

A Demand Theory of the Price Level

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Abstract

In this paper I propose a theory of a globally unique price level based on the simple idea that the price equates demand with supply in the goods market. Monetary policy through setting nominal interest rates, e.g. an interest rate peg, and fiscal policy, which satisfies the present value budget constraint at all times, jointly determine the price level. In contrast to the conventional view the long run inflation rate is, in the absence of output growth, equal to the growth rate of nominal government spending which is controlled by fiscal policy. This new theory where nominal government spending anchors aggregate demand and therefore current and future prices suggests a different perspective on the fiscal and monetary transmission mechanism, on policy coordination, on policies at the zero-lower bound and on U.S. inflation history.

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1 Introduction

This paper proposes a novel and to best of my knowledge the first theory of a globally determined price level. The idea is quite simple. Monetary policy is described through setting an arbitrary sequence of nominal interest rates, for example an interest rate peg. Fiscal policy sets sequences of nominal government spending, taxes and government debt which satisfy the present value government budget constraint. The price level is then determined such that demand equals supply in the goods market. Both monetary and fiscal policies aiming at increasing or decreasing the price level are effective in this endeavor only if they are effective in stimulating or contracting nominal aggregate demand.

The importance of a uniquely determined price level is best understood from considering the problems one runs into if prices are indetermined. The presumably most compelling illustration of these difficulties is in Cochrane (2015)'s policy analysis of new-Keynesian models during a liquidity trap. Cochrane (2015) shows that the policy predictions depend strongly on the choices made by the researcher to select a specific equilibrium. Depending on these choices, the fiscal multiplier can be arbitrarily large or negative. The root of this problem is clear. The price level is indetermined and therefore the researcher has to select one equilibrium where at the same time a continuum of other choices with very different policy predictions are possible. This lack of robust policy implications is clearly unsatisfactory.

The demand theory of the price level proposed in this paper to overcome these difficulties is motivated by the findings of a large empirical literature which rejects the permanent income hypothesis (Campbell and Deaton (1989), Attanasio and Davis (1996), Blundell et al. (2008), Attanasio and Pavoni (2011)). One of the striking empirical finding in this literature is that a permanent income gain - like a permanent tax rebate - does increase household consumption less than one-for-one and thus increases savings as well. This simple fact is the key to the determinacy result as it implies that taking prices as given, a permanent decrease in government spending by one dollar and a simultaneous permanent tax rebate of the same amount to private households lowers real total aggregate demand - the sum of private and government demand. The same logic also establishes why in a steady state real aggregate demand depends on the price level. Given monetary and fiscal policy, a higher steady state price level lowers real government consumption since government spending is (partially) fixed in nominal terms and at the same time lowers the tax burden for the private sector by the same amount. As private sector demand does not substitute one-for-one for the drop in government consumption but instead saves a fraction of the tax reduction, aggregate real

demand falls, establishing a downward sloping aggregate demand-price curve. The unique equilibrium steady state price level is then such that aggregate real demand equals real supply. It is important to point out that here the price level is determined whereas it would still be an unknown variable if only the inflation rate was determined, the typical case in new-Keynesian models with an active monetary policy. Moving beyond steady state results and establishing price determinacy globally requires to assume that the above empirical finding also holds outside the steady state, that is that a precautionary demand never vanishes.¹

It is important to emphasize, however, that it is the presence of precautionary savings which delivers the result and that prices are not determined in every model where Ricardian equivalence fails. For example the price level is not determined in an economy where one fraction of households are hand-to-mouth consumers whereas the remaining households act according to the permanent income hypothesis (PIH). The reason for the indeterminacy is the absence of precautionary savings. The PIH consumers increase their consumption one-for-one in response to a permanent tax rebate since this what a PIH household does and the hand-to-mouth consumers do the same just not as a result of optimization but by assumption. In such a model an increase in the price level also lowers real government consumption (since fixed in nominal terms) and increases private consumption but does not affect total consumption since the drop in government consumption is exactly offset by the increase in private demand. As a result aggregate demand equals supply for an infinite number of price levels each of them corresponding to a different size of the government.

To keep the model tractable, I use the simplest model which delivers the empirical finding on individual consumption and precautionary saving behavior discussed above. The key simplifying assumption is that households are members of a family which provides insurance such that the distribution of asset holdings across agent is degenerate. These families live in an infinite horizon endowment economy without capital which in terms of preferences, technology and trading arrangements is closer to an infinite horizon replication of a Diamond and Dybvig (1983) economy than to conventional macroeconomic models. It is however not difficult to integrate the framework into a standard framework such as the Aiyagari (1994) incomplete markets model and quantitatively explore policy implications. An advantage of using the simpler framework, besides enabling the researcher to better understand the monetary and fiscal transmission mechanism, is that it is not difficult to add a banking sector to the model. Although beyond the scope of this paper, such an extension will enable

¹In a standard incomplete markets model (Bewley-Imrohoroglu-Huggett-Aiyagari), the precautionary savings motive arises due to a potentially binding credit constraint and thus the assumption is trivially satisfied without an additional assumption as some households are always constrained.

the researcher to study the interaction of banking distress, policy, deflation and the real economy in a model where the price level is determinate.

The main result of the theoretical analysis is that the price level is globally uniquely determined and that it depends both on monetary and fiscal policy. To illustrate the workings of the model I then add some key features to the model to enable a meaningful numerical analysis: labor supply is endogenous, prices are sticky and only a small fraction of government spending is nominally fixed. I then numerically compute impulse responses to a monetary and a fiscal policy shock as well as to a technology and a discount factor shock. I find that all impulse responses are line with their empirical counterparts and the reason why is easily explained using the simple supply/demand logic which is the foundation of the determinacy result. An increase in nominal interest rates stimulates saving and therefore lowers consumption demand, implying a drop in prices. An increase in government spending stimulates aggregate demand, implying a rise in prices. An increase in technology raises supply and households incentives to save, implying a drop in prices. And finally an increase in the discount factor stimulates savings since households are more patient, implying a drop in prices. Quite remarkably, the model delivers these results not only for sticky prices but also when prices are flexible. In particular, price rigidities are not needed for monetary policy to have effects.

In the theory proposed in this paper fiscal policy provides a nominal anchor through setting nominal spending and nominal bonds making the treasury a key player in determining the price level and the key player in determining the long-run inflation rate. The steady state inflation is equal to the growth rate of nominal government spending (minus productivity growth) and is therefore controlled by the treasury. In contrast to conventional wisdom, a tough, independent central bank is not only not sufficient to guarantee price stability in the long-run, but a central bank has no direct control over long-run inflation. Through controlling the nominal anchor the treasury can always get its way when it comes to long-run inflation. However, if the treasury does not exercise its power, for example if government spending is fixed in real and not in nominal terms, the central bank takes over determining the steady-state inflation rate as conventional wisdom tells us but here in a model with a determinate price level since government debt is nominal.

The price level is always jointly controlled by fiscal and monetary policy as the impulse responses for an increase in spending and in the nominal interest rate already suggest. Fiscal policy can raise the price level and stimulate employment through actively increasing spending. But also an apparently passive fiscal policy, one that fixes nominal spending,

is actually automatically stabilizing the economy. Consider for example an increase in the discount factor which lowers prices and contracts employment. Lower prices automatically lead to higher real government spending since nominal spending is fixed and thus stimulate aggregate demand which partly offsets the fall in prices and employment. The effectiveness of expansionary fiscal policy in stabilizing the economy depends also on how it is financed, through higher deficits or higher taxes. Not surprisingly, using higher taxes is less effective than increasing deficits. Higher taxes lead to lower demand and thus partially offset the stimulative demand effects of higher government spending, an effect avoided when higher deficits are used.

Monetary policy can lower the price level and employment through increasing nominal interest rates and is quite effective in stabilizing the economy. Consider for example an increase in the discount factor. This shock can be fully neutralized through lowering nominal interest rates by the same amount as the increase in the discount factor keeping the effective discounting - given by the product of the discount factor and the nominal interest rate - by households unchanged. All variables, including employment, prices and consumption remain at their steady state values. In contrast, an expansion in government spending cannot at the same time stabilize employment and thus output and consumption, suggesting that monetary policy is the more effective option for stabilizing the economy.

However, the zero lower bound (ZLB) makes monetary policy ineffective and fiscal policy has to step in, what is the case if the increase in the discount factor is so large that stabilizing the economy only through nominal interest rates would require to set them at a negative value, what is impossible (the ZLB binds). In this scenario, expansionary fiscal policy can stabilize aggregate demand and completely offset the contractionary effects of the discount factor increase, such that prices and employment remain at their steady state values.

A stimulative policy initiated by the treasury to stimulate employment and raise prices requires coordination with monetary policy. In the absence of such a coordination, a central bank committed only to price stability, can raise nominal interest rates and successfully undo the price and employment effects of the fiscal stimulus. The result of this (attempted) stimulative fiscal policy and the response of monetary policy is: no change in employment and prices but higher government spending, taxes and debt. It is obvious that policy coordination extends beyond this example, simply because monetary and fiscal policy jointly determine demand and prices in this framework. Such political economy questions naturally come up in a framework where the price level is determinate and they can be answered because the price level is determinate.

Whereas monetary policy can neutralize short-run inflationary fiscal policy, taming inflation in the medium and long-run requires to constrain fiscal policy from running an inflationary spending plan. Although the central bank does not set the spending itself and therefore the treasury can control medium and long-run inflation if it wants to, control of the nominal interest rate is an effective tool to make an inflationary policy quite costly. Raising the nominal interest raises the interest payments on government debt which can sharply constrain government spending. High nominal interest rates may thus force the treasury into a less expansionary fiscal policy and thus indirectly lead to a lower inflation rate. Since there is no upper bound on nominal interest rates, there is no bound on the cost the central bank can inflict on the treasury. But the central bank can also support an expansionary fiscal policy through lowering nominal interest rates if it considers inflation to be too low. However the ZLB puts a limit on the budgetary support the central bank can provide. Independence of central banks guarantee that those interest rate decisions are taken by monetary and not by fiscal policy. As central banks arguably put more weight on price stability than the treasury which is more interested in taming deficits, independence leads to a more active interest rate policy to tame inflation than it would if the treasury controlled the interest rate.

This reasoning and the theory laid out in this paper also suggests a different perspective on two episodes of US inflation history. After experiencing high inflation rates in the 1970s, the 1980s were a success in keeping inflation rates low. The standard interpretation is that central banks eventually recognized that keeping inflation low is their primary objective and as a consequence were successful. The framework proposed in this paper suggests that it was not the change in the conduct of monetary policy but a switch to a less expansionary fiscal policy during the presidency of Ronald Reagan, maybe be forced to do so by prolonged high nominal interest rates set by central banks under chairman Paul Volcker and the resulting high deficits.

For pedagogical purposes, I start with a baseline model where labor is inelastically supplied and prices are flexible in Section 2. In Section 3 I prove price level determinacy for arbitrary sequences of monetary and fiscal policy. I then show that monetary and fiscal policy together determine the price level and the inflation rate and how these variables respond to policy changes and shocks. In Section 4, I describe the extension to the case of elastic labor supply and sticky prices and I will present some numerical exercises to illustrate the workings of the model, including computing impulse responses, government spending as automatic stabilizer, coordination of monetary and fiscal policy and policies at the ZLB. Section 5 concludes.

2 Model

Households in this economy are infinitely-lived and heterogenous in their spending needs. In terms of preferences and trading frictions each period resembles a Diamond and Dybvig (1983) economy. However, to keep the heterogeneity analytically tractable, households are members of large families which at the beginning of each period pool all family assets. As in the Bewley-Imrohoroglu-Huggett-Aiyagari incomplete market model households can acquire one interest-bearing liquid asset. I will not distinguish between money and bonds but show in Section 3.6, following the analysis of a cashless economy in Woodford (2003), that a demand for money can be added to the model, so that households can acquire cash and a liquid asset. The demand for money then determines the quantity of money that the central bank will need to supply in order to implement its nominal interest rate target. In the theoretical analysis in this and the next section prices are flexible and labor supply is inelastic since these assumptions by themselves do not imply determinacy or indeterminacy of the price level. To provide numerical illustrations of the workings of the model I will relax both assumptions in Section 4 where prices are sticky and labor supply is elastic.

2.1 Households

Time is discrete and extends from $t = 0, \dots, \infty$. There is a continuum of measure one of households. Each period $t \geq 0$ is divided into two distinct and successive sub-periods t_1 and t_2 . The only source of uncertainty is an idiosyncratic i.i.d. emergency expenditure shock in the spirit of Diamond and Dybvig (1983), which realizes only in period t_2 .

The timing of events is as follows: In subperiod t_1 , before the realization of the risk household h consumes C_t^h and consumes c_t^h in the second subperiod t_2 .

Households are exposed to liquidity risks at t_2 , which leads to heterogeneity in consumption and asset holdings. To keep the model tractable, I make the assumption that each household is a family which consists of a continuum of individuals of measure one. Each member of the household has a need for spending in t_2 which is governed by the i.i.d. shock

$$\theta \in [\underline{\theta}, \infty] \sim F, \tag{1}$$

where $\underline{\theta} \geq 0$ and corresponding pdf f . A household who experiences a shock θ and consumes

C_t^h in period t_1 and c_t^h at t_2 derives utility

$$u(C_t^h) + \theta v(c_t^h) \quad (2)$$

in period t . Each individual's demand for consumption at t_2 is increasing in the idiosyncratic value of θ . Because the household has a continuum of members the distribution of θ across the members of a household is given by the distribution F .

As I will later add elastic labor supply, I assume here already a linear technology which transforms each individual's inelastically supplied unit of labor $h^i = 1$ into Ah^i units of output y :

$$y = Ah, \quad (3)$$

so that each individual has A consumption goods in period t_1 .

As in Diamond and Dybvig (1983), because the expenditures needs at t_2 are sudden, I assume that a liquid asset (bonds) is necessary to make these expenditures beyond some level $\bar{b}$. The interpretation is that each member of the household has to acquire period t_2 consumption from the market and cannot obtain it from his or her own family. Up to some limit $\bar{b}$ this member can obtain a credit. For all additional expenditures the individual has to use savings (bonds). He or she cannot obtain any resources from the family at t_2 because they are spatially separated.

In period t_1 each household chooses consumption in period t_1 , C_t^h , consumption at t_2 as a function of θ , $c_t^h(\theta)$, and how many nominal bonds to acquire, b_t^h , so that b_t^h/P_t is the value of bonds in terms of consumption goods, where P_t is the price level at time t (same at t_1 and t_2 as goods are produced with the same technology). The return on bonds is R_{t+1} . Since all members of a household are identical each member has the same level of consumption at t_1 and enters period t_2 with the same amount of bonds. During period t_2 , each member has only access to his or her own bonds to be spend on consumption $c_t^h(\theta)$,

$$c_t^h(\theta) - \bar{b} \leq b_t^h/P_t. \quad (4)$$

Excess bonds not needed for emergency expenditures, $\min(b_t^h - P_t(c_t^h(\theta) - \bar{b}), b_t^h)$, are returned to the family.

The household's budget constraint at t_1 is:

$$P_t C_t^h + b_t^h = P_t A h_t - T_t + R_t b_{t-1}^h - P_t \bar{C}_t^h, \quad (5)$$

where T_t are nominal tax obligations of the household to be paid at t_1 , $\bar{C}_t^h$ is the sale of household consumption goods to members of other families who need consumption in period t_2 . Since θ is distributed according to F in all families, it follows that in a symmetric equilibrium expected spending on consumption in period t_2 is equal to sales in period t_2 :

$$E_\theta(c_t^h(\theta)) = \bar{C}_t^h, \quad (6)$$

so that the amount of bonds owned by a household at the end of period t_2 equals

$$b_{t_2}^h = E_\theta(b_t^h - P_t c_t^h(\theta)) + P_t \bar{C}_t^h = b_t^h, \quad (7)$$

Thus, the household's flow budget constraints simplifies to

$$P_t C_t^h + E_\theta(P_t c_t^h(\theta)) + b_t^h = P_t A h_t - T_t + R_t b_{t-1}^h, \quad (8)$$

The decision problem of a household with initial period bond holdings b_t is

$$V_t(b_{t-1}^h) = \max_{b_t^h, C_t^h, c_t^h(\theta)} \{u(C_t^h) + E_\theta \theta v(c_t^h(\theta)) + \beta E_t[V_{t+1}(b_t^h)]\} \quad (9)$$

subject to the flow budget constraint (8) and the liquidity constraint (4).

The optimal decision for $c_t(\theta)$ is described through threshold $\hat{\theta}_t$, which solves

$$\hat{\theta}_t v'(b_t^h/P_t + \bar{b}) = u'(C_t) \quad (10)$$

such that

$$\begin{aligned} i) \quad & c_t(\theta) \text{ solves } \theta v'(c_t(\theta)) = u'(C_t) \quad \text{if } \theta \leq \hat{\theta}_t, \text{ i.e. } v'(b_t^h/P_t + \bar{b}) \leq u'(C_t), \\ ii) \quad & c_t(\theta) = b_t^h/P_t + \bar{b} \quad \text{if } \theta > \hat{\theta}_t, \text{ i.e. } v'(b_t^h/P_t + \bar{b}) > u'(C_t). \end{aligned}$$

In case i), θ is not very high so that households do not have to use all their bonds for emergency expenditures at t_2 . In case ii), θ is so high that all bonds acquired at t_1 and

the full credit line $\bar{b}$ are used for consumption, so that overall consumption spending equals $b_t^h/P_t + \bar{b}$, the sum of the consumption value of bonds, b_t^h/P_t , and credit $\bar{b}$. The credit line expands the consumption possibilities of each individual at t_2 . Paying with savings or by credit is equivalent since the family has to pay back the credit in $t+1$ with a real interest rate $\frac{R_{t+1}P_t}{P_{t+1}}$. Note that in the absence of a binding liquidity constraint, households would choose the first best allocation $c_t^{FB}(\theta)$ which solves $\theta v'(c_t^{FB}(\theta)) = u'(C_t)$, which is not feasible for values of θ with $c_t^{FB}(\theta) > b_t^h/P_t + \bar{b}$.

To guarantee existence of an equilibrium I impose an Inada condition for bond holdings which ensure that the demand for liquidity is sufficiently strong²

$$v'(\bar{b} + \mathbf{0})E[\theta \mid \theta \geq \frac{u'(A - \bar{b})}{v'(\bar{b} + \mathbf{0})}] > u'(A - \bar{b}), \quad (11)$$

that is the marginal value of acquiring a bond (LHS) exceeds its cost (RHS) when the household has zero bonds.³

The remaining decision how much bonds to acquire is characterized through the first-order condition:

$$u'(C_t) = \int_{\hat{\theta}_t}^{\infty} \theta v'(b_t^h/P_t + \bar{b}) dF(\theta) + F(\hat{\theta}_t) E_t[\frac{R_{t+1}P_t}{P_{t+1}} \beta u'(C_{t+1})]. \quad (12)$$

Observe that if an individual's expenditure needs are strong enough, $\theta_t > \hat{\theta}_t$, what is the case with probability $1 - F(\hat{\theta}_t)$, there is a shortage of the liquid asset. The infinite support of θ ensures the existence of a finite $\hat{\theta}_t$ such that there is shortage of the liquid asset with positive, potentially small, probability.

2.2 Fiscal and Monetary Policy

The aim of the paper is to show how monetary policy and fiscal policy and their interaction determine the price level. A standard way to represent monetary policy is as setting a sequence of nominal interest rates,

$$\mathcal{R} = R_0, R_1, R_2, \dots, R_t, \dots \quad (13)$$

²This condition is for example satisfied if $F = \text{LogNormal}(0, 1)$ and $\frac{v'(\bar{b})}{u'(A - \bar{b})} > 1$.

³Note that in this case the LHS is not identical but smaller than the marginal value of a bond since the threshold $\hat{\theta} \leq \frac{u'(A - \bar{b})}{v'(\bar{b} + \mathbf{0})}$, ensuring existence of an equilibrium. At the cost of a more involved notation but without any substantive gain the Inada condition could be made exact.

Fiscal policy is represented by a sequence of nominal government spending

$$\mathcal{G} = G_0, G_1, \dots, G_t, \dots, \quad (14)$$

which need to be financed by levying nominal lump-sum taxes T_t ,

$$\mathcal{T} = T_0, T_1, \dots, T_t, \dots \quad (15)$$

The government's flow budget constraint has to be satisfied at any point in time, which implicitly defines a sequence of nominal bonds

$$B_{t+1} = R_t B_t + G_t - T_t, \quad (16)$$

such that the intertemporal government budget constraint is satisfied:

$$B_0 = \sum_{t=0}^{\infty} (T_t - G_t) \prod_{s=0}^t \frac{1}{R_s} \quad (17)$$

and

$$\lim_{t \rightarrow \infty} B_t \prod_{s=0}^{t-1} \frac{1}{R_s} = 0. \quad (18)$$

Since fiscal and tax policies are expressed in nominal terms, this constraint holds for *all* sequences of prices,

$$\mathcal{P} = P_0, P_1, \dots, P_t, \dots \quad (19)$$

In particular the price level is not determined such that the government budget constraint holds. Finally define the sequence of bonds

$$\mathcal{B} = B_0, B_1, \dots, B_t, \dots \quad (20)$$

2.3 Competitive Equilibrium

Definition 1. *Given sequences of nominal interest rates $\mathcal{R}$, nominal government spending $\mathcal{G}$, nominal taxes $\mathcal{T}$ and nominal bonds $\mathcal{B}$ a competitive symmetric equilibrium are sequences of consumption spending $\{C_t\}_{t=0}^{\infty}$ at t_1 and $\{c_t(\theta)\}_{t=0}^{\infty}$ at t_2 , bonds purchases $\{b_t^h\}_{t=0}^{\infty}$ and prices $\mathcal{P}$, such that for all t , the following holds:*

1. *Households take prices and policies as given and choose $\{C_t, c_t(\theta), b_t^h\}_{t=0}^{\infty}$ to maximize utility.*
2. *Given prices, firms choose $\{h_t\}_{t=0}^{\infty}$ to maximize profits.*
3. *The government budget constraint holds (17).*
4. *Market Clearing and Resource constraint:*

(a) *Bond Market :* $B_t = b_t^h$,

(b) *Resource Constraint* $C_t + E_{\theta}c_t(\theta) + G_t = Ah_t$,

(c) *Labor Market:* $h_t = 1$.

3 Prices, Inflation and Nominal Demand

This section shows that the price level is determined as the unique solution where supply equals demand. The main result which delivers determinacy is that aggregate demand - the sum of private and government demand - is decreasing in the price level. Key to this result is that households engage in precautionary savings. A fall in government consumption is then not offset one-for-one by an increase in private consumption of the same amount but instead households engage in precautionary savings, implying lower aggregate demand for a given price level.

This line of reasoning will be used throughout this section to first show the existence and uniqueness of a steady state price level, then to rule out a vanishing or exploding price level and to finally prove determinacy globally.

3.1 Price Level Determinacy: Steady State

Establishing that the steady state price level is determined proceeds in several steps. I first characterize the steady state and prove existence and uniqueness of the steady state price level. Next, I show that the nominal anchor provided by government spending prevents prices from converging to zero. Furthermore I establish that an exploding price level would lead to insufficient market demand as private consumption is not fully substituting for government demand but that households instead engage in precautionary savings, leading to a downward pressure on prices and thus ruling out such an explosive path. Finally, in Section 3.2, I use the same arguments used to rule out explosive and vanishing price paths to prove price level determinacy outside of steady states in Theorem 1.

In a *steady state* nominal interest rates are constant at R , nominal government spending, taxes and nominal bonds are all strictly positive growing at rate γ ,

$$G_t = G(1 + \gamma)^t, T_t = T(1 + \gamma)^t, B_t = B(1 + \gamma)^t, \quad (21)$$

consumption C at t_1 and $c(\theta)$ at t_2 are time-invariant, and prices are growing at a constant rate $\frac{P_{t+1}}{P_t} = \pi_{t+1} = \pi$,

$$P_t = P(1 + \pi)^t. \quad (22)$$

Since in a steady state C_t , B_t/P_t , $\hat{\theta}_t$ and $\frac{R_{t+1}}{1 + \pi_{t+1}}$ are constant, equation (12) implies that

$$\frac{\beta R}{1 + \pi} < 1. \quad (23)$$

Permanently higher nominal interest rates would not be consistent with a steady state and instead would lead to exploding asset demand as in the Bewley-Imrohoroglu-Huggett-Aiyagari incomplete markets model.

In a steady state real government spending is constant, implying that the inflation rate π equals the growth rate of nominal spending γ ,

$$\pi = \gamma. \quad (24)$$

Next, I characterize equilibrium consumption C_t at t_1 and how it depends on the price

level before I move to the main result on the price level,

$$C_t = A - \frac{G_t}{P_t} - E_\theta c_t(\theta), \quad (25)$$

where $E_\theta c_t(\theta)$ is the expected family consumption at t_2 . Equilibrium consumption C then equals output A minus government consumption G_t/P_t and consumption at t_2 . Clearly, G/P is falling in the price level, leaving more resources to the private economy and implying that consumption C at t_1 increases in P . Households' aim to also increase spending at t_2 is partly prevented by the credit constraint (4), which imposes an upper bound on spending at t_2 . As a result more households are credit constrained and the credit constraint becomes binding at a lower value of θ for higher price levels, that is the equilibrium threshold $\hat{\theta}$ falls in P . In the appendix I prove

Proposition 1. *Given a fixed sequence of government policies, consumption C_t in period t_1 can be written as a function of P_t only, $C_t = C(P_t)$, and is increasing in P_t and the threshold $\hat{\theta}_t$ is a function of P_t only, $\hat{\theta}_t = \hat{\theta}(P_t)$ and is decreasing in P_t .*

The steady state price level P^* then clears the good market as the solution to

$$u'(C(P^*)) = \int_{\hat{\theta}}^{\infty} \theta v'(B/P^* + \bar{b}) dF(\theta) + F(\hat{\theta}(P^*)) \frac{R}{(1 + \pi)} \beta u'(C(P^*)), \quad (26)$$

so that the price at time t , $P_t = P^*(1 + \pi)^t$.

Proposition 2. *A steady exists and is unique. In particular, the steady state price level is determined uniquely. The steady state inflation rate is equal to the growth rate of nominal government spending,*

$$\pi = \gamma,$$

which is set by fiscal policy only.

The preceding analysis implies that an equilibrium defines a difference equation in prices, relating prices in periods t and $t + 1$,

$$\Phi(\tilde{P}_t) := \frac{u'(C(\tilde{P}_t)) - \int_{\hat{\theta}(\tilde{P}_t)}^{\infty} \theta v'(B_t/\tilde{P}_t + \bar{b}) dF(\theta)}{\tilde{P}_t F(\hat{\theta}(\tilde{P}_t))} = \frac{R_{t+1}}{1 + \gamma} \beta \frac{u'(C(\tilde{P}_{t+1}))}{\tilde{P}_{t+1}} =: \frac{R_{t+1}}{1 + \gamma} \Gamma(\tilde{P}_{t+1}), \quad (27)$$

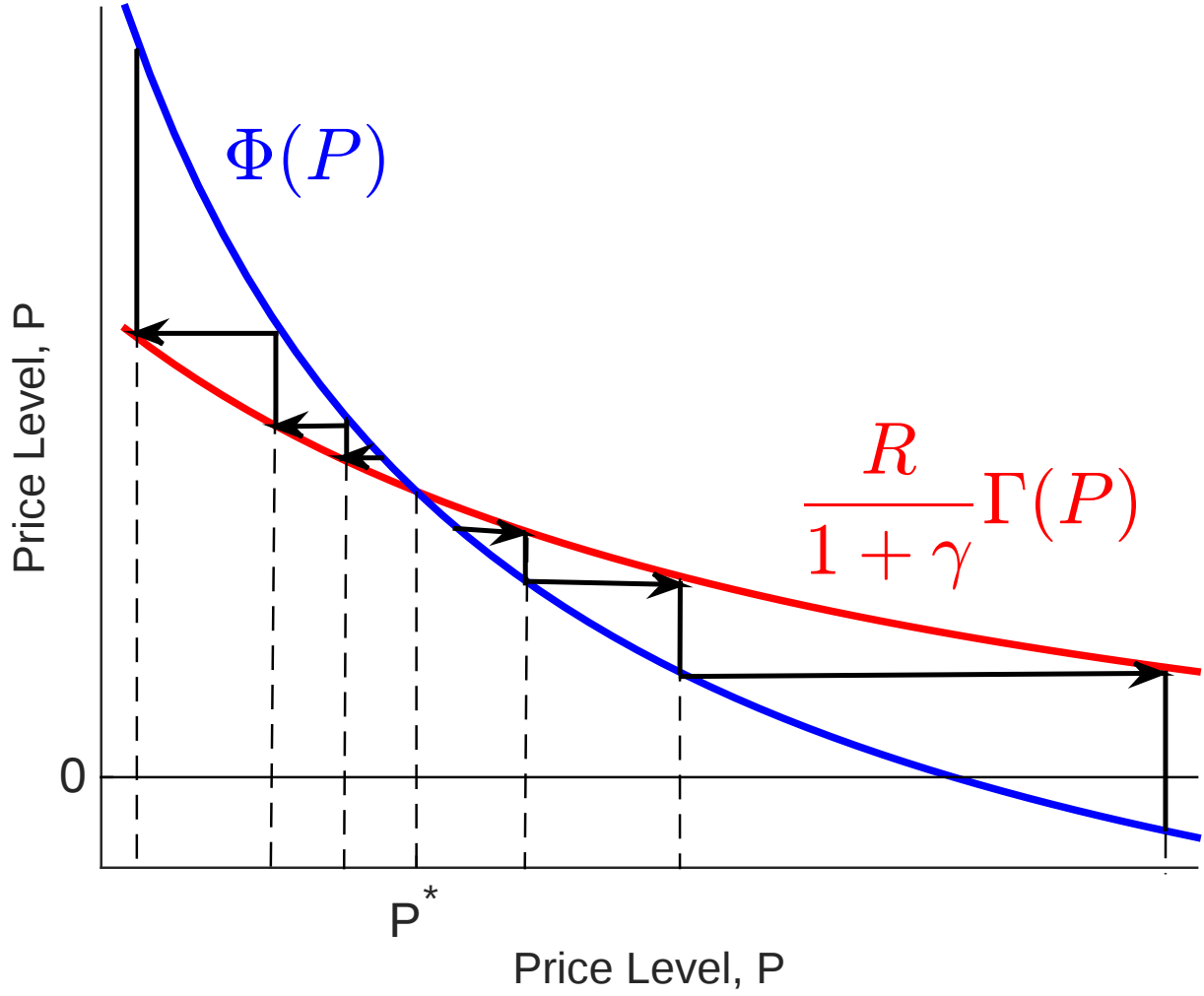


Figure 1: Dynamics of the Price Level

where I define the detrended price

$$\tilde{P}_t = \frac{P_t}{(1+\gamma)^t}. \quad (28)$$

In the steady state, $\tilde{P}_t = P^*$. I now show that this is the only solution by ruling out vanishing and explosive price paths. Since both functions Φ and Γ are decreasing in $\tilde{P}_t$ and $\tilde{P}_{t+1}$ respectively, this equation allows to solve $\tilde{P}_{t+1}$ as an increasing function of $\tilde{P}_t$,

$$\tilde{P}_{t+1} = \Gamma^{-1}\left(\frac{1+\gamma}{R_{t+1}}\Phi(\tilde{P}_t)\right). \quad (29)$$

Figure 1 illustrates the dynamics of the price level, $\tilde{P}$, that is implied by equations (27) and

(29) . Both functions Φ and Γ are downward sloping and from the diagram it is apparent that there is a unique steady state at P^* since Φ is steeper than Γ .

A price level $\tilde{P}_t$ higher than P^* implies lower real government spending and higher real private demand and savings. Goods market clearing requires an even higher price $\tilde{P}_{t+1}$ next period to decrease savings and increase demand to market clearing levels, which again leads by the same arguments to a higher price level in period $t+2$, $\tilde{P}_{t+2}$, and so on. Eventually the exploding price path will drive real government spending to zero and since precautionary savings do not disappear private demand will fall short of aggregate supply, establishing that this price sequence is not an equilibrium. The non-existence becomes apparent in the diagram as Φ eventually gets negative and the iteration breaks down since $\Gamma > 0$.

A price level $\tilde{P}_t$ lower than P^* implies higher real government spending and lower real private demand and savings. Goods market clearing requires an even lower price $\tilde{P}_{t+1}$ next period to increase savings and increase demand to market clearing levels, which again leads by the same arguments to a lower price level in period $t+2$, $\tilde{P}_{t+2}$, and so on. Eventually the price level will be so low that government demand exceeds output, which clearly cannot be an equilibrium.

The only price sequence which forms an equilibrium is the one where the price is constant and equal to the steady state price level P^* , that is the price at time t equals $P^*(1+\gamma)^t$ and aggregate demand equals aggregate supply.

Proposition 3. *For a constant nominal interest rates R , and strictly positive nominal government spending, taxes and nominal bonds which are growing at rate g , there is a unique equilibrium,*

$$\begin{aligned}\tilde{P}_t &= P^* \\ P_t &= P^*(1+\gamma)^t\end{aligned}$$

so that $C_t = C(P^*)$ and $\hat{\theta}_t = \hat{\theta}(P^*)$.

In particular the nominal anchor set by fiscal policy ensures that both exploding prices and vanishing prices are not an equilibrium.

3.2 Price Level Determinacy: Non Steady State

The analysis so far has considered stationary policies where the nominal interest rate is constant and fiscal policy is characterized by a constant growth rate. Since I also want to consider impulse responses to shocks to the nominal interest rate or to government spending, the analysis has to go beyond steady states. To this aim I now consider arbitrary sequences of nominal interest rates R_t , B_t , T_t and G_t which are assumed to be stationary only after time S , that is for $t \geq S$ $R_t = R$, $G_t = G(1+g)^{t-S}$, $T_t = T(1+g)^{t-S}$ and $B_t = B(1+g)^{t-S}$. The next proposition establishes that the determinacy result extends to these non-stationary policies with the difference that now the unique price sequence is not constant anymore but not stationary as well.

Theorem 1. *The price level is determined for arbitrary sequences of nominal interest rates and nominal government spending. In particular there is a unique price sequence.*

The theorem allows to iterate

$$\frac{u'(C(\tilde{P}_t))}{\tilde{P}_t} = \frac{\int_{\hat{\theta}(\tilde{P}_t)}^{\infty} \theta v'(B_t/\tilde{P}_t + \bar{b}) dF(\theta)}{\tilde{P}_t} + \frac{F(\hat{\theta}(\tilde{P}_t)) R_{t+1} \beta}{1+g} \frac{u'(C(\tilde{P}_{t+1}))}{\tilde{P}_{t+1}} \quad (30)$$

forward by recursively substituting $\frac{u'(C(\tilde{P}_t))}{\tilde{P}_t}$:

$$\frac{u'(C(\tilde{P}_t))}{\tilde{P}_t} = \sum_{s=0}^{\infty} \left\{ \left[\prod_{k=1}^s \frac{F(\hat{\theta}(\tilde{P}_{t+k-1})) R_{t+k} \beta}{1+g} \right] \frac{\int_{\hat{\theta}(\tilde{P}_{t+s})}^{\infty} \theta v'(B_{t+s}/\tilde{P}_{t+s} + \bar{b}) dF(\theta)}{\tilde{P}_{t+s}} \right\}, \quad (31)$$

which determines the current price level P_t . At this price level the benefits (RHS) and the cost (LHS) of saving are equalized. The cost of saving an additional unit is the standard utility loss from foregone consumption. The benefit from saving arises since at some point in time the bonds will be used for emergency expenditure, with probability $(\prod_{k=1}^s F(\hat{\theta}(\tilde{P}_{t+k-1}))) (1 - F(\hat{\theta}(\tilde{P}_{t+s})))$ in period $t+s$ with expected marginal utility $\frac{\int_{\hat{\theta}(\tilde{P}_{t+s})}^{\infty} \theta v'(B_{t+s}/\tilde{P}_{t+s} + \bar{b}) dF(\theta)}{\tilde{P}_{t+s}}$, discounted at $\prod_{k=1}^s \frac{R_{t+k} \beta}{1+g}$.

This iteration also serves to understand why steady state savings depend on the price level and why only one price level equalizes demand and supply. Households optimal savings decision as a function of the price level, $b(\tilde{P})$, solves:

$$u'(A + \frac{RB - T}{\tilde{P}} - \bar{C} - \frac{\mathbf{b}(\tilde{\mathbf{P}})}{\tilde{P}}) = \sum_{s=0}^{\infty} \left(\frac{F(\hat{\theta}(\tilde{P})) R \beta}{1+g} \right)^s \int_{\hat{\theta}(\tilde{P})}^{\infty} \theta v'(\frac{\mathbf{b}(\tilde{\mathbf{P}})}{\tilde{P}} + \bar{b}) dF(\theta), \quad (32)$$

such that real private consumption demand equals, using the steady state government budget constraint,

$$D(\tilde{P}) = A + \frac{RB - T}{\tilde{P}} - \frac{b(\tilde{P})}{\tilde{P}} = A + \frac{B - G}{\tilde{P}} - \frac{b(\tilde{P})}{\tilde{P}}. \quad (33)$$

Aggregate demand is the sum of private demand D and government consumption $G/\tilde{P}$

$$D(\tilde{P}) + \frac{G}{\tilde{P}} = A + \frac{B}{\tilde{P}} - \frac{b(\tilde{P})}{\tilde{P}} = A, \quad (34)$$

and is downward sloping in the price level,

$$\frac{\partial D(\tilde{P}) + \frac{G}{\tilde{P}}}{\partial \tilde{P}} < 0. \quad (35)$$

The unique price level $\tilde{P}$ then clears the good market such equilibrium real aggregate demand equals aggregate supply, A . Figure 2 illustrates the key features of the argument. The aggregate demand curve is downward sloping and intersects the aggregate supply curve at price level P^* . The two components of aggregate demand, government and private demand, add up to aggregate demand but the first one is decreasing and the latter one is increasing in the price level. For high price levels government consumption approaches zero but a non-vanishing precautionary demand prevents private demand from fully substituting for the fall in government consumption such that it always falls short of aggregate supply. This rules out prices higher than P^* . For low prices real government spending explodes and therefore private consumption falls such that government spending eventually exceeds output, ruling out prices lower than P^* as well.

Without a precautionary demand for savings consumption equals $C = A - G/\tilde{P}$ and aggregate demand is equal to aggregate supply $C + G/\tilde{P} = A$ for all price levels,

$$\frac{\partial D(\tilde{P}) + \frac{G}{\tilde{P}}}{\partial \tilde{P}} = 0. \quad (36)$$

In this case the price level would therefore not be determined as equating demand and supply but just determines the size of the government, $G/\tilde{P}$.

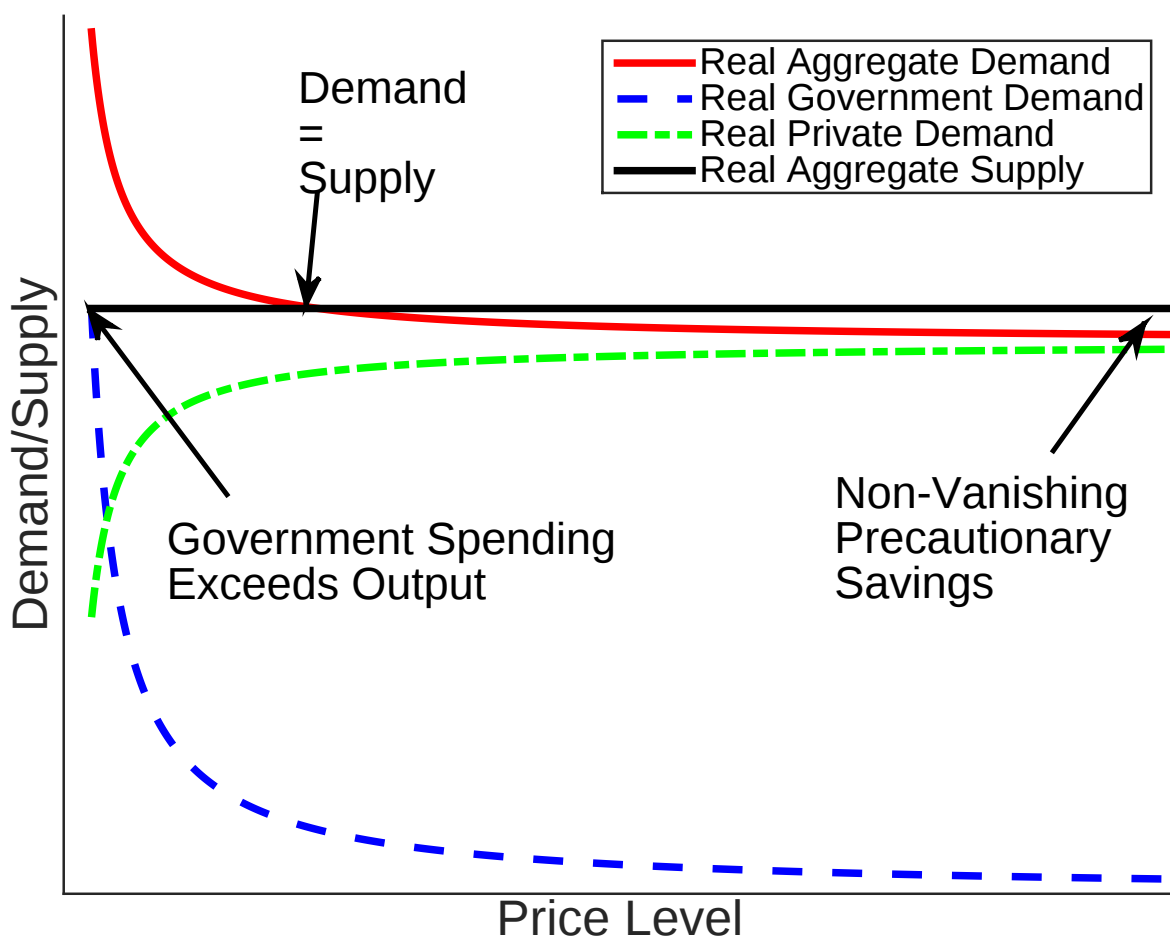


Figure 2: Aggregate Demand = Government Demand + Private Demand = Aggregate Supply

3.3 Example

I now consider a simplified economy that aims at providing the intuition for how the price level is determined and how it responds to policy changes. I am able to solve explicitly for the price level since I assume a particular form of the distribution of the idiosyncratic shock θ such that a large group exists without any need for emergency expenditures and the remaining small group is always constrained in equilibrium. Technically this avoids the recursiveness in the definition of the price level which depends on the threshold $\hat{\theta}$ which

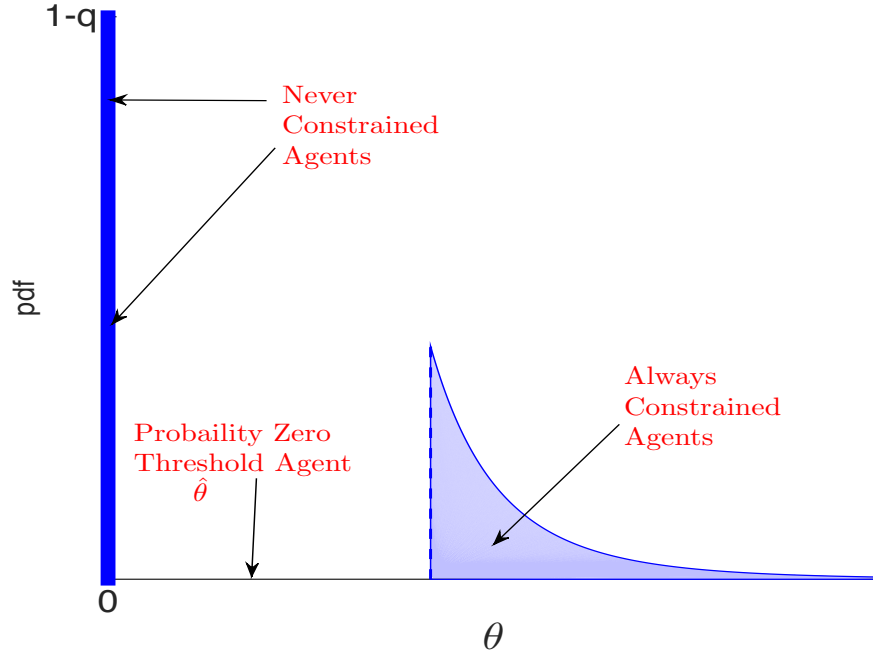


Figure 3: Example: Always or never constrained agents

itself depends on the price level. Assuming the cdf of θ to satisfy

$$F(\theta) = \begin{cases} 1 - q & \text{for } \theta \in [0, \tilde{\theta}] , \\ q \frac{G(\theta) - G(\tilde{\theta})}{1 - G(\tilde{\theta})} & \text{for } \theta \geq \tilde{\theta}, \end{cases}$$

ensures that no household is right at the credit constraint, where $1 - q$ is the probability of the mass-point $\theta = 0$, and g and G are the pdf and cdf of a lognormal distribution with parameters $\mu = 0$ and $\sigma = 1$, respectively.⁴ Figure 3 illustrates this simplifying assumption on the distribution.

The period t utility function is equal to

$$\log(C_t) + \theta \log(c_t).$$

and I set the credit limit at t_2 , $\bar{b} = 0$. All other model features remain unchanged. Households

⁴Since $\int_{\tilde{\theta}}^{\infty} \frac{\theta}{\tilde{\theta}} q \frac{g(\theta)}{(1 - G(\tilde{\theta}))} d\theta < 1 - (1 - q) \frac{\beta R}{1 + \pi}$ for small enough q ensures that $\hat{\theta} < \tilde{\theta}$, that is no household is right at the credit constraint.

optimal steady state consumption decision, taking prices as given, simplifies to

$$\frac{1}{\tilde{P}_t C_t} = q \frac{\kappa}{b} + (1 - q) \frac{R}{1 + g} \beta \frac{1}{\tilde{P}_{t+1} C_{t+1}}, \quad (37)$$

where $\kappa = \int_{\tilde{\theta}}^{\infty} \theta \frac{g(\theta)}{(1 - G(\theta))} d\theta$. Equivalently after recursively iterating households saving decision in a steady state as a function of the price level is characterized through

$$\frac{1}{\tilde{P}A + RB - T - \tilde{P}_t \bar{C} - b(\tilde{P})} = \frac{\kappa q}{b(\tilde{P})(1 - (1 - q) \frac{R}{1 + g} \beta)}. \quad (38)$$

Solving for the saving decision $b(\tilde{P})$,

$$b(\tilde{P}) = (\tilde{P}A + RB - T - \tilde{P}_t \bar{C}) \frac{\kappa q}{1 + \kappa q - (1 - q) \frac{R}{1 + g} \beta} \quad (39)$$

establishes that nominal savings are increasing in the price level, implying that real aggregate demand - the sum of private and government demand - equals, using $RB - T = B - G$,

$$D(\tilde{P}) + \frac{G}{\tilde{P}} = A + \frac{(1 - (1 - q) \frac{R}{1 + g} \beta) \frac{B}{\tilde{P}} + \kappa q \frac{G}{\tilde{P}}}{1 + \kappa q - (1 - q) \frac{R}{1 + g} \beta} - \frac{(A - qB) \kappa q}{1 + \kappa q - (1 - q) \frac{R}{1 + g} \beta}, \quad (40)$$

and is decreasing in $\tilde{P}$. The unique equilibrium price equalizes demand and supply,

$$D(\tilde{P}) + \frac{G}{\tilde{P}} = A, \quad (41)$$

and equals

$$\tilde{P} = \frac{G}{A} + \frac{B}{A} \left(q + \frac{1 - (1 - q) \beta \frac{R}{1 + g}}{\kappa q} \right). \quad (42)$$

The determinants of the steady state price level become clear. For a given price level, higher nominal government expenditures (a higher G) increase demand since private consumption is not reduced one-for one and savings are decreased implying a higher market clearing price level. Tighter monetary policy (higher R) increases savings and lowers private consumption and therefore aggregate demand, implying a lower market clearing price level. An increase in the discount factor (a higher β) has the same effect as tightening monetary policy: increases savings, lowers consumption and lowers prices. A technology improvement

(an increase in A) leads to larger increase in supply than in demand since savings increase as well, implying a lower market clearing price. Finally an increase in wealth (a higher B) leads to higher consumption demand, implying a higher market clearing price level.

3.4 Impulse Responses

Theorem 1 allows to consider the economy's impulse response to monetary and fiscal shocks. While a full analysis will be conducted in a model with sticky prices in Section 4, it is still instructive to compute the impulse responses in the flexible price version of the model.

A theoretical reason why this is usually not done, is that the price level is not determined in both standard sticky and flexible price models when monetary policy is described as setting the nominal interest rate. As this Section has established, this theoretical obstacle is overcome, allowing to conduct such policy experiments.

I consider two policy experiments, a persistent increase in the nominal interest rate and an increase in nominal government spending. I also consider the response of prices to a discount factor and a technology shock. Figures 4, 5 and 6 in Section 4 show not only the impulse responses in the sticky price model but also, as a benchmark, for the flexible price economy. A remarkable feature of these flexible price impulse responses is that prices adjust sluggishly and that monetary policy affects prices without the assumption of prices being sticky.

3.4.1 Monetary Policy

For the monetary policy experiment, I characterize the response of prices to an initial unexpected increase in interest rates $R_0 > R$, which then dies out over time and is back at the steady state level R from time S onwards, $R_t = R$ for $t \geq S$, that is the interest rate sequence equals

$$R_0 \geq R_1 \geq \dots \geq R_{S-1} \geq R_S = R, R, \dots \quad (43)$$

The resulting price sequence can be precisely characterized:

Proposition 4. *The detrended price sequence in response to a monetary policy as in (43) is*

$$\tilde{P}_0 \leq \tilde{P}_1 \leq \dots \leq \tilde{P}_{S-1} \leq \tilde{P}_S = P^*, \tilde{P}_{S+1} = P^*, \dots, \quad (44)$$

where the inequality between prices at t and $t + 1$ is strict whenever it is strict for nominal interest rates, $R_t > R_{t+1}$. For the non-detrended prices

$$\frac{P_{t+1}}{P_t} \geq (1 + g) \quad (45)$$

with strict inequality if $R_t > R_{t+1}$.

A more persistent or larger impulse to the nominal interest rate leads to uniformly lower prices.

Proposition 5. *Consider two interest rate sequences*

$$R_0^a \geq R_1^a \geq \dots \geq R_{S-1}^a \geq R_S^a = R, R, \dots, \quad (46)$$

$$R_0^b \geq R_1^b \geq \dots \geq R_{S-1}^b \geq R_S^a = R, R, \dots, \quad (47)$$

with $R_t^a \geq R_t^b$. Then the prices $\tilde{P}^a$ for policy R^a are uniformly lower than the prices $\tilde{P}^b$ for policy R^b ,

$$\tilde{P}_t^a \leq \tilde{P}_t^b. \quad (48)$$

3.4.2 Fiscal Policy

For the fiscal policy experiment, I characterize the response of prices to an initial unexpected increase in nominal government spending for S periods by x percent financed by an increase in taxation, so that

$$\begin{aligned} \hat{G}_t &= (1 + x)G_t & \text{for } 0 \leq t < S \\ \hat{G}_t &= G_t & \text{for } t \geq S. \end{aligned} \quad (49)$$

The response of prices can be precisely characterized:

Proposition 6. - *The detrended price sequence in response to a fiscal policy as in (49)*

is

$$\tilde{P}_0 \geq \tilde{P}_1 \geq \dots \geq \tilde{P}_{S-1} \geq \tilde{P}_S = P^*, \tilde{P}_{S+1} = P^*, \dots, \quad (50)$$

and for the non-detrended prices

$$\frac{P_{t+1}}{P_t} \leq (1 + g). \quad (51)$$

- A more expansive or persistent fiscal policy $G_t^a \geq G_t^b \geq G_t$ leads to stronger price increases

$$\tilde{P}_t^a \geq \tilde{P}_t^b. \quad (52)$$

3.4.3 Discount Factor Shock

I consider a discount factor shock which increases β for S periods:

$$\begin{aligned} \hat{\beta} &> \beta \quad \text{for } 0 \leq t < S \\ \hat{\beta} &= \beta \quad \text{for } t \geq S. \end{aligned} \quad (53)$$

The next proposition shows that prices persistently fall.

Proposition 7. - The detrended price sequence in response to a discount factor shock as in (53) is

$$\tilde{P}_0 \leq \tilde{P}_1 \leq \dots \leq \tilde{P}_{S-1} \leq \tilde{P}_S = P^*, \tilde{P}_{S+1} = P^*, \dots, \quad (54)$$

- A larger discount factor shock, $\hat{\beta}^a > \hat{\beta}^b$ leads to stronger price decreases

$$\tilde{P}_t^a < \tilde{P}_t^b. \quad (55)$$

With one important caveat, discount factor shocks can be fully neutralized by monetary policy through keeping $R\beta$ constant, that is decreasing R by the size of the shock $\hat{\beta}/\beta$. The caveat is that the zero lower bound prevents large cuts in the nominal interest rates in

response to large increases in β .

3.4.4 Productivity Shock

Finally I consider a persistent shock to the productivity A :

$$A_0 \geq A_1 \geq \dots \geq A_{S-1} \geq A_S = A, A, \dots \quad (56)$$

The response of prices to this productivity innovation can be precisely characterized as a persistent drop in prices:

Proposition 8. *The detrended price sequence in response to a technology shock as in (56) is*

$$\tilde{P}_0 \leq \tilde{P}_1 \leq \dots \leq \tilde{P}_{S-1} \leq \tilde{P}_S = P^*, \tilde{P}_{S+1} = P^*, \dots \quad (57)$$

3.5 How and when monetary policy controls the inflation rate

A main result of this paper is that it is fiscal policy which determines the steady state inflation rate through controlling nominal spending. If nominal government consumption, nominal taxation and nominal government debt grow at rate γ , then steady state inflation $\pi = \gamma$. In this case monetary policy has no control over the long-run inflation rate since fiscal policy does control the nominal anchor.

This does not mean that monetary policy is completely ineffective since, as shown above, it can still affect the price level. I also show in Section 4 that monetary policy can be quite effective in stabilizing prices around the steady state. An attempt by fiscal policy to stimulate the economy through generating above steady state temporary inflation and employment can be completely neutralized by monetary policy resulting in higher government spending but prices and employment remain at their steady state values.

In different scenarios monetary policy can affect or even fully control the long-run inflation rate. One such scenario is one where fiscal policy takes the initial level of nominal government debt, B , as given and fixes the level of real government consumption, G/P , and the real tax revenue, T/P . Some simple algebra can be used to illustrate how this scenario leads to a very different conclusion about long-run inflation. Let $s = \frac{T-G}{P} > 0$ denote the real constant primary surplus of the government and let $\hat{R}$ be the break-even nominal interest rate where

the government budget is balanced in a zero inflation steady state,

$$s = (\hat{R} - 1) \frac{B}{P}. \quad (58)$$

Assume now (and as it turns out correctly) that the real value of government debt, B/P , is the same across all steady states with different nominal interest rates. In a steady state with a constant nominal interest rate R and a fixed real surplus s , nominal debt B_t evolves as

$$B_{t+1} = RB_t - P_t s. \quad (59)$$

Monetary policy therefore determines the growth rate of nominal debt:

$$\frac{B_{t+1} - B_t}{B_t} = (R - 1) - \frac{P_t s}{B_t} = (R - 1) - (\hat{R} - 1) = R - \hat{R}, \quad (60)$$

implying that nominal debt and therefore prices are growing at the same rate in a steady state,

$$\pi = \frac{B_{t+1} - B_t}{B_t} = R - \hat{R}, \quad (61)$$

determined by monetary policy only.⁵

The reason why it is now monetary policy and not fiscal policy which determines the long-run inflation rate is simple. The general principle that it is fiscal policy which de facto determines steady state inflation still holds as it is the evolution of the nominal anchors which matter. However, a government which insists on a level of real expenditures and tax revenues has to finance all deficits through issuing debt with a return determined by monetary policy. An increase in the nominal interest rate will therefore lead to an increase in nominal government debt which with flexible prices materializes immediately in higher inflation rates. In this scenario it is therefore monetary policy which determines the growth rate of nominal government debt and therefore the growth rate of G , T and prices P . Future research will show whether and in which episodes monetary or fiscal policy determined the

⁵The real interest rate is (approximately) equal to $\hat{R} = R - \pi$ across all these steady states supporting the simplifying assumption that real bond demand is constant across these steady states. Obviously, the difference $\frac{R}{1+\pi} - (R - \pi)$ is economically not meaningful and I therefore ignored it for the clarity of the exposition.

inflation target.

But even if monetary policy cannot take over the control of steady state inflation, the reasoning illustrates that it still does have a large impact on government debt. This channel of monetary policy can be used to make a debt-financed expansionary fiscal policy very expensive for the government and in practice could be an effective way for monetary policy to prevent inflationary policies.

3.6 Robustness: Adding money to the model

This paper considers a cashless economy. I now extend the model to allow households to hold cash and show that this does not affect the results of this paper. The reason is simple. Setting nominal interest rates and fiscal policy is sufficient to determine the price level. Households' nominal money holdings are then endogenously determined to satisfy real money demand. To show this I assume that the transaction services provided by real money balances are represented as an argument of the utility function,

$$u(C_t^h) + \chi(M_t/P_t) + \theta v(c_t^h), \quad (62)$$

where M_t is a family's end of period t money holdings. The family's budget constraint then equals

$$P_t C_t + E_\theta(P_t c_t(\theta)) + b_t^h + M_t = P_t A h_t - T_t + R_t b_{t-1}^h + M_{t-1}, \quad (63)$$

where M_{t-1} is period t initial money holdings carried over from period $t-1$. The government collects seigniorage, which are rebated to households through lower taxation so that its flow budget constraint equals

$$B_{t+1} + M_{t+1} = R_t B_t + M_t + G_t - T_t. \quad (64)$$

The remaining features of the model remain unchanged.

The decision problem of a household with initial period bond holdings b_t^h and money holdings M_{t-1} is

$$V_t(b_{t-1}^h, M_{t-1}) = \max_{b_t^h, C_t, c_t(\theta), M_t} \{u(C_t^h) + \chi(M_t/P_t) + E_\theta \theta v(c_t^h) + \beta E_t[V_{t+1}(b_t^h, M_t)]\} \quad (65)$$

subject to the flow budget constraint (63) and the liquidity constraint (4).

Money holdings M_t appear only in the first-order condition of M_t (the small seignorage gains are rebated lump-sum to households),

$$\frac{u'(C_t) - \chi'(M_t/P_t)}{P_t} = \frac{u'(C_{t+1})}{P_{t+1}}, \quad (66)$$

implying that prices P_t and P_{t+1} and consumption C_t and C_{t+1} can be solved independently from M_t as the above analysis shows. The first order condition for money is then used to solve for M_t . The only purpose of adding a demand for money to the model is thus to determine the quantity of money that the central bank will need to supply in order to implement its nominal interest rate target.

4 Monetary and Fiscal Policy with Sticky Prices

This section adds two features to the basic model: sticky prices and elastic labor supply. These two features allow me to illustrate the workings of the model and to compute the response of the price level and employment to a tightening of monetary policy and to a fiscal demand stimulus.

4.1 A Sticky Price Model

I now assume that firms are constrained in their price setting to see whether the model can (qualitatively) produce standard impulse responses. I follow Mankiw and Reis (2002) and assume that information is sticky and also follow the literature in modeling a competitive final good and a monopolistically competitive intermediate sector.

Labor Supply Households provide h_t hours in a perfectly competitive labor market at a fully flexible real wage w_t , which they receive at t_1 . The disutility from working h hours is described through GHH preferences:

$$\log(C_t^h - \kappa h_t^{1+\phi}) + \theta \eta \log(c_t^h), \quad (67)$$

which implies that an increase in government spending has no negative wealth effect leading to an increase in labor supply. The benefit of this choice of preferences is that an effect of government spending is due to demand effects as in the previous sections and are not

confounded with well known wealth effects.

Final Goods

The perfectly competitive, representative, final good producing firm combines a continuum of intermediate goods $Y_t(j)$ indexed by $j \in [0, 1]$ using the technology

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{1}{1+\lambda}} dj \right)^{1+\lambda}. \quad (68)$$

Here $\lambda > 0$ and $\frac{1+\lambda}{\lambda}$ represents the elasticity of demand for each intermediate good. The final goods firm takes intermediate good prices $P_t(j)$ of $Y_t(j)$ and output prices P_t of the final output Y_t as given. Profit maximization of intermediate firms implies that the demand for intermediate goods is

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\frac{1+\lambda}{\lambda}} Y_t. \quad (69)$$

The relationship between intermediate goods prices and the price of the final good is

$$P_t = \left(\int_0^1 P_t(j)^{-\frac{1}{\lambda}} dj \right)^{-\lambda}. \quad (70)$$

Intermediate Goods.

Intermediate good j is produced by a monopolist who has access to a linear production technology which uses labor $H(j)$ as the only input:

$$Y_t(j) = AH_t(j), \quad (71)$$

Price Setting As in Mankiw and Reis (2002) prices adjust slowly since information disseminates slowly and processing information takes time. In each period, a fraction ω of firms obtains full information, so that their subjective expectation of the contemporaneous values of aggregate variables coincides with the actual values of these variables. The remaining firms base their expectations and decisions on outdated information.

Taking as given nominal wages, final good prices, the demand schedule for intermediate products and technological constraints, firm j chooses its labor inputs $H_t(j)$ and the price

$P_t(j)$ to maximize profits

$$\mathbb{E}_t^j \left(\frac{P_{t+s}(j)}{P_{t+s}} Y_{t+s}(j) - \frac{1}{P_{t+s}} W_{t+s} H_{t+s}(j) \right), \quad (72)$$

where the superscript j in the expectation operator $\mathbb{E}_t^j$ indicates that this expectation is formed using firm j 's information.

Combining the production technology and the demand schedule implies

$$H_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\frac{1+\lambda}{\lambda}} \frac{Y_t}{A_t}$$

Thus, the firm maximizes the following objective function with respect to $P_t(j)$:

$$\mathbb{E}_t^j \left(\frac{P_t(j)}{P_t} \left(\frac{P_t(j)}{P_t} \right)^{-\frac{1+\lambda}{\lambda}} Y_t - \frac{W_t}{P_t} \left(\frac{P_t(j)}{P_t} \right)^{-\frac{1+\lambda}{\lambda}} \frac{Y_t}{A_t} \right), \quad (73)$$

resulting in a price setting rule

$$\mathbb{E}_t^j \left(\frac{P_t(j)}{P_t} = \frac{W_t}{P_t A_t} (1 + \lambda) \right). \quad (74)$$

Each firm uses its information set to set the price as a markup $1 + \lambda$ over real marginal costs. As information is dispersed across firms and only a fraction ω of firms obtain updated information each period, prices are dispersed as well.

4.2 Numerical Example

The next goal is to provide some calibrated examples to illustrate the workings of the model through computing the same impulse responses as characterized analytically in section 3.4, but here prices are sticky. The results should however not be considered precise estimates due to the simplifying assumptions of the model, mainly when modeling the consumption and saving decisions.⁶

These numerical illustrations require to chose parametric forms and parameter values. I make the following choices. A model period is a quarter and therefore $\beta = 0.99$. I normalize steady state $A = 1$. The steady state nominal interest rate is 2% (annualized) and the steady

⁶To obtain analytical tractability I assumed large families with the consequence that the model is not even close to more realistic models of consumption behavior as for example in Kaplan and Violante (2014).

state inflation rate is zero. Following standard choices in the literature I assume $\lambda = 0.2$, $\omega = 0.25$ and $\phi = 2$. I allow households to use a credit line of 10% of their income to pay for emergency consumption, $\bar{b} = 0.1$. The distribution of the preference shocks θ is assumed to be lognormal with parameters $\mu = 0$ and $\sigma = 1$ (the mean and standard deviation of $\log(\theta)$). Kaplan and Violante (2014) document that the median household holds only a small amount of liquid assets, motivating a choice of $B = 0.25$, that is households' liquid assets to income ratio is 25%. These choices imply that only 1% of households are constrained in emergency expenditures, a fraction much lower than the number of between 17.5% and 35% hand to mouth consumers found in Kaplan and Violante (2014). The model therefore certainly does not overstate the importance of credit constraints. Government expenditures equal $G = 0.2$. Finally I pick the two preference scale parameters $\kappa = 0.833$ and $\eta = 0.079$ to obtain $h = 1$ and $P = 1$ in steady state.

I now use this calibrated model to compute model impulse responses to a monetary policy shock (an increase of R by one (annualized) percentage point), a fiscal policy shock (an increase in nominal government spending G by 1%), a technology shock (an increase of A by 1%) and a discount factor shock (an increase of β from 0.99 to 0.995). All shocks are unexpected. In all experiments except for the fiscal policy one I assume that 80% of government spending, 0.16, is fixed in real terms so that nominal spending equals $0.16P + 0.04$. Real spending thus equals $0.16 + 0.04/P$, which increases if the price level P falls. To the extent that government spending serves as a nominal anchor, real spending automatically increases in response to price decreasing shocks. I explore this automatic stabilization feature in more detail in the next section.

The monetary policy shock has the expected negative effects on prices and employment (panel a) of Figure 4) since higher nominal interest rates increase the incentives to save and thus decrease consumption. Interestingly, the aggregate price level response is very close with flexible and sticky prices whereas hours do not move if prices are flexible but drop quite a bit if prices are sticky. This is because with sticky prices many firms charge too high a markup and as a result hire too little.

The similar logic applies to the discount factor shock where again prices and hours fall.

The increase in government spending stimulates aggregate demand and therefore leads to an increase in prices both with flexible and sticky prices but hours only increase in the latter case since firms charge too low a markup and as a result hire more to satisfy the additional demand. With flexible prices firms always charge the right markup and as a result employment does not respond. In panel a) of Figure 5 the increase in spending is

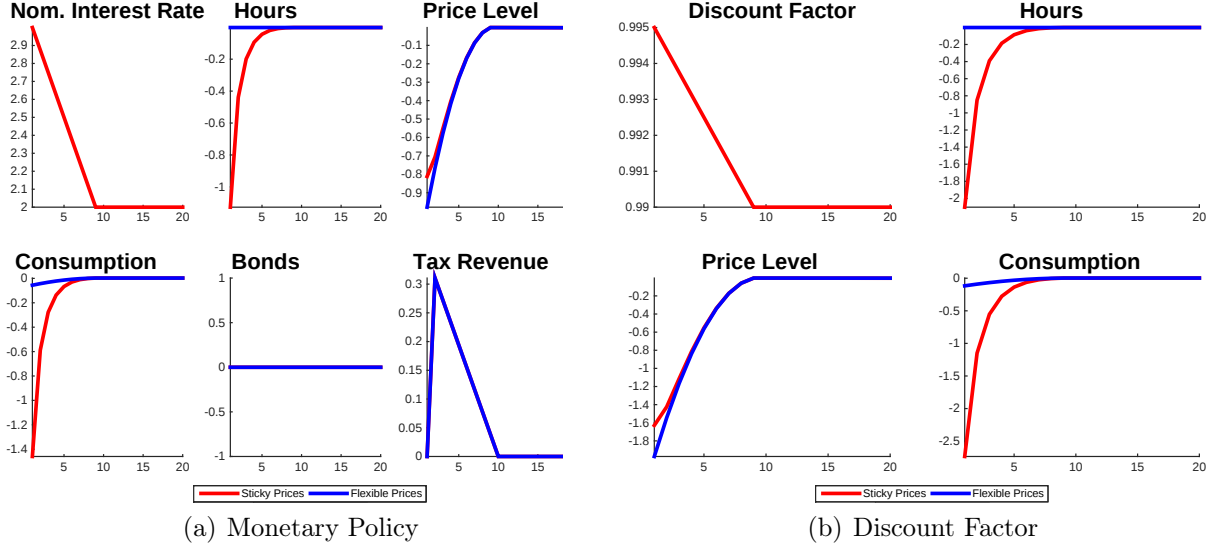


Figure 4: Impulse Responses for a) Monetary Policy and b) Discount Factor of hours, consumption C at t_1 and prices for both the sticky and flexible price economy.

financed with an increase in taxes only, which by itself contracts private demand. However the tax increase lowers private demand by less than the increase in government demand so that total demand still increases but the positive effects on prices and hours are muted. In panel b) of Figure 5 the increase in spending is financed by issuing government debt which is only paid back later through higher taxes. The initial deficit financing avoids the contractionary demand effect of the tax financing as shown in panel a). As a result the response of hours and prices is almost by an order of magnitude larger with deficit spending than with tax financing.

The technology policy shock has the negative effects on prices and employment (panel a) of figure 6). The aggregate price level response is very close with flexible and sticky prices whereas hours increase if prices are flexible but drop quite a bit if prices are sticky. This is again because with sticky prices many firms charge too high a markup and as result hire too little whereas for firms with flexible prices the increase in productivity leads to more employment. However the differences depend on the persistence of the technology shock as shown in panel b) of figure 6. In contrast, if the technology is only temporary consumption-smoothing households want to save more and as a result prices have to fall quite a bit to equalize demand and supply. If the technology innovation is persistent the consumption-smoothing incentives are muted and prices have to move only very little, so that markups are not distorted very much and the increase in productivity dominates, implying similar increases in hours if prices are flexible or sticky.

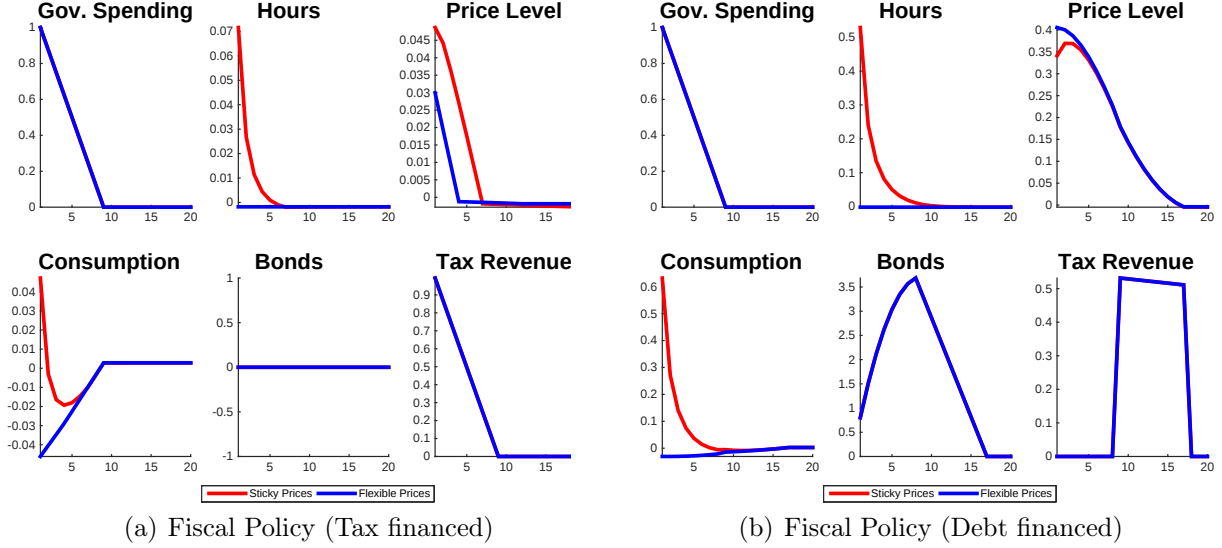


Figure 5: Fiscal Policy: Impulse Responses of hours, consumption C at t_1 and prices for both the sticky and flexible price economy.

4.3 Nominal Fiscal Anchor as Automatic Stabilizer

As explained above, when computing the impulse responses I assumed that 80% of government spending is real and only 20% is nominal, so that changes in the price level move only 20% of real government spending. The larger the fraction of nominal fixed spending is, the larger is the change in real spending as an automatic response to changes in prices. This means the larger is the nominal anchor the larger is the automatic demand response. Not surprisingly - I have established above that a fiscal policy stimulus increases prices and employment - the size of this automatic demand response matters for the reaction of the economy.

I now consider the same discount factor shock as above and again compute the impulse response of consumption, prices and hours but make one change. Government spending is now fully nominal, so that real spending as a function of the price level equals G/P . Since the discount factor induces a fall in prices, G/P increases automatically and significantly more than in the benchmark where only 20% of spending is nominal. This additional demand increase partly offsets the discount-factor shock induced drop in demand and therefore also alleviates the negative consequences on hours worked. The result is shown in Figure 7.

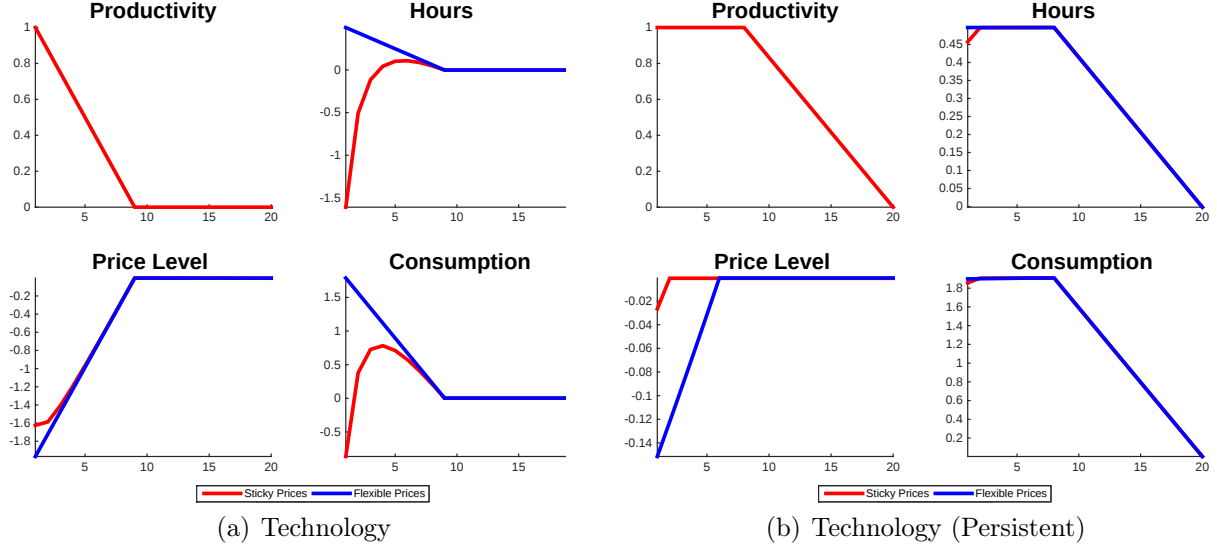


Figure 6: Technology Shock: Impulse Responses of hours, consumption C at t_1 and prices for both the sticky and flexible price economy.

4.4 Zero Lower Bound and Fiscal Policy

Even when government spending is fully nominal and therefore the automatic stabilizer is most effective, increases in the discount factor still lead to a contraction in employment. There are however multiple possibilities to neutralize the effect of a discount factor increase. As long as the zero lower bound is not binding the solution is particularly simple. An increase in the discount factor from β to $\hat{\beta}_t$ can be neutralized basically by monetary policy alone, by decreasing the nominal interest rate from R to $R_t = R \frac{\beta}{\hat{\beta}_t}$. Fiscal policy uses the, due to the lower R , lower interest rates payments on government debt to lower taxes. This policy is successful in stabilizing employment, consumption and prices at their steady state values.

If the shock to the discount factor is too large the zero lower bound renders this solution impossible since $R \frac{\beta}{\hat{\beta}_t} < 1$. In this scenario fiscal policy has to step in and increase spending to stabilize employment and prices. Obviously if the goal is employment stabilization output is stabilized as well and therefore private consumption has to fall. This suggests that monetary policy is in general the preferred policy tool as it allows to stabilize all three variables whereas fiscal policy cannot and fiscal actions are taken only if the zero lower bound makes monetary policy ineffective.

An increase of $\beta = 0.99$ to 1.01 implies that the zero lower bound is binding. Figure 8 shows the impulse responses when government spending is unchanged and only monetary policy decreases interest rates to zero as long as the zero lower bound is binding. Since

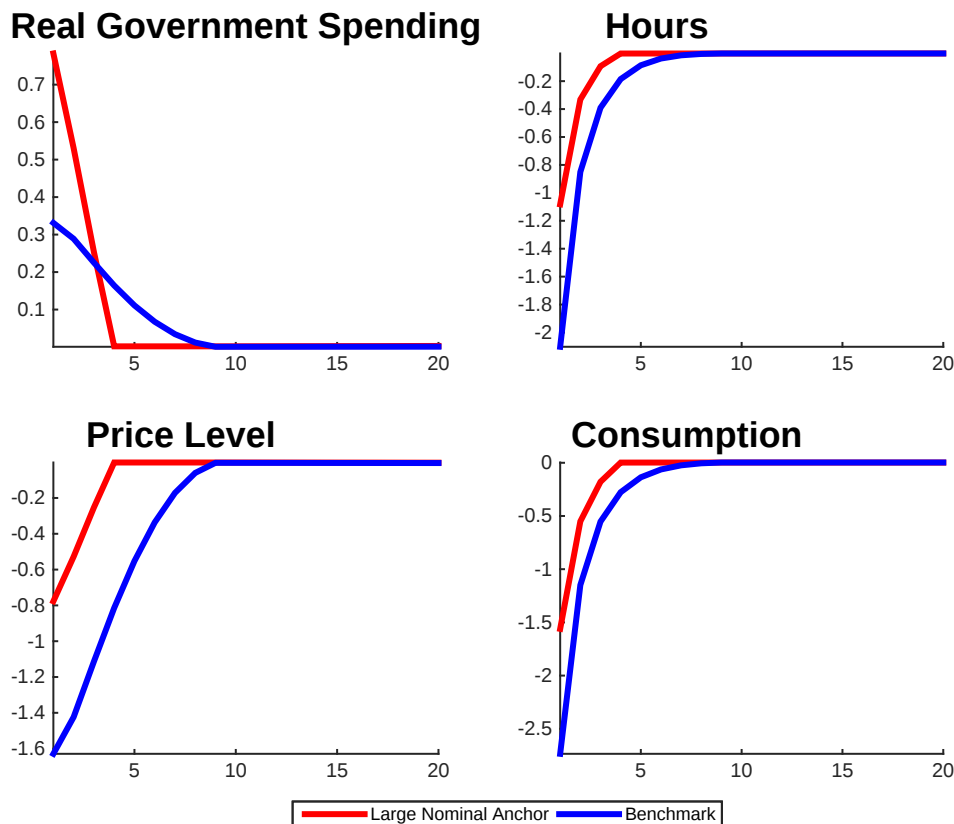


Figure 7: Response to Discount Factor Shock: a) Benchmark (Partial Nominal Anchor) and b) Full Nominal Anchor

monetary policy is not able to neutralize the increase in β , employment and prices fall. Under these circumstances the goal to stabilize employment and prices requires fiscal policy to become active and increase spending. This policy is successful. Employment and prices remain throughout at their steady state values. The path of government spending which achieves this is shown in the last panel of Figure 8

4.5 Monetary and Fiscal Policy: Coordination

A main result of this paper is that fiscal and monetary policy jointly determine the price level. Understanding how prices move therefore requires to understand the policy coordination of the treasury and the central bank. Since the price level is determined here, policy coordination problems are quite different here from those in Sargent and Wallace (1981)'s classic "monetarist arithmetic" or in Leeper (1991)'s active and passive monetary and fiscal policies. In particular the question is not which combination of fiscal and monetary policy leads to local determinacy of the price level and which combination does not. Instead prices

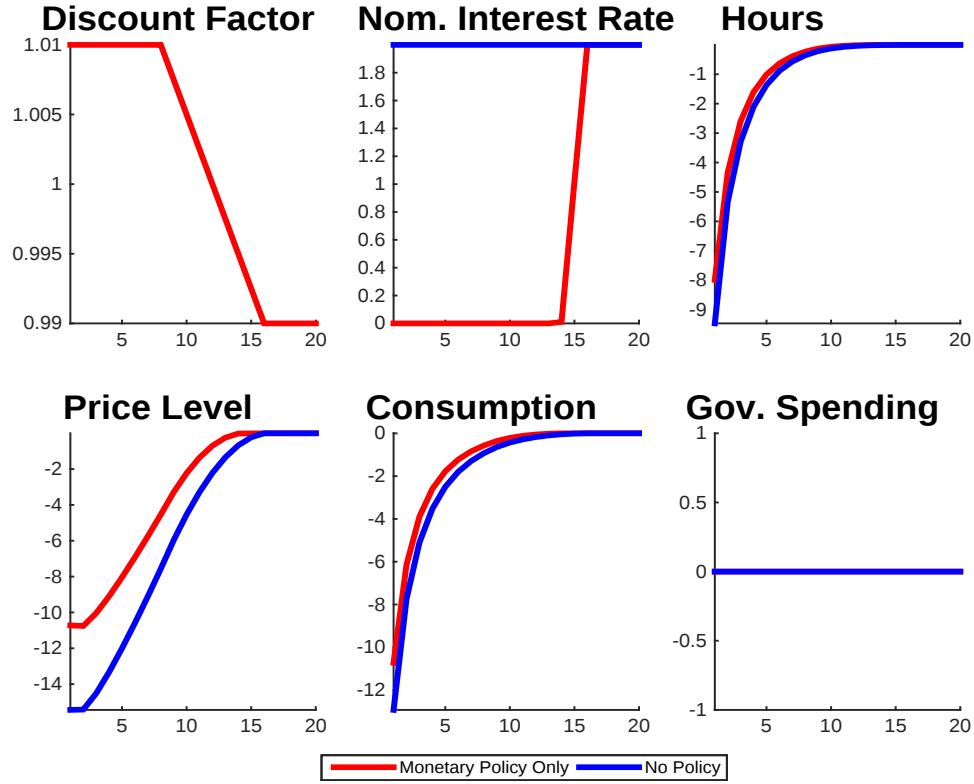
are globally unique for any combination of policies.

Policy coordination is therefore very different. Fiscal policy can control the long run inflation rate through controlling the nominal anchor. I have already shown in Section 3.5 that monetary policy takes over and sets the steady-state inflation rate when fiscal policy does not exercise its power to control long-run inflation.

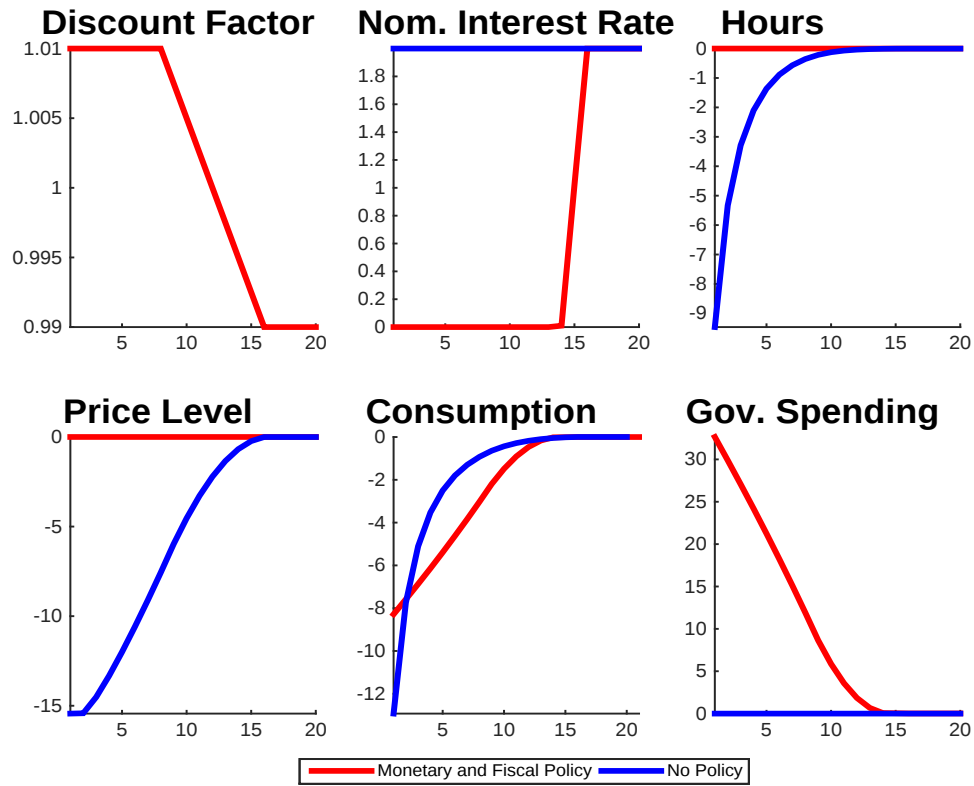
The analysis so far could give the impression that monetary policy is quite powerless and that fiscal policy can always get its way and not only determine fiscal policy but also set the inflation rate, which in every textbook model is under the control of monetary policy only.

This impression would however underestimate the power of a central bank. On the one hand I have shown the effectiveness of monetary policy in stabilizing the economy. Furthermore, I will now illustrate its power showing that the central bank can undo stimulative policies initiated by the treasury to raise inflation and boost employment. Any such short-run fiscal policy actions which do not find the central bank's approval, for example because they are conflicting with the objective of price stability, can be neutralized resulting in no change in prices and employment and only higher taxes or debt. Figure 9 shows the sequence of nominal interest rates, in panel a) for a tax-financing and in panel b) for deficit-financing, which undo the fiscal policy expansion considered in Figure 5. Since a deficit financed expansionary fiscal policy is more stimulative than a tax financed one the offsetting increase in nominal interest rates have to be higher in the first case.

This exercise reveals another unpleasant consequence of an increase in nominal interest rates for the treasury and this could be in practice the most effective way for the central bank to impose its will on fiscal policy. Increases in nominal interest rates raise the interest payments on government debt which leads to higher debt and eventually higher taxes.

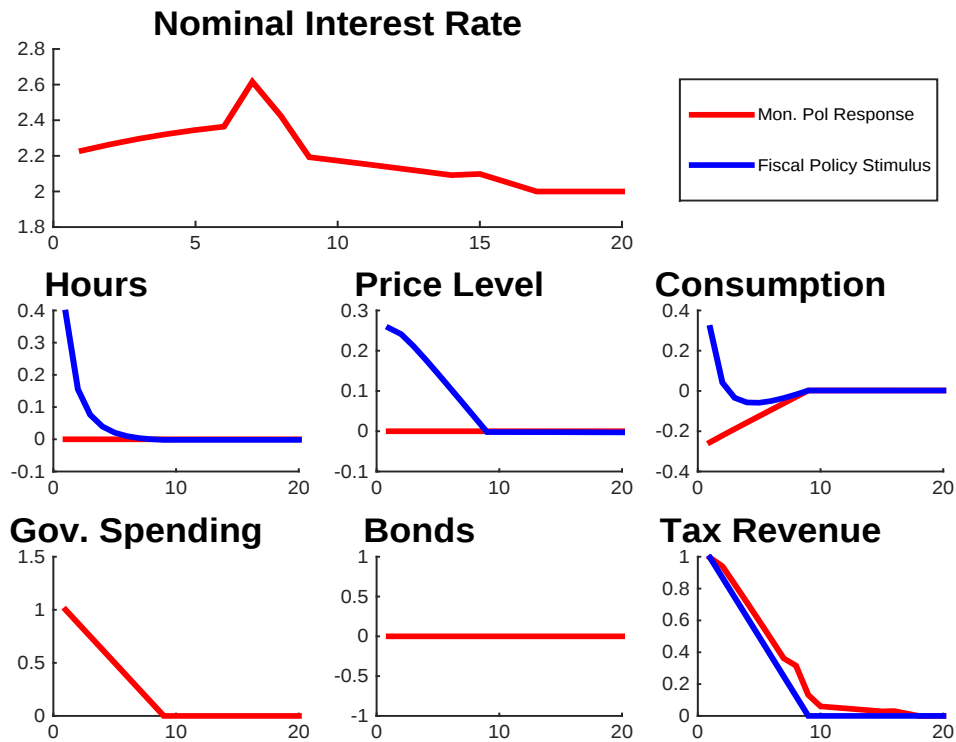


(a) ZLB: Monetary Policy Only

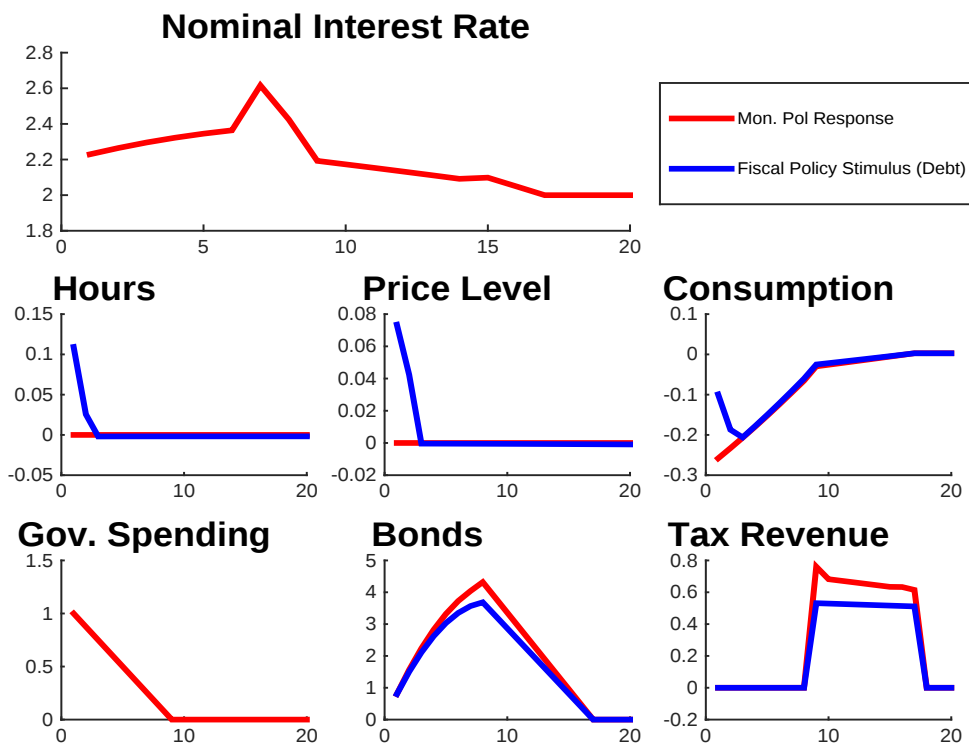


(b) ZLB: Fiscal and Monetary Policy

Figure 8: Zero Lower Bound : a) Monetary Policy Only b) Monetary and Fiscal Policy



(a) Monetary Policy Neutralizing Fiscal Policy



(b) Monetary Policy Neutralizing Fiscal Policy (Debt)

Figure 9: a) Fiscal Policy Stimulus (Tax financed) b) Fiscal Policy Stimulus (Debt financed)

5 Conclusion

In this paper I have shown that the price level is globally determined in a model which incorporates the simple empirical finding that a permanent income increase leads to a less than one-for-one increase in consumption and at the same time to an increase in precautionary savings.

The simplicity of the model keeps it tractable and enables the researcher to better understand the monetary and fiscal transmission mechanism. A key finding is that the price level is jointly determined by monetary and fiscal policy and that long-run inflation is determined by the growth rate of nominal government spending. The nominal anchor - nominal government spending - is controlled by fiscal policy which therefore has the power to set the long-run inflation rate.

In a numerical exercise, I show that impulse responses to monetary and fiscal policy shocks as well as to technology and discount factor shocks are in line with empirical evidence and conventional wisdom. I also establish how government spending serves as an automatic stabilizer, how monetary and fiscal policy interact and discuss stabilization policies at the ZLB.

The model used to conduct the quantitative exercises misses many elements which could be necessary to obtain precise estimates of policies. A full quantitative macroeconomic model would use the Aiyagari (1994) incomplete market model with capital and elastic labor supply as a starting point, would model the consumption behavior better such that the MPC is in line with the data, allow for distortionary taxation such that the cost of an expansionary policy is more realistic and allow for long-term government debt to discuss quantitative easing.

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Appendix

Proof of Proposition 1 For a given level of consumption C at t_1 and a given threshold level $\hat{\theta}$, consumption in period t_2 equals

$$\begin{aligned} i) \quad & c(\theta) = (v')^{-1}\left(\frac{u'(C)}{\theta}\right) \quad \text{if } \theta \leq \hat{\theta}, \\ ii) \quad & c(\theta) = B_t/P_t + \bar{b} \quad \text{if } \theta > \hat{\theta}. \end{aligned}$$

As a result consumption C at period t_1 solves

$$C = A - \frac{G}{P} - (1 - F(\hat{\theta}))\left(\frac{B}{P} + \bar{b}\right) + \int_{\underline{\theta}}^{\hat{\theta}} (v')^{-1}\left(\frac{u'(C)}{\theta}\right) dF(\theta). \quad (75)$$

The threshold level then solves

$$\hat{\theta} = \frac{u'(C)}{v'(\frac{B}{P} + \bar{b})} \quad (76)$$

This defines $\hat{\theta}$ as a function of P and C . Plugging this expression into the fixpoint equation (75) for C , shows that C is a function of P only and so is $\hat{\theta}$. To see that C is increasing in P , take derivatives of the fixpoint equation

$$C(P) = A - \frac{G}{P} - (1 - F(\hat{\theta}(P)))\left(\frac{B}{P} + \bar{b}\right) - \int_{\underline{\theta}}^{\hat{\theta}(P)} (v')^{-1}\left(\frac{u'(C(P))}{\theta}\right) dF(\theta). \quad (77)$$

w.r.t P ,

$$C'(P) = \frac{G}{P^2} + \left[\int_{\underline{\theta}}^{\hat{\theta}(P)} \left(\frac{u''(C(P))}{\theta v''((v')^{-1}(\frac{u'(C(P))}{\theta}))} \right) dF(\theta) \right] C'(P) \quad (78)$$

$$+ \hat{\theta}'(P) \underbrace{\left[\left(\frac{B}{P} + \bar{b} \right) f(\hat{\theta}(P)) - (v')^{-1}\left(\frac{u'(C(P))}{\theta}\right) f(\hat{\theta}(P)) \right]}_{=0}, \quad (79)$$

so that

$$C'(P) = \frac{\frac{G}{P^2}}{1 + \left[\int_{\underline{\theta}}^{\hat{\theta}(P)} \left(\frac{u''(C(P))}{\theta v''((v')^{-1}(\frac{u'(C(P))}{\theta}))} \right) dF(\theta) \right]} > 0, \quad (80)$$

since both $u'' < 0$ and $v'' < 0$. Having established that $C(P)$ is increasing in P immediately implies that

$$\hat{\theta}(P) = \frac{u'(C(P))}{v'(\frac{B}{P} + \bar{b})} \quad (81)$$

is decreasing in P since the numerator is decreasing in P and the denominator is increasing in P .

Proof of Proposition 2 Existence and uniqueness of the price level solving equation

(82) implies the existence and uniqueness of all other variables, which can all be expressed as functions of the price level. Rewriting equation (82) yields

$$u'(C(P)) - \int_{\hat{\theta}(P)}^{\infty} \theta v'(B/P + \bar{b}) dF(\theta) = F(\hat{\theta}(P)) \frac{R}{(1+\pi)} \beta u'(C(P)), \quad (82)$$

such that the derivatives of both the LHS and the RHS are negative,

$$\begin{aligned} \frac{\partial LHS}{\partial P} &= u''(C(P)) \frac{\partial C(P)}{\partial P} + \int_{\hat{\theta}(P)}^{\infty} \theta \frac{B}{P^2} v''(B/P + \bar{b}) dF(\theta) + f(\hat{\theta}(P)) \hat{\theta}(P) v'(B/P + \bar{b}) \frac{\partial \hat{\theta}(P)}{\partial P} \\ \frac{\partial RHS}{\partial P} &= F(\hat{\theta}(P^*)) \frac{R}{1+\pi} \beta u''(C(P)) \frac{\partial C(P)}{\partial P} + f(\hat{\theta}(P)) \frac{R}{1+\pi} \beta u'(C(P^*)) \frac{\partial \hat{\theta}(P)}{\partial P}, \end{aligned}$$

with

$$\frac{\partial LHS}{\partial P} < \frac{\partial RHS}{\partial P} < 0 \quad (83)$$

since $\frac{\partial \hat{\theta}(P)}{\partial P} < 0$, $\frac{\partial C(P)}{\partial P} > 0$ and $\hat{\theta}(P) v'(B/P + \bar{b}) = u'(C(P^*))$. Thus $LHS - RHS$ is a monotonically decreasing function of P and the intermediate value theorem implies that there is at most one P where $LHS(P) = RHS(P)$, that is there is at most one steady state price level.

To establish existence of such a price level, I now show that the LHS is larger than the RHS for small P and the RHS is larger than the LHS for large P , implying a unique P where the LHS is equal to the RHS.

To see this, note that if $P \rightarrow 0$, $u'(C(P)) \rightarrow \infty$ which since $F(\hat{\theta}(P)) \frac{R}{(1+\pi)} \beta < 1$ and all other terms are bounded implies that $LHS > RHS$.

If $P \rightarrow \infty$, C converges to a number larger than $A - \bar{b}$ and $\hat{\theta}$ converges to a number smaller than $\frac{u'(A-\bar{b})}{v'(\bar{b})}$. Thus

$$LHS \leq u'(A - \bar{b}) - \int_{\frac{u'(A-\bar{b})}{v'(\bar{b})}}^{\infty} \theta v'(\bar{b}) dF(\theta) < 0 < RHS \quad (84)$$

Proof of Proposition 3

Step 1

I first show that both the function

$$\Phi(\tilde{P}_t) = \frac{u'(C(\tilde{P}_t)) - \int_{\hat{\theta}(\tilde{P}_t)}^{\infty} \theta v'(B_t/\tilde{P}_t + \bar{b}) dF(\theta)}{\tilde{P}_t F(\hat{\theta}(\tilde{P}_t))} \quad (85)$$

and the function

$$\Gamma(\tilde{P}_{t+1}) = \beta \frac{u'(C(\tilde{P}_{t+1}))}{\tilde{P}_{t+1}} \quad (86)$$

are decreasing in $\tilde{P}_t$ and $\tilde{P}_{t+1}$ respectively.

This is obvious for Γ with derivative

$$\Gamma'(\tilde{P}) = \beta \frac{u''(C(\tilde{P}))C'(\tilde{P})\tilde{P} - u'(C(\tilde{P}))}{\tilde{P}^2} < 0 \quad (87)$$

since $u', C' > 0$ and $u'' < 0$.

The derivative of Φ equals

$$\begin{aligned} \Phi'(\tilde{P}) &= \frac{u''(C(\tilde{P}))C'(\tilde{P}) + \int_{\hat{\theta}(\tilde{P})}^{\infty} \theta \frac{B}{\tilde{P}^2} v''(B/\tilde{P} + \bar{b}) dF(\theta) + \hat{\theta}'(\tilde{P})\hat{\theta}(\tilde{P})v'(B/\tilde{P} + \bar{b})}{\tilde{P}F(\hat{\theta}(\tilde{P}))} \\ &- \frac{[u'(C(\tilde{P})) - \int_{\hat{\theta}(\tilde{P})}^{\infty} \theta v'(B/\tilde{P} + \bar{b}) dF(\theta)][F(\hat{\theta}(\tilde{P})) + \tilde{P}f(\hat{\theta}(\tilde{P}))\hat{\theta}'(\tilde{P})]}{(\tilde{P}F(\hat{\theta}(\tilde{P})))^2} \\ &= \frac{u''(C(\tilde{P}))C'(\tilde{P}) + \int_{\hat{\theta}(\tilde{P})}^{\infty} \theta \frac{B}{\tilde{P}^2} v''(B/\tilde{P} + \bar{b}) dF(\theta)}{\tilde{P}F(\hat{\theta}(\tilde{P}))} \\ &- \frac{u'(C(\tilde{P})) - \int_{\hat{\theta}(\tilde{P})}^{\infty} \theta v'(B/\tilde{P} + \bar{b}) dF(\theta)}{\tilde{P}^2 F(\hat{\theta}(\tilde{P}))} \\ &+ \hat{\theta}'(\tilde{P}) \frac{f(\hat{\theta}(\tilde{P}))}{F(\hat{\theta}(\tilde{P}))} \left[\hat{\theta}(\tilde{P})v'(B/\tilde{P} + \bar{b})F(\hat{\theta}(\tilde{P})) - u'(C(\tilde{P})) + \int_{\hat{\theta}(\tilde{P})}^{\infty} \theta v'(B/\tilde{P} + \bar{b}) dF(\theta) \right] \\ &< 0, \end{aligned} \quad (88)$$

since

$$\begin{aligned}
& \hat{\theta}(\tilde{P})v'(B/\tilde{P} + \bar{b})F(\hat{\theta}(\tilde{P})) - u'(C(\tilde{P})) + \int_{\hat{\theta}(\tilde{P})}^{\infty} \theta v'(B/\tilde{P} + \bar{b})dF(\theta) \\
& \geq \hat{\theta}(\tilde{P})v'(B/\tilde{P} + \bar{b})F(\hat{\theta}(\tilde{P})) - u'(C(\tilde{P})) + \int_{\hat{\theta}(\tilde{P})}^{\infty} \hat{\theta}(\tilde{P})v'(B/\tilde{P} + \bar{b})dF(\theta) \\
& = \hat{\theta}(\tilde{P})v'(B/\tilde{P} + \bar{b})F(\hat{\theta}(\tilde{P})) - u'(C(\tilde{P})) + (1 - F(\hat{\theta}(\tilde{P})))\hat{\theta}(\tilde{P})v'(B/\tilde{P} + \bar{b}) \\
& = \hat{\theta}(\tilde{P})v'(B/\tilde{P} + \bar{b}) - u'(C(\tilde{P})) \\
& = 0,
\end{aligned}$$

and $\hat{\theta}'(\tilde{P}) < 0$.

At the steady state $\Phi'(P^*) < \frac{R}{1+\gamma}\Gamma'(P^*)$:

$$\begin{aligned}
& \Phi'(P^*) - \frac{R}{1+\gamma}\Gamma'(P^*) \\
& = \frac{-1}{(P^*)^2 F(\hat{\theta}(P^*))} \left[u'(C(P^*)) - \int_{\hat{\theta}(P^*)}^{\infty} \theta v'(B/P^* + \bar{b})dF(\theta) - \frac{R}{1+\gamma} F(\hat{\theta}(P^*)) \beta u'(C(P^*)) \right] \\
& + \frac{u''(C(P^*))C'(P^*)}{P^*} \left[1 - \frac{R}{1+\gamma} F(\hat{\theta}(P^*)) \beta \right] \\
& + \frac{\int_{\hat{\theta}(P^*)}^{\infty} \theta \frac{B}{(P^*)^2} v''(B/P^* + \bar{b})dF(\theta)}{P^* F(\hat{\theta}(P^*))} \\
& + \hat{\theta}'(P^*) \frac{f(\hat{\theta}(P^*))}{F(\hat{\theta}(P^*))} \left[\hat{\theta}(P^*)v'(B/P^* + \bar{b})F(\hat{\theta}(P^*)) - u'(C(P^*)) + \int_{\hat{\theta}(P^*)}^{\infty} \theta v'(B/P^* + \bar{b})dF(\theta) \right] \\
& < 0,
\end{aligned}$$

since the first terms is zero at the steady state (the term in square brackets is the FOC and thus zero), the second term is negative since $[1 - \frac{R}{1+\gamma} F(\hat{\theta}(P^*)) \beta] > 0$ and $u'' < 0$, the third term is negative since $v'' < 0$ and the last term was shown above to be negative.

This implies that $\Phi(\tilde{P}) < \frac{R}{1+\gamma}\Gamma(\tilde{P})$ for $\tilde{P} > P^*$ and $\Phi(\tilde{P}) > \frac{R}{1+\gamma}\Gamma(\tilde{P})$ for $\tilde{P} < P^*$ since $\Phi(\tilde{P}) = \frac{R}{1+\gamma}\Gamma(\tilde{P})$ only if $\tilde{P} = P^*$.

Step 2

I now show that if $\tilde{P}_t > P^*$ then the subsequent sequence of prices is monotonically increasing, $\tilde{P}_t < \tilde{P}_{t+1} < \dots < \tilde{P}_{t+k}, \dots$ and if $\tilde{P}_t < P^*$ then the subsequent sequence of prices is monotonically decreasing, $\tilde{P}_t > \tilde{P}_{t+1} > \dots > \tilde{P}_{t+k}, \dots$ before I finally show in Step 3 that such price sequences do not form an equilibrium.

For every $\tilde{P}_s$ at time s the price at time $s + 1$, $\tilde{P}_{s+1}$, is defined as solving

$$\Phi(\tilde{P}_s) = \frac{R}{1 + \gamma} \Gamma(\tilde{P}_{s+1}) \quad (89)$$

If $\tilde{P}_s > P^*$ then

$$\frac{R}{1 + \gamma} \Gamma(\tilde{P}_s) > \Phi(\tilde{P}_s) = \frac{R}{1 + \gamma} \Gamma(\tilde{P}_{s+1}), \quad (90)$$

which implies that $\tilde{P}_{s+1} > \tilde{P}_s$ since $\Gamma'(\tilde{P}) < 0$.

If $\tilde{P}_s < P^*$ then

$$\frac{R}{1 + \gamma} \Gamma(\tilde{P}_s) < \Phi(\tilde{P}_s) = \frac{R}{1 + \gamma} \Gamma(\tilde{P}_{s+1}), \quad (91)$$

which implies that $\tilde{P}_{s+1} < \tilde{P}_s$ since $\Gamma'(\tilde{P}) < 0$.

Step 3

Step 2 shows that a price $\tilde{P}_t$ different from the steady state price P^* leads to either an monotonically increasing price sequence (if $\tilde{P}_t > P^*$) or a monotonically decreasing price sequence (if $\tilde{P}_t < P^*$).

The monotonically increasing price sequence is unbounded since otherwise the prices would converge. The limit would be a steady state, contradicting the uniqueness of a steady state established in Proposition 2.

For all high enough price levels $\tilde{P}$ it holds that $\hat{\theta}(\tilde{P}) < \frac{u'(A-\bar{b})}{v'(\bar{b})}$ and thus

$$u'(C(\tilde{P})) - \int_{\hat{\theta}(\tilde{P})}^{\infty} \theta v'(B/\tilde{P} + \bar{b}) dF(\theta) \leq u'(A - \bar{b}) - \int_{\frac{u'(A-\bar{b})}{v'(\bar{b})}}^{\infty} \theta v'(\bar{b}) dF(\theta) < 0, \quad (92)$$

implying that $\tilde{P}$ does not form an equilibrium since $\frac{R}{1+\gamma} \Gamma > 0$.

The monotonically decreasing price sequence is not bounded from below by some strictly positive number with a positive consumption level since otherwise the price sequence would converge to a positive price level. The limit would be a steady state, contradicting the uniqueness of a steady state established in Proposition 2.

But for a low enough price level $\tilde{P}$ it holds that $G/\tilde{P} > A$, that is real government expenditures exceed real output and consumption is non-positive, so that this $\tilde{P}$ is not an equilibrium either.

Together this implies that there is no $\tilde{P}_t \neq P^*$ since an $\tilde{P}_t \neq P^*$ would be consistent only with price expectations which are not an equilibrium.

Proof of Proposition 1 From time S on policies are stationary, i.e. for $t \geq S$, $R_t = R$, $G_t = G(1 + \gamma)^{t-S}$, $T_t = T(1 + \gamma)^{t-S}$ and $B_t = B(1 + \gamma)^{t-S}$. The results shown above imply that from time S the economy is characterized by a price level P^* such that prices for $t \geq S$, $P_t = P^*(1 + \gamma)^{t-S}$. As before consumption $C_t = C(P^*)$ and $\hat{\theta}_t = \hat{\theta}(P^*)$.

Prices before time S are defined recursively. Given a price P_t^* at time $t \leq S$, the price level at time $t - 1$ is

$$P_{t-1}^* = \Phi_t^{-1}(R_t \Gamma_t(P_t^*)), \quad (93)$$

where

$$\Phi_t(P_t) = \frac{u'(C(P_t, B_t)) - \int_{\hat{\theta}(P_t)}^{\infty} \theta v'(B_t/P_t + \bar{b}) dF(\theta)}{P_t F(\hat{\theta}(P_t))} \quad (94)$$

and the function

$$\Gamma_t(P_{t+1}) = \beta \frac{u'(C(P_{t+1}, B_{t+1}))}{P_{t+1}} \quad (95)$$

and Φ is strictly monotone and therefore invertible. Note that these are actual prices P and not detrended prices $\tilde{P}$ as the economy is not in steady state before time S .

This recursive definition yields a price sequence

$$(P_0^*, P_1^*, \dots, P_t^*, \dots, P_S = P^*, P_{S+1} = P^*(1 + \gamma), P_{S+2} = P^*(1 + \gamma)^2, \dots,) \quad (96)$$

I now show that this is the unique equilibrium sequence of prices. Suppose now to the contrary that there is another equilibrium price sequence $\hat{P}$ and let k be the first time when the price levels differ $\hat{P}_k \neq P_k^*$ (and $\hat{P}_m = P_m^*$ for $m < k$). The uniqueness of a steady state price level implies that $k < S$ since from period S onwards, the equilibrium price equals $P^*(1 + \gamma)^{t-S}$.

The price at time k , $\hat{P}_k$, uniquely pins down the full price sequence recursively,

$$\hat{P}_{m+1} = \Gamma_{m+1}^{-1}(\Phi_m(\hat{P}_m)/R_m), \quad (97)$$

for all $m \geq k$. Since both Γ_t and Φ_t are strictly decreasing functions, the concatenation $\Gamma_{l+1}^{-1}(\Phi_l(P_l^*)/R_l)$ is strictly increasing. This implies that if $\hat{P}_k > P_k^*$, then $\hat{P}_m > P_m^*$ for all $m \geq k$. In particular $\hat{P}_S > P_S^* = P^*$, which is not an equilibrium price.

Similarly, if $\hat{P}_k < P_k^*$, then $\hat{P}_m < P_m^*$ for all $m \geq k$. In particular $\hat{P}_S < P_S^* = P^*$, which is also not an equilibrium price.

Together this implies that $\hat{P}_k \neq P_k^*$ is proven wrong by contradiction, implying that $\hat{P}_t = P_t^*$ for all t is the only equilibrium price sequence.

Proof of Proposition 4 The proof proceeds by backwards induction. In period $S - 1$, the detrended price level $\tilde{P}_{S-1}$ solves

$$\Phi(\tilde{P}_{S-1}) = \frac{R_S}{1 + \gamma} \Gamma(P^*) \geq \frac{R}{1 + \gamma} \Gamma(P^*) = \Phi(P^*), \quad (98)$$

which implies that $\tilde{P}_{S-1} \leq P^*$ with strict inequality if $R_S < R$.

Now assume that $\tilde{P}_{t+1} \leq \tilde{P}_{t+2} \leq \dots$. The detrended price level $\tilde{P}_t$ solves

$$\Phi(\tilde{P}_t) = \frac{R_{t+1}}{1 + \gamma} \Gamma(\tilde{P}_{t+1}) \geq \frac{R_{t+2}}{1 + \gamma} \Gamma(\tilde{P}_{t+1}) \geq \frac{R_{t+2}}{1 + \gamma} \Gamma(\tilde{P}_{t+2}) = \Phi(\tilde{P}_{t+1}), \quad (99)$$

which implies that $\tilde{P}_t \leq P_{t+1}$ with strict inequality if $R_{t+1} < R_{t+2}$. The inequality uses that Γ is strictly decreasing and $\tilde{P}_{t+1} \leq P_{t+2}$.

This proves the statement for detrended prices by induction. This immediately implies

$$\frac{P_{t+1}}{P_t} \geq (1 + \gamma) \quad (100)$$

for non-detrended prices since the trend is $(1 + \gamma)$.

Proof of Proposition 5 The proof proceeds by backwards induction. In period $S - 1$, the detrended price levels $\tilde{P}_{S-1}^a$ and $\tilde{P}_{S-1}^b$ solve

$$\Phi(\tilde{P}_{S-1}^a) = \frac{R_S^a}{1 + \gamma} \Gamma(P^*) \geq \frac{R_S^b}{1 + \gamma} \Gamma(P^*) = \Phi(\tilde{P}_{S-1}^b). \quad (101)$$

which implies that $\tilde{P}_{S-1}^a \leq \tilde{P}_{S-1}^b$ with strict inequality if $R_S^a > R_S^b$ since Φ is decreasing.

Now assume that $\tilde{P}_{t+1}^a \leq \tilde{P}_{t+1}^b$. Then

$$\Phi(\tilde{P}_t^a) = \frac{R_{t+1}^a}{1+\gamma} \Gamma(\tilde{P}_{t+1}^a) \geq \frac{R_{t+1}^b}{1+\gamma} \Gamma(\tilde{P}_{t+1}^a) \geq \frac{R_{t+1}^b}{1+\gamma} \Gamma(\tilde{P}_{t+1}^b) = \Phi(\tilde{P}_t^b), \quad (102)$$

implying that $\tilde{P}_t^a \leq \tilde{P}_t^b$ with strict inequality if $R_{t+1}^a > R_{t+1}^b$.

Proof of Proposition 6

Define

$$\Phi(\tilde{P}_t, G) = \frac{u'(C(\tilde{P}_t, G)) - \int_{\hat{\theta}(\tilde{P}_t, G)}^{\infty} \theta v'(B/\tilde{P}_t + \bar{b}) dF(\theta)}{\tilde{P}_t F(\hat{\theta}(\tilde{P}_t))} \quad (103)$$

$$\Gamma(\tilde{P}_{t+1}, G) = \beta \frac{u'(C(\tilde{P}_{t+1}, G))}{\tilde{P}_{t+1}}, \quad (104)$$

where both functions are strictly decreasing in $\tilde{P}$ and strictly increasing in G .

The proof again proceeds by backwards induction. In period $S-1$, the detrended price level $\tilde{P}_{S-1}$ solves

$$\Phi(\tilde{P}_{S-1}, \hat{G}) = \frac{R}{1+\gamma} \Gamma(P^*, G) = \Phi(P^*, G) \leq \Phi(P^*, \hat{G}), \quad (105)$$

which implies that $\tilde{P}_{S-1} \geq P^*$ with strict inequality if $\hat{G} > G$ since Φ is decreasing in $\tilde{P}$.

Now assume that $\tilde{P}_{t+1} \geq \tilde{P}_{t+2} \geq \dots$. The detrended price level $\tilde{P}_t$ solves

$$\Phi(\tilde{P}_t, \hat{G}) = \frac{R}{1+\gamma} \Gamma(\tilde{P}_{t+1}, \hat{G}) \leq \frac{R}{1+\gamma} \Gamma(\tilde{P}_{t+2}, \hat{G}) = \Phi(\tilde{P}_{t+1}, \hat{G}), \quad (106)$$

which implies that $\tilde{P}_t \geq \tilde{P}_{t+1}$. The same arguments show that a more expansive fiscal policy leads to stronger price increases.

Proof of Proposition 7 The same arguments as in the proofs of Propositions 4 and 5 apply here.

Proof of Proposition 8 Define

$$\Phi(\tilde{P}_t, A) = \frac{u'(C(\tilde{P}_t, A)) - \int_{\hat{\theta}(\tilde{P}_t, A)}^{\infty} \theta v'(B/\tilde{P}_t + \bar{b}) dF(\theta)}{\tilde{P}_t F(\hat{\theta}(\tilde{P}_t))} \quad (107)$$

$$\Gamma(\tilde{P}_{t+1}, A) = \beta \frac{u'(C(\tilde{P}_{t+1}, A))}{\tilde{P}_{t+1}}, \quad (108)$$

where both functions are strictly decreasing in $\tilde{P}$ and A .

The proof again proceeds by backwards induction. In period $S - 1$, the detrended price level $\tilde{P}_{S-1}$ solves

$$\Phi(\tilde{P}_{S-1}, \hat{A}) = \frac{R}{1 + \gamma} \Gamma(P^*, A) = \Phi(P^*, A) \geq \Phi(P^*, \hat{A}), \quad (109)$$

which implies that $\tilde{P}_{S-1} \leq P^*$ with strict inequality if $\hat{A} > G$ since Φ is strictly decreasing in $\tilde{P}$.

Now assume that $\tilde{P}_{t+1} \leq \tilde{P}_{t+2} \leq \dots$. The detrended price level $\tilde{P}_t$ solves

$$\Phi(\tilde{P}_t, \hat{A}_t) = \frac{R}{1 + \gamma} \Gamma(\tilde{P}_{t+1}, \hat{A}_{t+1}) \geq \frac{R}{1 + \gamma} \Gamma(\tilde{P}_{t+2}, \hat{A}_{t+2}) = \Phi(\tilde{P}_{t+1}, \hat{A}_{t+1}), \quad (110)$$

which implies that $\tilde{P}_t \leq \tilde{P}_{t+1}$.