

# Optimal Taxation with Private Insurance <sup>\*</sup>

## *(Preliminary)*

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### **Abstract**

We derive a fully-nonlinear optimal income tax schedule in the presence of private insurance market. The optimal tax formula is expressed in terms of sufficient statistics—such as Frisch elasticity of labor supply, social preferences, and hazard rates of the income distributions—as in the standard Mirrleesian taxation without private insurance (e.g., Saez (2001)). However, in the presence of private market, the standard sufficient statistics are no longer sufficient to determine the exact shape of optimal tax schedule. The optimal tax rates also depends on how private savings interact with public insurance—through substitution and crowding in/out. Based on our formula, we compute the optimal tax schedule using a quantitative general-equilibrium model that is calibrated to reproduce the U.S. income distribution.

*Keywords:* Optimal Taxation, Private Insurance, Crowding Out, Mirrleesian Tax

*JEL Classification:*

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# 1 Introduction

What is the socially optimal shape of income tax schedule? This has been one of the classic and central questions in macroeconomics and public finance. Despite the significant progress in the literature, surprisingly little studies investigated the role of the private intermediation on the optimal tax system. Given that almost all households have access to financial markets, it is important to understand the interaction between the private and public insurance to determine the optimal tax. In this paper, we study optimal (fully) nonlinear income tax in the presence of private financial market.

In the classic Mirrleesian approach (Mirrlees (1971)) which studies how to design social insurance with information friction, it is commonly assumed that the government is the only provider of insurance. While Chetty and Saez (2010) is the exception in the static Mirrleesian literature, they assume a linear functional form for both tax schedule and private insurance. In the dynamic Mirrleesian literature (the so-called New Dynamic Public Finance), Golosov and Tsyvinski (2007) allow private insurance but with specific structure on the private market — competitive insurance firms with private information friction only. Given that the government and private firms face the same information friction, the role of the government is restricted to internalize the pecuniary externalities and thus there are limited implications on the optimal shape of tax schedule.

In the alternative Ramsey approach (Ramsey (1927)) which examines the optimal tax schedule within a class of functional form, many studies have provided quantitative answers on the optimal amount of redistributive taxation in the presence of self-insurance opportunities (e.g. Aiyagari and McGrattan (1998), Conesa and Krueger (2006), Conesa, Kitao, and Krueger (2009), Heathcote, Storesletten, and Violante (2014), Bhandari, Evans, Golosov, and Sargent (2016)). However, these studies assume a parametric form for tax schedule—either affine or log-linear. Moreover, they do not particularly focus on the role of private insurance—how the introduction of private savings affects the optimal tax schedule.

In this paper, we study *fully nonlinear* optimal income tax in the presence of private market. Different from the Mirrleesian which solves a mechanism design problem with information friction, we use a variational approach as in Saez (2001) and Golosov, Tsyvinski, and Werquin (2014). Different from the Ramsey, we do not impose a parametric functional form on the tax system. We study a fully nonlinear optimal income tax schedule within a class—i.e., age-independent and

time-invariant tax that is levied on the current income only. This allows us a direct comparison to the results in static Mirrleesian and Ramsey literatures.

We derive the optimal tax formula under a very general representation of private market intermediation. As in Saez (2001), the optimal tax formula is expressed in terms of standard *sufficient* statistics — such as Frisch elasticity of labor supply, hazard rate of income distribution, and marginal social welfare weights. In the presence of private market, however, these sufficient statistics are no longer sufficient. The optimal tax formula also includes additional statistics which capture the interaction between private and public insurances — such as the substitution between two insurance and crowding in/out of private intermediation.

More specifically, our formula provides transparent and intuitive insights about the role of private market in the optimal tax schedule. First, it captures the fact that public and private insurance are substitutes (which we call “substitution effect”)—a higher marginal private intermediation (e.g., high savings rate) leads to a lower marginal tax rate at a given level of income. Second, the original formula in Saez (2001) is modified to reflect the amplifying (or mitigating) factors that stem from private intermediation. Two components are important for this amplification (or mitigation) : (i) marginal propensity to consume (1- marginal private intermediation) and (ii) cross-sectional consumption inequality which depends on the distribution of wealth. The Saez formula is amplified if the marginal propensity to consume is larger than one (e.g., a negative marginal savings rate) and/or if the consumption inequality is larger (e.g., a large wealth inequality) in the presence of private intermediation. Third, it depends on the cross-sectional distribution of crowding in/out effects. More specifically, a tax reform is more effective when the response of private intermediation is aligned with the reform. For example, increasing the marginal tax rate is more desirable when the private insurance also becomes more progressive—e.g., the rich saves more in response to such tax reform.

Our optimal tax formula requires the cross-sectional distribution of crowding in/out elasticity which is very difficult to estimate from the data. We obtain these elasticities from a quantitative general equilibrium incomplete-markets model (Huggett (1993)) which is calibrated to resemble some salient features of U.S. economy (such as income distribution). This allows us to quantify the role of private insurance in the optimal tax system.

Our results show that the presence of private intermediation market is quantitatively important as the difference in optimal tax rates (with and without private insurance) can be as large as 10

percentage points. Moreover, the difference in tax rates with and without private intermediation market are not uniform. For the low and lower-middle income group, the optimal tax rate is higher in the presence of private market, mainly because of amplified Saez effects. For the middle to high income groups where marginal intermediation is positive, the substitution effect and mitigating effect in Saez formula dominate, resulting in lower tax rates in the presence of private market. At the very top income group, these forces almost cancel out each other, leaving the tax rate similar to that without private insurance. Our comparative statistic analysis shows that the various channels of private insurance—substitution, amplification/mitigation of Saez, and crowding in/out effects—depend on the fundamental parameters such as risk aversion, Frisch elasticity of labor supply, borrowing constraint, and the shape of the tail of productivity distribution.

The remainder of the paper is organized as follows. Section 2 provides a brief literature review. In section 3, we derive the optimal tax formula using a variation approach. Section 4 presents our benchmark quantitative analysis and Section 5 provides various comparative statistics. Section 6 concludes.

## 2 Relation to Literature

Our paper is related to a large literature on the optimal income tax-transfer system. In the Mirrleesian literature which solves the planner’s problem under private information, the optimal shape of nonlinear income tax has been studied in a static environment (Saez (2001) and Diamond (1998)). While the optimal labor wedge has been studied in a stochastic dynamic environment (Farhi and Werning (2013), Golosov, Troshkin, and Tsyvinski (2016) among others), the Mirrleesian literature largely abstracts from private insurance market by assuming that the government is the sole provider of insurance.

Chetty and Saez (2010) and Golosov and Tsyvinski (2007) are the exceptions that allow for private insurance market in the Mirrleesian literature. However, their questions are centered on the welfare gains from government intervention in the presence of private insurance. Chetty and Saez (2010) also points out that the optimal tax formula should take into account the interplay between the government and private market, especially when the private market suffers from the moral hazard or adverse selection. But they assume that both private and public insurance are linear, and thus have limited implications on the optimal tax schedule. Golosov and Tsyvinski (2007) examine a dynamic environment in the presence of competitive insurance market where the

private information is the only friction and show an important role of government intervention if households secretly trade risk-free bonds. We derive the optimal nonlinear tax formula under a very general representation of private market. Thus our formula can be applied to many different market structures regardless of the source of financial frictions.

In the Ramsey taxation literature (which studies an optimal tax schedule within a class of parametric functional form), Conesa and Krueger (2006), Conesa, Kitao, and Krueger (2009), and Heathcote, Storesletten, and Violante (2014), Heathcote and Tsujiyama (2015) among others are complementary to our paper. Imposing a parametric form on the tax function makes the analysis highly tractable even in a richer assumption about the private insurance market. While we allow for a fully nonlinear tax system, our analysis provides a transparent comparison to the results from the Ramsey literature. We also compute the optimal tax schedule in a general-equilibrium incomplete-markets economy—a workhorse model in macroeconomics. Our results show that the optimal tax schedule is very different from those commonly assumed—an affine or log-linear tax function.

Our paper is also related to the “variational approach” literature which builds on the perturbation method developed by Saez (2001) in a static economy and allows us to express the tax formula in terms of the so-called sufficient statistics. Recently Golosov, Tsyvinski, and Werning (2014) and Saez and Stantcheva (2016) have also extended this method to a dynamic environment.

Outside the optimal tax literature, several papers study the interaction between private and public insurance. Attanasio and Rios-Rull (2000) study how the compulsory public insurance against aggregate shock affects the private insurance against idiosyncratic shocks. Krueger and Perri (2011) study whether increasing the progressivity of income tax crowds out private risk sharing, under the limited commitment in the private market.

### **3 Optimal Nonlinear tax formula with private insurance**

#### **3.1 Restriction of tax system**

While we consider a fully nonlinear income tax system without assuming a functional form, we focus on a restrictive class of tax system. The class of tax system we consider is nonlinear labor-income

tax with a lump-sum transfer.<sup>1</sup> More precisely, (i) (Fully) nonlinear labor-income tax  $T(z)$  where  $z$  is the current labor income, (ii) The period- $t$  labor-income tax can be levied only on period  $t$ 's income, and (iii) Nonlinear tax function  $T(z)$  is age-independent and time invariant.

We impose these restrictions because they allow for a direct comparison to the static Mirrleesian taxation and Ramsey taxation literature. On one hand, in a static Mirrleesian analysis, the labor-income tax depends on income only (not on the productivity) because of information frictions. However, in a dynamic environment with stochastic productivity—which we are studying in this paper, the optimal allocation which solves the mechanism design problem with information friction (as in New Dynamic Public Finance) will depend on the history of incomes. Moreover, it is well known that the tax system that can implement the constrained-efficient allocation are highly complicated, and thus direct comparison of tax schedule between static and dynamic environment is not straightforward even without private market.<sup>2</sup> On the other hand, the Ramsey literature focuses on the tax system with particular functional forms. As in Ramsey literature, our analysis starts with a simple and implementable tax system, but allows for a fully nonlinear functional form. Thus, our analysis provides a transparent comparison to the theoretical results from the Mirrleesian as well as the Ramsey taxation.

### 3.2 Economic Environment with Private and Public Insurance

Consider an economy with continuum of workers with measure one. Workers face uncertainty about their labor productivity in the future. The individual productivity shock  $x_t$  follows a Markov process (which will be specified below) that has an invariant stationary (cumulative) distribution  $F(x)$  whose probability density is  $f(x)$ .

Individual workers have identical utility function  $\sum_{t=0}^{\infty} \beta^t E_0[U(c_t, l_t)]$ , where an instantaneous utility  $U(c, l)$  has the following form:

$$U(c, l) = u(c - v(l)),$$

where  $u(\cdot)$  is concave and increasing in consumption  $c$  and  $v(\cdot)$  is convex and increasing in labor supply  $l$ . We focus on households' preferences that has no wealth effect in labor supply (the so-called GHH preferences by Greenwood, Hercowitz, and Huffman (1988)). This assumption is often

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<sup>1</sup>We can allow capital income tax, but we only focus on optimal labor income tax for given capital income tax, without considering joint optimal taxation.

<sup>2</sup>Most of studies in New Dynamic Public Finance compares the implicit wedge from the dynamic economy to the marginal tax rate in the static economy.

adopted in the literature because it significantly simplifies the optimal tax formula. Earnings of a worker whose current productivity  $x_t$  is  $z_t = x_t l_t$ . The cumulative distribution of earnings is denoted by  $H(z)$  whose density function is  $h(z)$ .

Government provides insurance through a (time-invariant) nonlinear labor-income tax and lump-sum transfer system where the net payment schedule is denoted by  $T(z_t)$ . The after-tax labor income is  $y_t = z_t - T(z_t)$ . Workers can also participate a private market to self-insure against their income uncertainty. Denote the individual state in period  $t$  by  $(z_t, \mathbf{s}_t)$ <sup>3</sup>, where  $\mathbf{s}_t = (s_{1,t}, \dots, s_{M,t}) \in R^M$  is the vector of individual state variables other than labor income. For example, if the private insurance market is a Bewley-type incomplete markets and consumers can only self-insure themselves by savings and borrowing via non-contingent bond (e.g., Huggett (1993)), we only need one additional state variable, bond holdings  $a_t$ :  $\mathbf{s}_t = a_t$ .

We denote the net payment from the private insurance (payment - receipts) by  $P_t(z_t, \mathbf{s}_t; T)$ . Thus, the consumption is  $c_t(z_t, \mathbf{s}_t) = z_t - T(z_t) - P_t(z_t, \mathbf{s}_t; T)$ . This representation is very general. The only assumption we make is that the sum of the net payment in the private intermediation is zero:  $\int P(\cdot) = 0$ .<sup>4</sup> In a Huggett economy with self insurance only,  $P_t(z_t, a_t) = a_{t+1}(z_t, a_t) - (1+r)a_t$  where  $r$  is the rate of return on bond holdings.

The government chooses a tax/transfer schedule  $T(z)$  (labor-income tax plus lump-sum transfer) to maximize the following social welfare function (SWF):

$$SWF = E\left[\sum_{t=0}^{\infty} \beta^t G(U(c_t, l_t))\right],$$

where  $G(\cdot)$  is an increasing function which reflects the social preferences for redistribution.

The private insurance  $P(\cdot; T)$  depends on the tax/transfer schedule  $T$ . The government chooses the optimal  $T$  taking into account this interaction between the public and the private intermediation. To simplify the notation, from now on we often omit  $T$  in  $P(\cdot)$ , unless necessary. We also introduce the private intermediation as a function of after-tax income  $y$ :  $\tilde{P}(y, \mathbf{s}) = \tilde{P}(z - T(z), \mathbf{s}) = P(z, \mathbf{s}; T)$ .

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<sup>3</sup>Alternatively, the state can be expressed as  $(x_t, \mathbf{s}_t)$ . With no income effects on labor supply, labor income  $z_t$  and productivity  $x_t$  has one to one relationship and we can use them interchangeably.

<sup>4</sup>That is, we consider a pure insurance market where the aggregate transfer is exactly funded by the aggregate payment in each period.

### 3.3 Deriving Taxation Formula with Private Insurance

In deriving the optimal tax formula, we apply the variational approach (Saez (2001); Golosov, Tsyvinski, and Werquin (2014)). Instead of solving for a mechanism design problem, we optimize social welfare with respect to the tax function directly, under the restriction we impose on the class of tax system. That is, we consider a perturbation (a small deviation) from a given non-linear tax schedule. If there is no welfare improving perturbation in the given class of tax system, the given tax schedule is optimal.

For a given income tax schedule  $T(z)$ , the economy we consider converges to a steady state where the distribution of state variables  $\Phi(z, \mathbf{s})$  is stationary. We assume that in period 0 the economy starts from a steady state. Consider a (revenue-neutral) tax reform that increases the marginal tax rate  $T'(z)$  by  $\delta\tau$  on the income bracket  $[z^*, z^* + dz^*]$ , as in Saez (2001).

In order to analyze the welfare effect of a tax reform (in the presence of private market), it is important to examine its impacts on the household's payment in the *total* intermediation for insurance (government tax/transfer plus private savings). Denote the total intermediation by  $M_t(z, \mathbf{s}) = T(z) + P_t(z, \mathbf{s}; T)$ . When the government sets optimal tax schedule, it wants to optimize total insurance by considering the response of the private insurance to the tax schedule. Thus, the optimal marginal tax rate at income level depends on various factors: (i) the level of marginal private savings — which we call the “substitution” effect, (ii) the response (change) of private savings schedule — which we call the “crowding in/out” effect, and (iii) the response of labor supply — which we call “behavioral” effect. We discuss each of these effects.

#### 3.3.1 Effects on Total Intermediation

The total *marginal* intermediation at income level  $z$  is

$$M'_t(z, \mathbf{s}) = T'(z) + P'_t(z, \mathbf{s}).$$

Within the income band  $[z^*, z^* + dz^*]$ , where the marginal tax rate is changed, the change in the marginal total intermediation  $dM'$  reflects the change in marginal tax itself,  $dT'(z^*)$  and that in the marginal private intermediation,  $dP'(z^*, \mathbf{s})$ . The marginal private intermediation can be changed through the two channels, (i) change in after-tax income ( $-\delta\tau$ ) and (ii) change in the marginal

private payment schedule (change in  $\tilde{P}'$ ).<sup>5</sup> The sum of these two changes is:

$$d^o P'_t(z^*, \mathbf{s}) = -\tilde{P}'(y^*, \mathbf{s})\delta\tau + d^o \tilde{P}'_t \Big|_{y^*} \cdot (1 - T'(z^*)),$$

where  $d^o X$  represents the change in  $X$  due to the tax increase in its *own* income bracket—as opposed to the change of  $X$  due to the tax increase in other income brackets (which we will describe below).

Note that  $d^o \tilde{P}'|_{y^*}$  denotes the change in the marginal private payment schedule ( $\tilde{P}'(y, \mathbf{s})$ ) for a given after tax income  $y^* = z^* - T(z^*)$  due to change in its own tax rate  $T'(z^*)$ . Thus, the change in marginal total intermediation is:

$$d^o M'_t(z^*, \mathbf{s}) = (1 - \tilde{P}'(y^*, \mathbf{s}))\delta\tau + d^o \tilde{P}'_t \Big|_{y^*} \cdot (1 - T'(z^*))$$

It is easy to understand these terms in the context of Huggett (1993) economy with self insurance. The first term reflects an increase in marginal public savings (tax) at a given marginal propensity to save  $\tilde{P}'_y$ . The second term represents the change in the marginal private savings rate at a given disposable income level—which we refer as *own*-crowding in/out effect. We call this *own* crowding effect because this reflects the change in marginal private savings at the income level where the tax rate is changed. As we will discuss below, the private savings rate may change at income levels where the marginal tax rates remain unchanged because the tax reform affects the permanent income. Such changes in marginal savings rate which we refer as *cross*-crowding in/out effects.

Using the definition for the elasticity of the *own*-crowding out  $r_t^o(z, \mathbf{s})$ , as in Chetty and Saez (2010)<sup>6</sup> the change in total intermediation  $d^o M'_t(z^*, \mathbf{s})$  can be rewritten as follows:

$$\begin{aligned} d^o M'_t(z^*, \mathbf{s}) &= (1 - r_t^o(z^*, \mathbf{s}))(1 - \tilde{P}'_t(y^*, \mathbf{s}))\delta\tau, \\ r_t^o(z, \mathbf{s}) &= -\frac{d^o \log(1 - \tilde{P}'_t(y^z, \mathbf{s})) \Big|_{y^z}}{d \log(1 - T'(z))} \end{aligned}$$

To better understand  $d^o M'_t(z^*, \mathbf{s})$ , note also that the marginal propensity to consume out of *before-tax* income  $z$  is  $\frac{dc}{dz} = 1 - M'_t(z, \mathbf{s})$ . Thus, an increase in the marginal total intermediation is equivalent to the decrease in marginal consumption. If there is no crowding out of private intermediation ( $r^o = 0$ ), the decrease in marginal consumption is simply the marginal propensity to consume multiplied by the changes in tax rate  $(1 - \tilde{P}')\delta\tau$ . However, because of the crowding out of the private intermediation, the marginal total intermediation increases less than  $(1 - \tilde{P}')\delta\tau$ .

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<sup>5</sup>  $P'(z^*, \mathbf{s}) = \frac{dP}{dy} \frac{dy}{dz} = [\tilde{P}' \cdot (1 - T'(z^*))]$  where  $\tilde{P}(y, \mathbf{s}) = \tilde{P}(z - T(z), \mathbf{s}) = P(z, \mathbf{s}; T)$ .

<sup>6</sup> Chetty and Saez (2010) defined the degree of crowding out in terms of linear (public and private) savings.

In a dynamic environment where the individual productivity stochastically changes over time, there can be additional effects on private intermediation because the permanent (or future) income changes. For example, in an economy with self insurance, while the tax reform is confined to the income region of  $[z^*, z^* + dz^*]$  only, private savings in other income regions also changes. We refer this effect as *cross* crowding in/out effect. For each income level  $z$ , the change in marginal total intermediation via cross crowding out effect can be expressed as:

$$\begin{aligned} d^c M'_t(z, \mathbf{s}) &= \frac{d^c \tilde{P}'_t(y^z, \mathbf{s}) \Big|_{y^z}}{dT'(z^*)} (1 - T'(z)) \delta \tau dz^* \\ &= -r_t^c(z, \mathbf{s}) \frac{(1 - P'_t(z - T(z), \mathbf{s}))(1 - T'(z))}{1 - T'(z^*)} \delta \tau dz^*, \end{aligned}$$

where  $d^c P'_t(y^z, \mathbf{s})$  denotes the changes in the marginal private intermediation schedule due to *cross* crowding out for given after tax income  $y^z$  and  $r_t^c(z, \mathbf{s})$  is the elasticity of *cross* crowding out in marginal private intermediation:

$$r_t^c(z, \mathbf{s}) = - \frac{d^c \log(1 - \tilde{P}'_t(y^z, \mathbf{s})) \Big|_{y^z}}{d \log(1 - T'(z^*))}.$$

To summarize, in a income band where the tax reform takes place, the marginal total intermediation changes through the mechanical and the own crowding out effects. For all income level, the marginal total intermediation (which is equivalent to marginal private intermediation) changes due to cross crowding in/out of private savings.

More specifically, for an income level below  $z^*$ —where tax payments remain unchanged, the change in the level of total intermediation reflects the cross crowding out effect  $d^c M_t(z, \mathbf{s})$  only. For an income level above the band  $[z^*, z^* + dz^*]$ —where tax payments are actually changed, the total intermediation changes by  $d^o M_t(z, \mathbf{s}) + d^c M_t(z, \mathbf{s})$  with

$$\begin{aligned} d^o M_t(z, \mathbf{s}) &= (1 - r_t^o(z, \mathbf{s}))(1 - \tilde{P}'_t(y^z, \mathbf{s})) \delta \tau dz^*, \text{ for } z \geq z^* + dz^*, \\ d^c M_t(z, \mathbf{s}) &= d^c \tilde{P}_t(y^0, \mathbf{s}) + \delta \tau dz^* \int_0^z \frac{d^c \tilde{P}'_t(y^{\tilde{z}}, \mathbf{s})}{dT'(z^*)} (1 - T'(\tilde{z})) d\tilde{z}, \text{ for all } z, \end{aligned}$$

where  $d^c P_t(y^0, \mathbf{s})$  denotes the change in the intercept of private intermediation schedule due to cross-crowding out. Integrating the changes in the total intermediation for all households in the economy, the *aggregate* change in total intermediation in period  $t$ ,  $d\bar{M}_t^a$ , is:

$$\begin{aligned} d\bar{M}_t^a &= \int_{z \geq z^*, \mathbf{s}} d^o M_t(z, \mathbf{s}) h(z) d\Phi(z, \mathbf{s}) + \int d^c M_t(z, \mathbf{s}) d\Phi(z, \mathbf{s}) \\ &= \delta \tau dz^* (1 - H(z^*)) \int (1 - r_t^o(z^*, \mathbf{s}))(1 - \tilde{P}'_t(y^{z^*}, \mathbf{s})) d\Phi(\mathbf{s}|z^*) + \int d^c M_t(z, \mathbf{s}) d\Phi(z, \mathbf{s}). \end{aligned}$$

### 3.3.2 Behavioral Response of Labor Supply

With endogenous labor supply, there is an additional change in total intermediation we need to consider. The increased marginal tax rate on the small band  $[z^*, z^* + dz^*]$  affects the before-tax income through two channels: direct effects from an exogenous increase of  $\delta\tau$  and indirect effects due to the change in labor income along the tax schedule by  $dz$  which results in the change in the marginal tax rate by  $T''(z)dz$ . The increased of income is:

$$dz = -z^*e(z^*)\frac{\delta\tau}{1 - T' + z^*eT''},$$

where  $e(z^*)$  is the Frisch elasticity of labor supply at income level  $z^*$ . The change in total intermediation via these effects is:

$$\begin{aligned} & \int dz \cdot M'_t(z^*, \mathbf{s}) d\Phi(\mathbf{s}|z^*) h(z^*) dz^* \\ = & - \int \frac{T'(z^*) + \tilde{P}'_t(y^{z^*}, \mathbf{s})(1 - T'(z^*))}{1 - T' + z^*e(z^*)T''} d\Phi(\mathbf{s}|z^*) z^*e(z^*) h(z^*) \delta\tau dz^*. \end{aligned}$$

As in Saez (2001), we introduce the virtual density  $h^*(z)$  to simplify the presentation of optimal tax rates formula where  $h^*(z)$  is the density of incomes that would take place at  $z$  if the tax schedule  $T(\cdot)$  were replaced by the linear tax schedule tangent to  $T(\cdot)$  at level  $z$ . When there is no income effect, Lemma 1 of Saez (2001) still applies.

**Lemma 1** (Lemma 1 of Saez (2001)). *For any regular tax schedule  $T$  not necessarily optimal, the earnings function  $z_x$  is non-decreasing and satisfies the following differential equation,*

$$\frac{\dot{z}_x}{z_x} = \frac{1 + e}{x} - \dot{z}_x \frac{T'_{(x)}}{1 - T'_{(x)}} e.$$

Using Lemma 1 and the fact that  $f(x) = h(z)\dot{z}_x = h^*(z)\dot{z}^*_x$  where  $\dot{z}^*$  is the derivative when the linearized tax schedule is in place, we obtain:  $\frac{h^*(z)}{1 - T'(z)} = \frac{h(z)}{1 - T'(z) + ezT''(z)}$ . Then, the aggregate change in total intermediation via behavioral response of labor,  $d\bar{M}_t^b$ , is:

$$d\bar{M}_t^b = - \left\{ \frac{T'(z^*)}{1 - T'(z^*)} + \int \tilde{P}'_t(y^{z^*}, \mathbf{s}) d\Phi(\mathbf{s}|z^*) \right\} z^*e(z^*) h^*(z^*) \delta\tau dz^*.$$

In general, Frisch elasticity  $e(z^*)$  can be different for each income level  $z^*$ . From now on, however, we focus on constant elasticity, and we will simply denote the elasticity by  $e$ .

### 3.3.3 Optimal Tax Formula

In period  $t$ , a household pays an extra amount of  $dM_t(z, \mathbf{s}) = d^o M_t(z, \mathbf{s}) + d^c M_t(z, \mathbf{s})$  as a total intermediation. Using the envelope theorem, this leads to the change in social welfare of:

$$- \int dM_t(z, \mathbf{s}) G'(u(z, \mathbf{s})) u'(c(z, \mathbf{s})) d\Phi(z, \mathbf{s}).$$

Each period the increased aggregate extra payment of the total intermediation,  $d\bar{M}_t = d\bar{M}_t^a + d\bar{M}_t^b$ , will be rebated to everyone in a lump-sum fashion which results in the change in social welfare of:<sup>7</sup>

$$d\bar{M}_t \int G'(u(z, \mathbf{s})) u'(c(z, \mathbf{s})) d\Phi(z, \mathbf{s}).$$

The overall change in social welfare from the above tax reform is:

$$dSWF = \sum_{t=0}^{\infty} \beta^t d\bar{M}_t \int G'(u(z, \mathbf{s})) u'(c(z, \mathbf{s})) d\Phi(z, \mathbf{s}) - \sum_{t=0}^{\infty} \beta^t \int dM_t(z, \mathbf{s}) G'(u(z, \mathbf{s})) u'(c(z, \mathbf{s})) d\Phi(z, \mathbf{s}).$$

A tax schedule  $T(z)$  is optimal if  $dSWF = 0$ :

$$\begin{aligned} \sum_{t=0}^{\infty} \beta^t d\bar{M}_t &= \sum_{t=0}^{\infty} \beta^t \int dM_t(z, \mathbf{s}) g(z, \mathbf{s}) d\Phi(z, \mathbf{s}) \\ \text{where } g(z, \mathbf{s}) &= \frac{G'(u(z, \mathbf{s})) u'(c(z, \mathbf{s}))}{A}, \quad A = \int G'(u(z, \mathbf{s})) u'(c(z, \mathbf{s})) d\Phi(z, \mathbf{s}). \end{aligned}$$

By substituting out  $d\bar{M}_t$  and  $dM_t(z, \mathbf{s})$  and rearranging, we obtain the following optimal tax formula.

**Proposition 2.** *Optimal marginal tax rate at income  $z^*$  should satisfy the following formula :*

$$\frac{T'(z^*)}{1 - T'(z^*)} = (1 - \beta) \sum_{t=0}^{\infty} \beta^t \left[ - \int \tilde{P}'(y^{z^*}, \mathbf{s}) d\Phi(\mathbf{s}|z^*) + B_t(z^*) + C_t(z^*) \right] \quad (1)$$

where

$$\begin{aligned} B_t(z^*) &= \frac{1}{e} \frac{1 - H(z^*)}{z^* h^*(z^*)} \int_{z^*}^{\infty} \int (1 - r_t^o(z^*, \mathbf{s})) (1 - \tilde{P}'(y^{z^*}, \mathbf{s})) (1 - g(z, \mathbf{s})) d\Phi(\mathbf{s}|z) \frac{h(z)}{1 - H(z^*)} dz, \\ C_t(z^*) &= \frac{1}{e} \frac{1}{z^* h^*(z^*)} \int (1 - g(z, \mathbf{s})) d^c P_t(z, \mathbf{s}) d\Phi(z, \mathbf{s}), \end{aligned}$$

$$\text{and } d^c P_t(z, \mathbf{s}) = \left[ \frac{d^c \tilde{P}_t(y^0, \mathbf{s})}{\delta \tau dz^*} - \int_0^z r_t^c(\tilde{z}, \mathbf{s}) \frac{(1 - \tilde{P}'_t(y^{\tilde{z}}, \mathbf{s}))(1 - T'(\tilde{z}))}{1 - T'(z^*)} d\tilde{z} \right].$$

---

<sup>7</sup>More precisely, the change in aggregate total intermediation  $d\bar{M}_t$  is the sum of change in aggregate tax  $d\bar{T}$  and that in aggregate private intermediation  $d\bar{P}$ . The change in aggregate tax  $d\bar{T}$  is rebated back as a lump-sum transfer because we consider revenue-neutral tax reform, and the change in aggregate private intermediation  $d\bar{P}$  is zero because we consider a pure insurance private market where net payments sum to zero ( $\bar{P} = 0$ )

Note that the distribution are time invariant because we consider an economy starting from the steady state and the labor supply adjust instantaneously (no wealth effect in labor supply) in response to a tax reform. However, the private savings market may adjust slowly over time since the asset holdings may adjust slowly over time. Thus,  $r_t^o(\cdot)$  and  $r_t^c(\cdot)$  can be time varying.

### 3.3.4 Extension of Saez (2001)

[Need to write down proposition : under which condition the total intermediation with private market is equivalent to optimal tax without private market]

### 3.3.5 Role of Private Insurance Market

Our optimal tax formula (1) nests Saez (2001)'s formula without private insurance market. If there is no private insurance,  $P_t'(\cdot) = r_t^o(\cdot) = r_t^c(\cdot) = 0$  and  $\Phi(z, \mathbf{s}) = h(z)$ . Our formula goes back to that in Saez (2001):

$$\frac{T'(z^*)}{1 - T'(z^*)} = \frac{1}{e} \frac{1 - H(z^*)}{z^* h^*(z^*)} \int_{z^*}^{\infty} \left( 1 - \frac{G'(u(z))u'(c(z))}{A} \right) \frac{h(z)}{1 - H(z^*)} dz. \quad (2)$$

One of the nice features of Saez (2001) formula is that the optimal tax rate can be expressed in terms of “sufficient” statistics. The optimal tax rate ( $T'$ ) is decreasing in (i) Frisch elasticities of labor supply,  $e$ , (ii) hazard rate of the income distributions,  $\frac{z^* h(z^*)}{1 - H(z^*)}$ , and (iii) the average social marginal welfare weight of income above  $z^*$ ,  $E[g(z, \mathbf{s}) | z \geq z^*]$ . A larger Frisch elasticity implies a bigger efficiency cost from distorting labor supply and thus tax rate is decreasing in  $e$ . The cost of distortion is proportional to the number of workers ( $z^* h(z^*)$ ), while the revenue gain from the tax increase is proportional to the fraction of income higher than  $z^*$ :  $1 - H(z^*)$ . Thus, the optimal tax rate is decreasing in hazard rate ( $\frac{z^* h(z^*)}{1 - H(z^*)}$ ). Marginal social welfare weight  $g(z, \mathbf{s})$  measures the relative value of an additional dollar of consumption at each state  $(z, \mathbf{s})$  from the perspective of the government. A larger social welfare weights ( $g(z, \mathbf{s})$ ) above  $z^*$ , who pays extra tax, leads to a lower tax rate.

However, in the presence of private insurance market, the standard sufficient statistics are not sufficient to pin down the optimal tax schedule. The optimal tax schedule also depends on how the private insurance market interacts with public savings, such as the slope of private savings  $\tilde{P}'(\cdot)$  and crowding in/out effects,  $r_t^o(\cdot)$  and  $r_t^c(\cdot)$ .

To better understand the role of private insurance in the optimal tax rate, we re-arranged the formula as follows.<sup>8</sup>

$$\frac{T'(z^*)}{1 - T'(z^*)} = (1 - \beta) \sum_{t=0}^{\infty} \beta^t \left[ - \int \tilde{P}'(y^{z^*}, \mathbf{s}) d\Phi(\mathbf{s}|z^*) \right. \quad (3)$$

$$+ \frac{1}{e} \frac{1 - H(z^*)}{z^* h^*(z^*)} \int_{z^*}^{\infty} \int (1 - \tilde{P}'(y^{z^*}, \mathbf{s})) (1 - g(z, \mathbf{s})) d\Phi(\mathbf{s}|z) \frac{h(z)}{1 - H(z^*)} dz$$

$$+ \left. \frac{1}{e} \frac{1}{z^* h^*(z^*)} \int \frac{1}{e} \frac{1}{z^* h^*(z^*)} \int (1 - g(z, \mathbf{s})) \frac{d\tilde{P}_t(y^z, \mathbf{s})}{\delta \tau dz^*} d\Phi(z, \mathbf{s}) \right],$$

$$\text{where } d\tilde{P}_t(y^z, \mathbf{s}) = \underbrace{-r_t^o(z^*, \mathbf{s})(1 - \tilde{P}'_t(y^{z^*}, \mathbf{s}))\delta \tau dz^*}_{=d^o \tilde{P}_t(y^z, \mathbf{s})} + \underbrace{d^c M_t(z, \mathbf{s})}_{=d^c \tilde{P}_t(y^z, \mathbf{s})} \quad (4)$$

The first term in the bracket on the right hand side,  $-\int \tilde{P}'_t(z^* - T(z^*), \mathbf{s}) d\Phi(\mathbf{s}|z^*)$ , reflects the fact that two (public and private) insurances are substitutes from the household's perspective (which we refer as “substitution” effect). The optimal marginal tax rate is decreasing in marginal private savings  $\tilde{P}'_t(\cdot)$ .

The second term is identical to the original formula in Saez (2001)—Equation (2) above—except for two aspects: (i) the integration is now over the cross-sectional distribution of other state variables as well as that of incomes  $\Phi(z, \mathbf{s})$ , and (ii) it is now multiplied by  $(1 - \tilde{P}'_t(y^{z^*}, \mathbf{s}))$ . The original Saez' effects can be either amplified or mitigated depending on (i) the shape of  $\Phi(z, \mathbf{s})$  and (ii) the sign of the marginal private savings  $\tilde{P}'_t$ . The welfares are measured in terms of additional consumption units. For households who would like to borrow at the margin, one unit of additional disposable income is highly valuable—as the marginal propensity to consume  $(1 - \tilde{P}'_t)$  is greater than 1. Thus, when the marginal private saving is negative, the Saez' effects are amplified. On the other hand, when the marginal private saving  $\tilde{P}'_t$  is positive, one unit increase in disposable income increases the consumption by less than one  $((1 - \tilde{P}'_t) < 1)$ , and thus Saez' effects are mitigated.

The third term in the bracket reflects whether the tax reform is aligned with the change in the pattern of private savings. More precisely, this term captures the interaction of “progressivity”

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<sup>8</sup>We combine the terms associated with crowding out of private insurance. From  $B_t(z^*)$ ,  $(1 - r_t^o)(1 - \tilde{P}')$  measures how much total intermediation increases when tax rate increases by 1. The change in total intermediation can be decomposed into the mechanical increase of tax rate, 1, substitution of tax for private intermediation  $(-\tilde{P}')$ . The substitution effects only captures the decrease in private intermediation due to change in after-tax income without changing private intermediation schedule  $P_t$ .

between public and private insurances. To see this, note that the integral in the third term is:

$$\int \left\{ (1 - g(z, \mathbf{s})) \cdot d\tilde{P}_t(y^z, \mathbf{s}) \right\} d\Phi(z, \mathbf{s}) = Cov(1 - g(z, \mathbf{s}), d\tilde{P}_t(y^z, \mathbf{s}))$$

where  $g(z, \mathbf{s}) = \frac{G'(u(z, \mathbf{s}))u'(c(z, \mathbf{s}))}{A}$  is the marginal social welfare weights. In general (e.g., utilitarian welfare function), the marginal social welfare weights decreases with income.<sup>9</sup> Then, in general,  $1 - g(z, \mathbf{s})$  increases (from a negative to positive value) with income. The term  $d\tilde{P}_t(y^z, \mathbf{s})$  reflects the response of private savings with respect to a tax reform (crowding in or out). The optimal tax rate increases with the cross-sectional covariance,  $Cov(1 - g(z, \mathbf{s}), d\tilde{P}_t(y^z, \mathbf{s}))$ , and the covariance is positive when the tax reform makes the private insurance more progressive.

Consider a marginal tax increase—i.e., tax becomes more progressive. If  $d\tilde{P}_t(y^z, \mathbf{s})$  increases with income levels, the private intermediation becomes more “progressive” (the rich saves more in response to a tax reform). This will generate a positive  $Cov(1 - g(z, \mathbf{s}), d\tilde{P}_t(y^z, \mathbf{s}))$  which in turn results in a high optimal marginal tax. In other words, such tax reform is effective because private intermediation is aligned with the direction of tax reform. On the other hand, if private savings become more “regressive” in response to a tax increase (the poor saves more), the tax reform is not effective because progressive tax (for insurance) is partially undone by the private insurance market.

### 3.4 Rewriting optimal formula w.r.t. productivity distribution

The optimal tax formula (3) involves an endogenous income distribution. For quantitative analysis below, we would like to express the formula with respect to exogenous productivity productivity.

We know that income density and the skill density are related through the equation  $h^*(z)\dot{z}_x^* = f(x)$  and using lemma 1, we get

$$f(x) = (1 + e) \frac{z_x}{x} h^*(z_x). \quad (5)$$

With a slight abuse of notation we denote the joint distribution of skill and other state variables by  $\Phi(x, \mathbf{s})$  and its density by  $\phi(x, \mathbf{s})$ , where  $\phi(x, \mathbf{s}) = \phi(\mathbf{s}|x)f(x)$ . By combining (5) with (3), we obtain the following proposition.

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<sup>9</sup>These welfare weights decrease rapidly with income and assets under a social welfare function that has a strong preferences for redistribution  $G'' < 0$ .

**Proposition 3.** *The optimal marginal tax rate of the government should satisfy the following:*

$$\begin{aligned} \frac{T'(xl_x)}{1 - T'(xl_x)} &= (1 - \beta) \sum_{t=0}^{\infty} \beta^t \left[ - \int \tilde{P}'_t(xl_x - T(xl_x), \mathbf{s}) d\Phi(\mathbf{s}|x) \right. \\ &\quad + \frac{1 - F(x)}{xf(x)} \left( 1 + \frac{1}{e} \right) \int_x^{\infty} \int (1 - g(m, \mathbf{s})) (1 - \tilde{P}'_t(y_x), \mathbf{s}) d\Phi(\mathbf{s}|m) \frac{f(m)}{1 - F(x)} dm \\ &\quad \left. + \frac{1}{xf(x)} \left( 1 + \frac{1}{e} \right) \int (1 - g(m, \mathbf{s})) \frac{d\tilde{P}_t(y^m, \mathbf{s})}{\delta \tau dz^*} d\Phi(m, \mathbf{s}) \right], \end{aligned} \quad (6)$$

$$\text{where } g(m, \mathbf{s}) = \frac{G'(u(m, \mathbf{s}))u'(c(m, \mathbf{s}))}{A}$$

As we discussed above, the optimal formula has time-varying components because when there can be transition periods to a new steady state when the private market slowly adjustments (e.g., accumulation of assets). If we focus on the tax rate that maximizes the steady state social welfare only, the optimal formula is simplified to

$$\begin{aligned} \frac{T'(xl_x)}{1 - T'(xl_x)} &= - \int \tilde{P}'(xl_x - T(xl_x), \mathbf{s}) d\Phi(\mathbf{s}|x) \\ &\quad + \frac{1 - F(x)}{xf(x)} \left( 1 + \frac{1}{e} \right) \int_x^{\infty} \int (1 - g(m, \mathbf{s})) (1 - \tilde{P}(y^x, \mathbf{s})) d\Phi(\mathbf{s}|m) \frac{f(m)}{1 - F(x)} dm \\ &\quad + \frac{1}{xf(x)} \left( 1 + \frac{1}{e} \right) \int (1 - g(m, \mathbf{s})) \frac{d\tilde{P}(y^m, \mathbf{s})}{\delta \tau dz^*} d\Phi(m, \mathbf{s}), \end{aligned} \quad (7)$$

## 4 A Quantitative Analysis

According to our formula (3), the optimal tax schedule can be completely characterized by the standard sufficient statistics (Frisch elasticity of labor supply, marginal social welfare weights, and productivity distribution) and households' savings behavior such as marginal private intermediation  $P'(\cdot)$  and crowding in/out effects,  $r_t^o(\cdot)$  and  $r_t^c(\cdot)$ . The private intermediation (savings)  $P$  could be obtained from a cross-sectional data. However, the crowding in/out effects ( $r_t^o(\cdot), r_t^c(\cdot)$ ) are very difficult to measure empirically because it would require natural experiments of households at various income levels. In this section, we obtain these statistics from an incomplete-markets general-equilibrium model that is calibrated to reproduce the cross-sectional productivity distribution of U.S economy.

## 4.1 Model Economy: Huggett (1993)

We consider a variant of Huggett (1993) for two reasons. First, it is widely used in various macroeconomic analysis. Second, the proof of our formula requires aggregate private savings sum to zero ( $\int P = 0$ ): a pure insurance market. In Huggett (1993) economy, the private savings market is incomplete in two senses: (i) the only asset available for private insurance is a state-non-contingent bond  $a_t$ , and (ii) there is an exogenous borrowing limit:  $a_{t+1} \geq \underline{a} (< 0)$ .

In this economy, the individual state variables are asset holdings  $a$  and productivity  $x$ . The consumption of a worker with asset  $a$  and productivity  $x$  is:

$$c(a, x) = xl - T(xl) + (1 + r)a - a'(a, x).$$

where  $a'$  is the asset holdings in the next period. Private savings of a worker with asset  $a$  and productivity  $x$  is  $P(a, xl) = xl - T(xl) - c(a, x) = a'(a, x) - (1 + r)a$ . The aggregate private savings sum to zero in equilibrium:  $\int a' = 0$ .

In our model, the government spends its tax revenue on purchasing goods  $E$  (which does not enter into the households' utility) as well as lump-sum transfer. When we consider a revenue-neutral tax reforms, the government purchase  $E$  remains unchanged. The government budget is:

$$\int T(z)h(z)dz = \bar{E}.$$

We assume that productivity  $x$  can take values from a finite set of  $N$  grid points  $\{x_1, x_2, \dots, x_N\}$  and follows a Markov process which has an invariant distribution. We approximate an optimal nonlinear tax and private intermediation with a piecewise-linear so that:

$$\begin{aligned} T(z) &= T(0) + \sum_{k=1}^{i-1} T'_k(z_{x_k} - z_{x_{k-1}}) + T'_i(z - z_{x_{i-1}}), \quad z_{x_{i-1}} < z \leq z_{x_i} \\ \tilde{P}(y) &= \tilde{P}(0) + \sum_{k=1}^{i-1} \tilde{P}'_k(y_{x_k} - y_{x_{k-1}}) + \tilde{P}'_i(y - y_{x_{i-1}}), \quad y_{x_{i-1}} < y \leq y_{x_i}. \end{aligned}$$

Consider a tax reform of increasing the marginal tax rate on each grid point  $T'_i$ ,  $i = 1, \dots, N$ . For each tax reform, we compute a new equilibrium based on which an alternative tax rate is proposed according to our optimal tax formula. If the tax reform for every grid point no longer improves the social welfare, the optimal tax schedule is found. We calibrate the parameter values for our quantitative model as follows.

## 4.2 Calibration

*Preferences,  $\bar{E}$ , and  $\underline{a}$*

For our benchmark economy, we use the relative risk aversion ( $\sigma$ ) of 1.5 and the Frisch elasticity of labor supply ( $e$ ) of 0.5. We choose the discount factor ( $\beta$ ) so that the rate of return from asset holdings is 4% in the steady state. The government purchase  $\bar{E}$  is chosen so that the government expenditure-GDP ratio is 0.188 under the current U.S. income tax schedule (approximated by a log-linear functional form:  $T(z) = z - \lambda z^{1-\tau}$ ) as in Heathcote, Storesletten, and Violante (2014)).<sup>10</sup> The exogenous borrowing constraint ( $\underline{a} = -86.87$ ) is set to the average earnings of our model economy under the current U.S. tax schedule.<sup>11</sup> Under this borrowing limit, 9.7% of households are credit-constrained in the steady state. Finally, we assume that the social welfare function is utilitarian:  $G(\cdot)$  is linear. Table 1 summarizes the parameter values in our benchmark case. In Section 5, we consider various specifications of the model economy that differ with respect to the relative risk aversion, Frisch elasticity, borrowing constraints, and the shape of income distribution.

Table 1: Benchmark Parameter Values

| Parameter                   | Description                                           |
|-----------------------------|-------------------------------------------------------|
| $\sigma = 1.5$              | Relative Risk Aversion                                |
| $\beta = 0.9032$            | Discount factor                                       |
| $\gamma = 0.5$              | Frisch Elasticity of Labor Supply                     |
| $\underline{a} = -86.87$    | Borrowing Constraint                                  |
| $\frac{\bar{E}}{Y} = 0.181$ | Government Purchase to GDP Ratio under US tax         |
| $G''(\cdot) = 0$            | Utilitarian Social Welfare Function                   |
| $\rho_x = 0.92$             | Persistence of log productivity (before modification) |
| $\sigma_x = 0.561$          | S.D. of log productivity                              |
| $\frac{xf(x)}{1-F(x)} = 2$  | Hazard Rate at Top 5% of Wage (Income) Distribution   |

<sup>10</sup>The estimated value for the progressivity  $\tau^{US} = 0.161$  in Heathcote, Storesletten, and Violante (2014). We set  $\lambda$  to match the government expenditure-GDP ratio ( $\frac{\bar{E}}{Y}$ )

<sup>11</sup>This is largely in line with the consumer credit cards' limits (which is around 50% - 100% of average earnings) in the data. For example, according to Narajabad (2012), based on the 2004 Survey of Consumer Finance data, the mean credit limit of U.S. households is \$15,223 measured in 1989 dollars.

### Productivity Process

As shown in formula (3), the shape of income distribution (which is dictated by the stochastic process of productivity shock under our preferences with no wealth effect in labor supply) is crucial for the optimal marginal tax schedule. We generate an empirically plausible distribution of productivity as follows. Consider an AR(1) process for log productivity  $x$ :

$$\ln x' = (1 - \rho)\mu + \rho \cdot \ln x + \sigma_\epsilon \epsilon', \quad (8)$$

where  $\epsilon$  is distributed normally with mean zero and variance one. The cross-sectional standard deviation of  $\ln x$  is  $\sigma_x = \frac{\sigma_\epsilon}{\sqrt{1-\rho^2}}$ . While this process leads to a stationary log-normal distribution of productivity and earnings, it is well known that the actual distributions of wages and earnings have much fatter tails than a log-normal.<sup>12</sup>

We modify the Markov transition probability matrix to generate a fatter tail as follows. First, we set the persistence of productivity shock to be  $\rho = 0.92$  following Floden and Linde (2001) which is based on the PSID wages and largely consistent with other estimates in the literature. We obtain a transition matrix of  $x$  in a discrete space using the Tauchen (1986) with  $(\mu, \sigma_x) = (2.757, 0.5611)$ , which is Mankiw, Weinzierl, and Yagan (2009)'s estimates from the US wage distribution in 2007. In doing so, we set the end points of grid at  $(x_1, x_N) = (\exp(\mu - 3.4\sigma_x), \exp(\mu + 3.4\sigma_x))$ .<sup>13</sup> Second, in order to generate a fat right tail, we increase the transition probability  $\pi(x'|x)$  of the highest 3 grids so that the hazard rate of stationary distribution  $\frac{xf(x)}{1-F(x)} = 2$  for the top 5% of productivities. This hazard rate of 2 at the top 5% is based on the empirical wage distribution in Mankiw, Weinzierl, and Yagan (2009). Third, we also increase the transition probability of the lowest grid,  $\pi(x_1|x)$ , so that the stationary distribution has a little bit fatter left tail as in Mankiw, Weinzierl, and Yagan (2009).<sup>14</sup> As Figure 1 shows, the hazard rates of productivity distribution from our model almost exactly matches those in the wage distribution in the data from Mankiw, Weinzierl, and Yagan (2009). In Section 5, we also study the model economy under a simple log normal distribution of productivity to examine the impact of fat tails.

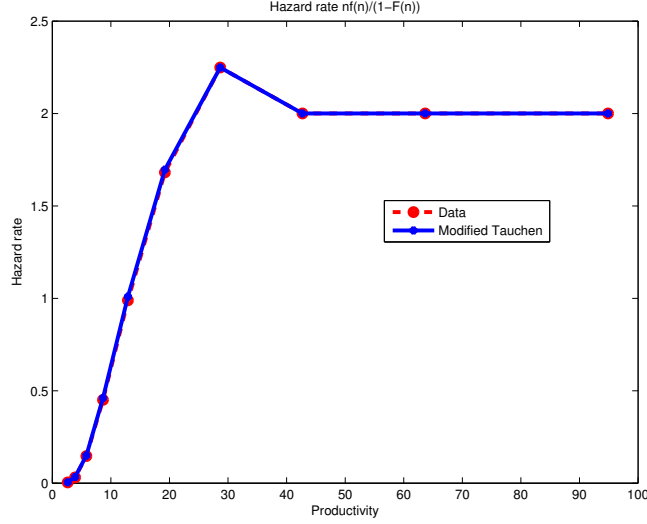
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<sup>12</sup>Saez (2001) and Heathcote, Storesletten, and Violante (2014) estimated the earning distribution and used tax data to obtain the skill distribution, while Mankiw, Weinzierl, and Yagan (2009) use the wage distribution as a proxy for productivity distribution.

<sup>13</sup>We set the highest grid point to 3.4 standard deviation of log normal so that the highest productivity is top 1% productivity of distribution in Mankiw, Weinzierl, and Yagan (2009)

<sup>14</sup>The bottom tail of the productivity distribution should take into account disabled workers or not-employed.

Figure 1: Hazard Rates of Wage (Productivity)



*Note:* The hazard rates for the are from Mankiw, Weinzierl, and Yagan (2009).

#### *Productivity, Income, and Asset Distributions*

Figures 2 and 3 show the stationary distributions of productivity and (labor) income, respectively, from our stochastic process of  $x$ . The productivity distribution exhibits fatter tails (at both ends) than a log-normal distribution, as we modified the transition probability matrix described above. As we use the preferences with no income effects in labor supply, the shape of income distribution is essentially identical to that of productivity. Figure 4 shows the asset distribution of the model under the current US tax system (approximated by the HSV functional form). As we discussed above, we set the borrowing constraint so that the fraction of credit constraint households are about 10%. It is well known that the asset distribution generated by Huggett (1993)-style model economy cannot successfully match a large wealth inequality in the data. Moreover, in our model with a pure insurance market, the aggregate asset holdings always sum to zero in equilibrium, which requires a large fraction of households hold negative asset holdings.

Figure 2: Productivity (Wage) Distribution

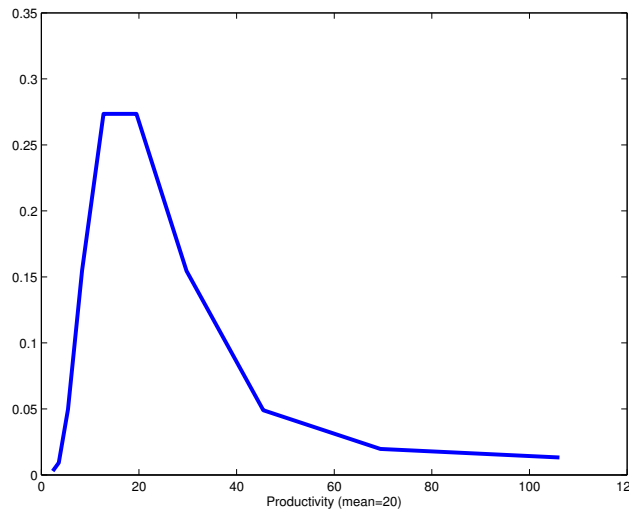


Figure 3: Income Distribution

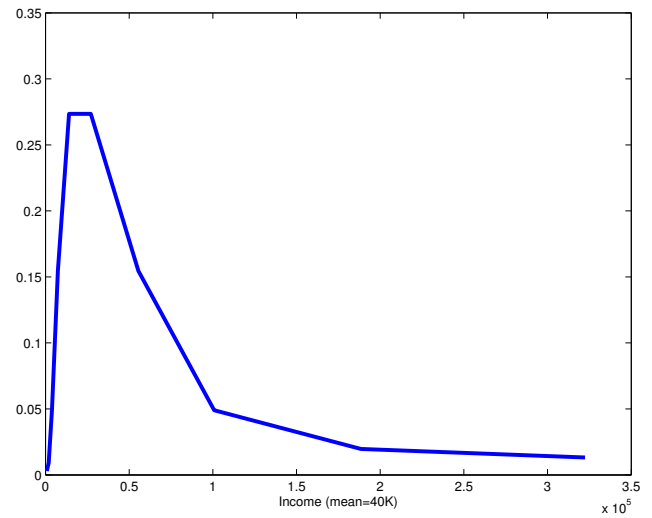
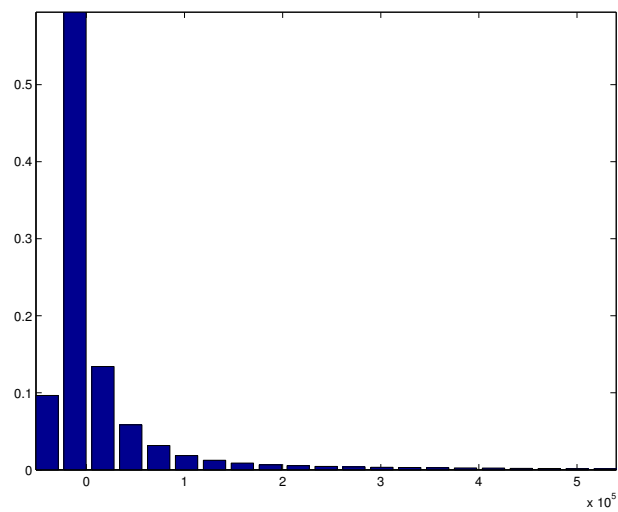


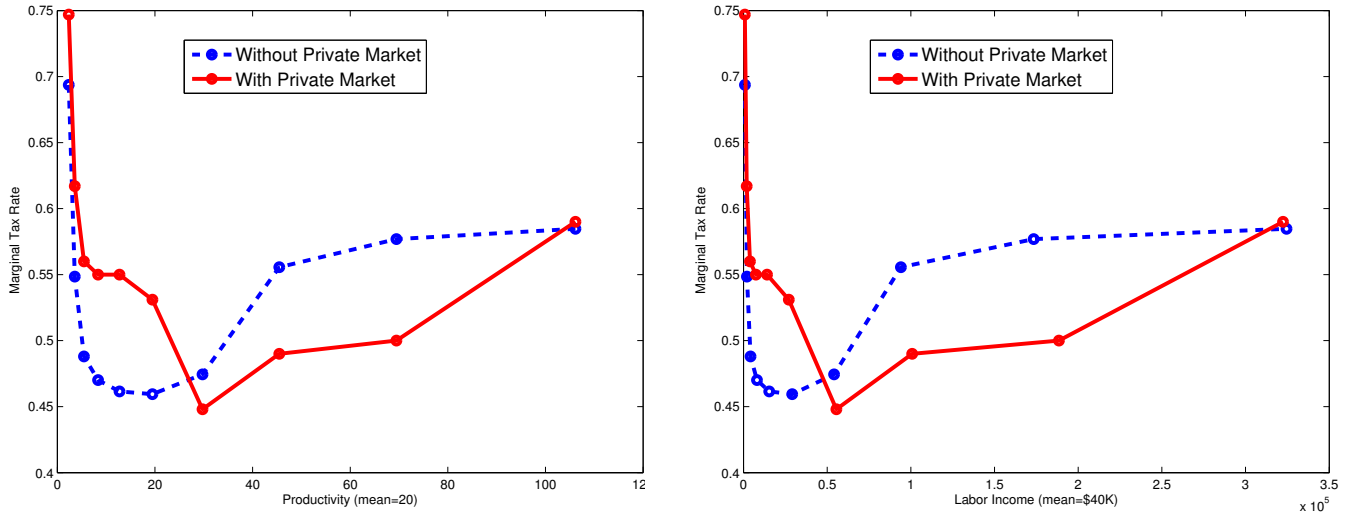
Figure 4: Asset Distribution under Current U.S. Tax Schedule



### 4.3 Quantitative Results

Figures 5 and 6 show the optimal marginal tax schedule across productivity and income, respectively, with and without private insurance market. We normalize the units of quantities in our model so that the average productivity (wage) is \$20 and the average labor income is \$40,000 (comparable to those in 2015 in U.S.). Without private insurance market (dotted line), the optimal marginal tax schedule exhibits a well-known U-shape as in the standard Mirrleesian taxation literature. High marginal tax rates at the very low income (or productivity) levels indicate that the net transfers to the low income households should quickly phase out. As seen in Figure 1, the hazard rate of productivity sharply increases, implying that the cost of distorting labor supply quickly increases (relative to the benefit): the optimal marginal tax rate should start decreasing with income. As the income increases, the marginal social welfare weight gradually diminishes—which eventually becomes a dominant factor and results in a higher marginal tax at the high income group.

Figure 5: Optimal Marginal Tax Rate by Productivity      Figure 6: Optimal Marginal Tax Rate by Income



While the same driving forces are operative in an economy with private insurance market, there are additional factors that make the optimal tax schedule different from that without private insurance market. Looking at Figures 5 and 6 again, the optimal tax rates in the presence of private insurance (solid line) are higher than those without private market (dotted line) at the low and middle income group (wage rates less than \$25). For the upper-middle and high income group (wage rates between \$25 and \$100), the optimal tax rates are higher than those without private insurance. For the income group at the top (wage rates above \$100 which is approximately 95th

percentile of the distribution), the tax rates with and without private insurance are similar.

We examine the factors that account for the difference in optimal tax rates with and without private insurance in details. Recall that the optimal tax formula in the presence of private insurance consists of three terms:

$$\begin{aligned} \frac{T'(z^*)}{1 - T'(z^*)} = & -E \left[ \tilde{P}'(z^* - T(z^*), a) | z^* \right] \\ & + \frac{1 - H(z^*)}{z^* h(z^*)} \frac{1}{e} E \left[ (1 - g(z, a)) (1 - \tilde{P}'(y^z, a)) | z \geq z^* \right] \\ & + \frac{1}{z^* h(z^*)} \frac{1}{e} E \left[ (1 - g(z, a)) \frac{d\tilde{P}(y^z, a)}{\delta \tau dz^*} \right]. \end{aligned}$$

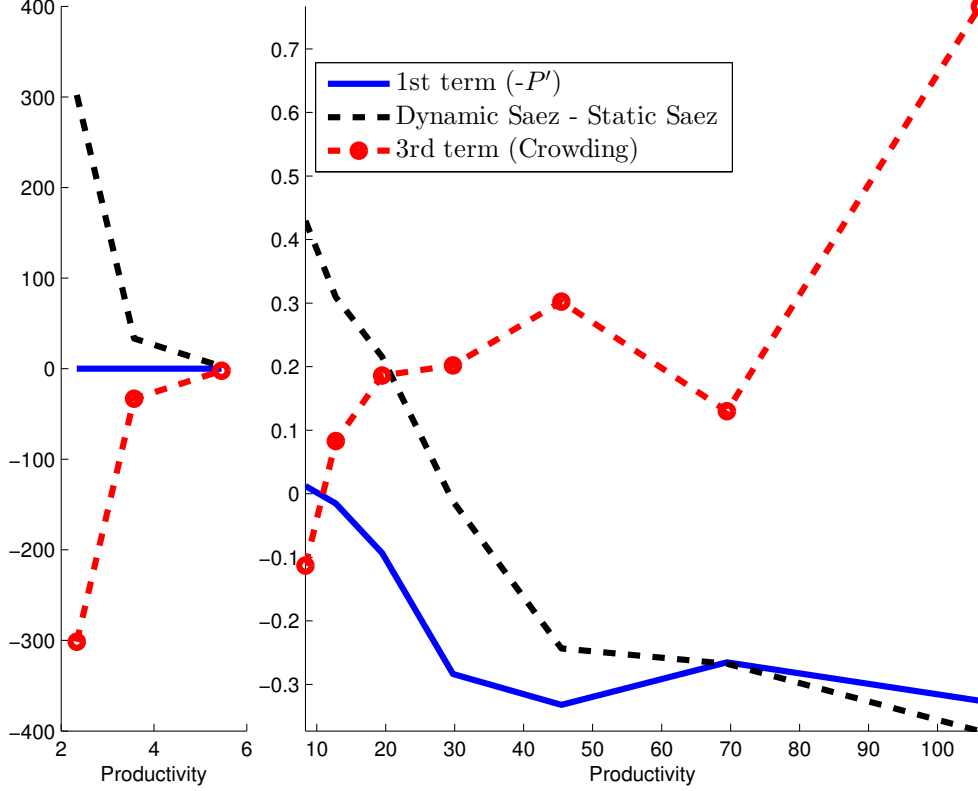
Comparing our optimal tax formula to that of Saez (2001), the difference consists of three components. The first component simply reflects the fact that private savings and tax are substitutes in insuring against future income uncertainty—the first term in our formula. The second component is the difference between the second term in our formula and the original Saez formula – i.e., “dynamic Saez vs. static Saez”. The third component is reflecting that a tax reform is more effective when the response of the private savings is aligned with such reform—the third term in our formula.

The sign and magnitude of these three components are determined by the marginal propensity to save (MPS)– $P'(\cdot)$ –and the crowding in/out of the private savings schedule through the following mechanisms. The MPS at its own income level enters negatively into the first and the second components—higher MPS implies that the tax is a substitute for private insurance (first term) and the Saez effects are amplified or mitigated (second term). The shape of the cross-sectional distribution of also MPS matters for the second and third components in the formula. In the incomplete-markets model, the asset distribution is dispersed (compared to the income distribution) which leads to a larger dispersion in consumption and  $1 - g$  (than the economy without private savings). Thus, the integrals in the second and third components become larger than the economy without private savings. The cross-sectional response of crowding in/out of private savings determines whether the response of the private savings is aligned with a tax reform. If an increase in marginal tax makes the private savings more progressive—i.e., crowds out the savings for the poor and crowds in the savings for the rich, the third component is positive and large. Such a tax reform (progressive tax) is more effective.

We now study these channels in more details. Figure 7 plots these three components. Since the

magnitude of the terms are huge at the very low productivity level, we plot these terms separately in two productivity groups: productivity below \$8 and above.

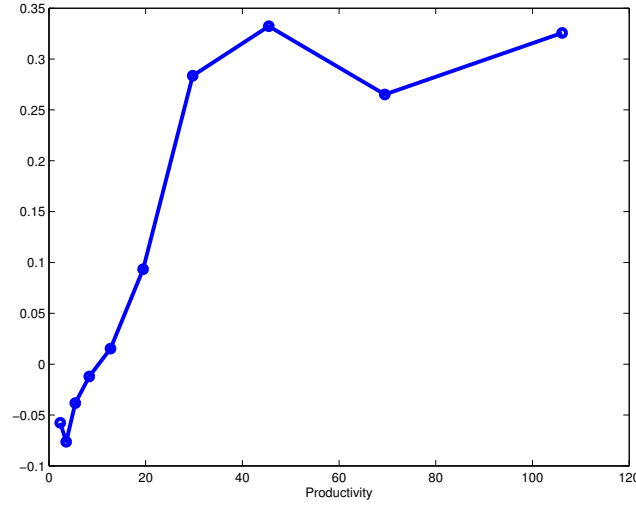
Figure 7: Decomposition of the Difference in  $\frac{T'}{1-T'}$  with and without private insurance



*Note:* The “1st term ( $-P'$ )” in solid line represents the first term in our optimal tax formula (3). The “Dynamic Saez - Static Saez” is the second term in our formula (3) minus the original Saez. The “3rd term (crowding)” represents the third term in our formula (3).

At the low income group (whose wages are less than \$8), the marginal private savings are negative ( $\tilde{P}' < 0$ ), as shown in Figure 8. The low-income households would like to borrow more at the margin. This has two impacts. First, the government increases the marginal tax rate to achieve the optimal total savings. The solid line (denoted by  $-P'$ ) in Figure 7 represents this substitution effect. Second, the negative marginal private savings rate amplifies the Saez effects—the dotted line denoted by ‘Dynamic Saez - Static Saez’, which also pushes the optimal tax rate up. While the third component (dotted line with circles) shows negative values—the cross-sectional private intermediation schedule becomes less progressive, the first and second components dominate this

Figure 8: Marginal Private Savings  $P'$



third component for this income group.

At the low-middle income group (whose wages are between \$8 and \$25) the marginal savings now becomes positive: workers now would like to save with an additional income. This would lower the optimal marginal tax. However, the second component (dynamic Saez - static Saez) is still positive and big due to higher consumption inequality generated by unequal asset distribution. The third component now turns from a negative to positive value —private insurance schedule becomes more “progressive”—which calls for a higher tax rate. Overall, the second and third components dominate the first component and thus the optimal tax rate is higher in the presence of private insurance.

At the upper-middle and high income groups (whose wages higher than \$25), the marginal private savings are always positive, which calls for a lower optimal tax rate as the private insurance is a good substitute for public insurance. Moreover, the difference between the dynamic and static Saez terms is now negative — the Saez effects are mitigated:  $(1 - P') < 1$ . While the third term (aligned private progressivity) is positive — the private insurance become more progressive, the first and second components dominate the third component except for the very top income group (wage rates higher than \$100) where two effects offset each other leaving the optimal tax rates similar.<sup>15</sup>

<sup>15</sup>At the productivity level (between \$50 and \$70), the marginal savings rate slightly falls. This pattern is due to our modification of the transition probability to match the fat tail at the top. Recall that we generated a fatter tail by increasing the probability of drawing higher productivity at the highest two productivity grids. This leads to a decrease in marginal private savings because once you reach this level of productivity, it is more likely to stay at a

In sum, the difference in tax rates with and without private insurance market are quantitatively important as the difference can be as large as 10 percentage points. According to our benchmark model, for the low and lower-middle income group, the optimal tax rate is higher in the presence of private market, mainly because of amplified Saez effects. Higher consumption inequality driven by asset inequality makes Saez effects stronger. In particular, for the very low income, households try to borrow at the margin ( $1 - P' > 1$ ), which amplifies Saez effects even more. For the middle to high income groups where marginal savings is positive, the substitution effect (the 1st term) and mitigating effect in Saez (the 2nd term) dominate the crowding in/out effects (the 3rd term), resulting in lower tax rates in the presence of private market. At the very top income distribution, these forces almost cancel out each other, leaving the tax rate with private savings close to that without.

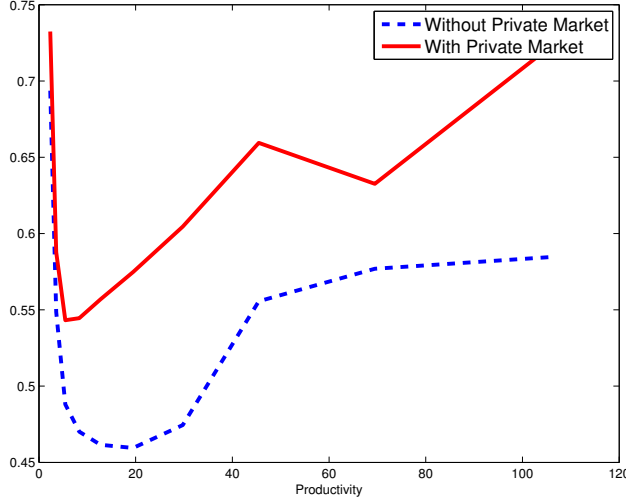
Figure 9 presents the marginal total intermediation across productivity levels: the average of marginal total intermediation conditional on income:  $E[M'(z, a)|z] = T'(z) + E[P'(z, a)|z]$ . This represents the overall insurance payment at the margin (tax rate plus marginal private savings). It clearly shows that the total marginal intermediation is higher in the presence of private market, which is mainly driven by the high marginal propensity to save from the precautionary savings motive in our model economy. This also implies that in the presence of private market the government cannot perfectly reproduce the optimal insurance in an economy without private market, because each individual with the history of productivity shocks have different precautionary motive of saving. Since we do not allow a history-dependent tax, the government cannot design a tax schedule that is perfectly in line with private savings incentive. This implies that the social welfare of the economy with private market can be lower than that of the economy without private market, which we discuss below.

**[Need formula for Welfare Comparison.]** We compare the social welfare under the US tax schedule, under the optimal tax with private market, and under the optimal tax without private market. We use the consumption-equivalent welfare loss associated with introducing private market. That is, if the steady state welfare under the optimal tax schedule without private market is  $U$ , we find a number  $\tau$ , that increasing consumption of each agent by  $\tau$  would make steady state welfare under optimal tax schedule with private market equal to  $U$ . We find that the welfare losses from losing control in private trading (welfare losses of introducing private insurance) is 9.2 percent. On the

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high productivity (less needs for savings). In Section 5 below, this pattern disappears when we use a pure log-normal productivity process.

Figure 9: Marginal Total Intermediation: (Tax + Savings)



other hand, the welfare loss of US tax system compare to the optimal tax with private market is 8.3 percent.

## 5 Comparative Statistics

Our formula shows that the optimal tax schedule depends on households' savings behavior as well as the standard sufficient statistics. In this section, we investigate how the optimal tax schedule varies with respect to different parameter values of relative risk aversion, Frisch elasticity, borrowing constraint, persistence of productivity shocks, and fat tails of income distribution. For each specification, we find new value of  $\beta$  to clear the private insurance market at the given real interest of  $r = 4\%$  in the steady and under the current U.S. tax schedule (approximately by a log-linear form as in HSV). Simultaneously, we also look for a new value of borrowing limit  $\underline{a}$  so that about 10% of households are credit-constrained in the steady (except for our comparative statics with respect to the borrowing limit itself).

### 5.1 Risk Aversion and Labor Supply Elasticity

We consider the relative risk aversion  $\sigma = 1$  (lower than the benchmark risk aversion:  $\sigma = 1.5$ ). Figure 10 shows the optimal tax rates without private insurance market. With a smaller risk aversion, the optimal tax rates are lower than those in our benchmark ( $\sigma = 1.5$ ) in all productivity

Figure 10: Optimal Tax without Private Insurance

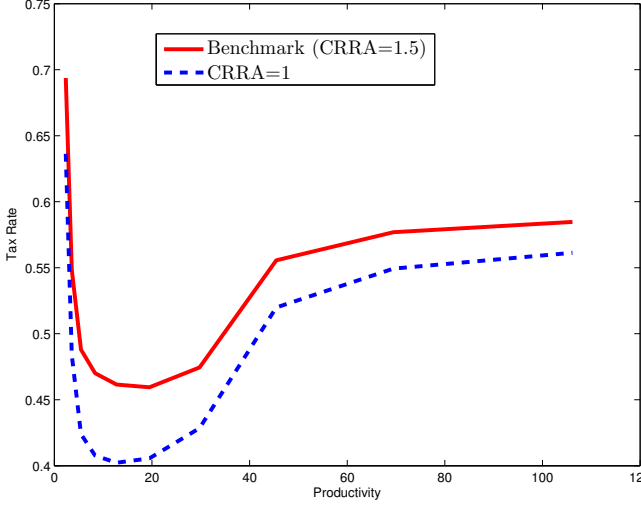
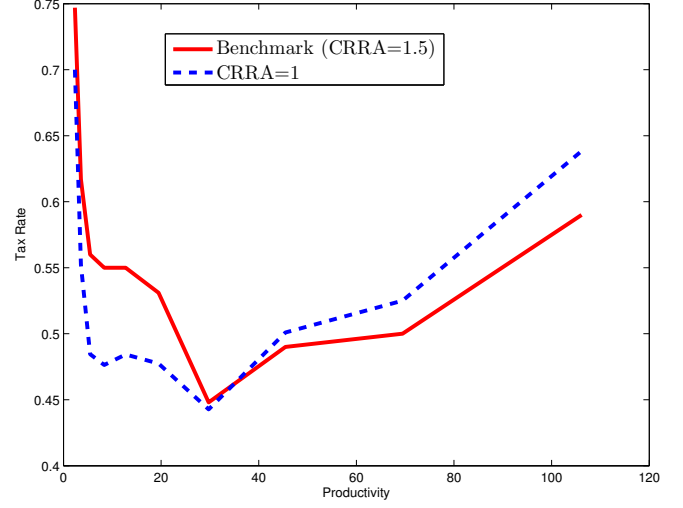


Figure 11: Optimal Tax with Private Insurance



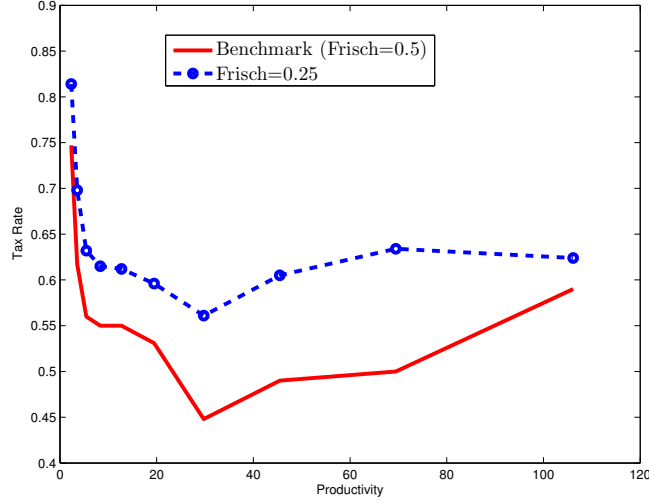
levels because there are less needs for insurance.<sup>16</sup> This driving force is still present in an economy with private market.

In the presence of private insurance market, however, the optimal tax rates at the upper-middle and high income groups are in fact higher when  $\sigma = 1$  (lower risk aversion) in Figure 11. This is because of the general equilibrium effect. With a smaller risk aversion, households save less (smaller precautionary savings motive). The real interest rate has to increase to clear the private savings market. For example, the real interest rate net of discounting  $\beta(1+r)$  increases from 0.9647 (under  $\sigma = 1.5$ ) to 0.9808 (under  $\sigma = 1$ ). This encourages the high-income households save more and makes the marginal private savings ( $P'$ ) schedule steeper, which leads to larger cross-sectional dispersion in assets and consumption. Thus, the second term and the third term in our formula (amplified Saez effects and crowding in/out effects) become larger. For upper-middle and high income groups (wages above \$30), this general equilibrium effect (larger second and third terms in the formula) dominates a weaker precautionary savings (smaller first term in the formula), resulting in higher marginal tax rates.

Next, we consider a smaller Frisch elasticity of labor supply ( $e = 0.25$ ). Figure 12 shows that for all income levels the optimal tax rate under  $e = 0.25$  are higher than those in our benchmark ( $e = 0.5$ ) because inelastic labor supply is associated with a smaller cost of distortion.

<sup>16</sup>Under the utilitarian social welfare, for example, the social welfare weights at high income levels ( $\frac{u'(c)}{A}$ ) are relatively high when the risk aversion is low.

Figure 12: Optimal Tax with Private Market

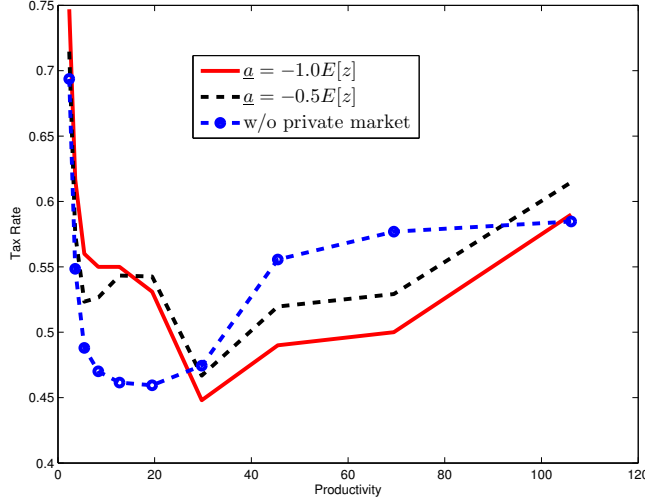


## 5.2 Borrowing Constraints

In the benchmark economy, we set the borrowing limit  $\underline{a} = -86.55$  which is the average annual earnings in the steady state under the current U.S. tax schedule (approximated by a log-linear form as in HSV). Under this borrowing limit, 9.7% of households are credit-constrained under the current U.S. tax schedule. We consider a tighter borrowing limit which is the half our benchmark case ( $\underline{a} = -43.3$ ) so that workers can borrow one-half of average earnings in the economy. With this tighter borrowing limit, about 34% of population are credit-constrained under the current U.S. tax schedule in our model. Figure 13 shows the optimal tax rate schedules under this tighter borrowing constraint. The optimal tax rates are roughly between those in the benchmark and those without private insurance market, except for the highest productivity group.

Under the tighter borrowing constraint, households tend to save more due to a stronger precautionary savings motive. To clear the private savings market, the real interest has to decrease. This increases the MPS of low income group and increases the MPS of high income group. This will lead to a lower marginal tax for low income group and higher marginal tax for high income group. In addition, the crowding in/out effects becomes larger under the tighter borrowing constraint as the tax reform induces a more progressive response of the private savings. At the top income group, this effect is strong enough to generate a pretty high optimal tax rate.

Figure 13: Optimal Tax under Tighter Borrowing Constraint



### 5.3 Persistence of Productivity Shock

Note that the persistence of productivity shock,  $\rho$ , does not appear in the optimal tax formula (7) because we restrict our tax system to be history non-contingent and the government maximizes steady state welfare only in the benchmark analysis. However, the persistence of shocks affects households' savings pattern and, as a result, the optimal tax rate in the presence of private insurance market. We examine the model with  $\rho = 0.8$  (lower persistence). We recalibrate the standard deviation to the innovation  $\sigma_\epsilon$  to obtain the same standard deviation of log productivity,  $\sigma_x = 0.561$ , in the benchmark. We also modify the transition probability matrix at both ends of productivity distribution to match the hazard rates in the data, as we did in our benchmark case.

Figure 14 shows that the optimal tax rates under  $\rho = 0.8$  are smaller than those in the benchmark model except for the very low income (productivity) group. This is mainly driven by the marginal private savings ( $P'$ ) shown in Figure 15. At the low income levels (wage rate below \$5), workers save less (borrow more) at the margin under  $\rho = 0.8$  because they expect that the productivity will increase in near future. This leads to higher tax rates via substitution effects and amplification of Saez effects. For higher income levels (wage rate above \$5), the optimal tax rates under  $\rho = 0.8$  is higher because these workers save more at the margin which leads to lower marginal tax via substitution effects and mitigation of Saez effects.

Figure 14: Optimal Tax with Private Market

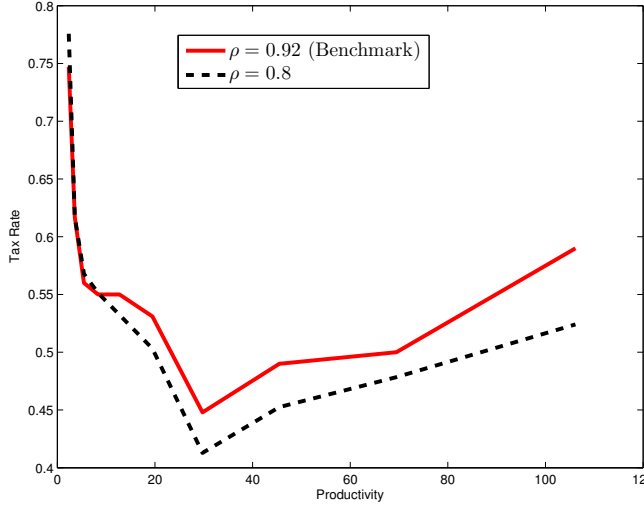
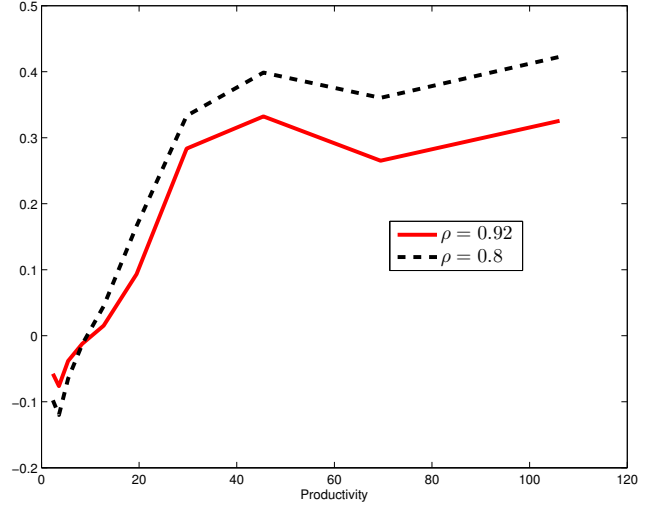


Figure 15: Marginal Private Savings



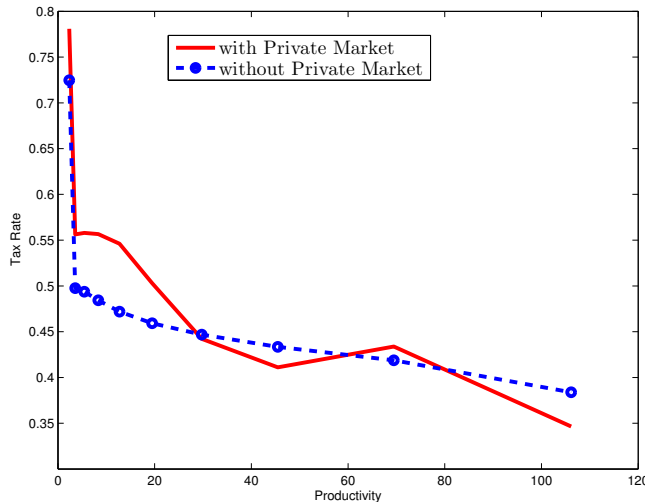
#### 5.4 Log-normal Distribution of Income: Effects of Fat Tails

We have modified the transition probability (from the discretized log normal distribution) to match the fat tail in income (and wage) distribution in the data. To examine the role of the fat tail, we compute the optimal tax under a pure log-normal productivity process without modification. The hazard rate  $\frac{xf(x)}{1-F(x)}$  of log-normal distribution monotonically increases. This results in a monotonically decreasing tax rates without private insurance market in Figure 16. The pattern prevails in the presence of private insurance, suggesting that the fat tail is crucial for the U-shaped optimal marginal tax schedule.

## 6 Conclusion

We study fully-nonlinear optimal income tax schedule in the presence of private market. As in Saez (2001), the optimal tax formula is expressed in terms of standard *sufficient* statistics such as Frisch elasticity of labor supply, hazard rate of income distribution, and marginal social welfare weights. In the presence of private market, however, these sufficient statistics are no longer sufficient. The optimal tax formula also depends on how private savings interact with public insurances. Our formula provides transparent and intuitive insights about the role of private market in the optimal tax schedule. First, the formula captures the fact that public and private insurance are substitutes—a higher marginal private savings leads to a lower marginal tax rate at that income level. Second,

Figure 16: Log-normal Distribution of Productivity



the original formula in Saez (2001) is modified to reflect the amplifying (or mitigating) effect by the private savings. Two components are important for this amplifying (or mitigating) factor: (i) marginal propensity to consume and (ii) consumption inequality which depends on the cross-sectional distribution of wealth. The Saez formula is amplified if the marginal propensity to consume is larger than 1 (i.e., a negative private savings) and/or if the consumption inequality is larger in the presence of private insurance. Third, it depends on the cross-sectional distribution of crowding out effects. A tax reform is more effective when the response of private insurance is aligned with the reform. For example, increasing the marginal tax rate is more desirable when the private savings (insurance) also becomes more progressive.

Using a quantitative general equilibrium model (Huggett (1993)) calibrated to resemble the shape of U.S. income distribution, we compute the optimal tax rates. Our results show that the presence of private market is quantitatively important as the difference in optimal tax rates (with and without private insurance) can be as large as 10 percentage points. For the low and lower-middle income group, the optimal tax rate is higher in the presence of private market, mainly because of amplified Saez effects. For the middle to high income groups where marginal savings is positive, the substitution effect and mitigating effect in Saez formula dominate, resulting in lower tax rates in the presence of private market. At the very top income distribution, these forces almost cancel out each other, leaving the tax rate similar to that without private insurance.

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Figure 17: Marginal Tax Rate: Current US vs. Optimal

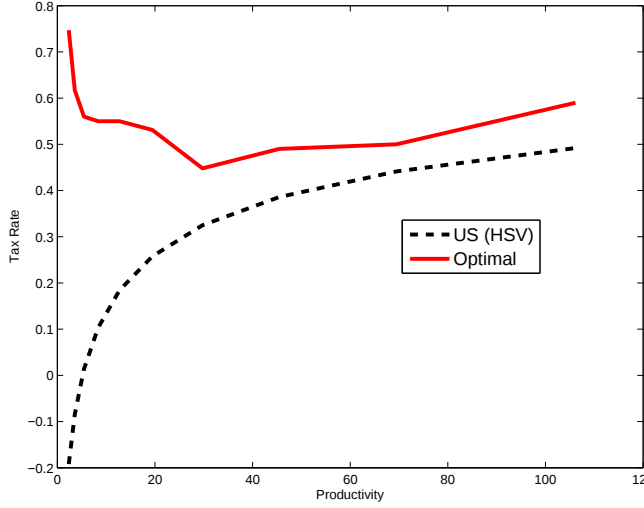
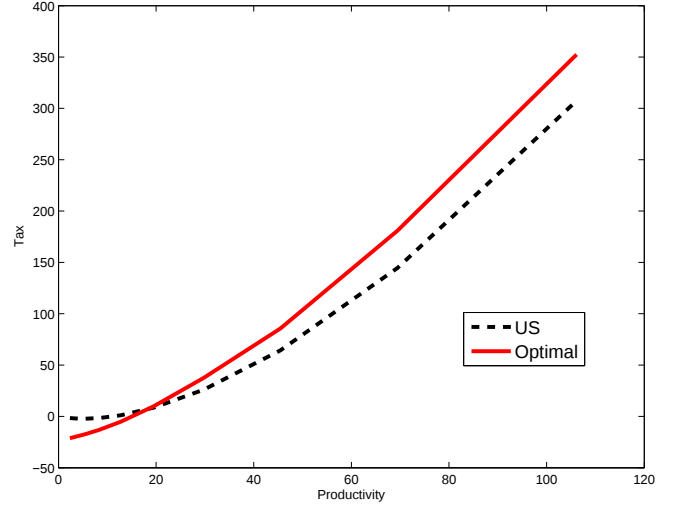


Figure 18: Tax Amount: US vs. Optimal



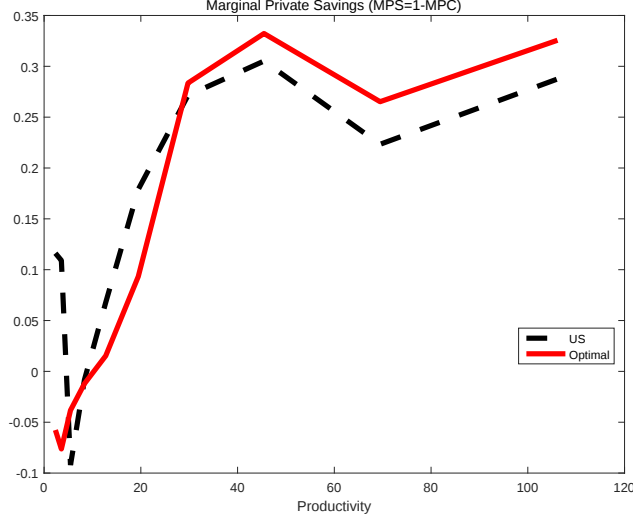
## Appendix A: Optimal tax schedule Vs US tax schedule

We now briefly compare our optimal tax schedule to the U.S. tax schedule. Figure 17 presents our optimal tax rates (solid line) to the current US tax rates approximated by the HSV functional form (dotted line). It clearly shows that the optimal tax rates are much higher than the current rates for all income levels, especially for the low income group. However, the government expenditure, the amount of the tax revenue which is not rebated back to the workers, is constant across two economies by assumption and thus this implies that lump-sum transfer under the optimal schedule is much higher under the optimal tax schedule. However, comparison of  $T(z)$ -schedule (tax payment net of the transfer) in Figure 18 illustrates that the low-income workers under the optimal schedule would receive more transfer under the optimal tax system.

## Appendix B: Computation Algorithm

We simulate the optimal tax schedule that satisfies the optimal formula under the private market structure assumption. Optimal tax formula maximizing social welfare considering the transition generated from the tax reform requires the time varying marginal private market intermediation  $P'_t(\cdot)$  and degree of crowding out  $r_t(\cdot)$  even though we assume that the economy starts from the steady state. As a first step, we focus on optimal tax schedule that only considers steady state social welfare. This analysis will provide a good comparison to the existing quantitative optimal tax studies following Ramsey tradition (such as HSV ..... need to check!!!!).

Figure 19: Marginal Private Savings: US vs. Optimal



We can simulate tax system numerically by the following algorithm.

1. Guess the marginal tax schedule  $\{T'(z)\}$ .
2. Given  $\{T'(z)\}$  Compute the stationary competitive equilibrium with incomplete market (under the assumption of Aiyagari or Kehoe-Levine).
3. Get  $T(0)$  so that  $\int_0^\infty T(z^x)f(x)dx = E$ .
4. Compute the marginal market intermediation  $\{P'(z - T(z), a)\}$  associated with assumed market structure using a competitive equilibrium allocation.
5. For a small change of marginal tax schedule for each productivity grid points, compute the change in private intermediation  $\{dP(z - T(z), a)\}$
6. Evaluate the RHS of optimal tax formula (7) using the elasticity estimate and statistics computed above. Update  $\{T'(z)\}$  and iterate until convergence.

Discuss how to compute a stationary competitive equilibrium under Huggett (1995).