

Expected Spot Prices and the Dynamics of Commodity Risk Premia^{*}

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Abstract

We analyse a novel time series of investors expectations on future commodity spot prices, and show that a model with adaptive learning can replicate investors' forecasts. We use this framework to back out the dynamics of the (ex-ante) risk premia for different commodities and maturities, and provide evidence that commodity risk premia are time-varying and their dynamics is predominantly due to the changing nature of risk sharing and appetite, as proxied by open interest, hedging pressure and time-series momentum. Finally, we show that the explanatory power of alternative factors is not constant over time, both across commodities and time horizons.

Keywords: Commodity Markets, Survey Expectations, Adaptive Learning, Risk Premia, Empirical Asset Pricing

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1 Introduction

The way in which investors form expectations about future commodity prices is of great interest to economists as well as market participants, and forward prices have been used extensively in economic models as an approximation of market expectations.¹ However, the forward curve includes not only the expectation of future spot prices, but also a component reflecting the compensation required by market participants for bearing the risk of uncertain future fluctuations in the price of the spot commodity, i.e. a risk premium.² Whether such risk premium is positive, negative, or time-varying and driven by changes in economic fundamentals has been controversial in the literature.³ This controversy stems from the fact that investors' expectations are not directly observable.

In this paper, we address this issue by using a novel survey provided by Bloomberg, which contains forecasts from professional analysts on future spot prices at multiple quarterly horizons and across different commodities. The survey includes analysts specialized in commodity markets, and is quantitative in nature as participants are asked to provide point forecasts on the average quarterly commodity price for specified futures contracts. These are key features of the survey composition as a deep knowledge of the commodity markets peculiarities from the survey respondent, coupled with a clear objective of the survey, allows to reduce the effect of potential biases, quality homogeneity issues, and limited information processing, which possibly characterizes the expectations formation mechanism of non-specialized, or retail, cross-markets investors (see, e.g. Cutler et al. 1990, Greenwood and Shleifer 2014 and Koijen et al. 2015).

Our first finding is that survey predictions tend to be extrapolative across commodities

¹For instance, futures-based forecasts for the Oil price play a role in the policy decision making process at the ECB, see e.g. Svensson (2005), at the Federal Reserve Board, see e.g. Bernanke (2004), and at the International Monetary Fund, see e.g. IMF World Economic Outlook 2005.

²Throughout the paper we use the terms *risk premium* and *expected payoff* interchangeably. In fact, all these terms identify a payoff expected at time t as a compensation for a risk which materializes at maturity $t + h$. Differently, a realized payoff, or realized risk premium, couples the risk premium with any unanticipated deviation of the future spot price from the expected future spot price.

³See, e.g. Keynes (1930), Hicks (1939), Kaldor (1939), Working (1949), Brennan (1958), Hsieh and Kulatilaka (1982), Fama and French (1987), Fama and French (1988), Gorton et al. (2013), Singleton (2014), Szymanowska et al. (2014) and Bakshi et al. (2015) just to cite a few.

and time horizons. Based on this evidence, we hypothesize that investor expectations are the results of an adaptive learning scheme in which expected future spot prices are revised in line with past prediction errors and are affected by changes in aggregate demand. In order to test such hypothesis, we first postulate the Perceived Law of Motion (PLM, henceforth) of commodity spot prices starting from an extended version of a market model with inventory speculations, which includes both predictable changes in aggregate demand and the presence of a futures market.⁴ In this setting, the learning dynamics implies that past forecast errors can potentially affect the extent to which expectations on future spot prices are revised. We compare the model-implied expectations with the average analysts' forecasts. Although with differences across commodities, our time series of adaptive expectations are consistent with the survey-based point forecasts from two to four quarters ahead.

The possibility to replicate observable investors' expectations is key for our purpose, as it allows to approximate the time-varying (ex-ante) risk premia implied by investors' beliefs for a reasonably long sample period. In order to understand the driving factors for the dynamics of risk premia, we compute the expected payoffs by taking the difference between the futures price (as of date t) for delivery at time $t+h$ and expectations at time t on future spot prices at time $t+h$. Our adaptive forecasts generate time varying risk premia similar to the ones filtered by using the survey expectations across alternative horizons and different commodities, as indicated by an average correlation between the two approaches for the overlapping sample ranging from 0.6 to 0.89 across commodities and maturities.

Reconciling the evidence on time-varying risk premia and the potential underlying

⁴We assume that shocks to aggregate supply are conditionally i.i.d. This assumption can be relaxed at the cost of having some reliable empirical proxy for aggregate supply shocks for agriculturals, e.g. Corn, and precious metals, e.g. Silver, to be used in modeling the dynamics of expected spot prices. Recently, Gilje et al. (2015) proposed a framework to identify the effect of shocks in the supply of Shale Oil on the aggregate stock market. A similar procedure would be interesting to be implemented in other commodity markets although might be prohibitive for precious metals and agriculturals. Also, while the i.i.d. assumption for supply shocks can be questionable for energy or industrial commodities such as oil and copper, the same assumption can be a fair approximation of supply shocks in agriculturals and precious metals, e.g. "harvest" can be thought as i.i.d. and storage of, say, corn is temporally limited.

risk factors that drives such dynamics poses significant challenges. First, the sensitivity of risk premia with respect to each risk factor is not constant over time. For example, the relative impact of emerging markets increased in the recent past mainly driven by the increasing weight of China on global economic growth. Similarly, a recent financialization of commodity markets and the current regime of zero nominal short term interest rates represent changes in the perception of risk for commodity investors which, by definition, were not considered ten years ago (see, e.g. Cheng and Xiong 2014). If so, it is fair to assume that the coefficients on the economic determinants of risk premia are changing over time. Second, the model relevant to understand the sources of time variation in commodity risk can be subject to structural changes. For instance, the set of economic variables that affect risk premia on WTI Oil is potentially different before and after periods of slowdown in the global economic growth or a radical change in the predominant monetary policy. This suggests that for $m = 1, \dots, M$ set of explanatory factors one have to consider all of the possible 2^M model specifications at each time $t = 1, \dots, T$. Even in relatively simple regression-based analysis with a limited number of variables, it would be infeasible to investigate pricing determinants by simply going through all of $K = 2^{MT}$ combinations.

To address these issues, we estimate a dynamic linear regression model, which features random-walk time-varying betas, for each of the 2^M models and use this approach to investigate ex-post the relative importance of each risk factor, as proxied by the marginal models probabilities computed at each time t . As far as the estimation strategy is concerned, we opt for a conjugate Bayesian framework, which allows to obtain robust finite-sample estimates that flexibly and explicitly accounts for different sources of uncertainty: uncertainty in the relative importance of covariates, uncertainty in the estimated coefficients and their degree of time-variation, and idiosyncratic risks.

Our main empirical results show that risk premia are time-varying, both across different horizons and commodities, and their dynamics is predominantly driven by risk sharing channels and the changing nature of commodity market participants, as proxied

by Open Interest (OI henceforth) and Hedging Pressure (HP henceforth), as well as by Value and time-series Momentum factors. This is true after controlling for other conditioning variables that have been recently studied in the empirical finance literature, e.g. changes in inventories. More generally, we provide evidence on the heterogeneity in the relative importance of different economic risk factors in the dynamics of risk premia.

The role of OI has been outlined by Hong and Yogo (2012). They show that the number of futures contracts outstanding represents a reliable signal of future economic activity and therefore could predict futures movements in asset prices. The role of HP in commodity risk premia has been first introduced by Keynes (1930) and Hicks (1939) in their Theory of Normal Backwardation, where a risk premium is accrued to speculators as a reward for bearing the risk of fluctuations in the price of spot commodity which hedgers sought to transfer.⁵ More recently, Carter et al. (1983), Bessembinder (1992), De Roon et al. (2000), Acharya et al. (2010) and Basu and Miffre (2013) show that HP can be interpreted as a systematic risk factor in the cross-section of commodity risk premia. Similarly, Hamilton and Wu (2014) show that risk sharing mechanism can give rise to an affine term structure model that explains the dynamics of futures prices. Our results from a full-scale dynamic regression model confirm the importance of HP in understanding the nature of commodity risk.⁶ Furthermore, the evidences from the marginal posterior model probabilities show that time-series Momentum plays a significant role in the dynamics of commodity risk premia, consistent with Asness et al. (2013) and Szymanowska et al. (2014).

Finally, we show that the role played by emerging economies, as proxied by the MSCI Emerging Market Index (MXEF), has sensibly increased over time for Oil and Copper. A possible explanation is the presence of spillover effects due to the increasing weight of the emerging economies in the global economic growth, especially the Chinese economy.

⁵Hirshleifer (1990) further extended the theory in a general equilibrium setting, linking backwardation to lower levels of HP and contango, which is the mainstay of the other competing theory of commodity risk premia proposed by Working (1949).

⁶Recently Gorton et al. (2013) argue that the null hypothesis that HP is an important determinant of commodity risk premia should be rejected. However, they effectively look at ex post risk premia on commodity futures.

As a matter of fact, although the direct impact of Chinese equity valuations is relatively low due to moderate foreign investments, financial turbulence in this stock market tend to be associated with concerns about a global economic slowdown.⁷

This paper builds on a number of works in the expectations formation literature such as Nerlove (1958), Evans and Honkapohja (2001), Sargent (2002), Sargent and Williams (2005), and Malmendier and Nagel (2015), who consider a model of adaptive learning to explain the dynamics of expectations on inflation and more general macroeconomic outcomes. Differently from them, we focus on commodity markets and use adaptive expectations as a tool to extract the real quantity of interest, namely the ex-ante risk premia across different products and time horizons. Conceptually, our work is also related to recent research that posits trading activity is the result of an adaptive process in which hedgers and speculators learns about economic fundamentals, both from public information and market prices (see, e.g. Singleton 2014).

Finally, this paper contributes to a growing literature that looks at survey forecasts as a way to approximate investors' expectations formation and test economic hypothesis across asset classes (see, e.g. Greenwood and Shleifer 2014, Gennaioli et al. 2015, and Koiien et al. 2015). Greenwood and Shleifer (2014) show that investors' expectations on future stock market returns are inconsistent with a rational expectations representative agent model of returns. Koiien et al. (2015) extended these results by investigating the implications for returns predictability and excess volatility across three asset classes: equity, fixed income and currency. They show that survey expectations tend to predict price changes but with the wrong sign and display a extrapolative features being influenced by past price levels and returns. Differently from them, we focus on commodity markets and formally postulate an adaptive learning scheme for future commodity spot prices which is consistent with a "learning from past errors".

The rest of the paper is organized as follows. Section 2 discusses the difference between

⁷China itself is the second largest economy and the second largest importer of both goods and commercial services.

ex-ante and realized risk premia, while Section 3 introduces the time series of investors' expectations and the data used in the empirical analysis. Next, in Section 4 we test the null hypothesis of adaptive expectations for the average investors' forecasts. The analytical framework with learning from past experience and the econometric methodology we use to understand the dynamics of the ex-ante risk premia are presented in Section 5. Finally, Section 6 reports our empirical results and Section 7 concludes.

2 Ex-Ante vs. Realized Risk Premium

In this section we briefly review the theory of normal backwardation of Keynes (1930) and Hicks (1939) and motivate the importance of using investors' expectations to identify commodity risk premia with a special emphasis on the dichotomy ex-ante vs. realized payoffs.

Let S_t and $F_t^{(h)}$ denote the spot and futures prices of a given commodity for delivery at time $t + h$. Define the basis to be the difference between the current futures and spot prices $F_t^{(h)} - S_t$. The theory of normal backwardation links the price of futures contracts and the commodity spot prices on the basis of risk insurance and appetite from hedgers and speculators. In this theory, the basis can be decomposed in two components;

$$F_t^{(h)} - S_t = E_t [\Delta S_{t+h}] + \underbrace{F_t^{(h)} - E_t [S_{t+h}]}_{y_t^{(h)}} \quad (1)$$

with $E_t [S_{t+h}]$ the market aggregate expected price at time $t + h$, $y_t^{(h)}$ the expected payoff at time t for the futures position, and $E_t [\Delta S_{t+h}]$ the expected change in spot prices. According to Keynes (1930), at least unconditionally, the risk premium is negative, which implies that hedgers are of net short and therefore willing to pay a premium to speculators that provide insurance against the risk of fluctuations in the spot market.⁸ Another

⁸Notice that equation (1) does not rule out the possibility of commodity markets to be in contango as current futures prices could still be lower than spot valuations.

interpretation of the basis is provided by the theory of storage of Kaldor (1939), Working (1949), and Brennan (1958), which derives a fundamental relationship between spot and futures prices based on an opportunity costs of forgone returns from a risk-less security and a convenience yield. Fama and French (1988) pointed out these two theories are not mutually exclusive. Indeed, variations in the ex-ante risk premium or the expected change in spot prices can be correlated with changes in interest rates, storage costs or convenience yields. Also, Szymanowska et al. (2014) showed that there is a mapping between $y_t^{(h)}$, which they call “spot premium”, and the risk premia implied by the theory of storage.

Few comments are in order. First, to the extent to which one want to investigate risk sharing and insurance mechanism in commodity markets, equation (1) offers the ideal setting since isolates the risk premium component in futures prices directly from investors’ expectations. Second, the risk premium $y_t^{(h)}$ represents the *ex-ante* payoff the average market participants expect from investing in a futures and holding the contract until maturity. This is fundamentally different from using the *realized* payoff of the contract, which also incorporates unexpected price changes in the spot market. Figures 1 makes this case in point.

[Insert Figure 1 about here]

We consider as an example a situation in which the price of the commodity at time t is equal to 50\$ and market expectations for the future spot price at time $t + h$ are equal to 47\$, i.e $E_t[S_{t+h}] = 47$. Let us assume also that in order to make investors willing to enter the market the current price of a futures contract at time t for delivery at time $t + h$ is equal to 43\$, which means futures are sold at a discount. The difference between the futures price and $E_t[S_{t+h}]$ at time t implies that the expected payoff of a futures short position, i.e. the risk premium, is equal to 4\$.

Top panel shows the case in which the commodity is effectively traded at 47\$ at

time maturity. In this case, given the no-arbitrage assumption that futures contracts at expiration trade at the spot price, and given unexpected price changes are zero, the ex-ante and the realized payoffs are equivalent. Now, consider instead a situation in which investors systematically make errors in forecasting future spot price realizations (see, e.g. Alquist and Kilian 2010 for a complete discussion on the predictability of nominal spot prices). The bottom panel shows an example in which the commodity is traded at a lower price of 45\$ at time $t + h$ on the spot market, which implies a forecast error $\hat{S}_{t+h|t} - S_{t+h} = 2$. Given that the value of the futures contract at maturity coincides with the spot at time $t + h$ the realized payoff is 2\$. In this case, ex-ante and the realized risk premia differ by the amount of the unexpected price realization.

Figure 1 shows that ex-ante and realized risk premia differ to the extent that investors' misjudge the level of future spot prices over time. The null hypothesis that investors' expectations are conditionally unbiased is mainly an empirical question that can be verified, at least preliminarily, by using average survey forecasts. Figure 2 shows the time series of the expectations error $\hat{S}_{t+h|t} - S_{t+h}$ for two different horizons, i.e. $h = 2, 4$ quarters ahead, and two alternative commodities, i.e. WTI Crude Oil and Silver. The aggregate forecast $\hat{S}_{t+h|t}$ is proxied by the average expected price from the Bloomberg survey of professional analysts. A complete discussion on how the survey is collected and structured, as well as a description of the data, is provided in Section 3.

[Insert Figure 2 about here]

Figure 2 makes clear the existence of a systematic and time-varying expectation error in predicting future spot prices up to four quarters ahead for both WTI Crude Oil and Silver. Unsurprisingly, unexpected depreciation for crude oil occurred over the great financial crisis of 2008/2009 and the recent collapse of late 2014/beginning of 2015. Similarly, unexpected appreciation of Silver occurred in the recovery of financial markets after 2009, consistent with the idea that the value of precious metals tend to be negatively

correlated with the business cycle.

To summarize, Figure 2 provides evidence that throughout the sample, the null hypothesis of either small or constant conditional unexpected price change can be sensibly rejected. Therefore the assumption that realized commodity risk premia can be used as a perfect substitute of ex-ante expected payoffs, which is the key component of the theory of normal backwardation, is not supported by the empirical evidence.

3 Data Description

We cover four main commodity futures which represent the energy, metals, and agricultural markets. We focus on these commodities as they are the most traded consumption commodities with the most complete sample of survey data. In this respect, the choice of the commodity to be included in the analysis is mostly dictated by the length of the corresponding survey and the number of professional analysts responding.⁹

Data are obtained from different resources. Futures prices data on Crude Oil (WTI) are from the New York Mercantile Exchange (NYMEX), prices are in U.S. dollars per barrel. Futures on Silver and Copper are obtained from the Commodity Exchange (COMEX). Silver is quoted in U.S. dollars per troy ounce while Copper is quoted in U.S. cents/pound. We convert the price of Copper futures contracts to USD/tonne to match the measurement unit of the survey forecasts, that instead refer to the London Metal Exchange (LME). Corn futures prices are from the Chicago Board of Trade (CBOT) with price quotation in USD cents per bushel. As in Szymanowska et al. (2014) the spot price for each commodity is approximated by using the nearest contract to maturity, and the futures price is the price of the next to the nearest futures contract for a given maturity.

We define the futures price at time t with average quarterly time to maturity h as $F_t^{(h)}$, where the definition of the average time to maturity is consistent with the average

⁹Including other commodities would come at a cost of using averages of few respondents or very short time series, e.g. from 2013.

forecasting horizon for the survey expectations. For example, the price of a future for delivery at four quarters ahead is computed interpolating the prices of the contracts between 10 and 12 months ahead. The sample period is monthly 01:1993-01:2016.

3.1 Measuring Investors' Expectations

We obtain individual price forecasts for different commodities and horizons from the Bloomberg's commodity price forecasts database. This database contains analysts' price expectations at multiple quarterly forecasting horizons and across diverse commodities from 2006 to 2016. This survey exclusively includes operators highly specialized in commodity markets mainly from banks and consulting firms. Yet, participants are asked to provide a point forecast on the average quarterly commodity price for a specified futures contract. A deep knowledge of commodity markets from the survey respondent and the clarity of the prediction allows to reduce the effect of potential biases, quality homogeneity issues, and limited information processing, which generally characterizes directional forecasts of non-specialized, or retail, cross-markets investors.¹⁰

There are two main objection on the use of survey expectations in empirical studies; first, the respondent may misunderstand the question which, for instance, can be posed in a simple directional way, e.g. do you expect prices increase, decrease or stay roughly constant. Second, respondent may intentionally hide their true expectations for strategic purposes. Our survey mitigates the effect of both of these sources of error as (1) the question is about giving a clear point estimate for future spot prices, and (2) survey participants are professional market participants who possibly have payoffs that directly depends on the precision of their estimates.¹¹

The database allows to retrieve for each analyst the historical price expectations and

¹⁰More specifically, the fact that only operators specialized in commodity markets are being surveyed increase the proportion of truly "informed" agents in the survey population compared to a case in which cross-market analysts are being surveyed (see, e.g. Cutler et al. 1990).

¹¹As we take the cross-sectional average of investors' forecast as our proxy for market expectations, any non-coordinated strategic bias/error at the individual level is mitigated (see, e.g. Bernhardt and Kutsoati 1999, Hong et al. 2000, and Hong and Kubik 2003)

the related publication date. The use of the survey for operational purposes involves some challenges as the quarterly analysts' forecasts submission are recorded daily and not evenly spaced in time. Analysts provides forecasts on spot prices in different days for fixed common maturities that corresponds to calendar quarters, namely survey respondents provide discontinued fixed-calendar maturity quarterly expectations.¹² In order to perform a standard time series analysis we need to transform analysts' responses in continued constant-horizon price forecasts. We aggregate responses at the monthly frequency in order to reduce the difference in the market information available between early and late submitters within a month. We then compute the forecasting horizon with respect to the end of the month of the last month in the quarter which is the object of the prediction. More specifically, at each point in time, we stack the forecasts with residual life that belongs to the following groups: 4 to 6; 7 to 9 and 10 to 12 months, then we approximate the aggregate expectations as the cross-sectional average prediction across analysts and time-horizons. In order to reduce moving average effects in the synthetic average expectations we discard the horizon between one and three months as the analysts take into account what has been the realized price over the first part of the quarter generating nowcasting dynamics which makes hard to disentangle the role of expectations versus current information in the dynamics of short-term risk premia.¹³

3.2 Explanatory Factors for Risk Premia

In order to study the sources of time variation in commodity risk premia, we collect diverse determinants that are considered to capture alternative sources of risk. As a

¹²Other surveys can be used to approximate the average investors' expectations, such as for instance the Energy & Metals Consensus Forecasts from Consensus Economics. However, Consensus forecasts do not count for agricultural and is lower frequency being collected every other month after April 2012 and on a quarterly basis before that date. Also, while Consensus forecasts are "contaminated" with forecasts from economists working in institutions that do not necessarily participate in commodity markets, Bloomberg's panel of respondents is uniquely composed by professional analysts' affiliated with banks and consultancy companies, which make the survey more suitable in approximating expectations by actual market participants.

¹³Also, contracts close to expiration are typically illiquid in commodity markets as futures traders do not want to take the risk of a physical delivering of the underlying.

proxy for the level of global commodity demand we use the index of world industrial production published by the Netherlands Bureau for Economic and Policy Analysis. This index aggregate information on industrial production from 81 countries worldwide, which account for about 97% of the global industrial production. The aggregate series starts in January 1991 and relate to import weighted, seasonally adjusted, industrial production.

Fluctuations in the global supply-demand imbalance for each commodity are captured by using inventory stocks. We collect data on Copper and Crude Oil inventories from the London Metal Exchange (LME) and Energy Information Administration (EIA) respectively. Copper inventory levels are recorded daily from June 1974 and relate the previous day closing stock of commodities held in LME. Crude Oil inventories are recorded weekly by the EIA and published monthly since January 1945. Stocks levels are measured in thousands of barrels and exclude strategic petroleum reserves.¹⁴ For Corn inventories, we use the U.S. ending stocks reported in thousands of metric tons. The time series is sampled at monthly frequency using the inventory level reported on the last business day of the month. Data are recorded from the United States Department of Agriculture (USDA) from January 1993. As far as Silver is concerned, we omit the inventory level variable as, similar to other precious metals, a considerable part of the existing reserves is privately held and therefore not reported in official statistics. In the regression specification we use the growth rate of inventories as the levels show the presence of a stochastic time trend.

Exchange rates is also a relevant risk factor as commodity trading takes place usually in U.S. Dollars, making the exchange rate a key factor for both producers and consumers, as it directly affect profits and costs denominated in domestic currency. In order to account for the risk of appreciation and depreciation in the U.S. Dollar, we include as additional risk factor the growth rate of Federal Reserve U.S. trade weighted exchange rate index, normalized to be equal to one hundred in March 1973. Together with exchange

¹⁴We include in the level of inventories those domestic and Customs-cleared foreign stocks held at, or in transit to, refineries and bulk terminals, and stocks in pipelines. Stocks include an adjustment of 10,630 thousand barrels (constant since 1983) to account for incomplete survey reporting of stocks held on producing leases.

rates, interest rates represents a key determinants of the cost of inter-temporal arbitrage strategies. For the analysis on the two- and four-quarter ahead risk premia we use the monthly LIBOR rate with 6 and 12-month maturity, respectively, matching the horizon of the hypothetical investment in the commodity futures and the risk-free asset.

Momentum in commodity futures has been widely documented in the empirical finance literature, e.g. Erb and Harvey (2006), Miffre and Rallis (2007), Asness et al. (2013) and Szymanowska et al. (2014) among others. Furthermore, we include a measure of time series momentum among the risk factors in our analysis as it can be directly linked to asset demand by momentum traders as shown in Cutler et al. (1990) and Kang et al. (2014). We construct time-series Momentum as the return over the past 12 months skipping the most recent month on each commodity future. In addition, we include a Value factor which is assumed to be intimately interrelated to the dynamics of commodity risk premia, as it affects the propensity of market participants to trade in backwardation or in contango and can proxy the trading activity of speculators following mean-reversion type trading strategies. We follow Asness et al. (2013) and define Value as the average of the log spot price from 4.5 to 5.5 years ago, divided by the most recent spot price, which is essentially the negative of the spot return over the last 60 months. In addition to time-series Value and Momentum, we also directly consider returns on the Standard and Poor's 500 and the MSCI Emerging Markets indexes as proxy for financial risk. Beyond direct effects on financial flows, we incorporate stock indexes as they likely capture spillover effects to the real economy. For instance, although shocks to equity valuations in emerging markets are possibly of moderate effect, financial turmoil in emerging economies are typically associated with uncertainty about global economic growth.

Finally, to capture risk sharing channels in the economic mechanism that drives commodity risk premia we consider the growth rate of OI and HP as variables to capture the changing nature of futures market participation, (see e.g. Baker and Routledge 2011 and Singleton 2014). OI is measured as the total number of outstanding contracts that are held by market participants at the end of the month. An outstanding contract is when a

seller and a buyer combine to create a single contract. For each seller of a futures there must be a buyer of that contract, therefore to determine the total OI for any given market we need to know the totals from one side or the other, buyers or sellers, not the sum of both. Increasing OI means that new cash is flowing into the marketplace, while declining activity means that the market is liquidating, which can be interpreted as a signal of a price turning point. HP represents a measure of net positions of hedgers in commodity futures markets which is the result of risks that market participants do not want, or cannot trade because of market frictions, information asymmetries and limited risk capacity (see, e.g. Hong and Yogo 2012 and Kang et al. 2014). We compute the level of HP for different commodities as the net excess in short futures positions by commercial traders, i.e. short minus long positions, divided by the amount of outstanding contracts. The data on commercial traders futures positions are from the Commodity Futures Trading Commission (CFTC).

4 Testing Extrapolative Expectations

At the outset of the paper we argue that one of the key issue we face is outline how best to describe the beliefs formation mechanism about future spot prices proxied by survey forecasts. Our aim is not to develop new hypothesis, but rather test the null hypothesis of adaptivity in the expectations formation process. The rational expectations model as proposed by Muth (1961) has obtained general acceptance as the benchmark model; it implies that decision makers know the true underlying model such that subjective beliefs are set to be equal to their objective counterparts. The assumption of complete knowledge of the data generating process is however fairly restrictive. This has led many researchers to propose a number of simple formulations based on weaker forms of the rational expectations that allow for model instability, uncertainty, and learning (see, e.g. Hsieh and Kulatilaka 1982, Frenkel and Froot 1987, Marcet and Sargent 1989, Evans and Honkapohja 2001, and Sargent 2002 in a general framework, and Singleton 2014 and

Sockin and Xiong 2015 relatively to commodity markets).

In their most general formulations, the model for adaptive expectations have a limited number of testable implications; the most important of which is the impact of past information on current forecasts (see, e.g. Frenkel and Froot 1987 and Pesaran and Weale 2006). We test for a general rule of updating by estimating the impact of current prices on expectations. Let $E_t [\Delta S_{t+h}]$ represents the investors' expectations at time t for a change in the future spot price from t to $t+h$. To test adaptivity we first estimate the following regression model;¹⁵

$$E_t [\Delta S_{t+h}] = \alpha + \beta \Delta S_t + e_t, \quad \text{for } h = 2, 3, 4, \quad \text{quarters}, \quad (2)$$

with $\Delta S_t = (S_t - S_{t-h})$ representing past changes in spot prices. We use past spot prices as, once become observable, they are assumed to summarize all the relevant current information which is readily available to professional analysts (see, e.g. Sockin and Xiong 2015). The regression equation (2) states that if a commodity has been recently depreciated, then it will be expected to depreciate in the near future as well. Strong rationality would imply the null hypothesis that there is no “learning” from past information, i.e. $H_0 : \beta = 0$. Panel A of Table 1 shows the results.

[Insert Table 1 about here]

Interestingly, the slope coefficients are all negative and strongly significant meaning that a recent depreciation of a commodity leads to an optimistic view on future spot prices, and vice versa. Such dynamics does not rule out the possibility of having positive autocorrelation in investors' expectations. Building on this result, we now test the further restriction that expectations are adaptive. Adaptive learning is the most prominent form of extrapolative expectations formation process (see, e.g. Nerlove 1958, Evans and

¹⁵We estimate the model by OLS with GMM corrected standard errors to account for autocorrelation and heteroschedasticity in the residuals.

Honkapohja 2001, Cho et al. 2002, Sargent 2002, Williams 2003, Sargent et al. 2004, Sargent and Williams 2005 and Malmendier and Nagel 2015 to cite a few). Under this model investors adjust their expectations in line with past prediction errors. In general, adaptive expectations need not be informationally efficient, and forecast errors can be serially correlated. We test the adaptive expectations hypothesis by regressing the expected price change on the lagged survey prediction error;

$$E_t [\Delta S_{t+h}] = \mu + \delta (E_{t-h} S_t - S_t) + \nu_t, \quad \text{for } h = 2, 3, 4, \quad \text{quarters}, \quad (3)$$

Panel B of Table 1 shows the results. The slope coefficients are positive and statistically significant across forecasting horizons and commodity markets. This implies that investors, on average, place positive weight on previous prediction errors. To summarize, investors' expectations on future spot prices are not static; in fact, the elasticity of the expected future spot prices with respect to past forecasting errors is positive and significant. Notably, the support for a form of adaptivity in the expectations formation process does not depend on the prediction horizon and the specific commodity market.

5 A Model of Adaptive Learning

Section 3 gives some insight on the nature of investors' expectations on commodity future spot prices. The candidate forecasting rule we examine have close resemblance to an adaptive learning scheme. To set up an analytical framework we need to specify the perceived law of motion that investors are trying to recursively estimate. A natural starting point is an extended Muth (1961)'s hog cycles "cobweb" model with the addition of a futures market (see, e.g. Turnovsky 1983, Kawai 1983, and Beck 1993). The market behaviour is characterized by an infinite horizon, discrete time model with both a spot and a futures market clearing condition that holds in each time period. By including a futures market we assume that suppliers, buyers and inventory holders now hedge their commodity positions by trading on futures, and so we explicitly consider the effect of

hedging in the decision-making process that leads to the PLM of spot prices. Also, we depart from a standard framework by allowing demand shocks to be predictable and potentially persistent.¹⁶ A unique reduced-form rational expectations equilibrium is defined as (see Appendix A)

$$S_{t+1} = \phi_0 + \phi_1 S_t + \phi_2 z_t + \eta_{t+1}, \quad (4)$$

with S_{t+1} the market commodity price at date $t + 1$, z_t the level of aggregate demand at time t , and η_{t+1} an unobservable random shock. Notably, a similar solution would be obtained by assuming market segmentation between spot and futures as originally proposed in Muth (1961)'s model, except that spot prices now depends also on aggregate demand.¹⁷

Figure 3 confirms that changes in aggregate demand represent an important source of fluctuations in commodity prices. In particular, this figure shows the year-on-year changes in the (log of) commodity spot prices (blue line) and aggregate demand as proxied by an index of world industrial production published by the Netherlands Bureau for Economic and Policy Analysis (magenta line).

[Insert Figure 3 about here]

With the only partial exception of Corn (bottom-left panel), which is an agricultural commodity and therefore less sensitive to business cycles, changes in spot commodity prices tend to align with changes in aggregate demand, especially after the beginning of

¹⁶In the original Muth (1961) framework demand shocks that induce changes in inventories quickly revert to their long-run equilibrium values. In that respect, inventories adjustments are perceived to have a stabilizing effect on prices. However, as recently showed by Dvir and Rogoff (2010) quick adjustments in inventories to demand shocks cannot explain the persistence in the time series of commodity prices and volatilities.

¹⁷One may also specify a model in which expectations of future aggregate demand rather than current levels enter in the equilibrium outcome. As far the unique reduced-form solution in Eq. (4) is concerned, the two things are essentially equivalent. Aggregate demand is specified as a potentially persistent AR(1), i.e. $z_{t+1} = bz_t + e_{t+1}$. This implies that $E_t z_{t+1} = bz_t$, which means that the structural coefficient b of the actual law of motion, although cannot be identified, is embedded in the reduced-form parameter ϕ_2 of the perceived law of motion.

2000s. Similar to this, Kilian and Hicks (2013) show that unexpected economic growth sensibly affect the dynamics of spot prices in the oil market. As a result, since in the adaptive learning framework expectations are revised in line with past prediction errors based on available information, aggregate demand affects investors' expectations as well.¹⁸

The key assumption to introduce learning is that agents do not know true values of the parameters $\phi = (\phi_0, \phi_1, \phi_2)$ and expectations are instead formed on the basis of current observations plus a constant $X_t = (1, S_t, z_t)$. Predictions of the parameters are updated over time; we follow Cho et al. (2002), Sargent (2002), and Sargent and Williams (2005) and explicit agents' recursive estimates in terms of a Bayesian prior that describes how coefficients drift at each time t ;¹⁹

$$\begin{aligned} S_{t+1} &= \phi'_{t+1} X_t + \eta_{t+1}, & \text{with } \eta_{t+1} &\sim N(0, \sigma^2), \\ \phi_{t+1} &= \phi_t + \xi_{t+1} & \text{with } \xi_{t+1} &\sim N(0, \Omega), \end{aligned} \quad (5)$$

with $\phi_t = (\phi_{0,t}, \phi_{1,t}, \phi_{2,t})'$ and $X_t = (1, S_t, z_t)'$. The shock η_{t+1} is uncorrelated with ξ_{t+1} , and $\Omega \ll \sigma^2 I$. The innovation covariance matrix Ω governs the perceived volatility of increments to the parameters (see, Sargent and Williams 2005). Agents' recursive optimal estimate of ϕ_{t+1} conditional on information available up to time t , $\gamma_{t+1} = \hat{\phi}_{t+1|t}$ are provided by a Kalman filter recursion;

$$\begin{aligned} \gamma_{t+1} &= \gamma_t + K_t (S_{t+1} - \gamma'_t X_t), \\ R_{t+1} &= R_t - \frac{R_t X_t X'_t R_t}{X'_t R_t X_t + 1} + \sigma^{-2} \Omega, \end{aligned} \quad (6)$$

where $K_t = R_t X_t (X'_t R_t X_t + \sigma^2)^{-1}$ determines the degree of updating of agents' beliefs

¹⁸An alternative explanation of why future spot prices can depend on information about economic fundamentals is provided by Singleton (2014), who argue that differences in beliefs can generate persistence in the dynamics of commodity spot prices. In fact, relatively to the Oil market, Singleton (2014) pointed out that "Perhaps more plausible is the assumption that participants [...] learn about the true mapping between changes in fundamentals and prices by conditioning on past fundamentals and prices".

¹⁹This random walk specification for the evolution of the parameters is widely used in applied work in macroeconomics and finance, e.g. Frühwirth-Schnatter (1994), West and Harrison (1997), Stock and Watson (1998), Primiceri (2005), Hansen (2007), and Leduc et al. (2015).

when faced with an unexpected commodity spot price $S_t - \gamma'_t X_t$. This learning dynamics represents a generalization of adaptive learning with constant gain. Although learning is perpetual in our model, the recursive estimates (6) converge to a steady-state solution for a given initial condition of the state covariance matrix Ω (see, Hamilton 1994 Proposition 13.1, pag. 390). We use the subscript $t + h|t$ to indicate a forecast for the $h > 0$ horizon made using information available to agents' at time t . The market price expected to prevail at time $t + 1$ given the information available through the t th period is obtained as

$$\hat{S}_{t+1|t} = \gamma'_{t+1} X_t, \quad (7)$$

Multi-period forecasts $\hat{S}_{t+h|t}$ are obtained by iterating forward the time- t estimates of the model parameters. Relatively simple recursive learning schemes as (6) are widely motivated in the adaptive learning literature by the fact that agents face constraints in cognitive abilities that limit their possibility to observe the true equilibrium parameters and use optimal, e.g. perfect foresight, forecasting rules (see, e.g. Carceles-Poveda and Giannitsarou 2008, Adam and Marcet 2011 and Malmendier and Nagel 2015).

Conditional forecasts from Eq. (6) allows to extract risk premia across predictive horizons and commodities. More specifically, let $\hat{S}_{t+h|t}$ be the model-implied expected future spot price of a given commodity at time t for the horizon $t + h$. The risk premium can be extracted from the price of a future contract at time t for delivery at time $t + h$, $F_t^{(h)}$, as;

$$y_t^{(h)} = F_t^{(h)} - \hat{S}_{t+h|t}, \quad (8)$$

Eq. (8) implies that it is not necessary for the investors to have private information for their actions to affect commodity risk premia. As a consequence, the latter may depend on the nature of agents' learning mechanism based on common signals.

5.1 Econometric Framework

Linking the time-variation of ex-ante risk premia to a set of observable risk factors poses two main challenges. First, the exposure of risk premia to a given economic variable is not necessarily constant over time. Consider for instance the increasing weight of China for the global economy. This possibly generates spillover effects due to shocks in Chinese stock valuations more sizable. Second, the optimal set of economic risk factors is arguably unknown ex-ante and potentially changes over time. As a result, for $m = 1, \dots, M$ set of explanatory factors one have to consider all of the possible 2^M model specifications at each time $t = 1, \dots, T$. Even in a relatively simple regression-based analysis with a limited number of variables it would be infeasible to investigate pricing determinants by simply going through all of $K = 2^{MT}$ combinations.

We use a dynamic regression modeling framework that explicitly allows for a time variation in the relationship between the risk premia $y_{t+1}^{(h)}$ over the interval $(t, t+1]$ and the realizations of the explanatory factors observed at time t , Z_t . This substantially reduces the curse of dimensionality as we now have to uniquely focus on exploring all of the possible 2^M models. In the following, for the ease of exposition we drop the superscript (h) that indicates the expectations horizon from the notation. Observable risk factors have a subscript that indicates the time at which they are known. Denoting these by $Z_{k,t}$ for $k \in K$, our set of models can be written as;

$$y_{t+1} = Z'_{k,t} \theta_{k,t} + v_{k,t+1}, \quad v_{k,t+1} \sim N(0, H_k), \quad (9)$$

$$\theta_{k,t+1} = \theta_{k,t} + \varepsilon_{k,t+1}, \quad \varepsilon_{k,t+1} \sim N(0, W_k), \quad (10)$$

The vector $\theta_{k,t}$ consists of unobservable, time-varying, regression coefficients that are specific for the model $k \in K$ (see West and Harrison 1997 for more details on dynamic linear models).²⁰ More prominently, the state-space formulation allows to explicitly account for

²⁰We specify the relationship between risk premia and economic risk factors in a predictive sense. However, our purpose is not to “predict” future expected payoffs in a pure out-of-sample sense.

different sources of uncertainty: uncertainty in the relative importance of predictors, uncertainty in the estimated coefficients and their degree of time-variation, and uncertainty on the “right” set of predictors. Also, the specification (9)-(10) allows to flexibly consider alternative model restrictions; for instance, if the state variance W_k is set to zero, the regression coefficients are made constant over time, which in turn reflect a standard unconditional least squares regression analysis. In that respect, the magnitude of W_k really gives the size of variability in the exposure of risk premia to factors.

A key advantage of our dynamic specification is that we can investigate the relative probability of inclusion for each risk factor in the time-series of ex-ante risk premia at each time t . Let $I_t = \{1, 2, \dots, K\}$ denote which model applies at each time period and $y^t = (y_1, \dots, y_t)$ the time series of expected risk premia for a given maturity. The posterior probability that the model $k \in K$ applies, conditional on the information on risk factors and premia at time t , can be computed as

$$\pi_{t|t,k} = \frac{\pi_{t|t-1,k} p(y_t | y^{t-1}, I_t = k)}{\sum_{l=1}^K \pi_{t|t-1,l} p(y_t | y^{t-1}, I_t = l)}, \quad (11)$$

with $p(y_t | y^{t-1}, I_t = k)$ be the marginal predictive density for the model $I_t = k$ evaluated at y_t and given past information y^{t-1} , and $\pi_{t|t-1,k} = p(I_t = k | y^{t-1})$ represents the conditional probability of model $k \in K$ (see Appendix B for details on how these quantities are computed).

Few comments are in order; first, in this framework we are interested in understanding the sources of risk that can explain risk premia rather than provide a framework to do optimal recursive forecast. In that respect, our approach is suboptimal for real-time out-of-sample forecasting as one should also compute the transition mechanism across models in order to achieve recursive model averaging or selection in a pure out-of-sample fashion. Second, the observational H_k and state variances W_k are estimated using the whole sample of observations of risk premia and factors. As such, although model probabilities

are estimated dynamically, the structural variances are considered constant over time.²¹ Third, we acknowledge that the state dynamics (10) implies that the elasticity of risk premia to given factors follows a random walk and that, at least asymptotically, this causes drift to deterministic high or low values of y_t , hence generating non-stationary expected risk premia.²²

Posterior model probabilities $\pi_{t|t,k}$ can be used to rank risk factors in terms of their relative contribution to explain the dynamics of commodity risk premia over time. More specifically, from $\pi_{t|t,k}$ we can compute the marginal probability of including a specific risk factor in the dynamic model at each point in time as;

$$\hat{\pi}_{m,t} = \sum_{k=1}^K \pi_{t|t,k} \mathbb{I}_{\{m \in k\}}, \quad t = 1, \dots, T, \quad \text{and} \quad m = 1, \dots, M, \quad (12)$$

with $\mathbb{I}_{\{m \in k\}}$ an indicator function that takes value one if a given risk factor is included in model $k = 1, \dots, K$ and zero otherwise. More specifically, $\hat{\pi}_{m,t}$ measures the importance of the m th factor in the dynamics of commodity expected risk premia at time t . The marginal probability for each model $p(y_t|y^{t-1}, I_t = k)$ is found by integrating the conditional density $p(y_t|y^{t-1}, I_t = k, \Theta_k)$ over the range of parameters $\Theta_k = (\theta_k^T, W_k, H_k)$. The posterior probability of $I_t = k$ is then updated according to (11).

The sequential model description in (9)-(10) requires that the defining quantities at time t be known at that time. Let $D_{k,0}$ contains the initial prior information about the elasticities and structural variances for the model $I_t = k$. We assume prior information about $\theta_{k,0}$ is vague and centered around the initial hypothesis of no effect of risk factors

²¹However, the framework could be easily extended by using an exponential weighted moving average recursion to obtain dynamic estimates for $H_{k,t}$ and $W_{k,t}$. We leave this for future research.

²²In a separate robustness check we extend Eq. (10) to be a more general AR(1) for the simple case of a model with all regressors. The estimation results show that the state parameters are highly persistence with low conditional variance. In that respect, the random walk assumption represents an attractive approximation because of its parsimony, ease of computation and the smoothness it induces in the estimated sensitivities over time. We share this findings with a large literature on returns predictability that assumes time variation in the predictive coefficients (see, Kilian and Taylor 2003, Ferreira and Santa-Clara 2011 and Dangl and Halling 2012). Similar to our argument they find that assuming parameters are random walks in predicting excess returns we benefit from a substantial reduction of estimation error without effectively increasing the precision in the estimated dynamics in a finite sample.

on premia, i.e. $\theta_{k,0}|D_{k,0} \sim N(c_{k,0}, C_{k,0})$, with $c_{k,0} = 0$ and $C_{k,0} = 10,000$. Also, we assume that the impact of risk factors is highly uncertain and volatile, as captured by an Inverse-Wishart distribution with small degrees of freedom and large scale parameter, i.e. $W_k|D_{k,0} \sim IW(a_{k,0}, A_{k,0})$ with $a_{k,0} = 3$ and $A_{k,0} = 10,000$. As a result we assume that when no historical information on expected risk premia and factors is available, elasticities are mainly driven by idiosyncratic risk as proxied by an Inverse-Gamma distribution with uninformative hyper-parameters, i.e. $H_k|D_0 \sim IG(n_{k,0}/2, n_{k,0}N_{k,0}/2)$ with $n_{k,0} = 0.001$ and $N_{k,0} = 0.001$. Notice priors are constant for all maturities $h = 2, 3, 4$ quarters. In Appendix B we fully describe in full details how parameters are estimated through a Gibbs sampler once historical information on expected risk premia is available.

6 Empirical Results

We first check the consistency of the model-implied expected future spot prices with survey forecasts. For the ease of exposition we report the results for dollar value expectations at maturities $h = 2, 4$ quarters.²³ The sample period is from 12:2006 to 01:2016 for the survey, and is from 01:1995 to 01:2016 for the model-implied expectations. The first 24 months of the model-based expectations are cut as burn-in sample for adaptive learning. Figure 4 reports the results for the WTI Crude Oil and Copper. The red markers represent the monthly observed survey forecast, and the light-blue marker shows our model expectations. The shaded area underlying the overlapping period between the survey and the model represents the difference between the two, i.e. a positive value means the model generates higher expected future spot prices than the survey and vice versa.

[Insert Figure 4 about here]

For a two-quarter horizon (top-left panels), the survey forecasts and the adaptive expectations line up fairly well across the overlapping sample for WTI Crude Oil. This holds

²³The empirical evidence for the intermediate horizon $h = 3$ are available upon request.

both during the dramatic rise and subsequent sharp fall in crude oil prices during the period 2008/2009, as well as during the market decline occurred since 2014.²⁴ The “spread” between the model and the survey increases as high as 20\$ across the great financial crisis, although is sensibly reduced over the remaining sample. Adaptive learning replicates survey expectations also over a four-quarter horizon (top right panel), although the similarity between the model and the survey partly deteriorates as indicated by a persistent gap throughout the sample.

Bottom panels show the results for Copper. Similar to Crude Oil, adaptive learning can mimic the drop in expected spot prices in the period 2008/2009, the subsequent rapid price recovery as well as the downward trend from 2011 until the end of the sample. Over a short-term horizon the model still generates higher expected prices compared to the survey, although the gap is small in magnitude after 2010. The ability of adaptive learning to replicate the survey average forecast partly deteriorates across the period 2006-2008 for a longer 4-quarter predictive horizon (bottom right panel). Finally, Figure 5 shows the results for Corn and Silver,

[Insert Figure 5 about here]

A comparison with observable expectations for Corn (top panel) is limited by the few observations available from the survey, which does not provide opinions from analysts in the period 2011-2013. The divergence around the great financial crisis is non-negligible as indicated by a 80 cents/bushel negative gap. However, over the last part of the sample adaptive learning closely replicates average survey forecasts. Results are stronger for Silver (bottom panel). The gap is fairly small, with the partial exception of a negative “spread” during the dramatic rise in spot prices occurred in the aftermath of the great financial crisis of 2008/2009.

Equation (8) allows to back out the dynamics of ex-ante risk premia from adaptive

²⁴Although the goodness of fit obtained from adaptive learning is remarkable, there is no perfect overlapping, and indeed there are no reasons why this should be the case.

learning. Panel A of Table 2 shows the in-sample descriptive statistics of the (annualized) risk premia, in percentages. Few comments are in order; first, the term structure of risk premia for Crude Oil and Copper is negatively sloped. Risk premia for these two commodities are negative and increasing over time in magnitude. This is consistent with the theory of Keynes (1930) and Hicks (1939), which posits that hedgers are net short and futures are set at a discount with respect to the future expected spot price, namely hedgers are willing to pay a premium for risk transfer to speculators that provide insurance against the unexpected changes in the spot market. Conversely, as far as Corn and Silver are concerned, risk premia are positive and increasing as a function of time horizon. This, according to the hedging pressure theory, is the result of hedgers predominantly being net-long with speculators willing to enter contracts with slightly negative payoff provided there are expectations of increasing future prices.

[Insert Table 2 about here]

Second, and with the only exception in the short-term risk for Crude Oil, risk premia are significantly different from zero, with a t-stat ranging from -1.96 ($h = 3$, Oil) to 6.64 ($h = 4$, Corn). Finally, the AR(1) coefficient provided in Table 2 shows that the persistence of risk premia tend to increase as the expectation horizon increases.

Panel B of Table 3 shows the in-sample correlation between the model- and survey-implied risk premia across commodities and horizons. Correlation estimates show that adaptive learning can possibly replicates the in-sample dynamics of the risk premia obtained from the survey, especially in the short-term, as indicated by an average correlation coefficient of 0.806. The model performance tend to deteriorate in the longer-term, where the correlation between the model- and the survey-implied risk premia decreases to an average value of 0.637, across commodities and time-horizons.

Table 3 shows that the correlation between Oil and Copper risk premia tend to be stronger in the shorter-term (0.361 at $h = 2$), and inversely related to the time horizon

(drops to 0.243 at $h = 4$). The cross-sectional correlation between Corn and Silver are relatively stable across products, although slightly declining with maturity.

[Insert Table 3 about here]

Tables 2-3 together, show that, except few nuances, commodity risk premia are more correlated in the time series than in the cross-section; with the size of cross-sectional (time-series) correlation decreasing (increasing) as the expectations horizon increases.

The structure of commodity markets, as well as the heterogeneity of the factors that drive risk sharing mechanisms, make unlikely that hedgers are permanently either of net short or net long, such that expected risk premia are inherently time-varying. The evidence for time variation in commodity risk premia is compelling in the finance literature (see, e.g. Fama and French 1987, Alquist and Kilian 2010, Hong and Yogo 2012, Gorton et al. 2013, Basu and Miffre 2013, Baumeister and Kilian 2014, Singleton 2014 and Szymanowska et al. 2014 among others). Top panels of Figure 6 confirm this evidence by showing the conditional mean of risk premia obtained from an exponential weighted moving average for $h = 2, 4$.²⁵ The fact that risk premia have their own dynamics im-

²⁵One may argue we could use standard rolling window estimates for the conditional mean rather than an exponential weighted average scheme. However, rolling window averages are inefficient in capturing fluctuations in the time series of risk premia. Such inefficiency can be better understood assuming the risk premium at time t for maturity h is originated by two orthogonal components, $y_t^{(h)} = \mu_t^{(h)} + \psi_t^{(h)}$, with $\psi_t^{(h)} \sim N(0, \sigma_\psi^{2,(h)})$. Rolling window estimates $\hat{\mu}_t$ exploit a limited amount of information assigning equal weight to each observation from $y_{t-n}^{(h)}$ to $y_t^{(h)}$;

$$\hat{\mu}_t^{(h)} = \sum_{i=0}^{n-1} \omega_i L^i y_t^{(h)}, \quad \text{with} \quad \omega_i = \frac{1}{n} \mathbb{I}\{i < n\},$$

where $L^i y_t^{(h)} = y_{t-i}^{(h)}$ the lag operator of order i , implicitly assuming that risk premia change (remain constant) across (within) sub-samples with probability one. By using recursive estimates for the conditional mean, instead, it is possible to keep track of information distant in the past, assigning most of the weight recent observations. Indeed, let us assume a simple random walk dynamics for the expected risk premium $\mu_t^{(h)} = \mu_{t-1}^{(h)} + \xi_t^{(h)}$, with $\xi_t^{(h)} \sim N(0, \sigma_\xi^{2,(h)})$. Optimal weights of past information are obtained as $\omega_i = (1 + \theta)(-\theta)^i$ where $\theta = [(\rho^2 + 4\rho)^{1/2} - 2 - \rho]/2$ and $\rho = \sigma_\xi^{2,(h)}/\sigma_\psi^{2,(h)}$ the signal-to-noise ratio. The weight assigned to past information now decrease exponentially rather than being constant, depending on how much past risk premia are informative about risks exposures, i.e. what is the value of ρ .

plies that the group of traders driving prices at any time t is determined by who has the strongest incentive to trade.²⁶ On average ex-ante expected payoffs for WTI turned from negative to positive after the recent great financial crisis of 2008/2009 until the end of 2013, which means that risk premia have been driven by consumers willing to hedge and financial traders who were expecting declining future prices. The sign of the risk premium for Copper tend to be positive in expansions and suffer during economic slowdown. Most drawdowns coincide with periods of slow global growth, which could explain the incentive of commercial producers to hedge against decreasing prices.

[Insert Figure 6 about here]

Bottom panels of Figure 6 show that the risk premia for Corn and Silver are predominantly positive over time, although steadily declining and turned to negative over the last few years of the sample. Unlike Oil and Copper, the average risk premium for both Corn and Silver does not show a strong correlation with business cycle fluctuations, which is coherent with being an agricultural commodity and a precious metal, respectively. Interestingly, the risk premia for all commodities turned to negative in the very last part of the sample.

6.1 Dissecting Commodity Risk Premia

As a preliminary analysis on the origins of risk premia we estimate a static version of the observation equation (9) in which we consider all of the economic risk factors Z_t outlined in Section 3. For the ease of interpretation, all economic predictors and risk premia are standardized by dividing by their respective sample standard deviation. Table 4 shows the coefficients and the corresponding robust standard errors in parenthesis.²⁷ From this

²⁶Time variation in commodity risk premia could be linked to time-varying risk risk-bearing capacity. Acharya et al. (2013), Etula (2013), and Cheng et al. (2015) emphasize that the amount of risk investors' are willing to take varies over time. This implies that the degree of risk sharing, and thus the risk premium required by hedgers and speculators is not constant over time.

²⁷We estimate the model by OLS with GMM corrected standard errors to account for autocorrelation and heteroschedasticity in the residuals ν_{t+1} . For the ease of exposition we report only the results for

table it can be concluded that HP and time-series Momentum explain a large fraction of the sample variation of the risk premia across commodities and investment horizons (see, e.g. De Roon et al. 2000, Basu and Miffre 2013, and Szymanowska et al. 2014).

[Insert Table 4 about here]

Except for futures on Corn at a two-quarter horizon, for each contract OI are positively and significantly related to the ex-ante risk premia, after controlling for net supply-demand imbalances among hedgers and spillover effects from emerging markets. The positive effect of OI on risk premia is consistent with the idea that increasing market activity signals changes in economic conditions. This, in turn, increases the marginal propensity of hedgers to take a net long/short position, generating price pressure on futures. This result is in line with Hong and Yogo (2012) who show that OI has a significant predictive power for realized payoffs in commodity markets in the presence of hedging demand and limited risk capacity. The results for HP and OI coupled together provide some indirect evidence on the financialization of commodity markets by which commodity prices, and therefore risk premia, are no longer simply determined by their supply-demand, but are also affected by aggregate risk appetite and investment behavior (see, e.g. Tang and Xiong 2012).

Similarly, a Value factor turns out to be negatively and significantly related to risk premia with the only exception of futures contracts on Copper, consistent with Asness et al. (2013). The negative effect of Value can be rationalized by mean-reversion in future spot prices; when current prices are low with respect to their anchoring value, there are expectations of increasing future valuations which rapidly reduces the risk premia paid by, for instance, net-short hedgers.

The regression results of Table 4 suggest, at least unconditionally, that risk sharing mechanism can sensibly explain the in-sample variation of the ex-ante risk premia.

two- and four-quarter ahead ex-ante risk premia. The results for the three-quarter ahead expectations are similar and therefore are not reported separately.

However, Figure 6 made clear that risk premia are not constant over time and possibly experience large swings in sign and magnitude. The fact that expected payoffs have their own dynamics could be the consequence of an heterogeneous exposure to different risk factors on a time scale. In this sense, the results of a static regression might be potentially incomplete, at best.

In the following, we use the dynamic linear regression model (9)-(10) and carefully investigate whether the time variation of ex-ante risk premia is mainly due to random unpredictable shocks, the nature of commodity market participants, or is the result of changes in market and economic conditions. Figures 7-8 shed light on which predictors are important at each time t by showing the factor-specific probability of inclusion obtained from (12). For the ease of exposition, we only show those posterior inclusion probabilities which exceeds a threshold value of 0.5 at least one point in time. Top panels of Figure 7 shows the results for WTI Crude Oil for both a two-quarter (top-left panel) and a four-quarter (top-right panel) horizon. The empirical evidence shows that the effect of financial risk in emerging markets, as proxied by the MSCI Emerging Market Index (MXEF), has become increasingly important especially in the aftermath of the great financial crisis of 2008/2009. A possible explanation is the presence of spillover effects due to the increasing weight of emerging economies in the global economic outlook, as growth in developing economies accounts for over 70 percent of global growth in 2016 (see, IMF Economic Outlook 2016). Indeed, although the direct impact of equity valuations in emerging markets is relatively low due to moderate foreign investments, financial turbulence in this area is often perceived as a negative factor increasing the probability of a slowdown in global economic growth.

[Insert Figure 7 about here]

Trading activity proxied by OI, also explains a considerable fraction of in-sample variation of risk premia in the period that coincides with the dramatic rise in oil prices between

2003 to the end of 2008. This period coincides with the Iraq invasion of March 20th 2003 and the overall higher volatility that affected the stock market afterwards, which increased the propensity of hedgers and speculators to trade on the futures market.

A large explanatory fraction of in-sample time variation in risk premia is also due to a time-series Momentum factor, which captures the short-term autocorrelation in returns, which possibly can be generated by psychological biases of market participants and informational frictions that delay their learning about fundamentals (see, e.g. Cutler et al. 1990, Greenwood and Shleifer 2014, and Singleton 2014). More importantly, time series momentum aims at capturing the changes in trading activity of feedback traders. In fact, as shown by Cutler et al. (1990), the demand for futures contracts by feedback (momentum) traders can depend on past market performances. By the same token, our results confirm the findings of Kang et al. (2014), that show the importance of speculators following momentum strategies in determining the market demand for liquidity.

The strong relevance of HP for the dynamics of expected payoffs confirms the primary relevance of futures as a risk insurance market place, as postulated by Keynes (1930) and Hicks (1939), and confirmed by our initial unconditional regression analysis in Table 4. As expected, the effect of the Value factor increases with the horizon of the investors expectations and becomes relevant especially after the financial crisis of 2008/2009. At its most basic, Value aims at capturing a premium from buying (selling) undervalued (overvalued) commodities, with the expectation that spot prices will increase (decrease) in the near future. As such, the increasing importance for the ex-ante risk premia four quarter ahead corroborates the hypothesis that Value can control for trading strategies based on mean-reversion arguments that aim at longer term returns.

Bottom panels of Figure 7 show the results for Copper for both a two-quarter (top-left panel) and a four-quarter (top-right panel) horizon. Much of the results for Oil hold also for Copper, which is not surprising as industrial metals and energy commodities are commonly sensitive to fluctuations over the business cycle and share most of the risk factors exposures and similar storage costs (see, e.g. Bhardwaj et al. 2015). Indeed,

similar to Oil, the price of Copper is primarily determined by demand for related goods and services as well as the ability of suppliers to extract and transport the product.

Moving to agricultural commodities, top panels of Figure 8 shows that again HP, Momentum and Value capture an important part of the dynamics of risk premia for $h = 2, 4$ quarters ahead. Momentum in agricultural markets can be generated by irregular production. Taking Corn as our example, consumer demand remains fairly stable throughout the year whilst production is seasonal and can vary hugely. For instance, a bad harvest in October/November in the U.S. (which represents around 40% of the global production) cannot be rebalanced until a good harvest occurs in the south hemisphere the next production cycle or in the U.S. the next year, increasing prices and possibly generating positive momentum as supply expectations are revised downward and stockpiles decrease.

[Insert Figure 8 about here]

Unlike WTI Oil and Copper, the ex-ante risk premia of Corn is increasingly related to the level of interest rates and the U.S dollar exchange rate. The effect of exchange rates is somewhat expected as the U.S. represents on itself 40% of the global production for Corn, and the U.S. dollar is the globally recognized currency upon which commodity trade is based. In that matter, a weak dollar generally leads to higher exports for the U.S. as a consequence of higher demand given more competitive prices, but it also means that the production of Corn will become less profitable (see, e.g. Hamilton 2009). Another possible explanation relies on the increasing financialization of the agricultural commodity markets. As shown by Tang and Xiong (2012), after 2004, agricultural commodities included in financial indexes such as the Goldman Sachs Commodity Index (GSCI) and the Dow Jones (DJ)-AIG, became much more responsive to shocks to the U.S. dollar exchange rate.

Similarly to Corn, bottom panels of Figure 8 show that in addition to time-series Momentum and HP, the risk premium on Silver is affected by variations in the U.S. dollar

exchange. This can be explained by the fact that usually Silver is internationally traded in dollars. In that respect, a falling dollar exchange rate makes Silver relatively cheap in foreign currencies. These capital inflows naturally promote the rise of Silver expected future spot price. Unlike energy and industrial-related commodities, such mechanism is exacerbated by the fact that precious metals are historically considered as safe assets, which are bought to protect against currency depreciation and corresponding increasing inflation. Finally, similar to WTI Crude Oil and Copper, trading activity as proxied by OI and a Value factor also explain the risk premium dynamics toward the end of the sample.

6.1.1 A Further Discussion on Hedging Pressure and Momentum

In this section we further discuss the dynamics of the betas on two main predictors which turned out to be significant in the empirical results: HP and Momentum. Figure 9 shows the posterior median of the betas for HP, i.e. $\hat{\beta}_{HP}$, across commodities for $h = 2, 4$ quarter-ahead expectations horizons.

[Insert Figure 9 about here]

There is a fair amount of heterogeneity in the dynamics of $\hat{\beta}_{HP}$, although some common features emerge. Except for Copper in 2006, the sign of the estimated coefficient is mostly positive, meaning that, on average throughout the sample, increasing net-excess short positions tend to be associated with higher risk premia across commodities. Also, the effect of HP tends to increase relatively after 2005 and to decrease after 2011, i.e. during the boom and the bust of the commodity super-cycle. A possible explanation lies on the fact that HP directly depends on constraints on the amount of capital different investor categories are willing to commit (see, e.g. Acharya et al. 2013 and Etula 2013). In this respect, HP typically increases when there are expectations of falling prices on the future spot market such as for instance during the crisis of 2008/2009. Therefore,

$\hat{\beta}_{HP}$ signals that when prices go down the marginal propensity to take the long side of the trade decreases, namely, traders to take long positions require a much higher risk premium ex-ante. These results are also in line with the findings of Kang et al. (2014), who show that the cost for speculators to provide liquidity increases when the positions of hedgers become more imbalanced.

As far as time series momentum is concerned, Figure 10 shows the posterior median of the corresponding betas $\hat{\beta}_{Mom}$ across commodities for $h = 2, 4$ quarter-ahead expectations horizons. Posterior estimates show that momentum plays a relevant role in the dynamics of risk premia even after investors' expectations are taken into account. This is consistent with the presence of momentum traders whose oscillating trading demand affects market liquidity. Indeed, as shown by Kang et al. (2014), momentum traders increase the demand for liquidity, which need to be absorbed by risk-averse market makers and hedgers who will require therefore appropriate risk compensation. Furthermore, the dynamic sensitivity of risk premia to past returns can be explained by the intrinsic fluctuating market exposure of momentum traders, whose aggregated portfolio size tend to increase in trending market. In other words, Momentum is intimately interrelated to hedging against sharp market corrections as postulated by the theory of normal backwardation of Keynes (1930) and Hicks (1939).

[Insert Figure 10 about here]

Except for Corn, the effect of Momentum decreases throughout the boom and bust of commodity prices over the period 2005-2011. On average over the sample and across commodities, increasing past returns tend to be associated with a lower ex-ante risk premia. The dynamics of $\hat{\beta}_{Mom}$ is consistent with the idea that increasing prices make hedgers more willing to take a long position; on the contrary speculators believing in mean-reversion are more willing to short when prices are high. This supply-demand mechanism lowers the risk premia embedded in futures contracts.

6.2 Model Assessment

One may argue that a full model in which all of the risk factors are given equal weight can equivalently shed light on the dynamics of commodity risk premia. To address this concern, we compare the in-sample goodness-of-fit of our dynamic model specification in which economic risk factors are weighted according to their probability of inclusion (12), with respect to two alternative mainstream specifications. The first benchmark is a model in which none of the economic risk factors is included and the dynamics of the ex-ante risk premia is determined uniquely by a time-varying intercept in addition to unpredictable idiosyncratic shocks. This specification is suitable to investigate the trade-off between including potentially irrelevant factors as opposed to consider the most possible parsimonious specification. The second benchmark model is a dynamic linear regression model with all risk factors included, which allows to investigate the contribution of the weighting the conditioning information by the relevance importance of each covariate as opposed to considering all of the risk factors as equally relevant for the dynamics of risk premia.

We assess the models performance by computing an in-sample Relative Root Mean Squared Error (RMSE) and a Bayes predictive factor. Gneiting (2011) showed that RMSE is a consistent evaluation measure when the point estimates equals the mean of the posterior distribution. As such we compute point predictions for each model as the average of the corresponding marginal posterior distribution of the ex-ante risk premia across commodities and horizons integrating out parameter uncertainty, see Appendix C for a more detailed explanation. Panel A of Table 5 shows the relative RMSE computed by taking the ratio between the RMSE and the competing specification. A number lower than one implies a better performance of our econometric specification;

[Insert Table 5 about here]

Panel A shows that there is value in our dynamic weighting scheme as suggested by

a reduction of the squared loss in the order of 50%, on average. More precisely, the RMSE of a model with only a time-varying intercept is more than doubled for eleven out of twelve cases. This implies that the risk factors effectively convey information in explaining the time-series variation of the ex-ante risk premia, across commodities and for different expectation horizons. Including all of the factors with equal weight throughout the sample, although clearly improves the goodness-of-fit with respect to the time-varying intercept specification, does not change the ranking. Indeed, for all of the cases the RMSE for the weighted model is lower and is less than half for three out of twelve cases. The closest RMSE with respect to the weighted model is for the pair (Corn, $h = 2$) which however still shows a RMSE which is 25% higher than our dynamic model specification.

Although the RMSE reveals interesting aspects of the in-sample goodness-of-fit implied by the posterior means, it cannot provide insight into the uncertainty that is associated with producing conditional mean estimates. In this respect, a direct evaluation of the marginal likelihood is a natural tool to assess the ability of our model specification to explain unusual developments in commodity expected payoffs, such as the likelihood of large drops or jumps in risk premia given current information. We now couple the evidence from the RMSE with a formal comparison of the marginal densities based on a Bayes factor that directly compares the marginal likelihood across models. In our setting, an analytical evaluation of the marginal likelihood is not possible. Gelfand and Dey (1994) and Newton and Raftery (1994) showed that a simulation consistent estimate of the marginal likelihood for a model $\mathcal{M}_i$ is obtained by the harmonic mean of the likelihood values, evaluated at each draw of the parameters sampled from the corresponding full conditional distributions (see Appendix C).²⁸ Panel B of Table 5 shows the Bayes' factors in $\log_{10}$ scale; a value greater than .5 would represents decisive evidence

²⁸A potential issue in using the harmonic mean of posterior-implied conditional likelihoods is that the inverse likelihood does not have finite variance (see, e.g. Chib 1995 for a detailed discussion). However, in our setting, the (log of) marginal likelihood can be efficiently computed through the Kalman filter recursions, mitigating potential concerns in using an harmonic mean approximation. In that respect, when computing the log-marginal likelihood, we checked the stability of the conditional likelihood for each draw from the posterior distributions. Results of the draw specific likelihood evaluation are available upon request.

against each benchmarking model (see, Kass and Raftery 1995). The empirical results show substantial evidence against a competing specification which only includes a time-varying intercept, with values in the range of 13.02 (Silver, $h = 3$) to 50.59 (WTI, $h = 4$). Similarly, the Bayes factors show that by including all regressors to explain the dynamics of risk premia reduces the possibility to capture efficiently anomalous realizations as suggested by a Bayes factor that ranges from 0.86 (Copper, $h = 2$) to 8.62 (WTI, $h = 4$).

7 Concluding Remarks

Our empirical analysis shows that investor expectations of future commodity spot prices can be rationalized by an adaptive learning scheme in which expected future spot prices are affected by past prediction errors and changes in aggregate demand, as proxied by an index of world industrial production. This expectations formation mechanism provides a framework to extract time-varying (ex-ante) risk premia.

By using a dynamic linear regression in which we accommodate uncertainty in; (1) the relative importance of alternative predictors, (2) the estimated coefficients and (3) their degree of time-variation, we show that time-variation in commodity risk premia is predominantly due to risk sharing channels and the changing nature of commodity market participants, as proxied by HP and partly by OI. In addition to trading activity, an important determinant for the dynamics of commodity risk premia is the persistence of past returns and their relationship with fundamental valuations, as proxied by time-series Momentum and Value factors.

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Appendix

A A Simple Model of Adaptive Expectations

We start from a simple rational expectations model which is closely related to the Muth (1961) market model with inventory speculation except demand shocks are predictable and not i.i.d. The market behavior is characterized by an infinite horizon, discrete time model with a market clearing condition that holds in each period, $t + 1$;

$$C_{t+1} + I_{t+1} = Q_{t+1} + I_t, \quad (\text{A.1})$$

where Q_{t+1} represents the output produced for a commodity in a period lasting as long as the production lag, C_{t+1} is the amount of commodity consumed in the same time period, and I_{t+1} the commodity inventories at the end of period $t + 1$. The standard Muth (1961) market model posits there are three categories of economic agents active in the market for commodities; the buyers, the producers and the inventory holders. The latter can capture speculation effects. The utility of price-taking consumers is declining in the current market price S_{t+1} and affected by an aggregate persistent demand shock z_t . On the other hand, the utility of risk-averse producers is positively related to expected spot prices $E_t S_{t+1}$, while inventories decisions depend on the expected capital gain of holding a unit of commodity. As a result, aggregate demand, supply and holding functions are defined as

$$C_{t+1} = A - \delta S_{t+1} + z_{t+1}, \quad (\text{A.2})$$

$$Q_{t+1} = \lambda E_t S_{t+1} + u_{t+1}, \quad (\text{A.3})$$

$$I_{t+1} = \nu (E_t S_{t+1} - S_{t+1}), \quad (\text{A.4})$$

with ν be a rescaled risk-aversion parameter. We extend the standard market model with inventory speculation assuming exogenous factors that affect aggregate demand are predictable and potentially persistent;

$$z_{t+1} = bz_t + e_{t+1}, \quad (\text{A.5})$$

with e_{t+1} and u_{t+1} zero-mean i.i.d. disturbance terms. Storage costs are assumed to be zero to simplify the model. These equations and assumptions are the same of the original Muth (1961) model, except for the predictability of demand shocks. Substituting (A.2)-(A.5) in the equilibrium condition (A.1), the spot market equilibrium can be expressed in terms of prices, price expectations, demand shocks and disturbances;²⁹

$$\begin{aligned} A - (\nu + \delta) S_{t+1} + bz_t + e_{t+1} &= \lambda E_t S_{t+1} + u_{t+1} - \nu S_t, \\ (\nu + \delta) S_{t+1} &= A + bz_t + e_{t+1} - \lambda E_t S_{t+1} - u_{t+1} + \nu S_t, \end{aligned} \quad (\text{A.6})$$

which can be rewritten as a simple linear model as follows

$$S_{t+1} = \mu + \beta E_t S_{t+1} + \theta S_t + \omega z_t + \eta_{t+1}, \quad (\text{A.7})$$

²⁹We assume there is a period distance in the future where the forward expectations are equivalent, i.e. $E_t S_{t+1} \equiv E_{t+1} S_{t+2}$, (see, e.g. Beck 1993).

By taking expectations on both sides and substituting back in (A.7), we can obtain a unique reduced-form Rational Expectations Equilibrium (REE) as

$$S_{t+1} = \phi_0 + \phi_1 S_t + \phi_2 z_t + \eta_{t+1}, \quad (\text{A.8})$$

with $\phi_0 = (1 - \beta)^{-1} \mu$, $\phi_1 = (1 - \beta)^{-1} \theta$, $\phi_2 = (1 - \beta)^{-1} \omega$ and $\eta_{t+1} = e_{t+1} - u_{t+1}$. This solution is the same as the original Muth (1961)'s model except that future commodity spot prices now depends on aggregate demand. Notice that for a given level of commodity prices, Eq. (A.8) implies that a positive (negative) shock to aggregate demand increases (decreases) future prices, while a positive (negative) shock in aggregate supply decreases (increases) prices.

A.1 Extension: Introducing a Futures Market

We now introduce a futures market upon the process of price formation and show that the functional form of the perceived law of motion under rational expectations is observationally equivalent to Eq.(A.8). By including a futures market we explicitly consider the effect of hedging in the decision-making process of a representative investor. We assume that suppliers, buyers and inventory holders now hedge their commodity positions by trading on futures. Following Turnovsky (1983), Kawai (1983), and Beck (1993), we start from the assumption that agents are making production, storage and hedging decisions simultaneously, depending on current futures and expectations of spot prices.³⁰ All agents are assumed to act as hedgers as well as speculators in the futures market. Assuming now futures prices and spot price expectations are linearly linked to each other, aggregate demand, supply and holding functions are defined as

$$C_{t+1} = A - \delta F_t + z_{t+1}, \quad (\text{A.9})$$

$$Q_{t+1} = \lambda F_t + u_{t+1}, \quad (\text{A.10})$$

$$I_{t+1} = \xi (F_t - S_t), \quad (\text{A.11})$$

$$-X_t^b = \chi^b [F_t - E_t S_{t+1}] - \tilde{C}_{t+1}, \quad (\text{A.12})$$

$$X_t^p = \chi^p [F_t - E_t S_{t+1}] + \tilde{Q}_{t+1}, \quad (\text{A.13})$$

$$X_t^i = \chi^i [F_t - E_t S_{t+1}] + \tilde{I}_t, \quad (\text{A.14})$$

where ξ represents the inverse of storage cost per unit of commodity, and X_t^b, X_t^p and X_t^i represent the speculative positions, i.e. excess supply of futures contracts, by buyers, producers and inventory holders, respectively. Planned levels of consumption, production and inventories denoted as $\tilde{C}_{t+1}, \tilde{Q}_{t+1}$ and $\tilde{I}_{t+1}$, indicate that commodity positions are completely hedged in the futures market. The market clearing condition on the futures market states that the aggregate excess supply of futures contract should be zero, i.e.

$$X_t^p + X_t^i - X_t^b = 0, \quad (\text{A.15})$$

³⁰More specifically, we assume buyers are intermediate producers, which therefore as well willing to reduce risk hedging their positions participating in the futures contract.

Substituting (A.9)-(A.11) in the market clearing condition (A.1), the spot market equilibrium can be expressed in terms of both futures and spot prices, demand shocks and disturbances, i.e.

$$A - \delta F_t + bz_t + e_{t+1} + \xi (F_{t+1} - S_{t+1}) = aF_t + u_{t+1} + \xi (F_t - S_t), \quad (\text{A.16})$$

Similarly, by substituting (A.12)-(A.14) in (A.15) we obtain;

$$\begin{aligned} \chi^f [F_t - E_t S_{t+1}] + \tilde{Q}_{t+1} + \chi^i [F_t - E_t S_{t+1}] + \tilde{I}_t + \chi^b [F_t - E_t S_{t+1}] - \tilde{C}_{t+1} &= 0, \\ \chi^f [F_t - E_t S_{t+1}] + \lambda F_t + \chi^i [F_t - E_t S_{t+1}] + \xi (F_t - S_t) + \chi^b [F_t - E_t S_{t+1}] - A + \delta F_t &= 0, \end{aligned}$$

Where $\tilde{Q}_{t+1}$, $\tilde{I}_t$ and $\tilde{C}_{t+1}$ are defined as (A.9)-(A.11) without the error terms. Solving for F_t we have that

$$F_t = \bar{A} + \bar{\chi} E_t S_{t+1} + \bar{\xi} S_t, \quad (\text{A.17})$$

with $\bar{A} = A/a$, $\bar{\chi} = \chi/a$ and $\bar{\xi} = \xi/a$, where $a = (\chi + \lambda + \xi - \delta)$ and $\chi = \chi^p + \chi^b + \chi^i$. Similarly, F_{t+1} can be obtained as a function of $E_{t+1} S_{t+2}$ and S_{t+1} , and substitute these values into (A.16) to obtain;

$$\begin{aligned} \xi (\bar{\xi} - 1) S_{t+1} &= \delta \bar{A} + \bar{\chi} (a + \delta) E_t S_{t+1} + \bar{\chi} (\chi + \delta) S_t + bz_t + e_{t+1} - u_{t+1}, \\ S_{t+1} &= \mu + \beta E_t S_{t+1} + \theta S_t + \omega z_t + \eta_{t+1}, \end{aligned} \quad (\text{A.18})$$

with $\mu = \delta \bar{A} / \xi (\bar{\xi} - 1)$, $\beta = \bar{\chi} (a + \delta) / \xi (\bar{\xi} - 1)$, $\theta = \bar{\chi} (\chi + \delta) / \xi (\bar{\xi} - 1)$, and $\omega = b / \xi (\bar{\xi} - 1)$, respectively. Equation (A.18) is analogous to (A.6) and can be solved in the same way. From (A.18), the solution procedure described above yields the same Perceived Law of Motion (PLM);

$$S_{t+1} = \phi_0 + \phi_1 S_t + \phi_2 z_t + \eta_{t+1}, \quad (\text{A.19})$$

with $\phi_0 = (1 - \beta)^{-1} \mu$, $\phi_1 = (1 - \beta)^{-1} \theta$, $\phi_2 = (1 - \beta)^{-1} \omega$ and $\eta_{t+1} = e_{t+1} - u_{t+1}$. To summarize, we show that by introducing a futures market in which different type of investors hedge their positions in physical commodities, the reduced form PLM has the same functional form of the case without a futures market. In the following section we introduce recursive learning on the reduced-form parameters ϕ_0, ϕ_1 and ϕ_2 in Eq.(A.19).

A.2 Adaptive Learning

The key assumption to introduce learning is that the expectations of economic agents $E_t [S_{t+1}]$ are not necessarily rational as agents do not know the structural parameters. Expectations are instead formed on the basis of current observations and predictions of parameters which are updated over time. There are two key building blocks to explicit the agents' learning dynamics. First, agents beliefs are described by means of a dynamic model. We assume the PLM as the same functional form of the REE (A.19), where the true values $\phi = (\phi_0, \phi_1, \phi_2)$ are not known. Second, we need to describe how agents obtain estimates for the parameters of the PLM. We explicit agents' recursive estimates in terms of a Bayesian prior that describes how coefficients in the PLM drift at each time t ;

$$\begin{aligned} S_{t+1} &= \phi'_{t+1} X_t + \eta_{t+1}, & \text{with} & \quad \eta_{t+1} \sim N(0, \sigma^2), \\ \phi_{t+1} &= \phi_t + \epsilon_{t+1} & \text{with} & \quad \epsilon_{t+1} \sim N(0, \Omega), \end{aligned} \quad (\text{A.20})$$

with $\phi_t = (\phi_{0,t}, \phi_{1,t}, \phi_{2,t})$ and $X_t = (1, S_t, z_t)$. The shock η_{t+1} is uncorrelated with ϵ_{t+1} , and $\Omega \ll \sigma^2 I$. The innovation covariance matrix Σ governs the perceived volatility of increments to the parameters, and is a key component of the model (see Sargent and Williams 2005). Agents' recursive optimal estimate of ϕ_{t+1} conditional on information available up to time t . $\gamma_{t+1} = \hat{\phi}_{t+1|t}$ are provided by the Kalman filter recursion;

$$\begin{aligned}\gamma_{t+1} &= \gamma_t + K_t (S_{t+1} - \gamma_t' X_t), \\ R_{t+1} &= R_t - \frac{R_t X_t X_t' R_t}{X_t' R_t X_t + 1} + \sigma^{-2} \Omega,\end{aligned}\tag{A.21}$$

where $K_t = R_t X_t (X_t' R_t X_t + \sigma^2)^{-1}$ determines the degree of updating of agents' beliefs when faced with an unexpected commodity spot price $S_t - \gamma_t' X_t$, i.e. Kalman gain. The recursive learning dynamics (A.20) represents a generalization of a recursive learning with constant gain as specified in Evans and Honkapohja (2001), Sargent (2002), Cho et al. (2002), and Williams (2003), among others.

B Estimation Strategy

In this section we provide details of the Gibbs sampler we use for the estimation of the dynamic linear model (9)-(10). For the ease of exposition, we report the updating scheme conditional on $L_t = \{1, 2, \dots, K\}$ and disregard the maturity super-script h . Let us denote $\mathbf{x}_{s:t} = (\mathbf{x}_s, \dots, \mathbf{x}_t)$, $s \leq t$, the set of vectors $\mathbf{x}_u$. The collections of parameters is defined as $\Theta = (\theta_{1:T}, W, H)$, respectively, where $\theta_{1:T}$ represents the $(T \times N)$ matrix of state parameters. Let θ_0 represents the initial value of the dynamic sensitivity to the k -dimensional vector of regressors. The complete likelihood function can be defined as

$$p(\mathbf{y}_{1:T}, \theta_{1:T} | \mathbf{Z}_{1:T}, W, H) = \prod_{t=1}^{T-1} p(\mathbf{y}_{t+1} | Z_t' \theta_t, H) p(\theta_t | \theta_{t-1}, W), \tag{A.22}$$

with $p(\mathbf{y}_{t+1} | Z_t' \theta_t, H) = N(Z_t' \theta_t, H)$ and $p(\theta_t | \theta_{t-1}, W) = N_k(\theta_t, W)$ two univariate and multivariate Gaussian distributions, respectively. The sequential model description in (9)-(10) requires that the defining quantities at time t be known at that time. We assume prior information about θ_0 is vague and centered around the initial hypothesis of no effect of risk factors on premia, i.e. $\theta_0 | D_0 \sim N(c_0, C_0)$, with $c_0 = 0$ and $C_0 = 10,000$. Also, we assume that the impact of risk factors is highly uncertain and volatile, as captured by an Inverse-Wishart distribution with small degrees of freedom and large scale parameter, i.e. $W | D_0 \sim IW(a_0, A_0)$ with $a_0 = 3$ and $A_0 = 10,000$. As a result we assume that when no historical information on expected risk premia and factors is available, elasticities are mainly driven by idiosyncratic risk as proxied by an Inverse-Gamma distribution with uninformative hyper-parameters, i.e. $H | D_0 \sim IG(n_0/2, n_0 N_0/2)$ with $n_0 = 0.001$ and $N_0 = 0.001$. Conditional on the latent states $\theta_{1:T}$ the complete likelihood can be factorized as the product of, as such combining the prior specification with the factorized completed likelihood (A.22), we obtain the posterior density

$$\begin{aligned}p(\theta_{1:T}, W, H | \mathbf{y}_{1:T}, \mathbf{Z}_{1:T}) &\propto p(\mathbf{y}_{1:T}, \theta_{1:T} | \mathbf{Z}_{1:T}, W, H) p(\theta_0, W, H), \\ &= p(\mathbf{y}_{1:T} | \theta_{1:T}, \mathbf{Z}_{1:T}, H) p(\theta_{1:T} | W) p(\theta_0, W, H),\end{aligned}$$

The joint posterior distribution of the states and parameters is not tractable analytically such that the estimator for the parameters cannot be obtained in closed form. The latent variables $\theta_{1:T}$ are simulated alongside the model parameters H and W . At each iteration, the sampler sequentially cycles through

the following steps:

1. Draw $\theta_{1:T}$ conditional on H , W and the data $\mathbf{y}_{1:T}, \mathbf{Z}_{1:T}$.
2. Draw W conditional on $\theta_{1:T}$.
3. Draw H conditional on $\mathbf{y}_{1:T}, \mathbf{Z}_{1:T}$, and $\theta_{1:T}$.

In what follows we provide details of each step of the Gibbs sampler.

B.1 Step 1. Sampling the Conditional Factor Sensitivities $\theta_{1:T}$

The full conditional posterior density for the time-varying factor loadings is computed using a Forward Filtering Backward Sampling (FFBS) approach as in Carter and Kohn (1994). The initial prior are sequentially updated via the Kalman filtering recursion. Conditionally on idiosyncratic risk H , state variance W , and assuming an initial distribution $\theta_0|y_0 \sim N(m_0, C_0)$, it is straightforward to show that the (see West and Harrison 1997 for more details)

$$\begin{aligned} \theta_t|Z_{1:t-1}, W &\sim N(a_t, R_t) && \text{Propagation Density} \\ Y_t|Z_{1:t-1}, H &\sim N(f_t, Q_t) && \text{Predictive Density} \\ \theta_t|Z_{1:t} &\sim N(m_t, C_t) && \text{Filtering Density} \end{aligned}$$

with

$$\begin{aligned} a_t &= m_{t-1} & R_t &= C_{t-1} + W \\ f_t &= Z_t' a_t & Q_t &= Z_t R_t X_t' + H \\ m_t &= a_t + K_t e_t & C_t &= R_t - K_t Q_t K_t' \end{aligned} \tag{A.23}$$

and $K_t = R_t X_t Q_t^{-1}$ and $e_t = y_t - f_t$. Conditional thetas are drawn from the posterior distribution which is generated by backward recursion (see Frühwirth-Schnatter 1994, Carter and Kohn 1994, and West and Harrison 1997), i.e. $p(\theta_t|\mathbf{y}_{1:T}) = N_k(m_t^b, C_t^b)$, with

$$\begin{aligned} m_t^b &= (1 - B_t) m_t + B_t m_{t+1}^b, \\ C_t^b &= (1 - B_t) C_t + B_t^2 C_{t+1}^b, \quad \text{with} \quad B_t = \frac{C_t}{C_t + W}, \end{aligned}$$

B.2 Step 2. Sampling the State Variance Parameters W

Conditional on the risk exposures, the estimate of the state variance covariance matrix coincide with the update of an Inverse-Wishart distribution. Posterior estimates are obtained by updating the prior structure as

$$W|\theta_{1:T} \sim IW(a_1, A_1) \tag{A.24}$$

with

$$\begin{aligned} a_1 &= a_0 + T \\ A_1 &= A_0 + \hat{\varepsilon} \hat{\varepsilon}' \end{aligned}$$

where $\hat{\varepsilon}' = (\hat{\varepsilon}_1, \dots, \hat{\varepsilon}_T)$ and $\hat{\varepsilon}_t = \hat{\theta}_t - \hat{\theta}_{t-1}$ given $\hat{\theta}_t = m_t^b$.

B.3 Step 3. Sampling the Idiosyncratic Risk H

For the posterior estimates of the idiosyncratic risk we exploit the fact that the prior and the likelihood are conjugate. The updating scheme is easily derived as

$$H|\theta_{1:T}, \mathbf{Z}_{1:T}, \mathbf{y}_{1:T} \sim IG(\nu_1/2, \nu_1 N_1/2) \quad (\text{A.25})$$

with

$$\begin{aligned} \nu_1 &= \nu_0 + T \\ \nu_1 N_1 &= \nu_0 N_0 + \hat{\mathbf{v}} \hat{\mathbf{v}}', \end{aligned}$$

where $\hat{\mathbf{v}}' = (\hat{v}_1, \dots, \hat{v}_T)$ and $\hat{v}_t = y_t - Z_t' \hat{\theta}_{t-1}$ given $\hat{\theta}_{t-1} = m_{t-1}^b$

C Marginal Likelihood Approximation

In our setting, an analytical evaluation of the marginal likelihood is not possible. Gelfand and Dey (1994) and Newton and Raftery (1994) showed that a simulation consistent estimate of the marginal likelihood for a model $\mathcal{M}_i$ is obtained by the harmonic mean of the likelihood values, evaluated at each draw of the parameters sampled from the corresponding full conditional distributions. The marginal likelihood is the probability that the model gives to the observed data, averaging over values of its parameters with respect to their prior distribution. If $y^t = (y_1, \dots, y_t)$ is the time series of the risk premia and $\Theta = (\theta^T, W, H)$ is the entire set of parameters for the model $\mathcal{M}_i$, then the marginal likelihood is

$$p(y^t | \mathcal{M}_i) = \int p(y^t | \Theta, \mathcal{M}_i) p(\Theta | y^t, \mathcal{M}_i) d(\Theta) \quad (\text{A.26})$$

The harmonic mean of the likelihood with respect to the posterior distribution, can be approximated by using the posterior draws from the Markov Chain Monte Carlo (MCMC) done to estimate parameters of each model;

$$p(y^t | \mathcal{M}_i) \approx \frac{1}{\frac{1}{N} \sum_{n=1}^N \frac{1}{p(y^t | \Theta^n, \mathcal{M}_i)}} \quad (\text{A.27})$$

with $p(y^t | \Theta^n, \mathcal{M}_i)$ the likelihood evaluated at the n th draw of the parameters from the posterior distribution. The Law of Large Numbers guarantees that this estimator is consistent

$$\begin{aligned} \int \frac{1}{p(y^t | \Theta, \mathcal{M}_i)} p(\Theta | y^t, \mathcal{M}_i) d(\Theta) &= \int \frac{1}{p(y^t | \Theta, \mathcal{M}_i)} \frac{p(y^t | \Theta, \mathcal{M}_i) p(\Theta | \mathcal{M}_i)}{p(y^t | \mathcal{M}_i)} d(\Theta) \\ &= \frac{1}{p(y^t | \mathcal{M}_i)} \int p(\Theta | \mathcal{M}_i) d(\Theta) \\ &= \frac{1}{p(y^t | \mathcal{M}_i)} \end{aligned}$$

C.1 Root Mean Squared Error and Predictive Bayes Factor

To compare the performance of alternative models we rely on two complementary measures of in-sample goodness-of-fit. We first compute a relative Root Mean Squared Error (RMSE) which allows to investigate the performance of alternative methodologies compared to our weighted dynamic linear model. For each alternative specification, we compute the relative value as the ratio of the RMSE implied by our model over the benchmark, so that values lower than one indicates that our model improves upon the alternative specification. More specifically, let us define the marginal distribution of the ex-ante risk premia at time t on a given commodity and for a given horizon as

$$p(y_t | \mathcal{M}_i) = \int p(y_t | \Theta, \mathcal{M}_i) p(\Theta | y^t, \mathcal{M}_i) d(\Theta),$$

with

$$p(\Theta | y^t, \mathcal{M}_i) \propto p(y^t | \Theta, \mathcal{M}_i) p(\Theta | \mathcal{M}_i),$$

and $p(y^t | \Theta, \mathcal{M}_i)$, $p(\Theta | \mathcal{M}_i)$ representing the conditional likelihood and the marginal prior probabilities, respectively. The RMSE of the i th model is defined as

$$RMSE_i = \sqrt{\frac{1}{T} \sum_{t=1}^T (y_t - E[y_t | \mathcal{M}_i])^2}, \quad (\text{A.28})$$

with

$$E[y_t | \mathcal{M}_i] = \int y_t p(y_t | \mathcal{M}_i) dy_t,$$

The relative measure is computed as $RMSE_i / RMSE_{weighted}$. Gneiting (2011) showed that RMSE is a consistent evaluation measure when the point estimate equals the mean of the posterior distribution. Although point estimates of ex-ante risk premia reveal interesting aspects of the explanatory power of our weighted dynamic linear model, such conditional means cannot provide insight into the uncertainty that is associated with producing these forecasts. In that respect, a direct evaluation of the marginal likelihood $p(y^t | \mathcal{M}_i)$ is a more natural tool to assess the ability of the weighted dynamic linear model to explain unusual developments in ex-ante risk premia, such as the likelihood of large drops or jumps in future realizations given current information. We therefore compare the alternative specifications as above, on the basis of a Bayes factor. The Bayes factor compares our model $\mathcal{M}_{weighted}$ against the alternative specifications and is defined as

$$\mathcal{BF}_{(\mathcal{M}_{weighted} \text{ vs } \mathcal{M}_i)} = \log_{10} \left[\frac{p(y^t | \mathcal{M}_{weighted})}{p(y^t | \mathcal{M}_i)} \right], \quad (\text{A.29})$$

The Bayes factor is computed in a $\log_{10}$ scale for the ease of exposition, so that values higher than 0.5 indicates substantial evidence in favour of our benchmark (see Kass and Raftery 1995 for more details). The complete marginal likelihood $p(y^t | \mathcal{M}_i)$ for each specification is computed as explained above.

Table 1. Testing Extrapolative Expectations

Adaptive expectations. This table shows the results of a test for extrapolative expectations as proxied by the average Bloomberg survey price forecasts. The sample period for the survey is 12:2006-01:2016, aggregated monthly, and collected for alternative commodities and time-horizons. The commodities considered are WTI Crude Oil, Copper, Silver and Corn, which are representative of the energy, industrial, agricultural and precious metals commodity markets. We exclude from the analysis the survey for Corn as the survey comprises lots of missing data which would make the sample size subject to potentially relevant small-sample biases. Regressions are estimated by GMM correcting standard errors to account for autocorrelation and heteroschedasticity in the residuals. **Panel A:** shows the results for a the null hypothesis that expectations are extrapolative in its general form. **Panel B:** shows the results for a the null hypothesis that expectations are revised in line with past prediction errors on future spot prices, i.e. adaptive expectations. Robust standard errors are in parenthesis, * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Horizon (Quarters)	Commodity	Panel A: Extrapolative Expectations				Panel B: Adaptive Expectations			
		α	β	adj R^2	DW	μ	δ	adj R^2	DW
$h = 2$	Crude Oil (WTI)	-0.002 (0.017)	-0.668*** (0.095)	0.466	0.567	0.003 (0.009)	0.244*** (0.038)	0.558	1.261
	Copper	-0.019 (0.018)	-0.491*** (0.193)	0.186	1.019	-0.004 (0.013)	0.226*** (0.056)	0.349	1.441
	Silver	0.027 (0.012)	-0.695*** (0.069)	0.513	1.123	0.020 (0.009)	0.290*** (0.042)	0.396	1.240
$h = 3$	Crude Oil (WTI)	0.020 (0.023)	-0.691*** (0.127)	0.324	0.391	0.0216 (0.009)	0.269*** (0.031)	0.669	1.199
	Copper	0.002 (0.019)	-0.497*** (0.199)	0.166	0.876	0.006 (0.015)	0.197*** (0.031)	0.334	1.140
	Silver	0.041 (0.013)	-0.723*** (0.079)	0.487	0.932	0.036 (0.013)	0.173*** (0.038)	0.201	1.013
$h = 4$	Crude Oil (WTI)	0.041 (0.027)	-0.802*** (0.131)	0.321	0.441	0.041 (0.013)	0.221*** (0.041)	0.512	0.677
	Copper	0.009 (0.021)	-0.537*** (0.161)	0.156	0.982	0.013 (0.019)	0.168*** (0.057)	0.197	1.051
	Silver	0.059 (0.015)	-0.769*** (0.059)	0.465	1.212	0.054 (0.017)	0.119*** (0.038)	0.106	1.251

Table 2. Descriptive Statistics

Descriptive statistics. This table reports the descriptive statistics for the risk premia for WTI Oil Crude, Copper, Corn and Silver, as well as the correlation between the model-implied and the survey based estimates. Model-implied risk premia are those obtained by filtering the expected future spot prices obtained from a model of adaptive learning for $h = 2, 3, 4$ quarters ahead. Similarly, the survey estimates are obtained by subtracting futures prices from the average survey forecasts across maturities and commodities. Futures prices data on Crude Oil (WTI) are from the New York Mercantile Exchange (NYMEX), prices are in U.S. dollars per barrel. Futures on Silver and Copper are obtained from the Commodity Exchange (COMEX). The sample period for the survey is 12:2006-01:2016. The sample period for the model-implied risk premia is 01:1995-01:2016, monthly; the first 24 months are cut as burn-in sample for the adaptive learning scheme. Risk premia are annualized and reported in percentages. **Panel A:** reports the mean, standard deviation, the corresponding t-stat and the AR(1) coefficients. **Panel B:** shows the correlation between the risk premia obtained by adaptive learning and the survey.

Panel A: Descriptive Statistics

	Horizon	Commodity			
		Crude Oil	Copper	Corn	Silver
Mean	$h = 2$	-0.115	-1.689	4.112	2.832
	$h = 3$	-1.431	-2.213	4.421	3.310
	$h = 4$	-2.023	-2.935	4.987	3.834
St.Dev	$h = 2$	3.443	3.512	3.518	3.513
	$h = 3$	3.748	3.507	3.502	3.498
	$h = 4$	3.452	3.498	3.492	3.486
t-stat	$h = 2$	-0.155	-2.238	5.440	3.752
	$h = 3$	-1.961	-2.937	5.875	4.407
	$h = 4$	-2.728	-3.917	6.647	5.118
AR(1)	$h = 2$	0.281	0.313	0.547	0.072
	$h = 3$	0.591	0.369	0.624	0.123
	$h = 4$	0.693	0.472	0.674	0.180

Panel B: Correlation Risk Premia Model vs Survey

Commodity	$h = 2$	$h = 3$	$h = 4$
Crude Oil (WTI)	0.815	0.787	0.682
Copper	0.721	0.672	0.612
Corn	0.891	0.721	0.652
Silver	0.798	0.723	0.602

Table 3. Risk Premia Cross-Sectional Correlations

Correlations across commodities. This table reports the cross-sectional correlations for the risk premia on WTI Oil Crude, Copper, Corn and Silver. Risk premia are those obtained by filtering out the model-implied expected future spot prices for $h = 2, 3, 4$ quarters ahead from the corresponding futures prices. Futures prices data on Crude Oil (WTI) are from the New York Mercantile Exchange (NYMEX), prices are in U.S. dollars per barrel. Futures on Silver and Copper are obtained from the Commodity Exchange (COMEX). The sample period is 01:1995-01:2016, monthly. The first 24 months are cut as burn-in sample for the adaptive learning scheme.

Panel A: Cross-Sectional Correlation for Model-Implied Risk Premia

Commodity	$h = 2$				$h = 3$				$h = 4$			
Crude Oil (WTI)	-				-				-			
Copper	0.361	-			0.282	-			0.243	-		
Corn	0.132	0.142	-		0.143	0.132	-		0.122	0.111	-	
Silver	0.263	0.271	0.232	-	0.221	0.253	0.211	-	0.192	0.191	0.183	-

Table 4. Static Regression Analysis

Static regressions. This table shows the estimates of a static version of the observation equation (9) in which we consider all of the economic risk factors Z_t outlined in Section 3. For the ease of interpretation, all of the economic predictors and risk premia are standardized by dividing by their respective sample standard deviation. Risk premia are those obtained by filtering out the model-implied expected future spot prices for $h = 2, 4$ quarters ahead from the corresponding futures prices. Futures prices data on Crude Oil (WTI) are from the New York Mercantile Exchange (NYMEX), prices are in U.S. dollars per barrel. Futures on Silver and Copper are obtained from the Commodity Exchange (COMEX). WLD represents the index of world industrial production published by the Netherlands Bureau for Economic and Policy Analysis and relate to import weighted, seasonally adjusted, industrial production. USD TW stands for the Federal Reserve U.S. trade weighted exchange rate index, normalized to be equal to one hundred in March 1973. Time-series Momentum and Value are constructed as in Asness et al. (2013). SPX and MXEF represent the Standard and Poor's 500 and the MSCI Emerging Markets indexes as proxy for financial risk. Open Interest (OIN) is defined as the total number of outstanding contracts that are held by market participants at the end of the month. Finally, hedging pressure (HP) is defined as the net excess in short futures positions by commercial traders, i.e. short minus long positions, divided by the amount of outstanding contracts. The data on commercial traders futures positions are from the Commodity Futures Trading Commission (CFTC). The sample period is 01:1995-01:2016, monthly. The first 24 months are cut as burn-in sample for the adaptive learning scheme. Robust standard errors are in parenthesis, $*p < 0.10$, $**p < 0.05$, $***p < 0.01$.

Explanatory Variables												
	Inventories	USD TW	WLD	SPX	MXEF	OIN	Libor 6m	HP	Momentum	Value	Adj R ²	
WTI	h=2	0.010 (0.045)	-0.078 (0.062)	0.103 (0.073)	-0.038 (0.081)	0.191** (0.099)	0.182*** (0.050)	0.084 (0.113)	0.154** (0.069)	-0.551*** (0.104)	-0.217* (0.120)	0.338
	h=4	0.022 (0.039)	-0.071 (0.061)	0.043 (0.050)	-0.011 (0.075)	0.169** (0.085)	0.146*** (0.046)	0.013 (0.147)	0.172** (0.073)	-0.623*** (0.115)	-0.138** (0.068)	0.379
Copper	h=2	-0.079 (0.184)	-0.096 (0.084)	-0.023 (0.065)	0.053 (0.048)	0.005 (0.054)	0.254*** (0.063)	0.081 (0.060)	0.164** (0.078)	-0.367*** (0.092)	0.058 (0.074)	0.341
	h=4	-0.347 (0.247)	0.035 (0.061)	-0.022 (0.060)	0.034 (0.052)	0.010 (0.061)	0.216*** (0.069)	0.063 (0.046)	0.270*** (0.098)	-0.332*** (0.091)	0.119** (0.061)	0.389
Corn	h=2	-0.491 (0.359)	-0.013 (0.065)	-0.090 (0.057)	0.014 (0.055)	0.071 (0.081)	0.115 (0.090)	0.153** (0.076)	0.324*** (0.113)	-0.423*** (0.126)	0.113 (0.111)	0.237
	h=4	-0.347 (0.247)	0.035 (0.061)	-0.057 (0.060)	0.103 (0.072)	-0.010 (0.061)	0.116* (0.069)	0.093** (0.046)	0.270*** (0.098)	-0.332*** (0.091)	0.099 (0.094)	0.389
Silver	h=2	-0.252*** (0.069)	-0.056 (0.043)	-0.056 (0.043)	-0.126 (0.070)	0.100 (0.076)	0.214*** (0.058)	0.104 (0.060)	0.225*** (0.060)	-0.208*** (0.067)	-0.151** (0.069)	0.325
	h=4	-0.283*** (0.069)	-0.045 (0.044)	-0.045 (0.044)	-0.088 (0.069)	0.060 (0.074)	0.193*** (0.056)	0.094 (0.065)	0.204*** (0.060)	-0.204*** (0.069)	-0.149** (0.074)	0.344

Table 5. In-Sample Diagnostics

Root mean squared errors and predictive Bayes factors. This table shows the results of in-sample goodness-of-fit of a dynamic model specification in which risk factors are weighted according to their probability of inclusion, against a model in which none of the economic risk factors is included and the dynamics of the ex-ante risk premia is determined by a time-varying intercept, as well as a model in which all the economic risk factors are included with equal weight. **Panel A:** shows the relative Root Mean Squared Errors (RMSE) for WTI Oil Crude, Copper, Corn and Silver for $h = 2, 3, 4$ quarters ahead. We report the ratio of the RMSE for the No-Factors and All Factors specification with respect to our model. **Panel B:** shows the Bayes factors in $\log_{10}$ scale for the two competing models. Bayes factors are obtained from the marginal likelihoods computed as the harmonic mean of the likelihood values, evaluated at each draw of the parameters sampled from the corresponding full conditional distributions (see Appendix C). The sample period is 01:1995-01:2016, monthly. The first 24 months are cut as burn-in sample for the adaptive learning scheme.

Panel A: Relative Root Mean Squared Error						
Commodity	$h = 2$		$h = 3$		$h = 4$	
	No Factors	All Factors	No Factors	All Factors	No Factors	All Factors
Crude Oil (WTI)	0.488	0.561	0.347	0.555	0.290	0.462
Copper	0.451	0.447	0.418	0.372	0.381	0.508
Corn	0.433	0.767	0.389	0.563	0.307	0.530
Silver	0.556	0.745	0.490	0.459	0.490	0.519

Panel B: Predictive Bayes Factors						
Commodity	$h = 2$		$h = 3$		$h = 4$	
	No Factors	All Factors	No Factors	All Factors	No Factors	All Factors
Crude Oil (WTI)	14.21	3.39	36.56	5.95	50.59	8.62
Copper	20.35	0.86	24.75	2.21	32.70	4.56
Corn	19.99	2.31	31.65	4.17	44.72	7.18
Silver	15.40	4.63	13.02	3.55	13.30	3.19

Figure 1. Expected vs Realized Risk Premia

Ex-ante risk premia. This figure shows the differences between the *expected* payoff, namely the ex-ante risk premium, and the *realized* payoff of a futures position. **Panel A:** shows the payoff structure of a futures position keeping the contract until maturity under no unexpected changes in spot prices. In this case the expected and the realized risk premia coincide. **Panel B:** shows the payoff structure of a futures position keeping the contract until maturity under a negative unexpected fluctuation in spot prices. In this case, the ex-ante and the realized risk premia diverge.

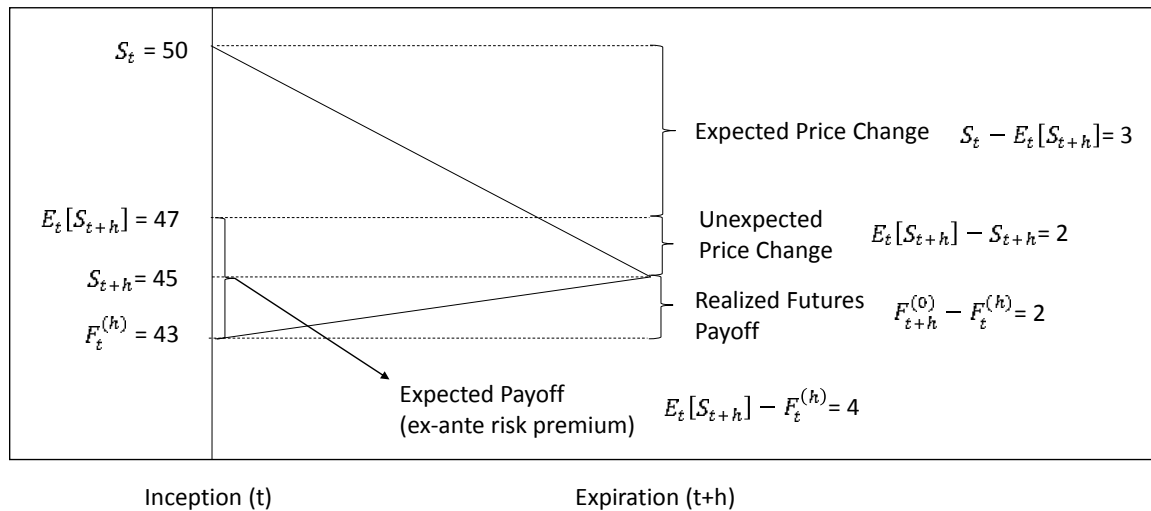
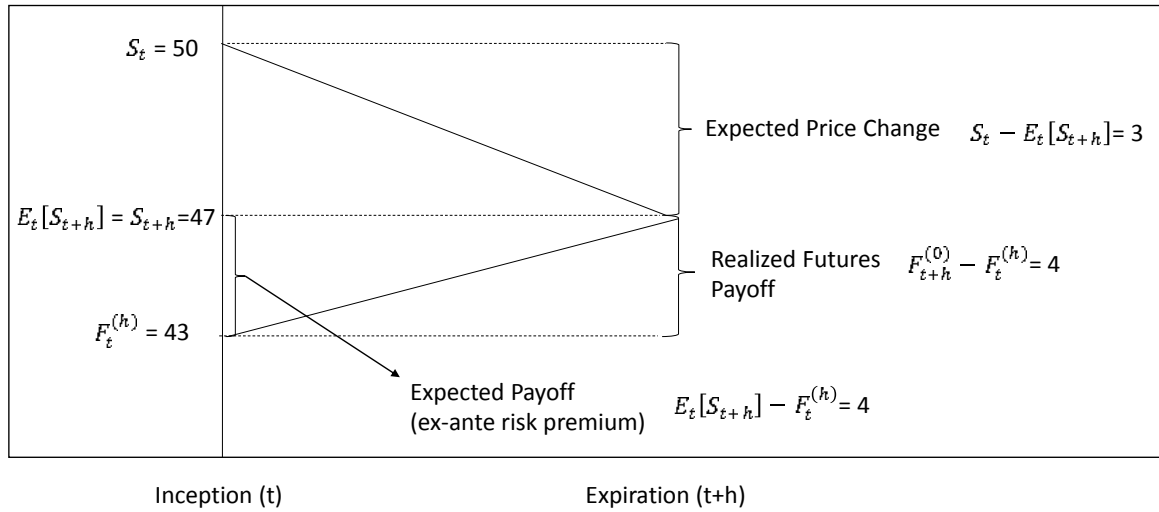


Figure 2. Expectations Errors for Future Spot Prices

Unexpected movements in spot prices. This figure shows the unexpected price realizations with respect to the survey forecasts, i.e. $E_t[S_{t+h}] - S_{t+h}$ for two different horizons, i.e. $h = 2, 4$. **Panel A:** shows the unexpected price changes for WTI Crude Oil (USD/Barrel). **Panel B:** shows the unexpected price changes for Silver (USD/Ounce). Data on Crude Oil (WTI) are from the New York Mercantile Exchange (NYMEX) and on Silver are obtained from the Commodity Exchange (COMEX). The sample period for the Survey is 12:2006-01:2016, aggregated monthly.

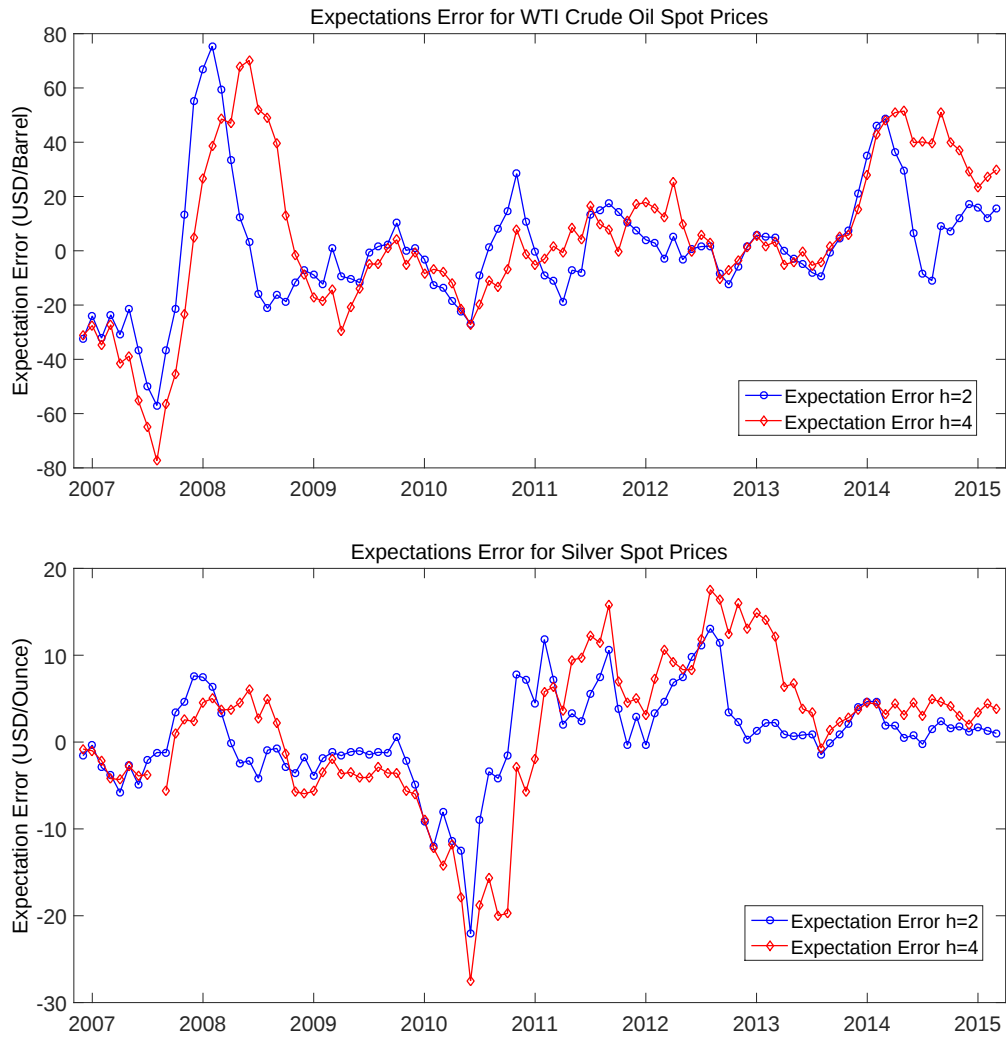


Figure 3. Changes in Commodity Spot Prices and World Industrial Production

Changes in spot prices and world industrial production. This figure shows the year-on-year changes in the (log of) commodity spot prices and the (log of) index of world industrial production. Top panels compare the changes in world industrial production to the variation in the WTI Crude Oil (top-left) and Copper (top-right) spot prices. Bottom panels compare the changes in world industrial production to the variation in the Corn (top-left) and Silver (top-right) spot prices. Spot prices data on Crude Oil (WTI) are from the New York Mercantile Exchange (NYMEX), prices are in U.S. dollars per barrel. Spot prices on Silver and Copper are obtained from the Commodity Exchange (COMEX). Silver futures are quoted in U.S. dollars per troy ounce while Copper is quoted in U.S. cents/pound. The index of world industrial production published by the Netherlands Bureau for Economic and Policy Analysis, and contains aggregate information on industrial production from 81 countries worldwide, which account for about 97% of the global industrial production. The sample period is 01:1995-01:2016.

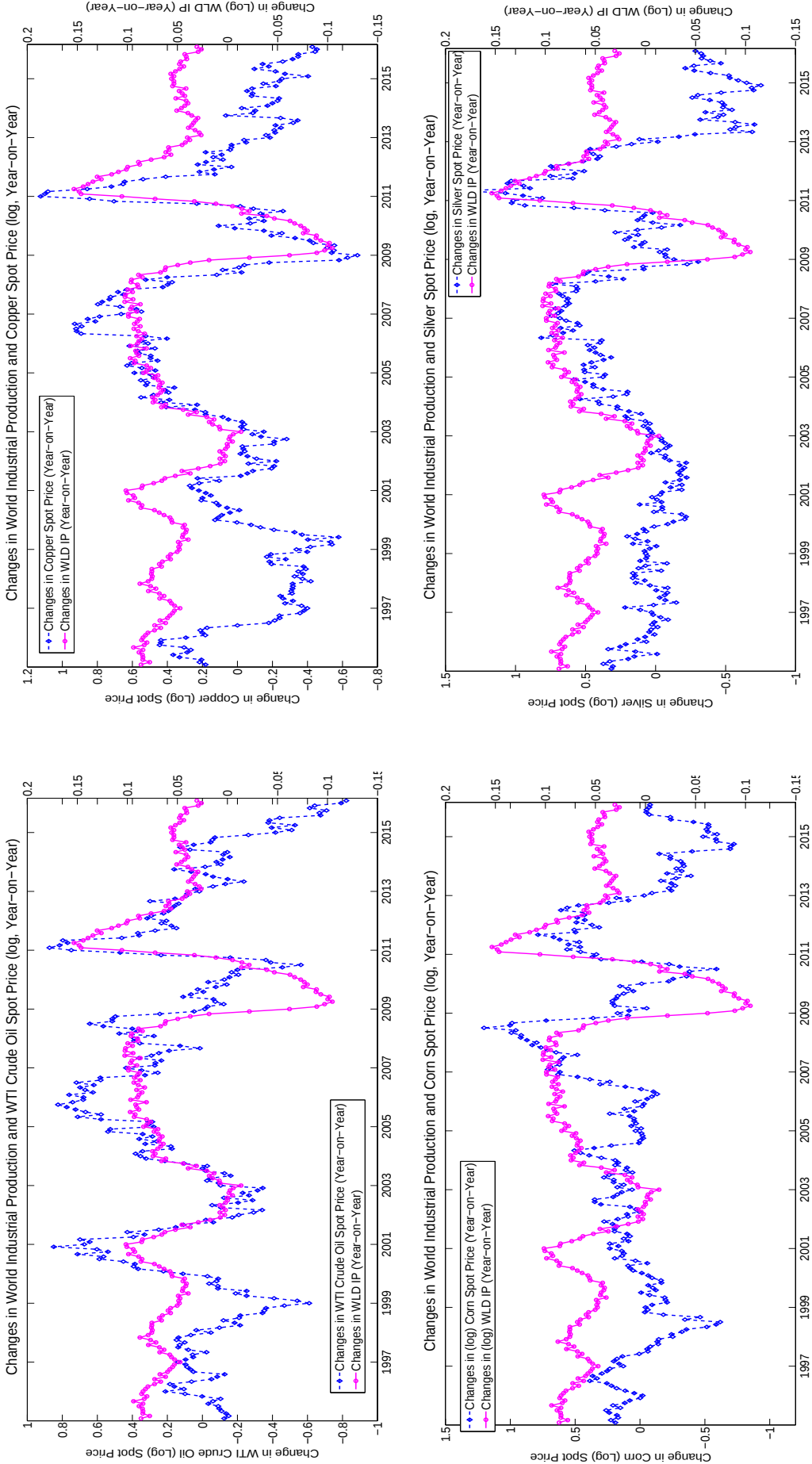


Figure 4. Survey Expectations and Model-Implied Expected Future Spot Prices: Crude Oil (WTI) and Copper

Expectations Formation. This figure compares the expected future spot prices implied by the adaptive learning with the Survey Price Forecasts on Crude Oil (WTI) and Copper come from Bloomberg's Commodity Price Forecasts Database for $h = 2, 4$ quarters ahead. WTI Crude Oil prices are in U.S. Dollars per barrel, whereas Copper prices are transformed from USD Cents/Pound USD/Tonne to match the measurement unit used in the survey forecasts. Data on Crude Oil (WTI) are from the New York Mercantile Exchange (NYMEX) and Copper are obtained from the Commodity Exchange (COMEX). The shaded area at the bottom shows the difference between expectations from adaptive learning and the survey for the overlapping periods. The shaded area at the bottom shows the difference between expectations from adaptive learning and the survey. The sample period of the model-implied expected future spot price is 01:1995-01:2016. The first 24 months are cut as burn-in sample for the adaptive learning scheme.

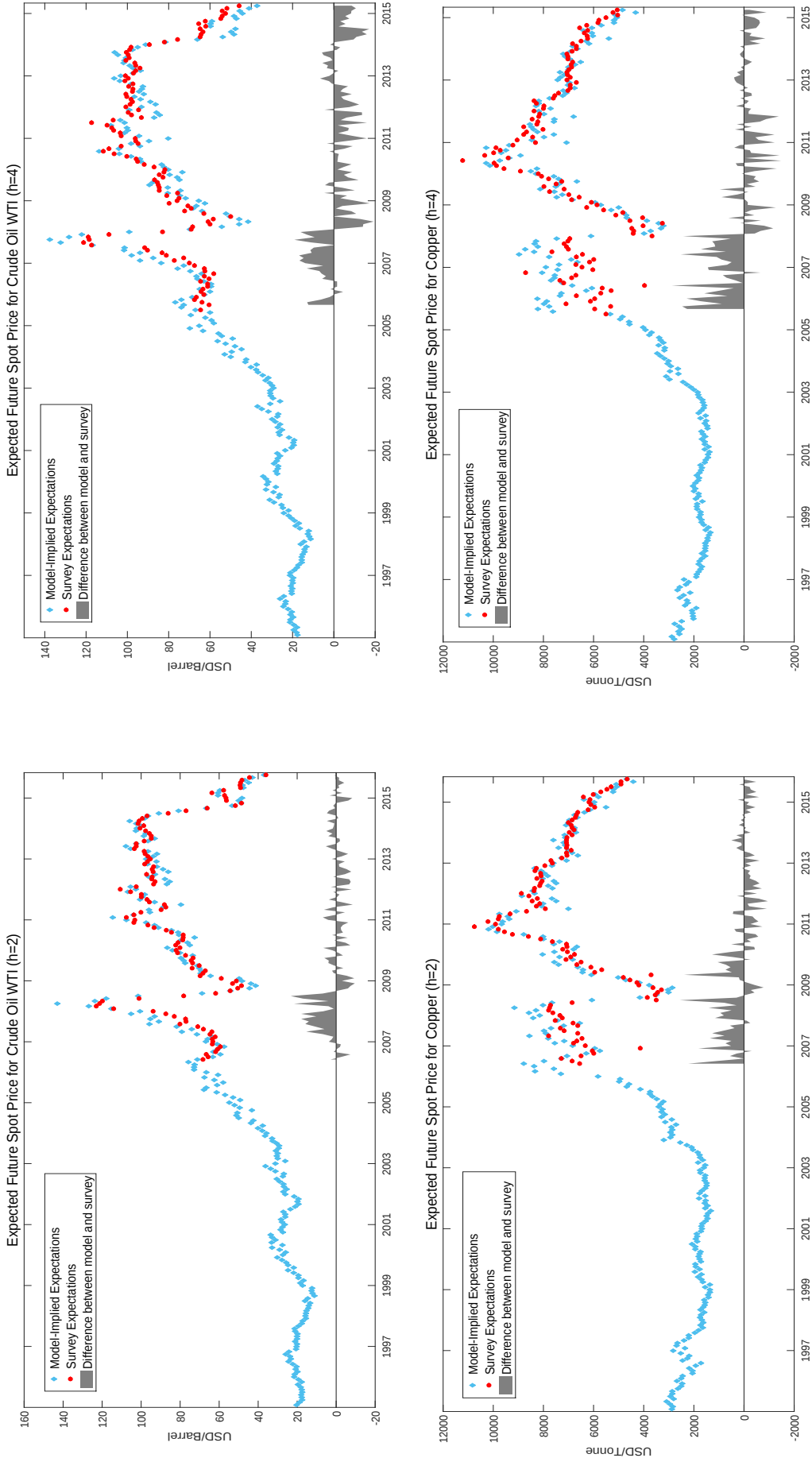


Figure 5. Survey Expectations and Model-Implied Expected Future Spot Prices: Corn and Silver

Expectations Formation. This figure compares the expected future spot prices implied by the adaptive learning with the Survey Price Forecasts on Corn and Silver come from Bloomberg's Commodity Price Forecasts Database for $h = 2, 4$ quarters ahead. Data on Silver are obtained from the Commodity Exchange (COMEX), and data for Corn are from the Chicago Board of Trade (CBOT) with price quotation in USD cents per bushel. The shaded area at the bottom shows the difference between expectations from adaptive learning and the survey for the overlapping periods. The sample period for the Survey is 12:2006-01:2016, aggregated monthly. The sample period of the model-implied expected future spot price is 01:1995-01:2016. The first 24 months are cut as burn-in sample for the adaptive learning scheme.

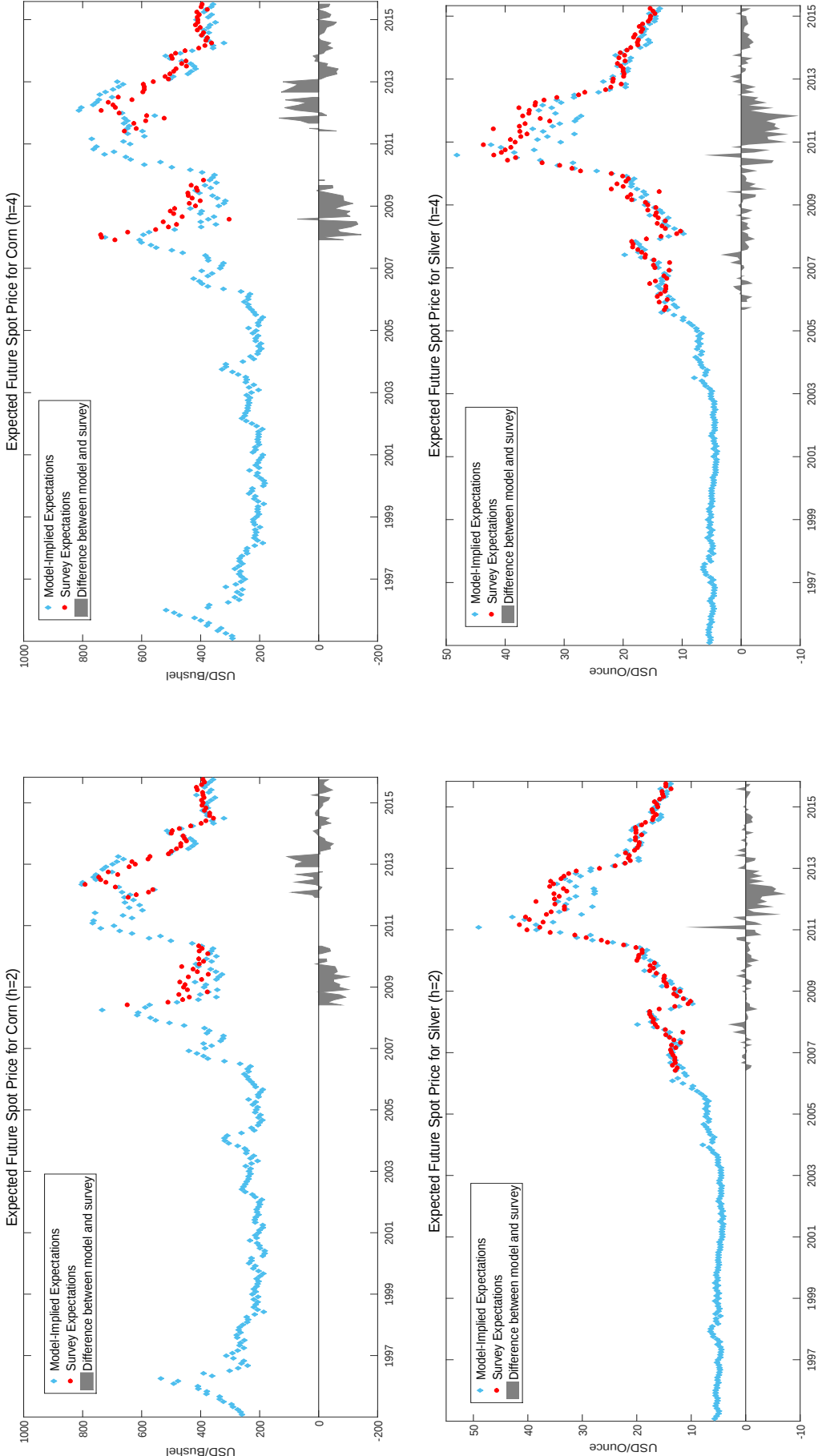


Figure 6. Recursive Averages of Commodity Risk Premia

Exponential weighted moving average of commodity risk premia. This figure shows the recursive average of the model-implied risk premia for WTI Crude Oil, Copper, Corn, and Silver for $h = 2, 4$ quarters ahead. Data on Crude Oil (WTI) are from the New York Mercantile Exchange (NYMEX), prices are in U.S. dollars per barrel. Data on Silver and Copper are obtained from the Commodity Exchange (COMEX). Copper prices are transformed from USD Cents/Pound USD/Tonne to match the measurement unit used in the survey forecasts. The sample period of the model-implied expected future spot price is 01:1995-01:2016. The first 24 months are cut as burn-in sample for the sequential learning scheme.

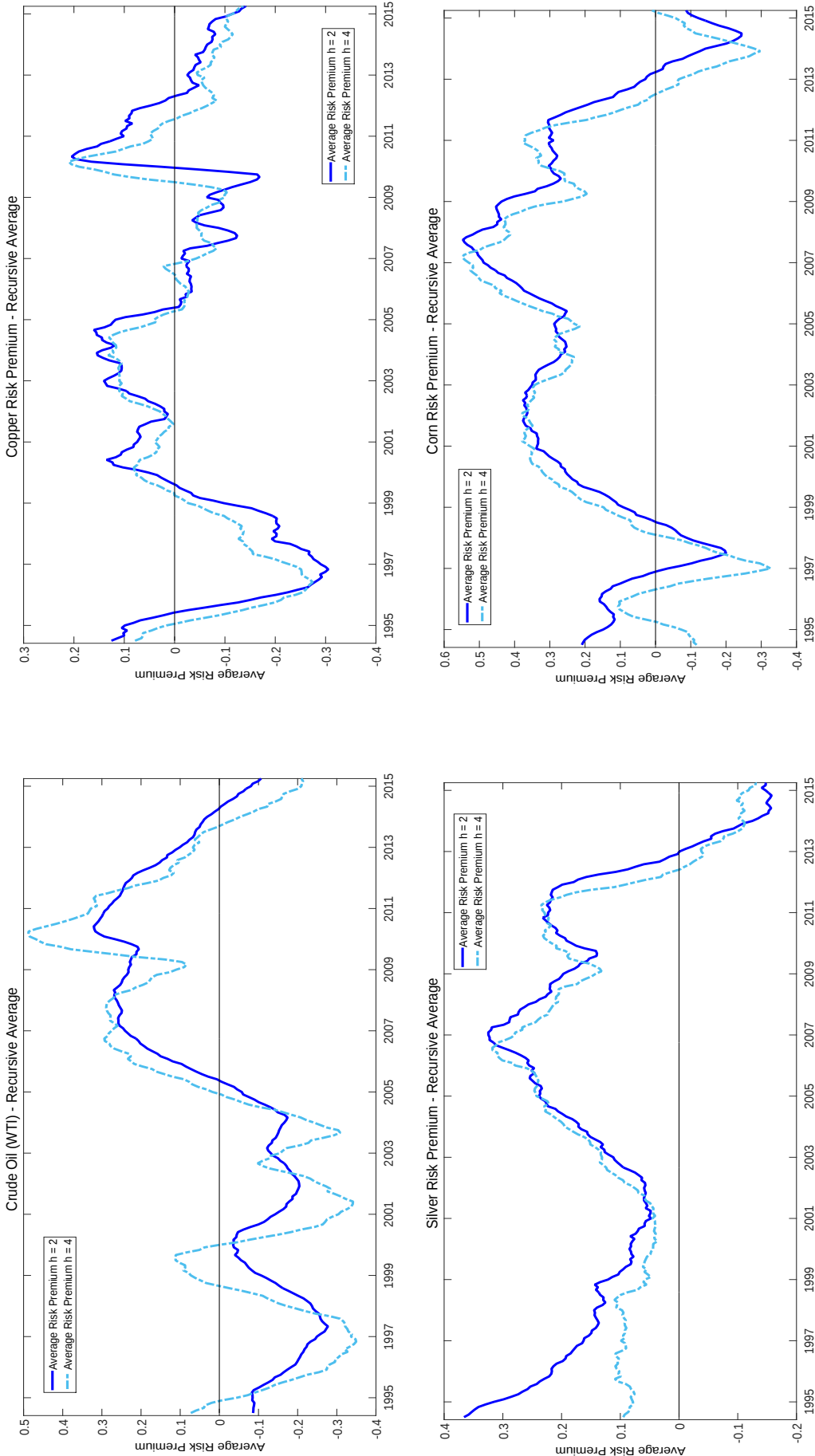


Figure 7. Probability of Inclusion of a Risk Factor in the Dynamics of Risk Premium: Crude Oil (WTI) and Copper

Dynamic inclusion probabilities for each risk factor. This figure shows the posterior dynamic inclusion probabilities for the risk factors driving the dynamics of WTI Crude Oil and Copper risk premia for $h = 2, 4$ quarters ahead. For the ease of exposition, we only show those posterior inclusion probabilities which exceeds a threshold value of 0.5 at least one point in time, that is, any predictor where the inclusion probability is never above 0.5 is not reported in the corresponding figure. Data on Crude Oil (WTI) are from the New York Mercantile Exchange (NYMEX), prices are in U.S. dollars per barrel. Data on Copper are obtained from the Commodity Exchange (COMEX), quoted in U.S. cents/pound, respectively. We convert the price of futures contracts to USD/tonne to match the measurement unit of the survey forecasts, that instead refer to the London Metal Exchange (LME) contract. The sample period of the model-implied expected future spot price is 01:1995-01:2016. The first 24 months are cut as burn-in sample for the sequential learning scheme.

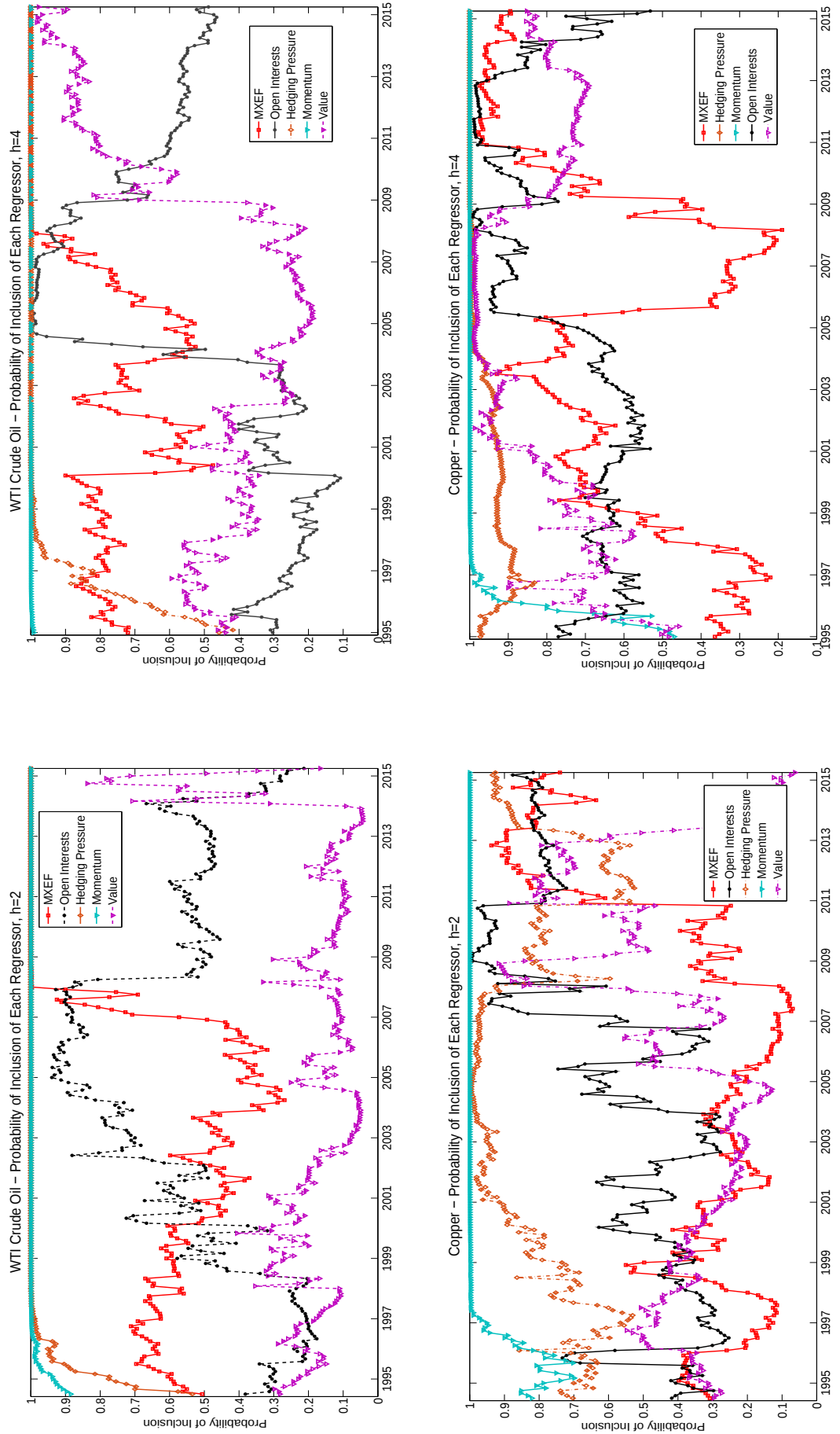


Figure 8. Probability of Inclusion of a Risk Factor in the Dynamics of Risk Premium: Corn and Silver

Dynamic inclusion probabilities for each risk factor. This figure shows the posterior dynamic inclusion probabilities for the risk factors driving the dynamics of WTI Crude Oil and Copper risk premia for $h = 2, 4$ quarters ahead. For the ease of exposition, we only show those posterior inclusion probabilities which exceeds a threshold value of 0.5 at least one point in time, that is, any predictor where the inclusion probability is never above 0.5 is not reported in the corresponding figure. Data on Silver are obtained from the Commodity Exchange (COMEX), quoted in U.S. dollars per troy ounce. Data on Corn are from the Chicago Board of Trade (CBOT) with price quotation in USD cents per bushel. The sample period of the model-implied expected future spot price is 01:1995-01:2016. The first 24 months are cut as burn-in sample for the sequential learning scheme.

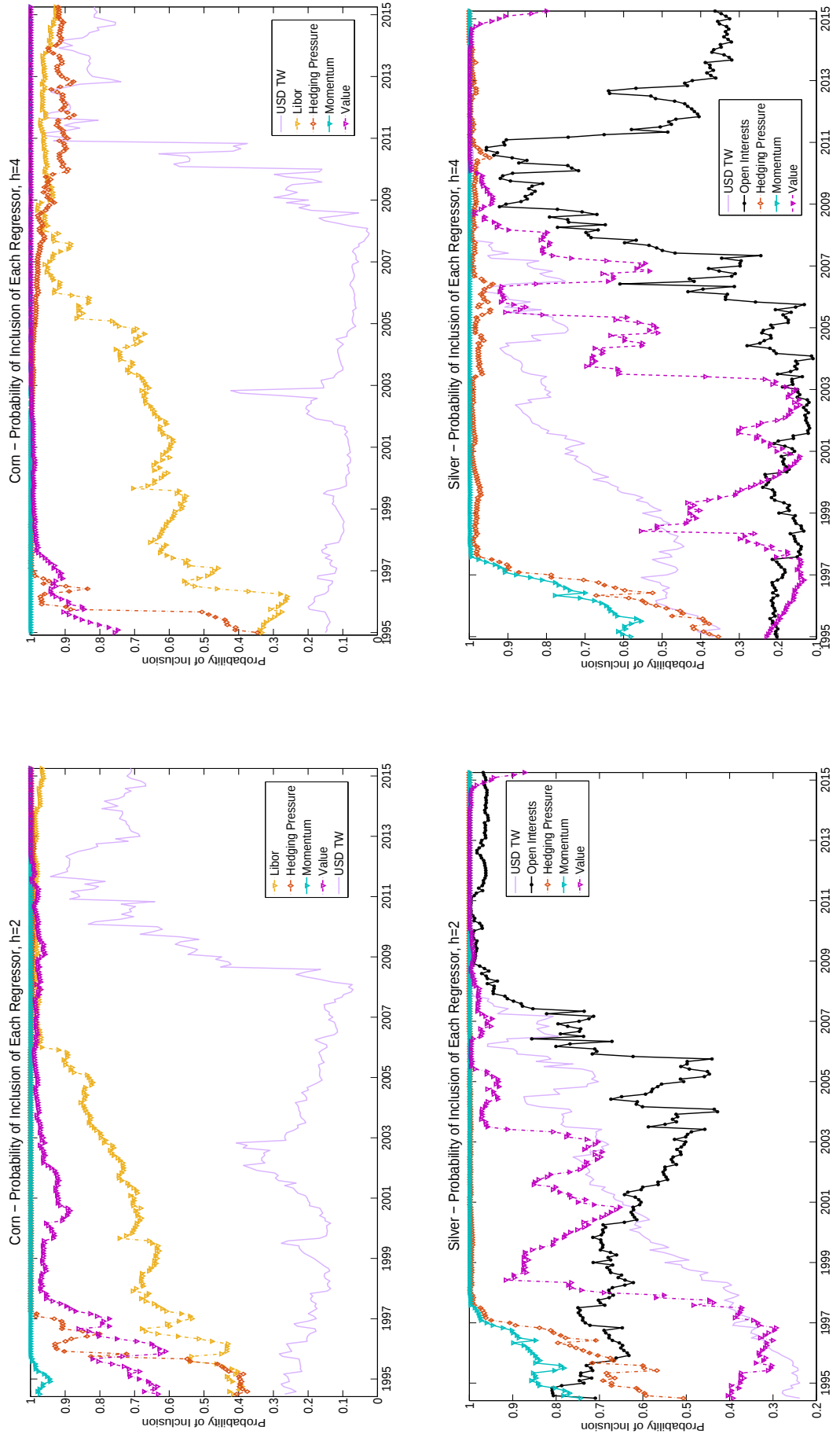


Figure 9. Hedging Pressure on Ex-Ante Risk Premia

Time-varying betas on Hedging Pressure. This figure shows the posterior median for the beta on Hedging Pressure, i.e. $\hat{\beta}_{HP}$, across commodities. The solid blue line represents the estimated beta for the two-quarter ahead risk premia, and the solid red line shows the estimated beta for the four-quarter ahead expected payoff. We compute the level of HP for different commodities as the net excess in short futures positions by commercial traders, i.e. short minus long positions, divided by the amount of outstanding hedging contracts. The data on commercial traders futures positions are from the Commodity Futures Trading Commission (CFTC). The sample period is 1993:01-2016:01. The first 24 months are cut as burn-in sample for the sequential learning scheme.

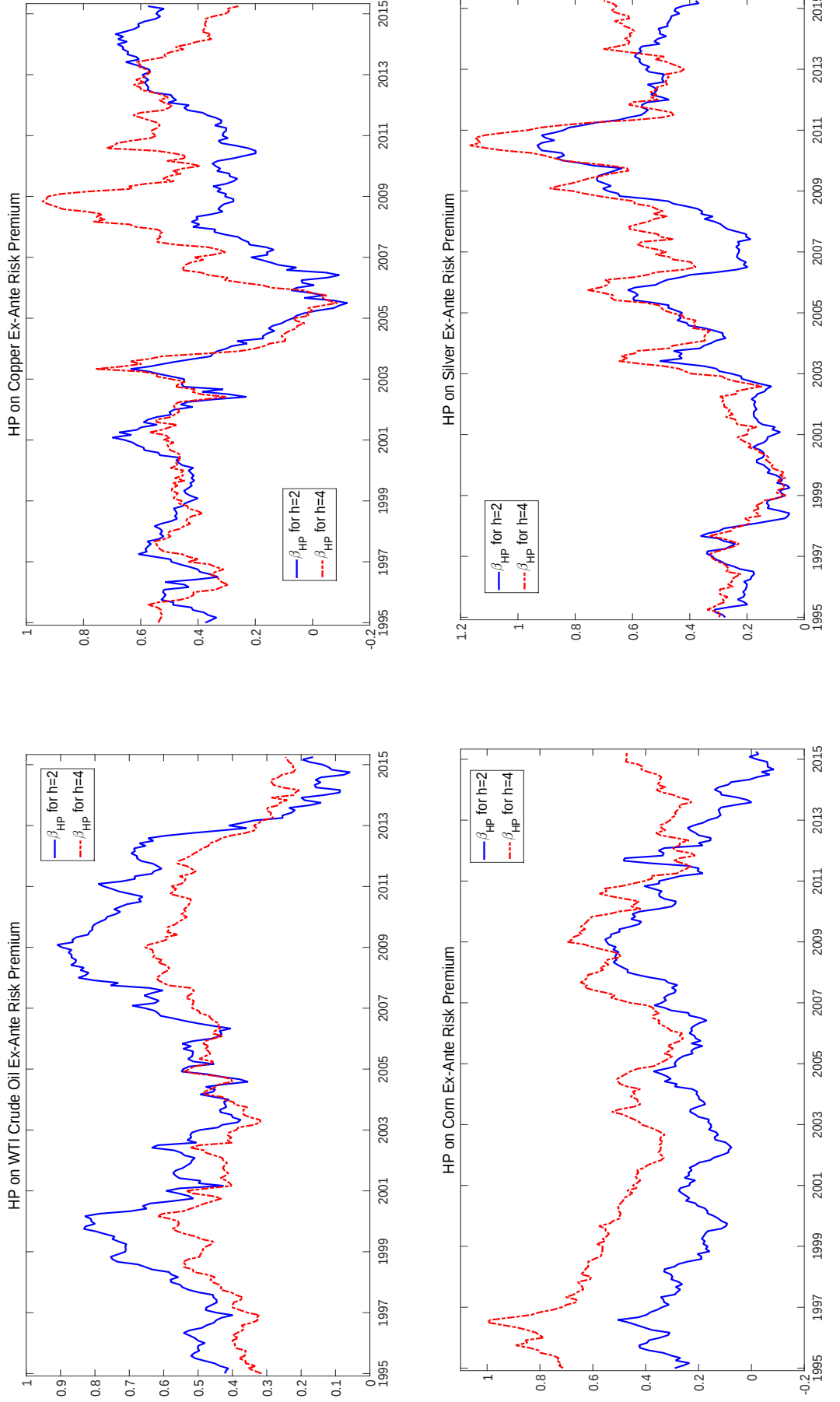


Figure 10. Time-Series Momentum on Ex-Ante Risk Premia

Time-varying betas on Momentum. This figure shows the posterior median for the beta on Momentum, i.e. $\hat{\beta}_{Mom}$, across commodities. The solid blue line represents the estimated beta for the two-quarter ahead risk premia, and the dashed-dot red line shows the estimated beta for the four-quarter ahead expected payoff. We construct time-series Momentum as the return over the past 12 months skipping the most recent month on each commodity future (see Asness et al. 2013). Data are obtained from different resources. Futures prices data on Crude Oil (WTI) are from the New York Mercantile Exchange (NYMEX), prices are in U.S. dollars per barrel. Futures on Silver and Copper are obtained from the Commodity Exchange (COMEX). Silver is quoted in U.S. dollars per troy ounce while Copper is quoted in U.S. cents/pound. We convert the price of Copper futures contracts to USD/tonne to match the measurement unit of the survey forecasts, that instead refer to the London Metal Exchange (LME). Corn futures prices are from the Chicago Board of Trade (CBOT) with price quotation in USD cents per bushel. The sample period 1993:01-2016:01. The first 24 months are cut as burn-in sample for the sequential learning scheme.

