

Of Cities and Slums*

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February, 2017

Abstract

The emergence of slums is a common feature in a country's path towards urbanization, structural transformation and development. Based on salient micro and macro evidence of Brazilian labor, housing and education markets, we construct a simple model to examine the conditions for slums to emerge. We then use the model to examine whether slums are barriers or stepping stones for lower skilled households and for the development of the country as a whole. We calibrate our model to explore the dynamic interaction between skill formation, income inequality and structural transformation with the rise (and potential fall) of slums in Brazil. We then conduct policy counterfactuals. For instance, we find that cracking down on slums could slow down the acquisition of human capital, the growth of cities (outside slums) and non-agricultural employment. The impact of reducing housing barriers to entry into cities and of different forms of school integration between the city and the slums is also explored.

Keywords: *Skill formation; Locations; Occupations; Structural transformation.*

JEL Codes: O15, O18, O64, R23, R31.

* *The views expressed here are those of the authors and do not necessarily reflect the opinion of the Federal Reserve Bank of St. Louis or the Federal Reserve System.*

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“What connexion can there have been between many people in the innumerable histories of this world, who, from opposite sides of great gulfs, have, nevertheless, been very curiously brought together!” Charles Dickens, BLEAK HOUSE

“The new residents brought garbage, bins, mongrel dogs... poverty to desire wealth...legs for waiting for buses, hands for hard work, pencils for state schools, courage to turn the corner and...asses for the police to kick...” Paulo Lins, CITY OF GOD: A NOVEL

1 Introduction

Structural transformation and urbanization are hallmarks in the development of countries.¹ Most developed countries have all but completely displaced their workers from agriculture –and other primary sectors– towards manufacturing and services sectors. Recent work pushes the importance of this reallocation further by highlighting the emergence of high-skill services in later stages of development.² Since agriculture is predominantly a rural sector, and manufacturing and many service sectors are predominantly urban activities, then, barriers to urbanization per se can easily translate into barriers to structural transformation and development, as emphasized by Lewis (1954) long ago. Likewise, forces that lead to structural transformation of countries can shape the urbanization patterns of countries, including the size and fragmentation of cities.

Urbanization, of course, has been rarely a smooth process. The story of any of today’s leading cities in the world –London, Paris, New York, Tokyo, etc– is in large measure the story of their slums. The emergence of these cities cannot be understood without putting considerable attention to slums, their rise and eventual fall, the lives of their dwellers, and the advancement of their descendants. More recently, since World War II, many developing countries have undergone transitions from rural to urban economies at a rapid pace. For instance, as shown in Figure 1, in Brazil, the fraction of the population living in urban areas increased from 36% in 1950 to 85% in 2010; by 1970 the country was already predominantly urban. Notice that the labor share in non-agricultural sectors closely follows the urban population, going from 36% in 1950 to 83% in 2010. Similar findings are observed in Mexico: between 1960 and 2010 the urban population rose from 51% to 88%, and the non-agricultural labor share went from 48% to 86%. South Korea was initially even more rural, but experienced a faster transformation. In the early 1960s, 72% of its population lived in rural areas and 62% of its workers were in agriculture. In 2010, the shares of urban population and non-agriculture labor were 93% and 82%, respectively.

Yet, underneath these common features of structural transformation, countries can exhibit drastic differences, in terms of output growth, forms of urbanization, social mobility and income disparities. For instance, since 1950, South Korea presents a sustained upward trend in its income per capita (and output per worker) relative to the U.S. Korean employment has shifted to high skill services. And in terms of urbanization, the slums in Seoul have all but disappeared. In contrast, Brazil, Mexico and quite a few other Latin American economies have stagnated since the early 1980s, falling further behind the U.S. in relative terms of output per capita. Non-agriculture labor productivity has been falling since 1980, because the employment expansion in those sectors has been mostly in low skill urban jobs, as we document below. Notably, the growth of the main cities in Brazil and Mexico has been driven in large part by the growth in the slum population. In these two countries, as well as in many others, migrants from rural areas have low levels of human capital and migrate to the urban areas to work in low skill jobs. Slums are the mechanism available to

¹See for example the Nobel lecture of Kuznets (1973).

²See Buera and Kaboski (2012) and Buera, Kaboski and Rogerson (2014.)

those migrants to avoid the housing costs of the formal city, and still gain access to the urban labor markets.

In this paper, we propose a dynamic model with endogenous skill formation and heterogeneity to understand the differences in the urbanization and structural transformation patterns of countries. In our model, individuals can choose to live in rural or urban locations. Along the country's equilibrium, urban locations can exhibit slums. The skill distribution in each location arises as the outcome of this endogenous sorting. We assume that the location's human capital distribution directly affects the human capital formation of children. Altruistic parents take into account the human capital formation of their children at the time they choose the location of residence. To live in the city people must buy a house, a form of fixed cost, while to live in slums they face costs and losses that are a function of their earnings. Therefore, the location decision depends on the skill of each adult. Individuals also choose what type of labor they offer, in labor markets that differ across locations. Non-homothetic preferences shape the demand for agricultural and non-agricultural goods, which in turn drive the demand for labor in rural and urban regions. We then analyze the equilibrium allocations in terms of the implied allocation of labor across sectors and occupations, the size of urban areas and their composition in terms of cities and slums. We then calibrate the model to the Brazilian economy, explore its ability to fit the key Brazilian data, and use it to explore the impact of alternative policies.

We guide the construction of the model by our own exploration of the Brazilian experience. We examine the evidence about urbanization, formation of slums, and human capital accumulation across different locations. Using the Brazilian Census we compute the evolution of population across rural and urban areas, and across slums and cities through the years. We also measure the population of migrants and non-migrants in slums and cities. Crucially, we explore what factors drive migration: the incomes, schooling attainment and inter-generational transitions of schooling attainment, conditional on locations.

We consider the simplest model that can be used to analytically examine: (i) structural transformation; (ii) urban development; (iii) income and skills distribution; (iv) social mobility. The key elements in the equilibrium are, first, how to allocate individuals and skills across locations, productive sectors and occupations and, second, the dynamic implications of those decisions for the skill formation of future generations. We consider a discrete-time, infinite-horizon economy populated by dynasties of two-period-lived overlapping generations (OLG) of individuals. In any period the population of the economy is described by a positive measure over all over all positive levels of skills. The population remains constant, but its skill composition evolves over time. The economy has two production sectors, *agriculture* and *non-agriculture* (manufacturing and services), three locations: *rural areas*, *favelas (slums)* and *city centers*. There are three occupations: rural occupations (low skill) and urban occupations, which are qualified or skilled. Qualified occupations required a minimum skill level to be productive, while the urban skill occupations, there can be two groups: low-skilled urban jobs or high skilled urban jobs.

We show that the equilibrium in the economy can have two different configurations: an equilibrium with only high skilled urban jobs and equilibrium with urban low skilled services jobs, and examine the conditions under which these configurations arise. In particular, we highlight the importance of initial skill disparities and preference non-homotheticities to generate low skill urban jobs equilibria. We also highlight the role of housing costs and education concerns in generating slums equilibria. Finally, we highlight the importance of segmentation in the formation of education and potential rising costs for the persistence of low skill urban jobs and slums.

A calibration of our simple model can reproduce the evolution in the distribution of the Brazilian population across occupations and locations from 1960 to 2010. Using the model as a basis for examining counterfactual policies, we explore the impact of rising housing costs and of cracking

down slums. We find that higher housing costs would increase the population in slums, but would not affect much the structural transformation of the country. In short, for low skill workers slums appear to be good substitutes to formal city dwelling. However, the complete prohibition of slums would have slowed down substantially the structural change and the urbanization of the country, as only the most skilled individuals would afford to live in the cities.

The reminder of the paper is organized as follows. In the next section, we briefly discuss existent related literature. In Section 3, we examine the case of Brazil and highlight a number of salient facts about its structural transformation, urbanization and the emergence of slums. Section 4 sets out our model and defines equilibrium. Section 5 characterizes the equilibrium allocations in the model, and explore the conditions for the emergence of slums, and the impact of slums on the overall urbanization, structural transformation and growth of a country. Section 6 calibrates our model to the Brazilian experience from 1960 to 2010. Section 7 uses the model to explore the implications of different policies: changes in the housing costs of cities, cracking down on slums, and school integration policies. Section 8 concludes. At the end of the paper, we include an appendix with additional data and analytical details.

2 Related Literature

This paper is connected to two broad and related areas in the development literature, namely structural transformation and urban development. Both branches are very extensive, and a comprehensive review would be well outside the limits of this paper. Hence, we will review only the most related papers and highlight the aspects most relevant for our analysis and findings.

With respect to the vast literature on structural transformation,³ our work is closest to papers that investigate episodes of accelerated growth, stagnation and decline, based on sectoral productivity differences and reallocation. Buera, Kaboski and Rogerson (2015) find that increases in GDP per capita are associated with a shift to sectors that are high-skill labor intensive and further development leads to an increase in the relative demand for skilled labor. Along the same lines, Duarte and Restuccia (2010) study the role of sectoral labor productivity in structural transformation and in the trajectory of aggregate productivity of 29 economies. They note that the catch-up process (relative to the United States.) in manufacturing productivity can account for about half of the productivity gains. As a counterpart, the low productivity –and lack of catching up– of the service sector explains the episodes of stagnation and decline. This work can be useful to understand the experience of countries that have stagnated. Indeed, Silva and Ferreira (2015) extend the analysis of Duarte and Restuccia (2010) for six Latin American economies in the period of 1950-2003. Using a four-sector model (agriculture, manufacturing, modern services and traditional services), these authors conclude that the poor performance of the traditional services sector is the main source of the slowdown in productivity growth after the mid-1970s in Latin America. Our paper here highlights that much of the expansion of non-agricultural production can occur in low-skills jobs, which would translated in observed low productivity, as we document for Brazil. Our simple model can be use to examine the conditions under which a country’s structural transformation is directed to high skills non-agricultural occupations or whether it will also be directed to low-skill ones.

At any rate, the reallocation from agriculture to non-agricultural sectors is strongly associated to the reallocation of workers from rural to urban areas. Indeed, the literature is strongly dominated by the view that urbanization, structural transformation and growth go together, partly because no single country has reached middle-income status without a significant population shift into cities.⁴

³See the Handbook chapter by Herrendorf, Rogerson and Valentiny (2014).

⁴See Figure 1.1 in Annez et al. (2009).

Many papers emphasize the role of agglomeration economies, from cost advantages for workers and firms inside a city, such as linkages between industries, better infrastructures, network externalities, thickness in labor and goods markets, etc. From this large and diverse literature, perhaps the closest to our model is Lucas (2004), which identifies cities as fertile places for the formation of skills because of the exposure of ideas. A fundamental difference between our model and Lucas (2004) is that in his model, cities are fully integrated. In our model, the skill formation inside urban areas can be segmented between a formal city and a marginal, informal areas, all of which we call slums.

In our model, the emergence of slums is driven by housing costs, a form of congestion costs. Therefore, we connect to a substantial literature on congestion costs (negative externalities) and their effect on urbanization affects growth. These congestion costs can be illustrated by higher cost of infrastructure (piped water, sewage, electricity, transportation), high real estate prices (low supply of housing), pollution, and bad quality of social services (education, health). The literature points to the evidence that urbanization takes place in the early stages of development, often before economies have reached middle incomes status. Therefore, rural-urban migrants with low human capital accumulation and low income usually settle in squatter areas, an aspect that is central to our model.

A crucial question in our paper is whether slums create a poverty trap or whether they are steeping stones. On one hand, the experience of developed countries would indicate that slums can be seen as temporary phase, and therefore, closer to a steeping stones.⁵ Indeed, slums were very common during the Industrial Revolution in European and American cities (e.g. London and New York.) Yet, most of these irregular settlements have disappeared. On the other hand, the more recent experience of developing countries since World War II, seem to indicate that slums may not be a transitory phenomenon, as they have been growing over the years, with many households seemingly stuck in low living standards for generations. Indeed, some observers have suggested that slums of developing countries have the same features of a poverty trap,⁶ driven by low human capital accumulation, low levels of public and private investments, and persistent policy neglect by governments. In fact, some studies⁷ claim that the persistence slums in developing countries is the result of policy failures that restrict the supply of affordable housing to the emerging urban population.

Empirically, Marx, Stocker and Suri (2013) discuss whether there is a relationship between economic growth, urban growth and slum growth in the developing world, and whether standards of living of slum dwellers improve over time, both within slums and across generations⁸. At a more micro level, Cavalcanti and Da Mata (2014) study how urban poverty, rural-urban migration and land use regulations can impact the growth of slums. They construct a structural general equilibrium model with heterogeneous agents that is able to measure the role of each determinant of growth of slums. With some counterfactual exercises, they show that those three factors explain much of the variation of slums dissemination in Brazil among the years 1980-2000. Our work complements that of Marx, Stocker and Suri (2013) by constructing a model in which slums arise endogenously in equilibrium, and use the model to examine the impact of housing restrictions, schooling policies and the interaction with income disparities over time. With respect to Cavalcanti and Da Mata (2014), our model is dynamic and can be used to explore the temporal interaction between slums and the factors that give rise to them.

⁵See Frankenhoff (1967), Turner (1969) and Glaeser (2011).

⁶See Marx et al. (2013).

⁷See Hammam (2013) and Lall et al. (2007).

⁸We also discuss these points later in this paper.

3 Brazil: Structural Change and the Rise of *Favelas*

As many other countries, Brazil went through a substantial structural transformation and urbanization from the years 1950 to 2010. In this section we explore the relationship in the patterns of development (growth) and urbanization, one question emerges. In this section, we use Brazilian data to review the common link between structural transformation and rural-urban migration and highlight the emergence of slums, or ‘favelas’ as a salient feature of the process of urbanization.

3.1 Data

Our data sources are explained in detail in Appendix A. First, our data on structural transformation is taken from the Groningen Growth and Development Centre (GGDC) database (Timmer et al. (2014)). The GGDC dataset includes series of value added, output deflators and persons employed for ten productive sectors.

Second, our demographic and income data comes from the Brazilian Census, conducted every ten years by the Brazilian Institute of Geography and Statistics (IBGE⁹). The census provides data on the population distribution between rural and urban areas, the levels of education, the average personal income and the labor distribution by productive sectors.

In addition, for the years 1991 and 2000, the Census provides an interesting variable, telling us if an household lives in a "subnormal agglomerate". The IBGE defines "subnormal agglomerate" as a set of 51 or more housing units characterized by absence of a proper ownership title and at least one of the following aspects: (i) Irregular traffic routes or irregular size (shape) of land plot; (ii) Lack of essential public services such as garbage collection, sewage system, electricity and public lighting. This description is almost equivalent to slums and very poor settlements as defined, for instance, by the UN Habitat¹⁰. Thus, we use here slums and "subnormal agglomerate" interchangeably.

We also use data from the Favela Census¹¹, conducted by the state government of Rio de Janeiro in 2010. This Census is a unique initiative of mapping and identifying the profile of residents who live in the three of the biggest slums (Alemão, Mangueiras and Rocinha) of Rio de Janeiro.

Finally, we use the social mobility supplement of PNAD (Pesquisa Nacional por Amostra de Domicílio¹²) for 1988 and 1996 to examine the intergenerational transition matrices across education levels to explore the dynamics of income disparities, urban development and the emergence of slums. The surveys for 1988 and 1996 have a special supplement which includes questions about parental education of the household head and the spouse.

3.2 Structural Transformation, Urbanization and Slums

Brazil, as many developing countries around the world, experienced a fast process of structural transformation and urbanization after World War II. Given housing prices, many rural -urban migrants could not afford to buy a spot in the cities and had to live in slums, that grew very fast in the period and do not seem to be reducing in size. Living in cities entails better work opportunities, even for those in slums, higher income and better education. For the residents of the favelas the expected schooling of their children, controlling for the education of the parents, is

⁹See www.ibge.gov.br/english/.

¹⁰The UN Habitat defines a slum household as a group of individuals living under the same roof and lacking one or more of the following conditions: (i) Access to improved water; (ii) Access to improved sanitation; (iii) Sufficient-living area; (iv) Durability of housing; (v) Security of tenure.

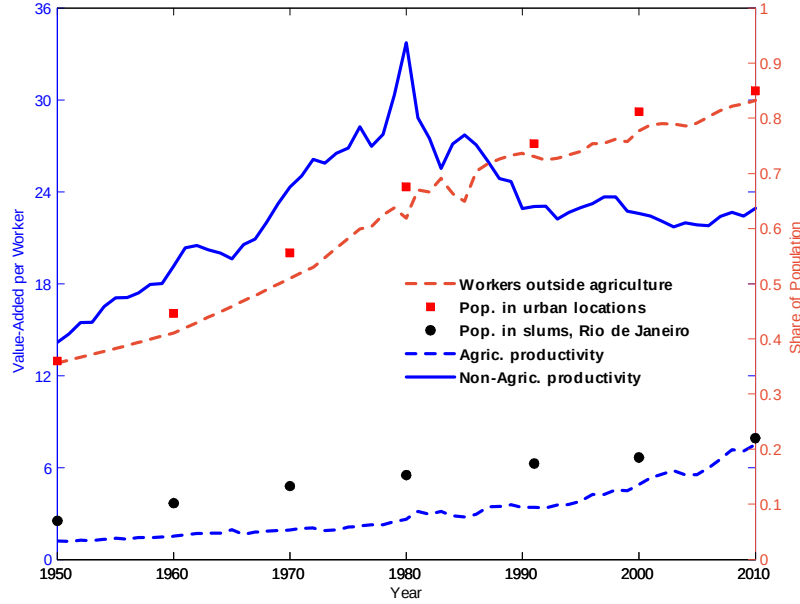
¹¹For more details see www.emop.rj.gov.br/trabalho-tecnico-social/censos-comunitarios.

¹²National Household Survey conducted every year in Brazil since 1976.

above the education of the children of the rural dwellers but smaller than the education of the city kids who live outside slums. Slum residents work mostly on low skill services and manufactures (in this case, construction), while non-slum urban workers are more evenly distributed among high and low skill services and manufacture. This section documents these facts.

In sixty years, Brazil went from a predominantly agricultural and rural economy to an urban economy with the services sector playing a dominant role. From Figure 1, we see that the share of labor employed in agriculture decreased steadily from 64% in 1950 to 16% in 2010, at the same time that urban and slum populations increased.

Figure 1: Value Added per Sector,
Population in Slums



Regarding the evolution of productivity (output per worker) agriculture presents a clear upward trend between 1950 and 2010 at the same time that one observes a rise followed by a fall in the non-agriculture productivity. For the period 1950-1980, the productivity in this sector grew by 2% per year, as well as in agriculture. The outstanding reallocation of labor from agriculture to non-agriculture (much more productive) during this period resulted in a convergence in direction to the U.S. economy and an acceleration of Brazilian aggregate productivity¹³. But in 1980 the picture changed. Since this year, non-agriculture productivity has fallen 1% per year. After the labor reallocation from lower to higher productive sectors, the next step would be to invest in human capital and give better opportunities to those rural-urban migrants and their offspring. That did not happen, and a very large uneducated workforce had to work in low skill sectors with poor growth prospects.

Since the early twentieth century, we observe waves of rural-urban migration and the emergence of the first slums in Rio de Janeiro (capital of Brazil during the years 1763 to 1960), but it was only after World War II that the process of urbanization and formation of slums became a national and widespread phenomenon. Pearlman (2010) notes that, before World War II, only 15% of Brazilians lived in cities. According to Census data, the urbanization rate was already 36% in 1950 and in the next thirty years a very large change in the rural-urban distribution of the population occurred. By

¹³Silva and Ferreira (2011) finds that 45% of the 1950-1980 growth in Brazil is due to structural transformation and labor reallocation.

the mid 1960s there were already more people living in cities than in the country-side: the urban population went from fewer than 13 million in 1940 to more than 50 million in 1970. By 2010 only 15% of the Brazilian population lived in rural areas.

Rural-urban migration is the most important force driving the urbanization process in developing countries¹⁴. World Bank (2008) estimates that around 40 million people left the countryside or poor regions for larger cities between the years 1960 and 1970, period of high economic growth in Brazil. This process continued in the following years. In 1960, as shown in Table 1, 40% of the people living in Rio de Janeiro were migrants, and this should be true for most large cities in the country. Although this process slowed down after the seventies, there were still 1.5 million migrants living in Rio de Janeiro in 2000, approximately 27% of its population.

Table 1: Migrants, (% of total population) Rio de Janeiro		
Year	Slums	City
1960	52.2	38.3
1991	29.8	27.7
2000	31.2	25.8

Source: Brazilian Census

The rural-urban migration resulted in a fast expansion of slums - a widespread reality across large cities - and the agglomeration of low skill labor in manufacturing and mainly in services sectors. From Table 1 we can see that in 1960 more than half the residents of slums in Rio de Janeiro were migrants¹⁵. São Paulo and Rio de Janeiro are the two richest cities in Brazil (in terms of GDP) and have some of largest shares of population living in slums, as shown in Table 2:

Table 2: Urban population living in slums (%)					
Year	Cities				
	Rio de Janeiro	São Paulo	Belo Horizonte	Belém	Salvador
1950	7.0	—	—	—	—
1960	10.2	—	—	—	—
1970	13.3	—	—	—	—
1991	17.4	9.2	14.2	25.8	10.1
2000	18.5	11.1	12.3	34.6	9.6
2010	22.0	23.2	—	—	—

Source: Brazilian Census

Rio de Janeiro is the only city for which we have information about urban and slum populations since 1940. The share of the Rio de Janeiro population living in slums went from 7% in 1950 to 22% in 2010. In São Paulo in 2010, 23% of total population was living in slums, more than doubling in twenty years. There were 2.1 and 1.7 million people living in slums in the metropolitan regions of São Paulo and Rio de Janeiro, respectively, in 2010. Thus, the slum phenomenon appears to be a reality that expands every year and is not a transitory phase.

Rural-urban migration depends on forces known in the literature as pull and push. The forces that pull migrants to their destinations are better economic opportunities in terms of jobs (due to agglomeration economies) and better amenities and public services such as piped water, electricity,

¹⁴For more details about urbanization and rural-urban migration in developing countries, see Brueckner and Lall (2015) and Lall et al. (2006).

¹⁵It is also true, according to the 1991 Census, that the share of migrants coming from rural areas is two times higher in slums than in the formal city.

hospitals and schools. And the forces which push migrants off their origin lands are low productivity in agriculture, environmental changes, pressures of population growth and lack of access to basic public services. Lall et al. (2009) study migration from lagging to leading regions in Brazil and the pull and push forces. They find that wage differences are the main factor driving migration. Access to basic public services matters a lot, they show that poor people are willing to accept lower wages in order to get access to better amenities and quantify how much is this willingness to pay for three public services (hospital, water access and electricity). Along the same lines, Dudwick et al. (2011) investigate why migrants are attracted to particular locations in Nepal. They show that destinations with better access to schools, hospitals and markets are the most preferred ones.

Here, we are interested in studying and focusing on two main pull forces: better economic opportunities and access to better education¹⁶. In the first case, migrants are looking for better jobs and higher income. In Table 3, we see that the total income of people living in urban areas, controlling by education, is significantly higher than of those who live in rural areas.

Table 3: Income Ratios by Education and Location (2000)

Education	Brazil	Rio de Janeiro		São Paulo	
	Urban/Rural	City/Rural	Slum/Rural	City/Rural	Slum/Rural
Average	2.5	4.2	1.3	4.5	1.4
0	1.3	2.1	1.6	2.5	2.0
1 to 3	1.4	1.9	1.4	2.1	1.6
4	1.3	1.6	1.1	1.8	1.2
5 to 8	1.3	1.6	1.0	1.7	1.0
9 to 11	1.4	1.6	0.8	1.8	0.9
12 or +	1.3	1.4	0.5	1.5	0.5

Source: Brazilian Census

On average, income in urban area is two and a half times larger than in rural areas, and this is also true for all years we have data (1970 to 2010). And that is not only due to a composite effect (there are less educated people in rural areas), but also due to the fact that at every education level mean income in the cities is higher than in the countryside. For those with four years or less of education, for instance, average income in 2000 in the rural region was only 74% of average income in the cities¹⁷.

Even when rural migrants move to cities and end up living in slums, their expected income is still higher than those who stayed in the rural areas, specially in the case of the less educated (the vast majority of rural and slum residents), as we can see from the figures in Table 3. While a typical resident in the slums in Rio or São Paulo makes less than someone with the same educational level living outside the slums, he makes considerably more than a corresponding person who lives in the countryside of Brazil¹⁸.

People migrate to cities, among other reasons, to have access to better jobs, even if they live in slums. From Table 4 we can see that the slums residents work mostly in cities (out of the slums),

¹⁶We are not saying that the other pull and push forces are not relevant, but following the literature that relates growth and human capital, we are interested in investigating how structural transformation, urbanization and education can explain the stages of economic growth.

¹⁷Ferreira, Menezes and Santos (2005) find that there is positive selection of migrants in Brazil, and this can partially explain the income difference between rural and urban areas. However, they estimated that this can only explain 10% of the income disparity, which is only a fraction of the numbers in Table 3. Other forces are contributing for the bulk of the income difference.

¹⁸This is also true for most large cities in Brazil, with the exception of Salvador, where incomes are about the same for people living inside slums and rural areas.

where the majority of opportunities and jobs are. And this is true even though they earn less than the cities residents, as we saw in Table 3.

Table 4: Job location of people living in three slums in Rio (%)

	Alemão	Manguinhos	Rocinha	Mean
Inside slums	22.70%	22.40%	22.00%	22.40%
In the close vicinity	15.70%	19.30%	6.90%	13.90%
Outside slums	61.60%	58.40%	71.10%	63.70%

Source: Favela Census of Rio de Janeiro

What are the sectors in which slum residents work, and are they very different from the sectors in which non-slum residents work? Data from the 1991 and 2000 Census (see Table 5) show some similarities and important differences. As one could expect from their lower education levels, slum residents are distributed more heavily across low skill sectors. In São Paulo and Rio, in 2000, around 50% of workers living in slums are in the "Low Skill Services" sector, which is mostly personal services (e.g., maids), retail and restaurants. The corresponding figures for non-slum residents are considerably smaller. There are proportionally fewer high skill workers living in the slums (and half of them are in the "Transportation, Storage and Communication" sector, which is partially a low skill sector). In contrast, the share of workers in the high skill sector living in the city is, in general, twice as large as those in the slums, and the concentration in the Health, Education, Government and Financial Services sub-sectors is much larger¹⁹. The proportion of workers in manufacturing is larger in slums, but this is mostly due to the construction²⁰ subsector.

Table 5: Labor distribution by sector and location (%)

	1991				2000			
	São Paulo		Rio de Janeiro		São Paulo		Rio de Janeiro	
	Slums	City	Slums	City	Slums	City	Slums	City
Agriculture	0.9	1.2	1.1	1.4	0.6	0.4	0.4	0.6
Manufacturing	41.2	31.1	29.3	18.5	32.1	24.4	22.9	15.5
Low Skill Services	42.8	33.8	48.7	34.0	47.7	39.6	53.6	39.5
High Skill Services	14.2	32.9	20.3	45.7	18.0	33.8	20.6	41.5
Not well defined	1.1	1.0	0.7	0.4	1.6	1.9	2.7	2.8

Source: Brazilian Census

Migrants also move to the cities looking for access to better education, because higher levels of education mean higher income on average and also because they care about their children's future. Average education is much lower in rural than in urban areas, as shown in Table 6. The mass of population in the first two levels of education is much higher in the countryside than in urban areas and the average years of schooling is much lower for all years we have data (1970 to 2000)²¹.

¹⁹ As for the labor distribution by sector in the rural area, between 87% and 71% of the workers are in the agriculture sector, depending on the Census year. In contrast, only 6% of urban residents worked in the agriculture in 2000.

²⁰ Without construction subsector, 17% of the labor force in São Paulo and around 10% in Rio de Janeiro worked in manufacturing in 2000.

²¹ In 1970, average schooling in the countryside was less than one year and 64% of the adult population had no education at all. The corresponding figures for urban areas were 3.4 years and 28%.

Table 6: Population distribution by years of schooling, 2000 (%)

Years	Brazil		São Paulo		Rio de Janeiro	
	Rural	Urban	Slums	City	Slums	City
0	31.3	11.6	14.4	5.4	12.4	3.8
1 to 4	50.2	33.3	42.7	27.5	39.1	21.5
5 to 8	12.5	23.3	30.2	23.6	31.0	21.0
9 to 11	4.9	20.9	10.4	23.2	15.4	29.1
12 or +	1.0	10.9	2.3	20.2	2.2	24.6
Average	2.9	6.5	4.8	8.1	5.3	9.0

Source: Brazilian Census

As one could expect, average education of slum residents is lower than of those living in formal cities. From Table 6 we can see that in Rio de Janeiro in 2000 the average adult education in the slum was almost 4 years smaller than in the city, and similar figures are true for São Paulo (and 1991). However, education levels in the slums of Rio and São Paulo are well above those in the rural areas of Brazil. On average, a slum resident of Rio de Janeiro in 2000 had two and a half years of schooling more than a typical resident of the rural areas. And while one third of the rural area residents in this same year had no education at all, only 12% of the slum residents of Rio had no education. Clearly, rural area populations are less educated than those in slums that are, in their turn, less educated than city dwellers.

Upward mobility is very pronounced in cities, specially for those with few years of schooling, when compared to rural areas. From PNAD 1996 supplement we can show that average years of schooling of a person whose parent had no education was almost two years smaller in the rural areas than in urban areas (2.65 and 4.48 years, respectively), and similar figures are obtained for those with parents with 1 to 4 years of education. Upward mobility is higher in formal cities than in slums, but for all metropolitan areas (that we have data) it is still higher in slums than in rural areas. For instance, in Rio de Janeiro and São Paulo the average schooling of someone whose father has no education was, in 1996, 5.58 and 3.94, respectively, way above the corresponding figure for rural areas we just saw. For the formal city, the values are even higher: 9.91 and 9.23, respectively. These numbers indicates that education mobility is higher outside than inside slums, and higher in slums than in the rural areas.

Another important evidence on different intergenerational mobility comes from transition matrices of individual education conditional on their parents education²². Figures 2a and 2b, constructed from transition matrices estimated using 1988 PNAD data for the Rio de Janeiro, show the likelihood of schooling attainment for rural, urban, slum and outside slum areas of children with parents with no education at all and with one to four years of schooling, respectively²³.

²²See Ferreira and Veloso (2003) for Brazil in 1996.

²³We assume that the households living in metropolitan areas with total income in the 35 percentile or lower are slum dwellers and the ones with income above the 35 percentile live in the cities.

Figure 2a: Education of a children with father with no schooling

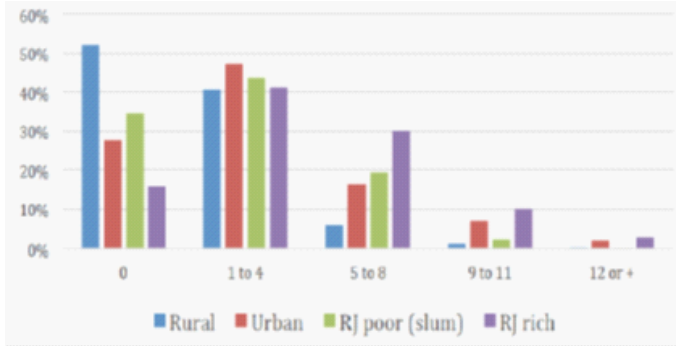
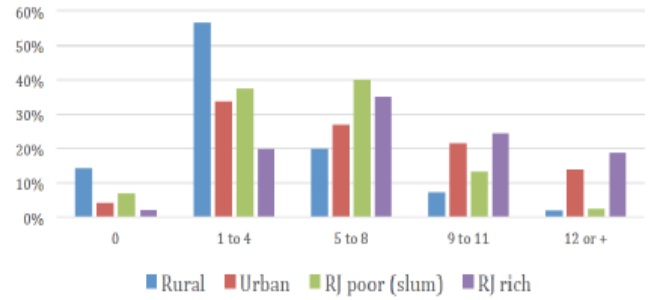


Figure 2b: Education of a children with father with 1-4 years of schooling



There is a great persistence of low educational levels in rural areas and upward mobility in urban areas. For example, in 1988, 52.08% of parents in the countryside with no schooling had children with the same level of education, compared with half this figure (27.58%) in cities. Similar results are found for the children with parents with 1 to 4 years of education: around 70% of them will have the same level or less education than their fathers, as opposed to less than 40% in the case of urban children. And in urban areas, 62.8% of parents with 12 or more years of schooling have children with the same level of education, while in rural areas the corresponding figure is only 30.6%.

The numbers in the transition matrices for slums are better than those for rural areas and worse than those for non-slum areas. In slums, 35% of parents with no education had children with no education too, 17 percentage points below rural areas in Brazil. The corresponding figure for city residents in Rio de Janeiro is only 16%. In the city 64% of parents with 12 or more years of schooling had children with the same level, but in the slum this was only 28%, although there were very few observations in this case. Thus, the persistence of low levels of education in slums is smaller than in the countryside but the upward mobility is worse than in the cities. Therefore, leaving the countryside is a first step to get better living conditions, but living in slums is not a good option compared to cities. The next step is to move away from slums.

Schools in slums are, in general, worse than in the formal cities as tests and quality of education measures tend to confirm. While parents benefit from jobs outside slums, education, in contrast, is mostly done inside the slums, or in their close vicinity. From Table 7 one can see that around three quarters of the students in these three slums attend schools inside the favela or less than 3 kilometers away. This may partly explain lower upward education mobility in slums.

Table 7: Percentage of students by location of school (living in these 3 slums)

	Alemão	Manguinhos	Rocinha	Mean
Inside slums	86.30%	55.90%	43.30%	61.80%
Outside but <1km away	8.90%	21.30%	0.50%	10.20%
Outside between 1-3km way	0.00%	12.30%	26.00%	12.80%
Outside >3km	1.50%	7.80%	30.20%	13.20%
Could not locate school	3.30%	2.70%	0.00%	2.00%

Source: Favela Census of Rio de Janeiro

One last important stylized fact is the difference of housing costs across locations. As shown in Table 8, rent prices in urban areas of Brazil was in 1991 more than twice as expensive than in rural areas on average, and renting 3 bedroom apartments was three times as expensive as in the countryside. Rents in the slums of Rio de Janeiro and São Paulo were also considerably more expensive than in the countryside of Brazil and this should be true everywhere, as slums are mostly located in large cities. But as expected, prices in formal cities are well above those in slums; in the

case of Rio and São Paulo around twice as much on average. The difference of rent prices between non-slums parts of these two cities and the rural areas of Brazil, is huge, more than four times on average in the case of São Paulo. Hence, living in the rural areas is considerably cheaper than living in the slums of Rio and São Paulo which are, in its turn, less expensive than living in non-slum parts of the city.

Table 8: Ratio of Monthly Rents (1991)

# Bedrooms	Brazil	Rio de Janeiro		São Paulo	
	Urb./Rur.	City/Rural	City/Slum	City/Rural	City/Slum
1	2.0	3.2	1.8	3.5	1.5
2	2.3	3.5	1.9	4.7	2.0
3	2.9	4.8	2.7	6.5	2.6
Average	2.3	3.6	2.3	4.1	1.9

Source: Brazilian Census

4 A Simple Model

We consider the simplest model that can be used to analytically examine: (i) structural transformation; (ii) urban development; (iii) income and skill distribution; (iv) social mobility. The key elements in the economy will be how to allocate individuals and skills across locations, productive sectors and occupations and the dynamic implications of those decisions for the skill formation of future generations.

We first lay out the environment and then define a competitive equilibrium.

4.1 The Environment

We consider a discrete-time, infinite-horizon economy populated by dynasties of two-period-lived overlapping generations (OLG) of individuals. Time periods are indexed by $t = 1, 2, 3, \dots$. In any period, individuals can differ in their skills. The population of the economy is described by a positive measure μ_t defined over all over all positive levels of skills $z \in \mathbb{R}_+$. We assume that the total mass of the population remains constant, and normalize $\int_0^\infty \mu_t(dz) = 1$ for all periods. However, the evolution over time of the distribution of skills, $\{\mu_t\}_{t=1}^\infty$ will be determined endogenously, as part of the equilibrium of the economy. Specifically, each adult at time $t + 1$ will draw a skill level from a distribution affected by the decisions of his parents in period t , as explained below.

Adults at t choose their occupations, locations and consumption of goods. In this economy, there are **three locations**: *rural areas*, *favelas (slums)* and *city centers*, which we index by $j = R, F, C$, respectively; there are **two sectors**, *agriculture* and *non-agriculture* (manufacturing and services), which we index by $i = A, M$; and, there are **three occupations** or types of labor services: *low-skilled*, *basic (qualified) skilled* and *amenable* labor. We index these jobs as $o = l, b, a$.

Preferences: The utility of a households at time t are defined over the consumption of goods and over the expected skill formation for the children. The utility of an adult at t is given by:

$$V_t = u(c_t) + \beta E_t [z_{t+1}],$$

where $u(\cdot)$ is the individual's utility. Here, the consumption vector, $c_t = (c_t^A, c_t^M)$, consists of consumption levels of agricultural and non-agricultural goods, respectively. As a driver of structural transformation, we assume the standard Stone-Geary non-homothetic preferences:

$$u(c_t) = \prod_{i \in \{A, M\}} (c_t^i - \bar{c}^i)^{\alpha_i},$$

where $\alpha_i > 0$ and $\sum_{i \in \{A, M\}} \alpha_i = 1$, and $\bar{c}^A > 0$ and $\bar{c}^M = 0$.

To simplify our analysis, we initially explore paternalistic preferences à la Fernandez and Rogerson (1998) with impure altruism (β), where z_{t+1} is the skill (human capital) of the child and $E_t(\cdot)$ is the expectation based on information at time t .

Supply of Labor: All workers can provide the different types of labor services depending on their skill level z . First, we assume that, regardless of z , everyone can provide the same units of low skilled labor. Second, the supply of amenable labor is proportional to the individual's skill z . Finally, non-negligible qualified skilled labor can only be provided by those workers with skills above a minimum qualification requirement, $z_{\min} > 0$. Specifically, Therefore, an individual with skill z can supply different types of work in different occupations:

$$\begin{aligned} h^l(z) &= 1 \text{ for all } z \in \mathbb{R}_+; \\ h^b(z) &= \begin{cases} 0 & \text{if } z < z_{\min}, \\ 1 & \text{otherwise;} \end{cases} \\ h^a(z) &= z^\phi \text{ for all } z \in \mathbb{R}_+, \phi > 0. \end{aligned}$$

Production of Goods: At any point in time, the three locations in the country have (exogenous) sectorial productivities described by $\{X_t^{i,j}\}$ for $i = A, M$, and $j = R, F, C$. The two consumption goods are produced using only labor. For simplicity, we assume that agriculture goods only uses unskilled labor. The aggregate output of production of agricultural goods, $Y_t^{A,j}$, is given by

$$Y_t^{A,j} = X_t^{A,j} L_t^{l,A,j}, \quad (1)$$

where $L_t^{l,A,j}$ is the aggregate units of low skill services.

We assume that non-agricultural goods uses both qualified and amenable labor. Specifically, the non-agricultural production in location j is given by:

$$Y_t^{M,j} = X_t^{M,j} \left(L_t^{b,M,j} \right)^\eta \left(L_t^{a,M,j} \right)^{1-\eta}, \quad (2)$$

where $0 \leq \eta \leq 1$. Here, $L_t^{b,M,j}$ and $L_t^{a,M,j}$ is the aggregate supply of qualified and amenable labor.

Locations I: Housing Costs and Occupations. Household have to decide whether to live in a rural or urban area. If they decide to live in the rural area, we assume, for simplicity, that the only occupations available are low-skill ones, $o = l$. If they live in an urban area, they can choose between providing qualified or amenable labor, b or a . Moreover, if the individual decides to live in an urban area, he has the choice of living in the city proper or in informal housing (slum).

Living in the city requires paying for housing. For simplicity, we assume that houses do not deliver utility directly, and simply gives access to living in the city. Each house requires $\xi > 0$ units of non-agriculture goods to be constructed. Hence, as describe in the equilibrium section, in a competitive equilibrium, housing prices p_t^h will be equal to ξp_t^M , where p_t^M i.e., for one house, it is need $\xi > 0$ units of non-agriculture goods. Therefore, in order to live in the city the household has to pay ξp_t^M .

Living in a slum circumvents the costs of paying for a proper house, but entails other costs (commuting, lack of property rights and protections, etc.), which we model as a fraction τ of the individuals consumption. Finally, as a normalization, we assume that living in the rural area entails no direct housing costs.

Next, we explain how we model the costs and benefits of the different locations in terms of the skill formation of the household's children.

Locations II: Skill Formation and Demographic Dynamics: At time t , individuals in the economy have skills z distributed with support of skills in the economy is $[0, \infty)$. The current distribution of skills in the population is μ_t , which we normalize to be a probability distribution, i.e., μ_t adds up to one for all t . The country's population $\mu_t(\cdot)$ is distributed across locations:

$$\mu_t(\cdot) = \sum_l \mu_t^j(\cdot),$$

where $\mu_t^j(\cdot)$ is the measure of individuals living in location j .

Consider a parent who lives in location j and has skill z . His child draws:

$$z' \sim Q(\cdot | Z_t^j),$$

where $Z_t^j \equiv \left[\int z^\rho \tilde{\mu}_t^j(dz) \right]^{1/\rho}$, $\tilde{\mu}_t^j \equiv \mu_t^j / \int \mu_t^j$ and the parameter ρ determines the degree of intergenerational persistence (social) mobility as well as the strength of human capital externalities.

Next period, the measure of population with skills in any Borel set $B \subset \mathbb{R}_+$ is given by:

$$\mu_{t+1}(B) = \sum_{j \in \{R, F, C\}} \int_0^\infty Q(B | Z_t^j) \mu_t^j(dz).$$

4.2 Equilibrium

In this section, we are going to define the competitive equilibria of this economy. The state variable is given by $X_t = \left(\mu_t, \{Z_t^j\}_{j \in \{R, F, C\}}, \{X_t^{i,j}\}_{i \in \{A, M\}, j \in \{R, F, C\}} \right)$, the measure over skills of the current generation, the vector of skills in each location j and the vector of productivities in each sector i and location j . The price systems are composed of the price of goods i in each location j , $\{p_t^{i,j}\}$, and the wages of low skilled, basic skilled and amenable workers, $\{w_t^l, w_t^b, w_t^a\}$, respectively. Given those prices, households of all skills $z \in \mathbb{R}_+$ in all periods $t \geq 0$, decide their consumption $c_t(z) = \{c_t^A(z), c_t^M(z)\}$, their selection of occupation, $\{\chi_t^l(z), \chi_t^b(z), \chi_t^a(z)\}$, and of location, $\{\chi_t^R(z), \chi_t^F(z), \chi_t^C(z)\}$.

We now lay out the equilibrium conditions:

Production: The two goods are produced in competitive markets and firms take the prices $\{p_t^{i,j}\}$ and wages $w_t^l(\cdot)$, $w_t^b(\cdot)$ and $w_t^a(\cdot)$ as given. Each firm maximizes profits by choosing low skilled, basic skilled and amenable labor units. Free entry and constant returns to scale imply that the size of firms is undetermined, so that the aggregate demand for each type of labor in each region j and each sector i can be solved by:

$$\max_{L_t^{l,i,j}, L_t^{b,i,j}, L_t^{a,i,j}} p_t^{i,j} Y_t^{i,j} - w_t^l L_t^{l,i,j} - w_t^b L_t^{b,i,j} - w_t^a L_t^{a,i,j},$$

subject, respectively, to the production function (1) or (2).

Note that how much low skilled, basic skilled and amenable labor is allocated across regions and sectors, is determined by the population of workers in each occupation and their decisions on consumption and place to live.

From our simplifying assumptions in section 4.5, we have $L_t^{l,M,j} = 0$, $L_t^{b,A,j} = 0$ and $L_t^{a,A,j} = 0$ for all $j \in \{R, F, C\}$. We also assume that agricultural goods are only produced in rural areas, i.e., $Y_t^{A,j} = 0$ for $j \in \{F, C\}$, and non-agricultural only in urban areas²⁴ (slums and cities), i.e.,

²⁴Note from Table 9 that the labor share employed by manufacturing in rural areas is very low, although it has been increasing across the years.

$Y_t^{M,R} = 0$. And in terms of tradeability, we suppose non-agricultural and agricultural goods are tradeable across regions, i.e., $p_t^{A,j} = p_t^A$ and $p_t^{M,j} = p_t^M$ for all j .

Taking agriculture as our numeraire ($p_t^A = 1$), maximization of firms' profits in the agricultural sector implies that the wage of low skilled labor is simply:

$$w_t^l = X_t^A. \quad (3)$$

And the profit maximization in non-agricultural sector gives us:

$$w_t^b = \eta p_t^M X_t^M \left(\frac{L_t^{a,M}}{L_t^{b,M}} \right)^{1-\eta}. \quad (4)$$

$$w_t^{a,U} = (1 - \eta) p_t^M X_t^M \left(\frac{L_t^{b,M}}{L_t^{a,M}} \right)^\eta. \quad (5)$$

Consumption of Goods: Consider a household with income y_t , he chooses the consumption of goods to solve:

$$[c_t^A, c_t^M] = \arg \max_{c_t^A, c_t^M} \prod_{i \in \{A, M\}} (c_t^i - \bar{c}^i)^{\alpha_i} \quad s.t. \quad \sum_{i \in \{A, M\}} p_t^i c_t^i \leq y_t.$$

The optimal consumption levels are given by:

$$c_t^i = \bar{c}^i + \frac{\alpha_i}{p_t^i} \left[y_t - \sum_{i \in \{A, M\}} p_t^i \bar{c}^i \right].$$

This implies that the intra-period indirect utility of someone with income y_t and facing prices $\{p_t^i\}_{i \in \{A, M\}}$ is given by:

$$v_t(y_t) = \prod_{i \in \{A, M\}} \left[\frac{\alpha_i}{p_t^i} \left(y_t - \sum_{i \in \{A, M\}} p_t^i \bar{c}^i \right) \right]^{\alpha_i}.$$

Occupation Choices: Conditional on the location decision, the household choose his occupation maximizing the income coming from labor supply. Thus, given his skill z and current wages $w_t^{l,j}$, $w_t^{b,j}$ and $w_t^{a,j}$, the income $e_t^j(z)$ can be simply written by:

$$e_t^j(z) = \max \left\{ w_t^{l,j} h^l(z), w_t^{b,j} h^b(z), w_t^{a,j} h^a(z) \right\}.$$

Denote by $o_t^j(z)$ the occupation choice for an individual with skills z at location j and time t .

Location Choices: Given prices $\{p_t^{i,j}\}$, optimal conditional occupation choice $o_t^j(\cdot)$ and optimal conditional income $e_t^j(\cdot)$, the expected discounted utility of a household with skill z at time t is defined by:

$$V_t(z) = \max_{j \in \{R, F, C\}} \prod_{i \in \{A, M\}} \left[\frac{\alpha_i}{p_t^{i,j}} \left(y_t^j(z) - \sum_{i \in \{A, M, S\}} p_t^{i,j} \bar{c}^i \right) \right]^{\alpha_i} + \beta E_t [z_{t+1} | Z_t^j],$$

where:

$$y_t^j(z) = \begin{cases} e_t^R(z) & \text{if } j = R; \\ (1 - \tau) e_t^F(z) & \text{if } j = F; \\ e_t^C(z) - p_t^h & \text{if } j = C; \end{cases}$$

is the income net of housing costs (income cost τ in slums and monetary cost p_t^h in cities).

Therefore, the optimal location decision is the solution of:

$$j_t^*(z) \in \arg \max_{j \in \{R, F, C\}} v_t(y_t^j(z)) + \beta E_t [z_{t+1} | Z_t^j].$$

Given the country's probability measure of skills $\mu_t(\cdot)$, the assignment (selection) of workers into regions defines the measure of skills $\mu_t^j(\cdot)$ for region j at time t as:

$$\mu_t^j(B) = \int_B \chi_{\{z: j_t^*(z)=j\}}(z) \mu_t(dz)$$

where $\chi_{\{z: j_t^*(z)=j\}}(\cdot)$ is an indicator function that takes a value equal to 1 if individuals of skills z are assigned into region j and zero otherwise; B is any Borel set on non-negative real numbers.

4.2.1 Definition of Equilibrium

Given the above definitions, we can write the following aggregates:

1. Production: The output levels of agriculture and non-agriculture goods are

$$Y_t^A = Y_t^{A,R} = A_t^A L_t^{l,A,R},$$

and

$$Y_t^M = Y_t^{M,U} = A_t^M \left(L_t^{b,M,U} \right)^\eta \left(L_t^{a,M,U} \right)^{1-\eta}.$$

2. Regional and country's aggregate incomes:

$$Y_t^j \equiv \int y_t^j(z) \mu_t^j(dz),$$

and

$$Y_t \equiv \sum_{j \in \{R, F, C\}} \int y_t^j(z) \mu_t^j(dz).$$

3. Aggregate Consumption (of each good):

$$C_t^i \equiv \int c_t^i(y_t(z)) \mu_t(dz) = \bar{c}^i + \frac{\alpha_i}{p_t^i} \left[Y_t - \sum_{i \in \{A, M, S\}} p_t^i \bar{c}^i \right];$$

4. Labor supply of low, amenable and basic skilled labor

$$\begin{aligned} L_t^l &= \int_{\mathcal{M}_t^{l,R}} \mu_t(dz), \\ L_t^{a,U} &= \int_{\mathcal{M}_t^{a,F}} \mu_t(dz) + \int_{\mathcal{M}_t^{a,C}} \mu_t(dz), \\ L_t^{b,U} &= \int_{\mathcal{M}_t^{b,F}} \mu_t(dz) + \int_{\mathcal{M}_t^{b,C}} \mu_t(dz). \end{aligned}$$

where $\mathcal{M}_t^{o,j}$ is the set of individuals who work at occupation o at location j .

With all above definitions in mind, now we can define the equilibrium of this model.

Definition 1 *Given an exogenous sequence of aggregate productivities $\{A_t^{i,j}\}$ and an initial skill distribution $\mu_0(\cdot)$, an **equilibrium** will be composed of:*

1. Individual location $j_t^*(z)$, occupation $o_t^j(z)$ and demand decisions $c_t^{i,j}(z)$ for each period and skill level z .
2. An endogenous sequence of probability distribution measures $\{\mu_t(\cdot)\}_{t=0}^\infty$ for the skills of the country;
3. Sequences $\{\mu_t^j(\cdot)\}_{t=0}^\infty$ of non-negative measures describing for each t how location choices allocate $\mu_t(\cdot)$ across locations j .
4. Sequences of aggregate outputs $Y_t^{i,j}$ and consumptions $C_t^{i,j}$;
5. A sequence of prices $\{p_t^{i,j}\}$ for the goods i in locations j at time t .

Such that:

1. **Individual optimization:** Given prices $\{p_t^{i,j}\}$, decisions $\{j_t^*(z), o_t^j(z), c_t^{i,j}(z)\}$ are optimal.
2. **Adding up:** For any t and for any Borel set $B \subset \mathbb{R}_+$

$$\mu_t(B) = \sum_j \mu_t^j(B) \quad .$$

3. **Aggregation Consistency:** For all t , given the pre-determined $\mu_t(\cdot)$, for all locations j :

$$\mu_t^j(B) = \int_B j_t^*(z) \mu_t(dz) \quad \text{for any Borel set } B.$$

4. **Market-Clearing conditions:**

- Goods markets (both tradeable)

$$\begin{aligned} \sum_{j \in \{R, F, C\}} C_t^{A,j} &= Y_t^A; \\ \sum_{j \in \{R, F, C\}} C_t^{M,j} &= Y_t^M. \end{aligned}$$

- Labor markets:

$$\begin{aligned} \text{Rural} &: L_t^{l,A,R} = L_t^{l,R}, \\ \text{Urban} &: L_t^{a,M,U} = L_t^{a,U}, \text{ and } L_t^{b,M,U} = L_t^{b,U}. \end{aligned}$$

5. **Law of Motion of Demography and Skills:** The country's population evolves according to:

$$\mu_{t+1}(B) = \sum_j \int Q_t(B | Z_t^j) \mu_t^j(dz),$$

for any Borel set $B \subset \mathbb{R}_+$.

In the next section, we derive some simple analytics on the conditions under which slums arise in equilibrium.

5 Equilibrium Allocations

We first provide a simple result on the occupation choices, conditional of location choices. The result is useful for describing equilibria in a number of contexts. We then use the result to explore the properties of the equilibrium location choices.

It can be easily shown that the equilibrium of this economy can be characterized by location and occupation thresholds. For locations, the support of μ_t , the population of skills in the country will be divided according to two thresholds $0 < z_t^R \leq z_t^F < \infty$, defining three groups: (a) *rural population*, $\mu_t [0, z_t^R]$, (b) *slum population*, $\mu_t (z_t^R, z_t^F]$ and (c) *city population* $\mu_t (z_t^F, \infty)$. For short, we will call those in (b) and (c) as the *urban population* $\mu_t (z_t^R, \infty)$.

For occupations, in our highly simplified model, there is only the choice of urban dwellers of offering qualified or amenable labor. To that end, we only need to define the threshold $z_t^H \geq \max \{z_t^R, z_{\min}\}$, so that the supply of basic qualified labor is $\mu_t [\max \{z_t^R, z_{\min}\}, z_t^H]$ and the rest of the urban population would be employed in amenable jobs. For future reference we will call an allocation to be **with urban low skill services jobs when** $z^R < z_{\min}$, i.e. when some urban dwellers do not qualify to provide basic skilled labor. Similarly, we say that an allocation is **high skill urban jobs only** when $z^R > z_{\min}$. In the first case, the marginal migrant from the country side enters the urban locations to work in low skill services. In the second case, the marginal worker would enter to work in qualified occupations and all the amenable service sector jobs will be provided by individuals who could have performed qualified jobs.

We now explore the behavior of these thresholds in our model economy, starting with the simplest cases and progressing towards the general model.

5.1 Conditional Occupation Choices

Conditional on any urban-rural divide, z_t^R , the equilibrium occupation choice z_t^H can be readily solved from the maximization of non-agricultural output Y^M . Denoting F_t the c.d.f. associated with μ_t , the aggregate supply of both forms of labor services can be written entirely in terms of the thresholds z^R and z^H :

$$L^b = F_t(z^H) - F_t(\max \{z^R, z_{\min}\})$$

and

$$L^a = \int_{z^R}^{\max \{z^R, z_{\min}\}} z \mu_t(dz) + \int_{z^H}^{\infty} z f(z) dz.$$

Then, the occupation divide z^H within the urban area can be simply written as:

$$\max_{z^H} Y^M(z^H; z^R) = Z^M [F(z^H) - F(z_{\min})]^\eta \left[\int_{z^R}^{\max \{z^R, z_{\min}\}} z \mu_t(dz) + \int_{z^H}^{\infty} z f(z) dz \right]^{1-\eta}.$$

Closed-form solutions for the optimal occupation split, $z^H(z^R)$, and for the resulting output of non-agricultural goods, $Y^N(z^R)$, cannot be provided. Instead, we can use the first order condition of this maximization implies that the solution is the fixed-point implied by the following equation:

$$z^H = \frac{\eta}{1-\eta} * \frac{\left[\int_{z^R}^{\max \{z^R, z_{\min}\}} z \mu_t(dz) + \int_{z^H}^{\infty} z \mu_t(dz) \right]}{[F_t(z^H) - F_t(\max \{z^R, z_{\min}\})]}. \quad (6)$$

While a closed form is not available, we can prove the following characterization:

Proposition 2 *Given a population μ_t and a rural-urban divide, $z^R > 0$, there exists a unique occupation threshold $z^H < \infty$. If the measure μ_t is continuous and with unbounded support, the threshold z^H is continuous. If (i) $z^R < z_{\min}$, then z^H is locally strictly decreasing in z^R , but if (ii) $z^R > z_{\min}$, then z^H is locally strictly increasing in z^R . Optimized non-agricultural output, $Y^M(z^R) \equiv \max_{z^H} Y^M(z^H; z^R)$ is always strictly decreasing in z^R .*

This simple proposition, whose proof is deferred to Appendix B, characterizes the occupation decisions inside a city. The most interesting aspect of this proposition is the non-monotonicity of the occupation threshold in terms of the size of the city. Only high skill city with low skill services. In particular, given the population distribution of skills μ_t , if the city grows (i.e. a lower z^R when initially $z^R < z_{\min}$), and this growth expands the supply of low-skill amenable labor, then the high-skill amenable sector becomes more selective (higher z^H .) An obvious implication is that the expansion of the city in this case would lead to more income inequality across its dwellers. On the other hand, if the city expansion (i.e. a lower z^R when initially $z^R > z_{\min}$), then the distribution of income inside the city becomes less disperse, as the high-skill sector becomes less selective (lower z^H) in order to expand.

The implication that $Y^M(z^R)$ is decreasing in z^R is straightforward.

5.2 Location Decisions

We now explore the location decisions. For exposition clarity, we do this exploration, starting with the simplest case of a static economy, and progressing to the more general specification of our model.

5.2.1 Equilibria in Static Economies

We first consider a case with myopic allocations, i.e. $\beta = 0$, when individuals are allocated across regions and occupations without considering the implications for their children's skill accumulation. We then examine the model when location decisions are also based on the skill formation of children.

To isolate the role of housing as a barrier to entry in the city, we first consider the case when housing is not an issue.

No Housing Costs, $p_t^h = 0$ We begin with the case in which $\xi = 0$, i.e. housing costs are negligible in the city. Obviously, slums would be empty, as they would be unnecessarily costly for anyone to opt for living there. In terms of our thresholds, this implies $z_t^F = z_t^R$. Here, the only location decision is the traditional rural-urban divide z_t^R . To solve for equilibrium locations, we need to solve for the prices of the goods produced in the different locations.

We use the price of agricultural goods as the numeraire, $p^A = 1$. Given, their income, individual household will maximize their utility. It is straightforward to show, that the optimal consumption levels (c^A, c^M) for all consumers satisfy

$$p^M = \frac{(1 - \alpha_A)}{\alpha_A} \left(\frac{c_t^A - \bar{c}^A}{c^M} \right). \quad (7)$$

It is straightforward to see that this condition carries over to the aggregate levels C_t^A and C_t^M . Hence with $C_t^A = Y_t^A(z^R) = X_t^A F_t(z_R)$ and $C_t^M = Y^M(z_R)$, we obtain

$$p_t^M(z^R) = \frac{(1 - \alpha_A)}{\alpha_A} \left[\frac{X_t^A F_t(z_t^R) - \bar{c}^A}{Y_t^M(z_t^R)} \right]. \quad (8)$$

Notice that the price of non-agricultural goods, $p_t^M(z^R)$ is strictly increasing in z^R (when the urban areas shrink). Moreover, the price of non-agricultural goods goes to zero if the rural areas becomes too small, i.e. z_t^R approaches $F_t^{-1}\left[\frac{\bar{c}^A}{Z_t^A}\right]$. Lastly, $p_t^M(z^R)$ goes to $+\infty$ when the city completely over disappears, i.e. if $z^R \rightarrow +\infty$. These simple properties of the price $p_t^M(z^R)$ will be useful to establish the equilibrium location decisions.

For brevity, we will define $\theta[p^M] \equiv [\alpha_A]^{\alpha_A} \left[\frac{1-\alpha_A}{p^M}\right]^{1-\alpha_A}$, a simple function that will be used to describe the utility levels of the different households in the different location options.

According to their skills z , the welfare of the households in each location is:

- *Rural Area:* The only option available is to work in agriculture, offering low skill services. Having normalized $p_t^A = 1$, the income of everyone is simply Z_t^A in units of agriculture goods. The attained utility of everyone in the country-side is:

$$V^R(z^R) = \theta[p_t^M(z^R)] [X_t^A - \bar{c}^A]. \quad (9)$$

It is independent of the individual's skill level and strictly decreasing in z^R , as smaller cities lead to more expensive non-agricultural goods.

- *Urban Area:* The attained utility of the households livings in cities depends both, on their occupation and skills levels. Unitary wage rates for amenable and basic qualified labor services are given by

$$w_t^a(z^R) = (1 - \eta) p_t^M(z^R) X_t^M \left(\frac{L_t^b(z^R)}{L_t^a(z^R)} \right)^\eta; \quad w_t^b(z^R) = \eta p_t^M(z^R) X_t^M \left(\frac{L_t^a(z^R)}{L_t^b(z^R)} \right)^{1-\eta}, \quad (10)$$

where the aggregate labor supplies are defined as $L^b(z^R) = F_t[z^H(z^R)] - F_t(\max\{z^R, z_{\min}\})$, and $L^a = \int_{z^R}^{\max\{z^R, z_{\min}\}} z \mu_t(dz) + \int_{z^H(z^R)}^\infty z f(z) dz$, and the function $z^H(z^R)$ is the optimal occupation split as defined in Proposition 1. Using these functions, the attainable utility, $V^U(z; z^R)$, of a household with skills z living in the urban area, conditional on an urban-rural divide z^R , is given by

$$V^U(z; z^R) = \begin{cases} \theta[p_t^M(z^R)] [w^b(z^R) - \bar{c}^A], & \text{if } z \in [\max\{z^R, z_{\min}\}, z^H] \\ \theta[p_t^M(z^R)] [w^a(z^R) - \bar{c}^A], & \text{otherwise} \end{cases}$$

The value of the marginal migrant, $MV^U(z^R)$, is simply $MV^U(z^R) \equiv V^U(z^R; z^R)$. Therefore, the equilibrium urban-rural threshold z^R is determined by the condition

$$MV^U(z^R) = V^R(z^R).$$

In the Appendix B we prove the following

Proposition 3 *Given a continuous distribution μ_t with support in $[0, \infty)$, if either α_A or $\bar{c}^A$ are low, there exists a unique threshold z^R that solves the equilibrium location decisions in a costless housing city economy.*

If the equilibrium threshold is such that $z^R < z_{\min}$, the economy will exhibit **urban low skill services jobs**. If so, the threshold z^R must solve the condition that the marginal worker in low skill urban jobs makes the same earnings as an agricultural worker in the rural area

$$z^R = \frac{X_t^A}{w^a(z^R)}.$$

If instead, the threshold is such that $z^R > z_{\min}$, the equilibrium will exhibit **only urban high skill service jobs**. Since qualified jobs have to be positive (because of the Inada condition on Y^M), then the marginal migrant from the rural area must be employed as qualified worker in the urban area. Hence, the threshold condition is given as the solution to

$$w^b(z^R) = X_t^A.$$

As we illustrate below, the actual form of equilibrium in the economy depends on the parameters of the economy and crucially on the initial skill heterogeneity μ_t . We discuss this further in the general setting below.

Housing Cost, $p_t^h > 0$, No slums, $\tau = 1$ Consider now the economy in which individuals face a non-zero housing cost to live (and work) in the city. By maintaining $\tau = 1$, for now we rule out the emergence of slums and urban areas are entirely composed of the formal city.

Occupation choices inside the city are given described by the same condition (6) and Proposition 1. As for the relative demand for of goods per households, it is also the same as in condition (7). The key difference is at the aggregate level, because some of the non-agricultural output has to be used for housing. Let $\Psi_t(z^R) \equiv [1 - F_t(z^R)]$ be the mass of people living in cities. Therefore, conditional on a urban-rural divide z^R , the equilibrium relative price of non-agricultural is given by

$$p^M(z^R) = \frac{(1 - \alpha_A)}{\alpha_A} \left[\frac{Z^A F(z^R) - \bar{c}^A}{Y^M(z^R) - \xi \Psi_t(z^R)} \right]. \quad (11)$$

It is straightforward to show that $p^M(z^R)$ is strictly increasing in z^R , that it goes to zero if z^R approaches $F^{-1} \left[\frac{\bar{c}^A}{Z^A} \right]$ and goes to $+\infty$ if $Y^M(z^R) \rightarrow \xi \Psi_t$ (i.e., if all non-agriculture production is consumed to produce houses in the cities).

Finding the equilibrium z^R is identical to the case with no housing.

- *Rural Area:* Remains exactly as before. The utility attain by all households in rural areas, $V^R(z^R)$, is given by the expression (9.)
- *Urban Area:* The wage rates for both urban occupations are as in (10). However, the utilities attained by city dwellers must account for the cost of housing:

$$V^U(z; z^R) = \begin{cases} \theta [p_t^M(z^R)] [w^b(z^R) - \xi p^M(z^R) - \bar{c}^A], & \text{if } z \in [\max\{z^R, z_{\min}\}, z^H], \\ \theta [p_t^M(z^R)] [w^a(z^R) z - \xi p^M(z^R) - \bar{c}^A], & \text{otherwise.} \end{cases}$$

The value of the marginal migrant, $MV^U(z^R)$, is simply $MV^U(z^R) \equiv V^U(z^R; z^R)$. Therefore, the equilibrium urban-rural threshold z^R is determined by the condition $MV^U(z^R) = V^R(z^R)$ as before. To extend Proposition 2 to the case of positive housing costs, we need to limit the cost of housing costs.

Proposition 4 *Given a continuous distribution μ_t with support in $[0, \infty)$, if (i) ξ is low and (ii) either α_A or $\bar{c}^A$ are low, there exists a unique threshold z^R that solves the equilibrium location decisions in a costless housing city economy.*

If the equilibrium threshold $z^R < z_{\min}$, the economy will exhibit **urban low skill services jobs**. With housing costs, the condition for this form of equilibrium is

$$z^R = \frac{X_t^A + \xi p^M(z^R)}{w^a(z^R)}. \quad (12)$$

If instead, the equilibrium threshold $z^R > z_{\min}$, the equilibrium will only exhibit urban high skill service jobs, and the threshold is given by

$$w^b(z^R) = X_t^A + \xi p^M(z^R). \quad (13)$$

As we illustrate below, the actual form of equilibrium in the economy depends crucially on the initial skill heterogeneity μ_t . We discuss this further in the general setting below.

Housing Costs, $p_t^h > 0$, and Slums, $\tau < 1$. We now allow for the possibility of working in the city but avoiding paying for housing, i.e. living in a slum. Needless to say, for this option not to be the one selected by every urban dweller, it has to come at some cost. In this paper, we have model such a cost as a loss $0 < \tau < 1$ of the household's gross-of-subsistence consumption. This loss is equivalent to an additional time cost, similar to an iceberg cost, that worker in a slum must supply to generate the same income.

Under our assumptions, the option to live in a slum or a city does not distort the occupation choices (and supplies of skills) for urban dwellers. Hence, the occupation choices remain characterized by condition (6) and Proposition 1, and, the individual demand of goods by condition (7).

To establish whether slums arise in equilibrium, we need to establish the existence threshold $z^F > z^R$, defining a set $\mu_t[z^R, z^F]$ of individuals that live and work in the urban area, but do not pay for housing. If so, we also need to explore the uniqueness of such equilibrium.

In the static (myopic) economy, the location decisions do not distort the occupation decisions inside the urban areas. In particular, the same condition (6) and *Proposition 1* characterize the occupation choices, given any urban-rural divide z^R . As for the relative price of goods, it is given by

$$p^M(z^R) = \frac{(1 - \alpha_A)}{\alpha_A} \left[\frac{Z^A F(z^R) - \bar{c}^A}{Y^M(z^R) - \xi \Psi_t(z^F)} \right], \quad (14)$$

where the only difference with (11) is that now the mass of housing is given by $\Psi_t(z^F)$, not $\Psi_t(z^R)$.

Finally, as for the inhabitants of the city, two dwelling options, lead to two possible values: the city

$$V^C(z; z^R) = \begin{cases} \theta [p_t^M(z^R)] [w^b(z^R) - \xi p^M(z^R) - \bar{c}^A], & \text{if } z \in [\max\{z^R, z_{\min}\}, z^H], \\ \theta [p_t^M(z^R)] [w^a(z^R) z - \xi p^M(z^R) - \bar{c}^A], & \text{otherwise.} \end{cases}$$

and the slum

$$V^F(z; z^R) = \begin{cases} \theta [p_t^M(z^R)] [(1 - \tau) w^b(z^R) - \bar{c}^A], & \text{if } z \in [\max\{z^R, z_{\min}\}, z^H], \\ \theta [p_t^M(z^R)] [(1 - \tau) w^a(z^R) z - \bar{c}^A], & \text{otherwise.} \end{cases}$$

Conditional on an urban-rural divide, z^R , for any value of skills z , the attainable utility of living in the urban area is the upper envelope of these two options,

$$V^U(z; z^R) = \max \{V^F(z; z^R), V^F(z; z^R)\}.$$

The conditions for the emergence of slums, in equilibrium, are very simple. First, by assuming that μ_t has an unbounded support, it is never the case that the cities are empty: There is always a mass of individuals for whom it is cheaper to pay the housing costs than the potentially unbounded cost of living in a slum. In consumption terms, the cost of living in the city is fixed, independent of z , and given by $\xi p^M(z^R)$. On the other hand, the cost of living in a slum, would be the same, $\tau [w^b(z^R) - \bar{c}^A]$, for all workers providing qualified skills and increasing in the worker's skill, $\tau [w^a(z^R) z]$ those in the amenable sector.

Except for a knife-edge case, the threshold z^F that defines the size of slums is given by

$$z^F = \frac{[\xi p^M(z^R) / \tau]}{w^a(z^R)}. \quad (15)$$

Conditional on z^R , the following result summarizes whether a slum will arise in equilibrium and, if so, the skill composition of its inhabitants:

Proposition 5 *Given a continuous distribution μ_t with support in $[0, \infty)$, and conditional on an urban-rural divide threshold z^R , there exists a unique configuration of the urban areas. The slum threshold z^F (15) and the occupation threshold z^H (6) define the population and skill composition of slums as follows: (i) **No slums:** if $z^F \leq z^R$, i.e. the slums are empty; (ii) **Slums with low-skill workers:** if $z^R < z^F < z_{\min}$, in which case, some low skill workers and all qualified workers and high skill workers live in the formal city; (iii) **Slums with qualified (and low) skilled workers:** if $\max\{z^R, z_{\min}\} < z^F < z^H$; in this case, all qualified workers will live in the slum, and, if the city exhibits low skill urban jobs, then, all of them will be in the slum; all the high skill workers will be in the formal city. Finally (iv) **Slums for all but the very high skill**, when $z^F > z^H$, in which all qualified workers, all low skilled workers and some of the high skill workers live outside the formal city.*

Conditional on z^R , once the internal urban locations and occupations decisions have been sorted out, we can solve for the equilibrium urban-rural decisions by the condition

$$V^R(z^R) = M^U(z^R),$$

where $M^U(z^R) \equiv V^U(z^R; z^R)$.

The conditions for the no-slum equilibrium are given by (12) or (13) with $z^F < z^R$. The conditions an equilibrium (ii), slums with only low-skilled urban workers is $z^R < z^F < z_{\min}$, the condition is,

$$\frac{X_t^A}{(1 - \tau) w^a(z^R)} < \frac{[\xi p^M(z^R) / \tau]}{w^a(z^R)} < z_{\min}.$$

Similarly, for case (iii) a slums with qualified and, possibly, low but not high skilled workers, the conditions are

$$\xi p^M(z^R) = \tau w^b(z^R), \text{ with } z^R < z^F \in [z_{\min}, z^H].$$

Finally, for case (iv), when the city is populated only by the high tail of the high skilled service workers, the equilibrium conditions are $z^H < z^F$, i.e.

$$\tau w^b(z^R) < \xi p^M(z^R).$$

We can show, in general, that the existence and uniqueness result of an economy with housing costs but no slums carries to this case. More interestingly, the possibility of living in a slum lead to larger urban areas, and to structural transformation (here, the expansion of the non-agricultural sector.) Formally:

Proposition 6 *Given a continuous distribution μ_t with support in $[0, \infty)$, if either α_A or $\bar{c}^A$ are low, there exists a unique threshold z^R that solves the equilibrium location decisions in a costless housing city economy. Consider two different values for the cost of living in slums, $0 \leq \tau_0 < \tau_1 < 1$, and let $z^R(\tau)$ and $z^F(\tau)$ be thresholds associated to the economy with τ . Then, lower costs τ lead to larger urban areas, $z^R(\tau_0) \leq z^R(\tau_1)$, smaller formal cities, $z^F(\tau_0) \leq z^F(\tau_1)$ and lower non-agricultural prices, $p^M[z^R(\tau_0)] < p^M[z^R(\tau_1)]$.*

Before highlighting the role of slums in facilitating urban growth and structural transformation, we must explore their impact in the dynamics of skills formation over time.

5.3 Dynamic Economies

We now retake the model with child and altruistic/paternalistic preferences, with $\beta > 0$. Recall that the utility of households is given by, $V_t = u(c_t) + \beta E_t[z_{t+1}|Z_t^j]$, where $Z_t^j \equiv \left[\int z^\rho \mu_t^j(dz) \div \int \mu_t^j(dz) \right]^{1/\rho}$, for locations $j = R, F, C$. That is, we are assuming that the average human capital of the child depends entirely on the average human capital of the parents in the location where he grows up, i.e. we are neither allowing for spillovers across regions, nor for their own parents to have a direct input. The first one is easy to generalize. The second one would lead to a very difficult computation problem of keeping up the next period distribution of skills, since the transition function will be household and location dependent.

5.3.1 Conditional Occupation Choices

Given μ_t and conditional on z^R , occupation choices in the dynamic model are exactly the same as in the static economy, i.e. the behavior of z^H is described by condition (6) and *Proposition 1*. Moreover, the relative price $p_t^M(z_t^R)$ will have exactly the same form. For brevity, let us define $e^*(z; z^R)$ the optimized income (with respect to occupations) for an individual with skills z living in the urban area:

$$e^*(z; z^R) = \begin{cases} w^a(z^R) \cdot z & \text{if } z < z_{\min} \text{ or } z \geq z^H(z^R), \\ w^b(z^R) & \text{if } z \in [z_{\min}, z^H(z^R)]. \end{cases}$$

This function applies regardless of housing costs and whether there slums or not in equilibrium.

No Housing Costs, $p_t^h = 0$ Consider first, the case with no housing costs. Given any z^R , the relative price $p_t^M(z_t^R)$ is given by expression (8.) The attainable utilities are

- *Rural Area:*

$$V_t^R(z^R) = \theta [p^M(z^R)] [X_t^A - \bar{c}^A] + \beta E_t \{ z_{t+1} | Z_t^R [0, z_t^R] \},$$

This function can be non-monotone in z_t^R . On one hand, from the consumption size, the higher z_t^R , the higher will be the price $p^M(z^R)$ and the lower the utility of consumption of goods. On the other hand, the higher z^R is, the higher is the average $Z_t^R [0, z_t^R]$ and the higher is the expected level of skills for children growing up in the rural area.

- *Urban Area:* The value of living in a city depends on z as before:

$$V_t^C(z; z^R) = \theta [p^M(z^R)] [e^*(z; z^R) - \bar{c}^A] + \beta E_t \{z_{t+1} | Z_t^C[z_t^R, \infty)\}.$$

Obviously, $V_t^C(\cdot; z^R)$ is increasing in z . More interestingly, the function $V_t^C(z, \cdot)$ is non-monotone. On the one hand, the consumption possibilities and utilities from consumption increases with z^R as the price of non-agricultural good, and the wages w^a, w^b increase with smaller cities. On the other hand, a higher z^R , implies a more selective city, translating into higher expected skills for the children. Then, the function $MV^C(z^R) = V_t^C(z^R; z^R)$ that defines the value for the marginal migrant into the city can be also non-monotone.

The equilibrium threshold is given by the condition $V_t^R(z^R) = MV^C(z^R)$, which can be solved as

$$e^*(z^R; z^R) = X_t^A - \frac{\beta [E_t \{z_{t+1} | Z_t^C[z_t^R, \infty)\} - E_t \{z_{t+1} | Z_t^R[0, z_t^R]\}]}{\theta [p^M(z^R)]}. \quad (16)$$

Two key implications arise. First, in any equilibrium, the *marginal* migrant, the household with $z = z^R$, living in the city would earn less than in the country side. This is because there is a premium to living in the city in terms of the children's education, i.e. the second term in the right hand side is compensating variation. This compensating variation is always positive but non-monotone, as it captures the non-monotonicities in $MV^C(z^R)$ and $V_t^R(z^R)$.

Housing Cost, $p_t^h > 0$, No slums, $\tau = 1$. Consider now the case with positive housing costs, but no slums because $\tau = 1$. Given any z^R , the relative price $p_t^M(z_t^R)$ is given by expression (11). The attainable utilities are

- *Rural Area:* Same as above.
- *Urban Area:*

$$V_t^C(z; z^R) = \theta [p^M(z^R)] [e^*(z; z^R) - \xi p^M(z^R) - \bar{c}^A] + \beta E_t \{z_{t+1} | Z_t^C[z_t^R, \infty)\}.$$

Therefore, the threshold for the rural-urban divide z^R is given by the condition:

$$e^*(z; z^R) = X_t^A + \xi p^M(z^R) + \frac{\beta E_t \{z_{t+1} | Z_t^R[0, z_t^R]\} - \beta E_t \{z_{t+1} | Z_t^C[z_t^R, \infty)\}}{\theta [p^M(z^R)]}.$$

The previous conclusions for the income of the marginal city dweller apply, once the income has been netted of housing costs. What is new here is that housing costs are endogenous to z^R , and, under our assumptions, larger cities lead to lower non-agricultural prices, and hence, lower housing costs, a force that adds an additional source of multiple equilibria.

Housing Costs, $p_t^h > 0$; Slums possible, $\tau < 1$. We first consider the decisions inside a city. Given z^R , the occupation decisions z^H are given by by condition (6) and the equilibrium price (14). The values functions for the rural area, $V_t^R(z^R)$ is exactly as in the previous section. If $z^F > z^R$, indicates the threshold below which an individual lives in a slum, then, the value of living in a slum or in a city are

$$\begin{aligned} V_t^F(z; z^R, z^F) &= \theta [p^M(z^R)] [(1 - \tau) e^*(z; z^R) - \bar{c}^A] + \beta E_t \{z_{t+1} | Z_t^F[z_t^R, z_t^F]\} \\ V_t^C(z; z^R, z^F) &= \theta [p^M(z^R)] [e^*(z; z^R) - \xi p^M(z^R) - \bar{c}^A] + \beta E_t \{z_{t+1} | Z_t^C[z_t^F, \infty)\}. \end{aligned}$$

Then, in the generic case in which $z^R < z^F$, i.e. these two threshold are given, respectively, by the conditions $V_t^R(z^R) = V_t^F(z^R; z^R, z^F)$ and $V_t^F(z^F; z^R, z^F) = V_t^C(z^F; z^R, z^F)$. These two conditions can be written as

$$e^*(z^R; z^R) = \frac{X_t^A}{(1-\tau)} - \frac{\beta [E_t \{z_{t+1} | Z_t^F [z_t^R, z_t^F]\} - E_t z_{t+1} | Z_t^R [0, z_t^R]]}{(1-\tau) \theta [p^M(z^R)]},$$

and

$$e^*(z^F; z^R) = \left(\frac{\xi}{\tau}\right) p^M(z^R) - \frac{\beta [E_t \{z_{t+1} | Z_t^C [z_t^F, \infty)\} - E_t \{z_{t+1} | Z_t^F [z_t^R, z_t^F]\}]}{\tau \theta [p^M(z^R)]}.$$

From the first expression, persons moving from the rural area to the slum, has to be compensated by the additional living costs τ , but is compensated by the better education prospects of their children in the slums relative to the country side. From the second expression, persons moving from the slum to the city will trade-off the housing costs of the city with the additional costs of the slum, and will also consider the better education prospects of the city.

6 Calibrating the Model to Brazil

6.1 Parameter Values and Initial Conditions

We need to determine the value of the preference parameters β , α and $\bar{c}^A$; the parameters of living costs in slums τ and in cities ξ ; and the technology parameters η , ϕ and $z_{\min}$. Moreover, we have to find the time series of productivity for each sector A_t^i , where $i \in \{A, M\}$. Regarding the skill distribution, we are going to assume that individual skills follow a Gamma distribution and each location $j \in \{R, F, C\}$ has a different initial scale parameter θ_0^j , but the same shape parameter k . We also have to pin down ρ , which determines the strength of human capital externalities. Table 25 shows us all the calibrated parameters.

The intertemporal preference parameter β is calibrated following the literature. Since one period time is 30 years in this model, we set β equal to 0.294 ($=0.96^{30}$). The parameter of agriculture weight on preference follow the calibration of Herrendorf et al. (2011). And the subsistence consumption level $\bar{c}^A$ is computed so that the model can reproduce the labor share in the agriculture sector for both periods (1951-1980 and 1981-2010).

Regarding the costs of living, we set the cost in slums for the second period (1981-2010) as 24.5% of the income ($\tau_{81-10} = 0.245$), following the numbers presented by the data²⁵. For the first period (1951-1980), since there is no data to support us, we choose $\tau_{51-80} = 0.19$ in order to reproduce the share of people living in slums. Following the same strategy, we set living costs in cities $\xi_{51-80} = 0.1$ and $\xi_{81-10} = 0.6$ in order to match the data.

The parameter η represents the share of basic labor in the total income of the non-agriculture sector. In the data, we interpret basic as the labor offered by individuals who have secondary schooling level (11 years of education). Using the Brazilian household survey, PNAD, we take the workers with 11 or more years of education and then multiply the number of individuals in this group by their average income to obtain the total income and divide it by the income of the entire non-agricultural sector. We find that this participation ranges from 0.35 to 0.41, using surveys from different years. Thus, we set $\eta = 0.6$. Since we take those individuals with secondary education as basic workers, we determine the minimum skill to offer basic labor as the years of education to complete secondary school, i.e., $z_{\min} = 11$. Regarding the curvature of amenable labor offer, we assume that the earnings for this labor type are linear on the individual skill, i.e., $\phi = 1$.

²⁵See Table 21.

Following the literature, productivity dispersion across sectors is crucial for the process of structural transformation. We measure the productivity in agriculture as the ratio between the value added and the total number of workers, using the data from GGDC. For the first period, the agricultural productivity is normalized to one, i.e., $A_{51-80}^A = 1$; and for the second period, $A_{81-10}^A = (1 + \gamma^A) A_{51-80}^A$, where γ^A is the growth rate of the average productivity between the periods 1951-1980 and 1981-2010. And the non-agricultural productivity for both periods is set in order to match the production share with the data.

Regarding the skill distribution, we assume that each location has its own Gamma distribution. But we set the same shape parameter $k = 2.4$ for all areas (rural, slum and city) and for both periods. The initial shape parameter θ_0^j for each location j is chosen in order to match the schooling distribution in each area for both periods. And ρ , which measures the strength of human capital externalities, is set to 1. This parameter is crucial to determine the human capital accumulation over years, so when ρ is high we see that human capital evolves very fast. Table A1 in the appendix presents the results of our calibration procedure.

6.2 Benchmark Economy

Table 9 presents the results for initial and final periods.

Table 9: Numerical Results

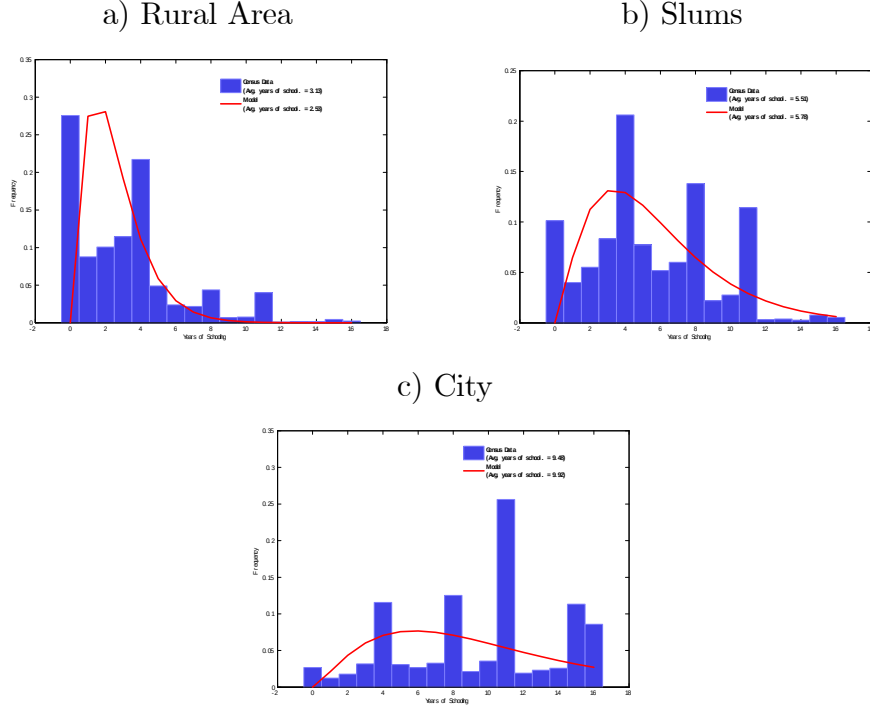
Variable	Brazil			
	1980		2010	
	Data	Model	Data	Model
Slum Pop.	10.34%	10.96%	18.70%	18.84%
City Pop.	57.26%	56.58%	66.30%	63.63%
L^A	38.15%	32.46%	16.70%	17.53%
Y^A/Y	6.85%	4.67%	5.72%	2.29%
Avg. years of school., Rural	1.46	2.20	3.13	2.53
Avg. years of school., Slums	NA	4.07	5.51	5.78
Avg. years of school., City	NA	4.27	9.48	9.92

The model closely reproduces the allocation of labor across the two productive sectors. For the first period, the model underestimates by around 6 p.p. the labor share in agriculture. And for the second period, the data is reproduced quite well, and the model overestimates by less than 1 p.p. Results regarding the production share in agriculture are not as good as the labor allocation. We can observe that there is an underestimation of this variable by 2.2 p.p initially and it increases to 3.4 p.p. at the final period. But in general, the process of structural transformation seems to be explained by the model.

Now talking about the urbanization process and the formation of slums, we observe that the model makes a really good job. The population share living in the cities is underestimated by only 0.7 p.p. in 1980 and by 2.7 p.p. in 2010. And looking to the share living in slums, the model underestimates this variable for both periods by less than 1 p.p..

Regarding the human capital distribution and accumulation across the years, we can only compare the results for the years 1981 - 2010 because we do not have data available for the first period (1951 - 2010). From Figure 3 we can observe the distribution of years of schooling across the population in the three different locations.

Figure 3: Human Capital Distribution (1980-2010)



Note that the Gamma distributions produced by our model follow the data quite well. And from Table 26 we see the average years of schooling (human capital) in each location.

7 Policy Counterfactuals

In this section we study two main counterfactual exercises: changes on living costs in the cities and in slums and education policies in slums. For the first class of these counterfactuals, we analyze what would happen to the economy if living in the cities became more expensive or cheaper in the first and second periods. Also, we run an experiment in which living in slums is prohibited or the costs in terms of income get higher. The second class is represented by a police in which we select kids from slums and put them to study in the cities. We can do that by adopting two strategies: select a random fraction of kids or choose the ones with the highest skills.

7.0.1 Increasing the Housing Costs in Cities

Here we are interested in checking how a rise in the housing cost in the city can impact the economy. In Table 10 we analyze the cases where the cost of living in the cities in terms of non-agriculture goods rises in the first and second period, maintaining constant cost for the other period.

Table 10: Counterfactual Results, Housing cost in the city

Variable	$\xi_1^C = 0.15$		$\xi_1^C = 0.2$		$\xi_2^C = 0.3$		$\xi_2^C = 0.9$	
	1980	2010	1980	2010	1980	2010	1980	2010
Slum Pop.	46.32%	24.50%	55.30%	24.90%	10.96%	1.44%	10.96%	31.63%
City Pop.	21.22%	57.46%	12.24%	57.50%	56.58%	81.03%	56.58%	50.84%
L^A	32.46%	18.03%	32.46%	17.60%	32.46%	17.53%	32.46%	17.53%
Y^A/Y	6.85%	4.67%	4.73%	2.28%	4.67%	2.34%	4.67%	2.26%
$E(z Z^R)$	2.20	2.53	2.20	2.54	2.20	2.53	2.20	2.53
$E(z Z^F)$	6.10	6.21	6.95	6.31	4.07	4.2	4.07	7.23
$E(z Z^C)$	6.69	10.66	8.14	10.63	4.27	8.32	4.27	11.44

When we rise ξ_1 from 0.1 (benchmark economy) to 0.15, the slum population increases substantially in the first period, going from 10.34%(benchmark model) to 46.32%, and in the second period there is also a rise but much smaller (going from 18.84% to 24.50%). At the same pace, the share of people living in cities changes from 57.26% to 21.22% in the first period, and from 63.63% to 57.46% in the second one. But the agricultural labor share remains the same, i.e., the process of structural transformation is not affected and the migration from rural to urban areas follows the benchmark economy. Since the rural population does not change for both periods, the human capital (average years of schooling, $E(z | Z^R)$) remains the same as well. And with more people living in slums, the average skill increases there for both periods. At the same time, only the most skilled individuals can afford to live in the city and because of that the human capital increases in this location. From Table 10 we observe the same pattern when living cost rises to 0.2.

For the second period, we first analyze the effects of a decrease of the living cost. When ξ_2 goes from 0.6 to 0.3, living in the city becomes cheaper and less skilled individuals can afford to buy a house. Therefore, people move from slums to legal settlements in urban areas and the cities become more populated. Also, the average human capital decreases in both urban locations (slum and city). It is noteworthy to mention that there is no change for rural area (population and human capital remain the same) and no change for all variables in the first period (since the living cost changes only for the second period).

We also study what happens to the economy when the living cost increases even more between the first and second period, i.e., when ξ_2 goes from 0.6 to 0.9. With a higher living cost in the city, only the most skilled individuals can afford a house and because of that the slum population increases. Thus, we have more skilled people living in slums and this results in a higher human capital accumulation in this location. In the cities, we have less people but they are more skilled than before, so the average years of schooling rises as well.

7.0.2 Cracking-down Slums

From Table 11, when we raise the cost of living in slums for the first period from 0.19 to 0.5, people choose to live in the cities, the population share in slums falls from 10.96% to 2.86% and in the cities, rises from 56.58% to 64.69% in 1980. Note that the population in rural areas stays the same. Because of this movement from slums to the cities, the human capital falls in both locations in 1980. Given this new configuration in 1980, even with all parameters constants for the second period, the share of people living in slums increase slightly and the share of those in the cities falls in 2010. We also observe that the rural population decreases. Regarding human capital accumulation, the average years of schooling falls slightly in slums and cities and stays the same in rural areas.

Table 11: Counterfactual Results, Housing cost in slums

Variable	$\tau_1^C = 0.5$		$\tau_1^C = 1$		$\tau_2^C = 0.5$		$\tau_2^C = 1$	
	1980	2010	1980	2010	1980	2010	1980	2010
Slum Pop.	2.86%	16.95%	0%	26.35%	10.96%	1.42%	10.96%	0%
City Pop.	64.69%	66.27%	16.66%	57.98%	56.58%	79.61%	56.58%	29.99%
L^A	32.46%	16.78%	82.66%	15.66%	32.46%	18.97%	32.46%	70.01%
Y^A/Y	4.65%	2.45%	1.43%	2.75%	4.67%	2.04%	4.67%	2.26%
$E(z Z^R)$	2.20	2.53	4.77	3.29	2.20	2.66	2.20	8.53
$E(z Z^F)$	3.72	5.67		7.42	4.07	4.44	4.07	
$E(z Z^C)$	3.95	9.62	7.33	10.21	4.27	8.44	4.27	14.88

We also analyze the case in which a public policy bans slums in urban areas at the first period, i.e., $\tau_1 = 1$. Getting out of rural areas becomes expensive with this policy, i.e., only the most skilled people can afford to live in urban areas. Because of that, the labor share in rural areas increases from 32.46% (benchmark economy) to 82.66% in 1980 and the urban population decreases from 67.54% to 16.66%. With more skilled people in rural areas and fewer less skilled ones in urban areas, the human capital increases in both locations in 1980. For the second period, we use the same parameters of the benchmark model and the formation of slums is allowed. Therefore, now the less skilled individuals can migrate from rural to urban areas and live in slums if they can afford to buy a house in the city. Because of that the slum population increases from 18.84% (benchmark model) to 26.35% and the labor employed in rural areas (agriculture) falls slightly (17.53% to 15.66%). Note that in the first period we have a big human capital accumulation in rural areas and even when we have a big mass of more skilled people moving from rural to urban areas, the average skills in rural locations decrease but they are still bigger (3.29) than in the benchmark economy (2.53). And with more people living in slums, the human capital increases there and in the cities if we compare to the benchmark economy.

We also make the experiment of raising the cost of living in slums for the second period from 0.245 to 0.5. First of all, note that the variables do not change in the first period because all parameters are the same as in the benchmark model. In the second period, the slum population falls from 18.84% to 1.44% and those living in the cities rise from 63.63% to 79.61%. The structural transformation, i.e., the labor share in agriculture, changes slightly, increasing from 17.53% to 18.97%. Regarding the human capital accumulation, the average skill rises in rural areas but falls in slums and cities.

And when we prohibits slums, living in urban areas become very expensive and only the most skilled individuals can afford to buy a house. Therefore, the city population falls from 63.63% to 29.99%, as people are moving out of cities to rural areas. And because of that, the labor share in agriculture increases from 17.53% to 70.01%. Since more skilled people are living in rural areas and only the most skilled ones can afford a house in the cities, the human capital raises in both locations.

8 Conclusions

After World War II, Brazil witnessed an episode of accelerated economic growth lasting until the early 1980's, a process of catching-up to the U.S. economy. The country experienced both high

productivity and output growth and a massive rural-urban migration. By the 1970's Brazil became predominantly urban, with people living in the cities and production concentrated in manufacturing and services. During the accelerated urbanization process (from 1950 to 1980), the formation of slums became a reality in big cities like Rio de Janeiro and São Paulo. Here we sustained the hypothesis that at first this rural-urban migration resulted in great economic growth (1950 - 1980) but after that Brazil experienced slowdown and stagnation. This first wave of rapid growth is due to the migration of workers from agriculture (rural areas) to non-agriculture sectors (urban areas), i.e., the reallocation of labor from a low productivity sector to the higher one. But these migrants had low skills and low income and when they got in the cities, they could not afford to buy a house and ended up living in slums and offering low skill labor in urban areas. This phenomenon resulted in a fall in non-agriculture productivity after 1980 and stagnation of the economy.

Here we have proposed a model which combines the elements of structural transformation literature to urbanization, formation of slums and human capital distribution. Furthermore, we considered in the model public policies of prohibition of slums and of housing costs in the cities. The model provided a good fit for the distribution of population across the three locations and the labor shares between the two sectors, as well as the human capital accumulation in each area through the periods. From counterfactual exercises, we concluded that the rise of housing costs in the cities makes people move to slums but at the same the human capital accumulation get higher in urban areas. So it is worth noting that the structural transformation process is not affected at all by the rise of housing costs. However, the public policy of prohibition of slums does affect the structural change and the urbanization process. People tend to stay in rural areas and only the most skilled individuals can afford to live in the city. So, the average skills in every location get higher. Therefore, using a structural transformation model blended with human capital distribution and housing decisions, we could explain the structural change experienced by the Brazilian economy, the urbanization process (and the formation of slums) and the human capital accumulation along two periods of clearly different growth paces.

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Table A1: Calibration

Parameter	Value	Source/Target
β	0.294	Based on literature
α	0.01	Based on literature
$\bar{c}^A$	0.245	Share of agricultural labor
ϕ	1	Normalized
$z_{\min}$	11	Data
η	0.6	Data
τ_1	0.19	Data
τ_2	0.245	Data
ξ_1	0.1	Share of city pop.
ξ_2	0.6	Share of city pop
A_{51-80}^A	1	Normalized
A_{81-10}^A	2.5	Data
A_{51-80}^M	10	Production share
A_{81-10}^M	11	Production share
$z \sim \text{Gamma Distribution}(\theta_0^j, k)$		
k	2.4	Human capital distribution
θ_0^R	0.8	Human capital distribution
θ_0^F	1	Human capital distribution
θ_0^C	2	Human capital distribution
ρ	1	Human capital accumulation

A Data

The sample covers Brazil and the years 1950 to 2010. The series of real value-added and employment by sectors were taken from the Groningen Growth and Development Centre (GGDC) database²⁶, which provides long-run internationally comparable statistics for 42 countries (Africa, Asia, Latin America, Europe and United States). The data set includes series of value added, output deflators and persons employed for ten productive sectors.

Although the data covers ten broad sectors, here they were grouped into two major sectors: agriculture and non-agriculture, following the structural transformation literature. The sectors are defined by the ISIC Rev. 3²⁷ and were grouped as follows: agriculture consists of agriculture, forestry and fishing (01-05); and non-agriculture is composed by mining (10-14), manufacturing (15-37), public utilities (40-41), construction (45), wholesale and retail trade, hotels and restaurants (50-55), transport, storage and communication (60-64), finance, insurance and real estate (65-74), and community, social, personal and governments services (75-99). Following the literature, the productivity series were constructed as the ratio between the real value added and the persons employed by each sector for the period 1950 to 2010.

From the Brazilian Census, we explore interesting characteristics and dynamics of the economy since 1950. The Census is a meticulous survey of all households in the country, conducted every ten years by the Brazilian Institute of Geography and Statistics (IBGE²⁸). In Appendix B and C, we can see tables and figures showing the population distribution between rural and urban areas, the

²⁶See Timmer et al. (2014).

²⁷International Standard Industrial Classification of All Economic Activities, Rev. 3.

²⁸See www.ibge.gov.br/english/.

levels of education, the average personal income and the labor distribution by productive sectors. In addition, for the years 1991 and 2000, the Census provides an interesting variable, telling us if an household lives in a "subnormal agglomerate"²⁹. The description is almost equivalent to slums and very poor settlements. Thus, following the Brazilian literature on this subject, we use here slums³⁰ and "subnormal agglomerate" interchangeably. With this variable, we are able to compute some facts about the slum population and compare them with the urban dwellers (city or metropolitan area³¹).

Regarding the characteristics of slums, we also have the Favela Census³², conducted by the state government of Rio de Janeiro³³ in 2010. This Census is a unique initiative of mapping and identifying the profile of residents who live in the three biggest slums (Alemão, Mangueiras and Rocinha) of Rio de Janeiro. This data allowed us to verify where the slum dwellers study and work (inside or outside these poor settlements³⁴), which will give empirical support to the assumptions made in our model.

In this paper we are also interested in investigating how the transmission of education between parents and children occurs in different areas (rural, urban and slums). We use the social mobility supplement³⁵ of PNAD (Pesquisa Nacional por Amostra de Domicílio³⁶) for 1988 and 1996 to compute the transition matrices and some interesting statistics³⁷. We do that for the whole economy and the rural and urban areas. Unfortunately, these surveys do not track the slum households. In order to solve this problem and have education data to compare slums and cities, we assume that the households living in metropolitan areas with total income in the 35 percentile or lower are slum dwellers and the ones with income above the 35 percentile live in the cities. This assumption is based on the Tables 6, where we can see that the total income in slums is on average 35% of the cities.

²⁹The IBGE defines "subnormal agglomerate" as a set of 51 or more housing units characterized by absence of a proper ownership title and at least one of the following aspects:

- Irregular traffic routes or irregular size (shape) of land plot;
- Lack of essential public services such as garbage collection, sewage system, electricity and public lighting.

³⁰The UN Habitat defines a slum household as a group of individuals living under the same roof and lacking one or more of the following conditions:

- Access to improved water;
- Access to improved sanitation;
- Sufficient-living area;
- Durability of housing;
- Security of tenure.

Comparing the IBGE and UN Habitat definitions, we see that the slum population in Brazil can be even bigger than that reported by IBGE, since this institute considers only slums with more than 51 households. For more details about the underestimation of the number of slum dwellers in Brazil, see Cavalcanti and Da Mata (2014).

³¹Groups of adjacent municipalities, in order to integrate the organization, planning and execution of public functions of common interest.

³²For more details see www.emop.rj.gov.br/trabalho-tecnico-social/censos-comunitarios.

³³Second richest Brazilian state (in terms of GDP) and where the first slums emerged.

³⁴See Tables 7 and 8.

³⁵The surveys for 1988 and 1996 have a special supplement which includes questions about parental education of the household head and the spouse.

³⁶National Household Survey conducted every year in Brazil since 1976.

³⁷See Tables 15, 16 and 17.

B Proofs

Proof of Proposition 1. Recall that the first order condition can be rearranged as

$$z^H = \frac{\eta}{1-\eta} * \frac{\left[\int_{z^R}^{\max\{z^R, z_{\min}\}} z \mu_t(dz) + \int_{z^H}^{\infty} z \mu_t(dz) \right]}{[F(z^H) - F(\max\{z^R, z_{\min}\})]}.$$

As functions of z^H we have the following: Obviously, the left-hand-side is increasing and runs from 0 to $+\infty$. The right-hand-side, for any given z^R , is strictly decreasing and goes from $+\infty$ (when z^H is close to $z_{\min}$) to $\frac{\eta}{1-\eta} * \frac{[\int_{z^R}^{z_{\min}} z \mu_t(dz)]}{[1-F(z_{\min})]} > 0$. Hence, there exist a single crossing. Moreover, if $z^R < z_{\min}$, then the right-hand-side boils down to

$$\frac{\eta}{1-\eta} * \frac{[\int_{z^R}^{z_{\min}} z \mu_t(dz) + \int_{z^H}^{\infty} z \mu_t(dz)]}{[F(z^H) - F(z_{\min})]}$$

which is decreasing in z^R . Hence, its intersection with z^H is also decreasing. On the other hand, if $z^R > z_{\min}$, the right-hand side becomes

$$\frac{\eta}{1-\eta} * \frac{[\int_{z^H}^{\infty} z \mu_t(dz)]}{[F(z^H) - F(z^R)]},$$

which is strictly increasing in z^R , and so it will be the intersection with z^H . ■

Proof of Proposition 2. Recall the definition

$$V^U(z; z^R) = \begin{cases} [\alpha_A]^{\alpha_A} \left[\frac{1-\alpha_A}{p^M(z^R)} \right]^{1-\alpha_A} [w^b(z^R) - \bar{c}^A], & \text{if } z \in [\max\{z^R, z_{\min}\}, z^H]; \\ [\alpha_A]^{\alpha_A} \left[\frac{1-\alpha_A}{p^M(z^R)} \right]^{1-\alpha_A} [w^a(z^R)z - \bar{c}^A], & \text{otherwise.} \end{cases}$$

For any given threshold z^R , the function $V^U(\cdot; z_R)$ is continuous and weakly increasing; there are flat segments when in the segments $z \in [\max\{z^R, z_{\min}\}, z^H]$, when the worker provides basic qualified skills. Outside those segments, $V^U(\cdot; z_R)$ is strictly increasing. On the other hand, if either $\alpha_A \rightarrow 0$ or $\bar{c}^A \rightarrow 0$, for any z , $V^U(z; \cdot)$ is strictly increasing in z^R and bounded from above when $w^a(z^R) > 0$. Under those conditions the function $MV^U(z^R) = V^R(z^R)$ is continuous, strictly increasing and unbounded. Since $V^R(z^R)$ is decreasing, the result follows. ■