Noise-Ridden Lending Cycles[☆]

[PRELIMINARY DRAFT]

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Abstract

In this paper, we use a neoclassical investment model to study the effects of imperfect information on the lending behaviour of financial intermediaries. We start by developing intuition in partial equilibrium. We model a rational financial intermediary with limited knowledge of the current state of the economy. In response to a noise shock, the intermediary lowers the interest rates on risky loans and extends relatively more credit, both of which are unaffected under perfect information. This credit boom is driven by informational rather than financial frictions and accompanied by higher aggregate default and decrease in credit spreads. We further show that these noise-ridden credit booms also survive in the general equilibrium version of the model.

Keywords: Credit Booms, Imperfect Information, Kalman Filter

JEL classification: D83, D84, E13, E44

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1. Introduction

Do banks' expectations about the current and future state of the economy play a role for the emergence of credit cycles? According to the Senior Loan Officer Opinion Survey (SLOOS) conducted by the Board of Governors of the Federal Reserve, commercial banks in the U.S. respond in the affirmative to this question. Figure 1 illustrates that "economic outlook" is an important reason for survey participants' decision to adjust their lending standards. In particular, current and future expected economic conditions seem to matter when tightening standards in a recession as well as when loosening standards during a boom or recovery.

A natural starting point to addressing the above question are diverging assumptions about the expectation-formation process of economic agents in the previous literature. Do banks behave rationally when deciding whether and under which conditions to grant a loan? Bordalo et al. (2016) build on insights from cognitive psychology, assuming that risk-neutral investors form so-called *diagnostic* rather than rational expectations.

Figure 1: The importance of "economic outlook" for changes in lending standards of commercial banks in the U.S.

Notes: Number of banks reporting that "economic outlook" was very important, somewhat important, or unimportant for tightening (positive) or loosening (negative) lending standards. **Source:** Senior Loan Officer Opinion Survey on Bank Lending Practices.

De Grauwe and Macchiarelli (2015) instead assume that, due to limited cognitive abilities, banks apply simple extrapolative rules. Although an appealing alternative explanation for the emergence of credit booms and busts, deviations from rational expectations often dispose of the modeling discipline associated with the latter. Moreover, irrational behavior comes in countless varieties, few of which make empirically testable predictions.

For this reason, we maintain the assumption of rational expectations, while departing from the frequently associated assumption of full or perfect information. Under *imperfect information*, rational economic agents are assumed to know the model environment, whereas they must "learn" the value of certain unobservable variables, such as total factor productivity, for example. These learning dynamics can alter the endogenous propagation of shocks by affecting the behavior of economic agents.

Given that the profitability of financial investments is intimately linked to future economic activity, the expectation-formation process of financial intermediaries is of particular interest. In what follows, we use survey data from the Blue Chip Economic Indicators to provide empirical evidence of informational frictions in banks' expectations.[1] We focus on the so-called *Blue Chip consensus*, i.e. the average expectation across all survey participants, of real GDP growth as a key variable describing the state o the economy.

Figure 2 compares the Blue Chip consensus *nowcast* of year-on-year real GDP growth to two realized-data benchmarks.[2] The $t+1$-benchmark refers to the so-called "advance release" of real-time data published by the Federal Reserve Bank of Philadelphia with a lag of one quarter. The 2016-benchmark refers to the most recent revised estimate of real GDP growth, currently dated 2016Q2. In general, Blue Chip consensus nowcasts seem to be relatively accurate in real time, closely tracking the $t+1$-benchmark. Nevertheless, we detect quantitatively large and sustained deviations from the advance release. In 1997-2000, for example, nowcasts are persistently more *pessimistic* about current real GDP growth than suggested by real-time data, whereas they are relatively more optimistic during 2004-2006. These tendencies are much more apparent in comparison with the 2016-benchmark. Moreover, Blue Chip consensus nowcasts were clearly *pessimistic* relative to 2016-benchmark for the majority of observations during 1984-2000.

[1]The Blue Chip Economic Indicators is a monthly survey of more than 50 economists employed by some of America's largest manufacturers, banks, insurance companies, and brokerage firms. It was established in 1976 and collects professional forecasts on macroeconomics aggregates for the U.S. economy. What makes this survey particularly interesting and suitable for our question is that the majority of participants are U.S. commercial and investment banks.

[2]We report Blue Chip consensus nowcasts and forecasts from the third month of a given quarter, i.e. from the March, June, September, and December releases, to measure expectations as of quarter I, II, III, and IV, respectively. Releases from the first and second month of each quarter yield similar results.

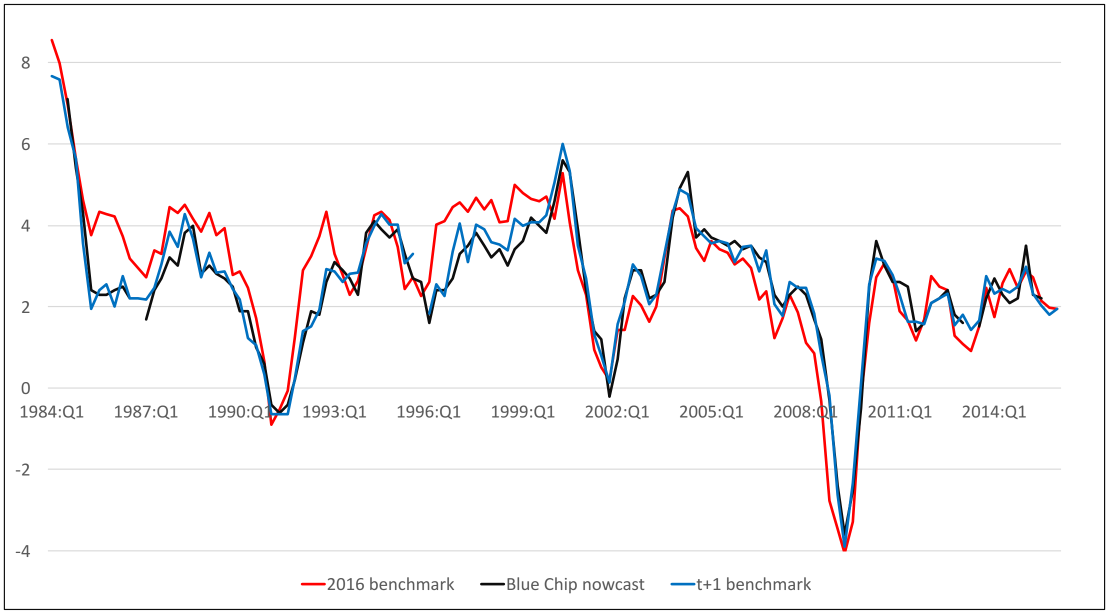

Figure 2: Blue Chip consensus nowcasts of real GDP growth and realized-data benchmarks

Figure 3 plots the corresponding *nowcast errors*, computed as the difference between the Blue Chip consensus nowcast and each of the realized-data benchmarks. Accordingly, positive values indicate relative optimism, while negative values indicate relative pessimism. Following the dot-com bubble, the dominating

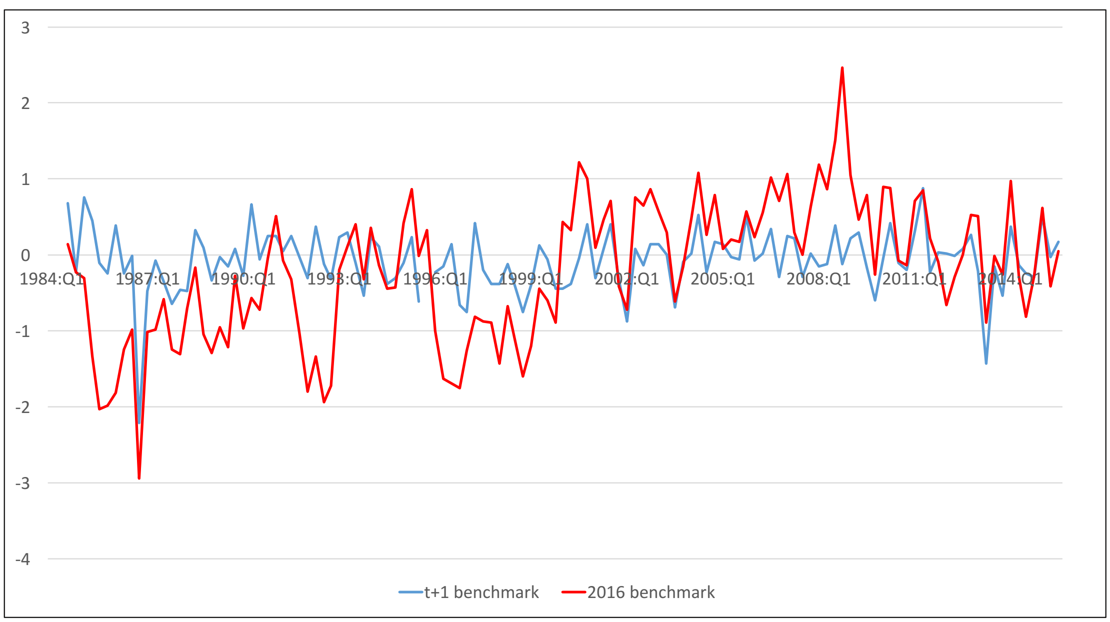

Figure 3: Blue Chip consensus nowcast errors of real GDP growth relative to realized data benchmarks

sign of nowcast errors changes from negative to positive, initiating a prolonged period of relative optimism. Note that the observed persistence of Blue Chip consensus nowcast errors is inconsistent with the assumption of full information and rational expectations, whereas these patterns might be explained by the presence of informational frictions and learning.

In this paper, we model the lending decision of a risk-neutral financial intermediary in the presence of imperfect information about the state of the economy, which determines the default or repayment of a given loan. In this environment, a positive signal about the current state of the economy induces the bank to lend to objectively riskier borrowers in expectation of higher future returns. If the signal is pure noise, however, the expansion of credit is unwarranted and would not have occurred under full information and rational expectations (FIRE). Adapting the partial equilibrium neoclassical investment model in Bordalo et al. (2016), we show that pure noise shocks indeed generate credit booms, if our profit-maximizing bank cannot observe the relevant state of the economy and must infer the riskiness of a given loan from other observables and a noisy signal. This credit boom is costly, as the bank's expectations turn out to be wrong ex post, leading to an increase in default. It is important to note that this credit boom is driven only by our assumption of imperfect information and does not rely on any kind of financial friction. Given that ex-ante lending sets a limit to ex-post productive capital, we find that symmetric positive and negative noise shocks have asymmetric effects on aggregate capital despite symmetric effects on aggregate credit.

We then estimate the model using a Kalman filter in order to quantify the importance of informational frictions. On the one hand, our maximum-likelihood estimates suggest that noise shocks are sizeable relative to fundamental shocks. On the other hand, the partial equilibrium model does not imply large quantitative effects of imperfect information and learning on aggregate credit volumes and default rates.

Finally, we present a tractable general equilibrium version of our partial equilibrium investment model and show that, even when accounting for feedback from bank behavior to funding costs, pure noise shocks can generate credit booms in the presence of imperfect information.

The rest of the paper is organized as follows. Section 2 uses a partial equilibrium model to illustrate how credit booms can emerge in the presence of imperfect information. Section 3 presents an empirical exercise, in which we estimate the partial equilibrium model using maximum-likelihood techniques and evaluate the importance of informational frictions based on the recovered estimates of parameters and shock processes. Section 4 presents a general equilibrium version of the model and discusses the implied model dynamics. Section 5 concludes and outlines directions for further work.

2. The Partial Equilibrium Model

We start by illustrating the intuition with a neoclassical partial equilibrium investment model similar to the one in Bordalo et al. (2016).

2.1. Partial Equilibrium Model under Full Information

Time $t = 1, 2, \ldots$ is discrete and the economy's state at time t, $\Omega_t \in \mathbb{R}$ with realization ω_t, follows a Markov process with a normal distribution conditional on Ω_{t-1}, as in the AR(1) case

$$\omega_t - \mu_\omega = b\,(\omega_{t-1} - \mu_\omega) + \epsilon_t^\omega, \tag{1}$$

with $\epsilon_t^\omega \sim N\left(0, \sigma^2\right)$ and $\mu_\omega \in \mathbb{R}$, $b \in [0, 1]$.

Credit demand by firms

A unit measure of atomistic firms uses capital to produce output, where productivity at time t depends on ω_t to a different extent for different firms. Each firm is identified by its *risk type*, $\rho \in \mathbb{R}$. Firms with higher ρ are less likely to be productive in any state ω_t and represent thus a riskier investment. The output of a type-ρ firm at time t is given by

$$y\,(k|\omega_t, \rho) = \begin{cases} k^\alpha & \text{if } \omega_t \geq \rho \\ 0 & \text{if } \omega_t < \rho \end{cases},$$

where $\alpha \in (0, 1)$.

The capital used for production at time $t + 1$ must be installed at time t already, before ω_{t+1} is known. For simplicity, we assume that capital depreciates fully in production. Each firm's risk type ρ is *common knowledge* and distributed across firms according to the continuous density function $f(\rho)$.

Each firm's capital investment is fully debt funded. A firm of type ρ borrows funds $D_{t+1}^f(\rho) = k_{t+1}(\rho)$ from a bank, taking the contractual interest rate $r_{t+1}(\rho)$ as given. It repays the loan only if the realized state of the economy allows the firm to be productive. Else, it defaults and repays nothing.

Assuming perfect competition in production, each firm type ρ borrows up to the point where the marginal product of capital equals the cost of borrowing from the bank, i.e.

$$k_{t+1}(\rho) = \left[\frac{\alpha}{r_{t+1}(\rho)}\right]^{\frac{1}{1-\alpha}}. \tag{2}$$

Credit supply by banks

Suppose that the credit market is *not* competitive, e.g. due to long-term lending relationships between banks and firms. In contrast to the representative household in Bordalo et al. (2016), which elastically supplies any amount of capital at the risk-free interest rate, the lenders in our model do *not* make optimal consumption-saving decisions. Instead, we assume that risk-neutral bankers invest all of their net worth, n_t, until they die (exogenously) and get to consume their accumulated net worth.

Given a predetermined value of n_t and a risk-free interest rate on deposits, r_t^d, determined by monetary policy or household preferences, banks maximize $E_t n_{t+1}$ by optimally choosing $r_{t+1}(\rho)$ for each firm type ρ, subject to the firm's downward-sloping demand curve for bank credit in (2):

$$
\begin{aligned}
&\max_{r_{t+1}(\rho)} \quad E_t n_{t+1} = Prob\left(\omega_{t+1} \geq \rho | \omega_t\right) r_{t+1}(\rho) k_{t+1}(\rho) - \left[k_{t+1}(\rho) - n_t(\rho)\right] r_t^d \\
\Leftrightarrow \quad & \frac{\partial E_t n_{t+1}}{\partial r_{t+1(\rho)}} = Prob\left(\omega_{t+1} \geq \rho | \omega_t\right) \alpha^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{1-\alpha}\right) r_{t+1}(\rho)^{-\frac{1}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}} \left(\frac{1}{1-\alpha}\right) r_{t+1}(\rho)^{-\frac{2-\alpha}{1-\alpha}} r_t^d = 0 \\
\Leftrightarrow \quad & r_{t+1}^*(\rho) = \frac{r_t^d}{\alpha \cdot Prob\left(\omega_{t+1} \geq \rho | \omega_t\right)}, \qquad (3)
\end{aligned}
$$

where $Prob(\bullet)$ is assumed to be independent of $r_{t+1}(\rho)$. As a result, we have a unique interior solution for the bank's optimal choice of $r_{t+1}(\rho)$ for each firm type ρ.

Equations (2) and (3) imply the following unique interior solution for the firm's optimal demand for capital:

$$
k_{t+1}^*(\rho) = \left[\alpha^2 \cdot Prob\left(\omega_{t+1} \geq \rho | \omega_t\right) / r_t^d\right]^{\frac{1}{1-\alpha}}. \qquad (4)
$$

From equations (3) and (4), a higher expected probability of repayment translates into a lower interest rate on bank credit, $r_{t+1}^*(\rho)$, and thus a higher demand for bank credit and capital, $k_{t+1}^*(\rho)$, for each firm type ρ.

Figure 4 plots the equilibrium interest rate in (3) and the demand for capital and bank credit in (4) *by firm type* ρ and illustrates that low-ρ types with a negligible risk of default pay an interest rate on bank credit that is close to the risk-free rate plus the bank's monopoly markup, $1/\alpha$. The figure represents a snapshot of the economy for a given expected value and variance of ω_{t+1}. The broken lines depict the effect of an increase in $E_t\omega_{t+1} = \mu_\omega$ without any change in $E_t\sigma_{\omega,t+1}$.

In contrast, Figure 5 plots the equilibrium interest rate in (3) and the demand for capital and bank credit in (4) *as a function of the expected repayment probability*. Changes in $E_t\omega_{t+1}$ affect the position of a given firm type ρ on the x-axis rather than shifting graphs (as in Figure 4). Figure 5 illustrates that symmetric increases

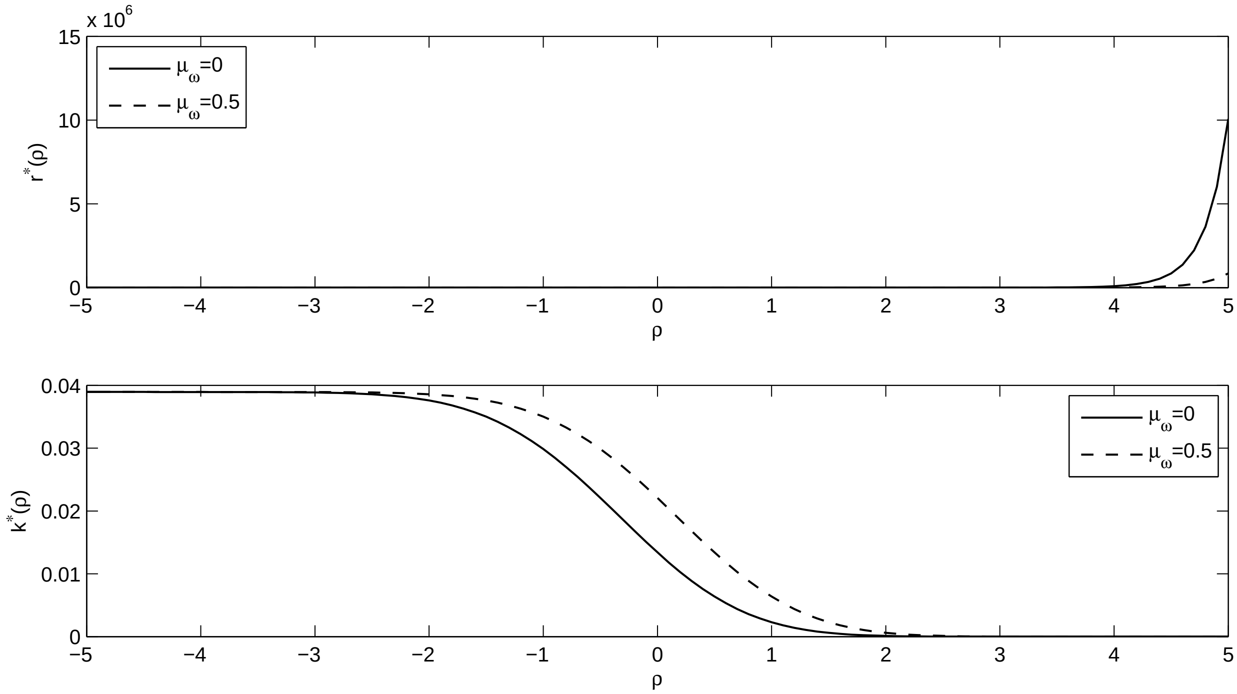

Figure 4: Equilibrium interest rates and demand for bank credit by firm type ρ

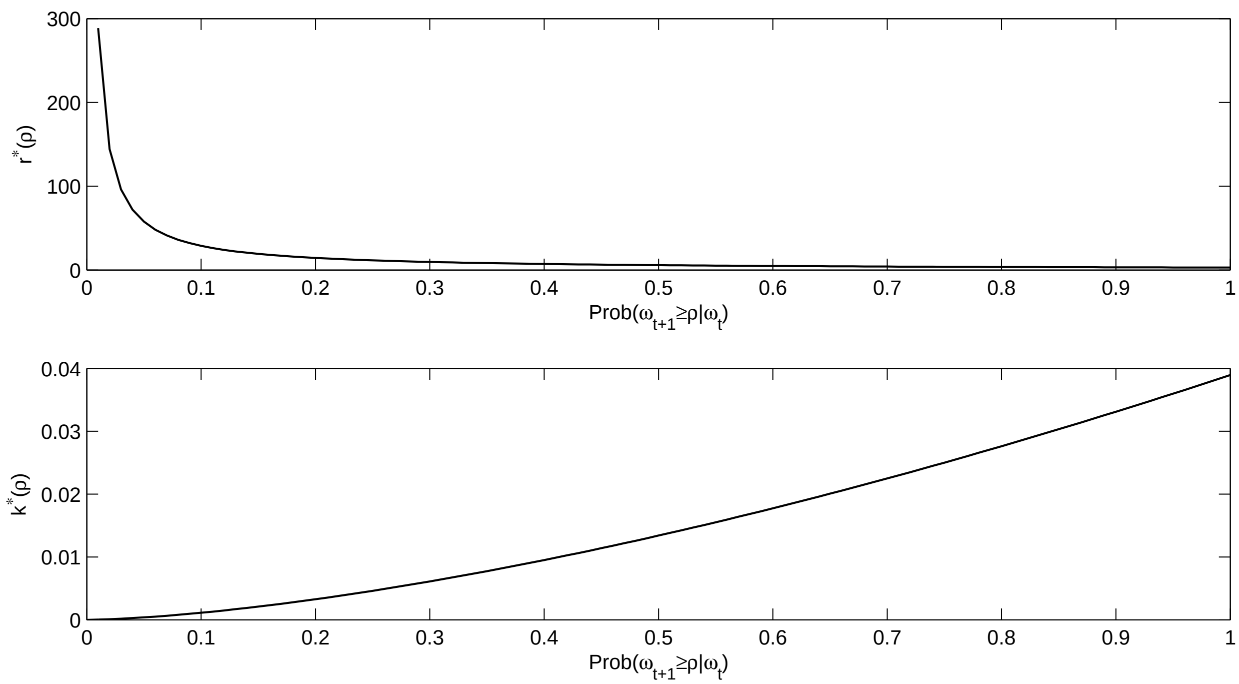

Figure 5: Equilibrium interest rates and demand for bank credit by expected repayment probability

and decreases in the expected repayment probability have *asymmetric effects* on the equilibrium interest rate and thus the demand for bank credit for a given firm type ρ. Due the presence of $Prob\,(\omega_{t+1} \geq \rho|\omega_t)$ in the denominator of (3), this asymmetry is more pronounced for low expected repayment probabilities, where a decrease in $Prob\,(\omega_{t+1} \geq \rho|\omega_t)$ raises the equilibrium interest rate by more than a symmetric increase lowers $r^*\,(\rho)$. We discuss below, whether t e effects of a change in $E_t\omega_{t+1}$ on *aggregate* bank credit and capital are more asymmetric in a recession, when the average probability of repayment is low, than in a boom.

2.1.1. Partial Equilibrium Model under Imperfect Information

In our partial equilibrium model, the bank optimally chooses

$$r^*_{t+1}(\rho) = \frac{r^d_t}{\alpha \cdot Prob\,(\omega_{t+1} \geq \rho|\omega_t, \omega_{t-1} \ldots)} \quad \forall \rho, \tag{5}$$

where the exogenous default threshold ω_t contains permanent and transitory components ν_t and η_t, respectively, which are both *unobservable*. Hence, the bank must form expectations about ω_{t+1} based on its nowcast of the aggregate state at time t, ν_t. Using the above notation, we formulate the bank's signal-extraction problem in state-space form as

$$\begin{aligned} \omega_t &= \nu_t + \eta_t, \\ \nu_t &= \nu_{t-1} + e_t, \quad e_t \sim N\left(0, \sigma^2_e\right), \\ \eta_t &= \phi\eta_{t-1} + \epsilon_t, \quad \epsilon_t \sim N\left(0, \sigma^2_\epsilon\right), \end{aligned} \tag{6}$$

where e_t and ϵ_t are assumed to be *contemporaneously* and *serially uncorrelated*.

Following Lorenzoni (2009), we assume that the bank receives a noisy public signal of the permanent component ν_t at time t:

$$\tilde{s}_t = \nu_t + \varepsilon_t, \quad \varepsilon_t \sim N\left(0, \sigma^2_\varepsilon\right), \tag{7}$$

where ε_t is assumed to be contemporaneously and serially uncorrelated with e_t and ϵ_t.

The disturbance term ε_t in (7) plays two roles. First, it prevents the bank from perfectly observing the permanent component of the aggregate state. Second, it generates an independent source of variation in the bank's beliefs about ν_t. Note that the disturbance terms in η_t and $\tilde{s}_t$ have very different interpretations. While ϵ_t is a shock to the transitory component of the aggregate state and affects thus ω_t, ε_t is a non-fundamental "noise shock", which propagates only through the bank's expectations about ω_{t+1}.

We can solve the measurement equation in (6) for the transitory or *cycle* component, η_t, and substitute into $\eta_t = \phi\eta_{t-1} + \epsilon_t$ in order to obtain

$$
\begin{aligned}
\omega_t &= \nu_t + \phi\left(\omega_{t-1} - \nu_{t-1}\right) + \epsilon_t, \\
\tilde{s}_t &= \nu_t + \varepsilon_t, \\
\nu_t &= \nu_{t-1} + e_t,
\end{aligned}
$$

where the first two are measurement equations while the third represents the state equation.

Defining the vector of states $\mathbf{s}_t \equiv [\nu_t, \nu_{t-1}]'$, we can also write the above system in matrix notation as

$$
\begin{aligned}
\mathbf{y}_t &= \mathbf{Z}\mathbf{s}_t + \mathbf{B}\mathbf{y}_{t-1} + \mathbf{u}_t, \\
\mathbf{s}_t &= \mathbf{T}\mathbf{s}_{t-1} + \mathbf{v}_t,
\end{aligned}
$$

where $\mathbf{y}_t \equiv [\omega_t, \tilde{s}_t]'$, $\mathbf{u}_t \equiv [\epsilon_t, \varepsilon_t]'$, $\mathbf{v}_t \equiv [e_t, 0]'$, and

$$
\mathbf{Z} \equiv \begin{bmatrix} 1 & -\phi \\ 1 & 0 \end{bmatrix}, \quad \mathbf{B} \equiv \begin{bmatrix} \phi & 0 \\ 0 & 0 \end{bmatrix}, \quad \mathbf{T} \equiv \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}.
$$

From the above system of equations, it is straightforward to derive Kalman filter *updating* expressions for the state and its variance conditional on information available at time t, $\mathbf{s}_{t|t}$ respectively $\Sigma_{t|t}$:

$$
\begin{aligned}
\mathbf{s}_{t|t} &= \mathbf{s}_{t|t-1} + \Sigma_{t|t-1}\mathbf{Z}'\left(\mathbf{Z}\Sigma_{t|t-1}\mathbf{Z}' + \mathbf{H}\right)^{-1}\left(\mathbf{y}_t - \mathbf{y}_{t|t-1}\right), \\
\Sigma_{t|t} &= \Sigma_{t|t-1} - \Sigma_{t|t-1}\mathbf{Z}'\left(\mathbf{Z}\Sigma_{t|t-1}\mathbf{Z}' + \mathbf{H}\right)^{-1}\mathbf{Z}\Sigma_{t|t-1},
\end{aligned}
\tag{8}
$$

where the corresponding Kalman filter *forecasting* expressions are given by

$$
\begin{aligned}
\mathbf{s}_{t+1|t} &= \mathbf{T}\mathbf{s}_{t|t}, \\
\Sigma_{t+1|t} &= \mathbf{T}\Sigma_{t|t}\mathbf{T}' + \mathbf{Q}, \\
\mathbf{y}_{t+1|t} &= \mathbf{Z}\mathbf{s}_{t+1|t} + \mathbf{B}\mathbf{y}_t,
\end{aligned}
\tag{9}
$$

and

$$\mathbf{H} \equiv \begin{bmatrix} \sigma_{\epsilon}^{2} & 0 \\ 0 & \sigma_{\varepsilon}^{2} \end{bmatrix}, \quad \mathbf{Q} \equiv \begin{bmatrix} \sigma_{e}^{2} & 0 \\ 0 & 0 \end{bmatrix}.$$

Note that the Kalman filter gain, $K \equiv \Sigma_{t|t-1}\mathbf{Z}'\left(\mathbf{Z}\Sigma_{t|t-1}\mathbf{Z}' + \mathbf{H}\right)^{-1}$, is increasing in σ_e while it is decreasing in σ_ϵ and σ_ε, i.e., the extent to which the bank optimally updates its *nowcast* of $\mathbf{s}_{t|t}$ depends on the so-called signal-to-noise ratio.

Applying standard Kalman filtering techniques, it can be shown that the dynamics of $\nu_{t|t}$ are given by

$$\nu_{t|t} = \rho\nu_{t-1|t-1} + (1-\rho)\left[\delta\omega_t + (1-\delta)\,\tilde{s}_t\right], \tag{10}$$

where

$$\rho \equiv \frac{1/\sigma_{\nu}^{2}}{1/\sigma_{\nu}^{2} + 1/\sigma_{\epsilon}^{2} + 1/\sigma_{\varepsilon}^{2}}, \quad \delta \equiv \frac{1/\sigma_{\epsilon}^{2}}{1/\sigma_{\epsilon}^{2} + 1/\sigma_{\varepsilon}^{2}}, \quad \sigma_{\nu}^{2} = \left(\frac{1}{\sigma_{\nu}^{2}} + \frac{1}{\sigma_{\epsilon}^{2}} + \frac{1}{\sigma_{\varepsilon}^{2}}\right)^{-1} + \sigma_{e}^{2}$$

(compare Lorenzoni, 2009). When updating its nowcast of the permanent component, $\nu_{t|t}$, the bank optimally weights the observable ω_t and the noisy signal $\tilde{s}_t$ according to their relative precision, δ. Moreover, it optimally attributes a weight of $(1-\rho)$, which is increasing in the variance of shocks to the permanent component of the observable state, to new information in ω_t and $\tilde{s}_t$.

Note that, while ϵ_t and ε_t are uncorrelated and σ_ε is *finite*, the noisy signal provides additional information and accelerates thus the Kalman filter learning process. As $\sigma_\varepsilon \to \infty$, the bank discards the signal as uninformative and attributes zero weight to $\tilde{s}_t$ in (10).

2.2. Impulse Response Functions under Imperfect Information

Based on the Kalman filter updating and forecasting expressions in (8) and (9), we can simulate the bank's expectation about ω_{t+1} and thus the expected repayment probability of each firm type ρ as of time t, $Prob\left(\omega_{t+1} \geq \rho|\omega_t, \omega_{t-1}, ...\right)$. Plugging these probabilities into the partial equilibrium model in equations (4) and (5), we can then derive the bank's profit-maximizing interest rate on a loan and the corresponding credit demand for each firm type ρ.

To compute the *ex-ante* aggregate demand for credit by firms at time t *before the realization of* ω_t, we integrate over the entire support of the probability density function of firm types, $f(\rho)$. To compute the *ex-post* aggregate amount of capital available for production *after the realization of* ω_t, we integrate over $f(\rho)$ from the lower bound of the support up to ω_{t+1}. Accordingly, the aggregate demand for credit is computed for *all* firm types ρ, whereas the aggregate amount of capital at time t is computed only for non-defaulting firm types $\rho < \omega_t$.

Given the non-linearity of the partial equilibrium model in (4) and (5) and the non-stationary process for ω_t in (6), we compute the generalized impulse responses (see Koop et al. (1996)) to a shock in the trend, cycle, and noise component, respectively, by simulation as follows:

1. Drawing e_t, ϵ_t, and ε_t from their stochastic distributions, we simulate equations (6) and (7) for $T = 250$ periods and save the resulting time series for ω_t and $\tilde{s}_t$. We then impose a unit shock on the trend, cycle, or noise component at time t_0, i.e. $e_{t_0} = 1$, $\epsilon_{t_0} = 1$, or $\varepsilon_{t_0} = 1$, and save the resulting time series for ω_t' and $\tilde{s}_t'$.
2. Given initial values for $\mathbf{s}_{0|0}$ and $\Sigma_{0|0}$, we compute $\mathbf{s}_{1|0}$, $\Sigma_{1|0}$, and $\mathbf{y}_{1|0}$ and simulate equations (8) and (9) recursively for $t = 1, ..., T$. Using the Kalman filter forecast of $Prob\,(\omega_{t+1} \geq \rho | \omega_t, \omega_t, ...)$, we compute the bank's optimal interest rate and the corresponding credit demand for each firm type ρ and integrate over all firm types.
3. The path-dependent impulse response function to a shock in the trend, cycle, or noise component can then be computed as the period-by-period difference between the path of a certain ρ-specific or aggregate variable with and without the respective shock for $t = 1, ..., T$.
4. We repeat steps 1-3 a large number (N) of times and take the period-by-period average across the path-dependent impulse response functions for all N replications.

In all simulations, we use $N = 5000$ replications and allow for a burn-in of 100 periods to be discarded when computing impulse response functions.

Regarding the parameters of our partial equilibrium model, we set the elasticity of output with respect to capital to $\alpha = 0.35$ and the gross risk-free interest rate on bank deposits to $r_t^d = r^d = 1.01$. We further assume that firm types ρ are normally distributed with zero mean and unit variance, i.e. $\rho \sim N\,(0, 1)$.[3] Regarding the exogenous processes in (6) and (7), we set $\phi = 0$, $\sigma_e = 0.07$, σ_ϵ, and $\sigma_\varepsilon = 0.2$ in order to illustrate the effects of imperfect information on the bank's optimal lending behavior and thus the dynamics of aggregate credit and capital in partial equilibrium (rather than to match the data). The impulse responses to a *positive unit shock* in the trend, cycle, and noise component, respect vely, are plotted in Figures 6-8.

[3] The assumption on the distribution of ρ is not crucial for the results. We get qualitatively similar results under the assumption of a uniform distribution.

Trend shocks

The upper left panel in Figure 6 illustrates that, in response to a unit shock to the trend component, ω increases by the amount of the disturbance *on impact*. The source of this increase is unobservable under imperfect information and rational expectations (Kalman), while ν_t and η_t can be observed separately under full information rational expectations (FIRE). Accordingly, the imperfectly informed bank initially attributes a substantial probability to the possibility that the observed increase in ω indicates a transitory (cycle) or noise shock rather than a trend shock.

As the observable remains high afterwards, the bank revises its nowcast of the trend component and thus its forecast of ω upwards, until they converge to the actual values for ν_t and ω_t. In contrast, the bank under FIRE immediately knows that the observed increase in ω is due an increase in ν.

For illustrative purposes, we report the responses of two ρ-specific lending rates set by the bank in line with equation (3). We choose $\rho_1 = -3$ and $\rho_2 = -1.5$ to represent a relatively safer and a relatively riskier firm type from $\rho \sim N(0, 1)$. In response to a positive shock to the trend component, the bank optimally lowers the lending rate for both ρ_1 and ρ_2. Given the necessity of learning about the actual state of the economy, however, the decrease in $r_t^*(\rho)$ is somewhat deferred under imperfect information (Kalman). Note that the impulse responses to a trend shock remain noisy even for $N = 250$ replications.

In contrast, the lower left panels in Figure 6 paint a unique picture of the effects of a trend shock on ex-ante credit and ex-post capital. The decrease in the lending rate induces all firm types ρ to purchase more capital financed by bank credit. This increase in the demand for credit is driven exclusively by the bank's expected probability of repayment in the lower right panel, which continues to increase as the bank learns about the actual state of the economy.

Finally, the lower next-to-right panel in Figure 6 illustrates that the share of loans to objectively riskier firm types in the bank's portfolio *increases* in response to a positive shock in the trend component. Due to the fact that a relatively riskier firm type, such as $\rho_2 = -1.5$, is generally more likely to default than a firm type $\rho_1 = -3$, a change in $E_t(\omega_{t+1}|\omega_t, \omega_{t-1}, ...)$ has a larger effect on $Prob(\omega_{t+1} \geq \rho_2)$ than on $Prob(\omega_{t+1} \geq \rho_1)$. Note that this effect is not driven by our assumptions about $f(\rho)$.

Cycle shocks

The upper left panels of Figure 7 illustrate that, although the effect of a shock to the cycle component is purely transitory, the bank still attributes part of the observed increase in ω to the trend component. Accordingly, the bank expects a higher probability of repayment and lowers the lending rate for both the

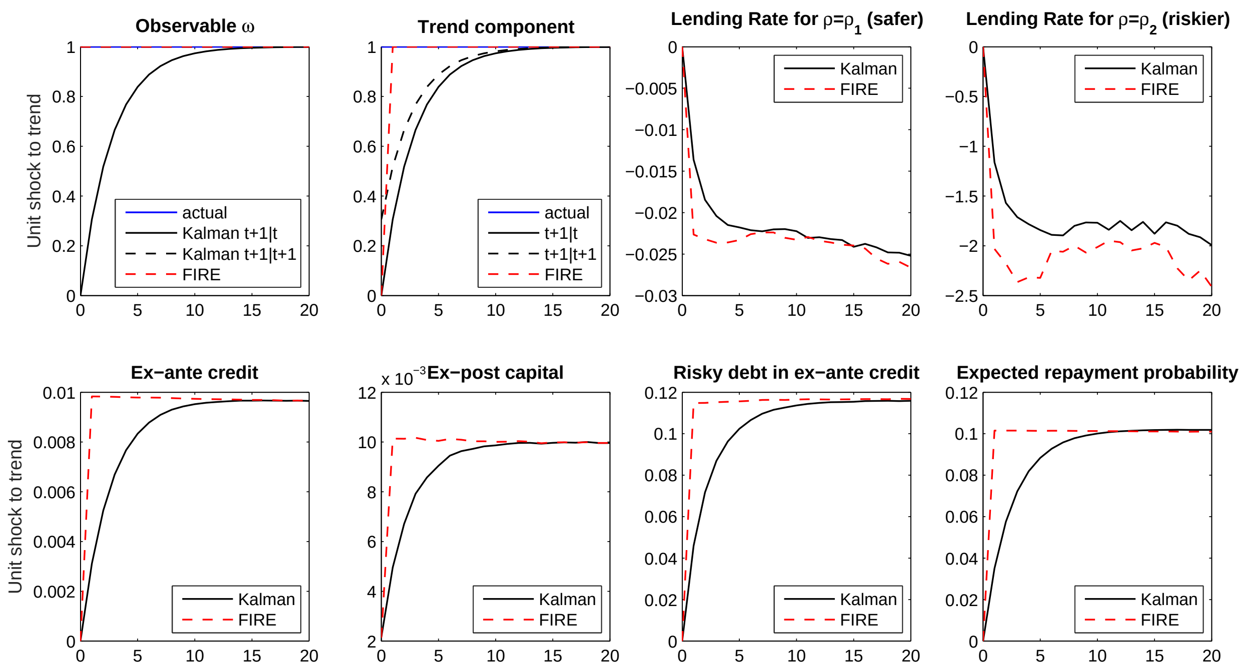

Figure 6: Impulse response functions to a unit shock in the trend component

relatively safer and the relatively riskier firm type. From the upper right panels, the decrease in r^*_{t+1} (ρ_2) is quantitatively more pronounced than the decrease in r^*_{t+1} (ρ_1). Importantly, the bank's slow learning process implies that its perceived probability of repayment for each firm type ρ remains elevated for an extended period before converging back to the baseline without the shock.

As a consequence, firms' aggregate demand for credit increases, replicating the pattern in ρ-specific lending rates with an opposite sign. In contrast, neither the lending rates nor aggregate credit respond to a cycle shock under FIRE. Once the agent observes the shock, its transitory nature ($\phi = 0$) implies that it is too late to lower lending rates and expand thus the supply of credit.

While ex-post capital increases on impact, due to the transitory increase in ω and the corresponding decrease in firm default, its impulse response remains quantitatively below that of ex-ante credit from period 2 onwards. The vertical distance between the responses of ex-ante credit and ex-post capital measures ex-post aggregate default of credit due to imperfect information.

From the lower right panels of Figure 7, a positive shock to the cycle component results in an increase in the share of loans to objectively riskier firm types as well as in the expected probability of repayment of the bank's loan portfolio.

Noise shocks

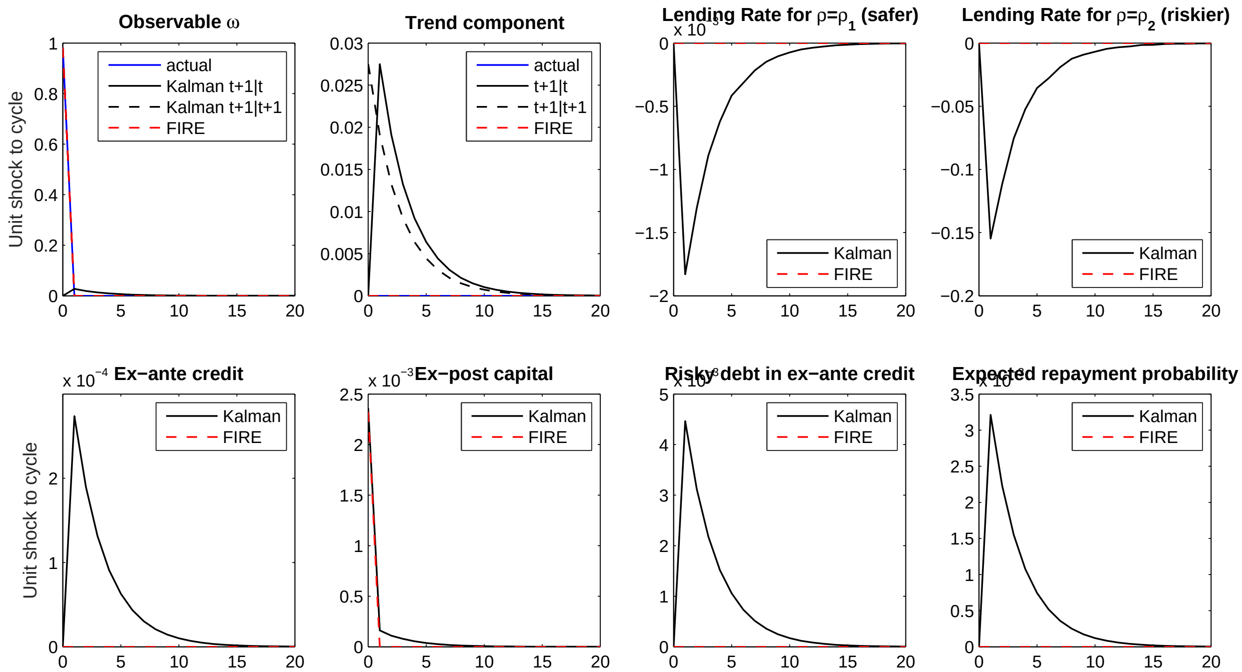

Figure 7: Impulse response functions to a unit shock in the cycle component

Figure 8 plots the impulse response functions after a positive unit shock to the signal in (7). From the upper left panel, a pure noise shock does not have any impact on the observable ω. Nevertheless, the positive signal induces the bank to attribute a nonzero probability to the possibility of an increase in the trend component under imperfect information.

Given its expectation of a higher value for ω_{t+1}, the bank revises its expected probability of repayment upwards and lowers the lending rate for relatively safer firm types, such as $\rho_1 = -3$, and even more so for relatively riskier firm types, such as $\rho = -1.5$.

As a consequence, firms' aggregate demand for credit increases, replicating the pattern in ρ-specific lending rates with an opposite sign. In contrast, neither the lending rates nor aggregate credit respond to a cycle shock under FIRE, where the bank realizes that the increase in $\tilde{s}_t$ represents pure noise.

In order to illustrate the role of imperfect information as a possible driver of credit booms in our partial equilibrium model, Figure 9 plots the impulse responses of ex-ante credit and ex-post capital to a pure noise shock and indicates ex-post aggregate default as the shaded area between the two responses. Accordingly, the learning process in this simple model implies that the bank might become more optimistic in response to misleading news about the state of the economy, lower its lending rates accordingly to attract borrowers, and cause thus an aggregate credit expansion accompanied by an increase in firm default. Note that a similar

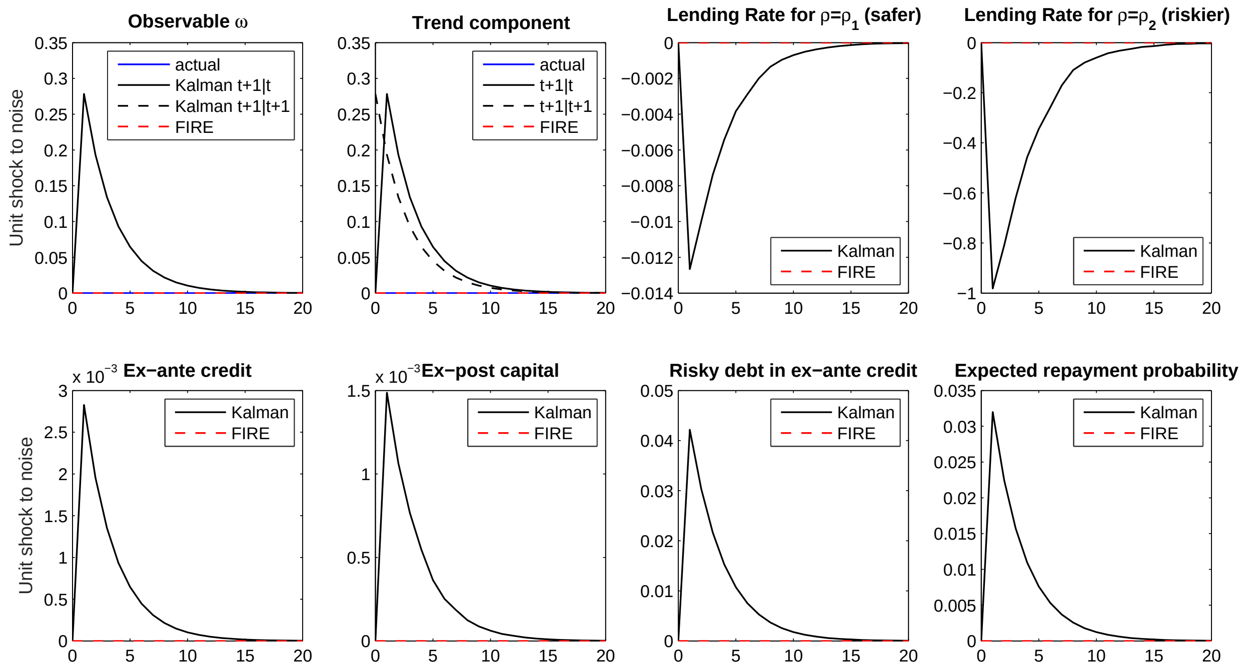

Figure 8: Impulse response functions to a unit shock in the signal

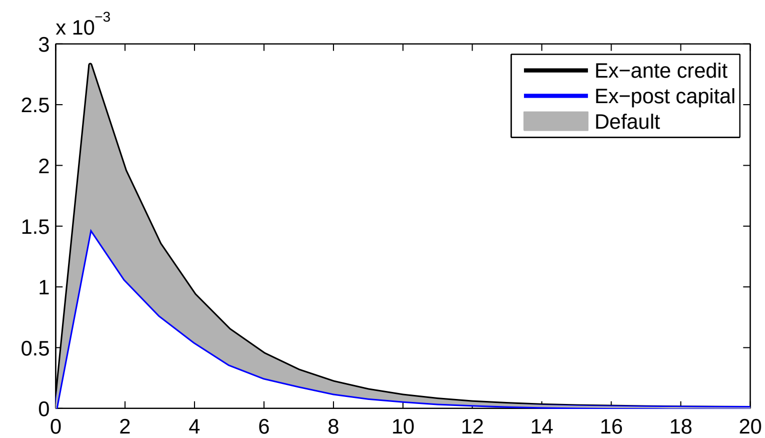

Figure 9: Impulse response function of ex-post credit default to a unit shock in the signal

noise-ridden credit boom does not occur under FIRE.

2.3. *Asymmetric Effects of Positive and Negative Shocks*

Note that Figures 4 and 5 do *not* require specifying a functional form for the distribution of firm types. Once we consider aggregate variables, however, we must assume a particular distribution for ρ. Figure 10 plots a uniform distribution with $\rho \in [-5, 5]$ against a standard normal distribution for $f(\omega)$, where time subscripts have been dropped. Given the symmetry of the uniform and the normal distribution about 0, it is

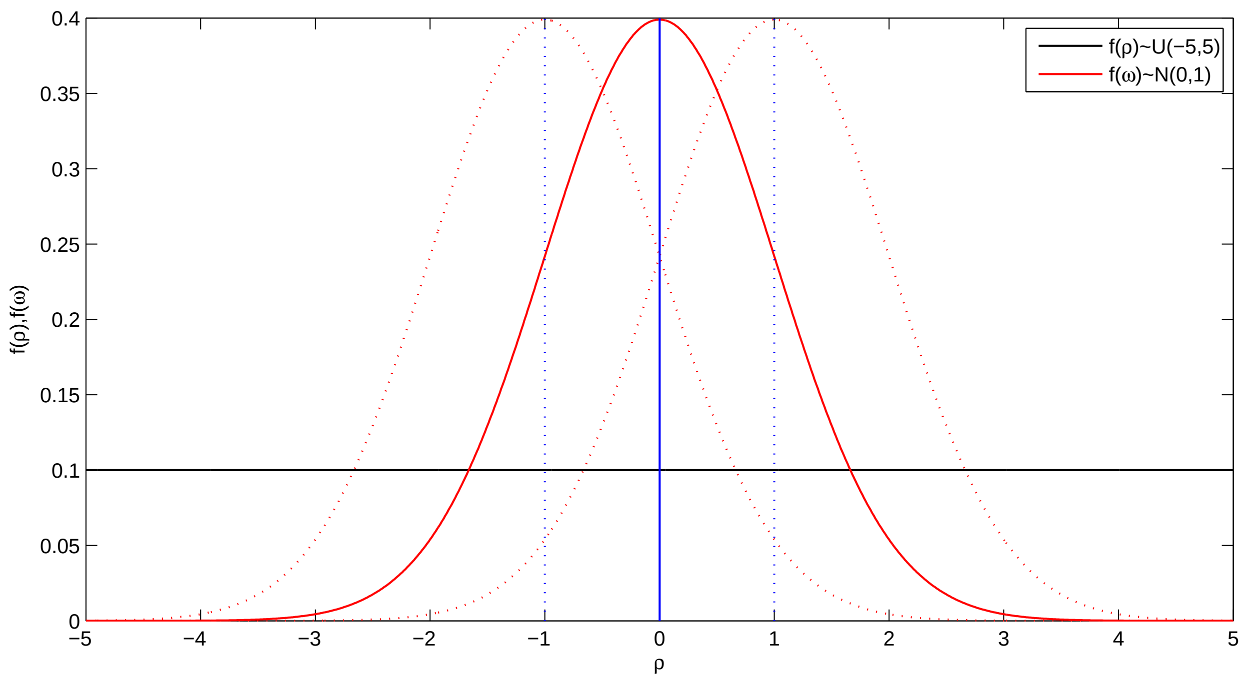

Figure 10: Uniform distribution of firm types and normal distribution of ω by firm type ρ

expected that half of all firm types, i.e. those firms with $\rho \leq E(\omega) = 0$, survive and repay their loans.

Moreover, symmetric shifts of $f(\omega)$ to the left and to the right have *symmetric effects* on the expected aggregate probability of repayment,

$$Pr(\omega_{t+1}) \equiv \int_{-\infty}^{\infty} [1 - F(\omega_{t+1})]\, U(\rho, -5, 5)\, d\rho.$$

Assuming the above functional forms, for example, $Pr(-1) = 0.4$, $Pr(0) = 0.5$, and $Pr(1) = 0.6$. Note that this symmetry applies to the aggregate repayment probability of firm types, whereas the asymmetry in the partial equilibrium model illustrated in Figures 4 and 5 suggests that aggregate economic variables, such as ex-ante bank credit and ex-post capital, might respond asymmetrically to symmetric shifts of $f(\omega)$.

In order to investigate whether this is the case, consider the following comparative statics experiment:

1. Suppose that we start from the situation depicted in Figure 10, where $f(\rho) \sim U(-5, 5)$ and $f(\omega) \sim N(0, 1)$. In the absence of shocks, the bank forms expectations about $Prob(\omega \geq \rho)$ for each firm type ρ, while $r^*(\rho)$ and $k^*(\rho)$ are determined by (3) and (4), respectively. For $\omega = 0$, aggregate ex-ante

credit and ex-post capital are then given by the following integrals:

$$L(0) \equiv \int_{-\infty}^{\infty} k^* (\rho, N(0,1))\, U(-5,5)\, d\rho = 0.0181,$$
$$K(0) \equiv \int_{-\infty}^{0} k^* (\rho, N(0,1))\, U(-5,5)\, d\rho = 0.0173.$$

2. Now suppose that the bank observes a *positive noise shock* and believes that $\hat{f}(\omega) \sim N(1,1)$, whereas in fact $f(\omega) \sim N(0,1)$, as before. While the bank forms expectations about $Prob(\omega \geq \rho)$ for each firm type ρ based on a different prior, the ex-post realization of ω is assumed to be the same as before. The corresponding aggregate ex-ante credit and ex-post capital are now given by the following integrals:

$$L(1) \equiv \int_{-\infty}^{\infty} k^* (\rho, N(1,1))\, U(-5,5)\, d\rho = 0.0219,$$
$$K(1) \equiv \int_{-\infty}^{0} k^* (\rho, N(1,1))\, U(-5,5)\, d\rho = 0.0190.$$

3. Finally, suppose that the bank observes a *negative noise shock*, which shifts its perceived distribution of ω, $\hat{f}(\omega) \sim N(-1,1)$, without affecting the actual $f(\omega)$. In this case, aggregate ex-ante credit and ex-post capital are given by

$$L(-1) \equiv \int_{-\infty}^{\infty} k^* (\rho, N(-1,1))\, U(-5,5)\, d\rho = 0.0142,$$
$$K(-1) \equiv \int_{-\infty}^{0} k^* (\rho, N(-1,1))\, U(-5,5)\, d\rho = 0.0141.$$

It is now straightforward to compare the effects of symmetric positive and negative noise shocks on aggregate ex-ante credit and ex-post capital as

$$L(1) - L(0) = 0.0219 - 0.0181 = 0.03894, \qquad L(-1) - L(0) = 0.142 - 0.0181 = -0.03894,$$
$$K(1) - K(0) = 0.0190 - 0.0173 = 0.00170, \qquad K(-1) - K(0) = 0.141 - 0.0173 = -0.00321.$$

Note that the effect on ex-ante credit is *symmetric*, whereas the effect on ex-post capital is *asymmetric*. In response to an unrealized positive and negative noise shock, respectively, aggregate ex-ante credit increases and decreases by exactly the same quantity. In contrast, aggregate ex-post capital falls by more in response to a negative than it rises in response to a positive noise shock. The reason is that an increase in aggregate

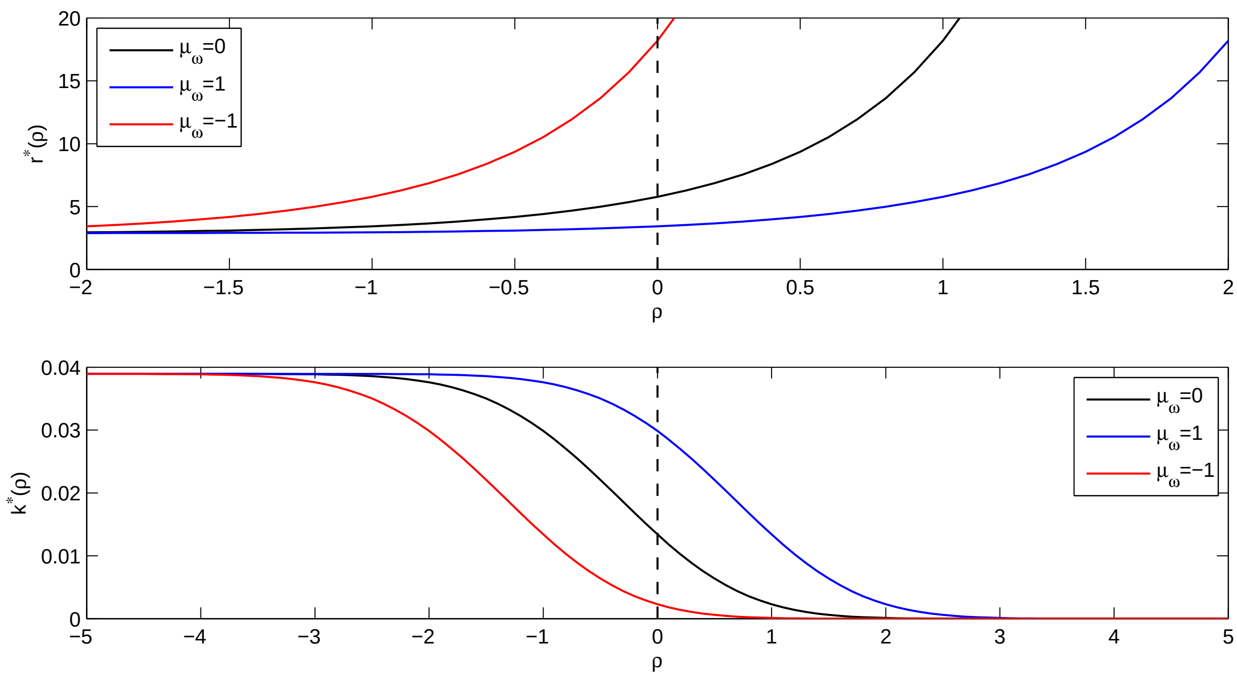

Figure 11: Effect of symmetric noise shocks on equilibrium interest rates and demand for bank credit by firm type ρ

ex-ante credit is a prerequisite for higher ex-post capital. In all three cases, the same firm types ρ default. However, the bank expects a higher repayment probability and raises the supply of credit in case 2, whereas it expects a lower repayment probability and reduces the supply of credit in case 3. Neither the former nor the latter expectation is realized. Instead, ex-post capital increases by the aggregate quantity of additional lending to firms with $\rho \leq 0$ in case 2, while it decreases by the aggregate reduction of lending to firms with $\rho \leq 0$ in case 3.

Figure 11 illustrates the above results graphically. The bank's unwarranted expectation of a higher and lower $E_t\omega_{t+1} = \mu_\omega$ has asymmetric effects on a given firm type ρ but symmetric effects on aggregate ex-ante credit, as measured by the areas between the black and blue and the black and red lines, respectively. Integrating over all firm types ρ, the bank raises and lowers ex-ante credit by the same aggregate quantities.

Given an ex-post realization of $\omega = 0$, indicated by the broken line in Figure 11, the same firm types $\rho \leq 0$ survive. However, these firm types have borrowed relatively more in response to a positive noise shock than in response to a positive noise shock, implying asymmetric effects on aggregate ex-post capital despite the symmetric effects on aggregate ex-ante credit. These results are robust to symmetric noise shocks around a different $f(\omega) = (\mu_\omega, \sigma_\omega)$, as long as the probability mass of ω beyond the support of ρ is small.

3. An Empirical Exercise

In this section, we conduct an empirical exercise based on the partial equilibrium investment model. Using data for the observable default threshold and signal, we estimate the joint process for ω_t and $\tilde{s}_t$ in equations (6) and (7). For the sake of generality, we do *not* impose a random walk without drift on the trend component ν_t and rather include the autocorrelation coefficient ρ_ν and the constant drift term c_ν in the vector of parameters to be estimated. Together with ϕ and the standard deviations of innovations to the trend, cycle, and noise component, σ_e, σ_ε, and σ_s, this yields a total of six unknown parameters.

Assuming that the error terms are Gaussian, $\mathbf{y}_t|\mathbf{y}_{t-1},\ldots,\mathbf{y}_1;\theta \sim N\left(\mathbf{y}_{t|t-1},\Sigma_{\mathbf{y},t|t-1}\right)$. We can thus estimate the vector of parameters $\theta = (\phi, c_\nu, \rho_\nu, \sigma_e, \sigma_\epsilon, \sigma_\varepsilon)$ by maximizing the log-likelihood function

$$\log f\left(\mathbf{y}_1,\ldots,\mathbf{y}_T;\theta\right) = -\frac{KT}{2}\log 2\pi - \frac{1}{2}\sum_{t=1}^{T}\log\left|\Sigma_{\mathbf{y},t|t-1}\right| - \frac{1}{2}\sum_{t=1}^{T}\left(\mathbf{y}_t - \mathbf{y}_{t|t-1}\right)'\Sigma^{-1}_{\mathbf{y},t|t-1}\left(\mathbf{y}_t - \mathbf{y}_{t|t-1}\right),$$

where $\mathbf{y}_t \equiv [\omega_t, \tilde{s}_t]'$ and $\Sigma_{\mathbf{y},t|t-1} = \mathbf{Z}\Sigma_{t|t-1}\mathbf{Z}' + \mathbf{H}$.

Recall that the observable ω_t determines which firm types $\rho \in [\underline{\rho}, \bar{\rho}]$ default, while $\tilde{s}_t$ is a noisy signal of the underlying trend component ν_t. Hence, we use the seasonally adjusted *charge-off rate* on all loans for all commercial banks in the U.S. as an empirical counterpart of ω_t and the Chicago Fed's *National Financial Conditions Index* (NFCI) as a proxy for $\tilde{s}_t$. Since higher charge-off rates and a higher NFCI indicate tighter financial conditions, whereas higher values of ω_t and $\tilde{s}_t$ imply a lower default probability of firm type ρ in the model, we take the negative of both time series. To facilitate the interpretation of σ_x, $x = \{e, \epsilon, \varepsilon\}$, we also standardize the data by subtracting the mean and dividing by the unconditional standard deviation.

Table 1 reports the maximum-likelihood point estimates of the parameters in equations (6) and (7). We find that the estimated trend component ν_t has virtually zero drift, while the autocorrelation coefficient ρ_ν is close to unity. At the same time, $\hat{\phi}_{\mathrm{ML}} = 0.9406$ suggests that there is substantial persistence also in the cycle component η_t. As a consequence, shocks to the trend or cycle component have similar effects on the observable ω_t, while only shocks to the trend component are reflected in the signal. Columns 4-6 in Table 1 indicate that noise shocks are more volatile than both trend and cycle shocks.

Table 1: Maximum-likelihood estimates of the parameters in equations (6) and (7)

Parameter	c_ν	ρ_ν	ϕ	σ_e	σ_ϵ	σ_ε	Loglik
Estimate	0.0071	0.9697	0.9406	0.1877	0.2284	0.5907	77.537

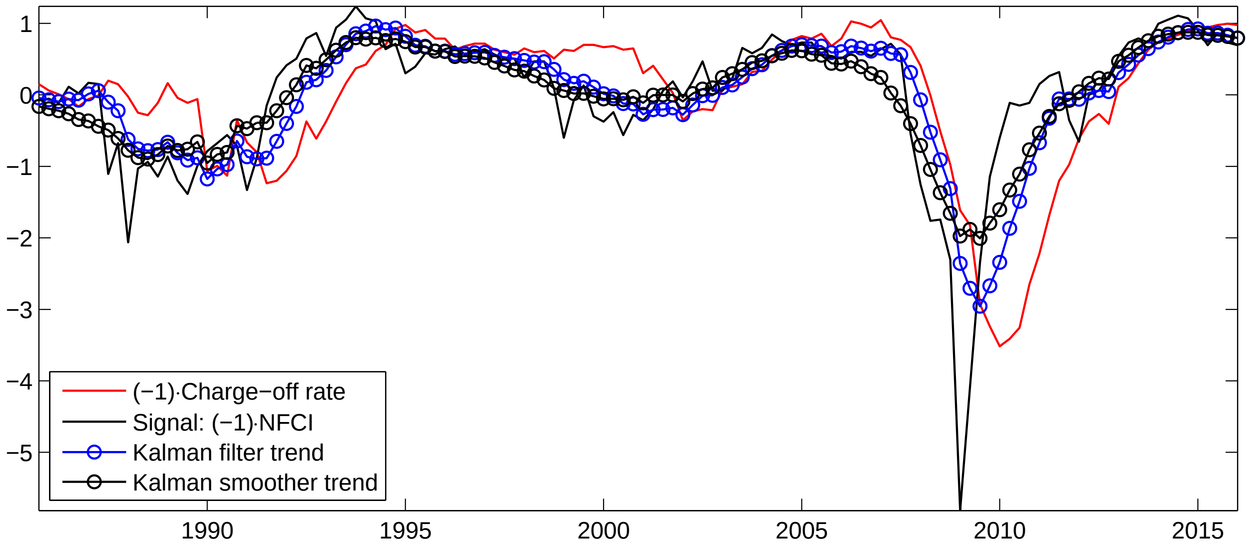

Figure 12: Kalman filter and smoother trends plotted against charge-off rate and NFCI data

When imposing a *strict unit root* on the trend component, i.e. $\bar{\rho}_\nu = 1$, the coefficient estimates in Table 1 are virtually unaffected and the log-likelihood decreases only marginally to 76.766. In contrast, imposing a *less persistent* cycle component drives the estimate of σ_ϵ towards zero. The corresponding log-likelihood for $\bar{\phi} = 0.5$ and $\bar{\phi} = 0.1$ equals 40.232 and 39.397, respectively, suggesting that a persistent cyclical component is strongly preferred to a transitory one.

Figure 12 plots the one-sided Kalman filter trend $\nu_{t|t}$ and the two-sided Kalman smoother trend $\nu_{t|T}$ against the (negative of the) charge-off rate and NFCI data, suggesting that the estimated trend component strikes a balance between the observable ω_t and the signal $\tilde{s}_t$. The figure also illustrates that the NFCI is leading ex-post charge-off rates on loans of U.S. banks by about three to four quarters.

Figure 13 plots the maximum-likelihood estimates of the trend, cycle, and noise shock series based on the Kalman filter and smoother, respectively. As expected, the financial crisis of 2007-2008 sticks out as a major event in all three shock series. While the Kalman *filter* credits the increase in charge-off rates and the tightening of financial conditions to a large negative shock in the trend component, the Kalman *smoother* instead attributes this mainly to a series of negative shocks in the cycle component. The peaking of NFCI in 2008Q4 is attributed mainly to a negative noise shock. This seems to be largely due to the time lag between a tightening of financial conditions reflected by the NFCI and an eventual increase in charge-off rates, which is not taken into account by the joint exogenous processes in (6) and (7).

As a final exercise, we investigate what the estimated trend, cycle, and noise decomposition based on

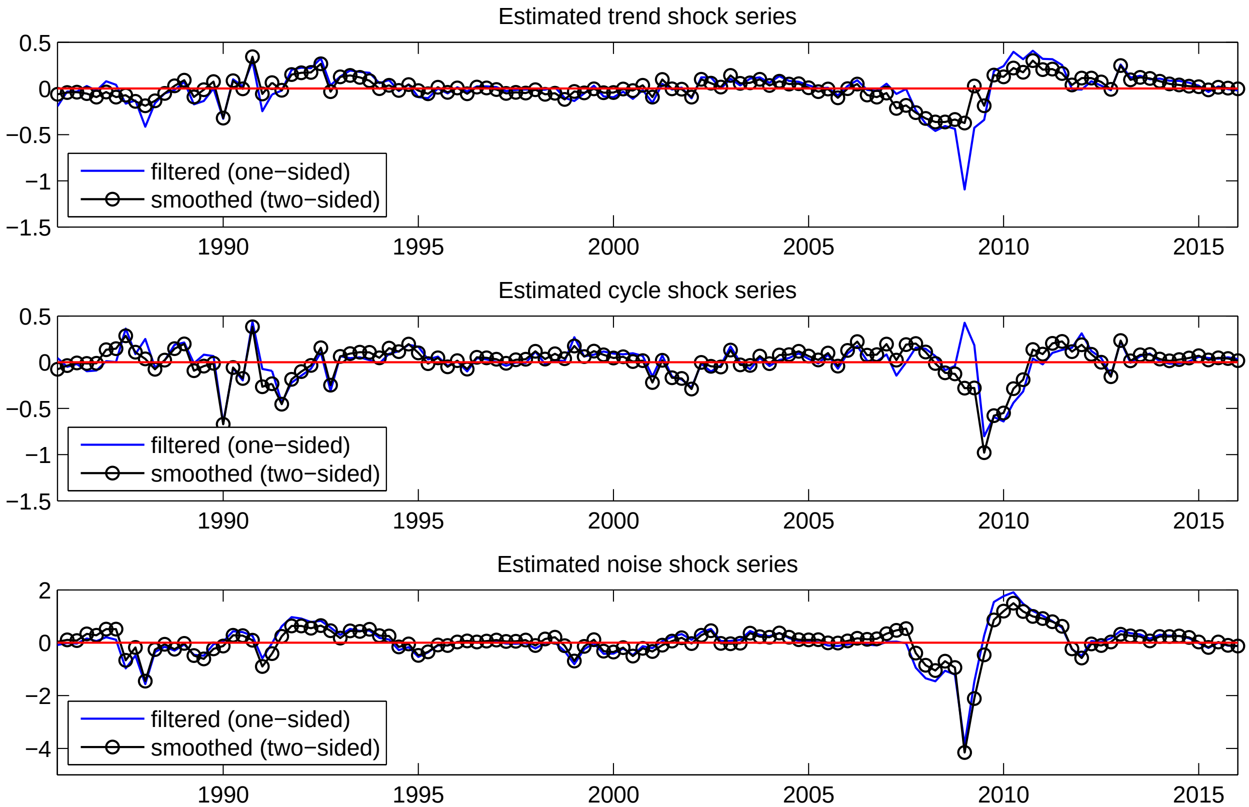

Figure 13: Estimated trend, cycle, and noise shock series based on the Kalman filter and smoother

the Kalman filter implies for the economic variables in our partial equilibrium model. The solid blue line in Figure 14 plots the simulated paths of aggregate ex-ante credit, ex-post capital, and the average repayment probability in the presence of imperfect information. The model implies a protracted contraction of credit and capital during 1985-1992, followed by an expansion peaking in 1995. After a relatively stable period, interrupted only by a moderate contraction following the dot-com bubble, ex-ante credit and ex-post capital plummet in the vague of the financial crisis. While the repayment probability on the bank's loan portfolio, which we compute by weighting the ex-ante probability of default by the corresponding amount of credit for each ρ, is largely stable prior to 2008, it sharply drops from around 98 to 93% during the Great Recession. In other words, the average probability of default more than triples within a year.

The broken black line in Figure 14 plots the simulated path of the same economic variables under full information and rational expectations (FIRE), assuming that the lender observes not just ω_t but also its decomposition in terms of ν_t and η_t. We treat the maximum-likelihood estimates of the trend and cycle components as realized data and assume that the bank can correctly attribute each innovation in ω_t to either

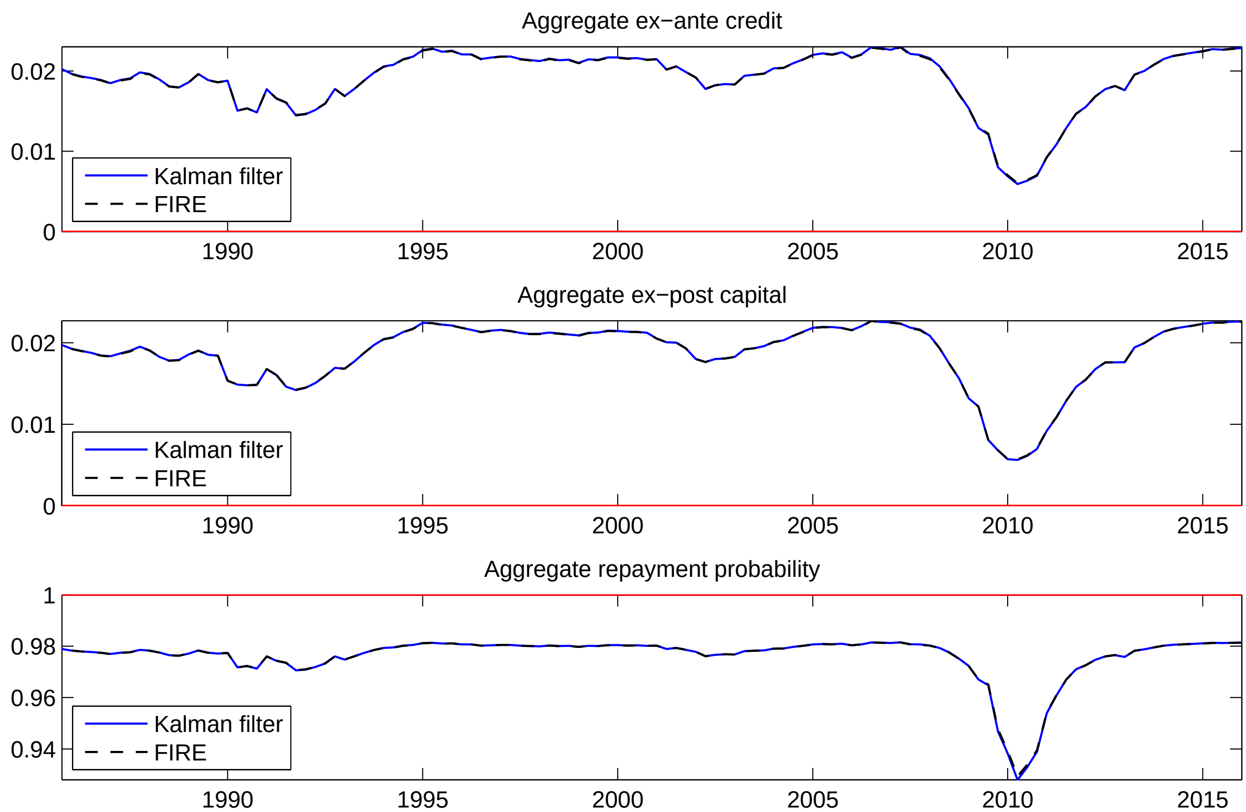

Figure 14: Simulated paths of economic variables based on estimated trend, cycle, and noise shock series in Figure 13

ν_t or η_t. Note that, in this counterfactual exercise, the signal $\tilde{s}_t$ becomes meaningless. Figure 14 illustrates that the effect of imperfect information on aggregate economic variables is generally small. Allowing for full information and rational expectations, the decrease in ex-ante credit, ex-post capital, and average repayment probability around 2010 is slightly less pronounced. While it is therefore fair to say that, through the lens of our partial equilibrium model, imperfect i formation and noise shocks contributed somewhat to the credit contraction during the Great Recession, the effect does not seem to be economically significant.

4. The General Equilibrium Model

4.1. A General Version

Suppose that lending to firms is undertaken by a continuum of perfectly competitive financial intermediaries, which are endowed with aggregate bank equity N_t and collect aggregate deposits D_t from a risk-averse representative household. Assuming that firm credit represents the only asset, while bank equity and deposits represent the only liabilities, the financial sector's aggregate balance sheet identity is given by

$$L_t \equiv N_t + D_t, \tag{11}$$

where L_t denotes ex-ante aggregate credit at the beginning of t. The sole purpose of bank equity is to shield depositors from unexpected fluctuations in the aggregate return on firm credit and guarantee them a risk-free rate of return on deposits in each state of the world, ω_t. To avoid that bank equity, N_t, grows without bound, each period a constant fraction δ is assumed to be consumed exogenously. Assuming a uniform distribution for firm types $\rho \in \left[\underline{\rho}, \bar{\rho}\right]$, aggregate bank equity evolves according to

$$N_{t+1} = (1-\delta)\left[\int_{\underline{\rho}}^{\omega_t} r_t(\rho)\, k_t(\rho)\, d\rho - r_t^d D_t\right] = (1-\delta)\left[\int_{\underline{\rho}}^{\omega_t} r_t(\rho)\, k_t(\rho)\, d\rho - r_t^d (L_t - N_t)\right].$$

The economy-wide resource constraint is given by

$$Y_t = C_t + L_{t+1}, \tag{12}$$

where C_t denotes aggregate consumption in period t. Making use of the production function for firm type ρ,

$$y_t^*(\rho) = \begin{cases} k_t^*(\rho)^\alpha & \text{if } \omega_t \geq \rho \\ 0 & \text{if } \omega_t < \rho \end{cases},$$

we can rewrite the resource constraint in (12) as

$$Y_t = \int_{\underline{\rho}}^{\omega_t} k_t^*(\rho)^\alpha\, d\rho = C_t + \int_{\underline{\rho}}^{\bar{\rho}} k_{t+1}^*(\rho)\, d\rho.$$

The representative household maximizes the present value of expected current and future utility from consumption subject to the budget constraint in period t, i.e.

$$\max_{C_{t+s}, D_{t+s+1}} \quad E_t \sum_{s=0}^{\infty} \beta^s u(C_{t+s}) \qquad \text{s. t.} \qquad D_{t+s+1} + C_{t+s} = r_{t+s}^d D_{t+s},$$

where r_t^d denotes the *gross* risk-free rate of return on bank deposits between period $t-1$ and period t, and the instantaneous utility function is assumed to satisfy $u' > 0$ and $u'' \leq 0$. The corresponding first-order conditions (FOCs) are given by

$$C_{t+s}: \qquad \beta^s u'(C_{t+s}) = \lambda_{t+s}.$$

$$D_{t+s+1}: \qquad \lambda_{t+s} = E_t \lambda_{t+s+1} r^d_{t+s+1},$$

which yields the following consumption Euler equation:

$$\frac{u'(C_{t+s})}{E_t u'(C_{t+s+1})} = \beta r^d_{t+s+1}, \qquad s \geq 0,$$

where r^d_{t+s+1} is assumed to be risk-free and thus predetermined in period $t+s+1$.

Together with the economy-wide resource constraint in (12), the Euler equation determines C_t and r^d_t. Note that, in the resource constraint, ex-ante aggregate credit, $L_t = \int_{\underline{\rho}}^{\bar{\rho}} k^*_t(\rho)\, d\rho$, is weakly larger than ex-post aggregate capital available for production in period t, $K_t = \int_{\underline{\rho}}^{\bar{\omega}_t} k^*_t(\rho)\, d\rho$.

Under these assumptions, we get the following set of equilibrium conditions for period t:

$$\frac{u'(C_t)}{E_t u'(C_{t+1})} = \beta r^d_{t+1}, \tag{13}$$

$$Y_t = C_t + L_{t+1}, \tag{14}$$

$$L_t = D_t + N_t, \tag{15}$$

$$K_t = \int_{\underline{\rho}}^{\omega_t} k^*_t(\rho)\, d\rho, \tag{16}$$

$$Y_t = \int_{\underline{\rho}}^{\omega_t} k^*_t(\rho)^\alpha\, d\rho, \tag{17}$$

$$L_{t+1} = \int_{\underline{\rho}}^{\bar{\rho}} k^*_{t+1}(\rho)\, d\rho, \tag{18}$$

$$N_{t+1} = (1-\delta)\left[\int_{\underline{\rho}}^{\omega_t} r^*_t(\rho)\, k^*_t(\rho)\, d\rho - r^d_t D_t\right], \tag{19}$$

where

$$k^*_{t+1}(\rho) = \left[\frac{\alpha}{r^*_{t+1}(\rho)}\right]^{\frac{1}{1-\alpha}} \qquad \text{and} \qquad r^*_{t+1}(\rho) = \frac{r^d_{t+1}}{Prob\,(\omega_{t+1} \geq \rho | \omega_t)}.$$

Equations (13)-(19) yield a system of seven equilibrium conditions in the seven endogenous variables C_t, Y_t, K_t, L_t, D_t, N_t, and r^d_t. Given exogenous processes for ω_t and $\tilde{s}_t$, we can investigate the general equilibrium implications of imperfect information for aggregate lending, capital, and output.

4.2. A Simplified Version with Two Risk Types

Suppose instead that there is a continuum of *less risky* and a continuum of *more risky* firms, both of mass 1/2, where the latter have a higher probability of default than the former in each state of the world ω_t. The corresponding variables carry a superscript "1" and "2", respectively. As before, ω_t consists of the permanent component $\nu_t = \nu_{t-1} + e_t$, $e_t \sim N\left(0, \sigma_e^2\right)$, and the transitory component $\eta_t = \phi\eta_{t-1} + \epsilon_t$, $\epsilon_t \sim N\left(0, \sigma_\epsilon^2\right)$, which are not separately observable, whereas the noisy signal $\tilde{s}_t = \nu_t + \varepsilon_t$, $\varepsilon_t \sim N\left(0, \sigma_\varepsilon^2\right)$, is publicly observable.

Given that ω_t is defined on the real line, whereas the probability of default is a percentage, we map the real line onto the interval (0, 1) by defining the aggregate repayment probabilities of type-1 and type-2 firms, respectively, as

$$Prob_t^1 \equiv \frac{4}{4 + e^{-\omega_t}} \qquad \text{and} \qquad Prob_t^2 \equiv \frac{1}{1 + e^{-\omega_t}}. \tag{20}$$

For $\omega_{ss} = 0$, this implies that $Prob_{ss}^1 = 0.8$ and $Prob_{ss}^2 = 0.5$, i.e., 80% of the less risky and 50% of the more risky loans are repaid in the steady state. Note that, while the mappings in (20) are chosen for illustrative purposes, their functional forms can be adjusted separately in order to target arbitrary repayment probabilities for the two risk types. Figure 15 plots the continuous mapping from $\omega_t \in \mathbb{R}$ onto $Prob_t^1 \in (0, 1)$ and $Prob_t^2 \in (0, 1)$, respectively.

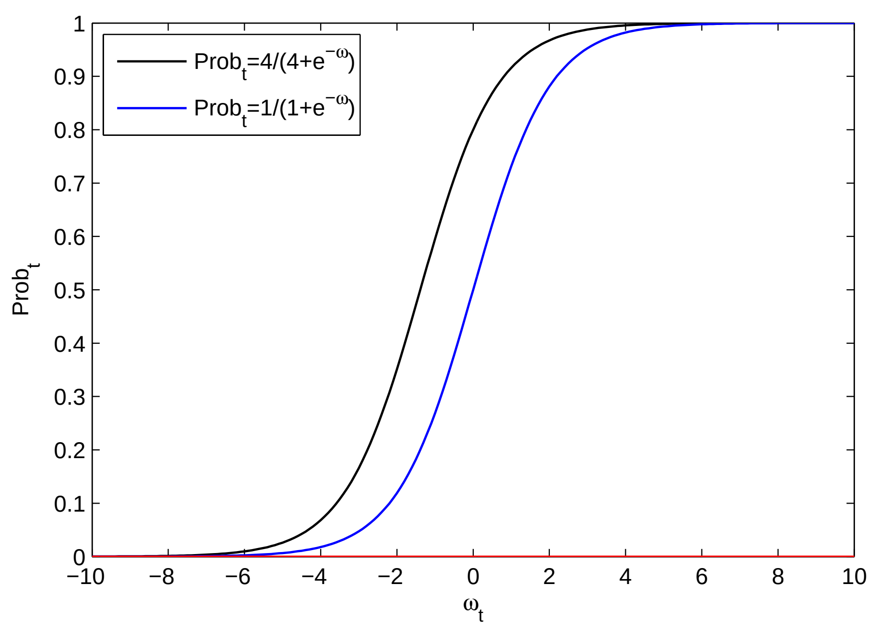

Figure 15: Continuous mappings from $\omega_t \in \mathbb{R}$ onto $Prob_t^1 \in (0, 1)$ and $Prob_t^2 \in (0, 1)$

The equilibrium conditions of the simplified model with two risk types can be summarized as follows:

$$\frac{u'(C_t)}{E_t u'(C_{t+1})} = \beta R_t^d, \tag{21}$$

$$R_t^1 = \frac{R_t^d}{E_t Prob_{t+1}^1}, \tag{22}$$

$$R_t^2 = \frac{R_t^d}{E_t Prob_{t+1}^2}, \tag{23}$$

$$L_t^1 = \frac{1}{2}\left(\frac{\alpha}{R_t^1}\right)^{\frac{1}{1-\alpha}}, \tag{24}$$

$$L_t^2 = \frac{1}{2}\left(\frac{\alpha}{R_t^2}\right)^{\frac{1}{1-\alpha}}, \tag{25}$$

$$L_t = L_t^1 + L_t^2, \tag{26}$$

$$K_t^1 = Prob_t^1 L_{t-1}^1, \tag{27}$$

$$K_t^2 = Prob_t^2 L_{t-1}^2, \tag{28}$$

$$K_t = K_t^1 + K_t^2, \tag{29}$$

$$Y_t = K_t^\alpha, \tag{30}$$

$$Y_t = C_t + L_t, \tag{31}$$

$$L_t = N_t + D_t, \tag{32}$$

$$N_t = (1-\delta)\left(R_{t-1}^1 K_{t-1}^1 + R_{t-1}^2 K_{t-1}^2 - R_{t-1}^d D_{t-1}\right). \tag{33}$$

Equations (21)-(33) yield a system of 13 equilibrium conditions in the 13 endogenous variables C_t, Y_t, K_t, K_t^1, K_t^2, L_t, L_t^1, L_t^2, D_t, N_t, R_t^d, R_t^1, and R_t^2, where the aggregate default probabilities $Prob_t^1$ and $Prob_t^2$ are defined in (20). Given exogenous processes for ω_t and $\tilde{s}_t$, we can investigate the general equilibrium implications of imperfect information for aggregate lending, capital, and output.[4] Moreover, we define the aggregate lending rate R_t as the average of type-specific interest rates, weighted by the respective lending volumes in (34), and the credit spread as the ratio of R_t to the risk-free rate on deposits in (35):

$$R_t \equiv \frac{R_t^1 L_t^1 + R_t^2 L_t^2}{L_t^1 + L_t^2}, \tag{34}$$

$$spread_t \equiv \frac{R_t}{R_t^d}. \tag{35}$$

[4]Appendix B summarizes the log-linearized equilibrium conditions of the simplified model with two risk types in (21)-(33).

4.3. Impulse Response Functions

It is straightforward to solve the simplified general equilibrium model and compute impulse response functions of the variables of interest. In what follows, we set $\alpha = 0.35$ and $R^d = 1/\beta = 1.01$, as in our partial equilibrium model, while 10% of the financial sector's net worth is consumed each period, i.e. $\delta = 0.1$. The relative size of trend, cycle, and noise shocks is taken from our parameter estimates in Table 1, i.e. $\sigma_e = 0.1877$, $\sigma_\epsilon = 0.2284$, and $\sigma_\varepsilon = 0.5907$.

Trend shocks

Figure 16 plots the impulse response functions based on the simplified GE model with just two risk types after a positive one-standard-deviation innovation in the trend component. In period 1, ν_t and thus both ω_t and $\tilde{s}_t$ increase by the same amount as e_t, whereas the Kalman filter nowcast of the trend component, $\nu_{t|t}$ takes time to adjust to its new value, as the economic agents learn that the change in ω_t is permanent rather than transitory.

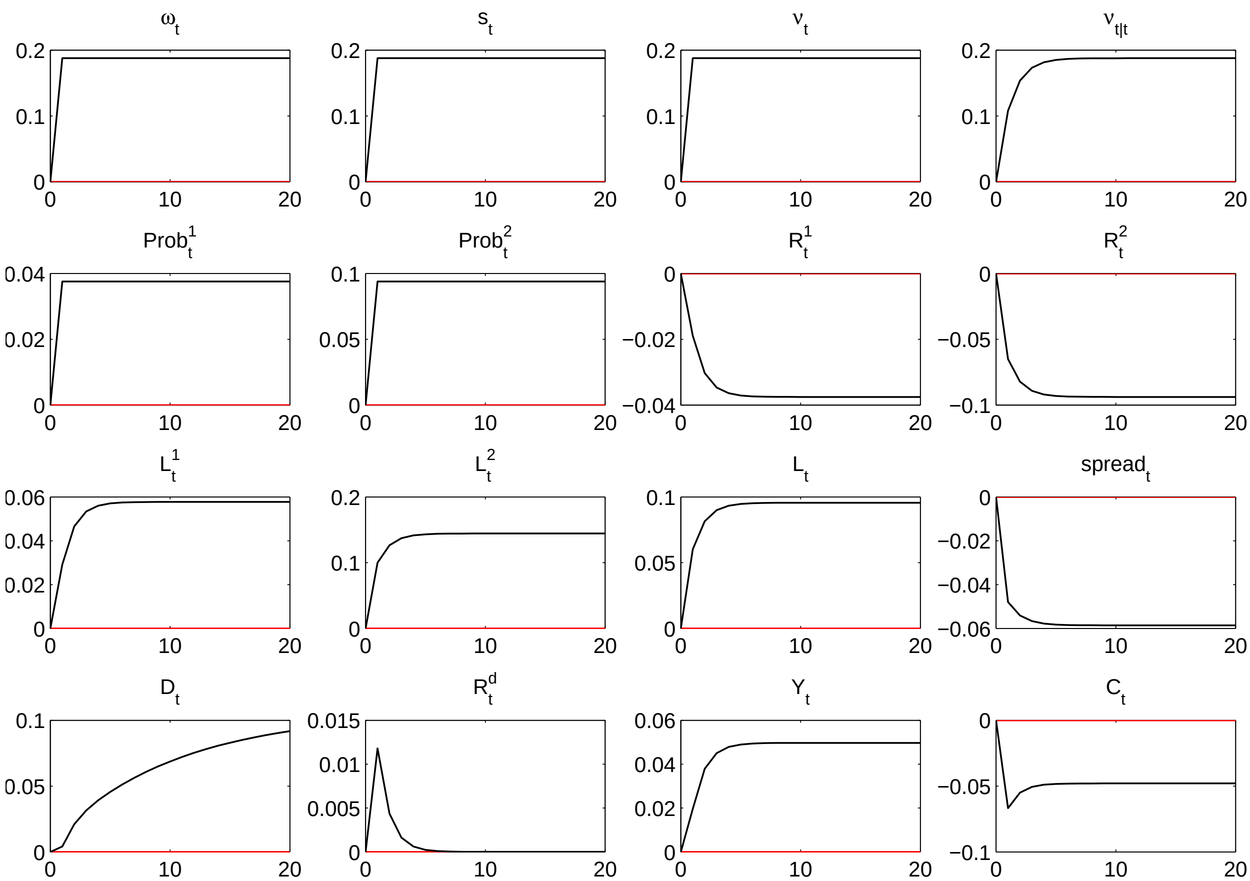

Figure 16: Impulse responses to a shock in the trend component based on the simplified general equilibrium model in (21)-(33)

Accordingly, the "objective" repayment probabilities of both risk types defined in (20) increase on impact, whereas the corresponding lending rates R_t^1 and R_t^2, which depend on the Kalman filter forecasts of $E_t Prob_{t+1}^1$ and $E_t Prob_{t+1}^2$, respectively, take time to adjust downwards. As a consequence, type-specific and aggregate credit volumes expand slowly. The reduction in the spread between the aggregate lending rate and the risk-free rate on deposits illustrates that this expansion is driven by a lower cost of credit associated with a lower default probability of both firm types.

The expansion on the asset side of the financial sector's aggregate balance sheet requires an identical expansion on the liability side. Hence, both household deposits and bank net worth (not shown) increase in order to fund the additional lending to both firm types, while consumption decreases, as private households are more patient than financial markets.

Cycle shocks

Consider now the impulse response functions to a positive one-standard-deviation shock in the cycle component in Figure 17. Given our assumption that $\phi = 0$ in (6), ω_t increases in period 1 by the amount of the shock, falling back to zero from period 2 onwards, whereas the trend component ν_t and the public signal $\tilde{s}_t$ are equal to zero throughout. Nevertheless, the uncertainty about the origin of the observed increase in ω_t under imperfect information implies that $\nu_{t|t}$ also increases. While the "objective" probabilities of repayment are therefore back to their steady-state values from period 2 onwards, the corresponding lending rates remain below their steady-state values for an extended period. Simultaneously, the type-specific and individual credit volumes expand temporarily.

Importantly, household deposits fall while consumption increases in response to a shock to the transitory component. This is only consistent with a reduction in the risk-free rate on deposits. Nevertheless, the credit spread decreases, indicating that the reduction in lending rates (weighted by the respective volumes) is larger than the reduction in the deposit rate and that the expansion of credit is driven by the supply side.

Noise shocks

While the impulse responses in Figures 16 and 17 arise from shocks to the fundamental determinants of credit risk, pure noise shocks can also affect the financial intermediaries' lending behavior and thus real economic activity in the presence of imperfect information. Figure 18 plots the impulse response functions for a positive one-standard-deviation shock to the public signal $\tilde{s}_t$ *without any change in fundamentals*.

Although the observable ω_t and its latent trend component ν_t remain unaffected, the positive signal in period 1 induces the Kalman filter to revise the nowcast of the trend component $\nu_{t|t}$ upwards. While the

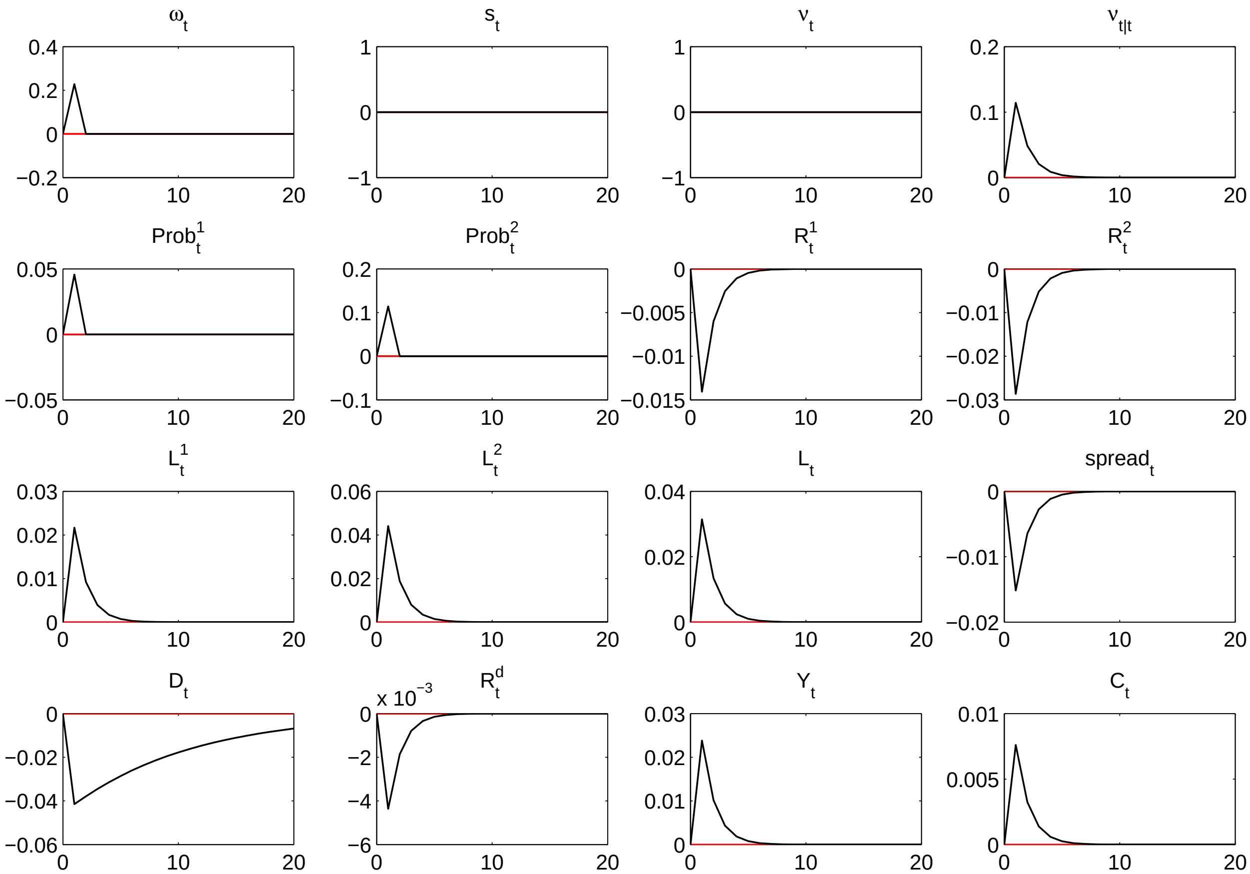

Figure 17: Impulse responses to a shock in the cycle component based on the simplified general equilibrium model in (21)-(33)

"objective" repayment probabilities are therefore unchanged, the interest rate on loans to *less risky* firms increases on impact before falling below its steady-state value. In contrast, the interest rate on loans to *more risky* firms falls on impact before converging to its steady-state value from below. This difference is a direct consequence of going from partial to general equilibrium.

In response to a pure noise shock, output does not change in the period of the shock. As a consequence, any additional lending must be backed by an increase in deposits at a higher risk-free rate, which raises financial intermediaries' funding costs for *all* loans. Type-specific lending rates fall only if the increase in the expected repayment probability offsets the increase in R_t^d, which is the case for the more risky but not the less risky firm type. Accordingly, lending to the former increases on impact, whereas lending to the latter initially decreases before increasing relative to its steady-state value.

Given that the expansion of credit to more risky firms dominates the initial contraction of credit to less risky firms, aggregate lending increases while the weighted average of lending rates decreases on impact.

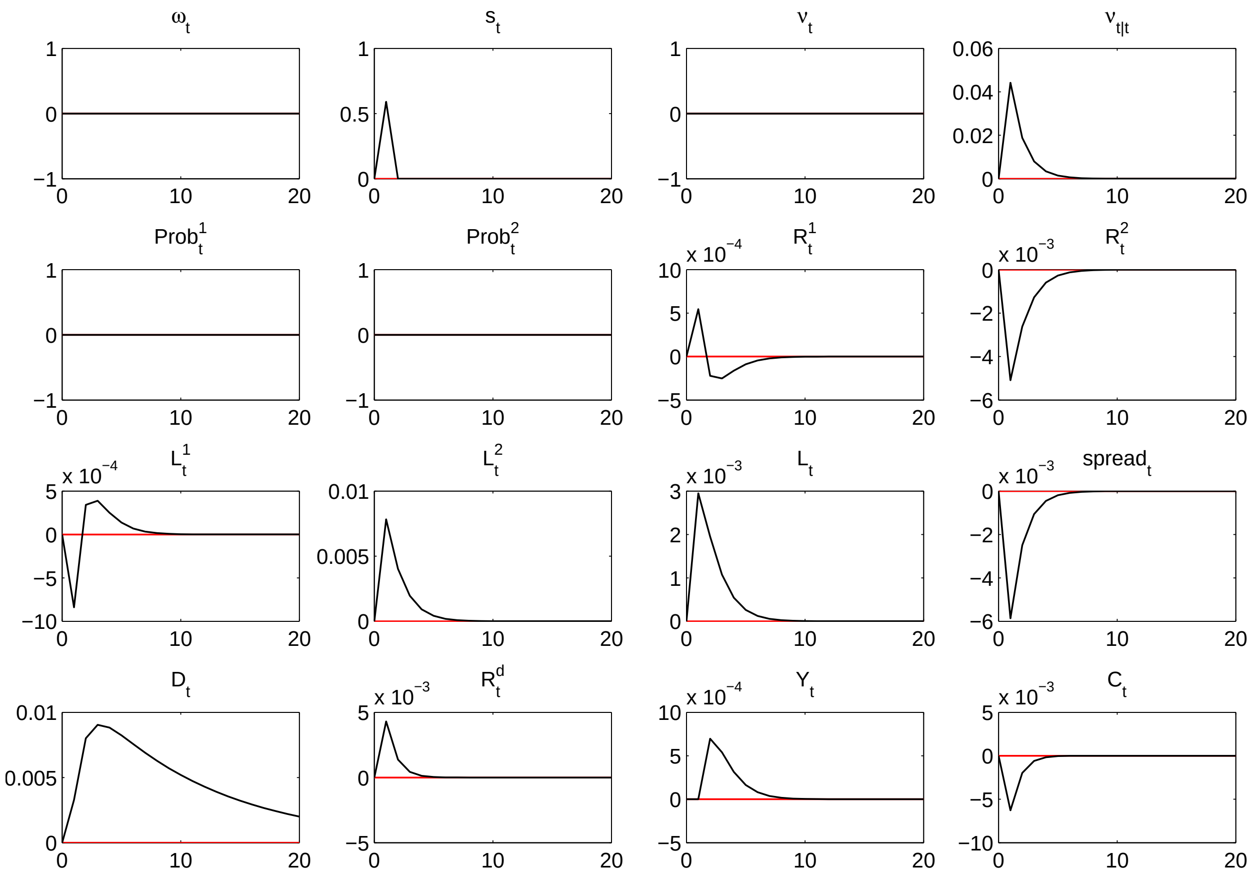

Figure 18: Impulse responses to a shock in the signal based on the simplified general equilibrium model in (21)-(33)

As before, the reduction in the credit spread indicates that this expansion is driven by the supply side. This time, however, it arises from a pure noise shock rather than from a change in fundamentals.

5. Concluding Remarks

In this paper, we investigate to which extent banks' imperfect information about the state of the economy can cause lending cycles. We start by analyzing a partial equilibrium neoclassical investment model, where financial intermediaries must solve a signal-extraction problem in order to learn the fundamental and non-fundamental components that determine the state of the economy. The model suggests that credit booms can arise simply based on informational frictions and that such credit booms are associated with increased default rates relative to the perfect-information benchmark. Taking our partial equilibrium model to the data, we find that, although the standard deviation of noise shocks is large relative to that of fundamental shocks, the effects of informational frictions on aggregate credit and default rates seems economically small.

We intend to build on our analysis along the following dimensions. While the partial equilibrium model already provides the economic intuition and first empirical insights, it does not allow for potential feedback effects from aggregate macroeconomic variables to banks' expectations, thus limiting the role for informational frictions by construction. In a next step, we will therefore use the general equilibrium version of the model to identify trend, cycle, and noise shocks based on aggregate macroeconomic and financial data. We will further include Blue Chip consensus forecasts in our empirical exercises in order to quantify the role for informational frictions and identify the shocks of interest more precisely. Finally, we want to match cross-sectional survey data on their expectations with the actual lending behavior of U.S. banks to corroborate the relevance of informational frictions for the emergence of credit cycles at the microeconomic level.

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Appendix A. The Log-Linearized GE Model

It is straightforward to log-linearize the Euler equation and the market-clearing conditions in (13)-(15):

$$E_t\hat{C}_{t+1} - \hat{C}_t = \hat{r}^d_{t+1}, \tag{A.1}$$

$$Y_{ss}\hat{Y}_t = C_{ss}\hat{C}_t + L_{ss}\hat{L}_{t+1}, \tag{A.2}$$

$$L_{ss}\hat{L}_t = D_{ss}\hat{D}_t + N_{ss}\hat{N}_t, \tag{A.3}$$

where X_{ss} denotes the steady-state value of variable X and $\hat{X}_t$ the percentage deviation of variable X in period t from its steady-state value, i.e. $\hat{X}_t \equiv (X_t - X_{ss})/X_{ss}$.

Note that equations (16)-(19) are nonlinear functions of the endogenous variables ω_{t+1}, $E_t\omega_{t+1}$, and R^d_t. Accordingly, their log-linear approximations around the steady state are given by

$$K_{ss}\hat{K}_t = \left(\frac{\partial K_t}{\partial \omega_t}\right)_{ss}\omega_{ss}\hat{\omega}_t + \left(\frac{\partial K_t}{\partial E_{t-1}\omega_t}\right)_{ss}\omega_{ss}E_{t-1}\hat{\omega}_t + \left(\frac{\partial K_t}{\partial r^d_t}\right)_{ss} r^d_{ss}\hat{r}^d_t, \tag{A.4}$$

$$Y_{ss}\hat{Y}_t = \left(\frac{\partial Y_t}{\partial \omega_t}\right)_{ss}\omega_{ss}\hat{\omega}_t + \left(\frac{\partial Y_t}{\partial E_{t-1}\omega_t}\right)_{ss}\omega_{ss}E_{t-1}\hat{\omega}_t + \left(\frac{\partial Y_t}{\partial r^d_t}\right)_{ss} r^d_{ss}\hat{r}^d_t, \tag{A.5}$$

$$L_{ss}\hat{L}_{t+1} = \left(\frac{\partial L_{t+1}}{\partial \omega_{t+1}}\right)_{ss}\omega_{ss}\hat{\omega}_{t+1} + \left(\frac{\partial L_{t+1}}{\partial E_t\omega_{t+1}}\right)_{ss}\omega_{ss}E_t\hat{\omega}_{t+1} + \left(\frac{\partial L_{t+1}}{\partial r^d_{t+1}}\right)_{ss} r^d_{ss}\hat{r}^d_{t+1}, \tag{A.6}$$

$$N_{ss}\hat{N}_{t+1} = (1-\delta)\left[\left(\frac{\partial N_{t+1}}{\partial \omega_t}\right)_{ss}\omega_{ss}\hat{\omega}_t + \left(\frac{\partial N_{t+1}}{\partial E_{t-1}\omega_t}\right)_{ss}\omega_{ss}E_{t-1}\hat{\omega}_t + \left(\frac{\partial N_{t+1}}{\partial r^d_t}\right)_{ss} r^d_{ss}\hat{r}^d_t - r^d_{ss}D_{ss}\hat{D}_t\right] \tag{A.7}$$

where $(\bullet)_{ss}$ denotes the partial derivative of variable X with respect to variable Z evaluated at the steady state:

$$\frac{\partial K_t}{\partial \omega_t} = k^*_t(\omega_t), \qquad \frac{\partial K_t}{\partial E_{t-1}\omega_t} = \frac{\alpha}{1-\alpha}\frac{1}{r^d_t}\int_{\underline{\rho}}^{\omega_t}[k^*_t(\rho)]^{\alpha}\,\phi\left(\rho, E_{t-1}\omega_t, E_{t-1}\sigma^2_t\right)d\rho, \qquad \frac{\partial K_t}{\partial r^d_t} = -\frac{1}{1-\alpha}\frac{1}{r^d_t}K_t,$$

$$\frac{\partial Y_t}{\partial \omega_t} = [k^*_t(\omega_t)]^{\alpha}, \qquad \frac{\partial Y_t}{\partial E_{t-1}\omega_t} = \frac{\alpha^2}{1-\alpha}\frac{1}{r^d_t}\int_{\underline{\rho}}^{\omega_t}[k^*_t(\rho)]^{2\alpha-1}\,\phi\left(\rho, E_{t-1}\omega_t, E_{t-1}\sigma^2_t\right)d\rho, \qquad \frac{\partial Y_t}{\partial r^d_t} = -\frac{\alpha}{1-\alpha}\frac{1}{r^d_t}Y_t,$$

$$\frac{\partial L_{t+1}}{\partial \omega_{t+1}} = 0, \qquad \frac{\partial L_{t+1}}{\partial E_t\omega_{t+1}} = \frac{\alpha}{1-\alpha}\frac{1}{r^d_{t+1}}\int_{\underline{\rho}}^{\bar{\rho}}\left[k^*_{t+1}(\rho)\right]^{\alpha}\phi\left(\rho, E_t\omega_{t+1}, E_t\sigma^2_{t+1}\right)d\rho, \qquad \frac{\partial L_{t+1}}{\partial r^d_{t+1}} = -\frac{1}{1-\alpha}\frac{1}{r^d_{t+1}}L_{t+1},$$

$$\frac{\partial N_{t+1}}{\partial \omega_t} = (1-\delta)\,r^*_t(\omega_t)\,k^*_t(\omega_t), \qquad \frac{\partial N_{t+1}}{\partial E_{t-1}\omega_t} = (1-\delta)\frac{\alpha}{1-\alpha}\int_{\underline{\rho}}^{\omega_t}\frac{r^*_t(\rho)\,k^*_t(\rho)}{Prob\,(\omega_t \geq \rho|\omega_{t-1})}\phi\left(\rho, E_{t-1}\omega_t, E_{t-1}\sigma^2_t\right)d\rho,$$

$$\frac{\partial N_{t+1}}{\partial r^d_t} = (1-\delta)\left[-\frac{\alpha}{1-\alpha}\frac{1}{r^d_t}\int_{\underline{\rho}}^{\omega_t} r^*_t(\rho)\,k^*_t(\rho)\,d\rho - D_t\right],$$

where $\phi(\bullet)$ denotes the probability density function (pdf) of the normal distribution.

Appendix B. The Log-Linearized Simplified GE Model with Two Risk Types

Assuming $u(C_t) = \log C_t$, it is straightforward to log-linearize the equilibrium conditions in equations (21)-(35) around the non-stochastic steady state:

$$E_t\hat{C}_{t+1} - \hat{C}_t = \hat{R}_t^d, \tag{B.1}$$

$$\hat{R}_t^1 = \hat{R}_t^d - \frac{e^{-\omega_{ss}}}{4 + e^{-\omega_{ss}}} E_t\widehat{Prob}_{t+1}^1, \tag{B.2}$$

$$\hat{R}_t^2 = \hat{R}_t^d - \frac{e^{-\omega_{ss}}}{1 + e^{-\omega_{ss}}} E_t\widehat{Prob}_{t+1}^2, \tag{B.3}$$

$$\hat{L}_t^1 = -\frac{1}{1-\alpha}\hat{R}_t^1, \tag{B.4}$$

$$\hat{L}_t^2 = -\frac{1}{1-\alpha}\hat{R}_t^2, \tag{B.5}$$

$$L_{ss}\hat{L}_t = L_{ss}^1\hat{L}_t^1 + L_{ss}^2\hat{L}_t^2, \tag{B.6}$$

$$\hat{K}_t^1 = \widehat{Prob}_t^1 + \hat{L}_{t-1}^1, \tag{B.7}$$

$$\hat{K}_t^2 = \widehat{Prob}_t^2 + \hat{L}_{t-1}^2, \tag{B.8}$$

$$K_{ss}\hat{K}_t = K_{ss}^1\hat{K}_t^1 + K_{ss}^2\hat{K}_t^2, \tag{B.9}$$

$$\hat{Y}_t = \alpha\hat{K}_t, \tag{B.10}$$

$$Y_{ss}\hat{Y}_t = C_{ss}\hat{C}_t + L_{ss}\hat{L}_t, \tag{B.11}$$

$$L_{ss}\hat{L}_t = D_{ss}\hat{D}_t + N_{ss}\hat{N}_t, \tag{B.12}$$

$$N_{ss}\hat{N}_t = (1-\delta)\left[R_{ss}^1K_{ss}^1\left(R_{t-1}^1 + K_t^1\right) + R_{ss}^2K_{ss}^2\left(R_{t-1}^2 + K_t^2\right) - R_{ss}^dD_{ss}\left(R_{t-1}^d + D_{t-1}\right)\right], \tag{B.13}$$

$$R_{ss}\left(L_{ss}^1\hat{L}_t^1 + L_{ss}^2\hat{L}_t^2\right) = R_{ss}^1L_{ss}^1\left(\hat{R}_t^1 + \hat{L}_t^1\right) + R_{ss}^2L_{ss}^2\left(\hat{R}_t^2 + \hat{L}_t^2\right) - R_{ss}\left(L_{ss}^1 + L_{ss}^2\right)\hat{R}_t, \tag{B.14}$$

$$\widehat{spread}_t = \hat{R}_t - \hat{R}_t^d. \tag{B.15}$$

where X_{ss} denotes the steady-state value of variable X and $\hat{X}_t$ the percentage deviation of X in period t from its steady-state value, i.e. $\hat{X}_t \equiv (X_t - X_{ss})/X_{ss}$.