Structural Transformation of Occupation Employment*

Georg Duernecker†
(University of Mannheim)
Berthold Herrendorf‡
(Arizona State University)

February 15, 2017

Abstract

We provide evidence on structural transformation from censuses covering three quarters of the world population. As countries develop, the standard patterns of labor reallocation hold for broad categories of both industries (”sectors”) and occupations while the employment shares of the service occupations rise in all sectors. We propose a model of structural transformation with sectors and occupations that is consistent with these patterns. The key ingredient of our model is uneven, *occupation–specific* technological progress. We show that our model is useful for predicting changes in the occupation composition and for understanding why sectoral labor productivity growth has slowed.

Keywords: Occupations; Structural Transformation; Technological Change.

JEL Classification: O11; O14.

---

*For helpful comments and suggestions, we would like to thank Doug Gollin, Bart Hobijn, Rachel Ngai, Edward Prescott, B. Ravikumar, Todd Schoellman, Ákos Valentinyi, Gustavo Ventura, and the audiences at Arizona State University, the Central Bank of Mexico, the Christmas Meeting of the German Expat Economists in München, the Frankfurt–Mannheim Macro Workshop, the IMF Development Conference and the St. Andrews Workshop on Structural Transformation. Herrendorf thanks the Spanish Ministry of Education for research support (Grant ECO2012-31358). All errors are our own.

†Address: Department of Economics, University of Mannheim, 68131 Mannheim, Germany. E-mail: duernecker@uni-mannheim.de

‡Address: Department of Economics, W.P. Carey School of Business, Arizona State University, Tempe, AZ 85287–9801, USA. E-mail: berthold.herrendorf@asu.edu

1 Introduction

Kuznets (1973) included the reallocation of labor across broad sectors as one of the main features of modern economic growth. A large recent literature studies this so–called structural transformation and shows that it is a crucial force behind the behavior of aggregate variables like hours worked and labor productivity and behind phenomena like regional convergence and urbanization.[1] The typical approach in this literature is to define sectors as broad categories of *industries*, which produce either tangible value added (goods industries) or intangible value added (service industries). In this paper, we explore an alternative approach that distinguishes between broad categories of *occupations*. The guiding principle for categorizing occupations is the same as for industries: goods occupations produce tangible value added whereas service occupations produce intangible value added. To give concrete examples, according to the alternative approach, farm workers and miners are in goods occupations whereas clerks and managers are in service occupations.

Focusing on broad occupation categories has several pay offs. To begin with, it connects two literatures that have developed largely independently: the macro literature about the reallocation of labor at the level of broad sectors and the micro literature about the role of occupations for important labor–market outcomes. Examples from the micro–labor literature that we have in mind are Kambourov and Manovskii (2008, 2009a,b), who use detailed micro data to study the role of occupations for worker mobility and the accumulation of human capital and skills. To avoid misunderstandings, we do not mean to imply that the literature on structural transformation should abandon the industry classification and switch over to the occupation classification. If the focus of the analysis is on value added instead of employment, then forming sectors in the usual way as broad industry categories is the natural route to take, because NIPA measures value added by industries and not by occupations. We will therefore study the forces behind changes in the composition of both sectors and occupations together.

Focusing on broad occupation categories also has the pay off that we can speak to the common view that an important part of structural transformation is mere relabelling which occurs when service occupations are outsourced from goods to service industries; see for example the discussion in Berlingieri (2014). An example of how such relabelling may occur is if a manufacturing firm stops employing its own cleaners and starts purchasing cleaning services from an independent contractor. According to the industry classification, this would reduce the employment of the goods industries and increase the employment of the service industries. According to the occupation classification, in contrast, this would not affect the employment of either goods or service occupations, as cleaners are a service occupation in both cases. Consequently,

[1] Herrendorf et al. (2014) provide a review of the literature. Key contributions to it include Echevarria (1997), Laitner (2000), Caselli and Coleman (2001), Kongsamut et al. (2001), Gollin et al. (2007), Ngai and Pissarides (2007), Rogerson (2008), Duarte and Restuccia (2010), Buera and Kaboski (2012), Herrendorf et al. (2013), and Boppart (2016).

measuring structural transformation by the reallocation among broad occupation categories is not affected by outsourcing.[2]

We document stylized facts about structural transformation using data from 182 censuses of 67 countries from the Integrated Public Use Microdata Series (IPUMS) International, Minnesota Population Center (2015). These 67 countries cover more than two thirds of world GDP and more than three quarters of the world population. We emphasize that in contrast to the usual evidence from currently rich countries, this evidence comes from countries of all income levels including many countries from Sub–saharan Africa. It is crucial in this context that IPUMS harmonized the census information so that it contains both industry and occupation identifiers that are comparable across countries and across time.

We find that the standard patterns of structural transformation hold for both industry and occupational employment: as GDP per capita increases, goods employment decreases and service employment increases. This is not an obvious but a novel and surprising observation, because, as it turns out, many occupations are used in different sectors. We also find that the goods sector is more intensive in the goods occupations than the service sector while the employment share of the service occupations increases in both sectors as GDP per capita increases. If outsourcing was the major force behind structural transformation, then one would observe the exact opposite, that is, the employment share of the service occupations would decrease in the goods sector. That this is not the case suggests that either outsourcing is not important quantitatively or is offset by other economic forces.

To understand the economic forces behind these patterns in the data, we propose a new model of structural transformation that connects the behavior of the goods and service sectors with the behavior of the goods and service occupations. We require that our model be consistent with the standard stylized facts: as GDP per capita increases, labor is reallocated from the goods sector to the service sector and the value added share of the service sector increases; the price of value added from the goods sector relative to value added from the service sector increases; labor productivity increases more in the goods sector than in the service sector. In addition, we require that our model be consistent with the new stylized facts that we have described above: the goods sector is more intensive in the goods occupation than the service sector; as GDP per capita increases, labor is reallocated from the goods occupations to the service occupations on the aggregate and in both sectors.

Our model has three sectors, which produce investment, consumption goods, and consumption services. There is a measure one of identical households which have a CES utility function over goods and services. Note that by choosing a CES utility function we assume away income effects, which allows us to focus on the technological forces behind structural transformation. The investment sector has an AK technology and the consumption sectors produce value added

[2]Note that given the nature of the census data we are using, we can only speak to outsourcing within the same country and cannot speak to outsourcing to other countries ("offshoring").

from capital and labor according to Cobb–Douglas production functions. This implies that the reallocation of labor between and within the two consumption sectors of the model will represent structural transformation in the data. In each consumption sector labor from the goods and service occupations provide distinct labor services that are imperfectly substitutable. We aggregate these labor services through a CES aggregator with sector–specific relative weights and a common elasticity of substitution. We assume that technological progress is occupation specific, but is not sector specific. All households can supply their labor endowments to both sectors and both occupations. The assumption that households can supply their labor to both sectors is as in most of the literature on structural transformation. The assumption that households can supply their labor to both occupations is a useful first step towards understanding the forces behind the reallocation of labor between occupations. It implies that, in this paper, we will not have anything to say about wage differences between occupations.

We prove that our model is consistent with the stylized facts if and only if the following conditions hold: consumption goods and services are complements in the utility function, which is as in Ngai and Pissarides (2007); the relative weight of the goods occupations is larger in the goods sector than in the service sector; occupations are complements in the CES aggregator of labor; goods–occupation–specific technological progress grows more strongly than service–occupation–specific technological progress. The last condition is in the spirit of Baumol (1967) who suggested that there is little or no technological progress in the production of many services for which output essentially equals the labor input they contain. To study the quantitative implications of our model, we calibrate it to replicate the reallocation of labor between goods and service sectors and goods and service occupations in the U.S. economy during 1950–2000. Our model has no trouble generating the patterns in the data including the quantitatively important reallocation of occupation labor within each sector that the standard model misses. It is critical for achieving this that occupation–specific technological progress is highly uneven: our calibration implies that goods–occupation–specific technological progress grows four times more strongly than service–occupation–specific technological progress.

We illustrate the usefulness of our model by showing that it performs well also along several non–targeted dimensions. To this end, we assume that technological progress grows at the same constant annual rates that we calibrated for the U.S. 1950–2000. Our model then accounts for most of the reallocation of the occupation employment in the U.S. during 1850–1950 and of the occupation employment in our sample of censuses from around the world. Our model does well also when used to make out–of–sample predictions about the changes in the composition of broad categories of industry and occupation employment in the U.S. during 1972–2014. Indeed, in most cases that we study it outperforms the BLS forecasts, which are among the most downloaded statistics of the BLS. Lastly, our model accounts for most of the observed decline in the growth rates of sectoral labor productivity in the U.S. during 1950–2000. In contrast, the standard model has constant growth rates of sectoral labor productivity, which is counterfactual. The reason for why our model does better than the standard model along this dimension is that as GDP per capita grows, employment in both sectors is reallocated non–linearly towards the service occupations which experience the slower technological progress.

We emphasize that we obtain all of these results while using the constant annual rates of *occupation–specific* technological progress that we have calibrated for the U.S. during 1950–2000. The broad success of this approach suggests to reevaluate the usual approach of the structural–transformation literature which assumes constant growth rates of *sector–specific* technological progress.

The paper is organized as follows. In the next section, we document the stylized facts of the structural transformation of occupation and sector employment. Afterwards, we develop a new model that accounts for them. In Section 4 we derive restrictions under which our model is qualitatively consistent with the stylized facts. The next two Sections contain the quantitative results and the discussion of the relationship to the literature. In Section 7, we conclude. The proofs of our analytical results are in the Appendix.

2 Evidence on Structural Transformation of Occupation Employment

2.1 Data

IPUMS International reports census data that are comparable across countries and over time. For our purpose here, it is crucial that the censuses have harmonized information about employment by both occupation and industry at a sufficiently disaggregate level so that we can form broad occupation categories and broad industry categories ("sectors").[3] This is the case for 182 censuses from 67 countries, which are listed in Appendix A. The sample has 21 countries from America and the Caribbean, 19 from Africa (including many Sub–saharan ones), 14 from Europe, and 13 from Asia. In 1990, the countries in the sample covered three quarters of the world population; included seven of the ten most populous countries (namely, China, India, U.S.A., Indonesia, Brazil, Pakistan, and Nigeria); represented more than two thirds of world output. The countries in the sample are from all income levels and the largest income difference exceeds a factor fifty (the richest country is the US in 2000 with $\$30,491$ and the poorest country is Guinea in 1990 with $544, both in 1990 international $'s).

We construct the broad categories of industry employment ("sectors") in the standard way:

- goods sector:

[3]The level of disaggregation varies for occupations between one and four digits. For most countries including the US the level of disaggregation is three digits.

  - agriculture sector: agriculture, fishing, and forestry;
  - industry sector: construction; electricity, gas and water; manufacturing; mining;[4]

- service sector: education; financial services and insurance; health and social work; hotels and restaurants; other services; private household services; public administration and defense; real estate and business services; services not specified; transportation and communications; wholesale and retail trade.

We construct the broad categories of occupation employment by following the principle that goods occupations produce, process or transform tangible output whereas service occupations produce intangible output. Goods occupations are related to, but not equal to blue–collar or brawn–intensive occupations whereas service occupations are related to but not equal to white–collar or brain–intensive occupations. This principle lets us construct the broad categories of occupation employment in the following way:

- goods occupations:
  - agriculture occupations: elementary agricultural occupations; skilled agricultural and fishery workers;
  - industry occupations: crafts and related trades workers; elementary industry occupations; plant and machine operators and assemblers;

- service occupations: armed forces; clerks; elementary service occupations; legislators, senior officials and managers; professionals; service workers and shop and market sales; technicians and associate professionals.

There are several potential problems with using the occupation classification. To begin with, although IPUMS puts a great deal of effort into harmonizing the data so that it is comparable over time, some occupation classifications may change over time. Since our analysis is at a broad level, this should not constitute a problem for us here. A second problem comes from the fact that most censuses list the elementary occupations by industry but do not disaggregate them further. We assign the employment of elementary occupations in each industry to agricultural, industry, and service occupations by using the proportions of these three occupation categories in the rest of the employment in the same industry. We establish for the U.S. during 1850–2010 that this proportional disaggregation of elementary occupations is a good approximation. Like many other censuses, the U.S. censuses have much more detailed information than is reported in harmonized form, permitting us to disaggregate the elementary occupation employment of each sector into agriculture, industry, and service occupations. The detailed results are in part A

[4]For lack of better terminology, industry in our paper may refer to a generic industry or to the broad sector industry. To avoid this double usage, some authors have used the term manufacturing sector, but that is not ideal either because manufacturing is only one subsector of the industry sector.

of the Appendix. A third potential problem is that some broad categories of goods occupations may contain some service occupations and vice versa. We use the U.S. censuses during 1850–2010 to establish that this is not a quantitatively important issue. The detailed results are again in part A of the Appendix.

We report the facts of structural transformation of employment for the working–age population (age 15–64). We plot the employment by sector or broad occupation category against GDP per capita, which is from Maddison's Groningen database and is in 1990 international $'s. We start with the standard broad categories agriculture, industry, and services.

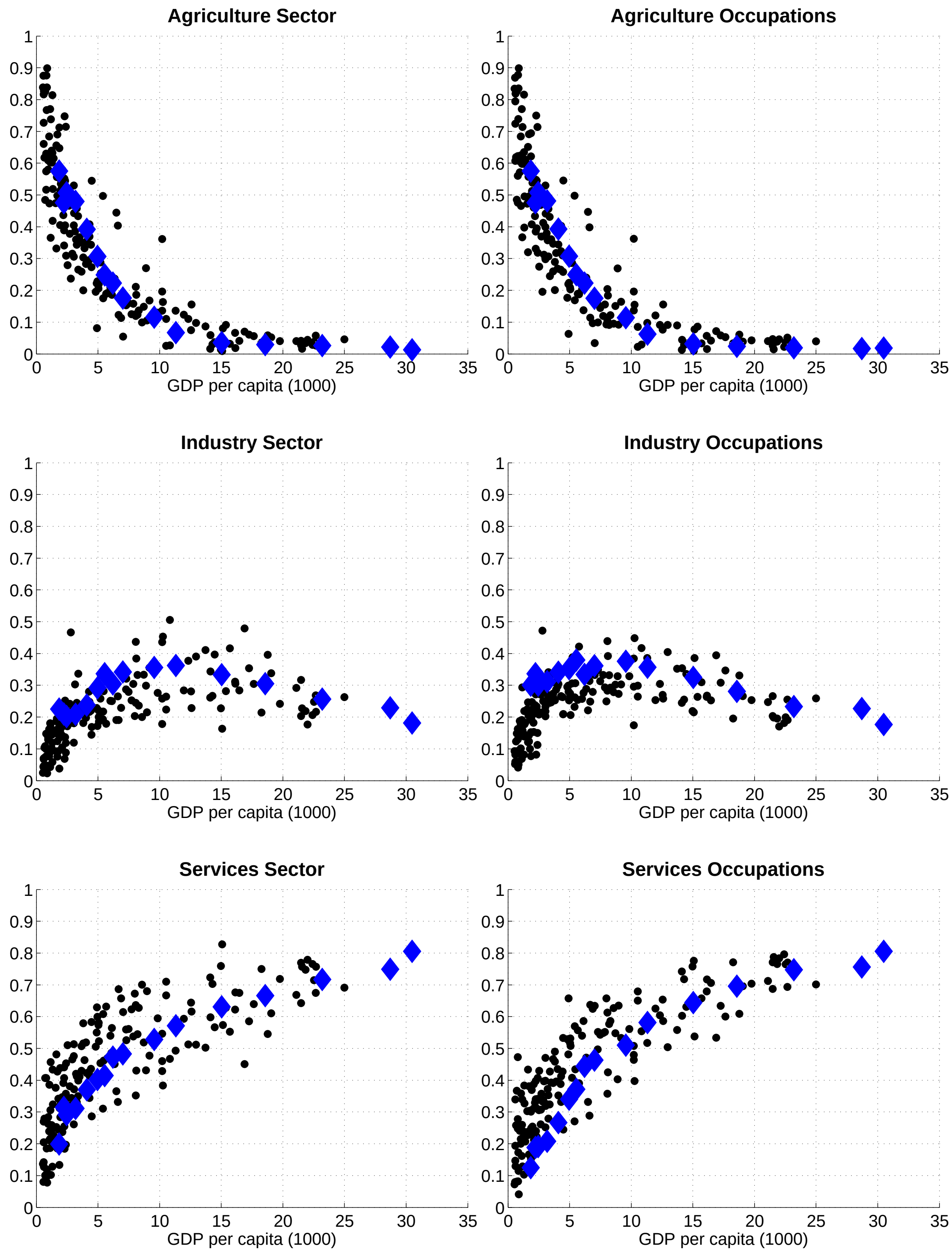
2.2 Facts for Agriculture, Industry, and Services

The left panel of Figure 1 shows that for sectoral employment the standard patterns of structural transformation hold: the share of agriculture employment decreases, the share of industry employment first increases and then decreases ("hump shape"), and the share of service employment increases. The right panel of Figure 1 shows that very similar patterns also hold for occupation employment. That the patterns of structural transformation with occupation employment are so similar to those with industry data is remarkable in light of the fact that many occupations are not sector specific (think for example of clerks or managers).

Figure 1 also plots the US time series 1850–2010 in blue diamonds, showing that the patterns in the US time series are very similar to the patterns in the cross–country data. This provides support for the notion that when the U.S.A. was poor it had a similar sectoral composition as currently poor countries have. Many authors have conjectured that this is the case, and in fact several have made this assumption for lack of data from currently poor countries. However, to the best of our knowledge, we are the first to provide hard supporting evidence from high quality census data for a broad set of currently rich and poor countries that covers the vast majority of world population and world production. Note that although we find no evidence that the structural transformation in currently poor countries follows systematically different patterns, the service and industry shares of poor and middle–income countries show a lot of variation that is unrelated to GDP per capita. This is consistent with the evidence presented by Bah (2011) for a smaller subset of observations. An important task for future research is to understand the reasons for this large variation. One possibility is that international trade, and in particular offshoring of tasks to foreign countries, may lead to different industry employment shares for countries with similar GDP per capita. Given the limitations of census data, we are not be able to assess the effects of offshoring on structural transformation in this paper.

The evidence presented above also implies that structural transformation must reflect a fundamental shift of economic activity across sectors, as opposed to just a relabeling that would result from outsourcing of services. The possibility that outsourcing of services may be a quantitatively important force behind structural transformation has been raised frequently. An

Figure 1: Structural Transformation in our Panel of Countries and in the U.S. Time Series
(black dots are country–year observations, blue diamonds are US observations)



example of how outsourcing may seemingly lead to structural transformation is that janitorial services provided in–house by a car manufacturer are counted as manufacturing value added, whereas janitorial services that are outsourced and purchased on the market by the same car manufacturer are counted as services. This possibility is worrying for the structural transformation literature because of the wave of outsourcing that took place since the 1980s, especially regarding business services.[5] Our evidence implies that outsourcing is not a quantitatively important force behind structural transformation, as occupation employment clearly shows structural transformation although it is invariant to outsourcing (janitors employed by car manufacturers and cleaning firms are both janitors).

That outsourcing is not a quantitatively important force behind structural transformation is a much stronger conclusion than what the literature has reached, which has almost exclusively talked about the postwar US whereas our evidence is for a very large number of rich and poor countries. Moreover, the evidence in the literature has been rather indirect. For example, Herrendorf et al. (2013) observed that outsourcing does not affect the composition of final expenditure, and since there is ST in final expenditure, it cannot be the case that all structural transformation is due to outsourcing. Another example is Berlingieri (2014), who found that changes in the input–output structure have increased service employment by 36%, with outsourcing of business services being a crucial driver of changes in the input–output structure.

2.3 Facts for Goods and Services

In this subsection, we report the stylized facts of structural transformation of employment for the coarser two–sector split of goods and services. Considering these two broad categories is of interest in situations in which the focus is on tangible versus intangible value added. As we will see in a moment, it also implies sharp patterns of reallocation that are a natural starting point for building a model that articulates the forces behind the structural transformation of sector and occupation employment.

Table 1: Reallocation between sectors and occupations

GDP per capita (1990 int. $'s)	1,000	15,000	30,000
Employment share of			
goods sector	74.3	38.0	19.9
service sector	25.7	62.0	80.1
goods occupations	77.1	36.6	16.3
services occupations	22.9	63.4	83.7

[a] Shares are in percent and are from fitted curves.

[5] Abraham and Taylor (1996) provided evidence on determinants of outsourcing. Abramovsky and Griffith (2006) evidence on ICT and outsourcing.

Table 1 reports how the employment shares by sector and broad occupation category depend on GDP per capita. All numbers are in percentages and are from locally weighted splines (LOWESS) that are fitted through the data points. Two stylized facts (SF's for short) emerge:

SF 1: As GDP per capita increases, labor is reallocated from the goods sector to the service sector.

SF 2: As GDP per capita increases, labor is reallocated from goods occupations to service occupations.

Moreover, a simple shift–share analysis reveals that moving from GDP per capita of $\$1,000$ to GDP per capita of $\$30,000$, roughly half of the reallocation from goods–occupation labor to service–occupation labor is due to the reallocation of labor between the goods and the service sector and the other half is due to the reallocation of labor from the goods to the service occupations within the two sectors.

Table 2: Reallocation of occupations within sectors

	Goods sector			Service sectors		
GDP per capita (1990 int. $'s)	1,000	15,000	30,000	1,000	15,000	30,000
Employment share of						
goods occupations	97.3	73.5	42.1	18.7	13.8	9.9
service occupations	2.7	26.5	57.9	81.3	86.2	90.1

[a] Shares are in percent and are from fitted curves.

Table 2 reports the shares of occupations in sector employment for different GDP per capita, again from fitted curves. Two additional stylized facts emerge:

SF 3: The goods sector is more intensive in the goods occupation than the service sector; the service sector is more intensive in the service occupation than the goods sector.

SF 4: As GDP per capita increases, labor is reallocated from goods to service occupations in both sectors.

Stylized Fact 4 implies that the share of service occupation employment increases with GDP per capita not only in the service sector, but also in the goods sector. This is the exact opposite of what outsourcing would imply, which would decrease the share of service occupation employment in goods sector employment and leave the share of service occupation employment in total employment constant. Hence, there must be an important force other than outsourcing behind the four stylized facts. We now build a model to study how this force works.

3 Model

3.1 General remarks

To impose discipline, we ask our model to match seven stylized facts. The first four are SF 1–4 from above and the last three are from the previous literature (see Herrendorf, Valentinyi, and Rogerson, 2014 for a summary of the evidence):

SF 5: As GDP per capita increases, the price of value added from the goods sector relative to value added from the service sector decreases.

SF 6: As GDP per capita increases, labor productivity increases more in the goods sector than in the service sector.

SF 7: As GDP per capita increases, the expenditure share of the service sector increases (and that of the goods sector decreases).

To account for these stylized facts, we will need three key features: value added is disaggregated into broad categories of industries ("sectors"); labor in each sector is disaggregated into broad categories of occupations; technological progress augments occupation labor. In what follows, we will extend the canonical model of structural transformation developed by Ngai and Pissarides (2007) by these features.

3.2 Environment

Time is discrete and runs forever. There are three sectors which produce investment X; consumption goods C_G; consumption services C_S. In each period, the investment good is the numeraire.

The investment technology is of the AK form:

$$Y_{Xt} = A_X K_{Xt} \tag{1}$$

where A_X is the TFP of producing investment goods from capital K_X. We will use upper–case letters to index sectors and lower–case letters to index occupations.

The consumption technologies are of the Cobb–Douglas form:

$$Y_{Jt} = K_{Jt}^{\theta} L_{Jt}^{1-\theta} \tag{2}$$

where $J \in \{G, S\}$ is the consumption–sector index; $\theta \in (0, 1)$ is the capital share parameter;[6] L_J

[6]While Valentinyi and Herrendorf (2008) show that in the data $\theta_g \neq \theta_s$, Herrendorf et al. (2015) show that Cobb–Douglas production functions with equal θ do a reasonable job at capturing the technological forces behind the postwar structural transformation in the US. Acemoglu and Guerrieri (2008) explore what happens when θ_j are sector specific.

is a CES aggregator of labor from the two occupations:

$$L_{Jt} = \left[(\alpha_J)^{\frac{1}{\sigma}} (A_{gt} N_{Jgt})^{\frac{\sigma-1}{\sigma}} + (1 - \alpha_J)^{\frac{1}{\sigma}} (A_{st} N_{Jst})^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{3}$$

where $\alpha_J \in [0, 1]$ is the intensity of labor from the goods occupations; $j \in \{g, s\}$ is the occupation index; N_{Jj} is the labor from occupation j employed in sector J; A_j is occupation–specific labor–augmenting technological progress (which is not sector specific); σ is the elasticity of substitution where $\sigma \to 0$ is Leontief; $\sigma \to 1$ is Cobb–Douglas; $\sigma \to \infty$ is perfect substitutes. Note that the standard model of structural transformation in which labor is homogeneous is a special case for $\alpha_J = 0$, or for $\alpha_J = 1$, or for $\sigma = \infty$ together with $A_g = A_s$.

The way we model the aggregation of labor from different occupations has similarities to the canonical model of skill–biased technological change as described for example by Acemoglu and Autor (2011), which also assumes that technological progress is specific to broad categories of labor. The assumption that the intensity of occupation labor depends on the sector but labor–augmenting technological progress is independent of the sector can be viewed as a reduced–form of a more elaborate production process that involves three stages: value added in each sector is produced from different tasks and the intensity of each task differs across sectors; each tasks is produced from labor of different occupations and other inputs according to a technology that is common to all industries. Goos et al. (2014) develop an example of such a model. The way we model the aggregation of labor from different occupations also has similarities with Ngai and Petrongolo (2014) and Bárány and Siegel (2014). They embedded a Roy model into a structural transformation model, assuming that there are time–invariant CES functions that aggregate the different categories of labor to sector labor and that sector–specific technological progress augments all sector labor. The novelty of our paper is that labor–augmenting technological progress is occupation specific instead of sector specific.

There is a continuum of measure one of identical households. The present discounted life-time utility takes the standard separable form:

$$\sum_{t=0}^{\infty} \beta^t \log(C_t) \tag{4}$$

where $\beta < 1$ is the discount factor, the consumption aggregator of goods and services is

$$C_t = \left[(\alpha_U)^{\frac{1}{\varepsilon}} (C_{Gt})^{\frac{\varepsilon-1}{\varepsilon}} + (1 - \alpha_U)^{\frac{1}{\varepsilon}} (C_{St})^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{5}$$

and $\alpha_U \in [0, 1]$ is a relative weight and ε is the elasticity of substitution.

The representative household is endowed with a positive initial capital stock, $K_0 > 0$, which can be used in all sectors. Moreover, it is endowed with one unit of labor in each period, which can be used in both sectors and in both occupations. The usual assumption in the canonical model of structural transformation is that workers can use their labor endowment in all sectors, implying that in equilibrium real wages are equalized across sectors. We make the same assumption also for occupations, implying that in equilibrium real wages will also be equalized across occupations. We view this as a useful first step towards studying the structural transformation of occupation employment.[7]

The resource constraints and market–clearing conditions are:

$$K_{t+1} = (1-\delta)K_t + X_t \tag{6}$$

$$K_t = K_{Xt} + (K_{Gt} + K_{St}) \tag{7}$$

$$N_{Jt} \equiv N_{Jgt} + N_{Jst} \tag{8}$$

$$N_{jt} \equiv N_{Gjt} + N_{Sjt} \tag{9}$$

$$1 = N_t = N_{Gt} + N_{St} = N_{gt} + N_{st} \tag{10}$$

$$Y_{Xt} = X_t, \quad Y_{Gt} = C_{Gt}, \quad Y_{St} = C_{St} \tag{11}$$

The first equation is the standard law of motion for capital. The second equation is the adding up constraint for capital in each period. The third, fourth, and fifth equations are the adding up constraints for sectoral labor, occupation labor, and total labor. The last equations are the market clearing constraints for investment, consumption goods, and consumption services.

4 Analytical Results

4.1 Solving the individual problems

The household problem is:

$$\max_{\{K_{t+1}, C_{Gt}, C_{St}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \log\left(\left[(\alpha_U)^{\frac{1}{\epsilon}}(C_{Gt})^{\frac{\epsilon-1}{\epsilon}} + (1-\alpha_U)^{\frac{1}{\epsilon}}(C_{St})^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}}\right) \tag{12}$$

$$\text{s.t.} \quad P_{Gt}C_{Gt} + P_{St}C_{St} + K_{t+1} = (1 + r_t - \delta)K_t + w_t$$

[7] For recent structural transformation models in which wages are not equalized, see Bárány and Siegel (2014), Ngai and Petrongolo (2014), and Buera et al. (2015).

The first–order conditions to the household problem are standard:

$$\frac{C_{t+1}P_{t+1}}{C_tP_t} = \beta(1 + r_{t+1} - \delta) \tag{13}$$

$$\lim_{t\to\infty}\beta^t \frac{K_{t+1}}{C_tP_t} = 0 \tag{14}$$

$$\frac{P_{St}C_{St}}{P_{Gt}C_{Gt}} = \frac{1-\alpha_U}{\alpha_U}\left(\frac{P_{St}}{P_{Gt}}\right)^{1-\epsilon} \tag{15}$$

where

$$P_t = \left[\alpha_U(P_{Gt})^{1-\epsilon} + (1-\alpha_U)(P_{St})^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$$

Note that (15) implies that an increase in the price of services relative to goods (SF 6) goes along with an increase in the expenditure share of services (SF 5) if and only if $\epsilon < 1$. This was a key point of Ngai and Pissarides (2007), who asked for which parameter restrictions the multi–sector growth model with CES preferences is consistent with the stylized facts of structural transformation.

The problem of the firm in the investment sector is:

$$\max K_{Xt}(A_X - r_t)$$

The first–order condition to the investment–sector firm problem is:

$$r_t = A_X \tag{16}$$

The problems of the firms in the consumption sectors are:

$$\max \left(K_{Jt}\right)^{\theta}\left(\left[(\alpha_J)^{\frac{1}{\sigma}}(A_{gt}N_{Jgt})^{\frac{\sigma-1}{\sigma}} + (1-\alpha_J)^{\frac{1}{\sigma}}(A_{st}N_{Jst})^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}\right)^{1-\theta} \tag{17}$$
$$- r_tK_{Jt} - w_t(N_{Jgt} + N_{Jst})$$

The first–order conditions to the problems of the consumption–sector firms imply:[8]

$$\frac{K_{Gt}}{N_{Gt}} = \frac{K_{St}}{N_{St}} \tag{18}$$

$$\frac{Y_{Gt}/N_{Gt}}{Y_{St}/N_{St}} = \left(\frac{L_{Gt}/N_{Gt}}{L_{St}/N_{St}}\right)^{1-\theta} \tag{19}$$

$$\frac{Y_{Gt}/N_{Gt}}{Y_{St}/N_{St}} = \frac{P_{St}}{P_{Gt}} \tag{20}$$

$$\frac{N_{Jst}}{N_{Jgt}} = \frac{1-\alpha_J}{\alpha_J}\left(\frac{A_{gt}}{A_{st}}\right)^{1-\sigma} \tag{21}$$

$$\frac{N_{Sst}}{N_{Gst}} = \frac{1-\alpha_S}{1-\alpha_G}\left(\frac{L_{Gt}/N_{Gt}}{L_{St}/N_{St}}\right)^{\sigma}\frac{L_{St}}{L_{Gt}} \tag{22}$$

(18) is the usual result that the capital–labor ratios are equalized if the sectoral production functions are Cobb–Douglas with equal exponents. (19) shows that, as a result, the ratio of the labor productivities depends only on the ratio of the sector–labor aggregators per unit of sector–labor input.[9] (20) implies that the price of services relative to goods increases (SF 6) if and only if labor productivity increases more strongly in the goods than in the service sector (SF 7). (21) implies that service occupations have a larger employment share in the service sector and goods occupations have a larger employment share in the goods sector (SF 3) if and only if $\alpha_G > \alpha_S$; labor is reallocated from goods to service occupations in both sectors (SF 4) if A_g/A_s increases and $\sigma < 1$ or if A_g/A_s decreases and $\sigma > 1$. (22) describes how labor from service occupations is allocated between the two sectors.

4.2 Analytical results on generalized balanced growth and structural transformation

Since there is reallocation of labor between the consumption sectors, there is no balanced growth path along which all ratios are constant. We follow Kongsamut et al. (2001) and study generalized balanced growth path (GBGP) which is an equilibrium along which the real interest rate is constant while sectoral ratios may change. A GBGP trivially exists here because of the AK technology in the investment sector. Given this, we have two propositions, which are proven in the Appendix.

Assumption 1.
Suppose that $\gamma \equiv \beta(1 + A_x - \delta) > 1$.

[8]The derivations can be found in the Appendix.
[9]This shows that in terms of reallocation of labor our model behaves like a simpler model without capital.

Proposition 1.

There is a unique GBGP. Along the GBGP, the following variables grow at factor γ: aggregate capital, capital in each sector, expenditure on total consumption, GDP, investment, wage.

Proposition 2.

The GBGP is consistent with the seven stylized facts if and only if the parameters satisfy: (i) $\alpha_S < \alpha_G$; (ii) $\sigma < 1$; (iii) $\varepsilon < 1$; (iv) $A_{gt}/A_{st} \uparrow$.

Condition (i) says that the goods sector is more intensive in the goods occupation than the service sector and the service sector is more intensive in the service occupation than the goods sector. Condition (ii) says that the inputs into the production function are complements (they are less substitutable than Cobb Douglas).[10] Condition (iii) says that the inputs into the utility function are complements.[11] Condition (iv) says that technological progress is faster for the goods than the service occupation. We stress that Proposition 2 needs uneven occupation-specific technological progress, but it does not need uneven sector-specific technological progress. In fact, our model is qualitatively consistent with the seven stylized facts although it does not feature technological progress at the sector level at all.

We now turn to providing intuition for Proposition 2. For convenience, we state the seven stylized facts again:

SF 1: As GDP per capita increases, labor is reallocated from the goods sector to the service sector.

SF 2: As GDP per capita increases, labor is reallocated from goods occupations to service occupations.

SF 3: The goods sector is more intensive in the goods occupation; the service sector is more intensive in the service occupation.

SF 4: As GDP per capita increases, labor is reallocated from goods to service occupations in both sectors.

SF 5: As GDP per capita increases, the price of value added from the goods sector relative to value added from the service sector decreases.

SF 6: As GDP per capita increases, labor productivity increases more in the goods sector than in the service sector.

SF 7: As GDP per capita increases, the expenditure share of the service sector increases.

[10] Note that this is different from the canonical model with unskilled and skilled labor inputs described by Acemoglu and Autor (2011), in which it is empirically plausible to choose an elasticity of substitution larger than one.

[11] The evidence from Herrendorf, Rogerson, Valentinyi, AER, 2013 suggests that $\epsilon \approx 0$. In other words, imposing $\epsilon < 1$ is not restrictive.

SF 3 holds in our model because of Assumption (i). SF 4 holds because of Assumption (iv) and $\sigma < 1$. SF 5 holds because of Assumptions (i) and (iv). SF 6 holds because of (20) and SF 5 holds. SF 7 holds because of Assumption (iii) and the relative price of services increases. SF 1 holds because SF 7 holds and (20) implies that

$$\frac{N_{St}}{N_{Gt}} = \frac{P_{St}C_{St}}{P_{Gt}C_{Gt}}$$

SF 2 holds because SF 4 implies that the share of service occupations increases in both sectors and SF's 1 and 3 imply labor gets reallocated to the sector with the higher share of service occupation employment.

In other words, the intuition for Proposition 2 is that the relative price of services increases because services are intensive in service occupation employment which experiences slower labor–augmenting technological change than goods occupation employment. Since value added from the goods and service sector are complements in the utility function, the increase in the relative price of services implies that expenditures and labor get reallocated from the goods to the service sector. Since occupations are complements in the production functions, the slower technological change of the service occupations implies that labor gets allocated from the goods to the service occupations in each sector. Labor gets allocated from the goods to service occupations in the whole economy because not only does that happen in each sector but also does labor get reallocated from the goods sector, which is less intensive in service occupations, to the service sector, which is more intensive in service occupations. In reality, part of the changes in the employment composition results from the different degrees in which capital replaces labor from the goods occupations and the service occupations. Our model captures these changes by assigning stronger labor–augmenting technological progress to the category that is more strongly replaced by capital. Since $\sigma < 1$, this category then loses employment relative to the other category.

In sum, two forces generate reallocation of labor from the good to the service occupations: substitution between occupations within each sector; substitution of labor between sectors. In our model, both effects result from uneven technological progress at the occupation level. Note that the latter effect would also occur with even technological progress when preferences are non–homothetic (see Kongsamut et al. (2001)).

4.3 Discussion

An obvious question to ask at this point is whether there are plausible examples for the notion that technological change is occupation specific and uneven. A first supportive example comes from Goldin and Katz (2008) who pointed out that during the 19. century manufacturing technologies tended to replace skilled artisans. This is consistent with our model because skilled artisans are in the goods occupations. A second supportive example come from Baumol (1967) who argued that the increasing relative prices of many services (his famous "cost disease") is related to the lack of technological progress in the production of these services. Specifically, on page 415 and following he wrote: "... economic activities can ... be grouped into two types: technologically progressive activities in which innovations, capital accumulation, and economies of large scale all make for a cumulative rise in output per man hour and activities which, by their very nature, permit only sporadic increases in productivity. ... The basic source of differentiation resides in the role played by labor in the activity. In some cases labor is primarily an instrument – an incidental requisite for the attainment of the final product, while in other fields of endeavor, for all practical purposes the labor is itself the end product. Manufacturing encompasses the most obvious examples of the former type of activity. ... On the other hand there are a number of services in which the labor is an end in itself, in which quality is judged directly in terms of amount of labor. Teaching is a clear-cut example, where class size (number of teaching hours expended per student) is often taken as a critical index of quality." Baumol's distinction between the two types of labor is closely related to our distinction between goods occupations and service occupations.

A third supportive example comes from the recent labor literature on job polarization. Autor et al. (2006) and Autor and Dorn (2013) documented that job polarization has happened in the non–farm sector of the U.S. since the 1970s: middle–wage occupations have experienced declines in their relative employment and relative wages compared to low–wage and high–wage occupations during recent decades. Goos et al. (2014) documented that the same phenomena happened in Western Europe during 1993–2000. These papers argue that the main force behind job polarization is routine–biased technological change, which has increasingly replaced routine tasks that tend to be produced by middle–wage occupations, but has hardly affected non–routine tasks that tend to be produced by low–wage and high–wage occupations. Routine–biased technological progress is one reason for why the goods occupations have experienced stronger labor–augmenting technological progress than the service occupation in recent decades. Specifically, while service occupations perform mostly non–routine tasks (e.g., managers) or routine tasks (e.g. clerks), goods occupations tend to perform mostly routine tasks. Hence, routine–biased technological change affects the goods occupations more strongly than the service occupations.

We conclude this subsection with a brief discussion of three additional aspects of what we have achieved thus far. First, although our model generates the qualitative patterns of *nominal* expenditure shares, it cannot generate the qualitative patterns of *real* expenditure. In the model the *real* share of services decreases whereas in the data it decreases. This is a common problem in the literature which is due to the fact that for CES utility functions the real expenditure shares move opposite to relative prices, except in the extreme Leontief case in which the real expenditure shares stay constant. The recent work of Boppart (2016) and Comin et al. (2015) uses non–homothetic CES utility functions that, under some restrictions, are able to account for the patterns of real shares. While that work is important for understanding the forces behind structural transformation that arise on the preference side, we use a homothetic CES utility function here because our focus is on the forces that come from the technology side.

Second, instead of occupation–specific technological progress, the literature assumes that sectoral labor is homogenous and uses sector–specific, labor–augmenting technological progress:

$$Y_{Jt} = K_{Jt}^{\theta}(A_{Jt}L_{Jt})^{1-\theta} \tag{23}$$

where :

$$L_{Jt} = \left[(\alpha_J)^{\frac{1}{\sigma}} N_{Jgt}^{\frac{\sigma-1}{\sigma}} + (1-\alpha_J)^{\frac{1}{\sigma}} N_{Jst}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{24}$$

It is easy to show that with sector–specific technological progress the model is consistent with SFs 1–3 and 5–7 if and only if Assumption (i) and (ii) hold and $A_{Gt}/A_{St} \uparrow$. Because N_{Jgt}/N_{Jst} is constant in equilibrium for each sector in the model used by the literature, it cannot match SF 4. As we documented above, SF 4 is a quantitatively important part of the reallocation of occupation employment, and so we will focus our attention on occupation–specific technological change.

Third, whether technological progress happens at the sector level or at the occupation level has an important implication for the behavior of sectoral labor productivity growth, where labor productivity is defined as $LP_{Jt} \equiv Y_{Jt}/N_{Jt}$. If we assume that technological progress happens at the sector level and sectoral technological progress grows at a constant rate, then sectoral labor productivity grows at constant rate. This is the common case analyzed in the literature. If, instead, we assume that technological progress happens at the occupation level and grows at a constant rate, then sectoral labor productivity will not grow at constant rates. The intuitive reason for this is that structural transformation implies a *non–linear* relationship between sectoral value added and the sectoral occupation–labor composition. The next Proposition formalizes this intuition.

Proposition 3.
If $0 < \alpha_S < \alpha_G < 1$*;* $0 < \sigma < 1$*;* $A_{gt}/A_{st} \uparrow$ *with constant growth factors* $\gamma_j \equiv A_{jt}/A_{jt-1}$*, then:*

- $\lim_{t\to\infty}[\Delta \log(LP_{St}) - \Delta \log(LP_{Gt})] = 0$*;*
- $\exists\, \bar{t} > 0\ \forall t > \bar{t}\ \Delta \log(LP_{St}) - \Delta \log(LP_{Gt}) \leq 0$ *and increases over time.*

While Proposition 3 implies that in at least one sector the growth rate of labor productivity changes over time, the quantitative analysis that follows next shows that the growth rates of both sectoral labor productivities change over time.

5 Quantitative Results

Proposition 2 specifies under what conditions our model is qualitatively consistent with the seven stylized facts. In this section, we show that our model is successful quantitatively as well. To establish this, we will calibrate it to the composition of occupation employment in the U.S. in 1950 and in 2000. We find that the model has no trouble matching this episode, and that it performs well along several dimensions that we have not targeted. We will then show that the calibrated model accounts for most of the structural transformation of occupation employment in both the U.S. during 1850–1950 and in our sample of countries from around the world.

5.1 Calibration

We need to calibrate the following parameters: the discount factor β; the depreciation rate δ; the elasticities ϵ, σ, θ; the relative weights $\alpha_G, \alpha_S, \alpha_U$; the technological–progress parameters A_g, A_s, A_X.

We choose standard parameter values to the extent possible and calibrate the remaining parameters jointly by matching salient features of the U.S. in 1950 and 2000.[12] Table 3 lists the resulting parameter values. Specifically, we choose the standard values $\beta = 0.96$ and $\delta = 0.05$. We choose a low value $\epsilon = 0.05$, which is based on the evidence provided by Herrendorf et al. (2013) that the elasticity of substitution between broad consumption categories is close to zero. We choose $\theta = 0.17$, which implies an aggregate capital share of 1/3.[13] We choose $A_X = 0.10$, which implies a net real interest rate $r - \delta = 0.05$. We make the following normalizations: $A_{g,1950} = A_{s,1950} = K_{1950} = 1$. We jointly calibrate $\sigma, \alpha_G, \alpha_S, \alpha_U, A_{g,2000}, A_{s,2000}$ to match six U.S. targets: we use that, according to Maddison, real GDP per capita increased by a factor of three during 1950–2000; from the population censuses we use five shares in total employment: the goods occupations working in the goods sector in 1950 and in 2000, the service occupations working in the goods sector in 1950 and in 2000, the service occupations working in the service sector in 1950. Note that since shares add up to 100, these targets imply the share in total employment of the service occupation working in the service sector in 1950 and the share in total employment of both occupations working in the service sector in 2000.

Table 4 shows that we match our targets well and that our model also performs reasonably well along several dimensions that we did not target, including the capital–output ratio, the

[12] Although we have data for 2010 we deliberately do not calibrate the model to 210 because we want to avoid the Great Recession.

[13] Note that in our model the capital share parameter in the consumption sectors is lower than usual because the capital share in the investment sector equals one.

Table 3: Calibrated Parameters

					1950	2000	
β	0.96	α_U	0.47				
δ	0.05	α_G	0.80	A_g	1	20.15	(6.1% average growth p.a.)
ϵ	0.05	α_S	0.22	A_s	1	2.14	(1.5% average growth p.a.)
σ	0.56	θ	0.17	A_X	0.10	0.10	

Table 4: Targets (in boldface and blue) and Model Predictions

	Model		U.S. Data	
Increase in per capita GDP (in 1990 prices) 1950 to 2000	3		**3**	
Capital share in total income	1/3		**1/3**	
Capital–to–output ratio	3.33		~ 3	
Investment–to–output ratio	0.19		0.23	
	1950	**2000**	**1950**	**2000**
Employment share of services occupations in goods sector	0.10	0.10	**0.10**	**0.10**
goods occupations in goods sector	0.38	0.15	**0.38**	**0.15**
services occupations in services sector	0.41	0.68	**0.41**	0.66
goods occupations in services sector	0.12	0.07	0.12	0.08
Relative labor productivity of goods to services	1	2.8	1	2.2
price of goods to services	1	0.36	1	0.53
Nominal expenditure share of goods	47.2	25.3	60.8	36.1

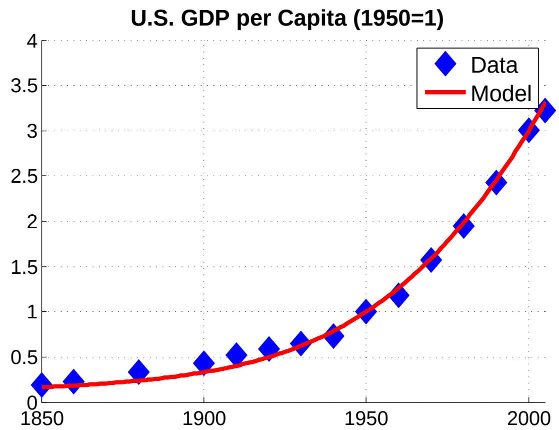
[a] Employment numbers are from population censuses. All other targets are standard parameter values or from the BEA.

investment–output ratio, the evolution of the labor productivity of goods relative to services, and the price of goods to services.

To understand better how the model works, it is instructive to assess how the model fit changes when we vary the key parameters α_U, α_G, σ, ϵ, A_{gt}/A_{st} one by one. The results can be found in Table 5. Note that we do not report what happens when we change α_S, because the effects are the opposite of those when we change α_G.

If we change the relative weights α_U and α_G, then we get straightforward level effects in the expected directions. Specifically, increasing the relative weight of goods in the utility function, $\alpha_U \uparrow$, implies that a higher share of consumption expenditures goes to goods; see the last row in the table and Equation (15). This pushes up the relative demand for the value added of the goods sector. Since the technologies are unchanged with respect to the baseline, the increase in demand leads to a higher labor demand for both occupations in the goods sector. As a result, N_{Gj} go up compared to the baseline case and N_{Sj} decline. Decreasing the relative weight of goods

Figure 2: US GDP per capita: Model versus Data



occupations in the goods sector, while maintaining that the goods sector is more intensive in goods occupations, $\alpha_G \downarrow > \alpha_S$, implies – according to Equation (21) – an increase in the employment share of N_{Gs} and a decrease in the employment share of N_{Gg} compared to the baseline case. The initial employment in the service sector is unaffected and the initial levels of N_{Sg} and N_{Ss} are unchanged. A lower α_G also triggers a dynamic effect. The increasing A_g/A_s leads to the usual within–sector reallocation of employment to the service occupations but since the initial value of N_{Gg} is lower than in the baseline case, there is less room for reallocation. As a result, the total labor reallocation between sectors is smaller. The employment share of the goods sector shrinks moderately from 0.47 in 1950 to 0.46 in 2000. And since the new α_G is only marginally higher than α_S (0.25 vs 0.22), the lack of labor reallocation within and between sectors implies that the relative labor productivity, the relative price of goods as well as the expenditure shares remain almost unchanged.

If we increase $\epsilon \uparrow$, then we reduce the changes in the relative expenditure shares in response to price changes; see Equation (15). This in turn reduces the changes in relative sector labor in response to price changes; this follows by rearranging Equation (20). As a result, between–sector reallocation is too low as the economy develops and the relative price of goods falls. This implies that the employment shares of the intensive occupations change too little with GDP per capita.

If we increase the elasticity of substitution in the CES aggregators of labor, $\sigma \uparrow < 1$, then the initial labor allocation remains unchanged and only a dynamic effect occurs. According to (21), σ governs how changes in relative technological progress affect the within–sector allocation of employment. A higher value of σ implies that the exponent in the term $(A_g/A_s)^{1-\sigma}$ shrinks and, hence, the within–sector labor allocation reacts less to changes in relative technological progress. For example, the ratio of service to goods employment in the service sector, N_{Ss}/N_{Sg}

increases by a factor 1.2 between 1950 and 2000 whereas in the benchmark case it went up by a factor of 2.9. At the same time, the sectoral labor allocation is unaffected by the change in σ and is identical to that of the baseline case.

If we increase the difference between the relative growth rates of technological progress, $\%\Delta A_{gt}/A_{st} \uparrow$, then according to Equation (21) we get more within–sector reallocation of labor. At the same time, Equations (19), (20), and (35) imply that the difference between the labor productivities of goods and services increases faster and, as a consequence, the relative price of goods declines faster than in the benchmark case. This leads to a decrease in the expenditure share of goods – compare Equation (15) – and an increase in the demand for labor in the service sector. Hence N_S/N_G goes up by more than in the baseline case (from 0.53 to 0.84 instead of to 75).

5.2 Experiments

In this subsection, we use our model to conduct several experiments. First, we explore how well it accounts for the reallocation of occupation employment in cases that we have not targeted: the U.S. during 1850–1950 and our full sample of censuses from around the world. Then we explore how well it does if we calibrate it for subperiods and use it for out–of–sample predictions. Lastly, we document by how much the growth rates of sectoral labor productivity change over time and compare it with the data.

To obtain the model predictions for the U.S. during 1850–1950 and for our sample of censuses from around the world, we impose that technological progress grows at the constant average annual growth rates that we calibrated for the U.S. during 1950–2000, namely, $\Delta A_g/A_g = 6.1\%$ and $\Delta A_s/A_s = 1.5\%$. We then simulate the model over time while leaving all model parameters unchanged and report the labor allocations that it creates for each level of GDP er capita. We calculate the GDP per capita in the model in constant model prices, which we choose as the model prices from the U.S. in 1990.[14]

Figure 2 shows that the assumption of constant annual growth rates of A_{jt} yields a good time series fit of U.S. GDP per capita during 1850–2010. Figure 3 reports the relationship between GDP per capita and the shares of occupation employment by sector in total employment in the model and the data. We can see that our simple model captures the general trends in the evolution of the occupation shares for the U.S. during 1850–1950 as well as for our sample of censuses from around the world. We stress that this is not a foregone conclusion because we have imposed many restrictive assumptions, including: the average growth rates of occupation–labor–augmenting technological progress that we calibrated from the U.S. during 1950–2000 also applies during 1840–1950 and to the sample of countries; the elasticity of substitution

[14]The Groningen database calculates GDP in constant international dollars from 1990. These prices are not equal to those in the U.S.A., but since the U.S.A. has a large GDP weight, they are not far off either.

Table 5: Changing the Values of Key Parameters

Baseline: $\alpha_U = 0.47$; $\alpha_G = 0.80$; $\epsilon = 0.05$; $\sigma = 0.56$; $\%\Delta\frac{A_g}{A_s} = 4.6$			
		1950	2000
Share of goods occ. in goods sector	N_{Gg}	0.38	0.15
Share of service occ. in goods sector	N_{Gs}	0.10	0.10
Share of goods occ. in service sector	N_{Sg}	0.12	0.08
Share of service occ. in service sector	N_{Ss}	0.41	0.68
Rel. labor prod. of goods to services	$\frac{Y_G/L_G}{Y_S/N_S}$	1	2.8
Rel. price of goods to services	$\frac{P_G}{P_S}$	1	0.36
Nominal expenditure share of goods	$\frac{P_G Y_G}{PC}$	47.2	25.3

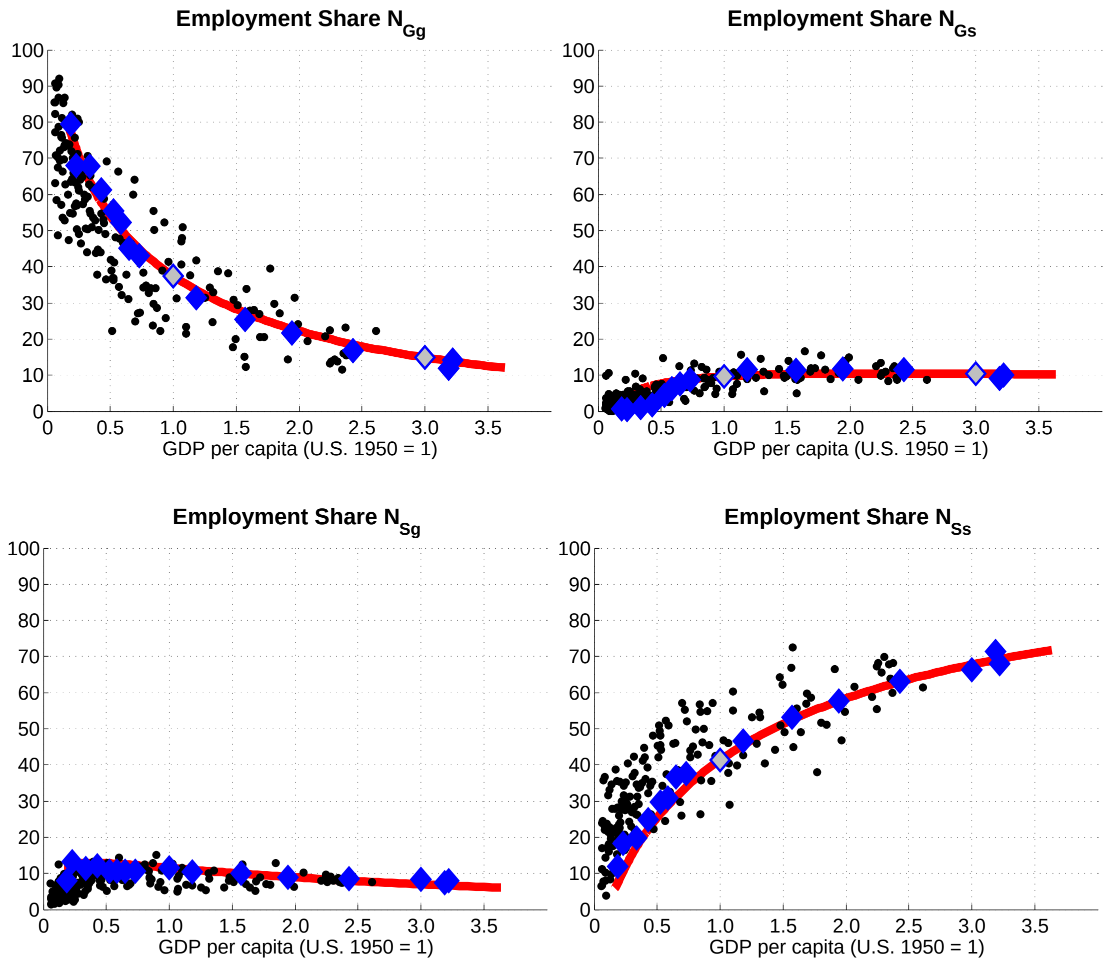
New:	$\alpha_U = 0.75$		$\alpha_G = 0.25$		$\epsilon = 0.75$		$\sigma = 0.95$		$\%\Delta\frac{A_g}{A_s} = 8.0$	
	1950	2000	1950	2000	1950	2000	1950	2000	1950	2000
N_{Gg}	0.60	0.31	0.12	0.05	0.38	0.24	0.38	0.19	0.38	0.07
N_{Gs}	0.15	0.22	0.35	0.41	0.10	0.17	0.10	0.06	0.10	0.10
N_{Sg}	0.05	0.04	0.12	0.05	0.12	0.06	0.12	0.15	0.12	0.04
N_{Ss}	0.20	0.42	0.41	0.49	0.41	0.54	0.41	0.61	0.41	0.80
$\frac{Y_G/L_G}{Y_S/N_S}$	1	2.8	1	1.1	1	2.8	1	2.9	1	4.9
$\frac{P_G}{P_S}$	1	0.36	1	0.96	1	0.36	1	0.34	1	0.20
$\frac{P_G Y_G}{PC}$	75.0	53.2	47.2	46.1	47.2	40.9	47.2	24.4	47.2	16.4

between goods– and service–occupation labor is the same in both sectors; there is no sector–specific, labor–augmenting technological progress, which features prominently in most models of structural transformation.

The previous results suggest that our model should do well at forecasting changes in the composition of broad industry and occupation categories. To establish that is the case, we compare forecasts from our model with actual data and forecasts published by the BLS for several subperiods of 1972–2014. We choose the BLS forecasts because they are among the most downloaded BLS statistics which are commonly used to form expectations about changes in the occupation composition.[15] While the BLS provides its forecasts at a fairly disaggregate level,

[15]According to the BLS, "The Occupational Outlook Handbook is one of the nations most widely used sources

Figure 3: Structural Transformation of Occupation Employment: Model versus Data
(black dots are country–year data, blue diamonds are US data, grey–blue diamonds are targets, red solid line is model)



one can aggregate them to the two broad occupation and industry categories that we have studied here. We focus on seven subperiods that have end dates five years apart; see Table 6.[16] To obtain model forecasts (abbreviated D–H below) for these subperiods, we calibrate our model to U.S. data from 1950 until the first year of the subperiod and then simulate it forward while keeping all parameters including the growth rates of occupation–labor–augmenting technological progress unchanged. Table 6 reports the results. We can see that our model does really well compared to the actual data. In fact, it actually outperforms the BLS forecasts except during the 1990s. This suggests to us that our model captures quantitatively important non–linearities of structural transformation that standard model–free forecasting techniques miss. While our model does not imply forecasts at the same disaggregate level as those provided by the BLS, we conjecture that the BLS would improve upon its disaggregate forecasts if it took the aggregate implications of our model into account.

Table 6: Data vs. Forecasts of Changes in Employment Shares of Goods Occupations and Goods Sector (in percentage points)

	72–85	**78–90**	**82–95**	**88–00**	**90–05**	**00–10**	**04–14**
	Change in employment share of goods occupations						
Data	-6.6	-6.2	-5.3	-3.2	-3.4	-3.7	-2.7
BLS forecast	-3.9	-1.8	-1.5	-3.0	-2.9	-1.0	-1.0
D–H forecast	-7.9	-6.6	-6.7	-5.7	-6.7	-3.8	-3.5
	Change in employment share of goods sector						
Data	-5.3	-5.8	-5.5	-4.0	-4.2	-4.1	-3.1
BLS	-3.4	-2.1	-0.4	-3.3	-4.2	-1.9	-2.4
D–H	-5.4	-4.6	-4.8	-4.2	-4.9	-3.1	-2.9

We conclude this subsection by returning to Proposition 3, in which we proved that at least one growth rate of sectoral labor productivity changes over time. The simplest way to assess the quantitative implications of our model in this regard is to regress the model–generated annual changes in sectoral labor productivity in constant prices on a constant and a linear time trend. This regression gives a time trend of -0.016 for the goods sector and -0.009 for the service sector. In comparison, for actual U.S. data 1950–2010 the coefficients are -0.027 for the goods sector and -0.014 for the service sector. All of these coefficient come out significant at the 5 or 1% level. These estimates suggest that our model offers a simple first step towards understanding why sectoral growth rates have declined. In a companion paper, Duernecker et al. (2016), we show that this has happened more widely across countries and make the con-

of career information. It provides details on hundreds of occupations and is used by career counselors, students, parents, teachers, jobseekers, career changers, education and training officials, and researchers."

[16]The last subperiod is only four years long because we don't yet have data for 2015.

nection to Baumol's disease, that is, the decline in aggregate growth that results when labor is reallocated to the service sector and service occupations, which both experience slower growth than the goods sector and the goods occupations.

6 Related Literature

As we pointed out above, our work is related to the large literature on job polarization; see for example Autor et al. (2006), Autor and Dorn (2013), and Goos et al. (2014). We emphasize that in several dimensions our perspective is broader than in that literature. To begin with, the literature on job polarization is focused on non–farm employment of currently rich western countries in which agriculture plays a negligible role. In contrast, our sample of 67 countries covers many currently poor countries from Africa, Asia, and Latin America in which the agriculture sector is sizeable or even the largest sector. We have therefore included the agricultural sector in our analysis as part of the goods sector. With regards to the U.S., we find very similar reallocation patterns also for more than a century starting in 1850 when ICT technology advancements did not yet play a large role. This suggests there must be other reasons for why technological progress is biased towards goods occupations.

Bárány and Siegel (2014) and Lee and Shin (2015) studied the relationship between structural transformation and job polarization. The main difference between Bárány and Siegel (2014) and our work is that they embedded a Roy model into a structural transformation model. Specifically, they assumed that there are time–invariant CES functions that aggregate the different categories of labor to sector labor and that sector–specific technological progress augments all sector labor. In contrast, our results suggest that labor–augmenting, occupation–specific technological progress that is independent of the sectors is the main force behind the reallocation of occupation labor. The main differences between Lee and Shin (2015) and our work is that they are focused on the U.S. and emphasize the role that managers play in the process of U.S. structural transformation.[17] In contrast, we have studied a very sample and have treated all service occupations including managers as one category. An obvious, and potentially fruitful, extension of our model would be to disaggregate the service occupations into high–skilled and low–skilled service occupations. Managers would then be an important, albeit not the only, part of the high–skilled service category.

In a related project, Duernecker and Herrendorf (2015), we pursue this extension and study how barriers to entry have reduced employment in Germany compared to the U.S. The idea of this project is that while job polarization has happened in both Germany and the U.S., in Germany barriers to the entry into low–skilled jobs have prevented many workers who were

[17] Our comparison with Lee and Shin's work is based on a set of slides. We have not had access to a completed paper.

displaced by routine–biased technological change from finding new employment. We show that this difference between the labor markets of Germany and the U.S. is a quantitatively important reason for the lower aggregate hours worked per working–age population in Germany than in the U.S.

There is interesting recent work that studies how structural transformation affects relative wages between subgroups of the labor force like the gender gap in wages or the skill premium; see for example Bárány and Siegel (2014), Ngai and Petrongolo (2014), and Buera et al. (2015). By construction, our model cannot speak to changes in relative wages, because we assumed for simplicity that everyone can supply every occupation, which means that wages have to be equalized across occupation. While that is useful first step that parallels the assumption that is made in most of the literature on structural transformation between sector labor, the next step on our agenda is to break this assumption. For example, we plan to introduce different entry costs into occupations. This will allow us to study questions related to relative wage differences, like the gender gap in wages or the skill premium.

7 Conclusion

We have established that the standard facts of structural transformation hold for both sector and occupation employment and that in all sectors the employment share of the service occupations rises with GDP per capita. We have built a model of structural transformation that accounts quantitatively for these facts. The key novelty of our model has been that it features broad categories of both industries and sectors and has labor–augmenting technological progress that is specific to occupations but independent of the sector in which occupation labor is used. We have shown that this model is not only able to account for the structural transformation of sector and occupation employment, but also is useful for predicting changes in the composition of broad occupations and for understanding why sectoral labor productivities are declining.

Two steps are next on our agenda: extend our analysis to more than two sectors/occuatpions; model differences in entry costs into occupations and study the implied differences in wages.

References

Abraham, Katharine G. and Susan K. Taylor, "Firms' Use of Outside Contractors: Theory and Evidence," *Journal of Labor Economics*, 1996, *107*, 394–424.

Abramovsky, Laura and Rachel Griffith, "Outsourcing and Offshoring of Business Services: How Important is ICT?," *Journal of the European Economic Association*, 2006, *4*, 594–601.

Acemoglu, Daron and David H. Autor, "Skills, Tasks and Technologies: Implications for Employment and Earnings," in Orley Ashenfelter and David E. Card, eds., *Handbook of Labor Economics*, Vol. 4B, Amsterdam: Elsevier B.V., 2011, pp. 1043–1071.

—— and Veronica Guerrieri, "Capital Deepening and Non–Balanced Economic Growth," *Journal of Political Economy*, 2008, *116*, 467–498.

Autor, David H. and David Dorn, "The Growth of Low–Skilled Service Jobs and the Polarization of the US Labor Market," *American Economic Review*, 2013, *103*, 1553–1597.

——, Lawrence F. Katz, and Melissa S. Kearney, "The Polarization of the U.S. Labor Market," *American Economic Review*, 2006, *96*, 1553–1597.

Bah, El–hadj, "Structural Transformation Paths Across Countries," *Emerging Markets Finance and Trade*, 2011, *47*, 5–19.

Bárány, Zsófia and Christian Siegel, "Job Polarization and Structural Change," Manuscript, Sciences Po and University of Exeter 2014.

Baumol, William J., "Macroeconomics of Unbalanced Growth: The Anatomy of the Urban Crisis," *American Economic Review*, 1967, *57*, 415–426.

Berlingieri, Giuseppe, "Outsourcing and the Rise in Services," Manuscript, London School of Economics 2014.

Boppart, Timo, "Structural Change and the Kaldor Facts in a Growth Model with Relative Price Effects and Non–Gorman Preferences," *Econometrica*, 2016, pp. 2167–2196.

Buera, Francisco J. and Joseph P. Kaboski, "The Rise of the Service Economy," *American Economic Review*, 2012, *102*, 2540–2569.

——, ——, and Richard Rogerson, "Skill Biased Structural Change," Working Paper 21165, NBER 2015.

Caselli, Francesco and Wilbur John Coleman, "The U.S. Structural Transformation and Regional Convergence: A Reinterpretation," *Journal of Political Economy*, 2001, *109*, 584–616.

Comin, Diego, Martí Mestieri, and Danial Lashkari, "Structural Transformations with Long–run Income and Price Effects," Manuscript, Northwestern University 2015.

Duarte, Margarida and Diego Restuccia, "The Role of the Structural Transformation in Aggregate Productivity," *Quarterly Journal of Economics*, 2010, *125*, 129–173.

Duernecker, Georg and Berthold Herrendorf, "Occupations, Barriers to Entry and Hours Worked in the U.S. and Germany," Manuscript, University of Mannheim and Arizona State University 2015.

——, ——, and Ákos Valentinyi, "How Sickening is Baumol's Disease?," Manuscript, University of Mannheim, Arizona State University, and Cardiff Business School 2016.

Echevarria, Cristina, "Changes in Sectoral Composition Associated with Economic Growth," *International Economic Review*, 1997, *38*, 431–452.

Goldin, Claudia and Laurence F. Katz, *The Race between Education and Technology*, Cambridge: Harvard University Press, 2008.

Gollin, Douglas, Stephen L. Parente, and Richard Rogerson, "The Food Problem and the Evolution of International Income Levels," *Journal of Monetary Economics*, 2007, *54*, 1230–1255.

Goos, Maarten, Alan Manning, and Anna Salomons, "Explaining Job Polarization: Routine–Biased Technological Change and Offshoring," *American Economic Review*, 2014, *104*, 2509–2526.

Herrendorf, Berthold, Christopher Herrington, and Ákos Valentinyi, "Sectoral Technology and Structural Transformation," *American Economic Journal – Macroeconomics*, 2015, *7*, 1–31.

——, Richard Rogerson, and Ákos Valentinyi, "Two Perspectives on Preferences and Structural Transformation," *American Economic Review*, 2013, *103*, 2752–2789.

——, ——, and ——, "Growth and Structural Transformation," in Philippe Aghion and Steven N. Durlauf, eds., *Handbook of Economic Growth*, Vol. 2, Elsevier, 2014, pp. 855–941.

Kambourov, Gueorgui and Iouri Manovskii, "Rising Occupational and Industry Mobility in the United States: 1968–97," *International Economic Review*, 2008, *49*, 41–79.

—— and ——, "Occupation Mobility and Wage Inequality," *Review of Economic Studies*, 2009, *76*, 731–751.

—— and ——, "Occupation Specificity of Human Capital," *International Economic Review*, 2009, *50*, 63–115.

Kongsamut, Piyabha, Sergio Rebelo, and Danyang Xie, "Beyond Balanced Growth," *Review of Economic Studies*, 2001, *68*, 869–882.

Kuznets, Simon, "Modern Economic Growth: Findings and Reflections," *Amercian Economic Review*, 1973, *63*, 247–258.

Laitner, John, "Structural Change and Economic Growth," *Review of Economic Studies*, 2000, *67*, 545–561.

Lee, Tim and Yongseok Shin, "Managing a Polarized Structural Change," Manuscript, Washington University 2015.

Minnesota Population Center, "Integrated Public Use Microdata Series, International: Version 6.4," Machine–Readable Database, Minneapolis: University of Minnesota 2015.

Ngai, L. Rachel and Christopher A. Pissarides, "Structural Change in a Multisector Model of Growth," *American Economic Review*, 2007, *97*, 429–443.

Ngai, Rachel and Barbara Petrongolo, "Gender Gaps and the Rise of the Service Economy," Manuscript, London School of Economics 2014.

Rogerson, Richard, "Structural Transformation and the Deterioration of European Labor Market Outcomes," *Journal of Political Economy*, 2008, *116*, 235–259.

Valentinyi, Ákos and Berthold Herrendorf, "Measuring Factor Income Shares at the Sectoral Level," *Review of Economic Dynamics*, 2008, *11*, 820–835.

Appendix A: Data

Census observations

Armenia (2011); Argentina (1970, 1980); Austria (1971, 1981, 1991, 2001); Bolivia (1976, 1992, 2001); Brazil (1960, 1970, 1980, 1990, 2000, 2010); Burkina Faso (1996); Cambodia (1998, 2008); Cameroon (2005); Canada (1971, 1981, 1991, 2001); Chile (1960, 1970, 1982, 1992, 2002); China (1982, 1990); Colombia (1964, 1973); Costa Rica (1973, 1984, 2000, 2011); Dominican Republic (1960, 1970, 1981); Ecuador (1962, 1982, 1990, 2001, 2010); Egypt (2006); El Salvador (1992); France (1962, 1968, 1975, 1982, 1990, 1999, 2006, 2011); West Germany (1970, 1987); Ghana (1984, 2000, 2010); Greece (1971, 1981, 1991, 2001); Guinea (1983); Haiti (1982, 2003); Hungary (2001); India (1983, 1987, 1993, 1999, 2004); Indonesia (1971, 1976, 1980, 1985, 1990, 1995); Iran (2006); Ireland (1971, 1981, 1991, 1996, 2002, 2006, 2011); Italy (1997); Jamaica (1982, 1991); Kyrgistan (2004); Liberia (2008); Malawi (1987, 1998, 2008); Malaysia (1970, 1980, 1991, 2000); Mali (1987, 1998, 2009); Mexico (1970, 1990, 1995, 2000, 2010); Mongolia (2000); Morocco (1982, 1994, 2004);

Mozambique (1997, 2007); Netherlands (2001); Nicaragua (1971, 1995, 2006); Nigeria (2008, 2009, 2010); Pakistan (1973); Panama (1960, 1970, 1980, 1990, 2000, 2010); Paraguay (1962, 1972, 1982, 1992, 2002); Peru (1993, 2007); Philippines (1990); Portugal (1981, 1991, 2001, 2011); Puerto Rico (1990, 2000, 2005, 2010); Romania (1992, 2002); Rwanda (2002); Senegal (1998); Sierra Leone (2004); Slovenia (2002); South Africa (2007); Spain (1981, 1991, 2001); Sudan (2008); Switzerland (1970, 1980, 1990, 2000); Tanzania (2002); Turkey (1985, 1990, 2000); Uganda (2002); U.K. (1991, 2001); Uruguay (1963, 1996, 2006); U.S.A. (1960, 1970, 1980, 1990, 2000, 2010); Venezuela (1981, 1990, 2001); Vietnam (1999, 2009); Zambia (1990, 2000, 2010).

Elementary occupations

To determine the validity of our way of assigning elementary occupations to agriculture, industry, and service occupations, we use that U.S. census data have detailed occupation information that allows us to decompose the broad category elementary occupations into finer subcategories that we can assign to services and goods. The first two panels in Tables 7 and 8 compare the employment shares for the U.S. obtained from our approximation (upper panel) with the actual employment shares. By and large, the approximation is accurate and, more importantly, the trends in the data are fully preserved.

Table 7: Aggregate Employment Shares in the U.S. 1850–2000

Employment share of	1850	1900	1950	2000
	Baseline			
goods occupations	87.5	73.3	49.0	23.2
service occupations	12.5	26.7	51.0	76.8
	Re–classify elementary			
goods occupations	87.7	73.0	50.9	25.2
service occupations	12.3	27.0	49.1	74.8
	Re-classify all			
goods occupations	84.4	68.1	44.4	21.1
service occupations	15.6	31.9	55.6	78.9
service occupations re–classified as goods	0.0	0.0	0.0	0.0
goods occupations re–classified as services	3.1	4.6	6.5	4.0

Broad versus fine occupation groups

The information provided by IPUMS International about a worker's occupation is encoded in the variable OCCISCO. This variable captures 10 broad occupation groups. An important question is whether the classification that IPUMS applied to obtain the 10 groups is consistent with our classification of occupations into goods and services. For our purpose it is important that each of the 10 IPUMS groups are sufficiently homogenous and do not contain a mixture of service and goods occupations. We again use U.S. census data from IPUMS International to find out whether there are any occupations in the IPUMS goods occupation groups which are, in fact, service occupations and vice versa. The advantage of the U.S. censuses is that they come with a fine 3–digit occupation classification (in addition to broad IPUMS occupation classification), which allows us to determine the composition of each broad IPUMS occupation group.

Out of the 269 3–digit occupations in the U.S. censuses, we re–classify 25 occupations. All of them are service occupations that are part of a broad IPUMS group which we have classified as goods. The total employment share of the re–classified occupations is shown below in Table 9. The same table also shows the largest occupations (in terms of their 2000 employment share) which we re–classify. The lower panels in Tables 7 and 8 show the employment shares of goods and service occupations in the U.S. economy under under the adjusted classification. As before, there are level effects but the trends are unaffected.

Table 8: Sectoral Employment Shares in the U.S. 1850–2000

Employment share of	Goods sector				Service sector			
	1850	1900	1950	2000	1850	1900	1950	2000
				Baseline				
goods occupations	99.3	97.1	79.5	58.9	40.4	32.8	21.8	11.1
service occupations	0.7	2.9	20.5	41.1	59.6	67.2	78.2	88.9
				Re–classify elementary occupations				
goods occupations	99.3	97.5	81.7	62.3	41.3	31.3	23.5	12.6
service occupations	0.7	2.5	18.3	37.7	58.7	68.7	76.5	87.4
				Re-classify all				
goods occupations	99.3	97.0	77.7	58.5	24.7	19.1	14.7	8.5
service occupations	0.7	3.0	22.3	41.5	75.3	80.9	85.3	91.5

Table 9: Reclassified Occupations for the U.S. 1850–2000

	1850	1900	1950	2000
Truck and tractor drivers	0.70	1.90	2.41	2.36
Linemen and servicemen	0.00	0.05	0.39	0.39
Bus drivers	0.08	0.02	0.29	0.36
Laundry and dry cleaning operatives	0.00	0.35	0.75	0.22
Taxicab drivers and chauffers	0.08	0.23	0.36	0.18
Stationary engineers	0.00	0.13	0.38	0.15
Stationary firemen	0.01	0.09	0.22	0.04
Locomotive engineers	0.01	0.25	0.13	0.04
Deliverymen and routemen	0.02	0.18	0.42	0.04
Power station operators	0.00	0.00	0.04	0.03
Sailors and deck hands	1.54	0.21	0.08	0.02
Rest	0.66	1.19	1.02	0.22
Total	3.08	4.59	6.50	4.04

8 Appendix B: Proofs

First–Order Conditions to the Firm Problem

- We need to show (18)–(22).
- We drop the time indexes when this does not cause confusion.
- The first–order conditions to problem (17) are:

$$r = \theta P_J K_J^{\theta-1} L_J^{1-\theta} = \theta P_J \left(\frac{K_J}{N_J}\right)^{\theta-1} \left(\frac{L_J}{N_J}\right)^{1-\theta} \tag{25}$$

$$w = (1-\theta) P_J K_J^{\theta} L_J^{-\theta} L_J^{\frac{1}{\sigma}} \alpha_J^{\frac{1}{\sigma}} A_g^{\frac{\sigma-1}{\sigma}} N_{Jg}^{-\frac{1}{\sigma}} \tag{26}$$

$$w = (1-\theta) P_J K_J^{\theta} L_J^{-\theta} L_J^{\frac{1}{\sigma}} (1-\alpha_J)^{\frac{1}{\sigma}} A_s^{\frac{\sigma-1}{\sigma}} N_{Js}^{-\frac{1}{\sigma}} \tag{27}$$

- (21) follows by dividing (26) and (27) by each other.
- Multiplying (26)–(27) with the respective labor and adding up gives that:

$$w = (1-\theta) P_J \left(\frac{K_J}{N_J}\right)^{\theta} \left(\frac{L_J}{N_J}\right)^{1-\theta} \tag{28}$$

- Dividing (28) by (25), we find:

$$\frac{w}{r} = \frac{1-\theta}{\theta}\frac{K_J}{N_J} \tag{29}$$

- Hence,

$$\frac{K_G}{N_G} = \frac{K_S}{N_S} \tag{30}$$

which was (18).

- (28) and (30) imply (19) and (20):

$$\begin{aligned}\frac{Y_G/N_G}{Y_S/N_S} &= \left(\frac{L_G/N_G}{L_S/N_S}\right)^{1-\theta}\\ \frac{Y_G/N_G}{Y_S/N_S} &= \frac{P_S}{P_G}\end{aligned}$$

- It remains to show (22). Using (27) for $J = G, S$, we obtain:

$$P_G K_G^{\theta} L_G^{-\theta} L_G^{\frac{1}{\sigma}} (1-\alpha_G)^{\frac{1}{\sigma}} A_s^{\frac{\sigma-1}{\sigma}} N_{Gs}^{-\frac{1}{\sigma}} = P_S K_S^{\theta} L_S^{-\theta} L_S^{\frac{1}{\sigma}} (1-\alpha_S)^{\frac{1}{\sigma}} A_s^{\frac{\sigma-1}{\sigma}} N_{Ss}^{-\frac{1}{\sigma}}$$

Using (20) and (30), this can be simplified to (22):

$$\frac{N_{Ss}}{N_{Gs}} = \frac{1-\alpha_S}{1-\alpha_G}\left(\frac{L_S}{L_G}\right)^{1-\sigma}\left(\frac{N_S}{N_G}\right)^{\sigma}$$

- QED

Proof of Proposition 1

- Need to show that there is a unique GBGP.
- (16) implies that $r_t = r = A_x$.
- (13) implies that

$$\frac{C_{t+1}P_{t+1}}{C_t P_t} = \gamma \equiv \beta(1 + A_X - \delta)$$

- $w_t = w_t N_t = (1-\theta) C_t P_t$ implies that w_t grows at the same factor as $C_t P_t$, i.e., γ.
- $r_t K_{Ct} = A_X K_{Ct} = \theta C_t P_t$ implies that K_{Ct} grows at the same factor as $C_t P_t$, i.e., γ.
- The consumer budget constraint can be rewritten to:

$$\frac{\theta C_t P_t}{K_t} + \frac{K_{t+1}}{K_t} = 1 + A_x - \delta$$

Hence, if K_t grows at a constant factor, then that factor must be γ.

- $K_t = K_{Xt} + K_{Ct}$ implies that K_{Xt} grows at factor γ too.
- QED

Proof of Proposition 2

Proof of "$\Longrightarrow$"

Derivation of SF 2

- We need to show that N_S/N_G increases.
- (21) implies:

$$N_J = N_{Jg} + N_{Js} = \left[\frac{N_{Jg}}{N_{Js}} + 1\right] N_{Js} = \left[\frac{\alpha_J}{1-\alpha_J}\left(\frac{A_g}{A_s}\right)^{\sigma-1} + 1\right] N_{Js}$$

- Hence, the ratio of sectoral labor satisfies:

$$\frac{N_S}{N_G} = \left[\frac{1 + \left[\alpha_S/(1-\alpha_S)\right]\left(A_g/A_s\right)^{\sigma-1}}{1 + \left[\alpha_G/(1-\alpha_G)\right]\left(A_g/A_s\right)^{\sigma-1}}\right]\frac{N_{Ss}}{N_{Gs}} \tag{31}$$

- Combining (15) and (20) and rearranging gives:

$$\frac{P_S}{P_G} = \left(\frac{\alpha_U}{1-\alpha_U}\frac{N_S}{N_G}\right)^{\frac{1}{1-\varepsilon}}$$

Substituting this into (22), we obtain

$$\frac{N_{Ss}}{N_{Gs}} = \frac{1-\alpha_S}{1-\alpha_G}\left(\frac{1-\alpha_U}{\alpha_U}\right)^{\frac{1-\sigma}{(1-\varepsilon)(1-\theta)}}\left(\frac{N_S}{N_G}\right)^{1-\frac{1-\sigma}{(1-\varepsilon)(1-\theta)}}$$

- Substituting this into (31) gives:

$$\frac{N_S}{N_G} = \left(\frac{1-\alpha_S}{1-\alpha_G}\right)^{\frac{(\varepsilon-1)(1-\theta)}{\sigma-1}}\frac{1-\alpha_U}{\alpha_U}\left[\frac{1 + \left[\alpha_S/(1-\alpha_S)\right]\left(A_g/A_s\right)^{\sigma-1}}{1 + \left[\alpha_G/(1-\alpha_G)\right]\left(A_g/A_s\right)^{\sigma-1}}\right]^{\frac{(\varepsilon-1)(1-\theta)}{\sigma-1}} \tag{32}$$

- Define

$$f(x) \equiv \left[\frac{1 + \tilde{\alpha}_S x^{\sigma-1}}{1 + \tilde{\alpha}_G x^{\sigma-1}}\right]^{\frac{1}{\sigma-1}}$$

where $\tilde{\alpha}_J \equiv \alpha_J/(1-\alpha_J)$.

- It is straightforward to show that given Assumption (i) we have $f'(x) < 0$.
- Fact 1 now follows from (32), $f'(x) < 0$, and Assumption (iii) and (iv).
- QED

Derivation of SF 4

- We need to show N_{Js}/N_{Jg} increases.
- This follows directly from (21) and Assumptions (ii) and (iv).
- QED

Derivation of SF 3

- We need to show $N_{Ss} > N_{Sg}$ and $N_{Gg} > N_{Gs}$.
- (i) and (21) imply the claim.
- QED

Derivation of SF 1

- N_s/N_g increases.
- To see this, note that:

$$N_s = N_{Gs} + N_{Ss} = N_G \frac{N_{Gs}}{N_G} + N_S \frac{N_{Ss}}{N_S}$$

- Hence,

$$\Delta N_s = \Delta N_G \frac{N_{Gs}}{N_G} + N_G\, \Delta \frac{N_{Gs}}{N_G} + \Delta N_S \frac{N_{Ss}}{N_S} + N_S\, \Delta \frac{N_{Ss}}{N_S}$$

- Using that $N_S = 1 - N_G$, this becomes:

$$\Delta N_s = N_G \Delta \frac{N_{Gs}}{N_G} + N_S \Delta \frac{N_{Ss}}{N_S} + \Delta N_S \left(\frac{N_{Ss}}{N_S} - \frac{N_{Gs}}{N_G} \right)$$

- SF 2 implies $\Delta N_S > 0$;
  SF 3 implies $N_{Ss}/N_S > N_{Gs}/N_G$;
  SF 4 implies $\Delta N_{Js}/N_J > 0$;

- Hence, the right–hand side is positive and N_s grows.

- Since $N_g = 1 - N_s$, this implies that N_g falls.
- QED

Derivation of SF 7

- We need to show that $(P_S Y_S)/(P_G Y_G)$ grows.
- To see this, note that (20) implies that

$$\frac{Y_S P_S}{Y_G P_G} = \frac{N_S}{N_G} \tag{33}$$

- Fact 5 therefore follows from Fact 1.
- QED

Derivation of SF's 5 and 6

- We need to show that P_S/P_G increases and $(Y_S/N_S)/(Y_G/N_G)$ decreases.
- (20) implies that either one of these statements is true iff the other one is true.
- We therefore only show that $(Y_S/N_S)/(Y_G/N_G)$ decreases.
- (19) implies that this is equivalent to showing that $(L_S/N_S)/(L_G/N_G)$ decreases.
- To see that $(L_S/N_S)/(L_G/N_G)$ decreases, rewrite (3) while using (21):

$$\begin{aligned}
\frac{L_S}{L_G} &= \left[\frac{1+\left[\alpha_S/(1-\alpha_S)\right]^{\frac{1}{\sigma}}\left[(A_g N_{Sg})/(A_s N_{Ss})\right]^{\frac{\sigma-1}{\sigma}}}{1+\left[\alpha_G/(1-\alpha_G)\right]^{\frac{1}{\sigma}}\left[(A_g N_{Gg})/(A_s N_{Gs})\right]^{\frac{\sigma-1}{\sigma}}}\right]^{\frac{\sigma}{\sigma-1}}\left(\frac{1-\alpha_S}{1-\alpha_G}\right)^{\frac{1}{\sigma-1}}\frac{N_{Ss}}{N_{Gs}} \\
&= \left(\frac{1-\alpha_S}{1-\alpha_G}\right)^{\frac{1}{\sigma-1}}\left[\frac{1+\left[\alpha_S/(1-\alpha_S)\right]\left(A_g/A_s\right)^{\sigma-1}}{1+\left[\alpha_G/(1-\alpha_G)\right]\left(A_g/A_s\right)^{\sigma-1}}\right]^{\frac{\sigma}{\sigma-1}}\frac{N_{Ss}}{N_{Gs}}
\end{aligned} \tag{34}$$

- Dividing this by (31) gives:

$$\frac{L_S/N_S}{L_G/N_G} = \left(\frac{1-\alpha_S}{1-\alpha_G}\right)^{\frac{1}{\sigma-1}}\left[\frac{1+\left[\alpha_S/(1-\alpha_S)\right]\left(A_g/A_s\right)^{\sigma-1}}{1+\left[\alpha_G/(1-\alpha_G)\right]\left(A_g/A_s\right)^{\sigma-1}}\right]^{\frac{1}{\sigma-1}} \tag{35}$$

- Using that $f'(x) < 0$ and Assumption (iv), it follows that $(L_S/N_S)/(L_G/N_G)$ decreases.
- QED

Proof of "$\Longleftarrow$"

- To get the SF 1–7 conditions (i)–(iv) must hold.
- (21) implies that we need (i) to get SF 3.
- (15) implies that given SF 5, we need (iii) to get SF 7.
- (32) and $f'(x) < 0$ implies that to get SF 1 given (iii) holds, we need (iv).
- (21) implies that given (iv), we need (ii) to get SF 4.
- QED

Proof of Proposition 3

We need to show that at least one of the two growth rates

$$\Delta \log(LP_{Jt}) \equiv \log\left(\frac{Y_{Jt}}{N_{Jt}}\right) - \log\left(\frac{Y_{Jt-1}}{N_{Jt-1}}\right)$$

changes over time. We are going to achieve this by showing that their difference changes over time. The difference of the two growth rates is given by:

$$\begin{aligned}
&\Delta \log(LP_{St}) - \Delta \log(LP_{Gt}) \\
&= \log\left(\frac{Y_{St}/N_{St}}{Y_{Gt}/N_{Gt}}\right) - \log\left(\frac{Y_{St-1}/N_{St-1}}{Y_{Gt-1}/N_{Gt-1}}\right) \\
&= \log\left(\frac{K_{St}/N_{St}}{K_{Gt}/N_{Gt}}\right)^{\theta}\left(\frac{L_{St}/N_{St}}{L_{Gt}/N_{Gt}}\right)^{1-\theta} - \log\left(\frac{K_{St-1}/N_{St-1}}{K_{Gt-1}/N_{Gt-1}}\right)^{\theta}\left(\frac{L_{St-1}/N_{St-1}}{L_{Gt-1}/N_{Gt-1}}\right)^{1-\theta} \\
&= (1-\theta)\left[\log\left(\frac{L_{St}/N_{St}}{L_{Gt}/N_{Gt}}\right) - \log\left(\frac{L_{St-1}/N_{St-1}}{L_{Gt-1}/N_{Gt-1}}\right)\right]
\end{aligned}$$

where we have used that in each period the capital–labor ratio is equalized across sectors. Recalling (35), we can rewrite the difference of the growth rates:

$$\begin{aligned}
&\Delta \log(LP_{St}) - \Delta \log(LP_{Gt}) \\
&= \frac{1-\theta}{\sigma-1}\left[\log\left(\frac{1+\frac{\alpha_S}{1-\alpha_S}\left(\frac{A_{gt}}{A_{st}}\right)^{\sigma-1}}{1+\frac{\alpha_G}{1-\alpha_G}\left(\frac{A_{gt}}{A_{st}}\right)^{\sigma-1}}\right) - \log\left(\frac{1+\frac{\alpha_S}{1-\alpha_S}\left(\frac{A_{gt-1}}{A_{st-1}}\right)^{\sigma-1}}{1+\frac{\alpha_G}{1-\alpha_G}\left(\frac{A_{gt-1}}{A_{st-1}}\right)^{\sigma-1}}\right)\right] \\
&= \frac{1-\theta}{\sigma-1}\left[\log\left(1+a_S\gamma^t\right) - \log\left(1+a_G\gamma^t\right) - \log\left(1+a_S\gamma^{t-1}\right) + \log\left(1+a_G\gamma^{t-1}\right)\right]
\end{aligned}$$

where $a_J \equiv (A_{g0}/A_{s0})^{\sigma-1}\alpha_J/(1-\alpha_J)$ and $\gamma \equiv (\gamma_g/\gamma_s)^{\sigma-1}$. After taking time derivatives and doing some tedious algebra, we obtain:

$$\frac{\partial}{\partial t}[\Delta \log(LP_{St}) - \Delta \log(LP_{Gt})]$$
$$= \frac{\gamma^{t-1}\log(\gamma)(1-\theta)(1-\gamma)(a_G - a_S)}{1-\sigma} \frac{a_G a_S \gamma^{2t-1} - 1}{(1+a_S\gamma^t)(1+a_G\gamma^t)(1+a_S\gamma^{t-1})(1+a_G\gamma^{t-1})}$$

Since $\gamma < 1$, the first ratio is negative and converges to zero. Since $\gamma < 1$, γ^{2t-1} is monotonically decreasing towards zero, the second ratio becomes negative to the right of a finite threshold value of t. Hence, the overall derivative is positive from that threshold value onwards. Going from $t-1$ to t to the right of the threshold value, the growth rate difference at t must be larger than the growth rate difference at $t-1$ to the right of the threshold value. Since the growth rate difference goes to zero that must mean that the growth rate difference is negative all the time while becoming less and less negative, approaching zero from below. QED

Appendix C: Calibration

Equilibrium conditions

- **Household**'s optimality conditions (μ is the Lagrange multiplier on the budget constraint)

$$\begin{array}{ll} (I) & \mu P_G = C^{\frac{1-\epsilon}{\epsilon}} \alpha_U^{\frac{1}{\epsilon}} C_G^{-\frac{1}{\epsilon}} \\ (II) & \mu P_S = C^{\frac{1-\epsilon}{\epsilon}} (1-\alpha_U)^{\frac{1}{\epsilon}} C_S^{-\frac{1}{\epsilon}} \\ (III) & \mu_t = \beta[r_{t+1} + 1 - \delta]\mu_{t+1} \end{array}$$

- Firm's optimality conditions (**goods sector**)

$$\begin{array}{ll} (IV) & w = (1-\theta)P_G C_G L_G^{\frac{1-\sigma}{\sigma}} \alpha_G^{\frac{1}{\sigma}} A_g^{\frac{\sigma-1}{\sigma}} N_{Gg}^{-\frac{1}{\sigma}} \\ (V) & w = (1-\theta)P_G C_G L_G^{\frac{1-\sigma}{\sigma}} (1-\alpha_G)^{\frac{1}{\sigma}} A_s^{\frac{\sigma-1}{\sigma}} N_{Gs}^{-\frac{1}{\sigma}} \\ (VI) & r_t = \theta \frac{P_G C_G}{K_G} \end{array}$$

- Firm's optimality conditions (**services sector**)

$$\begin{array}{ll} (VII) & w = (1-\theta)P_S C_S L_S^{\frac{1-\sigma}{\sigma}} \alpha_S^{\frac{1}{\sigma}} A_g^{\frac{\sigma-1}{\sigma}} N_{Sg}^{-\frac{1}{\sigma}} \\ (VIII) & w = (1-\theta)P_S C_S L_S^{\frac{1-\sigma}{\sigma}} (1-\alpha_S)^{\frac{1}{\sigma}} A_s^{\frac{\sigma-1}{\sigma}} N_{Ss}^{-\frac{1}{\sigma}} \\ (IX) & r_t = \theta \frac{P_S C_S}{K_S} \end{array}$$

- Firm's optimality condition (**investment sector**)

$$(X) \qquad r_t = A_X$$

Solving for the labor allocation

We transform the system of equations to solve for the labor allocation $N_{Gg}, N_{Gs}, N_{Sg}, N_{Ss}$:

- Divide (I) by (II)
- Divide (IV) by (V)
- Divide the production function for C_G by that for C_S and use $K_G/N_G = K_S/N_S$
- Divide (VI) by (IX) and use $K_G/N_G = K_S/N_S$

where we use the following definitions: $N_G = N_{Gg} + N_{Gs}$ and $N_S = N_{Sg} + N_{Ss}$. That gives the following system:

$$\begin{array}{ll}
(I) & \frac{P_G}{P_S} = \left(\frac{\alpha_U}{1-\alpha_U}\right)^{\frac{1}{\epsilon}} \left(\frac{C_G}{C_S}\right)^{-\frac{1}{\epsilon}} \\
(II) & 1 = \left(\frac{\alpha_G}{1-\alpha_G}\right)^{\frac{1}{\sigma}} \left(\frac{A_g}{A_s}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{N_{Gg}}{N_{Gs}}\right)^{-\frac{1}{\sigma}} \\
(III) & 1 = \left(\frac{\alpha_S}{1-\alpha_S}\right)^{\frac{1}{\sigma}} \left(\frac{A_g}{A_s}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{N_{Sg}}{N_{Ss}}\right)^{-\frac{1}{\sigma}} \\
(IV) & 1 = N_{Gg} + N_{Gs} + N_{Sg} + N_{Ss} \\
(V) & \frac{C_G}{C_S} = \left(\frac{N_G}{N_S}\right)^{\theta} \left(\frac{L_G}{L_S}\right)^{1-\theta} \\
(VI) & 1 = \frac{P_G}{P_S}\frac{C_G}{C_S}\left(\frac{N_G}{N_S}\right)^{-1} \\
(VII) & \frac{L_G}{L_S} = \left[\frac{\alpha_G^{\frac{1}{\sigma}}\left(\frac{A_g}{A_s}N_{Gg}\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha_G)^{\frac{1}{\sigma}} N_{Gs}^{\frac{\sigma-1}{\sigma}}}{\alpha_S^{\frac{1}{\sigma}}\left(\frac{A_g}{A_s}N_{Sg}\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha_S)^{\frac{1}{\sigma}} N_{Ss}^{\frac{\sigma-1}{\sigma}}}\right]^{\frac{\sigma}{\sigma-1}}
\end{array}$$

Next, we insert (VI) into (I) to get rid of P_G/P_S and use (V) to substitute for C_G/C_G. We get the following system of equations which – given A_g/A_s – we can solve for the labor allocation $N_{Gg}, N_{Gs}, N_{Sg}, N_{Ss}$:

$$\begin{aligned}
1 &= \left(\frac{\alpha_U}{1-\alpha_U}\right)^{\frac{1}{\epsilon}} \left[\left(\frac{N_G}{N_S}\right)^{\theta-\frac{\epsilon}{\epsilon-1}} \left(\frac{L_G}{L_S}\right)^{1-\theta}\right]^{\frac{\epsilon-1}{\epsilon}} \\
1 &= \left(\frac{\alpha_G}{1-\alpha_G}\right)^{\frac{1}{\sigma}} \left(\frac{A_g}{A_s}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{N_{Gg}}{N_{Gs}}\right)^{-\frac{1}{\sigma}} \\
1 &= \left(\frac{\alpha_S}{1-\alpha_S}\right)^{\frac{1}{\sigma}} \left(\frac{A_g}{A_s}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{N_{Sg}}{N_{Ss}}\right)^{-\frac{1}{\sigma}} \\
1 &= N_{Gg} + N_{Gs} + N_{Sg} + N_{Ss}
\end{aligned} \tag{36}$$

where

$$\frac{L_G}{L_S} = \left[\frac{\alpha_G^{\frac{1}{\sigma}}(\frac{A_g}{A_s}N_{Gg})^{\frac{\sigma-1}{\sigma}} + (1-\alpha_G)^{\frac{1}{\sigma}} N_{Gs}^{\frac{\sigma-1}{\sigma}}}{\alpha_S^{\frac{1}{\sigma}}(\frac{A_g}{A_s}N_{Sg})^{\frac{\sigma-1}{\sigma}} + (1-\alpha_S)^{\frac{1}{\sigma}} N_{Ss}^{\frac{\sigma-1}{\sigma}}}\right]^{\frac{\sigma}{\sigma-1}}$$

Once we have obtained $N_{Gg}, N_{Gs}, N_{Sg}, N_{Ss}$, we can solve for P_G/P_S and C_G/C_S.

Solving for the growth rate and more ratios

Next, we use the Euler equation from above to obtain:

$$\frac{P_{t+1}C_{t+1}}{P_tC_t} = \frac{K_{t+1}}{K_t} = \frac{K^G_{t+1}}{K^G_t} = \frac{K^S_{t+1}}{K^S_t} = \gamma \equiv \beta\Big[1 + A_X - \delta\Big]$$

Then, we divide the law of motion $K_{t+1} = (1 - \delta)K_t + X_t$ by K_t and use $K_{t+1}/K_t = \gamma$ to obtain:

$$\gamma = 1 - \delta + A_X\left[1 - \frac{K_G + K_S}{K}\right]$$

Given β, A_X, δ, we obtain the growth rate γ and from the second equation we get $(K_G + K_S)/K = K_C/K$ and $K_X/K = 1 - K_C/K$. Moreover, we get that $X/K = A_X\big(1 - K_C/K\big)$. Finally, to obtain K_G/K_S, we combine $rK_G = \theta P_G C_G$ and $rK_S = \theta P_S C_S$ to obtain $K_G/K_S = (P_G/P_S)(C_G/C_S)$.

To summarize, given values for the parameters:

- ϵ, σ, θ (elasticities)
- $\alpha_U, \alpha_G, \alpha_S$ (shares)
- A_g/A_s (technology)
- β, A_X, δ (others)

we can solve the model for the:

- $N_{Gg}, N_{Gs}, N_{Sg}, N_{Ss}$ (labor allocation)
- P_G/P_S (relative price of goods to services)
- C_G/C_S (ratio of goods over services consumption)
- $K_G/K_S, K_C/K, K_X/K, X/K$ (capital ratios)
- γ (growth rate of CP, K, K_C, X, w)

We cannot solve for the growth rate of real GDP per capita (per worker, actually) which is given by:

$$\gamma = \frac{P_G(0)C_G(1) + P_S(0)C_S(1) + X(1)}{P_G(0)C_G(0) + P_S(0)C_S(0) + X(0)}$$

where $P_J(0)$ are the period–0 prices and $C_J(0), C_J(1)$ are the period–0 and period–1 quantities. To compute γ, we need to know the level of prices and quantities in each period. Those can be computed as explained below.

Calibration strategy

The elasticity between goods and services ϵ is not identified from the employment shares. The calibration of ϵ requires information on real expenditure shares or the relative price of services.

We set $\epsilon = 0.05$, that is, goods and services enter as strong complements in the consumption basket. We normalize the initial relative technologies and set $A_g(0)/A_s(0) = 1$. In the next step, we calibrate α_G, α_S and α_U so that the model matches the employment shares $N_{Gg}(0) = 0.375$, $N_{Gs}(0) = 0.097$, $N_{Ss}(0) = 0.413$. We obtain: $\alpha_G = 0.795$, $\alpha_S = 0.218$, $\alpha_U = 0.472$. The model matches the data targets exactly.

Next, we calibrate the remaining parameters $A_g/A_s(1)$ and σ to match two targets: $N_{Gg}(1) = 0.149$, $N_{Gs}(1) = 0.104$. The model can match the targets exactly and we obtain: $A_g(1)/A_s(1) = 9.42$ and $\sigma = 0.557$.

Next, we calibrate the level of occupation–specific technology. First, we set $A_s(0) = 1$. Since, $A_g(0)/A_s(0) = 1$, we get that $A_g(0) = 1$. Then, we set $A_s(1)$ so that the increase in real GDP per capita between periods 0 and 1 that is implied by the model matches the increase in the data. In the data, GDP per capita has increased by a factor 3. In the model, we can derive neither the level of nor the change in real GDP without knowing the level of relative prices (P_G, P_S) and quantities (C_G, C_S, X). We proceed as follows to compute the levels:

- First, we normalize the aggregate capital stock in period–0 to 1 ($K(0) = 1$). Since the model is AK, this normalization does not result in a loss of generality.
- Given $K(0)$, we can use K_C/K from above to compute $K_C(0)$. Next, we use K_G/K_S from above and $K_C = K_G + K_S$ to compute $K_G(0)$ and $K_S(0)$. Since $K_X = X/A_X = K - K_C$, we also get $X(0)$.
- Using the period–0 labor allocation, the technology level $A_g(0) = A_s(0) = 1$ and initial capital $K_G(0), K_S(0)$, we can compute $C_G(0)$ and $C_S(0)$ from the sectoral production functions.
- Moreover, using $rK_G(0) = \theta P_G(0)C_G(0)$ and $rK_S(0) = \theta P_S(0)C_S(0)$, we compute $P_G(0)$ and $P_S(0)$.

Now we have everything to compute real GDP in period 0:

$$Y(0) = P_G(0)C_G(0) + P_S(0)C_S(0) + X(0)$$

Real U.S. GDP per capita increased by a factor 3 between 1950 and 2005. We know that the aggregate capital stock K grows at an annual rate equal to γ. Hence after 50 years, the capital stock is $K(1) = \gamma^{50}K(0)$. To derive γ, we assume that $\beta = 0.96$, $\delta = 0.05$ and $A_X = 0.10$. The implied net real interest rate is $A_X - \delta = 0.05$ and $\gamma = 1.008$. Moreover, we calibrate θ so that the capital share in the model is equal to 1/3. The implied $\theta = 0.17$. Once we have computed $K(1)$, we follow the same steps as above to obtain $C_G(1), C_S(1)$ and $X(1)$ and real per–capita GDP:

$$Y(1) = P_G(0)C_G(1) + P_S(0)C_S(1) + X(1)$$

Finally, we search for the value of $A_s(1)$ so that $Y(1)/Y(0) = 3$.