

Optimal Asset Division Rules for Dissolving Partnerships

Preliminary and Very Incomplete

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Abstract

We study the optimal design of the bankruptcy code in dynamic production economies with limited commitment, where investment and the accumulation of (real) assets takes place. In particular, we will focus our attention on economies where the accumulation of assets is a collective rather than an individual decision. To find the optimal bankruptcy rule, specifying how to allocate existing assets among different partners, we need to balance risk sharing between the partners and output efficiency when new profitable opportunities may arise in the future. We consider both the case of private or public (verifiable) information on outside options. One of the key insight is that under private information, defaulting agents will get a smaller proportion of accumulated assets, even though we will have typically more (and some socially inefficient) separations. We also study how the option of default affects ex ante asset accumulation under private and public information. This environment has several applications including business partnerships, couples and economic unions.

1 Introduction

There is a large literature studying default (bankruptcy) in an environment where there is clear distinction between a lender and a borrower. If the borrower also accumulates physical asset, then in most cases, the ex ante optimal allocation do not provide any assets for the defaulters at the time of the default. Another literature studies risk sharing among (possibly symmetric) economic agents who face idiosyncratic shocks. This literature may also consider real assets accumulated by the "partnership" but typically does not consider the dissolution of this arrangement along the equilibrium path. In section 1.1, we discuss these literatures in more detail.

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Our main departure from these strands of literature is motivated by the observation that, in many situations, the accumulation of assets is a collective rather than an individual decision problem: consider, for instance, investment decisions in partnerships, in joint ventures among firms, but also the provision of public goods in communities or regions/countries, or the choice of household savings in married couples. The inability of the partners of the group to commit in these situations raises some novel and interesting issues. There is in fact an additional commitment problem, besides the one towards lenders discussed so far in the literature, and is towards the other partners or members of the group involved in the collective activity. Each member or partner can in fact decide at any point in time to quit the group. The specification of the rules concerning the division of the assets among the members of a partnership/group in the event that some of the members want to separate - in short of what we can still refer to as the 'bankruptcy rule' - is the main instrument available to address this commitment problem.

Furthermore, it is natural to think that in these situations the opportunity cost of any individual of remaining part of the group varies over time, possibly in a stochastic way. In particular, new investment opportunities may arise for members outside the partnerships which are so profitable that a dissolution of the partnership may be optimal even in the absence of any commitment problem. That is, when we face the limited commitment problem in partnerships we have to take into account the fact that dissolution - or default - may arise in equilibrium. Hence the design of the optimal bankruptcy rule in these environments may feature less extreme punishments for default and the possibility that default actually occurs in equilibrium. These rules have important effects on the work of partnerships and on their incentives to accumulate assets. In this paper, we analyze the design of the bankruptcy rules for partnership that allows to achieve the highest possible welfare. To this end we will abstract, at least in a first instance, from the possible other commitment problem regarding external lenders, by considering situations where partnerships do not borrow and consider situations where the accumulation of assets happens using only the member's assets and the revenue generated by the partnership's production activity.

Our environment features two risk averse agents who can form a business partnership/firm to engage in some production activity. The firm operates a linear technology and the two partners confer to it the resources/capital they are initially endowed with (which we can assume to be the same for both). The firm operates each period by using the accumulated capital to produce, distributing part of the output as dividends to the partners and using the rest to accumulate new capital. As an alternative, each agent can form an individual firm, that operates with a different (linear) technology which has a lower productivity. At any point in time each partner receives, with some positive probability a new investment opportunity, given by an improvement

in the productivity of the technology he can operate individually. This new productivity level is itself a random variable. This new opportunity can only be seized by breaking the partnership and forming an (individual) firm to operate the improved technology. This would force the partner to also form an individual firm, which has a lower productivity in the absence of technological improvements.

A trade-off may then arise between production efficiency and risk sharing: when the productivity of the improved individual technology is higher than the one of the technology available to the partnership, breaking the firm and allocating all capital to the partner with the improved technology allows to increase total output, but this happens at the expense of extreme inequality, since the other partner is left with no assets and no transfers of resources are clearly enforceable between the two partners after the break-up of the firm, given agents' inability to commit. Note that the breaking of the partnership can always be prevented by adopting the bankruptcy rule that gives no capital to the partner who chooses to quit. The punishment given by the loss of all accumulated assets suffices to make quitting never attractive (under the assumption that the only resources available to agents are given by these assets). However, when the technological improvement is sufficiently high, social welfare may be maximized by breaking the partnership so as to capture this new opportunity. Hence the optimal contractual arrangement between the partners may feature the break-up of the relationship with positive probability. This would require a bankruptcy rule that leaves a positive fraction of the available resources to the partner who decides to quit (after receiving the new opportunity). Naturally, the higher the level of the assets left to the partner who quits, the higher is total output but also the higher the inequality between the partners. To limit this inequality, thus providing some insurance to risk averse partners against the risk of being forced to revert to the less productive individual technology, a sufficiently high amount of assets should also be left to the partner who did not receive the improvement. The design of the optimal bankruptcy rule will then have to find the optimal solution of this trade-off, taking also into account the effects on the incentives to accumulate assets in the partnership.

It is useful to consider initially the situation where the productivity of the new investment opportunity is commonly observable by all partners, in which case the bankruptcy rule may specify a division of resources which depends on the value of this opportunity. For the realizations of the productivity for which a break-up is not desirable we can always, as argued above, specify a rule that prevents the break-up in these cases. In contrast, for the realizations for which a break-up is desirable, a first finding we obtained is that at an optimum the share of the accumulated capital that goes to the partner who asks to dissolve the partnership is decreasing in the level of the productivity of his new investment opportunity. Moreover under CRRA utility functions, the share the defaulting agents receive is always below one

half as long as the coefficient of relative risk aversion is bigger than one. The log utility case is special, because in this case risk sharing and output efficiency is always balanced with an equal asset division rule independently of the productivity of the outside option.

There may important implications for the properties of capital accumulation in the partnership. Under our assumption of homothetic utility and linear technology, the (ex post optimal) division rules and separation thresholds, do not depend on the level of assets. At the same time, the frequency of separations and the division rules affect capital accumulation. Consumption becomes risky and that leads to precautionary savings. At the same time, on expectation, consumption goes up because separations can only happen if at least one agent have significant consumption benefits. This potential consumption increases may provide incentives for agents to front-load consumption and reduce asset accumulation. It is a quantitative question which effect dominates.

The picture changes significantly in what we can argue is the empirically more relevant situation where neither the presence nor the value of the new investment opportunity of a partner are observable by the other partners. In this case, the division of the accumulated assets cannot be contingent on the level of the productivity of this outside opportunity.

In the log case, we can prove two sets of important results for the private information case. First, the optimal asset division rule is lower than $1/2$ in this case. This result is intuitive. Under private information with equal sharing there is too much separation and reducing the sharing rule marginally is costless from $1/2$ by the usual envelope condition argument. We can also show that as long as separations are sufficiently desirable, we will have excessive amount of separations. In other words, given the asset division rule, it would be socially beneficial to keep the marginal defaulting agent in the contract. However, when at the observable information case separations are relatively low and they only deliver modest benefits, another allocation emerges: under private information it is preferable to allocate zero assets and hence eliminate separations altogether. The intuition is as follows, under private information more separations occurs for a given division rule and this is socially costly independently of the potential gains. When the gains are small these losses cannot be offset and better to eliminate separations altogether. To sum up the results for the log case, private information always leads lower than efficient asset division for defaulters. If separation is socially sufficiently valuable, we still see inefficiently many separations in this case. In the other case where separations have lower social value, under private information it is optimal to eliminate them. The exact value of the asset division rule, critically depends on the distribution of productivities. Whenever separations are likely and they are large social benefits from separations, the optimal separation rule tend to stay close to one half. As separations get less likely, the optimal division rule drops.

The more general case of CRRA ($\sigma > 1$ preferences) is more complicated as the optimal asset division rule under full information itself depends on the productivity of the outside option. Here, we need to consider a new aspect of optimality. Now, optimality would require a decreasing asset division rule across productivity levels, hence the constant asset division rule we consider need to find on *on average* the optimal balance between risk sharing and output efficiency. Both of these forces imply that the optimal asset division rule is lower than under private information at the separation threshold. In this environment, depending on the distribution of output shocks, it is even possible that separations are too low compared to fully information case where the asset division rule depends on output. This is the case if there is a big mass of possible output realizations that is far from the separation threshold under full information.

The arguments above are related to the work of Hopenhayn and Werning (2008). They study a situation where the accumulation of assets is still an individual problem, as there is a risk neutral lender and a risk averse borrower seeking funds to get capital for his production activity, but where the borrower can default and take the capital with him. The borrower's utility after defaulting - constituting his outside option - is uncertain. Hopenhayn and Werning show that, whenever the outside option of the borrower is not observable/verifiable, inefficient defaults can occur in the optimal contract along the equilibrium path. This is in contrast with the case where the outside option is observable (see e.g. Albuquerque and Hopenhayn 2004). In our framework private information also leads to inefficiencies in the dissolution decision. However, there are some important differences. First of all, we analyze an environment where the accumulation of assets is collective decision problem, hence the situation is symmetric with lack of commitment of all partners, while they study an environment where the accumulation of assets is an individual decision and the commitment problem is inherently asymmetric, since only the borrower has incentives to leave the contract. Second and most importantly, in their environment default is never socially desirable as opposed to our environment, hence no trade-off between output efficiency and risk sharing arises. This implies that in their set-up private information always leads to "too much" separation occurring, while as explained above the distortion may go in both ways and we may have inefficiently low separation. Finally, they only consider an extreme sharing rule of the accumulated capital upon default, where all the capital goes to the borrower. In contrast, we allow for all possible sharing rules: the core of our research is in fact to compare alternative sharing rules so as to identify the optimal one. (In fact, in their environment, the optimal sharing rule upon default would always allocate all the capital to the lender implying full risk sharing.)

1.1 Theoretical Background

The incentives faced by parties in credit arrangements, in particular the incentive of borrowers to repay their loans, play an important role in determining the operation of financial markets and their role in fostering the process of accumulation of capital and more generally of assets and hence the growth of an economy. An important role in shaping these incentives is played by the legal arrangements specifying procedures to follow in the event of default by the lender, more generally of termination of the credit relationship. These issues have been fairly investigated in a recent literature. The main focus has been on pure exchange economies, where there is no accumulation of real assets and financial markets offer instruments allowing individuals to transfer wealth over time and insure risks. Both the case where debt is secured and where it is unsecured have been considered. In the first case the borrower must post some assets as collateral, and the loss of the collateral constitutes the loss of the borrower, and the benefit of the lender in the event of default. In the second case, the punishment for a borrower who defaults is given by the seizure of part of his assets (up to the exemption level as specified by the bankruptcy code), of part of his future earnings and loss in access to future credit (as it happens, for instance, as a consequence of the drop in the borrower's credit score).

The lessons emerging from this literature are that, when financial markets are complete (that is, no limitations on the set of securities available for trade exist, besides the limited commitment problem), it is optimal that the bankruptcy code, specifying punishment and other provisions in the event of default, is as harsh as possible, that is the level of exemption granted by the code is as low as possible and the exclusion from future credit as severe as possible. In fact, the most severe subgame perfect punishment in this context is complete future exclusion and no exemptions (see Kocherlakota (1996) for a discussion). In other words, this is the optimal penal code in the sense of Abreu (1988). Similar considerations apply for the amount of assets to be posted as collateral for any short position taken by borrowers. The optimal bankruptcy code entails a sufficiently high penalty for defaulting and/or a sufficiently high collateral level, so that in equilibrium borrowers never default, that is lending relationships are never terminated. Also, when the bankruptcy code is sufficiently harsh and the amount of assets that can be seized in the event of bankruptcy (as specified by the bankruptcy code and/or collateral requirements) sufficiently large, the equilibrium allocation is first best efficient, that is, the frictions imposed by the limited commitment of borrowers do not bind (Gottardi and Kubler (2015), Kehoe and Levine (1993)).

In contrast, when financial markets are (exogenously) incomplete a more lenient bankruptcy code may be preferable to a harsher one as better risk sharing properties may be attainable if borrowers default in their lower in-

come states. Similarly, welfare may be higher if the collateral requirements are set at a lower level than the one required to exclude default in all circumstances (that is, in some states, borrowers may end up having negative equity in their mortgages and optimally choose to default). (See e.g. Araujo et al. (2012), Chatterjee et al. (2007)).

We extend these analyses to the case of production economies, where investment and the accumulation of (real) assets takes place. This generates an additional reason for agents' borrowing, to finance the accumulation of assets. As long as the process of asset accumulation takes place at the individual level, that is assets are held by individuals for the services they provide (as in the case of housing) or to fund their future consumption, the analysis of equilibrium allocations can proceed along similar lines to the ones above. One novel feature is that the amount of accumulated assets which can be used as collateral is now endogenously determined and this additional benefit of asset holdings - to facilitate future borrowing needs - provides an additional incentive for agents to accumulate assets. In addition, the presence of an endogenously determined amount of accumulated assets that can also be seized in the event of default allows to enhance the punishments that can be inflicted by seizing the borrower's assets in the event of default, something that, as argued above, is always beneficial when financial markets are complete. At the same time, there is an effect that points in the opposite direction, as the threat of expropriation of the accumulated assets in the event of default may reduce the agents' incentives to accumulate assets.

When assets are accumulated individually, the first effect prevails. In fact welfare is maximized with a harsh bankruptcy code, where all the accumulated assets of borrowers are seized in the event of their default. The threat of the loss of assets in the event of default does not discourage individuals from accumulating assets. On the contrary, in a competitive equilibrium with limited commitment and such harsh default penalties agents will typically accumulate more assets compared to the case where borrowers can fully commit to repay their debt obligations. In this sense, these economies tend to exhibit overaccumulation of assets compared to the first-best. In particular, agents may accumulate enough assets along the equilibrium path, so as to reach a point where the limited commitment/default constraints no longer bind and the equilibrium allocation, from then onward, is first best optimal. In particular, Ábrahám and Laczó (2015) show in an economy with a linear production technology that accumulating assets may be socially optimal even though their return is inferior to the rate of time preference when full insurance is not implementable due to binding enforceability constraints. This is the case because (i) asset accumulation facilitates better individual consumption smoothing and (ii) exclusion from future asset trades becomes a harsher penalty whenever the economy's production capacity has been increased by asset accumulation. Ábrahám and Carceles-Poveda (2006) study limited commitment in an economy with neoclassical production functions.

In this environment, increasing asset (capital) accumulation also implies increasing wages and hence also increases the value of defaulting. Consequently, whether capital is over or underaccumulated will depend on the relative strength of these two forces. In the numerical examples considered, the overaccumulation effect was always stronger.

In both of the above cases, the constrained efficient allocation, that maximizes social welfare subject to the constraint that borrowers are unable to commit, can be implemented as a competitive equilibrium when agents trade in a complete set of financial markets, subject to appropriately specified borrowing constraints, and can accumulate real assets. Welfare is maximized if the punishment in the case of default is as harsh as possible, that is, all assets are seized. A similar situation should arise in models where assets are used as collateral, and seizure of collateral is the only punishment for default. Hence the implications for the design of the bankruptcy code are similar to the case of pure exchange economies, where there is no accumulation of assets: as harsh as possible penalties are optimal, and no default occurs in equilibrium, when financial markets are complete.

It is interesting to contrast these findings with the ones obtained by Marcet and Marimon (1992) and Kehoe and Perri (2002) in environments where agents can still accumulate assets, but political or institutional constraints make it impossible to seize the borrower's assets in the event of default (for instance because they belong to agents who are in a different country), that is impose some limitations on the harshness of the punishment for default. As a consequence, in this case the accumulation of assets makes the limited commitment problem more severe, as it increases the benefits of a borrower of defaulting.

2 Model

Set-up There is a continuum of ex ante identical agents initially endowed with the same amount k_0 of capital at the initial date $t = 0$. Each agent is infinitely lived and has preferences described by a CRRA utility function $u(\cdot) = c^{1-\sigma}/(1-\sigma)$ over consumption at each date and by a common discount factor $\beta < 1$. Any pair of these agents can form a partnership at the initial date. The partnership runs a linear technology that produces each period t $R > 1$ units of the consumption good per unit of capital, that is $y_t = Rk_t$. Current output y_t can then be allocated to dividend payments - and hence consumption of the partners - and to capital accumulation:¹

$$k_{t+1} + c_t^1 + c_t^2 = y_t.$$

¹We implicitly assume that agents cannot save and borrow.

Individual optimization It is useful to consider first the optimal path of capital accumulation and consumption of an agent who is running a technology with productivity A and initial capital K_0 .

Note that the optimality condition implies, for each $t \geq 0$:

$$\begin{aligned}\frac{c_{t+2}}{c_{t+1}} &= (\beta A)^{1/\sigma}, \\ c_{t+1} + k_{t+1} &= Ak_t\end{aligned}$$

with $k_0 = K_0$. Hence we get

$$\frac{Ak_{t+1} - k_{t+2}}{Ak_t - k_{t+1}} = (\beta A)^{1/\sigma},$$

consumption grows at a constant rate, equal to $(\beta A)^{1/\sigma}$, while capital satisfies the following law of motion:

$$k_{t+2} = \left(A + (\beta A)^{1/\sigma}\right) k_{t+1} - (\beta A)^{1/\sigma} Ak_t.$$

There is then one degree of freedom, given by the value of k_1 . If we conjecture that, at a solution, capital also grows at a constant rate, $k_{t+1} = \gamma k_t$ then we obtain that the growth rate γ satisfies:

$$\gamma^2 = \left(A + (\beta A)^{1/\sigma}\right) \gamma - (\beta A)^{1/\sigma} A,$$

that is

$$\gamma = \frac{A + (\beta A)^{1/\sigma} \pm \left(A - (\beta A)^{1/\sigma}\right)}{2} = \begin{cases} A \\ (\beta A)^{1/\sigma} \end{cases}$$

The first solution entails a zero consumption level, hence should be discarded. The second solution entails a growth rate for capital equal to the growth rate of consumption, and hence a level of consumption at $t > 0$

$$c_t = (A - (\beta A)^{1/\sigma})k_{t-1} = (A - (\beta A)^{1/\sigma})K_0 (\beta A)^{(t-1)/\sigma},$$

which is positive, and hence an admissible solution, as long as $A > (\beta A)^{1/\sigma}$. Whenever this condition is satisfied, the optimal allocation is indeed characterised by constant growth.

The life time utility associated to the solution with a constant growth rate of capital (when $A > (\beta A)^{1/\sigma}$) is given by

$$\begin{aligned}V(A, K_0) &= \sum_{t=0}^{\infty} \beta^t \frac{\left[(\beta A)^{t/\sigma} \left(A - (\beta A)^{1/\sigma}\right) K_0\right]^{1-\sigma}}{1-\sigma} = \\ &= \frac{\left[\left(A - (\beta A)^{1/\sigma}\right) K_0\right]^{1-\sigma}}{1-\sigma} \sum_{t=0}^{\infty} \beta^t (\beta A)^{\frac{t(1-\sigma)}{\sigma}} =\end{aligned}$$

$$\begin{aligned}
&= \frac{\left[\left(A - (\beta A)^{1/\sigma} \right) K_0 \right]^{1-\sigma}}{1-\sigma} \sum_{t=0}^{\infty} \beta^{\frac{t}{\sigma}} A^{\frac{t(1-\sigma)}{\sigma}} = \\
&= \frac{\left[\left(A - (\beta A)^{1/\sigma} \right) K_0 \right]^{1-\sigma}}{1-\sigma} \frac{1}{1 - \beta^{\frac{1}{\sigma}} A^{\frac{(1-\sigma)}{\sigma}}} = \\
&= \frac{\left[\left(A - (\beta A)^{1/\sigma} \right) K_0 \right]^{1-\sigma}}{1-\sigma} \frac{1}{1 - \frac{(\beta A)^{1/\sigma}}{A}} = \\
&= \frac{\left[\left(A - (\beta A)^{1/\sigma} \right) K_0 \right]^{1-\sigma}}{1-\sigma} \frac{A}{A - (\beta A)^{1/\sigma}} = \frac{A \left(A - (\beta A)^{1/\sigma} \right)^{-\sigma} K_0^{1-\sigma}}{1-\sigma}.
\end{aligned}$$

The above computation implicitly assumes that $\beta^{\frac{1}{\sigma}} A^{\frac{(1-\sigma)}{\sigma}} < 1$. Note that this inequality is equivalent to the condition $A > (\beta A)^{1/\sigma}$ imposed above. The inequalities $\beta < 1$, $A \geq 1$ and $\sigma \geq 1$ are sufficient to guarantee that such condition holds.

First-Best with no Outside Options Suppose that R is such that $\beta R = 1$. In this case, with no outside options, the optimal allocation of the partnership entails no capital accumulation ($k_t = k_0$ for all t) and - if we implicitly assume equal Pareto weights of the two partners - the same level of consumption of the two partners $c_t^i = (R - 1)K_0/2 \forall i, t$.

The lifetime utility of each partner (at the solution with constant growth rate of capital, equal to zero in this case) is

$$V(1/\beta, K_0/2) = \sum_{t=0}^{\infty} \beta^t \frac{[(1/\beta - 1) K_0/2]^{1-\sigma}}{1-\sigma} = \frac{2^{\sigma-1} [(1/\beta - 1) K_0]^{1-\sigma}}{(1-\sigma)(1-\beta)}$$

The life-time utility of keeping the partnership together with initial total capital K_0 is then

$$W(K_0) = 2V(1/\beta, K_0/2) = \frac{2^\sigma [(1/\beta - 1) K_0]^{1-\sigma}}{(1-\sigma)(1-\beta)}.$$

First-Best with stochastic outside Options Next suppose that each individual at $t = 0$ has also access to an individual technology, with productivity $1 \leq L < R$. Also, at each date each individual receives with positive probability π access to a technological improvement, so that the productivity of the individual technology becomes A , where A is a random variable with support $[L, \bar{A}]$ and distribution $f(A)$. We assume that $\bar{A} > R$. Also, for simplicity we consider for now the case where such technological improvement can only happen once in an agent's lifetime, and that if an agent receives it, his partner will not receive it in the future. Given this, when an offer

arrives to one of the partners, the following two decision needs to be taken: The partnerships stay together or separates. We assume that the partnership contracts specifies the division of capital for the cases when separation occurs. Here, we implicitly assume that after separations no enforceable transfers between the two partners are possible². The decision faced by the two partners when an outside option of value A arrives to one of them is then the following one:

$$\max \left\{ W(K_0), \max_{0 \leq k \leq K_0} V(A, k) + V(L, K_0 - k) \right\}.$$

The second term in the above expression describes the optimal value of the partnership under separation: this is given by the sum of the continuation utilities of both partners (with equal weights) when the existing amount of capital K_0 is optimally allocated among them, i.e. so as to maximize value under separation. It is convenient to denote this value by $\widetilde{W}(A, K_0)$, obtained by solving the following problem:

$$\widetilde{W}(A, K_0) = \max_{0 \leq k \leq K_0} V(A, k) + V(L, K_0 - k)$$

The optimality condition with respect to the level of capital k allocated to the partner with the investment opportunity is given by:

$$A \left(A - (\beta A)^{1/\sigma} \right)^{-\sigma} k^{-\sigma} = L \left(L - (\beta L)^{1/\sigma} \right)^{-\sigma} (K_0 - k)^{-\sigma},$$

or

$$\frac{k}{K_0 - k} = \frac{L - (\beta L)^{1/\sigma}}{A - (\beta A)^{1/\sigma}} \left(\frac{A}{L} \right)^{1/\sigma} = \frac{L^{\frac{\sigma-1}{\sigma}} - \beta^{1/\sigma}}{A^{\frac{\sigma-1}{\sigma}} - \beta^{1/\sigma}} = Z(A).$$

Given our assumptions on A , L and β , the above expression for $Z(A)$ implies that, whenever we have $\sigma > 1$, the share of capital allocated to the defaulting agent (the one with the outside opportunity) is less than $1/2$, that is $Z(A) < 1$. Moreover this share is decreasing with the level of productivity A . Moreover the optimal share does not depend on the level of initial capital K_0 . Also, note that under log utility ($\sigma = 1$), we have $Z(A) = 1$, that the share of capital going to the defaulter is equal to $1/2$. To simplify notation, we will denote by $\eta(A)$ the share of capital that optimally goes to the defaulting agent. Note that

$$\eta(A) = \frac{Z(A)}{1 + Z(A)}$$

²Otherwise the solution is trivial: whenever $A > R$ the partnership separates, all the capital is allocated to the new project and there is full consumption sharing between the two partners.

Next, we characterise the threshold level of productivity A^* which triggers "efficient" separations. This is obtained at the value at which the value of the partnership under separation is the same as with no separation, that is:

$$2^\sigma \frac{[(1/\beta - 1)K]^{1-\sigma}}{(1-\sigma)(1-\beta)} = V(A^*, \eta(A^*)K) + V(L, (1 - \eta(A^*))K).$$

By using the value functions and simplifying, we obtain

$$\begin{aligned} & 2^\sigma \frac{\beta^{\sigma-1}}{(1-\beta)^\sigma} \\ = & \left[\left((A^*)^{\frac{\sigma-1}{\sigma}} - \beta^{1/\sigma} \right)^{-\sigma} (\eta(A^*))^{1-\sigma} + \left(L^{\frac{\sigma-1}{\sigma}} - \beta^{1/\sigma} \right)^{-\sigma} (1 - \eta(A^*))^{1-\sigma} \right]. \end{aligned}$$

Hence the threshold A^* is independent of the level of aggregate capital. Since $\widetilde{W}(A, K_0)$ is increasing in A it then follows that for all $A \geq A^*$ separation is efficient.

Private information regarding the receipt of an investment opportunity Let us introduce now private information regarding the arrival of the new investment opportunity. Let us consider first the case where the agent who receives the opportunity can not disclose it, but if he does disclose it then the value of A becomes known to the other partner. We show that in this case the incentive to disclose the new opportunity never binds, that is the same allocation as the one obtained above is still implementable.

The incentive to disclose the investment opportunity whenever separation is efficient is satisfied if the agent who receives this opportunity has a (weakly) higher utility by disclosing than by not disclosing.

We show next that, at the threshold level A^* :

$$V(A^*, \eta(A^*)K) > V(L, (1 - \eta(A^*))K).$$

The above inequality is equivalent to

$$\frac{K^{1-\sigma} (\eta(A^*))^{1-\sigma}}{1-\sigma} \left((A^*)^{\frac{\sigma-1}{\sigma}} - \beta^{1/\sigma} \right)^{-\sigma} > \frac{K^{1-\sigma} ((1 - \eta(A^*)))^{1-\sigma}}{1-\sigma} \left(L^{\frac{\sigma-1}{\sigma}} - \beta^{1/\sigma} \right)^{-\sigma},$$

that is, since $\sigma > 1$,

$$\left((A^*)^{\frac{\sigma-1}{\sigma}} - \beta^{1/\sigma} \right)^{-\sigma} Z(A^*)^{1-\sigma} < \left(L^{\frac{\sigma-1}{\sigma}} - \beta^{1/\sigma} \right)^{-\sigma},$$

or

$$\begin{aligned} \left(\frac{L^{\frac{\sigma-1}{\sigma}} - \beta^{1/\sigma}}{(A^*)^{\frac{\sigma-1}{\sigma}} - \beta^{1/\sigma}} \right)^{-\sigma} & > Z(A^*)^{1-\sigma} = \left(\frac{L^{\frac{\sigma-1}{\sigma}} - \beta^{1/\sigma}}{A^{\frac{\sigma-1}{\sigma}} - \beta^{1/\sigma}} \right)^{1-\sigma} \\ \Leftrightarrow 1 & > Z(A^*), \end{aligned}$$

always satisfied, as we showed above.

This in turn implies, since at the threshold A^* we have

$$V(1/\beta, K/2) = \frac{V(A^*, \eta(A^*)K) + V(L, (1 - \eta(A^*))K)}{2}.$$

Then, at A^* :

$$V(A^*, \eta(A^*)K) > V(1/\beta, K_0/2) = \frac{2^{\sigma-1} [(1/\beta - 1) K_0]^{1-\sigma}}{(1 - \sigma)(1 - \beta)}$$

the agent who receives the outside investment opportunity has an incentive to truthfully reveal the receipt of such offer. (Note that given that under our parametrisation the share of the partner not receiving outside offers is bigger than 1/2, so he does not have an incentive to misreport.)

Next we show that $V(A, \eta(A)K)$ is strictly increasing in A for $\sigma > 1$, which implies that the agent who receives an outside investment opportunity has always the incentive to report it when separation is efficient (that is for all $A \geq A^*$). We have

$$\begin{aligned} V(A, \eta(A)K) &= \frac{K^{1-\sigma}}{1 - \sigma} \left((A^*)^{\frac{\sigma-1}{\sigma}} - \beta^{1/\sigma} \right)^{-\sigma} (\eta(A))^{1-\sigma} \\ &= \frac{K^{1-\sigma}}{1 - \sigma} \left((A^*)^{\frac{\sigma-1}{\sigma}} - \beta^{1/\sigma} \right)^{-\sigma} \left(\frac{L^{\frac{\sigma-1}{\sigma}} - \beta^{1/\sigma}}{L^{\frac{\sigma-1}{\sigma}} - \beta^{1/\sigma} + A^{\frac{\sigma-1}{\sigma}} - \beta^{1/\sigma}} \right)^{1-\sigma} \\ &= \frac{K^{1-\sigma}}{1 - \sigma} \frac{\left(L^{\frac{\sigma-1}{\sigma}} - \beta^{1/\sigma} + A^{\frac{\sigma-1}{\sigma}} - \beta^{1/\sigma} \right)^{\sigma-1}}{\left(A^{\frac{\sigma-1}{\sigma}} - \beta^{1/\sigma} \right)^{\sigma}} \left(L^{\frac{\sigma-1}{\sigma}} - \beta^{1/\sigma} \right)^{1-\sigma} \end{aligned}$$

and hence

$$\begin{aligned} \frac{\partial V(A, \eta(A)K)}{\partial A} &\sim \frac{\frac{(\sigma-1)^2}{\sigma} A^{-\frac{1}{\sigma}} \left(L^{\frac{\sigma-1}{\sigma}} - \beta^{1/\sigma} + A^{\frac{\sigma-1}{\sigma}} - \beta^{1/\sigma} \right)^{\sigma-2} \left(A^{\frac{\sigma-1}{\sigma}} - \beta^{1/\sigma} \right)^{\sigma}}{\left(A^{\frac{\sigma-1}{\sigma}} - \beta^{1/\sigma} \right)^{2\sigma}} \frac{K^{1-\sigma}}{1 - \sigma} \\ &\sim \left[\left(A^{\frac{\sigma-1}{\sigma}} - \beta^{1/\sigma} \right) (\sigma - 1) - \sigma \left(L^{\frac{\sigma-1}{\sigma}} - \beta^{1/\sigma} + A^{\frac{\sigma-1}{\sigma}} - \beta^{1/\sigma} \right) \right] \frac{K^{1-\sigma}}{1 - \sigma} \\ &= - \left[\left(A^{\frac{\sigma-1}{\sigma}} - \beta^{1/\sigma} \right) + \sigma \left(L^{\frac{\sigma-1}{\sigma}} - \beta^{1/\sigma} \right) \right] \frac{K^{1-\sigma}}{1 - \sigma} > 0 \end{aligned}$$

for $\sigma > 1$.

For the *log* case ($\sigma = 1$), we have $Z(A) = 1$ for all A , hence both agents are allocated the same amount of capital for all values of A . In this case incentive constraints are also always satisfied when separation is efficient, since the utility of the agent who receives the outside opportunity is clearly always higher than the utility of the other agent and is always increasing in A .

Private Information regarding the value of the offers. In this case, it is obvious that in order to be incentive compatible, the sharing rule after separation cannot depend on A . In order to simplify notation, we will denote as η the share of capital that goes to the agents who leaves the partnership because of a better opportunity. This implies that we can derive the threshold level of offers for each asset distribution rule as $\hat{A}(\eta)$ using an indifference condition of staying together and defaulting.

We continue with presenting some results for the log specification.

Result 1: $\hat{A}(1/2) = 1/\beta < A^*$

Proof. The indifference condition determining $\hat{A}(\eta)$ is given by

$$V(1/\beta, K/2) = V(\hat{A}(\eta), \eta K).$$

This implies that $\hat{A}(1/2) = 1/\beta$. ■

Result 2: $A^{*PI} > 1/\beta$

Proof. At $\eta = 1/2$ due to the envelope condition are no marginal losses to reduce η , but there are positive marginal gains of reducing separations. This implies that $\eta^{PI} < 1/2$ and hence $A^{*PI} > 1/\beta$. ■

Let's define $A^*(\eta)$ as the efficient separation threshold given η . If

$$2V(1/\beta, K/2) > V(\bar{A}, \eta K) + V(L, (1 - \eta)K)$$

$A^*(\eta) = \bar{A}$ otherwise it is given by

$$2V(1/\beta, K/2) = V(A^*(\eta), \eta K) + V(L, (1 - \eta)K).$$

Result 3: $A^{*PI} < A^*(\eta^*)$

Proof: Depending on L and the distribution of A may or may not exist a threshold $\tilde{\eta}$ such that $\hat{A}(\eta^*) = A^*(\tilde{\eta})$. If it does not exist Result 3 trivially holds. Otherwise, at $\tilde{\eta}$, the marginal loss of increasing η in terms of higher separations is zero because of the indifference given by the definition of by the definition of $A^*(\tilde{\eta})$. At the same time, there is a positive gain for all realisations when separation happens because we are moving to the direction of the efficient level $\eta = 1/2$.

This result implies that there are always inefficient separation given this η .

Now we turn to the general case where $\sigma > 1$. In this case we have shown that, in the full information case, that the asset division rule depends on A . In particular, we have shown that $\eta^*(A)$ is an a strictly decreasing function and $\eta^*(A) < 1/2$ for all A such that separations are efficient. First of all, let us define as $\hat{\eta}$ as $\max_{A \geq A^*} \eta(A) = \eta(A^*)$.

Result 4: $\eta^{PI} < \hat{\eta}$.

Proof. Suppose that $\eta^{PI} = \hat{\eta}$. At this point, we will have inefficient separations because $\hat{A}(\eta) < A^*$. Moreover, for all $A > A^*$ we have inefficient division of capital that can be improved by decreasing η . ■

TO BE COMPLETED

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