Multiple Contracting in Insurance Markets*

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Abstract

We study a model of insurance markets in which multiple contracting endogenously emerges in equilibrium. Different layers of coverage are fairly priced according to the types of consumers who purchase them, giving rise to cross-subsidies between types, but not between contracts. Riskier consumers demand greater total coverage at an increasing unit price, but the contracts offered by firms exhibit quantity discounts. We emphasize the need to regulate the supply side of insurance markets, while consumers can be left free to choose their coverage level.

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1 Introduction

Insurance markets face the threat of adverse selection: insurees may be better informed about their riskiness than insurers are. Because insurees' riskiness directly affects insurers' profit, trade on these markets can be impeded, as emphasized by the well-known theoretical analyses of Akerlof (1970) and Rothschild and Stiglitz (1976). This in turn raises the question of public intervention, and each reform of an insurance market is soon followed by discussions on how to best tackle the adverse-selection problem. To take but one example, the recent reform wave of health insurance has led to a revival of important policy debates related to adverse selection: on whether basic coverage should be made mandatory; and on how competition between insurance providers should be organized, especially when it comes to policies that propose optional, additional coverage.

In this paper, we argue that answering such questions requires taking into account the possibility that a given insuree buys several insurance policies from several distinct insurers. Indeed, multiple contracting turns out to be a widespread phenomenon. In health-insurance markets, it is quite common for consumers in Europe and in the US to complement a basic coverage with an additional insurance policy. For example, 92% of the French population have both a mandatory coverage and a private coverage,[1] while in the US, 10 million out of the 42 million individuals covered by Medicare opt for buying an additional, private coverage (Medigap).[2] The US life-insurance market also typically allows for multiple contracting,[3] and so does the UK annuity market.[4]

In spite of the empirical relevance of multiple contracting, the theoretical literature has so far mostly relied on models à la Rothschild and Stiglitz (1976) and Wilson (1977), which postulate exclusive relationships between insurers and insurees, so that multiple contracting is a priori ruled out. The literature on nonexclusive competition in insurance did not so far provide any justification for multiple contracting either. Indeed, as shown by Attar, Mariotti, and Salanié (2014a), existence of pure-strategy equilibria requires demanding assumptions, such equilibria when they exist do not feature the empirically prevalent distinction between basic and additional coverage, and, moreover, equilibrium allocations can often be sustained by exclusive relationships on the equilibrium path.

This paper proposes a model of an insurance market in which multiple contracting is

[1] See Thompson, Osborne, Squires, and Jun (2013).

[2] See http://www.protectmedigap.org.

[3] As stated in Cawley and Philipson (1999), "multiple contracting is highly prevalent" and represents roughly one quarter of consumers with at least one life-insurance policy.

[4] See Finkelstein and Poterba (2004), in particular Footnote 2: "[...] we view the exclusivity condition as unlikely to be satisfied in annuity markets," and the end of Section IV.A.

allowed for, and in which it indeed emerges as an equilibrium phenomenon. As in the Rothschild–Stiglitz–Wilson setting, first contracts are offered by insurers, and then each type of consumer chooses which contract(s) to accept—the key difference in our model being that a consumer may accept several contracts from several distinct insurers. As in the original Rothschild and Stiglitz (1976) model, each insurer is restricted to offer a single contract; menus are considered subsequently, in relation with regulation. We consider the two-type case, and we show that pure-strategy equilibria exists under reasonably general assumptions; moreover, the equilibrium aggregate allocation is unique. This model may thus be helpful for evaluating market reforms. It also yields several insights that we now detail.

In models that postulate exclusivity, competition bears on the pricing of the aggregate coverage bought by each insuree. Allowing for nonexclusivity changes the focus: from the insuree's viewpoint, each additional contract represents an additional layer of coverage, and the study naturally focuses on the pricing of such additional layers. The unique equilibrium aggregate allocation has a nice and natural structure: both low- and high-risk types buy the same basic coverage, and in addition the high-risk type buys a complementary coverage. Competition ensures that each contract makes zero expected profit: being sold to both low- and high-risk types, basic coverage is priced at the average cost, whereas complementary coverage is priced at marginal cost, reflecting that it is only sold to the high-risk type. In short, our equilibrium allocation offers a natural generalization of Akerlof (1970) pricing to divisible coverage, as each additional layer is priced at the expected cost associated to the set of insurees who buy this layer. That this allocation, originally studied by Jaynes (1978), Hellwig (1988), and Glosten (1994), naturally emerges when insurers compete in single-contract offers, is a novel contribution of our analysis.[5]

It is worth noting that multiple contracting must take place in equilibrium for the high-risk type: no insurer would be ready to sell her aggregate coverage at the equilibrium price, as this would make a loss. Thus the high-risk type type must buy basic coverage from one insurer and complementary coverage from another insurer. As a result, the contracts bought by this type are qualitatively different: multiple contracting emerges because competition deals with layers of coverage that are sold independently.[6]

In equilibrium, the low-risk type subsidizes the high-risk type: this is in contrast to the

[5]Unlike Jaynes (1978, 2011) and Hellwig (1988), our model does not postulate explicit communication between insurers. Unlike Glosten (1994), our analysis does not make use of free-entry arguments. Properties of the Jaynes–Hellwig–Glosten allocation are also studied in Attar, Mariotti, and Salanié (2014a, 2014b).

[6]Notice also that, in our setting, multiple contracting is not related to the insuree splitting her aggregate demand between identical insurers. See Biais, Martimort, and Rochet (2000, 2013) and Back and Baruch (2013) for common-value environments in which multiple contracting arises for precisely this reason.

Rothschild and Stiglitz (1976) allocation, in which the profit extracted from each type is exactly zero, but is reminiscent of the Wilson (1977), Miyazaki (1977), and Spence (1978) allocations. The fact that cross-subsidies survive cream-skimming deviations is intriguing. Recall nevertheless that the associated tariff for aggregate coverage has been shown by Glosten (1994) to be the only tariff that makes entry unprofitable: that is, no inactive insurer can make a profit by offering an additional collection of contracts, taking into account that the original tariff remains available. We reinforce this result by showing that the aggregate allocation resulting from this tariff can be uniquely supported as the equilibrium outcome of an oligopoly game played by finitely many strategic insurers. Our construction relies on a latent contract (Arnott and Stiglitz (1993)), which is inactive on the equilibrium path but makes cream-skimming deviations unfeasible.

In our model, each insurer is only allowed to offer a single contract. In practice, insurers can offer menus of such contracts; yet, at the same time, insurance markets are heavily regulated, constraining the offers that insurers can make. In that respect, it is interesting to notice that our results extend to the case where insurers are allowed to offer menus of contracts, provided cross-subsidies between contracts are prohibited; we thus see the single-contract restriction as a parsimonious way of capturing the impact of such a policy.[7] Overall, our analysis suggests that regulation has a role to play on the supply side of insurance markets, to deter insurers from dumping high-risk consumers on their competitors. In Section 6.3, we discuss to which extent the organization of health insurance systems in developed countries can help dealing with this issue. It should be noted, however, that there is no need to constrain insurees' choices in our setting, for instance by requiring basic coverage to be mandatory. This contrasts with standard policy recommendations from exclusive-competition models of insurance markets under adverse selection.

An important feature of our setting is that, compared to models that postulate exclusivity, it allows to draw a distinction between the set of contracts offered by the insurers and the set of equilibrium aggregate coverage levels chosen by an insuree by combining some of these contracts. Under a standard single-crossing condition, the latter set has a familiar structure that it shares with exclusive-competition models: a riskier type buys a higher aggregate coverage, at a unit price which is increasing with coverage. Yet, the former set has a very

[7] In the absence of regulation, and as shown by Attar, Mariotti, and Salanié (2014a), an incumbent insurer could profitably deviate by offering two different contracts. The first contract would make a profit by attracting only the low-risk type on a basic coverage. The second contract would offer additional coverage at a loss, so as to attract only the high-risk type, the key point being that this type would find it profitable to buy the basic coverage from other insurers. As we show in Section 6.2, cross-subsidies between contracts are the essence of any such deviation.

different structure; indeed, we show that, under the very same assumptions that ensure the existence of an equilibrium, the supply of coverage involves a unit price that is decreasing with coverage. That is, the contracts offered by insurers exhibit quantity discounts. The empirical predictions of our model are thus richer than in the exclusive case, and may be more in line with empirical findings; we comment on these points at more length in Section 6.4. In particular, both the predictions of a unit price that increases with coverage, and of a positive correlation between riskiness and coverage,[8] may be reversed when multiple contracting takes place, depending on whether data originate from the demand side or the supply side of the market.

The paper is organized as follows. Section 2 describes the model. Section 3 characterizes equilibrium aggregate trades. Section 4 shows how to construct equilibria sustaining such trades for a special class of preferences. Section 5 shows that the equilibria constructed in Section 4 are robust in the sense that similar equilibria exist for any preferences in a neighboring class. Section 6 draws the main normative, theoretical, policy, and empirical lessons from our analysis.

2 The Model

In this section, we describe a general model of trade under adverse selection, in which a buyer can make simultaneous purchases of a divisible good from several sellers. The key difference with the nonexclusive-competition models studied in Biais, Martimort, and Rochet (2000, 2013) and Attar, Mariotti, and Salanié (2011, 2014a) is that sellers are restricted to make single-contract offers to the buyer. A typical example is an insurance market in which a privately informed risk-averse consumer can buy coverage from several insurers, and each insurer can only offer a single level of coverage, as in Rothschild and Stiglitz (1976), Wilson (1977), and Hellwig (1987).

2.1 The Buyer

The buyer is privately informed of her preferences. She may be of two types, $i = 1, 2$, with positive probabilities m_1 and m_2 such that $m_1 + m_2 = 1$. Type i has preferences over aggregate quantity-transfer bundles (Q, T) in some consumption set $X \subset \mathbb{R}_+ \times \mathbb{R}$, the precise specification of which depends on the interpretation of the model. We require that X contain the no-trade point $(0, 0)$, that it be convex with a nonempty interior, and that

[8] See Chiappori and Salanié (2000), and Chiappori, Jullien, Salanié, and Salanié (2006) for general results.

it be comprehensive in the sense that $(Q,T') \in X$ if $(Q,T) \in X$ and $T' < T$. Type i's preferences over X are taken to be representable by a function U_i defined over an open, convex, and comprehensive neighborhood V of X.[9] We assume that U_i is twice continuously differentiable, with $\partial U_i/\partial T < 0$, and that U_i is strictly quasiconcave.[10] Hence, type i's marginal rate of substitution of the good for money,

$$\tau_i \equiv -\frac{\partial U_i/\partial Q}{\partial U_i/\partial T}, \tag{1}$$

is well defined over V and strictly decreasing along her indifference curves. The following single-crossing property is key to our results.

Assumption SC *For each* $(Q,T) \in V$, $\tau_2(Q,T) > \tau_1(Q,T)$.

Geometrically, an indifference curve for type 2 crosses an indifference curve for type 1 only once, from below. Consequently, type 2 is more eager to increase her purchases than type 1 is.

2.2 The Sellers

There are n identical sellers, with n large but finite. If a seller provides type i with a quantity q and obtains a transfer t in return, he earns a profit $t - v_i q$, where v_i is the cost of serving type i. The following assumption will be maintained throughout the analysis.

Assumption CV $v_2 > v_1$.

Along with Assumption SC, Assumption CV creates adverse selection: whereas type 2 is more willing to trade at the margin than type 1 is, she faces sellers who are more reluctant to trade with her than with type 1. We let $v \equiv m_1 v_1 + m_2 v_2$ be the average cost of serving the buyer, so that $v_2 > v > v_1$.

2.3 The Trading Game

As in Biais, Martimort, and Rochet (2000, 2013) and Attar, Mariotti, and Salanié (2011, 2014a), no seller can control and, a fortiori, contract on the trades that the buyer makes with his competitors. A distinguishing feature of our analysis is that sellers compete to serve the buyer by proposing bilateral contracts, that is, quantity-transfer bundles, rather than menus of such contracts. Therefore, the timing of our trading game is as follows:

[9] This last assumption is a standard technical trick that allows us to define marginal rates of substitution over the boundary of X (see, for instance, Mas-Colell (1985, Definition 2.3.17)).

[10] We do not assume that U_i is monotone in quantities, as, indeed, need not be the case for the quadratic specification of Section 4.1.2.

1. Each seller k proposes a contract $(q^k, t^k) \in \mathbb{R}_+ \times \mathbb{R}$.[11]

2. After privately learning her type, the buyer selects which contracts to trade with the sellers, if any.

Given a vector of contract offers $((q^1, t^1), \ldots, (q^n, t^n))$, type i's problem is then

$$\max \left\{ U_i \left(\sum_{k \in K} q^k, \sum_{k \in K} t^k \right) : K \subset \{1, \ldots, n\} \text{ and } \left(\sum_{k \in K} q^k, \sum_{k \in K} t^k \right) \in X \right\}, \tag{2}$$

with $\sum_{\emptyset} = 0$ by convention. Note that, because, by assumption, $(0,0) \in X$, the feasible set in type i's problem is always nonempty. We use perfect Bayesian equilibrium as our equilibrium concept. Throughout the paper, we focus on pure-strategy equilibria.

3 Equilibrium: Necessary Conditions

In this section, we show that any equilibrium aggregate allocation of our trading game is of the form predicted by Jaynes (1978), Hellwig (1988), and Glosten (1994). We give necessary conditions under which equilibria feature multiple contracting, in that both types first trade the same basic quantity, which type 2 complements by conducting additional trades. All contracts are fairly priced given the types who trade them. As a result, equilibria involve zero expected profit for the sellers and cross-subsidies between types. We also provide necessary conditions for the existence of an equilibrium.

3.1 Jaynes–Hellwig–Glosten Pricing

Let us fix an equilibrium, and let (Q_1, T_1) and (Q_2, T_2) be the equilibrium aggregate trades of types 1 and 2. According to Assumption SC together with the fact that marginal rates of substitution are well defined for each type, we know that $Q_2 \geq Q_1$. Our first result describes the price structure of equilibrium.

Theorem 1 *Suppose that there are at least three sellers. Then, in any equilibrium,*

$$T_1 = vQ_1, \tag{3}$$

$$T_2 - T_1 = v_2(Q_2 - Q_1). \tag{4}$$

Moreover, any traded contract is issued at unit price v or v_2 and makes zero expected profit.

Hence, in the aggregate, type 2 first trades a quantity Q_1 at unit price v, just as type 1

[11] The null contract is $(0,0)$. A contract (q^k, t^k) with $q^k > 0$ has unit price t^k/q^k.

does, and then, on top of this, a quantity $Q_2 - Q_1$ at unit price v_2. In the case of insurance, this corresponds to a situation in which the consumer can purchase basic coverage at a relatively low premium rate v, which she can complement by further coverage at a relatively high premium rate v_2. Premium rates reflect in a fair way the composition of the pool of types trading each policy. This implies that each seller and, therefore, each contract earn zero expected profit. Theorem 1 thus essentially describes a competitive outcome, which can be interpreted as a marginal version of Akerlof (1970) pricing. At the same time, this competitive outcome is consistent with cross-subsidies between types: the quantity Q_1 is sold at an average unit price, reflecting the fact that type 1 subsidizes type 2.

It is useful to compare this outcome to that which would arise in a monopolistic setting. Standard arguments then imply that, in the profit-maximizing, incentive-compatible menu of contracts $\{(Q_1^m, T_1^m), (Q_2^m, T_2^m)\}$, type 1 is indifferent between trading (Q_1^m, T_1^m) and not trading at all, and type 2 is indifferent between trading (Q_2^m, T_2^m) and trading (Q_1^m, T_1^m).[12] That is, according to Wilson's (1993, Chapter 4) demand profile interpretation, the marginal quantities Q_1^m and $Q_2^m - Q_1^m$ are sold at the profit-maximizing price, given the types who buy them. By contrast, the marginal quantities Q_1 and $Q_2 - Q_1$ in Theorem 1 are sold at the expected cost of providing them, given the types who buy them. This means, in particular, that type 1 is strictly better off trading Q_1 than not trading at all as long as $Q_1 > 0$, and that type 2 is strictly better off trading the additional quantity $Q_2 - Q_1$ on top of Q_1 as long as $Q_2 - Q_1 > 0$.

The price structure of equilibria derived in Theorem 1 is identical to that obtained by Jaynes (1978), Hellwig (1988), and Glosten (1994) in related contexts. Our characterization, however, differs from these earlier contributions in several ways. Unlike in Jaynes (1978) and Hellwig (1988), it does not rely on the possibility of inter-seller communication or on a specific timing of the sellers' offers. Unlike in Glosten (1994), it does not result from the derivation of an entry-proof tariff, but rather from the analysis of the sellers' deviations. In that respect, a novel insight of Theorem 1 is that one only needs to consider single-contract deviations to obtain Jaynes–Hellwig–Glosten pricing in equilibrium.

3.2 The Jaynes–Hellwig–Glosten Allocation

Hereafter, and to focus on the most interesting case, we restrict parameter values to be such that type 1 is ready to buy a positive quantity at unit price v.

Assumption 1-*v* $\tau_1(0, 0) > v$.

[12] See, for instance, Stiglitz (1977), in the case of insurance.

If Assumption 1-v did not hold, then, according to Theorem 1, we would have $Q_1 = 0$, so that type 1 would be excluded from trade in any equilibrium.[13] We show below that, under Assumption 1-v, a corollary of Theorem 1 is that type 1 trades the optimal quantity at unit price v, which pins down the value of Q_1:

$$\tau_1(Q_1, vQ_1) = v. \tag{5}$$

We also restrict parameter values to be such that type 2, having already bought a quantity Q_1 at unit price v, is ready to buy an additional positive quantity at unit price v_2.

Assumption 2-v_2 *$\tau_2(Q_1, vQ_1) > v_2$.*

If Assumption 2-v_2 did not hold, then, according to Theorem 1, we would have $Q_1 = Q_2$, so that a pooling allocation would emerge. We show below that, under Assumption 2-v_2, a corollary of Theorem 1 is that type 2 trades, on top of (Q_1, T_1), the optimal quantity complement at unit price v_2, which pins down the value of Q_2:

$$\tau_2(Q_2, vQ_1 + v_2(Q_2 - Q_1)) = v_2. \tag{6}$$

It should be noted that Assumptions 1-v and 2-v_2 were implicit in the insurance models of Jaynes (1978) and Hellwig (1988). The following corollary summarizes the aggregate features of candidate equilibria.

Corollary 1 *Under Assumptions 1-v and 2-v_2, any equilibrium satisfies (3)–(6).*

The corresponding Jaynes–Hellwig–Glosten allocation is illustrated in Figure 1.

3.3 Indispensability and Incentive Compatibility

A key feature of equilibrium is that, as in standard Bertrand competition, no seller can be indispensable in providing either type 1 or type 2 with their equilibrium trades; otherwise, he could earn a strictly positive expected profit by slightly increasing his price. This means, in particular, that the buyer has the opportunity to trade more than the quantity Q_1 at the relatively low unit price v. Whereas, according to (5), this opportunity is of no value for type 1, it could attract type 2, thereby destabilizing the equilibrium. Hence, a necessary condition for the existence of equilibrium is that these additional trades be of such a magnitude that type 2 is not willing to make them, given the strict convexity of her preferences. An additional condition is that it be impossible for a deviating buyer to profitably exploit these trades. These two conditions can be formulated as follows.

[13] It should be noted that this somewhat degenerate outcome is the only one consistent with equilibrium when sellers can compete through menus of contracts (Attar, Mariotti, and Salanié (2014a)).

Corollary 2 *Under Assumptions 1-v and 2-v_2, any equilibrium satisfies*

$$U_2(Q_2, T_2) \geq U_2(2Q_1, 2T_1), \tag{7}$$

$$2Q_1 > Q_2. \tag{8}$$

Conditions (7)–(8) are most easily understood when only two sellers issue contracts at unit price v. Then, by the dispensability property, each of them must offer a contract equal to type 1's entire equilibrium aggregate trade (Q_1, T_1). Thus the incentive-compatibility condition (7) must hold, expressing that type 2 is not willing to trade (Q_1, T_1) twice on the equilibrium path. Now, if condition (8) were not satisfied, then some other seller could attract type 2 by proposing her to trade the quantity $Q_2 - 2Q_1$ at a unit price slightly above v_2. Indeed, combined with $(2Q_1, 2T_1)$, such a contract would allow type 2 to buy the same quantity Q_2 as in equilibrium, in exchange for a transfer decreased by almost $(v_2 - v)Q_1$. Such a deviation would clearly be profitable, thereby upsetting the equilibrium.[14] This logic easily extends when more than two sellers issue contracts at unit price v. The economic implications of condition (8) are discussed at greater length in Section 6.4.

Geometrically, conditions (7)–(8) state that the aggregate trade $(2Q_1, 2T_1)$ is located in the lower contour set of (Q_2, T_2) for type 2, to the right of (Q_2, T_2). As $T_1 = vQ_1$ according to Theorem 1, this implies that

$$v > \tau_2(2Q_1, 2T_1). \tag{9}$$

Conditions (7)–(8) are satisfied whenever the complement $Q_2 - Q_1$ that type 2 wants to trade at unit price v_2 is sufficiently small relative to the basic quantity Q_1 that both types want to trade at unit price v. In the case of insurance, this holds whenever type 1 wants to purchase some insurance at the premium rate v (Assumption 1-v) and type 1 and type 2's riskinesses are not too far apart.

4 Equilibrium: Sufficient Conditions

In Section 3, we characterized the basic structure of aggregate trades in any candidate equilibrium. We now investigate whether and how these trades can effectively be sustained in equilibrium through appropriate contract offers. In this section, we focus on a family of specifications of our general model for which equilibrium existence is guaranteed as soon

[14] This argument presumes that $2Q_1 \neq Q_2$, which is necessarily the case, for, otherwise, type 2 would trade $(2Q_1, 2T_1)$ instead of (Q_2, T_2) on the equilibrium path, because one would then have $2T_1 = 2vQ_1 < vQ_1 + v_2(Q_2 - Q_1) = T_2$.

as the necessary conditions (7)–(8) are satisfied. Our construction relies on two kinds of contracts: first, contracts that are indeed traded in equilibrium, and, second, contracts that are not traded in equilibrium and the sole role of which is to deter deviations by the sellers.

4.1 A Class of Buyer's Preferences

The existence result that we provide in this section relies on two further restrictions on the buyer's preferences.

First, each type i has quasilinear preferences,

$$U_i(Q,T) = u_i(Q) - T, \tag{10}$$

where the utility function u_i is twice continuously differentiable, with $\partial^2 u_i < 0$. The marginal rate of substitution $\tau_i(Q,T) = \partial u_i(Q)$ of type i is then independent of T, so that all her indifference curves are vertical translates of each other. Assumption SC amounts to $\partial u_2(Q) > \partial u_1(Q)$ for all Q.

Second, and less standardly, there is a positive constant Q_0 such that

$$\partial u_2(Q + Q_0) = \partial u_1(Q) \tag{11}$$

for all $Q \geq 0$. Hence, in terms of the buyer's preferences, everything happens as if type 1 were identical to type 2, except that she had already traded a quantity Q_0.[15]

Geometrically, properties (10)–(11) imply that any pair of indifference curves for types 1 and 2 are, over the relevant domain, oblique or horizontal translates of each other. The translating vector connects the points of these indifference curves where type 1 and type 2 have equal marginal rates of substitution, as illustrated in Figure 2. This vector defines a contract stipulating a positive quantity and a transfer.

We shall illustrate these properties by means of two examples.

4.1.1 The CARA Example

Our first example is an insurance model in line with Rothschild and Stiglitz (1976). A risk-averse consumer can purchase coverage from several insurers. She faces a binomial risk on her wealth, which can take two values (W_B, W_G), with probabilities $(v_i, 1 - v_i)$ that define her type. Here $W_G - W_B$ is the positive monetary loss that the consumer incurs in the bad state and v is the average probability of a loss. The consumer's preferences have an

[15] Note, however, that, unlike in a private-value environment, this is not the only difference between types 1 and 2, for, by Assumption CV, the costs of serving them are not the same.

expected-utility representation with constant absolute risk aversion α.[16]

An insurance contract specifies an indemnity q in the bad state, along with a premium t, implying an expected profit $t - v_i q$ for the insurer that trades it with the consumer. In the aggregate, the consumer purchases at a price $T \equiv \sum_k t^k$ an indemnity $Q \equiv \sum_k q^k$ in the bad state. Type i's preferences over aggregate quantity-transfer bundles $(Q,T) \in X \equiv \mathbb{R}_+ \times \mathbb{R}$ are then represented by (10), with

$$u_i(Q) \equiv -\frac{1}{\alpha} \ln(v_i \exp(-\alpha(W_B + Q)) + (1 - v_i) \exp(-\alpha W_G)). \tag{12}$$

For each type i, the marginal rate of substitution (1) of indemnities for premia is

$$\partial u_i(Q) = \frac{1}{1 + [(1 - v_i)/v_i] \exp(-\alpha(W_G - W_B - Q))}. \tag{13}$$

By Assumption CV, type 2 has a higher probability of incurring a loss than type 1. This, in turn, implies that Assumption SC holds: type 2 is more eager to buy larger amounts of insurance than type 1. According to (13), condition (11) holds for

$$Q_0 \equiv \frac{1}{\alpha} \ln\left(\frac{(1 - v_1)/v_1}{(1 - v_2)/v_2}\right) > 0.$$

The first-best level of trade is the same for type 1 as for type 2 and entails full insurance, $Q_1^* = Q_2^* = W_G - W_B$. Assumption 1-$v$ amounts to $W_G - W_B > \ln([(1-v_1)/v_1]/[(1-v)/v])/\alpha$. The Jaynes–Hellwig–Glosten allocation is such that

$$Q_1 = W_G - W_B - \frac{1}{\alpha} \ln\left(\frac{(1 - v_1)/v_1}{(1 - v)/v}\right) < Q_1^*, \quad Q_2 = W_G - W_B = Q_2^*,$$

reflecting that type 1 purchases less than full coverage at the rate $v > v_1$. Assumption 2-v_2 is then automatically satisfied because type 2 wants to, and in equilibrium indeed does, obtain full coverage at the fair premium rate v_2.

4.1.2 The Quadratic Example

Our second example is a pure-trade model in line with Biais, Martimort, and Rochet (2000, 2013) and Back and Baruch (2013). In these market-microstructure models, a risk-averse insider with constant absolute risk aversion α trades shares of an asset with several market makers partly for informational and partly for hedging purposes, while facing Gaussian noise with variance σ^2. Here v_i is the expected value of the asset conditional on the insider's type i. Type i's preferences over aggregate quantity-transfer bundles $(Q,T) \in X \equiv \mathbb{R}_+ \times \mathbb{R}$ are

[16] We thus abstract from income effects associated to price changes. A similar assumption underlies the estimation of the welfare cost of adverse selection proposed by Einav, Finkelstein, and Cullen (2010).

then represented by (10), with

$$u_i(Q) \equiv \theta_i Q - \frac{\alpha\sigma^2}{2} Q^2. \tag{14}$$

For each type i, the marginal rate of substitution (1) of shares for money is

$$\partial u_i(Q) = \theta_i - \alpha\sigma^2 Q, \tag{15}$$

so that Assumption SC requires that $\theta_2 > \theta_1$. According to (15), condition (11) holds for

$$Q_0 \equiv \frac{\theta_2 - \theta_1}{\alpha\sigma^2} > 0.$$

When dealing with this example, we assume that $\theta_2 - v_2 > \theta_1 - v_1 > 0$. That is, there are always gains from trade between the insider and the market makers, and these gains are higher for type 2 than for type 1. As a result, the first-best level of trade is higher for type 2 than for type 1,

$$Q_2^* = \frac{\theta_2 - v_2}{\alpha\sigma^2} > \frac{\theta_1 - v_1}{\alpha\sigma^2} = Q_1^*.$$

Given Assumption SC, this is a standard responsiveness condition (Caillaud, Guesnerie, Rey, and Tirole (1988)) that ensures that first-best quantities are implementable. Assumption 1-v amounts to $\theta_1 > v$. The Jaynes–Hellwig–Glosten allocation is such that

$$Q_1 = \frac{\theta_1 - v}{\alpha\sigma^2} < Q_1^*, \quad Q_2 = \frac{\theta_2 - v_2}{\alpha\sigma^2} = Q_2^*.$$

Hence, type 2 purchases her first-best quantity, while type 1 purchases less than her first-best quantity, reflecting that she trades at a unit price v strictly higher than the cost v_1 of serving her. Assumption 2-v_2 amounts to $\theta_2 - v_2 > \theta_1 - v$, which is automatically satisfied under the above responsiveness condition.

4.2 Basic and Complementary Contracts

Let us first describe the contracts we use to reach the Jaynes–Hellwig–Glosten allocation that must prevail in equilibrium. Our construction involves two such contracts, a basic contract $c \equiv (Q_1, T_1)$, and a complementary contract $c' \equiv (Q_2 - Q_1, T_2 - T_1)$. In line with Theorem 1, c has unit price v, whereas c' has higher unit price v_2. As shown in Figure 1, type 1 reaches her equilibrium aggregate trade (Q_1, T_1) by trading a single contract c, while type 2 reaches her equilibrium aggregate trade (Q_2, T_2) by trading a contract c along with a contract c'. Thus types 1 and 2 do not trade the same contracts and type 2 trades two different contracts. As no seller can be indispensable in providing either of these contracts, we begin our construction of an equilibrium by imposing that two sellers offer the contract c and that two sellers offer the contract c'.

Lemma 1 *Suppose that Assumptions 1-v and 2-v_2 and conditions (7)–(8) are satisfied. Then, if two sellers offer the contract c and two sellers offer the contract c', it is a best response for both types to trade a contract c with the same seller, and for type 2 to trade in addition a contract c' with some other seller.*

Although the contracts c and c' lead to the desired aggregate trades, they are in general not sufficient to sustain an equilibrium. To clarify this point, consider the configuration illustrated in Figure 3. We have assumed that the only available contracts are c and c', with two sellers offering each of them, and that the trade $2c$ is strictly less preferred by type 2 than $c + c'$. Now, consider the contract $\tilde{c}$ close to c as shown. Contract $\tilde{c}$ allows the buyer to purchase a quantity less than Q_1 at a unit price lower than v. This contract certainly attracts type 1, and it yields a strictly positive profit to a deviating seller proposing it if it does not attract type 2 along the way. To see that this is indeed the case, observe that combining $\tilde{c}$ with c, c', or any combination of these contracts, leaves type 2 with a strictly lower utility than trading a contract c along with a contract c', which remains feasible following any seller's unilateral deviation. This is because $\tilde{c} + c$ is close to $2c$ and thus is strictly less preferred by type 2 than $c + c'$, just as $2c$, and because $\tilde{c} + c'$ is below the line of slope v_2 passing through c and thus is strictly less preferred by type 2 than $c + c'$. Hence, for any seller, offering contract $\tilde{c}$ is a successful cream-skimming deviation: it cannot be blocked by the contracts c and c' and it is profitable.

4.3 Deterring Cream-Skimming Deviations

To construct an equilibrium, we must supplement the contracts c and c' by further contracts preventing sellers from offering profitable deviations such as $\tilde{c}$. This role will be played by a single contract, denoted c'', defined as the contract which, combined with c, allows type 2 to reach a point on her equilibrium indifference curve $\mathcal{I}_2$ where her marginal rate of substitution is equal to v; that is, $U_2(c + c'') = U_2(c + c')$ and $\tau_2(c + c'') = v$. This contract exists and is unique if conditions (7)–(8) are satisfied. One can verify that c'' stipulates a quantity strictly larger than $Q_2 - Q_1$, at a unit price strictly between v and v_2.[17]

As c'' is not meant to be traded in equilibrium, we first check that its introduction does not cause the buyer to modify her trades on the equilibrium path.

[17] The quantity stipulated by c'' must be larger than $Q_2 - Q_1$ because any point on $\mathcal{I}_2$ to the left of (Q_2, T_2) is such that the marginal rates of substitution for type 2 is higher than $\tau_2(Q_2, T_2) = v_2$ and thus, a fortiori, higher than v. For the same reason, the unit price of c'' must be strictly lower than v_2. Finally, the unit price of c'' must be strictly higher than v; otherwise, $\mathcal{I}_2$ would lie entirely above the line with slope v going through (Q_1, T_1) and thus could not go through (Q_2, T_2).

Lemma 2 *Suppose that Assumptions 1-v and 2-v_2 and conditions (7)–(8) are satisfied. Then, if two sellers offer the contract c and two sellers offer the contract c', and if the other sellers offer either the contract c'' or the null contract, it remains a best response for both types to trade a contract c with the same seller, and for type 2 to trade in addition a contract c' with some other seller.*

The intuition for this result is simple. As for type 1, she cannot increase her utility by trading contracts other than c, as these contracts are issued at a unit price higher than v and she can buy her optimal quantity at unit price v by trading a contract c. Turning to type 2, observe first that she is indifferent between trading a contract c along with a contract c' and trading a contract c along with a contract c'' . Moreover, because her marginal rate of substitution at $c + c''$ is v, the optimal way for type 2 to trade c'' along with some of the offered contracts consists in combining it with a contract c. Thus type 2 is not made strictly better off when contract c'' is introduced on top of contracts c and c'.

We next investigate the sellers' deviations. We first show that no profitable deviation can attract type 2; that is, only cream-skimming deviations may create a problem for equilibrium existence.

Lemma 3 *Suppose that Assumptions 1-v and 2-v_2 and conditions (7)–(8) are satisfied. Then, if two sellers offer the contract c and two sellers offer the contract c', and if the other sellers offer either the contract c'' or the null contract, any deviation that attracts type 2 is unprofitable.*

To understand this result, observe first that no profitable deviation can attract both types, as such a contract would need to have a unit price higher than v and type 1 can buy her optimal quantity at unit price v by trading a contract c. Furthermore, no profitable deviation can only attract type 2, as such a contract would need to have a unit price higher than v_2 and type 2 can trade, on top of c, the optimal complement c' at unit price v_2. Moreover, as noted above, type 2 would be strictly worse off combining such a contract with a contract c''.

Remark It should be emphasized that Lemmas 1–3 do not depend on whether the buyer's preferences satisfy conditions (10)–(11). For instance, they would go through in a standard insurance setting à la Rothschild and Stiglitz (1976) with nonconstant absolute risk aversion. This observation is crucial for the robustness analysis that we develop in Section 5.

It follows from Lemma 3 that the only remaining possibility for a profitable deviation is a cream-skimming deviation that only attracts type 1, as in the example illustrated in Figure

3. The following result shows that, when sufficiently many sellers offer the contract c'', such deviations are ruled out for preferences in the class defined in Section 4.1.

Lemma 4 *Suppose that the buyer's preferences satisfy (10)–(11) and that Assumptions 1-v and 2-v_2 and conditions (7)–(8) are satisfied. Then, if two sellers offer the contract c and two sellers offer the contract c', and if sufficiently many sellers offer the contract c'', there is a best response for the buyer such that no deviation by a seller only attracts type 1.*

To understand how the contract c'' succeeds in deterring cream-skimming deviations, recall that, for preferences that satisfy (10)–(11), the buyer's indifference curves, whatever her type, are, over the relevant range, all translates of each other. This is in particular true of the equilibrium indifference curves $\mathcal{I}_1$ and $\mathcal{I}_2$ of types 1 and 2; moreover, as $\tau_2(c + c'') = v = \tau_1(c)$, the vector that translates $\mathcal{I}_1$ into $\mathcal{I}_2$ corresponds to the contract c''. Now, consider a potential cream-skimming deviation such as $\tilde{c}$ in Figure 4. This contracts certainly attracts type 1, like in Figure 3. However, because of the translation property, type 2 would also increase her utility by trading $\tilde{c}$ along with c''. Thus $\tilde{c}$ attracts both types. Because $\tilde{c}$ must have a unit price at most equal to v to attract type 1, this deviation cannot be profitable. More generally, any attempt by a seller at attracting and making a profit with type 1 while leaving the other sellers to trade with type 2 is doomed to fail because, if type 1 can increase her utility by trading with the deviator, so can type 2 by mimicking type 1 and trading an additional contract c''. This is why Lemma 4 requires that there be enough sellers offering the contract c''; in any case, at least two of them. Indeed, c'' must remain available for type 2 to trade following a deviation, taking into account that some contracts c'' might be traded by type 1 in those circumstances.

The central result of this section is a direct implication of Lemmas 1–4.

Theorem 2 *Suppose that the buyer's preferences satisfy (10)–(11), that Assumptions 1-v and 2-v_2 and conditions (7)–(8) are satisfied, and that there are sufficiently many sellers. Then an equilibrium exists.*

We know from Theorem 1 and Corollary 1 that, under Assumptions 1-v and 2-v_2, the Jaynes–Hellwig–Glosten allocation is the only candidate equilibrium aggregate allocation. Moreover, according to Corollary 2, conditions (7)–(8) are necessary for an equilibrium to exist. What Theorem 2 shows is that these conditions are actually sufficient provided there are enough sellers in the market and the buyer's preferences satisfy conditions (10)–(11). The next section shows that these last two conditions can both be somewhat relaxed, thus extending Theorem 2 to more general preferences.

5 Robustness

In this section, we investigate to which extent Theorem 2 can be extended to a more general class of preferences for the buyer than those satisfying conditions (10)–(11). We shall restrict attention to equilibria that only rely on the contracts c, c', and c'' introduced above. Because Lemmas 1–3 do not require conditions (10)–(11) to hold, the analysis can focus on Lemma 4, that is, on the possibility of deterring cream-skimming deviations through the single contract c''. It is easy to convince oneself that a sufficient condition for c'' to deter any profitable deviation that would attract type 1 is that the translate of the upper contour set of c for type 1 along the vector c'' lies in the upper contour set of $c + c'$ for type 2. Preference specifications that satisfy conditions (10)–(11) correspond to the knife-edge case where these two sets coincide. We first provide, in the quasilinear case, an economically meaningful condition on the demand function of the two types of buyer such that this property is upheld. In a second step, we relax the quasilinearity assumption and show that there is, in an appropriate sense, an open set of preferences for the buyer such that an equilibrium involving the same strategies as in our main examples exists.

5.1 Quasilinear Preferences

The desired translation property for upper contour sets is a special case of the property that, if type 1 is ready to trade a bundle (q, t), given that she can already buy any quantity at unit price p, then type 2 facing the same options would also choose to trade the bundle (q, t).[18] Theorem 3 below provides, in the quasilinear case, a condition on both types' demand functions that ensures that this is so, and thus that an equilibrium can be constructed along the lines of Theorem 2.

Formally, let type i's preferences over $X \equiv \mathbb{R}_+ \times \mathbb{R}$ be represented by (10). Then, for each p in the relevant range, the demand of type i at price p is given by

$$D_i(p) \equiv (\partial u_i)^{-1}(p).$$

Because $\partial^2 u_i < 0$, the demand functions D_i are strictly decreasing, that is, $\partial D_i < 0$ as long as $D_i > 0$. Moreover, because, by Assumption SC, $\partial u_2 > \partial u_1$, they are strictly ordered, that is, $D_2 > D_1$ as long as $D_2 > 0$. The following result shows that a strengthening of this property is sufficient to ensure the existence of an equilibrium.

Theorem 3 *Suppose that the buyer's preferences satisfy (10), that Assumptions 1-v and 2-v$_2$ and conditions (7)–(8) are satisfied, and that there are sufficiently many sellers. Then,*

[18]Observe that the quantity q could be negative, so that the bundle (q, t) need not be a contract.

if, for any price p in the relevant range,

$$|\partial D_2(p)| > |\partial D_1(p)|, \tag{16}$$

an equilibrium can be constructed along the lines of Theorem 2.

Observe that (16) represents a strengthening of the single-crossing property: not only is type 2 always more willing to buy than type 1 at any price p, but the demand of type 2 is always more sensitive to price increases than that of type 1. The primary use of Theorem 3 is to show that perturbations of our leading examples admit similar equilibria. For instance, in the insurance example of Section 4.1.1, we maintain that the consumer has constant absolute risk aversion and, thus, quasilinear preferences, but we now can perturb her preferences by making the low-risk consumer more risk averse than the high-risk consumer.

Example 1 *Type i has preferences represented by (10), with*

$$u_i(Q) \equiv -\frac{1}{\alpha_i} \ln(v_i \exp(-\alpha_i(W_B + Q)) + (1 - v_i) \exp(-\alpha_i W_G)),$$

where $v_2 > v_1$, $\alpha_1 > \alpha_2$, and $[(1 - v_1)/v_1]/[(1 - v_2)/v_2] > \exp((\alpha_1 - \alpha_2)(W_G - W_B))$.

In the pure-trade example of Section 4.1.2, we now can perturb the seller's quadratic cost function.

Example 2 *Type i has preferences represented by (10), with*

$$u_i(Q) \equiv \theta_i Q - C(Q),$$

where $\theta_2 > \theta_1$, $\partial^2 C > 0$, $\partial^3 C < 0$, and $\lim_{Q\to\infty} \partial C(Q) = \infty$.

A noticeable feature of the functions (u_1, u_2) satisfying condition (16) is that they form a contractible subspace of the pairs of twice continuously differentiable and strictly concave utility functions over $\mathbb{R}_+$; that is, it can be continuously shrunk into a point. This property notably implies that this subspace is simply connected: it has no "holes." This means that, between two examples such as Examples 1–2, one can construct one, and essentially only one, path of similar examples connecting them.

5.2 General Preferences

Examples 1–2 show that the existence of an equilibrium of the kind constructed in Section 4 is not confined to preference specifications for the buyer that satisfy (10)–(11). We now establish that these examples are robust, in the sense that the desired translation property for upper contour sets is satisfied for any choice of preferences that are "close enough" to those in these examples. That is, we show that Theorem 2 holds for an open set of preferences, including preferences that are not quasilinear.

5.2.1 A Geometrical Condition

We first provide a geometrical interpretation of condition (16). Recall from Debreu (1972) or Mas-Colell (1985, Proposition 2.5.1) that the (Gaussian) curvature κ_i of type i's indifference curve at an arbitrary point of V is

$$\kappa_i \equiv \frac{1}{\|\partial U_i\|^3} \begin{vmatrix} -\partial^2 U_i & \partial U_i \\ -\partial U_i' & 0 \end{vmatrix}. \tag{17}$$

For quasilinear preferences represented by (10), simple algebra using $\tau_i(D_i(p), pD_i(p)) = p$ and $D_i(p) = (\partial u_i)^{-1}(p)$ yields that, at type i's optimal demand at price p,

$$\kappa_i(D_i(p), pD_i(p)) = -\frac{\partial^2 u_i(D_i(p))}{(1+p^2)^{3/2}} = \frac{1}{|\partial D_i(p)|(1+p^2)^{3/2}}. \tag{18}$$

Hence, an alternative way of stating (16) is that

$$\tau_1(Q,T) = \tau_2(Q',T') \quad \text{implies} \quad \kappa_1(Q,T) > \kappa_2(Q',T'). \tag{19}$$

That is, the curvature of any indifference curve of type 1 is strictly higher than that of any indifference curve of type 2 at any pair of points where they have equal slopes. Note that (19), unlike (16), does not require preferences to be quasilinear. This property has a natural interpretation in terms of variations of marginal rates of substitution. Indeed, from (17),

$$\left.\frac{d\tau_i}{dQ}\right|_{U_i=\text{const}} = -\,(1+\tau_i^2)^{3/2}\kappa_i. \tag{20}$$

Together with (19), this identity implies that, at any pair of points (Q,T) and (Q',T') where type 1's and type 2's marginal rates of substitution coincide, type 1's marginal rate of substitution at (Q,T) decreases faster than type 2's marginal rate of substitution at (Q',T') along the corresponding indifference curves. As a result, the demand of type 1 reacts less to changes in prices than the demand of type 2, which in the quasilinear case is expressed by (16). It is clear from (20) that preferences which satisfy (19) also satisfy the desired property for deterring cream-skimming deviations: the translate of the upper contour set of c for type 1 along the vector c'' lies in the upper contour set of $c+c'$ for type 2.

5.2.2 The Openness Result

To establish our openness result, we first specify a set $\mathbf{P}_{sc}$ of strictly convex preferences for each type of the buyer, endowed with a suitable topology. This set, which we construct in

Appendix B, can be identified to a subspace of $C^2(V)$, the space of real-valued C^2 functions over V endowed with the topology of uniform convergence over compact subsets of V of functions and of their derivatives up to the order 2. In line with the assumptions made in Section 2.1, any preference in $\mathbf{P}_{sc}$ can be represented by a strictly quasiconcave utility function $U \in C^2(V)$ such that $\partial U / \partial T < 0$.

Then the following openness result holds.

Theorem 4 *For buyer's preferences in an open subset of $\mathbf{P}_{sc} \times \mathbf{P}_{sc}$, an equilibrium can be constructed along the lines of Theorem 2.*

The proof proceeds by showing that, around the preferences constructed in Examples 1–2, there is an open set of preferences in $\mathbf{P}_{sc} \times \mathbf{P}_{sc}$ such that condition (19) uniformly holds over a large enough compact subset of V. Key to this logic is that condition (19) involves a strict comparison of curvatures, and that marginal rates of substitution and curvatures vary continuously with preferences in the topology of $\mathbf{P}_{sc}$.

Theorem 4 in turn sheds light on the main examples of Section 4.1: as the indifference curves for type 1 and type 2 are then translates of each other, preferences in these examples do not satisfy condition (19), but instead a limit condition, namely, that

$$\tau_1(Q, T) = \tau_1(Q', T') \quad \text{implies} \quad \kappa_1(Q, T) = \kappa_2(Q', T').$$

This suggests that such preferences lie on the boundary of the set of preferences in $\mathbf{P}_{sc}$ such that the contract offers considered in Theorem 2 are consistent with equilibrium. Indeed, it can be shown that a necessary condition for preferences to belong to that set is that

$$\kappa_1(c) \geq \kappa_2(c + c''). \tag{21}$$

Condition (21) provides a simple test to rule out certain equilibrium configurations.

Example 3 *Consider preferences as in Example 2, where $\theta_2 > \theta_1$, $\partial^2 C > 0$, and $\partial^3 C > 0$. Then an equilibrium cannot be constructed along the lines of Theorem 2.*

Thus Theorem 2 applies to $C(Q) = Q^\gamma/\gamma$ for $1 < \gamma < 2$, but not for $\gamma > 2$. This, of course, does not prejudge the possibility of alternative equilibrium constructions.

6 Discussion

6.1 Constrained Efficiency

In this section, we adopt a normative viewpoint to discuss the efficiency properties of our equilibrium outcome. Suppose that a planner in our setting has to design a budget-balanced informationally constrained trading system. Because the planner cannot observe the buyer's type, the best he can do is to propose a tariff that specifies a transfer $T(Q)$ to be paid as a function of the aggregate quantity Q chosen by the buyer (Rochet (1985)). Each type i then chooses a quantity Q_i that maximizes $U_i(Q, T(Q))$ with respect to Q, and in such a case we say that the tariff T implements the allocation $((Q_1, T(Q_1)), (Q_2, T(Q_2))$. The planner's problem is to design a tariff that yields the highest possible utility to all types, in the usual Pareto sense, under the budget constraint. The latter can be written as

$$T(Q_1) - vQ_1 + m_2[T(Q_2) - T(Q_1) - v_2(Q_2 - Q_1)] \geq 0. \tag{22}$$

So far, this planning problem is the one defining constrained efficiency when the planner can perfectly control communication between agents, as in Myerson (1979, 1982). This requirement fits well the exclusive-competition case; in the context of insurance, it is key to the characterization of constrained-efficient allocations provided by Crocker and Snow (1985a). However, non-exclusive competition violates this requirement by construction, as trades between agents cannot be observed nor, a fortiori, controlled for. We, therefore, have to impose additional constraints to the planner's problem; yet, there is no consensus on what these constraints should be.[19] We choose to follow Glosten (1994) and to focus on tariffs that are robust to entry, in the sense that no third party can propose another tariff that would profitably attract some types, taking into account that the original tariff is still proposed by the planner. Glosten (1994) allowed the entrant to offer arbitrary menus of contracts; to make this normative analysis more in line with our model, we impose instead that the entrant can only offer a single contract. Interestingly, this condition is strong enough to characterize a unique allocation, which turns out to be our equilibrium allocation.

Property 1 *Any tariff robust to entry with a single contract must implement the Jaynes–Hellwig–Glosten allocation.*

The reasoning is as follows. First, one must have

$$U_1(Q_1, T(Q_1)) \geq \max\{U_1(Q, vQ) : Q \geq 0\},$$

for, otherwise, an entrant could propose a contract with unit price slightly above v that would attract type 1, and that would be profitable even if it also were to attract type 2.

[19] Facing a similar difficulty, Farhi, Golosov, and Tsyvinski (2009) allow agents to retrade on an anonymous lending market. Similarly, Labadie (2009) allows anonymous retrading on a complete set of markets. In both cases, the equilibrium prices on these markets are endogenous and thus depend on the planner's initial offer. Note that, in the second paper, the postulated set of markets faces the same adverse-selection problem as the planner does, whereas this complication is not present in the first paper.

Second, and similarly, one must have

$$U_2(Q_2, T(Q_2)) \geq \max\{U_2(Q_1 + Q, T(Q_1) + v_2 Q) : Q \geq 0\},$$

for, otherwise, an entrant could propose a contract with unit price slightly above v_2 that would profitably attract type 2, and that would be even more profitable if it also were to attract type 1. These two inequalities imply that

$$T(Q_1) \leq v Q_1, \quad T(Q_2) \leq T(Q_1) + v_2(Q_2 - Q_1),$$

so that the budget constraint (22) can be satisfied if and only if all these inequalities are in fact equalities; we are then back to the Jaynes–Hellwig–Glosten allocation, as characterized in (5)–(6).

Property 1 shows that the possibility of entry drastically reduces what the planner can implement. There remains to show that there exist tariffs that are robust to entry.

Property 2 *Suppose that the buyer's preferences satisfy (10)–(11). Then the tariff built from the three contracts c, c', and c'' is robust to entry with a single contract.*

The proof of this fact is very simple. To make a profit with a single contract, an entrant must attract type 1, so that the unit price of the contract must be less than v. But, as c'' remains available, type 2 is attracted by the entrant's contract whenever type 1 is, and entry cannot be profitable.

At this point, one may wonder whether these properties can be extended to the case when an entrant could use menus of contracts. Because this change can only reduce the set of tariffs that are robust to entry, Property 1 is still valid. On the other hand, Property 2 does not hold anymore. The reason is that an entrant facing the three contracts c, c', and c'' could propose two contracts: one designed to profitably attract type 1, and one designed to limit his loss with type 2.[20] Interestingly, Glosten (1994) shows that, under the quasilinearity condition (10), the piecewise-linear convex tariff defined by

$$T(Q) \equiv 1_{\{Q \leq Q_1\}} v Q + 1_{\{Q > Q_1\}} [v Q_1 + v_2(Q - Q_1)] \tag{23}$$

is robust to entry. The key here is that all quantities below Q_1 are offered, even though

[20] The argument follows Attar, Mariotti, and Salanié (2014a, Proposition 3). In the case of quasilinear preferences, observe that, under the assumptions of Theorem 2, any contract $\tilde{c}_1$ that allows type 1 to increase her utility by a positive amount ε also allows type 2, through the use of the latent contract c'', to increase her utility by ε. However, the entrant may also propose a contract $\tilde{c}_2 = c' - (0, 2\varepsilon)$, which, coupled to a contract c offered by an incumbent, allows type 2 to increase her utility by 2ε. Choosing $\tilde{c}_1$ close enough to c and ε close enough to zero then enables the entrant to reap almost the entire aggregate profit $T_1 - v_1 Q_1$ associated with type 1, while making an arbitrarily small loss when trading with type 2.

they are not meant to be traded. A contract offered by an entrant and designed to attract type 1 must also attract type 2, because she can trade an additional quantity at unit price v to ensure she is better off doing so. Hence, those additional contracts are latent contracts that are offered by the planner to make entry with two contracts unprofitable. This tariff, however, cannot sustained in an equilibrium of a competitive nonlinear pricing game.[21]

6.2 Menus and Equilibrium Existence

In the trading game studied so far, sellers can propose at most one contract. In practice, however, sellers can offer menus of such contracts; for instance, insurers typically offer several levels of deductible. An alternative trading game in which sellers compete in menus of contracts is the extensive-form game studied, among others, by Biais, Martimort, and Rochet (2000, 2013) and Attar, Mariotti, and Salanié (2011, 2014a, 2014b). In the first stage of this game, each seller k proposes a menu of contracts, that is, a compact set C^k of quantity-transfer bundles that contains at least the no-trade contract $(0, 0)$. Then, after privately learning her type, the buyer selects one contract from each of the menus C^k. Players' preferences are specified as in Sections 2.1 and 2.2. Observe that the strategies available to sellers in the single-contract game of Section 2.3 remain available in this menu game.

Now, suppose that, in this menu game, sellers play the strategies used in Theorem 2 to construct an equilibrium of the single-contract game. That is, together with the no-trade contract $(0, 0)$, two sellers offer the contract c, two sellers offer the contract c', and sufficiently many sellers offer the contract c''. According to Lemma 2, it is a best response for both types to trade a contract c with the same seller, and for type 2 to trade in addition a contract c' with some other seller.

The question is whether such strategies for the sellers remain equilibrium strategies in the menu game. The answer is negative if we can find a seller and a deviation for this seller that is profitable no matter the buyer's best response.[22] It follows from Attar, Mariotti, and Salanié (2014a, Proposition 3) that, in the situation under scrutiny, at least one seller has such a "strongly profitable" deviation. Moreover, according to Theorem 2, a strongly profitable deviation must separate the buyer's types on two different contracts other than the null contract. The following result further characterizes these deviations.

Corollary 3 *Suppose that the buyer's preferences satisfy (10)–(11), that Assumptions 1-v*

[21]As shown by Attar, Mariotti, and Salanié (2014b), this negative result also holds when sellers are restricted to offer convex tariffs such as (23).

[22]Otherwise, the deviation may be trivially made unprofitable by specifying the buyer's strategy so that she chooses among her best responses one that implies a nonpositive expected profit for the deviating seller.

and 2-v_2 and conditions (7)–(8) are satisfied, and that there are sufficiently many sellers. Then, if the sellers play the strategies used in Theorem 2 to construct an equilibrium of the single-contract game, any deviation by a seller that is profitable no matter the buyer's best response makes a profit with type 1 and a loss with type 2.

The proof relies on a very simple observation. Consider the contracts that are offered in the absence of a deviation. Type 2 can buy c and c', and get utility $U_2(Q_2, T_2)$; in particular, she would not be interested by an additional contract with unit price no less than v_2. The same properties hold after the deviation, because c and c' remain available. Therefore, as a strongly profitable deviation must increase type 1's utility beyond $U_1(Q_1, T_1)$ and hence, in the presence of the latent contract c'', type 2's utility beyond $U_2(Q_2, T_2)$, it must be that the deviating seller makes a loss with type 2.

The key insight of Corollary 3 is that any strongly profitable deviation features cross-subsidies between contracts. For the deviation to be strongly profitable, the profit that the deviating seller earns on type 1 must thus compensate the loss he makes with type 2. One may, therefore, describe the deviator's behavior as a dumping practice, designed to boost the profit he makes with type 1. If a suitable regulatory mechanism were set in place that prohibited such cross-subsidies, then Corollary 3 would prove that it is possible to sustain the Jaynes–Hellwig–Glosten allocation in equilibrium, even in the menu game. The feasibility of such a mechanism will be discussed in Section 6.3.

In the absence of such a regulation, we have to acknowledge that an equilibrium exists only when sellers are restricted to single-contract offers. This assumption is often made for simplicity, as in Rothschild and Stiglitz's (1976) canonical analysis of exclusive insurance markets. Indeed, under exclusivity, whether sellers compete through single contracts or through menus of those does not alter the characterization of equilibrium in a fundamental way: the same equilibrium allocation emerges in both cases, although existence conditions become more stringent in the latter case.[23] This insight does not carry over to nonexclusive competition. Attar, Mariotti, and Salanié (2014a) provide a characterization of equilibrium aggregate allocations in a generalized version of Rothschild and Stiglitz's (1976) model in which sellers can offer menus of nonexclusive contracts. They show that, in any pure-strategy equilibrium, a type of the buyer may trade only if the other type does not trade at all, a type of market breakdown reminiscent of that emphasized by Akerlof (1970).[24] By contrast, the present paper shows that, when each seller is restricted to offer a single contract, there

[23] See, for instance, Hahn (1978), Fagart (1996), and Farinha Luz (2012).

[24] This result applies to the version of their model in which the buyer is restricted to trade nonnegative quantities, as in the present paper (see Attar, Mariotti, and Salanié (2014a, Section 5.1)).

exists an equilibrium in which both types trade. In that respect, nonexclusive competition is qualitatively different from exclusive competition.

That a restriction on the sellers' contractual possibilities is instrumental to sustain the Jaynes–Hellwig–Glosten allocation in equilibrium is also a novel contribution of our analysis. In contrast with Hellwig (1988) and Jaynes (2011), who rely on extensive forms with multiple rounds of communication between sellers to sustain this equilibrium outcome, we show that an equilibrium exists in a standard competitive game without communication. Explicit communication about a seller attempting to deal with type 1 only is replaced by the single latent contract c'', the presence of which makes it also profitable for type 2 to trade with this seller, thereby defeating any cream-skimming deviation.

From a game-theoretical viewpoint, our work is closely related to the common-agency literature, in which principals compete through mechanisms to trade with an agent. In that literature, a well-known result, sometimes presented as a failure of the Revelation Principle, is that some outcomes are sustainable in equilibrium in the game relative to an arbitrary set of indirect mechanisms, but not in the corresponding direct-mechanism game.[25] In the present paper, sellers (principals) make single-contract offers to a buyer (agent) who can be of two types. That is, the set of mechanisms available to each principal is even smaller than the set of direct mechanisms. Despite this restriction, existence of a pure-strategy equilibrium obtains. Moreover, expanding the sellers' strategy sets by allowing for arbitrary indirect mechanisms would lead, under Assumption 1-v, to equilibrium nonexistence, as shown by Attar, Mariotti, and Salanié (2014a). The intuition for this counterintuitive result is that, when sellers are restricted to single-contract offers, none of them can cross-subsidize between contracts at the deviation stage, which, as shown by Corollary 3, is an essential feature of any strongly profitable menu deviation. As a consequence, a larger set of equilibrium allocations can be sustained.

6.3 Policy Implications for Insurance Markets

We suggested in the previous section that, even when sellers can offer menus of contracts, it is possible to sustain the Jaynes–Hellwig–Glosten allocation in equilibrium provided a suitable regulatory mechanism is set in place. According to Corollary 3, regulation only has to prohibit cross-subsidies between contracts, so as to deter dumping practices. The key issue here is the amount of information needed by a regulatory authority. We shall discuss this issue in the context of insurance markets.

[25] See, among others, Peck (1997), Peters (2001), and Martimort and Stole (2002).

To eliminate cross-subsidies, it seems at first that one needs to observe the profits made by each insurer on each contract. Although this task does not appear to be out of reach in developed countries,[26] we are not aware of any existing regulation relying on such audits, especially if performed on a contract-by-contract basis.[27]

Our analysis can be exploited to reduce the information needed by regulators. Recall that any profitable deviation towards a menu of contracts must separate the low-risk type from the high-risk type. In such a case, the contract chosen by the low-risk type would make a positive profit and exhibit a low riskiness, that is, it would involve a low level of indemnities per insuree. Intuitively, the regulatory authority should thus aim at identifying and penalizing sellers of contracts with a riskiness significantly below the average riskiness of insurees. Informational requirements are now much less demanding: one only needs to observe, for each contract, the number of clients and the total indemnities paid.

As a matter of fact, in the past two decades several countries[28] have implemented "risk equalization schemes," which are ways of pooling and redistributing costs among insurers who sell a standardized basic coverage contract.[29] Such schemes clearly reduce the incentives to attract only good risks and to dump bad risks on other insurers. In Switzerland, the basic coverage contract is defined at the national level, and then insurers compete over prices to provide the corresponding amount of insurance. Yet, an additional rule specifies that costs are pooled and redistributed among insurers. In Germany, the basic coverage contract is also defined at the national level, and is offered by 134 not-for-profit, nongovernmental "sickness funds." Insurees contribute a fixed fraction of their wealth; these contributions are then centrally pooled and redistributed to sickness funds according to a rather precise

[26] At the beginning of the 21st century, no International Financial Reporting Standard (IFRS) for insurance contracts existed. Since then, the International Accounting Standards Board (IASB) has introduced several measures to provide a unified treatment of the principles that an entity should apply to report information to users of its financial statements about the nature, amount, timing, and uncertainty of cash flows from insurance contracts. On June 20, 2013, the IASB published a document attempting at setting up a standard for consistent accounting for all insurance contracts (IASB (2013)). The aim is to improve the transparency of the effects of insurance contracts on an entity's financial position and financial performance and to reduce diversity in the accounting for insurance contracts. Specifically, any entity is required to "present insurance contract revenue to depict the transfer of promised services arising from an insurance contract in an amount that reflects the consideration to which the entity expects to be entitled in exchange for those services, and to present expenses as the entity incurs them" (IASB (2013, page 13)).

[27] A natural explanation is that auditing costs might be quite high, especially if one has to take into account the difficult question of the repartition of joint production costs among the different contracts sold by the same insurer. Note, additionally, that, by penalizing profitable contracts, such a regulation would undermine cost-reduction efforts.

[28] The information that follow are drawn from the surveys of Thomson and Mossialos (2009), which is mainly about private health insurance, and Thomson, Osborne, Squires, and Jun (2013), which is about health care in general.

[29] See Thomson and Mossialos (2009, page 84). Besides Switzerland and Germany, which we discuss below, other countries using such schemes include Australia, Ireland, the Netherlands, and Slovenia.

risk-adjusted capitation formula.[30]

Observe that such schemes rely on the existence of a standardized contract: this makes the riskiness and the average riskiness easy to compute and to compare. But the idea of penalizing the issuer of a contract with an observed low riskiness is more general and could be applied even in the absence of standardized contracts.[31] In either case, we suggest that a regulatory authority does not need to observe profits, but only coarser pieces of information such as riskiness and average riskiness.

Despite their limited informational requirements in terms of the observability of insurers' profits, these policies rely on a fair amount of information on the supply side of the market. In particular, they assume some ability to monitor the offers of each of the insurers. This could in principle conflict with our general view of nonexclusive competition, which typically prevents one from writing contracts based on the full profile of each agent's trades. If this last informational constraint is binding, it is still possible to advocate a simple regulation that would allow to reach our candidate allocation. To explain why, suppose that a public agency were able to determine the average riskiness in the population, v, and the optimal value of the basic coverage, Q_1, as defined in (5). Then this agency would be able to behave as an insurer by posting the contract (Q_1, vQ_1). Competition on the complementary layer of coverage would then lead other insurers to propose complementary insurance at unit price v_2. The resulting offers do not form an equilibrium, however: as discussed in Footnote 20, it would be possible for an insurer to profitably separate the two types. This might help to explain why some countries make basic coverage both universal and mandatory. This is the case in France, where the basic coverage is publicly financed. All private insurers, and some public agencies, are then allowed to propose additional coverage as they wish, provided they do not cover services already included in the basic coverage.

To avoid this difficulty, the public agency must issue latent contracts. Adding the contract c'' to its offer is not enough, as the resulting tariff is not immune to entry by a cross-subsidizing offer with two contracts. An alternative possibility is for the public agency to offer any level of coverage up to Q_1 at unit price v,

$$T(q) \equiv vq, \quad q \leq Q_1.$$

[30]However, the German system also allows insurees to opt out from this mandatory system and instead buy a basic coverage contract designed and priced by private for-profit insurers. According to our analysis, this may make the system unstable: in line with this, Thomson, Osborne, Squires, and Jun (2013, page 57) note that "Especially for young people with a good income, [...] [privately offered basic coverage] [...] is attractive, as the insurance may offer contracts with more extensive ranges of services and lower premiums."

[31]In the absence of a standardized contract, one has to identify consumers who trade several contracts, in order to avoid double counting.

It is easy to show that this offer from the public agency induces other insurers to subsequently propose complementary coverage at unit price v_2. The resulting tariff resists any kind of deviation. In fact, it coincides with the Jaynes–Hellwig–Glosten tariff, for which entry-proofness has been argued to hold at the end of Section 6.1. We thus have identified a regulation that implements the desired outcome, even when trades, contracts, or riskiness are all unobservable. At the same time, the agency is supposed to know well enough the preferences of low-risk consumers, as well as the average riskiness in the population.

To summarize, in the context of insurance markets, and especially of health insurance, our analysis suggests that, when multiple contracting is allowed, coverage can be provided by the private sector in a constrained-efficient way. The only requirement is that insurers should not be allowed to reduce prices below costs on a complementary coverage contract, in order to convince bad risks to buy the basic coverage elsewhere. We examined above at least two manners to proceed, each with different informational requirements. In any case, it should be emphasized that there is no need to make basic coverage mandatory: regulation need not interfere with the choices of consumers, who can remain sovereign in their decisions to purchase insurance. Neither are taxes or subsidies needed. This contrasts with policy recommendations from exclusive models of competitive insurance markets under adverse selection:[32] competition is enough to select a unique equilibrium in which prices efficiently reflect costs—though this rule applies to successive layers of insurance, and not to the aggregate coverage bought by each type of insuree. In a nutshell, we conclude that public intervention in nonexclusive insurance markets should simply aim at preventing cross-subsidies between contracts at the firm level so as to sustain cross-subsidies between types at the industry level.

6.4 Testable Predictions

In this section, we discuss more precisely the testable predictions of our analysis. Recall that, under exclusivity, the standard predictions of the Rothschild and Stiglitz (1976) model are that the unit price of coverage should increase with coverage, and that there should be a positive correlation between the aggregate coverage bought by a consumer and this consumer's risk. Empirically, these properties may be tested by surveying consumers to get

[32] See Crocker and Snow (1985b) for a study of taxes and subsidies under exclusivity. Mandatory insurance is evoked in Akerlof (1970), and has been the focus of much empirical work (Finkelstein (2004), Einav, Finkelstein, and Cullen (2010), Einav and Finkelstein (2011)). Wilson (1977), Dahlby (1981), and Crocker and Snow (1985a) show that making basic coverage mandatory and simultaneously allowing private insurers to compete on an extended coverage allows one to reach an constrained-efficient outcome. In the nonexclusive case, Villeneuve (2003) performs a similar analysis, although in a model that assumes linear pricing.

data on their total coverage and total insurance premium. One may alternatively choose to gather information on the contracts offered by firms. Under exclusivity, these two approaches make no difference, as the aggregate demand of a consumer must be supplied by a single contract offered by a single firm. However, under nonexclusivity, the second approach yields strikingly different results, as we now argue.

Recall that three contracts are offered in equilibrium: a basic contract $c = (Q_1, T_1)$, a complementary contract $c' = (Q_2 - Q_1, T_2 - T_1)$, and a latent contract $c'' = (q'', t'')$. Under conditions (7)–(8), which are necessary for an equilibrium to exist, it is easily checked that one must have

$$Q_1 > q'' > Q_2 - Q_1.$$

Now, the unit price of coverage Q_1 is low, as it is bought by both types; the unit price of coverage $Q_2 - Q_1$ is high, as it is bought by the high-risk type only; last, as can be seen in Figure 4, the unit price of coverage q'' is intermediate. A testable prediction of our model is thus that the contracts offered by firms exhibit quantity discounts, whereas consumers end up paying quantity premia. This is a striking result that contrasts with a natural intuition, namely, that nonexclusivity should push consumers towards splitting their demands between firms (Chiappori (2000)). The reason why, in our competitive setting, firms end up proposing quantity discounts, is that basic coverage must be larger than complementary coverage to prevent high-risk consumers from purchasing several basic policies from different firms.[33] Each consumer then finds in her own interest to concentrate her trades on a minimum number of contracts. The key is that firms together only propose a few contracts, which the consumer may combine. The low-risk type then ends up trading a single contract, while the high-risk type ends up buying two different contracts.

Importantly, according to the above analysis, one should observe a positive correlation between risk and coverage when considering total coverage for each consumer: indeed, the single-crossing property is enough to ensure that riskier types buy more coverage. On the other hand, with data originating from a single firm, one should now observe a negative correlation between risk and coverage: the relatively small complementary coverage $Q_2 - Q_1$ is bought only by the riskiest type. Such remarks are useful when considering the empirical evidence, as exemplified by the work of Cawley and Philipson (1999) on life insurance or the work of Finkelstein and Poterba (2004) on annuities. Because the reference model in those papers is Rothschild and Stiglitz's (1976), the above distinction between demand- and

[33] This explanation for quantity discounts differs from that proposed by Chade and Schlee (2012), who consider a monopolistic insurer as in Stiglitz (1977).

supply-side approaches is overlooked. As a result, evidence of quantity discounts, or the absence of a positive correlation property, is interpreted as rejecting the presence of adverse selection on life-insurance or annuity markets. However, such markets being nonexclusive, one must be very careful when testing for the existence of quantity discounts, as one needs to observe the total insurance coverage and the total insurance premium paid by consumers. In particular, checking only the contracts offered by firms, or the contracts sold by a given firm, may be insufficient and even misleading. Such an empirical inquiry is beyond the scope of the present paper, but it would certainly be worth proceeding to this task, while taking all precautions to ensure that data are comprehensive.[34]

7 Conclusion

This work has focused on insurance markets under adverse selection. One of our findings is the fruitfulness of interpreting these markets as offering layers of coverage, rather than as merely offering insurance contracts. Indeed, the game we consider has a unique equilibrium outcome, in which any such layer is fairly priced according to the identity of the buyers who purchase it. As a result, there are no cross-subsidies between contracts, but there are cross-subsidies between types. When sellers can post menus of contracts, regulation is necessary to block deviations that cross-subsidize between contracts. Finally, we discussed new empirical implications of our setting that may prove useful when testing for the presence of adverse selection on nonexclusive insurance markets.

[34] At least one of the econometric treatments performed in Cawley and Philipson (1999) seems to escape this criticism, as it is based on a consumer survey (AHEAD) that includes information on aggregate demand. For a recent and positive test for adverse selection using the same data, see He (2009).

Appendix A (For Online Publication)

Proof of Theorem 1. Subscripts i and j refer throughout to the buyer's possible types, with $i \neq j$ by convention. For all i and k, let (q_i^k, t_i^k) be the equilibrium trade of type i with seller k, with $(q_i^k, t_i^k) \equiv (0,0)$ by convention if type i does not trade with seller k. Denote by $b_i^k \equiv t_i^k - v_i q_i^k$ the corresponding profit for seller k; his expected profit then writes as $b^k \equiv m_1 b_1^k + m_2 b_2^k$. We let $B_i \equiv \sum_k b_i^k$ and $B \equiv \sum_k b^k$ be the corresponding aggregate type-by-type and expected profits. Define $s_i^k \equiv t_i^k - t_j^k - v_i(q_i^k - q_j^k)$ to be the profit from trading $(q_i^k - q_j^k, t_i^k - t_j^k)$ with type i, and similarly define $S_i \equiv \sum_k s_i^k = T_i - T_j - v_i(Q_i - Q_j)$ to be the profit from trading $(Q_i - Q_j, T_i - T_j)$ with type i. Because of the accounting identity $B = T_1 - vQ_1 + m_2 S_2$, establishing (3)–(4) amounts to proving that

$$B = S_2 = 0, \tag{24}$$

an implication of which, because sellers cannot make negative expected profits, is that they all earn zero expected profit in equilibrium, $b^k = 0$ for all k.

The proof of (24) goes through a series of steps, corresponding to various deviations for the sellers. We will thereby establish a number of intermediate results that are explicitly labelled as formulas (25)–(39) below.

As a preliminary remark, note that $S_i = B_i - B_j + (v_i - v_j)Q_j$, which, because $Q_2 \geq Q_1 \geq 0$ by Assumption SC and $v_2 > v_1$ by Assumption CV, implies that

$$S_2 \geq B_2 - B_1 \tag{25}$$

and

$$S_1 + S_2 \leq 0. \tag{26}$$

We can now proceed to the bulk of the argument.

Step 1 The first deviation we examine consists for any seller k in offering a contract $(Q_i, T_i - \varepsilon)$ for some positive number ε. This attracts type i for sure. If this only attracts type i, then one must have $b^k \geq m_i(B_i - \varepsilon)$ or, equivalently, $m_j B_j \geq B - b^k - m_i \varepsilon$. If this attracts both types, then one must have $b^k \geq T_i - vQ_i - \varepsilon$ or, equivalently, $m_j S_j \geq B - b^k - \varepsilon$. Letting ε go to zero, we get that, for each k,

$$m_1 \max\{B_1, S_1\} \geq B - b^k, \tag{27}$$

$$m_2 \max\{B_2, S_2\} \geq B - b^k. \tag{28}$$

Note that $B - b^k = \sum_{k' \neq k} b^{k'} \geq 0$ for all k. Several consequences follow.

First, the sellers' aggregate profits on type 1 are nonnegative,

$$B_1 \geq 0. \tag{29}$$

To see why, observe that, if $B_1 < 0$, then $S_1 \geq 0$ by (27), so that $S_2 \leq 0$ by (26), and hence $B_2 < 0$ by (25). But then the aggregate profits $B = m_1 B_1 + m_2 B_2$ are negative, a contradiction.

Second, $B_1 \geq S_1$, so that (27) reduces to

$$b^k \geq m_2 B_2 \tag{30}$$

for all k. To see why, observe that, if $B_1 < S_1$, then $S_1 > 0$ by (29), so that $S_2 < 0$ by (26), and hence $B_2 \geq 0$ by (28) and $S_2 + B_1 \geq 0$ by (25). But then, because, by assumption, $B_1 < S_1$, we get that $S_1 + S_2 > 0$, which contradicts (26).

Third, the profit from trading $(Q_2 - Q_1, T_2 - T_1)$ with type 2 is nonnegative,

$$S_2 \geq 0. \tag{31}$$

To see why, observe that, if $S_2 < 0$, then $B_2 \geq 0$ by (28), which reduces to $b^k \geq m_1 B_1$. Summing this inequality to (30), we get that $2b^k \geq B$. In turn, summing these inequalities over k, we get that $2B \geq nB$, which, as $n \geq 3$, implies that $B = 0$. But then, from (29) and $B_2 \geq 0$, we get that $B_1 = B_2 = B = 0$ and thus, by (25), that $S_2 \geq 0$, a contradiction.

Fourth, the profit from trading $(Q_1 - Q_2, T_1 - T_2)$ with type 1 is nonpositive,

$$S_1 \leq 0. \tag{32}$$

This follows at once from (26) and (31).

Fifth, $S_2 \geq B_2$, so that (28) reduces to

$$m_2 S_2 \geq B - b^k \tag{33}$$

for all k. To see why, observe that, if $S_2 < B_2$, summing the inequalities (28) over k yields $nm_2 B_2 \geq (n-1)B$, whereas summing the inequalities (30) over k yields $B \geq nm_2 B_2$. It follows that $nm_2 B_2 \geq n(n-1)m_2 B_2$ and thus, as $n \geq 3$, that $B_2 \leq 0$. But then, we get from (31) that $S_2 \geq B_2$, a contradiction.

Step 2 The second deviation we examine consists for any seller k in offering a contract $(q_i^k, t_i^k - \varepsilon)$ for some positive number ε. This attracts type i for sure, for instance along with the contracts $(q_i^{k'}, t_i^{k'})$, $k' \neq k$. If this only attracts type i, then one must have $b^k \geq m_i(b_i^k - \varepsilon)$ or, equivalently, $b_j^k \geq -(m_i/m_j)\varepsilon$. If this attracts both types, then one must have $b^k \geq t_i^k - vq_i^k - \varepsilon$ or, equivalently, $m_j s_j^k \geq -\varepsilon$. Letting ε go to zero, we get that, for each k,

$$\max\{b_1^k, s_1^k\} \geq 0, \tag{34}$$

$$\max\{b_2^k, s_2^k\} \geq 0. \tag{35}$$

A consequence of this is that sellers make nonnegative profits with type 1,

$$b_1^k \geq 0 \tag{36}$$

for all k. To see why, observe that, if $b_1^k < 0$ for some k, then $b_2^k > 0$ because seller k must earn nonnegative expected profit. But then we get that $s_1^k = b_1^k - b_2^k + (v_1 - v_2)q_2^k < 0$, which contradicts (34).

Step 3 The third deviation we examine consists for any seller k in offering a contract $(q_1^k + Q_2 - Q_1, t_1^k + T_2 - T_1 - \varepsilon)$ for some positive number ε. This attracts type 2 for sure, for instance along with the contracts $(q_1^{k'}, t_1^{k'})$, $k' \neq k$. If this only attracts type 2, then one must have $b^k \geq m_2[t_1^k + T_2 - T_1 - \varepsilon - v_2(q_1^k + Q_2 - Q_1)]$ or, equivalently, $m_1 b_1^k \geq m_2(S_2 - s_2^k - \varepsilon)$. If this attracts both types, then one must have $b^k \geq t_1^k + T_2 - T_1 - \varepsilon - v(q_1^k + Q_2 - Q_1)$ or, equivalently, $m_1 S_1 \geq m_2(S_2 - s_2^k) - \varepsilon$. Letting ε go to zero, we get that, for each k, $m_1 \max\{b_1^k, S_1\} \geq m_2(S_2 - s_2^k)$ and thus

$$m_1 b_1^k \geq m_2(S_2 - s_2^k) \tag{37}$$

by (32) and (36). Two consequences follow.

First, the sellers' aggregate profits on type 2 are nonpositive,

$$B_2 \leq 0. \tag{38}$$

To see why, observe that summing (37) over k yields $m_1 B_1 \geq (n-1)m_2 S_2$, whereas summing (33) over k yields $nm_2 S_2 \geq (n-1)B$ or, equivalently, $m_1 B_1 \leq -m_2 B_2 + [n/(n-1)]m_2 S_2$. Chaining these inequalities yields $m_2 B_2 \leq [n/(n-1) - n + 1]m_2 S_2$, from which (38) follows as $n \geq 3$ and $S_2 \geq 0$ by (31).

Second, the profit from trading $(q_2^k - q_1^k, t_2^k - t_1^k)$ with type 2 is nonnegative,

$$s_2^k \geq 0 \tag{39}$$

for all k. To see why, observe that, by (37), $m_2 b_2^k = b^k - m_1 b_1^k \leq b^k - m_2 S_2 + m_2 s_2^k$. Applying (33) to some $k' \neq k$ yields $m_2 S_2 \geq B - b^{k'} \geq b^k$ and hence $b^k - m_2 S_2 \leq 0$. Combining these inequalities yields $b_2^k \leq s_2^k$, from which (39) follows by (35).

Step 4 In this step, we investigate the structure of equilibrium in the hypothetical case where $S_2 > 0$. Then $Q_2 > Q_1$ and, as each seller offers a single contract, some seller k must only trade with type 2. For any such k, $b_1^k = 0$ and $b_2^k = s_2^k$; hence, by (37), $b_2^k \geq S_2$.

Now, in this situation, it is not possible that all the sellers who trade with type 2, only trade with type 2. Indeed, in that case, we would have $B_2 \geq S_2$ according to the previous argument. But then, as $S_2 > 0$ by assumption, we would get that $B_2 > 0$, which contradicts (38). Thus, if $S_2 > 0$, at least one seller k must trade with both types, which implies that $Q_1 > 0$.

Let K_2 be the set of sellers who trade with type 2 but not with type 1, and let K_2^c be its complement. According to the above argument, if $S_2 > 0$, we know that $K_2 \neq \emptyset$, that $K_2^c \neq \emptyset$, and that $q_1^k = q_2^k > 0$ for some $k \in K_2^c$. We also have $s_2^k = b_2^k \geq S_2$ for all $k \in K_2$ and, therefore,

$$S_2 \geq S_2 - \sum_{k \in K_2^c} s_2^k \geq |K_2| S_2,$$

where the first inequality follows from (39). Hence, if $S_2 > 0$, there must be a single seller in K_2 who earns S_2 from trading with type 2. From the above inequality along with (39) and $|K_2| = 1$, we then get that $s_2^k = 0$ and thus $b^k = t_1^k - v q_1^k$ for all $k \in K_2^c$.

Let $k \in K_2^c$, so that either $q_1^k > 0$ or $q_1^k = q_2^k = 0$. The latter case, in which $b_1^k = s_2^k = 0$, is not possible, as we would have $m_2 S_2 \leq 0$ by (37), in contradiction with the assumption that $S_2 > 0$. Thus, if $S_2 > 0$, then $q_1^k > 0$ for all $k \in K_2^c$: all the sellers are thus active on the equilibrium path.

Write $K_2^c = K_1 \cup K_{12}$, where K_1 is the set of sellers who only trade with type 1, and K_{12} is the set of buyers who trade with both types. For each $k \in K_1$, we have $b^k = t_1^k - v q_1^k = m_1(t_1^k - v_1 q_1^k)$ and thus $t_1^k = v_2 q_1^k$; that is, whether type 2 is attracted by the contract (q_1^k, t_1^k) is irrelevant for any seller $k \in K_1$.

We already know that $K_{12} \neq \emptyset$. By (38), $B_2 \leq 0$. Hence, as the single seller in K_2 earns $S_2 > 0$ from trading with type 2, one must have $b_2^k < 0$ for some seller $k \in K_{12}$. Any seller $k \in K_{12}$ such that $b_2^k < 0$ can deviate by proposing $(q_1^k + \sum_{k' \in K_1} q_1^{k'}, t_1^k + \sum_{k' \in K_1} t_1^{k'} - \varepsilon)$ for some positive number ε. This attracts type 1 for sure, for instance along with the contracts $(q_1^{k'}, t_1^{k'})$, $k' \in K_{12}$, $k' \neq k$. Because $b_2^k = t_1^k - v_2 q_1^k < 0$ and, as observed in the previous paragraph, $t_1^{k'} = v_2 q_1^{k'}$ for all $k' \in K_1$, at worst this also attracts type 2. Letting ε go to zero, we get that

$$b^k \geq t_1^k - v q_1^k + \sum_{k' \in K_1} (t_1^{k'} - v q_1^{k'}).$$

As $b^k = t_1^k - vq_1^k$ and $t_1^{k'} = v_2 q_1^{k'}$ for all $k' \in K_1$, this implies that

$$(v_2 - v) \sum_{k' \in K_1} q_1^{k'} \leq 0$$

and thus $\sum_{k' \in K_1} q_1^{k'} = 0$. It follows that $K_1 = \emptyset$ and $K_{12} = K_2^c$.

For any seller $k \in K_{12}$, we have $b^k \geq B - m_2 S_2$ by (33). But $m_2 S_2$ is precisely the expected profit of the single seller in K_2. As $K_{12} = K_2^c$, we get that $b^k \geq \sum_{k' \in K_{12}} b^{k'}$ for all $k \in K_{12}$. Because $|K_{12}| = n - 1$ and $n \geq 3$, this implies that each seller in K_{12} earns zero expected profit and thus trades at unit price v.

Under the assumption that $S_2 > 0$, we now have a clear picture of the contracts offered and traded in equilibrium: $n - 1$ sellers offer contracts stipulating positive quantities at unit price v, which make up the aggregate trade (Q_1, T_1) of type 1 and are traded by both types, while the remaining seller offers a contract stipulating an additional positive quantity $Q_2 - Q_1$ at a unit price strictly higher than v_2, which is only traded by type 2 to overall reach her aggregate trade (Q_2, T_2).

Step 5 We now derive a contradiction from the assumption that $S_2 > 0$. We start with two standard observations.

As in Attar, Mariotti, and Salanié (2014a, Proof of Lemma 4), one can show that no seller in K_{12} can be indispensable in providing type 1 with her equilibrium utility. Otherwise, some seller $k \in K_{12}$ is indispensable and can, therefore, propose the quantity q_1^k at a unit price slightly above v. This attracts type 1 for sure, because trading only with the sellers other than k would yield her a lower utility. Moreover, this deviation yields seller k a positive profit even in the worst-case scenario where it attracts both types.

As in Attar, Mariotti, and Salanié (2014a, Proof of Lemma 3), one can also show that $\tau_1(Q_1, T_1) = v$. Otherwise, some seller $k \in K_{12}$ can deviate by proposing a quantity close to $Q_1 > 0$ at a unit price slightly above v. This attracts type 1 for sure. Moreover, this deviation yields seller k a positive profit even in the worst-case scenario where it attracts both types.

The desired contradiction follows from these two observations. Indeed, because, as shown in Step 4, all contracts are proposed at a unit price at least equal to v, the marginal condition $\tau_1(Q_1, T_1) = v$ implies that the equilibrium indifference curve of type 1 intersects his budget set $\{(\sum_{k \in K} q^k, \sum_{k \in K} t^k) : K \subset \{1, \ldots, n\}\} \cap X$ only at the point (Q_1, T_1). Because no seller in K_{12} is indispensable in providing type 1 with her equilibrium utility, this means that the aggregate trade (Q_1, T_1) remains available if any such seller k withdraws his contract; but this seller may then propose to trade the quantity $Q_2 - Q_1$ at a unit price between v_2 and

$(T_2 - T_1)/(Q_2 - Q_1)$, the latter number being strictly larger than v_2 if $S_2 > 0$. This attracts type 2 for sure, for instance along with the aggregate trade (Q_1, T_1). Moreover, this deviation yields seller k a positive profit even in the worst-case scenario where it only attracts type 2. This, however, is impossible as the sellers in K_{12} earn zero expected profit in equilibrium according to Step 4.

We are now ready to complete the proof of Theorem 1. That $S_2 = 0$ follows from (31) along with Step 5. Together with (33), this implies that $b^k = B = 0$ for all k and hence that (24) holds. That all traded contracts must be issued at unit price v or v_2 follows from considering the families of sellers K_1, K_2, and K_{12} introduced in Step 4. Because sellers earn zero expected profit, sellers in K_2 and K_{12} must respectively trade at unit prices v_2 and v. Now, if one had $K_1 \neq \emptyset$, then type 1 should trade the corresponding contract(s) at unit price $v_1 < v$. However, as $T_1 = vQ_1$, this implies that she should trade some contract(s) at a unit price strictly above v. But the only contracts satisfying this property are traded at unit price v_2 between type 2 and sellers in K_2. Therefore, $K_1 = \emptyset$. Hence the result. ■

Proof of Corollary 1. To show that the marginal condition (5) holds under Assumption 1-v, one can follow Attar, Mariotti, and Salanié (2014a, Proof of Lemma 3) as we did in Step 4 of the proof of Theorem 1. The only thing to be checked is that $Q_1 > 0$. To see why, observe that, otherwise, any seller can deviate and issue a contract stipulating a small quantity at a unit price between v and $\tau_1(0, 0)$. This attracts type 1 for sure. Moreover, this deviation yields a positive profit even in the worst-case scenario where it attracts both types.

The proof that (6) holds under Assumption 2-v_2 is similar. Arguing as in Step 4 of the proof of Theorem 1, we first get that the aggregate trade (Q_1, T_1) remains available if any seller withdraws his contract. Assumption 2-v_2 then implies that there are gains from trade between type 2 and any seller, who can propose additional trades to type 2 on top of the aggregate trade (Q_1, T_1) at a unit price between v_2 and $\tau_2(Q_1, vQ_1)$. As above, this implies that $Q_2 - Q_1 > 0$. The marginal condition (6) then follows from a standard argument along the lines of Attar, Mariotti, and Salanié (2014a, Proof of Lemma 6). Hence the result. ■

Proof of Corollary 2. We argued in the proof of Corollary 1 that the aggregate trade (Q_1, T_1) remains available if any seller withdraws his contract. One can more precisely show that the way in which (Q_1, T_1) thus remains available involves only contracts with unit price v. To see why, observe that, otherwise, some seller k offers a contract (q^k, t^k) stipulating a quantity $q^k < Q_1$ at unit price $t^k/q^k < v$. (This contract cannot be traded in equilibrium, according to Theorem 1.) But then any seller $k' \neq k$ can deviate and issue a contract stipulating a quantity $Q_1 - q^k$ at a unit price $v + \varepsilon$ for some positive number ε. Because $t^k/q^k < v$, we have

$$t^k + (v + \varepsilon)(Q_1 - q^k) < vQ_1 = T_1$$

when ε is small enough. Hence seller k''s offer attracts type 1 for sure, because combining it with the contract (q^k, t^k) allows her to purchase her equilibrium quantity Q_1 in exchange for a transfer lower than T_1. Moreover, this deviation yields a positive profit even in the worst-case scenario where it attracts both types.

Let K_v be the set of sellers issuing contracts (q^k, t^k) at unit price $t^k/q^k = v$ and, for each $k \in K_v$, let $\alpha^k \equiv q/Q_1$. Fix some $k \in K_v$ who is active in equilibrium, so that, in particular, $0 < \alpha^k \leq 1$. It follows from the previous reasoning that there exists $K_v^{-k} \subset K_v \setminus \{k\}$ such that $\sum_{k' \in K_v^{-k}} \alpha^{k'} = 1$ and, therefore,

$$1 < \sum_{k' \in K_v^{-k}} \alpha^{k'} + \alpha^k = 1 + \alpha^k \leq 2.$$

Note that the aggregate trade $(1 + \alpha^k)(Q_1, vQ_1)$ is available on the equilibrium path, and cannot be strictly more preferred by type 2 than $c + c'$. Moreover, $k'' \notin \{k\} \cup K_v^k$ for some seller k'' because some sellers must issue contracts at unit price v_2 according to Theorem 1 and Corollary 1.

Fix k as above. To conclude the proof, we only need to show that

$$(1 + \alpha^k)Q_1 > Q_2. \tag{40}$$

Indeed, along with the fact that the aggregate trade $(1 + \alpha^k)(Q_1, vQ_1)$ is not strictly more preferred by type 2 than $c + c'$ and that $1 + \alpha^k \leq 2$, (40) implies that the aggregate trade $(2Q_1, 2vQ_1) = (2Q_1, 2T_1)$ is also not strictly more preferred by type 2 than $c + c'$, which is (7), and that $2Q_1 > Q_2$, which is (8). To establish (40), let us first remark that we cannot have $(1 + \alpha^k)Q_1 = Q_2$, for, otherwise, type 2 could purchase the quantity Q_2 in exchange for a transfer vQ_2 strictly lower than her equilibrium aggregate transfer $T_2 = vQ_1 + v_2(Q_2 - Q_1)$, a contradiction. Let us then suppose that $(1 + \alpha^k)Q_1 < Q_2$. Then any seller $k'' \notin \{k\} \cup K_v^k$ can deviate by offering a contract stipulating a quantity $Q_2 - (1 + \alpha^k)Q_1$ at a unit price $v_2 + \varepsilon$ for some positive number ε. As $1 + \alpha^k > 1$, we have

$$v(1 + \alpha^k)Q_1 + (v_2 + \varepsilon)[Q_2 - (1 + \alpha^k)Q_1] < vQ_1 + v_2(Q_2 - Q_1) = T_2$$

Hence seller k'''s offer attracts type 2 for sure, because combining it with the contracts (q^k, t^k) and $(q^{k'}, t^{k'})$ for $k' \in K_v^{-k}$ allows her to purchase her equilibrium quantity Q_2 in exchange for a transfer strictly lower than T_2. Moreover, this deviation yields a positive profit even in the worst-case scenario where it only attracts type 2. Hence the result. ■

Proof of Lemma 1. Consider first type 1. Because $\tau_1(c) = v$ by (5), c is her most preferred contract with unit price v. As all offered contracts have unit prices at least equal to v, trading a single contract c is thus optimal for type 1. Consider next type 2. Because $\tau_2(c+c') = v_2$ by (6) and the unit price v_2 of c' is strictly higher than the unit price v of c, she is strictly worse off trading only contracts c' than trading a contract c along with a contract c'. Thus type 2 optimally trades at least one contract c. By (6) again, if she trades exactly one contract c, it is optimal for her to trade in addition exactly one contract c'. Hence we only need to prove that she cannot be strictly better off trading c twice. To see why, note that, according to (9), $\tau_2(2c)$ is lower than v and thus, a fortiori, lower than the unit price v_2 of c'. This, together with (7), implies that trading c twice, possibly along with one or two contracts c', cannot yield type 2 a higher utility than trading a contract c along with a contract c'. The result follows. ■

Proof of Lemma 2. As for type 1, the proof follows along the lines of Lemma 1, observing that the unit price of c'' is strictly higher than the unit price v of c. Consider next type 2. As $U_2(c+c'') = U_2(c+c')$, it is sufficient to check that, if type 2 trades c'' once, the optimal thing for her to do is to combine this contract c'' with exactly one contract c. Indeed, because $\tau_2(c+c'') = v$, c is, among all contracts with unit price v, the best that type 2 can combine with c''. As all offered contracts have unit prices at least equal to v, trading a single contract c is, therefore, the unique optimal choice for type 2 once she has traded c''. The result follows. ■

Proof of Lemma 3. First, we show that there is no profitable deviation for a seller that attracts both types. Indeed, to be profitable, the corresponding contract $\tilde{c}$ would need to have a unit price strictly higher than v. However, recall that $\tau_1(c) = v$ by (5) and that all offered contracts have unit prices at least equal to v. Trading $\tilde{c}$ would then yield type 1 a strictly lower utility than trading a single contract c, which remains feasible following any seller's unilateral deviation. Such a deviation is thus not possible.

Second, we show that there is no profitable deviation for a seller that only attracts type 2. Indeed, to be profitable, the corresponding contract $\tilde{c}$ would need to have a unit price strictly higher than v_2. However, recall that $\tau_2(c+c') = v_2$ by (6) and that, by (9), type 2 cannot gain from combining any contract with unit price strictly higher than v_2 with $2c$. Trading $\tilde{c}$, possibly along with some contracts c or c', would then yield type 2 a strictly lower utility than trading a contract c along with a contract c', which remains feasible following any seller's unilateral deviation. The possibility remains that type 2 combines c'' with $\tilde{c}$. However, as all offered contracts have unit prices at least equal to v, and strictly so for $\tilde{c}$, trading a single contract c is the unique optimal choice for type 2 once she has traded c'', see the proof of Lemma 2. Such a deviation is thus not possible. The result follows. ■

Proof of Lemma 4. The result requires that sufficiently many sellers offer the contract c''. A simple upper bound on the number of required sellers can be obtained as follows. Specifically, define

$$A_1 \equiv \{(q,t) \in \mathbb{R}_+ \times \mathbb{R} : q \leq Q_2 \text{ and } vq \geq t \geq v_1 q\},$$
$$A_2 \equiv \{Nc + N'c' : (N,N') \in \{0,1,2\} \times \{0,1,2\}\},$$
$$A_3 \equiv \{(Q,T) \in \mathbb{R}_+ \times \mathbb{R} : U_1(Q,T) \geq U_1(c)\}.$$

To interpret A_1, observe that if type 1 were attracted by a contract (q,t) with $q \geq Q_2$ issued by a deviating seller, then this would mean that by trading (q,t), possibly along with other available contracts, she could reach an aggregate trade (Q,T) with $Q \geq Q_2$, which she would weakly prefer to the aggregate trade $c + c'$ that remains available following any unilateral deviation. Because $c+c'$ is the equilibrium aggregate trade of type 2 and involves an aggregate quantity Q_2, it would follow from Assumption SC that type 2 would strictly prefer (Q,T) to her equilibrium aggregate trade $c+c'$, and thus would be strictly attracted by the contract (q,t). Therefore, we can safely restrict our quest for potential cream-skimming deviation to the set of contracts (q,t) such that $q \leq Q_2$. In addition, the contracts in A_1 imply no loss for the sellers when only traded by type 1 and have a unit price lower than v, so that they are potentially attractive for type 1, either per se or combined with other available contracts. Next, A_2 is the set of aggregate trades that can be made with four sellers, two of whom offer the contract c and two of whom offer the contract c'. Last, A_3 is the upper contour set of c for type 1. Then

$$N'' \equiv \max\{N \in \mathbb{N} : (A_1 + A_2 + Nc'') \cap A_3 \neq \emptyset\} \tag{41}$$

is the maximum number of contracts c'' type 1 may ever want to trade, if she were proposed a contract in A_1, which she could complement by aggregate trades in A_2 and as many contracts c'' as she wishes. Because A_1 is compact, $c \in A_1 \cap A_3$, $(0,0) \in A_2$, and $\tau_1(c) = v$ is strictly lower than v_2 and the unit price of c'', N'' is well defined and finite. Suppose now that two sellers offer the contract c, two sellers offer the contract c', and $\max\{N''+1, 2\}$ sellers offer the contract c''. Consider now a deviation that attracts type 1. Trading the corresponding contract $\tilde{c}$, possibly along with contracts c, c', and c'', must yield type 1 at least her equilibrium utility. However, because $\tau_2(c+c'') = \tau_1(c)$, under conditions (10)–(11) the equilibrium indifference curve $\mathcal{I}_2$ of type 2 is the translate of the equilibrium indifference curve $\mathcal{I}_1$ of type 1 along the vector c''. Thus type 2 could also weakly increase her utility by trading the same contracts as type 1, plus one additional contract c''. The definition (41) of N'' ensures that type 1 will never trade more than N'' contracts c'' following the deviation. If, as postulated, $\max\{N''+1, 2\}$ sellers offer the contract c'', a contract c'' remains available for type 2 to trade even after mimicking type 1. As a result, one can construct the buyer's best response in such a way that both types trade $\tilde{c}$ with the deviating seller, which, by Lemma 3, cannot be profitable for him. The result follows. ■

Proof of Corollary 3. Let $(q^{k'}, t^{k'})$, $k' \neq k$, be the contracts offered by the sellers other than k in the equilibrium of the single-contract game constructed along the lines of Theorem 2. Given these offers, type i's indirect utility from trading a contract (q, t) with buyer k is given by

$$z_i^{-k}(q,t) = \max\left\{U_i\left(q + \sum_{k' \in K^{-k}} q^{k'}, t + \sum_{k' \in K^{-k}} t^{k'}\right) : K^{-k} \subset \{1, \ldots, n\} \setminus \{k\}\right\}.$$

Clearly, $z_i^{-k}(q,t)$ is continuous in (q,t) and strictly decreasing in t. Now, let $\hat{C}^k$ be a strongly profitable menu deviation for seller k and let $(\hat{q}_2^k, \hat{t}_2^k)$ be an optimal choice of type 2 in $\hat{C}^k$ given the contracts offered by the sellers other than k. Given that $z_2^{-k}(0,0) = U_2(c+c')$, it follows along the lines of Lemmas 1–3 that trading with seller k a contract with unit price v_2 cannot make type 2 strictly better off. (Note that this remark does not require that the buyer's preference satisfy (10)–(11).) That is,

$$z_2^{-k}(0,0) \geq \sup\{z_2^{-k}(q, v_2 q) : q \geq 0\}. \tag{42}$$

Now, observe that $z_2^{-k}(\hat{q}_2^k, \hat{t}_2^k) \geq z_2^{-k}(0,0)$, for, otherwise, $(\hat{q}_2^k, \hat{t}_2^k)$ would not be an optimal choice for type 2 in $\hat{C}^k$ given the contracts offered by the sellers other than k. This, along with (42) and the fact that $z_2^{-k}(\hat{q}_2^k, t)$ is strictly decreasing in t, implies that $\hat{t}_2^k \leq v_2 \hat{q}_2^k$. We must show that this inequality is strict. If it were not, we would, following the above logic, have $z_2^{-k}(\hat{q}_2^k, \hat{t}_2^k) = z_2^{-k}(0,0)$ and thus type 2 would not be strictly better off trading with seller k. For $\hat{C}^k$ to be a strongly profitable deviation for seller k, an optimal choice for type 1 in $\hat{C}^k$ given the contracts offered by the sellers other than k should thus make her strictly better off, for, otherwise, she might as well abstain from trading with seller k. Yet, using now conditions (10)–(11), reasoning along the lines of the proof of Lemma 4 implies that type 2 would be strictly better off trading with seller k any contract that attracts type 1 in this way. It then follows that $\hat{C}^k$ is not a strongly profitable deviation, a contradiction. Hence the result. ■

Proof of Theorem 3. According to our preliminary discussion, it is sufficient to show that, for all $p < \partial u_1(0)$ and $(q,t) \in (-D_1(p), \infty) \times \mathbb{R}$,

$$u_1(D_1(p)+q) - pD_1(p) - t \geq u_1(D_1(p)) - pD_1(p)$$

implies that

$$u_2(D_2(p)+q) - pD_2(p) - t \geq u_2(D_2(p)) - pD_2(p),$$

or, equivalently, that, for all $p < \partial u_1(0)$ and $q > -D_1(p)$,

$$\Delta(p,q) \equiv [u_1(D_1(p)) - u_1(D_1(p)+q)] - [u_2(D_2(p)) - u_2(D_2(p)+q)] \geq 0. \tag{43}$$

It follows from the definitions of the functions Δ, D_1, and D_2 that $\Delta(p,0) = 0$ and $(\partial\Delta/\partial q)(p,0) = 0$ for all $p < \partial u_1(0)$. A sufficient condition for $\Delta(p,\cdot)$ to reach a global minimum over $(-D_1(p), \infty)$ at $q = 0$, as required by (43), is thus that, for each $q > -D_1(p)$, $(\partial\Delta/\partial q)(p,q) = 0$ implies that $(\partial^2\Delta/\partial q^2)(p,q) > 0$. That is, for all $p < \partial u_1(0)$ and $q > -D_1(p)$,

$$\partial u_1(D_1(p)+q) = \partial u_2(D_2(p)+q)$$

implies that

$$\partial^2 u_2(D_2(p)+q) > \partial^2 u_1(D_1(p)+q).$$

But the first term of this implication simply says that there exists a price $p' < \partial u_1(0)$ such that $D_1(p)+q = D_1(p')$ and $D_2(p)+q = D_2(p')$. Hence a sufficient condition for (43) to hold is that, for each $p' < \partial u_1(0)$,

$$\partial^2 u_2(D_2(p')) > \partial^2 u_1(D_1(p')). \tag{44}$$

Differentiating the identity $\partial u_i(D_i(p')) = p'$ with respect to p' and using the fact that, for each i, $\partial^2 u_i < 0$ and $\partial D_i < 0$, shows that (16) and (44) are equivalent. Hence the result. ■

Proof of Example 1. The parameter restrictions imposed on v_1, v_2, α_1, and α_2 ensure that Assumption SC is satisfied. A direct computation yields

$$D_i(p) = \max\left\{W_G - W_B + \frac{1}{\alpha_i}\ln\left(\frac{(1-p)/p}{(1-v_i)/v_i}\right), 0\right\}.$$

Thus (16) holds over the relevant range if $\alpha_1 > \alpha_2$. ■

Proof of Example 2. A direct computation yields

$$D_i(p) = \max\left\{(\partial C)^{-1}(\theta_i - p), 0\right\}.$$

Thus (16) holds over the relevant range if $\partial^2 C \circ (\partial C)^{-1}$ is decreasing, that is, because $\partial^2 C > 0$, if $\partial^3 C < 0$. ■

Proof of the Contractibility of the Set of Preferences Satisfying (16). Observe first that the equation $D(p) = (\partial u)^{-1}(p)$ and its inverse $u(Q) = \int_0^Q D^{-1}(q)\, dq$ define, over the relevant range, a homeomorphism between the set of twice continuously differentiable and strictly concave utility functions u over $\mathbb{R}_+$ and the space of continuously differentiable and strictly decreasing demand functions D. The latter space is itself contractible, as is easily seen by taking convex combinations $\lambda D + (1 - \lambda)\overline{D}$ of demand functions, for some fixed demand function $\overline{D}$. Because condition (16) is, over the relevant range, linear in the strictly negative functions ∂D_1 and ∂D_2, it follows that the set of pairs (D_1, D_2) satisfying (16) is contractible and, hence, by homeomorphism, that so is the corresponding space of utility functions (u_1, u_2). ■

Proof of (20). It follows from the implicit function theorem that the analytic expression for $d\tau_i/dQ|_{U_i=\text{const}}$ is

$$-\frac{[(\partial^2 U_i/\partial Q^2) + (\partial^2 U_i/\partial Q\partial T)\tau_i](\partial U_i/\partial T) - [(\partial^2 U_i/\partial Q\partial T) + (\partial^2 U_i/\partial T^2)\tau_i](\partial U_i/\partial Q)}{(\partial U_i/\partial T)^2}. \quad (45)$$

Moreover, using the fact that $\partial U_i/\partial T < 0$, it is straightforward to check from (17) that

$$\kappa_i = \frac{(\partial^2 U_i/\partial Q^2) + 2(\partial^2 U_i/\partial Q\partial T)\tau_i + (\partial^2 U_i/\partial T^2)\tau_i^2}{(\partial U_i/\partial T)(1 + \tau_i^2)^{3/2}}. \quad (46)$$

Using (46) to simplify (45) yields (20). ■

Proof of Theorem 4. According to Lemmas 1–3, we only need to show that Assumptions 1-v and 2-v_2 and conditions (7)–(8) and (19) hold for preferences for types 1 and 2 in an open subset of $\mathbf{P}_{sc} \times \mathbf{P}_{sc}$. We focus on the last condition, the proof for the first set of assumptions and conditions following in a similar way. Because a cream-skimming deviation must not attract type 2, it is sufficient to prove that (19) uniformly holds over a large enough compact subset Y of V, independent of the particular pair of preferences chosen in the required set. To do so, take any pair of preferences $(\succeq_1, \succeq_2) \in \mathbf{P}_{sc} \times \mathbf{P}_{sc}$ with representation (10) over $X \equiv \mathbb{R}_+ \times \mathbb{R}$ as in Examples 1–2. (For instance, one may take $C(Q) = Q^\gamma/\gamma$ for $1 < \gamma < 2$

in Example 2, assuming in addition that the inequality in (7) is strict.) Then condition (19) is satisfied for this pair of preferences. Suppose, by way of contradiction, that condition (19) is not satisfied over Y for preferences in a neighborhood of $(\succeq_1, \succeq_2)$ in $\mathbf{P}_{sc} \times \mathbf{P}_{sc}$. Then there exists a sequence $\{(\succeq_1^n, \succeq_2^n)\}$ converging to $(\succeq_1, \succeq_2)$ in $\mathbf{P}_{sc} \times \mathbf{P}_{sc}$ and sequences $\{(Q_n, T_n)\}$ and $\{(Q'_n, T'_n)\}$ in Y such that, for each n,

$$\tau_{1,n}(Q_n, T_n) = \tau_{2,n}(Q'_n, T'_n) \quad \text{and} \quad \kappa_{1,n}(Q_n, T_n) \leq \kappa_{2,n}(Q'_n, T'_n). \tag{47}$$

Because $\{(\succeq_1^n, \succeq_2^n)\}$ converges to $(\succeq_1, \succeq_2)$ in $\mathbf{P}_{sc} \times \mathbf{P}_{sc}$, it follows from the definition of the topology of $\mathbf{P}_{sc}$ that, for each i, the sequences $\{\tau_{i,n}\}$ and $\{\kappa_{i,n}\}$ converge uniformly to τ_i and κ_i over Y. Because Y is compact, we can assume without loss of generality that the sequences $\{(Q_n, T_n)\}$ and $\{(Q'_n, T'_n)\}$ converge in Y to some (Q, T) and (Q', T'). Taking limits in (47) then yields that property (19) is not satisfied for $(\succeq_1, \succeq_2)$ at (Q, T) and (Q', T'), a contradiction. Hence the result. ■

Proof of the Necessity of (21). Suppose that the contracts offered are those of Theorem 2, and that $\kappa_2(c+c'') > \kappa_1(c)$. We show that a profitable cream-skimming deviation consists in offering a contract $\tilde{c}$ in the upper contour set of c for type 1, close enough to c, and above the line with slope v_2 that passes through c and $c+c'$. As each type's utility remains available if any seller deviates unilaterally, we only need to show that type 1 is strictly better off trading $\tilde{c}$ and that type 2 would be strictly worse off trading c. The first point is clear. To show the second point, observe first that, if $\tilde{c}$ is close enough to c and (7) is strict, type 2 would be strictly worse off trading $\tilde{c}$ along with one (or two) c contract(s) instead of trading c along with c'. Second, as $\tilde{c}$ is above the line with slope v_2 that passes through c and $c+c'$, and as $\tau_2(c+c') = v_2$, type 2 would be strictly worse off combining $\tilde{c}$ with c' contract(s) only. It thus only remains to show that type 2 would be strictly worse off trading $\tilde{c}$ along with c''; indeed, because $\tilde{c}$ is close to c and $c+c''$ gives type 2 her maximum utility if she only trades contracts other than $\tilde{c}$, and because $\tau_2(c+c'') = v$ and no contract other than $\tilde{c}$ is issued at a unit price lower than v, trading further contracts would clearly be suboptimal for type 2. To show this, it is enough to show that, for any small enough ε,

$$\tau_1(Q_1 + \varepsilon, \mathcal{T}_1(Q_1 + \varepsilon)) \gtrless \tau_2(Q_1 + q'' + \varepsilon, \mathcal{T}_2(Q_1 + q'' + \varepsilon)) \quad \text{if} \quad \varepsilon \gtrless 0,$$

where the mappings $Q \mapsto \mathcal{T}_1(Q)$ and $Q \mapsto \mathcal{T}_2(Q)$ stand for the analytic expressions of the indifference curves $\mathcal{I}_1$ and $\mathcal{I}_2$, and q'' is the quantity specified by c''. A sufficient condition for this to hold is that

$$\left.\frac{d\tau_1}{dQ}\right|_{U_1=U_1(c)} (Q_1) > \left.\frac{d\tau_2}{dQ}\right|_{U_2=U_2(c+c'')} (Q_1 + q''),$$

which, according to (20) and to the fact that $\tau_2(c+c'') = \tau_1(c)$, amounts to $\kappa_2(c+c'') > \kappa_1(c)$, as postulated. Thus (21) is a necessary condition to construct an equilibrium along the lines of Theorem 2. ■

Proof of Example 3. By construction, $\theta_1 - \partial C(Q_1) = \tau_1(c) = v = \tau_2(c + c'') = \theta_2 - \partial C(Q_1 + q'')$, where q'' is the quantity specified by c''. By quasilinearity, $D_1(v) = Q_1$ and $D_2(v) = Q_1 + q''$. Hence, according to (18) and to the fact that, for each i, $-\partial^2 u_i = \partial^2 C$, comparing the curvature of $\mathcal{I}_1$ at c to that of $\mathcal{I}_2$ at $c + c''$ just amounts to compare $\partial^2 C(Q_1)$ to $\partial^2 C(Q_1 + q'')$. Because $\partial^3 C > 0$ and $q'' > 0$, one has $\partial^2 C(Q_1 + q'') > \partial^2 C(Q_1)$, so that $\kappa_2(c + c'') > \kappa_1(c)$ and (21) is violated. ■

Appendix B (For Online Publication)

In this appendix, we formally construct the preference space $\mathbf{P}_{sc}$. In line with Section 2.1, we consider regular preference relations $\succeq$ over an open, convex, and comprehensive set V that contains the no-trade point $(0,0)$. We first impose the following restrictions on $\succeq$:

(i) $\succeq$ is closed relative to $V \times V$.

(ii) $\succeq$ is strictly monotone in transfers: if $(Q,T) \in V$ and $T' > T$, then $(Q,T) \succ (Q,T')$.

(iii) $\succeq$ is convex: if $(Q,T) \succeq (Q',T')$ and $\lambda \in [0,1]$, then $\lambda(Q,T)+(1-\lambda)(Q',T') \succeq (Q',T')$.

(iv) $\succeq$ has closed upper contour sets relative to $\mathbb{R} \times \mathbb{R}$.

(v) $\succeq$ has a boundary in $V \times V$ that is a C^2 manifold.

Conditions (i) and (iii) are standard. Condition (ii) requires monotonicity of preferences in transfers, but not necessarily in quantities. Condition (iv) is a convenient boundary condition. Condition (v) is our basic regularity condition.

Our first task is to characterize the set $\mathbf{P}$ of preferences $\succeq$ that satisfy conditions (i)–(v). The following notation will be useful. Let $\mathcal{U}_{(Q,T)}$ and $\mathcal{L}_{(Q,T)}$ be the upper and lower contour sets of (Q,T) for $\succeq$, and let $\mathcal{I}_{(Q,T)} \equiv \mathcal{U}_{(Q,T)} \cap \mathcal{L}_{(Q,T)}$ be the indifference set of (Q,T) for $\succeq$. Also denote by cl and ∂ the closure and boundary operators relative to V or $V \times V$, depending on the context. We start with two technical lemmas.

Lemma 5 *If $\succeq$ satisfies (i)–(ii), then, for each $(Q,T) \in V$,*

- *$\mathcal{U}_{(Q,T)}$ has a nonempty interior relative to $\mathbb{R} \times \mathbb{R}$.*
- *$\mathcal{I}_{(Q,T)} = \partial \mathcal{U}_{(Q,T)}$.*

Proof. To prove the first claim, observe that $V \setminus \mathcal{L}_{(Q,T)}$ is open relative to V by (i), and thus relative to $\mathbb{R} \times \mathbb{R}$ as V is an open subset of $\mathbb{R} \times \mathbb{R}$. Hence, as $V \setminus \mathcal{L}_{(Q,T)}$ is nonempty by (ii), $\mathcal{U}_{(Q,T)} \supset V \setminus \mathcal{L}_{(Q,T)}$ has a nonempty interior relative to $\mathbb{R} \times \mathbb{R}$. To prove the second claim, observe that, as $\succeq$ is closed relative to $V \times V$ by (i), $\mathcal{U}_{(Q,T)}$ and $\mathcal{L}_{(Q,T)}$ are closed relative to V. Therefore, we have

$$\partial \mathcal{U}_{(Q,T)} \equiv \mathrm{cl}(\mathcal{U}_{(Q,T)}) \cap \mathrm{cl}(V \setminus \mathcal{U}_{(Q,T)}) = \mathcal{U}_{(Q,T)} \cap \mathrm{cl}(V \setminus \mathcal{U}_{(Q,T)}) \subset \mathcal{U}_{(Q,T)} \cap \mathcal{L}_{(Q,T)} = \mathcal{I}_{(Q,T)}.$$

The reverse inclusion holds if $\mathcal{I}_{(Q,T)} \subset \mathrm{cl}(V \setminus \mathcal{U}_{(Q,T)})$, which is obviously true because, for each $(Q',T') \in \mathcal{I}_{(Q,T)}$, $(Q',T'+\varepsilon) \in V$ for any small enough $\varepsilon > 0$ by openness of V, and $(Q',T') \succ (Q',T'+\varepsilon)$ for any such ε by (ii). The result follows. ■

Lemma 6 *If $\succeq$ satisfies (i)–(iv), then, for each $(Q,T) \in V$, $\partial \mathcal{U}_{(Q,T)}$ is connected.*

Proof. We first show that $\mathcal{U}_{(Q,T)}$ does not contain a vertical line $\{(Q',T') : T' \in \mathbb{R}\}$. Indeed, if this were the case then, by (iii), $\mathcal{U}_{(Q,T)}$ would contain all the points of the form $(1-1/T')(Q,T) + (1/T')(Q_1,T')$ for $T' > 1$. Letting T' go to infinity, this would imply by (i) that $\mathcal{U}_{(Q,T)}$ contains the point $(Q, T+1)$, which is ruled out by (ii). Hence the claim. We next show that, if $\mathcal{U}_{(Q,T)}$ contains a nonvertical line $\{(Q',T') : T' = aQ' + b\}$, then $\mathcal{U}_{(Q,T)}$ is a union of closed half-spaces of the form

$$\bigcup_{(\tilde{Q},\tilde{T}) \in \mathcal{U}_{(Q,T)}} \{(Q',T') \in \mathbb{R} \times \mathbb{R} : T' \leq \tilde{T} + a(Q' - \tilde{Q})\}. \tag{48}$$

Indeed, in that case, it follows from (iii) that, for each $(\tilde{Q},\tilde{T}) \in \mathcal{U}_{(Q,T)}$ and for each $(Q',Q'') \in \mathbb{R} \times \mathbb{R}$ such that $(Q'-\tilde{Q})(Q''-\tilde{Q}) > 0$ and $|Q'-\tilde{Q}| < |Q''-\tilde{Q}|$, $\mathcal{U}_{(Q,T)}$ contains the point $[(Q''-Q')/(Q''-\tilde{Q})](\tilde{Q},\tilde{T}) + [(Q'-\tilde{Q})/(Q''-\tilde{Q})](Q'', aQ''+b)$. Letting $|Q''|$ go to infinity and taking advantage of (i), we obtain that $\mathcal{U}_{(Q,T)}$ contains the point $(Q', \tilde{T} + a(Q'-\tilde{Q}))$, and hence, by (ii), all the points (Q',T') with $T' \leq \tilde{T} + a(Q'-\tilde{Q})$. Hence the claim as $(\tilde{Q},\tilde{T})$ is arbitrary. It follows from (48) that $\mathcal{U}_{(Q,T)}$ is either the entire space $\mathbb{R} \times \mathbb{R}$, or a closed half-space in $\mathbb{R} \times \mathbb{R}$, and thus also relative to V. The former alternative is impossible as $\mathcal{U}_{(Q,T)}$ does not contain a vertical line. Thus the latter alternative holds, so that $\partial \mathcal{U}_{(Q,T)}$ is a line and, therefore, is connected. Finally, suppose that $\mathcal{U}_{(Q,T)}$ does not contain a line. By Lemma 5, $\mathcal{U}_{(Q,T)}$ has a nonempty interior in $\mathbb{R} \times \mathbb{R}$. Moreover, because V is comprehensive, $\mathcal{U}_{(Q,T)}$ is unbounded by (ii) and, according to (iv), $\mathcal{U}_{(Q,T)}$ is closed relative to $\mathbb{R} \times \mathbb{R}$. Thus, as $\mathcal{U}_{(Q,T)}$ does not contain a line, its boundary in $\mathbb{R} \times \mathbb{R}$ is homeomorphic to $\mathbb{R}$ and, therefore, is connected (Bourbaki (2003, Chapter II, §2, Exercise 19) a)). Because V is open in $\mathbb{R} \times \mathbb{R}$ and $\mathcal{U}_{(Q,T)}$ is closed in $\mathbb{R} \times \mathbb{R}$ by (iv), the boundary of $\mathcal{U}_{(Q,T)}$ in $\mathbb{R} \times \mathbb{R}$ is nothing but $\partial \mathcal{U}_{(Q,T)}$. The result follows. ■

Lemmas 5–6 imply the following representation result.

Lemma 7 *$\succeq$ satisfies (i)–(v) if and only if it admits a quasiconcave C^2 utility function U such that $\partial U / \partial T < 0$ and such that $U^{-1}((-\infty, \upsilon])$ is closed in $\mathbb{R} \times \mathbb{R}$ for all $\upsilon \in \mathbb{R}$.*

Proof. (Direct part.) Suppose that $\succeq$ is representable by U. Then $\succeq$ trivially satisfies (i)–(iii). Next, because $U^{-1}((-\infty, \upsilon])$ is closed in $\mathbb{R} \times \mathbb{R}$ for all $\upsilon \in \mathbb{R}$, $\succeq$ satisfies (iv). Finally, because U clearly has no critical point, it follows as in Mas-Colell (1985, Proposition 2.3.5) that $\succeq$ satisfies (v).

(Indirect part.) By (ii), $\succeq$ is locally nonsatiated, and by (v), $\partial \succeq$ is a C^2 manifold in $V \times V$. Hence $\succeq$ is of class C^2 (Mas-Colell (1985, Definition 2.3.4)). Moreover, by Lemmas 5–6, $\succeq$ has connected indifference sets $\mathcal{I}_{(Q,T)}$. Hence it admits a C^2 utility function U over V with no critical point, that is, $\partial U \neq 0$ over V (Mas-Colell (1985, Proposition 2.3.9)). That U is quasiconcave follows from (iii). To show that $\partial U/\partial T < 0$, observe first from (ii) that $\partial U/\partial T \leq 0$. Now, if $(\partial U/\partial T)(Q,T) = 0$, then $(\partial U/\partial Q)(Q,T) \neq 0$ as U has no critical point. Thus the line through (Q,T) orthogonal to $\partial U(Q,T)$ which supports the convex set $\mathcal{U}_{(Q,T)}$ is vertical. It follows then that the strict upper contour set of (Q,T) for $\succeq$, $\mathcal{U}_{(Q,T)} \setminus \mathcal{L}_{(Q,T)}$, lies either to the left or to the right of this line, which violates (ii). Finally, that $U^{-1}((-\infty, \upsilon])$ is closed in $\mathbb{R} \times \mathbb{R}$ for all $\upsilon \in \mathbb{R}$ follows from (iv). The result follows. ∎

Let $\mathbf{U}$ be the set of quasiconcave C^2 functions $U : V \to \mathbb{R}$ such that $\partial U/\partial T < 0$ and such that $U^{-1}((-\infty, \upsilon])$ is closed in $\mathbb{R} \times \mathbb{R}$ for all $\upsilon \in \mathbb{R}$. We know from Lemma 7 that any preference relation in $\mathbf{P}$ can be represented by some function in $\mathbf{U}$ and, conversely, that any function $\mathbf{U}$ represents a preference relation in $\mathbf{P}$. For each $U \in \mathbf{U}$, let $P(U) \in \mathbf{U} \times \mathbf{U}$ be the preference relation represented by U. In line with Mas-Colell (1985, Chapter 2, Section 4), a topology on $\mathbf{P}$ can be constructed as follows. Note that $\mathbf{U}$ is a subspace of $C^2(V)$, the Polish space of real-valued C^2 functions over V endowed with the topology of uniform convergence over compact subsets of V of functions and of their derivatives up to the order 2 (Mas-Colell (1985, Chapter 1, K.1.2)). Then endow $\mathbf{P}$ with the identification topology from P, that is, let O be open in $\mathbf{P}$ if $P^{-1}(O)$ is open in $\mathbf{U}$. It will be convenient to work with a normalized space of utility functions, $\mathbf{U}_d \equiv \{U \in \mathbf{U} : u(0,T) = -T \text{ for all } T \text{ such that } (0,T) \in V\}$. We are now ready to complete the characterization of $\mathbf{P}$.

Lemma 8 $\mathbf{U}_d$ *and* $\mathbf{P}$ *are homeomorphic under the natural map* P*.*

Proof. We must prove that P restricted to $\mathbf{U}_d$ is one-to-one, onto, continuous, and open.

(One-to-one.) Let U and U' in $\mathbf{U}_d$ such that $P(U) = P(U')$. Then $U = \xi \circ U'$, where $\xi : U'(V) \to \mathbb{R}$ is C^2, increasing, and regular (Mas-Colell (1985, Proposition 2.3.11)). But for each $\upsilon \in U'(V)$, $\xi(\upsilon) = \xi(U'(0, -\upsilon)) = U(0, -\upsilon) = \upsilon$, so that $U = U'$.

(Onto.) Let $\succeq \, \in \mathbf{P}$, and let $U \in \mathbf{U}$ such that $\succeq \, = P(U)$ and $\text{range}(U(0,\cdot)) = \mathbb{R}$. Define $U' : V \to \mathbb{R}$ implicitly by $U(Q,T) = U(0, -U'(Q,T))$. Clearly $P(U') = \, \succeq$. We now check that $U' \in \mathbf{U}_d$. As $\partial U/\partial T < 0$, $U'(0,T) = -T$ for all T such that $(0,T) \in V$. That U' is quasiconcave follows from the observation that $\{(Q,T) \in V : U'(Q,T) \geq \upsilon\} = \{(Q,T) \in V : U(Q,T) \geq U(0,\upsilon)\}$ for all $\upsilon \in \mathbb{R}$; this also implies that $(U')^{-1}((-\infty, \upsilon])$ is closed in $\mathbb{R} \times \mathbb{R}$ for any such υ. That U' is C^2 follows from the implicit function theorem along with the fact that $\partial U/\partial T \neq 0$. That $\partial U'/\partial T < 0$ follows from $(\partial U/\partial T)(Q,T) = -(\partial U/\partial T)(0,-U'(Q,T))(\partial U'/\partial T)(Q,T)$, using again the fact that $\partial U/\partial T < 0$. Hence $U' \in \mathbf{U}_d$, as claimed.

(Continuous.) This follows from the definition of the topology of $\mathbf{P}$.

(Open.) Mimic the proof of Mas-Colell (1985, Proposition 2.4.2)). The result follows. ■

Preferences in $\mathbf{P}$ are not necessarily strictly convex. Thus we must add this as a further restriction:

(vi) $\succeq$ is strictly convex: if $(Q,T) \succeq (Q',T')$, $(Q,T) \neq (Q',T')$, and $\lambda \in (0,1)$, then $\lambda(Q,T) + (1-\lambda)(Q',T') \succ (Q',T')$.

Finally, to obtain a topologically complete space of preferences, we require preferences to be nonlinear, even in a local sense. To do so, observe that because a utility function $U \in \mathbf{U}_d$ representing a preference $\succeq \in \mathbf{P}$ has no critical point, the curvature $\kappa(Q,T)$ of the indifference curve passing through any point (Q,T) of V is well defined and given by formula (17). The last restriction we impose on preferences is that this curvature nowhere vanishes.

(vii) Any point of V is regular for $\succeq$, that is, $\kappa \neq 0$ over V.

Preferences that satisfy conditions (vi)–(vii) are said to be differentiably strictly convex (Mas-Colell (1985, Definition 2.6.1)). We can now define our fundamental space of preferences as the space $\mathbf{P}_{sc}$ of preferences over V that satisfy conditions (i)–(vii). According to Lemma 8, $\mathbf{P}_{sc}$ can be seen as a subset of $\mathbf{U}_d$ and, hence, of $C^2(V)$. Our final result is that $\mathbf{P}_{sc}$ is topologically complete, as desired, and that it is contractible.

Lemma 9 *$\mathbf{P}_{sc}$ is a contractible Polish space.*

Proof. We first prove that $\mathbf{P}_{sc}$ is a Polish space. Let $\{T_n\}$ be a sequence in $\mathbb{R}$ increasing to $\sup\{T \in \mathbb{R} : (0,T) \in V\}$, and let $\{Y_n\}$ be a countable collection of compact convex sets covering V. Then $\mathbf{P}_{sc}$ is the intersection of the following countable families of open sets:

$$\left\{U \in C^2(V) : \frac{\partial U}{\partial T}(Q,T) < 0 \text{ for all } (Q,T) \in Y_n\right\},$$

$$\Big\{U \in C^2(V) : \text{ there exists } \varepsilon > 0 \text{ such that } U(Q,T) < U(0,T_n)$$
$$\text{if } (Q,T) \in Y_n \text{ and } \inf\{\|(Q',T') - (Q,T)\| : (Q',T') \in \mathbb{R} \times \mathbb{R} \setminus V\} \leq \varepsilon\Big\},$$

$$\left\{U \in C^2(V) : \max\{|U(0,T) + T| : T \in [-n,n] \text{ and } (0,T) \in V\} < \frac{1}{n}\right\},$$

$$\Big\{U \in C^2(V): \text{ there exists } \xi : U(V) \to \mathbb{R} \text{ such that } \partial\xi > 0 \text{ over } U(V)$$

$$\text{and } \partial^2(\xi \circ U) \text{ is negative definite over } Y_n\Big\}.$$

The first family deals with the monotonicity in transfers (condition (ii)), the second family with the boundary condition (condition (iv)), the third family with the normalization, and the fourth family with the differential strict convexity of preferences (conditions (vi)–(vii)), bearing in mind that differentiably strictly convex preferences that can be represented by a C^2 function with no critical point can be represented over any compact convex set Y by a C^2 utility function U with no critical point such that $\partial^2 U$ is negative definite over Y (Mas-Colell (1985, Proposition 2.6.4)). Hence $\mathbf{P}_{sc}$ is a G_δ in the Polish space $C^2(V)$ and thus, by Alexandrov's lemma (Mas-Colell (1985, Chapter 1, A.3.4)), a Polish space itself in the relative topology.

To prove that $\mathbf{P}_{sc}$ is contractible, we exhibit a contraction $h : \mathbf{P}_{sc} \times [0,1] \to \mathbf{P}_{sc}$, that is, we show that the identity function on $\mathbf{P}_{sc}$ is homotopic to a constant function. The proof follows Mas-Colell (1985, Proposition 2.6.7), with some adjustments. Pick an arbitrary $\overline{\succeq} \in \mathbf{P}_{sc}$ with corresponding utility function $\overline{U} \in \mathbf{U}_d$. To each $(U, \xi) \in \mathbf{U}_d \times [0,1]$ we associate a utility function $U_\xi \in \mathbf{U}_d$ as follows. We first let $U_0 \equiv U$ and $U_1 \equiv \overline{U}$. For all $\xi \in (0,1)$ and $(Q,T) \in V$, we then let $\mu_\xi(Q,T) \in (0, 1/\xi)$ be the unique solution to

$$\overline{U}(Q, \mu_\xi(Q,T)T) = U\left(Q, \left[\frac{1 - \xi\mu_\xi(Q,T)}{1-\xi}\right]T\right)$$

and we let $U_\xi(Q,T) \equiv \overline{U}(Q, \mu_\xi(Q,T)T)$. What this transformation does is that, to each T' such that $(0,T') \in V$, it assigns an indifference curve $U_\xi^{-1}(-T')$, which is the vertical convex combination of $\overline{U}^{-1}(-T')$ and $U^{-1}(-T')$ with weights ξ and $1-\xi$. (Bear in mind that, by normalization, $\overline{U}(0,T') = U(0,T') = -T'$.) One can then verify that the mapping $h : (U, \xi) \mapsto U_\xi$ is the desired contraction. The result follows. ■

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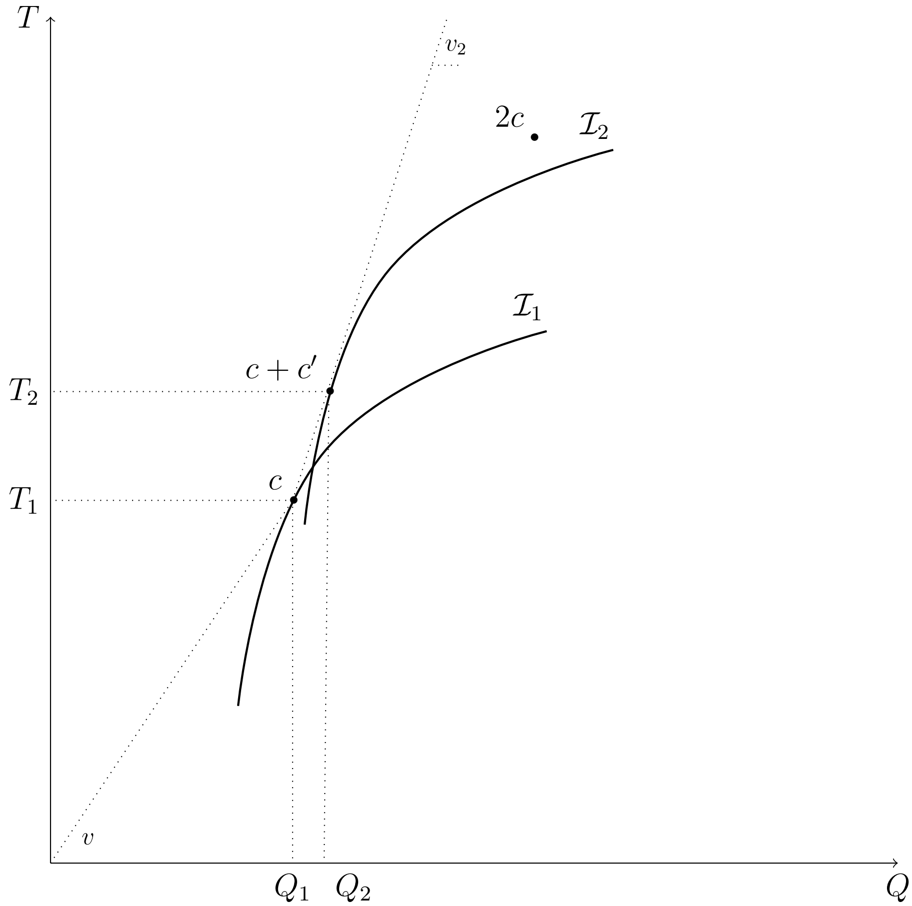

Figure 1 The Jaynes–Hellwig–Glosten outcome.

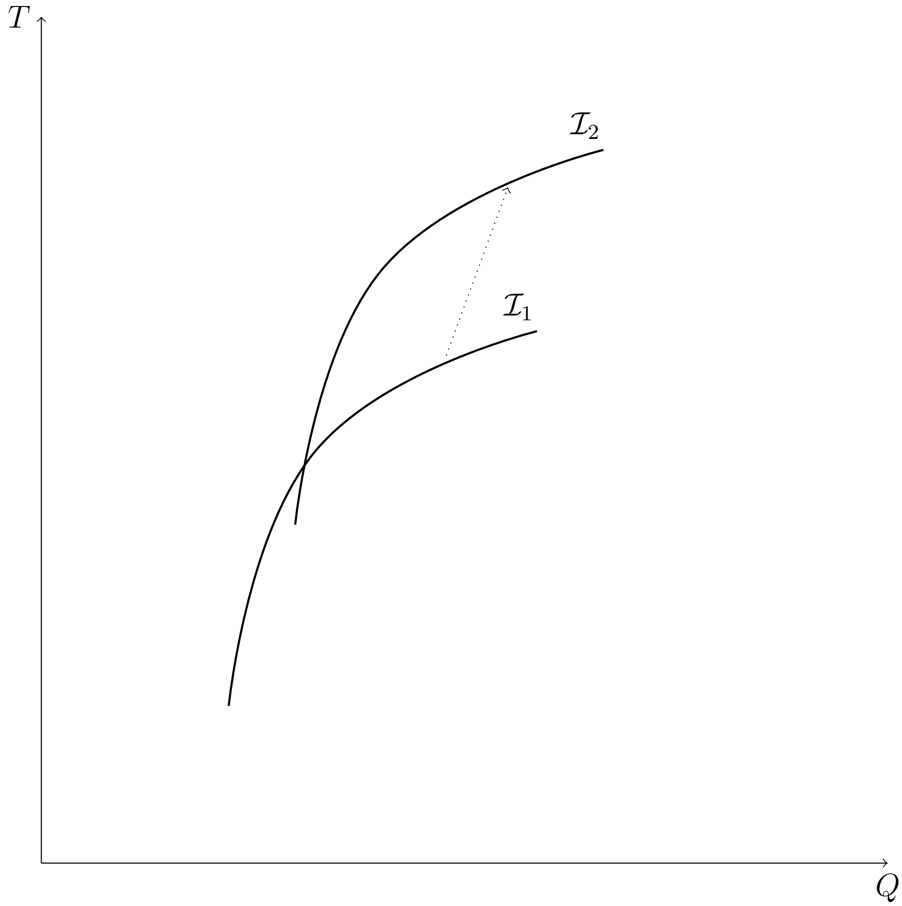

Figure 2 The translation property.

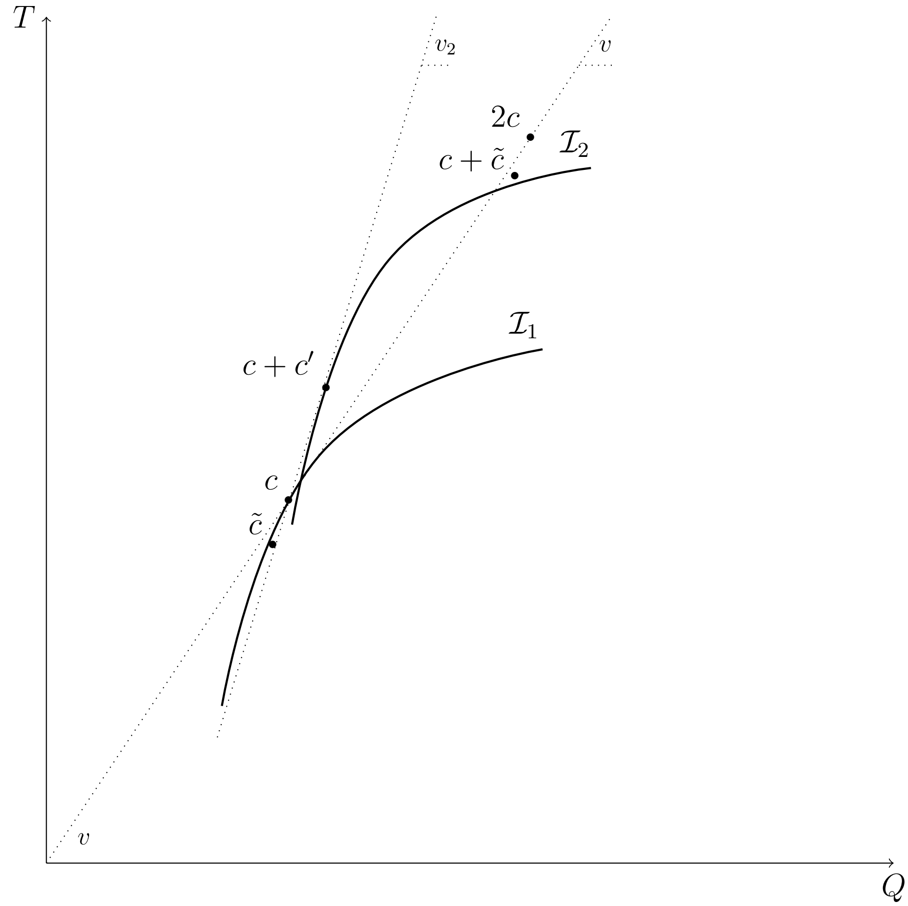

Figure 3 A cream-skimming deviation.

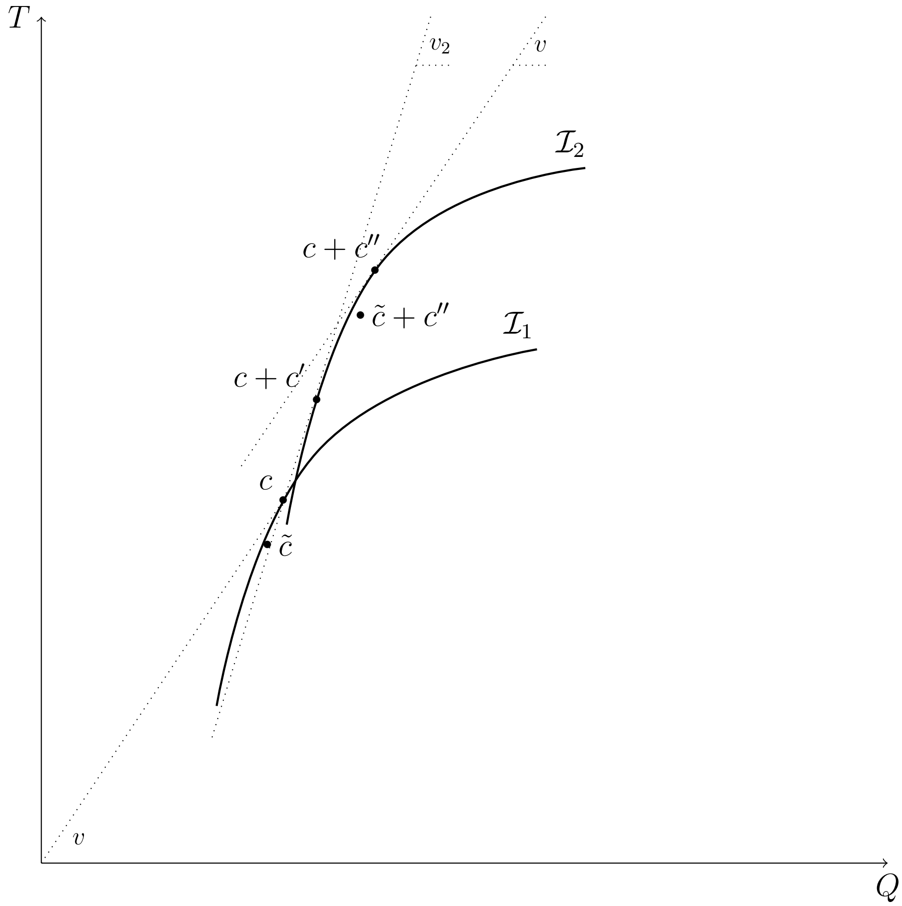

Figure 4 Deterring cream-skimming deviations with the contract c''.