The Effect of Firm Entry on Capacity Utilization and Macroeconomic Productivity

Anthony Savagar[*]
Huw Dixon

August 31, 2016

Abstract

This paper argues that firm entry causes endogenous fluctuations in macroeconomic productivity through its effect on incumbent firms' capacity utilization. The analysis shows that imperfect competition causes long-run excess entry leading to many small firms each with excess capacity. Since entry occurs slowly, macroeconomic shocks are initially borne by these incumbents who respond by altering their capacity utilization. As they vary utilization efficiency changes because of non-constant returns to scale and this aggregates to affect the economy's productivity. In the long run, entry occurs and new firms dissipate the shock, which alleviates incumbents' alteration in capacity. Therefore the endogenous productivity effect is temporary.

How does industry competition affect firm entry and consequently macroeconomic productivity? We argue that under imperfect competition firm entry causes endogenous productivity fluctuations in the short run as firms adjust. Since Chamberlin 1933, economists have understood that in a monopolistically competitive economy there is excess capacity. This means that each firm produces below their cost-minimizing full capacity because underproduction earns monopoly profits. In relation to firm entry, underproduction and monopoly profits translate to excessive entry of small firms. This paper's

[*]Corresponding author a.savagar@kent.ac.uk. Thanks to seminar participants: Strasbourg Dynamic Macro, MMF Durham, Leicester, UCL RES, Warwick, York. Notably Leo Kaas, Frédéric Dufourt, Raouf Boucekkine, Vito Polito, Ilaf Scheikh Ilard, and Akos Valentinyi. Thanks to Luis Costa and Paulo Brito at University Lisbon ISEG for hospitality and comments.

focus is the transition toward this excess capacity steady state. During transition the capacity utilization of firms fluctuates which causes endogenous productivity variations. The productivity variations arise because with non-constant returns to scale capacity utilization affects productivity. The aim of the paper is to provide a tractable theory to connect firm entry, capacity utilization and productivity dynamics. Although the main scope is theoretical, we note that contemporary 'productivity puzzles' (see Pessoa and Van Reenen 2014 or Haskel, Goodridge, and Wallis 2015) provide excellent motivation to better understand the theoretical underpinnings of endogenous productivity movements, and the testable implications of our model should interest applied researchers. For example, our theory implies that a negative supply-side shock to the economy is not immediately dissipated by firms exiting, and thus worsens capacity utilization temporarily which exacerbates negative productivity until firms have time to exit. Furthermore, following the shock more competitive economies suffer a weaker fall in productivity than less competitive economies.[1]

The paper's main result is that short-run measures of productivity are misleading if firm entry (or exit) has not had time to adjust. We show that imperfect competition leads to excessive entry in the long run, so there are too many firms each with excess capacity. Then a movement away from equilibrium causes firms to increase (reduce) capacity so they temporarily produce closer to (further from) full capacity in the short run whilst other firms gradually enter to arbitrage monopoly profits (exit to avoid negative profits). For example, a positive production shock causes a short-run productivity gain. Incumbents raise output as other firms cannot enter in the short run to absorb the positive shock. On raising output, increasing returns improve incumbents' productivity which aggregates to the macroeconomy. Since the output of the incumbents exceeds equilibrium and yields monopoly profits, prospective firms gradually enter to arbitrage the profits back to zero profit long-run equilibrium. Therefore the productivity gain due to increased capacity utilization is only temporary, as equilibrium excess capacity returns in the long run. The business cycle implications of this theory are that capacity utilization and net entry are procylical, and productivity is positively correlated with capacity utilization. We show these relationships in figure 1 for quarterly US data 1994-2013[2], where the correlation between GDP and

[1]Interestingly the highly competitive US economy has been relatively resilient to the productivity downturns experienced in Europe since 2007.

[2]Data are logged and HP-filtered at a quarterly frequency. Raw data is taken from Federal Reserve FRED database, and the unique MNEMONICS are GDP and TCU. The definition of TCU is *"Capacity utilization is the percentage of resources used by corporations and factories to produce goods in manufacturing, mining, and electric and gas utilities for*

net entry is 0.64 and between GDP and capacity utilization is 0.84. In addi-

Figure 1: US Procylical Capacity Utilization and Net Entry

tion to our simple indicators, there is substantial empirical research to support these relationships. Portier 1995 and Dos Santos Ferreira and Dufourt 2006 show procyclicality of French business formation, and studies including Lee and Mukoyama 2015, Campbell 1998 and Ambler and Cardia 1998 find procyclicality in the US. Morrison 1992 and Berndt and Fuss 1986 emphasize the positive relationship between productivity and capacity utilization which Caves, Christensen, and Swanson 1981 tie to scale economies. More recently Rossi and Chini 2016 find that entry is procyclical and sluggish in US data 1977-2013.

The focus on theory and analytical dynamics distinguishes this paper from related research (discussed below) that typically focuses on quantitative replication of empirical results. These quantitative works are stochastic and the dynamics are simulated, as opposed to our qualitative deterministic study that avoids specifying functional forms or solving parameterized, path dependent models. The advantage of our approach is to give an analytical understanding of the effect entry has on the model variables, particularly

all facilities". Net entry is calculated from quarterly firm birth and death figures taken from the Bureau of Labor Statistics BED program.

productivity. The model includes important features from the main papers in the literature so far: endogenous labour and capital accumulation (Bilbiie, Ghironi, and Melitz 2012); imperfect competition (Etro and Colciago 2010) and entry congestion effects that Lewis 2009 finds empirically important and Berentsen and Waller 2015 model structurally[3]. Imperfect competition is our focus since it is common to these models but its relation to entry and mapping to dynamics has not been investigated. Whilst Aloi and Dixon 2003 follow a similar line of argument in an open economy model without capital, we focus on relating this insight to a business cycle framework with capital. Excess capacity is a standard feature of models with imperfect competition, but its relationship to entry is often overlooked, since three features are necessary for their to be a capacity utilization effect on productivity. They are imperfect competition, non-constant returns to scale and slow firm entry. The counterfactuals of these three assumptions clarifies their necessity. 1) Without slow entry, there is standard instantaneous free entry[4], so there is no short-run variation in capacity utilization. 2) If there were perfect competition, then firms produce efficiently at minimum average cost. Therefore they do not have excess capacity and utilization has no first order effect on productivity. 3) If there were constant returns to scale, entry has no effect: there is no difference between a few large firms or many small firms producing all output.

Related Literature The paper bridges two groups of literature: quantitative endogenous entry DSGE models and qualitative dynamical systems analyses of determinacy, returns to scale, capacity utilization, and imperfect competition. A growing group of endogenous entry DSGE papers show promising quantitative results from including richer industrial organization features in standard RBC models. Notable papers are Jaimovich and Floetotto 2008, Etro and Colciago 2010, Lewis and Poilly 2012 and Bilbiie, Ghironi, and Melitz 2012. The former two provide initial forays into qualitative dynamics of the systems they simulate. These are complementary exercises to establish saddlepath stability, all in discrete time due to simulation. A simplicity trade-off they employ is to reduce their models' dimensionality to one for qualitative analysis, and this involves dropping a state variable (capital) from the system leaving only number of firms as a state variable. This rules out the most empirically appealing dynamics of their simulations: those that exhibit nonmonotone impulse responses.

Our model is closely related to Brito and Dixon 2013. Their model is a

[3]Congestion means entry costs increase with number of entrants.

[4]The two extreme entry cases–instantaneous free entry and no entry fixed number of firms–are special cases the model.

perfectly competitive Cass-Koopmans model with labour-leisure choice, capital accumulation and the Datta and Dixon 2002 entry model. They show that with perfect competition, firm entry causes empirically plausible macroeconomic dynamics in a Ramsey-Cass-Koopmans model. The main result is that entry is sufficient to give nonmonotone deviations from equilibrium under fiscal shocks. We add imperfect competition in the product market, so firms control output price. This addition creates the additional capacity utilization mechanism and excess entry.[5] In the aforementioned papers, firms have U-shaped average cost curves or monotone decreasing AC that endogenously give rise to locally increasing returns to scale under imperfect competition. Our paper also assumes an endogenous entry cost, driven by a congestion effect, where the sunk entry cost rises with the number of entrants: Lewis 2009 offered initial empirical support for congestion effects of firm entry. This VAR study showed congestion effects can account for observed lags in monetary policy. Congestion effects are also popular in industrial organization literature such as Das and Das 1997 and Ericson and Pakes 1995.

Section 1 develops a model of firm entry in the macroeconomy. Section 2 analyses comparative statics and comparative dynamics. Section 3 is the main result which builds upon the the analytical results to show that productivity varies endogenously. Section 4 concludes with a comparison of our results to the standard RBC model with capital utilization.

1 Endogenous Entry Model with Imperfect Competition

The model follows a Ramsey-Cass-Koopmans setup with additions of imperfect competition, firm entry, and capital accumulation. The model is deterministic, and labour is endogenous as developed by Brock and Turnovsky 1981. We solve the model as a decentralised equilibrium because imperfect competition distorts the optimising behaviour of the firm which does not coincide with centralised equilibrium of a planner optimising behaviour of the economy.

1.1 Household

The economy is inhabited by a continuum of infinitely-lived identical households. A household seeks policy functions of consumption $\{C(t)\}_0^\infty \in \mathbb{R}$ and

[5] Etro and Colciago 2010 also note that Cournot competition causes inefficiency through excess entry. Etro 2009 provides an excellent survey of macroeconomic models with endogenous entry and endogenous market structures.

labour supply $\{L(t)\}_0^\infty \in [0,1]$ that maximise lifetime utility. Aggregate utility $\mathrm{U}: \mathbb{R}^2 \to \mathbb{R}$ is composed of individual utility $u : \mathbb{R} \times [0,1] \to \mathbb{R}$ which is jointly concave and differentiable in both of its arguments. It is strictly increasing in C and strictly decreasing in L, so $u_C > 0, u_L < 0$. Thus u_L is labour disutility, or read $-u_L$ as labour utility. Both goods are normal $u_{CC},\ u_{LL} < 0$, so marginal utility of consumption and disutility of labour are diminishing, and utility is additively separable $u_{CL} = 0$.

A representative household solves the utility maximisation problem

$$\max \mathrm{U} := \int_0^\infty \mathrm{u}(C(t), 1 - L(t)) \exp^{-\rho t} \mathrm{d}t \tag{1}$$

$$\text{s.t.} \quad \dot{K}(t) = rK(t) + wL(t) + \Pi(t) - C(t) \tag{2}$$

where exp is the exponential function and $\rho \in (0,1)$ is the discount factor over time $t \in \mathbb{R}_+$.[6] The household owns capital $K \in \mathbb{R}$ and takes equilibrium rental rate r and wage rate w as given by the market rate. Households own firms and receive firm profits $\Pi \in \mathbb{R}$. Solving the optimization problem simplifies to three conditions for optimal consumption and labour.[7] They are the intertemporal consumption Euler equation (3), intratemporal labour-consumption trade-off (4) and the resource constraint (2).

$$\dot{C}(r,\rho) = \frac{C}{\sigma(C)}(r - \rho), \quad \text{where } \sigma(C) = -C\frac{u_{CC}(C)}{u_C(C)} \tag{3}$$

$$w = -\frac{u_L(L)}{u_C(C)} \tag{4}$$

The solution of the dynamic optimization problem for household consumption will be one of the solutions of this system of two differential equations that satisfy the initial condition $K(0) = K_0$. To complete the boundary value problem we add two transversality conditions on the upper boundary. This completes the unique solution for the boundary value problem: three variables (K, L, C), three equations (2)-(4), three boundary conditions (5). $\lambda(t)$ is the costate variable associated with the state variable K in the Maximum Principle.

$$\lim_{t\to\infty} K(t)e^{-\rho t} \geqslant 0\ , \quad \lim_{t\to\infty} K(t)\lambda(t)\exp^{-\rho t} = 0\ , \quad K_0 = K(0) \tag{5}$$

[6]For clarity, we follow the continuous time literature e.g. Benhabib, Nishimura, and Shigoka 2008 by suppressing time dependence $X(t)$ to X after initial introduction. To be sure, time dependent variables are $C, e, K, n, L, \Pi, Z, q, r, w, Y, y, k, l, \pi, \mathfrak{u}, \mathcal{P}$ and constant parameters are in table 1.

[7]Appendix A solves the Hamiltonian problem.

Equations (2)-(5) characterize optimal paths of consumption and labour. In equilibrium these equations continue to hold, with factor prices w and r at their market value, which arises from firms profit maximisation under imperfect competition as we show in section (1.2).

1.2 Firm Production and Strategic Interactions

There is imperfect competition in the product market and perfect competition in the factor market. Product market imperfect competition allows firms to control output price, whereas factor market perfect competition means firms take factor prices (wage and interest rate) as determined by the market. Since each firm faces the same factor prices resources are divided equally among firms.

Definition 1.
We denote a per firm variable in lower case where $n(t) \in \mathbb{R}_+$ is the number of firms.

$$k(t) = \frac{K(t)}{n(t)}, \quad l(t) = \frac{L(t)}{n(t)}, \quad y(t) = \frac{Y(t)}{n(t)}$$

$Y(t)$ is the final good and is a CES aggregate of each $i \in n$ firms' output. A firm is a 1-firm industry, so $\theta \in (1, \infty)$ is intersector substitutability which is a Cobb-Douglas aggregator in the imperfectly substitutable limit $\theta \to 1$.[8] The $n^{\frac{1}{1-\theta}}$ component removes love-of-variety.

$$Y = n^{\frac{1}{1-\theta}} \left[\int_0^n y(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{1-\theta}} \tag{6}$$

With the unit price of the aggregate good as the numeraire the sectoral demand $y(i)$ directed at each 1-firm industry takes constant elasticity form

$$y(i) = p(i)^{-\theta} Y, \quad \forall i \in (0, n) \tag{7}$$

with inverse demand for industry i given by $p(i)$. Firms have the same production technology

$$y(t) = \max\{A\mathrm{F}(k, l) - \phi, 0\} \tag{8}$$

[8]Sector, firm, industry and product are synonyms in this model since a sector is a 1-firm industry.

where $F : \mathbb{R}^2_+ \supseteq (k, l) \to \mathbb{R}_+$ is a firm production function with continuous partial derivatives which is homogenous of degree $\nu < 1$ (hod-ν) on the open cone $\mathbb{R}^2_+$, and $\phi \in \mathbb{R}_+$ a fixed cost denominated in output. F has concavity properties $F_k, F_l, F_{kl} = F_{lk} > 0, F_{kk}, F_{ll} < 0, F_{kk}F_{ll} - F^2_{kl} > 0$. A is a technology parameter. This production function gives a U-shaped average cost curve because there are initally increasing returns from the fixed cost, but these diminish due to decreasing returns to scale from the production technology $F(k, l)$ which is convex in capital and labour and leads to increasing marginal cost. The fixed cost ϕ is the nonconvexity which prevents some firms producing, and it occurs each period, which is different to the sunk entry cost that is paid once to enter (see Entry section 1.3)[9]. The increasing returns that ϕ causes is a standard outcome of the firm entry in macroeconomics literature (e.g. Bilbiie, Ghironi, and Melitz 2012 and Lewis and Poilly 2012) and literature on new international economics (Melitz 2003). There is also empirical evidence by Hall 1989 and Diewert and Fox 2008 for strongly increasing returns in US manufacturing. Inada's conditions hold so that marginal products of capital and labour are strictly positive which rules out corner solutions.

Aggregate output Y is the integral across each industry that produces according to the firm level production function, and under symmetry this simplifies to

$$Y(K, L, n) = \int_0^n y(i)di = ny = n\left[AF(k, l) - \phi\right] \tag{9}$$

Therefore we can view the number of firms as a quasi input in production, since it determines the allocation of capital and labour across firms. From (8), the firm level of production is homogeneous of degree 0 (hod-0) in inputs (K, L, n), whereas the aggregate production function (9) is hod-1 in (K, L, n). For example, consider a doubling of all inputs (double capital, labour and number of firms): at the firm level output and productivity are unaffected, so that aggregate output doubles since there are now twice as many firms.

Since there is a fixed cost and increasing marginal cost, the average cost curve is U-shaped, and there is an optimal efficient level of production at minimum average cost where average cost and marginal cost intersect. These efficient levels of production and corresponding capital and labour per firm are given by the optimal firm size condition $Y_n = 0$[10] for a given level of

[9]As in Jaimovich 2007 and Rotemberg and Woodford 1992 the role of this parameter is to reproduce the apparent absence of pure profits despite market power. It allows zero profits in the presence of market power

[10]Under symmetry aggregate output is $Y = ny$ so given (K, L) we have $Y_n = y + ny_n = y + n \cdot \frac{-\nu}{n}(y - \phi)$, where subscript denotes derivative e.g. Y_n is $\frac{\partial Y}{\partial n}$.

(K, L). Therefore the efficient levels, denoted with an e superscript, are

$$F(k^e, l^e) = \frac{\phi}{A(1-\nu)} \tag{10}$$

$$y^e = AF(k^e, l^e) - \phi = \frac{\phi\nu}{1-\nu} \tag{11}$$

Technology A does not affect optimal firm output, but it reduces variable production $F(k, l)$. From (10) $\left(\frac{1}{n}\right)^{\nu} F(K, L)^e = \frac{\phi}{A(1-\nu)}$ whence $n^e = \left(\frac{A(1-\nu)}{\phi} F(K, L)\right)^{\frac{1}{\nu}}$ so number of firms is hod-1 in capital and labour. A rise in capital and labour by some proportion will raise number of firms by the same proportion, so capital and labour per firm remain unchanged. Hence the result in (11) that optimal output per firm is a fixed level, independent of model variables.

Under imperfect competition the efficient outcomes do not arise in equilibrium but are a limiting case of zero market power. Under monopolistic competition a firm maximises profits subject to sectoral demand (7) given real wage w and interest rate r by choosing labour and capital such that the following factor market equilibrium holds

$$AF_k(k, l)(1 - \zeta) = r \tag{12}$$

$$AF_l(k, l)(1 - \zeta) = w \tag{13}$$

Factor market equilibrium shows that the marginal revenue product of capital equates to the cost of capital and the marginal revenue product of labour equates to the wage. Wage is implicitly defined as a function of (K, L, n, ζ). Standard derivation[11] shows that the Lerner Index is the inverse of intersector substitutability $\zeta = \frac{1}{\theta} \in (0, 1)$. The Lerner Index is the difference between price and marginal cost as a proportion of price $(\frac{P-MC}{P})$ the limits capture no market power $\zeta = 0$ when goods are highly substitutable and total market power $\zeta = 1$ when goods are completely differentiated.[12]

[11] Maximise firm profits subject to sectoral demand 7.

[12] Macroeconomists often express the equivalent idea in terms of a 'price-over-marginal-cost' markup usually denoted $\mu \in (1, \infty)$. This markup is defined as price over marginal costs $(\frac{P}{MC})$ so its relationship to the Lerner Index is $\mu = \frac{1}{1-\zeta} = \frac{\theta}{\theta-1}$.

1.2.1 Zero Profit Outcome and Excess Capacity

Market power ζ suppresses factor prices which raises profits π relative to the perfect competition case $\pi(\zeta) > \pi(\zeta = 0)$[13]

$$\pi = (1 - (1 - \zeta)\nu)AF(k, l) - \phi \tag{14}$$

The extra profit $\zeta\nu AF(k, l)$ from imperfect competition is crucial to understanding the entry-imperfect-competition relationship. Profits offer entry incentives, so monopoly profits encourage excess entry. In section (2.1) we show that zero profits arise naturally as a fixed point of the economy, but now take zero profits as given and steady-state output per firm becomes

$$F(k^*, l^*) = \frac{\phi}{A(1 - (1 - \zeta)\nu)} \tag{15}$$

$$y^*(\phi, \nu, \zeta) = AF(k^*, l^*) - \phi = \frac{\nu(1 - \zeta)\phi}{1 - (1 - \zeta)\nu} \tag{16}$$

where an asterix denotes steady-state under monopolistic competition. Therefore zero-profit output per firm is increasing in both fixed cost and returns to scale and is decreasing in market power. More market power raises marginal revenue products of inputs, so less needs to be produced in order to cover fixed costs and attain zero profits. Importantly y^* is unaffected by A, so firms always produce the same long-run level of output regardless of technology. Comparing the outcome under imperfect competition to the efficient outcome ((10) and (11) with (15) and (16)) demonstrates excess capacity and its link to increasing returns.

Proposition 1 Excess Capacity.
In zero-profit steady state firms competing under monopolistic competition have excess capacity. That is, they produce less than the technically efficient level of output $y^* < y^e$.

As ζ increases long-run firm output y^* decreases. Hence imperfect competition causes firms to produce less than the efficient level y^e, so average cost exceeds marginal cost and average cost is decreasing, giving locally increasing returns to scale. Increasing output would improve firm productivity, and with a gradual entry response we shall see that in the short run, firms may use up some excess capacity whilst other firms adjust. The excess capacity result implies that in equilibrium the trade-off between an additional firm

[13] See appendix B for derivation. It follows from substituting factor prices into the profit definition $\pi(t) = y - wl - rk$.

bringing an extra fixed cost (which is underutilized, reducing overall production possibility frontier) outweighs the efficiency gain from smaller firms producing with lower marginal cost[14]. Note that we do not have love-of-variety in our model. If there were, then the technical inefficiency in terms of excess capacity can be counterbalanced by additional product diversity as in Dixit and Stiglitz 1977 and Tirole 1988.

1.2.2 Intratemporal Labour

Now we have determined wage $w(L, K, n)$ at market equilibrium (13), the intratemporal condition (4) defines optimal labour choice in terms of the other variables $L(C, K, n)$. This reduces the model's dimensionality by one variable.

$$w(L, K, n) = AF_l(k, l)(1 - \zeta) = -\frac{u_L(L)}{u_C(C)}$$

By applying the implicit function theorem to this updated intratemporal condition we can determine that labour supply increases in capital and number of firms and decreases in consumption.[15]

$$L_K > 0, L_n > 0, L_C < 0$$

The intratemporal condition shows that the marginal rate of substitution between consumption and labour equates to the wage (negative because labour decreases utility). Labour is decreasing in consumption because a rise in consumption causes the marginal utility of consumption to fall (consumption is a normal good) therefore marginal disutility of labour must decrease to maintain the marginal rate of substitution, hence labour decreases which reduces disutility. Capital causes an increase in the labour supply through an increase in the marginal product of labour and hence real wage. Number of firms increases labour supply as lower employment per firm raises the marginal product of labour and hence wage.

1.3 Firm Entry

The number of firms at time t is determined by an endogenous sunk cost of entry and an arbitrage condition that equates entry cost with incumbency profits.

[14] Zero profit equilibrium only exists if the denominator is positive $\nu < \frac{1}{1-\zeta}$ which will always hold under our assumption of decreasing returns $\nu < 1$.

[15] See Appendix C for derivation. Subscripts denote derivative e.g. L_K is $\frac{\partial L}{\partial K}$.

Assumption 1 Sunk Entry Cost (congestion effect).
Entry cost $q \in \mathbb{R}$ increases with the number of entrants $\dot{n}$ in t.

$$q(t) = \gamma \dot{n}, \qquad \gamma \in (0, \infty) \tag{17}$$

The process is symmetric, a prospective firm pays q to enter; an incumbent firm pays $-q$ to exit.[16] If there is exit $\dot{n} < 0$, so $q < 0$ and $-q > 0$ this means an incumbent must pay a fee to exit, for example redundancy payments or legal fees. $\dot{n}$ is the change in the stock of firms so represents net business formation; later we define this as 'entry' (definition 2). γ is the marginal cost of entry, and its bounds capture the limiting cases of entry. $\gamma \to 0$ implies instantaneous free entry because the sunk cost is vanishingly small so the outcome is similar to the static case, and $\gamma \to \infty$ implies fixed number of firms because the sunk cost is so high that it prohibits entry.

Assumption 2 Entry Arbitrage.
Gain from entry equals return from investing the cost of entry at the market rate.

$$\dot{q} + \pi = rq \tag{18}$$

A firm's gain from entry is the value of incumbency which is operating profits π plus any change in the original sunk cost $\dot{q}$.

The two assumptions are well supported. The congestion effect assumption is common in industrial organization literature for example Das and Das 1997. It is justified by increased advertising costs, or competition for a fixed resource, and variously called "negative network effects" or "entry adjustment cost". In terms of the macroeconomy, Lewis 2009 statistically models congestion externalities in entry and concludes that the mechanism improves model fit because it reduces impact responses of entry. Berentsen and Waller 2015 also model a congestion effect in a DSGE model.

The arbitrage assumption implies that the return to investing in a firm is equal to the return of that investment at the risk free rate r. And an implication of this is that the value of a firm is equal to present discounted value of future profits as in Bilbiie, Ghironi, and Melitz 2012 and Datta and Dixon 2002.

[16] Sunk cost symmetry is not a necessary feature of the model, but it eases exposition as we only focus on deterministic shocks in a single direction. To generalize the process for asymmetric costs would involve different γ for entry and exit. We thank a referee for pointing this out.

The two assumptions form a dynamical system in number of firms and cost of entry $\{n, q\}$ which reduces to a second-order ODE in number of firms

$$\gamma\ddot{n} - \gamma r\dot{n} + \pi = 0 \tag{19}$$

To interpret this second-order ODE consider that if profits are high, then to maintain equilibrium the speed of net business formation $\dot{n}$ is high which translates to higher sunk entry cost thus discouraging future entry so net business formation decelerates $\ddot{n} < 0$ to maintain equilibrium. Defining entry can help this intuition, and by defining entry, this second-order ODE is separable into two first-order ODEs

Definition 2 Entry and Exit.
Entry is measured by net entry which is the change in the number of firms. Thus negative entry is exit.

$$e(t) = \dot{n} \tag{20}$$

Hence our model of industry dynamics, which determines the number of firms, is defined by two ODEs

$$\dot{n} = e \tag{21}$$

$$\dot{e} = -\frac{\pi}{\gamma} + re \tag{22}$$

The endogenous entry cost causes a non-instantaneous adjustment path to steady state, which provides an analytical framework to understand short-run dynamics. To observe the importance of the endogenous entry cost consider the contradiction that entry cost is fixed $q = \gamma$. In which case the the second-order ODE becomes static $\pi = r\gamma$ so there is no dynamic entry.

Integrating the sunk cost of entry across all entrants in a period gives the aggregate cost of entry which forms a part of aggregate profits

$$Z = \gamma \int_0^e i \, di = \gamma \frac{e^2}{2} \tag{23}$$

Aggregate profits are aggregate operating profits (14) less the aggregate sunk cost

$$\Pi = n\pi - Z = n[AF(k, l)(1 - (1 - \zeta)\nu) - \phi] - \gamma \frac{e^2}{2} \tag{24}$$

1.4 Derivation of Measured Productivity

Aggregate output provides a natural derivation of measured productivity $\mathcal{P}$ since it is hod-1 in capital and labour. In the long run $Y^* = n^* y^*$ and if we substitute in zero profit output per firm (16) long-run output is

$$Y^* = n^* \left[AF\left(\frac{K^*}{n^*}, \frac{L^*}{n^*}\right) \nu(1-\zeta) \right] = {n^*}^{1-\nu} AF(K^*, L^*)\nu(1-\zeta) \tag{25}$$

then substitute out $n^* = \frac{Y^*}{y^*} = \frac{Y^*}{\frac{\phi\nu(1-\zeta)}{1-\nu(1-\zeta)}}$

$$Y^* = F(K^*, L^*)^{\frac{1}{\nu}} A^{\frac{1}{\nu}} \nu(1-\zeta) \left(\frac{1-\nu(1-\zeta)}{\phi}\right)^{\frac{1-\nu}{\nu}} \tag{26}$$

Since $\nu < 1$, free entry causes long-run increasing returns to scale between $F(K, L)$ and Y (i.e. Y is hod-$\frac{1}{\nu} > 1$ in F). This derivation shows Y is hod-1 in (K, L) so it provides a scale adjusted measure of productivity. Accounting for the scale effect ensures that an economy is not considered more productive simply because it has more K, L resources, thus measured productivity is[17]

$$\mathcal{P}^* = \frac{Y^*}{F(K^*, L^*)^{\frac{1}{\nu}}} = A^{\frac{1}{\nu}} \nu(1-\zeta) \left(\frac{1-\nu(1-\zeta)}{\phi}\right)^{\frac{1-\nu}{\nu}} \tag{27}$$

Measured productivity $\mathcal{P}$ is an efficiency parameter which reduces to the standard Solow residual measure of total factor productivity (TFP) A under constant returns to scale ($\nu = 1$) and perfect competition ($\zeta = 0$), from (27) $\mathcal{P}^* = A$.[18] Measured productivity is decreasing in market power ζ because it allows firms to suppress output more, so they benefit less from returns to scale offered by the fixed cost. And in the perfect competition limit $\zeta \to 0$ measured productivity is at its efficient level $\mathcal{P}^e = A^{\frac{1}{\nu}} \nu \left(\frac{\phi}{1-\nu}\right)^{\frac{\nu-1}{\nu}}$, which can also be verified by substituting the efficient levels of production (11) into $\mathcal{P}^e = \frac{y^e}{F(k^e, l^e)^{\frac{1}{\nu}}}$. The expression for measured productivity $\mathcal{P}$ is increasing in technology A.

[17] Basu and Fernald 2001 give a detailed discussion of scale adjusted productivity, whilst Harrison 1994 derives a similar measure for regression purposes. Jaimovich and Floetotto 2008 call our 'measured productivity' 'measured TFP' in a model with constant returns, imperfect competition and free entry, so $\mathcal{P} = A(1-\zeta)$ in every period.

[18] Using this definition of productivity the long-run level also follows from the firm level zero profit condition which implies $y^* = wl^* + rk^* = (1-\zeta)A\nu F(k^*, l^*)$ thus $\mathcal{P}^* = \frac{y^*}{F(k^*, l^*)^{\frac{1}{\nu}}} = F(k^*, l^*)^{\frac{\nu-1}{\nu}} A\nu(1-\zeta)$, so the measures at aggregate and firm level are equivalent.

1.5 General Equilibrium

General equilibrium determines prices, consumption and labour given the current capital stock and number of firms. The aggregate equation of motion for capital follows from the household resource constraint (2), which equates expenditure to income $C + \dot{K} + Z = rK + wL + n\pi$, with factor prices and aggregate profit substituted in this becomes

$$\dot{K} = n(1-\zeta)\nu AF(k,l) + n\left[AF(k,l)\left(1-(1-\zeta)\zeta\right) - \phi\right] - \gamma\frac{e^2}{2} - C$$

which then simplifies to

$$\dot{K} = Y - C - \gamma\frac{e^2}{2} \tag{28}$$

Rearranging the result shows that output is divided among consumption and investment $Y = C + I$ where investment is either in capital or new firms $I(t) = \dot{K} + Z(t)$.

Definition 3.
Competitive equilibrium is the 'equilibrium' paths of aggregate quantities and prices $\{C(t), L(t), K(t), n(t), e(t), w(t), r(t)\}_{t=0}^{\infty}$, with prices strictly positive, such that $\{C(t), L(t)\}_{t=0}^{\infty}$ solve the household problem. $\{K(t)\}_{t=0}^{\infty}$ satisfies the law of motion for capital. Labour and capital $\{L(t), K(t)\}_{t=0}^{\infty}$ maximise firm profits given factor prices. The flow of entry causes the arbitrage condition on entry to hold. State variables $\{K(t), n(t)\}_{t=0}^{\infty}$ satisfy transversality. Factor prices are set according to (12) and (13), which ensures goods and factor markets clear.

2 Dynamical System of Endogenous Entry Economy

The model economy is a system of four ordinary differential equations (ODEs) in four variables (C, e, K, n): two equations dictate the law of motion of capital and number of firms and two equations restrict these evolutions so that consumption maximises utility and entry arbitrages profit opportunities. There is also a fifth condition, the intratemporal condition, which is static and determines labour as a function of $\{C, K, n\}$. The system is sufficient to qualitatively analyse the economy. Thus for a given parameter set $\Omega \in \mathbb{R}^{\mathbb{P}}$, we find the set of solutions that satisfy the system given an initial state on the manifold $m_0 := (K_0, n_0) \in \mathbb{M} \subset \mathbb{X}$, where $\mathbb{X} \in \mathbb{R}^4$ is the four dimensional space of the dynamical system, and $\mathbb{M} \in \mathbb{R}^2$ is the manifold: a subset defined by states (capital and number of firms). Therefore at a point in time $t \in (0, \infty)$ the economy is entirely described by its level of capital and the number of firms. For now we do not specify initial conditions or functional forms that would give specific trajectories of capital and number of firms. The primitives of our boundary value problem are the state of the system defined on an open set $\mathbb{M} := K \times n \in \mathbb{R}^2$, the time $t \in \mathbb{I}$ defined on an open interval of $\mathbb{R}$, the parameterization defined on an open set $\Omega \in \mathbb{R}^{\mathbb{P}}$ and the nonlinear C^1 function $g : \mathbb{M} \times I \times \Omega \supseteq \mathbb{R}^{3+\mathbb{P}} \to \mathbb{R}^2$, or *"time evolution law"*, that maps a given state, time and parameterization into a new state. This rule determines the state of the system at each moment in time from its state at all previous times. For most of the analysis we work with a system where the state space includes C and e, although these variables can be defined by their *policy functions* which map $K \times n$ into $C \times e$, so the system equations $\dot{K}$ and $\dot{n}$ describe the evolution of the state variables along the stable manifold.

Definition 4 Nonlinear system.
The dynamical system is a pair $(\mathbb{X}, g)$, where $\mathbb{X} = (\mathbb{R}^4, \psi)$ is Euclidean space and metric. It defines at a point in time $t \in \mathbb{R}$ the state of the system $x(t) \in \mathbb{X} \subseteq \mathbb{R}^4$. The motion of the system is described by a C^1 vector valued transition map $g : \mathbb{R}^{5+\mathbb{P}} \supseteq \mathbb{X} \times \mathbb{R} \times \Omega \longrightarrow \mathbb{R}^4$

Throughout the system labour is implicitly defined by the static intratemporal condition $w = \frac{-u_L}{u_C}$ and the system is given by the following four ODEs.

$$\dot{K} = Y - \frac{\gamma}{2}e^2 - C, \qquad Y = n(AF(k,l) - \phi) \tag{29}$$

$$\dot{n} = e \tag{30}$$

$$\dot{C} = \frac{C}{\sigma(C)}(r - \rho), \qquad \sigma(C) = -\frac{Cu_{CC}}{u_C} \tag{31}$$

$$\dot{e} = re - \frac{\pi}{\gamma}, \qquad \pi = AF(k,l)(1 - (1-\zeta)\nu) - \phi \tag{32}$$

with factor prices

$$r = (1-\zeta)AF_k, \qquad w = (1-\zeta)AF_l \tag{33}$$

The *system equations*, (29) and (30), explain how the state of the system evolves. The *optimization conditions*, (31) and (32), restrict the state evolutions. They impose that households maximise utility and potential entrants maximise profits.

2.1 General Steady State Behaviour

In steady state capital, number of firms, consumption and entry are stationary $\dot{K} = \dot{n} = \dot{C} = \dot{e} = 0$[19] which implies $Y^* = C^*$, $e^* = 0$, $r^* = \rho$ and $\pi^* = 0$. Then substituting out π^*, r^*, Y^* and remembering per firm definition 1 gives steady state conditions in terms of $(C^*, K^*, L^*, n^*, e^*)$

$$\dot{K} = 0: \qquad n^* \left[AF\left(\frac{K^*}{n^*}, \frac{L^*}{n^*}\right) - \phi \right] = C^* \tag{34}$$

$$\dot{n} = 0: \qquad e^* = 0 \tag{35}$$

$$\dot{C} = 0: \qquad (1-\zeta)AF_k\left(\frac{K^*}{n^*}, \frac{L^*}{n^*}\right) = \rho \tag{36}$$

$$\dot{e} = 0: \qquad F\left(\frac{K^*}{n^*}, \frac{L^*}{n^*}\right) = \frac{\phi}{A(1-(1-\zeta)\nu)} \tag{37}$$

with static intratemporal condition

$$(1-\zeta)AF_l\left(\frac{K^*}{n^*}, \frac{L^*}{n^*}\right) = -\frac{u_L(L^*)}{u_C(C^*)} \tag{38}$$

The system determines (C^*, K^*, n^*, e^*), where $e^* = 0$ is immediate, and the static intratemporal condition (38) defines $L^*(C^*, K^*, n^*)$. The results confirm that the zero-profit outcome we analyzed in section 1.2.1 is a steady state outcome of the system. The entry arbitrage condition (37) is this zero-profit condition in steady state and, as we have shown, this determines steady-state firm output y^*. Then given y^*, (34) determines steady-state consumption per firm $C^*(n^*)$, which gives labour $L^*(C^*(n^*), K^*, n^*)$ in K^*, n^* terms. Hence we can graphically analyze the steady-state through two equations, (36) and (37), in (K^*, n^*) space as depicted in figure 2.

In figure 2 (36) defines the $r = \rho$ curve, which gives the combinations of (K, n) for which consumption is unchanging ($\dot{C} = 0$) and equation (37) defines the $\pi = 0$ curve, which gives the combinations of (K, n) for which profits are zero and hence $\dot{n} = 0$. The dotted line shows that an increase in imperfect competition $\zeta_A \to \zeta_B$ shifts both the curves upwards which moves equilibrium from point A to B which represents lower capital per firm $k < k^*$. The $r = \rho$ curve shifts up because a rise in imperfect competition cuts the interest rate below the discount rate, thus capital falls to raise the marginal product and interest rate back to parity. Similarly the $\pi = 0$ curve shifts up because imperfect competition raises profits given K, n, so entry increases. The unambiguous fall in capital per firm can be shown analytically

[19] Ignore the trivial steady state that arises when the state vector is the zero vector.

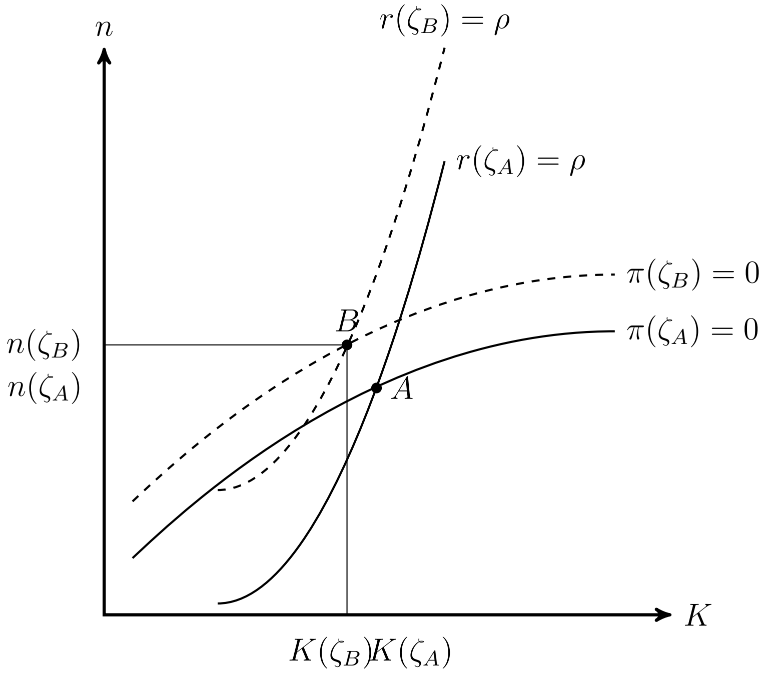

Figure 2: $\pi = 0$ and $r = \rho$ in K, n space

Proposition 2.
In steady state market power decreases capital per firm, and ambiguously affects labour per firm

$$k_\zeta < 0, \quad l_\zeta \lesseqgtr 0 \tag{39}$$

Proof. Appendix (D) □

To understand the ambiguity in labour response, figure 3 uses the same graphical intuition to plot the remaining conditions (34) and (38) which determine the household consumption labour choice in general equilibrium. Since (36) and (37) determine K^*, n^*, then this gives the intratemporal condition (38) in L, C space

$$w^* = (1-\zeta)AF_l\,(k^*, l^*) = -\frac{u_L(L^*)}{u_C(C^*)} = MRS \tag{IEP}$$

This condition is the income expansion path (IEP) (or income-consumption curve) since it describes optimal L, C bundles as income changes given w^*. Income and utility expand north-west on the IEP, and it is downward sloping because labour is a bad and consumption is a good.[20] To determine equilibrium L^*, C^* graphically, the steady state budget constraint (34) in L, C space is

$$C^* = n^* y^* \tag{BC}$$

In (L, C) space the rise in imperfect competition shifts both curves down

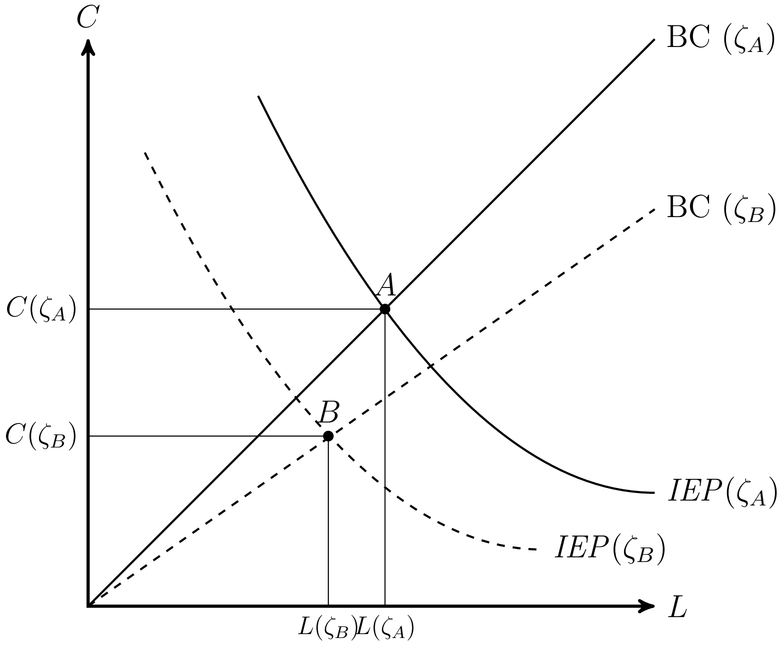

Figure 3: IEP and BC in L, C space

which results in a fall in consumption, but an ambiguous change in labour. The IEP shifts down because market power ζ reduces wage, so for a given L income is lower and C falls to achieve lower marginal rate of substitution (MRS). This is a substitution effect: labour is more expensive relative to consumption. The BC slope becomes shallower because each unit of L attains less wage, which creates an income effect as more labour is necessary to attain a given consumption level. Both substitution and income effects reduce consumption, but the substitution effect reduces labour whereas the income effect increases labour. Figure 3 shows the case where substitution effect dominates the income effect so that labour supply falls.

[20]The slope is convex because utility from consumption diminishes and disutility from labour grows. Under quasi-homothetic preferences the IEP is a straight line; if preferences are homomethic IEP intersects (1,0).

2.2 Numerical Steady State Behaviour

In this section we specify functional forms and parameterizations which illustrate the bifurcations discussed above as imperfect competition changes. As in the baseline RBC model we assume isoelastic utility and Cobb-Douglas production.

$$U(C,L) = \frac{C^{1-\sigma}-1}{1-\sigma} - \xi\frac{L^{1+\eta}}{1+\eta} \tag{40}$$

$$F(k,l) = k^{\alpha}l^{\beta} = F(K,L)n^{-(\alpha+\beta)} \tag{41}$$

where α and β are capital and labour shares. σ is a curvature parameter which we assume to be unity implying fixed labour supply, and η is inverse Frisch elasticity ($\eta = 0$ is indivisible labour). Table 1 summarizes the parameter values we use for simulation exercises[21] notice $\nu = \alpha + \beta = 0.8$ satisfies

ζ	α	β	ϕ	γ	A	σ	ρ	ξ	η
$(0,1)$	0.3	0.5	0.3	3.0	1.0	1.0	0.025	0.01	0.5

Table 1: Parameter Values for Numerical Exercises

the homogeneity requirement $\nu < 1$. Under this function specification the dynamical system (29)-(32) becomes

$$\dot{C} = \frac{C}{\sigma}\left[(1-\zeta)A\alpha K^{\alpha-1}L^{\beta}n^{1-(\alpha+\beta)} - \rho\right] \tag{42}$$

$$\begin{aligned}\dot{e} &= (1-\zeta)A\alpha K^{\alpha-1}L^{\beta}n^{1-(\alpha+\beta)}e \\ &\quad - \frac{1}{\gamma}\left(AK^{\alpha}L^{\beta}n^{-(\alpha+\beta)}(1-(1-\zeta)\nu) - \phi\right)\end{aligned} \tag{43}$$

$$\dot{K} = n\left[AK^{\alpha}L^{\beta}n^{-(\alpha+\beta)} - \phi\right] - \frac{\gamma}{2}e^2 - C \tag{44}$$

$$\dot{n} = e \tag{45}$$

and from the static relationship $L(C,K,n)$ takes the form[22]

$$L = \left(\frac{(1-\zeta)AK^{\alpha}\beta n^{1-(\alpha+\beta)}}{\xi C^{\sigma}}\right)^{\frac{1}{1+\eta-\beta}} \tag{46}$$

[21] Since our interest in simulation is on illustrating the dynamics we show analytically and not the quantitative magnitudes of the effects this benchmark specification suffices and is replicated from Brito and Dixon 2013.

[22] Appendix E shows labour's behaviour, which corroborates the earlier theoretical results.

Solving for steady state gives

$$n^* = \left[\frac{\beta}{\xi\nu^\sigma}\left\{\left(A\left(\frac{\alpha}{\rho}\right)^\alpha\right)^{1+\eta}(1-\zeta)^{\alpha(1+\eta)+\beta(1-\sigma)}\right.\right.$$
$$\left.\left.\left(\frac{1-(1-\zeta)\nu}{\phi}\right)^{1-\nu+\eta(1-\alpha)+\sigma\beta}\right\}^{\frac{1}{\beta}}\right]^{\frac{1}{\eta+\sigma}} \tag{47}$$

With n^* solved all the steady state variables (C^*, e^*, K^*, L^*) follow

$$C^* = n^*\frac{\phi(1-\zeta)\nu}{1-(1-\zeta)\nu} \tag{48}$$

$$e^* = 0 \tag{49}$$

$$K^* = n^*\frac{\phi\alpha(1-\zeta)}{(1-(1-\zeta)\nu)\rho} \tag{50}$$

$$L^* = n^*\left[\frac{1}{A}\left(\frac{\rho}{\alpha(1-\zeta)}\right)^\alpha\left(\frac{\phi}{1-(1-\zeta)\nu}\right)^{1-\alpha}\right]^{\frac{1}{\beta}} \tag{51}$$

Figure 4 illustrates figure 2 for this particular numerical scenario. It shows steady state (K^*, n^*) combinations as market power ζ increases. The values of ζ are marked next to the equilibrium points, and for two of these equilibrium points the associated $\pi = 0$ (curved line) and $r = \rho$ (straight line) lines are plotted. This emphasizes the curves upward shift as market power increases which determines a new equilibrium with lower capital per firm. The nonlinearity in $\pi = 0$ is responsible for relatively fewer, and eventually decreasing number of firms, as market power rises. At low levels of capital, small changes in capital cause large changes in number of firms. A higher σ risk aversion would flatten the profit nullcline (to maintain zero profits a rise in n requires a larger rise in K) so that number of firms continues to increases as the curve shifts upwards.

Figure 5 plots the household Income Expansion Path (IEP) and Budget Constraint (BC) in steady state which corresponds to figure 3. Since we have set unitary risk aversion $\sigma = 1$ then there is log utility in consumption, and this causes income and substitution effects to cancel out meaning the rise in imperfect competition (solid lines to dashed lines) has no effect on labour. Clearly consumption falls unambiguously.

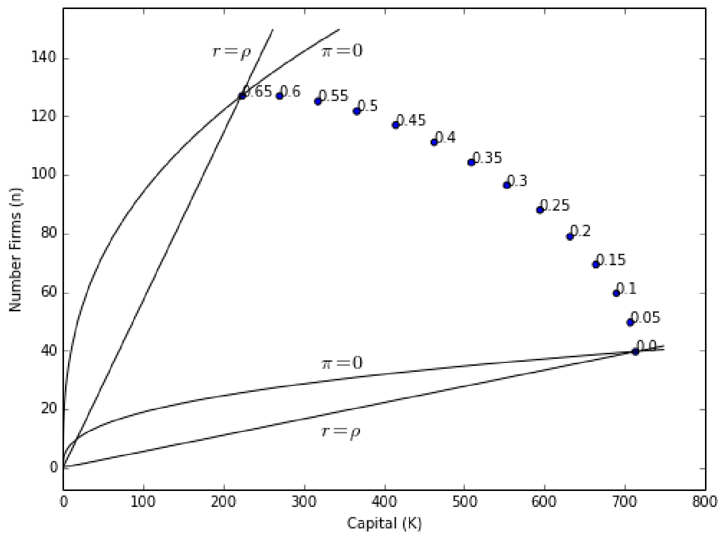

Figure 4: Capital and No. Firms Equilibrium as Market Power Changes

2.3 General Dynamic Behaviour

The system's dynamics explain how the economy transitions to excess capacity steady state. Our interest is how imperfect competition affects this transition, specifically through its effect on firm capacity utilization. To understand this we solve the four dimensional system which gives trajectories of the variables over t. The system is in nonlinear form $\dot{x} = g(x)$, as given in definition 4, but we linearize it to $\dot{x} = \mathbf{J}(x - x^*)$ and analyse the $\mathbf{J}$ matrix. $\mathbf{J}$ is the Jacobian matrix where each element is a respective derivative. The derivatives treat $\{K, n, C, e\}$ as independent, and labour is a function of these variables through the intratemporal condition $L(C, K, n)$.

Definition 5 Linearized System and Jacobian Matrix.
In linearized form the system is $\dot{x} = \mathbf{J}(x - x^*)$, *where the Jacobian matrix* $\mathbf{J} : \mathbb{R}^4 \to \mathbb{R}^4$ *is the nonlinear system matrix evaluated at steady state.*

$$\begin{bmatrix} \dot{C} \\ \dot{e} \\ \dot{K} \\ \dot{n} \end{bmatrix} = \begin{bmatrix} \frac{C^*}{\sigma} r_C^* & 0 & \frac{C^*}{\sigma} r_K^* & \frac{C^*}{\sigma} r_n^* \\ -\frac{\pi_C^*}{\gamma} & \rho & -\frac{\pi_K^*}{\gamma} & -\frac{\pi_n^*}{\gamma} \\ Y_C^* - 1 & 0 & Y_K^* & Y_n^* \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} C - C^* \\ e - e^* \\ K - K^* \\ n - n^* \end{bmatrix} \tag{52}$$

In the linearized model, the state vector is deviation from equilibrium point. Superficially, the elements of the Jacobian appear the same as the perfect competition case in Brito and Dixon 2013, for example element 3×3

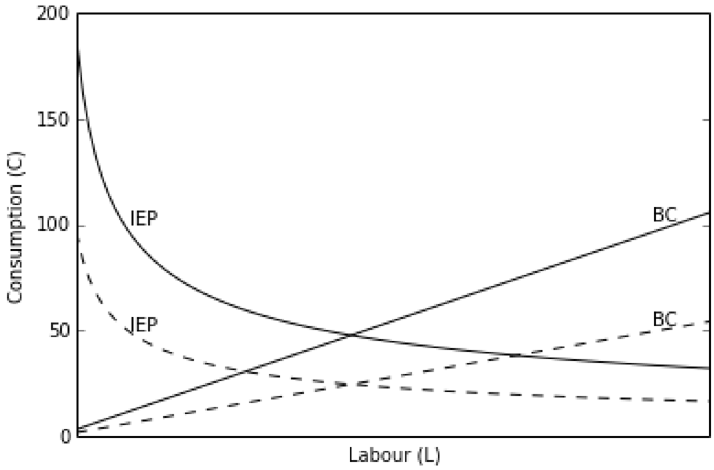

Figure 5: Determining Household Consumption-Labour Choice

is Y_n^* in both cases. But their magnitudes change for any given point due to the mark-up ζ, and as the steady state is also different the matrix is evaluated at different points. This is clear when we substitute in functional forms to get the Jacobian associated with system (42 - 45)

$$\begin{bmatrix} -\frac{\rho\beta}{1+\eta-\beta} & 0 & \frac{\rho^2\nu((1+\eta)(\alpha-1)+\beta)}{\sigma\alpha(1+\eta-\beta)} & \frac{\phi(1-\zeta)\nu\rho(1-\nu)(1+\eta)}{(1-(1-\zeta)\nu)\sigma(1+\eta-\beta)} \\ \frac{(1-(1-\zeta)\nu)\beta\sigma}{\gamma n^*(1+\eta-\beta)(1-\zeta)\nu} & \rho & \frac{-(1-(1-\zeta)\nu)\rho(1+\eta)}{\gamma n^*(1-\zeta)(1+\eta-\beta)} & \frac{\phi(\nu(1+\eta)-\beta)}{\gamma n^*(1+\eta-\beta)} \\ \frac{-\beta\sigma}{(1+\eta-\beta)(1-\zeta)\nu}-1 & 0 & \frac{\rho(1+\eta)}{(1-\zeta)(1+\eta-\beta)} & \frac{\phi[-\zeta\nu(1+\eta-\beta)+\beta(1-\nu)]}{(1-(1-\zeta)\nu)(1+\eta-\beta)} \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{53}$$

for clarity we leave n^* in the second row since it is a large expression (see (47)).[23] Importantly we are able to discern the sign structure of the general Jacobian matrix with functional forms, and prove that it will always have two strictly positive and two strictly negative eigenvalues, thus it has saddle dynamics. A corollary to the proof is that the Jacobian determinant is strictly positive, and therefore the system is always determinate. Furthermore, this robustly verifies that our subsequent analysis of the linearized dynamics are topologically equivalent to the nonlinear system via the Hartman-Grobman theorem.

[23] Appendix F derives the parameterized Jacobian.

2.4 Numerical Dynamic Behaviour

Substituting the table 1 values into the analytical Jacobian (53) gives a numerical version which supports the proof of saddlepath dynamics. Figure 6 shows that the Jacobian evaluated at steady state is a saddle point (2 positive eigenvalues, 2 negative eigenvalues) for different values of market power ζ. The plot shows 20 values of market power $\zeta \in (0, 0.05...0.95)$ on the x-axis and the corresponding 4 eigenvalues at each of these levels of market power on the y-axis. Despite the appearance of convergence to the zero axis[24], there are always two unstable (positive) and two stable (negative) eigenvalues. This will allow us to show that the model is able to generate endogenous productivity movements which the next section explains.

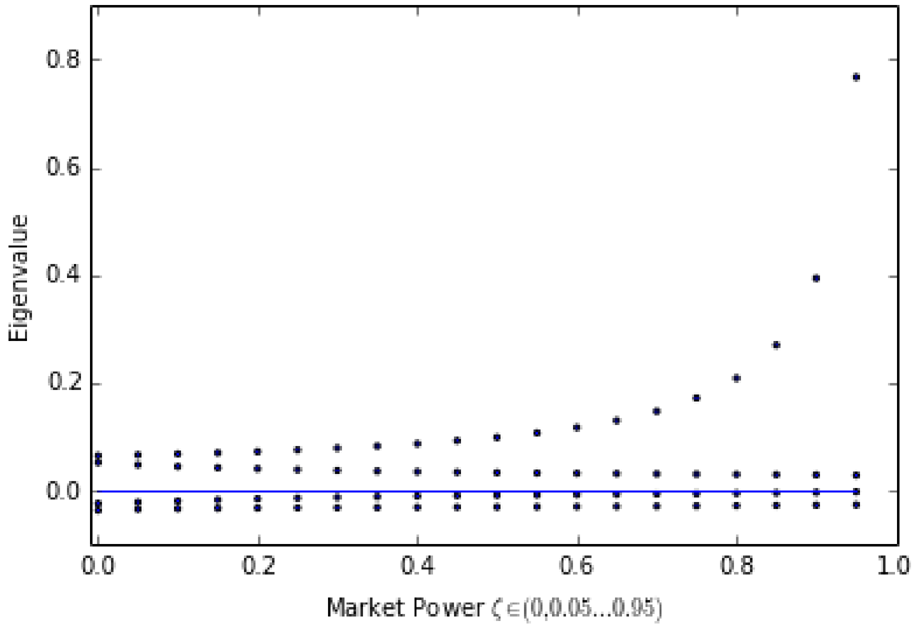

Figure 6: Sets of 4 Eigenvalues for 20 Values of Market Power

3 Main Result: Capacity Utilization

The previous section showed that the four dimensional system has two positive and two negative eigenvalues, so the system is a saddle which is locally

[24]We show determinacy in appendix F which rules out zero eigenvalues. An interesting observation is that the stable eigenvalues decline in value as market power increases which implies that increasing market power slows the rate of convergence of the economy.

asymptotically unstable. It is well-known (e.g. Caputo 2005, p.426) that with saddle dynamics variables on the system's stable manifold are predetermined and do not respond on impact, whereas jump variables change instantaneously causing subsequent movement in the states. In our model the stable manifold is defined in terms of K, n which implies that these do not move on impact of a perturbation from steady state, whereas C, e are instantaneously effected after a perturbation.[25] The crucial implication for what follows is that the number of firms does not adjust on impact $t = 0$, so in response to a technology shock $n_A(t = 0) = 0$. This captures the idea of sluggish firm entry, and allows incumbents to raise capacity on impact at $t = 0$. Labour on the other hand is a free variable that will jump instantaneously after a shock. The timing difference between immediate labour adjustment and slow capital and firm adjustment exacerbates the capacity utilization effect which causes short-run measured productivity to exceed underlying productivity.

In steady state firms have excess capacity. Since entry is a slow process, a positive production shock prompts incumbent firms to increase their capacity toward full capacity, making better use of their fixed costs, which improves productivity. In the long run, firms will enter until zero profit returns each firm to producing a long-run level of output which is unchanged. This leads to the following main result

Theorem 3.1 Measured Productivity Overshooting.
A change in technology causes a greater change in measured productivity than underlying productivity. Thus there is amplification (overshooting) in measured productivity.

$$\left|\frac{d\mathcal{P}(0)}{dA}\right| \geqslant \left|\frac{d\mathcal{P}^*}{dA}\right|$$

Proof. (Full derivation Appendix G) □

The proof reduces to the following relationship

$$\begin{aligned}\frac{d\mathcal{P}(0)}{dA} - \frac{d\mathcal{P}^*}{dA} &= \left(\frac{\zeta}{1-\zeta}\right)\left[\frac{d\mathcal{P}^*}{dA} + \frac{\mathcal{P}^* F_L L_A}{\nu F}\right] \\ &= \zeta\left(\frac{\phi}{A(1-(1-\zeta)\nu)}\right)^{1-\frac{1}{\nu}}\left(1 + \frac{AF_L L_A}{F}\right) \geqslant 0, \quad \nu < 1\end{aligned} \tag{54}$$

The degree of measured productivity overshooting depends on the level of industry competition ζ, the fixed cost of production ϕ, and the response

[25] Appendix F.5 defines the stable manifold which shows that K, n do not respond on impact $t = 0$, whereas C, e respond instantaneously. This will also be obvious in the numerical simulations figure 8 that follow.

of endogenous labour L_A. Variable production returns to scale ν are not a necessary condition as the result still holds with constant returns to scale $\nu \to 1$. But importantly the result is undefined if there are constant returns and perfect competition $\zeta = 0$ since equilibrium cannot exist in the presence of a fixed cost, returns are globally decreasing, and therefore we do not have an efficient outcome to benchmark against. In the constant returns limit, it is clear that amplification is increasing in ζ, but the result does not hold for the general case. The following corollary outlines the main implications of the theorem.

Corollary 1 Nature of Productivity Overshooting.
Imperfect competition, fixed costs in production and labour response all reinforce the overshooting effect.

- *Imperfect competition $\zeta > 0$ is necessary for measured productivity overshooting. As proof consider that if $\zeta = 0$ then $\frac{d\mathcal{P}(0)}{dA} = \frac{d\mathcal{P}^*}{dA}$.*
- *The fixed cost $\phi > 0$ necessary for productivity overshooting because it endogenously creates locally increasing returns to scale that incumbent firms exploit on impact.*
- *The positive elasticity of labour term $\frac{AF_L L_A}{F}$ caused by endogenous labour strengthens overshooting. This arises because labour jumps immediately on impact, unlike the slow response of the state variables (K, n).*

A positive once-and-for-all technology shock immediately increases output per firm. Firms do not have time to adapt, so incumbents absorb the new technology and produce more output given their existing capital. Producing more reduces their excess capacity. Profits increase (π is increasing in A), and gradually capital and number of firms adjust, returning production to y^* and profits to zero. Endogenous labour strengthens the impact response, and causes more overshooting.

3.1 Numerical Illustration of Productivity Overshooting

This section provides a numerical simulation of the general results of the previous section.[26] In addition to the table 1 parameter values we set the Lerner Index to $\zeta = 0.2$.[27] Figure 7 illustrates the numerical analog to

[26] Under isoelastic utility and Cobb-Douglas production, we have $\frac{AF_L L_A}{F} = \frac{\beta}{1+\eta-\beta}$ so the $\left(1 + \frac{AF_L L_A}{F}\right)$ component of our measured productivity theorem becomes $\frac{1+\eta}{1+\eta-\beta} \geqslant 1$.

[27] This corresponds to a price over marginal cost markup of $\frac{1}{1-\zeta} = 1.25$. In general estimates of markups in value added data range from 1.2 to 1.4, and in gross output they

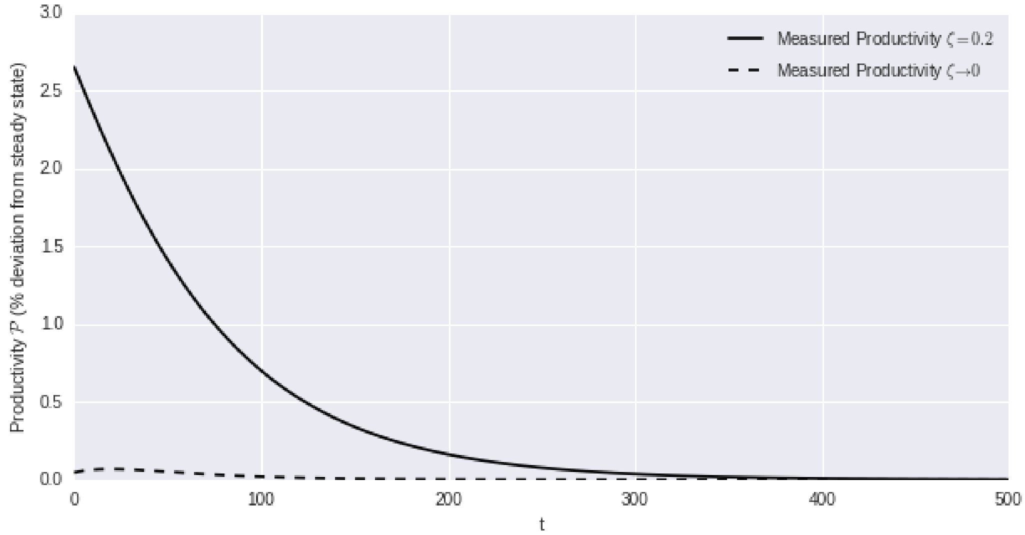

Figure 7: Measured Productivity Amplification

theorem 3.1. It shows that after a once-and-for-all technology shock $A = 1$ to $A = 1.1$ measured productivity $\mathcal{P}$ overshoots its new long-run level $\mathcal{P}^*(A = 1.1)$ by 2.7%. Also as market power diminishes $\zeta \to 0$ the effect disappears which verifies the corollary to theorem. Figure 8 shows the transmission of the positive deterministic shock through the model's underlying variables, where the y-axis is in absolute terms since our the focus is on shape of dynamics. As predicted by the theory, the positive technology shock causes capital and number of firms to begin at their initial pre-shock steady state and start increasing over time as they converge to the new steady state with improved technology. It is this slow response of number of firms to the shock which leads to the productivity overshooting shown in figure 7. We can also see that the number of firms increases at a decreasing rate, reflecting that entry is initially high to arbitrage large profits, but diminishes over time as congestion effects increase. In the long-run entry is zero as the number of firms is fixed at its new long-run level.

4 Capital Utilization Versus Capacity Utilization

We have shown that slow firm responses cause variations in capacity utilization which lead to endogenous fluctuations in productivity in the short-

vary between 1.05 and 1.15, see Basu and Fernald 2001 and Morrison 1992

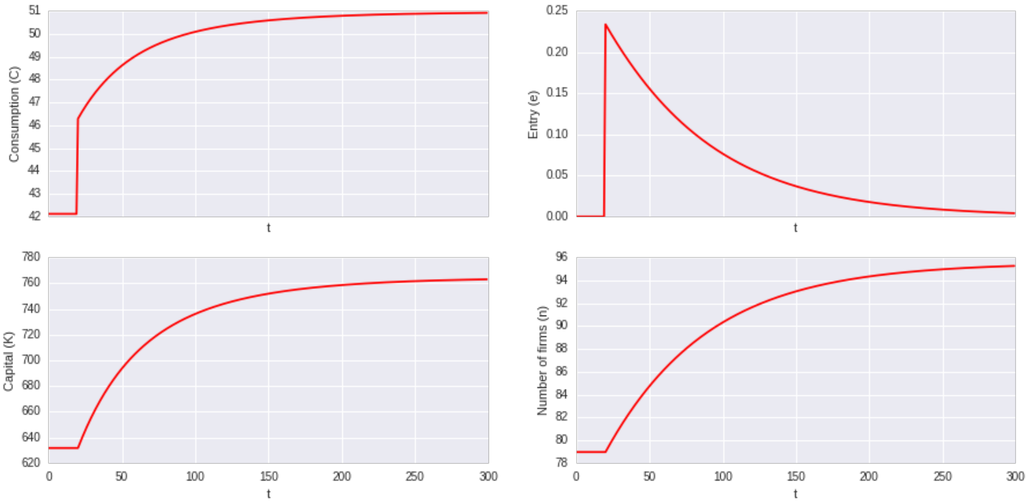

Figure 8: Positive Shock Dynamics

run as firm dynamics adjust. However RBC literature[28] emphasizes that *'capital utilization'* can account for endogenous variations in measured TFP, and can amplify exogenous technology shocks. In this section we include this additional capital utilization mechanism in conjunction with our capacity utilization mechanism to show that our model generalizes the standard model, strengthening its productivity amplifying effects. The results show that with capital utilization imperfect competition is no longer necessary for overshooting (i.e. the standard result that capital utilization is sufficient), but when there is imperfect competition the overshooting is strengthened. We isolate the capacity utilization effect by showing that overshooting remains even when measured productivity is adjusted according to the King and Rebelo 1999 "modified Solow Residual" which removes capital utilization amplification.

First it is important to distinguish *capital utilization* from *capacity utilization*. We use capacity utilization in its traditional microeconomic sense[29] to refer to the distance of firm actual output (or variable cost) y from the firm's optimal long-run equilibrium point (minimum average cost in our case) which is full capacity y^e. Hence excess capacity is $y^e - y$, and entry-induced changes in y cause variations in capacity utilization. Traditionally many RBC papers[30] use capacity utilization to mean the more specific concept of

[28] For example, Kydland and Prescott 1988, Kydland and Prescott 1991, Shapiro 1993, Bils and Cho 1994, Cooley, Hansen, and Prescott 1995, Burnside, Eichenbaum, and Rebelo 1995, Burnside, Eichenbaum, and Rebelo 1996, King and Rebelo 1999

[29] Chamberlin 1933, Vives 1999, Tirole 1988, Basu and Fernald 2001.

[30] For example, Greenwood, Hercowitz, and Huffman 1988, Wen 2001, Benhabib,

capital utilization. Capital utilization is the endogenous depreciation of capital based on its usage. King and Rebelo 1999 were early adopters of the preciser term capital utilization, and Basu and Fernald 2001 also emphasized the distinction.

Capital utilization nests a new functional in the firm production function. The utilization function $\mathfrak{u}(t) : K \times n \to (0,1)$ reflects the intensity of capital usage, so the capital utilization production function is

$$y = AF(\mathfrak{u}k, l) - \phi \tag{55}$$

In addition to the modified production function, capital utilization will affect depreciation in the budget constraint (or equivalently the income identity). This creates a difference between the household's market return from lending capital $r(t)$ and the firm's cost of renting capital $R(t)$, which are equivalent in the no depreciation case. The relationship is that the return to lending capital is the rental paid by firms less depreciation $r(t) = R(t) - \delta(\mathfrak{u}, t)$.

We assume that the rate of capital depreciation $\delta(\mathfrak{u}, t) \in (0,1)$ is an increasing convex function of the rate of utilization $\mathfrak{u} \in (0,1)$ given by

$$\delta = z\mathfrak{u}^{\vartheta} \tag{56}$$

where $\vartheta > 1$[31] and $z \in R_+$. Therefore $z = 0$ and $\mathfrak{u} = 1$ cause production (55) and depreciation (56) to collapse to the no utilization setup. Since utilization is endogenous it cannot be exogenously set to full utilization $\mathfrak{u} \to 1$, but the exogenous convexity parameter can be made large to achieve this effect, since $\lim_{\vartheta \to \infty} \mathfrak{u} = 1$. The intuition is that ϑ is the elasticity of $\delta_{\mathfrak{u}}$[32]. When it is large the marginal cost of utilization (replacement rate) $\delta_{\mathfrak{u}}$ responds elastically to utilization which encourages full utilization. It is calibrated to 1.1 in King and Rebelo 1999, 1.4 in Wen 1998, and between 1.25 and 2.0 in Benhabib, Nishimura, and Shigoka 2008.

Figure (9) shows that overshooting is 5.3% which comprises 2.7% from capacity utilization and 2.6% from capital utilization. This adds $\vartheta = 1.4, z = 1.0$ to the table 1 numerical values. Notably the 2.7% overshooting is exactly the same as the section without considering utilization, this shows that King and Rebelo 1999 suggested 'modified Solow Residual' works well at eradicating the capital utilization bias, leaving only the capacity utilization bias.

Nishimura, and Shigoka 2008, Jaimovich 2007

[31]This parametric restriction imposes a convex cost structure on capital utilization so there is an interior solution for $\mathfrak{u}$ in steady state. If $\vartheta \leqslant 1$ then the optimal rate of capital utilization is always $\mathfrak{u} = 1$ i.e. full utilization.

[32] $\frac{\delta_{\mathfrak{u}\mathfrak{u}}}{\delta_{\mathfrak{u}}}\mathfrak{u} = \vartheta - 1$. In King and Rebelo 1999 they call $\vartheta - 1 = \xi$ and calibrate to 0.1.

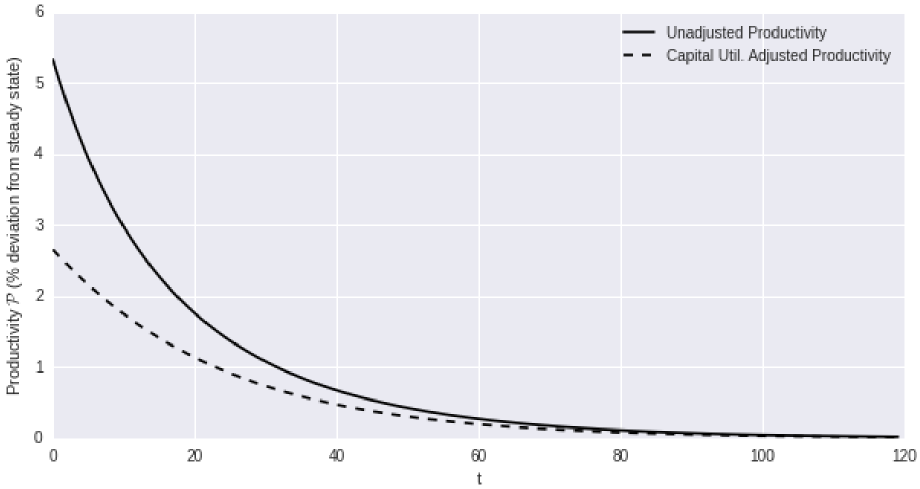

Figure 9: Capacity Utilization and Capital Utilization Amplification

The unadjusted measure fails to account for $\mathfrak{u}$ in the denominator of the measured productivity definition $\mathcal{P}^{\text{unadj.}} = \frac{y}{F(k,l)^{\frac{1}{\nu}}}$ whereas the adjusted measure ensures the denominator is correctly specified $\mathcal{P}^{\text{adj.}} = \frac{y}{F(\mathfrak{u}k,l)^{\frac{1}{\nu}}}$, where for both definitions y includes utilization as in (55).

5 Summary

This paper shows that non-instantaneous entry and exit of firms to macroeconomic conditions causes endogenous productivity dynamics over the business cycle. The result is that productivity shocks are amplified in the short run relative to their long-run level.

We propose the theory through an endogenous firm entry dynamic general equilibrium model with imperfect competition and endogenous entry costs. Imperfect competition creates monopoly profits which cause "too many" entrants in steady state. Each incumbent has excess capacity defined by the amount it underproduces relative to a cost minimizing level of output. Incumbents' excess capacity varies in the short-run because firms cannot enter/exit so incumbents vary production in response to economic shocks. Through increasing returns to scale, variations in production (and therefore capacity utilization) cause variations in firm productivity which aggregate to the macroeconomy. The effect is temporary because over time firms enter/exit which dissipates the shock, causing incumbents to return to their original production.

A Household Optimization Problem

To obtain the necessary conditions for a solution to the household's utility maximisation problem I use the Maximum Principle. The current value Hamiltonian is

$$\begin{aligned}\hat{\mathcal{H}}(t) = u(C(t), L(t)) + \\ \lambda(t)(w(t)L(t) + r(t)K(t) + \Pi(t) - C(t))\end{aligned} \tag{57}$$

The costate variable λ_t is the shadow price of wealth in utility units. The Pontryagin necessary conditions are

$$\hat{\mathcal{H}}_C(K, L, C, \lambda) = 0 \implies u_C - \lambda = 0 \tag{58}$$

$$\hat{\mathcal{H}}_L(K, L, C, \lambda) = 0 \implies u_L + \lambda w = 0 \tag{59}$$

$$\begin{aligned}\hat{\mathcal{H}}_K(K, L, C, \lambda) = \rho\lambda - \dot{\lambda} &\implies \lambda r = \rho\lambda - \dot{\lambda} \\ &\implies \frac{\dot{\lambda}}{\lambda} = -(r - \rho)\end{aligned} \tag{60}$$

$$\hat{\mathcal{H}}_\lambda := \dot{K}_t \implies \dot{K} = rK + wL + \Pi - C \tag{61}$$

Equations (58)-(60) reduce to two equations. These are a differential equation we call the consumption Euler equation (intertemporal condition) and a static relationship between labour and consumption (intratemporal condition).

B Costs and Operating Profit

Total costs are wages paid on labour and interest on capital

$$\begin{aligned}wl + rk = AF_l(1-\zeta)l + AF_k(1-\zeta)k = A(1-\zeta)[F_l l + F_k k] \\ = (1-\zeta)\nu AF(k, l)\end{aligned} \tag{62}$$

Operating profit is output less total costs

$$\pi = y - wl - rk = (AF(k, l) - \phi) - ((1-\zeta)\nu AF(k, l)) \tag{63}$$

$$\pi(L, K, n; A, \zeta, \phi) = AF(k, l)(1 - (1-\zeta)\nu) - \phi \tag{64}$$

C Optimal Labour Derivatives

Partially differentiate the intratemporal condition with respect to each variable treating labour as an implicit function. Then by implicit function theorem get labour responses $L(\underset{-}{C}, \underset{+}{K}, \underset{+}{n}; \underset{+}{A}, \underset{-}{\zeta})$

$$u_L + u_C(1-\zeta)AF_l\left(\frac{K}{n}, \frac{L}{n}\right) = 0 \tag{65}$$

Assumptions:

$$F_{ll}\left(\frac{K}{n}, \frac{L}{n}\right), u_{CC}(C), u_{LL}(L) < 0 \tag{66}$$

$$u_C(C), F_l\left(\frac{K}{n}, \frac{L}{n}\right), F_{lk}\left(\frac{K}{n}, \frac{L}{n}\right) > 0 \tag{67}$$

where $l = \frac{L}{n}$ and $k = \frac{K}{n}$

$$u_{LL}L_C + u_{CC}(1-\zeta)AF_l + u_C(1-\zeta)AF_{ll}\frac{L_C}{n} = 0, \tag{68}$$

$$L_C = \frac{-u_{CC}(1-\zeta)AF_l}{u_{LL} + u_C(1-\zeta)A\frac{F_{ll}}{n}} < 0 \tag{69}$$

$$u_{LL}L_K + u_C(1-\zeta)AF_{lk}\frac{1}{n} + u_C(1-\zeta)AF_{ll}\frac{L_K}{n} = 0, \tag{70}$$

$$L_K = \frac{-u_C(1-\zeta)A\frac{F_{lk}}{n}}{u_{LL} + u_C(1-\zeta)A\frac{F_{ll}}{n}} > 0 \tag{71}$$

$$u_{LL}L_n + u_C(1-\zeta)A\left[F_{lk}\frac{-K}{n^2} + F_{ll}\left(\frac{-L}{n^2} + \frac{L_n}{n}\right)\right] = 0, \tag{72}$$

$$L_n = \frac{u_C(1-\zeta)A(\nu-1)\frac{F_l}{n}}{u_{LL} + u_C(1-\zeta)A\frac{F_{ll}}{n}} > 0 \tag{73}$$

$$u_{LL}L_A + u_C(1-\zeta)F_l + u_C(1-\zeta)AF_{ll}\frac{L_A}{n} = 0, \tag{74}$$

$$L_A = \frac{-u_C(1-\zeta)F_l}{u_{LL} + u_C(1-\zeta)A\frac{F_{ll}}{n}} > 0 \tag{75}$$

Assumptions on the functions, given above, are sufficient to determine the signs in all cases except L_n, which depends on returns to scale of the technology ν. The denominator is the same in each derivation. It is the intratemporal condition differentiated with respect to L, and it is negative. The negativity reflects that utility is decreasing in labour. Therefore the numerator distinguishes signs. Labour behaves as the analytical results show if we specify isoelastic utility and Cobb-Douglas production.

$$L_n = \frac{1-(\alpha+\beta)}{1+\eta-\beta}\frac{L}{n} \tag{76}$$

$$L_K = \frac{\alpha}{(1+\eta-\beta)}\frac{L}{K} \tag{77}$$

$$L_C = \frac{\xi\sigma}{(1-\zeta)A\beta(\beta-1-\eta)}\frac{C^{\sigma-1}}{K^{\alpha}L^{\beta-2-\eta}n^{1-(\alpha+\beta)}} \tag{78}$$

$$= -\frac{\sigma}{(1+\eta-\beta)}\frac{L}{C} \tag{79}$$

$$L_\zeta = -\frac{1}{(1-\zeta)(1+\eta-\beta)}L \tag{80}$$

$$L_A = \frac{1}{A(1+\eta-\beta)}L \tag{81}$$

D Comparative statics

From (36) and (37)

$$F_k(k,l) = \Upsilon, \quad F(k,l) = \Xi \tag{82}$$

$$\text{where, } \Upsilon = \frac{\rho}{A\,(1-\zeta)} \quad \Xi = \frac{\phi}{A\,(1-(1-\zeta)\,\nu)} \tag{83}$$

Use Cramer's rule to determine the effect of a change in ζ. Differentiate with respect to ζ

$$F_{kk}k_\zeta + F_{kl}l_\zeta = \Upsilon_\zeta \tag{84}$$

$$F_k k_\zeta + F_l l_\zeta = \Xi_\zeta \tag{85}$$

$$\overbrace{\begin{bmatrix} F_{kk} & F_{kl} \\ F_k & F_l \end{bmatrix}}^{\mathbf{A}} \begin{bmatrix} k_\zeta \\ l_\zeta \end{bmatrix} = \begin{bmatrix} \Upsilon_\zeta \\ \Xi_\zeta \end{bmatrix} \tag{86}$$

$$\begin{bmatrix} k_\zeta \\ l_\zeta \end{bmatrix} = \underbrace{\frac{1}{det(\mathbf{A})}\begin{bmatrix} F_l & -F_{kl} \\ -F_k & F_{kk} \end{bmatrix}}_{\mathbf{A}^{-1}} \begin{bmatrix} \Upsilon_\zeta \\ \Xi_\zeta \end{bmatrix} \tag{87}$$

$$det(\mathbf{A}) = F_{kk}F_l - F_{kl}F_k < 0, \tag{88}$$

$$\Upsilon_\zeta = \frac{\rho}{A(1-\zeta)^2} > 0 \tag{89}$$

$$\Xi_\zeta = -\frac{\phi}{A(1-(1-\zeta)\nu)^2\nu} < 0 \tag{90}$$

Hence,

$$k_\zeta = \frac{1}{det(\mathbf{J})}\left(F_l \Upsilon_\zeta - F_{kl}\Xi_\zeta\right) < 0 \tag{91}$$

$$l_\zeta = \frac{1}{det(\mathbf{J})}\left(-F_k \Upsilon_\zeta + F_{kk}\Xi_\zeta\right) \gtrless 0$$

$$\text{if} \quad \left|\frac{F_k}{F_{kk}}\right| \lessgtr \left|-\frac{\phi(1-\zeta)^2}{\rho(1-(1-\zeta)\nu)^2\nu}\right| \tag{92}$$

E Parameterized Steady State

To derive steady state outcomes under CES utility and Cobb-Douglas production it is easier to use per firm variables k and l. Thus, the intratemporal condition is

$$C^* = \left(\frac{\beta(1-\zeta)Ak^{*\alpha}l^{*\beta-1-\eta}n^{*-\eta}}{\xi}\right)^{\frac{1}{\sigma}} \tag{93}$$

the dynamical system is

$$C^* = \frac{\phi(1-\zeta)\nu}{1-(1-\zeta)\nu}n^* \tag{94}$$

$$e^* = 0 \tag{95}$$

$$\alpha k^{*\alpha-1}l(k^*,n^*)^\beta = \frac{\rho}{A(1-\zeta)} \tag{96}$$

$$k^{*\alpha}l(k^*,n^*)^\beta = \frac{\phi}{A(1-(1-\zeta))\nu)} \tag{97}$$

Therefore substituting $C^*(n^*)$ into the intratemporal condition, where $\nu = \alpha + \beta < 1$, gives

$$l^*(k^*,n^*) = \left(\frac{(1-\zeta)^{1-\sigma}Ak^{*\alpha}\beta n^{*-\sigma-\eta}}{\xi\left(\frac{\phi\nu}{1-(1-\zeta)\nu}\right)^\sigma}\right)^{\frac{1}{1+\eta-\beta}} \tag{98}$$

Solving (96) and (97) gives k^* and l^*

$$k^* = \frac{\phi\alpha(1-\zeta)}{(1-(1-\zeta)\nu)\rho} \tag{99}$$

$$l^* = \left[\frac{1}{A}\left(\frac{\rho}{\alpha(1-\zeta)}\right)^\alpha\left(\frac{\phi}{1-(1-\zeta)\nu}\right)^{1-\alpha}\right]^{\frac{1}{\beta}} \tag{100}$$

These results capture that capital per firm is decreasing in market power, whilst labour per firm is ambiguous. This verifies the general result 2. Furthermore it emphasizes that fixed cost raises both inputs as does returns to scale both raise the output level that minimizes costs.

Rearranging the intratemporal condition (98) gives[33]

$$n^* = \left(\left(\frac{\alpha}{\rho\nu}\right)^{\sigma} \frac{(1-\zeta)A\beta}{\xi k^{*\sigma-\alpha} l^{*1+\eta-\beta}}\right)^{\frac{1}{\sigma+\eta}} \tag{101}$$

where k^* and l^* are in terms of model parameters defined in (99) and (100). Since k^* and l^* are increasing in fixed cost ϕ, then the number of firms is decreasing in fixed cost, which the next equation shows. By substituting in k^* and l^*, simplifying gives an expression for the steady state number of firms entirely in exogenous parameters

$$\begin{aligned}
n^* &= \left[\frac{\beta}{\xi\nu^{\sigma}}\left(A\left(\frac{1-(1-\zeta)\nu}{\phi}\right)^{1-\alpha}\left(\frac{\alpha(1-\zeta)}{\rho}\right)^{\alpha}\right)^{\frac{1+\eta}{\beta}}\right. \\
&\qquad \left.\left(\frac{\phi(1-\zeta)}{(1-(1-\zeta)\nu)}\right)^{1-\sigma}\right]^{\frac{1}{\eta+\sigma}} \\
&= \left[\frac{\beta}{\xi\nu^{\sigma}}\left\{A\left(\frac{\phi}{1-(1-\zeta)\nu}\right)^{\frac{\beta(1-\sigma)}{1+\eta}-(1-\alpha)}(1-\zeta)^{\frac{\beta}{1+\eta}(1-\sigma)+\alpha}\left(\frac{\alpha}{\rho}\right)^{\alpha}\right\}^{\frac{1+\eta}{\beta}}\right]^{\frac{1}{\eta+\sigma}} \\
&= \left[\frac{\beta}{\xi\nu^{\sigma}}\left\{A\left(\frac{1-(1-\zeta)\nu}{\phi(1-\zeta)}\right)^{\frac{1-\nu+\eta(1-\alpha)+\beta\sigma}{1+\eta}}(1-\zeta)\left(\frac{\alpha}{\rho}\right)^{\alpha}\right\}^{\frac{1+\eta}{\beta}}\right]^{\frac{1}{\eta+\sigma}}
\end{aligned} \tag{102}$$

Which simplifies to equation (47) in the paper

$$\begin{aligned}
n^* = \left[\frac{\beta}{\xi\nu^{\sigma}}\left\{\left(A\left(\frac{\alpha}{\rho}\right)^{\alpha}\right)^{1+\eta}(1-\zeta)^{\alpha(1+\eta)+\beta(1-\sigma)}\right.\right. \\
\left.\left.\left(\frac{1-(1-\zeta)\nu}{\phi}\right)^{1-\nu+\eta(1-\alpha)+\sigma\beta}\right\}^{\frac{1}{\beta}}\right]^{\frac{1}{\eta+\sigma}}
\end{aligned}$$

[33]Intermediate step from rearranging intratemporal condition is $n^* = \left(\frac{l^{*1+\eta-\beta}\xi\left(\frac{\phi\nu}{(1-(1-\zeta)\nu)}\right)^{\sigma}}{k^{*\alpha}(1-\zeta)^{1-\sigma}A\beta}\right)^{-\frac{1}{\sigma+\eta}}$ and by noting $\left(\frac{\phi\nu}{(1-(1-\zeta)\nu)}\right)^{\sigma} = \left(\frac{\nu\rho k^*}{\alpha(1-\zeta)}\right)^{\sigma}$ simplifies to above.

F Jacobian

The Jacobian is defined as follows with each element evaluated at steady state

$$\begin{bmatrix} \dot{C}_C & \dot{C}_e & \dot{C}_K & \dot{C}_n \\ \dot{e}_C & \dot{e}_e & \dot{e}_K & \dot{e}_n \\ \dot{K}_C & \dot{K}_e & \dot{K}_K & \dot{K}_n \\ \dot{n}_C & \dot{n}_e & \dot{n}_K & \dot{n}_n \end{bmatrix} \tag{103}$$

F.1 General Jacobian

Before evaluating at steady state the general version of the Jacobian is

$$\begin{bmatrix} \frac{C^*}{\sigma}(1-\zeta)\frac{A}{n}F_{kl}L_C & 0 & \frac{C^*}{\sigma}(1-\zeta)\frac{A}{n}[F_{kk}+F_{kl}L_K] & \frac{C^*}{\sigma}(1-\zeta)\frac{A}{n}[(1-\nu)F_k+F_{kl}L_n] \\ (1-\zeta)\frac{A}{n}F_{kl}L_Ce-\frac{AF_lL_C(1-(1-\zeta)\nu)}{\gamma n} & (1-\zeta)AF_k & (1-\zeta)\frac{A}{n}[F_{kk}+F_{kl}L_K]e-\Xi & (1-\zeta)\frac{A}{n}[(1-\nu)F_k+F_{kl}L_n]e-\Psi \\ AF_lL_C-1 & -\gamma e & A(F_k+F_lL_K) & (1-\nu)AF-\phi+AF_lL_n \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{104}$$

where $\Xi = \frac{A(F_k+F_lL_K)(1-(1-\zeta)\nu)}{n\gamma}$ and $\Psi = \frac{A(-\nu F+F_lL_n)(1-(1-\zeta)\nu)}{n\gamma}$. Then in steady state we have $e=0$, $F_k = \frac{\rho}{A(1-\zeta)}$, and $F = \frac{\phi}{A(1-(1-\zeta)\nu)}$. Giving the unparameterized Jacobian in steady state in terms of K^* and n^* (where function domain is in per firm form (e.g. F_x is $F_x(k,l)$) and $\nu = \alpha+\beta$) as

$$\begin{bmatrix} \frac{C^*}{n^*\sigma}(1-\zeta)AF_{kl}L_C & 0 & \frac{C^*}{n^*\sigma}(1-\zeta)A(F_{kk}+F_{kl}L_K) & \frac{C^*}{n^*\sigma}(1-\zeta)\left[(1-\nu)\frac{\rho}{1-\zeta}+AF_{kl}L_n\right] \\ -\frac{(1-(1-\zeta)\nu)}{\gamma n^*}AF_lL_C & \rho & -\frac{(1-(1-\zeta)\nu)}{\gamma n^*}\left(\frac{\rho}{1-\zeta}+AF_lL_K\right) & \frac{1}{\gamma n^*}(\nu\phi-(1-(1-\zeta)\nu)AF_lL_n) \\ AF_lL_C-1 & 0 & \frac{\rho}{1-\zeta}+AF_lL_K & \frac{-\zeta\nu\phi}{1-(1-\zeta)\nu}+AF_lL_n \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{105}$$

Then the signs of the Jacobian may be determined as

$$\begin{bmatrix} - & 0 & - & + \\ + & + & - & + \\ - & 0 & + & \pm \\ 0 & + & 0 & 0 \end{bmatrix} \tag{106}$$

F.2 Parameterized Jacobian

Under the parameterized model, the elements of the Jacobian can be determined entirely in terms of model parameters $\Omega := \{\alpha,\beta,\phi,\gamma,\xi,\rho,\eta,\zeta,\sigma\}$.

We need to substitute the following from the general Jacobian

$$\frac{C^*}{n^*} = \frac{\phi(1-\zeta)\nu}{1-(1-\zeta)\nu}, \tag{107}$$

$$F_l = k^{*\alpha}\beta l^{*\beta-1}, \tag{108}$$

$$F_{kl} = \alpha k^{*\alpha-1}\beta l^{*\beta-1}, \tag{109}$$

$$F_{kk} = \alpha(\alpha-1)k^{*\alpha-2}l^{*\beta}, \tag{110}$$

$$L_C^* = -\frac{\sigma(1-(1-\zeta)\nu)}{(1+\eta-\beta)(\phi(1-\zeta)\nu)}l^*, \tag{111}$$

$$L_n^* = \frac{1-(\alpha+\beta)}{(1+\eta-\beta)}l^*, \tag{112}$$

$$L_K^* = \frac{\alpha}{(1+\eta-\beta)}\frac{l^*}{k^*}, \tag{113}$$

So the parameterized Jacobian is as follows, and conforms to our general remark on the Jacobian signs.

$$\begin{bmatrix} -\frac{\rho\beta}{1+\eta-\beta} & 0 & \frac{\rho^2\nu((1+\eta)(\alpha-1)+\beta)}{\sigma\alpha(1+\eta-\beta)} & \frac{\phi(1-\zeta)\nu\rho(1-\nu)(1+\eta)}{(1-(1-\zeta)\nu)\sigma(1+\eta-\beta)} \\ \frac{(1-(1-\zeta)\nu)\beta\sigma}{\gamma n^*(1+\eta-\beta)(1-\zeta)\nu} & \rho & \frac{-(1-(1-\zeta)\nu)\rho(1+\eta)}{\gamma n^*(1-\zeta)(1+\eta-\beta)} & \frac{\phi(\nu(1+\eta)-\beta)}{\gamma n^*(1+\eta-\beta)} \\ \frac{-\beta\sigma-(1+\eta-\beta)(1-\zeta)\nu}{(1+\eta-\beta)(1-\zeta)\nu} & 0 & \frac{\rho(1+\eta)}{(1-\zeta)(1+\eta-\beta)} & \frac{\phi[-\zeta\nu(1+\eta-\beta)+\beta(1-\nu)]}{(1-(1-\zeta)\nu)(1+\eta-\beta)} \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{114}$$

For clarity, we have not substituted out n^* using (47) in row two.

F.3 Principal Minors

The characteristic polynomial associated with the Jacobian can be determined through the principal minors of the Jacobian matrix. Where M_k is the sum of principal minors of order k.

$$c(\lambda) = \lambda^4 - M_1\lambda^3 + M_2\lambda^2 - M_3\lambda + M_4 \tag{115}$$

The sum of principal minors of order 1 is the trace

$$M_1 = \frac{\rho\left[2(1+\rho-\beta)+\zeta(1+\eta-2\beta)\right]}{(1-\zeta)(1+\eta-\beta)} \tag{116}$$

The sum of principal minors of order 2

$$M_2 = M_{(1,1),(2,2)} + M_{(1,1),(3,3)} + M_{(1,1),(4,4)} + M_{(2,2),(3,3)} + M_{(2,2),(4,4)} + M_{(3,3),(4,4)} \tag{117}$$

$$\begin{aligned}&= 0 + \frac{-\phi(\nu(1+\eta)-\beta)}{\gamma n^*(1+\eta-\beta)} + \frac{\rho^2(1+\eta)}{(1-\zeta)(1+\eta-\beta)} \\ &\quad + 0 + \frac{-\rho^2\left[\beta\sigma + \nu(1-\zeta)(1-\nu+(1-\alpha)\eta)\right]}{(1-\zeta)(1+\eta-\beta)\alpha\sigma} + \frac{-\rho^2\beta}{1+\eta-\beta}\end{aligned} \tag{118}$$

The sum of the principal minors of order 3

$$M_3 = M_{(1,1)} + M_{(2,2)} + M_{(3,3)} + M_{(4,4)} \tag{119}$$

$$= \frac{-\rho(1+\eta)\phi\nu}{\gamma n^*(1+\eta-\beta)} + 0 + \frac{\rho\beta\phi}{\gamma n^*(1+\eta-\beta)} \tag{120}$$

$$+ \frac{-\rho^3\left[\beta\sigma + \nu(1-\zeta)(1-\nu+(1-\alpha)\eta)\right]}{(1-\zeta)(1+\eta-\beta)\sigma\alpha} \tag{121}$$

The sum of principal minors of order 4 is the determinant

$$M_4 = \frac{\rho^2\phi\beta\nu(\eta+\sigma)}{(1+\eta-\beta)\gamma\sigma\alpha n^*} \tag{122}$$

F.4 Signs of Principal Minors

It is clear that $M_1, M_4 > 0$, in other words the trace and the determinant are positive. It is also easily shown that $M_3 < 0$. Lastly M_2 is more difficult to sign. Numerical calculations make it negative, but regardless it is unnecessary to determine the number of sign changes in the characteristic polynomial. Hence the signs of the coefficients in the characteristic polynomial go

$$+ \quad - \quad \pm \quad + \quad +$$

We can see counting left to right there are two sign changes, regardless of whether M_2 is positive or negative. Thus there are two positive eigenvalues, and the remaining two are negative since the zero case is ruled out by a strictly positive determinant. The determinant (M_4) is the product of the eigenvalues, and the trace (M_1) is their sum.

F.5 Solving the Model

That is having shown the system is a saddlepath, then by the stable manifold theorem, we show saddlepath stability by setting the constant of integration on the two explosive eigenvalues to zero, thus reducing attention to the stable set. This gives a system of saddle-path conditions that may be solved for C and e. These conditions for C and e describe the stable manifold $\mathbb{M} \in \mathbb{R}^2$ of the system that ensures the constants on the explosive eigenvalues are zero, and thus solutions asymptotically reach steady state.

The solution in *open-loop* form which is in terms of initial values (K_0, n_0), t, and the parameters that comprise the eigenvectors $P^i_{j,k}$ and eigenvalues λ_j in steady state is as follows:

$$\begin{bmatrix} c(t) \\ e(t) \\ K(t) \\ n(t) \end{bmatrix} = \begin{bmatrix} c^* \\ e^* \\ K^* \\ n^* \end{bmatrix} +$$
$$\frac{1}{P^s_{1,3}P^s_{2,4} - P^s_{2,3}P^s_{1,4}} \begin{bmatrix} P^s_{1,1}P^s_{2,4}e^{\lambda_1 t} - P^s_{2,1}P^s_{1,4}e^{\lambda_2 t} & -P^s_{1,1}P^s_{2,3}e^{\lambda_1 t} + P^s_{2,1}P^s_{1,3}e^{\lambda_2 t} \\ P^s_{1,2}P^s_{2,4}e^{\lambda_1 t} - P^s_{2,2}P^s_{1,4}e^{\lambda_2 t} & -P^s_{1,2}P^s_{2,3}e^{\lambda_1 t} + P^s_{2,2}P^s_{1,3}e^{\lambda_2 t} \\ P^s_{1,3}P^s_{2,4}e^{\lambda_1 t} - P^s_{2,3}P^s_{1,4}e^{\lambda_2 t} & -P^s_{1,3}P^s_{2,3}e^{\lambda_1 t} + P^s_{2,3}P^s_{1,3}e^{\lambda_2 t} \\ P^s_{1,4}P^s_{2,4}e^{\lambda_1 t} - P^s_{2,4}P^s_{1,4}e^{\lambda_2 t} & -P^s_{1,4}P^s_{2,3}e^{\lambda_1 t} + P^s_{2,4}P^s_{1,3}e^{\lambda_2 t} \end{bmatrix} \begin{bmatrix} K(0) - K^* \\ n(0) - n^* \end{bmatrix} \tag{123}$$

Therefore on impact $t = 0$ elements $(2, 3)$ and $(1, 4)$[34] are 0, and elements $(1, 3)$ and $(2, 4)$ are 1. In other words states do not move on impact, jump variables do, which is vital for the endogenous productivity dynamics derivation.

G Productivity effect

Denote $F(K, L)$ by F. On impact $t = 0$ the state variables K, n are predetermined so do not adjust; however, L adjusts.[35] By the quotient rule differentiate $\mathcal{P} = \frac{Y}{F^{\frac{1}{\nu}}} = \frac{An^{1-\nu}F+n\phi}{F^{\frac{1}{\nu}}}$ with K, n fixed

$$\begin{aligned} \frac{d\mathcal{P}(0)}{dA} = F^{-\frac{1}{\nu}}(n^{1-\nu}F + An^{1-\nu}F_L L_A) - \\ (An^{1-\nu}F + n\phi)\frac{1}{\nu}F^{-\frac{1}{\nu}-1}F_L L_A \end{aligned} \tag{124}$$

Evaluate impact effect beginning in steady state $\left.\frac{\partial \mathcal{P}(0)}{\partial A}\right|_{K^*,n^*}$. In equilibrium

$$\begin{aligned} \mathcal{P}^* &= (1-\zeta)A\nu {n^*}^{1-\nu} F(K^*, L^*)^{1-\frac{1}{\nu}} \\ &= (1-\zeta)A^{\frac{1}{\nu}}\nu \left(\frac{\phi}{1-(1-\zeta)\nu}\right)^{1-\frac{1}{\nu}} \end{aligned} \tag{125}$$

[34] Which may first be simplified to $P^s_{1,3}P^s_{2,3}(e^{\lambda_2 t} - e^{\lambda_1 t})$ and $P^s_{1,4}P^s_{2,4}(e^{\lambda_1 t} - e^{\lambda_2 t})$

[35] See Caputo 2005 p.426

so $\frac{\partial \mathcal{P}^*}{\partial A} = (1-\zeta)A^{\frac{1}{\nu}-1}\left(\frac{\phi}{1-(1-\zeta)\nu}\right)^{1-\frac{1}{\nu}} = \frac{\mathcal{P}^*}{A\nu}$. Therefore

$$\frac{d\mathcal{P}(0)}{dA} = n^{1-\nu}F^{1-\frac{1}{\nu}} + \frac{\mathcal{P}^*}{(1-\zeta)F}F_L L_A - \mathcal{P}^*\frac{1}{\nu F}F_L L_A \tag{126}$$

$$= \frac{\mathcal{P}^*}{(1-\zeta)A\nu} + \frac{\mathcal{P}^* F_L L_A}{\nu F}\left(\frac{\zeta}{1-\zeta}\right) \tag{127}$$

$$= \frac{1}{(1-\zeta)}\frac{d\mathcal{P}^*}{dA} + \frac{\mathcal{P}^* F_L L_A}{\nu F}\left(\frac{\zeta}{1-\zeta}\right) \tag{128}$$

$$\frac{d\mathcal{P}(0)}{dA} - \frac{d\mathcal{P}^*}{dA} = \left(\frac{\zeta}{1-\zeta}\right)\frac{d\mathcal{P}^*}{dA} + \frac{\mathcal{P}^* F_L L_A}{\nu F}\left(\frac{\zeta}{1-\zeta}\right) \tag{129}$$

Substitute out $\mathcal{P}^*$ and $\frac{\partial \mathcal{P}^*}{\partial A}$

$$\frac{d\mathcal{P}(0)}{dA} - \frac{d\mathcal{P}^*}{dA} = \zeta\left(\frac{\phi}{A(1-(1-\zeta)\nu)}\right)^{1-\frac{1}{\nu}}\left(1 + \frac{AF_L L_A}{F}\right) \tag{130}$$

References

Aloi, Marta and Huw Dixon (2003). “Entry Dynamics, Demand Shocks and Induced Productivity Fluctuations”. In: *Ekonomia* 6.2, pp. 115–146.

Ambler, Steve and Emanuela Cardia (1998). “The Cyclical Behaviour of Wages and Profits under Imperfect Competition”. In: *Canadian Journal of Economics* 31.1, pp. 148–164.

Basu, Susanto and John Fernald (2001). “Why Is Productivity Procyclical? Why Do We Care?” In: *New Developments in Productivity Analysis*. NBER Chapters. National Bureau of Economic Research, Inc, pp. 225–302.

Benhabib, Jess, Kazuo Nishimura, and Tadashi Shigoka (2008). “Bifurcation and sunspots in the continuous time equilibrium model with capacity utilization”. In: *International Journal of Economic Theory* 4.2, pp. 337–355.

Berentsen, Aleksander and Christopher Waller (2015). “Optimal Stabilization Policy with Search Externalities”. In: *Macroeconomic Dynamics* 19 (03), pp. 669–700.

Berndt, Ernst R and Melvyn A Fuss (1986). “Productivity measurement with adjustments for variations in capacity utilization and other forms of temporary equilibrium”. In: *Journal of econometrics* 33.1, pp. 7–29.

Bilbiie, Florin O., Fabio Ghironi, and Marc J. Melitz (2012). “Endogenous Entry, Product Variety, and Business Cycles”. In: *Journal of Political Economy* 120.2, pp. 304 –345.

Bils, Mark and Jang-Ok Cho (1994). “Cyclical factor utilization”. In: *Journal of Monetary Economics* 33.2, pp. 319–354.
Brito, Paulo and Huw Dixon (2013). “Fiscal policy, entry and capital accumulation: Hump-shaped responses”. In: *Journal of Economic Dynamics and Control* 37.10, pp. 2123–2155.
Brock, William A and Stephen J Turnovsky (1981). “The analysis of macroeconomic policies in perfect foresight equilibrium”. In: *International Economic Review*, pp. 179–209.
Burnside, A Craig, Martin S Eichenbaum, and Sergio T Rebelo (1996). “Sectoral Solow residuals”. In: *European Economic Review* 40.3, pp. 861–869.
Burnside, Craig, Martin Eichenbaum, and Sergio Rebelo (1995). “Capital utilization and returns to scale”. In: *NBER Macroeconomics Annual 1995, Volume 10*. MIT Press, pp. 67–124.
Campbell, Jeffrey R (1998). “Entry, exit, embodied technology, and business cycles”. In: *Review of economic dynamics* 1.2, pp. 371–408.
Caputo, M.R. (2005). *Foundations of Dynamic Economic Analysis: Optimal Control Theory and Applications*. Cambridge University Press.
Caves, Douglas W, Laurits R Christensen, and Joseph A Swanson (1981). “Productivity Growth, Scale Economies, and Capacity Utilization in U.S. Railroads, 1955-74”. In: *American Economic Review* 71.5, pp. 994–1002.
Chamberlin, Edward (1933). *The theory of monopolistic competition*. Volume 38 of Harvard economic studies. Harvard University Press.
Cooley, Thomas F, Gary D Hansen, and Edward C Prescott (1995). “Equilibrium business cycles with idle resources and variable capacity utilization”. In: *Economic Theory* 6.1, pp. 35–49.
Das, Sanghamitra and Satya P Das (1997). “Dynamics of entry and exit of firms in the presence of entry adjustment costs”. In: *International Journal of Industrial Organization* 15.2, pp. 217–241.
Datta, Bipasa and Huw Dixon (2002). “Technological Change, Entry, and Stock-Market Dynamics: An Analysis of Transition in a Monopolistic Industry”. In: *American Economic Review* 92.2, pp. 231–235.
Diewert, W. Erwin and Kevin J. Fox (2008). “On the estimation of returns to scale, technical progress and monopolistic markups”. In: *Journal of Econometrics* 145.1-2, pp. 174–193.
Dixit, Avinash K. and Joseph E. Stiglitz (1977). “Monopolistic Competition and Optimum Product Diversity”. English. In: *The American Economic Review* 67.3, pp. 297–308.
Dos Santos Ferreira, Rodolphe and Frédéric Dufourt (2006). “Free entry and business cycles under the influence of animal spirits”. In: *Journal of Monetary Economics* 53.2, pp. 311–328.

Ericson, Richard and Ariel Pakes (1995). “Markov-Perfect Industry Dynamics: A Framework for Empirical Work”. English. In: *The Review of Economic Studies* 62.1, pp. 53–82.

Etro, Federico (2009). *Endogenous market structures and the macroeconomy.* Springer Science & Business Media.

Etro, Federico and Andrea Colciago (2010). “Endogenous Market Structures and the Business Cycle*”. In: *The Economic Journal* 120.549, pp. 1201–1233.

Greenwood, Jeremy, Zvi Hercowitz, and Gregory W Huffman (1988). “Investment, capacity utilization, and the real business cycle”. In: *The American Economic Review*, pp. 402–417.

Hall, Robert E (1989). *Invariance properties of Solow's productivity residual.* Tech. rep. National Bureau of Economic Research.

Harrison, Ann E. (1994). “Productivity, imperfect competition and trade reform : Theory and evidence”. In: *Journal of International Economics* 36.1-2, pp. 53–73.

Haskel, J, P Goodridge, and G Wallis (2015). *Accounting for the UK productivity puzzle: a decomposition and predictions.* Working Papers 21167. Imperial College, London, Imperial College Business School.

Jaimovich, Nir (2007). “Firm dynamics and markup variations: Implications for sunspot equilibria and endogenous economic fluctuations”. In: *Journal of Economic Theory* 137.1, pp. 300–325.

Jaimovich, Nir and Max Floetotto (2008). “Firm dynamics, markup variations, and the business cycle”. In: *Journal of Monetary Economics* 55.7, pp. 1238 –1252.

King, Robert G. and Sergio T. Rebelo (1999). “Resuscitating real business cycles”. In: *Handbook of Macroeconomics.* Ed. by J. B. Taylor and M. Woodford. Vol. 1. Handbook of Macroeconomics. Elsevier. Chap. 14, pp. 927–1007.

Kydland, Finn and Edward Prescott (1991). “Hours and Employment Variation in Business Cycle Theory”. In: *Economic Theory* 1.1, pp. 63–81.

Kydland, Finn E and Edward C Prescott (1988). “The workweek of capital and its cyclical implications”. In: *Journal of Monetary Economics* 21.2-3, pp. 343–360.

Lee, Yoonsoo and Toshihiko Mukoyama (2015). “Entry and exit of manufacturing plants over the business cycle”. In: *European Economic Review* 77, pp. 20 –27.

Lewis, Vivien (2009). “Business Cycle Evidence On Firm Entry”. In: *Macroeconomic Dynamics* 13 (05), pp. 605–624.

Lewis, Vivien and Céline Poilly (2012). "Firm entry, markups and the monetary transmission mechanism". In: *Journal of Monetary Economics* 59.7, pp. 670–685.

Melitz, Marc J (2003). "The impact of trade on intra-industry reallocations and aggregate industry productivity". In: *Econometrica* 71.6, pp. 1695–1725.

Morrison, Catherine J (1992). "Unraveling the productivity growth slowdown in the United States, Canada and Japan: the effects of subequilibrium, scale economies and markups". In: *The Review of Economics and Statistics*, pp. 381–393.

Pessoa, João Paulo and John Van Reenen (2014). "The UK productivity and jobs puzzle: does the answer lie in wage flexibility?" In: *The Economic Journal* 124.576, pp. 433–452.

Portier, Franck (1995). "Business Formation and Cyclical Markups in the French Business Cycle". English. In: *Annales d'Économie et de Statistique* 37/38, pp. 411–440.

Rossi, Lorenza and Emilio Zanetti Chini (2016). *Firms' Dynamics and Business Cycle: New Disaggregated Data*. Tech. rep. University of Pavia, Department of Economics and Management.

Rotemberg, Julio J and Michael Woodford (1992). "Oligopolistic Pricing and the Effects of Aggregate Demand on Economic Activity". In: *Journal of Political Economy* 100.6, pp. 1153–1207.

Shapiro, Matthew D (1993). "Cyclical productivity and the workweek of capital". In: *The American Economic Review* 83.2, pp. 229–233.

Tirole, J. (1988). *The Theory of Industrial Organization*. Cambridge, Mass.

Vives, X. (1999). *Oligopoly Pricing: Old Ideas and New Tools*. MIT Press.

Wen, Yi (1998). "Capacity Utilization under Increasing Returns to Scale". In: *Journal of Economic Theory* 81.1, pp. 7 –36.

— (2001). "Understanding self-fulfilling rational expectations equilibria in real business cycle models". In: *Journal of Economic Dynamics and Control* 25.8, pp. 1221 –1240.