# Housing Market Freezes, Deleveraging, and Aggregate Demand*

*Preliminary and Incomplete*

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**Abstract**

This paper develops a general equilibrium model of incomplete markets, liquid paper assets and illiquid housing. Housing liquidity fluctuates significantly over time and systematically so over the business cycle. A decrease in the liquidity of housing leads to an increased demand for liquid paper assets and a decrease in demand for houses (as assets). We show that the model generates substantial business cycle effects of fluctuations in housing liquidity on house prices, employment and output, while being in line with relatively small fluctuations in rental rates of housing. We find that low housing liquidity during the Great Recession offers a novel explanation for the sharp decline in interest rates on government bonds.

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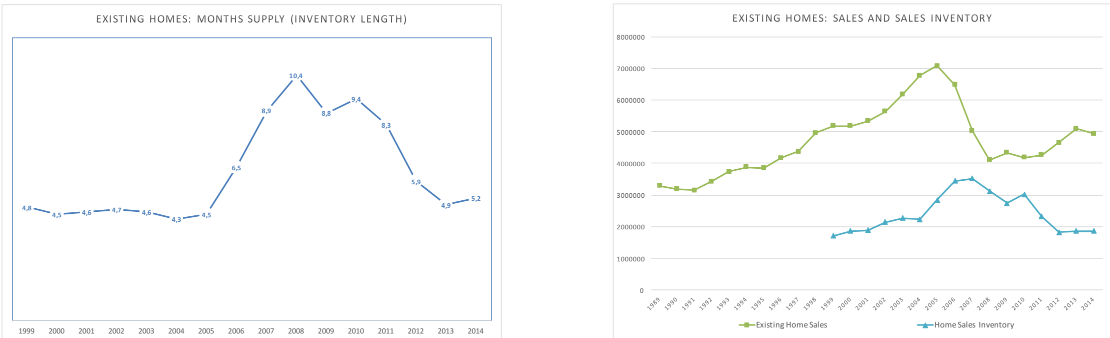
# 1 Introduction

The US housing market has undergone a tremendous boom-bust cycle over the past decade with stark aggregate repercussions as Mian and Sufi (2011) argue. Importantly for the present paper, the boom-bust cycle in housing not only regards fluctuations in the number of transactions and house prices, but also the time between listing and sale, c.f. Figure 1. During the housing boom in the early 2000s the average time to sell a house was 4 months. This time-to-sell more than doubled to over 10 months at the peak of the crisis. In fact, listings were still rising in 2006 and 2007 while the number of transactions drastically fell. It took until 2012 for the time on sale to return to pre-crisis levels.

Figure 1: Home Sales in the US




Source: FRED Database.

Given that housing wealth is the most important asset for the overwhelming fraction of US households, one can expect that the freeze in the housing market observed during the Great Recession affects portfolio allocations, house prices, and aggregate activity beyond its pure price effect on wealth. We assess the effects of fluctuations in the liquidity of the housing market in a quantitative DSGE model. This model captures changes in the liquidity of housing as time-varying time-to-sell in a framework of wealthy hand-to-mouth consumers developed in Kaplan and Violante (2014) and the corresponding general equilibrium model of Bayer et al. (2015).

In this framework, households face uncertain labor productivity and decide on labor supply and consumption. They can only self-insure on incomplete markets against their productivity shocks. We assume that there are two types of assets: Houses that are illiquid and can only be traded every period with a certain probability and liquid real bonds that can be traded each period and may be issued by households up to a borrowing limit. Households hence not only face a consumption-savings and labor supply decision, but also an important decision regarding their portfolio composition in terms of illiquid houses and liquid bonds.

Here is where fluctuations in the liquidity of housing come into play. As houses become less liquid, households want to increase their savings in liquid assets to be able to smooth their consumption in the future. They do so by increasing their overall savings as well as by liquidating part of their housing position. This leads to a sharp and simultaneous drop in consumption and investment demand - a fact reminiscent of the Great Recession - and observationally close to a simultaneous shock to investment productivity and time preferences in representative agent models, which is exactly what a business cycle accounting of the great recession produces Fratto and Uhlig (2014). This drop in demand meets monopolistically competitive firms as producers that use labor for production. As monopolistic competitors they charge a markup over marginal costs, but to introduce a nominal friction, we assume them to be subject to a Rotemberg (1982) price setting friction.

This element of shocks to housing liquidity brings an important new aspect to our understanding of the role of the housing market in the Great Recession. Recent work by Justiniano et al. (2015) rejects the previously put forward micro-foundation of the zero-lower bound that relies on deleveraging by households. The authors point out that shocks to the loan-to-value ratio as in Guerrieri and Lorenzoni (2011) should only apply to new houses. Hence, the adjustment of the loan-to-value ratio is asymmetric in that all households participate in the loosening of borrowing constraints, but only new sales are subject to tighter borrowing constraints. Shocks to housing liquidity overcome this problem as they affect all households.

In our model, we find that an increase in the expected time to liquidate a housing position from 2 to almost 4 quarters, as observed in the Great Recession, leads to a sharp fall in the real interest rate from 2% to -2% annualized. Without imposing the zero-lower bound, this fall in the interest rate increases investment and, thereby, generates a basically flat response of output. Fiscal policy can stabilize the real interest rate by running deficits and matching the increased demand for liquid savings vehicle. Calibrating the model to a joint monetary and fiscal response similar to the Great Recession, we find that in many scenarios the interest rate still hits the zero-lower bound. What is more, our model generates fluctuations in house prices much in excess of fluctuations in the returns on housing. Real rental rates of houses moved relatively little over the crisis and also have not been fallen much since nor are they much below their pre-crisis trend.

## 2 Model

We model an economy inhabited by a continuum of households that face idiosyncratic income risk. Households self-insure against income fluctuations to smooth consumption. They can invest into liquid real bonds and into houses which are illiquid. Real bonds offer a real return, houses offer a return by being rented out on market for housing services. Illiquidity is understood in the spirit of Kaplan and Violante's (2014) model of "wealthy hand-to-mouth" consumers, where trading the illiquid asset is subject to a friction. We model this friction as a transaction cost of trading houses and a probability of trading in a given period. From those households who decide to pay the entry cost into the market for houses, a fraction of households is randomly selected to trade houses every period. All other households may only adjust their real bonds holdings.

The idiosyncratic income risks, which households face, stem from two sources. First, households stochastically transition between a "worker" and an "entrepreneur" state. Entrepreneurs obtain income from monopoly profits as they hire labor to produce and supply differentiated products. Due to their market power, they set prices above marginal costs subject to a price setting friction à la Rotemberg (1982) so that price adjustment on differentiated goods is sluggish. We calibrate the entrepreneur income to be high such that a transition from entrepreneur to worker state implies a substantial income decline. The second source of income risk regards workers, who supply labor, but are subject to idiosyncratic shocks to their idiosyncratic labor productivity. Therefore workers are heterogenous in their incomes.

Besides this private sector, there is a government in form of a fiscal and a monetary authority. The fiscal authority collects taxes, issues liquid real debt, and spends on government consumption and interest expenses. Government consumption follows a simple rule that stabilizes debt (c.f. Bi et al., 2013). The monetary authority, the central bank, sets the interest rate on real bonds according to a Taylor (1993)-type rule.

### 2.1 Households

The continuum of ex-ante identical households has measure one and is indexed by $i$. A fraction $\theta$ of them are entrepreneurs and obtain profit income while the fraction $1-\theta$ are workers obtaining labor income. Both types may in addition earn income from renting out houses and from holding government bonds. Transition between these two household types is exogenous and reflects an event in which a household by chance invents an improved version of a differentiated product. This allows the households to take over the market power from the incumbent. It remains in this position until the product is replaced by an again better version invented by another household. Households are infinitely lived, have time-separable preferences with time-discount factor $\beta$, and derive felicity from consumption $c_{it}$, housing $h_{it}$, and suffer a disutility from work.

### 2.1.1 Labor Supply

We assume that entrepreneurs have zero productivity on the labor market and thus will not offer labor services. Instead they obtain profit income $\Pi_t$, which is the same across all entrepreneurs. Worker households by contrast are heterogeneous in their productivity and hence all obtain different incomes on the labor market. Their labor income is $w_t \chi_{it} n_{it}$, composed of the wage rate per efficiency unit, $w_t$, hours worked, $n_{it}$, and idiosyncratic labor productivity, $\chi_{it}$. Conditional on not transiting to the entrepreneur state, log productivity follows and AR(1)-process:

$$\chi_{it} = \begin{cases} \exp\{\rho_h \log \chi_{it-1} + \epsilon_{it}\}, & \text{if } \chi_{it-1} > 0 \text{ with prob. } (1-\lambda) \\ \exp(\epsilon_{it}), & \text{if } \chi_{it-1} = 0 \text{ with prob. } \zeta \\ 0, & \text{if } \chi_{it-1} > 0 \text{ with prob. } \lambda \\ 0, & \text{if } \chi_{it-1} = 0 \text{ with prob. } (1-\zeta) \end{cases}, \tag{1}$$

where $\epsilon_{it} \sim N(0, \sigma_h)$. The first case is a worker remaining a worker. The second case captures the transition from entrepreneur to worker; the third case the reverse transition from worker to entrepreneur. Finally, the fourth case captures households that remain in the entrepreneur state.

Households have Greenwood-Hercowitz-Huffman (GHH) preferences and maximize the discounted sum of felicity :

$$V = E_0 \max_{\{c_{it}, h_{it}, n_{it}\}} \sum_{t=0}^{\infty} \beta^t u\left[c_{it}^{\alpha} h_{it}^{1-\alpha} - \chi_{it} G(n_{it})\right]. \tag{2}$$

The felicity function has constant relative risk aversion (CRRA) with risk aversion $\xi$:

$$u(x_{it}) = \frac{1}{1-\xi} x_{it}^{1-\xi}, \quad \xi > 0,$$

For notational simplicity, we define an aggregated consumption-housing-leisure good $x_{it} := z_{it} - \chi_{it} G(n_{it})$, where $z_{it}$ itself is an aggregator over housing and goods consumption, $z_{it} := c_{it}^{\alpha} h_{it}^{1-\alpha}$.

With these definitions, the intratemporal first order condition for this composite housing-consumption good, $u'(x_{it}) = P_{z,t}\Lambda_{it}$, equates the marginal felicity to the price, $P_{z,t}$ of the aggregated housing-consumption good $z$ times the Lagrangian multiplier on the budget constraint, $\Lambda_{it}$.

Making use of this first-order condition for $z$, the first-order condition on labor supply can be expressed as:

$$G'(n_{it}) = (1-\tau)w_t, \quad w_t := W_t/P_{z,t}, \tag{3}$$

such that under our assumption of GHH-preferences with a disutility of work proportional to productivity, a household's labor supply decision only depends on the aggregate real wage $w_t$ (in composite goods) after taxes $\tau$. Neither does it depend on the individuals productivity, nor its consumption. Scaling the disutility from work by a worker's productivity reflects that more productive, hence better paid, jobs can be more demanding and stressful, see Bayer et al. (2015).[1]

Since all workers supply the same amount of labor, we can drop the household-specific index $i$, and set $n_{it} = N_t$. Total labor input supplied is given by:

$$\tilde{N}_t = N_t E \chi_{it}.$$

With respect to the disutility from labor, we assume a constant Frisch elasticity of aggregate labor supply with $\gamma$ being the inverse elasticity:

$$G(N_t) = \frac{1}{1+\gamma} N_t^{1+\gamma}, \quad \gamma > 0,$$

and use this to simplify the expression for the composite consumption good $x_{it}$. Exploiting the first-order condition on labor supply, the disutility of working can be expressed in terms of the wage rate:

$$\chi_{it} G(N_t) = \chi_{it} \frac{N_t^{1+\gamma}}{1+\gamma} = \frac{\chi_{it} G'(N_t) N_t}{1+\gamma} = \frac{(1-\tau) w_t \chi_{it} N_t}{1+\gamma}.$$

This allows us to express the demand for the composite consumption-housing-leisure good, $x_{it}$, as:

$$x_{it} = z_{it} - \chi_{it} G(N_t) = z_{it} - \frac{(1-\tau) w_t \chi_{it} N_t}{1+\gamma}.$$

[1] While this assumption is to no extent central for any of our findings, it avoids the counterfactual prediction of a large cross-sectional dispersion of hours worked despite the convenient GHH preference specification. In addition it ensures a balanced growth path in the presence of labor augmenting technological change in our model.

### 2.1.2 Consumption and Housing

Regarding the consumption of goods, we assume that there is a continuum of differentiated products over which households form their preferences according to a Dixit-Stiglitz aggregator:

$$c_{it} = \left( \int c_{ijt}^{\frac{\eta-1}{\eta}} dj \right)^{\frac{\eta}{\eta-1}}.$$

Each of these differentiated goods is offered at price $p_{jt}$ so that the demand for each of the varieties is given by

$$c_{ijt} = \left( \frac{p_{jt}}{P_{c,t}} \right)^{-\eta} c_{it},$$

where $P_{c,t} = \left( \int p_{jt}^{1-\eta} dj \right)^{\frac{1}{1-\eta}}$ is the average price level of consumption goods.

The market for housing services by contrast is assumed to be perfectly competitive and frictionless. The rental rate of housing services (expressed in units of the consumption good) is $R_{h,t}P_{c,t}$ and is such that it clears the market, i.e.,

$$R_{h,t} = \frac{1-\alpha}{\alpha} \frac{C_t}{H_t}, \tag{4}$$

where $C_t$ and $H_t$ are the aggregate supply of consumption goods and housing services respectively.

The composite housing-consumption good price $P_{z,t}$ is, due to the Cobb-Douglas utility specification, given by the aggregator

$$P_{z,t} = P_{c,t}^{\alpha} (R_{h,t} P_{c,t})^{1-\alpha} = P_{c,t} (R_{h,t})^{1-\alpha}.$$

### 2.1.3 Savings decisions and household portfolios

Since households can only trade in two assets not contingent on their income state – real bonds, $b_{it}$, and houses (as assets), $k_{it}$, they face incomplete markets. Households may borrow in bonds while holdings of houses have to be non-negative. Importantly, the holdings of housing assets do not need to coincide with housing consumption but any access or shortage can be rented on the rental market for houses.

The illiquidity of houses is captured by a two-stage participation friction. First, households that would like to participate in the asset market for houses need to pay some utility cost in case of a transaction. This results in not all houses being up "for sale" every period. Second, households need to find a suitable trade which we model as an exogenous trading probability, $\nu_t$. Therefore, only those households may trade housing assets that, first, are willing to pay the transaction costs in case of trade, and, second, find a trading option. Only these households can fully adjust their portfolio positions of houses and bonds. All other households obtain incomes from renting out houses, but they may adjust only their bonds holdings. For those households participating in both markets, the budget constraint, expressed in units of the consumption good, reads for workers:

$$\begin{aligned} x_{it} + b_{it+1} + q_t k_{it+1} &= R^B_{t-1} b_{it} + (R^\alpha_{h,t} - \delta) k_{it} + q_t k_{it} + (1-\tau)\frac{\gamma \chi_{it}}{1+\gamma} w_t N_t, \\ k_{it+1} &\geq 0, \quad b_{it+1} \geq -\phi, \end{aligned} \tag{5}$$

and for entrepreneurs:

$$\begin{aligned} x_{it} + b_{it+1} + q_t k_{it+1} &= R^B_{t-1} b_{it} + (R^\alpha_{h,t} - \delta) k_{it} + q_t k_{it} + (1-\tau)\Pi_t, \\ k_{it+1} &\geq 0, \quad b_{it+1} \geq -\phi, \end{aligned} \tag{6}$$

where $b_{it}$ are real bonds holdings (all expressed in composite housing-consumption goods), $k_{it}$ are is the amount of houses owned, $\chi_{it}$ is the stochastic endowment with efficiency units of labor, $q_t$ is the relative price of houses, $w_t$ is the wage rate, $R_{h,t}$ is the rental rate on housing services (in consumption goods), $\delta$ is the maintenance cost of housing, i.e. the fraction of the housing stock that has to be replaced by new composite-consumption goods every period, $R^B_{t-1}$ is the real return on bonds. We denote real bonds holdings of household $i$ at the end of period $t$ by $b_{it+1}$ relative to the price level in period $t$. Note that we assume all housing-capital income to go untaxed.

For those households that cannot trade in the market for houses the budget constraint simplifies for workers to:

$$x_{it} + b_{it+1} = R^B_{t-1} b_{it} + (R^\alpha_{h,t} - \delta) k_{it} + (1-\tau)\frac{\gamma \chi_{it}}{1+\gamma} w_t N_t, \quad b_{it+1} \geq -\phi, \tag{7}$$

and for entrepreneurs analogously.[2]

We assume that the probability to find a trade conditional on paying the participation cost is common across households and follows a logit-transformed AR(1) process:

$$\nu_t = \frac{\exp \nu^*_t}{1 + \exp \nu^*_t}, \quad \nu^*_t = (1-\rho_\nu)\bar{\nu} + \rho_\nu \nu^*_{t-1} + \epsilon^\nu_t. \tag{8}$$

[2]While in principle it is possible that a sufficiently strong surprise deflation pushes for $\phi \neq 0$ the household over the natural borrowing limit, we decided to allow households to take on debt. In the numerical simulations households are always able to repay their debt.

#### 2.1.4 Recursive formulation of the households planning problem

Since a household's saving decision will be some non-linear function of that household's wealth and productivity; asset prices and asset supply will therefore be functions of the joint distribution $\Theta_t$ of $(b_t, k_t, \chi_t)$. This makes $\Theta_t$ a state variable of the households' planning problem. We assume that the participation frequency $\nu$ evolves over time according to a first order Markov chain. This lets $\Theta_t$ fluctuate as well.

With this setup, the dynamic planning problem of a household is then characterized by three Bellman equations – $V_a$ in case the household participates and can adjust its housing assets, $V_{an}$ if the household pays the participation costs, but finds no trading partner, and $V_{nn}$ if the household prefers not to pay trading costs. Since the cost of participation are irrelevant as long as no trade occurs, $V_{an} = V_{nn} =: V_n$ holds. The two value functions are given by:

$$\begin{aligned} V_a(b,k,\chi;\nu,\Theta) &= \max_{k',b'_a} u[x(b,b'_a,k,k',\chi)] + \beta E\bar{V}(b'_a,k',\chi';\nu',\Theta') \\ V_n(b,k,\chi;\nu,\Theta) &= \max_{b'_n} u[x(b,b'_n,k,k,\chi)] + \beta E\bar{V}(b'_n,k,\chi';\nu',\Theta') \\ \bar{V}(b,k,\chi;\nu,\Theta) &= E\left\{\max\left[V_n, +\nu(V_a-\kappa) + (1-\nu)V_n\right]\right\} \end{aligned} \tag{9}$$

Since $V_{an} = V_{nn}$ we can simplify the problem, expressing $\bar{V}$ as

$$\bar{V}(b,k,\chi;\nu,\Theta) = (1-\nu)V_n + \nu E\left\{\max\left[V_n, V_a-\kappa\right]\right\}. \tag{10}$$

For tractability, we assume that the participation costs are i.i.d. with distribution function $F_\kappa$, which we assume to be logit, and define $\nu^*(b,k,\chi;\nu,\Theta) := \nu F_\kappa[(V_a - V_n)]$ as the effective probability to trade, where $F_\kappa[(V_a - V_n)]$ is the faction of households participating in the housing market.

In line with this notation, we define the optimal consumption policies for the adjustment and non-adjustment cases as $x^*_a$ and $x^*_n$, the bonds holding policies as $b^*_a$ and $b^*_n$, and the housing policy as $k^*$.

### 2.2 Entrepreneurial Income

Entrepreneurs hire labor at wage rate $w_t$ to produce a differentiated product according to a linear production technology

$$Y_t = \tilde{N}_t,$$

where $\tilde{N}$ are efficiency units of labor. They set the prices for these differentiated products subject to an adjustment cost á la Rotemberg (1982), i.e. log-quadratic adjustment costs and symmetric price setting. For tractability, we assume that price setting is delegated to a mass-zero group of households (managers) that are risk neutral and compensated by a share in profits. They do not participate in any asset market and have to pay a quadratic price adjustment cost if they adjust prices. Under this assumption, price setting is carried out with a time-constant discount factor.

The managers therefore maximize the present value of real profits under this constant discount factor:

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{P_{c,t}}{P_{z,t}} Y_t \left\{ \eta^{-1} \left( \frac{p_{jt}}{P_{c,t}} - \frac{W_t}{P_{c,t}} \right) \left( \frac{p_{jt}}{P_{c,t}} \right)^{-\eta} - \frac{\kappa}{2} \left[ \log \left( \frac{p_{jt}}{p_{jt-1}} \right) \right]^2 \right\}. \tag{11}$$

The corresponding first-order condition reads

$$\begin{aligned} 0 = \quad & \frac{P_{c,t}}{P_{z,t}} Y_t \left\{ P_{c,t}^{-1} \left( \frac{p_{jt}}{P_{c,t}} \frac{1-\eta}{\eta} - \frac{W_t}{P_{c,t}} \right) \left( \frac{p_{jt}}{P_{c,t}} \right)^{-\eta-1} - \kappa \left[ \log \left( \frac{p_{jt}}{p_{jt-1}} \right) \right] p_{jt}^{-1} \right\} \\ + \quad & \beta \kappa E_t \frac{P_{c,t+1}}{P_{z,t+1}} Y_{t+1} \left[ \log \left( \frac{p_{jt+1}}{p_{jt}} \right) \right] p_{jt}^{-1}. \end{aligned} \tag{12}$$

A symmetric equilibrium ($p_{jt} \equiv P_{c,t}$), i.e., an equilibrium in which price-setting is homogeneous, simplifies this first-order-condition to the following Phillips curve:

$$\begin{aligned} 0 = \quad & \frac{P_{c,t}}{P_{z,t}} Y_t \left\{ \left( \frac{1-\eta}{\eta} - \frac{W_t}{P_{c,t}} \right) - \kappa \log(1+\pi_t) \right\} \\ + \quad & \beta \kappa E_t \frac{P_{c,t+1}}{P_{z,t+1}} Y_{t+1} \log(1+\pi_{t+1}), \end{aligned} \tag{13}$$

such that

$$\log(1+\pi_t) = \beta E_t \left[ \log(1+\pi_{t+1}) \frac{Y_{t+1}}{Y_t} \left( \frac{R_{h,t}}{R_{h,t+1}} \right)^{1-\alpha} \right] + \kappa^{-1} \left( w_t R_{h,t}^{1-\alpha} - \frac{\eta - 1}{\eta} \right), \tag{14}$$

where $\pi_t$ is the net inflation rate (of consumption goods), $\pi_t := \frac{P_{c,t} - P_{c,t-1}}{P_{c,t-1}}$, and $w_t R_{h,t}^{1-\alpha}$ is the real wage in consumption goods. The price adjustment then creates real costs $\frac{\kappa \log(1+\pi_t)^2}{2} Y_t$.[3]

[3] For the numerical solution, we parameterize the expectation function $E_t \left[ \log(1+\pi_t) \frac{Y_{t+1}}{Y_t} \left( \frac{R_{h,t}}{R_{h,t+1}} \right)^{1-\alpha} \right]$ log-linearly as we do for the other price expectations when solving for a Krusell-Smith equilibrium.

Since managers are a measure-zero group in the economy all profits net of adjustment costs go to the entrepreneur households (whose $\chi = 0$). In addition to profits from monopolistic competition in final goods, these households also obtain profit income from adjusting the aggregate housing stock. They can transform $I_t$ consumption goods into $\Delta K_{t+1}$ additional units of housing goods (and back) according to the transformation function

$$I_t = \frac{\phi}{2}\left(\frac{\Delta K_{t+1}}{K_t}\right)^2 K_t + \Delta K_{t+1}.$$

Since they are facing perfect competition in this market, entrepreneurs will adjust the stock of housing until the following first-order condition holds:[4]

$$q_t = 1 + \phi\frac{\Delta K_{t+1}}{K_t}. \tag{15}$$

### 2.3 Government

The government operates a monetary and a fiscal authority. The monetary authority controls the real interest rate on liquid assets, while the fiscal authority emits government bonds to finance government deficits and adjusts government expenditures to stabilize debt in the long run.

We assume that monetary policy sets the real interest rate on bonds following a Taylor (1993)-type rule with interest rate smoothing

$$\log R^b_{t+1} = (\theta_1 + \theta_2 \log \pi_t)(1 - \theta_3) + \theta_3 \log R^b_t. \tag{16}$$

The coefficient $\theta_1 \geq 0$ implicitly determines the steady-state real interest rate. The coefficient $\theta_2 \geq 0$ the extent to which the central bank attempts to stabilize inflation around its steady-state value: the larger $\theta_2$ the stronger is the reaction of the central bank to deviations from the inflation target. When $\theta_2 \to \infty$ inflation is perfectly stabilized at its steady-state value. $\theta_3 \geq 0$ captures interest rate smoothing.

The emission of real government bonds is governed by the government's intertemporal

[4]Note that we assume housing-capital adjustment costs only on new housing-capital (or on the active destruction of old housing-capital) but not on the replacement of depreciation. Depreciated housing-capital is assumed to be replaced at the cost of one-to-one in consumption goods, and replacement is forced before the housing-capital stock is adjusted at a cost. This differential treatment of depreciation and net investment simplifies the equilibrium conditions substantially, because the user cost of housing-capital and hence the dividend paid to households do not depend on the next period's stock of housing-capital, and the decisions of non-adjusters are not influenced by the price of housing-capital $q_t$. Quantitatively, the fluctuations in dividends that maintenance at price $q_t$ would bring about are negligible.

budget constraint:

$$B_{t+1} = (1 + R_t^b)B_t + G_t - T_t.$$

Total tax revenues are given by

$$T_t = \tau(w_t \tilde{N}_t + \Pi_t).$$

We assume that government expenditures follow a rule that stabilizes government debt in the long run around $\bar{B}$:

$$G_t = \gamma_1 - \gamma_2(B_t - \bar{B}) + \gamma_3 T_t$$

Together these imply for the law of motion of government debt

$$B_{t+1} = (1 + R_t^b)B_t + \gamma_1 - \gamma_2(B_t - \bar{B}) - (1 - \gamma_3)T_t \tag{17}$$

The coefficient $\gamma_2$ captures whether and how fast the government seeks to stabilize its debt. For $\gamma_2 > 0$ the government seeks to actively stabilize government debt, for $\gamma_2 = 0$ debt will only be stable if the monetary authority violates the Taylor principle ($\theta_2 < 1$) such that inflation can stabilize debt ("fiscal dominance"). The coefficient $\gamma_3$ captures the cyclicality of government expenditure: for $\gamma_3 = 0$ government expenditures do not react to tax income and are acyclical, for $\gamma_3 < 0$ expenditures are countercyclical for $\gamma_3 > 0$ they are procyclical.

### 2.4 Goods, Bonds, Capital, and Labor Market Clearing

The labor market clears at the competitive wage. The bonds market clears, whenever the following equation holds:

$$\begin{aligned} B_{t+1} = & \int \nu^*(b,k,\chi;q_t,\pi_t,\nu_t) \quad b_a^*(b,k,\chi;q_t,\pi_t,\nu_t)\Theta_t(b,k,\chi)dbdkd\chi \\ + & \int [1-\nu^*(b,k,\chi;q_t,\pi_t,\nu_t)] \quad b_n^*(b,k,\chi;q_t,\pi_t,\nu_t)\Theta_t(b,k,\chi)dbdkd\chi. \end{aligned} \tag{18}$$

Last, the asset market for housing has to clear:

$$\begin{aligned} q_t &= 1 + \phi\frac{K_{t+1} - K_t}{K_t} = 1 + \nu\phi\frac{K^*_{t+1} - K_t}{K_t} \qquad (19) \\ K_{t+1} &= \int k^*(b,k,\chi;q_t,\pi_t,\nu_t)\nu^*(b,k,\chi;q_t,\pi_t,\nu_t)\Theta_t(b,k,\chi)dbdkd\chi \\ &+ \int k\left[1-\nu^*(b,k,\chi;q_t,\pi_t,\nu_t)\right]\Theta_t(b,k,\chi)dbdkd\chi. \end{aligned}$$

where the first equation stems from competition in the production of houses, the second equation defines the aggregate supply of funds from households trading houses, and the third equation defines the law of motion of the aggregate housing stock. The goods market then clears due to Walras' law, whenever both, bonds and the house-asset markets, clear.

### 2.5 Recursive Equilibrium

A *recursive equilibrium* in our model is a set of policy functions $\{x_a^*, x_n^*, b_a^*, b_n^*, k^*\}$, value functions $\{V_a, V_n\}$, pricing functions $\{r, w, \pi, q\}$, aggregate housing and labor supply functions $\{K, N\}$, distributions $\Theta_t$ over individual asset holdings and productivity, and a perceived law of motion $\Gamma$, such that

1. Given $V$, $\Gamma$, prices, and distributions, the policy functions $\{x_a^*, x_n^*, b_a^*, b_n^*, k^*\}$ solve the households' planning problem, and given the policy functions $\{x_a^*, x_n^*, b_a^*, b_n^*, k^*\}$, prices and distributions, the value functions $\{V_a, V_n\}$ are a solution to the Bellman equation (9).

2. The markets for labor, consumption-goods, and housing services, as well as the asset markets for bonds, and houses clear, i.e. (4), (14), (18), and (19) hold.

3. The actual law of motion and the perceived law of motion $\Gamma$ coincide, i.e. $\Theta' = \Gamma(\Theta, v')$.

## 3 Numerical Implementation

The dynamic program (9) and hence the recursive equilibrium is not computable, because it involves the infinite dimensional object $\Theta_t$.

### 3.1 Krusell-Smith Equilibrium

To turn this problem into a computable one, we assume that households predict future prices only on the basis of a restricted set of moments, as in Krusell and Smith (1997, 1998). Specifically, we make the assumption that households condition their expectations only on last period's aggregate real bond holdings and return, $B_t$ and $R_t^B$, the aggregate stock of housing-capital, $K_t$, and the liquidity state, $v_t$. The reasoning behind this choice goes as follows: (18) determines inflation, which in turn depends on the beginning of period bond stock. Once inflation is fixed, the Phillips curve (14) determines markups and hence wages and dividends. These will pin down asset prices by making the marginal investor indifferent between bonds and housing-capital. If asset-demand functions, $b_{a,n}^*$

and $k^*$, are sufficiently close to linear in human capital, $h$, and in non-human wealth, $b, k$, at the mass of $\Theta_t$, we can expect approximate aggregation to hold. For our exercise, the four aggregate states – $v_t$, $B_t$, $R_t^B$, and $K_t$ – are sufficient to describe the evolution of the aggregate economy.

While the law of motion for $v_t$ is pinned down by (8), households use the following log-linear forecasting rules for current inflation, the price of housing, the rental rate of housing, and the expectations term in the Phillips curve, where the coefficients depend on the liquidity state:

$$\begin{aligned}
\log \pi_t &= \beta_\pi^1(v_t) + \beta_\pi^2(v_t)\log B_t + \beta_\pi^3(v_t)\log K_t + \beta_\pi^4(v_t)R_t^b, &(20)\\
\log q_t &= \beta_q^1(v_t) + \beta_q^2(v_t)\log B_t + \beta_q^3(v_t)\log K_t + \beta_q^4(v_t)R_t^b, &(21)\\
\log R_{h,t} &= \beta_R^1(v_t) + \beta_R^2(v_t)\log B_t + \beta_R^3(v_t)\log K_t + \beta_R^4(v_t)R_t^b, &(22)
\end{aligned}$$

$$\begin{aligned}
\left[\log(1+\pi_t)\frac{Y_{t+1}}{Y_t}\left(\frac{R_{h,t}}{R_{h,t+1}}\right)^{1-\alpha}\right] &= \beta_{E\pi}^1(v_t) + \beta_{E\pi}^2(v_t)\log B_t + \beta_{E\pi}^3(v_t)\log K_t\\
&+ \beta_{E\pi}^4(v_t)R_t^b + \beta_{E\pi}^5(v_{t+1}). &(23)
\end{aligned}$$

The law of motion for real bond holdings, $B_t$, follows from (17) and rhe law of motion for $K_t$ results from (19).

Technically, finding the equilibrium is similar to Krusell and Smith (1997), as we need to find market clearing prices within each period. Concretely, this means the posited rules, (20), (21), (22), (23), are used to solve for households' policy functions. Having solved for the policy functions conditional on the forecasting rules, we then simulate $n$ independent sequences of economies for $t = 1, \dots, T$ periods, keeping track of the actual distribution $\Theta_t$. In each simulation the sequence of distributions starts from the stationary distribution implied by our model without aggregate shocks. We then calculate in each period $t$ the optimal policies for market clearing inflation rates and house prices assuming that households resort to the policy functions derived under forecasting rules from period $t+1$ onward. Having determined the market clearing prices, we obtain the next period's distribution $\Theta_{t+1}$. In doing so, we obtain $n$ sequences of equilibria. The first 250 observations of each simulation are discarded to minimize the impact of the initial distribution. We next re-estimate the parameters of (20) and (21) from the simulated data and update the parameters accordingly. By using $n = 20$ and $T = 750$, it is possible to make use of parallel computing resources and obtain 10.000 equilibrium observations. Subsequently, we recalculate policy functions and iterate until convergence in the forecasting rules.

The forecasting rules, (20), (21), (22), (23), approximate the aggregate behavior of the economy fairly well. The minimal within sample $R^2$ is above 99%.

### 3.2 Solving the Household Planning Problem

In solving for the households' policy functions we apply an endogenous gridpoint method as originally developed in Carroll (2006) and extended by Hintermaier and Koeniger (2010), iterating over the first-order conditions, as in Bayer et al. (2015). We approximate the idiosyncratic productivity process by a discrete Markov chain with 4 states. The liquidity volatility process is approximated in the same vein using 5 states and the Rouwenhorst method.

## 4 Calibration

We calibrate the model to the U.S. economy. Where possible we identify parameters from the behavior of the model in steady state without fluctuations in liquidity. The aggregate data used for calibration spans 1980 to 2007. One period in the model refers to a quarter of a year. The choice of parameters as summarized in Table 1 is explained next. We present the parameters as if they were individually changed in order to match a specific data moment, but all calibrated parameters are determined jointly.

### 4.1 Preferences and Technology

Table 1 summarizes our calibration. In detail, we choose the parameter values as follows.

For the felicity function, $u = \frac{1}{1-\xi}x^{1-\xi}$, we set the coefficient of relative risk aversion $\xi = 2$. The time-discount factor, $\beta$, the distribution of adjustment costs, $logit1$ and $logit2$, and the probability of trading, $\nu$, are jointly calibrated to match the ratios of liquid and illiquid assets to output and the liquidity of housing. We equate illiquid assets to housing capital. This implies for the total value of illiquid assets relative to nominal GDP a capital-to-output ratio of about 250%. In our baseline calibration, this implies an annual real return for illiquid assets of 3%. We equate liquid assets to claims of the private sector against the government. Specifically, we look at average U.S. federal debt for the years 1980 to 2007 held by domestic private agents plus the monetary base. This yields an annual bond-to-output ratio of 20%.

Table 1: Calibrated parameters

| Parameter | Value | Description | Target |
|---|---|---|---|
| **Households** | | | |
| $\beta$ | 0.98 | Discount factor | $K/Y = 286\%$ (annual) |
| $\nu$ | 50% | Probability of sale | Pre-Great Recession |
| $logit1$ | 17.5 | Location parameter | Average adj. prob. |
| $logit2$ | 5 | Shape parameter | $B/Y = 20\%$ (annual) |
| $\xi$ | 2 | Coefficient of rel. risk av. | Standard value |
| $\gamma$ | 1 | Inverse of Frisch elasticity | Standard value |
| $\rho_h$ | 0.976 | Persistence of income | Standard value |
| $\bar{\sigma}$ | 0.078 | STD of innovations to income | Standard value |
| $\alpha$ | 0.8 | Share of non-housing | NIPA |
| $\delta$ | 1% | Depreciation rate of housing | NIPA |
| **Final Goods** | | | |
| $\kappa$ | 0.09 | Price stickiness | Mean price duration of 4 quarters |
| $\mu$ | 0.05 | Markup | 5% markup (standard value) |
| **Capital Goods** | | | |
| $\phi$ | 100 | Capital adjustment costs | |
| **Monetary Policy** | | | |
| $\theta_1$ | 1.005 | Steady state rate | 2% p.a. |
| $\theta_2$ | 1.25 | Reaction to inflation deviations | Standard value |
| $\theta_3$ | 0.7 | Inertia | Standard value |
| **Fiscal Policy** | | | |
| $\gamma_1$ | 1.44 | Steady state spending | 20% of output |
| $\gamma_2$ | 0.1 | Reaction to the debt | Standard value |
| $\gamma_3$ | 2.5 | Reaction to the cycle | Fiscal response in 2009 |

We take a standard value for the inverse Frisch elasticity of labor supply, $\gamma = 1$, which follows the New Keynesian literature (Chetty et al. (2011)). We assume standard parameters for the idiosyncratic income process. We think of household income as pre-tax labor income plus private and public transfers minus all taxes. We calibrate the probability of becoming an entrepreneur by matching the Gini of networth in the Survey of Consumer Finances, following Castaneda et al. (1998). The resulting fraction of entrepreneurs is 1%, which remain in that role for an average of 16 quarters. The probability for a worker to become an entrepreneur is 0.0005%.

# 5 Quantitative Effects of Liquidity Shocks

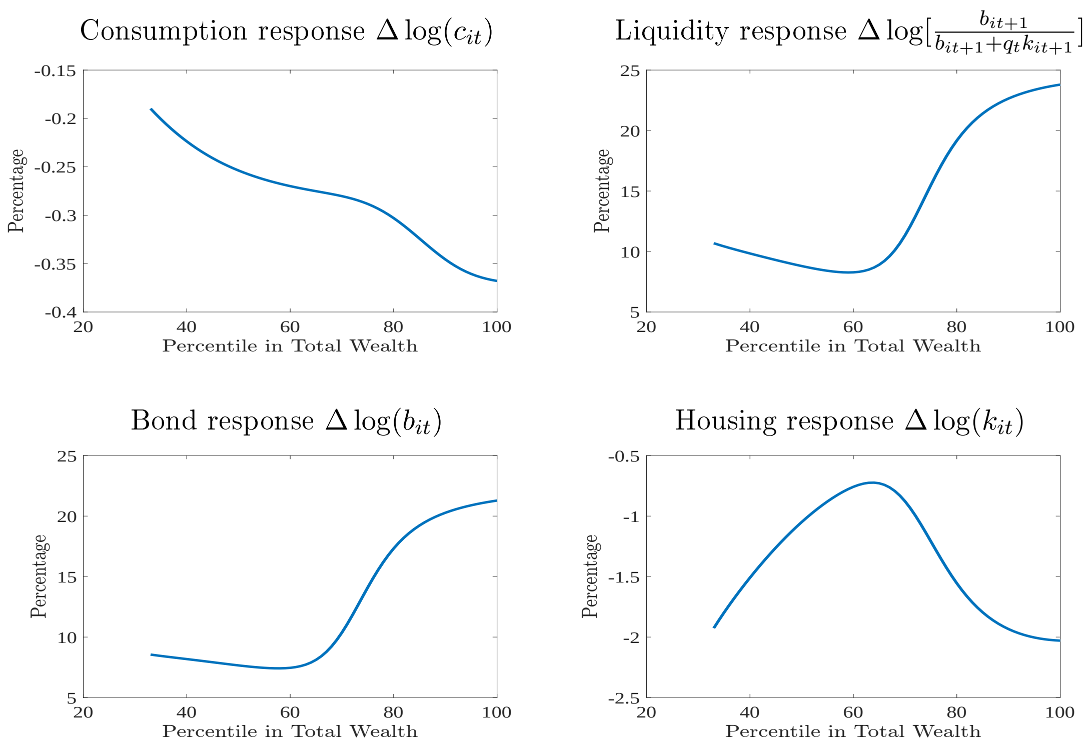
## 5.1 Household Portfolios and the Individual Response

In our model, households hold bonds because they provide better short-term consumption smoothing than housing-capital, as the latter can only be traded infrequently. This value of liquidity decreases in the amount of bonds a household holds, because a household rich in liquid assets will likely be able to tap into its illiquid wealth before running down all liquid wealth. For this reason, richer households, who typically hold both more liquid and illiquid assets, hold less liquid portfolios. The poorest households, on the contrary, hold almost all their wealth in the liquid asset. This holds true in the actual data as well as in our model.

So what happens to total savings and its composition when liquidity decreases? In response to the decrease in liquidity, households aim for a larger portfolio share of liquid assets in their portfolio. Those households active in the market for illiquid assets do so by trading, those that are inactive simply save more liquid assets. Figure 2 shows how households' portfolio composition and consumption policy react to a drop in liquidity without imposing any market clearing. The top panels displays the relative change in the consumption and portfolio liquidity compared to the average liquidity state. For this exercise, we evaluate households' consumption policies and the portfolio choice of adjusters and non-adjusters after moving from the average liquidity state to the low liqidity state. We here perform a partial equilibrium analysis and compute the policies under the expectation that all prices are at their steady-state values isolating hence the direct effect of liquidity. The bottom panels of Figure 2 display the change in demand for bonds and housing-capital separately.

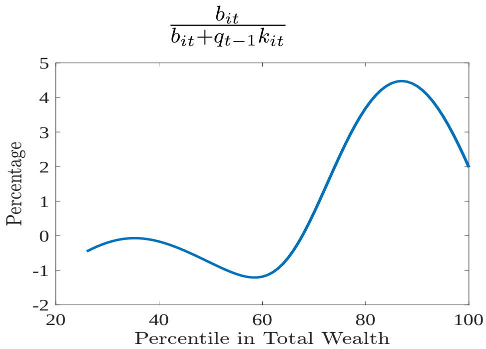
The change in the liquidity of household portfolios in general equilibrium, when monetary and fiscal policy responds, is displayed in Figure 3; the left-hand panel shows the

Figure 2: Partial equilibrium response – Change in individual policy upon an negative liquidity shock keeping prices and expectations constant at steady-state values




*Notes:* Reaction to low liquidity of individual consumption, portfolio liquidity, bond holdings and housing at constant prices and price expectations relative to the respective counterpart at average liquidity. The policies are averaged using frequency weights from the steady-state wealth distribution and reported conditional on a household falling into the x-th wealth percentile. Low liquidity corresponds to a 50% increase in time to sell.

Figure 3: General equilibrium response – Change in the liquidity of household portfolios




*Notes:* Change in the distribution of liquidity at all percentiles of the wealth distribution at equilibrium prices and price expectations after a 50% increase in time to sell. The liquidity of the portfolios is averaged using frequency weights from the steady-state wealth distribution and reported conditional on a household falling into the x-th wealth percentile.

change in value terms; the right-hand panel shows the change in quantities, i.e., at constant prices.

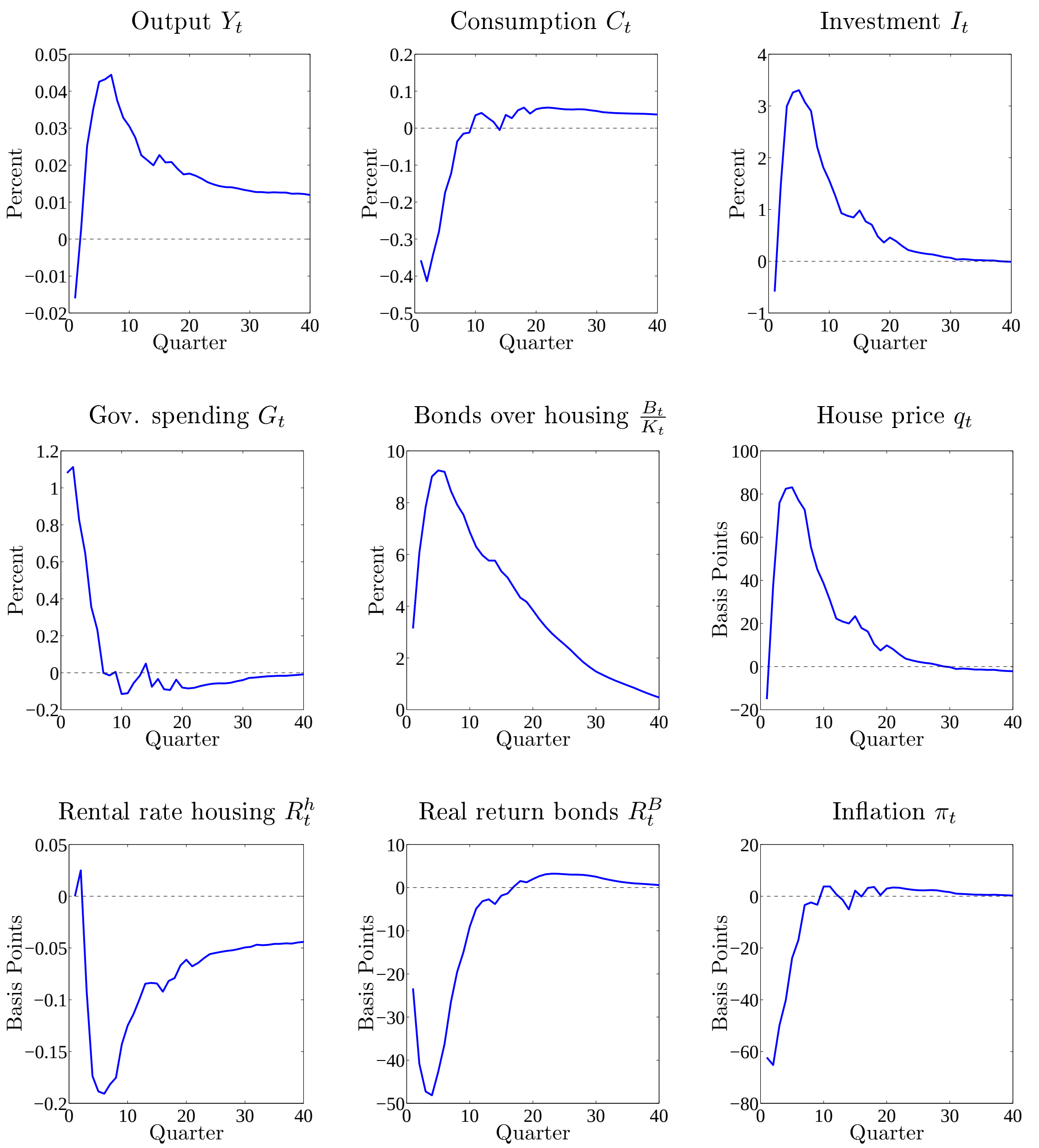
## 5.2 Aggregate Consequences

### 5.2.1 Baseline with Monetary and Fiscal Stabilization

The drop in demand for housing (as an asset) leads to a decline in the price of houses and creates an excess demand for liquid bonds. Consequently, the real interest rate on government bonds has to fall, in order to get the economy back to equilibrium. The central bank's Taylor rule, however, only allows the interest rate to fall when inflation falls simultaneously. This leads to a decline in output as it pushes up markups and drives down real wages. As the economy falls into a recession also the rental rates for existing houses fall.

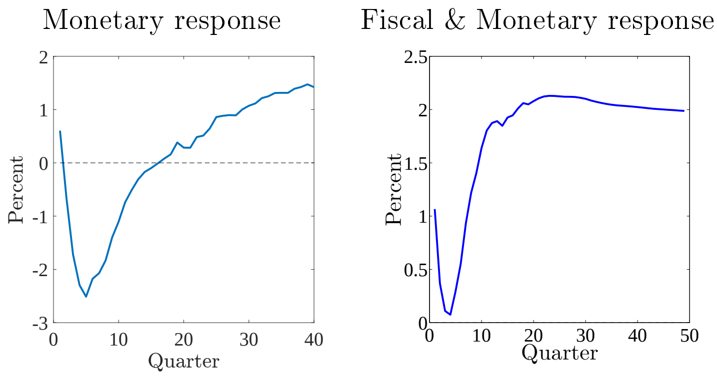
In the baseline scenario, c.f. Figure 4, we consider a joint fiscal and monetary response that stabilizes the economy similar to the Great Recession ($\theta 2 = 1.25, \gamma 3 = 2.5$). The central bank follows a Taylor rule and the fiscal authority a counter-cyclical spending rule. In response to the liquidity shock, goverment spending increases by 1%. This fiscal response together with the fall in the real interest rate by 2 percentage point

Figure 4: Liquidity shock with fiscal and monetary stabilization




*Notes:* Impulse responses to a 50% increase in time to sell of housing capital. All rates (inflation, returns, etc.) are *not* annualized.

Figure 5: Real return on bonds




*Notes:* Annualized times series after a 50% increase in time to sell of housing capital.

(annualized) stabilizes output. The fiscal authority satisfies the higher demand for liquid saving vehicles by running a deficit. This increases the amount of liquid assets in the economy. The ratio of liquid to illiquid assets goes up by 9%.

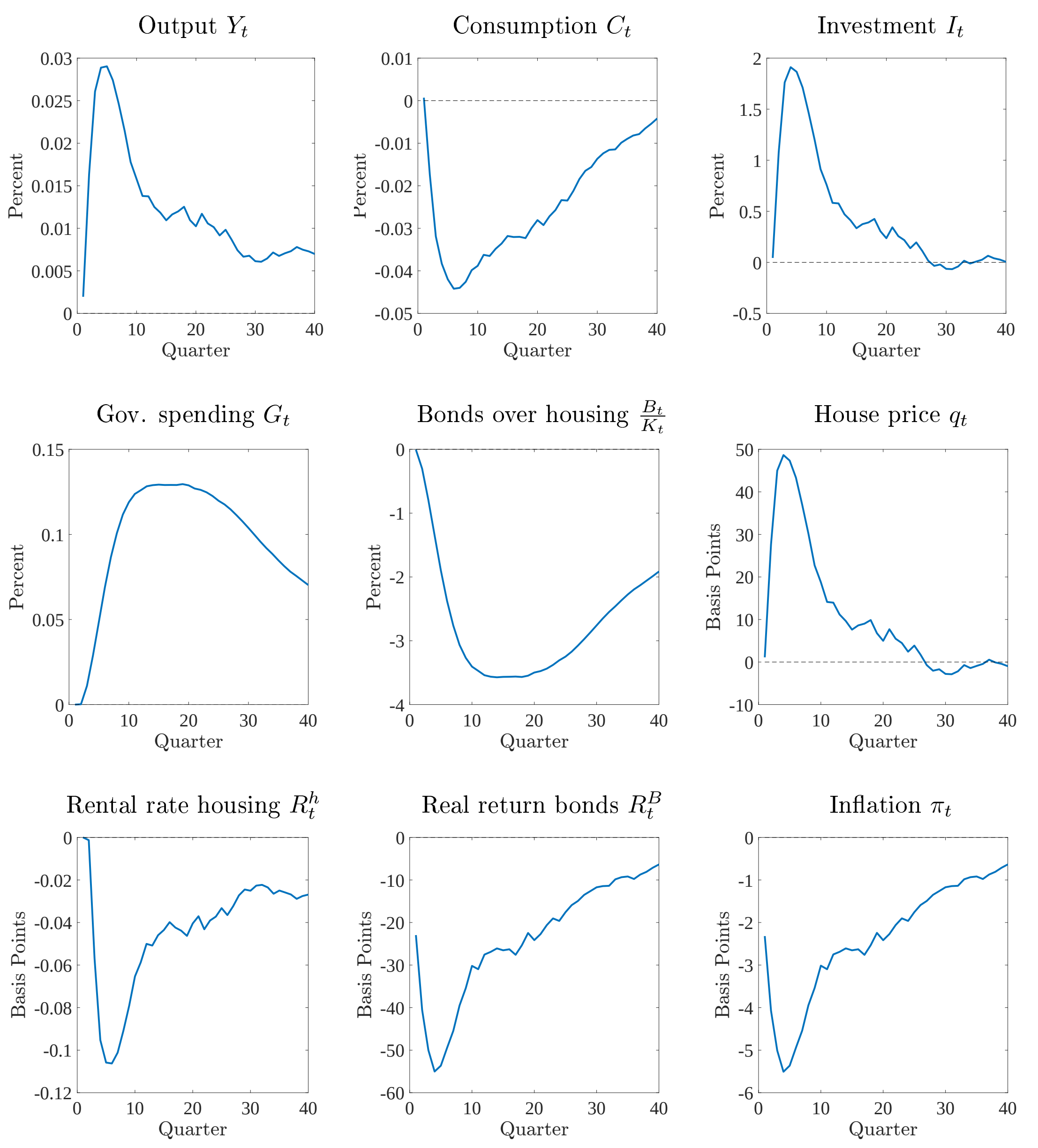
The strong fiscal expansion, therefore, stabilizes the real interest rate on government bonds. Nonetheless, the interest rate remains low for 12 quarters and is close to the zero-lower bound during the first 4 quarters. The next section examines the response of the economy to liquidity shocks, when only the monetary authority responds to the shock and fiscal policy is passive.

### 5.2.2 Monetary Stabilization

In Figure 6, the monetary authority responds to inflation deviations from target, while the fiscal authority does not react to inflation ($\theta_2 = 10, \gamma_3 = 0$). Government spending only reacts to government debt to bring back the debt level to its steady state value in the long run.

The real interest rate falls by more than 4 percentage points (annualized) and, thus, becomes negative at -2%. In response, output and consumption move very little, whereas investment in housing increases by 2 percent. The low return on liquid assets leads to a decline in the ratio of liquid to illiquid assets by about 3.5%. Figure 5 compares the responses of the real rate and aggregate liquidity with only monetary stabilization to the baseline.

Figure 6: Liquidity shock with monetary stabilization




*Notes:* Impulse responses to a 50% increase in time to sell of housing capital. All rates (inflation, returns, etc.) are *not* annualized.

### 5.2.3 Fiscal Stabilization

In Figure 7, we look at the reverse case. Fiscal policy responds strongly to inflation, whereas the monetary authority keeps the real rate of interest constant. In this case, government spending increases by more than 1% and stays high for about 8 quarters. This perfectly stabilizes output.

## 6 Conclusion

This paper examines how variations in housing liquidity affect the macroeconomy through portfolio re-adjustments. For this purpose we develop a tractable framework that combines nominal rigidities and incomplete markets in which households choose portfolios of liquid paper and illiquid housing assets. In this model, lower liquidity of housing triggers a flight to liquidity and, thus, puts downwards pressure on interest rates on government bonds. This reduces not only consumption but also investment and hence depresses economic activity.

Calibrating the model to match the increased time-on-market of houses during the Great Recession, we find that monetary policy quickly hits the zero-lower bound in its attempt to stabilize the economy. Fiscal policy can alleviate this problem by providing additional liquid assets in the form of goverment bonds.

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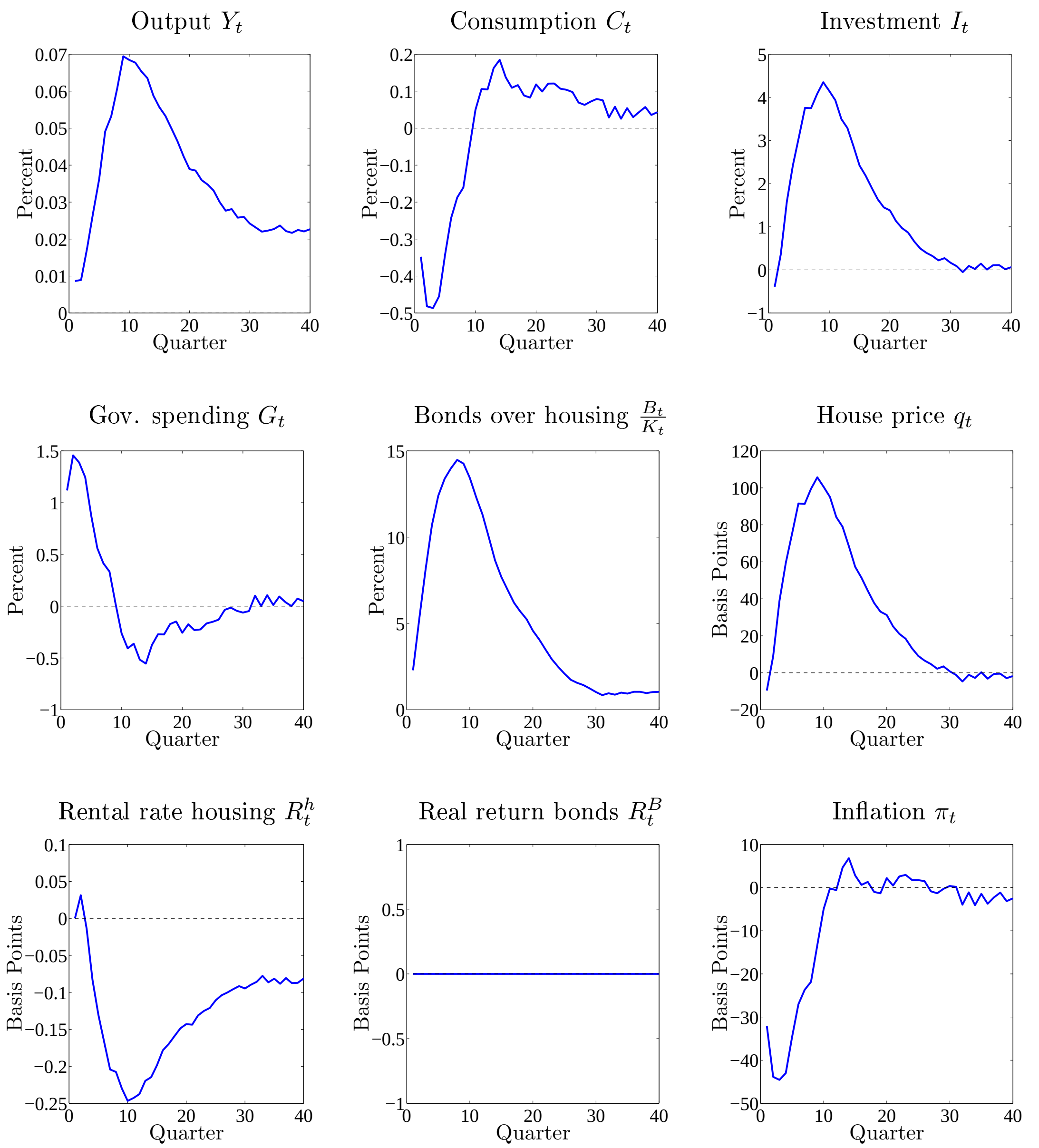
Figure 7: Liquidity shock with fiscal stabilization




*Notes:* Impulse responses to a 50% increase in time to sell of housing capital. All rates (inflation, returns, etc.) are *not* annualized.

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