Commodity Prices and Sovereign Default: A New Perspective on The Harberger-Laursen-Metzler Effect*

PRELIMINARY AND INCOMPLETE

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Abstract

This paper starts by documenting the main facts about the relationship between country risk and default episodes in oil producing economies. We find that oil prices are strongly negatively correlated with risk premia. Furthermore, the ability to extract oil (oil production), which enhances a country's ability to repay reduces risk premia, while countries with a large stock of oil reserves exhibit higher risk premia. The latter reflects the fact that having a large stock of oil increases a country's outside option, augmenting the probability of default. We then develop a small open economy model, where there is a sovereign government who owns a stock of oil and delegates extracting activities to an oil producing company. The sovereign can trade non-state contingent bonds with risk neutral competitive foreign lenders in international financial markets and cannot commit to repaying its debt. We show that the model can rationalize the main empirical findings and explore the following questions: How does the introduction of default risk affect the correlation between the trade balance and the terms of trade? How does it affect the co-movement between that correlation and the persistence of terms of trade shocks? What happens in the model if we do conditional versus unconditional comparisons to evaluate this relationship?

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1 Introduction

This paper documents the main facts about the relationship between country risk, default episodes and oil production for the 35 biggest oil producing economies in the world. Country risk data is taken from the Institutional Investor's country credit ratings, which contains information for the 1979-2010 period on sovereign debt risk that is published biannually in the March and September issues of Institutional Investor magazine. Those ratings are based on a survey of leading international bankers, who are asked to rate each country on a scale from 0 to 100 (where 100 represents maximum creditworthiness).

First and in line with common sense, a given oil producing country is perceived by investors as less risky, the larger the share of oil in GDP and the higher the oil prices, allowing its public sector to support higher levels of public debt. Second, and contrary to common sense, in the long run, country risk perception increases the higher the level of oil reserves of the country. This reflects the fact that having a large stock of oil increases a country's outside option, making default more appealing. Moreover, during default episodes, the median of oil producing countries increases oil production. This evidence suggests that a country in default and excluded from international financial markets, raises its extraction rate to withstand the consequences of financial autarky. We explore the channels through which oil abundance and oil dependency affect country risk in a small, open and naturally rich economy. To explain these empirical regularities, we build a small open economy model, where there is a sovereign government who owns a stock of oil and delegates extracting activities to an oil producing company. The sovereign can trade non-state contingent bonds with risk neutral competitive foreign lenders in international financial markets and cannot commit to repaying its debt.

Long-run reserves of the resource have two opposing effects in determining the long-term sovereign risk premium. Higher stock of reserves allow the country to have a higher extraction rate to support debt repayments, lowering default risk. However, they also allow the country to use the resource during default times, making the value of default more attractive. The tension between these two forces determines the long run default risk premium. In addition, we show how alternative penalty schemes commonly used in sovereign default models (other than the exclusion from access to international financial markets) affect the equilibrium default risk premium.

We finish by exploring how does the introduction of default risk affect the correlation between the trade balance and the terms of trade, how does it affect the co-movement between that correlation and the persistence of terms of trade shocks, and what happens in the model if we do conditional versus unconditional comparisons to evaluate this relationship.

2 Empirical Evidence

2.1 Data

We have collected data for oil GDP, non-oil GDP, oil reserves, oil consumption, oil net exports, total public debt, total external public debt, net foreign assets, default episodes and country risk, considering the following countries: Saudi Arabia, Canada, Iran, Iraq, Kuwait, Venezuela, United Arab Emirates, Russian Federation, Libya, Nigeria, Kazakhstan, Qatar, United States, China, Brazil, Algeria, Mexico, Angola, Azerbaijan, Norway, Ecuador, India, Oman, Sudan, Malaysia, Indonesia, Egypt, Australia, United Kingdom, Yemen, Argentina, Syrian Arab Republic, Gabon, Colombia and Vietnam. The main feature among these countries is that they are those with the largest proven oil reserves in the world.

The data we use come from different separate sources. First, country risk data is taken from the Institutional Investor's country credit ratings, which contains information for the 1979-2010 period of sovereign debt risk that is published biannually in the March and September issues of Institutional Investor magazine. Those ratings are based on a survey of leading international bankers, who are asked to rate each country on a scale from 0 to 100 (where 100 represents maximum creditworthiness). The answers are then weighted in accordance with the particular bank's global exposure and the level of sophistication for that country's analysis systems. A second source of information we use is the US Energy Information Administration dataset (EIA), which reports oil reserves, oil production, oil consumption and oil net exports information (in thousand barrels per day) for 219 countries, as well as Brent spot prices (in USD per barrel), from 1980 to 2013.

Total public debt data comes from World Development Indicators tables (WDI) and World Economic Outlook database (WEO). We have information, covering 1979-2010 period, for direct government fixed-term contractual obligations to others outstanding on a particular date. It includes domestic and foreign liabilities such as currency and money deposits, securities other than shares, and loans. In this case, information is not reported for Irak and Lybia. Total public external debt data is taken from the World Bank Global Development Finance database (GDF), which has annual data for over 130 countries on total external debt by maturity and type of debtor (private non-guaranteed debt and publicly guaranteed debt). The data goes back as far as 1970 and is collected on the basis of public and publicly-guaranteed debt reported in the World Bank's Debtor Reporting System by each of the countries. This information is not available for Saudi Arabia, Canada, Iraq, Kuwait, United Arab Emirates, Libya, Qatar, United States, Oman, Australia and United Kingdom. In the case of Russia, we use data reported by the Ministry of Finance[1].

Furthermore, we use the updated and extended version of "External Wealth of Nations" dataset, constructed by Lane and Milesi-Ferreti (2007) to obtain information of net foreign assets positions. It contains data for the 1970-2011 period and for 188 countries (including those of our sample), plus the euro area as a whole. Specifically, net foreign assets series are based on three alternative measures: i.) the cumulated current account, adjusted to reflect the impact of capital transfers, valuation changes, capital gains and losses on equity and Foreign Direct Investment (FDI), and debt reduction and forgiveness; ii.) the net external position, reported in the International Investment Positions section of the International Monetary Fund's Balance of Payments Statistics (BOPS), and net of gold holdings; iii.) the sum of net equity and FDI positions (both adjusted for valuation effects), foreign exchange reserves and the difference between cumulated flows of "debt assets", and the stock of debt measured

[1]Russian total public external debt data is available for a monthly frequency. Since GDP information is reported only at the end of each year, we use debt information in December, in order to construct public external debt-to-GDP ratios.

by the World Bank (or the OECD).

Added to this, we obtained default data from Borensztein and Panizza (2006) paper, considering 1980-2010 period. The authors use Standard and Poor's (S&P) information in order to date sovereign default episodes over the last two hundred years. As pointed in the paper, S&P generally defines sovereign default as the failure to meet a principal or interest payment on the due date (or within the specified grace period) contained in the original terms of the debt issue. In particular, each issuer's debt is considered in default in any of the following circumstances: i.) For local and foreign currency bonds, notes and bills, when either scheduled debt service is not paid on the due date, or an exchange offer of new debt contains terms less favorable than the original issue; ii.) for central bank currency, when notes are converted into new currency of less than equivalent face value; iii.) for bank loans, when either scheduled debt service is not paid on the due date, or a rescheduling of principal and/or interest is agreed to by creditors at less favorable terms then the original loan. Such rescheduling agreements covering short and long term debt are considered defaults even where, for legal or regulatory reasons, creditors deem forced rollover of principal to be voluntary.

In this case, we end up with a sample of 24 countries, because there is not information available for Saudi Arabia, Kuwait, Libya, Qatar, Azerbaijan, Oman, Malaysia, Australia, United Kingdom and Syrian Arab Republic. Furthermore, there are 6 countries that have not had any default episode: Canada, United Arab Republic, Kazakhstan, United States, China and India.

2.2 Stylized Facts

We perform different graphical exercises to illustrate some relationships across countries between the Institutional Investor Index (III, henceforth) and various key variables. The next figures were constructed considering only data for oil net exporters countries, and excluding those countries that have not had any default episode. Furthermore, OPEC countries are displayed in blue.

A quick look at the importance of oil prices in understanding the dynamics of this country risk index is presented in Figure 1, in which correlation coefficients greater than 0.5 are displayed in red. Each dot corresponds to one year:

Figure 1: Institutional Investor Index (X-Axis) and oil price (Real USD, Y-Axis).

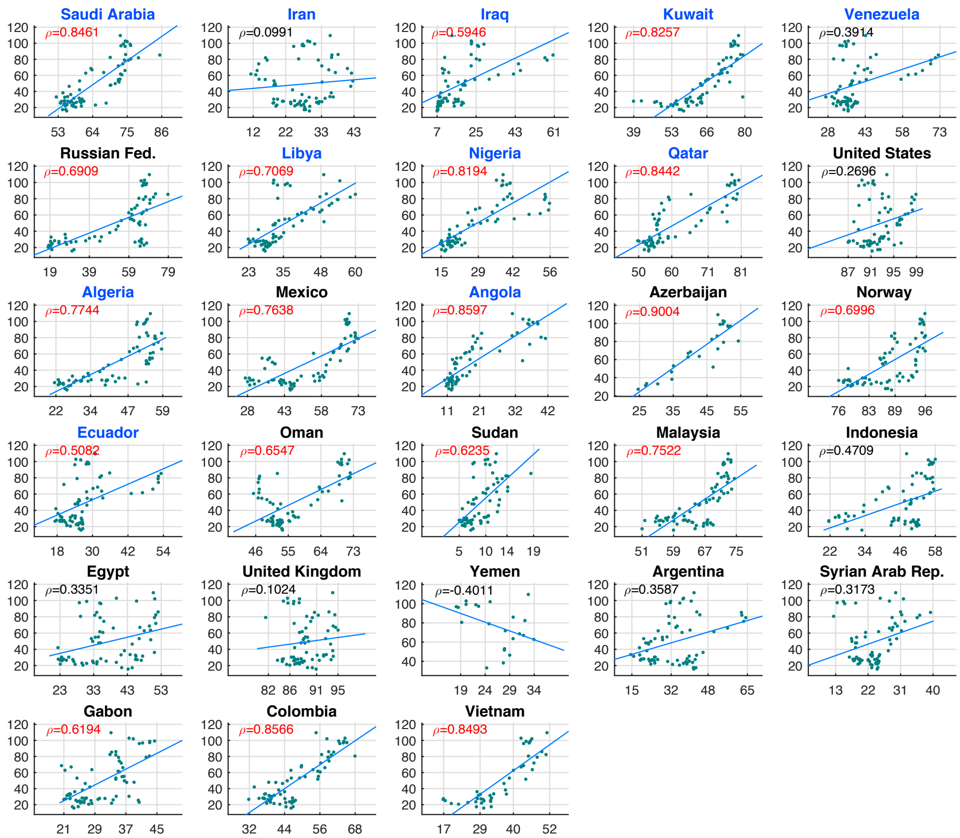
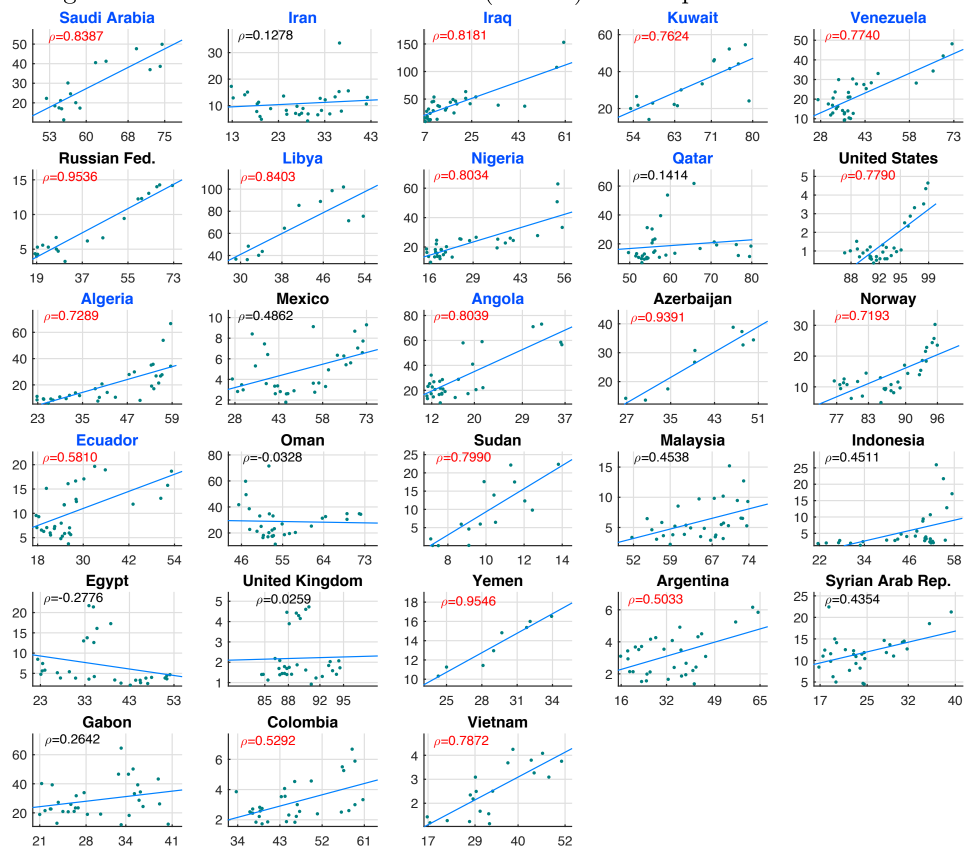


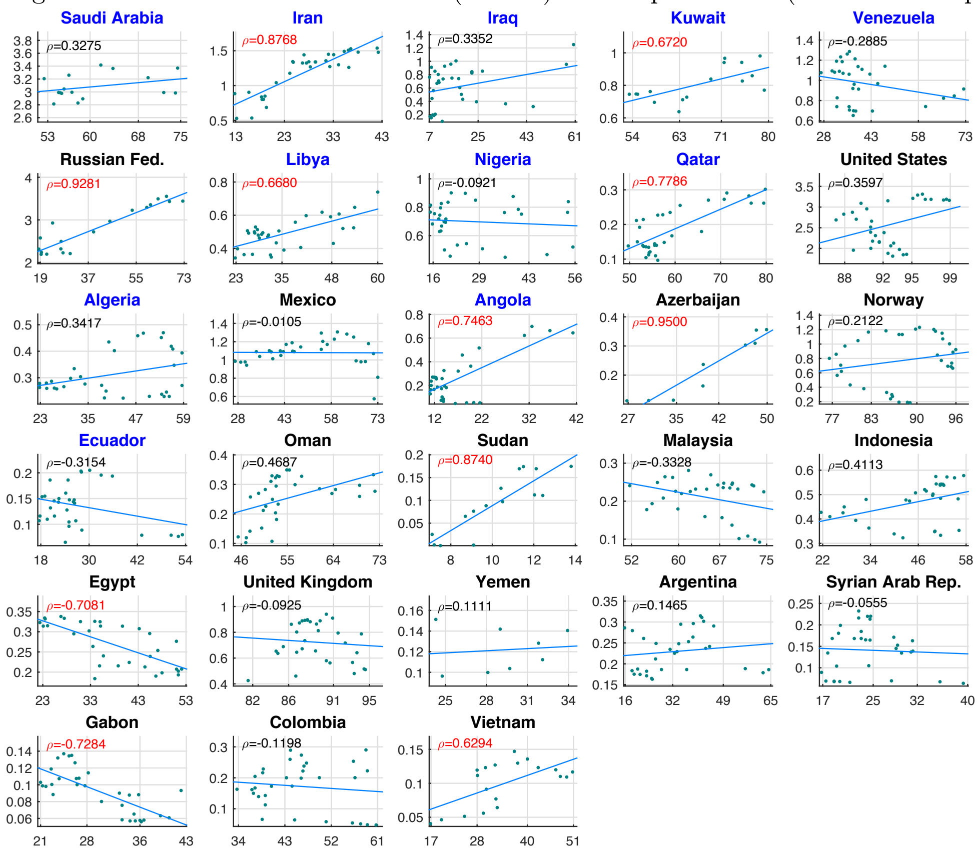
Figure 1 shows that oil prices are, in general, strongly positively correlated with the III. For 19 of 28 countries considered here, the correlation coefficient is greater than 0.5. Added to this, Figure 2 plots the relationship between this index and oil production value to GDP ratio, for each country, over the period 1979-2010 (in most cases):

Figure 2: Institutional Investor Index (X-Axis) and oil production value to GDP (%, Y-Axis).



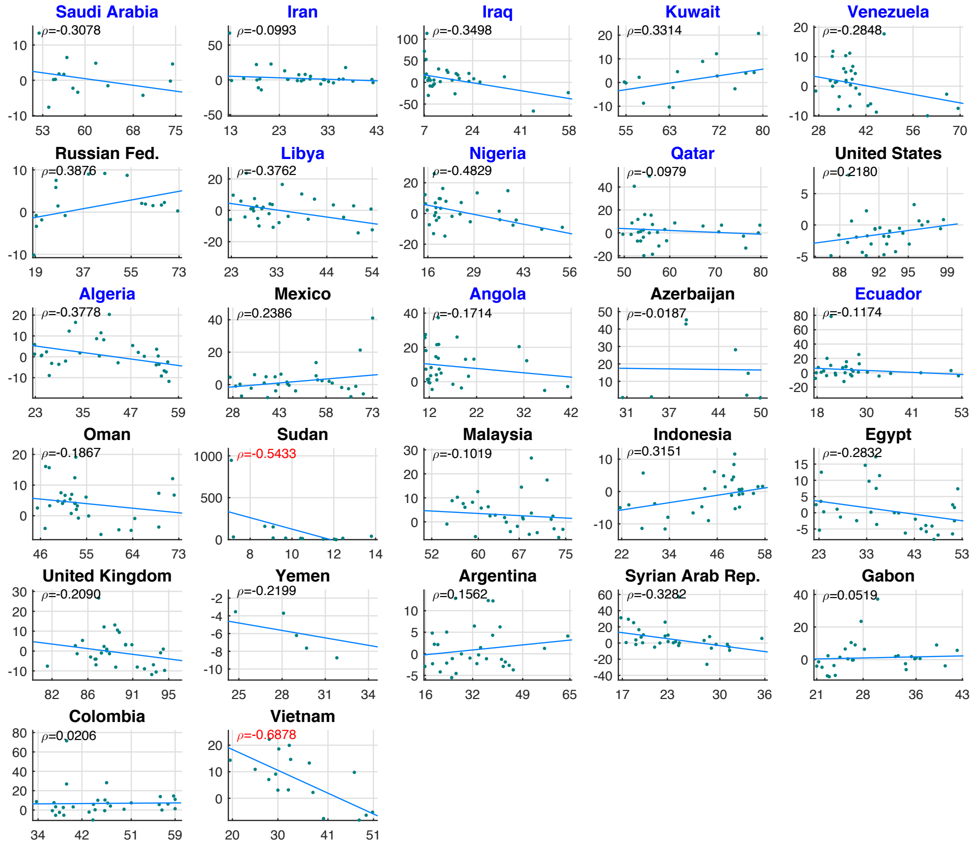
One feature stands out from Figure 2: when oil production value to GDP ratio is high, the country risk index tends to improve. Note that there are countries where the correlation is not significant, such as Iran, United Kingdom, Egypt or Gabon. Besides, Figure 3 presents the III versus oil production (in billion barrels per year):

Figure 3: Institutional Investor Index (X-Axis) and oil production (billion barrels per year, Y-Axis).



In this figure, absolute value of correlation coefficients greater than 0.5 are displayed in red. As we can see, there is not a clear pattern, since there are some countries for which the relationship is clearly positive, while for others it is negative or zero. This suggests that oil price is the "main driving force" behind changes in the country risk index (and not oil production). In Figure 4 we document the association between III and the oil production growth rate.

Figure 4: Institutional Investor Index (X-Axis) and oil production growth rate (%, Y-Axis).



In this case, correlation coefficients lower than -0.5 are displayed in red. The results point in the direction that there is not any association between these two variables, although a negative relationship is observed for Sudan and Vietnam. Additionally, Figures 5 and 6 show the importance of total public debt and total public external debt to GDP ratios, respectively:

Figure 5: Institutional Investor Index (X-Axis) and total public debt to GDP (%, Y-Axis).

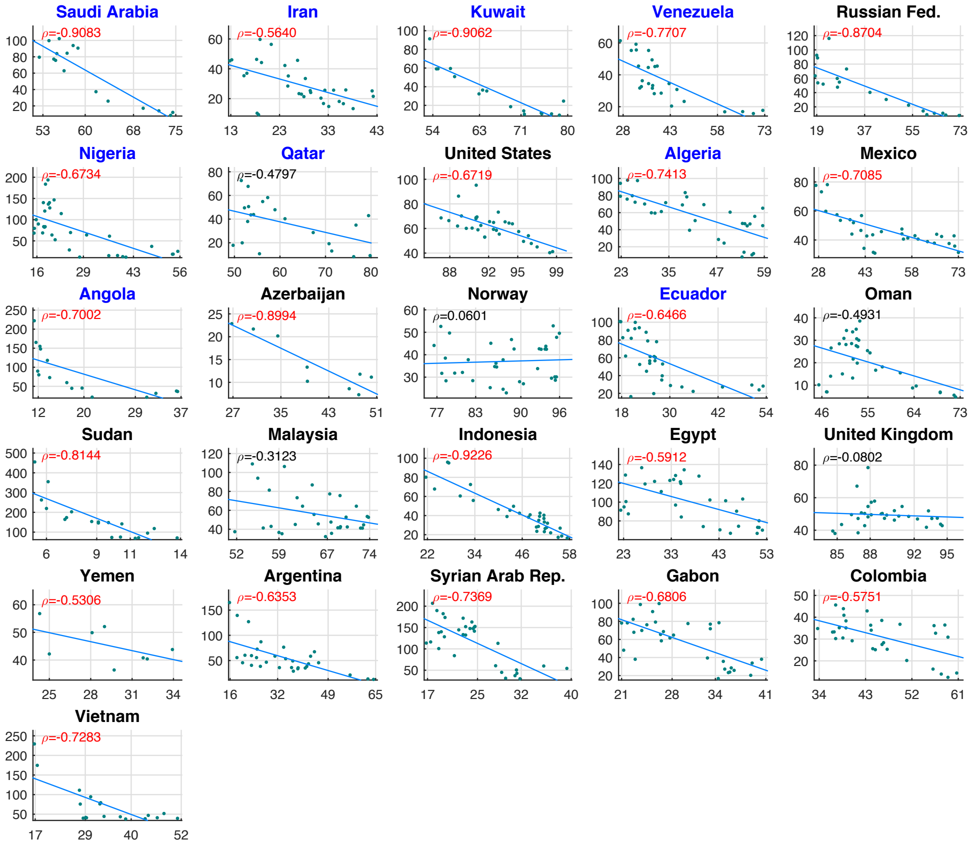
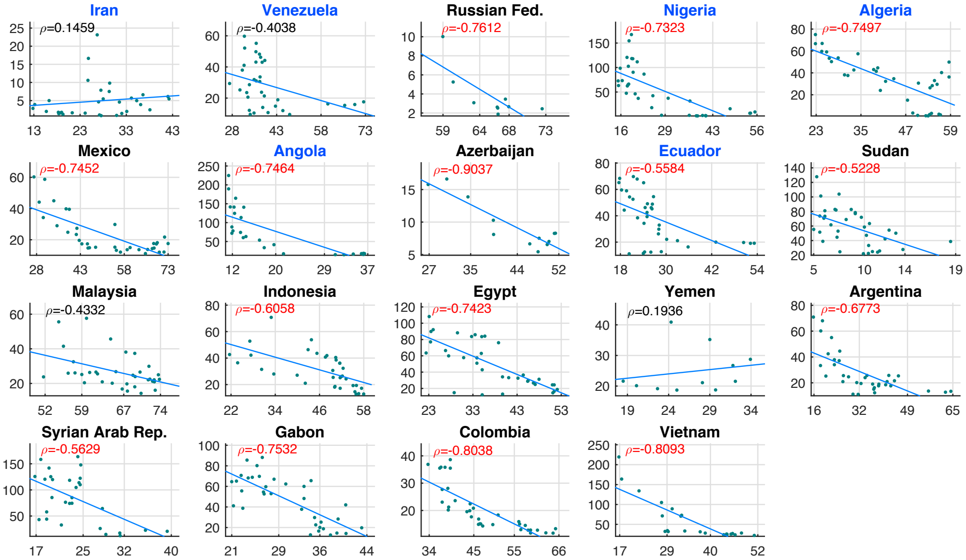
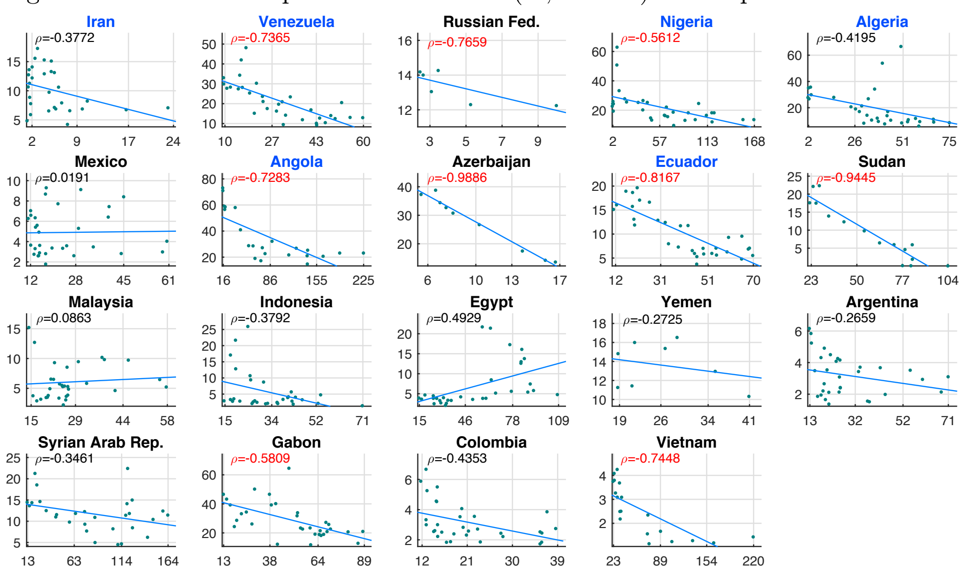


Figure 6: Institutional Investor Index (X-Axis) and total external public debt to GDP (%, Y-Axis).



Note that for most countries, correlation coefficients are displayed in red, which means that these are lower than -0.5. As we can see, III goes down when total public debt (or total public external debt) increases. Additionally, Figure 7 show the association between total external public debt to GDP ratio and oil production value to GDP ratio:

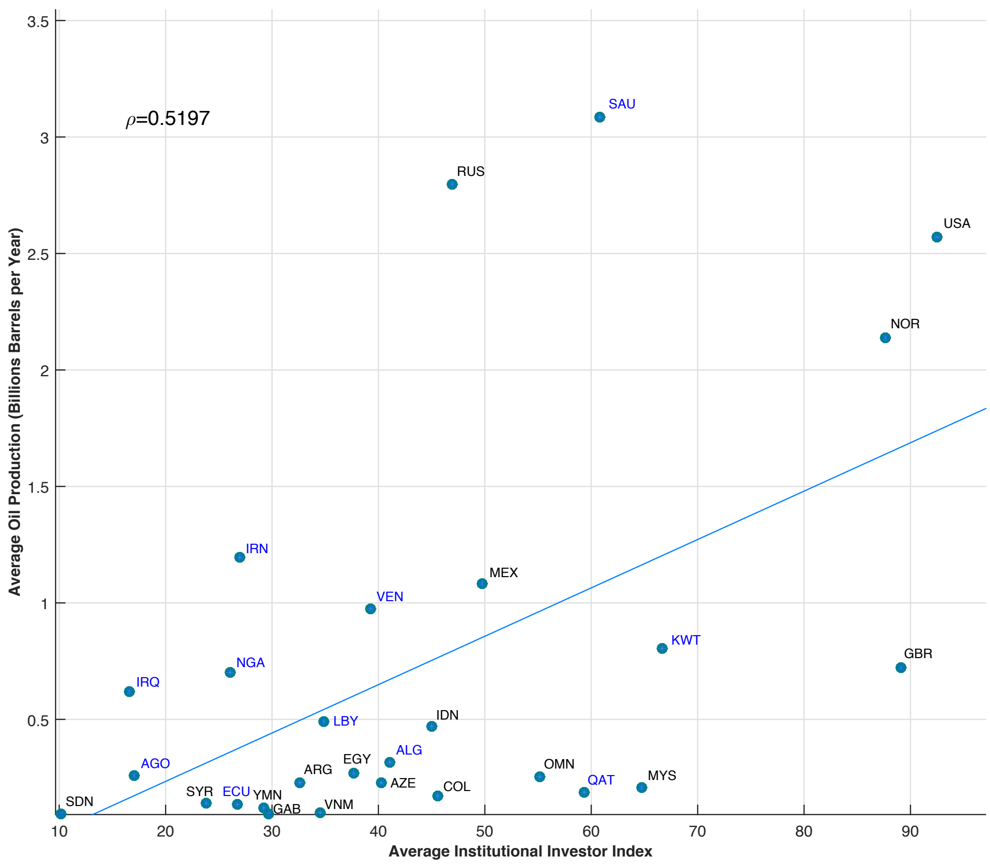
Figure 7: Total external public debt to GDP (%, X-Axis) and oil production value to GDP (%, Y-Axis).



As we can see, for 9 countries there is a negative correlation, which imply than when oil production value to GDP is high, total public external debt tends to be low. Nevertheless, such a contention is not reinforced by the rest of countries in the sample, since no significance is observed. Moreover, in the case of Egypt, the estimated coefficient shows almost a positive and statistically strong effect.

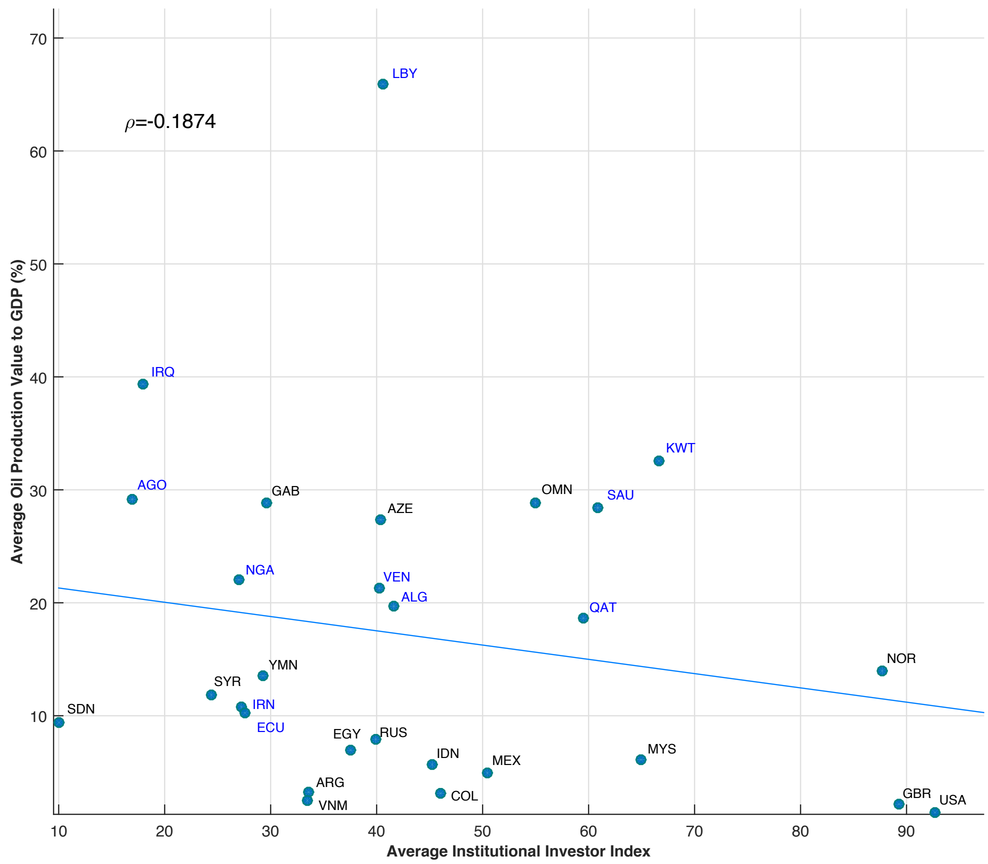
We now perform a long-run analysis between oil production and III, based on the following scatter plot:

Figure 8: Average Institutional Investor Index (X-Axis) and average oil production (Y-Axis): 1979-2010.



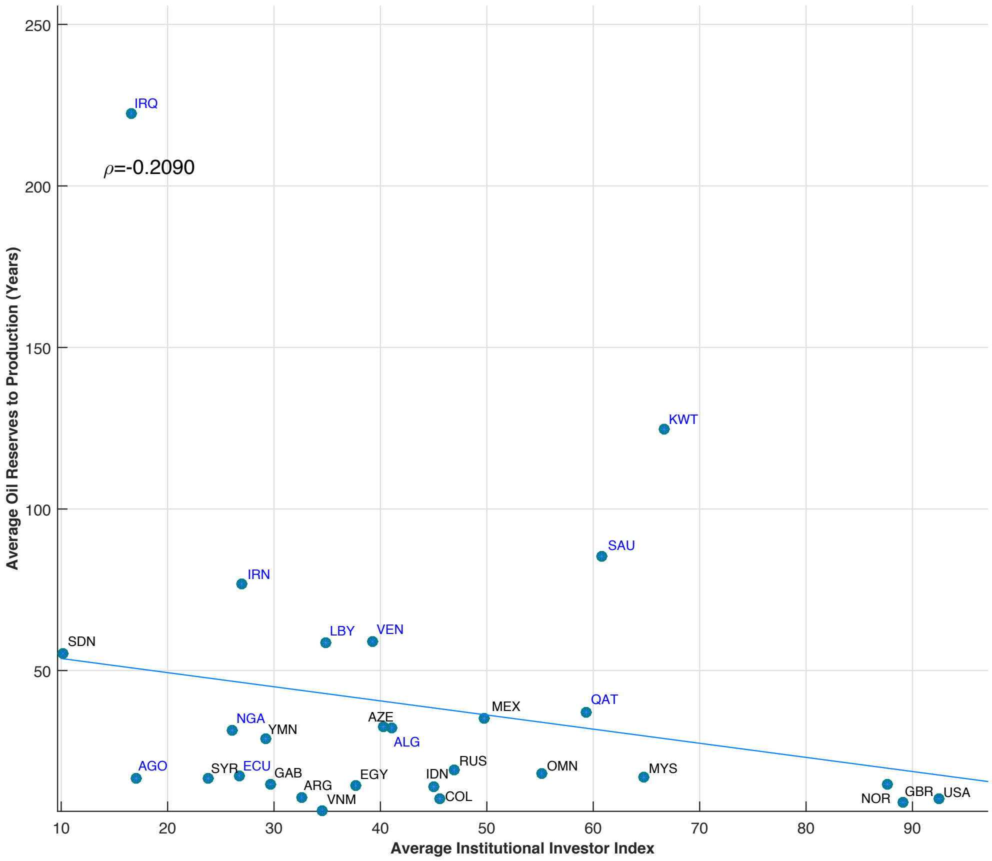
Each dot represents the average over time. Historically, countries that have maintained high oil production have been characterized by a high average credit rating. Note that the correlation coefficient is not small (0.519). For example, Norway has shown oil production levels above 2 billion barrels per year, and its average III is close to 90; something similar happens with USA, and in turn, the opposite is observed in Sudan, Angola or Syria. Moreover, Figure 9 plots the average III against average oil production value to GDP:

Figure 9: Average Institutional Investor Index (X-Axis) and average oil production value to GDP (Y-Axis): 1979-2010.



In this case, we compute a low correlation coefficient (-0.187). The negative trend indicates that countries with high oil production value to GDP over time show a high country risk (or a low average III). It is important to mention that average oil production value to GDP may be low because historical GDP is very high when compared with the historical oil production value, such as in USA or Norway. Furthermore, this negative relationship may also be driven by exceptional cases such as Libya or Iraq, which have average oil production value to GDP of about 67 and 39 percent, and average III of about 41 and 17, respectively. Added to this, in Figure 10 we show the association between average III and average reserves to production:

Figure 10: Average Institutional Investor Index (X-Axis) and average oil reserves to production (Years, Y-Axis): 1979-2010.



Considering the figure above, we obtain a correlation coefficient of about -0.2, which indicates that historically, countries with low oil reserves to production tend to have a higher average III. However, this conclusion may also be influenced by Iraq, Iran or Kuwait, which hold average oil reserves to production of about 225, 75 or 125 years, respectively, with a not so high average III.

After determining the most appropriate econometric method for the sample, we focus on estimation results. Table 1 reports the outcome of DFE estimation of the long-run, short-run and convergence coefficients. Due to data limitations, in Model (1) Yemen was excluded from the sample. In Model (2) Irak, Lybia, Yemen and Azerbaijan were excluded, and in Model (3) Irak, Russian Federation, Yemen and Azerbaijan. Consequently, the estimation is performed taking into account 887, 785 and 828 observations, respectively.

A quick inspection of these results leads us to the observation that in each model the convergence coefficient has the expected sign and is statistically significant at the 1% level. Since the estimated coefficient takes, on average, the value of -0.2, convergence in III runs at an annual rate of about 0.2%, which means that each year III covers about 0.2% of its distance from the "steady state". It should

Table 1: Dynamic Fixed Effects Regression Results for Institutional Investor Index

	Δ Inst. Investor Index		
	Model (1)	Model (2)	Model (3)
Convergence coefficient			
Inst. Investor Index (-1)	-0.218***	-0.207***	-0.196***
	(0.017)	(0.019)	(0.017)
Short-run coefficients			
Δ Oil GDP	0.012	0.020	0.012
	(0.012)	(0.012)	(0.011)
Δ Non-oil GDP	0.051**	0.270***	0.062***
	(0.025)	(0.053)	(0.024)
Δ Oil reserves	0.028*	0.035**	0.029*
	(0.016)	(0.016)	(0.015)
Δ Tot. pub. debt		-0.085***	
		(0.016)	
Δ NFA			0.005
			(0.010)
Long-run coefficients			
Oil GDP	0.283***	0.136***	0.245***
	(0.029)	(0.034)	(0.032)
Non-oil GDP	0.165***	0.270***	0.220***
	(0.047)	(0.053)	(0.050)
Oil reserves	-0.071**	-0.069**	-0.054
	(0.035)	(0.034)	(0.036)
Tot. pub. debt		-0.196***	
		(0.036)	
NFA			0.067***
			(0.022)
Constant	-0.180	-0.002	-0.225**
	(0.115)	(0.128)	(0.110)
Observations	887	785	828

Standard errors in parentheses
*** $p<0.01$, ** $p<0.05$, * $p<0.1$

Note: Due to data limitations, in Model (1) Yemen was excluded from the sample. In Model (2), Irak, Lybia, Yemen and Azerbaijan were excluded, and in Model (3) Irak, Russian Federation, Yemen and Azerbaijan.

also be noted that convergence is slightly slower in Model (3), when considering net foreign assets-to-GDP ratio as a factor that explains country risk. Furthermore, focusing on short-term dynamics, we observe that oil GDP coefficient is positively signed in the 3 models, although not significant at the conventional significance levels. Nonetheless, the opposite happens with non-oil GDP, because its effect on the dynamics of III is robust to the 3 specifications, and strongly relevant. In general, an increase in non-oil GDP tends to improve III in the short-run.

Additionally, there is statistical evidence that suggests that when oil reserves grow the country improves its risk indicator in the short run. Notice that the estimated coefficient is always significant, and its sign does not change through models. Now, Model (2) regression results show that increases in total public debt result in a worsening of the risk rating. This coefficient is especially statistically relevant given that its value is 5 times its estimated standard error. On the other hand, considering Model (3), the outcome of the estimation allows us to conclude that there is insufficient statistical evidence to show that changes in NFA affect III behavior in the short term, since its standard error is twice the estimated coefficient. At the first sight, this result reflects a lack of a short-run relationship between NFA and III, when controlling for oil reserves.

Now, taking into account long-run coefficients we highlight the following facts. First, we obtain 1% significance in most cases. Second, as shown in Pesaran *et al.* (1999), the usual interpretation given to these coefficients, when series are in logs, is that of an elasticity. Then, the long-run oil GDP elasticity is 0.28 in the first model, 0.13 in the second, and 0.24 in the third, which means that when oil GDP increases by 1%, III is 0.2% higher in the long-run. With respect to non-oil GDP, long-run elasticities are positive, and significantly different from zero; other things equal, country risk rating rises on average 2.2% because of a 10% increment in non-oil GDP. Therefore, we can see that III improves in long-run not only due to greater oil GDP, but also because of increases in non-oil GDP.

Moreover, a significant negative relationship between oil reserves and III was detected, and it is maintained when we incorporate total public debt as a regressor. Contrary to what we find in the short term, it is observed that a rise in oil reserves worsen our measure of country risk in the long term. Thus, an oil exporting economy is perceived as more risky in the future when it boost its reserves today. However, when we include NFA (as in Model (3)) oil reserves elasticity is no longer statistically different from zero, although it remains negative; apparently, its relevance is now captured by NFA. Consider now the estimated parameter of total public debt. A larger level of the government contractual obligations translates into a lower III. In particular, when these obligations increase by 10%, III deteriorates by an amount of about 2% in the long-run. Again, this finding is statistically strong in the sense that the estimated value is 5.4 times its standard error.

3 Two period model

The model consists of a small open economy who's sovereign owns a stock of oil that seeks to exploit by delegating extracting activities to an oil producing company. The sovereign can trade non-state contingent bonds with risk neutral competitive foreign lenders in international financial markets. We start by describing the lenders problem, then the oil company's one and end with the sovereign's problem.

International financial markets are competitive and populated by risk-neutral lenders. These lenders can trade one-period zero-coupon risk-free bonds at a price $\bar{q} \in [0,1]$ and one-period zero-coupon risky sovereign bonds at the price q. Sovereign bonds are risky because the sovereign can default with probability $\delta \in [0,1]$. Thus the no-arbitrage condition is:

$$q = \bar{q}\left(1-\delta\right). \tag{1}$$

As we will see later, the probability of default (and therefore the price of the sovereign bonds) is endogenous and will be obtained as an equilibrium outcome. The price of the sovereign bonds $q \in [0,1]$ will also be determined in equilibrium.

There are two periods: $t = 1, 2$, and there are two types of goods: a tradable non-storable consumption good and oil. There are y_t units of the consumption good available in fixed supply every period, while oil must be extracted at a cost. In any given period, the relative price of oil is p_t units of the consumption good per unit of oil. Oil prices are set in a competitive international oil market. We assume p_1 is known while p_2 is drawn from a known distribution.

The oil producing company decides how much oil to extract optimally from the available stock of proven oil reserves, considering the present and future extraction costs as well as the international oil price pattern and transfers the profits from operation to the sovereign.

The economy is endowed with an initial stock of oil reserves, $s_0 > 0$. Oil is in the ground and the oil company can extract $x_t \geq 0$ units of oil every period. The total cost of extraction during period t is a function of the stock of reserves at the beginning of every period and the amount extracted during the period. It is conventional in the natural resource economics literature to assume that the extraction cost function, e, has the following properties: $e_x > 0$, $e_{xx} \geq 0$, $e_s \leq 0$, $e_{xs} \leq 0$ and $e\left(0,s\right) = 0$. For simplicity, we assume that there are no new oil discoveries and that oil reserves will be extracted in either of the two periods, $x_1 + x_2 = s_0$. Note that under these assumptions the cost function of the second period, depends only on x_2.

The problem of the oil company is to choose x_1 and x_2 to maximize:

$$\max_{x_1 \geq 0, x_2 \geq 0} p_1 x_1 - e\left(x_1, s_0\right) + q E_{p_2}\left[p_2 x_2 - e\left(x_2\right)\right]$$

subject to

$$x_1 + x_2 = s_0,$$

$$x_1 \geq 0,$$

$$x_2 \geq 0.$$

Let $x_1^*\left(p1, s_0; q\left(B_1\right)\right)$ and $x_2^*\left(p1, s_0; q\left(B_1\right)\right)$ be the optimal extraction policies. Note that we have stressed the dependency of the optimal policies on p_1, s_0 and B_1.

The sovereign takes as given the optimal extraction policies x_1^* and x_2^* which imply a pair of optimal profits π_1^* and π_2^*.

As mentioned earlier, in period 1 there is no uncertainty. In period 2, however, the timing of events is as follows: at the beginning of the second period p_2 is drawn from a known probability density h over the random variable p_2 with support $\mathcal{P}$. Then the sovereign makes the decision to default or not after x_2^* is observed. If the government defaults, it does not pay back debt B_1 but at the cost of paying a trade penalty ϕ on oil exports. The default decision is thus:

$$d = \begin{cases} 1 & \text{if } u\left(y_2 + \phi p_2 x_2^* - e\left(x_2^*\right)\right) \geq u\left(y_2 + p_2 x_2^* - e\left(x_2^*\right) + B_1\right) \\ 0 & \text{otherwise} \end{cases}$$

Thus, for a given level of B_1, there is a threshold oil price in period 2, $\tilde{p_2}$, such that oil prices below this level will induce the sovereign to default. This threshold satisfies the condition:

$$\tilde{p_2} x_2^*\left(p_1, \tilde{p_2}, s_0; q\right) = -\frac{B_1}{1-\phi}.$$

Note that the future oil price threshold depends on debt, the default cost, the current level of prices and the stock of oil reserves.

The default set is then defined by those levels of debt for which the sovereign finds convenient to default,

$$\mathcal{D}\left(B_1, p_1, s_0\right) = \left\{p_2 \in \mathcal{P}_2 : d\left(B_1; p_1, p_2, s_0\right) = 1\right\}$$

and the probability of default is:

$$\delta\left(B_1, p_1, s_0\right) = \int_0^{\tilde{p_2}} d\left(B_1; p_1, p_2, s_0\right) h\left(p_2\right) dp_2.$$

The optimal choice of B_1 is then:

$$\begin{aligned} \max_{B_1} \quad & u\left(y_1 + p_1 x_1^* - e\left(x_1^*\right) - q\left(B_1, p_1, s_0\right) B_1\right) \\ & +\beta \left\{ \delta\left(B_1, p_1, s_0\right) \int_0^{\tilde{p_2}\left(B_1, p_1, s_0\right)} u\left(y_2 + \phi p_2 x_2^* - e\left(x_2^*\right)\right) h(p_2) dp_2 \right. \\ & \left. +\beta \left[1 - \delta\left(B_1, p_1, s_0\right)\right] \int_{\tilde{p_2}\left(B_1, p_1, s_0\right)}^{\infty} u\left(y_2 + p_2 x_2^* - e\left(x_2^*\right) + B_1\right) h(p_2) dp_2 \right\}. \end{aligned} \tag{2}$$

3.1 Equilibrium Example with log utility and quadratic costs

In this subsection we assume some specific functional forms for the utility function and costs of extraction, and solve for the equilibrium of the model.

Consider the problem of an oil company that faces quadratic costs of extraction: $e\left(x_t, s_{t-1}\right) = \kappa \frac{x_t^2}{s_{t-1}}$. This cost function satisfies the conditions stated above: $e_x > 0$, $e_{xx} \geq 0$, $e_s \leq 0$, $e_{xs} \leq 0$ and $e\left(0, s\right) = 0$. Note that with this function: $e\left(x_1, s_0\right) = \kappa \frac{x_1^2}{s_0}$ and $e\left(x_2, s_1\right) = \kappa x_2$, because $s_1 = s_0 - x_1 =$

x_2. p_2 is a random variable distributed uniformly over the interval $\left[0, p_2^H\right]$. The density function is:

$$h(p_2) = \begin{cases} \frac{1}{p_2^H} & \text{if } 0 < p_2 \leq p_2^H \\ 0 & \text{otherwise} \end{cases}$$

and its cdf is:

$$H(\omega) = \begin{cases} 0 & \text{for } \omega \leq 0 \\ \frac{\omega}{p_2^H} & \text{for } 0 < \omega \leq p_2^H \\ 1 & \text{for } \omega > p_2^H \end{cases}$$

The problem of the oil company is to choose x_1 and x_2 such that:

$$\max_{x_1 \geq 0, x_2 \geq 0} p_1 x_1 - \kappa \frac{x_1^2}{s_0} + q\left[\frac{p_2^H}{2} x_2 - \kappa x_2\right].$$

subject to

$$x_1 + x_2 = s_0,$$

$$x_1 \geq 0,$$

$$x_2 \geq 0.$$

The Lagrangian is

$$\mathcal{L} = p_1 x_1 - \kappa \frac{x_1^2}{s_0} + q\left[\frac{p_2^H}{2} x_2 - \kappa x_2\right] + \mu_0 \left[s_0 - x_1 - x_2\right] + \mu_1 x_1 + \mu_2 x_2$$

where the μ's are the Lagrange multipliers.

The first order conditions are given by

$$\begin{aligned} p_1 - \frac{2\kappa}{s_0} x_1 + \mu_1 &= q\left\{\frac{p_2^H}{2} - \kappa\right\} + \mu_2 \\ x_1 + x_2 &= s_0 \\ \mu_1 x_1 = 0; &\quad x_1 \geq 0 \\ \mu_2 x_2 = 0; &\quad x_2 \geq 0. \end{aligned}$$

Using the optimality conditions from above one can solve for the optimal extraction policies. Assuming an interior solution these are given by:

$$
\begin{aligned}
x_1^\star &= \max\left\{\min\left[\frac{p_1 - q\left(\frac{p_2^H}{2} - \kappa\right)}{2\kappa} s_0, s_0\right], 0\right\} \\
x_2^\star &= \max\left\{\min\left[\left(1 - \frac{p_1 - q\left(\frac{p_2^H}{2} - \kappa\right)}{2\kappa}\right) s_0, s_0\right], 0\right\}.
\end{aligned}
$$

Note that for $x_t \in (0, s_0)$, today's optimal extraction, x_1^*, is increasing in today's price, p_1, and the stock of available oil reserves, s_0, but decreasing in future prices, p_2 and the parameter that determines marginal extraction cost, κ. If $p_2 > \kappa$, then x_1^* is decreasing sovereign bond prices, q, or equivalently, increasing in interest rates. In addition, there is a level of today's price $\hat{p}_1$ (keeping p_2, q and κ constant) such that at prices higher than this level it is optimal to extract all the available oil reserves in period 1:

$$\hat{p}_1 = q\left(\frac{p_2^H}{2} - \kappa\right) + 2\kappa.$$

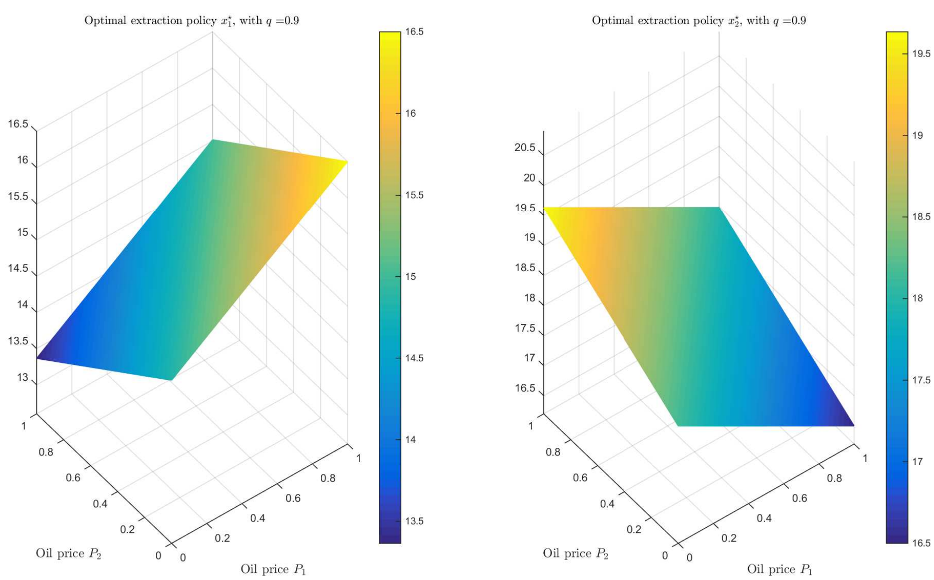
For $x_t \in (0, s_0)$, future optimal extraction, x_2^*, is increasing in future prices, p_2, and o_0, but decreasing in p_1 and increasing in κ. Similarly, there is a level of the future expected price $E\hat{p}_2$ (keeping p_1, q and κ constant) such that it at prices higher than this level is optimal to extract all the available oil reserves in period 2:

$$E\hat{p}_2 = \frac{p_1}{q} + \kappa.$$

If future prices are higher than the future marginal cost, that is $p_2 > \kappa$, then x_1^* current oil extraction is decreasing on sovereign bond prices, q (or equivalently, increasing in interest rates) and x_2^* is increasing sovereign bond prices, q (or equivalently, decreasing in interest rates).

Figure 11 illustrates the optimal extraction policies for a given q.

Figure 11: Optimal extraction policies



1

Note that if $p_2 > \kappa$, an exogenous fall in q (as a result of higher foreign interest rates, for example) would increase extraction today and reduce it in the future.

Note that given the optimal extraction policies, the optimal profits for $x_t \in (0, s_0)$ are:

$$\pi_1^* = p_1 \frac{[p_1 - q(p_2 - \kappa)]}{2\kappa} s_0 - \kappa \left(\frac{[p_1 - q(p_2 - \kappa)]}{2\kappa} \right)^2 s_0,$$

$$\pi_2^* = (p_2 - \kappa) \left(1 - \frac{[p_1 - q(p_2 - \kappa)]}{2\kappa} \right) s_0.$$

The oil price pattern affects the profile of the oil company's profits. For $x_t \in (0, s_0)$, it can be shown that π_1^* is increasing in p_1 and s_0, but decreasing in q and p_2. π_2^* is increasing in p_2, q and s_0, but decreasing in p_1. The relationship between π_1^* and π_2^* with κ is uncertain. However, if $\kappa > p_2$, π_1^* decreases and π_2^* increases as κ grows.

We denote by $\tilde{p}_2(B_1, o_0, p_1)$ the threshold price below which the sovereign will default, and it is give by:

$$\tilde{p_2} x_2^* (p_1, \tilde{p_2}, s_0; q) = -\frac{B_1}{1 - \phi}.$$

Substituting for the optimal extraction policy in the second period,

$$\begin{aligned}
\tilde{p}_2 &= \frac{-\frac{B_1}{(1-\phi)s_0}}{\left(1-\frac{p_1-q(Ep_2-\kappa)}{2\kappa}\right)} \\
&= \frac{-\frac{B_1}{(1-\phi)s_0}}{\left(\frac{2\kappa-[p_1-q(Ep_2-\kappa)]}{2\kappa}\right)} \\
&= \frac{-\frac{2\kappa B_1}{(1-\phi)s_0}}{2\kappa-[p_1-q\left(Ep_2-\kappa\right)]} \\
&= \frac{-\frac{2\kappa B_1}{(1-\phi)s_0}}{q\left(Ep_2-\kappa\right)-(p_1-2\kappa)}.
\end{aligned}$$

Let H be the cdf of p_2 and compute the probability of default $H\left(\tilde{p}_2\left(B_1,s_0,p_1\right)\right)$, which in the case of the uniform distribution of future oil prices is:

$$\delta=\tilde{H}=\frac{\tilde{p}_2}{p_2^H}=\frac{-\frac{2\kappa B_1}{p_2^H[(1-\phi)s_0]}}{q\left(\frac{p_2^H}{2}-\kappa\right)-(p_1-2\kappa)}.$$

Since in equilibrium the price of sovereign bonds must satisfy the arbitrage equation, $q=\bar{q}\left(1-\tilde{H}\right)$ then,

$$\begin{aligned}
q &= \bar{q}\left(1-\frac{-\frac{2\kappa B_1}{p_2^H[(1-\phi)o_0]}}{q\left(\frac{p_2^H}{2}-\kappa\right)-(p_1-2\kappa)}\right) \\
&= \bar{q}\left(1+\frac{\frac{2\kappa B_1}{p_2^H[(1-\phi)s_0]}}{q\left(\frac{p_2^H}{2}-\kappa\right)-(p_1-2\kappa)}\right).
\end{aligned}$$

Let $\varphi=\frac{2\kappa B_1}{p_2^H[(1-\phi)s_0]}$ then:

$$\begin{aligned}
q &= \bar{q}\left(1+\frac{\varphi}{q\left(\frac{p_2^H}{2}-\kappa\right)-(p_1-2\kappa)}\right) \\
&= \bar{q}\left(\frac{q\left(\frac{p_2^H}{2}-\kappa\right)-(p_1-2\kappa)+\varphi}{q\left(\frac{p_2^H}{2}-\kappa\right)-(p_1-2\kappa)}\right).
\end{aligned}$$

Solving for q

$$q\left(q\left(\frac{p_2^H}{2}-\kappa\right)-(p_1-2\kappa)\right)-\bar{q}\left(q\left(\frac{p_2^H}{2}-\kappa\right)+(p_1-2\kappa)+\varphi\right)=0$$

or equivalently:

$$\underbrace{\left[\frac{p_2^H}{2}-\kappa\right]}_{a}q^2\underbrace{-\left[(p_1-2\kappa)+\bar{q}\left(\frac{p_2^H}{2}-\kappa\right)\right]}_{b}q+\underbrace{\left[\bar{q}\left(p_1-2\kappa\right)-\bar{q}\varphi\right]}_{c}=0$$

This is a quadratic equation in q and the solution is:

$$\begin{aligned} q_i &= \frac{-b\pm\sqrt{b^2-4ac}}{2a} \\ &= \frac{\left[p_1-2\kappa+\bar{q}\left(\frac{p_2^H}{2}-\kappa\right)\right]\pm\sqrt{\left(-\left[(p_1-2\kappa)+\bar{q}\left(\frac{p_2^H}{2}-\kappa\right)\right]\right)^2-4\left[\frac{p_2^H}{2}-\kappa\right]\left[\bar{q}\left(p_1-2\kappa\right)-\bar{q}\varphi\right]}}{2\left[\frac{p_2^H}{2}-\kappa\right]}. \end{aligned}$$

For a solution to exist we need at least one positive root: $\frac{-b+\sqrt{b^2-4ac}}{2a}>0$. Lets assume for now that $a>0$. In this case for there to be a positive root, we need $-b+\sqrt{b^2-4ac}>0$:

$$(p_1-2\kappa)+\bar{q}\left(\frac{p_2^H}{2}-\kappa\right)+\sqrt{\left(-\left[(p_1-2\kappa)+\bar{q}\left(\frac{p_2^H}{2}-\kappa\right)\right]\right)^2-4\left[\frac{p_2^H}{2}-\kappa\right]\bar{q}\left(p_1-2\kappa-\varphi\right)}>0$$

$$\left(-\left[(p_1-2\kappa)+\bar{q}\left(\frac{p_2^H}{2}-\kappa\right)\right]\right)^2<\left(-\left[(p_1-2\kappa)+\bar{q}\left(\frac{p_2^H}{2}-\kappa\right)\right]\right)^2-4\left[\frac{p_2^H}{2}-\kappa\right]\bar{q}\left(p_1-2\kappa-\varphi\right)$$

$$0<-4\left[\frac{p_2^H}{2}-\kappa\right]\bar{q}\left(p_1-2\kappa-\varphi\right)$$

$$0>4\left[\frac{p_2^H}{2}-\kappa\right]\bar{q}\left(p_1-2\kappa-\varphi\right). \tag{3}$$

Equation 3 will be true in two cases:

1. If $\frac{p_2^H}{2}-\kappa<0$ & $p_1-2\kappa-\varphi>0$, or
2. If $\frac{p_2^H}{2}-\kappa>0$ & $p_1-2\kappa-\varphi B_1<0$.

Case (1):

$$\frac{p_2^H}{2}<\kappa, \text{ can be impossed} \tag{4}$$

$$p_1-2\kappa>\varphi\Longrightarrow p_1-2\kappa>\frac{2\kappa B_1}{p_2^H\left[(1-\phi)\,s_0\right]}. \tag{5}$$

Because $B_1<0$, condition 5 is always true as long as $p_1>2\kappa$. However if 4 holds, $a<0$, which is a contradiction because this would imply $\frac{-b+\sqrt{b^2-4ac}}{2a}<0$.
Case (2):

$$\frac{p_2^H}{2}>\kappa,\ can\, be\, impossed \tag{6}$$

$$p_1 - 2\kappa < \varphi \Longrightarrow 2\kappa - p_1 > \frac{-2\kappa B_1}{p_2^H \left[(1-\phi)\, s_0\right]}$$

$$1 - \frac{p_1}{2\kappa} > \frac{-B_1}{p_2^H \left[(1-\phi)\, s_0\right]}. \tag{7}$$

If 6 and 7 are satisfied then there is at least one internal solution.

For this solution to be unique we need $\frac{-b-\sqrt{b^2-4ac}}{2a} < 0$, and from existence we know that $\frac{p_2^H}{2} - \kappa > 0$, $p_1 - 2\kappa - \varphi < 0$ and $a > 0$ which implies that we need:

$$-b - \sqrt{b^2 - 4ac} < 0.$$

Substituting in,

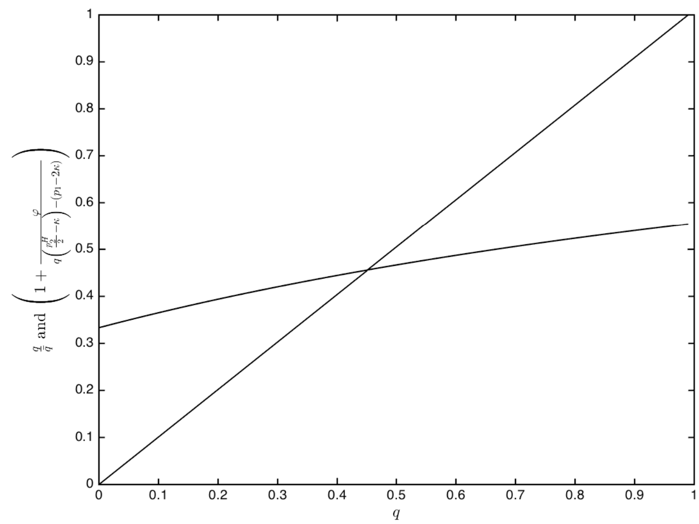
$$(p_1 - 2\kappa) + \bar{q}\left(\frac{p_2^H}{2} - \kappa\right) < \sqrt{\left(-\left[(p_1 - 2\kappa) + \bar{q}\left(\frac{p_2^H}{2} - \kappa\right)\right]\right)^2 - 4\left[\frac{p_2^H}{2} - \kappa\right]\bar{q}\,(p_1 - 2\kappa - \varphi)}$$

$$\left[(p_1 - 2\kappa) + \bar{q}\left(\frac{p_2^H}{2} - \kappa\right)\right]^2 < \left(-\left[(p_1 - 2\kappa) + \bar{q}\left(\frac{p_2^H}{2} - \kappa\right)\right]\right)^2 - 4\left[\frac{p_2^H}{2} - \kappa\right]\bar{q}\,(p_1 - 2\kappa - \varphi)$$

$$0 > 4\left[\frac{p_2^H}{2} - \kappa\right]\bar{q}\,(p_1 - 2\kappa - \varphi)\,. \tag{8}$$

Equation 8 has to be satisfied for there to be a unique solution. We know that it is satisfied from the existence conditions, so there is an internal solution that is unique. Figure shows this result graphically:

Figure 12: Bond price implied by the arbitrage condition



3.2 Comparative statics

Figure 13: Debt and Risk Premia

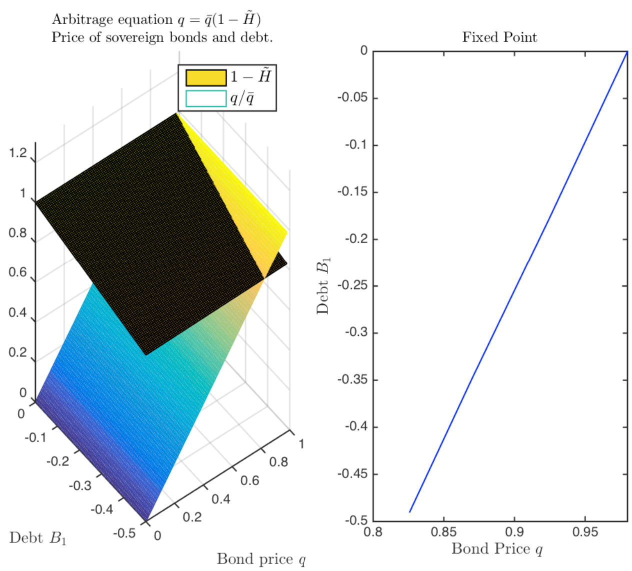


Figure 14: Oil Reserves and Risk Premia

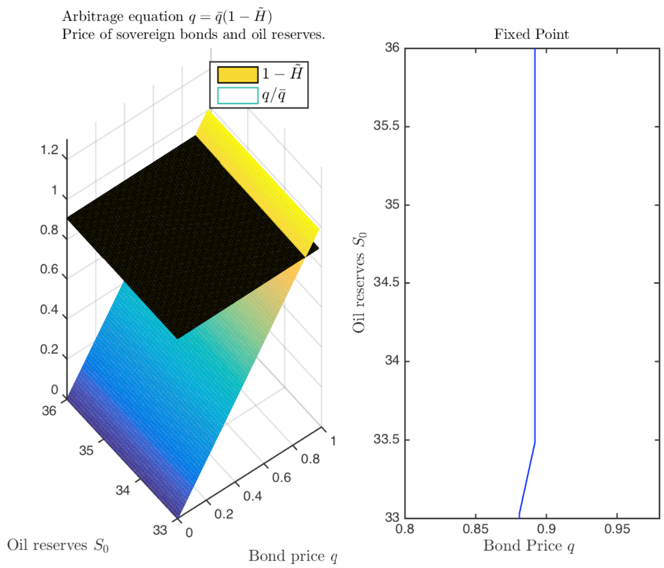


Figure 15: First Period Price and Risk Premia

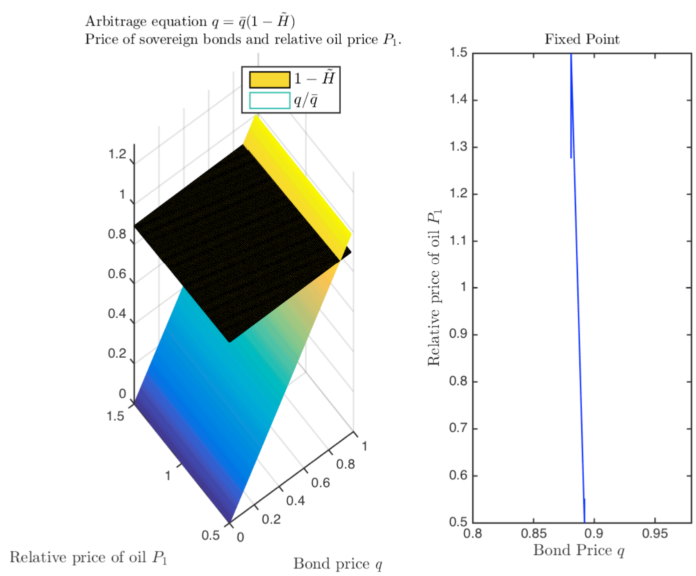
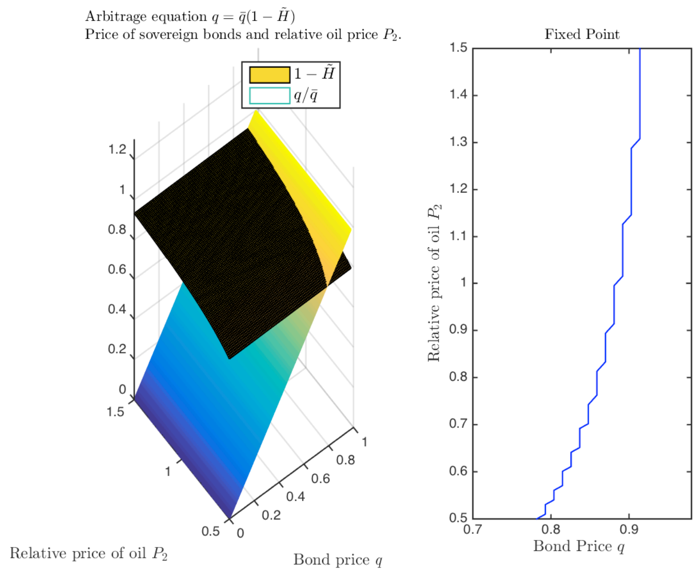


Figure 16: Second Period Price and Risk Premia



4 Dynamic model

The model consists of a small open economy that has two types of goods: a tradable non-storable consumption good and oil. Time is discrete and infinite and it is denoted by $t = 1, 2, \ldots, \infty$. There are y_t units of the consumption good available in fixed supply every period t, while oil must be extracted at a cost. In any given period, the relative price of oil is p_t units of the consumption good per unit of oil. This price is stochastic and set in the international oil market.

There is a sovereign government that owns and operates an oil producing company and it decides how much oil to extract optimally from the available stock of proven oil reserves, considering the present and future extraction costs as well as the random pattern of prices and a fixed discovery rate.

The sovereign government receives an income flow y_t and the revenue generated by extracting oil

$p_t x_t$. As both the endowment and oil revenue vary randomly, uncertain shifts in earnings can be mitigated by borrowing in international financial markets by issuing one-period, non-state contingent discount bonds b_{t+1} to be repaid in period $t+1$—markets of contingent claims are incomplete—and the sovereign cannot commit to repaying its debt. When the country borrows $b_{t+1} < 0$. The set of bond face values is $B = [b_{min}, b_{max}] \subset R$ where $b_{min} \leq b_{max} = 0$.

In period t there is a stock of oil $o_t \in O = \{0, 1, 2, \ldots, \bar{o}\}$, where $\bar{o}$ denotes the maximum number of units of oil reserves in the ground. Let n_o be the number of elements in the set O. The sovereign extracts x_t units to be exported and sold in the international oil market at the international oil price p_t. Every period an amount k of oil is discovered, such that oil reserves evolve according to:

$$o_{t+1} = o_t - x_t + k, \tag{9}$$

and $x_t \in X = \{0, 1, 2, \ldots, o\}$. The total cost of extracting x_t units of oil, given that there are o_t, is denoted by $\chi(o_t, x_t)$. The total cost function is expressed in units of the consumption good and has the following properties: $\chi_o < 0$, $\chi_x > 0$ and $\chi_o(o, 0) = 0$. Total extraction costs fall with oil reserves but increase with the extraction rate. Also, the marginal cost of an additional unit of reserves, conditional on not extracting oil is zero. We assume that $\chi(o_t, x_t)$ is given by:

$$\chi(o_t, x_t) = \kappa \frac{x_t}{(1 + o_t)} \tag{10}$$

where κ is a parameter that determines the total cost elasticity to changes in the rate of extraction.

As mentioned earlier the sovereign cannot commit to repay its debt. As in the Eaton-Gersovitz model, when the country defaults it does not repay at date t and the punishment is exclusion from world credit markets in the same period. Next period the country can re-enter world financial markets with a probability ϕ.

The planner's payoff is given by:

$$V(b_t, o_t, y_t, p_t) = \max \left\{ v^{nd}(b_t, o_t, y_t, p_t), v^d(o_t, y_t, p_t) \right\},$$

where $v^{nd}(b_t, o_t, y_t, p_t)$ is the value of no-default and $v^d(o_t, y_t, p_t)$ is the value of default.

The value of no-default solves the following constrained maximization problem:

$$v^{nd}(b_t, o_t, y_t, p_t) = \max_{\{c_t, x_t, b_{t+1}, o_{t+1}\}} \{u(c_t) + \beta E\left[V\left(b_{t+1}, o_{t+1}, y_{t+1}, p_{t+1}\right)\right]\} \tag{11}$$

subject to

$$c_t - b_t + \chi(o_t, x_t) = y_t + p_t x_t - q_t\left(b_{t+1}, o_{t+1}, y_t, p_t\right) b_{t+1} \tag{12}$$

$$o_{t+1} = o_t - x_t + k, \tag{13}$$

$$x_t \leq o_t, \; x_t \geq 0 \tag{14}$$

where q_t is a bond pricing function.

The value of default solves the following constrained optimization problem:

$$v^d(o_t, y_t, p_t) = \max_{\{c_t, x_t\}} \left\{ u(c_t) + \beta (1-\phi) EV^d (o_{t+1}, y_{t+1}, p_{t+1}) + \beta\phi EV (0, o_{t+1}, y_{t+1}, p_{t+1}) \right\} \quad (15)$$

$$c_t + \chi(o_t, x_t) = y_t + p_t x_t \quad (16)$$

$$o_{t+1} = o_t - x_t + k, \quad (17)$$

$$x_t \leq o_t, \; x_t \geq 0 \quad (18)$$

The definition of the default set and the probability of default are as in the Eaton-Gersovitz model. For a debt position $b_t < 0$, default is optimal for the set of realizations of $\{y_t, p_t\}$ for which $v^d(o_t, y_t, p_t)$ is at least as high as $v^{nd}(b_t, o_t, y_t, p_t)$:

$$D(b_t, o_t) = \left\{ \{y_t, p_t\} : v^{nd}(b_t, o_t, y_t, p_t) \leq v^d(o_t, y_t, p_t) \right\}. \quad (19)$$

The probability of default at $t+1$ perceived as of date t, $d(b_{t+1}, o_{t+1}, y_t, p_t)$, can be calculated from the default set and the transition probability functions of the endowment $z_y(y_{t+1} \mid y_t)$ and oil prices $z_p(p_{t+1} \mid p_t)$ as follows:

$$d_t(b_{t+1}, o_{t+1}, y_t, p_t) = \int \int_{D(b_{t+1}, o_{t+1})} dz_y(y_{t+1} \mid y_t)\, dz_p(p_{t+1} \mid p_t). \quad (20)$$

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Appendix

Estimation Approach

Before proceeding to dynamic panel data models, we need to verify that all variable are integrated of the same order. In doing so, we have used the test of the panel unit root of Im. *et al.* (2003, IPS henceforth), which is based on averaging individual unit root test statistics for panels. Specifically, they proposed a test based on the average of augmented Dickey-Fuller statistics (ADF hencefoth) computed for each group in the panel. In accordance with some survey on panel unit root tests (such as those discussed in Banerjee (1999)), this test is less restrictive and more powerful than others that do not allow for heterogeneity in the autoregresive coefficient. IPS test permit solving serial correlation problem by assuming heterogeneity between units (in this case, countries) in a dynamic panel framework, as considered here. The basic equation of IPS test is as follows:

$$\Delta y_{it} = \alpha_i + \beta_i y_{it-1} + \sum_{j=1}^{p} \phi_{ij} \Delta y_{it-j} + \epsilon_{it} \tag{21}$$

for $i = 1, 2, ..., N$ and $t = 1, 2, ..., T$, where N refers to the number of countries in the panel and T refers to the number of observations over time. In this case, y_i stands for each variable under consideration in our model (for example, III, total public debt, oil GDP or non-oil GDP), α_i is the individual fixed effect and p is the maximum number of lags included in the test. The null hypothesis then becomes $\beta_i = 0$ for all i, against the alternative hypothesis, which is that $\beta_i < 0$ for some $i = 1, ..., N_1$ and $\beta_i = 0$ for $i = N_1 + 1, ..., N$, where N_1 denote the number of stationary panels. Therefore, IPS statistic can be written as follows:

$$\bar{t} = \frac{1}{N} \sum_{i=1}^{N} t_i^{ADF} \tag{22}$$

where t_i^{ADF} is the ADF t-statistic for country i, taking into account the country specific ADF regression, given by (21). The $\bar{t}$ statistic has been shown to be normally distributed under H_0. Table 1 reports the outcome for the global sample of this test.

As we can see, each variable is integrated of order one. Once the order of stationary has been defined, we estimated a country risk equation on the basis of cross-country panel data. In particular, we focus on three estimation methods which are consistent when both T and N are large. At one extreme, the usual practice is either to estimate N separate regressions and compute the mean of the estimated coefficients across countries, which is called the Mean Group (MG) estimator. Pesaran and Smith (1995) show that the MG estimator will produce consistent estimates of the average of the parameters, but ignores the fact that certain parameters are the same across countries.

At the other extreme are the traditional pooled estimators (such as dynamic fixed effects estimators), where the intercepts are allow to differ across countries while all other coefficients and error

Table 2: Im *et al.* (2003) panel unit root test outcome: 1979-2010

	Levels		Logs	
	t-statistic	P-value	t-statistic	P-value
Inst. Inv.	-1.396	0.635	-1.356	0.741
Δ Inst. Inv.	-3.600	0.000	-3.613	0.000
Oil GDP	-0.603	1.000	-1.208	0.949
Δ Oil GDP	-3.486	0.000	-3.745	0.000
Non-oil GDP	0.630	1.000	-1.150	0.981
Δ Non-oil GDP	-2.839	0.000	-3.249	0.000
Oil reserves	-0.974	0.999	-1.234	0.940
Δ Oil reserves	-3.812	0.000	-3.871	0.000
Tot. pub. debt	-1.397	0.572	-1.277	0.855
Δ Tot. pub. debt	-2.926	0.000	-2.940	0.000
NFA	-1.227	0.943	-	-
Δ NFA	-3.044	0.000	-	-

Note: When computing NFA outcome, we excluded Irak because of data limitations.

variances are constrained to be the same. In this case, the model controls for all time-invariant differences between countries, so the estimated coefficient cannot be biased because of omitted time-invariant characteristics. An intermediate technique is the Pooled Mean Group (PMG) estimator, proposed by Pesaran *et al.* (1999), which relies on a combination of pooling and averaging of coefficients, allowing the intercepts, short-run coefficients and error variances to differ freely across countries, but the long-run coefficients are constrained to be the same.

Therefore, for the implementation of these methods we consider the following model:

$$III_{it} = \theta_{0i} + \theta_{1i}OilGDP_{it} + \theta_{2i}NonOilGDP_{it} + \theta_{3i}OilR_{it} + \theta_{4i}X_{it} + \mu_i + \epsilon_{it} \qquad (23)$$

Again, each observation is subscripted for the country i and the year t. In this case, $X \in \{TotPubD, NFA\}$. The variable III is the log of Institutional Investor's country credit ratings, $OilGDP$ is the log of oil GDP, $NonOilGDP$ is the log of non-oil GDP, $OilR$ is the log of oil reserves stock, $TotPubD$ is the log of total public debt, and NFA corresponds to net foreign assets to GDP ratio. Additionally, μ_i is a set of country fixed effects (such as geographical or institutional factors) and ϵ_{it} is the idyosincratic error term.

Now, with a maximum lag of one for all variables, we construct the autorregresive distributive lag (ARDL) (1,1,1,1,1) dynamic panel specification of (23):

$$\begin{aligned}III_{it} = \lambda_i III_{i,t-1} + \delta_{10i} OilGDP_{it} + \delta_{11i} OilGDP_{i,t-1} + \delta_{20i} NonOilGDP_{it} + \\ \delta_{21i} NonOilGDP_{i,t-1} + \delta_{30i} OilR_{it} + \delta_{31i} OilR_{i,t-1} + \delta_{40i} X_{it} + \delta_{41i} X_{i,t-1} + \mu_i + \epsilon_{it} \quad (24)\end{aligned}$$

Then, the error correction equation of (24) is:

$$\begin{aligned}\Delta III_{it} = \phi_i \left(III_{i,t-1} - \hat{\theta_{0i}} - \hat{\theta_{1i}} OilGDP_{it} - \hat{\theta_{2i}} NonOilGDP_{it} - \hat{\theta_{3i}} OilR_{it} - \hat{\theta_{4i}} X_{it} \right) - \\ \delta_{11i} \Delta OilGDP_{it} - \delta_{21i} \Delta NonOilGDP_{it} - \delta_{31i} \Delta OilR_{it} - \delta_{41i} \Delta X_{it} + \epsilon_{it} \quad (25)\end{aligned}$$

where

$$\hat{\theta_{0i}} = \frac{\mu_i}{1-\lambda_i}; \hat{\theta_{1i}} = \frac{\delta_{10i} + \delta_{11i}}{1-\lambda_i}; \hat{\theta_{2i}} = \frac{\delta_{20i} + \delta_{21i}}{1-\lambda_i}$$
$$\hat{\theta_{3i}} = \frac{\delta_{30i} + \delta_{31i}}{1-\lambda_i}; \hat{\theta_{4i}} = \frac{\delta_{40i} + \delta_{41i}}{1-\lambda_i}; \phi_i = -(1-\lambda_i)$$

In this case, ϕ_i is the error correction speed of adjustment parameter, and we would expect ϕ_i to be negative if the variables exhibit a return to long-run equilibrium[2].

[2]Replacing $\hat{\theta}_i$-parameters and ϕ_i in equation (23) we get:

$$\begin{aligned}\Delta III_{it} = -(1-\lambda_i)\Big(III_{i,t-1} - \frac{\mu_i}{1-\lambda_i} - \frac{\delta_{10i}+\delta_{11i}}{1-\lambda_i} OilGDP_{it} - \frac{\delta_{20i}+\delta_{21i}}{1-\lambda_i} NonOilGDP_{it} - \frac{\delta_{30i}+\delta_{31i}}{1-\lambda_i} OilR_{it} - \\ \frac{\delta_{40i}+\delta_{41i}}{1-\lambda_i} X_{it} \Big) - \delta_{11i}\Delta OilGDP_{it} - \delta_{21i}\Delta NonOilGDP_{it} - \delta_{31i}\Delta OilR_{it} - \delta_{41i}\Delta X_{it} + \epsilon_{it}\end{aligned}$$

Removing similar terms, the above expression is as follows:

$$\begin{aligned}\Delta III_{it} = -(1-\lambda_i) III_{i,t-1} + \mu_i + (\delta_{10i}+\delta_{11i}) OilGDP_{it} + (\delta_{20i}+\delta_{21i}) NonOilGDP_{it} + (\delta_{30i}+\delta_{31i}) OilR_{it} + \\ (\delta_{40i}+\delta_{41i}) X_{it} - \delta_{11i}\Delta OilGDP_{it} - \delta_{21i}\Delta NonOilGDP_{it} - \delta_{31i}\Delta OilR_{it} - \delta_{41i}\Delta X_{it} + \epsilon_{it}\end{aligned}$$

Rewriting:

$$\begin{aligned}III_{it} - III_{i,t-1} = &- (1-\lambda_i) III_{i,t-1} + \mu_i + (\delta_{10i}+\delta_{11i}) OilGDP_{it} + (\delta_{20i}+\delta_{21i}) NonOilGDP_{it} + (\delta_{30i}+\delta_{31i}) OilR_{it} \\ &+ (\delta_{40i}+\delta_{41i}) X_{it} - \delta_{11i}(OilGDP_{it} - OilGDP_{i,t-1}) - \delta_{21i}(NonOilGDP_{it} - NonOilGDP_{i,t-1}) \\ &- \delta_{31i}(OilR_{it} - OilR_{i,t-1}) - \delta_{41i}(X_{it} - X_{i,t-1}) + \epsilon_{it}\end{aligned}$$

Again, simplifying this equality we obtain:

$$\begin{aligned}III_{it} = \lambda_i III_{i,t-1} + \delta_{10i} OilGDP_{it} + \delta_{11i} OilGDP_{i,t-1} + \delta_{20i} NonOilGDP_{it} + \\ \delta_{21i} NonOilGDP_{i,t-1} + \delta_{30i} OilR_{it} + \delta_{31i} OilR_{i,t-1} + \delta_{40i} X_{it} + \delta_{41i} X_{i,t-1} + \mu_i + \epsilon_{it}\end{aligned}$$

Note that this expression is equivalent to (24). For a long-run relationship to exist, we require that $\phi \neq 0$.

Estimation results

In this subsection we estimate the PMG, MG and DFE estimators for model (25). In order to obtain reliable estimators and seeking to maintain a large data sample, we include information for United States, China, India, Brazil and Australia, since these countries have large proven oil reserves, although these have not been oil net exporters in the time interval considered here. When deciding about model selection, we apply the Hausman test to see whether there are significant differences among these three estimators. The null of this test is that the difference between DFE and MG, DFE and PMG or PMG and MG is not significant. Consider, for example, the test between DFE and PMG. If the null is not rejected, the DFE estimator is recommended since it is efficient. The alternative is that there is a significant difference between PMG and DFE, and the null is rejected. Specifically, the Hausman statistic is:

$$H = (\beta_{DFE} - \beta_{PMG})' \left[\text{var}(\beta_{DFE}) - \text{var}(\beta_{PMG})\right]^{-1} (\beta_{DFE} - \beta_{PMG}) \sim \chi^2$$

where β_j is the vector of coefficients and var(β_j) is the covariance matrix of β_j, estimated using the $j-$technique, for j =DFE, PMG. Under the null hypothesis, H has asymptotically the χ^2 distribution. Table 2 reports the results of Hausman test, in which Model (1) corresponds to equation (25), excluding X_i from regressors, while Model (2) and Model (3) represent the same equation, considering $X_i = TotPubD$ and $X_i = NFA$, respectively.

Table 3: Hausman test outcome: 1979-2010

	Model (1)		Model (2)		Model (3)	
	χ^2-stat.	P-value	χ^2-stat.	P-value	χ^2-stat.	P-value
DFE vs. MG	0.01	0.999	0.00	1.000	0.01	1.000
DFE vs. PMG	0.02	0.999	0.10	0.998	0.03	0.999
PMG vs. MG	1.68	0.640	2.87	0.579	3.50	0.477

Under the current specification, the hypothesis that the country risk equation (equation (25)) is adequately modeled by a PMG or MG model is resoundingly rejected. In general, when considering Model (1) the results in the table above suggest that it is not possible to reject the null hypothesis of the homogeneity restriction on regressors (in the short and long run), since P-values are both 0.999, which indicates that DFE is more efficient estimator than MG and PMG, respectively. Notice that this conclusion holds for Model (2) and Model (3), because P-values associated to these tests are close to 1. Because of this, we choose to employ the DFE estimator.