

Corporate Finance and Monetary Policy*

Guillaume Rocheteau

University of California, Irvine

Randall Wright

University of Wisconsin, Madison; FRB Chicago and FRB Minneapolis

Cathy Zhang

Purdue University

This version: January 2016

Abstract

This paper provides a theory of external and internal finance where entrepreneurs finance random investment opportunities with fiat money, bank liabilities, or trade credit. Loans are distributed in an over-the-counter credit market where the terms of the loan contract, including size, rate, and down payment, are negotiated in a decentralized fashion subject to pledgeability constraints. The model has implications for the cross-sectional distribution of corporate loan rates and loan sizes, interest rate pass-through, and the transmission of monetary policy (described either as money growth or open market operations) with or without liquidity requirements.

JEL Classification Numbers: D83, E32, E51

Keywords: Money, Credit, Interest rates, Corporate Finance, Pledgeability.

*We thank Sebastien Lotz for his input and comments on a preliminary version of the paper, and many people for input on this and related work. We also thank conference participants at the 2015 WAMS-LAEF Workshop in Sydney for useful feedback and comments. Wright acknowledges support from the Ray Zemon Chair in Liquid Assets at the Wisconsin School of Business. The usual disclaimers apply.

1 Introduction

It is commonly thought (and taught) that monetary policy influences the real economy by setting short-term nominal interest rates that affect the real rate at which households and firms borrow. While perhaps appealing heuristically, it is not easy to model this rigorously. One reason is that it arguably requires, among other things, an environment where money, credit, and government bonds coexist. This is challenging, in theory, because the same frictions that make money essential—commitment and information frictions—can make credit infeasible, and because one has to face classic thorny issues concerning the coexistence of money and interest-bearing securities. Understanding the transmission mechanism from monetary policy to investment, we believe, also calls for a sound theory of corporate finance and firms’ liquidity management.

The goal of this paper is to develop a novel approach to corporate finance, building on recent advances in the study of money, credit, and asset markets in the New Monetarist literature.¹ This allows explicit analysis of the channels through which monetary policy affects firms’ liquidity, trade and bank credit, loan rates, loan sizes, and investment.

1.1 Preview

We describe an economy where entrepreneurs receive random opportunities to invest using either retained earnings held in liquid assets (internal finance) or loans from banks who issue short-term liabilities (external finance). In our benchmark model, entrepreneurs cannot get trade credit directly from suppliers of investment goods, because they would renege on repayment. (We relax this assumption in one extension to allow for trade credit.) This creates a need for either outside liquidity, in the form of cash, or inside liquidity, provided by banks. Banks in the model have two distinctive features: they can monitor entrepreneurs and enforce repayment to some extent; and their liabilities are recognizable and hence able to serve as payment instruments. Importantly and realistically, our credit market is an over-the-counter (OTC) market, with search and bargaining, where some trades involve intermediation.

¹A detailed literature review follows below; in these preliminary remarks, we cite only a few papers that are directly relevant by way of motivation or explanation.

The determination of loan contracts involves an entrepreneur and a bank who negotiate the loan size, down payment, and interest rate. The terms of contracts are subject to limited enforcement, with only a fraction of the returns from investment being pledgeable, as in Holmstrom and Tirole (1999) or Kiyotaki and Moore (1997). Alternatively, entrepreneurs cannot commit to pay their debts, but can be monitored to some extent, and excluded from future credit in case of default, as in Kehoe and Levine (1993). We study both interpretations and show they generate closely related outcomes. Additionally, finding someone to extend a loan is a time-consuming process, as in Wasmer and Weill (2004). As a result, credit in our framework has both an extensive margin (acceptability of loan applications) and an intensive margin (loan size), matching salient features of actual corporate credit markets.

In the benchmark model with only external finance, the efficient level of investment can be financed provided pledgeability is sufficiently high and banks' bargaining power is sufficiently low. When the repayment constraint binds, investment and loan size depend on pledgeability, bargaining power and technology. If the source of heterogeneity among entrepreneurs and banks is due to differences in bargaining powers, then the model predicts a negative correlation between loan sizes and lending rates in the cross section. If the source of the heterogeneity is in terms of pledgeability of investment projects, then the model generates no correlation between loan sizes and lending rates. To illustrate the flexibility of our approach, we consider several applications of the benchmark model. In one extension, entrepreneurs can obtain trade credit directly from suppliers, and also banks, which affects loan contracts with banks. In equilibrium, some investment is financed by trade credit, and some by bank credit, very much consistent with conventional wisdom and empirical research. In another extension, we endogenize the frequency at which entrepreneurs access banks, by allowing free entry, and show how this margin is affected by policy. Finally, we describe an extension where the market for capital goods is decentralized with search and bargaining. In that case, the loan contract is determined through a trilateral negotiation between an entrepreneur, a supplier, and a bank.

We also introduce internal finance by letting entrepreneurs accumulate cash to pay for random investment opportunities. The cost of holding money is the nominal interest

rate on illiquid bonds, which is a policy variable. Money held by firms serves two roles: an insurance function, where entrepreneurs can finance investment if they lack sufficient external finance; and a strategic function, where they can negotiate lower loan rates by making larger down payments. In accordance with the evidence, firms' money demand increases with idiosyncratic risk and decreases with the pledgeability of output (captured empirically by assets being more tangible). By lowering the nominal rate, a central bank encourages entrepreneurs to hold more liquidity, allowing them to finance larger investments and negotiate better terms in their loan contracts. However, it also reduces banks' incentives to participate in the credit market, thereby reducing entrepreneurs' access to loans. Moreover, the ability to self-finance raises pledgeable output, and this creates an amplification mechanism for monetary policy. In addition, the relationship between loan size and the policy rate is non-monotone. For high nominal rates, it is negative, since inflation reduces down payments, pledgeable output, and lending. For low nominal rates, bank lending increases to substitute for internal finance. As the nominal rate is driven to zero (the Friedman rule), the fraction of investment financed internally is maximized and the real rate on short-term loans is minimized.

A key finding of our model is that it generates a pass-through from the nominal policy rate to the real loan rate. We obtain closed-form expressions for this pass-through and show it prevails even in the absence of nominal rigidities or reserve requirements. Moreover, there is a positive pass-through even when borrowing constraints are slack. The lending rate is more responsive to changes in the policy rate when banks have more bargaining power and entrepreneurs have better access to loans.

We also consider short-term government bonds and regulatory requirements on reserves and liquidity, to study how open market operations (OMOs) affect investment and loan rates. To satisfy regulatory requirements, banks can access a competitive interbank market, akin to the Fed Funds Market, where they can trade reserves and government bonds overnight. Our model generates money demand from entrepreneurs and banks, and a structure of interest rates composed of a short-term lending rate in the interbank market, a corporate loan rate, an interest rate on government bonds, and an interest rate on illiquid bonds. In the case of a strict reserve requirement, an OMO that purchases bonds

on the interbank market raises banks' reserves, thereby reducing their borrowing cost in the interbank market, and promotes bank lending. The increase in the money supply leads to a proportional increase in the price level, which reduces entrepreneurs' real balances and their ability to self-finance their investment opportunities. As a result, there is a redistribution of liquidity across entrepreneurs and banks, which alters the composition of corporate finance towards bank lending. In the aggregate, investment increases.

We then turn to a broader liquidity requirement that can be satisfied with money and bonds. If the supply of government bonds is sufficiently low, their nominal yield hits zero, and the economy falls into a liquidity trap where changes in the supply of bonds are ineffective. If the supply of these bonds is large, but not too large, so that they still entail a liquidity premium, changes in the supply of bonds have real effects. A permanent increase in the supply of government bonds lowers the loan rate, which generates a redistribution of entrepreneurs' financing. As a result, investment financed by credit increases, while investment financed internally decreases. However, an OMO in the interbank market has no effect because money and bonds are substitutes to fulfill regulatory requirements. We think these kinds of results put monetary policy in a new light, based on theory with relatively explicit microfoundations for the notion of liquidity.

1.2 Theory Literature

We build on the New Monetarist framework discussed in surveys by Williamson and Wright (2010), Nosal and Rocheteau (2011) and Lagos et al. (2015), except our focus is on entrepreneurs financing investment, while most of the other work emphasizes households financing consumption.² Like most of these papers, we incorporate search frictions. Nosal and Rocheteau (2011) and, more recently, Gu et al. (2015) provide discussions of search-based (and other) models of credit.³ Our main emphasis is on credit intermediated by

²Exceptions with firms trading inputs include Silveira and Wright (2011,2015), Chiu and Meh (2011) and Chiu et al. (2015). Those models have trade credit and bank credit, where banks reallocate liquidity, as in Berentsen et al. (2007), which can be understood as a general equilibrium monetary version of Diamond and Dybvig (1983). However, those banks do not issue assets that facilitate third-party transactions.

³Early search-based models include Diamond (1987) and Shi (1996); more recent work includes Telyukova and Wright (2008), Sanches and Williamson (2010), Hu and Rocheteau (2013), Lagos (2013), Bethune et al. (2014), Bethune et al. (2015), Araujo and Hu (2015), Carapella and Williamson (2015) and

banks that issue short-term liabilities that can be used as means of payment, in the spirit of Cavalcanti and Wallace (1999a,b), He et al. (2005,2008) and Gu et al. (2013a).

Somewhat related is work by those following Duffie et al. (2005), Lagos and Rocheteau (2009) and others, who study intermediation fees (bid-ask spreads) in decentralized asset markets. Our OTC credit market is similar to Wasmer and Weil (2004), and related papers by Petrosky-Nadeau and Wasmer (2013), Petrosky-Nadeau (2014), Betsi et al. (2005, 2013) and Den Haan et al. (2003). However, we formalize the role of money and monetary policy explicitly, have both internal and external financing, endogenize loan size and are more explicit about frictions like limited commitment and monitoring.

Obviously the approach is related to Kiyotaki and Moore (1997, 2005) and Holmstrom and Tirole (1998, 2011), emphasizing limited pledgeability of output as a key constraint in credit arrangements. See also DeMarzo and Fishman (2003), Inderst and Mueller (2003) and Biais et al. (2007), where entrepreneurs can divert profit flows. Bernanke et al. (1996) and Holmstrom and Tirole (1998,2011) rationalize limited pledgeability from moral hazard problems.⁴ A key difference is that pledgeability here is endogenous, and interacts with the loan contract, generating multiplier effects. Moreover, we provide alternative microfoundations for pledgeability as arising from commitment issues along the lines of Kehoe and Levine (1993), Alvarez and Jermann (2000) and Gu et al. (2013b).

Bolton and Freixas (2006) also provide a setting for analyzing monetary policy and corporate finance, without modeling money explicitly— they simply take the interest rate on T-bills as a policy instrument. We generate a richer and more realistic structure of interest rates, including rates on illiquid bonds, liquid bonds, corporate loans and inter-bank loans. Like Williamson (2012), Rocheteau and Rodriguez (2014), and Rocheteau et al. (2015), we study monetary policy, including OMOs, but we propose a novel theory of the determination of the lending rate and the dual role of banks in issuing liabilities and providing loans to firms. Moreover, we formalize the interbank market, access to which is

Lotz and Zhang (2015).

⁴See also Aghion and Bolton (1992) and Hart and Moore (1994) in the context of corporate finance, or Rocheteau (2011) and Li et al. (2012) in monetary theory. Pledgeability is the focus of several more or less applied New Monetarist models, including Ferraris and Watanabe (2008), Lagos (2010), Williamson (2012,2015), Nosal and Rocheteau (2013), Venkateswaran and Wright (2013), Rocheteau and Rodriguez-Lopez (2014) and He et al. (2015).

restricted to banks. This is related to Alvarez et al. (2001,2002) and Khan and Thomas (2015), e.g., where only some agents have access to asset markets. While we also feature market segmentation, our description of credit markets with search and bargaining is very different, and generates new insights into the effects of OMOs on loan sizes and rates.

There is a large macro literature on credit frictions and monetary policy– e.g., see surveys by Bernanke and Gertler (1995) and Bernanke et al. (1999). They emphasize the effect of policy on borrowers’ balances sheets and on the supply of loans, and the impact of policy on the cost of borrowing amplified through the so-called financial accelerator (see Bianchi and Bigio 2014 for a recent version of such a model). We deliver similar results, although the transmission is different, and works through the role of money in the determination of loan contracts. Monetary policy here also affects borrowers’ balance sheets– noticeably, their precautionary holdings of liquid assets– and the availability of credit through a channel tightly linked to the OTC structure of credit markets, with endogenous search frictions and decentralized negotiations of terms.

1.3 Empirical Support

On the importance of corporate liquidity management, in general, see Campello (2015). To mention a few key aspects, firms’ cash balances here are explained by idiosyncratic opportunities and limited pledgeability, consistent with the evidence. Sánchez and Yurdagul (2013), e.g., document that firms in 2011 held \$1.6 trillion in money, defined broadly as short-term investments easily transferable into cash. A main reason for this is a precautionary motive, given uncertainty and credit constraints.⁵ Similarly, Bates et al. (2009) and Opler et al. (1999) link firms’ money demand to idiosyncratic risk, R&D and growth opportunities.⁶

Our firms use both money and credit, consistence with ample evidence discussed by

⁵Another motive is linked to the taxation of repatriated funds, which we do not capture here. They provide support for the precautionary motive by comparing cash holdings across firms of different sizes, with the presumption that small firms are more likely to face credit constraints, especially those with more R&D activity. Relatedly, Mulligan (1997) argues that large firms hold less cash as a percentage of sales because of scale economies in liquidity demand.

⁶Bates et al. (2009) argue R&D investment is subject to tighter constraints because of lower asset tangibility. Mulligan (1997) argues large firms hold less cash as a percentage of sales because of scale economies in liquidity demand.

Mach and Wolken (2006). In 2003, e.g., they report that around 95% of firms had checking accounts, while 22.1% had savings accounts. Also, 60.4% of all firms had a line of credit or some other form of relatively easily accessible liquidity. Small businesses also use credit cards (47% personal and 48% business cards), consistent with versions of our model. Our model can also have both bank credit and trade credit. Trade credit was used by 60% of small businesses in 2003, and 40% of all firms use both bank and trade credit (SBA, 2010). Consistent with evidence from Petersen and Rajan (1997), our firms use trade credit more when credit from financial institutions tightens.

A key feature is the two margins for bank credit: an intensive margin, capturing loan size; and an extensive margin, capturing the frequency at which firms obtain credit. This is consistent with the Joint Small Business Credit Survey Report (2014) from the Federal Reserve Banks of New York, Atlanta, Cleveland and Philadelphia. Among the participants in the survey who applied for a loan, 33% received the amount they asked for, 21% received less, and 44% were denied. Credit is denied if firms have no relationship with a lender (14%) or have low credit scores (45%). Also, in support of our formalization of a frictional credit market, Dell’Ariccia and Garibaldi (2005) document sizable gross credit flows for the U.S. banking system between 1979 and 1999, using a similar methodology as the one commonly used for job flows.

We think it is fair to say that actual credit markets are characterized by price dispersion and bargaining power by lenders, which are characteristics of markets with bilateral relationships and search frictions. For instance, Mora (2014, p.102) documents a considerable dispersion in loan rates across banks and argues this is explained by lender pricing power. In the mortgage market at the end of 2012, the 5th to 95th percentile range for mortgage rates was 3.17% to 4.92%, respectively. Several sources cited in Silveira and Wright (2015) argue that private equity markets are also well described as frictional markets. Generally, informational and limited commitment frictions that impinge on credit market are easier to understand in the context explicit meetings between lenders and borrowers.

The loan rate in our model is related to the notion of net interest margin, which is a measure of the difference between the interest income generated by banks and the amount

of interest paid out to their lenders (e.g., depositors), relative to their assets. Saunders and Schumacher (2000) document interest rate margins vary widely across countries— in 1995, 4.264% for the U.S. and 1.731% for Switzerland. Here these differences can be explained by different market structures, with banks having more bargaining power in some economies than others, or different reserve or liquidity requirements.

There is much empirical work quantifying the relative importance of the money and lending channels.⁷ Romer and Romer (1990), Ramey (1993), Bernanke and Gertler (1995) and Ashcraft (2006) provide examples, but the evidence is largely inconclusive. Kashyap et al. (1993) find evidence that tighter monetary policy leads to a shift in firms' mix of external and internal financing, as is the case here. Bernanke and Gertler (1995) argue there is little evidence the cost of capital matters for investment, as is related to the absence of correlation between loan rates and loan sizes in our benchmark model. This is due to the fact that in OTC markets loan rates are determined after matches are formed and, hence have little allocative role.

2 Environment

Time is denoted by $t \in \mathbb{N}_0$. Each period is divided into two stages. In the first stage, there is a Walrasian market for capital goods (productive inputs) and an OTC market for banking services (provision of loans and means of payment) with search and bargaining. In the second stage, there is a frictionless centralized market where agents settle debts and trade a final good and assets. As in the New Monetarist literature, we label the first stage DM (decentralized market) and the second stage CM (centralized market). The capital good k is storable across stages but not across periods. A numéraire consumption good c is produced and traded in the CM. Good c is not storable.

There are three types of agents indexed by $j \in \{e, s, b\}$. Type e represents *entrepreneurs* that need capital k ; type s represents *suppliers* that can produce k ; and type b represents *banks* whose role is to finance the acquisition of capital by entrepreneurs as

⁷Here monetary policy affects investment through the cost of holding cash, broadly in accordance with the so-called money view. It also has an impact on lending through by affecting incentives to hold precautionary balances that can be used for down payments, and by affecting banks' portfolios and their cost of complying with regulations, broadly in line with the credit view. See Bernanke and Blinder (1988).

explained below. The population of entrepreneurs is normalized to one. Given CRS for the production of capital goods (see below), the population size of suppliers is irrelevant. The population size of banks will be captured by the matching probability between entrepreneurs and banks in the DM. All agents have linear preferences, $U(c, h) = c - h$, where c is the consumption of the numéraire and h is hours of work. They discount across periods according to $\beta = 1/(1 + \rho)$, $\rho > 0$.⁸

Entrepreneurs have two technologies to produce numéraire. They can transform k into $f(k)$ units of c , where $f(0) = 0$, $f'(0) = \infty$, $f'(\infty) = 0$ and $f'(k) > 0 > f''(k) \forall k > 0$. Here k is capital brought into the CM, having been acquired by the entrepreneur in the previous DM. Entrepreneurs can also produce c using their own CM labor according to a linear technology, $c = h$. Capital k is produced by suppliers in the DM with a linear technology, $k = h$. Banks cannot produce c nor k .

In the DM, each entrepreneur receives an investment opportunity with probability λ , in which case they can operate the technology f .⁹ There is an OTC banking sector where each entrepreneur meets a bank at random with probability α . Given an independence assumption, the probability an entrepreneur has an investment opportunity and is matched with a bank is $\alpha\lambda$. With probability $\lambda(1 - \alpha)$, an entrepreneur has an investment opportunity but no access to a bank. To summarize, entrepreneurs face two types of idiosyncratic uncertainty: one related to the timing of investment opportunities, as in Kiyotaki-Moore-type (1997, 2005) models, and one related to access to banks, as in Wasmer and Weil (2004).

We now turn to the enforcement technology in the CM for debts incurred by entrepreneurs in the DM. Consider an entrepreneur with k units of capital goods and liabilities $\ell_b \geq 0$ and $\ell_s \geq 0$ toward banks and suppliers, respectively. Repayment of these liabilities

⁸Without changing the key results we could adopt quasi-linear preferences of the form $U(c) - h$ with $U'' < 0$, or CRS utility as in Wong (2015), or any utility function as long as we impose indivisible labor, as in Rocheteau et al. (2008). We also know how to depart from these restrictions to allow for ex-post heterogeneity and a distribution of asset holdings as in the theoretical analysis of Rocheteau et al. (2015) or the numerical analysis of Molico (2006).

⁹An equivalent interpretation, consistent with the literature on bilateral credit, is that entrepreneurs meet suppliers at random and suppliers have no bargaining power. See Section 7.2 for details.

can be enforced if:

$$\begin{aligned}\ell_s &\leq \chi_s f(k) \\ \ell_b + \ell_s &\leq \chi_b f(k),\end{aligned}$$

where $0 \leq \chi_s \leq \chi_b \leq 1$. Intuitively, entrepreneurs can renege on any promised payment in the next CM. In general, suppliers and banks have some recourse after the entrepreneur defaults on some obligation, which involves seizure of a fraction χ_j of CM output $f(k)$, while the entrepreneur walks away with the rest. Suppliers can recover up to a fraction χ_s while banks can secure a larger fraction, χ_b , net of the repayment to sellers.¹⁰ As a benchmark, χ_j is taken as a primitive, but can be endogenized (see the Appendix) using limited commitment and monitoring. As is standard, only entrepreneur's capital income, and not labor, is pledgeable. For much additional discussion of these kinds of constraints, see the references in fn. 4.

Limited enforcement can generate a need for liquid assets. We consider two types of liquid assets: outside fiat money and banks' short-term liabilities. Fiat money is storable, and it evolves over time according to $A_{m,t+1} = (1 + \pi) A_{m,t}$. Here π is the rate of monetary expansion, or contraction if $\pi < 0$, implemented by lump sum transfers (or taxes) to entrepreneurs in the CM. The price of money in terms of numéraire is $q_{m,t}$. In stationary equilibrium, $q_{m,t} = (1 + \pi)q_{m,t+1}$ so π is also the rate of inflation (or deflation). We assume $\pi > \beta - 1$.

Banks issue intra-period liabilities in the DM, called notes, and can commit to redeem them in the following CM. Notes can be authenticated at no cost in the period issued but can be counterfeited costlessly in subsequent periods. Hence, banks' notes cannot circulate across periods since they would not be accepted.¹¹ There is also a fixed supply,

¹⁰Implicitly here, the debt toward suppliers has higher seniority than the debt toward banks, but as will be clear later, our results are robust to alternative specifications. Moreover, the assumption $\chi_b \geq \chi_s$ means banks have a comparative advantage in enforcing debt repayment. In Gu et al. (2013), this kind of banking is an endogenous arrangement that arises due to explicit commitment and monitoring frictions. See also Donaldson et al. (2015) and Huang (2015).

¹¹For an explicit formalization of the counterfeiting interpretation, see Nosal and Wallace (2007), Lester et al. (2012), and Li et al. (2012). This assumption is made to simplify the analysis, but it would be an interesting extension to allow banks' liabilities to circulate across CMs since they could acquire a liquidity premium that would affect the terms of the bank loans.

A_g , of one-period government bonds that promise one unit of numéraire each to its bearer in the next CM. Government bonds are non-pledgeable assets that cannot be used as media of exchange by entrepreneurs.¹² The price of a newly-issued bond in the CM is q_g . The real rate of return of government bonds is $r_g = 1/q_g - 1$, and the nominal interest rate is $i_g = (1 + \pi)/q_g - 1$.

Banks can trade money and bonds in a competitive interbank market that opens in the DM. Asset purchases in the interbank market can be financed with intra-period credit as banks can commit to repay their debt to other banks. This description is in accordance with the functioning of the Federal Funds Market where overnight loans are unsecured. We let $\hat{q}_g$ denote the price of bonds and $\hat{q}_m$ the price of money in the interbank market, both in terms of numéraire. The interbank market only plays a role when we introduce regulatory requirements in Section 6.

3 Preliminaries

First, we derive some general properties of agents' value functions. Consider an entrepreneur at the beginning of the CM with k units of capital goods purchased in the previous DM and financial wealth ω denominated in units of numéraire. Financial wealth is composed of real balances, a_m , and government bonds, a_g , net of debt obligations. The entrepreneur's lifetime expected utility solves:

$$\begin{aligned} W^e(k, \omega) &= \max_{c, h, \hat{a}_m, \hat{a}_g} \{c - h + \beta V^e(\hat{a}_m, \hat{a}_g)\} \\ \text{s.t. } c &= f(k) + h + \omega + T - (1 + \pi) \hat{a}_m - q_g \hat{a}_g, \end{aligned}$$

where T corresponds to CM transfers minus taxes and $V^e(\hat{a}_m, \hat{a}_g)$ is the entrepreneur's continuation value in the DM with $\hat{a}_m$ real balances and $\hat{a}_g$ bonds (expressed in terms of the numéraire). The budget constraint requires the change in financial wealth, $(1 + \pi) \hat{a}_m + q_g \hat{a}_g - \omega$, is paid with retained earnings, $f(k) + h + T - c$. Substituting $c - h$ into W^e

¹²Rocheteau et al. (2015) study OMOs in a New Monetarist model with either short-term real bonds, long-term real bonds, and nominal bonds. They also allow for partially-pledgeable bonds. They show the outcome of OMOs is robust to these different characteristics of bonds.

yields

$$W^e(k, \omega) = f(k) + \omega + T + \max_{\hat{a}_m, \hat{a}_g \geq 0} \{-(1 + \pi) \hat{a}_m - q_g \hat{a}_g + \beta V^e(\hat{a}_m, \hat{a}_g)\}. \quad (1)$$

So W^e is linear in total wealth, $f(k) + \omega + T$, and the DM portfolio, $(\hat{a}_m, \hat{a}_g)$, is independent of (k, ω) . By a similar reasoning, the CM lifetime expected utility of a supplier or bank, $j \in \{b, s\}$, with wealth ω is

$$W^j(\omega) = \omega + \max_{\hat{a}_m, \hat{a}_g \geq 0} \{-(1 + \pi) \hat{a}_m - q_g \hat{a}_g + \beta V^j(\hat{a}_m, \hat{a}_g)\}. \quad (2)$$

As before, W^j is linear in ω and $(\hat{a}_m, \hat{a}_g)$ is independent of ω .

Consider next the problem of suppliers at the beginning of the DM:

$$V^s(\hat{a}_m, \hat{a}_g) = \max_{k \geq 0} \{-k + W^s(\hat{a}_m + \hat{a}_g + q_k k)\}, \quad (3)$$

where q_k is the DM price of capital goods expressed in numéraire. According to (3), a supplier produces k at a linear cost in exchange for a payment $q_k k$. Using the linearity of W^s , the supplier's problem reduces to $\max_k \{-k + q_k k\}$. If the market for capital goods is active, $q_k = 1$ and $V^s(\hat{a}_m, \hat{a}_g) = W^s(\hat{a}_m + \hat{a}_g)$. From (2), his portfolio choice solves:

$$\max_{\hat{a}_m, \hat{a}_g \geq 0} \{-(1 + \pi) \hat{a}_m - q_g \hat{a}_g + \beta(\hat{a}_m + \hat{a}_g)\}.$$

Provided $\pi > \beta - 1$, $\hat{a}_m = 0$. Suppliers hold no real balances since they have no liquidity needs. Similarly, suppliers hold bonds only if $q_g = \beta$.

An entrepreneur's lifetime expected utility at the beginning of the DM is:

$$V^e(\hat{a}_m, \hat{a}_g) = \mathbb{E}[W^e(k, \hat{a}_m + \hat{a}_g - \psi - \phi)]. \quad (4)$$

The entrepreneur purchases k at total cost $\psi = q_k k$ and compensates the bank for its intermediation services with a fee, ϕ . The total payment, $\psi + \phi$, is subtracted from the entrepreneur's financial wealth in the CM. Notice (k, ϕ) is a random variable that depends on whether the entrepreneur receives an investment opportunity (if not, $k = \psi = \phi = 0$) and whether he is matched with a bank (if not, $\phi = 0$). Combining (1) and (4) and using the linearity of W^e , the entrepreneur's choice of real balances is

$$\max_{\hat{a}_m \geq 0} \{-i \hat{a}_m + \mathbb{E}[f(k) - k - \phi]\}, \quad (5)$$

where $i \equiv (1 + \pi)(1 + \rho) - 1$ and (k, ϕ) is a function of $\hat{a}_m$. So the entrepreneur maximizes his expected surplus from an investment opportunity net of the cost of holding real balances. By assumption, government bonds are not pledgeable and hence (k, ϕ) is independent of $\hat{a}_g$. As a result, entrepreneurs hold bonds only if $q_g = \beta$.

Finally, the lifetime expected utility of a bank is:

$$V^b(\hat{a}_m, \hat{a}_g) = \max_{a_m, a_g \geq 0} \mathbb{E} [W^b(\omega)] \quad (6)$$

$$\text{s.t. } \omega = a_m + a_g - \frac{\hat{q}_m}{q_m}(a_m - \hat{a}_m) - \hat{q}_g(a_g - \hat{a}_g) + \Pi, \quad (7)$$

where Π represents net profits from extending a loan in the DM. Without regulatory requirements, $\Pi = \phi$. According to (6), the bank that enters the DM with a portfolio $(\hat{a}_m, \hat{a}_g)$ chooses (a_m, a_g) , which is its portfolio in excess of regulatory requirements, and it promises to repay $\hat{q}_m(a_m - \hat{a}_m)/q_m$ and $\hat{q}_g(a_g - \hat{a}_g)$ in the next CM, where $\hat{q}_m/q_m$ and $\hat{q}_g$ are the relative prices of money and bonds in the interbank market. Accordingly, $-q_g + \beta\hat{q}_g \leq 0$, with an equality if $\hat{a}_g > 0$. Similarly, $-(1 + \pi)q_m + \beta\hat{q}_m \leq 0$, with an equality if $\hat{a}_m > 0$. Moreover, $-\hat{q}_g + 1 \leq 0$, with an equality if $a_g > 0$, and $-\hat{q}_m + q_m \leq 0$, with an equality if $a_m > 0$.

It is easy to check that $a_m > 0$ implies $\hat{q}_m = q_m$ and hence $\hat{a}_m = 0$, which is inconsistent with market clearing in the interbank market. Hence, banks do not hold money in excess of regulatory requirements, $a_m^b = 0$. The cost of holding bonds in the DM, denoted τ_g , is the difference between the price of bonds in the DM and their price in the subsequent CM, $\tau_g \equiv \hat{q}_g - 1$. Provided the interbank market is active, $\hat{a}_g > 0$,

$$\tau_g = \frac{i - i_g}{1 + i_g}.$$

The cost of holding government bonds is approximately equal to the spread between the rate of return of an illiquid bond, i , and the rate of return of a government bond, i_g . Without regulatory requirements, $\hat{q}_g = 1$ and $q_g = \beta$. In that case, $i_g = (1 + \pi)(1 + \rho) - 1 = i$ and $\tau_g = 0$. The cost of holding a unit of real balances from the interbank market to the next CM is $\tau_m \equiv (\hat{q}_{m,t} - q_{m,t})/q_{m,t}$. If $\hat{a}_m > 0$, then

$$\tau_m = (1 + \pi)(1 + \rho) - 1 = i.$$

The cost of holding money is the nominal interest rate on an illiquid bond.

4 External finance

In this section, we study outcomes where trades occur with external finance only, which consists either of bilateral credit between the entrepreneur and supplier or intermediated credit where the bank issues short-term debt to be used by the entrepreneur to pay for k . For now we consider nonmonetary equilibrium, where fiat currency is not valued, $q_m = 0$.

4.1 Trade credit

To illustrate the theory as simply as possible, consider first an economy without banking. This means the entrepreneur must rely on trade credit extended directly by the supplier.¹³ The left panel of Figure 1 depicts such trade credit where the entrepreneur gets k from the supplier in exchange for a promise of ψ in the next CM.

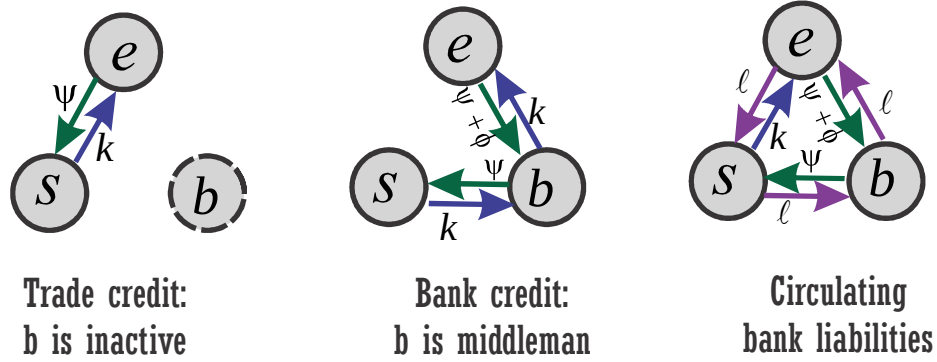


Figure 1: Transactions patterns

The payment to the supplier is subject to a liquidity constraint, $\psi = k \leq \chi_s f(k)$, where the entrepreneur cannot repay more than a fraction χ_s of his output in the CM. Hence, the entrepreneur with ω^e financial wealth solves:

$$\max_{k, \psi} W^e(k, \omega^e - \psi) \quad \text{s.t.} \quad \psi = k \leq \chi_s f(k). \quad (8)$$

Using the linearity of W^e , (8) simplifies to:

$$\Delta(\chi_s) \equiv \max_{k \geq 0} \{f(k) - k\} \quad \text{s.t.} \quad k \leq \chi_s f(k). \quad (9)$$

¹³In practice, trade credit refers to short-term loans extended by suppliers to customers purchasing their products. According to Rajan and Zingales (1991), trade credit to customers represented 17.8% of total assets for U.S. firms. Similarly, Kohler et al. (2000) find 55% of total short-term credit received by U.K. firms took the form of trade credit.

There are two cases. If on the one hand $k \leq \chi_s f(k)$ is slack, then it is easy to see that $\psi = k = k^*$ where $f'(k^*) = 1$. This is the first-best outcome, and it obtains when $\chi_s \geq \chi_s^* = k^*/f(k^*)$. If on the other hand $\psi \leq \chi_s f(k)$ is binding then $\psi = k$ where k is the largest non-negative solution to $\chi_s f(k) = k$. This is the second-best outcome, and it obtains if $\chi_s < \chi_s^*$. The solution is continuous and increasing in χ_s with $k(0) = 0$ and $k(\chi_s^*) = k^*$.

4.2 Bank credit

Now suppose $\chi_s = 0$, so the supplier has no ability to enforce payment from the entrepreneur, and let us reintroduce banks. Suppose the entrepreneur receives an investment opportunity and meets a bank. There are gains from trade since the bank can credibly promise a payment to the supplier, and enforce payments from the entrepreneur up to the limit implied by χ_b . We refer to this as bank credit, and let ϕ be a payment from the entrepreneur to the bank for intermediation services.

As illustrated by the middle and right panels of Figure 1, there are several ways to achieve the same allocation. In the middle panel, the bank gets k from the supplier in exchange for a promise of ψ , then give k to the entrepreneur in exchange for a promise of $\psi + \phi$, with both promises due in the next CM. An alternative implementation is shown in the right panel, where the bank extends a loan to the entrepreneur by crediting a deposit account in his name for the amount ℓ . The deposit claims are liabilities of the bank that can be transferred from the entrepreneur to the supplier (e.g., by writing a check) in exchange for k . In the next CM, the supplier redeems the claim on the bank for ψ , and the entrepreneur settles his debt by returning $\psi + \phi$ to the bank. The arrangement in the right panel monetizes the transaction between the entrepreneur and supplier using deposits as inside money, consistent with the notion that a salient feature of banks is that their liabilities facilitate third-party transactions.¹⁴

The terms of the loan contract is a pair, (ψ, ϕ) , where $\psi = q_k k$, determined through bilateral negotiation. If an agreement is reached, the entrepreneur's payoff is $W^e(k, \omega^e -$

¹⁴For some issues, the difference between this and the middle panel may not be crucial, but there are scenarios where it matters, e.g., if physical transfers of k are spatially or temporally separated. Then having a transferable asset can be essential. For additional discussion, see Gu et al. (2013).

$\psi - \phi$) and the bank's payoff is $W^b(\omega^b + \phi)$. Hence, the surpluses from an agreement are:

$$\begin{aligned} S^e &\equiv W^e(k, \omega^e - \psi - \phi) - W^e(0, \omega^e) = f(k) - \psi - \phi \\ S^b &\equiv W^b(\omega^b + \phi) - W^b(\omega^b) = \phi. \end{aligned}$$

The total surplus is $S^e + S^b = f(k) - k$. In Figure 2, we represent the Pareto frontier in both the utility space (right panel) and in terms of the two dimensions of the loan contract, $\psi = k$ and ϕ (left panel). In the Appendix, we show the maximum surplus the bank can obtain is $\chi_b f(\hat{k}) - \hat{k} \leq f(\hat{k}) - \hat{k}$, where $\hat{k}$ solves $\chi_b f'(\hat{k}) = 1$. As a result, the bargaining set is not convex for all $\chi_b < 1$. We will see this non-convexity is inconsequential under the Nash solution (but could matter under alternative bargaining solutions). Moreover, from the left panel, k cannot be less than $\hat{k}$ since otherwise one could raise both the intermediation fee and the entrepreneur's surplus, thereby generating a Pareto improvement.

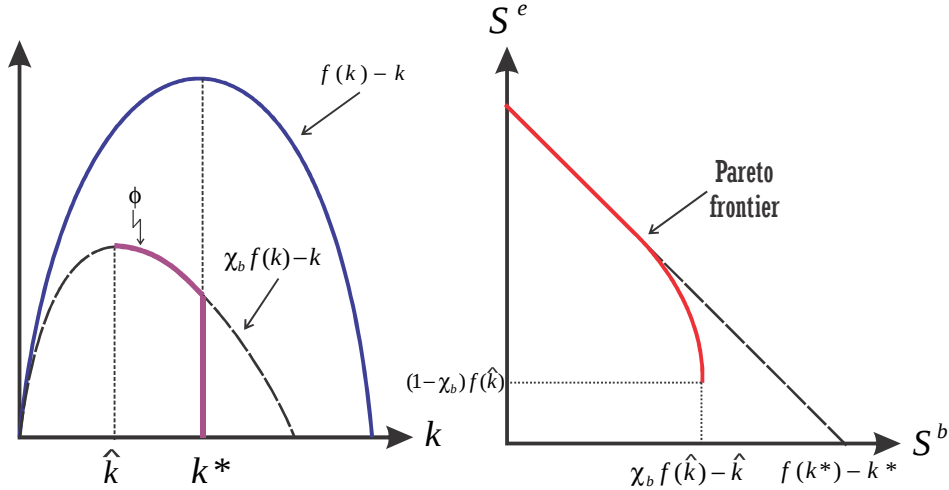


Figure 2: Negotiation of a bank loan: Pareto frontier

The solution to the bargaining problem is given by the generalized Nash bargaining solution where $\theta \in (0, 1)$ is the bank's bargaining power.¹⁵ The outcome solves

$$(k, \phi) \in \arg \max [f(k) - k - \phi]^{1-\theta} \phi^\theta \text{ s.t. } k + \phi \leq \chi_b f(k). \quad (10)$$

If the liquidity constraint does not bind, then $k = k^*$ and

$$\phi = \theta [f(k^*) - k^*]. \quad (11)$$

¹⁵In the Appendix, we provide strategic foundations by adopting an alternating offer bargaining game.

According to (11), ϕ is independent of χ_b but increases with θ and the gains from trade, $f(k^*) - k^*$. In addition, the lending rate is

$$r = \frac{\phi}{\psi} = \frac{\theta [f(k^*) - k^*]}{k^*}. \quad (12)$$

From (12), the lending rate is proportional to capital's average return, $f(k^*)/k^* - 1$. The threshold for χ_b below which the liquidity constraint binds is:

$$\chi_b^* \equiv \frac{(1 - \theta)k^* + \theta f(k^*)}{f(k^*)}.$$

If $\chi_b < \chi_b^*$,

$$\phi = \chi_b f(k) - k. \quad (13)$$

Substituting ϕ from (13) into (10) and taking the FOC, k solves

$$\frac{k}{f(k)} = \frac{\chi_b f'(k) - \theta}{(1 - \theta) f'(k)}. \quad (14)$$

Provided that $\chi_b < \chi_b^*$, there is a unique solution $k \in (\hat{k}, k^*)$ to (14).¹⁶ This solution is continuous and increasing with χ_b , $k(0) = 0$ and $k(\chi_b^*) = k^*$. Also, $\partial k / \partial \theta < 0$, $\partial \phi / \partial \theta > 0$ and $\partial \phi / \partial \chi_b > 0$. Intuitively, a bank with more bargaining power can ask for a larger interest payment, which reduces investment and pledgeable output, thereby tightening the liquidity constraint.

The lending rate is

$$r = \frac{\chi_b f(k)}{k} - 1 = \frac{\theta [1 - \chi_b f'(k)]}{\chi_b f'(k) - \theta}. \quad (15)$$

Since k decreases with θ , and the middle term in (15) is decreasing in k , $\partial r / \partial \theta > 0$. Finally, $\partial r / \partial \chi_b$ is ambiguous, as both k and ϕ increase with pledgeability. If $f(k) = zk^\gamma$, where $z > 0$ and $\gamma \in (0, 1)$, then these two effects cancel out as the lending rate, $r = \theta(1 - \gamma)/\gamma$, is independent of χ_b . Hence, the model makes several predictions about how loan size and interest rates depend on parameters. Suppose $1 - \theta$ varies across entrepreneurs. In this case, the theory predicts a negative correlation between k and

¹⁶The left side of (14) is increasing in k from zero to ∞ (where the limits are obtained by applying L'Hopital's rule). The right side of (14) is decreasing in k for all k such that the numerator is positive, and the right side evaluated at k^* , $(\chi_b - \theta)/(1 - \theta)$, is smaller than the left side provided that $\chi_b < \chi_b^*$. Moreover, at $k = \hat{k}$ the right side of (14) is equal to $1/f'(\hat{k}) = 1/\chi_b$ which is greater than the left side, $\hat{k}/f(\hat{k})$.

r . Alternatively, the source of variation across loans could come from χ_b due to e.g. tangibility of different investment projects. Then there is no correlation between r and k since the lending rate is independent of pledgeability.

4.3 Coexistence of bank and trade credit

We now relax the assumption $\chi_b\chi_s = 0$. Instead, we assume $0 < \chi_s < \chi_b$. In the absence of banks, the entrepreneur can pledge a positive fraction, χ_s , to the supplier. If the entrepreneur is in contact with a bank, then the entrepreneur can pledge a larger fraction. If the entrepreneur has debt obligations to both the supplier and bank, his total obligations cannot be greater than $\chi_b f(k)$ and his obligations to the supplier cannot be greater than $\chi_s f(k)$.

In order for bank credit to play an essential role, we need $\chi_s < \chi_s^* = f(k^*)/k^*$, i.e., trade credit alone does not implement the first best. Hence, if the entrepreneur meets a bank, there are gains from trade. A measure $\lambda(1 - \alpha)$ of investment projects are financed with trade credit while the remaining measure, $\lambda\alpha$, is financed with bank credit. So, there is coexistence between the two forms of credit, and we will see that the terms of the loan contract with the bank depend on χ_s .

Consider the negotiation between the entrepreneur and the bank. In case of disagreement, the entrepreneur can obtain a direct loan from the supplier. Hence $\Delta(\chi_s)$ is the disagreement point for the entrepreneur. An equivalent interpretation is for the entrepreneur to take a direct loan from the supplier and to supplement the loan with a second one from the bank. The terms of the loan contract, (k, ϕ) , are given by:

$$\max_{k, \phi} [f(k) - k - \phi - \Delta(\chi_s)]^{1-\theta} \phi^\theta \quad \text{s.t.} \quad k + \phi \leq \chi_b f(k).$$

The solution is $k = k^*$ and $\phi = \theta [f(k^*) - k^* - \Delta(\chi_s)]$ if

$$\chi_b \geq \chi_b^*(\chi_s) \equiv \frac{(1 - \theta)k^* + \theta [f(k^*) - \Delta(\chi_s)]}{f(k^*)}.$$

The lending rate is $r = \phi/k^*$, or

$$r = \frac{\theta [f(k^*) - k^* - \Delta(\chi_s)]}{k^*}.$$

The lending rate decreases with $\Delta(\chi_s)$, i.e., $\partial r / \partial \chi_s < 0$. Moreover, as χ_s approaches χ_s^* , r approaches zero. Intuitively, the outside option provided by trade credit allows the entrepreneur to negotiate better terms for his loan with a bank, which also reduces the threshold for χ_b above which k^* can be financed.

If $\chi_b < \chi_b^*(\chi_s)$, the liquidity constraint binds and (k, ϕ) solves:

$$\frac{(1 - \chi_b)f'(k)}{(1 - \chi_b)f(k) - \Delta(\chi_s)} = \frac{\theta}{1 - \theta} \frac{1 - \chi_b f'(k)}{\chi_b f(k) - k}, \quad (16)$$

$$\phi = \chi_b f(k) - k. \quad (17)$$

There is a unique k solution to (16) and it increases with χ_s . Indeed, if the entrepreneur's output becomes more pledgeable in trade credit, ϕ falls, which allows the entrepreneur to finance a larger investment. If χ_s varies across investment opportunities, then the model predicts a negative correlation between k and r . Moreover, as χ_s approaches χ_b , ϕ tends to zero and k approaches the positive solution to $\chi_b f(k) = k$.

5 Internal and external finance

So far, the only way for entrepreneurs to finance investment opportunities is through external finance by obtaining a loan from a bank or directly from a supplier. Now we introduce internal finance by allowing entrepreneurs to retain earnings and accumulate real balances to purchase k on the spot or to use as a down payment on a loan.¹⁷ To simplify the exposition, we set $\chi_s = 0$.

Consider an entrepreneur in the DM with an investment opportunity but no access to a bank. Since $\chi_s = 0$ feasibility requires $k \leq a_m^e$ and the surplus from investing is

$$\Delta^m(a_m^e) = f(k^m) - k^m \quad \text{where } k^m = \min\{a_m^e, k^*\}. \quad (18)$$

The function $\Delta^m(a_m^e)$ is increasing and strictly concave for all $a_m^e < k^*$ with $\Delta^{m'}(a_m^e) = f'(k^m) - 1 > 0$.

¹⁷Internal finance refers to a firm's use of its own profits as way to fund new investment. Our model captures the salient features of internal finance as described by Hubbard et al. (1995) and Bernanke et al. (1996); namely, internal finance provides an immediate form of funding, does not have interest payments, and sidesteps the need to interact with a third party.

Suppose next that the entrepreneur is in contact with a bank. The terms of the contract specify: (i) the investment level, k ; (ii) the down payment, d ; (iii) the bank's fee, ϕ . If the negotiation with the bank is unsuccessful, the entrepreneur purchases k with his real balances and achieves a surplus $\Delta^m(a_m^e)$. Hence, his surplus from a bank loan is $f(k) - k - \phi - \Delta^m(a_m^e)$.¹⁸ Accordingly, (k, d, ϕ) solves

$$\max_{k, d, \phi} [f(k) - k - \phi - \Delta^m(a_m^e)]^{1-\theta} \phi^\theta \text{ s.t. } k - d + \phi \leq \chi_b f(k) \text{ and } d \leq a_m^e. \quad (19)$$

Suppose first the liquidity constraint does not bind. The solution to (19) is

$$\begin{aligned} k^c &= k^* \\ \phi^c &= \theta [f(k^*) - k^* - \Delta^m(a_m^e)]. \end{aligned}$$

It follows that $\partial k^c / \partial a_m^e = 0$ and $\partial \phi^c / \partial a_m^e < 0$. By accumulating real balances the entrepreneur can reduce interest payments to the bank thereby increasing his profits. The constraint does not bind if

$$a_m^e + \theta \Delta^m(a_m^e) \geq (1 - \theta)k^* + (\theta - \chi_b)f(k^*). \quad (20)$$

If $\chi_b \geq \chi_b^*$, (20) holds for all $a_m^e \geq 0$. However, if $\chi_b < \chi_b^*$, then financing k^* requires a down payment equal to $d^* > 0$, the value of a_m^e such that (20) holds at equality.

If the liquidity constraint binds, then the solution to (19) is:

$$\frac{a_m^e + \chi_b f(k^c) - k^c}{(1 - \chi_b)f(k^c) - a_m^e - \Delta^m(a_m^e)} = \frac{\theta}{1 - \theta} \frac{1 - \chi_b f'(k^c)}{(1 - \chi_b)f'(k^c)} \quad (21)$$

$$k^c + \phi^c = a_m^e + \chi_b f(k^c). \quad (22)$$

There is a unique $k^c > \hat{k}$ solution to (21) and $\partial k^c / \partial a_m^e > 0$. Hence, $\partial [a_m^e + \chi_b f(k^c)] / \partial a_m^e > 1$, i.e., by accumulating real balances the entrepreneur raises his financing capacity by more than one since pledgeable output increases.

The lending rate, $r \equiv \phi^c / (k^c - a_m^e)$, is

$$r = \begin{cases} \frac{\theta [f(k^*) - k^* - \Delta^m(a_m^e)]}{k^* - a_m^e} & \text{if } a_m^e \in [d^*, k^*] \\ \frac{\chi_b f(k^c)}{k^c - a_m^e} - 1 & \text{if } a_m^e < d^* \end{cases} \quad (23)$$

¹⁸Alternatively, we could interpret $\Delta^m(a_m^e)$ as an outside option. For a discussion of the distinction between disagreement points and outside options, see Osborne and Rubinstein (1990, Section 3.12) and Muthoo (1999, Chapter 5). See also the Appendix for an alternating-offer bargaining games that delivers the same outcome as in (19).

where we assume that whenever $a_m^e \in [d^*, k^*)$, $d = a_m^e$. It is easy to check that $dr/da_m^e < 0$ for all $a_m^e \in [d^*, k^*)$ and $\lim_{a_m^e \nearrow k^*} r = \Delta^{m'}(k^*) = 0$.

From (5), the entrepreneur's choice of real balances solves

$$\max_{a_m^e \geq 0} \{-ia_m^e + \lambda(1 - \alpha)\Delta^m(a_m^e) + \alpha\lambda\Delta^c(a_m^e)\}, \quad (24)$$

where $\Delta^c(a_m^e) \equiv f(k^c) - k^c - \phi^c$ takes the following expressions:

$$\Delta^c(a_m^e) = \begin{cases} (1 - \theta)[f(k^*) - k^*] + \theta\Delta^m(a_m^e) & \text{if } a_m^e \geq d^* \\ (1 - \chi_b)f(k^c) - a_m^e & \text{otherwise.} \end{cases}$$

If $a_m^e \geq k^*$, the entrepreneur can finance k^* without resorting to bank credit. In that case, he appropriates the full gains from trade. If $a_m^e \in [d^*, k^*)$, he can still finance k^* , but now has to resort to bank credit. The bank captures a fraction θ of the increase in the surplus generated by its loan, $f(k^*) - k^* - \Delta^m(a_m^e)$. Finally, if $a_m^e < d^*$, then the liquidity constraint binds and the entrepreneur's surplus is equal to the non-pledgeable output net of his real balances.

A monetary equilibrium with internal and external finance is a list, (k^m, k^c, a_m^e, r) , that solves (18), (19), (23), and (24). Equilibrium has a recursive structure. Equation (24) determines a $a_m^e \in [0, k^*]$.¹⁹ Knowing a_m^e , (18) and (19) determines k^m and k^c . Then, one can compute r from (23).

Consider first $k^c = k^*$. Such equilibria occur if $\chi_b \geq \chi_b^*$ or if i is small enough so that $a_m^e \geq d^*$. The FOC associated with (24) is

$$i = \lambda[1 - \alpha(1 - \theta)][f'(k^m) - 1].$$

Accordingly, $\partial k^m / \partial i < 0$, $\partial k^m / \partial \lambda > 0$, $\partial k^m / \partial \alpha < 0$, and $\partial k^m / \partial \theta > 0$. In words, as bank credit becomes less readily available or more expensive, entrepreneurs reduce their reliance on credit by holding more real balances.

The lending rate becomes

$$r = \frac{\theta\{[f(k^*) - k^*] - [f(k^m) - k^m]\}}{k^* - k^m}. \quad (25)$$

¹⁹For all $i > 0$, the solution to (24) is such that $a_m^e \in [0, k^*]$ since for all $a_m^e \geq k^*$, the entrepreneur's expected surplus is constant and equals $f(k^*) - k^*$ while the cost of holding real balances, ia_m^e , is increasing in a_m^e . Since the objective in (24) is continuous and maximized over a compact set, a solution exists. Even though the objective is not necessarily concave, we can use the argument from Gu and Wright (2015) to show the solution is generically unique.

It is increasing with i . Monetary policy, by controlling the cost of holding liquid assets, affects the real rate at which entrepreneurs can borrow. This transmission mechanism operates even though there is no credit rationing, no reserve requirement, and no sticky prices. When i is close to zero, the rate of pass-through is approximated by

$$\frac{\partial r}{\partial i} \approx \frac{\theta}{2\lambda[1 - \alpha(1 - \theta)]}.$$

Some authors argue that the interest rate pass-through has been significantly weaker since 2008.²⁰ This is consistent with new regulations that reduce banks' market power, tighter lending standards that reduce the acceptance rate of loan applications, and more frequent investment opportunities.

We now turn to $k^c < k^*$. We consider first special cases where banks have either no bargaining power, $\theta = 0$, or all the bargaining power, $\theta = 1$. If $\theta = 0$, k^c solves $a_m^e + \chi_b f(k^c) = k^c$. Hence,

$$\Delta^c(a_m^e) = \frac{f'(k^c) - 1}{1 - \chi_b f'(k^c)}. \quad (26)$$

If the entrepreneur borrows an additional unit of numéraire, he can purchase an additional unit of k , which raises his surplus by $f'(k^c) - 1$. The numerator in (26) is a financing multiplier that says one unit of k raises pledgeable output by $\chi_b f'(k^c)$. As pledgeable output increases so does borrowing, which generates a multiplier effect. From (24), the choice of a_m^e when the liquidity constraint binds is

$$(1 - \alpha) f'(k^m) + \alpha \frac{(1 - \chi_b) f'(k^c)}{1 - \chi_b f'(k^c)} = 1 + \frac{i}{\lambda}. \quad (27)$$

Both terms on the LHS of (27) are decreasing in a_m^e . Hence, there is a unique solution to (27). It can be checked that $\partial a_m^e / \partial i < 0$, $\partial k^m / \partial i < 0$, and $\partial k^c / \partial i < 0$. So our model is consistent with the view that an increase in the nominal interest rate reduces aggregate investment.

Suppose $\theta = 1$. Banks maximize ϕ subject to

$$\begin{aligned} \phi + k &\leq \chi_b f(k) + d, \\ f(k) - k - \phi &\geq \Delta^m(a_m^e) \quad \text{and} \quad d \leq a_m^e. \end{aligned}$$

²⁰See Illes and Lombardi (2013) and Mora (2014) for literature reviews on the transmission of monetary policy to loan rates.

There is a solution, $\underline{a}$, to $\Delta^m(\underline{a}) + \underline{a} = (1 - \chi_b)f(\hat{k})$ such that for all $a_m^e \leq \underline{a}$, the bank offers $k = \hat{k}$, $\phi = \chi_b f(\hat{k}) - \hat{k} + a_m^e$, and $\Delta^c(a_m^e) = (1 - \chi_b)f(\hat{k}) - a_m^e$. The entrepreneur's surplus decreases with a_m^e since the more real balances he brings in a match, the larger the payment the bank can obtain. For i large enough, $a_m^e \leq \underline{a}$ is optimal, in which case $\partial k^c / \partial i = 0$ and $\partial a_m^e / \partial i < 0$. Moreover, $r = \chi_b f(\hat{k}) / (\hat{k} - a_m^e) - 1$ is increasing in a_m^e . So our model generates a negative interest rate pass-through when inflation is high.

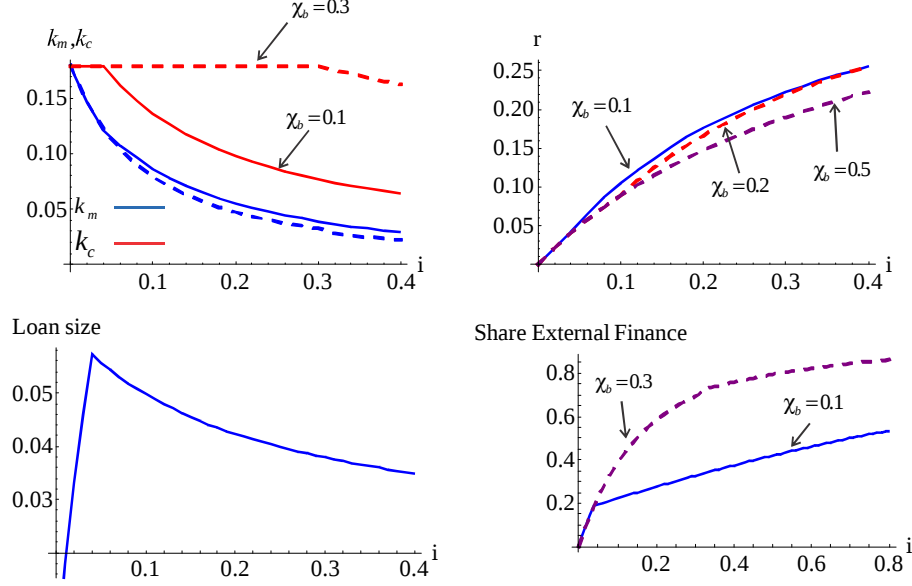


Figure 3: internal and external finance: a numerical example

In Figure 3, we provide an example where $f(k) = k^{0.3}$, $\theta = 0.3$, $\lambda = 0.2$, $\alpha = 0.5$, and $i = 0.05$. The top right panel illustrates the interest-rate pass-through. The pass-through decreases with χ_b since for a given i , r is larger for lower values of χ_b . The top left panel illustrates the transmission mechanism of monetary policy according to which k^m and k^c are decreasing with i . As χ_b increases, k^c rises but k^m falls.

The bottom left panel shows the loan size varies non-monotonically with i . An increase in i has two opposite effects on loan sizes. There is a substitution effect whereby an increase in i raises external finance and loan sizes in order to economize on the cost of holding real balances. There is also a financing multiplier effect whereby a reduction in a_m^e tends to reduce pledgeable output and loan sizes. For low inflation, the substitution effect dominates whereas the financing multiplier effect dominates for high inflation. The

bottom right panel plots the share of external finance, defined as $1 - k_m / [(1 - \alpha)k^m + \alpha k^c]$, as a function of i . At the Friedman rule, $i = 0$, the share of external finance is zero since entrepreneurs can finance their investment opportunities with money only. As the cost of holding money increases, so does the share of external finance because entrepreneurs with access to banks supplement their lower real balances with a loan.

6 Reserve requirements and monetary policy

We now study the mechanism through which monetary policy affects corporate finance and investment when banks are subject to reserve requirements. These requirements take the following form. A fraction $\nu_g \in [0, 1]$ of notes issued by banks must be backed by liquidity defined broadly as government bonds or fiat money. In addition, a fraction $\nu_m \leq \nu_g$ of this requirement must be satisfied with fiat money only.²¹ We interpret ν_m as a strict reserve requirement and ν_g as a broad liquidity requirement. Hence, given a loan size $\ell = k - d$, the bank must hold $\nu_m \ell$ real balances and $(\nu_g - \nu_m)\ell$ broad liquidity at the start of the CM.

The regulatory cost imposed on the bank is $\bar{\tau}\ell$, and its profits are $\Pi = \phi - \bar{\tau}\ell$ where $\bar{\tau} \equiv \tau_m \nu_m + \tau_g (\nu_g - \nu_m)$. Assuming $f'(a_m^e) \geq 1 + \bar{\tau}$, so that there are gains from trade, the terms of the loan contract solve:

$$\max_{k, \phi, d} \{f(k) - k - \phi - \Delta^m(a_m^e)\}^{1-\theta} \{\phi - \bar{\tau}(k - d)\}^\theta \quad (28)$$

$$\text{s.t. } k + \phi \leq d + \chi_b f(k), \quad d \leq \min\{k, a_m^e\}. \quad (29)$$

If the liquidity constraint does not bind,

$$f'(k^c) = 1 + \bar{\tau}, \quad (30)$$

$$\phi^c = (1 - \theta)\bar{\tau}(k^c - a_m^e) + \theta[f(k^c) - k^c - \Delta^m(a_m^e)]. \quad (31)$$

²¹These constraints can capture cash reserve requirements as described by Carlson (2011) and Calomiris et al. (2012) or a liquidity coverage ratio as described by the Basel Committee (2013). Historically, in the free-banking era, banks were required to purchase government bonds to be able to operate. See Goodhart (1995) and Wicker (2000) for a historical account. Our formalization of reserve requirements is similar to Romer (1985) and Freeman (1987) where reserve requirements work as a tax on deposits. See also Bech and Monnet (2015) for a more recent treatment.

There are two novelties. First, $\bar{\tau}$ acts as a proportional tax on investment. If $\bar{\tau} > 0$, then $k^c < k^*$ and $\partial k^c / \partial \bar{\tau} < 0$. Second, $\partial \phi^c / \partial \bar{\tau} > 0$ for all $\theta < 1$. The constraint does not bind if a_m^e is larger than some threshold d^* (which depends on $\bar{\tau}$ and χ_b). If the constraint binds, (k^c, ϕ^c) solves:

$$\frac{(1 + \bar{\tau})(a_m^e - k^c) + \chi_b f(k^c)}{(1 - \chi_b)f(k^c) - a_m^e - \Delta^m(a_m^e)} = \frac{\theta}{1 - \theta} \frac{(1 + \bar{\tau}) - \chi_b f'(k^c)}{(1 - \chi_b)f'(k^c)} \quad (32)$$

$$\phi^c = a_m^e + \chi_b f(k^c) - k^c. \quad (33)$$

It can be checked that $\partial k^c / \partial \bar{\tau} < 0$ and $\partial k^c / \partial a_m^e > 0$.

The supply of bonds in the interbank market is A_g since it is optimal for banks to hold government bonds. The supply of real balances in the interbank market is equal to $\hat{A}_m^b = \hat{a}_m^b$. The demand for bonds and real balances arises from regulatory requirements: a measure $\alpha\lambda$ of banks demand $(\nu_g - \nu_m)\ell$ in broad liquidity and $\nu_m\ell$ in real balances, where $\ell = k^c - a_m^e$. Market clearing implies:

$$\hat{A}_m^b \begin{cases} = \\ = \\ \geq \end{cases} \alpha\lambda\nu_m\ell \text{ and } A_g + \hat{A}_m^b \begin{cases} \geq \\ = \\ = \end{cases} \alpha\lambda\nu_g\ell \text{ if } \tau_g \begin{cases} = 0 < \tau_m \\ \in (0, \tau_m) \\ = \tau_m > 0 \end{cases}. \quad (34)$$

If $\tau_g = 0$, then banks are willing to hold liquidity in excess of the regulatory requirements. If $\tau_g = \tau_m = i$, money and bonds are perfect substitutes to fulfill the broad liquidity requirement. Finally, if $\tau_g \in (0, \tau_m)$, banks hold the exact amounts of real balances and bonds implied by reserve and liquidity requirements.

An equilibrium is now a list, $(k^m, k^c, a_m^e, \hat{a}_m^b, r, i_g)$, that solves (19), (24), (28), (29), and (34). In the following, we describe equilibria under two types of regulatory requirements: strict reserve requirements where $\nu_g = \nu_m$ and broad liquidity requirements where $\nu_g > \nu_m = 0$.

6.1 Strict reserve requirements

From (34), $i_g = i$ since bonds are not eligible to satisfy regulatory requirements. If the the liquidity constraint does not bind, $\partial k^c / \partial i < 0$ and

$$r = \nu_m i + \theta \left[\frac{f(k^c) - k^c - \Delta^m(a_m^e)}{k^c - a_m^e} - \nu_m i \right].$$

The first component of the lending rate is the cost due to the reserve requirement while the second term is the bank's surplus per unit of loan. For small values of i ,

$$r \approx \left\{ \nu_m + \frac{\theta(1 - \lambda\nu_m)}{2\lambda[1 - \alpha(1 - \theta)]} \right\} i.$$

The reserve ratio, ν_m , raises the interest rate pass-through. The responses of allocations and prices to monetary policy are qualitatively similar to the ones obtained without reserve requirement as illustrated by Figure 4 using the same parametrization as in Figure 3. The plain lines correspond to $\nu_m = 10\%$ while the dashed lines are obtained for $\nu_m = 100\%$.

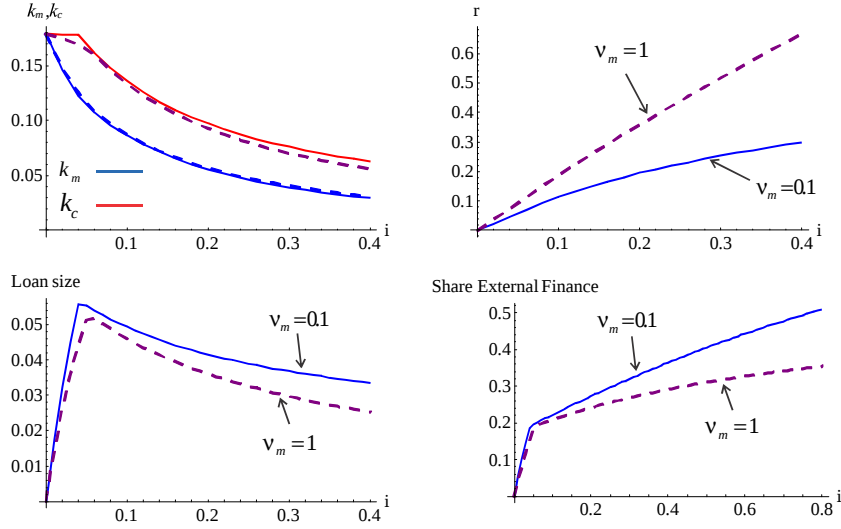


Figure 4: Reserve requirements: a numerical example

We now describe the effects of a one-time, unanticipated OMO in the interbank market that raises the supply of money by μA_m , where $\mu > 0$, while reducing A_g . Since bonds play no regulatory role, only the change in A_m is relevant. We focus on equilibria where the economy returns to its steady state in the following CM where q_m is scaled down by $1 + \mu$. As a result, $a_m^{e'} = a_m^e / (1 + \mu)$, where a prime denotes a variable at the time of the money injection. By money neutrality, $a_m^{e'} + \hat{A}_m^{b'} = a_m^e + \hat{A}_m^b$. It follows that

$$\hat{A}_m^{b'} = \frac{\mu}{1 + \mu} a_m^e + \hat{A}_m^b. \quad (35)$$

The first term on the RHS of (35) corresponds to the increase in banks' real balances financed by the inflation tax on entrepreneurs' real balances. In equilibria with $\tau'_m > 0$,

banks hold no excess reserves, $\hat{A}_m^{b'} = \alpha \lambda \nu_m (k^{c'} - a_m^{e'})$. From (35),

$$k^{c'} - k^c = \frac{(1 - \alpha \lambda \nu_m) \mu}{\alpha \lambda \nu_m (1 + \mu)} a_m^e.$$

Hence, if $\lambda \nu_m < 1$, $\partial k^c / \partial \mu > 0$, $\partial k^m / \partial \mu < 0$, and $\partial \tau_m / \partial \mu < 0$. The OMO reduces the cost of borrowing reserves, which induces banks to offer larger loans, but it also reduces $a_m^{e'}$ and k^m by redistributing liquidity from entrepreneurs to banks. The overall change in investment is:

$$\alpha \lambda (k^{c'} - k^c) + \lambda (1 - \alpha) (a_m^{e'} - a_m^e) = \left(\frac{1 - \lambda \nu_m}{\nu_m} \right) \frac{\mu}{1 + \mu} a_m^e.$$

If $\lambda \nu_m < 1$, the change in aggregate investment is positive. Intuitively, money is more effective at financing investment when held by banks because banks can leverage their liquid assets by issuing their own liabilities and, through the interbank market, they can reallocate liquidity towards those banks with lending opportunities.

Figure 5 describes the effects of an OMO for $f(k) = k^{0.3}$, $\theta = 0.3$, $\chi_b = 0.1$, $\lambda = 0.5$, $\alpha = 0.5$, $\nu_m = 0.5$, and $i = 0.1$. The top left panel shows that a money injection in the interbank market reduces the cost of borrowing reserves. The top right panel shows that OMOs have both real and redistributive effects: k^c increases while k^m decreases with μ . The pass-through to the lending rate is illustrated in the bottom right panel where r decreases with μ as long as $\tau_m > 0$. A liquidity trap occurs when τ_m is driven to zero in which case reserves are abundant and $k^c = k^*$. However, a_m^e and k^m keep falling with μ . Moreover, both r and loan sizes (bottom left panel) increase. So OMOs have real effects in the liquidity trap but those effects are opposite to the ones obtained when $\tau_m > 0$.

Now suppose OMOs occur every period. In order to set τ_m independently from i , OMOs are neutralized in the following CM. If $\beta \hat{q}_{m,t} < q_{m,t-1}$, then $\hat{a}_m^b = 0$ and all reserves are provided by OMOs. Following the same reasoning as above,

$$r \approx (1 - \theta) \nu_m \tau_m + \frac{\theta (\lambda \nu_m \tau_m + i)}{2\lambda [1 - \alpha(1 - \theta)]}.$$

This formulation allows us to disentangle the effects of τ_m and i on r . Notice even when τ_m is driven to zero, r remains positive when $i > 0$.

Finally, consider a policy of paying interest on reserves. For each unit of money a bank holds at the beginning of the DM (CM), it receives i_m^1 (i_m^2) additional units of

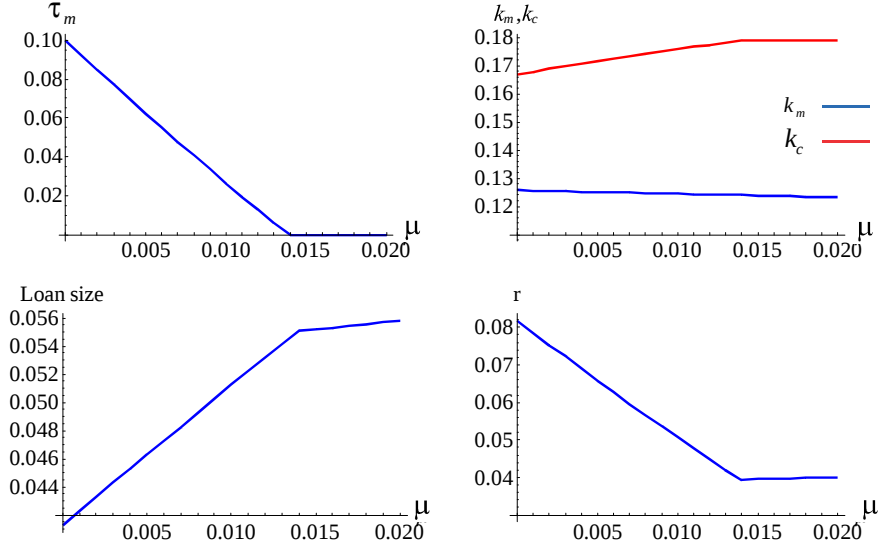


Figure 5: OMO in the interbank market: a numerical example

money. The effects of such payments on A_m are neutralized in the CM through lump-sum taxation. The arbitrage condition between the CM and the following interbank market gives $(1 + \pi)q_m = \beta \hat{q}_m(1 + i_m^1)$. Moreover,

$$\tau_m = \frac{i - i_m}{1 + i_m^1},$$

where $1 + i_m = (1 + i_m^1)(1 + i_m^2)$. So paying interest on reserves promote lending and investment by making it less costly to hold reserves.

6.2 Broad liquidity requirements

Suppose first liquidity constraints do not bind. The FOC for the entrepreneur's real balances is:

$$\frac{i - \alpha \lambda \theta \nu_g \tau_g}{\lambda [1 - \alpha(1 - \theta)]} = \Delta^{m'}(a_m^e). \quad (36)$$

Hence, if $\tau_g > 0$ then $\partial a_m^e / \partial \nu_g > 0$. We distinguish three regimes. Suppose first $\tau_g = 0$. Allocations and prices are identical to the ones without regulatory constraints in Section 5. In such equilibria, changes in A_g are neutral. From (34), this regime occurs if $A_g \geq \bar{A}_g \equiv \alpha \lambda \nu_g (k^* - a_m^e)$.

Next, consider the liquidity-trap regime where $i_g = 0$. From (30) and (36),

$$f'(k^c) = 1 + \nu_g i \quad (37)$$

$$f'(k^m) = 1 + \frac{1 - \alpha \lambda \theta \nu_g}{\lambda [1 - \alpha(1 - \theta)]} i. \quad (38)$$

Hence, $\partial k^c / \partial i < 0$, $\partial k^m / \partial i < 0$ but $\partial k^c / \partial A_g = \partial k^m / \partial A_g = 0$. From (31),

$$r = \theta \nu_g i + \theta \left[\frac{f(k^c) - f(k^m)}{k^c - k^m} - 1 \right]. \quad (39)$$

Differentiating (37)-(39), $\partial r / \partial i > 0$. From (34), the liquidity trap occurs if $A_g \leq \underline{A}_g \equiv \alpha \lambda \nu_g (k^c - k^m)$ where $\underline{A}_g > 0$ if $[1 - \alpha(1 - 2\theta)] \lambda \nu_g < 1$. Moreover, $\underline{A}_g < \bar{A}_g$.

Finally, there is an intermediate regime with $i_g \in (0, i)$. From (30), (34), and (36):

$$k^c - k^m = \frac{A_g}{\alpha \lambda \nu_g} \quad (40)$$

$$f'(k^m) = 1 + \frac{i - \alpha \lambda \theta \nu_g \tau_g}{\lambda [1 - \alpha(1 - \theta)]} \quad (41)$$

$$f'(k^c) = 1 + \nu_g \tau_g \quad (42)$$

From (41)-(42), $\partial k^m / \partial \tau_g > 0$ and $\partial k^c / \partial \tau_g < 0$, which implies from (40), $\partial \tau_g / \partial A_g < 0$ and $\partial i_g / \partial A_g > 0$. In accordance with the evidence from Krishnamurthy and Vissing-Jorgensen (2012), the spread between government bonds and illiquid bonds, τ_g , increases if A_g goes up. In addition, $\partial k^m / \partial i < 0$, $\partial k^c / \partial i < 0$ and $\partial \tau_g / \partial i > 0$. The expression for r is given by (39) where i is replaced with τ_g .

We now turn to the case where the liquidity constraint binds. If $\theta = 0$, k^c solves:

$$a_m^e - k^c + \frac{\chi_b}{1 + \tau_g \nu_g} f(k^c) = 0. \quad (43)$$

The last term on the LHS shows the liquidity requirement is formally equivalent to a reduction in χ_b . The entrepreneur's choice of real balances is given by

$$(1 - \alpha) f'(k^m) + \alpha \frac{\left(1 - \frac{\chi_b}{1 + \tau_g \nu_g}\right) f'(k^c)}{1 - \frac{\chi_b}{1 + \tau_g \nu_g} f'(k^c)} = 1 + \frac{i}{\lambda}. \quad (44)$$

In the regime where $i_g \in (0, i)$, the pair (k^m, k^c) is determined by (44) and $k^c - k^m = A_g / (\alpha \lambda \nu_g)$. It can be checked that $\partial k^c / \partial A_g > 0$, $\partial k^m / \partial A_g < 0$, and $\partial i_g / \partial A_g > 0$.

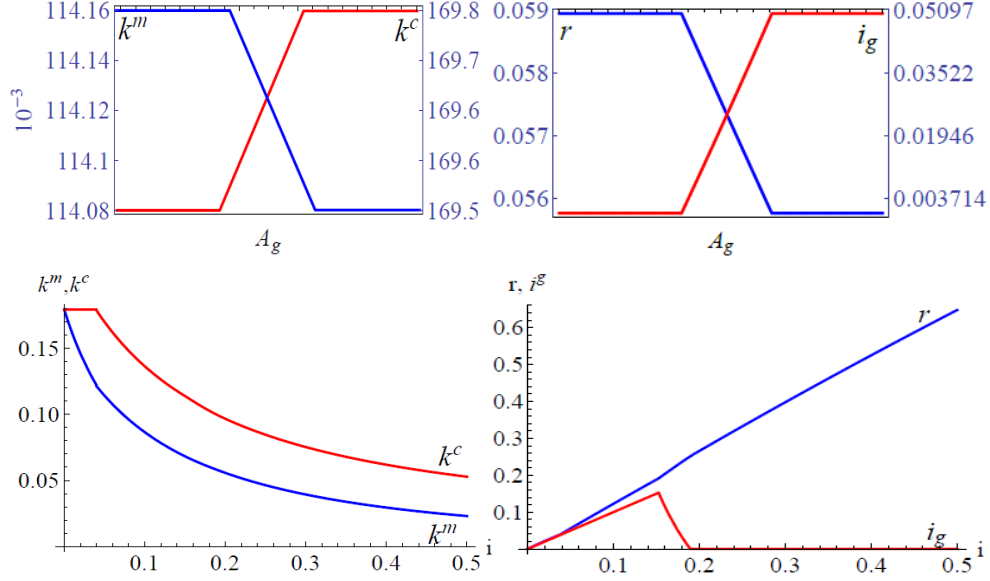


Figure 6: Monetary policy under broad liquidity requirements: a numerical example

We now turn to an example with the same parametrization as in Figure 3. The top left panel in Figure 6 illustrates the three regimes described earlier. For intermediate values of A_g , an increase in A_g leads to a decrease in k^m and an increase in k^c , and hence a change in the composition between external and internal finance. According to the top right panel, i_g (red line) increases with A_g since the regulatory premium for bonds decreases while r (blue line) decreases since τ_g decreases. So changes in A_g generate a negative correlation between i_g and r .

From the bottom left panel, k^c and k^m decrease with i . The bottom right panel shows a non-monotone relationship between i_g and i . For low inflation, liquidity is abundant and there is no regulatory premium on government bonds, $i_g = i$. For intermediate inflation, liquidity is scarce and government bonds acquire a positive regulatory premium. As a result, i_g decreases with inflation. When i is sufficiently large, the economy falls in a liquidity trap, $i_g = 0$. The lending rate increases with i since a_m^e decreases and τ_g increases.

Finally, we describe an unanticipated OMO in the interbank market similar to the one in Section 6.1. The central bank increases the money supply by μA_m by purchasing

$A_g - A'_g$ bonds where $\mu \hat{q}'_m A_m = \hat{q}'_g (A_g - A'_g)$. In the following periods the supply of bonds is kept at its initial level, A_g . Because money and bonds are equally suitable to satisfy regulatory requirements, $\tau'_g = \tau'_m$. It follows that $\hat{q}'_m / q'_m = \hat{q}'_g$ and $\mu q'_m A_m = A_g - A'_g$. So the overall liquidity in the interbank market, $\mu q'_m A_m + A'_g = A_g$, is unaffected by the OMO. If the initial equilibrium is such that liquidity is scarce, $i_g < i$, then bank lending is unaffected since $\alpha \lambda (k^{e'} - a_m^{e'}) = \nu_g A_g$. However, $a_m^{e'} = a_m^e / (1 + \mu)$ so that $k^{e'} < k^c$ and $k^{m'} < k^m$. If the initial equilibrium is such that liquidity is abundant, $i_g = i$, then bank lending increases to compensate for the fall in $a_m^{e'}$.

7 Extensions

We now succinctly describe two extensions of the model. In the first extension, we allow free entry of banks to endogenize entrepreneurs' access to credit, α , which can be interpreted as the extensive margin of credit. In the second extension, we add to the matching process between entrepreneurs and banks a random matching process between entrepreneurs and suppliers thereby allowing for trilateral matches.

7.1 Access to credit and entry of banks

We endogenize entrepreneurs' access to credit by allowing for entry of banks. In order to participate in the DM, a bank must pay a flow cost $\varsigma \geq 0$, denominated in units of numéraire. The measure of banks that enter the DM is b and the measure of contacts between banks and entrepreneurs is $\alpha(b)$. We assume $\alpha(b)$ is increasing and concave, $\alpha(0) = 0$, $\alpha'(0) = 1$, $\alpha(+\infty) = 1$, and $\alpha'(+\infty) = 0$. The matching probability for an entrepreneur is $\alpha(b)$ and for a bank is $\alpha(b)/b$. For simplicity, we assume there is no liquidity requirement.

The lifetime expected utility of a bank that participates in the DM solves:

$$V^b = \frac{\alpha(b)}{b} \lambda \phi + \beta V^b.$$

Free entry of banks means they make zero expected profits from entering the DM, or

$V^b = \varsigma$. As a result, the bank solves $\lambda\phi\alpha(b)/b = \varsigma$, or

$$\frac{b}{\alpha(b)} = \frac{\lambda\phi}{\varsigma}. \quad (45)$$

Hence, banks enter the DM until their expected surplus of participating in DM trades, $[\alpha(b)/b]\lambda\phi$, equals the entry cost, ς . In addition, entry occurs if and only if $\lambda\phi > \varsigma$, which requires $\theta > 0$. Intuitively, banks participate in the market so long as they make profits from their loans to entrepreneurs to recover their ex ante entry cost. From Section 5, the interest payments to the bank, ϕ , decrease with a_m^e . By holding larger real balances, entrepreneurs can negotiate lower interest payments. As a result, (45) gives a negative relationship between b and a_m^e , represented by the curve BE in Figure 7. If entrepreneurs hold more real balances, then fewer banks enter the market. As a_m^e approaches k^* , then ϕ tends to zero and b tends to zero as well.

Suppose i is not too large so that $k^c = k^*$. The entrepreneur's demand for real balances is

$$f'(a_m^e) = 1 + \frac{i}{\lambda[1 - \alpha(b)(1 - \theta)]}. \quad (46)$$

So the entrepreneur's demand for real balances decreases with b , which is represented by the curve MD in Figure 7. As b approaches zero, a_m^e tends to its level in a pure monetary economy, i.e., $f'(a_m^e) = 1 + i/\lambda$. As b tends to infinity, a_m^e approaches the solution to $f'(a_m^e) = 1 + i/\lambda\theta$. So there exists a solution (b, a_m^e) to (45)-(46) such that (45) intersects (46) by above in the (b, a_m^e) -space. As i increases MD shifts downward so that a_m^e decreases but b increases. So the entry margin amplifies the effect of monetary policy on entrepreneur's real balances since a higher $\alpha(b)$ reduces a_m^e further. As i approaches zero, then a_m^e tends to k^* and b tends to zero.

7.2 Trilateral bargaining

So far we have assumed that upon receiving an investment opportunity, with probability λ , entrepreneurs could purchase capital goods on a Walrasian market. Alternatively, we could assume that the market for capital goods is decentralized and features search and bargaining. An entrepreneur is matched with a supplier with probability α_s and with a bank with probability α_b . Assuming independent matching processes, an entrepreneur

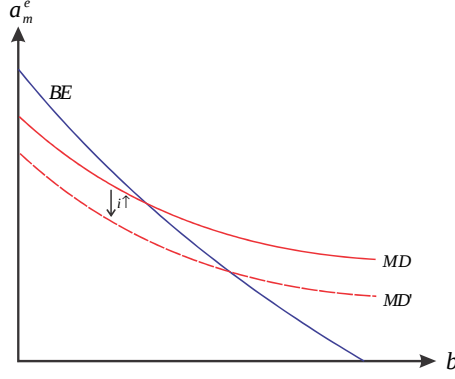


Figure 7: Equilibrium with entry of banks

meets both a supplier and a bank with probability $\alpha_b \alpha_s$. If suppliers have no bargaining power, then the payment they receive is equal to their disutility cost, $\psi = k$, and the analysis is identical to the one done so far. In the following, we describe the bank loan contract when suppliers have some bargaining power, and for simplicity, we focus on equilibria with $q_m = 0$.

In a trilateral match, the solution to the bargaining problem is given by the generalized Nash bargaining solution where θ_b is the bank's bargaining power, θ_s is the supplier's bargaining power, and $1 - \theta_b - \theta_s$ is the entrepreneur's bargaining power. The bargaining outcome solves

$$(k, \psi, \phi) \in \arg \max [f(k) - \psi - \phi]^{1-\theta_b-\theta_s} [\phi]^{\theta_b} [-k + \psi]^{\theta_s}, \quad (47)$$

subject to $\psi + \phi \leq \chi_b f(k)$. Suppose the liquidity constraint is slack. The solution to (47) is $k = k^*$, $\phi = \theta_b [f(k^*) - k^*]$, and $\psi = k^* + \theta_s [f(k^*) - k^*]$. In addition to being compensated for their disutility of production suppliers receive a fraction θ_s of the surplus generated by the investment opportunity. The lending rate is:

$$r = \frac{\phi}{\psi} = \frac{\theta_b [f(k^*) - k^*]}{k^* + \theta_s [f(k^*) - k^*]}.$$

As θ_s increases, ψ increases but r decreases. The threshold for χ_b above which the first best is achieved is:

$$\chi_b^* = \frac{k^* + (\theta_b + \theta_s) [f(k^*) - k^*]}{f(k^*)}.$$

So χ_b^* increases if banks or suppliers have more bargaining power.

Consider next the case where the liquidity constraint binds. The solution is:

$$\frac{k}{f(k)} = \frac{\chi_b f'(k) - (\theta_s + \theta_b)}{[1 - (\theta_b + \theta_s)] f'(k)} \quad (48)$$

$$\phi = \frac{\theta_b}{\theta_s + \theta_b} [\chi_b f(k) - k] \quad (49)$$

$$\psi = \frac{\theta_s}{\theta_s + \theta_b} \chi_b f(k) + \frac{\theta_b}{\theta_s + \theta_b} k \quad (50)$$

Equation (48) that determines k is identical to (14) where θ is replaced with $\theta_s + \theta_b$. So if either the bank or the supplier has more bargaining power, k decreases. Given k (49) determines ϕ and (50) determines ψ .

8 Conclusion

We provided a theory of internal and external corporate finance and its relation to monetary policy using a New Monetarist approach. We considered an environment where entrepreneurs who receive random investment opportunities can access banks in a decentralized credit market featuring limited enforcement and search frictions. The terms of the loan contract, including loan rates, loan size, and down payment, are outcome of a negotiation between entrepreneurs and banks. In this negotiation, the entrepreneur is subject to an endogenous borrowing constraint that arises due to the limited pledgeability of the returns of investment projects. A key contribution is to provide a framework to study the effects of monetary policy, both in terms of money growth rates and open market operations, on corporate finance, firms' liquidity management, and investment in an environment that is explicit about the coexistence of money, credit, and government bonds.

References

- Fernando Alvarez, Andrew Atkeson, and Patrick Kehoe (2002) . Money, Interest Rates, and Exchange Rates with Endogenously Segmented Markets. *JPE*, 110, 73-112.
- Fernando Alvarez and Urban Jermann (2000) . Efficiency, Equilibrium, and Asset pricing with Risk of Default. *Econometrica*, 68, 775-97.
- Fernando Alvarez, Robert Lucas, and Warren Weber (2001) . Interest Rates and Inflation. *AER P&P*, 91, 219-225.
- Luis Araujo and Tai-Wei Hu (2015) . Optimal Monetary Intervention in Credit Markets, Mimeo.
- Adam Ashcraft (2006) . New Evidence on the Lending Channel. *Journal of Money, Credit, and Banking*, 38, 751-775.
- Basel Committee on Banking Supervision (2013) . Basel III: The Liquidity Coverage Ratio and Liquidity Risk Monitoring Tools. *BIS*.
- Thomas Bates, Kathleen Kahle, and Rene Stulz (2009) . Why Do U.S. Firms Hold So Much More Cash than They Used To? *JF*, 64, 5.
- Zsolt Becsi, Victor Li, and Ping Wang (2005) . Heterogeneous Borrowers, Liquidity, and the Search for Credit. *JEDC*, 29, 1331-1360.
- Zsolt Becsi, Victor Li, and Ping Wang (2013) . Credit Mismatch and Breakdown. *EER*, 59, 109-125.
- Aleksander Berentsen, Gabriele Camera, and Christopher Waller (2007) . Money, Credit and Banking. *JET*, 135, 171-95.
- Ben Bernanke and Alan Blinder (1988) . Credit, Money, and Aggregate Demand. *AER*, 78, 435-439.
- Ben Bernanke and Mark Gertler (1995) . Inside the Black Box: the Credit Channel of Monetary Policy Transmission. *JEP*, 9, 27-48.
- Ben Bernanke, Mark Gertler, and Simon Gilchrist (1996) . The Financial Accelerator and Flight to Quality. *RESTAT*, 78, 1-15.
- Ben Bernanke, Mark Gertler, and Simon Gilchrist (1999) . The Financial Accelerator in a Quantitative Business Cycle Framework. *Handbook of Macroeconomics*, ed. J. Taylor and M. Woodford, 1342-1393.
- Zachary Bethune, Tai-Wei Hu, and Guillaume Rocheteau (2014) . Dynamic Indeterminacy and Welfare in Credit Economies. Mimeo.
- Zachary Bethune, Guillaume Rocheteau, and Peter Rupert (2015) . Aggregate Unemploy-

- ment and Household Unsecured Debt. *RED*, 18, 77-100.
- Bruno Biais, Thomas Mariotti, Guillaume Plantin, and Jean-Charles Rochet (2007) . Dynamic Security Design: Convergence to Continuous Time and Asset Pricing Implications. *RES*, 74, 345-390.
- Javier Bianchi and Saki Bigio(2015) . Banks, Liquidity Management and Monetary Policy, *Mimeo*.
- Saki Bigio and Jennifer La'O (2015) . Financial Frictions in Production Networks, *Mimeo*.
- Patrick Bolton and Xavier Freixas (2006) . Corporate Finance and the Monetary Transmission Mechanism. *RFS*, 19, 829-870.
- Francesca Carapella and Stephen Williamson (2015) . Credit Markets, Limited Commitment, and Government Debt. *RES*, 82, 963-90.
- Mark Carlson (2014) . Lessons from the Historical Use of Reserve Requirements in the United States to Promote Bank Liquidity. *Mimeo*.
- Ricardo Cavalcanti and Neil Wallace (1999a) . Inside and Outside Money as Alternative Media of Exchange. *JMCB*, 31, 443-57.
- Ricardo Cavalcanti and Neil Wallace (1999b) . A Model of Private Bank-Note Issue. *RED*, 2, 104-36.
- Jonathan Chiu and Cesaire Meh (2011) . Financial Intermediation, Liquidity, and Inflation. *Macroeconomic Dynamics*, 15, 83-118.
- Jonathan Chiu, Cesaire Meh, and Randall Wright (2015) . Innovation and Growth with Financial and Other Frictions. *IER*, in press.
- Giovanni Dell'Ariccia and Pietro Garibaldi (2005) . Gross Credit Flows. *RES*, 72, 665-685.
- Wouter del Haan, Garey Ramey, and Joel Watson (2003) . Liquidity Flow and Fragility of Business Enterprises. *JME*, 50, 1215-1241.
- Peter DeMarzo and Michael Fishman (2003) . Optimal Long-Term Financial Contracting with Privately Observed Cash Flows, *Mimeo*.
- Peter Diamond (1987) . Multiple Equilibria in Models of Credit. *AER*, 77, 82-86.
- Jason Donaldson, Giorgia Piacentino, Anjan Thakor (2015) . Warehouse Banking, *Mimeo*.
- Darrell Duffie, Nicolae Garleanu, and Lasse Pedersen (2005) . Over-the-Counter Markets. *Econometrica*, 73, 1815-47.
- Leo Ferraris and Makoto Watanabe (2008) . Collateral Secured Loans in a Monetary Economy. *JET*, 143, 405-424.
- Scott Freeman (1987) . Reserve requirements and optimal seigniorage. *JME*, 19.

- Mark Gertler and Nobuhiro Kiyotaki (2011) . Financial Intermediation and Credit Policy in Business Cycle Analysis, *Handbook of Monetary Economics*, vol. 2, B. Friedman and M. Woodford, eds. North-Holland, Amsterdam.
- Charles Goodhart (1995) *The Central Bank and the Financial System*. MIT Press.
- Chao Gu, Fabrizio Mattesini, Cyril Monnet, and Randall Wright (2013a) . Banking: A New Monetarist Approach. *RES*, 80, 636-66.
- Chao Gu, Fabrizio Mattesini, Cyril Monnet, and Randall Wright (2013b) . Endogenous Credit Cycles. *JPE*, 121, 940-65.
- Chao Gu, Fabrizio Mattesini, and Randall Wright (2015) . Money and Credit Redux. *Econometrica*, forthcoming.
- Chao Gu and Randall Wright (2015) . Monetary Mechanisms, Mimeo.
- Chao He, Randall Wright, and Yu Zhu (2015) . Housing and Liquidity. *RED*, 18, 435-455.
- Ping He, Lixin Huang, and Randall Wright (2005) . Money and Banking in Search Equilibrium. *IER*, 46, 637-670.
- Ping He, Lixin Huang, and Randall Wright (2008) . Money, Banking, and Monetary Policy. *JME*, 55, 1013-1024.
- Bengt Holmstrom and Jean Tirole (1998) . Private and Public Supply of Liquidity. *JPE*.
- Bengt Holmstrom and Jean Tirole (2011) . *Inside and Outside Liquidity*. MIT Press.
- Angela Huang (2015) . On the Number and Size of Banks: Efficiency and Equilibrium, *Mimeo*.
- Roman Inderst and Holger Muller (2003) . Internal versus External Financing: an Optimal Contracting Approach. *JF*, 3, 1033-1062.
- Joint Small Business Credit Survey Report (2014) . Federal Reserve Banks of New York, Atlanta, Cleveland and Philadelphia.
- Aubhik Khan (2006) . The Role of Segmented Markets in Monetary Policy. *FRB Philadelphia Business Review*.
- Aubhik Khan and Julia Thomas (2015) . Revisiting the Tale of Two Interest Rates with Endogenous Asset Market Segmentation. *RED*, forthcoming.
- Elhud Kalai (1977) . Proportional Solutions to Bargaining Situations: Interpersonal Utility Comparisons. *Econometrica*, 45, 1623-1630.
- Anil Kashyap, Jeremy Stein, and David Wilcox (1993) . Monetary Policy and Credit Conditions: Evidence From the Composition of External Finance, *AER*, 83, 78-98.
- Timothy Kehoe and David Levine (1993) . Debt-Constrained Asset Markets. *RES*, 60, 865-888.

- Nobuhiro Kiyotaki and Ricardo Lagos (1997) . A Model of Job and Worker Flows. *JPE*, 115, 770-819.
- Nobuhiro Kiyotaki and John Moore (1997) . Credit Cycles. *JPE*, 105, 211-48.
- Nobuhiro Kiyotaki and John Moore (2005) . Liquidity and Asset Prices. *IER*, 46, 317-49.
- Narayana Kocherlakota (1998) . Money is Memory. *JET* , 232-51.
- Arvind Krishnamurthy and Annette Vissing-Jorgensen (2012) . The Aggregate Demand for Treasury Debt. *JPE*.
- Ricardo Lagos (2010) . Asset Prices and Liquidity in an Exchange Economy. *JME*, 57, 913-930.
- Ricardo Lagos and Guillaume Rocheteau (2008) . Money and Capital as Competing Media of Exchange. *JET*, 142, 247-258.
- Ricardo Lagos and Guillaume Rocheteau (2009) . Liquidity in Asset Markets with Search Frictions. *Econometrica*, 77, 403-426.
- Ricardo Lagos, Guillaume Rocheteau, and Randall Wright (2015). “Liquidity: A New Monetarist Perspective,” *JEL*, in press.
- Ricardo Lagos and Randall Wright (2005) . A Unified Framework for Monetary Theory and Policy Analysis. *JPE*, 113, 463-484.
- Yiting Li (2011) . A Search Model of Money and Circulating Private Debt with Applications to Monetary Policy. *IER*, 42, 925-46.
- Yiting Li and Ying-Syuan Li (2012) . Liquidity and Asset Prices: A New Monetarist Approach. *JME*, 60, 426-38.
- Yiting Li, Guillaume Rocheteau, and Pierre-Olivier Weill (2012) . Liquidity and the Threat of Fraudulent Assets. *JPE*, 120, 815-46.
- Sébastien Lotz and Cathy Zhang (2015) . Money and Credit as Means of Payment: A New Monetarist Approach. *JET*, in press.
- Traci Mach and John Wolken (2006) Financial Services Used by Small Businesses: Evidence from the 2003 Survey of Small Business Finances. *FRB Bulletin*, A170.
- Merton Miller and Daniel Orr (1966) A Model of the Demand for Money by Firms. *QJE*, 80, 413-435.
- Nada Mora (2014) . The Weakened Transmission of Monetary Policy to Consumer Loan Rates. *FRB Kansas Economic Review*.
- Casey Mulligan (1997) Scale Economies, the Value of Time, and the Demand for Money: Longitudinal Evidence from Firms. *JPE*, 105, 1061-1079.
- Abhinav Muthoo (1999) . *Bargaining Theory with Applications*. Cambridge University

Press.

Ed Nosal and Guillaume Rocheteau (2011) . *Money, Payments, and Liquidity*. MIT Press.

Ed Nosal and Guillaume Rocheteau (2013) . Pairwise Trades, Asset Prices, and Monetary Policy. *JEDC*, 37, 1-17.

Ed Nosal and Neil Wallace (2007) . A Model of (the Threat of) Counterfeiting. *JME*, 54, 994-1001.

Ed Nosal, Yuet-Yee Wong, and Randall Wright (2015) . More on Middlemen: Equilibrium Entry and Efficiency in Intermediated Markets. *JMCB*, 47, 7-36.

Nicolas Petrosky-Nadeau (2014) . Credit, Vacancies, and Unemployment Fluctuations. *RED*, 17(2).

Nicolas Petrosky-Nadeau and Etienne Wasmer (2013) . Macroeconomic Dynamics in a Model of Goods, Labor, and Credit Market Frictions. *AEJ: Macro*, 72.

Tim Opler, Lee Pinkowitz, Rene Stulz, and Rohan Williamson(1999) . The Determinants and Implications of Corporate Cash Holdings. *JFE*, 52, 3-46.

Martin Osborne and Ariel Rubinstein (1994) *A Course in Game Theory*. MIT Press.

Mitchell Petersen and Raghuram Rajan (1997) . Trade Credit: Theories and Evidence. *RFS*, 10, 661-691.

Valerie Ramey (1990) . How Important is the Credit Channel in the Transmission of Monetary Policy? *Carnegie-Rochester Conference Series on Public Policy*, 1-45.

Guillaume Rocheteau (2011) . Payment and Liquidity Under Adverse Selection. *JME*, 58, 191-205.

Guillaume Rocheteau and Antonio Rodriguez-Lopez (2014) . Liquidity Provision, Interest Rates, and Unemployment. *JME*, 65, 80-101.

Guillaume Rocheteau and Randall Wright (2005) . Money in Search Equilibrium, in Competitive Equilibrium, and in Competitive Search Equilibrium. *Econometrica*, 73, 175-202.

Guillaume Rocheteau, Randall Wright, and Sylvia Xiaolin Xiao (2015) . Open Market Operations, Mimeo.

David Romer (1985) . Financial intermediation, reserve requirements, and inside money. *JME*, 16, 175-194.

Christina Romer and David Romer (1990) . New Evidence on the Monetary Transmission Mechanism. *Brookings Papers on Economic Activity*.

Ariel Rubinstein and Asher Wolinsky (1987) . Middlemen. *QJE*, 102, 581-594.

Daniel Sanches and Stephen Williamson (2010) . Money and Credit with Limited Commitment and Theft. *JET*, 145, 1525-1549.

- Juan Sanchez and Emircan Yurdagul (2013). Why Are U.S. Firms Holding So Much Cash? An Exploration of Cross-Sectional Variation. *FRB St. Louis Review*.
- Anthony Saunders and Liliana Schumacher (2000). The Determinants of Bank Interest Rate Margins. *JIMF*, 19, 813-832.
- Shouyong Shi (1996). Credit and Money in a Search Model with Divisible Commodities. *Review of Economic Studies*, 63, 627-52.
- Ali Shourideh and Ariel Zetlin-Jones (2014). External Financing the Role of Financial Frictions over the Business Cycle: Measurement and Theory, *Mimeo*.
- Rafael Silveira and Randall Wright (2010). Search and the Market for Ideas. *JET*, 145, 550-1573.
- Rafael Silveira and Randall Wright (2015). Venture Capital: a Model of Search and Bargaining. *RED*, forthcoming.
- Irina Telyukova and Randall Wright (2008). A Model of Money and Credit, with Application to the Credit Card Debt Puzzle. *RES*, 75, 629-47.
- William Thomson (1989). Cooperative Models of Bargaining. *Handbook of Game Theory*, 2, edited by R. Aumann and S. Hart.
- Robert Townsend (1989). Currency and Credit in a Private Information Economy. *JPE*, 97, 1323-1344.
- Venky Venkateswaran and Randall Wright (2013). Pledgeability and Liquidity: a New Monetarist Model of Macroeconomic and Financial Activity. *NBER Macro Annual*.
- Etienne Wasmer and Phillipe Weill (2004). The Macroeconomics of Labor and Credit Market Frictions. *AER*, 94(4): 944-963.
- Elmus Wicker (2000). *Banking Panics of the Gilded Age*, New York. Cambridge University Press.
- Stephen Williamson (1986). Costly Monitoring, Financial Intermediation, and Equilibrium Credit Rationing. *JME*, 159-179.
- Stephen Williamson (2012). Liquidity, Monetary Policy, and the Financial Crisis: A New Monetarist Approach. *AER*, 102, 2570-2605.
- Stephen Williamson (2015). Scarce Collateral, the Term Premium, and Quantitative Easing. *JET*, in press.
- Stephen Williamson and Randall Wright (2010). New Monetarist Economics I: Models, *Handbook of Monetary Economics*, vol. 2, B. Friedman and M. Woodford, eds. North-Holland.

Appendices

A1. Pareto frontier of the bargaining set

In the following we characterize the Pareto frontier of the bargaining set in a match between an entrepreneur and a bank. The surplus of the entrepreneur is $S^e = f(k) - k - \phi$ while the surplus of the bank is $S^b = \phi$. Provided the liquidity constraint does not bind, the match surplus is maximum and equal to $f(k^*) - k^*$. In that case, the Pareto frontier is linear and given by $S^e + S^b = f(k^*) - k^*$, as illustrated in the right panel of Figure 2. The pledgeability constraint is not binding if $S^b = \phi \leq \chi_b f(k^*) - k^*$. Hence, the Pareto frontier has a linear portion if and only if $\chi_b \geq k^*/f(k^*)$ and is entirely linear if $S^b = f(k^*) - k^* \leq \chi_b f(k^*) - k^*$, which only occurs when output is fully pledgeable, $\chi_b = 1$.

If $S^b > \chi_b f(k^*) - k^*$, the pledgeability constraint binds, in which case $\phi = \chi_b f(k) - k$, since otherwise one could keep ϕ constant and raise k so as to increase the entrepreneur's surplus. This point is illustrated in the left panel of Figure 2. Take a pair (ϕ, k) located underneath the curve $\chi_b f(k) - k$ such that $k < k^*$. By raising k , the entrepreneur's surplus, which is the difference between $f(k) - k$ and ϕ , increases. Moreover, $k \geq \hat{k} = \arg \max [\chi_b f(k) - k]$, since otherwise one could raise $\phi = \chi_b f(k) - k$ and increase both the bank's and the entrepreneur's surpluses. Hence, the Pareto frontier when the liquidity constraint binds is characterized by the following set:

$$\left\{ (S^e, S^b) \in \mathbb{R}_{2+} : S^e = (1 - \chi_b)f(k), S^b = \chi_b f(k) - k, k \in [\hat{k}, \bar{k}] \right\},$$

where $\bar{k} = k^*$ if $\chi_b f(k^*) - k^* \geq 0$ or $\bar{k}$ is the largest solution to $\chi_b f(\bar{k}) - \bar{k} = 0$ otherwise. In addition, the Pareto frontier is downward sloping:

$$\frac{dS^e}{dS^b} = \frac{(1 - \chi_b)f'(k)}{\chi_b f'(k) - 1} < 0.$$

As $k \rightarrow \hat{k}$, $dS^e/dS^b \rightarrow -\infty$. If $\chi_b f(k^*) - k^* \geq 0$, then as $k \rightarrow k^*$, $dS^e/dS^b \rightarrow -1$. Importantly, the bargaining set is not convex since the point on the Pareto frontier that gives the maximum surplus to the bank, $\chi_b f(\hat{k}) - \hat{k}$, is located above the horizontal axis. Hence, the entrepreneur enjoys a positive surplus, $(1 - \chi_b)f(\hat{k})$, due to the limited pledgeability of output.

A2. External finance under proportional bargaining

As has been shown throughout the New Monetarist literature, the choice of the trading mechanism is key for both positive and normative analysis. A commonly used alternative to the Nash solution is the proportional solution from Kalai (1977). We define the proportional bargaining solution as:

$$\max_{\phi, k} S^b = \phi \quad \text{s.t.} \quad S^e = f(k) - k - \phi \geq \frac{1-\theta}{\theta} S^b \quad \text{and} \quad k + \phi \leq \chi_b f(k).$$

Under proportional bargaining, the bank chooses the terms of the contract, (ϕ, k) , in order to maximize his surplus subject to the constraint that the entrepreneur obtains at least a fraction $1-\theta$ of the total match surplus.²² Provided $\chi_b \geq \chi_b^*$, the liquidity constraint does not bind, in which case the proportional solution coincides with the Nash solution, i.e., $k = k^*$, $\phi = \theta[f(k^*) - k^*]$. If the liquidity constraint binds, k solves

$$(\chi_b - \theta) f(k) = (1 - \theta)k \quad \text{if } \chi_b > \theta \text{ and } k \geq \hat{k} \quad (51)$$

$$k = \hat{k} \quad \text{otherwise.} \quad (52)$$

According to (51), the proportional solution, $k \geq \hat{k}$, splits the match surplus so as assign a share θ of the surplus to the bank while satisfying the liquidity constraint. If the solution to (51) is such that $k < \hat{k}$, the solution is not Pareto optimal. Indeed, by increasing k to $\hat{k}$, the bank's surplus reaches its maximum, $S^b = \chi_b f(\hat{k}) - \hat{k}$, while the entrepreneur's surplus, $(1 - \chi_b)f(k)$, increases. In that case, we select $k = \hat{k}$, in accordance with the Lexicographic proportional solution (Thomson, 1994, p. 1253).

The interest rate on the loan when the pledgeability constraint binds is

$$\begin{aligned} r &= \frac{\theta(1 - \chi_b)}{\chi_b - \theta} \quad \text{if } \theta \leq \hat{\theta} \equiv \frac{\chi_b f(\hat{k}) - \hat{k}}{f(\hat{k}) - \hat{k}} \\ &= \frac{\hat{\theta}(1 - \chi_b)}{\chi_b - \hat{\theta}} \quad \text{if } \theta > \hat{\theta}. \end{aligned}$$

Provided the bank's bargaining power is not too large, the interest rate is decreasing with the pledgeability of the entrepreneur's output. In the case of a Cobb-Douglas production technology, $f(k) = zk^\gamma$, the interest rate and production of investment goods are given

²²The strict proportional bargaining solution requires $f(k) - k - \phi = \frac{1-\theta}{\theta} S^b$. We write this constraint as an inequality to guarantee existence despite the non-convexity of the bargaining set arising from the endogenous pledgeability constraint. This formulation corresponds to the Lexicographic proportional solution as described by Thomson (1994, p. 1253).

by

$$r = \frac{\frac{\theta(1-\gamma)}{\gamma}}{\frac{\chi_b - \theta}{\gamma}} \quad \text{and} \quad k = \frac{(\gamma z)^{\frac{1}{1-\gamma}}}{\left[\frac{(\chi_b - \theta)z}{1-\theta} \right]^{\frac{1}{1-\gamma}}} \quad \text{if} \quad \chi_b \in \left[\frac{\theta}{1-\gamma(1-\theta)}, (1-\theta)\gamma + \theta \right)$$

$$< \frac{\theta}{1-\gamma(1-\theta)}$$

There are three different regimes depending on the value of the χ_b . For low pledgeability levels, the lending rate reaches a maximum and is independent of both χ_b and θ . In this regime, the pledgeability constraint binds and the equilibrium level of investment maximizes the bank's surplus. For intermediate pledgeability levels, the lending rate is decreasing in χ_b and increasing with the θ . For high pledgeability levels, entrepreneurs are no longer liquidity constrained, in which case both the equilibrium levels of k and r are constant and independent of the level of pledgeability.

A3. Limited commitment under public monitoring

In the main text we have assumed that the entrepreneur's borrowing constraint was given by his pledgeable output defined as a fraction χ_b of his final output, $f(k)$. This pledgeability constraint was motivated by a moral hazard problem where the entrepreneur can walk away in the CM with a fraction $1 - \chi_b$ of his final output, which means the bank can seize at most $\chi_b f(k)$. Alternatively, we can rationalize the entrepreneur's borrowing constraint with a limited commitment and enforcement problem, along the lines of Kehoe and Levine (1993) and Alvarez and Jerman (2000), that places an endogenous upper bound on the entrepreneur's payment capacity.

We now assume banks can no longer seize the output produced by the entrepreneur: in the CM, the entrepreneur can choose to walk away with all his output and default on the repayment of the loan amount and interest payment. However, banks have access to a public record of entrepreneurs' loans and repayment histories. This access to monitoring provides a mechanism for individual borrowers to be punished if they do not deliver on their promise to repay their loans. We focus on equilibria where the entrepreneur's punishment for default is permanent exclusion from accessing future loans. As before, we assume banks can commit to always honor their own debt obligations to suppliers.

Following Alvarez and Jermann (2000), the equilibrium borrowing constraint is determined to ensure the entrepreneur voluntarily repays his debt in the CM.²³ Under this specification, how much an entrepreneur can borrow now depends on the lifetime value of an entrepreneurship, defined as $\bar{W}^e = W^e(0, 0)$, or

$$\bar{W}^e = \beta \{ \alpha \lambda [f(k) - k - \phi] + \bar{W}^e \}.$$

An entrepreneur who enters the CM with no wealth has an investment opportunity in the next period with probability $\alpha \lambda$ in which case he enjoys a surplus equal to $f(k) - k - \phi$. Solving for $\bar{W}^e$, we obtain

$$\bar{W}^e = \frac{\alpha \lambda [f(k) - k - \phi]}{\rho}. \quad (53)$$

According to (53), the value of being an entrepreneur in the CM is simply the discounted sum of profits, $f(k) - k$, net of the interest payment, ϕ , where the discount rate is the rate of time preference, ρ . Since any default by the entrepreneur is publicly recorded,

²³See Bethune et al. (2014) for an explicit game-theoretic analysis of pairwise credit under limited commitment and monitoring. Gu et al. (2013a,b) and Bethune et al. (2015) consider imperfect monitoring where default is detected only probabilistically. In that case, the debt limit is only a fraction of the agent's value.

there is an equilibrium where no bank will agree to lend to him in the future. Hence by defaulting, the entrepreneur is banished to permanent autarky and loses the future value of an entrepreneurship, $\bar{W}^e$. Consequently, the entrepreneur's borrowing constraint is

$$\psi + \phi \leq \bar{W}^e. \quad (54)$$

By defaulting, the entrepreneur saves the cost of repaying $\psi + \phi$ but he loses the continuation value of being an entrepreneur with access to credit, $\bar{W}^e$. The borrowing constraint, (54), corresponds to the "not-too-tight" solvency constraint in Alvarez and Jermann (2000).²⁴ It is also similar to the collateral constraint in Bernanke et al. (1996) where the maximum an entrepreneur can borrow is determined by his net worth.

We now determine the terms of the loan contract. Under generalized Nash bargaining the bargaining outcome solves

$$(k, \phi) \in \arg \max [f(k) - k - \phi]^{1-\theta} \phi^\theta \quad \text{s.t.} \quad k + \phi \leq \bar{W}^e. \quad (55)$$

In contrast with Section 4 under limited pledgeability, the bargaining problem with limited commitment is now convex since the payment capacity of the entrepreneur, $\bar{W}^e$, is independent of the choice of k in a match. The equation of the Pareto frontier is

$$\begin{aligned} S^e + S^b &= f(k^*) - k^* \quad \text{if } S^b \leq \bar{W}^e - k^* \\ \Delta^{-1}(S^e + S^b) + S^b &= \bar{W}^e \quad \text{otherwise,} \end{aligned}$$

where $\Delta(k) \equiv f(k) - k$ denotes the total match surplus when the entrepreneur's borrowing constraint binds, or $S^b > \bar{W}^e - k^*$. Relative to Figure 2, the Pareto frontier under limited commitment intersects the horizontal axis where $S^e = 0$.

Under Nash bargaining, $k = k^*$ and $\phi = \theta [f(k^*) - k^*]$ if $\bar{W}^e \geq k^* + \phi$. Substituting ϕ into (53), the value of being an entrepreneur who is not borrowing constrained is $\bar{W}^e = \alpha\lambda(1 - \theta) [f(k^*) - k^*] / \rho$. Accordingly, a regime where entrepreneurs are not borrowing constrained occurs if

$$\rho \leq \rho^* \equiv \frac{\alpha\lambda(1 - \theta) [f(k^*) - k^*]}{(1 - \theta)k^* + \theta f(k^*)}.$$

Entrepreneurs voluntarily repay k^* so long as they are sufficiently patient. In addition, how patient agents must be for the borrowing constraint not to bind depends on θ , λ , and α . The threshold, ρ^* , decreases with θ but increases with $\alpha\lambda$.

²⁴Bethune et al. (2014) show that equilibria with "not-too-tight" solvency constraints are only a small subset of all perfect Bayesian equilibria. Given our focus on steady states "no-too-tight" solvency constraints generate the highest debt limits.

Next consider a regime where the entrepreneur's borrowing constraint binds. The solution to (55) is

$$\bar{W}^e = \frac{\theta f(k) + (1 - \theta)f'(k)k}{(1 - \theta)f'(k) + \theta}. \quad (56)$$

The investment choice, k , is such that the entrepreneur's borrowing limit, $\bar{W}^e$, is a weighted average of the entrepreneur's output, $f(k)$, and the supplier's cost, k . We now use (53) and (54) at equality to express the borrowing limit of an entrepreneur as

$$\bar{W}^e = \frac{\alpha\lambda}{\rho + \alpha\lambda} f(k). \quad (57)$$

Interestingly, the borrowing limit from (57) is analogous to the pledgeability constraint from Section 4 where the pledgeability coefficient is $\chi_b = \alpha\lambda/(\rho + \alpha\lambda)$. So the pledgeability of output depends negatively on ρ and positively on λ and α . A subtle difference relative to our earlier formulation with limited pledgeability is that the output on the RHS of (57) is the output in future periods if the entrepreneur has access to credit and not his current output. Substituting $\bar{W}^e$ from (57) into (56), k solves

$$\frac{k}{f(k)} = \frac{\alpha\lambda(1 - \theta)f'(k) - \rho\theta}{(\rho + \alpha\lambda)(1 - \theta)f'(k)}. \quad (58)$$

Notice $k = 0$ is always a solution to (58). Intuitively, if banks believe entrepreneurs cannot obtain credit in the future, they cease lending in the current period. Since entrepreneurs can only pay with credit, $k = 0$. In addition, there is another solution with $k > 0$, which is uniquely determined since the left side of (58) is increasing in k from zero when $k = 0$ to ∞ when $k = \infty$, while the right side is decreasing in k for all k such that $\alpha\lambda(1 - \theta)f'(k) > \rho\theta$. The positive solution to (58) increases with α and λ and decreases with ρ and θ .

The lending rate is

$$r = \frac{\bar{W}^e - k}{k} = \frac{\alpha\lambda}{\rho + \alpha\lambda} \frac{f(k)}{k} - 1.$$

In accordance with (15), r increases with θ . However now, r depends on ρ since the borrowing limit is determined by the discounted sum of future surpluses. The lending rate also depends on λ and α . If both α and λ are large, entrepreneurs are more trustworthy since the value of being in good standing with banks is high. In turn, the scale of the trade is also high, which reduces r .

Assuming $f(k) = zk^\gamma$, when the borrowing constraint is slack, $k^* = (\gamma z)^{\frac{1}{1-\gamma}}$ and $r = \theta \left(\frac{1-\gamma}{\gamma} \right)$, which are identical to the solutions obtained in Section 4 under limited

pledgeability. When the borrowing constraint binds,

$$\begin{aligned} k &= \left[\frac{\chi_b(1-\theta)z\gamma}{(1-\theta)\gamma + (1-\chi_b)\theta} \right]^{\frac{1}{1-\gamma}} \\ r &= \frac{(1-\chi_b)\theta}{(1-\theta)\gamma}, \end{aligned}$$

where we define the pledgeability of output as $\chi_b = \alpha\lambda/(\rho + \alpha\lambda)$. So investment increases with output pledgeability while the lending rate decreases with pledgeability.

A4. Strategic foundations for the axiomatic bargaining game

We describe a bargaining game with alternating offers between an entrepreneur and a bank. The game, which takes place within a period, has no discounting but it features exogenous risks of breakdown.

Timing of the game In the initial stage, the entrepreneur makes an offer, (k^e, d^e, ϕ^e) , that the bank can either accept or reject. If it says yes, then the offer is implemented. If it rejects the offer, then the bargaining game continues. With probability δ^e the negotiation ends, in which case the entrepreneur can purchase k from suppliers with money only. With probability $1 - \delta^e$ it is the bank's turn to make an offer, (k^b, d^b, ϕ^b) , that the entrepreneur can either accept or reject. If he says yes, then the offer is implemented. If he rejects the offer, then the bargaining game continues. With probability δ^b the negotiation ends, in which case the entrepreneur purchases k with money in the Walrasian market for capital goods. With probability $1 - \delta^b$ the negotiation continues as in the initial stage where it is the entrepreneur's turn to make an offer.

We provide a graphical illustration of the game tree in Figure 8 where a thick dot indicates that it is a player's turn to take an action and players' identities are indicated by the letters E (entrepreneur) and B (bank), and S. A node with two players corresponds to a simultaneous move. The risk of exogenous breakdown is represented by a move by Nature (N) where the probabilities of the events are given by the numbers between squared brackets. The bases of the grey triangles represent the set of all possible offers that a player can make when it is his turn to make an offer.

Characterization of equilibria We restrict our attention to stationary equilibria where the entrepreneur and the bank make constant offers, (k^e, d^e, ϕ^e) and (k^b, d^b, ϕ^b) , respectively, whenever it is their turn to make an offer and they adopt a constant acceptance rule whenever it is their turn to accept or reject an offer. We restrict our attention to equilibria where the acceptance rules take the form of reservation surpluses, R^e and R^b , that specify the minimum surplus that a player must receive in order to accept an offer. Hence, entrepreneurs accept all offers such that

$$f(k) - \psi - \phi \geq R^e, \quad (59)$$

The left side of (59) is the entrepreneur's surplus defined as his output, $f(k)$, net of the payment to the supplier, ψ , and net of the interest payment to the bank, ϕ . The right

i.e., the liquidity constraint does not bind. From (64) if the reservation surplus of the bank is larger than a threshold, then the liquidity constraint binds and the entrepreneur asks for the largest quantity that satisfies the liquidity constraint. If the reservation surplus is too large then the entrepreneur cannot satisfy the constraint $\phi \geq R^b$ and make a positive surplus. It can be checked that $S^e(R^b)$ is decreasing (strictly so when $S^e(R^b) > 0$) and concave with $S^e(0) > 0$. Similarly, the bank's surplus when it is his turn to make an offer:

$$S^b(R^e) = \max_{k, \phi} \{ \phi \mathbb{I}_{\{f(k) - k - \phi \geq R^e\}} \} \quad (65)$$

$$\text{s.t. } k + \phi \leq \chi_b f(k) + a_m^e. \quad (66)$$

According to (65)-(66), the bank chooses an offer in order to maximize his interest payment subject to the acceptance rule of the entrepreneur and the liquidity constraint. The solution to (65)-(66) is:

$$S^b(R^e) = f(k^*) - k^* - R^e \quad \text{if } R^e \in [(1 - \chi_b)f(k^*) - a_m^e, f(k^*) - k^*] \quad (67)$$

$$= \chi_b f(\hat{k}) - \hat{k} + a_m^e \quad \text{if } R^e \leq (1 - \chi_b)f(\hat{k}) - a_m^e \quad (68)$$

$$= f(k) - k - R^e \quad \text{otherwise,} \quad (69)$$

where k solves $(1 - \chi_b)f(k) = R^e + a_m^e$. From (67) if the entrepreneur's reservation surplus is sufficiently large (but not so large that the bank would not want to participate) then the bank offers to finance the efficient investment level. From (68) and (69) if the reservation surplus of the entrepreneur is low then the bank finds it optimal to ask for sufficiently large interest payments so that the borrowing constraint binds. Below a threshold for R^e the investment level is chosen so as to maximize the pledgeable output net of the investment cost, $\chi_b f(k) - k$. It can also be checked that $S^b(R^e)$ is non-decreasing, concave, and such that $S^b(R^e) > 0$.

Let us now turn to the endogenous reservations surpluses of the entrepreneur and the bank. They solve:

$$R^e = (1 - \delta^b)S^e(R^b) + \delta^b \Delta^m(a_m^e) \quad (70)$$

$$R^b = (1 - \delta^e)S^b(R^e). \quad (71)$$

According to (70), the reservation surplus of the entrepreneur, R^e , is the surplus that makes the entrepreneur indifferent between accepting or rejecting an offer. If he accepts the bank's offer such that $f(k) - k - \phi = R^e$ then the entrepreneur gets R^e . If he rejects the offer, then the negotiation ends with probability δ^e in which case the entrepreneur

enjoys a surplus equal to $\Delta^m(a_m^e)$ by purchasing k with money only. With complement probability, $1 - \delta^b$, the negotiation continues with an offer from the entrepreneur that allows him to obtain a surplus equal to $S^e(R^b)$. Equation (71) has a similar interpretation. Note that in the event of an exogenous breakdown the bank receives no surplus.

One can show that the system (70)-(71) has a unique solution. Equation (70) is represented in Figure 9 by a blue curve while (71) is represented by a red curve. We already showed that both curves are downward-sloping and concave. We first establish existence. Let $\bar{R}^e > 0$ denote the value for R^e such that $S^b(R^e) = 0$. By the duality of the entrepreneur's and bank's problems, $\bar{R}^e = S^e(0)$. Moreover, provided that $a_m^e < k^*$ then $\Delta^m(a_m^e) < S^e(0)$ (since $\Delta^m(a_m^e) = S^e(0)$ when $\chi_b = 0$). Hence, the blue curve is located underneath the red one at $R^b = 0$. Moreover, the red curve reaches a maximum $(1 - \delta^e)S^b(0) < \chi_b f(\hat{k}) - \hat{k} + a_m^e$. So at $R^b = \chi_b f(\hat{k}) - \hat{k} + a_m^e$ the blue curve is located to the right of the red curve. Hence, the two curves intersect, i.e., a solution to (70)-(71) exists. Uniqueness follow from the concavity of the two relationships and the fact that when they are linear they have different slopes.

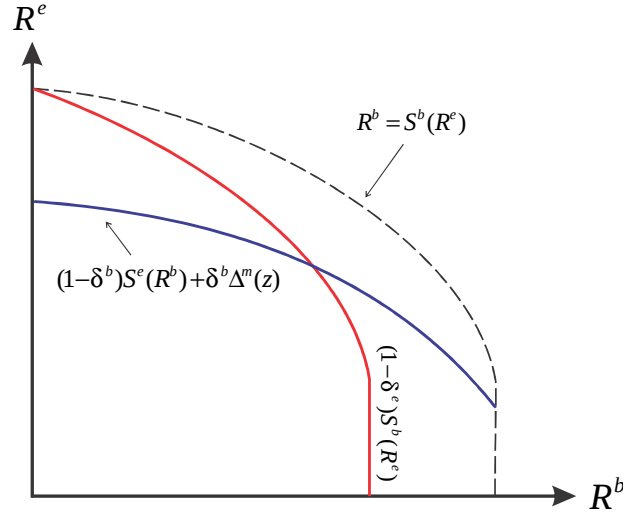


Figure 9: Determination of (R^b, R^e)

We are now in position to define a stationary, subgame perfect equilibrium of our alternating offer bargaining game. Such an equilibrium is composed of two offers, (k^e, d^e, ϕ^e) and (k^b, d^b, ϕ^b) , and two reservation surpluses, R^e and R^b , that solve (61)-(62), (65)-(66), and (70)-(71). One can check that strategies are individually optimal by applying the one-stage-deviation principle. Given acceptance rules, offers are optimal. And given of-

fers, acceptance rules are optimal. The existence and uniqueness of equilibrium follows from the uniqueness of the solution to (70)-(71).

Limiting equilibria when the risk of breakdown vanishes In the following, we consider limiting equilibria where the exogenous risk of breakdown of the negotiation is small. Rewrite the probabilities of breakdown as $\delta^e = \varepsilon \bar{\delta}^e$ and $\delta^b = \varepsilon \bar{\delta}^b$ where ε is small. From (70) and (71) as ε approaches 0, $S^e(R^b) - R^e \rightarrow 0$ and $S^b(R^e) - R^b \rightarrow 0$. The problems of the entrepreneur and the bank are dual problems leading to the same offer. In other words, when the risk of breakdown goes to zero, the first-mover advantage vanishes. Graphically, the equilibrium reservation values at the intersection of the blue and red curves in Figure 9 converge to a point on the dashed curve.

Consider ε small and suppose that offers are such that the borrowing constraint does not bind. From (61)-(62) and (65)-(66), $S^e(R^b) = f(k^*) - k^* - R^b$ and $S^b(R^e) = f(k^*) - k^* - R^e$. Both the entrepreneur and the bank offer the efficient investment size, k^* , and they use the interest payment, ϕ , to satisfy the acceptance rules of the responders. Plugging these expressions into (70) and (71) and taking the limit as ε goes to 0:

$$R^e \rightarrow \frac{\bar{\delta}^e [f(k^*) - k^*] + \bar{\delta}^b \Delta^m(a_m^e)}{\bar{\delta}^e + \bar{\delta}^b} \quad (72)$$

$$R^b \rightarrow \frac{\bar{\delta}^b}{\bar{\delta}^e + \bar{\delta}^b} [f(k^*) - k^* - \Delta^m(a_m^e)]. \quad (73)$$

From (73), the surplus of the bank approaches a fraction $\bar{\delta}^b / (\bar{\delta}^e + \bar{\delta}^b)$ of the whole surplus. This solution coincides with the axiomatic solution in the text when the bank's bargaining power is $\theta = \bar{\delta}^b / (\bar{\delta}^e + \bar{\delta}^b)$.

Consider next offers such that the liquidity constraint binds. From (61)-(62) and (65)-(66) $S^e(R^b) = f(k^e) - k^e - R^b$ where k^e is the highest solution to $k^e + R^b = \chi_b f(k^e) + a_m^e$ and $S^b(R^e) = f(k^b) - k^b - R^e$ where k^b is the solution to $(1 - \chi_b) f(k^b) = R^e + a_m^e$. Hence, from (70) and (71) the list (k^e, k^b, R^e, R^b) is determined by the following system:

$$R^e = (1 - \bar{\delta}^b \varepsilon) [f(k^e) - k^e - R^b] + \bar{\delta}^b \varepsilon \Delta^m(a_m^e) \quad (74)$$

$$R^f = (1 - \bar{\delta}^e \varepsilon) [f(k^b) - k^b - R^e] \quad (75)$$

$$R^b = \chi_b f(k^e) - k^e + a_m^e \quad (76)$$

$$R^e = (1 - \chi_b) f(k^b) - a_m^e \quad (77)$$

Rearranging (74) and (75) we obtain:

$$R^e = \frac{(1 - \bar{\delta}^b \varepsilon) \{f(k^e) - k^e - (1 - \bar{\delta}^e \varepsilon) [f(k^b) - k^b]\} + \bar{\delta}^b \varepsilon \Delta^m(a_m^e)}{1 - (1 - \bar{\delta}^b \varepsilon)(1 - \bar{\delta}^e \varepsilon)}.$$

Taking the limit as ε goes to zero and applying L'Hopital's rule:

$$R^e = \frac{\bar{\delta}^e [f(k) - k] + [f'(k) - 1] \left(\frac{dk^e}{d\varepsilon} - \frac{dk^b}{d\varepsilon} \right) + \bar{\delta}^b \Delta^m(a_m^e)}{\bar{\delta}^b + \bar{\delta}^e}. \quad (78)$$

The terms $dk^e/d\varepsilon$ and $dk^b/d\varepsilon$ are obtained by differentiating (74)-(77) in the neighborhood of $\varepsilon = 0$:

$$\frac{dk^e}{d\varepsilon} - \frac{dk^b}{d\varepsilon} = \frac{\bar{\delta}^e [f(k) - k - R^e]}{1 - \chi_b f'(k)}. \quad (79)$$

Substitute (79) into (78) and replaces R^e by $(1 - \chi_b)f(k) - a_m^e$:

$$\left(\frac{\bar{\delta}^b}{\bar{\delta}^e} \right) \frac{1 - \chi_b f'(k)}{(1 - \chi_b)f'(k)} = \frac{\chi_b f(k) - k + a_m^e}{(1 - \chi_b)f(k) - a_m^e - \Delta^m(a_m^e)}. \quad (80)$$

Equation (80) corresponds to the first-order condition from the Nash solution when $\theta = \bar{\delta}^b/(\bar{\delta}^e + \bar{\delta}^b)$. Hence, the subgame perfect equilibrium of the alternating offer bargaining game with exogenous risk of breakdown generates the same allocation as the axiomatic Nash solution.

A5. Technological linkages

In order to introduce linkages between entrepreneurs' borrowing constraints and provide an amplification mechanism from credit frictions, we now consider an environment where competitive firms produce a final consumption good by purchasing input factors from many monopolistically-competitive entrepreneurs.²⁵ For simplicity, we assume fiat money is not valued. The technology for producing the final consumption good takes the following CES form:

$$Y = \left[\int_0^1 (y_i)^\sigma di \right]^{\frac{\gamma}{\sigma}}, \quad \sigma < \gamma, \quad \gamma < 1 \quad (81)$$

where $i \in [0, 1]$ denotes the identity of an individual entrepreneur, y_i denotes the input produced by the entrepreneur in a match with a supplier, and $1/(1 - \sigma)$ is the elasticity of substitution between input factors. In addition, we assume the entrepreneur's technology to produce the secondary input from the supplier's primary intermediate good is linear, $y_i = k_i$. The chain of production is illustrated in Figure 10.

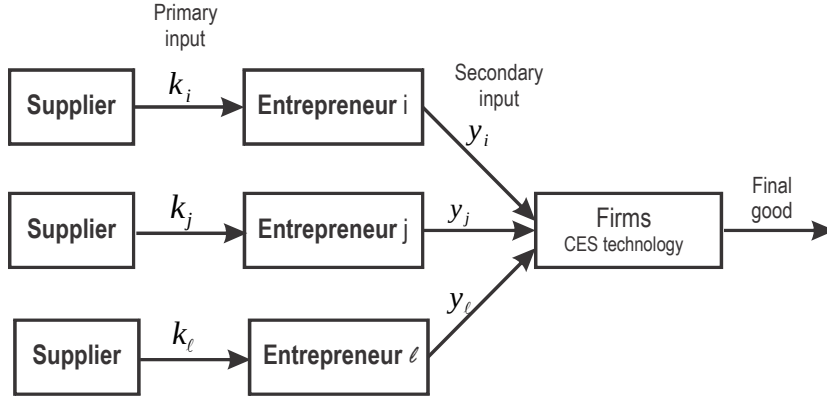


Figure 10: Model with technological linkages

The problem of a firm purchasing input factors from monopolistically competitive entrepreneurs is:

$$\max_{y_i \geq 0} \left\{ \left[\int_0^1 (y_i)^\sigma di \right]^{\frac{\gamma}{\sigma}} - \varpi_i y_i \right\}, \quad (82)$$

where ϖ_i is the price of the input factor in terms of the numéraire, which firms take as given. The FOC associated with (82) gives the inverse demand function for the input

²⁵There are recent studies that consider how non-financial linkages across firms impede interfirm trade and amplify the effects of financial frictions. See e.g. Bigio and La'O (2015) and Shourideh and Zetlin-Jones (2014) for models where complementarities between firms' input choices amplify the effects of financial frictions.

factor:

$$\varpi_i = \frac{\partial Y}{\partial y_i} = \gamma y_i^{\sigma-1} Y^{\frac{\gamma-\sigma}{\gamma}}.$$

Provided that $\sigma < \gamma$, the price of input i , ϖ_i , depends positively on the output of other entrepreneurs, Y . This provides a channel for credit frictions to be amplified. Since an entrepreneur with a tight borrowing constraint can only supply a small quantity of input, this reduces total output and affects the demand for other entrepreneurs' inputs. Using that entrepreneur i can transform k_i units of primary input into y_i units of secondary input, the total revenue of entrepreneur i from acquiring k_i units of primary input from a supplier is

$$f(k_i; Y) \equiv \varpi_i k_i = \gamma Y^{\frac{\gamma-\sigma}{\gamma}} k_i^\sigma. \quad (83)$$

The revenue function in (83) is formally equivalent to the Cobb-Douglas technology introduced in Section 4, except the total factor productivity term is now endogenous and equal to $\gamma Y^{\frac{\gamma-\sigma}{\gamma}}$. As a result, the DM bargaining problem is the same Nash bargaining problem as (10) where z from Section 4 is replaced with $\gamma Y^{\frac{\gamma-\sigma}{\gamma}}$ and γ is replaced with σ . The solution is

$$k_i = (\sigma\gamma)^{\frac{1}{1-\sigma}} Y^{\frac{\gamma-\sigma}{\gamma(1-\sigma)}} \text{ if } \chi_{b,i} \geq \chi_b^* \equiv (1-\theta)\sigma + \theta \quad (84)$$

$$= \left[\frac{\chi_{b,i}}{(1-\theta)\sigma + \theta} \right]^{\frac{1}{1-\sigma}} (\sigma\gamma)^{\frac{1}{1-\sigma}} Y^{\frac{\gamma-\sigma}{\gamma(1-\sigma)}} \text{ otherwise,} \quad (85)$$

where pledgeability, $\chi_{b,i}$, is entrepreneur specific. From (84)-(85), individual investment increases with aggregate output due to technological linkages between entrepreneurs' inputs. Let $G(\chi_b)$ denote the cumulative distribution of pledgeability coefficients with density $g(\chi_b)$. From (81), aggregate output is $Y = \left[\int_0^1 \alpha \lambda(k_i)^\sigma di \right]^{\frac{\gamma}{\sigma}}$ since each entrepreneur i has an opportunity to invest with probability $\alpha\lambda$. From (84) and (85), aggregate output is

$$Y = (\alpha\lambda)^{\frac{\lambda(1-\sigma)}{\sigma(1-\lambda)}} (\sigma\gamma)^{\frac{\gamma}{1-\gamma}} \left[\frac{\int_0^{\chi_b^*} (\chi_b)^{\frac{\sigma}{1-\sigma}} dG(\chi_b)}{[(1-\theta)\sigma + \theta]^{\frac{\sigma}{1-\sigma}}} + [1 - G(\chi_b^*)] \right]^{\frac{\gamma(1-\sigma)}{\sigma(1-\gamma)}}, \quad (86)$$

which depends on the entire distribution of pledgeability coefficients. For instance, if $\sigma = 1/2$ then aggregate output simplifies to

$$Y = \left[\frac{\alpha\lambda\gamma\mathbb{E}[\chi_b]}{1+\theta} \right]^{\frac{\gamma}{1-\gamma}},$$

which depends positively on the expected pledgeability across all entrepreneurs. Substi-

tuting Y from (86) into (84) and (85), the expression for k_i becomes

$$k_i = \min \left\{ 1, \left[\frac{\chi_{b,i}}{(1-\theta)\sigma + \theta} \right]^{\frac{1}{1-\sigma}} \right\} (\alpha\lambda)^{\frac{\gamma-\sigma}{\sigma(1-\gamma)}} (\sigma\gamma)^{\frac{1}{1-\gamma}} \left[\frac{\int_0^{\chi_b^*} (\chi_b)^{\frac{\sigma}{1-\sigma}} dG(\chi_b)}{[(1-\theta)\sigma + \theta]^{\frac{\sigma}{1-\sigma}}} + [1 - G(\chi_b^*)] \right]^{\frac{\gamma-\sigma}{\sigma(1-\gamma)}}. \quad (87)$$

According to (87), the investment decision of an individual entrepreneur depends on the credit conditions of other entrepreneurs in the economy. If a shock reduces χ_b for all other entrepreneurs, then entrepreneur i will reduce his own investment provided that $\gamma > \sigma$. For instance, assuming $\sigma = 1/2$, the elasticity of entrepreneur i 's investment size relative to the average pledgeability of other firms is

$$\frac{\partial \ln k_i}{\partial \ln \mathbb{E}[\chi_b]} = \frac{2\gamma - 1}{1 - \gamma} > 0 \text{ for all } \gamma > \frac{1}{2}.$$

Notice the closer the production function is to a constant-returns-to-scale technology, the larger is the effect of a change in average pledgeability on the entrepreneur's investment decision.