

The Macroeconomic Impact of Money Market Freezes*

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Abstract

We build a general equilibrium model featuring unsecured and secured interbank markets, and collateralized central bank funding. The model accounts for some key facts about the European money markets since 2008: i) the decline in the ratio of interbank liabilities in total bank assets since the onset of the global financial crisis; ii) the reduced ability of banks to access the unsecured market during the sovereign crisis, and their shift to secured market funding; iii) the increased reliance on central bank funding, particularly for banks in countries with a vulnerable sovereign. Using the calibrated model, we find that a decline in the share of unsecured to secured interbank market transactions, as observed during the crisis, generates a sizeable macroeconomic impact.

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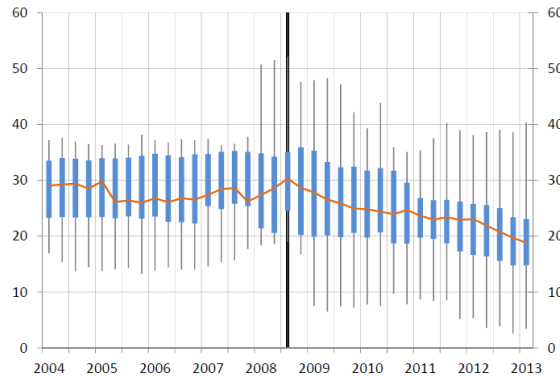
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1 Introduction

In this paper, we construct a dynamic general equilibrium model featuring heterogeneous banks, interbank money markets for both secured and unsecured credit, and a central bank providing funding against collateral. Interbank markets are essential to banks' liquidity management. They also play a key role in the transmission of monetary policy. These markets came under severe stress during the global financial crisis of 2007-2009 as well as during the euro area sovereign debt crisis of 2010-2012.¹ We use our model to assess the macroeconomic impact of the observed interbank market tensions.

Our modelling approach is motivated by three stylized facts about euro area money markets. First, interbank money markets are an important funding source for banks in the euro area but their share in total interbank funding has been diminishing since 2008 (see Figure 1). In 2008, the ratio of interbank liabilities to total assets was about 30%. This ratio started to decline with the onset of the global financial crisis, dipping below 20% by 2013. This declining trend reflects tensions in money markets, with some market segments “freezing” or drying-up altogether.

Figure 1: Interbank liabilities in total assets



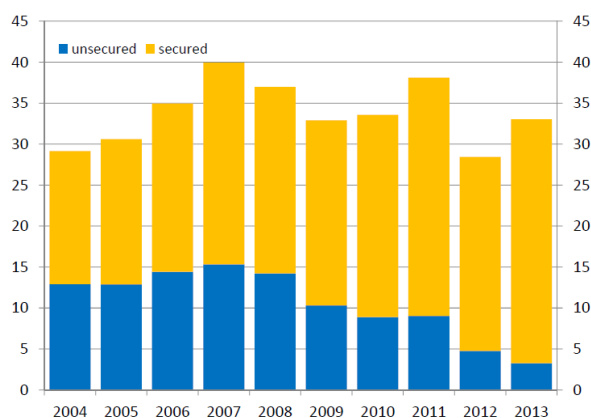
Notes: Interbank liabilities in total assets, percentages, Q1 2004 - Q1 2013; maximum, minimum and interquartile distribution across euro area countries; black vertical line in Q3 2008 indicates the Lehman collapse and the intensification of the global financial crisis. Source: ECB dataset, own calculations.

Second, there was a dramatic shift away from unsecured and towards secured money market funding since 2008 (see Figure 2). The secured money market segment was nearly double that of the unsecured segment in 2008 in terms of the transaction volumes. During the financial

¹The failure of the interbank market to redistribute liquidity was highlighted in a number of accounts of the recent crisis (see, for example, Allen and Carletti, 2008, and Brunnermeier, 2009).

crisis, the share of secured funding grew, as some banks became unable to borrow in the unsecured markets due to perceptions of increased counterparty risk and shifted to secured borrowing instead. By 2013, the secured segment was ten times bigger compared to the unsecured segment.

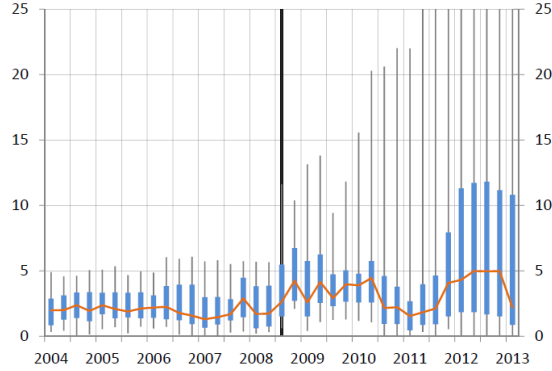
Figure 2: Quarterly turnover in the euro money market



Notes: Cumulative quarterly turnover in the euro money market (EUR trillion); unsecured market (blue bars) and secured market (yellow bars). Based on the panel comprising 98 euro area credit institutions. Source: Euro Area Money Market Survey.

Third, with private money markets malfunctioning, banks increasingly turned to the central bank for refinancing (see Figure 3). Reliance on central bank funding gradually rose in the euro area, particularly with the onset of the sovereign debt crisis. Banks located in euro area countries with vulnerable sovereigns started borrowing larger amounts from the ECB and pledging riskier collateral, taking advantage of the more favourable haircuts on risky assets imposed by the ECB relative to the secured market (Drechsler, Drechsel, Marques-Ibanez and Schnabl, 2016).

Figure 3: Eurosystem funding in total deposit liabilities



Notes: Eurosystem funding in total deposit liabilities, percentages, Q1 2004 - Q1 2013; maximum, minimum and interquartile distribution across euro area countries; black vertical line in Q3 2008 indicates the Lehman collapse and the intensification of the global financial crisis. Source: ECB dataset, own calculations.

In our model, banks can refinance in the secured or unsecured market, or at the central bank. Banks face an exogenous probability of being “connected,” defined as the ability to borrow in the unsecured market. To access the secured market and central bank funding banks need to hold government bonds which can be pledged as collateral. Collateralized borrowing is subject to a haircut, which can be different in the private secured market and at the central bank. If a bank loses access to the unsecured market but its government bond holdings are sufficiently valuable, it can replace unsecured funding with secured funding. Our model can therefore capture the shift from unsecured to secured funding observed in the recent years. However, if private counterparties become reluctant to accept a particular government bond as collateral (due to, e.g., concerns about that sovereign’s health), access to the secured market will become impaired as well.

Banks can always access central bank funding in exchange for pledging sufficient collateral. We allow not only for possible differences between haircuts in the private secured market and those charged by the central bank but also for differences in the type of securities that are accepted as collateral at the central bank and in the private secured market. The latter feature of our model allows us to capture the fact that, in crisis times, haircuts set by the central bank can be relatively unfavorable compared to the private market for high-quality collateral, the opposite being true for low-quality collateral (Drechsler, Drechsel, Marques-Ibanez and Schnabl, 2016). In such circumstances, banks in stressed countries may replace their lost unsecured and secured funding with borrowing from the central bank.

We calibrate our model using euro area data to quantify the impact of money market tensions on the economy. We analyze the impact of 1) reduced access to the unsecured money market; 2) increased haircuts in the secured market, and 3) increased customer withdrawals, reported in some euro area countries.²

We find that money market malfunctioning and customer “runs” generate a sizeable macroeconomic impact. For example, as haircuts in the secured markets increase, the model economy moves between two regions. When private haircuts are low and collateral valuable, the economy is in the region that is reminiscent of “normal times,” whereby the large majority of banks have access to the private funding markets (either secured or unsecured), and there is no recourse to central bank funding (in excess of what is used to cover expected liquidity needs, e.g., reserve requirements). When private haircuts increase beyond a certain threshold, the economy moved to the region that corresponds to a period of financial stress. Banks relying on secured markets for funding become unable to cover their unexpected liquidity needs as the value of their collateral drops. They either have to self-insure against liquidity shocks by holding precautionary cash buffers or - if haircuts on central bank funding are more favorable than in the private market - can turn to the central bank for their refinancing needs. Quantitatively, results from our calibrated model show that an increase in private haircuts from 5% to 20% generates a reduction in GDP of around 3 percent. During the crisis, some securities (and securities from some countries) faced a far more dramatic increase in haircuts. Some assets seized to be accepted as collateral altogether, which is equivalent to a 100% haircut.

This paper is related to the literature on interbank markets and on the impact of sovereign risk on the macroeconomy. There is an extensive literature in banking on the role of interbank markets in banks’ liquidity management, starting with Bhattacharya and Gale (1987). A number of recent papers focused on analyzing frictions that prevent interbank markets from distributing liquidity efficiently within the banking system. Frictions include asymmetric information about banks’ assets (Flannery, 1996, Freixas and Jorge, 2007, Heider, Hoerova and Holthausen, 2015), imperfect cross-border information (Freixas and Holthausen, 2005), banks’ free-riding on liquidity provision by the central bank (Repullo, 2005), and multiplicity of Pareto-ranked equilibria (Freixas, Martin and Skeie, 2011). Papers in this strand of literature tend to be partial equilibrium and static, with links to the real economy modeled in a reduced-form fashion.

²See, e.g., “Worrying about a Greek bank run,” Reuters, April 15, 2010, <http://blogs.reuters.com/felix-salmon/2010/04/15/worrying-about-a-greek-bank-run/>.

Several recent papers build dynamic general equilibrium models which include interbank trade. For example, Afonso and Lagos (2015) develop an OTC model of the federal funds market and use it to study the intraday evolution of the distribution of reserve balances. Atkeson, Eisfeldt, and Weill (2015) present a model to study trading decisions of banks in an OTC market and draw implications for policy. Bianchi and Bigio (2014) develop a framework to study the implementation of monetary policy through the banking system. Our paper contributes to this literature by considering both unsecured and secured interbank markets, and collateralized lending from the central bank.

Since lending from the central bank is backed by government bonds, and haircuts on sovereign bonds can increase in our model (capturing in a reduced-form the impact of sovereign default risk), our paper is related to the literature on the impact of sovereign default risk on financial intermediation and the macroeconomy. Recent contributions to this strand of literature study the impact of sovereign risk on the funding ability of banks and their lending decisions (Boccola, 2016) as well as the link between government default and financial fragility, including the question of why the banking system may become exposed to government bonds (e.g., Genaioli, Martin, and Rossi, 2014). We do not model sovereign default risk explicitly, focusing instead on the implications of increased haircuts on government bonds for banks' ability to borrow in secured and unsecured interbank markets, as well as on how central bank policies can help alleviate bank funding problems arising due to increased private market haircuts.

The paper proceeds as follows. In section 2, we describe the model. In section 3, we define the equilibrium. In section 4, we characterize the system of equilibrium conditions. In section 5, we describe the steady state and present some analytical results. Section 6 illustrates the model predictions through a numerical analysis. Section 7 concludes.

2 The model

The economy is inhabited by a continuum of households, firms and banks. There is a government and a central bank.

Time is discrete, $t = 0, 1, 2, \dots$. We think of a period as composed of two sub-periods (“morning” and “afternoon”). Let us describe each in turn.

At the beginning of each period (in the morning), aggregate shocks occur. Households receive payments from financial assets and allocate their nominal wealth among money, long-term government bonds, and deposits at banks. Households also supply labor to firms in their

country, receiving wages in return. The government taxes the labor income of the households in its country, makes payments on its debt and may change the stock of outstanding debt. Banks accept deposits from households and the central bank and make dividend payments to households. After accepting the deposits, banks learn their afternoon type in the morning. This latter can be either “connected,” in which case banks can borrow in the unsecured interbank market, or “not connected,” in which case they cannot, and the only possibility is to borrow by pledging assets in the secured interbank market. Banks then lend to firms (more precisely, finance their capital) and they hold government bonds and reserves (“cash”). The central bank provides funding to banks that wish to borrow against collateral. As an additional policy tool, the central bank can choose “haircuts” on the collateral pledged to access those funds.

During the afternoon, firms use labor and capital to produce a homogeneous output good which is consumed by households. Banks experience idiosyncratic deposit withdrawal shocks which average out to zero across all banks. Conceptually, these relate to random idiosyncratic consumption needs, additional economic activity and immediate payment for these services, which we shall refrain from modelling. Banks can accommodate those shocks by using their existing reserves, by selling government bonds or by borrowing in the unsecured market from other banks. They can only access the unsecured market, however, if they are “connected”. Banks are assumed to always position themselves so as to meet these liquidity withdrawals, i.e., bank failures are considered too costly and not an option. All banks meet as “one big banker family” at the end of the period. One can think of it as follows. First, the same bank-individual liquidity shock happens “in reverse”, so that banks enter the banker-family meeting in the same state they were in at the beginning of the afternoon. However, there would then still be bank heterogeneity left. Thus, banks all equate their positions at that point and restart the next period with the same portfolio. Alternatively, and equivalently, one can think that there are securities markets which open at the end of the period and allow banks to equate their portfolios. Banks during the period therefore are only concerned with the marginal value of an additional unit of net worth they can produce for the next period.

Firms and banks are owned by households. Similar to Gertler and Kiyotaki (2011) and Gertler and Karadi (2011), banks are operated by bank managers who run a bank on behalf of their owning households. We deviate from those papers in that we assume that banks pay a fixed fraction of their net worth to households as a dividend in the morning of every period.

2.1 The households

There is a representative household, indexed by $i \in (0, 1)$. In each period t , the household invests in three assets: cash, M_t^H , one period deposits at banks, D_t , and government bonds issued by the government with maturity $1/\kappa$, B_t^H .

At the beginning of time t , banks repay deposits opened in the previous period gross of the due interest, $R_{t-1}^D D_{t-1}$. The governments make payments on a fraction of the bonds held by investors, i.e. a household receives cash payments κB_{t-1}^H . The remaining government bonds in the hands of the households after cash payments are thus $(1 - \kappa) B_{t-1}^H$. The household then changes its bond position by purchasing amounts ΔB_t^H at current market prices Q_t , using cash. The position can be increased, if ΔB_t^H is positive, or decreased, if negative. Thus, the household's bond holdings at the end of period t are $B_t^H = (1 - \kappa) B_{t-1}^H + \Delta B_t^H$.

The timing of households' decisions is as follows. In the morning, holding an amount H_t of nominal wealth at hand, each household chooses how to allocate it among existing nominal assets, namely money, M_t^H , bonds in the amount ΔB_t^H , and deposits, D_t .

During the day, beginning-of-period money balances are increased by the value of households' revenues and decreased by the value of their expenses. The amount of nominal balances brought by household i into period $t + 1$, $\widetilde{M}_t^H$, is thus

$$\widetilde{M}_t^H = M_t^H + (1 - \tau_t) W_t l_t + E_t - P_t c_t, \quad (1)$$

where P_t is the price of the consumption good, l_t is hours worked, τ_t is the labor tax rate, W_t is the nominal wage level, and E_t is the profit payout ("earnings") by banks.

The nominal wealth available at the beginning of period $t + 1$ for investment in nominal assets is given by

$$H_{t+1} = R_t^D D_t + \widetilde{M}_t^H + \kappa B_t^H. \quad (2)$$

The household then solves the problem

$$\max_{\{c_t > 0, l_t > 0, D_t \geq 0, M_t^H \geq 0, B_t \geq 0\}} E_t \sum_{t=0}^{\infty} \beta^t \left[u(c_t, l_t) + v\left(\frac{M_t^H}{P_t}\right) \right] \quad (3)$$

subject to (2) and

$$D_t + M_t^H + Q_t (B_t^H - (1 - \kappa) B_{t-1}^H) \leq H_t.$$

2.2 Firms

A final-good firm j uses capital $k_{t-1,j}$ and labor $l_{t,j}$ to produce a homogeneous final output good $y_{t,j}$ according to the production function

$$y_{t,j} = \gamma_t k_{t-1,j}^\theta l_{t,j}^{1-\theta}$$

where γ_t is a country-specific productivity shock. It receives revenues $P_t y_{t,j}$ and pays wages $W_t l_{t,j}$. Capital is owned by the firms, which are in turn owned by banks: effectively then, the banks own the capital, renting it out to firms and extracting a real “rental rate” r_t per unit of capital or total nominal rental rate payments $P_t r_t k_{t-1,j}$ from firm j on their capital $k_{t-1,j}$.

Capital-producing firms buy old capital k_{t-1} from the banks and combine it with final goods I_t to produce new capital k_t , according to

$$k_t = (1 - \delta) k_{t-1} + I_t.$$

New capital is then sold back to banks. Alternatively and equivalently, one may directly assume that the banks undertake the investments.

3 The government

The government has some outstanding debt with face value $\bar{B}_{t-1}$. It needs to purchase goods g_t and pays for it by taxing labor income as well as issue discount bonds with a face value $\Delta \bar{B}_t$ to be added to the outstanding debt next period, obtaining nominal resources $Q_t \Delta \bar{B}_t$ for it in period t . We assume that some suitable no-Ponzi condition holds. The government discount bonds are repaid at a rate κ .

The outstanding debt at the beginning of period $t + 1$ will be

$$\bar{B}_t = (1 - \kappa) \bar{B}_{t-1} + \Delta \bar{B}_t \tag{4}$$

The government budget balance at time t is

$$P_t g_t + \kappa \bar{B}_{t-1} = \tau_t W_t l_t + Q_t \Delta \bar{B}_t + S_t \tag{5}$$

where S_t are seignorage payments from the central bank. We assume that the government expenditures g_t is an exogenously given process. The government follows the policy rule

$$\tau_t W_t l_t = \alpha \bar{B}_{t-1} \quad (6)$$

determining the tax rate τ_t and newly issued debt $\Delta \bar{B}_t$.

3.1 Central Bank

The central bank chooses the total money supply $\bar{M}_t$ and interacts with banks in the “morning”, providing them with funds. Banks come into the period with total liabilities (F=“funds from the central bank”) at face value $\bar{F}_{t-1}$. Banks make payments $\kappa^F \bar{F}_{t-1}$ on these liabilities and obtain new funds, at face value $\Delta \bar{F}_t$. Thus,

$$\bar{F}_t = (1 - \kappa^F) \bar{F}_{t-1} + \Delta \bar{F}_t \quad (7)$$

Banks obtain funds $Q_t^F \Delta \bar{F}_t$ for these new liabilities, at the common price or discount factor Q_t^F . This discount factor is a policy parameter set by the central bank. The central bank furthermore buys and sells government bonds outright. Let B_{t-1}^C be the stock of government bonds held by the central bank (“C”) at the beginning of period t . The government makes payments on a fraction of these bonds, i.e., the central bank receives cash payments κB_{t-1}^C . The remaining government bonds in the hands of the central banks are $(1 - \kappa) B_{t-1}^C$. The central bank then changes its stock to B_t^C , at current market prices Q_t , using cash. Thus,

$$B_t^C = (1 - \kappa) B_{t-1}^C + \Delta B_t^c$$

The central bank balance sheet looks as follows at time t :

Assets	Liabilities
$Q_t^F \bar{F}_t$ (loans to banks)	M_t^H (currency held by HH)
$Q_t B_t^C$ (bond holdings)	M_t (bank reserves)
	S_t (seignorage)

Let

$$\bar{M}_t = M_t^H + M_t$$

be the total money stock before seignorage is paid. Note that the seignorage is paid to the government at the end of the period and therefore becomes part of the currency in circulation next period. The flow budget constraint of the central bank is given by:

$$\begin{aligned}\overline{M}_t - \overline{M}_{t-1} &= S_{t-1} + Q_t^F (\overline{F}_t - (1 - \kappa^F) \overline{F}_{t-1}) - \kappa^F \overline{F}_{t-1} \\ &\quad + Q_t (B_t^C - (1 - \kappa) B_{t-1}^C) - \kappa B_{t-1}^C.\end{aligned}\tag{8}$$

Seignorage can then be calculated as the residual balance sheet profit,

$$S_t = Q_t^F \overline{F}_t + Q_t B_t^C - \overline{M}_t.\tag{9}$$

Using equation (9) to substitute S_{t-1} in (8) implies the following equation for the money stock, which we shall use instead of the flow equation above, in our calculations,

$$\begin{aligned}\overline{M}_t &= Q_{t-1}^F \overline{F}_{t-1} + Q_{t-1} B_{t-1}^C + Q_t^F (\overline{F}_t - (1 - \kappa^F) \overline{F}_{t-1}) - \kappa^F \overline{F}_{t-1} \\ &\quad + Q_t (B_t^C - (1 - \kappa) B_{t-1}^C) - \kappa B_{t-1}^C\end{aligned}\tag{10}$$

This equation essentially says that the outstanding central bank money liabilities equal the cash given out for the stock of all loans and bonds held last period plus the cash needed this period for open market operations, i.e. changes in these stocks, minus the repayments made on the loans and bonds.

Substituting out $\overline{M}_t$ once again with equation (10) and solving for S_t yields

$$\begin{aligned}S_t &= (Q_t^F - Q_{t-1}^F) (1 - \kappa^F) \overline{F}_{t-1} + (1 - Q_{t-1}^F) \kappa^F \overline{F}_{t-1} \\ &\quad + (Q_t - Q_{t-1}) (1 - \kappa) B_{t-1}^C + (1 - Q_{t-1}) \kappa B_{t-1}^C\end{aligned}$$

Here, profits are expressed as the change in the valuation for the remaining part of the last-period stock of loans and bonds plus the difference between the cash payments received today and their last-period-valuation on these stocks.

3.2 Banks

There is a continuum of banks (“Lenders”), indexed by $l \in (0, 1)$. Consider a bank l .

3.2.1 Assets and liabilities

At the end of the morning, after earning income on its assets, paying interest on its liabilities and re trading, but just before paying dividends to share holders, the bank holds four type of assets. It additionally and briefly holds an asset in the afternoon, for a total of five. As an overview, the end-of-morning balance sheet of that bank is

Assets	Liabilities
$P_t k_{t,l}$ (capital held)	$D_{t,l}$ (deposits by HH)
$Q_t B_{t,l}$ (bond holdings)	$Q_t^F F_{t,l}$ (secured loans)
$E_{t,l}$ (cash dividends)	$N_{t,l}$ (net worth)
$M_{t,l}$ (cash reserves)	

In detail:

1. Capital $k_{t,l}$ of firms, or, equivalently, firms, who in turn own the capital. Capital can only be acquired and traded in the morning. Capital evolves according to

$$k_{t,l} = (1 - \delta) k_{t-1,l} + \Delta k_{t,l}$$

where $\Delta k_{t,l}$ is the gross investment of bank l in capital.

2. Bonds with a nominal face value $B_{t,l}$. A fraction κ of the government debt will be repaid. The bank changes its government bond position per market purchases or sales ("-") $\Delta B_{t,l}$ in the morning, so that

$$B_{t,l} = (1 - \kappa) B_{t-1,l} + \Delta B_{t,l}$$

at the end of the morning. If the bank purchases (sells) bonds on the open market, it pays (receives) $Q_t \Delta B_{t,l}$. As a baseline, we allow $\Delta B_{t,l}$ to be negative, indicating a sale. In the afternoon and after the first bank-individual liquidity shock, once again, the bank can change its government bond position per market purchases or sales ("-") $\widetilde{\Delta B}_{t,l}$, so that

$$\widetilde{B}_{t,l} = B_{t,l} + \widetilde{\Delta B}_{t,l}$$

When the second reverse liquidity shock hits, the trade is reversed as well, resulting in

$$B_{t,l} = \widetilde{B}_{t,l} - \widetilde{\Delta B}_{t,l}$$

One can instead think of this as a secured repo market, vis-a-vis other banks. To that end, it is useful to introduce haircut parameters $0 \leq \tilde{\eta}_t \leq 1$, imposed by other lending banks. The bank then receives the cash amount $\tilde{\eta}_t Q_t \widetilde{\Delta B}_{t,l}$ in the first of these two transactions, repaying the same amount in the second. Taken literally, there is no risk here that this haircut could reasonably insure against, but this is just due to keeping the model simple. The interest rate is zero.

3. Cash $E_{t,l}$ earmarked to be distributed to shareholders (E = “earmarked” or “earnings”) at the end of the morning. Note that this does not mean that the households end up being forced to hold money, as everything happens “simultaneously” in the morning. If they want to hold those extra earnings as extra deposits, then D_t would simply already be higher before they receive the earnings from the banks, in “anticipation” of these earning payments.
4. Reserves (M=“money”) $M_{t,l} \geq 0$. They may add to cash (not earmarked for paying shareholders) in the morning,

$$M_{t,l} = M_{t-1,l} + \Delta M_{t,l} \geq 0$$

as well as in the afternoon,

$$\widetilde{M}_{t,l} = M_{t,l} + \widetilde{\Delta M}_{t,l} \geq 0$$

reversing the first-liquidity-shock transaction when the reverse liquidity shock hits,

$$M_{t,l} = \widetilde{M}_{t,l} - \widetilde{\Delta M}_{t,l}$$

5. Unsecured claims on other banks at face value, obtained during the first liquidity shock in the afternoon. They are repaid at zero interest rate during the second reverse-liquidity shock.

Bank l has four types of liabilities:

1. Deposits $D_{t,l}$. This is owed to household and subject to aggregate withdrawals and additions $\Delta D_{t,l}$ in the morning, so that

$$D_{t,l} = R_{t-1}^D D_{t-1,l} + \Delta D_{t,l}$$

where R_{t-1}^D is the return on one unit of deposits, agreed at time $t - 1$. Additionally, there are idiosyncratic withdrawals and additions in the afternoon, to be described.

2. Secured loans (F=“funding”) from the central bank at face value $F_{t,l}$. Secured loans require collateral. A bank l with liabilities $F_{t,l}$ to the central banks needs to pledge an amount $0 \leq B_{t,l}^F \leq B_{t,l}$ of government bonds $B_{t,l}$ satisfying the collateral constraint

$$F_{t,l} \leq \eta_t Q_t B_{t,l}^F \quad (11)$$

where η_t is a haircut parameter and is set by the central bank. The collateral constraints are set in terms of the market value of securities, as is the case in ECB monetary policy operations. Secured loans from the central bank are obtained in the morning. The change in the secured loans $\Delta F_{t,l}$ provide the banks with change in liquidity (“cash”) of $Q_t^F \Delta F_{t,l}$, in addition to the liquidity carried over from the previous period. Liquidity is needed in the afternoon. Therefore, the discount rate Q_t^F will not only relate to an intertemporal trade-off, as is common in most models, but importantly also to the intratemporal tradeoff of obtaining potentially costly liquidity in the morning in order to secure sufficient funding in the afternoon.

3. Outstanding unsecured liabilities to other banks issued at the time of the first liquidity shock in the afternoon. Only “connected” banks can issue them. They are repaid at zero interest rate at the time of the reverse liquidity shock.
4. Net worth $N_{t,l}$.

The sum of assets equals the sum of liabilities, at any point in time.

3.2.2 Liquidity needs in the afternoon

At the beginning of the afternoon, households hold total deposits D_t with banks. We seek to capture the daily churning of deposits at banks, due to cross-household and firm-household payment activities with inside money, as follows. At the start of the afternoon in period t , deposits get reshuffled across banks, so that bank l with pre-shuffle end-of-morning deposits $D_{t,l}$ experiences a withdrawal $\omega_{t,l} D_{t,l}$. Here, $\omega = \omega_{t,l} \in (-\infty, \omega^{\max}]$, with $0 \leq \omega^{\max} \leq 1$, is a random variable, which is iid across banks l and is distributed according to $F(\omega)$. The

remaining post-shuffle beginning-of-afternoon deposits $\tilde{D}_{t,l}$ are thus

$$\tilde{D}_{t,l} = (1 - \omega_{t,l}) D_{t,l}$$

In order to meet withdrawals, banks need to have enough reserves at hand to cover them. We assume that banks will always find defaulting on the withdrawals worse than any precautionary measure they can take against it, and thus rule out withdrawal caps and bank runs by assumption. Reserves can be obtained in the morning by various trades, resulting in bank holdings $M_{t,l}$. In the afternoon, additional reserves can be obtained only by new unsecured loans from other banks, maturing at the end of the afternoon, or by selling government bonds. Implicitly, we are assuming that the discount window of the central bank is not open in the afternoon, i.e., that banks need to obtain central bank funding in the morning in precaution to withdrawal demands in the afternoon. This captures the fact that the discount window is rarely used for funding liquidity needs and that these liquidity transactions happen “fast”, compared to central bank liquidity provision.

The withdrawal shock is exactly reversed with a second reverse liquidity shock, so that banks exit the period with the original level of deposits $D_{t,l}$ and can thus repay their unsecured loans or buy back the government securities originally sold. The same holds if the signs are reversed. Thus, the first liquidity shock creates only a very temporary liquidity need that banks must satisfy.

New unsecured loans can only be obtained by “connected” banks. We assume that banks face an exogenous iid probability ξ_t of being connected and being able to borrow on the unsecured loan market. We assume this probability to be iid across banks and time. The draw of the type of the bank (i.e., “connected” or “not connected”, with probability ξ_t) happens early in the morning: thus, banks know in the morning, whether they are able to potentially borrow in the afternoon or whether they need to potentially sell government bonds instead. Every bank can lend unsecured, if they so choose.

If banks do not have access to the unsecured loan market, they will need to sell government bonds, in case of liquidity needs. They can only do so with the portion that has not yet been pledged to the central bank. With $\omega^{\max}$ as the maximal withdrawal shock, non-connected banks therefore have to hold government securities satisfying

$$\omega^{\max} D_{t,l} - M_{t,l} \leq \tilde{\eta}_t Q_t (B_{t,l} - B_{t,l}^F)$$

where $0 \leq \tilde{\eta}_t \leq 1$ is a haircut parameters imposed by other lending banks, if we interpret this sale of government bonds as a private repo or private secured borrowing, and where the constraint is in terms of the unpledged portion of the government bond holdings $B_{t,l} - B_{t,l}^F$.

As all the afternoon transactions are reversed at the end of the afternoon and since all within-afternoon interest rates are zero, banks will be entirely indifferent between using any of the available sources of liquidity: what happens in the afternoon stays in the afternoon. The only impact of these choices and restrictions is that banks need to plan ahead of time in the morning to make sure that they have enough funding in the afternoon, in the worst case scenario. If a bank is unconnected, that worse-case scenario is particularly bad, as it needs to have enough of cash reserves plus unpledged bonds to meet the maximally conceivable afternoon deposit withdrawal.

3.2.3 Objective function and leverage constraints

Banks are owned by households in their country. If net worth is nonnegative, they repay a portion ϕ of their net worth to households each period

$$E_{t,l} = \phi N_{t,l}$$

In terms of aggregate bank equity N_t and resulting dividend payments, the profit payments by banks are $E_t = \phi N_t$, if $N_t \geq 0$. If net worth is negative, banks declare bankruptcy. In that case, all assets are sold, and the proceeds are returned pro rata to the holders of bank liabilities. We shall consider only shocks and scenarios, so that net worth remains positive.

The net worth of bank l before payments to shareholders satisfies

$$\begin{aligned} N_{t,l} &= \max\{0, P_t(r_t + 1 - \delta)k_{t-1,l} + M_{t-1,l} + ((1 - \kappa)Q_t + \kappa)B_{t-1,l} - R_{t-1}^D D_{t-1,l} - \kappa^F F_{t-1,l}\} \\ &= \max\{0, P_t k_{t,l} + Q_t B_{t,l} + M_{t,l} - D_{t,l} - Q_t^F F_{t,l} + E_{t,l}\} \end{aligned}$$

where the first equation is the net worth calculated on the balance of assets and their earnings and payments before the bank makes its portfolio decision, while the second equation exploits the equality of assets to liabilities after the portfolio decision.

From these two equations, one can calculate

$$\Delta M_{t,l} = M_{t,l} - M_{t-1,l}.$$

Given the draw of the type according to $\xi_t = P(\text{“connected”})$, bank l can be either “connected” or “unconnected” (denoted with the subscript “ c ” or “ u ”, respectively).

Aggregate net worth at the beginning of the period is

$$\begin{aligned} N_t = \max \{0, & \quad P_t (r_t + 1 - \delta) (\xi_{t-1} k_{t-1,c} + (1 - \xi_{t-1}) k_{t-1,u}) \\ & + (\xi_{t-1} M_{t-1,c} + (1 - \xi_{t-1}) M_{t-1,u}) \\ & + ((1 - \kappa^F) Q_t^F + \kappa^F) (\xi_{t-1} F_{t-1,c} + (1 - \xi_{t-1}) F_{t-1,u}) \\ & + ((1 - \kappa) Q_t + \kappa) (\xi_{t-1} B_{t-1,c} + (1 - \xi_{t-1}) B_{t-1,u}) \\ & - R_{t-1}^D (\xi_{t-1} D_{t-1,c} + (1 - \xi_{t-1}) D_{t-1,u}) \} \end{aligned}$$

which implies that

$$N_t = \xi_t N_{t,c} + (1 - \xi_t) N_{t,u}.$$

In principle, the second expression could be negative and aggregate net worth could become zero, in which case the banking sector of an entire country becomes insolvent. For those cases, it would be important to specify what happens to the assets and liabilities, and the economy overall. To keep the analysis manageable, we shall entirely focus on shocks and equilibria for now, where this does not happen along the equilibrium paths.

We shall impose that sub-banks get the same net worth, regardless of type (“connected”, “unconnected”), effectively assuming that the net worth is assigned before the type is known³,

$$N_{t,c} = N_{t,u} = N_t,$$

where $N_{t,c}$ is the net worth per connected bank, i.e., the total net worth in all connected banks is $\xi_t N_{t,c}$, the total net worth in all unconnected banks is $(1 - \xi_t) N_{t,u}$. Correspondingly, all assets and liabilities are likewise distributed equally, regardless of type (again, assuming that this redistribution is done before the new type is drawn for each sub-bank).

Summing this and imposing the two previous equations shows that total net worth is N_t , as it should be. Therefore, we shall drop the distinction between $N_{t,c}$, $N_{t,u}$ and N_t . The sub-bank budget constraint is

$$P_t k_{t,l} + Q_t B_{t,l} + M_{t,l} + \phi N_t = D_{t,l} + Q_t^F F_{t,l} + N_t \quad (12)$$

³If the net worth could be assigned after the type is known, obviously only connected banks would get any net worth, and the model would become rather uninteresting.

As in Gertler and Kiyotaki (2011) and Gertler and Karadi (2011), we assume that there is a moral hazard constraint in that bank managers may run away with a fraction of their assets in the morning, after their asset trades are completed and after dividends are paid to the household. The constraint is

$$\lambda (P_t k_{t,l} + Q_t B_{t,l} + M_{t,l}) \leq V_{t,l}$$

where $0 \leq \lambda \leq 1$ is a leverage parameter. Implicitly, we assume that the same leverage parameter holds for all assets, and that bankers can run away with all assets, including government bonds that may have been pledged as collateral vis-a-vis the central bank⁴.

4 Equilibrium

An equilibrium is a vector of sequences such that:

1. Given $P_t, \tau_t, W_t, R_{t-1}^D, E_t$, the representative household chooses $c_t > 0, l_t > 0, D_t \geq 0, M_t^H \geq 0, B_t^H \geq 0$ to maximize their objective function

$$\max_{\{c_t > 0, l_t > 0, D_t \geq 0, M_t^H \geq 0, B_t^H \geq 0\}} E_t \left[\sum_{t=0}^{\infty} \beta^t \left[u(c_t, l_t) + v\left(\frac{M_t^H}{P_t}\right) \right] \right]$$

subject to

$$D_t + M_t^H + Q_t (B_t^H - (1 - \kappa) B_{t-1}^H) \leq H_t$$

where

$$H_{t+1} = R_t^D D_t + M_t^H + (1 - \tau_t) W_t l_t + E_t - P_t c_t + \kappa B_t^H.$$

2. Final good firms choose capital and labor to maximize their expected profits from production, which makes use of the technology

$$y_{t,j} = \gamma_t k_{t-1,j}^\theta l_{t,j}^{1-\theta}.$$

⁴Alternatively, one may wish to impose that banks cannot run away with assets pledged to the central bank as collateral. In that case, the collateral constraint would be

$$\lambda \left[P_t (k_{t,l} - k_{t,l}^F) + Q_t (B_{t,l} - B_{t,l}^F) + M_{t,l} \right] \leq V_{t,l}$$

or a version in between this and the in-text equation. Since collateral pledged to the ECB remains in the control of banks, we feel that the assumption used in the text is more appropriate.

3. Capital-producing firms choose how much old capital k_{t-1} to buy from banks and to combine with final goods I_t to produce new capital k_t , according to the technology

$$k_t = (1 - \delta) k_{t-1} + I_t.$$

4. Bank families aggregate the assets and liabilities of the individual family members:

$$V_t = \xi_t V_{t,c} + (1 - \xi_t) V_{t,u} \quad (13)$$

$$k_t = \xi_t k_{t,c} + (1 - \xi_t) k_{t,u} \quad (14)$$

$$D_t = \xi_t D_{t,c} + (1 - \xi_t) D_{t,u} \quad (15)$$

$$B_t = \xi_t B_{t,c} + (1 - \xi_t) B_{t,u} \quad (16)$$

$$F_t = \xi_t F_{t,c} + (1 - \xi_t) F_{t,u} \quad (17)$$

$$M_t = \xi_t M_{t,c} + (1 - \xi_t) M_{t,u} \quad (18)$$

5. Given the stochastic paths for the endogenous variables c_t , l_t , r_t , P_t , Q_t , Q_t^F , η_t , and stochastic exogenous sequence for $\tilde{\eta}_t$ and the draw of the type according to ξ_t , the representative date- t connected bank chooses $k_{t,c}$, $B_{t,c}$, $B_{t,c}^F$, $F_{t,c}$, $D_{t,c}$, $M_{t,c}$ and the representative date- t unconnected bank chooses $k_{t,u}$, $B_{t,u}$, $B_{t,u}^F$, $F_{t,u}$, $D_{t,u}$, $M_{t,u}$ to maximize the banks' objective function, i.e. to maximize

$$V_{t,l} = P_t E \left[\phi \sum_{s=0}^{\infty} (\beta (1 - \phi))^s \frac{u_c(c_{t+s}, l_{t+s})}{u_c(c_t, l_t)} \frac{N_{t+s}}{P_{t+s}} \right] \quad (19)$$

where

$$\begin{aligned} N_t = \max\{0, & \quad P_t (r + 1 - \delta) (\xi_{t-1} k_{t-1,c} + (1 - \xi_{t-1}) k_{t-1,u}) \\ & + (\xi_{t-1} M_{t-1,c} + (1 - \xi_{t-1}) M_{t-1,u}) \\ & + ((1 - \kappa^F) Q_t^F + \kappa^F) (\xi_{t-1} F_{t-1,c} + (1 - \xi_{t-1}) F_{t-1,u}) \\ & + ((1 - \kappa) Q_t + \kappa) (\xi_{t-1} B_{t-1,c} + (1 - \xi_{t-1}) B_{t-1,u}) \\ & - R_{t-1}^D (\xi_{t-1} D_{t-1,c} + (1 - \xi_{t-1}) D_{t-1,u}) \} \end{aligned} \quad (20)$$

s.t. for $l = c, u$,

$$\begin{aligned}
V_{t,l} &\geq \lambda (P_t k_{t,l} + Q_t B_{t,l} + M_{t,l}) \\
0 &\leq B_{t,l} - B_{t,l}^F \\
P_t k_{t,l} + Q_t B_{t,l} + M_{t,l} + \phi N_t &= D_{t,l} + Q_t^F F_{t,l} + N_t \\
F_{t,l} &\leq \eta_t Q_t B_{t,l}^F
\end{aligned}$$

as well as

$$\omega^{\max} D_{t,u} - M_{t,u} \leq \tilde{\eta}_t Q_t (B_{t,u} - B_{t,u}^F)$$

for the unconnected banks.

6. The central banks chooses the total amount of money supply $\overline{M}_t$, the haircut parameter η_t , the discount factor on central bank funds Q_t^F , the bond purchases B_t^C as well as the seignorage payment S_t . It satisfies the balance sheet constraint

$$S_t = Q_t^F \overline{F}_t + Q_t B_t^C - \overline{M}_t \quad (21)$$

and the budget constraint

$$\begin{aligned}
\overline{M}_t &= Q_{t-1}^F \overline{F}_{t-1} + Q_{t-1} B_{t-1}^C + Q_t^F (\overline{F}_t - (1 - \kappa^F) \overline{F}_{t-1}) \\
&\quad - \kappa^F \overline{F}_{t-1} + Q_t (B_t^C - (1 - \kappa) B_{t-1}^C) - \kappa B_{t-1}^C
\end{aligned} \quad (22)$$

7. The government satisfies the debt evolution constraint, the budget constraint and the tax rule

$$\overline{B}_t = (1 - \kappa) \overline{B}_{t-1} + \Delta \overline{B}_t \quad (23)$$

$$P_t g_t + \kappa \overline{B}_{t-1} = \tau_t W_t l_t + Q_t \Delta \overline{B}_t + S_t \quad (24)$$

$$\tau_t W_t l_t = \alpha \overline{B}_{t-1}. \quad (25)$$

8. Markets clear:

$$c_t + g_t + I_t = y_t \quad (26)$$

$$\overline{B}_t = B_t + B_t^C + B_t^H \quad (27)$$

$$\overline{F}_t = F_t \quad (28)$$

$$\overline{M}_t = M_t + M_t^H \quad (29)$$

5 Analysis

We characterize the decision of households, firms and banks in turn.

5.1 Households

The household budget constraint at time t writes as

$$\begin{aligned} & D_t + M_t^H + Q_t (B_t^H - (1 - \kappa) B_{t-1}^H) \\ & \leq R_{t-1}^D D_{t-1} + M_{t-1}^H + \kappa B_{t-1}^H + (1 - \tau_{t-1}) W_{t-1} l_{t-1} + E_{t-1} - P_{t-1} c_{t-1} \end{aligned} \quad (30)$$

Note that the household's problem is subject to a set of non-negativity conditions,

$$B_t^H \geq 0, \quad (31)$$

$$M_t^H \geq 0, \quad (32)$$

Note also that $c_t > 0$, $l_t > 0$, and $D_t \geq 0$. We do not list these constraints separately for the following reasons. For $c_t > 0$ and $l_t > 0$, we can assure nonnegativity with appropriate choice for preferences and per the imposition of Inada conditions. We constrain the analysis a priori to $D_t > 0$, despite the possibility in principle that it could be zero or negative when allowing for more generality⁵.

Let μ_t^{HH} denote a Lagrange multiplier on the period- t household budget constraint (30), and $\hat{\mu}_{t,h}^B$ and $\hat{\mu}_{t,h}^M$ the multipliers on the constraint (31) and (32). The optimality conditions are

⁵We have not yet fully analyzed this matter for the dynamic evolution of the economy. It may well be that net worth of banks temporarily exceeds the funding needed for financing the capital stock, and that therefore deposits ought to be negative, rather than positive. For now, the attention is on the steady state analysis, however, and on returns to capital exceeding the returns on deposits.

given by:

$$\begin{aligned}
-\frac{u_l(c_t, l_t)}{u_c(c_t, l_t)} &= (1 - \tau_t) \frac{W_t}{P_t} \\
v_M(m_t^h) &= u_c(c_t, l_t) (R_t^D - 1) - \mu_{t,h}^M \\
\frac{u_c(c_{t-1}, l_{t-1})}{P_{t-1}} &= \beta R_t^D \left[\frac{u_c(c_t, l_t)}{P_t} \right] \\
R_t^D &= E_t \left[\frac{\mu_{t+1}^{HH}}{E_t \mu_{t+1}^{HH}} \frac{Q_{t+1} (1 - \kappa) + \kappa}{Q_t} \right] + \mu_{t,h}^B
\end{aligned}$$

where $\mu_{t,h}^B = \frac{R_t^D}{Q_t} \frac{\hat{\mu}_{t,h}^B}{\mu_t^{HH}}$ and $\mu_{t,h}^M = P_t \hat{\mu}_{t,h}^M$.

5.2 Firms

First-order conditions arising from the problem of the firms are

$$\begin{aligned}
y_t &= \gamma_t k_{t-1}^\theta l_t^{1-\theta}, \\
W_t l_t &= (1 - \theta) P_t y_t, \\
r_{t,A} k_{t-1,A} &= \theta y_t, \\
k_t &= (1 - \delta) k_{t-1} + I_t.
\end{aligned}$$

5.3 Banks

The run-away constraint (assuming it always binds) is

$$V_{t,l} = \lambda (P_t k_{t,l} + Q_t B_{t,l} + M_{t,l}) \quad (33)$$

The value of the mother bank is V_t , which is given by

$$V_t = \xi_t V_{t,c} + (1 - \xi_t) V_{t,u} \quad (34)$$

Proposition 1 *The problem of bank l is linear in net worth and*

$$V_{t,l} = \psi_t N_{t,l} \quad (35)$$

for any bank l and some factor ψ_t . In particular, $V_{t,l} = 0$ if $N_{t,l} = 0$.

Proof: Since there are no fixed costs, a bank with twice as much net worth can invest twice as much in the assets. Furthermore, if a portfolio is optimal at some scale for net worth, then doubling every portion of that portfolio is optimal at twice that net worth. Thus the value of the bank is twice as large, giving the linearity above.

We need to calculate $V_{t,l}$. The proposition above implies

$$V_t = \psi_t N_t \quad (36)$$

giving us a valuation of a marginal unit of net worth at the beginning of period t , for a representative bank.

Suppose, at the end of the period, the representative “mother bank” has various assets, k_t , B_t , and M_t , brought to it by the various sub-banks as they get together again at the end of the period. The end-of-period value $\tilde{V}_t$ of the “mother bank” then satisfies

$$\begin{aligned} \tilde{V}_t &= \phi\beta(1-\phi)E_t \left[\frac{u_c(c_{t+1}, l_{t+1})}{u_c(c_t, l_t)} \frac{P_t}{P_{t+1}} \psi_{t+1} N_{t+1} \right] \\ &= \tilde{\psi}_{t,k} k_t + \tilde{\psi}_{t,B} B_t + \tilde{\psi}_{t,M} M_t - \tilde{\psi}_{t,D} D_t - \tilde{\psi}_{t,F} F_t \end{aligned} \quad (37)$$

Per inspecting (20) as well (35), we obtain

$$\tilde{\psi}_{t,k} = \phi\beta(1-\phi)E_t \left[\frac{u_c(c_{t+1}, l_{t+1})}{u_c(c_t, l_t)} \frac{P_t}{P_{t+1}} \psi_{t+1} P_{t+1} (r_{t+1} + 1 - \delta) \right] \quad (38)$$

$$\tilde{\psi}_{t,B} = \phi\beta(1-\phi)E_t \left[\frac{u_c(c_{t+1}, l_{t+1})}{u_c(c_t, l_t)} \frac{P_t}{P_{t+1}} \psi_{t+1} ((1-\kappa)Q_{t+1} + \kappa) \right] \quad (39)$$

$$\tilde{\psi}_{t,D} = \phi\beta(1-\phi)E_t \left[\frac{u_c(c_{t+1}, l_{t+1})}{u_c(c_t, l_t)} \frac{P_t}{P_{t+1}} \psi_{t+1} R_t^D \right] \quad (40)$$

$$\tilde{\psi}_{t,F} = \phi\beta(1-\phi)E_t \left[\frac{u_c(c_{t+1}, l_{t+1})}{u_c(c_t, l_t)} \frac{P_t}{P_{t+1}} \psi_{t+1} ((1-\kappa^F)Q_{t+1}^F + \kappa^F) \right] \quad (41)$$

$$\tilde{\psi}_{t,M} = \phi\beta(1-\phi)E_t \left[\frac{u_c(c_{t+1}, l_{t+1})}{u_c(c_t, l_t)} \frac{P_t}{P_{t+1}} \psi_{t+1} \right] \quad (42)$$

For the sub-banker of type l , write

$$V_{t,l} = \phi N_{t,A} + \tilde{V}_{t,l} \quad (43)$$

The sub-bankers contribute to $\tilde{V}_t$ per

$$\tilde{V}_{t,l} = \tilde{\psi}_{t,k} k_{t,l} + \tilde{\psi}_{t,B} B_{t,l} + \tilde{\psi}_{t,M} M_{t,l} - \tilde{\psi}_{t,D} D_{t,l} - \tilde{\psi}_{t,F} F_{t,l} \quad (44)$$

The run-away constraint for bank l can then be rewritten as

$$\phi N_{t,A} + \tilde{V}_{t,l} \geq \lambda (P_t k_{t,l} + Q_t B_{t,l} + M_{t,l}) \quad (45)$$

Banks will pledge just enough collateral to the central bank to make the collateral constraint binding, nothing more (even if indifferent between that and pledging more: then, “binding” is an assumption). For both types of banks,

$$F_{t,l} = \eta_t Q_t B_{t,l}^F \quad (46)$$

with

$$0 \leq B_{t,l} - B_{t,l}^F \quad (47)$$

There are also nonnegativity constraints for investing in cash, bonds, loans and for financing from the central bank, for both types of banks:

$$0 \leq M_{t,l} \quad (48)$$

$$0 \leq B_{t,l} \quad (49)$$

$$0 \leq k_{t,l} \quad (50)$$

$$0 \leq F_{t,l}. \quad (51)$$

Note that we are interested in cases where banks choose to raise deposits and to extend loans. The former requirement ensures that banks have liquidity shocks in the afternoon and thus provides a meaningful role for interbank markets. The latter requirement generates an active link between financial intermediation and real activity in our economy.

We can have cases, however, when banks decide not to raise central bank finance, as in the case of connected banks that can always get afternoon zero-interest rate unsecured loans from other banks, if the need arises (this is assuming that $Q_t^F \leq 1$, otherwise there would be arbitrage possibilities for banks!). Similarly, banks can decide not to hold cash, if they have access to afternoon unsecured or secured finance, and if the expected return on capital is higher than the expected return on money.

To simplify the analysis, we assume (and verify in appendix A) that the economy is in an interior equilibrium for $D_{t,l}$, and $k_{t,l}$ in all the interesting cases we consider. In light of the considerations above, we explicitly allow for corner solutions for $F_{t,l}$, $B_{t,l}$ and $M_{t,l}$.

As for the afternoon, there is no need to keep track of trades, except to make sure that the afternoon funding constraints for the unconnected banks hold,

$$\omega^{\max} D_{t,u} - M_{t,u} \leq \tilde{\eta}_t Q_t (B_{t,u} - B_{t,u}^F). \quad (52)$$

Banks $l = u$ and $l = c$ who are given N_t maximize (44) subject to the sub-bank budget constraint (12) and the run-away constraint (45), the collateral constraints (46), (47), as well as (52) only for the unconnected banks. Let μ^{BC} denote the Lagrange multiplier on the budget constraint (12), μ^{RA} the Lagrange multiplier on the run-away constraint (45), $\mu_{t,l}^{CC}$ the Lagrange multiplier on the collateral constraint (46), $\mu_{t,l}^M \geq 0$, $\mu_{t,l}^F \geq 0$, $\mu_{t,l}^C \geq 0$ and $\mu_{t,l}^B \geq 0$ the Lagrange multipliers on the constraints $M_{t,l} \geq 0$, $F_{t,l} \geq 0$, the collateral constraint (47), and the non-negativity constraint for bonds $B_{t,l}$, respectively, and $\mu_{t,u}$ the Lagrange multiplier on the afternoon funding constraint of the unconnected banks.

The first-order conditions characterizing banks' asset choices are

$$\begin{aligned} (1 + \mu_{t,l}^{RA}) \frac{\tilde{\psi}_{t,k}}{P_t} &= \mu_{t,l}^{BC} + \lambda \mu_{t,l}^{RA} \\ \left(1 + \mu_{t,l}^{RA}\right) \frac{\tilde{\psi}_{t,B}}{Q_t} &= \mu_{t,l}^{BC} + \mu_{t,l}^{RA} \lambda - \mu_{t,l}^C \quad \text{for } l = c \\ \left(1 + \mu_{t,l}^{RA}\right) \frac{\tilde{\psi}_{t,B}}{Q_t} &= \mu_{t,l}^{BC} + \mu_{t,l}^{RA} \lambda - \mu_{t,l}^C - \mu_{t,u} \tilde{\eta}_t \quad \text{for } l = u \\ \left(1 + \mu_{t,l}^{RA}\right) \tilde{\psi}_{t,M} + \mu_{t,c}^M &= \mu_{t,l}^{BC} + \mu_{t,l}^{RA} \lambda \quad \text{for } l = c \\ \left(1 + \mu_{t,l}^{RA}\right) \tilde{\psi}_{t,M} + \mu_{t,c}^M &= \mu_{t,l}^{BC} + \mu_{t,l}^{RA} \lambda - \mu_{t,u} \quad \text{for } l = u \end{aligned}$$

Those characterizing banks' liability choices are

$$\begin{aligned} \left(1 + \mu_{t,l}^{RA}\right) \tilde{\psi}_{t,D} &= \mu_{t,l}^{BC} \quad \text{for } l = c \\ \left(1 + \mu_{t,l}^{RA}\right) \tilde{\psi}_{t,D} &= \mu_{t,l}^{BC} - \omega^{\max} \mu_{t,u} \quad \text{for } l = u \end{aligned} \quad (53)$$

$$(1 + \mu_{t,l}^{RA}) \tilde{\psi}_{t,F} = \mu_{t,l}^{BC} Q_t^F - \mu_{t,l}^{CC} + \mu_{t,c}^F \quad (54)$$

$$\mu_{t,l}^{CC} \eta_t^K = \mu_{t,l}^{CK} \quad (55)$$

$$\mu_{t,l}^{CC} \eta_t = \mu_{t,l}^C \quad \text{for } l = c \quad (56)$$

$$\mu_{t,l}^{CC} \eta_t = \mu_{t,u} \tilde{\eta}_t + \mu_{t,l}^C \quad \text{for } l = u$$

The complementary slackness conditions are

$$\mu_{t,l}^F F_{t,l} = 0 \quad (57)$$

$$\mu_{t,l}^M M_{t,l} = 0 \quad (58)$$

$$\mu_{t,l}^C (B_{t,l} - B_{t,l}^F) = 0 \quad (59)$$

$$\mu_{t,l}^B B_{t,l} = 0 \quad (60)$$

Note these are linear programming problem, maximizing a linear objective subject to linear constraints. So, the solution is either a corner solution or there will be indifference between certain asset classes, resulting in no-arbitrage conditions.

6 Steady state analysis

We characterize a stochastic steady state where prices grow at the rate π and all shocks are zero except for the idiosyncratic liquidity shock ω faced by banks. The steady state is characterized by the set of conditions reported in Appendix A.

6.1 Analytical characterization of the bank problem

In this section, we provide some analytical results for the bank problem. We focus on the set of parameters such that:

1. Both bank types choose to extend loans and to raise deposits, $k_l > 0$ and $d_l > 0$. The requirement $k_l > 0$ ensures an active link between activity of all banks and the real activity. This requires capital to be sufficiently productive compared to the cost of deposits, $\tilde{\psi}_k > \tilde{\psi}_D$, which after substituting for $\tilde{\psi}_k$ and $\tilde{\psi}_D$ yields:

$$\theta \frac{y}{(\xi k_c + (1 - \xi) k_u)} + 1 - \delta > \frac{1}{\beta}. \quad (61)$$

The requirement that $d_l > 0$ means that both bank types will be subject to liquidity shocks in the afternoon and thus liquidity management will play an important role for both bank types. Different bank types may still choose to manage their liquidity differently (through interbank markets and/or by borrowing from the central bank and saving cash for the afternoon). For

households to deposit with banks we need that $R^D > 1$ or, equivalently,

$$\frac{\pi}{\beta} > 1. \quad (62)$$

2. The central bank conducts monetary policy by conducting open market operations, i.e. by changing the amount of bonds b_C held on its balance sheet. It also sets the price of central bank funding, Q^F , and the haircut η .

3. Connected banks do not borrow from the central bank, $\mu_c^F > 0$ and $f_c = 0$. Note: In reality, when banks can easily borrow unsecured, they use central bank funding only to manage their expected liquidity needs, like reserve requirements. Those are set to zero in the model. In our model, banks will only access central bank funding when their access to interbank markets is impaired. Indeed, historically, banks have made precautionary use of central bank funding to satisfy (unexpected) liquidity needs only in crisis periods.

A sufficient condition for $\mu_c^F > 0$ and $f_c = 0$ is

$$\frac{(1 - \kappa^F) Q^F + \kappa^F}{Q^F} > \frac{\pi}{\beta} = R^D. \quad (63)$$

Note that for $\kappa^F = 1$, this condition is equivalent to

$$\frac{1}{Q^F} > \frac{\pi}{\beta}.$$

The condition is intuitive: if the interest rate on central bank funding is higher than the rate on deposits, central bank funding will not be used. It is both more expensive in terms of the interest rate and it requires collateral.

When conditions (61)-(63) hold, we can characterize decisions of connected banks as follows (the proof is in the Appendix).

Proposition 2 *Suppose conditions (61)-(63) hold. Then, a connected bank does not borrow from the central bank. A connected bank does not hold any cash. Moreover, if the afternoon constraint of unconnected banks binds, $\mu_u > 0$, then a connected bank does not hold any bonds, i.e., $b_c = 0$.*

Connected banks have access to the unsecured market in which they can smooth out liquidity shocks without a need for collateral. Given condition (63), central bank funding is more expensive than the cost of deposits so connected banks will not use it for funding purposes.

Similarly, connected banks will not hold any precautionary cash reserves since holding cash carries an opportunity cost. Whenever the afternoon constraint of unconnected banks binds, physical return on bonds is lower than the return on capital as bonds command a collateral premium. However, since connected banks do not need any collateral, they prefer to invest solely in capital.

Decisions of unconnected banks are as follows (the proof is in the Appendix).

Proposition 3 *Suppose conditions (61)-(63) hold. If the afternoon constraint is slack, $\mu_u = 0$, then an unconnected bank does not borrow from the central bank, $\mu_u^F > 0$ and $f_u = 0$. Also, if condition*

$$\tilde{\eta} \geq \eta Q^F \omega^{\max} \quad (64)$$

holds, then an unconnected bank does not borrow from the central bank. If the afternoon constraint binds and condition

$$\tilde{\eta} < \frac{\tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q}}{\tilde{\psi}_k - \tilde{\psi}_M} \quad (65)$$

holds, then an unconnected bank does not borrow in the secured market and instead it borrows only from the central bank and holds money, $m_u > 0$.

If the afternoon constraint is slack, unconnected banks are unconstrained in their afternoon borrowing in the secured market. Therefore, they do not borrow from the central bank. Similarly, they do not borrow from the central bank whenever (64) holds, as private haircut $\tilde{\eta}$ on bonds is favorable compared to the cost of central bank funding ηQ^F weighted by the maximum afternoon withdrawals $\omega^{\max}$. Since $\eta Q^F \leq 1$ holds, an even simpler sufficient condition for (64) to hold is $\tilde{\eta} \geq \omega^{\max}$. By contrast, whenever (65) holds, private haircut $\tilde{\eta}$ is so unfavorable that unconnected banks do not use secured market and borrow from the central bank instead. Since unconnected banks borrow from the central bank, their afternoon constraint binds, $\mu_u > 0$. It follows that money holdings are positive

$$m_u = \omega^{\max} d_u > 0.$$

7 Numerical analysis [preliminary and incomplete]

Throughout the numerical analysis, we assume the following functional form for utility:

$$u(c_t, l_t) + v\left(\frac{M_t^H}{P_t}\right) = \log c_t + \chi \log\left(\frac{M_t^H}{P_t}\right) - l_t.$$

Notice that we use the log specification also for real balances, so that the ratio $\frac{c_t}{m_t^H}$ depends on $(R_t^D - 1)$, consistent with a transaction technology specification.

We first describe our calibration strategy. We then use a numerical analysis to illustrate the properties of our model under specific disruptions of the funding markets.

7.1 Calibration

In order to evaluate the macroeconomic impact of disruptions in money markets, we calibrate the model to capture main features of the euro area economy in normal times.

In the model, each period is a quarter. We set the discount factor at $\beta = .997$ and the depreciation rate at $\delta = .025$, corresponding to an annual rate of 10 percent. The capital income share θ takes on the value of .33. We set g to 0.88, so that the ratio of government expenditure to GDP is 0.4.

Two novel parameters of our model, which capture frictions in the funding markets and are key to determining banks' choices, are the share of "connected" banks, ξ , and the maximum fraction of deposits that households can withdraw at any single point in time, $\omega^{\max}$. We compute the average pre-crisis value of ξ using data from Euro Area Money Market Survey (2013), which covers a panel of 98 euro area credit institutions.⁶ We set $\xi = .42$, where .42 is the average ratio of the annual cumulative quarterly turnover in the secured market segment over the sum of the annual cumulative turnover in the secured and unsecured segments, over the period 2004-2007 (where 2004 is the first year with an observation in the survey, and 2007 is the last year before the crisis). When we assess the impact of money market freezes, we also compute the same average for the crisis period, i.e. over the period 2008-2013 (where 2013 is the last available observation in the survey). The average value for that period is .24. We calibrate $\omega^{\max}$ in normal times using the information embedded in the liquidity coverage ratio (LCR) - a macroprudential instrument that requires banks to hold high-quality liquid assets

⁶The survey provides information on annual cumulative quarterly turnover in the secured and unsecured market segments, as reported at the end of each year's second quarter. The unsecured segment comprises all unsecured transactions (total maturities and total turnover). The secured market segment is the repo market (and also includes total maturities and turnover).

(HLQA) in an amount that allows financial institutions to meet 30-days liquidity outflows under stress. We set $\omega^{\max} = .34$, where this latter is the ratio of the liquidity outflows (per quarter, and as approximated by the HLQA) to total banks assets provided in EBA (2014).

We are left with six free parameters that we calibrate to jointly minimize the squared log-deviation of the model-based predictions from their empirical counterparts for six variables. The calibrated parameters are the share of net worth distributed by banks to households as dividends, ϕ , the share of assets bankers can run away with, λ , the coefficient in the fiscal rule that determines the reaction of the income tax rate to debt, α , the inverse of the average maturity of government bonds, κ , the coefficient determining the utility from money holdings for households, χ , and the amount of government bonds purchased by the central bank (one of the policy parameters), B^C . The targeted variables are: i) an average debt to GDP ratio of 0.57; ii) a bank leverage of 6 ; iii) a lending spread of 2.4 percent per annum; iv) a share of households' bond holdings in total debt of 0.51; v) a share of banks' bond holdings in total debt of .45; and vi) an average inflation of 2 percent per annum.⁷ The calibration procedure delivers the following values of the parameters: $\phi = .032$, $\lambda = .151$, $\alpha = .111$, $\kappa = .1$, $\chi = .0015$, and $B^C = .181$. Table 1 reports the average values in the data and the model-implied values for the six targeted variables. Table 2 summarizes all parameter values.

7.2 Comparative statics

We assess the macroeconomic impact of disruptions in money markets by means of a comparative statics analysis.

Shift from unsecured to secured interbank market funding The first exercise we conduct aims at analyzing the effects on real activity of the drying up of the euro area money markets during the initial phase of the financial crisis. At the end of 2008, banks located in the

⁷The average debt to GDP ratio is computed for the euro area as the ratio of debt securities to GDP over the period 1999Q1-2006Q4, using data from the Annual Financial Accounts of Eurostat. The value of bank leverage is taken from Andrade et al. (2016), who compute it as the ratio of monetary and other financial institution and NFC assets over equity weighted by their relative assets, between 1999Q1 and 2014Q2, based on euro area sectoral accounts. The average value of the lending spread is also taken from Andrade et al. (2016), where it is computed as the spread between the euro area long-term composite cost of borrowing indicator and the overnight Eonia rate. The share of households' bond holdings in total debt is computed as the ratio of government debt held by the non-financial sector (residents and non residents) of the euro area to all debt (all maturities) over the period 1999-2006, using data from the ECB Statistical Data Warehouse. The share of banks' bond holdings in total debt is computed, using the same source and over the same sample, as the ratio of government debt held by other euro area monetary financial institutions (residents and non residents) to total debt (all maturities). Average inflation is computed as annual percentage change in the HICP of the euro area, over the period 1999-2006, using data from the ECB Statistical Data Warehouse.

euro area became increasingly unable to borrow in the unsecured markets due to perceptions of increased counterparty risk, and shifted to secured funding (as documented in figure 2).

Figure 4 shows the results of a comparative statics exercise where the share of connected banks, ξ , is increased from 0.1 to 0.5. The figure reports with a solid red line the share of connected banks (i.e. those with access to the secured market) under our benchmark calibration, i.e. $\xi = 0.42$, and with a dashed black line the share of connected banks in the pre-crisis period, $\xi = .24$. To understand the exercise, it is useful to start considering the points to the right of the figure, i.e. where $\xi = 0.5$, and then move to the left. When half of the banks have access to the unsecured market, and all other parameters are at the values set in our benchmark calibration, the collateral premium (given by $\tilde{\psi}_{t,k} - \frac{\tilde{\psi}_{t,B}}{Q_t}$) is positive, indicating that the afternoon constraint is binding for unconnected banks. Unconnected banks need to hold enough bonds and/or cash to satisfy the maximum possible withdrawal of deposits in the afternoon. Given the prevailing inflation rate and private haircut in the secured market, banks decide to hold enough bonds, b_u , to be able to finance any possible afternoon liquidity need by borrowing in the secured market rather than to hold money, i.e. m_u is zero. As shown analytically, connected banks hold neither bonds, nor money (not reported in the figure), i.e. $b_c = m_c = 0$. Those banks combine deposits and net worth to finance the maximum possible amount of high return loans to firms, k_c .

As the share of connected banks is decreased (moving to the left of the figure), the afternoon constraint becomes progressively more binding for unconnected banks. This is because a larger share of banks now need to satisfy the possible afternoon liquidity withdrawals by holding bonds. The stock of bonds accepted as collateral in the secured market is in given supply, $\bar{b}$, and held by three types of agents: households, unconnected banks and the central bank. The central bank chooses its bond holdings to be constant at b^C . Households, however, hold bonds to the point where their return equals the return on money and on deposits. As the demand for bonds from unconnected banks rises, households substitute out of bond holdings, b^H , into money holdings, m^H , and deposits, d . This is made possible by a reduction in inflation, which increases the attractiveness of cash. Although lower inflation decreases the return on deposits, these latter become more attractive than bonds due to the increase in the bond price, which reflects a rising collateral premium. When the share of unconnected banks becomes really small ($\xi = .12$, corresponding to a money market "freeze"), households' bond holdings reach zero. At that point, unconnected banks cannot relax the increasingly binding afternoon constraint by

raising the amount of bonds they hold. The price Q and the collateral premium rise sharply. For unconnected banks, holding money is still not the optimal choice. They prefer to satisfy the afternoon constraint by decreasing deposits. The reduction in their deposit liabilities implies that they also need to reduce their loans to firms, k_u . Households provide a link from the choice of unconnected banks to the choice of connected ones. Below $\xi = .12$, they find it optimal to allocate their wealth to either money or deposits. As inflation keeps falling, increasing the return to money and decreasing the return on deposits, households choose to raise cash and to reduce the amount they deposit at banks. As a consequence, the deposit liabilities of connected banks fall too, reducing the funds those banks can intermediate.

As the share of connected banks is decreased from 0.5 to 0.12, output falls. The fall reflects the lower labor supply, induced by the reduction in the real wage, which more than offsets the increase in loans (capital) extended by both connected and unconnected banks. The contraction in real activity is substantial, i.e. the decline in output from the pre-crisis level $\xi = .42$ to the crisis one, $\xi = .24$, is 2.6 percent.

Tension on secured interbank market due to increased sovereign risk During the sovereign debt crisis, the euro area money market faced renewed tensions. Banks that were excluded from the unsecured segment found themselves with limited access to the secured market as well, due to increased haircuts applied to eligible collateral (those typically consisting of sovereign bonds).

We analyse the impact of increased tensions on sovereign debt markets by changing the haircut parameter on collateral in the secured market. The results are shown in figure 5, where we let $\tilde{\eta}$ move from 0.5 to 0.95. The red solid line denotes the value of $\tilde{\eta}$ under our benchmark calibration, i.e. $\tilde{\eta} = .94$. At that point, the afternoon constraint binds for unconnected banks. These banks, however, can pledge collateral in the secured market at reasonable haircuts in order to satisfy their liquidity needs. As the private haircut is increased (or $\tilde{\eta}$ is reduced), unconnected banks face a tighter afternoon constraint. They need to increase their holdings of bonds, because each bond provides less liquidity when used as collateral in the private market. The larger demand for bonds by unconnected banks is satisfied through the reduction in households' bond holdings. This latter is made possible by an increase in the collateral premium which makes deposits more attractive relative to bonds for households. The increase in deposits implies that both connected and unconnected banks can raise their liabilities. Their choices on the asset side are different, though. While connected banks use the higher deposit

liabilities to finance more loans (k_c increases as $\tilde{\eta}$ falls), unconnected banks reduce their loans. Because of higher haircuts, unconnected banks need to use a higher fraction of their net worth to buy bonds and have therefore to reduce their investment in productive assets (capital). When households' bond holdings reach zero, unconnected banks are unable to increase their bond holdings. They satisfy the afternoon constraint by reducing deposits and by bringing money from the morning. The decline in loans (capital) of unconnected banks accelerates.

For all values of $\tilde{\eta}$ for which households' bonds remain positive, the increase in private haircuts depresses the economy. The mechanism is different relative to the case where the share of connected banks is changed. Here, capital of connected banks increase, but this increase is more than offset by the decline in capital of unconnected banks. The combination of lower aggregate capital stock and declining labor, due to a reduction in the real wage, explains the observed fall in output.

8 Conclusions

We presented a general equilibrium model where banks can finance their liquidity needs in the unsecured or secured interbank markets, and where they have also access to collateralized central bank funding. The model accounts for the decline in the ratio of interbank liabilities in total bank assets observed since the onset of the global financial crisis, the reduced ability of banks to access the unsecured market during the financial and sovereign crisis, and their shift to secured market funding. It also accounts for the increased reliance on central bank funding, particularly for banks in countries with a vulnerable sovereign.

Results from our calibrated model show that money market freezes can have a sizeable macroeconomic impact. A decline in the share of secured interbank market transactions, similar to the one observed during the crisis, generates a reduction in GDP of around 3 percent.

Our model can be used as a laboratory to assess the impact of a range of unconventional monetary policy measures that were undertaken in the euro area since 2008, namely standard interest rate policy, collateral policy (changes in the composition of the collateral pool or in the haircuts applied to specific classes of assets), main refinancing operations (including changes in their maturity), targeted long-term refinancing operations, and asset purchases. We leave this for future work.

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Table 1: Calibration targets

Variable	Data	Model
Debt/GDP	0.57	0.90
Bank leverage	6.00	6.26
Loan spread (annual)	0.021	0.024
Household's share bond holdings	0.51	0.36
Banks' share bond holdings	0.46	0.62
Inflation	0.020	0.018

Table 2: Parameter values

Parameter	Description	Value
θ	Capital share in income	0.33
δ	Capital depreciation rate	0.025
β	Discount rate households	0.997
χ	Coefficient in households' utility	0.0015
g	Government spending	0.885
κ^{-1}	Average maturity bonds	10
α	Coefficient on debt in fiscal rule	0.1115
ϕ	Fraction of net worth paid as dividends	0.032
ξ	Fraction of banks with access to unsecured market	0.42
$\tilde{\eta}$	Haircut on bonds set by banks	0.97
η	Haircut on bonds set by central bank	0.96
λ	Share of assets bankers can run away with	0.1512
$\omega^{\max}$	Max possible withdrawal as share of deposits	0.34
B_C	Bonds held by central bank	0.1807

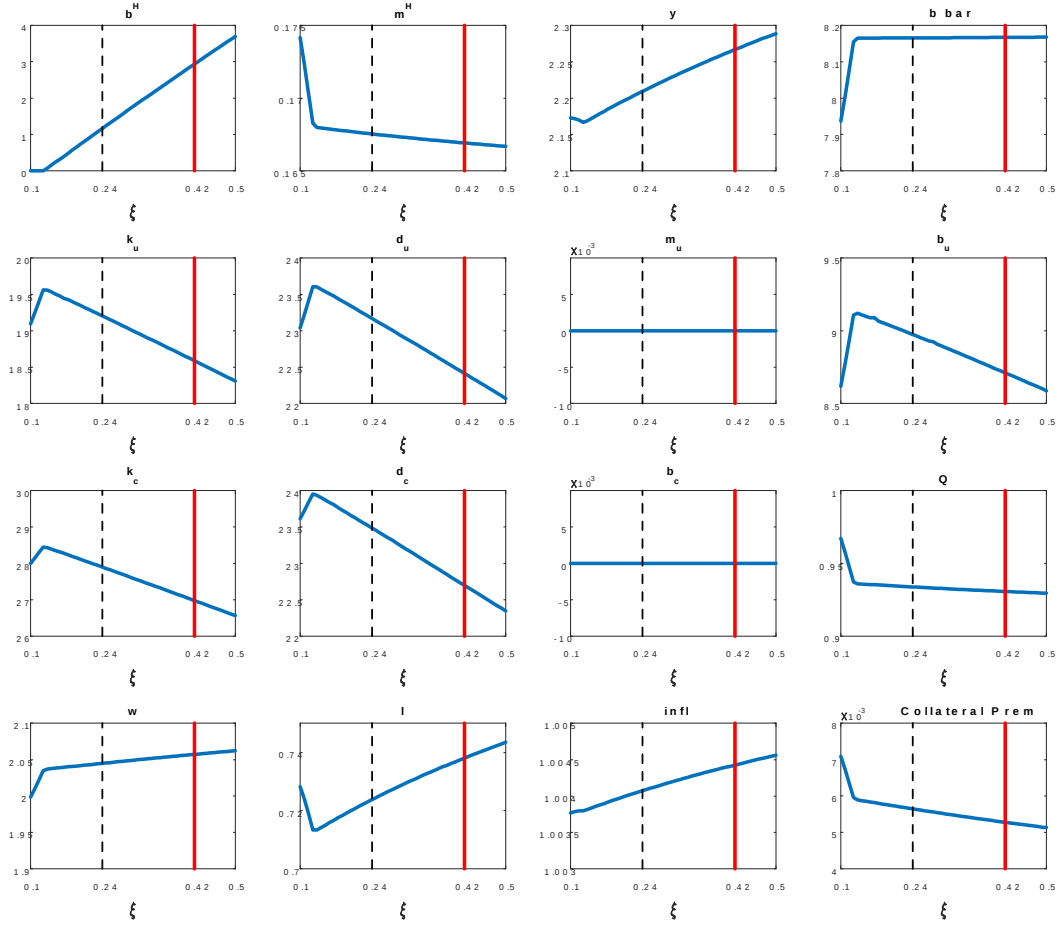


Figure 4: Shift from unsecured to secured interbank market funding (changing ξ).

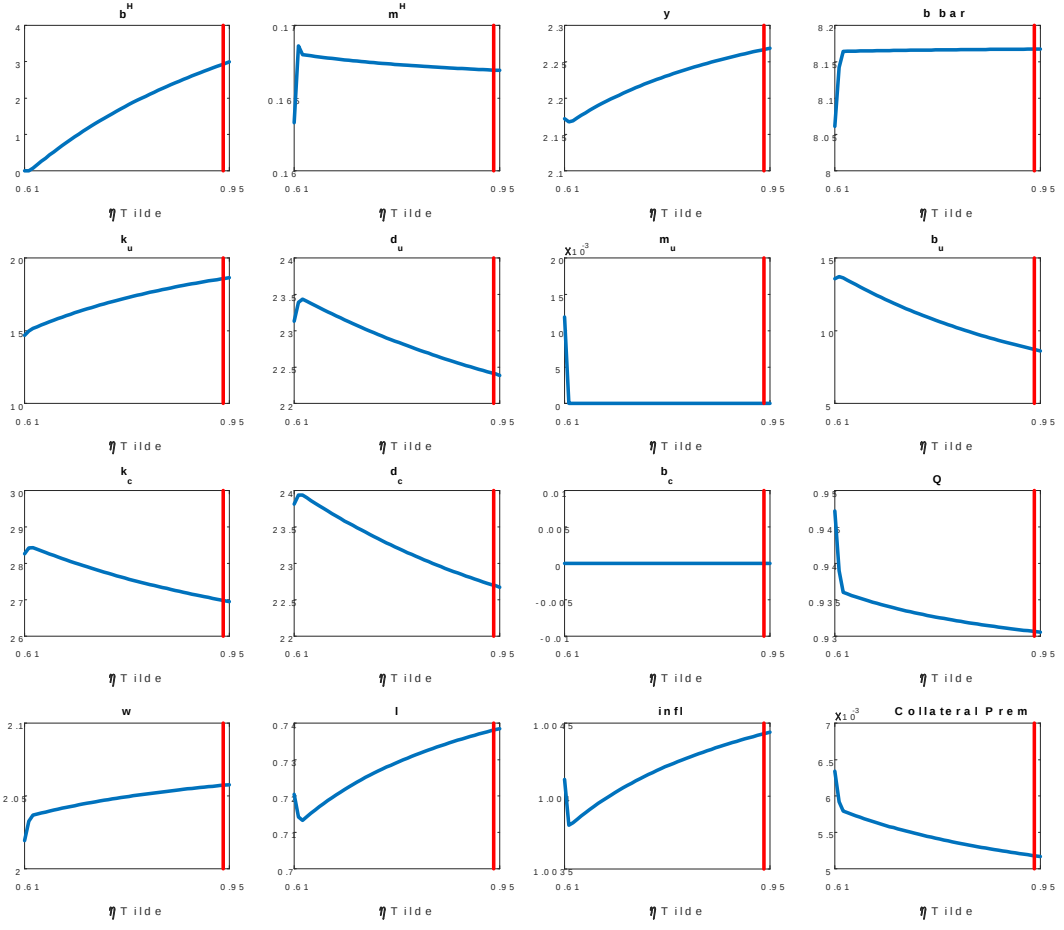


Figure 5: Tension on the secured interbank market due to increased sovereign risk (changing $\tilde{\eta}$).

A The equations characterizing the steady state

We characterize the steady state of the model. For simplicity, we focus on the case when capital is not accepted as collateral at the central bank, $\eta^K = 0$. Recall that we already assume that capital is not accepted as collateral in the private secured market, $\tilde{\eta}^k = 0$.

Define a generic variable as the corresponding capital letter variable, divided by the contemporaneous price level, i.e. $x_t = \frac{X_t}{P_t}$. The steady state is characterized by the following conditions:

1. 5 household equations:

$$\begin{aligned}
 R^D &= \frac{\pi}{\beta} \\
 R^D &= \frac{Q(1-\kappa) + \kappa}{Q} + \mu_H \\
 -\frac{u_l(c, l)}{u_c(c, l)} &= (1-\tau)w \\
 \frac{v_M(m^h)}{u_c(c, n)} &= R^D - 1 \\
 c &= (1-\tau)wl + \left(\frac{1}{\beta} - 1\right)\pi d + (1-\pi)m^h + (\kappa - Q(\pi - 1 + \kappa))b_H + \phi n
 \end{aligned} \tag{66}$$

2. 4 firms' equations:

$$\begin{aligned}
 y &= \gamma k^\theta l^{1-\theta} \\
 wl &= (1-\theta)y \\
 rk &= \theta y
 \end{aligned}$$

and

$$I = \delta k.$$

3. 5 central bank equations: 2 equations

$$\begin{aligned}
 s &= Q^F \bar{f} + Q b_C - \bar{m} \\
 \bar{m} &= \left[Q^F - \kappa^F \frac{1}{\pi} (1 - Q^F) \right] \bar{f} + \left[Q - \kappa \frac{1}{\pi} (1 - Q) \right] b_C
 \end{aligned}$$

plus the value of 3 variables (policy instruments):

$$\eta, Q^F, b_C.$$

Note that the seigniorage revenue of the central bank is given by the interest rate payments on its assets:

$$s = \kappa^F \frac{1}{\pi} (1 - Q^F) \bar{f} + \kappa \frac{1}{\pi} (1 - Q) b_C.$$

4. 3 government equations:

$$\begin{aligned} \bar{b} &= (1 - \kappa) \bar{b} \frac{1}{\pi} + \Delta \bar{b} \\ g + \kappa \bar{b} \frac{1}{\pi} &= \tau w l + \frac{1}{\pi} \Delta \bar{b} + \frac{1}{2} s \\ \tau w l &= \alpha \bar{b} \frac{1}{\pi}, \end{aligned}$$

where g is exogenous.

5. 4 market clearing equations:

$$\begin{aligned} \bar{f} &= f \\ \bar{m} &= m + m^h \\ \bar{b} &= b + b_C + b_H \\ y &= c + g + I \end{aligned}$$

where the market clearing condition for the goods market (last equation above) is redundant due to the Walras law.

6. 45 (8+18+7+6+6) bank equations:

8 equations common to c and u banks,

$$\nu = \psi n$$

$$\begin{aligned}
n = & \max \left\{ 0, (r+1-\delta) (\xi k_c + (1-\xi) k_u) \right. \\
& + (\xi m_c + (1-\xi) m_u) \frac{1}{\pi} \\
& + ((1-\kappa) Q + \kappa) (\xi b_c + (1-\xi) b_u) \frac{1}{\pi} \\
& - ((1-\kappa^F) Q^F + \kappa^F) (\xi f_c + (1-\xi) f_u) \frac{1}{\pi} \\
& \left. - \frac{1}{\beta} (\xi d_c + (1-\xi) d_u) \right\},
\end{aligned}$$

$$\tilde{v} = \tilde{\psi}_k k + \tilde{\psi}_B b + \tilde{\psi}_M m - \tilde{\psi}_D d - \tilde{\psi}_F f$$

$$\begin{aligned}
\tilde{\psi}_k &= \beta (1-\phi) \psi (r+1-\delta) \\
\tilde{\psi}_B &= \beta (1-\phi) \frac{1}{\pi} \psi [(1-\kappa) Q + \kappa] \\
\tilde{\psi}_D &= \beta (1-\phi) \frac{1}{\beta} \psi \\
\tilde{\psi}_F &= \beta (1-\phi) \frac{1}{\pi} \psi ((1-\kappa^F) Q^F + \kappa^F) \\
\tilde{\psi}_M &= \beta (1-\phi) \frac{1}{\pi} \psi
\end{aligned}$$

9*2=18 equations for $l = c, u$:

$$k_l + Qb_l + m_l + \phi n = d_l + Q^F f_l + n$$

$$\phi n + \tilde{v}_l = \lambda (k_l + Qb_l + m_l)$$

$$v_l = \phi n + \tilde{v}_l$$

$$\tilde{v}_l = \tilde{\psi}_k k_l + \tilde{\psi}_B b_l + \tilde{\psi}_M m_l - \tilde{\psi}_D d_l - \tilde{\psi}_F f_l$$

$$f_l = \eta Q b_l^F$$

$$\mu_l^F f_l = 0$$

$$\mu_l^M m_l = 0$$

$$\mu_l^C (b_l - b_l^F) = 0$$

$$\mu_l^B b_l = 0$$

7 equations for unconnected banks:

$$(1 + \mu_u^{RA}) \tilde{\psi}_k = \mu_u^{BC} + \lambda \mu_u^{RA} \quad (67)$$

$$(1 + \mu_u^{RA}) \frac{\tilde{\psi}_B}{Q} + \mu_u^B = \mu_u^{BC} + \lambda \mu_u^{RA} - \mu_u^C - \mu_u \tilde{\eta} \quad (68)$$

$$(1 + \mu_u^{RA}) \tilde{\psi}_M + \mu_u^M = \mu_u^{BC} + \lambda \mu_u^{RA} - \mu_u \quad (69)$$

$$(1 + \mu_u^{RA}) \tilde{\psi}_D = \mu_u^{BC} - \omega^{\max} \mu_u \quad (70)$$

$$(1 + \mu_u^{RA}) \frac{\tilde{\psi}_F}{Q^F} = \mu_u^{BC} - \mu_u^C \frac{1}{Q^F} + \mu_u^F \frac{1}{Q^F} \quad (71)$$

$$\mu_u^C = \mu_u^{CC} \eta - \mu_u \tilde{\eta} \quad (72)$$

$$\mu_u [\omega^{\max} d_u - m_u - \tilde{\eta} Q (b_u - b_u^F)] = 0 \quad (73)$$

6 equations for connected banks:

$$(1 + \mu_c^{RA}) \tilde{\psi}_k = \mu_c^{BC} + \lambda \mu_c^{RA} \quad (74)$$

$$(1 + \mu_c^{RA}) \frac{\tilde{\psi}_B}{Q} + \mu_c^B = \mu_c^{BC} + \lambda \mu_c^{RA} - \mu_c^C \quad (75)$$

$$(1 + \mu_c^{RA}) \tilde{\psi}_M + \mu_c^M = \mu_c^{BC} + \lambda \mu_c^{RA} \quad (76)$$

$$(1 + \mu_c^{RA}) \tilde{\psi}_D = \mu_c^{BC} \quad (77)$$

$$(1 + \mu_c^{RA}) \frac{\tilde{\psi}_F}{Q^F} = \mu_c^{BC} - \mu_c^C \frac{1}{Q^F} + \mu_c^F \frac{1}{Q^F} \quad (78)$$

$$\mu_c^C = \mu_c^{CC} \eta \quad (79)$$

6 bank aggregation equations:

$$v = \xi v_c + (1 - \xi) v_u$$

$$k = \xi k_c + (1 - \xi) k_u$$

$$d = \xi d_c + (1 - \xi) d_u$$

$$b = \xi b_c + (1 - \xi) b_u$$

$$f = \xi f_c + (1 - \xi) f_u$$

$$m = \xi m_c + (1 - \xi) m_u.$$

These are 21+45=66 equations (one redundant by the Walras law). The endogenous variables are 32+30+3=65:

32:

$$\left\{ \begin{array}{c} y, k, c, l, d, n, m^h, b, b_H, f, m, v, \tilde{v}, \bar{b}, \Delta \bar{b}, \tau, \\ \psi, \tilde{\psi}_k, \tilde{\psi}_B, \tilde{\psi}_M, \tilde{\psi}_D, \tilde{\psi}_F, \mu_u, w, r, Q, R^D, \pi, I, s, \bar{f}, \bar{m} \end{array} \right\}$$

plus 15*2=30:

$$\{k_l, m_l, f_l, b_l, b_l^F, d_l, v_l, \tilde{v}_l, \mu_l^F, \mu_l^M, \mu_l^{RA}, \mu_l^{BC}, \mu_l^{CC}, \mu_l^C, \mu_l^B\},$$

plus the value of 3 variables (policy instruments)

$$\eta^A, Q^F, b_C$$

and of the following exogenous variables:

$$g, \xi.$$