it requires to incorporate observed marginal propensities to consume which imply a substantial deviation from the permanent income hypothesis.

In this paper we calculate the size of the fiscal multiplier in a dynamic equilibrium model with these two elements. We extend the standard Bewley-Imrohoroglu-Huggett-Aiyagari model to include New-Keynesian style nominal price rigidities. Introducing incomplete markets allows the model to match the rich joint distribution of income, earnings and wealth. Further, such heterogeneity is crucial in generating a realistic distribution of MPCs. The price rigidities ensure that the model has a meaningful demand channel operating. Furthermore, "the fiscal multiplier" is not a single number—as its size depends very much on how it is financed (debt, distortionary taxation, reduction of transfers), how persistent fiscal policy is, what households and firms expect about future policy changes, and whether spending is increased or transfers are directed to low-income households. Important details which we incorporate in our model but are difficult to control for in empirical studies. Maybe because of these difficulties and despite the importance of this research question no consensus on the size of the multiplier has been reached and findings come with a lot of uncertainty (see Ramey (2011) for a survey).<sup>1</sup>

We are not the first who in response to the difficulties faced in empirical work, use a more theoretical approach to address this classic question. But we are the first to allow both for a demand channel and a realistic consumption response to changes in income within the same model.<sup>2</sup>

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<sup>1</sup>Most of the empirical studies use aggregate data to measure the strength of the fiscal multiplier, which range from around 0.6 to 1.8, although "reasonable people can argue, however, that the data do not reject 0.5 or 2.0" (see Ramey (2011)). Another more recent strand of the literature looks at cross-state evidence and typically finds larger multipliers. However, as Ramey (2011) and Farhi and Werning (2013) have pointed out correctly, the size of the local multipliers found in those studies may be not very informative about the magnitude of aggregate multipliers. For example the local multiplier could be 1.5 whereas the aggregate multiplier is 0.

<sup>2</sup>There is a growing literature which incorporates nominal rigidities into incomplete markets models, for example Oh and Reis (2012), Guerrieri and Lorenzoni (2015), Gornemann et al. (2012), Kaplan et al. (2016), Auclert (2016) and Lütticke (2015), McKay and Reis (2016), McKay et al. (2015), Bayer et al. (2015), Ravn

One strand of the literature assumes flexible prices and thus eliminates the demand channel. An early example is Baxter and King (1993) in a representative agent model, and later contributions with heterogeneous agents and incomplete markets include Heathcote (2005) and Brinca et al. (2016). This framework is limited in its ability to provide a full assessment as only the supply but not the demand channel is operative, that is the first essential element is not present. Another strand of the literature overcomes this limitation as it uses New Keynesian models with sticky prices and sticky wages to compute the fiscal multiplier. Price rigidities provide a role for the demand channel but now the the second essential element is missing. A main limitation of the New Keynesian models used so far for policy analysis is that the households in the model economies are representative agents. The households behave exactly like permanent-income households—as a result there is no heterogeneity in the marginal propensity to consume (MPC). Further, the MPC in response to temporary shock is small, which stands in the face of the findings of a large empirical literature that has documented substantial MPC heterogeneity and large consumption responses to transitory income and transfer payments. More generally the consumption model embedded in the New Keynesian model focuses on intertemporal substitution of consumption only whereas the data assign only a small role to such considerations (Kaplan and Violante (2014), Kaplan et al. (2016)).

While allowing for heterogeneity will overcome one shortcoming over the existing New Keynesian literature, a second problem was pointed out by Cochrane (2015). He studies fiscal policy in new-Keynesian models during a liquidity trap and shows that the policy predictions depend strongly on the choices made by the researcher to select a specific equilibrium. Depending on these choices, the fiscal multiplier can be arbitrarily large or even negative. The root of this problem is clear. The price level is indeterminate and therefore the researcher has to select one equilibrium where at the same time a continuum of other choices with very different policy predictions are possible. This renders this class of models not very useful for a policy analysis.

To overcome this second challenge, we leverage the insight of Hagedorn (2016) who shows that incomplete markets combined with fiscal policy specified partially in nominal terms delivers a globally determined price level—so that the researcher does not have to pick a specific

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and Sterk (2013) and Den Haan et al. (2015), but we are not aware of any contribution in this literature which considers fiscal multipliers.

equilibrium. Instead the price level is uniquely and jointly determined by fiscal and monetary policy. This determinacy result enables the researcher to study arbitrary policies and how they affect the economy, including a constant nominal interest rate that would lead to indeterminacy in the standard approach.

Thus, our paper is the first to quantify the size of the fiscal multiplier in the presence of significant household heterogeneity and in a model with a uniquely determined price level which predicts a unique equilibrium in response to a stimulus. Further, using an incomplete markets model also allows us to conduct a meaningful analysis of transfer multipliers, which is an important objective as many stimulus policies take the form of transfers and not an increase in spending. We also use the theoretical model to compute the welfare consequences of temporary increases in government spending and in transfer payments. This exercise is more interesting than in a complete markets environment since the welfare gains of high MPC households may outweigh the losses of low MPC (rich) households.

Our preliminary findings indicate that it is important to distinguish between the impact multiplier and the cumulated multiplier which is the discounted average multiplier over time. In the benchmark the multiplier is on impact quite high, equal to 2.28 if spending is tax financed and 2.8 if it is deficit financed. The multiplier however dies out quite quickly and consumption at some point falls even below its steady state level such that the cumulative multiplier is quite low, equal to 0.19 if spending is tax financed and 0.3 if it is deficit financed. These findings suggest that fiscal spending is quite effective in providing a short stimulus which might be beneficial if the economy is hit by a bad shock now so that the current gains outweigh the future losses.

The benchmark also shows that using deficit spending is the more effective stimulative policy. This is not quite surprising since increasing spending and taxes at the same time first stimulates demand but then offsets it to a large degree through raising taxes which also affects high MPC households. In contrast with deficit financing, the newly issued debt is largely bought by low MPC households whereas high MPC households largely consume additional income. Deficit financing thus implicitly redistributes from asset-rich households with low MPC who finance their consumption more from asset income to low-asset households with high MPC who rely more on labor income so that the aggregate MPC increases.

We also consider the effect of a pre-announced anticipated spending increase and find

this “forward-spending” to be less effective than an unexpected stimulus. The main reason being that firms raise prices immediately in anticipation of future higher demand which leads to output losses before the actual policy is implemented. The benchmark analysis assumes a nominal interest fixed at zero. However, we corroborate our benchmark findings when we deviate from this assumption and assume that monetary policy is described by some interest rate feedback rule instead. This is because prices do not move much. Firms anticipate correctly that the long-run price level returns eventually to its pre-stimulus level which dampens the incentives to increase prices substantially. This together with strong price rigidities makes prices move only little such that the interest rate rule implies little movement in interest rates as well.

The question on the size of the fiscal multiplier received renewed interest in the aftermath of the Great Recession. We also consider the multiplier in this scenario where we not only peg the nominal interest rate to zero but in addition also engineer a liquidity trap where the natural real interest rate falls below zero. The results from the benchmark analysis are amplified. The impact multiplier is now above 3 and stays at this level for a few periods. However, later it falls to negative values such that the cumulative multiplier is only about 0.5. This suggest, consistent with our findings for the benchmark, that a fiscal stimulus is quite effective in the short-run in particular when the economy is hit by a bad shock but that eventually negative effects kick in (at which point the economy might have already recovered though). We also find that the multiplier in a liquidity trap gets smaller if prices become more flexible in contrast to the opposite findings in the New Keynesian literature. Our results also indicate some limits to scaling up the stimulus since the multiplier is decreasing in the size of the spending stimulus.

To obtain a better understanding of the aggregate consumption response we decompose it into four components: the direct impact of spending on employment and consumption, the effects of changes in tax and transfers, the indirect multiplier effect of higher labor income and the effect of changing prices and interest rates. The initial consumption response is 0.19% of output with tax financing and 0.26% with deficit financing.

The direct impact of spending accounts for 49% and 36% of the total response respectively. The increase in spending translates one-to-one into higher demand and higher employment since prices respond only very little initially, leading to a aggregate consumption response of

0.093% of output independent of the type of financing. When the negative effects of higher taxes are taken into account the consumption response drops to 0.066% and 0.091% respectively, accounting for about 35% of the total response in both cases. The consumption response is larger with deficit than with tax financing since only in the latter case higher taxes partly offset the initial demand stimulus. As a result of this, the indirect multiplier effects are also larger with deficit financing, 0.17% relative to 0.13%, both accounting for about 65% of the total response. The effect of changes in prices and interest rates is negligible. Together our analysis reveals that the fiscal multiplier is larger with deficit financing, that about a third of the consumption response is due to the direct impact of spending and tax changes and two thirds are due to indirect multiplier effects.

The paper is organized as follows. Section 2 presents our incomplete markets model with price rigidities. In Section 3 we study the size of the government-spending multiplier both for an interest rate peg and when monetary policy is described by a Taylor rule. Section 4 extends the basic model and we add capital accumulation and allow for habit persistence and sticky wages.

## 2 Model

The model is a standard New Keynesian model with one important modification: Markets are incomplete as in Aiyagari (1994, 1995) whereas they are complete in a standard New Keynesian model. We add the standard features of new Keynesian models to an incomplete markets model. Price setting faces some constraints as price adjustments are costly as in Rotemberg (1982) leading to price rigidities. As is standard in the New Keynesian literature, final output is produced in several intermediate steps. Final good producers combine the intermediate goods to produce a competitive goods market. Intermediate goods producers are monopolistically competitive. They set a price they charge to the final good producer to maximize profits taking into account the price adjustment costs they face. The intermediate goods producer buy the input, labor, in competitive markets. Later when we allow for sticky wages as well, we assume that differentiated labor is monopolistically supplied as well.

## 2.1 Households

The economy consists of a continuum of agents normalized to measure 1 with identical GHH preferences for leisure nested within constant relative risk aversion (CRRA) preferences for non-durable consumption:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, h_t)$$

where:

$$u(c, h) = \begin{cases} \frac{(c-g(h_t))^{1-\sigma}-1}{1-\sigma} & \text{if } \sigma \neq 1 \\ \log(c - g(h_t)) & \text{if } \sigma = 1 \end{cases}$$

and  $\beta \in (0, 1)$  is the discount factor. Agents' labor productivity  $\{s_t\}_{t=0}^{\infty}$  is stochastic and is characterized by an  $N$ -state Markov chain that can take on values  $s_t \in \mathcal{S} = \{s_1, \dots, s_N\}$  with transition probability characterized by  $p(s'|s)$  and  $\int s = 1$ . Agents rent their labor services,  $hs$ , to firms for a real wage  $w_t$  and their nominal assets  $a$  to the bond market for a nominal rent  $i$  and a real return  $(1+r) = \frac{1+i}{1+\pi}$ , where  $1+\pi = \frac{P'}{P}$  is the inflation rate.

Thus, at time  $t$  an agent faces the following budget constraint:

$$P_t c_t + a_{t+1} = (1+i_t)a_t + (1-\tau_t)P_t w_t h_t s_t + T_t$$

where  $\tau_t$  is a proportional labor tax and  $T_t$  is a nominal lump sum transfer. Agents are price takers. Thus, we can rewrite the agent's problem recursively as follows:

$$\begin{aligned} V(a, s; \Omega) &= \max_{c \geq 0, h \geq 0, a' \geq 0} u(c, l) + \beta \sum_{s' \in \mathcal{S}} p(s'|s) V(a', s'; \Omega') \\ \text{subj. to } & P c + a' = (1+i)a + P(1-\tau)w h s + T \\ & \Omega' = \Gamma(\Omega) \end{aligned} \tag{1}$$

where  $\Omega(a, s) \in \mathcal{M}$  is the distribution on the space  $X = A \times S$ , agents asset holdings  $a \in A$  and labor endowment  $s \in S$ , across the population, which will together with the policy variables determine the equilibrium prices.  $\mathcal{H}$  is an equilibrium object that specifies the evolution of the wealth distribution.



## 2.2 Production

**Final Good Producer** A competitive representative final goods producer aggregates a continuum of intermediate goods indexed by  $j \in [0, 1]$  and with prices  $p_j$ :

$$Y_t = \left( \int_0^1 y_{jt}^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}.$$

where  $\epsilon$  is the elasticity of substitution across goods. Given a level of aggregate demand  $Y$ , cost minimization for the final goods producer implies that the demand for the intermediate good  $j$  is given by

$$y_{jt} = y(p_{jt}; P_t, Y_t) = \left( \frac{p_{jt}}{P_t} \right)^{-\epsilon} Y_t, \quad (2)$$

where  $P$  is the (equilibrium) price of the final good and can be expressed as

$$P_t = \left( \int_0^1 p_{jt}^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}.$$

**Intermediate good producer** Each intermediate good  $j$  is produced by a monopolistically competitive producer using labor input  $n_j$ . Production technology is linear.

$$y_{jt} = Z_t n_{jt}^{1-\alpha}$$

where  $Z_t$  is aggregate productivity. Intermediate producers hire labor at the nominal wage  $P_t w_t$  in a competitive labor market. With this technology, the marginal cost of a unit of intermediate good is

$$mc_{jt} = \frac{w_t}{Z_t(1-\alpha)n_{jt}^\alpha}.$$

Each firm chooses its price to maximize profits subject to price adjustment costs as in Rotemberg (1982). These adjustment costs are measured in units of aggregate output and are given by a quadratic function of the change in prices above and beyond steady state inflation  $\bar{\Pi}$ ,

$$\Theta(p_{jt}, p_{jt-1}; Y_t) = \frac{\theta}{2} \left( \frac{p_{jt}}{p_{jt-1}} - \bar{\Pi} \right)^2 Y_t$$

Given last period's individual price  $p_{jt-1}$  and the aggregate state  $(P_t, Y_t, w_t, r_t)$ , the firm

chooses this period's price  $p_{jt}$  to maximize the present discounted value of future profits. The firm satisfies all demand,

$$y(p_{jt}; P_t, Y_t) = Z_t n(y(p_{jt}; P_t, Y_t))^{1-\alpha} \quad (3)$$

by hiring the necessary amount of labor,

$$n_{jt} = n(y(p_{jt}; P_t, Y_t)) = \left( \frac{y(p_{jt}; P_t, Y_t)}{Z_t} \right)^{\frac{1}{1-\alpha}} = \left( \left( \frac{p_{jt}}{P_t} \right)^{-\epsilon} \frac{Y_t}{Z_t} \right)^{\frac{1}{1-\alpha}} \quad (4)$$

The firm's pricing problem is

$$V_t(p_{jt-1}) \equiv \max_{p_{jt}} \frac{p_{jt}}{P_t} y(p_{jt}; P_t, Y_t) - w_t \left( \frac{y(p_{jt}; P_t, Y_t)}{Z_t} \right)^{\frac{1}{1-\alpha}} - \frac{\theta}{2} \left( \frac{p_{jt}}{p_{jt-1}} - \bar{\Pi} \right)^2 Y_t - \Phi + \frac{1}{1+r_t} V_{t+1}(p_{jt}),$$

where  $\Phi$  are fixed operating costs. In equilibrium all firms choose the same price, and thus, aggregate consistency implies  $p_{jt} = P_t$  for all  $j$  and  $t$ . Thus,  $\frac{p_{jt}}{p_{jt-1}} = \frac{P_t}{P_{t-1}} = \pi_t$  and  $\frac{p_{jt+1}}{p_{jt}} = \frac{P_{t+1}}{P_t} = \pi_{t+1}$ .

Some algebra (delegate to the appendix) yields the New Keynesian Phillips Curve

$$(1 - \epsilon) + \frac{\epsilon}{1 - \alpha} w_t Z_t^{\frac{1}{\alpha-1}} \left( \frac{Y_t}{P_t} \right)^{\frac{\alpha}{1-\alpha}} - \theta (\pi_t - \bar{\Pi}) \pi_t + \frac{1}{1+r_t} \theta (\pi_{t+1} - \bar{\Pi}) \pi_{t+1} \frac{Y_{t+1}}{Y_t} = 0$$

The equilibrium real profit of each intermediate goods firm is then

$$d_t = Y_t - \Phi - w_t \left( \frac{Y_t}{Z_t} \right)^{\frac{1}{1-\alpha}} - \frac{\theta}{2} (\pi_t - \bar{\Pi})^2 Y_t$$

## 2.3 Government

The government obtains revenue from taxing labor income, issuing bonds and taxing profits at 100%. Household labor income  $wsl$  is taxed progressively with a nominal lump-sum transfer  $T_t$  and a proportional tax  $\tau$ :

$$\tilde{T}(wsh) = -T + \tau Pwsh.$$

The government issues nominal bonds denoted by  $B^g$ , with negative values denoting government asset holdings and fully taxes profits away, obtaining nominal revenue  $Pd$ .

The government uses the revenue to finance exogenous nominal government expenditures,

$G_t$ , interest payments on bonds and transfers to households.

The government budget constraint is therefore given by:

$$B_{t+1}^g = (1 + i_t)B_t^g + G_t - P_t d_t - \int \tilde{T}_t(w_t s_t h_t) d\Omega. \quad (5)$$

## 2.4 Equilibrium

Market clearing requires that the labor demanded by the firm is equal to the aggregate labor and that the bonds issued by the government equals the amount of assets provided by households:

$$B_{t+1} = \int \sum_{a_t} \sum_{s_t \in \mathcal{S}} a_{t+1}(a_t, s_t) d\Omega_t(a_t, s_t) \quad (6)$$

$$H_t = \int \sum_{a_t} \sum_{s_t \in \mathcal{S}} s_t h_t(a_t, s_t) d\Omega_t(a_t, s_t) \quad (7)$$

where we have abused notation slightly here,  $a_{t+1}(a_t, s_t)$  is the asset choice of an agent with asset level  $a_t$  and period labor endowment  $s_t$ , and similarly  $h_t(a_t, s_t)$  is the associated labor policy function.

**Definition:** A monetary competitive equilibrium is a sequence of prices  $P_t$ , tax rates  $\tau_t$ , nominal transfers  $T_t$ , nominal government spending  $G_t$ , bonds  $B_t^g$ , a value functions  $v_t : X \times \mathcal{M} \rightarrow \mathcal{R}$ , policy functions  $a_t : X \times \mathcal{M} \rightarrow \mathcal{R}_+$  and  $c_t : X \times \mathcal{M} \rightarrow \mathcal{R}_+$ ,  $h_t : X \times \mathcal{M} \rightarrow \mathcal{R}_+$ ,  $H_t : A \rightarrow \mathcal{R}_+$ , pricing functions  $r_t : A \rightarrow \mathcal{R}$  and  $w_t : A \rightarrow \mathcal{R}_+$ , and law of motion  $\Gamma : A \rightarrow A$ , such that:

1.  $v_t$  satisfies the Bellman equation with corresponding policy functions  $a_t, c_t, h_T$  given price sequences  $r_t(), w_t()$
2.  $w_t()$  and  $h_t()$  satisfy:

$$w_t(\Omega_t)s(1 - \tau_t) = g'(h_t(s, \dots))$$

3. For all  $\Omega \in \mathcal{M}$ :

$$\begin{aligned} B_{t+1} &= \int a_{t+1}(a, s; \Omega_t) d\Omega_t \\ H_t(\Omega_T) &= \int s h_t d\Omega_t \\ Y_t = Z_t H_t^{1-\alpha} &= \int c(a, s; \Omega) d\Omega_t + \frac{G}{P} + \Phi + \frac{\theta}{2} (\Pi_t - \bar{\Pi})^2 Y_t \end{aligned}$$

4. Aggregate law of motion  $\Gamma$  generated by  $a'$  and  $p$

### 3 The Fiscal Multiplier

In this Section we calculate the fiscal multiplier in our model with incomplete markets, conducting the following experiment. Assume that the economy is in steady state with nominal bonds  $B_{ss}$ , government spending  $G_{ss}$ , transfers  $T_{ss}$  and a tax rate  $\tau_{ss}$  and where the price level is  $P_{ss}$ . The real value of bonds is then  $B_{ss}/P_{ss}$ , the real value government expenditure is  $G_{ss}/P_{ss}$  and so on. We then consider an M.I.T. (unexpected and never-again-occurring) shock to government expenditures and compute the impulse response to this persistent innovation in  $G$ . Eventually the economy will reach the new steady state characterized by government bonds  $B_{ss}^{new}$ , government spending  $G_{ss}^{new}$ , transfers  $T_{ss}^{new}$  and a tax rate  $\tau_{ss}^{new}$  the price level is  $P_{ss}^{new}$ .

#### 3.1 The Fiscal Multiplier in Incomplete Market Models

Before computing the fiscal multiplier, it is instructive to first recall how the size of the multiplier is determined in New Keynesian models with complete markets. We then explain the differences for the fiscal multiplier between complete markets and incomplete markets, focusing on the two main differences. First, building on the results in Hagedorn (2016) we argue that the price level is determinate in our model and explain the consequences for the size of the fiscal multiplier. Second a fiscal stimulus has distributional consequences only if markets are incomplete, which impact aggregate demand and therefore the output response.

### 3.1.1 Fiscal Multiplier with Complete Markets

Farhi and Werning (2013) show that if the nominal interest rate is pegged, the size of the multiplier  $m$  is determined by the response of the inflation rate only. They start from the Consumption Euler equation for a utility function,  $\frac{C^{1-\sigma}}{1-\sigma} + \dots$

$$C_t^{-\sigma} = \beta \frac{1 + i_{ss}}{1 + \pi_{t+1}} C_{t+1}^{-\sigma}. \quad (8)$$

Iterating this equation and assuming that consumption is back to the steady state level at time  $T$ ,  $C_T = C_{ss}$ , we obtain for consumption at time  $t = 1$  when spending is increased,

$$C_1^{-\sigma} = \left( \prod_{t=1}^{T-1} \left( \beta \frac{1 + i_{ss}}{1 + \pi_{t+1}} \right) \right) C_T^{-\sigma}, \quad (9)$$

so that the initial percentage increase in consumption and thus the fiscal multiplier equals

$$m = \frac{C_1}{C_{ss}} = \left( \prod_{t=1}^{T-1} \frac{1 + \pi_{t+1}}{\beta(1 + i_{ss})} \right)^{\frac{1}{\sigma}} = \left( \prod_{t=1}^{T-1} \frac{1 + \pi_{t+1}}{1 + \pi_{ss}} \right)^{\frac{1}{\sigma}} \quad (10)$$

where we have used that  $\beta \frac{1+i_{ss}}{1+\pi_{ss}} = 1$ . As a result the size of the fiscal multiplier is one-to-one related to the accumulated response of inflation which is induced by the fiscal stimulus,

$$m^{\sigma} = \prod_{t=1}^{T-1} \frac{1 + \pi_{t+1}}{1 + \pi_{ss}}. \quad (11)$$

However, as Cochrane (2015) has pointed out, the model does not predict a unique outcome for the path of inflation in response to a fiscal stimulus but instead a continuum of equilibria exists. What renders this problematic for a study of fiscal policy is that the size of the fiscal multiplier can be very different across these equilibria, ranging from arbitrarily large to even negative. In other words the researcher has to select a specific equilibrium and thus implicitly chooses the size of the multiplier. The root of this problem is well known. The price level is indeterminate that is a continuum of price levels satisfies the equilibrium restrictions.

### 3.1.2 Unique Equilibrium with Incomplete Markets

We now show that first the results in Hagedorn (2016) imply that these problems are overcome in models with incomplete market models such as ours since the price level is determinate in

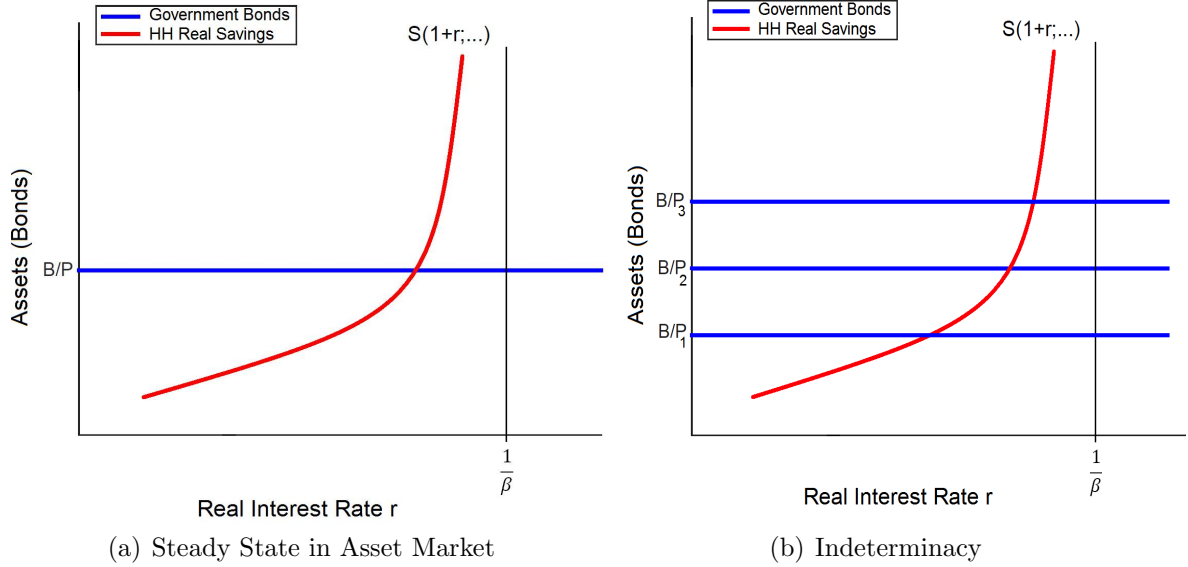


Figure 1: Steady State Asset Market Clearing

these type of models. We then illustrate how a fiscal stimulus affects the price level in the new steady state,  $P_{ss}^{new}$ .

We consider steady state and use that asset market clearing implies an equilibrium. The asset market clears iff aggregate asset supply (households' savings) equals real aggregate asset demand (real government bonds), as illustrated in the left panel of Figure 1. Households' savings  $S(1 + r, \dots)$  is an upward sloping function of the real interest rate  $1 + r$  and real asset supply equals  $B_{ss}/P_{ss}$ . The equilibrium condition is

$$S(1 + r, \dots) = \frac{B}{P}. \quad (12)$$

This is one equation with two unknowns, the real interest rate  $1 + r$  and the price level  $P$ , such that the price level is not determinate yet. Indeed, as illustrated in the right panel of Figure 1, different price levels  $P_1, P_2, P_3$ , each associated with a different real value of bonds and a different real interest rate, are consistent with asset market clearing.

Using the arguments in Hagedorn (2016), we will now argue that this equation nevertheless determines the price level since the real interest rate is determined by monetary and fiscal policy. In both complete and incomplete market models a Fisher relation between the steady state nominal interest  $i_{ss}$ , real interest rate  $r_{ss}$  and inflation  $\pi_{ss}$  holds:

$$(1 + i_{ss}) = (1 + r_{ss})(1 + \pi_{ss}), \quad (13)$$

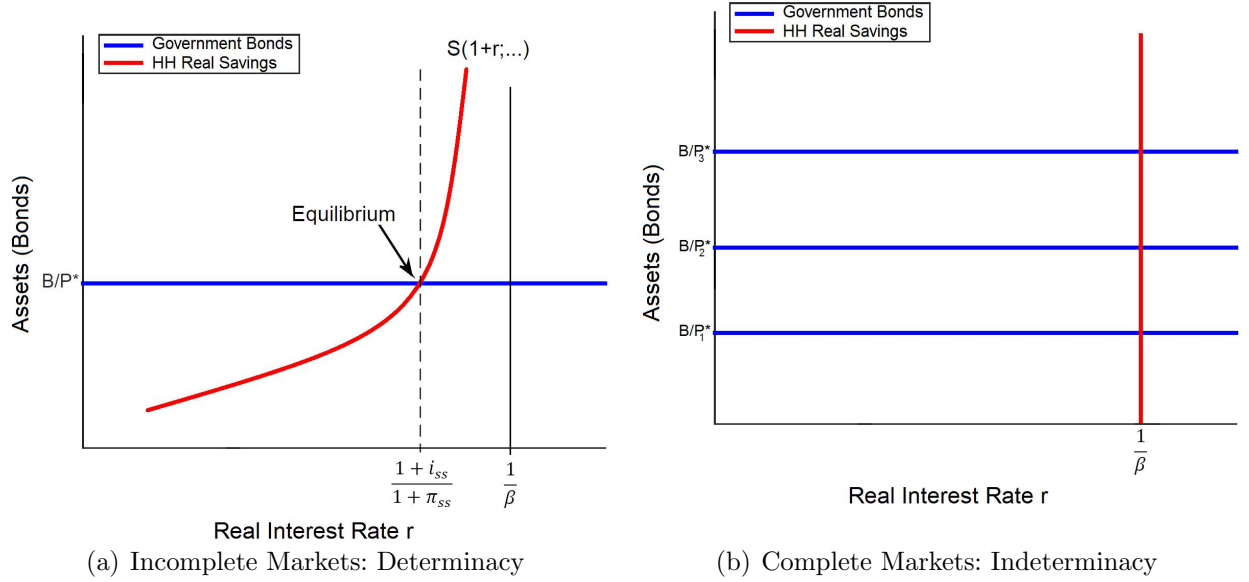


Figure 2: Price Level (In)determinacy with (In)complete Markets

with an important difference between complete and incomplete markets. If markets were complete the real interest rate is determined by the discount factor only,  $(1 + r_{ss})\beta = 1$ , whereas in incomplete market models the real interest rate depends on virtually all model primitives such as preferences, uncertainty, etc. and also on fiscal policies  $G_{ss}/P_{ss}$ ,  $T_{ss}/P_{ss}$  and  $B_{ss}/P_{ss}$ , so that the above equation in our incomplete markets models can be written as

$$\frac{1 + i_{ss}}{1 + \pi_{ss}} = (1 + r_{ss}(G_{ss}/P_{ss}, T_{ss}/P_{ss}, B_{ss}/P_{ss}, \dots)). \quad (14)$$

This equation together with the steady state condition that the growth rate of nominal spending, nominal taxes and nominal debt is equal to the inflation rate (in the absence of economic growth),

$$1 + \pi_{ss} = \frac{G' - G}{G} = \frac{T' - T}{T} = \frac{B' - B}{B}, \quad (15)$$

determine, given monetary and fiscal policy, the two unknown variables, the inflation rate  $\pi_{ss}$  and the price level  $P_{ss}$ . Note that equation (15) implies that the inflation rate is set by fiscal policy and is equal to the growth rate of nominal government spending. The price level then adjust such that equation (14) holds, that is that the real interest rate  $1 + r_{ss}$  is equal to  $\frac{1+i_{ss}}{1+\pi_{ss}}$ . The left panel of Figure 2 shows the equilibrium in the asset market and how the steady-state price level  $P^*$  is determined. The right panel of Figure 2 establishes why we need incomplete markets. With complete markets a continuum of price levels such as  $P_1^*, P_2^*, P_3^*$  are consistent

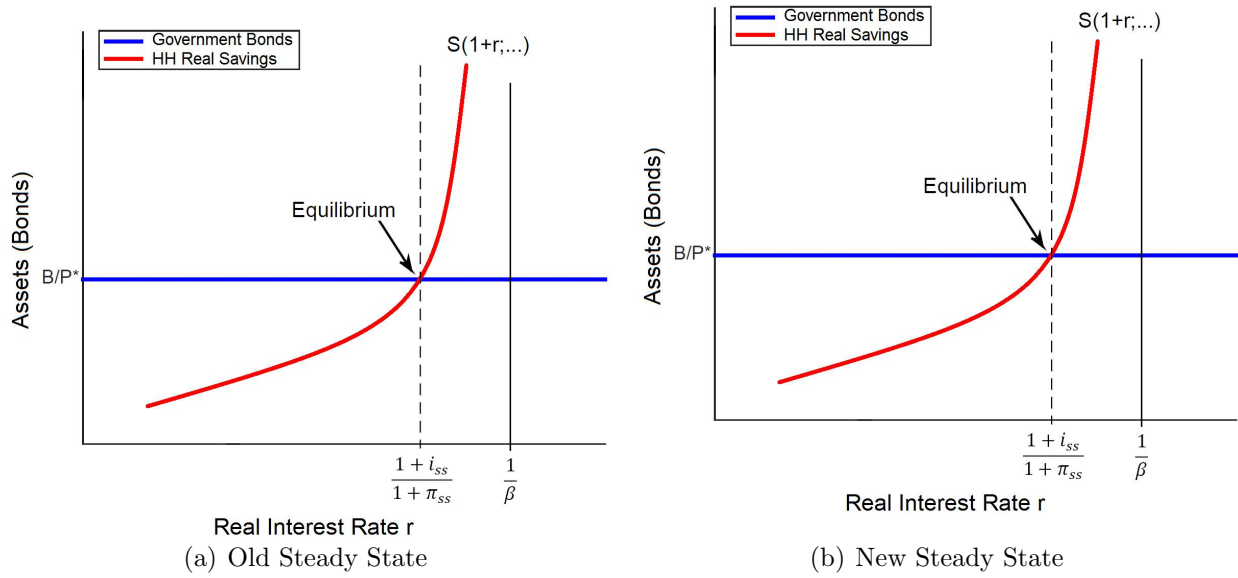


Figure 3: Price Level and Unchanged Long-run Policy

with all equilibrium conditions, that is the price level is indeterminate.

The underlying reason for these findings is the failure of the permanent income hypothesis and that agents, as a result of this failure, engage in precautionary savings. The intuition is straightforward. A higher steady state price level lowers real government consumption since fiscal policy is nominal and at the same time lowers the tax burden for the private sector by the same amount. Households however do not spend all of the tax rebate on consumption but instead use some of the tax rebate to increase their precautionary savings. This less than one-for-one substitution of private sector demand for government consumption implies a drop in aggregate demand, requiring an adjustment of the real interest rate to stimulate demand such that it equals supply. As explained above, however, the steady state real interest rate cannot adjust to equate supply and demand as it is pinned down by the nominal interest rate set by monetary policy and the inflation rate which is equal to the growth rate of nominal government spending. Therefore the price level has to adjust such that demand equals supply when the real interest rate equals  $r_{ss}$ .

This determinacy result implies that both the price level in the steady state before the fiscal stimulus,  $P_{ss}$ , and the price level in the new steady state,  $P_{ss}^{new}$  to which the economy converges after the stimulus has died out are uniquely and jointly determined by fiscal and monetary policies. In Figure 3 the long-run policies are unchanged



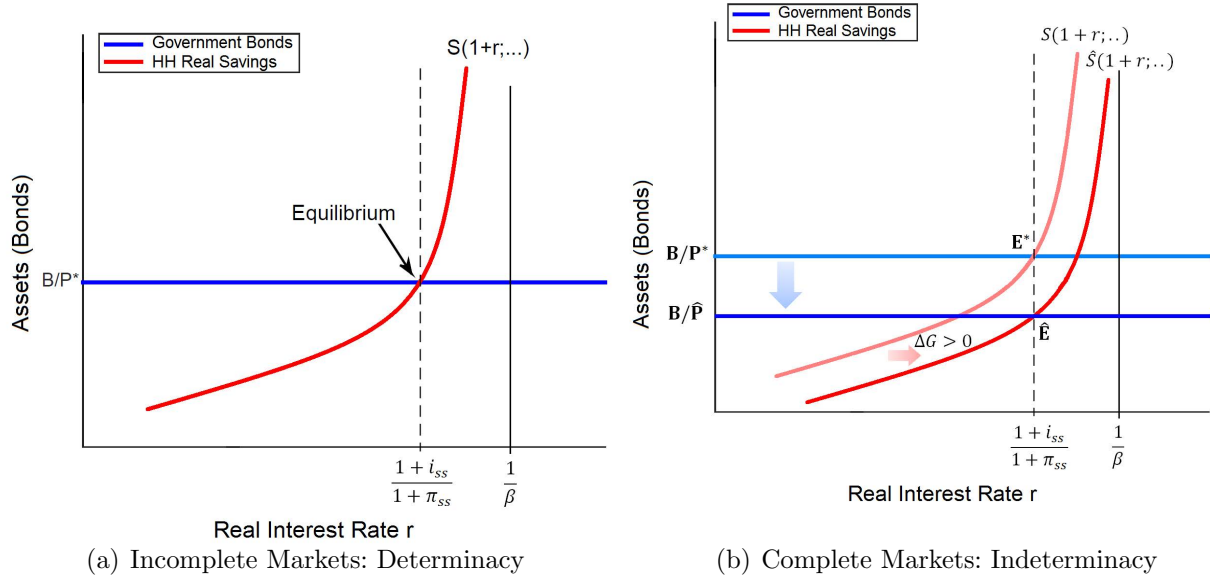


Figure 4: Price Level and Permanent Higher Spending,  $\Delta G > 0$ .

$$B_{ss}^{new} = B_{ss}, G_{ss}^{new} = G_{ss}, T_{ss}^{new} = T_{ss}. \quad (16)$$

As a result the identical equations,

$$\frac{1+i_{ss}}{1+\pi_{ss}} = (1+r_{ss}(G_{ss}/P_{ss}, T_{ss}/P_{ss}, B_{ss}/P_{ss}, \dots)) \quad (17)$$

$$= (1+r_{ss}(G_{ss}^{new}/P_{ss}^{new}, T_{ss}^{new}/P_{ss}^{new}, B_{ss}^{new}/P_{ss}^{new}, \dots)) \quad (18)$$

determine the price levels  $P_{ss}$  and  $P_{ss}^{new}$  which are therefore identical,  $P_{ss} = P_{ss}^{new}$ . As explained above, the accumulated inflation rate is linked one-to-one to the size of the fiscal multiplier in complete market models but is indeterminate. But we can still ask what the price path response in our incomplete markets model would imply for the fiscal multiplier in a model with complete markets. In the scenario shown in Figure 3 where the price level eventually returns to its pre-stimulus steady state level,  $P_{ss}^{new} = P_{ss}$ , the accumulated inflation rate equals (assuming  $\pi_{ss} = 0$ )

$$\prod_{t=1}^{T-1} (1 + \pi_{t+1}) = \frac{P_{ss}^{new}}{P_1} = \frac{P_{ss}}{P_1} < 1, \quad (19)$$

as the price level jumps from  $P_{ss}$  to  $P_1$  in the initial period. Then the as-if complete markets multiplier would be smaller than one.

If on the other hand nominal government expenditure is a higher by  $\Delta G > 0$  in the new

steady state, then the new price level will be higher as well,  $P_{ss}^{new} > P_{ss}$ , as is illustrated in Figure 4. The left panel shows the old steady state. In the right panel, government expenditure and therefore taxes are permanently higher so that households save less. This downward shift the savings curve requires an increase in the price level such that the real value of bonds falls and is equal to the now lower real aggregate savings. As a result, the as-if complete markets multiplier would be higher in scenario Figure 4 than in Figure 3.

What is important for our aim to compute the fiscal multiplier is that in our model the accumulated inflation rate is also uniquely determined by policy. Furthermore the model delivers unique responses of prices, output, consumption and employment although monetary policy is not following an active rule but instead is stuck at the ZLB, a necessary requirement for a locally determined equilibrium in New Keynesian models with complete markets.

But in incomplete markets models, the accumulated inflation rate is not linked one-to-one to the size of the multiplier as the derivation for complete markets does not apply here. Since wealth is heterogenous and credit constraints are binding occasionally, one cannot iterate using the consumption Euler equation of the representative household. Instead the stimulus policy has distributional consequences which since marginal propensities to consume are heterogenous affect aggregate consumption and thus also employment. We discuss those now.

### 3.1.3 Distributional Consequences of a Stimulus

An increase in spending, the necessary adjustments in taxes and transfers and the resulting responses of prices and hours operate through various distributional channels. Changes in the tax code naturally deliver winners and losers. An increase in the price level and of labor income leads to a redistribution from households who finance their consumption more from asset income to households who rely more on labor income. Changes in interest rates also redistribute between debtors and lenders.

These redistributions matter due to the endogenous heterogeneity in the MPCs in the data and in our incomplete markets model. This heterogeneity together with the redistribution determines the aggregate consumption response, and since output is demand determined due to price rigidities, also determines output. Individual household consumption  $c_t$  depends on transfers  $T$ , tax rates  $\tau$ , labor income  $wh$ , prices  $P$  and nominal interest rates  $i$ , so that

aggregate private consumption

$$C_t(\{T_t, \tau_t, w_t h_t, P_t, i_t\}_{t \geq 0}) = \int c_t(a, s; \{T_t, \tau_t, w_t h_t, P_t, i_t\}_{t \geq 0}) d\Omega_t. \quad (20)$$

In our model hours is a household choice variable but demand determined as well. Of course consumption and hours worked are jointly determined in equilibrium but to understand the demand response of the fiscal stimulus it turns out to be useful to consider  $wh$  as exogenous for consumption decisions here. In particular it allows us to distinguish between the initial impact, “first round”, demand impulse due to the policy change and “second, third ... round” due to equilibrium responses. Those arise in our model since an initial policy-induced demand stimulus leads to more employment by firms, and so higher labor income which in turn implies more consumption demand, which again leads to more employment and so on until an equilibrium is reached where all variables are mutually consistent. Denoting pre stimulus variables by a bar, we can now decompose the aggregate consumption response,

$$(\Delta C)_t = C_t(\{T_t, \tau_t, w_t h_t, P_t, i_t\}_{t \geq 0}) - C_t(\{\bar{T}, \bar{\tau}, \bar{w} \bar{h}, \bar{P}, \bar{i}\}_{t \geq 0}) \quad (21)$$

into its different channels:

$$(\Delta C)_t = \underbrace{C_t(\{T_t, \tau_t, \bar{w} \bar{h}, \bar{P}, \bar{i}\}_{t \geq 0}) - C_t(\{\bar{T}, \bar{\tau}, \bar{w} \bar{h}, \bar{P}, \bar{i}\}_{t \geq 0})}_{\text{Direct Impact of Transfers and Taxes}} \quad (22)$$

$$+ \underbrace{C_t(\{T_t, \tau_t, w_t h_t, \bar{P}, \bar{i}\}_{t \geq 0}) - C_t(\{T_t, \tau_t, \bar{w} \bar{h}, \bar{P}, \bar{i}\}_{t \geq 0})}_{\text{Indirect Equilibrium Effect: Labor Income}} \quad (23)$$

$$+ \underbrace{C_t(\{T_t, \tau_t, w_t h_t, P_t, i_t\}_{t \geq 0}) - C_t(\{T_t, \tau_t, w_t h_t, \bar{P}, \bar{i}\}_{t \geq 0})}_{\text{Price and Interest rate Adjustment}} \quad (24)$$

Total demand is the sum of private consumption demand  $C$  and real government consumption  $g = G/P$ , which both determine output. The private consumption response does not directly depend on  $G/P$  but it does indirectly. First, transfers  $T$  and taxes  $\tau$  have to adjust to balance the intertemporal government budget constraint. Second, increases in  $G/P$  translate one-for-one into increases in demand. On impact an increase by  $\Delta g$  increases demand by  $\Delta g$  and thus our worked from  $h_{ss}$  to  $h_{ss} + \Delta g$ . As before, this increase in labor income stimulates private demand which in turn leads to higher employment, then again higher consumption and so

on until convergence. We therefore decompose the total demand effect  $\Delta D$  of an increase in government spending by  $\Delta g$  as

$$(\Delta D)_t = \underbrace{(\Delta g)_t}_{\text{Direct Govt' Spending Response}} + \underbrace{(\Delta C)_t}_{\text{Private Consumption Response}} \quad (25)$$

$$= (\Delta \mathbf{g})_t + \underbrace{C_t(\{\bar{T}, \bar{\tau}, \bar{w}(\bar{h} + \Delta \mathbf{g}), \bar{P}, \bar{i}\}_{t \geq 0}) - C_t(\{\bar{T}, \bar{\tau}, \bar{w}\bar{h}, \bar{P}, \bar{i}\}_{t \geq 0})}_{\text{Direct Impact on Private Consumption}} \quad (26)$$

$$+ \underbrace{C_t(\{T_t, \tau_t, w_t h_t, P_t, i_t\}_{t \geq 0}) - C_t(\{\bar{T}, \bar{\tau}, \bar{w}(\bar{h} + \Delta \mathbf{g}), \bar{P}, \bar{i}\}_{t \geq 0})}_{\text{Indirect Private Consumption Response}} \quad (27)$$

A fiscal stimulus, in addition to the immediate impact on government demand, also leads to higher employment and labor income and thus stimulates private consumption, the *Direct Impact on Private Consumption*. The remainder of the private consumption is as above the sum of the direct impact of transfers and taxes, the indirect equilibrium effects of labor income and and price and interest rate adjustment, such that the full decomposition of the total demand effect  $\Delta D$  is

$$(\Delta D)_t = \underbrace{(\Delta g)_t}_{\text{Direct G Impact}} \quad (28)$$

$$+ \underbrace{C_t(\{\bar{T}, \bar{\tau}, \bar{w}(\bar{h} + \Delta g), \bar{P}, \bar{i}\}_{t \geq 0}) - C_t(\{\bar{T}, \bar{\tau}, \bar{w}\bar{h}, \bar{P}, \bar{i}\}_{t \geq 0})}_{\text{Direct G Impact on C}} \quad (29)$$

$$+ \underbrace{C_t(\{T_t, \tau_t, \bar{w}(\bar{h} + \Delta g), \bar{P}, \bar{i}\}_{t \geq 0}) - C_t(\{\bar{T}, \bar{\tau}, \bar{w}(\bar{h} + \Delta g), \bar{P}, \bar{i}\}_{t \geq 0})}_{\text{Indirect Tax/Transfer Impact}} \quad (30)$$

$$+ \underbrace{C_t(\{T_t, \tau_t, w_t h_t, \bar{P}, \bar{i}\}_{t \geq 0}) - C_t(\{T_t, \tau_t, \bar{w}(\bar{h} + \Delta g), \bar{P}, \bar{i}\}_{t \geq 0})}_{\text{Indirect Labor Income Impact}} \quad (31)$$

$$+ \underbrace{C_t(\{T_t, \tau_t, w_t h_t, P_t, i_t\}_{t \geq 0}) - C_t(\{T_t, \tau_t, w_t h_t, \bar{P}, \bar{i}\}_{t \geq 0})}_{\text{Indirect Price and Interest Impact}} \quad (32)$$

### 3.1.4 Multiplier: Definition

As we can now be sure that the fiscal multiplier is well defined in our economy, we now follow Farhi and Werning (2013) in computing the response of the economy to a fiscal stimulus.

Concretely, we compute the response of the economy to an unexpected increase in the path of nominal government spending to  $G_1 = G_{ss}, G_2, \dots, G_t, \dots, G_{ss}$ , where  $G_{ss}$  is the

steady nominal spending level and  $G_t \geq G_{ss}$ .

We summarize the effects of spending on output in several ways. First, we compute the path of dynamic multipliers as the sequence of

$$m_t^D = \frac{\frac{Y_t}{Y_{ss}} - 1}{\frac{G_0 P_{ss}}{P_0 G_{ss}} - 1} \frac{Y_{ss}}{G_{ss}/P_{ss}}, \quad (33)$$

and the path of static multipliers as

$$m_t^S = \frac{\frac{Y_t}{Y_{ss}} - 1}{\frac{G_t P_{ss}}{P_t G_{ss}} - 1} \frac{Y_{ss}}{G_{ss}/P_{ss}}, \quad (34)$$

and the present value multipliers as

$$m_t^{PV} = \frac{\sum_{k=0}^t \beta^k (\frac{Y_k}{Y_{ss}} - 1)}{\sum_{k=0}^t \beta^k (\frac{G_k P_{ss}}{P_k G_{ss}} - 1)} \frac{Y_{ss}}{G_{ss}/P_{ss}}, \quad (35)$$

where all three statistics coincide at  $t = 0$  and represent the impact multiplier. A useful statistic is then the long-run present value multiplier, which represents the discounted percentage change in real output to the discounted percentage change in real government spending for any path of government spending:

$$\bar{M} = m_\infty^{PV} = \frac{\sum_{t=0}^{\infty} \beta^t (\frac{Y_t}{Y_{ss}} - 1)}{\sum_{t=0}^{\infty} \beta^t (\frac{G_t P_{ss}}{P_t G_{ss}} - 1)} \frac{Y_{ss}}{G_{ss}/P_{ss}}, \quad (36)$$

where  $P_{ss}, G_{ss}, Y_{ss}$  are the steady state price level, nominal spending and nominal output respectively and  $\frac{Y_t}{P_t}$  and  $\frac{G_t}{P_t}$  are real output and real government spending respectively. For comparison with the complete markets case we also compute the as-if dynamic complete markets multiplier,  $m_t^{CM}$ , using the price path we obtain from our model:

$$\left( \frac{1}{\frac{G_0 P_{ss}}{P_0 G_{ss}} - 1} \frac{G_{ss}/P_{ss}}{C_{ss}} \right) m_t^{CM} = \frac{C_t}{C_{ss}} = \prod_{s=t}^{T-1} (1 + \pi_{s+1}) = \frac{P_{ss}^{new}}{P_t}.$$

### 3.2 Calibration

To quantitatively assess the size of the fiscal multiplier we now calibrate the model.

**Preferences** Households have GHH preferences over labor nested within constant relative risk aversion preferences for consumption. We set the risk-aversion parameter,  $\sigma$ , equal to 2. We choose the discount factor,  $\beta$ , to target a quarterly risk-free rate of 25 BP. We assume the functional form for  $g$ :

$$g(h, s) = s\psi \frac{h^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \quad (37)$$

This formulation implies that households do not adjust their labor supply in response to changes in their idiosyncratic productivity, but only movements in the wage rate per unit of labor efficiency. We set the Frisch elasticity,  $\varphi = 0.5$ , following micro estimates. We choose  $\psi = 0.6$  such that in steady state  $h = 1$ .

**Productivity Process** We follow Krueger et al. (2016) who use data from the Panel Survey of Income Dynamics to estimate a stochastic process for labor productivity. They estimate that log income consists of a persistent and transitory component. They estimate that the persistent shock has an annual persistence of 0.9695 and variance of innovations of 0.0384. We treat the transitory shock as measurement error. We follow Krueger et al. (2016) in converting these annual estimates into a quarterly process. We discretize the persistent shock into a seven state Markov chain using the Rouwenhorst method.

**Production Technology** We assume constant returns to scale so  $\alpha = 1$ . We choose the elasticity of substitution between intermediate goods,  $\epsilon = 10$ , to match an average markup of 10%. The adjustment cost parameter on prices,  $\theta = 300$ , to match a slope of the NK Philips curve,  $\epsilon/\theta = 0.03$ . We set the firm operating cost  $\Phi$  equal to 80% of the steady state markup such that steady state profits equal 0.2% (Basu and Fernald (1997)). These profits are fully taxed and are distributed to households as lump-sum transfers in the benchmark.

**Government** We set the proportional labor income tax,  $\tau$  equal to 25%. We set nominal government spending,  $G$  in steady state equal to 15% of output. The value of lump-sum transfers  $T$  is set to 7.25% of output to generate a steady-state government debt to annual GDP ratio of 0.8.

Table I: Calibrated Parameters

| Parameter  | Interpretation     | Internally Calibrated | Value          |
|------------|--------------------|-----------------------|----------------|
| $\sigma$   | Risk-aversion      | N                     | 2              |
| $\beta$    | Discount Factor    | Y                     | 1.3            |
| $\varphi$  | Frisch Elasticity  | N                     | 0.5            |
| $\psi$     | Labor disutility   | Y                     | 0.6            |
| $\epsilon$ | Elas. substitution | N                     | 10             |
| $\theta_p$ | Price adjustment   | N                     | 300            |
| $\theta_w$ | Wage adjustment    | N                     | 300            |
| $\Phi$     | Firm Fixed Cost    | Y                     | 0.1            |
| $\tau$     | Labor tax          | N                     | 25%            |
| $T$        | Transfer           | Y                     | 7.5% of income |
| $\theta_w$ | Wage adjustment    | N                     | 300            |

**Monetary Policy** For the benchmark specification we assume that the monetary authority operates a constant interest rate peg of  $i = 0$ . Note that the results in Hagedorn (2016) imply that there is a unique response of prices, output, consumption and employment although monetary policy is not following an active rule, a necessary requirement for a locally determined equilibrium. For purposes of comparison, we will also solve for transitions where we assume that the monetary authority follows a Taylor rule, which sets the nominal interest rate according to:

$$i_{t+1} = \max(X_{t+1}, 0) \quad (38)$$

where

$$X_{t+1} = \left(\frac{1}{\zeta}\right) \left(\frac{P_t}{P_{ss}}\right)^{\phi_1(1-\rho_R)} \left(\frac{Y_t}{Y_{ss}}\right)^{\phi_2(1-\rho_R)} [\zeta(1+i_t)]^{\rho_R} - 1.$$

We follow the literature in setting  $\rho_R = 0.8$ ,  $\phi_1 = 1.5$ ,  $\phi_2 = 0$  and  $\zeta = 1/(1+r_{ss})$ . Fiscal monetary coordination will be carried out under various schemes listed in the next section.

**Price and Wage Philips Curves** We set the slopes of both the NK price and wages Philips curve to 0.03, which is at the lower end of available estimates.

## Parameter Values

Table II: Net Worth Distributions: Data vs Model

| % Share held by: | Data (SCF 07) | Model |
|------------------|---------------|-------|
| Q1               | -0.2          | 0.3   |
| Q2               | 1.2           | 1.3   |
| Q3               | 4.6           | 3.4   |
| Q4               | 11.9          | 10.3  |
| Q5               | 82.5          | 84.7  |
| 90-95            | 11.1          | 16.7  |
| 95-99            | 25.3          | 31.1  |
| Top 1%           | 33.5          | 18.5  |

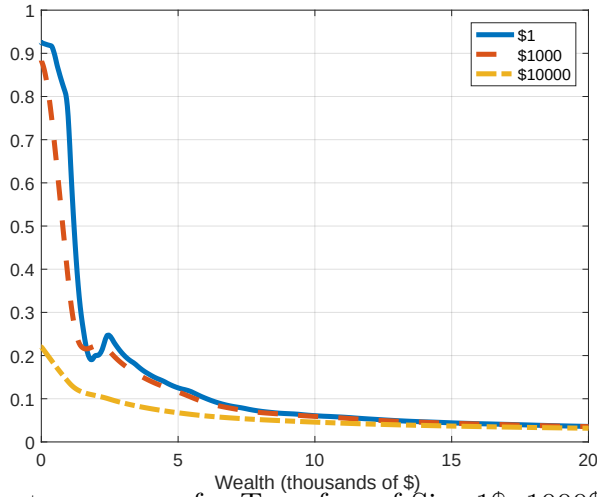


Figure 5: Propensity to consume for Transfers of Size 1\$, 1000\$ and 10000\$.

**Steady State Model Fit** Table II shows that we match the distribution of net worth as well as the Gini coefficient quite well.

In the model 2% of agents have 0 wealth, and 14% of agents less than \$1000. The annual MPC out of transitory income equals 0.4, which is in the middle range of empirical estimates 0.2-0.6 (e.g. Johnson, Parker and Souleles) Figure 5 shows the distribution of first period MPCs as function of households assets for transfers of various sizes, 1\$, 1000\$ and 10000\$.

Figure 6 shows the dynamic response of aggregate consumption  $\{C_t\}_{t=0,1,2,\dots}$  to transfers of various sizes, 1\$, 1000\$ and 10000\$ paid once at periods 0, 4, 8 and 12.



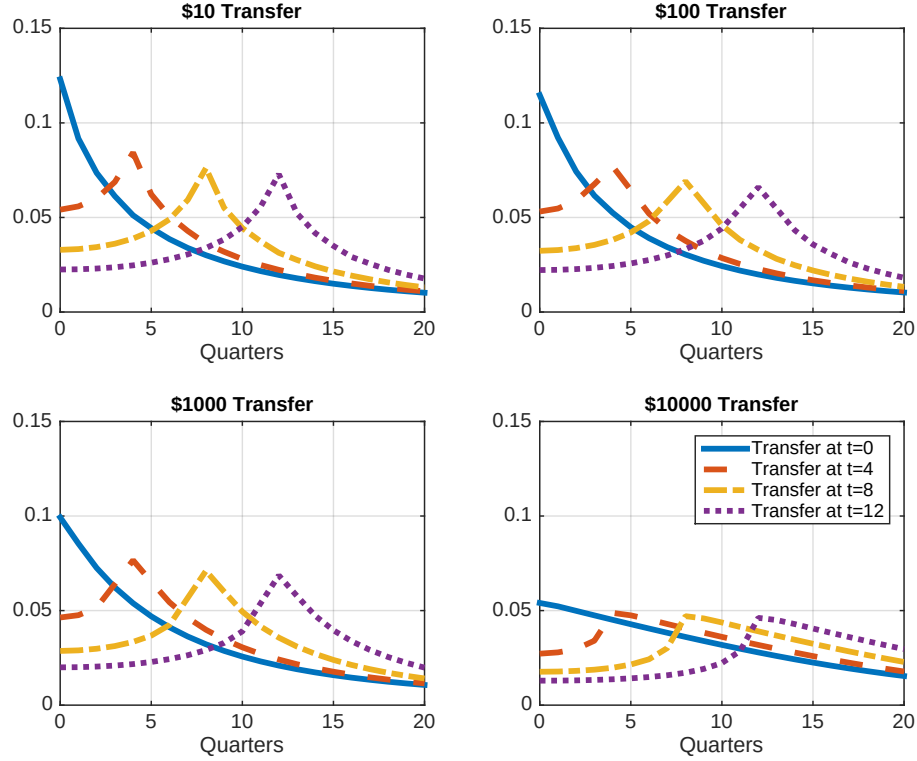


Figure 6: Propensity to consume for Transfers of Size 1\$, 100\$, 1000\$ and 10000\$.

### 3.3 Results

We can now compute the response of prices, employment, output and consumption to a persistent increase in nominal government consumption by one percent where spending follows an AR(1) process with parameter  $\rho_g = 0.7$  after the initial innovation. Balancing the government budget when government spending is increased requires to adjust taxes or debt or both. As Ricardian equivalence does not hold in our model different assumptions on the path of taxes and debt will have different implications for the path of aggregate consumption and therefore prices and the change in output. We consider two scenarios:

1. Transfer are adjusted period by period to keep nominal debt constant.
2. Deficit financing and delayed transfers to pay back debt after 12 quarters.

For each of the two scenarios we report the dynamic response of hours, consumption, output, prices, tax revenue and debt as well as the of the path of dynamic and static multipliers

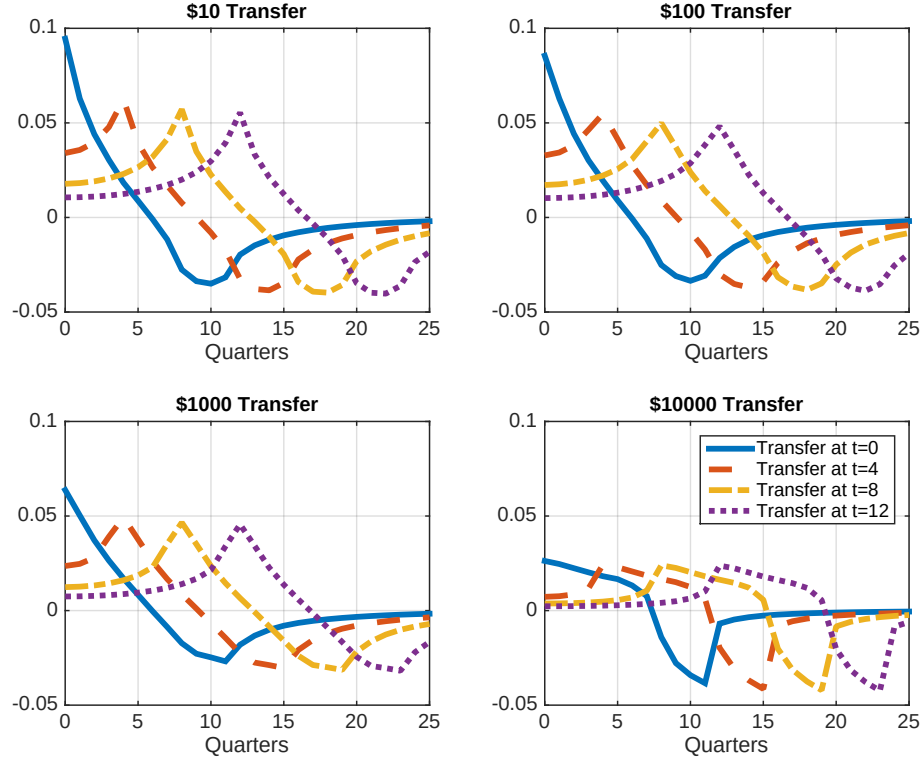


Figure 7: Propensity to consume for Transfers of Size 1\$, 100\$ 1000\$ and 10000\$ that are repaid after two years.

$m_t^D, m_t^S$  and of the as-if complete markets multiplier  $m_t^{CM}$  and the summary multiplier  $\overline{M}$ .

### 3.3.1 Tax Financing: Constant nominal debt

Under the first financing scheme, we assume that the government adjusts lump-sum transfers period by period to keep the level of nominal debt constant. The four panels of Figure 8 show the results for the aggregate consumption and output response, the different multipliers, the decomposition of aggregate consumption, and government bonds.

The level of government bonds is unchanged since the stimulus is tax-financed. On impact  $G$  increases by 1% that is 0.15% of output and consumption increases by 0.19% of output leading to an impact multiplier of 2.29. The consumption response weakens over time and gets negative from period 6 onwards such that the static multiplier, which relates the period  $t$  consumption response to the period  $t$  government spending increase, becomes negative. The dynamic multiplier converges to zero since the consumption response although negative slowly

dies out and becomes small relative to initial government spending increase. The decomposition of the total consumption response reveals the quantitative importance of the direct, the indirect and the price effects. The stimulus of 0.15% directly increases households labor supply by the same amount, leading to a aggregate consumption response of 0.093% of output. (equation 29). The contemporaneous cut in transfers lowers aggregate consumption by 0.027% on impact (equation 30) , implying a total initial effect of 0.066%. This effect is positive since the government spending has a ‘MPC of one’ whereas the transfer cut of the same size has a smaller MPC, such that households income increases. This increase leads to more consumption demand, which in turn leads to higher labor demand, higher labor income and again higher consumption demand until an equilibrium is reached. These indirect multiplier effects sum up to 0.012% (equation 31) even larger than the initial impact of spending. Finally, the decomposition shows that the price increase (and the unchanged interest rate) effects are small (equation 32).

The impulse response of the remaining variables to a 1% innovation in government spending are plotted in Figure A-1 in the appendix. The cumulated multiplier, reported in Table III, is only 0.19.

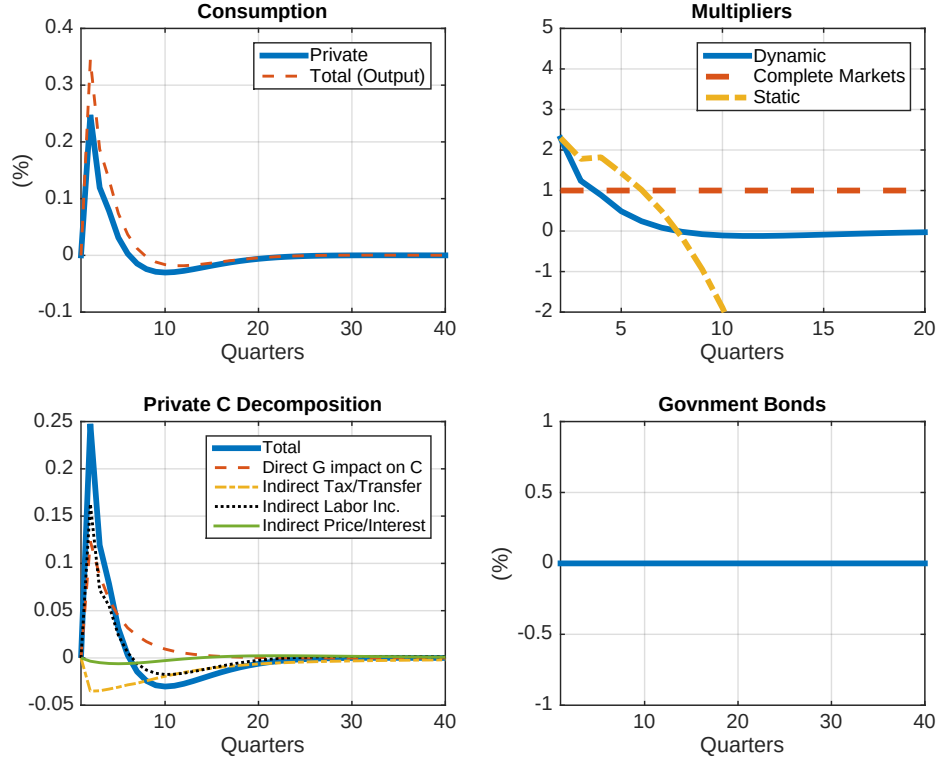
### 3.3.2 Deficit financing

Under this scenario we assume that real transfers are kept constant during the first 20 quarters after the innovation to government spending. Then, the government is assumed to adjust transfers linearly over eight quarters, keep them constant for eight quarters, and then allow transfers to revert back to the real steady state level with an autocorrelation parameter of 0.8. Thus, under this timing scheme, the government chooses only the level of adjustment to transfers to guarantee that nominal government debt returns to its original level.

The four panels of Figure 9 summarize the main results for the aggregate consumption and output response, different multipliers, the decomposition of aggregate consumption, and government bonds.

Deficit instead of tax financing increases the initial multiplier from 2.29 to 2.80 and the initial aggregate consumption response from 0.19% to 0.26%. The decomposition of the consumption responds makes clear why. The direct impact of the spending stimulus is basically identical (0.093%) but now there is no initial offsetting effect from contemporaneously higher

Figure 8: Fiscal Multiplier and Aggregate Consumption: Tax Financing

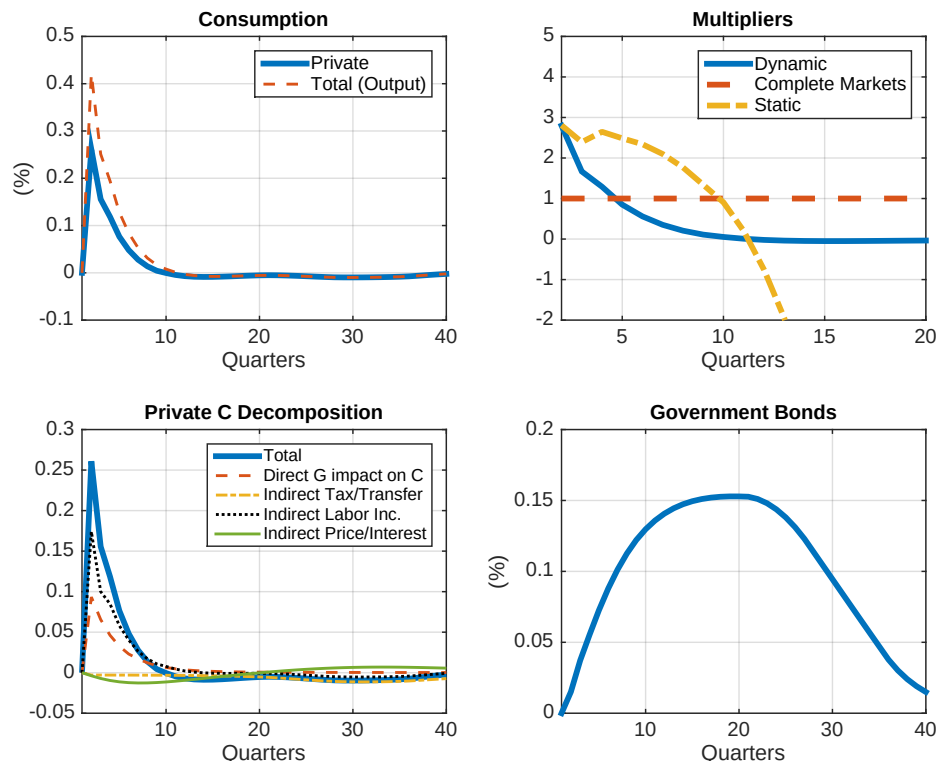


taxes. The total initial effect thus equals 0.091 (0.066% before), almost identical to the direct spending impact, leading to a larger increase in labor demand and households income. The indirect multiplier effects now accumulate to 0.017%. The deficit financing leads to an increase in government bonds and the consumption response becomes negative only from period 9 onwards. However, the increase in government spending is ultimately financed through a future reduction in transfers, which results in a contraction in future output. Thus, despite the large impact multiplier, the cumulated discounted multiplier is only around 0.31. The impulse responses of the remaining variables are plotted in Figure A-2 in the appendix.

### 3.4 Further Analysis

We now extend the analysis in various directions. First we investigate in Section 3.4.1 how the size of the fiscal multiplier depends on the MPC by considering identical economies but with lower MPCs due to relaxed credit constraints. We then use a Taylor interest rate rule to

Figure 9: Fiscal Multiplier and Aggregate Consumption: Deficit Financing



describe monetary policy instead of a nominal interest rate fixed at the ZLB in Section 3.4.2. We then ask how the size of the fiscal multiplier depends on the timing of spending (“forward spending”) and on the persistence of the stimulus in Sections 3.4.3 and 3.4.4. So far we have focused - as does the literature - on the effects of an increase in government spending. Another stimulus policy is to increase transfers and we consider such policies in Section 3.4.5. Finally, we consider spending and transfer policies in a liquidity trap in Sections 3.4.6, 3.4.7 and 3.4.8. We also investigate how the size of the multiplier depends on the scale of the stimulus and on the degree of price and wage rigidities.

### 3.4.1 Different MPCs

Here we consider in more detail how the fiscal multiplier depends on the MPC. In our benchmark analysis the annual aggregate MPC equals 0.4. We now redo the experiments from the previous Section but with lower MPCs. To obtain lower MPPs we loosen households credit

Table III: Cumulated Multiplier  $\overline{M}$

| Tax Financing | Deficit Financing |
|---------------|-------------------|
| 0.19          | 0.31              |

constraints. In the benchmark households constraint is zero, that is they cannot obtain any credit. We now consider credit constraints  $xxx$  including the natural borrowing limit which implies annual aggregate MPCs of  $xxx$ .

The four panel in Figure xxx show the results for the aggregate consumption response and its components and the fiscal multiplier for all the MPCs we consider. For each MPC we have one Figure which includes consumption, its components and the multiplier which is just the sum of the aggregate consumption response and the increase in government expenditures

We find ...

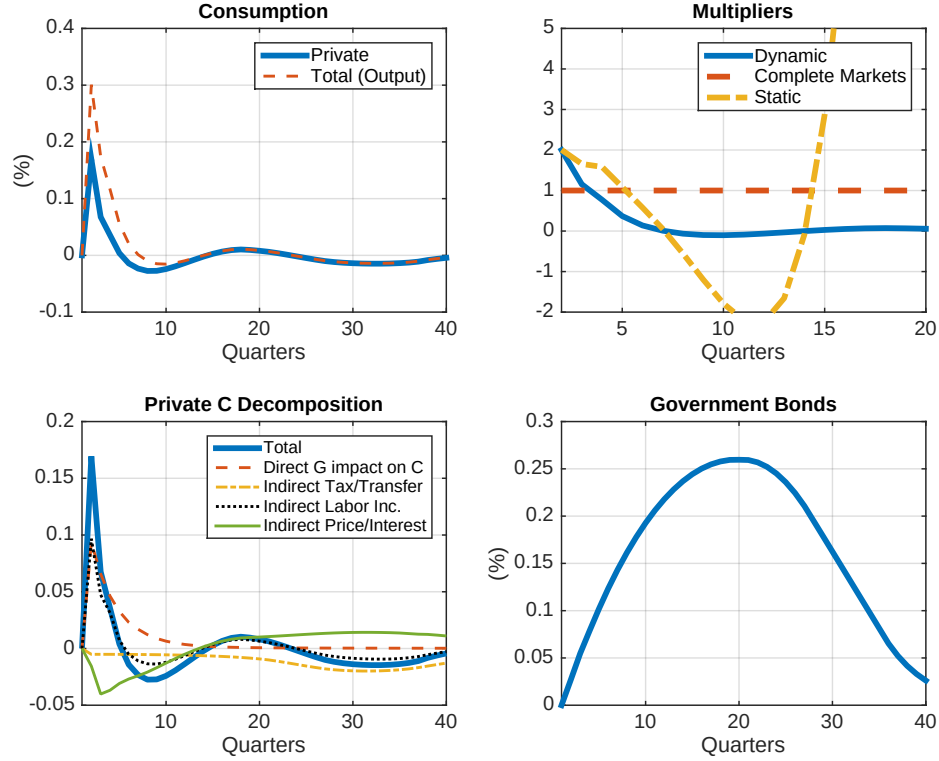
### 3.4.2 Taylor Rule

We find similar results if instead of an interest rate peg, the monetary authority follows a Taylor rule. This is not surprising since the prices respond only very little when the interest is pegged at zero. The four panels of Figure 9 summarize the main results. The impulse response are plotted in Figure A-3. The same conclusion is reached for tax instead of deficit financing as the impulse responses in Figure A-4 show.

### 3.4.3 Forward Spending

The multiplier gets smaller if the spending is pre-announced to occur at a future date, 8 quarters from now. The additional spending is deficit financed. The price level now increases gradually in anticipation of the future increase in government spending such that Initially output falls before it increases at the time of the spending increase 8 quarters in the future. However, the increase in consumption as well as the multiplier at that time are smaller than the corresponding multiplier in the case when the stimulus occurs immediately and is deficit

Figure 10: Fiscal Multiplier and Aggregate Consumption: Taylor Rule and Deficit Financing



financed. The impulse responses to a spending increase 8 quarters in the future are plotted in Figure A-6 in the appendix.

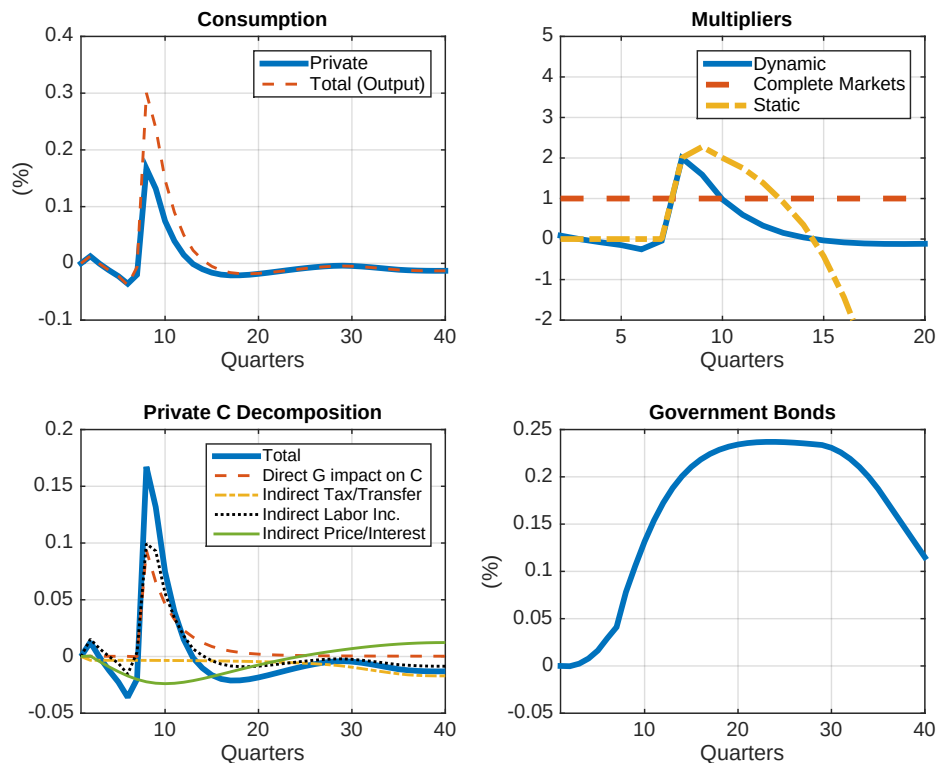
#### 3.4.4 More Persistent Spending

We now again compute the response of prices, employment, output and consumption to a persistent increase in nominal government consumption by one percent where spending follows an AR(1) process but now with a higher persistence parameter  $\rho_g = 0.9 > 0.7$ . Figure A-7 in the appendix shows the impulse responses.

#### 3.4.5 Transfer Multiplier

In this section we consider the multiplier in response to a one percent increase in government transfers. We assume that nominal government spending adjusts to keep real government spending constant in response to the innovation in transfers. We allow the government to

Figure 11: Future (+8 quarters) spending: Deficit Financing



finance the increase in transfers by first increasing government debt, but by increasing future transfers as in the previous section to pay back the debt.

The impulse response is plotted in Figure A-5. The impulse response is qualitatively and quantitatively similar to the impulse response to an increase in government spending with delayed repayment. Output rises more, however, when transfers increase than when spending increases. This can be understood because, in addition to an increase in spending coming from an increase in the price level and a decline in the real rate, the heterogeneity in marginal propensities to consume means that some households will increase their spending by even more than would be implied from the fall in the real rate in a representative agent model. However, the cumulative multiplier ends up being around -0.1. As the future decrease in transfers needed to return nominal government debt to its steady state level are sufficiently contractionary to offset the contemporaneous gains.



Table IV: Main Results Consumption and Multipliers

|                                     | Baseline    |         | Taylor Rule | Forward | Persistence | Transfer | MPC |     |
|-------------------------------------|-------------|---------|-------------|---------|-------------|----------|-----|-----|
|                                     | Tax-Finance | Deficit | Deficit     | Deficit | Deficit     | Deficit  | 0.x | 0.x |
|                                     | (1)         | (2)     | (3)         | (4)     | (5)         | (6)      | (7) | (8) |
| Impact Mult.                        | 2.29        | 2.80    |             |         |             |          |     |     |
| Cumul. Mult.                        | 0.19        | 0.31    | 0.15        |         |             |          |     |     |
| $\Delta C_0$                        | 0.19        | 0.26    |             |         |             |          |     |     |
| <u>Decomposition of Consumption</u> |             |         |             |         |             |          |     |     |
| Direct G on C                       | 0.093       | 0.093   |             |         |             |          |     |     |
| Tax/Transfers                       | -0.027      | -0.002  |             |         |             |          |     |     |
| Indirect Income                     | 0.012       | 0.017   |             |         |             |          |     |     |
| Prices                              | -0.003      | -0.004  |             |         |             |          |     |     |

Note - The table contains the impact and the cumulated multiplier  $\bar{M}$  as well as the initial consumption response  $\Delta C_0$ . The last four rows show the decomposition of the initial aggregate consumption response into the direct  $G$  impact on  $C$  (equation 29), the effect of taxes/transfers (equation 30), indirect income effects (equation 31) and the price and interest rate effects (equation 32).

### 3.4.6 Liquidity Trap

In this section we explore the extent to which the size of the multiplier may vary with other shocks hitting the economy. In particular, we consider what the government multiplier is after a demand shock. We therefore first have to generate a liquidity trap in the model, where the ZLB on nominal interest rates is binding. In doing so we follow Cochrane (2015) and construct a series of discount factors  $\{\beta_t\}_{(t=1,2,\dots)}$  such that the natural real rate of interest - the real interest rate in a world with flexible prices and wages - equals  $-2\%$  for 5 years and then returns to zero afterwards. All other parameters are unchanged.

We then feed the series of discount factors  $\{\beta_t\}_{(t=1,2,\dots)}$  into our model with price and wage rigidities and calculate the response of the economy, which is shown in Figure 12. The resulting recession is quite large as output initially drops by about 5 percent. We solve for the impulse response to the demand shock under two scenarios previously considered, one - tax financing - where real government debt is kept constant and the other - deficit financing - where we adjust transfers in the future to return back to steady state nominal debt.

Under these two scenarios, we also compute the effect of a simultaneous (at the same time as the liquidity trap starts) 1% increase in nominal government spending. Thus, we can compute the fiscal multiplier as the percent increase in output under this scenario, relative to the benchmark with no increase in spending, divided by the relative percent differences in government spending. The multipliers are plotted in the left panel of Figure 13. The right panel of Figure 13 shows the transfer multiplier, where again only deficit financing is meaningful.

### 3.4.7 Scale Effects

We consider the size of the multiplier in the liquidity trap described above and how it depends on the scale of the government spending and transfer stimulus. The left panel of Figure 14 shows the government spending multiplier for a 1%, 2%, 5%, 10% increase. The right panel of Figure 14 shows the same for the transfer multiplier again for 1%, 2%, 5%, 10% increases.

### 3.4.8 The degree of price and wage rigidities

Show liquidity trap multiplier for various degrees of price and wage rigidities. The left panel of Figure 15 shows the government spending multiplier for various degrees of price rigidities,

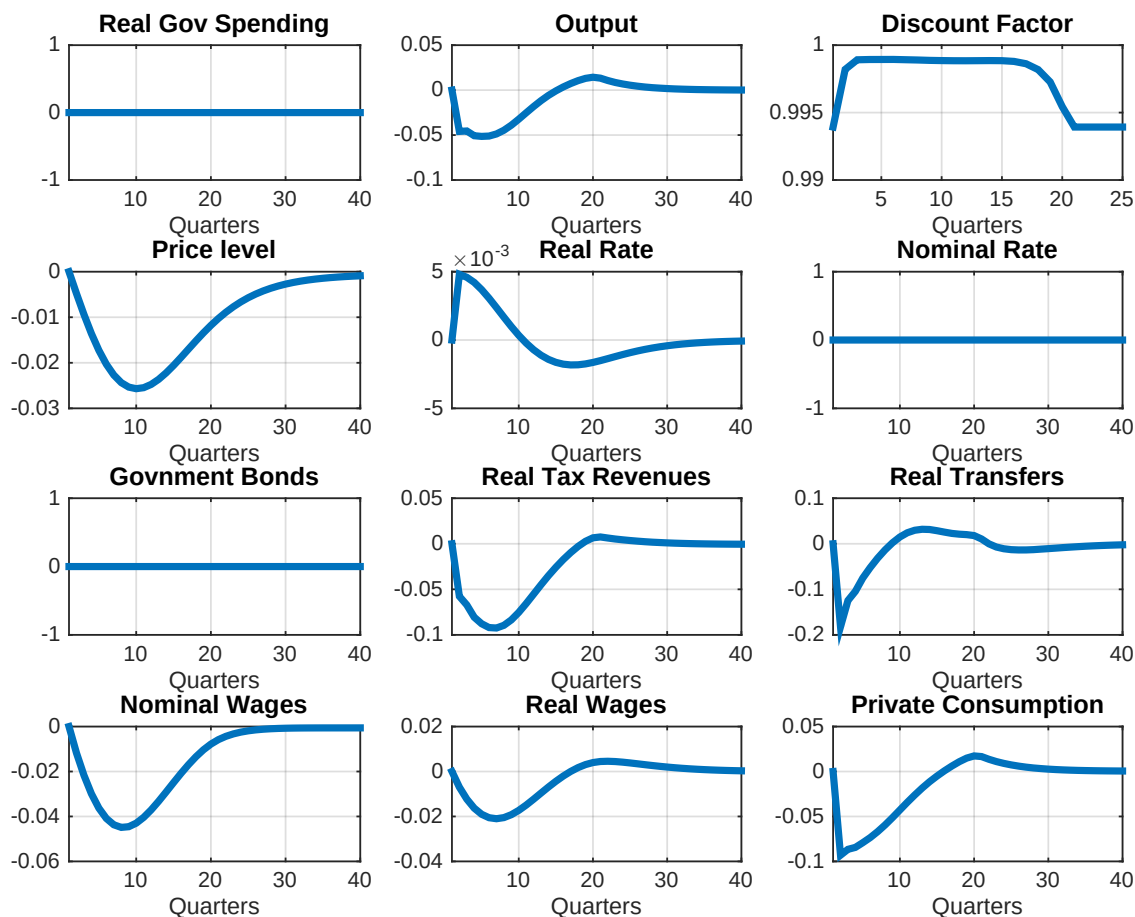


Figure 12: Economy in a Liquidity Trap

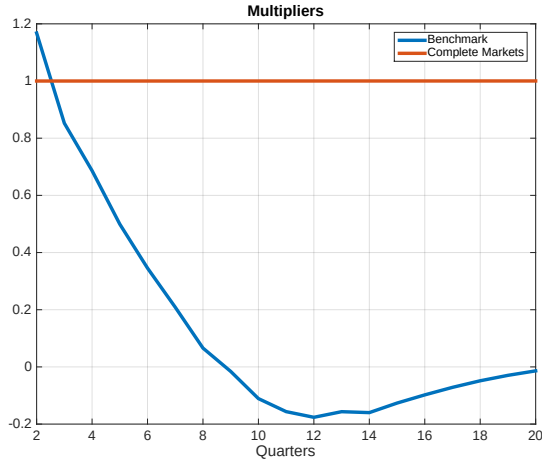
including fully flexible prices. The right panel of Figure 15 shows the same for wage rigidities.

## 4 The Fiscal Multiplier in the Generalized Model with Capital

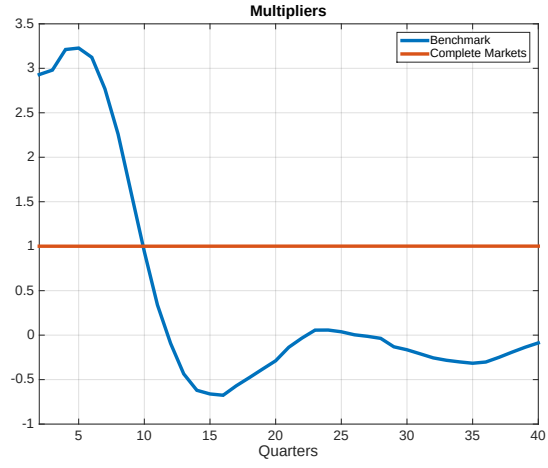
In this section we consider a generalized version of the model considered in the previous sections where we allow for capital accumulation (with adjustment costs and capital utilization) and for habit persistence in consumption. In addition not only prices but also wages are sticky.

### 4.1 The generalized Model

The government and the final good sector are as in the simple model. What changes are the household sector and the intermediate goods sector. In addition we add another sector, which transforms households savings into investment goods.

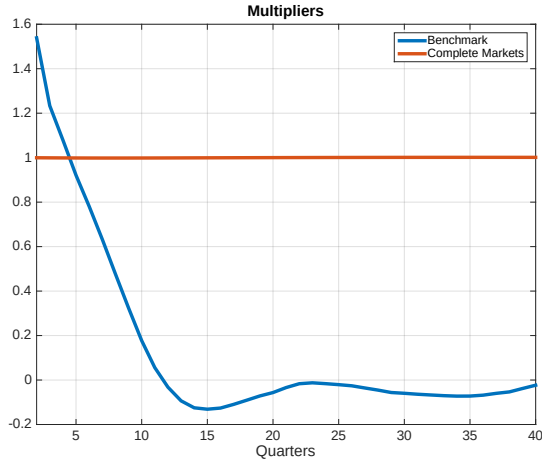


(a) Spending Multiplier

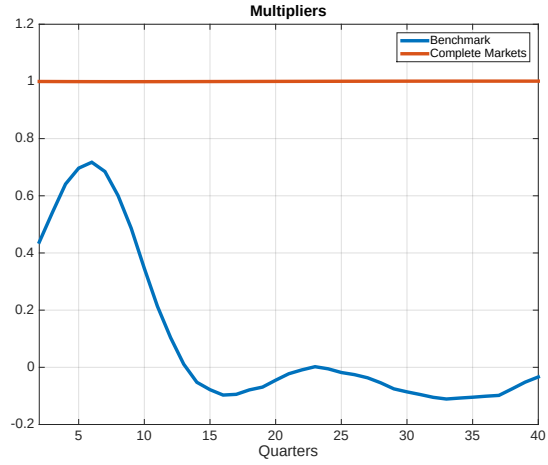


(b) Transfer Multiplier

Figure 13: Fiscal Multipliers in a Liquidity Trap



(a) Scale of Spending



(b) Scale of Transfers

Figure 14: Multiplier in a Liquidity Trap and Scale of Spending/Transfers (Deficit Financing)

#### 4.1.1 Households

We add habit persistence in consumption so that households utility function is

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t - \kappa c_{t-1}, h_t, s_t)$$

where:

$$u(c - \kappa c_{-1}, h, s) = \begin{cases} \frac{(c - \kappa c_{-1} - sg(h))^{1-\sigma} - 1}{1-\sigma} & \text{if } \sigma \neq 1 \\ \log(c - \kappa c_{-1} - sg(h)) & \text{if } \sigma = 1 \end{cases}$$

To allow for sticky wages we follow the literature and assume that each household  $j$  provides

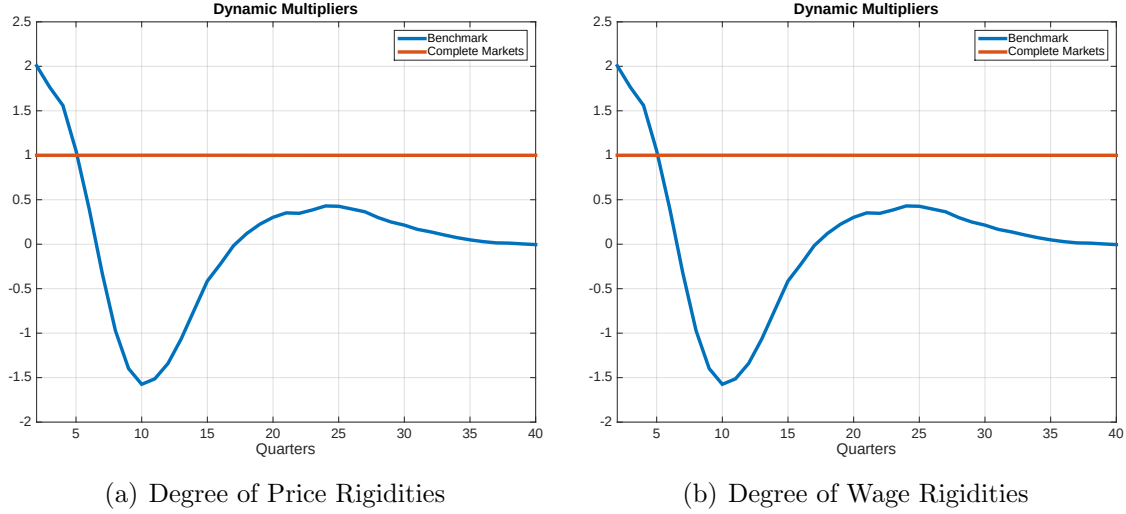


Figure 15: Multiplier in a Liquidity Trap and Degree of Rigidities

differentiated labor services,  $h_{jt}$ . These differentiated labor services are transformed by a representative, competitive labor packer firm into an aggregate effective labor input,  $H_t$  using the following technologies:

$$H_t = \left( \int_0^1 s_{jt} (h_{jt})^{\frac{\epsilon_w - 1}{\epsilon_w}} dj \right)^{\frac{\epsilon_w}{\epsilon_w - 1}}, \quad (39)$$

where  $\epsilon_w$  is the elasticity of substitution across labor services.

A middleman firm (e.g. a union) sells households labor services to the labor packer, which given aggregate labor demand  $H$  by the intermediate goods sector, minimizes costs

$$\int_0^1 W_{jt} s_{jt} h_{jt} dj, \quad (40)$$

implying a demand for the labor services of household  $j$ :

$$h_{jt} = h(W_{jt}; W_t, H_t) = \left( \frac{W_{jt}}{W_t} \right)^{-\epsilon_w} H_t, \quad (41)$$

where  $W_t$  is the (equilibrium) nominal wage which can be expressed as

$$W_t = \left( \int_0^1 s_{jt} W_{jt}^{1-\epsilon_w} dj \right)^{\frac{1}{1-\epsilon_w}}.$$

The middleman sets a nominal wage  $\hat{W}_t$  for an effective unit of labor so that  $W_{jt} = \hat{W}_t$

to maximize profits subject to wage adjustment costs similar to the price adjustment costs as in Rotemberg (1982). These adjustment costs are proportional to idiosyncratic productivity  $s$  and are measured in units of aggregate output and are given by a quadratic function of the change in wages above and beyond steady state wage inflation  $\bar{\Pi}^w$ ,

$$\Theta \left( s_{jt}, W_{jt} = \hat{W}_t, W_{jt-1} = \hat{W}_{t-1}; Y_t \right) = s_{jt} \frac{\theta_w}{2} \left( \frac{W_{jt}}{\hat{W}_{t-1}} - \bar{\Pi}^w \right)^2 H_t = s_{jt} \frac{\theta_w}{2} \left( \frac{\hat{W}_t}{\hat{W}_{t-1}} - \bar{\Pi}^w \right)^2 H_t.$$

The middleman's wage setting problem is to maximize<sup>3</sup>

$$\begin{aligned} & V_t^w \left( \hat{W}_{t-1} \right) \\ \equiv & \max_{\hat{W}_t} \int \left( \frac{s_{jt}(1-\tau_t)\hat{W}_t}{P_t} h(\hat{W}_t; W_t, H_t) - s_{jt}g(h(\hat{W}_t; W_t, H_t)) \right) dj - \int s_{jt} \frac{\theta_w}{2} \left( \frac{\hat{W}_t}{\hat{W}_{t-1}} - \bar{\Pi}^w \right)^2 H_t dj. \\ & + \frac{1}{1+r_t} V_{t+1}^w \left( \hat{W}_t \right), \end{aligned} \quad (42)$$

Some algebra (delegated to the appendix) yields, using  $h_{jt} = H_t$  and  $\hat{W}_t = W_t$  and defining the real wage  $w_t = \frac{W_t}{P_t}$ , the wage inflation equation

$$\theta_w \left( \pi_t^w - \bar{\Pi}^w \right) \pi_t^w = (1-\tau_t)(1-\epsilon_w)w_t + \epsilon_w g'(h(\hat{W}_t; W_t, H_t)) + \frac{1}{1+r_t} \theta_w \left( \pi_{t+1}^w - \bar{\Pi}^w \right) \pi_{t+1}^w \frac{H_{t+1}}{H_t}. \quad (43)$$

The costs of wage adjustment as well as the disutility of work are borne by the households directly and wages are directly paid to households, so that middlemen make no profits.

Households budget constraint equals

$$P_t c_t + b_{t+1}^h + P_t k_{t+1}^h + P_t \Phi(k_{t+1}^h, k_t^h) + s_t \frac{\theta_w}{2} \left( \frac{\hat{W}_t}{\hat{W}_{t-1}} - \bar{\Pi}^w \right)^2 H_t = (1+i_t)b_t^h + (1+r_t^a)P_t k_t + (1-\tau_t)P_t w_t h_t s_t + T_t,$$

where  $k_t^h$  is the (beginning of period  $t$ ) stock of physical capital with a real return  $r_t^a$  and  $\Phi(k_{t+1}^h, k_t^h)$  are real portfolio adjustment costs which we specify (for now) as

$$\Phi(k_{t+1}^h, k_t^h) = \frac{\phi_k}{2} \left( \frac{k_{t+1}^h - k_t^h}{k_t^h} \right)^2 k_t^h.$$

---

<sup>3</sup>Equivalently one can think of a continuum of middlemen each setting the wage for a representative part of the population with  $\int s = 1$  at all times.

Household supply of capital in period  $t$  is then

$$K_t^h = \int k_t^h(b, k, s_{t-1}; \Omega) d\Omega_{t-1}$$

#### 4.1.2 Investment-Goods Firms

The investment-good sector transforms households capital holdings into capital services  $\bar{K}_t$  and sells them to the intermediate goods sector which now uses both capital and labor. Capital services,  $\bar{K}_t$ , are related to the physical stock of capital by  $\bar{K}_t = u_t K_t$ . Here,  $u_t$  denotes the utilization rate of capital and capital depreciates at rate  $\delta(u)$ , which we specify as

$$\delta(u) = \bar{\delta} + \frac{r_{ss}^k}{1 + 1/\delta_u} (u^{1+1/\delta_u} - 1)$$

HANK specifies  $\bar{\delta}$  equal to 10% per annum, and  $\delta_u = 5$  implying an elasticity of depreciation to utilization of 1.2. Investment-good firms engage in two activities. They rent capital  $K_t$  at a real rate  $1 + r_t^a$  from households and choose the utilization of capital  $u_t$  and rent the resulting capital services for intermediate goods at rate  $1 + r_t^k$ . They also collect households investment  $K_{t+1}^h - K_t^h$  and transform it into new capital  $K_{t+1}$ . Thus the investment-good firms choose  $K$ ,  $I$  and  $u$  to maximize profits

$$\sum_{t=1}^{\infty} \beta^t D_t,$$

where

$$D_t = K_{t+1}^h - K_t^h + r_t^k u_t K_t - r_t^a K_t - I_t$$

and capital accumulates as

$$K_{t+1} = K_t(1 - \delta(u)) + I_t.$$

Equivalently the firm chooses the path of capital and utilization to maximize

$$\sum_{t=1}^{\infty} \beta^t \left( K_{t+1}^h - K_t^h + r_t^k u_t K_t - r_t^a K_t - K_{t+1} + K_t(1 - \delta(u)) \right),$$

which gives first order conditions

$$r_t^a = r_t^k u_t - \delta(u) + 1 - 1/\beta \quad (44)$$

$$r_t^k = \delta'(u). \quad (45)$$

The profits of firms in the investment goods sector are also fully taxed by the government.

Note that

$$D_t = K_{t+1}^h - K_t^h + (\delta(u) - 1 + 1/\beta)K_t - I_t \quad (46)$$

$$= K_{t+1}^h - K_t^h - (K_{t+1} - K_t) + (1/\beta - 1)K_t, \quad (47)$$

which in equilibrium equals  $D_t = (1/\beta - 1)K_t$ .

#### 4.1.3 Intermediate-Goods Firms

A monopolist produces intermediate good  $j \in [0, 1]$  using the following technology:

$$Y_{jt} = \begin{cases} Z_t \bar{K}_{jt}^\alpha H_{jt}^{1-\alpha} - Z_t \Phi & \text{if } \geq 0 \\ 0 & \text{otherwise} \end{cases}, \quad (48)$$

where  $0 < \alpha < 1$ ,  $\bar{K}_{jt}$  is capital services rented,  $H_{jt}$  is labor services rented and the fixed cost of production are again denoted  $\Phi > 0$ .

Intermediate firms rent capital and labor in perfectly competitive factor markets. Profits are fully taxed by the government. A firm's real marginal cost is  $mc_{jt} = \partial S_t(Y_{jt})/\partial Y_{jt}$ , where

$$S_t(Y_{jt}) = \min_{\bar{K}_{jt}, H_{jt}} r_t^k \bar{K}_{jt} + w_t H_{jt}, \text{ where } Y_{jt} \text{ is given by (48)} \quad (49)$$

Given our functional forms, we have

$$mc_t = \left(\frac{1}{\alpha}\right)^\alpha \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \frac{(r_t^k)^\alpha (w_t)^{1-\alpha}}{Z_t} \quad (50)$$

and

$$\frac{\bar{K}_{jt}}{H_{jt}} = \frac{\alpha w_t}{(1-\alpha)r_t^k} \quad (51)$$

Prices are sticky as there are the same Rotemberg (1982) price adjustment costs as in the simple model.

Given last period's individual price  $p_{jt-1}$  and the aggregate state  $(P_t, Y_t, Z_t, w_t, r_t)$ , the firm chooses this period's price  $p_{jt}$  to maximize the present discounted value of future profits,



satisfying all demand. The firm's pricing problem is

$$V_t(p_{jt-1}) \equiv \max_{p_{jt}} \frac{p_{jt}}{P_t} y(p_{jt}; P_t, Y_t) - S(y(p_{jt}; P_t, Y_t)) - \frac{\theta}{2} \left( \frac{p_{jt}}{p_{jt-1}} - \bar{\Pi} \right)^2 Y_t - Z_t \Phi + \frac{1}{1+r_t} V_{t+1}(p_{jt}),$$

where  $\Phi$  are fixed operating costs.

The same algebra as in the simple model yields the New Keynesian Phillips Curve

$$(1 - \epsilon) + \epsilon mc_t - \theta (\pi_t - \bar{\Pi}) \pi_t + \frac{1}{1+r_t} \theta (\pi_{t+1} - \bar{\Pi}) \pi_{t+1} \frac{Y_{t+1}}{Y_t} = 0$$

The equilibrium real profit of each intermediate goods firm is then

$$d_t = Y_t - Z_t \Phi - S(Y_t) - \frac{\theta}{2} (\pi_t - \bar{\Pi})^2 Y_t$$

## 4.2 Computation

Computation could be done similarly to what we do in the simple model with the modification/complication that not only prices but also a sequence of wages needs to be guessed.

Algorithm for computing:

1. Guess path for the price level  $\{P_t\}_{t=0}^T$  and real wages  $\{w_t\}_{t=0}^T$
2. Use the wage Phillips curve, eq (91), to back out the sequence for  $\{H_t\}_{t=0}^T$
3. Note that we can express contemporaneous output,  $Y_t$  as a function of  $P_t, w_t, H_t$  and  $mc_t$ :

(a) If we know  $mc_t$  and  $w_t$  then from equation (50):

$$\Rightarrow r_t^k = \alpha (1 - \alpha)^{\frac{1-\alpha}{\alpha}} w_t^{\frac{\alpha-1}{\alpha}} mc_t^{\frac{1}{\alpha}} Z_t^{\frac{1}{\alpha}} \quad (52)$$

(b) Knowing  $H_{jt} = H_t$ ,  $w_t$  and  $r_t^k$  and from equation

$$\frac{\bar{K}_{jt}}{H_{jt}} = \frac{\alpha w_t}{(1 - \alpha) r_t^k} \quad (53)$$

we get capital services  $\bar{K}_{jt} = \bar{K}_t$ .

(c) This allows to compute output using the production function in equation (48).

4. The only unknown in the price Phillips curve, equation (52), is marginal costs (since we know  $Y_t$  from the previous step), so we can use it to back out the sequence for  $\{mc_t\}_{t=0}^T$ .
5. Using the same arguments as above allows us to infer the series of interest rates  $r_t^k$ , capital services  $\bar{K}_t$  and output  $Y_t$ .
6. Then from eq. (45) we back out utilization:

$$u = \delta'^{-1}(r_t^k) \quad (54)$$

7. And from eq. (44) we back out households return on capital :

$$r_t^a = r_t^k u_t - \delta(u) + 1 - 1/\beta \quad (55)$$

$$(56)$$

8. One then has all the prices - wages  $w$ , interest rates  $r^a$  on capital, prices  $P_t$  nominal interest  $i$  on bonds as well as policies - tax rates  $\tau_t$  and transfers  $T_t$  to compute the HH problem delivering a sequence of capital supply, bond supply and labor supply.
9. Iterate guessing sequences of prices  $P_t$  and wages  $w_t$  until the bond market, capital market and the labor market clear.

**Solving the household problem:** We proceed to solve the household problem by iterating on the first order conditions for the optimal bond and capital choices for the households. The household problem can be written recursively as:

$$\begin{aligned}
V(\chi, b, k, s; \Omega) &= \max_{c, b', k', h} u(c, \chi, h, s) + \beta \mathbb{E}V(\chi', b', k', s'; \Omega') \\
&\text{s.t.} \\
&Pc + b' + Pk' + P\Phi(k', k) + s \frac{\theta_w}{2} \left( \frac{\hat{W}}{\hat{W}_{-1}} - \bar{\Pi}^w \right)^2 H \\
&= (1+i)b + (1+r^a)Pk + (1-\tau)Pwhs + T \\
\chi' &= (1-\lambda)\chi + \lambda c \\
\Omega' &= \Gamma(\Omega)
\end{aligned}$$

where  $\Omega$  is the distribution over households.

Let  $\xi$  be the lagrange multiplier on the budget constraint. Then we have

FOC (c):

$$(u_1(c, \chi, h, s) + \beta \lambda \mathbb{E}V_1(\chi', b', k', s'; \Omega')) = P\xi \quad (57)$$

FOC (b'):

$$\beta \mathbb{E}V_2(\chi', b', k', s'; \Omega') = \xi \quad (58)$$

FOC (k'):

$$\beta \mathbb{E}V_3(\chi, b', k', s'; \Omega') = P\xi(1 + \Phi_1(k', k)) \quad (59)$$

We use FOC (c) to solve out for  $\xi$ , giving us:

FOC (b'):

$$\beta \mathbb{E}V_2(\chi', b', k', s'; \Omega') = \frac{1}{P} (u_1(c, \chi, h, s) + \beta \lambda \mathbb{E}V_1(\chi', b', k', s'; \Omega')) + \quad (60)$$

FOC (k'):

$$\beta \mathbb{E}V_3(\chi, b', k', s'; \Omega') = (1 + \Phi_1(k', k)) [u_1(c, \chi, h, s) + \beta \lambda \mathbb{E}V_1(\chi, b', k', s'; \Omega')] \quad (61)$$

We can now take the envelope conditions

EC ( $\chi$ ):

$$V_1(\chi, b, k, s; \Omega) = u_2(c, \chi, h, s) + (1-\lambda)\beta \mathbb{E}V_1(\chi', b', k', s'; \Omega') \quad (62)$$

[Note, I don't think there is any lambda here, but it's important that the derivative is  $u_2$ ]  
 EC (b):

$$V_2(\chi, b, k, s; \Omega) = (1 + i)\xi \quad (63)$$

$$= \frac{1+i}{P}(u_1(c, \chi, h, s) + \beta\lambda\mathbb{E}V_1(\chi', b', k', s'; \Omega')) \quad (64)$$

EC (k):

$$V_3(\chi, b, k, s; \Omega) = (1 + r^a - \Phi_2(k', k))P\xi \quad (65)$$

$$= (1 + r^a - \Phi_2(k', k)) [u_1(c, \chi, h, s) + \beta\lambda\mathbb{E}V_1(\chi', b', k', s'; \Omega')] \quad (66)$$

In order to solve this, we first guess the derivatives of the value functions  $\{V_1^0, V_2^0, V_3^0\}$ . Then, we use the two FOCs to back out the policy functions for  $c, b', k'$ . Then, we can use the three envelope conditions to update our guesses  $\{V_1^1, V_2^1, V_3^1\}$ . If they are sufficiently close, we've found the solution and we stop. Otherwise, we continue iterating.

Taking first order and using envelope conditions we obtain for  $b$  and  $b'$ :

$$V_b - (1 + i)\xi = 0 \quad (67)$$

$$\xi + \beta\mathbb{E}V_{b'} = \xi + \beta(1 + i')\mathbb{E}\xi' = 0, \quad (68)$$

where  $\xi$  is the multiplier on the constraint. For  $c_{-1}$  and  $c$  we get:

$$V_{c_{-1}} = -\kappa u_1(c - \kappa c_{-1}, h, s) \quad (69)$$

$$u_1(c - \kappa c_{-1}, h, s) + \beta\mathbb{E}V_c + P\xi = u_1(c - \kappa c_{-1}, h, s) - \beta\kappa\mathbb{E}u_1(c' - \kappa c, h, s) + P\xi = 0 \quad (70)$$

Combining the FOC for  $c$  with the one for  $\xi$  we obtain the FOC:

$$u_1(c - \kappa c_{-1}, h, s) - \beta\kappa\mathbb{E}u_1(c' - \kappa c, h, s) = P\beta(1 + i)\mathbb{E}\xi' = \quad (71)$$

$$\frac{P}{P'}\beta(1 + i')\mathbb{E}\{u_1(c' - \kappa c, h, s) - \beta\kappa\mathbb{E}u_1(c'' - \kappa c', h, s)\} = \quad (72)$$

$$\beta\frac{1 + i'}{1 + \pi'}\mathbb{E}\{u_1(c' - \kappa c, h, s) - \beta\kappa u_1(c'' - \kappa c', h, s)\}. \quad (73)$$

FOC for  $k$  and  $k'$ :

$$V_k = P\xi\Phi_2(k', k) - P\xi(1 + r^a)(74)$$

$$P\xi + P\xi\Phi_1(k', k) + \beta\mathbb{E}V_{k'} = P\xi + P\xi\Phi_1(k', k) + \beta\mathbb{E}\{P'\xi'\Phi_2(k'', k') - P'\xi'(1 + (r^a)')\} = 0.(75)$$

Combining with the FOC for  $c$  we obtain:

$$(u_1(c - \kappa c_{-1}, h, s) - \beta\kappa\mathbb{E}u_1(c' - \kappa c, h, s))(1 + \Phi_1(k', k)) = (76)$$

$$\beta\mathbb{E}\{u_1(c' - \kappa c, h, s) - \beta\kappa u_1(c'' - \kappa c', h, s)\}((1 + (r^a)') - \Phi_2(k'', k'))\} (77)$$

Specifying the functional form for adjustment costs as

$$\Phi(k_{t+1}^h, k_t^h) = \frac{\phi_k}{2} \left( \frac{k_{t+1}^h - k_t^h}{\tilde{k}_t^h} \right)^2 \tilde{k}_t^h.$$

where  $\tilde{k}_t^h = \max\{\underline{k}, k_t^h\}$  implies

$$\Phi_1(k', k) = \begin{cases} \phi_k \left( \frac{k'}{k} - 1 \right) & \text{if } k \geq \underline{k} \\ \phi_k \left( \frac{k' - k}{\underline{k}} \right) & \text{otherwise} \end{cases} (78)$$

For the case where  $k \geq \underline{k}$ :

$$\begin{aligned} \Phi_2(k', k) &= -\phi_k \left( \frac{k'}{k} - 1 \right) - \frac{\phi_k}{2} \left( \frac{(k' - k)^2}{k^2} \right) \\ &= \phi_k \left( \frac{k'}{k} - 1 \right) \left( \frac{-k' - k}{2k} \right) \\ &= -\frac{\phi_k}{2} \left( \frac{k'^2 - k^2}{k^2} \right). \end{aligned}$$

Oterhwise:

$$\Phi_2(k', k) = -\phi_k \left( \frac{k' - k}{\underline{k}} \right)$$

Thus:

$$\Phi_2(k', k) = \begin{cases} -\frac{\phi_k}{2} \left( \frac{k'^2 - k^2}{k^2} \right) & \text{if } k \geq \underline{k} \\ -\phi_k \left( \frac{k' - k}{\underline{k}} \right) & \text{otherwise} \end{cases} (79)$$

Using this in the FOC above yields

$$(u_1(c - \kappa c_{-1}, h, s) - \beta \kappa \mathbb{E} u_1(c' - \kappa c, h, s))(1 + \Phi_1(k', k)) = \quad (80)$$

$$\beta \mathbb{E} \left\{ \{u_1(c' - \kappa c, h, s) - \beta \kappa u_1(c'' - \kappa c', h, s)\} \frac{(1 + (r^a)') + \frac{\phi_k}{2} \left( \frac{k'^{r^2} - k'^2}{k'^2} \right)}{1 + \phi_k \left( \frac{k'}{k} - 1 \right)} \right\}. \quad (81)$$

### 4.3 Steady State

The following equations characterize a steady state:

Households

$$(u_1(c - \kappa c_{-1}, h, s) - \beta \kappa \mathbb{E} u_1(c' - \kappa c, h, s))(1 + \Phi_1(k', k)) = \quad (82)$$

$$\beta \mathbb{E} \left\{ \{u_1(c' - \kappa c, h, s) - \beta \kappa u_1(c'' - \kappa c', h, s)\} \frac{(1 + (r^a)') + \frac{\phi_k}{2} \left( \frac{k'^{r^2} - k'^2}{k'^2} \right)}{1 + \phi_k \left( \frac{k'}{k} - 1 \right)} \right\}. \quad (83)$$

$$(u_1(c - \kappa c_{-1}, h, s) - \beta \kappa \mathbb{E} u_1(c' - \kappa c, h, s))(1 + \Phi_1(k', k)) = \quad (84)$$

$$\beta \mathbb{E} \{u_1(c' - \kappa c, h, s) - \beta \kappa u_1(c'' - \kappa c', h, s)\} ((1 + (r^a)') - \Phi_2(k'', k')) \quad (85)$$

Aggregated HH budget constraint:

$$PC + B^h + PK^h + P \int \Phi(k_{+1}^h, k^h) + \frac{\theta_w}{2} \left( \frac{\hat{W}}{\hat{W}_{-1}} - \bar{\Pi}^w \right)^2 H = (1+i)B^h + (1+r^a)PK^h + (1-\tau)PwH + T,$$

Profits

$$d = Y - Z\Phi - S(Y) - \frac{\theta}{2} (\Pi - \bar{\Pi})^2 Y \quad (86)$$

$$= Y - Z\Phi - r^k \bar{K} - wH - \frac{\theta}{2} (\Pi - \bar{\Pi})^2 Y \quad (87)$$

$$D = (1/\beta - 1)K. \quad (88)$$

Resource Constraint

$$Y = Z\bar{K}^\alpha H^{1-\alpha} - Z\Phi = C + I + G/P = C + K\delta(u) + G/P + \frac{\theta_w}{2} \left( \frac{\hat{W}}{\hat{W}_{-1}} - \bar{\Pi}^w \right)^2 H + \int \Phi(k_{+1}^h, k^h) + \frac{\theta}{2} (\Pi - \bar{\Pi})^2 Y \quad (89)$$

Government budget

$$B^g = (1+i)B^g + G - P(d+D) - \int \tilde{T}(wsH)d\Omega. \quad (90)$$

Wage Setting and Employment

$$\theta_w (\pi_t^w - \bar{\Pi}^w) \pi_t^w = (1 - \tau_t)(1 - \epsilon_w)w_t + \epsilon_w g'(h(\hat{W}_t; W_t, H_t)) + \frac{1}{1 + r_t} \theta_w (\pi_{t+1}^w - \bar{\Pi}^w) \pi_{t+1}^w \frac{H_{t+1}}{H_t}. \quad (91)$$

Price Setting

$$(1 - \epsilon) + \epsilon mc - \theta (\pi - \bar{\Pi}) \pi + \frac{1}{1 + r} \theta (\pi_{+1} - \bar{\Pi}) \pi_{+1} \frac{Y_{+1}}{Y} = 0$$

$$mc = \left( \frac{1}{\alpha} \right)^\alpha \left( \frac{1}{1 - \alpha} \right)^{1-\alpha} \frac{(r^k)^\alpha (w)^{1-\alpha}}{Z} \quad (92)$$

Investment

$$u = \delta'^{-1}(r^k) \quad (93)$$

$$r^a = r^k u - \delta(u) + 1 - 1/\beta \quad (94)$$

$$\bar{K} = uK \quad (95)$$

Market clearing

Capital, Bonds, Hours (we impose this already as everybody provides the same number of hours  $H$ ).

$$K^h = \int k^h(b, k, s_{t-1}; \Omega) d\Omega \quad (96)$$

$$B = \int b^h(b, k, s_{t-1}; \Omega) d\Omega \quad (97)$$

Check Consistency of HH budget, resource and government budget constraint (Walras law). Start with HH budget and derive resource constraint using government budget and

other equations:

$$PC + B^h + PK^h + P \int \Phi(k_{+1}^h, k^h) + \frac{\theta_w}{2} \left( \frac{\hat{W}}{\hat{W}_{-1}} - \bar{\Pi}^w \right)^2 H \quad (98)$$

$$= (1+i)B^h + T + (1+r^a)PK^h + (1-\tau)PwH, \quad (99)$$

Substituting in the government budget constraint:

$$iB + T - \tau PwH = P(d + D) - G, \quad (100)$$

we get

$$PC + P \int \Phi(k_{+1}^h, k^h) + \frac{\theta_w}{2} \left( \frac{\hat{W}}{\hat{W}_{-1}} - \bar{\Pi}^w \right)^2 H \quad (101)$$

$$= P(d + D) - G + r^a PK + PwH. \quad (102)$$

Now use

$$r^a PK + PwH = r^k uK - \delta(u)PK + (1 - 1/\beta)PK + PwH \quad (103)$$

$$= Y - Z\Phi - Pd - \frac{\theta}{2} (\Pi - \bar{\Pi})^2 Y - \delta(u)PK + (1 - 1/\beta)PK \quad (104)$$

we get (now in real terms):

$$C + \int \Phi(k_{+1}^h, k^h) + \frac{\theta_w}{2} \left( \frac{\hat{W}}{\hat{W}_{-1}} - \bar{\Pi}^w \right)^2 H \quad (105)$$

$$= D - G/P + Y - Z\Phi - \frac{\theta}{2} (\Pi - \bar{\Pi})^2 Y - \delta(u)K + (1 - 1/\beta)K \quad (106)$$

$$= Y - G/P - Z\Phi - \frac{\theta}{2} (\Pi - \bar{\Pi})^2 Y - \delta(u)K, \quad (107)$$

which is equivalent to the resource constraint:

$$C + G/P + \delta(u)K + \frac{\theta_w}{2} \left( \frac{\hat{W}}{\hat{W}_{-1}} - \bar{\Pi}^w \right)^2 H \quad (108)$$

$$= Y - Z\Phi - \int \Phi(k_{+1}^h, k^h) - \frac{\theta}{2} (\Pi - \bar{\Pi})^2 Y. \quad (109)$$



### 4.3.1 Calibration Strategy

In the calibration we will target (all values are quarterly)  $r_{ss}^a = 0.01$ ,  $i_{ss} = 0.005$ ,  $K_{ss}/Y_{ss} = 10.26$ ,  $G_{ss} = 0.15Y_{ss}$  and  $B_{ss}^g/Y_{ss} = 1$  in the steady state. We will also target  $u_{ss} = 1$  in steady state, such that  $\bar{K}_{ss} = K_{ss}$  and  $\delta(u_{ss}) = \bar{\delta}$ .

We normalize steady state productivity,  $Z_{ss} = 1$ .

We will use  $\beta$  and  $\phi_k$  to ensure market clearing in the bond and capital (and by construction, goods) markets. Thus, we need to figure out  $\bar{\delta}(\beta)$  and  $T(\beta)$ , such that  $r^a = 0.01$ ,  $K_{ss}/Y_{ss} = 10.26$  and  $B_{ss}^g/Y_{ss} = 1$ .

In steady state, first, we solve for marginal cost:

$$mc_{ss} = \frac{\epsilon - 1}{\epsilon} \quad (110)$$

Next, we can express the wage as a function of  $r_{ss}^k$  and  $mc_{ss}$ :

$$w_{ss} = \left( \frac{r_{ss}^k}{\alpha} \right)^{\frac{\alpha}{\alpha-1}} (1 - \alpha) mc_{ss}^{\frac{1}{1-\alpha}} \quad (111)$$

Then, some additional algebra and substitution yields:

$$r_{ss}^k = \frac{\alpha mc_{ss}}{K_{ss}/Y_{ss}} \quad (112)$$

where all quantities on the right hand side are given from the calibration targets. Then we can solve for  $w_{ss}$  using the above equation and  $\bar{\delta}(\beta)$ :

$$\bar{\delta}(\beta) = r_{ss}^k - \frac{1}{\beta} + 1 - r_{ss}^a \quad (113)$$

we then obtain  $H_{ss}$  from the wage philips curve:

$$H_{ss} = (g')^{-1} \left( \frac{\epsilon_w - 1}{\epsilon_w} (1 - \tau) w_{ss} \right) \quad (114)$$

Then we can compute the profits of the intermediate and investment firms:

$$d_{ss} = Y_{ss} - w_{ss}H_{ss} - r_{ss}^k \overline{K}_{ss} - \Phi \quad (115)$$

$$D_{ss} = (1/\beta - 1)K_{ss} \quad (116)$$

We can then compute total output from the production function and solve for  $T_{ss}$  (as a fraction of output):

$$T_{ss} = -i \frac{B_{ss}^g}{Y_{ss}} - \frac{G_{ss}}{Y_{ss}} + \frac{d_{ss} + D_{ss}}{Y_{ss}} + \tau w_{ss} \frac{H_{ss}}{Y_{ss}} \quad (117)$$

Then, we iterate on  $\beta, \phi_k$  until we achieve market clearing (hopefully a solution exists).

#### 4.4 Estimation

#### 4.5 Results

### 5 Conclusions

[TO BE COMPLETED]

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## APPENDICES

# I Figures

## I.1 Impulse Responses: Main Results (Section 3.3)

Figure A-1: Impulse response to a 1% increase in nominal government spending: **Tax Financing** (Constant Nominal Debt).

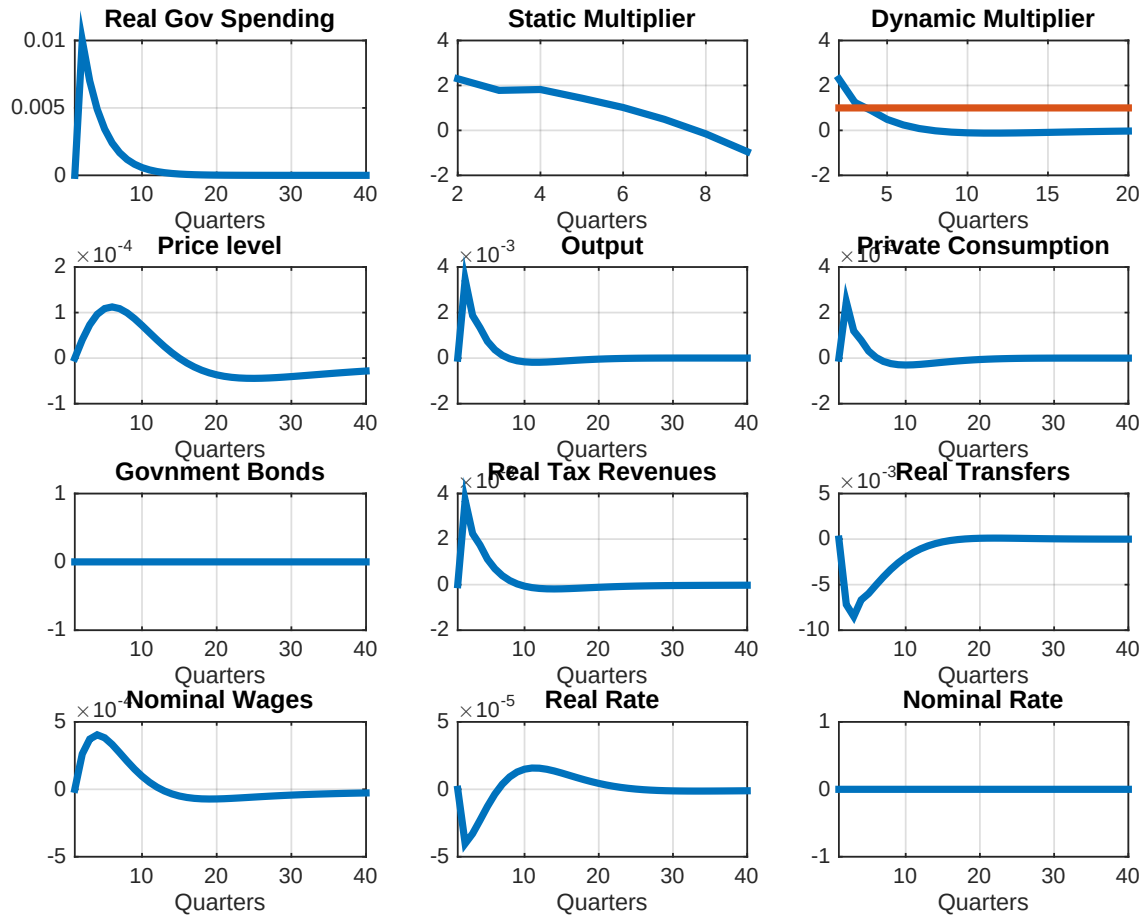
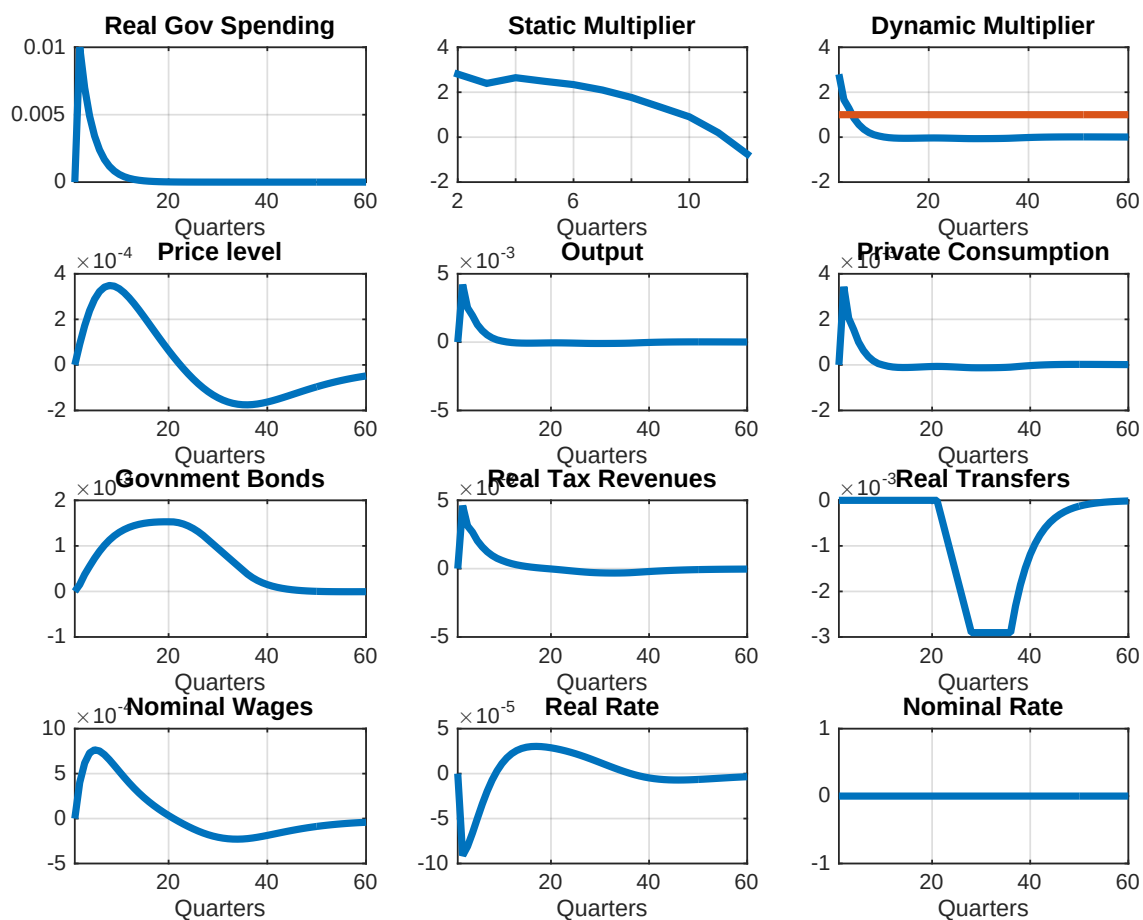


Figure A-2: Impulse response to a 1% increase in nominal government spending: **Deficit Financing**



## I.2 Impulse Responses: Taylor Rule

Figure A-3: Impulse response to a 1% increase in nominal government spending: **Deficit Financing, Taylor Rule**

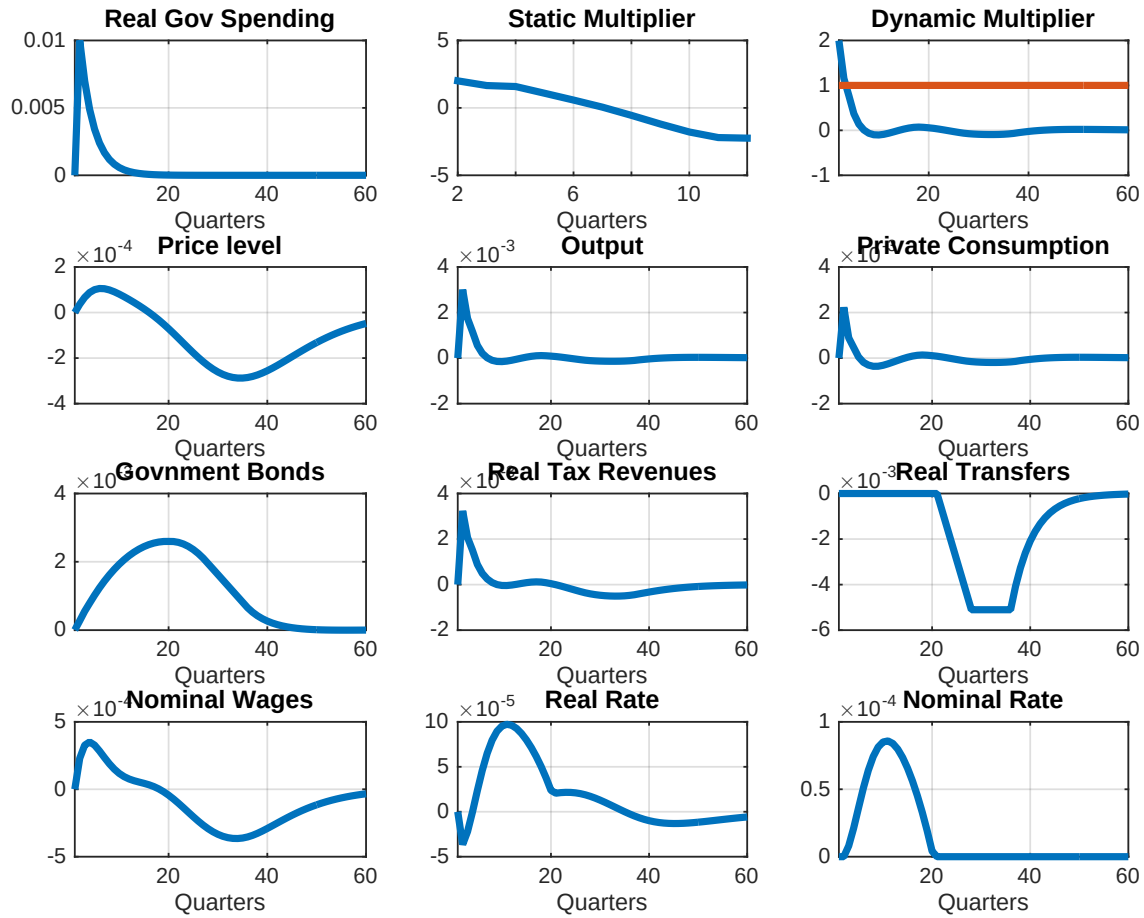
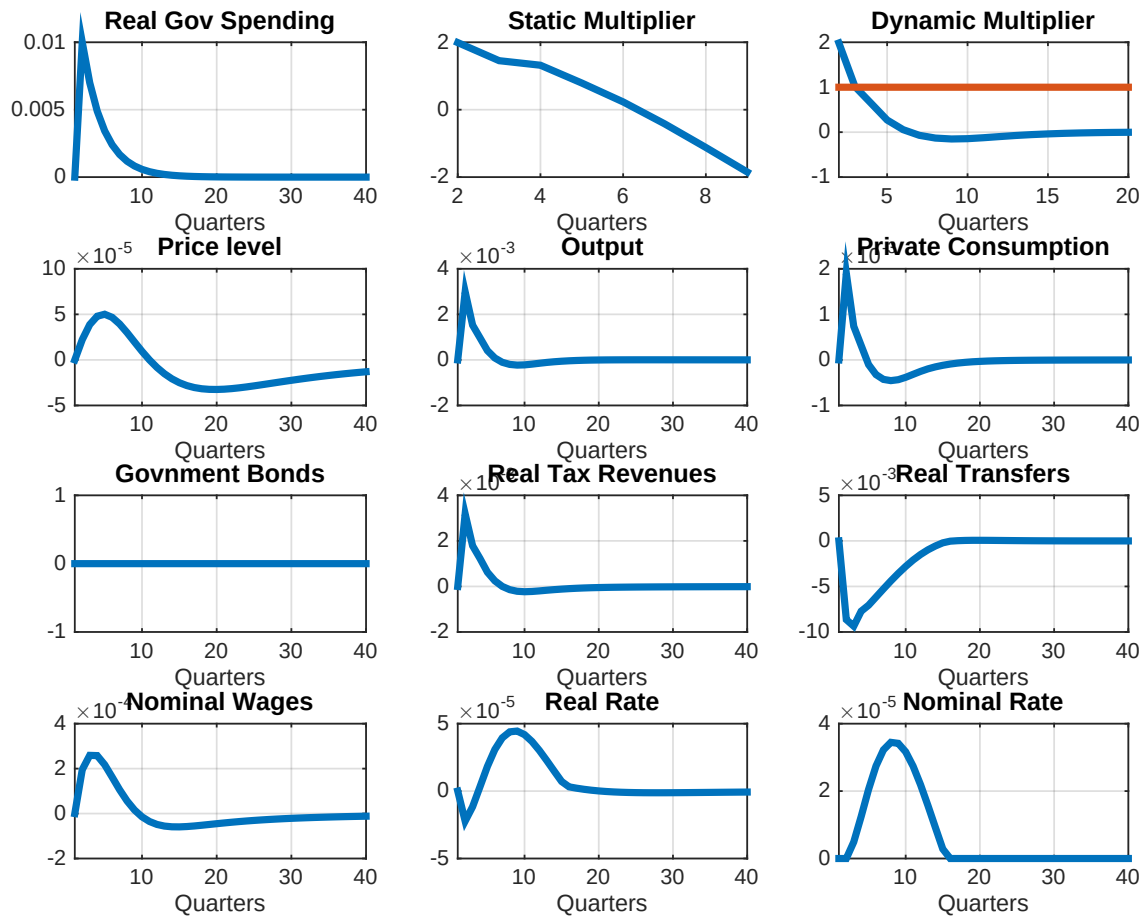


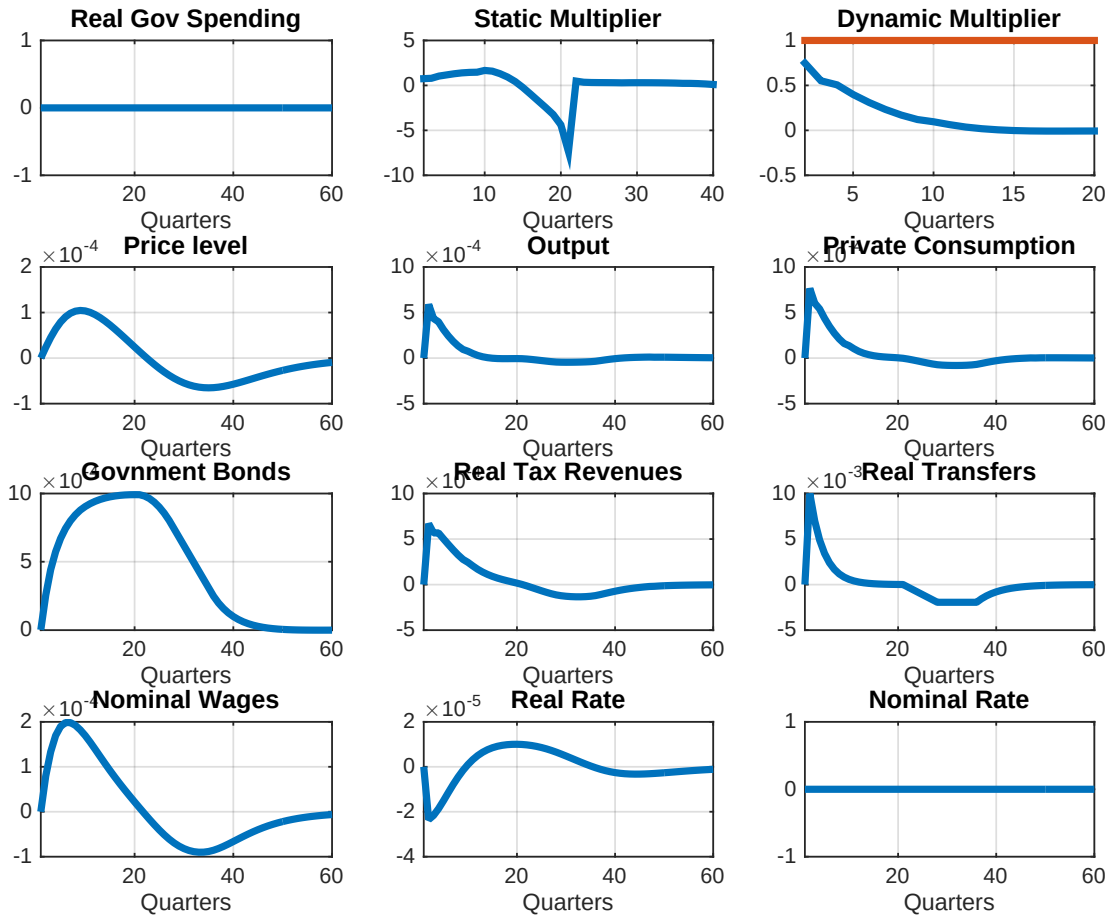
Figure A-4: Impulse response to a 1% increase in nominal government spending: **Tax Financing, Taylor Rule**





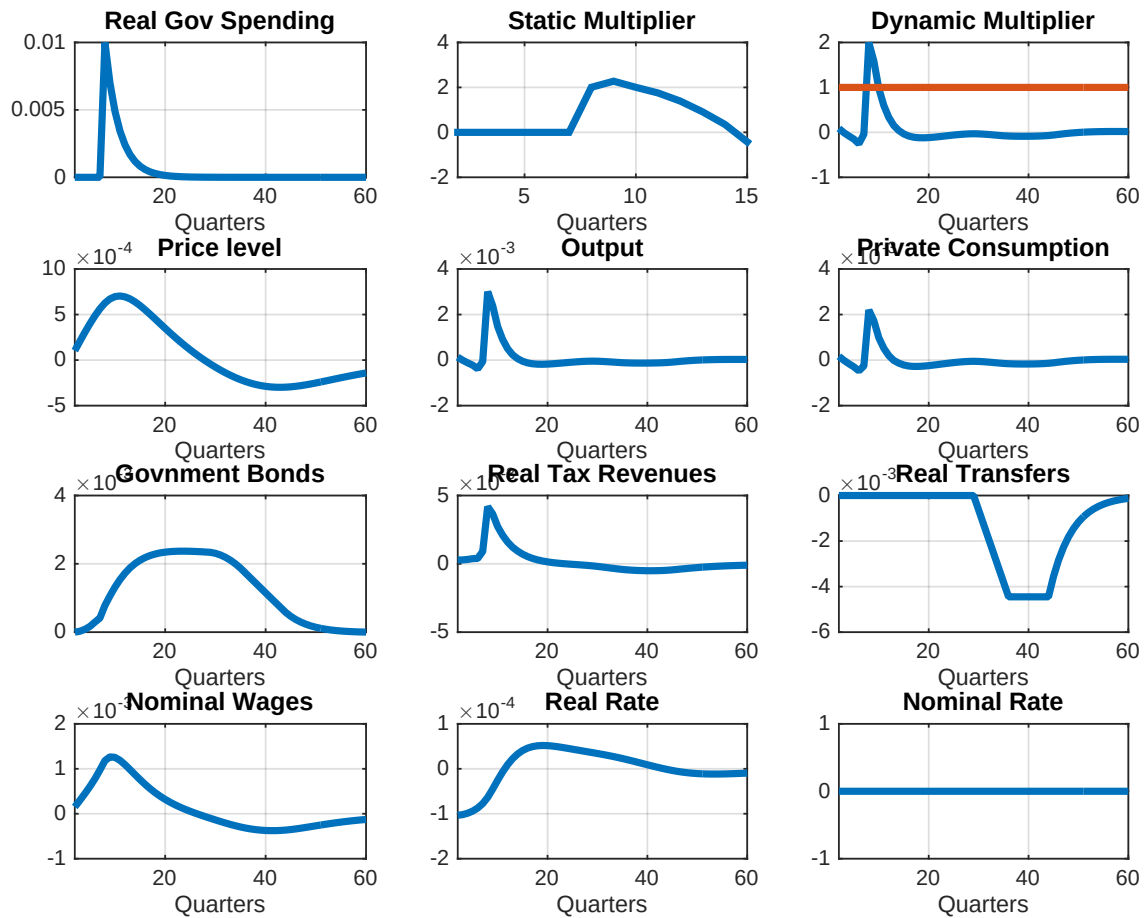
### I.3 Impulse Responses: Transfer Multiplier

Figure A-5: Impulse response to a 1% increase in nominal government **Transfers: Deficit Financing**



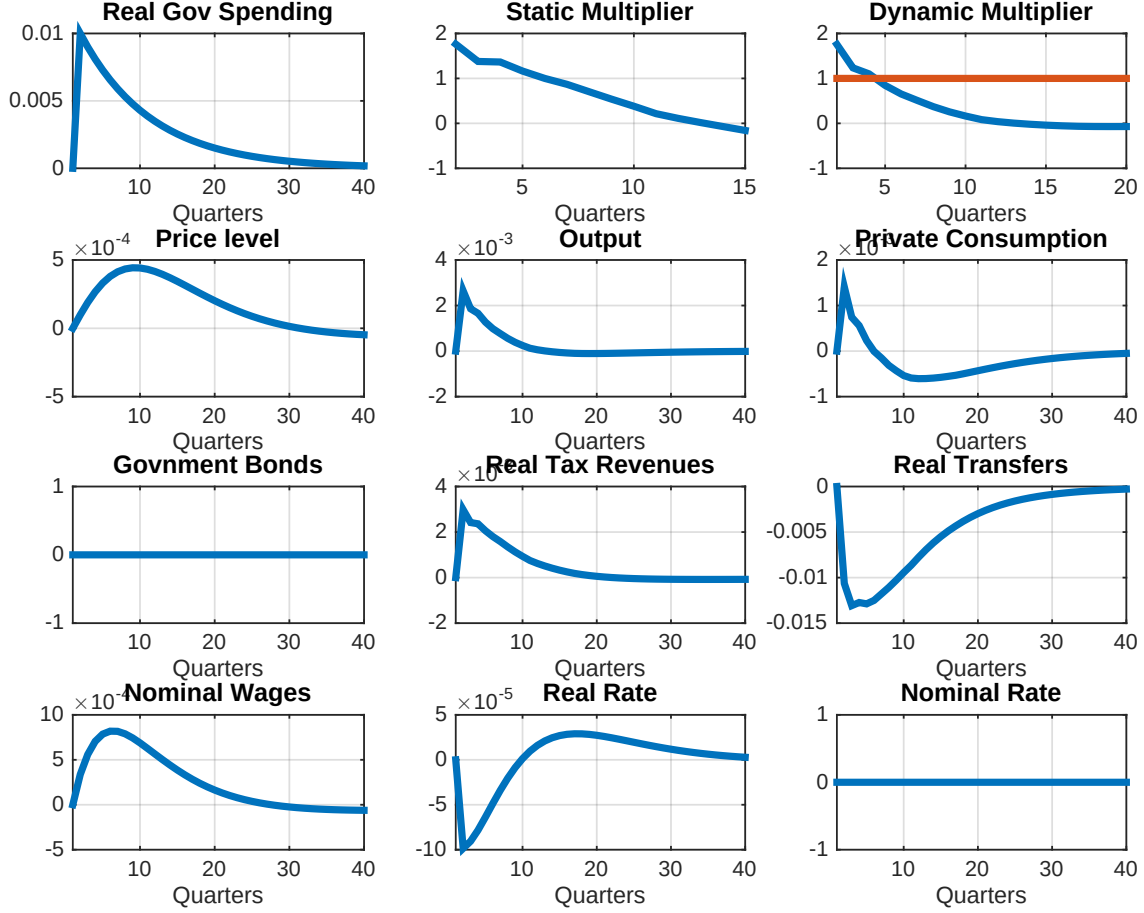
## I.4 Impulse Responses: Forward Spending

Figure A-6: Impulse response to a **future** (+8 quarters) 1% increase in nominal government spending: **Deficit Financing**



## I.5 Impulse Responses: Higher persistence

Figure A-7: Impulse response to a 1% increase in nominal government spending (**Persistence 0.9**) ): **Deficit Financing**



## II Derivations and Proofs

### II.1 Derivation Pricing Equation

The firm's pricing problem is

$$V_t(p_{jt-1}) \equiv \max_{p_{jt}} \frac{p_{jt}}{P_t} y(p_{jt}; P_t, Y_t) - w_t \left( \frac{y(p_{jt}; P_t, Y_t)}{Z_t} \right)^{\frac{1}{1-\alpha}} - \frac{\theta}{2} \left( \frac{p_{jt}}{p_{jt-1}} - \bar{\Pi} \right)^2 Y_t + \frac{1}{1+r_t} V_{t+1}(p_{jt}),$$

subject to the constraints  $n_{jt} = \left(\left(\frac{p_{jt}}{P_t}\right)^{-\epsilon} \frac{Y_t}{Z_t}\right)^{\frac{1}{1-\alpha}}$  and  $y(p_{jt}; P_t, Y_t) = \left(\frac{p_{jt}}{P_t}\right)^{-\epsilon} Y_t$ .  
Equivalently

$$V_t(p_{jt-1}) \equiv \max_{p_{jt}} \frac{p_{jt}}{P_t} \left(\frac{p_{jt}}{P_t}\right)^{-\epsilon} Y_t - w_t \left(\left(\frac{p_{jt}}{P_t}\right)^{-\epsilon} \frac{Y_t}{Z_t}\right)^{\frac{1}{1-\alpha}} - \frac{\theta}{2} \left(\frac{p_{jt}}{p_{jt-1}} - \bar{\Pi}\right)^2 Y_t + \frac{1}{1+r_t} V_{t+1}(p_{jt}),$$

The FOC w.r.t  $p_{jt}$

$$(1-\epsilon) \left(\frac{p_{jt}}{P_t}\right)^{-\epsilon} \frac{Y_t}{P_t} + \frac{\epsilon}{1-\alpha} w_t \left(\frac{p_{jt}}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}-1} \left(\frac{Y_t}{Z_t P_t}\right)^{\frac{1}{1-\alpha}} - \theta \left(\frac{p_{jt}}{p_{jt-1}} - \bar{\Pi}\right) \frac{Y_t}{p_{jt-1}} + \frac{1}{1+r_t} V'_{t+1}(p_{jt}) = 0 \quad (\text{A1})$$

and the envelope condition

$$V'_{t+1} = \theta \left(\frac{p_{jt+1}}{p_{jt}} - \bar{\Pi}\right) \frac{p_{jt+1}}{p_{jt}} \frac{Y_{t+1}}{p_{jt}}. \quad (\text{A2})$$

Combining the FOC and the envelope condition

$$(1-\epsilon) \left(\frac{p_{jt}}{P_t}\right)^{-\epsilon} \frac{Y_t}{P_t} + \frac{\epsilon}{1-\alpha} w_t \left(\frac{p_{jt}}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}-1} \left(\frac{Y_t}{Z_t P_t}\right)^{\frac{1}{1-\alpha}} - \theta \left(\frac{p_{jt}}{p_{jt-1}} - \bar{\Pi}\right) \frac{Y_t}{p_{jt-1}} + \frac{1}{1+r_t} \theta \left(\frac{p_{jt+1}}{p_{jt}} - \bar{\Pi}\right) \frac{p_{jt+1}}{p_{jt}} \frac{Y_{t+1}}{p_{jt}} = 0 \quad (\text{A3})$$

Using that all firms choose the same price in equilibrium

$$(1-\epsilon) + \frac{\epsilon}{1-\alpha} w_t Z_t^{\frac{1}{\alpha-1}} \left(\frac{Y_t}{P_t}\right)^{\frac{\alpha}{1-\alpha}} - \theta (\pi_t - \bar{\Pi}) \pi_t + \frac{1}{1+r_t} \theta (\pi_{t+1} - \bar{\Pi}) \pi_{t+1} \frac{Y_{t+1}}{Y_t} = 0 \quad (\text{A4})$$

## II.2 Derivation Wage Equation

$$\Theta(s_{jt}, W_{jt}, W_{jt-1}; Y_t) = s_{jt} \frac{\theta_w}{2} \left(\frac{\hat{W}_t}{\hat{W}_{t-1}} - \bar{\Pi}^w\right)^2 H_t.$$

The middleman's wage setting problem is to maximize

$$\begin{aligned} & V_t^w(\hat{W}_{t-1}) \\ \equiv & \max_{\hat{W}_t} \int \left(\frac{s_{jt}(1-\tau_t)\hat{W}_t}{P_t} h(\hat{W}_t; W_t, H_t) - s_{jt} g(h(\hat{W}_t; W_t, H_t))\right) dj - \int s_{jt} \frac{\theta_w}{2} \left(\frac{\hat{W}_t}{\hat{W}_{t-1}} - \bar{\Pi}^w\right)^2 H_t dj \\ & + \frac{1}{1+r_t} V_{t+1}^w(\hat{W}_t), \end{aligned} \quad (\text{A5})$$

where  $h_{jt} = h(W_{jt}; W_t, H_t) = \left(\frac{W_{jt}}{W_t}\right)^{-\epsilon_w} H_t$ .

The FOC w.r.t  $\hat{W}_t$

$$(1 - \tau_t)(1 - \epsilon_w) \left(\frac{\hat{W}_t}{W_t}\right)^{-\epsilon_w} \frac{H_t}{P_t} + \epsilon_w g'(h(\hat{W}_t; W_t, H_t)) \left(\frac{\hat{W}_t}{W_t}\right)^{-\epsilon_w - 1} \frac{H_t}{W_t} \quad (\text{A6})$$

$$- \theta_w \left(\frac{\hat{W}_t}{\hat{W}_{t-1}} - \overline{\Pi^w}\right) \frac{H_t}{\hat{W}_{t-1}} + \frac{1}{1 + r_t} V'_{t+1}(\hat{W}_t) = 0 \quad (\text{A7})$$

and the envelope condition

$$V'_{t+1} = \theta_w \left(\frac{\hat{W}_{t+1}}{\hat{W}_t} - \overline{\Pi^w}\right) \frac{\hat{W}_{t+1}}{\hat{W}_t} \frac{H_{t+1}}{\hat{W}_t}, \quad (\text{A8})$$

where we have used that  $\int s = 1$ .

Combining the FOC and the envelope condition

$$\begin{aligned} & (1 - \tau_t)(1 - \epsilon_w) \left(\frac{\hat{W}_t}{W_t}\right)^{-\epsilon_w} \frac{H_t}{P_t} + \epsilon_w g'(h(\hat{W}_t; W_t, H_t)) \left(\frac{\hat{W}_t}{W_t}\right)^{-\epsilon_w - 1} \frac{H_t}{W_t} \\ & - \theta_w \left(\frac{\hat{W}_t}{\hat{W}_{t-1}} - \overline{\Pi^w}\right) \frac{H_t}{\hat{W}_{t-1}} + \frac{1}{1 + r_t} \theta_w \left(\frac{\hat{W}_{t+1}}{\hat{W}_t} - \overline{\Pi^w}\right) \frac{\hat{W}_{t+1}}{\hat{W}_t} \frac{H_{t+1}}{\hat{W}_t} = 0 \end{aligned} \quad (\text{A9})$$

Using that  $\hat{W}_t = W_t$ ,  $\pi_t^w = \frac{W_t}{\hat{W}_{t-1}} = \frac{\hat{W}_t}{\hat{W}_{t-1}}$  and  $h_{jt} = H_t$ :

$$\begin{aligned} & (1 - \tau_t)(1 - \epsilon_w) \frac{W_t}{P_t} + \epsilon_w g'(h(\hat{W}_t; W_t, H_t)) \\ & - \theta_w (\pi_t^w - \overline{\Pi^w}) \pi_t^w + \frac{1}{1 + r_t} \theta_w (\pi_{t+1}^w - \overline{\Pi^w}) \pi_{t+1}^w \frac{H_{t+1}}{H_t} = 0 \end{aligned} \quad (\text{A10})$$