Labor Market Frictions and Aggregate Employment

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Abstract

What are the aggregate employment effects of labor market frictions? Canonical labor market models share a common theme in exploring the implications of adjustment frictions. We use this shared microeconomic structure to devise an empirical diagnostic that allows one to bound the effects of this class of frictions on the path of aggregate employment. Application of this diagnostic to rich establishment microdata for the United States suggests that canonical labor market frictions are unable to explain the majority of observed employment dynamics. This result can be traced to the failure of canonical models to account for the dynamics of the firm size distribution observed in establishment microdata.

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What are the aggregate employment effects of labor market frictions? In this paper, we present a new approach to this question for a popular class of labor market frictions. This class shares a common theme in exploring the aggregate implications of (non-convex) adjustment frictions. Notable examples include canonical models of lumpy adjustment (Caballero, Engel and Haltiwanger 1997), per-worker hiring and firing costs (Bentolila and Bertola 1990; Hopenhayn and Rogerson 1993), and search and matching (Diamond 1982; Pissarides 1985; Mortensen and Pissarides 1994).

Our approach is motivated by two themes. First, the pattern of microeconomic adjustment in the economy leaves a clear imprint on the dynamics of the firm-size distribution—that is, cross-sectional flows of employment across firms. Canonical frictions impede these flows, and it is through this microeconomic channel that frictions shape the path of aggregate employment.

Second, many components of the implied firm-size dynamics can be measured, admitting a clear mapping from theoretical concepts to empirical analogues. We show how establishment-level microdata on employment shed light on these cross-sectional employment dynamics, and thereby inform an analysis of the aggregate consequences of canonical models of labor market frictions.

In section 1, we show that conventional models of labor market frictions imply cross-sectional employment dynamics that take on an intuitive and simple structure. The change in the mass of establishments of a given size n reflects the difference between the inflow of establishments to size n, and the outflow from that mass. Frictions act to reduce the rates of adjustment to and from the mass at each point n in the firm-size distribution.

We show how this structure can be used to motivate a *diagnostic* that has a clear relationship to the level of aggregate employment that would emerge in an economy without frictions. We define the *flow steady state* density of employment as the distribution that would be ground out if current rates of adjustment to and from each n were held constant. Our diagnostic is defined as the mean of this distribution, aggregate flow steady state employment. We prove that, under this class of frictions, the path of aggregate flow steady state employment *bounds* the path of aggregate *frictionless* employment in canonical models of lumpy adjustment, and is approximately *equal to* aggregate frictionless employment in models of linear and search and matching frictions. As a result, the behavior of the diagnostic sheds light on the path of aggregate employment that would emerge in the absence of frictions.

To see the intuition behind our result, consider the effect on impact of an increase in aggregate labor productivity. In the absence of frictions, all establishments would wish to hire more workers. *Conditional on adjusting*, this increase in desired employment will imply an increase (decrease) in the mass of large (small) establishments in flow steady state: the flow steady state density shifts right. In addition, however, the *probability of adjustment* reacts: Firms will become more (less) likely to adjustment toward versus away from large (small) employment levels. This *extensive margin* effect induces a further rightward shift in the flow steady state density, overshooting the response of its frictionless counterpart. In this way, the difference between the paths of aggregate flow steady state and actual employment is an *upper bound* on the effect of this class of frictions.

A crucial feature of the diagnostic is that it also can be *measured* in establishment microdata on employment. Accordingly, in section 2 we use rich microdata underlying the Quarterly Census of Employment and Wages for the period 1992Q1 through to 2014Q2 to measure the diagnostic empirically. These microdata report the employment distribution, the inflow of establishments to each size in the distribution, and the probability of outflow from each size. These allow one to infer the flow steady state density and thus our diagnostic, aggregate flow steady state employment.

We find that the empirical properties of the diagnostic resemble those of the empirical path of *actual* employment. Specifically, we contrast the paths of (detrended) actual and flow steady state aggregate employment. Aside from a brief period around the trough of the Great Recession, the two series are remarkably similar: The median (mean) absolute deviation between the two series is just 0.5 (0.8) of a log point over the sample period. Viewed through the lens of canonical models, the inference is that this class of frictions accounts for little of observed employment dynamics—the path of aggregate employment would have looked similar even in the absence of these frictions.

1. Labor market frictions and firm size dynamics

Canonical models of labor market frictions share a common theme in exploring the implications of microeconomic adjustment frictions. Models of discrete, or "lumpy," adjustment in the tradition of Caballero, Engel and Haltiwanger (1997) study the effects of *fixed costs* of adjustment. Seminal analyses of the effects of firing costs, such as Bentolila and Bertola (1990) and Hopenhayn and Rogerson (1993), invoke the presence of per-worker, *linear costs* of adjustment. Similarly, recent models of search costs that extend the standard Diamond-Mortensen-Pissarides model to incorporate a notion of firm size have underscored its interpretation as the presence of a per-worker, linear hiring cost (Acemoglu and Hawkins 2014; Elsby and Michaels 2013).

In this section we describe the economic channels through which labor market frictions shape aggregate employment in this canonical class of models. A key observation is that these frictions affect aggregate employment by impeding the flow of firms across different firm sizes. We show that canonical models in turn imply a specific structure to these firm size dynamics. We use this structure to motivate a diagnostic for the aggregate effects of these frictions. A virtue of this diagnostic that we take up in later sections of this paper is that it can be measured directly from establishment microdata.

1.1 Fixed costs

A leading model of labor market frictions postulates the presence of a fixed cost of adjusting employment that is independent of the scale of adjustment. In what follows we review the wellunderstood distortions of firms' labor demand policies induced by this friction. More importantly for our purposes, we use this to infer the (less-widely-understood) implications for firm size flows, and thereby aggregate employment.

With regard to the structure of labor demand, the key implication of a fixed cost is that employment will be adjusted only intermittently and, upon adjustment, discretely—hence "lumpy" adjustment. Thus, labor demand takes the form of a threshold "*Ss*" policy, illustrated in Figure 1A:

$$n = \begin{cases} n^* & \text{if} \quad n^* > U(n_{-1}), \\ n_{-1} & \text{if} \quad n^* \in [L(n_{-1}), U(n_{-1})], \\ n^* & \text{if} \quad n^* < L(n_{-1}). \end{cases} \tag{1}$$

Here n^* is the level of employment that a firm would choose if it adjusted**.** Under the *Ss* policy, a firm's current employment n is adjusted away from its past level n_{-1} whenever n^* deviates sufficiently from n_{-1}, as dictated by the adjustment triggers $L(n_{-1}) < n_{-1} < U(n_{-1})$.

Caballero, Engel and Haltiwanger (1995, 1997) refer to n^* as *mandated* employment. This can be interpreted as what the firm would choose if the friction were suspended for the current period. It is, in principle, distinct from *frictionless* employment, which emerges if the fixed cost is set to zero now *and forever.* However, in the Appendix, we verify that for a reasonably calibrated model within this canonical class, the dynamics of mandated and frictionless employment are very similar.[2] Henceforth, then, we will interpret n^* as frictionless employment and sometimes use "mandated" and "frictionless" interchangeably.[3]

The dynamics of *aggregate* employment implied by the firm behavior in equation (1) can be inferred from its implications for firm size flows. Imagine the economy enters the period with a density of past employment, $h_{-1}(n)$, and that realizations of idiosyncratic and aggregate shocks induce a density of mandated employment $h^*(n)$. Our strategy is to infer a law of motion for the current-period density $h(n)$ implied by equation (1). This in turn will imply a path for aggregate employment in the economy, which we denote by N, since the latter is captured by the mean of the density, $N \equiv \int nh(n)dn$.

The adjustment policy in Figure 1A suggests a straightforward approach to constructing a law of motion for the firm-size density $h(n)$. Consider first the outflow of mass from employment level n. Among the $h_{-1}(n)$ mass of firms that enter the period with n workers, only the fraction whose desired employment n^* lies outside the inaction region $[L(n), U(n)]$ will choose to incur the adjustment cost and leave the mass. Symmetrically, now consider the inflow of mass to employment level n. Among the $h^*(n)$ mass of firms whose desired employment is equal to n, only the fraction whose inherited employment n_{-1} lies outside of the inverse inaction

[2] This has also been proven analytically for the case of a plausibly small fixed cost (Gertler and Leahy, 2008; Elsby and Michaels, 2014).

[3] To conserve space, Figure A in the Appendix displays impulse responses of all models for the parameterization that yields an inaction rate in the middle of the range that we explore in our quantitative analysis. This parameterization still implies considerably more inaction than in the data.

region $[U^{-1}(n), L^{-1}(n)]$ will choose to incur the adjustment cost and flow to n. Thus, the change in the mass at employment level n follows the law of motion

$$\Delta h(n) = \tau(n)h^*(n) - \phi(n)h_{-1}(n), \tag{2}$$

where $\tau(n)$ and $\phi(n)$ are respectively the probabilities of adjusting to and from an employment level n,

$$\begin{aligned} \tau(n) &= \Pr(n_{-1} \notin [U^{-1}(n), L^{-1}(n)] | n^* = n) \text{, and} \\ \phi(n) &= \Pr(n^* \notin [L(n), U(n)] | n_{-1} = n). \end{aligned} \tag{3}$$

Formal derivations of equations (2) and (3) are provided in the appendix.

1.2 An empirical diagnostic

With this theoretical law of motion in hand, our next step is to consider which of its components can be *measured* empirically using available data. As we shall see, establishment-level panel data allow one to observe much of equation (2): One can measure the mass at each employment level at each point in time, $h_{-1}(n)$ and $h(n)$; one can also observe the fraction of establishments at each employment level that adjusts away, $\phi(n)$. Crucially, however, we observe only the *total* inflow, $\tau(n)h^*(n)$, rather than its constituent parts. And, of course, this is precisely the identification problem that we are attempting to surmount: If we could measure both $\tau(n)$ and $h^*(n)$, the latter would allow us to infer a measure of aggregate *mandated* employment $N^* \equiv \int nh^*(n)dn$. Comparison of N^* with the observed path of actual aggregate employment N would then indicate the wedge between these two induced by the adjustment friction.

Our approach is instead to ask whether we can infer useful information about the path of desired aggregate employment N^* without observing directly the density of mandated employment $h^*(n)$. Our point of departure is to note that, for fixed adjustment rates $\tau(n)$ and $\phi(n)$, the law of motion (2) implies that the firm size density will converge to what we shall refer to as a *flow steady state*,

$$\hat{h}(n) \equiv \frac{\tau(n)}{\phi(n)} h^*(n), \tag{4}$$

$\hat{h}(n)$ is useful for several reasons. First, it can be measured straightforwardly, since it requires knowledge only of the total inflow, $\tau(n)h^*(n)$, and the probability of outflow $\phi(n)$, both of which are observed in establishment panel data.

Second, we argue in what follows that the flow steady state conveys important information on the path of mandated employment, and thereby on the role of adjustment frictions in shaping the dynamics of aggregate employment. Specifically, note that the aggregate employment level implied by the flow steady state, $\hat{N} \equiv \int n\hat{h}(n)dn$, can be written as

$$\hat{N} = N^* + cov_{h^*}\left(n, \frac{\tau(n)}{\phi(n)}\right), \tag{5}$$

where cov_{h^*} denotes a covariance taken with respect to the distribution of mandated employment, $h^*(n)$.

Equation (5) reveals that aggregate flow steady-state employment $\widehat{N}$ will *bound* the path of aggregate mandated employment N^* under a monotonicity condition—namely that firms on average are more likely to adjust to versus from high (low) employment levels following positive (negative) innovations to aggregate mandated employment. Under this condition, $\widehat{N}$ will rise more than N^* when the latter rises, and fall more than N^* when it falls.

The monotonicity condition that underlies this intuition is closely related to the *selection effect* that has been emphasized in the literature on adjustment frictions (Caballero and Engel 2007; Golosov and Lucas 2007). This refers to a property shared by state-dependent models of adjustment whereby the firms that adjust tend to be those with the greatest desired adjustment. By the same token, firms in these models also will adjust in the *direction* of the desired adjustment.

The forgoing intuition is captured especially clearly in standard models of fixed adjustment frictions, such as that set out in Caballero and Engel (1999). In this environment, firms face an isoelastic production function $y = pxn^\alpha$ that is subject to idiosyncratic shocks x. Firms thus face the following decision problem

$$\Pi(n_{-1}, x) \equiv \max_n \{pxn^\alpha - wn - C\mathbb{I}[n \neq n_{-1}] + \beta\mathbb{E}[\Pi(n, x')|x]\}, \tag{6}$$

where p denotes (fixed) aggregate productivity, w the wage, and C the fixed adjustment cost.

Caballero and Engel show that, if idiosyncratic shocks follow a geometric random walk, and the adjustment cost C is scaled to be proportional to the firm's frictionless labor costs, the resultant homogeneity of a firm's labor demand problem has two tractable implications, summarized in the following Lemma:

Lemma 1 (Caballero and Engel 1999) *Consider the firm's problem in* (6)*. If (i)* $\ln x' = \ln x + \varepsilon'_x$ *with* ε'_x *i.i.d., and (ii)* $C = \Gamma \cdot (\alpha x / w^\alpha)^{1/1-\alpha}$*, then (a) the adjustment triggers in* (1) *are linear and time invariant,* $L(n) = L \cdot n$ *and* $U(n) = U \cdot n$ *for constants* $L < 1 < U$*; and (b) desired (log) employment adjustments,* $\ln(n^*/n_{-1})$*, are independent of initial firm size* n_{-1}.

Proposition 1 uses these properties of the canonical model to formalize the heuristic claim above that changes in aggregate flow steady state employment bound changes in aggregate mandated employment. Because of the model's loglinear structure, the result is most simply derived in terms of aggregate *log* mandated employment, which we shall denote by $\mathcal{N}^*$, and its aggregate *log* flow steady state counterpart $\widehat{\mathcal{N}}$.

Proposition 1 *In the canonical model of fixed adjustment costs, relative to a prior constant-*$\mathcal{N}^*$ *state, a small change in aggregate log mandated employment,* $\Delta\mathcal{N}^*$*, induces on impact a larger change in aggregate log flow steady state employment,* $\Delta\widehat{\mathcal{N}}$*. That is,*

$$\Delta\widehat{\mathcal{N}} = A \cdot \Delta\mathcal{N}^*, \text{ where } A > 1. \tag{7}$$

Proposition 1 has a number of virtues. First, it holds irrespective of whether adjustment is symmetric or asymmetric. Second, it holds in the presence of changes in (market) wages w. Intuitively, the latter simply dampen changes in mandated employment $\Delta\mathcal{N}^*$. Since the change in the aggregate flow steady state $\Delta\widehat{\mathcal{N}}$ is *proportional to* $\Delta\mathcal{N}^*$, changes in wages affect both symmetrically, leaving the result in Proposition 1 unimpaired.

There are also limitations to Proposition 1, however. It relies on the homogeneity of the canonical model implied by the random walk assumption on shocks. It is also a comparative statics result, describing the response of the economy to a change in aggregate labor demand, indexed by p, that is expected to occur with zero probability from the firms' perspectives. For these reasons, in the next subsection, we explore the robustness of the bounding result anticipated above in numerical simulations that relax these assumptions.

1.3 Quantitative illustrations

In what follows, we illustrate the dynamics of fixed costs models that resemble the canonical model described above, but with two differences. First, we relax the random walk assumption on idiosyncratic shocks which we allow to follow a geometric AR(1),

$$\ln x' = \rho_x \ln x + \varepsilon_x', \text{ where } \varepsilon_x' \sim N(0, \sigma_x^2). \tag{8}$$

Second, we allow for the presence of aggregate productivity shocks, and for their stochastic process to be known to firms in the model. The evolution of these aggregate shocks also is assumed to follow a geometric AR(1),

$$\ln p' = \rho_p \ln p + \varepsilon_p', \text{ where } \varepsilon_p' \sim N\left(0, \sigma_p^2\right). \tag{9}$$

To mirror the timing of the data we use later in the paper, a period is taken to be one quarter. Based on this, we set the discount factor β to 0.99, consistent with an annual interest rate of around 4 percent. To parameterize the remainder of the model, we appeal to the empirical literature that estimates closely related models of firm dynamics. Thus, the returns to scale parameter α is set to 0.64, as in the estimates of Cooper, Haltiwanger, and Willis (2007, 2015).

The parameters of the idiosyncratic productivity shock process (8) are based on estimates reported by Abraham and White (2006). Abraham and White's results imply a quarterly persistence parameter ρ_x of approximately 0.7 and a standard deviation of the idiosyncratic innovation ε_x' of around $\sigma_x = 0.15$.[4]

The parameters of the process for aggregate technology in (9) are chosen so that aggregate frictionless employment in the model exhibits a persistence and volatility comparable to aggregate employment in U.S. data. This yields $\rho_p = 0.95$ and $\sigma_p = 0.0026$. Although frictions augment persistence, and dampen volatility, the intent is for the model environment to resemble

[4] The Appendix derives these parameters and contrasts them with other estimates reported in related literature. It also shows that the implied dynamics of aggregate employment implied by reasonable changes in these parameters are qualitatively similar to those described here.

broadly the U.S. labor market with respect to these *un*conditional moments. Importantly, the approach does not build in any persistence in employment *conditional* on technology.

Finally, with respect to the adjustment cost C, we explore three parameterizations that successively raise the friction. The first is chosen to replicate the observed fraction of firms that do not adjust from quarter to quarter. In the data used later in the paper, this inaction rate averages 52.5 percent. However, the latter inaction rate is implied by a fixed cost equal to 1.3 percent of quarterly revenue, which is at the lower end of available estimates (Bloom 2009; Cooper, Haltiwanger, and Willis 2007, 2015). For this reason, we also explore fixed costs that imply quarterly inaction rates of 67 percent and 80 percent, corresponding to adjustment costs of 2.7 percent and 5.2 percent of quarterly revenue, respectively.

We solve the labor demand problem for 250,000 firms via value function iteration on an integer-valued employment grid, $n \in \{1,2,3 \dots\}$. The latter mirrors the integer constraint in the data, allowing one to construct the flow steady state density $\hat{h}(n)$ in the simulated data in the same way as we later implement in the real data.

Figure 2 plots simulated impulse responses of aggregate employment N, together with its mandated and flow steady state counterparts, N^* and $\widehat{N}$ respectively. The bounding result anticipated in the previous subsection, and in Proposition 1 in particular, is clearly visible in the model dynamics. For all three parameterizations of the adjustment cost our proposed diagnostic, flow steady state employment $\widehat{N}$, responds more aggressively to the aggregate shock than mandated employment N^*. Moreover, the magnitude of the overshooting of $\widehat{N}$ relative to N^* is substantial in the model, responding on impact around twice as much to the impulse.

1.4 Linear costs

Prominent alternative models of labor market frictions appeal instead to linear costs of adjustment in which the friction is discrete *at the margin*, and rises with the scale of adjustment. As noted above, simple models of per-worker firing and hiring costs fit directly into this mold. But we will also include canonical models of search frictions in this class, since they are models of *time-varying* per-worker hiring costs (Elsby and Michaels 2013).

Relative to the fixed costs case examined above, linear frictions alter the structure of both labor demand and firm size dynamics. Although labor demand will continue to feature intermittent adjustment a key difference is that, conditional on adjusting, firms will no longer implement their desired mandated employment n^*. Rather, they will reduce the magnitude of hires and separations, shedding fewer workers when they shrink, and hiring fewer workers when they expand. Formally, the policy rule for separations, which we shall denote by $l(\cdot)$, will differ from the policy rule used for hiring, denoted $u(\cdot)$, inducing the continuous *Ss* policy illustrated in Figure 1B,

$$n = \begin{cases} u^{-1}(n^*) & \text{if} \quad n^* > u(n_{-1}), \\ n_{-1} & \text{if} \quad n^* \in [l(n_{-1}), u(n_{-1})], \\ l^{-1}(n^*) & \text{if} \quad n^* < l(n_{-1}). \end{cases} \tag{10}$$

The key distinction, that the *direction* of adjustment must be taken into account in the presence of linear costs, also leaves its imprint on the law of motion for the firm size distribution. As before, the labor demand policy in Figure 1B motivates the form of this law of motion. This reveals that the structure of outflows is qualitatively unchanged—of the $h_{-1}(n)$ density of firms currently at employment level n, only those with mandated employment outside the inaction region $[l(n_{-1}), u(n_{-1})]$ will adjust away. But inflows are now differentiated by the direction of adjustment. The inflow of mass adjusting down to n is comprised of firms whose past employment n_{-1} is greater than n, and whose mandated employment n^* is equal to $l(n) < n$. Likewise, the inflow of mass flowing up to n consists of firms with $n_{-1} < n$ and $n^* = u(n) > n$.

Piecing this logic together yields the following law of motion for the firm size density,

$$\Delta h(n) = \tau_l(n) h_l^*(n) + \tau_u(n) h_u^*(n) - \phi(n) h_{-1}(n), \tag{11}$$

where $h_l^*(n) = l'(n) h^*\big(l(n)\big)$ and $h_u^*(n) = u'(n) h^*\big(u(n)\big)$ are the densities of employment that would emerge if all firms adjusted, respectively, according to the separation rule, $l(n)$, and hiring rule, $u(n)$, and the adjustment probabilities take the form

$$\begin{aligned} \tau_l(n) &= \Pr\big(n_{-1} > n \mid n^* = l(n)\big), \\ \tau_u(n) &= \Pr\big(n_{-1} < n \mid n^* = u(n)\big), \text{ and} \\ \phi(n) &= \Pr(n^* \notin [l(n), u(n)] \mid n_{-1} = n). \end{aligned} \tag{12}$$

Extending the interpretation of the fixed costs case above, $\tau_l(n)$ is the probability that a firm adjusts *down to* n, while $\tau_u(n)$ is the probability that a firm adjusts *up to* n.

To construct the flow steady state density for the linear costs case note that, for fixed adjustment rates $\tau_l(n)$, $\tau_u(n)$ and $\phi(n)$, the law of motion (11) implies that the firm size density will converge to

$$\hat{h}(n) \equiv \frac{\tau_l(n)}{\phi(n)} h_l^*(n) + \frac{\tau_u(n)}{\phi(n)} h_u^*(n). \tag{13}$$

Once again, the behavior of aggregate flow steady state employment implied by linear frictions can be formalized most straightforwardly in a canonical linear cost model[5] in which firms face isoelastic production $y = pxn^\alpha$, and idiosyncratic shocks that follow a geometric random walk. The key difference is that the adjustment friction is now scaled by the magnitude of adjustment, so that firms face the decision problem:

$$\Pi(n_{-1}, x) \equiv \max_n \{pxn^\alpha - wn - c^+ \Delta n^+ + c^- \Delta n^- + \beta \mathbb{E}[\Pi(n, x') | x]\}. \tag{14}$$

[5] Nickell (1978, 1986) was the first to formalize the linear-cost model in the context of labor demand. Bentolila and Bertola (1990) introduced uncertainty into Nickell's continuous-time formulation. Equation (14) can be seen as the discrete-time analogue to Bentolila and Bertola (though we do not have to assume Gaussian innovations).

A simple extension of Caballero and Engel's (1999) homogeneity results can be used to show that, if per-worker hiring and firing costs are proportional to wages, the same tractable properties of Lemma 1 hold for the case of linear frictions:

Lemma 1$'$ *Consider the firm's problem in (*14*). If (i)* $\ln x' = \ln x + \varepsilon_x'$ *with* ε_x' *i.i.d., and (ii)* $c^+ = \gamma^+ w$ *and* $c^- = \gamma^- w$*, then (a) the adjustment triggers in (*10*) are linear and time invariant,* $l(n) = l \cdot n$ *and* $u(n) = u \cdot n$ *for constants* $l < 1 < u$*; and (b) desired (log) employment adjustments,* $\ln(n^*/n_{-1})$*, are independent of initial firm size* n_{-1}.

As in Proposition 1 above for the fixed costs case, the latter properties allow one to relate the response of aggregate flow steady state log employment $\widehat{\mathcal{N}}$ to the response of aggregate frictionless log employment $\mathcal{N}^*$ following a change in aggregate productivity.

Proposition 2 *In the canonical model of linear adjustment costs, relative to a prior constant-*$\mathcal{N}^*$ *state, a small change in aggregate log mandated employment,* $\Delta\mathcal{N}^*$*, induces on impact approximately the same change in aggregate log flow steady state employment,* $\Delta\widehat{\mathcal{N}}$*. That is,*

$$\Delta\widehat{\mathcal{N}} \approx \Delta\mathcal{N}^*. \tag{15}$$

Proposition 2 shares the same virtues and limitations as Proposition 1: It holds for asymmetric frictions $c^+ \neq c^-$, and in the presence of changes in wages w; but it requires the homogeneity assumptions in Lemma 1$'$, and is a comparative statics result.

Proposition 2 also suggests that the flow steady state is again informative with respect to the path of frictionless employment. A key difference relative to the fixed costs case, however, is that the response of $\widehat{\mathcal{N}}$ no longer *bounds* that of $\mathcal{N}^*$, as in the fixed costs case, but is approximately *equal* to it.

The key difference relative to the fixed costs case is that firms adjust only *partially* toward their frictionless employment under linear frictions. A rise in aggregate frictionless employment places more firms on the hiring margin, where employment is set below its frictionless counterpart, and fewer firms on the separation margin, where employment exceeds its frictionless level. Both forces serve to attenuate the response of flow steady state aggregate employment relative to the fixed costs case. Proposition 2 shows that, to a first order, this attenuation offsets exactly the overshooting of the flow steady state in the fixed costs case.

Figures 3 and 4 show that the result of Proposition 2 is mirrored in numerical simulations of models that allow the presence of more plausible idiosyncratic shocks x, and fully stochastic aggregate shocks p. These numerical results are based on the same methods and baseline parameterization described in section 1.3.

Figure 3 illustrates impulse responses of actual, frictionless and flow steady state aggregate employment in the presence of *symmetric* linear frictions where $c^+ = c^-$. As before, each panel of Figure 3 successively raises the friction to produce increasingly higher average rates of inaction in employment adjustment. As foreshadowed by Proposition 2, although the response of actual employment becomes progressively more sluggish as the friction rises, the response of flow steady state employment lies very close to the frictionless path.

Figure 4 in turn reveals that this result is unimpaired by the presence of *asymmetric* frictions, as suggested by Proposition 2. Its first three panels report results for successively higher hiring costs, $c^{+} > 0$ and $c^{-} = 0$; the latter three panels do the same for firing costs, $c^{-} > 0$ and $c^{+} = 0$. Strikingly, it is hard to discern differences between the impulse responses for hiring and firing costs, and between these and the impulse response for the symmetric case in Figure 3.

The message of Figures 3 and 4, then, is that the insight of Proposition 2 is robust to empirically reasonable parameterizations of canonical models of linear frictions. This reinforces the message of section 1 that the flow steady state is indeed a useful diagnostic for the path of frictionless employment, and thereby for the aggregate effects of canonical frictions.

However, Proposition 2 does not allow the adjustment triggers to vary, since these are independent of $\Delta\mathcal{N}^{*}$ under the time-invariant linear frictions we have consider thus far. This is a key distinction with respect to models of search frictions, to which we now turn.

1.5 Search costs

The canonical Diamond-Mortensen-Pissarides (DMP) model of search frictions, in which a single firm matches with a single worker, can be extended to a setting with "large" firms that operate a decreasing-returns-to-scale production technology (Acemoglu and Hawkins 2014; Elsby and Michaels 2013). The presence of search frictions implies two modifications to the canonical linear cost model studied above.

First, search frictions induce a *time-varying* per-worker hiring cost. Hiring is mediated through vacancies, each of which is subject to a flow cost c, and is filled with a probability q that depends on the aggregate state of the labor market. Under a law of large numbers, the effective per-worker hiring cost is thus c/q, which varies over time with variation in the vacancy-filling rate q. The typical firm's problem therefore takes the form:

$$\Pi(n_{-1}, x) \equiv \max_{n}\left\{pxn^{\alpha} - w(n,x)n - \frac{c}{q}\Delta n^{+} + \beta\mathbb{E}[\Pi(n,x')|x]\right\}. \tag{16}$$

Second, search frictions induce *ex post* rents to employment relationships over which a firm and its workers may bargain. In an extension of the bilateral Nash sharing rule invoked in standard one-worker-one-firm search models, Elsby and Michaels (2013) show that a marginal surplus-sharing rule proposed by Stole and Zwiebel (1996) implies a wage equation of the form

$$w(n,x) = \eta\frac{px\alpha n^{\alpha-1}}{1-\eta(1-\alpha)} + (1-\eta)\omega. \tag{17}$$

Here $\eta \in [0,1]$ indexes worker bargaining power, and ω is the annuitized value of the threat point faced by workers. Bruegemann, Gautier and Menzio (2015) show that the marginal surplus-sharing rule underlying (17) can be derived from an alternating-offers bargaining game between a firm and its many workers in which the strategic position of each worker in the firm is symmetric.

As before, we consider a version of the search model with a tractable homogeneity property. Specifically, we study the case in which the friction, embodied in the vacancy cost, is proportional to the workers' outside option ω.[6] The normalized posting cost is $\gamma \equiv c/\omega$. Under these assumptions, the Appendix shows that the homogeneity properties of Lemma $1'$ continue to hold, with one exception: although the adjustment triggers remain linear, they no longer are time-invariant, for the simple reason that the friction varies with the aggregate state.

The presence of time-varying adjustment triggers modifies Proposition 2 as follows:

Proposition 3 *Assume (i) firms are sufficiently patient, $\beta \approx 1$; (ii) frictions are sufficiently small, $\gamma^2 \approx 0$; and (iii) the distribution of ε_x is symmetric. Then, relative to a prior constant-$\mathcal{N}^*$ state, a small change in aggregate log mandated employment, $\Delta\mathcal{N}^*$, induces on impact the following change in aggregate log flow steady state employment,*

$$\Delta\widehat{\mathcal{N}} \approx \frac{1-\epsilon_\omega}{1-\epsilon_{w^*}}\Delta\mathcal{N}^*, \tag{18}$$

where ϵ_ω and ϵ_{w^*} are the elasticities of ω and frictionless wages w^* to aggregate productivity p.

Proposition 3 shows that the response of aggregate log flow steady-state employment $\widehat{\mathcal{N}}$ still approximates the response of aggregate log mandated employment $\mathcal{N}^*$, under a few additional restrictions. We argue in what follows that these restrictions are plausible.

The first two restrictions—that firms are patient, and that frictions are sufficiently small—are quantitative. We address their plausibility by examining results from a numerical model that does not impose these restrictions. As above, this model sets the discount factor β to match an annual interest rate of 4 percent, and sets the vacancy cost c to replicate the average probability of employment adjustment. The numerical results will thus address the extent to which β is close enough to one, and the friction sufficiently small, for the insight of Proposition 3 to hold.

The third restriction concerns the symmetry of the distribution of idiosyncratic shocks. This can be justified along two grounds. First, as foreshadowed in the quantitative analyses of preceding sections, it is conventional to implement shock processes with symmetrically distributed—typically Normal—innovations. In addition to its being commonplace, it is also consistent with the observed pattern of employment adjustment, which is close to symmetric (see Davis and Haltiwanger 1992, and Elsby and Michaels 2013, among others).

These three restrictions aid the proof of Proposition 3, which is based on symmetry. If the firm is sufficiently patient ($\beta \approx 1$), the cost of hiring in the current period implies an equal cost of firing in the subsequent period. As a result, one can show that the optimal policy is symmetric, to a first-order approximation around $\gamma = 0$, as long as the driving force ε_x is symmetric. In

[6] This can be motivated through the presence of a dual labor market in which recruitment is performed by workers hired in a competitive market, who are paid according to the annuitized value of unemployment ω.

terms of the notation of Lemma $1'$, the upper and lower adjustment triggers satisfy $\ln u \approx -\ln l$, and move by approximately the same amount in response to a shift in aggregate productivity.

The final restriction necessary for the change in aggregate log flow steady-state employment $\Delta\widehat{\mathcal{N}}$ to approximate its frictionless counterpart $\Delta\mathcal{N}^*$ is that the flexibility of frictionless wages w^* mirrors the flexibility of workers' outside option in the presence of frictions ω, $\epsilon_{w^*} = \epsilon_\omega$.

We parameterize these elasticities as follows. In the frictionless case, ϵ_{w^*} is related to the Frisch elasticity of labor supply. Kimball and Shapiro (2003) use a model of household labor supply to infer the latter from long-run labor supply responses to wealth changes.[7] We apply their estimate and set the Frisch labor supply elasticity equal to one. This implies $\epsilon_{w^*} = 1/(2-\alpha) \approx 0.735$ when α is set to equal 0.64.

To calibrate ϵ_ω, we take the model with search frictions as the data-generating process for observed short-run fluctuations. Once account is taken of biases associated with the shifting composition of employment over the business cycle, microdata-based estimates suggest that real wages are about as cyclical as employment (Solon, Barksy, and Parker 1994; Elsby, Solon, and Shin 2015). Accordingly, we set ϵ_ω to match an elasticity of average real wages with respect to aggregate employment approximately equal to one. We find that $\epsilon_\omega = 2/3$ hits this target.[8]

Note that Proposition 3 implies that the response of aggregate flow steady-state employment should bound that of frictionless employment under this parameterization, since $(1-\epsilon_\omega)/(1-\epsilon_{w^*}) \approx 1.25$. Figure 5 shows that this prediction of Proposition 3 is visible in numerical simulations of the model based on the same methods and baseline parameterization described in section 1.3—that is, with stationary idiosyncratic shocks x, and fully stochastic aggregate shocks p. The impulse responses in Figure 5 suggest that flow steady-state employment reacts slightly more on impact to the aggregate shock than its frictionless counterpart.

2. Empirical implementation

The previous section gave a theoretical rationale for an empirical diagnostic, flow steady state employment $\widehat{N}$, which provides a bound for the aggregate effects of a canonical class of labor market frictions. A key virtue of this diagnostic is that it can be measured with access to establishment panel data on employment. In this section, we apply these insights to a rich source of microdata from the United States.

2.1 Data

The data we use are taken from the Quarterly Census of Employment and Wages (QCEW). The QCEW is compiled by the Bureau of Labor Statistics (BLS) in concert with State Employment

[7] An advantage of Kimball and Shapiro's focus on long-run responses is that they are likely to be less sensitive to search frictions.

[8] The level of ω is chosen to induce an average establishment size of 20, consistent with data from County Business Patterns. The other new parameter to set is η. This is chosen to be 0.4 to induce a realistic labor share.

Security Agencies. The latter collect data from all employers in a state that are subject to the state's Unemployment Insurance (UI) laws. Firms file quarterly UI Contribution Reports to the state agency, which provide payroll counts of employment in each month. These are then aggregated by the BLS, which defines employment as the total number of workers on the establishment's payroll during the pay period that includes the 12th day of each month. Following BLS procedure, we define quarterly employment as the level of employment in the third month of each quarter.[9]

From the cross-sectional QCEW data, the BLS constructs the Longitudinal Database of Establishments (LDE), which we use in what follows. Although data are available for the period 1990Q1 to 2014Q2, we restrict attention to data from 1992Q1 due to difficulty in matching establishments in the first two years of the sample.

Sample restrictions. The QCEW data are a near-complete census of workers in the United States, covering approximately 98 percent of employees on non-farm payrolls. The dotted line in Figure 6 plots the time series of log aggregate employment in private establishments in the full QCEW sample. Relative to this full sample we apply three further sample restrictions, illustrated by the successive lines in Figure 6.

First, access for outside researchers to QCEW/LDE microdata is restricted to a subset of forty states that approve access onsite at the BLS for external research projects. As a result, our sample excludes data for Florida, Illinois, Massachusetts, Michigan, Mississippi, New Hampshire, New York, Oregon, Pennsylvania, Wisconsin, and Wyoming.

Second, we restrict our sample to continuing establishments with positive employment in consecutive quarters. Specifically, we construct a set of overlapping quarter-to-quarter balanced panels that exclude births and deaths of establishments *within* the quarter. Note that we do not balance *across* quarters, so births in a given panel will appear as incumbents in the subsequent panel (if they survive). We focus on continuing establishments because the canonical models of adjustment frictions analyzed above are intended to describe adjustment patterns among incumbent firms.[10]

Our final sample restriction is to exclude establishments with more than 1000 employees in consecutive quarters. We do this for practical reasons. To measure the flow steady state employment distribution in equations (4) and (13), and hence the diagnostic suggested by the theory, we require measures of establishment flows between points in the firm size distribution—specifically, inflows of mass to each employment level, and the probability of outflow. To measure the latter with sufficient precision requires sufficient sample sizes at all points in the

[9] The count of workers includes all those receiving any pay during the pay period, including part-time workers and those on paid leave.

[10] An additional, more practical reason for focusing on continuers is that the matching of establishments over time is more subject to error when identifying births and deaths in the QCEW. Among other steps, the latter involves identification of predecessor and successor establishments to detect spurious births or deaths. By excluding establishment births and deaths, our sample excludes establishments that require linkage using predecessor or successor information. The latter is a very small subset of establishments, around 0.1 percent in 2014Q2.

distribution. Since establishments with more than 1000 employees comprise a very small fraction of U.S. establishments—less than 0.1 percent in 2014Q2—sample sizes become impracticably thin beyond 1000 employees, inducing substantial noise in implied estimates of our diagnostic.

Recall that our goal is to understand whether canonical models of labor market frictions can account for the dynamics of aggregate employment. A worry, then, is that the forgoing sample restrictions might alter the dynamics of aggregate employment in our sample relative to the full sample.

Figure 6 reveals that this is not the case. In terms of levels, the largest loss of sample size occurs because we are unable to access data for all states, accounting for around 30 percent of total employment in the United States. The further exclusion of non-continuing establishments and large establishments accounts, respectively, for around 2 percent and 10 percent of employment. However, Figure 6 shows that the path of aggregate employment in our sample resembles, in both trend and cycle, the path of aggregate employment in the full QCEW sample. The correlation between log aggregate employment in the published QCEW series for all states and that in our final microdata sample is 0.99.

Measurement. To estimate our diagnostic, we require first an estimate of what we refer to as the flow steady state distribution of employment, $\hat{h}(n)$. Rearranging equations (4) and (13), we can write the latter as

$$\hat{h}_t(n) = h_{t-1}(n) + \frac{\Delta h_t(n)}{\phi_t(n)}, \tag{19}$$

where t indexes quarters, $h_{t-1}(n)$ is the previous quarter's mass of establishments with employment n, $\Delta h_t(n) \equiv h_t(n) - h_{t-1}(n)$ is the quarterly change in that mass, and $\phi_t(n)$ is the fraction of establishments that adjusts away from an employment level of n in quarter t. Thus, estimation of $\hat{h}_t(n)$ requires only an estimate of the outflow adjustment probability $\phi_t(n)$, in addition to measures of the evolution of the firm size distribution $h_t(n)$.

The simplest approach to measuring $\phi_t(n)$ is to use our microdata to compute the fraction of establishments with n workers in quarter t that reports employment different from n in quarter $t+1$. As alluded to above in motivating our sample restrictions, however, a practical issue that arises is that sample sizes become small as n gets large, inducing sampling variation in estimates of $\phi_t(n)$.

We address this issue by discretizing the employment distribution at large n. An advantage of the substantial sample sizes in the QCEW/LDE microdata is that we can be relatively conservative in this regard. In particular, we allow individual bins for each integer employment level up to 250 workers. In excess of 99 percent of establishments lie in this range, and so sample sizes in each bin are large, between about 100 and 1.3 million establishments. For establishment sizes of 250 through 500 workers we use bins of length five, allowing us to maintain sample sizes above about 80 establishments in each quarter. Further up the distribution, of course, sample sizes get smaller, so we extend our bin length to ten for employment levels between 500 and 999 workers. In this range, sample sizes are at least 15 establishments in each quarter.

Denoting these bins by b, we estimate the firm size mass and the outflow probability as

$$h_t(b) = \sum_i \mathbb{I}[n_{it} \in b]\text{, and } \phi_t(b) = \frac{\sum_i \mathbb{I}[n_{it} \notin b | n_{it-1} \in b]}{\sum_i \mathbb{I}[n_{it-1} \in b]}, \tag{20}$$

where i indexes establishments. We use these measures to compute the flow steady state mass in each bin according to equation (19) as $\hat{h}_t(b) = h_{t-1}(b) + [\Delta h_t(b)/\phi_t(b)]$. Finally, we compute aggregate employment and its flow steady state counterpart by taking the inner product of h_t and $\hat{h}_t$ with the midpoints of each bin, denoted m_b,

$$N_t = \sum_b m_b h_t(b)\text{, and } \hat{N}_t = \sum_b m_b \hat{h}_t(b). \tag{21}$$

2.2 Inferring the aggregate effects of frictions

With this estimate of flow steady state aggregate employment $\hat{N}_t$ in hand, we implement in this section three explorations of its empirical behavior in contrast to the path of actual aggregate employment N_t, and to the predictions of canonical frictions summarized in section 1.

Empirical time series. Our first, and simplest approach is to compare the paths of actual and flow steady state aggregate employment in the data. Under the class of frictions analyzed in section 1, the time series of $\hat{N}_t$ will *bound* the path of mandated aggregate employment N_t^* under fixed adjustment frictions, and will *approximate* N_t^* in the case of linear or search frictions. Thus a simple comparison of the empirical time paths of N_t and $\hat{N}_t$ provides, at least, a bound on the aggregate effects of these canonical frictions.

Figure 7 plots the time series of N_t and $\hat{N}_t$ derived from application of equation (21) to the QCEW/LDE microdata. Both series are expressed in log deviations from a quadratic trend.[11] Figure 7 reveals that $\hat{N}_t$ is a leading indicator of actual employment N_t, and is also more volatile. Specifically, the standard deviation of N_t is 0.025, whereas the standard deviation of $\hat{N}_t$ is 0.031.

However, on the whole, the differences between the two series appear pretty modest. The median (average) absolute difference is just 0.5 (0.8) log points. As Figure 7 indicates, the only substantial "daylight" between the series emerges in the Great Recession. For instance, in the 5 quarters that bracket the trough (2009q2) of the recession, the average difference between the series is about 3 log points. However, this difference is shortlived. Since 2010, the two series have moved in tandem: employment has increased 11.6 log points, whereas steady state has increased 11.9 log points.

[11] We use a quadratic trend rather than the HP filter, because it is consistent with how we detrend the same series to estimate impulse responses. As Ashley and Verbrugge (2006) have warned, the HP filter is problematic in this context because it uses *future* realizations to derive the cyclical component in the present period. As a result, lags of HP deviations included as regressors in a VAR are not predetermined with respect to the outcome variable.

Together these observations give a first suggestion that, even when viewed through the lens of canonical models of labor market frictions, the aggregate effects of such frictions account for a modest part of aggregate employment dynamics.

Dynamics of aggregate employment. Our quantitative illustrations in section 1 compared impulse responses of actual versus flow steady state employment in models with a variety of adjustment frictions. These emphasized that canonical frictions imply quite distinctive dynamics of flow steady state employment $\widehat{N}$. Even when actual employment N moves sluggishly, $\widehat{N}$ jumps on impact of an aggregate shock, and bounds the response of frictionless employment N^*.

In this section, we confront these theoretical predictions with the empirical dynamics of our measures of N and $\widehat{N}$. Our goal is not to use the data to estimate impulse responses of these variables to identified structural shocks. Instead, we undertake a descriptive analysis of the persistence properties of aggregate employment. A commonly used gauge for the latter is a comparison of the dynamics of employment relative to output-per-worker.

We formalize this comparison as follows. Denote log output-per-worker by y_t. In a first stage, we estimate innovations in y_t that are unforecastable conditional on lags of y, and lags of log aggregate employment N. Specifically, we use quarterly data on output-per-worker in the nonfarm business sector from the BLS Productivity and Costs release and our measure of actual employment from the QCEW to estimate the following AR(L) specification:

$$y_t = \alpha^y + \sum_{s=1}^{L} \beta_s^y y_{t-s} + \sum_{s=1}^{L} \gamma_s^y N_{t-s} + \delta_1^y t + \delta_2^y t^2 + \varepsilon_t^y. \tag{22}$$

Note that we account for trends in these series using a quadratic time trend.

The estimated residuals from this first-stage regression, $\hat{\varepsilon}_t^y$, are then used as the innovations to output-per-worker from which we derive impulse responses of actual and flow steady state employment in a second stage,

$$\begin{aligned} N_t &= \alpha^N + \sum_{s=0}^{L-1} \beta_s^N \hat{\varepsilon}_{t-s}^y + \sum_{s=1}^{L} \gamma_s^N N_{t-s} + \delta_1^N t + \delta_2^N t^2 + \varepsilon_t^N, \text{ and} \\ \widehat{N}_t &= \alpha^{\widehat{N}} + \sum_{s=0}^{L-1} \beta_s^{\widehat{N}} \hat{\varepsilon}_{t-s}^y + \sum_{s=1}^{L} \gamma_s^{\widehat{N}} \widehat{N}_{t-s} + \delta_1^{\widehat{N}} t + \delta_2^{\widehat{N}} t^2 + \varepsilon_t^{\widehat{N}}. \end{aligned} \tag{23}$$

Note that the timing in the lag structure of innovations to output-per-worker permits a *contemporaneous* relationship between these innovations and employment, as suggested by the model-based impulse responses described in section 1.

The estimates from the regressions in equations (22) and (23) allow us to trace out the dynamic relationship between each measure of log aggregate employment and a one-log-point innovation in output-per-worker. In practice, we use a lag order of $L = 4$ in both stages, (22) and (23), and obtain estimates using the data for the full sample period, 1992Q2 to 2014Q2.[12]

[12] The precise peak of the hump shaped impulse responses vary slightly across different lag lengths, but Figure 8 is representative of results across a number of specifications.

Panel A of Figure 8 plots the results. The dynamic response of aggregate employment takes a familiar shape, rising slowly after the innovation with a peak response of around 1 log point after five quarters. These hump-shaped dynamics mirror similar results found using different methods elsewhere in the literature (Blanchard and Diamond 1989; Fujita and Ramey 2007; Hagedorn and Manovskii 2011). This is one representation of the persistence of aggregate employment.

As suggested by the time series in Figure 7, the dynamics of the flow steady state diagnostic $\widehat{N}$ share many of these properties. Although its peak response occurs earlier—after three quarters—reinforcing the impression of Figure 7 that $\widehat{N}$ is a leading indicator of the path of N, it exhibits a similar volatility, and a clear hump-shape.

To contrast the empirical dynamics illustrated in Figure 8A with those implied by canonical models of frictions, we rerun the regressions in equations (22) and (23) using model-generated data. Specifically, we maintain the baseline parameterization from the quantitative illustrations in section 1, but with one important difference: We allow for a hybrid of fixed and linear adjustment frictions, and choose the configuration of these that minimizes the (sum of squares) distance between the empirical dynamics of actual employment N in Figure 8A and those implied by the model.

As foreshadowed by the theoretical results of Figures 2 and 3, we find that a pure linear cost model is best able to generate the substantial empirical persistence of aggregate employment.[13] To achieve this, however, the degree of inaction implied by the model must exceed its empirical analogue. Each quarter around 85 percent of firms do not adjust employment in the model, compared to 52 percent in the data. We are thus being generous to the model in allowing it to violate this moment of the data.

Panel B of Figure 8 reveals that this parameterization of the model is able to generate a dynamic relationship between employment and output-per-worker that is comparable to the data. Although the model overstates the impact response, the amplitude and persistence of employment are similar to their empirical counterparts.

A key result of Figure 8B, however, is that the model-implied dynamics of flow steady state employment are profoundly different from those seen in the data. Confirming the impression of the theoretical impulse responses in Figure 3, $\widehat{N}$ *jumps* in response to innovations in output-per-worker in the model, deviating substantially from the path of actual employment N. In marked contrast, the empirical dynamics of $\widehat{N}$ in Figure 8A are much more sluggish, bearing a closer resemblance to the empirical path of actual employment than its model-implied counterpart.

The substantial discrepancy between the implied and observed dynamics of flow steady state employment is an important failure of canonical models of frictions. Under these models, the path of flow steady state employment is closely related to the path of frictionless employment. The gap between the paths of N and $\widehat{N}$ is therefore indicative of the aggregate effects of these

[13] We do not pursue the effects of asymmetries in the adjustment costs here: the theoretical results of Propositions 1 and 2, and the quantitative results of Figure 4, suggest that any such asymmetries affect neither the dynamics of aggregate employment, nor its flow steady state counterpart.

canonical frictions. That this gap is relatively small in the data therefore implies that canonical frictions play a minor role in generating the observed dynamics of aggregate employment.

Dynamics of the firm size distribution. To examine the origins of this failure of canonical models, recall that the link between our diagnostic flow steady state employment $\widehat{N}$ and frictionless employment N^* is mediated through the behavior of firm size flows—the τs and ϕs of equations (4) and (13)—and that canonical frictions have strong predictions regarding the dynamics of these flows by establishment size.

As we have emphasized, a key benefit of the data is that we can measure aspects of these flows using the longitudinal dimension of the QCEW microdata—specifically the total inflow to, and the probability of outflow from, each employment level. Our next exercise, therefore, is to contrast the dynamics of the firm size distribution in the data to those implied by canonical models of frictions.

To do this, we first split establishments in the data into three size classes. We choose these to correspond to the lower quartile (fewer than 15 employees), interquartile range (16 to 170 employees), and upper quartile (171 employees and greater) of establishment sizes. We then estimate descriptive impulse responses that mirror equations (22) and (23) for the total inflow to, and probability of outflow from, each size class.[14] As in our previous analysis of the dynamics of aggregate employment, we repeat these same steps using data simulated from the model underlying Figure 8B that is calibrated to match as closely as possible the empirical dynamics of aggregate employment.

Panels C through F of Figure 8 illustrate the results of this exercise. The empirical and model-implied dynamics share a qualitative property, namely that positive aggregate shocks render small (large) establishments more (less) likely to adjust away from their current employment, and induce fewer (more) establishments to adjust to low (high) employment levels.

Aside from this broad qualitative similarity, the quantitative dynamics reveal striking contrasts. The empirical behavior of firm size flows exhibits an inertia not only in the sense that their *levels* are retarded relative to a frictionless environment, but also in the sluggishness of their *responses* to aggregate disturbances.

We highlight three manifestations of this general observation. First, note that the empirical responses of the firm size flows in Figures 8C and 8E are an order of magnitude smaller than their theoretical counterparts in Figures 8D and 8F. Second, the dynamics of the flows in the data are much more sluggish than implied by canonical frictions. Firm size dynamics in the model respond aggressively on impact of the aggregate shock. In the data, the response is mild and delayed. Third, the empirical dynamics reveal an establishment size gradient in the magnitude of the response of firm size flows: Flows to and from smaller establishments respond less than their counterparts for larger establishments.

[14] To aggregate within a quartile range, we take a weighted average across establishment sizes, where the weight is the size's share of all establishments in the range.

The upshot of this exercise is that canonical models of labor market frictions do a poor job of capturing the empirical dynamics of the firm size distribution. Since the latter is the key channel through which canonical frictions are supposed to impede aggregate employment dynamics, this is an important limitation of this class of model.

Matching the time series of aggregate employment. Our final exercise is inspired by an approach devised by King and Rebelo (1999) and Bachmann (2012). They show that it is possible to find a sequence of aggregate shocks that generates a path for aggregate model-generated outcomes—in our case employment—that matches an empirical analogue. In what follows, we use this technique to contrast the time series of flow steady state employment in model and data when the path of aggregate employment in each is constructed to be the same.

The procedure relies on the ability to summarize the dynamics of aggregate employment implied by the model using a simple aggregate law of motion. In a related adjustment cost model, Bachmann shows that an AR(1) specification does an excellent job of summarizing the dynamics of (log) aggregate employment. We find that the same property holds for our model.

Specifically, we initiate an algorithm with the model underlying Panels B, D and F of Figure 8. Following Bachmann, in a first step we use this model to generate 89 quarters of simulated data (the same time span as in the data). Second, we estimate via OLS the following AR(1) process that relates log aggregate employment to its lag and current total factor productivity p_t,

$$N_t = \hat{v}_0 + \hat{v}_1 N_{t-1} + \hat{v}_2 p_t. \tag{24}$$

With estimates of equation (24) in hand, it is possible to back out a series for productivity $\{p_t\}$ that replicates the path of log aggregate employment $\{N_t\}$. Since the resultant sequence $\{p_t\}$ may not be consistent with the data generating process assumed, in a final step we re-estimate the productivity process and re-initialize the model with this updated process. These steps are repeated until the moments of the implied productivity series are consistent with the parameterization assumed. In practice, the AR(1) specification in (24) fits the data closely (the R-squared of the regression is 0.9985), and so the algorithm converges quite quickly, after just a few iterations.[15]

Figure 9 illustrates the results. The model yields an aggregate steady state employment notably more variable than its empirical counterpart. By the trough of the two NBER-dated recessions, the model-generated steady state employment has fallen 5 log points more, for instance. In the wake of the downturns, the model's steady state employment also recovers more quickly. In the 6-8 quarters after the Great Recession, for instance, the model's steady state employment rises almost 10 log points. Its empirical counterpart increases by less than half that in that period.

[15] The i.i.d. innovations of the implied productivity process have standard deviation 0.0062, which is more than twice that used in the process that underlies Figures 2-4. But, throughout Figures 2-4, we wanted to hold the aggregate productivity process fixed, and a standard deviation of 0.0062 would yield much larger fluctuations in model-generated employment than ever seen in the data.

3. Summary and discussion

[To be completed.]

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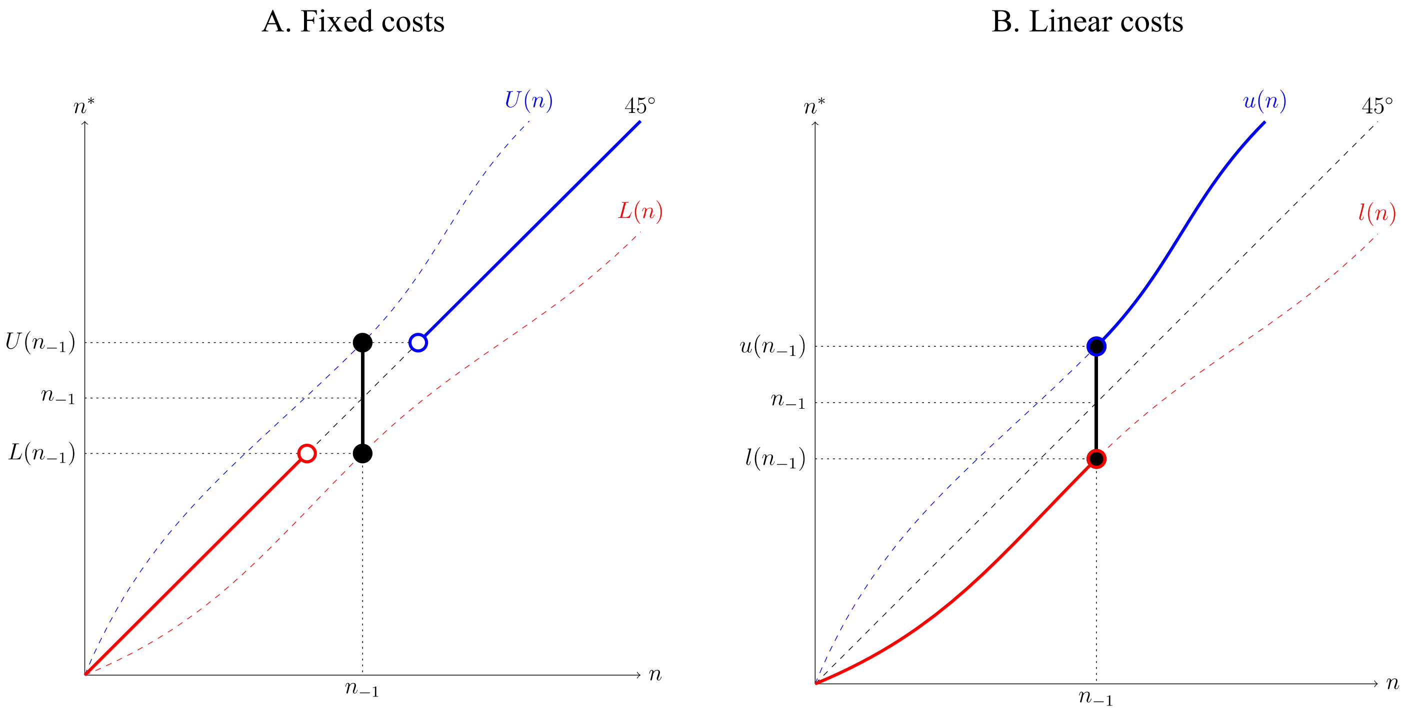
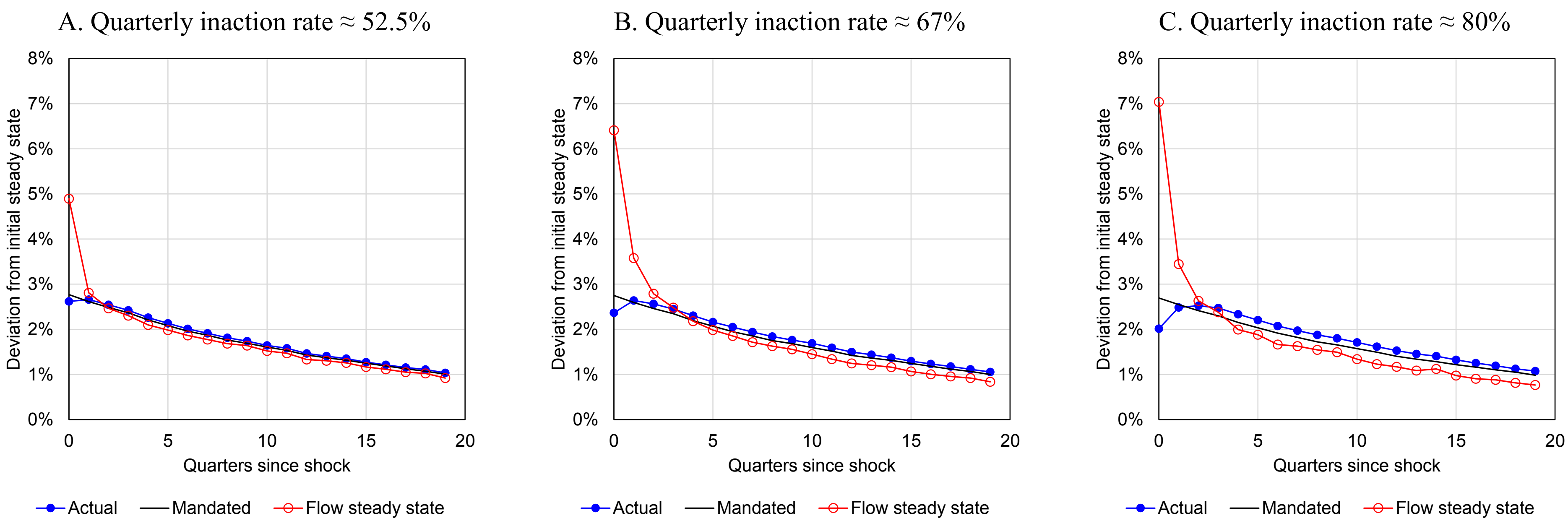
Figure 1. *Ss* policies in the presence of fixed and linear adjustment frictions



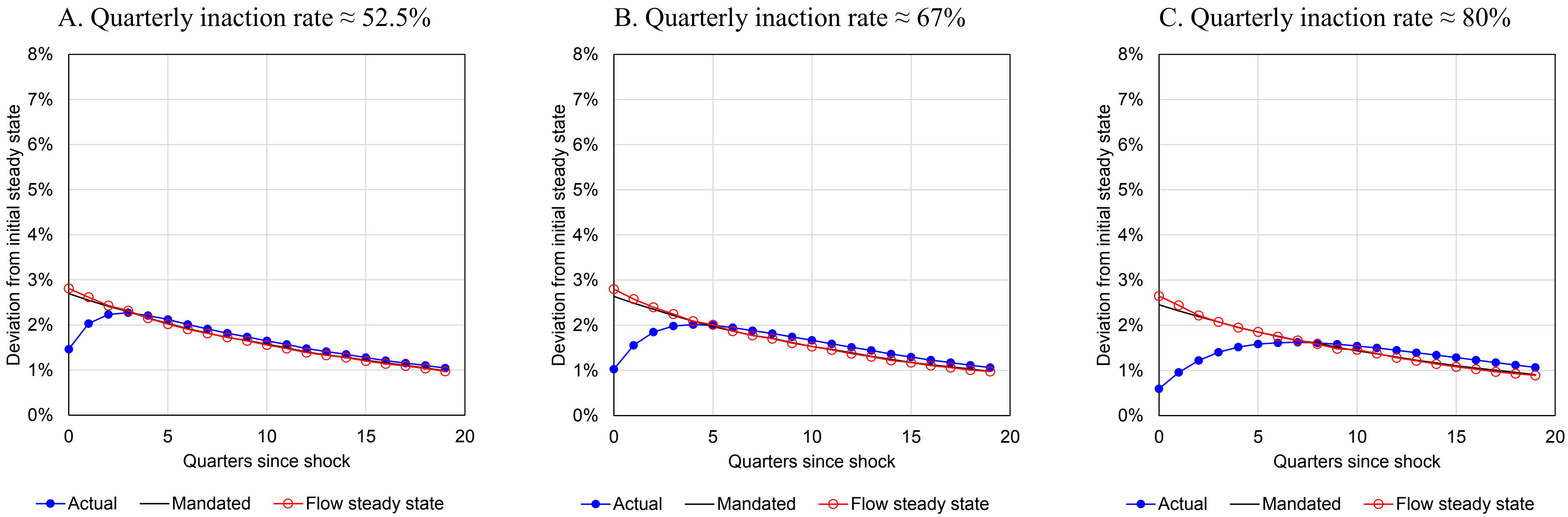
Notes:

Figure 2. Impulse responses of aggregate employment in selected parameterizations: Fixed costs



Notes:

Figure 3. Impulse responses of aggregate employment in selected parameterizations: Linear costs

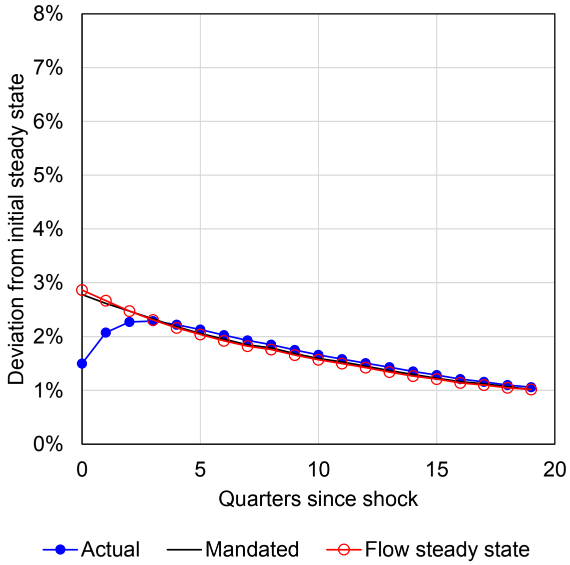


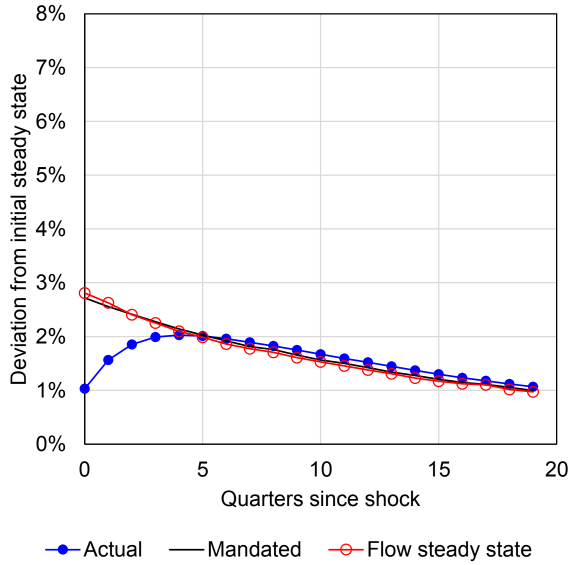
Notes:

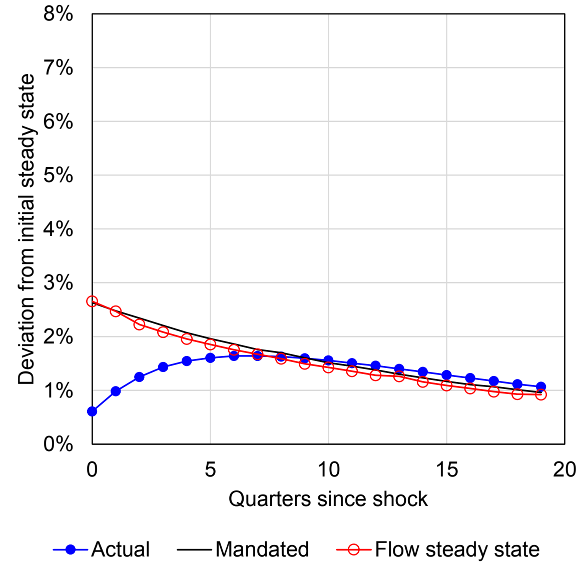
Figure 4. Impulse responses of aggregate employment in selected parameterizations: Asymmetric linear costs

A. Quarterly inaction rate ≈ 52.5% | B. Quarterly inaction rate ≈ 67% | C. Quarterly inaction rate ≈ 80%

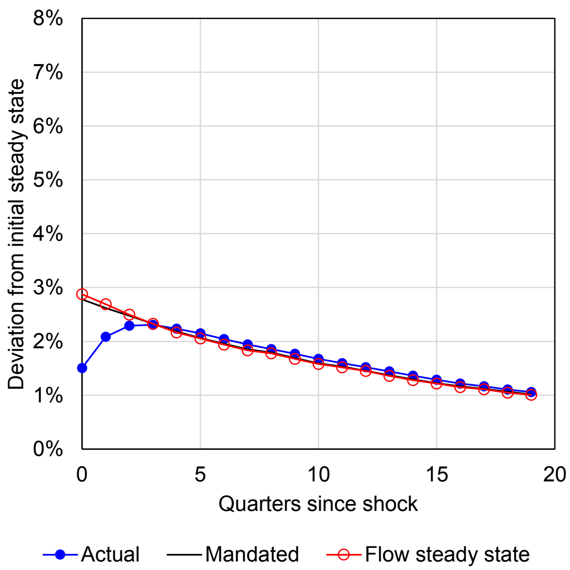
i. Pure hiring cost

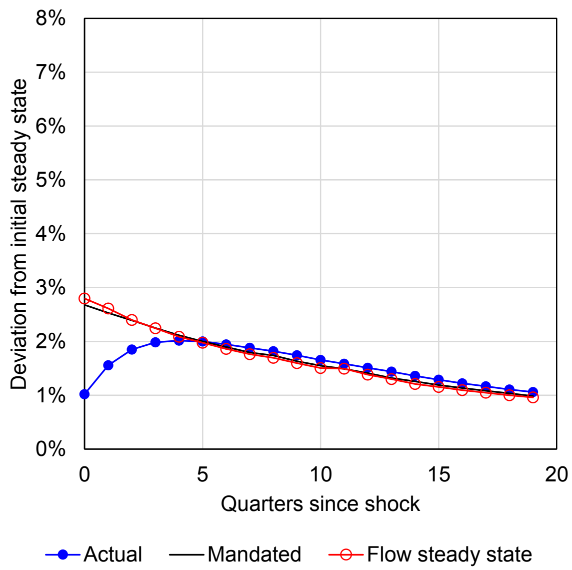






ii. Pure firing cost





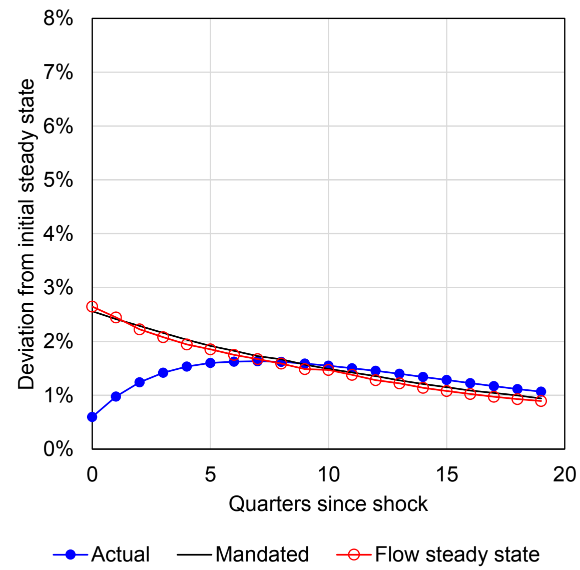
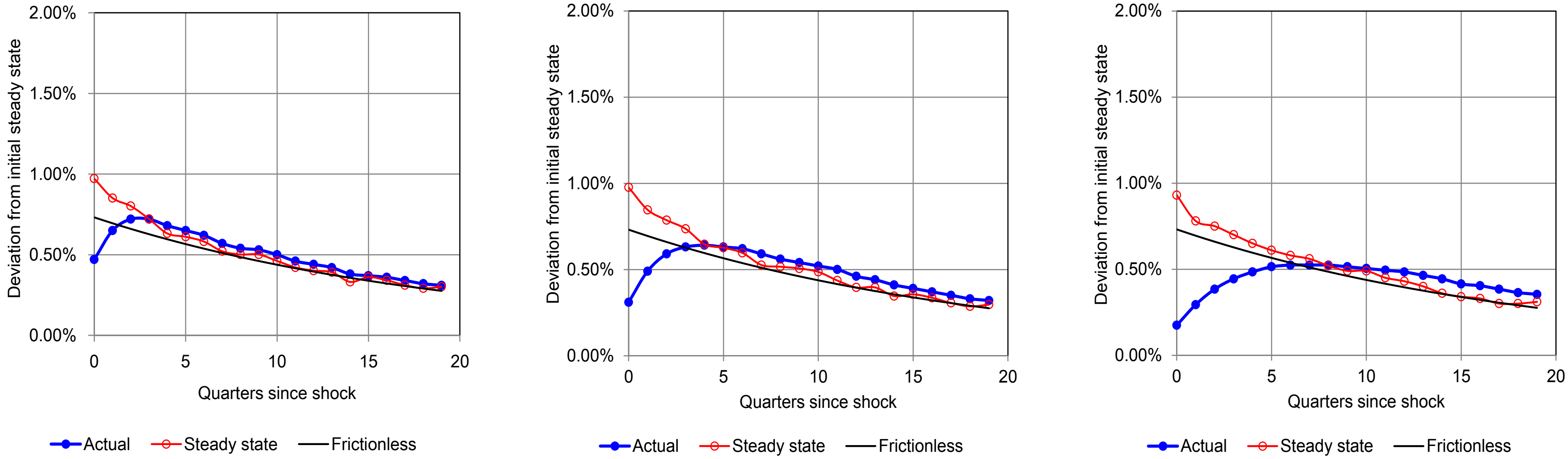


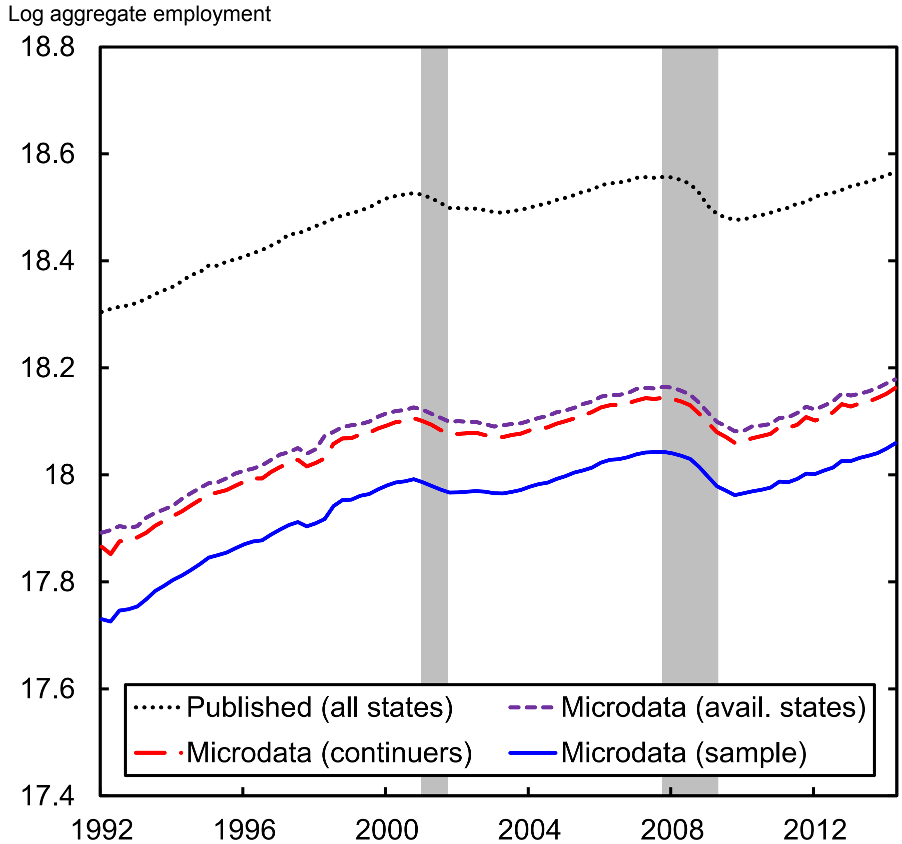
Figure 5. Impulse responses of aggregate employment in selected parameterizations: Search costs

A. Quarterly inaction rate ≈ 52.5%   B. Quarterly inaction rate ≈ 67%   C. Quarterly inaction rate ≈ 80%



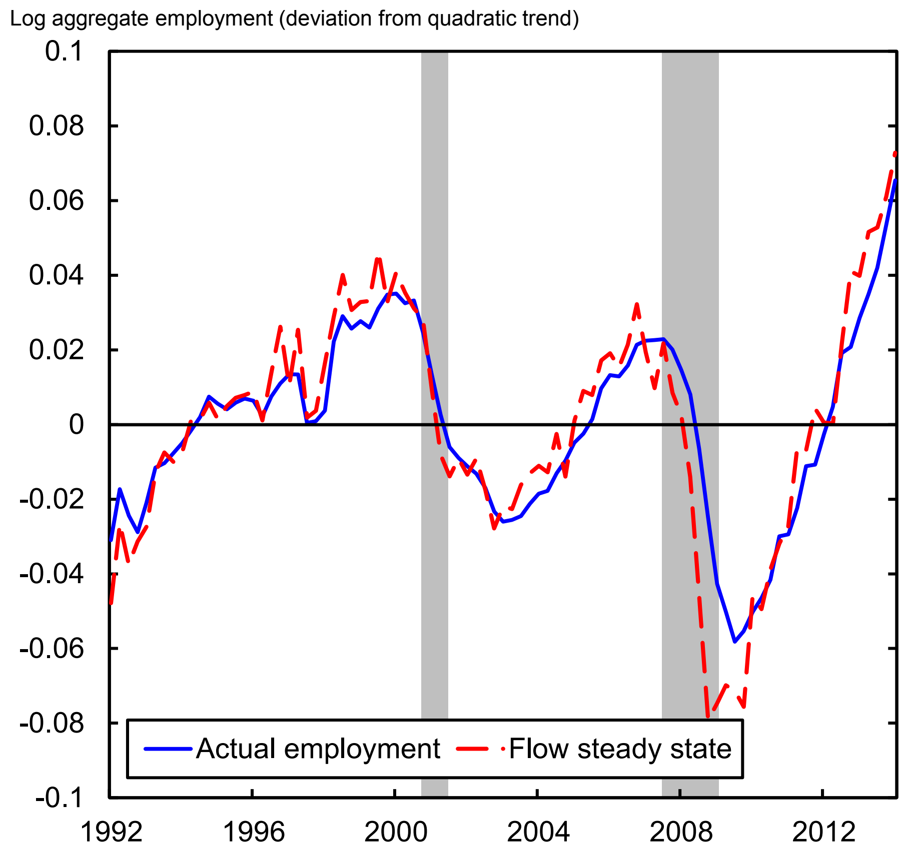
Notes:

Figure 6. Aggregate employment in the QCEW by sample restriction



Notes:

Figure 7. Actual and flow steady state log aggregate employment



Notes:

Figure 8. Descriptive impulse responses of employment and firm size flows: Data versus model

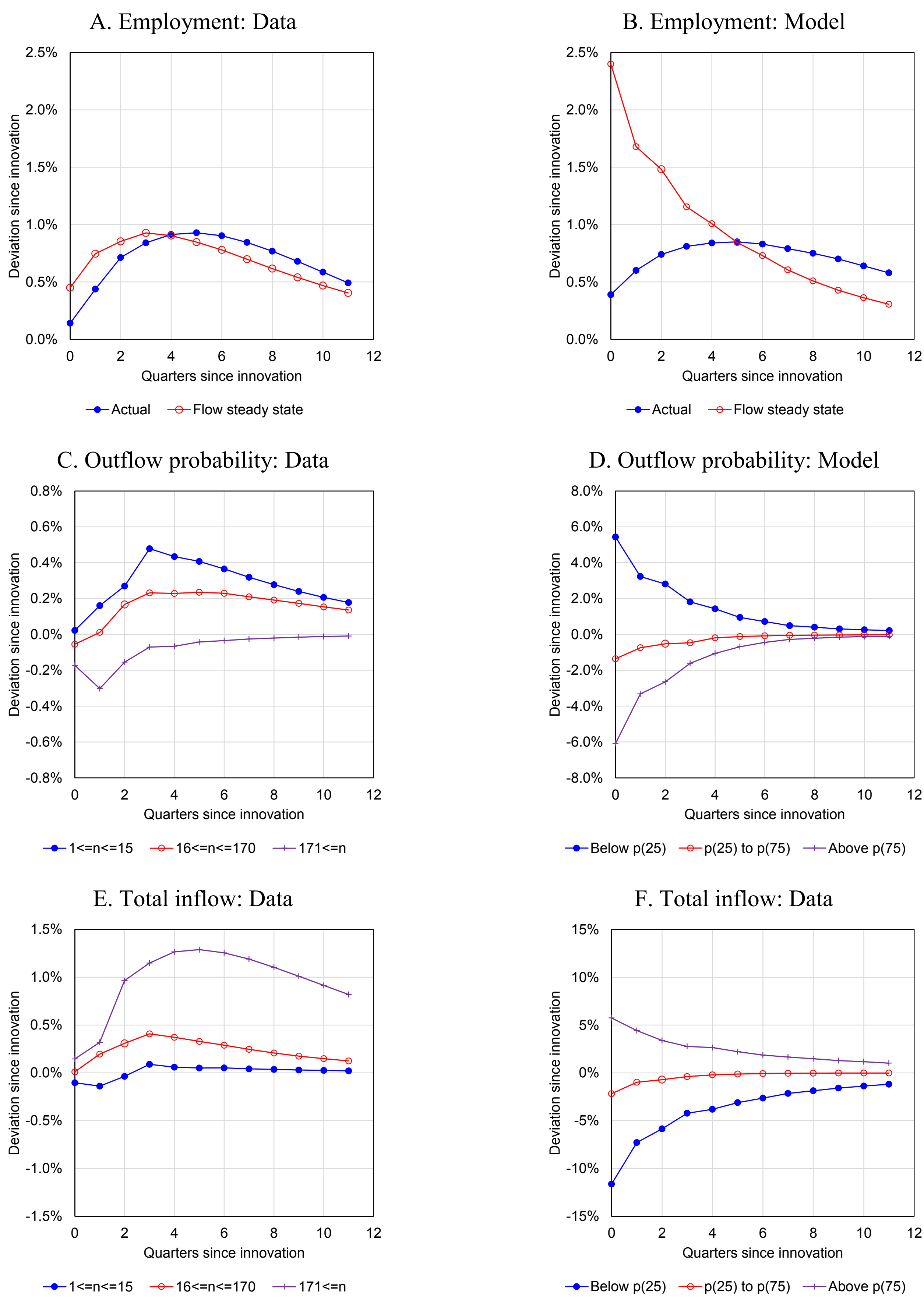
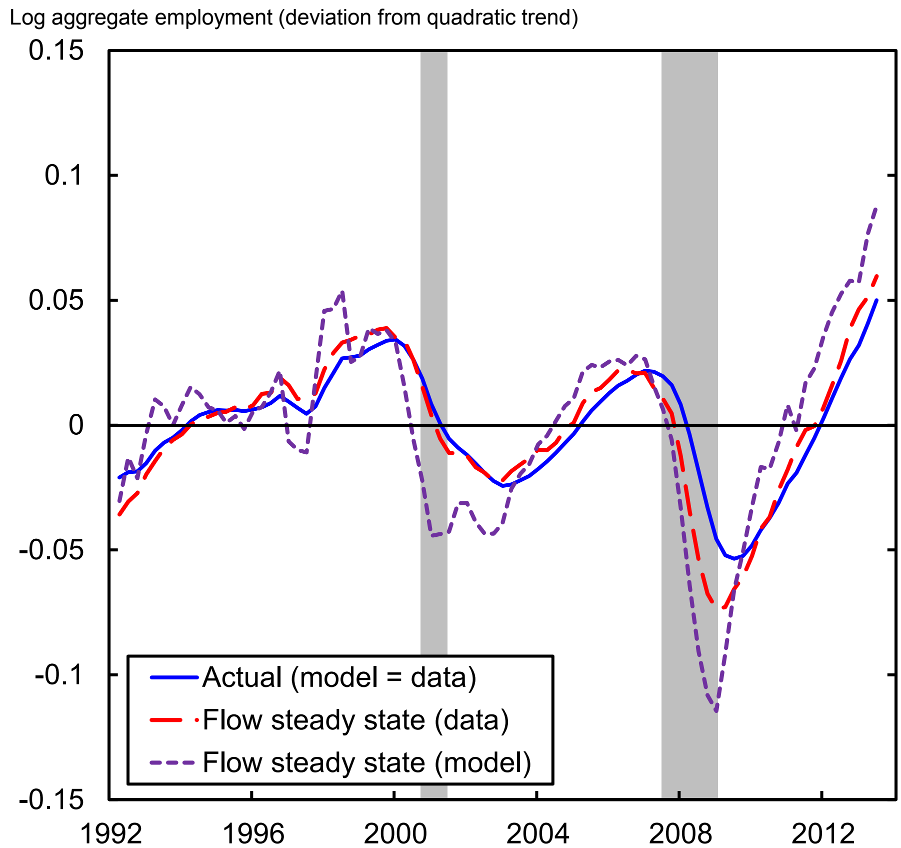


Figure 9. Model-implied time series for flow steady state log aggregate employment



Appendices

A. Laws of motion for the firm size distribution

To derive the laws of motion for the density of employment across firms stated in the main text for the fixed and linear cost models, we require notation for several distributions. As in the main text, we denote the densities of employment, lagged employment and mandated employment by h, h_{-1} and h^*. Conventionally, we will refer to their respective distribution functions by analogous upper-case letters, H, H_{-1} and H^*. In addition, however, we require notation for the distributions of mandated employment conditional on lagged employment, which we denote by $\mathcal{H}^*(\xi|\nu) = \Pr(n^* < \xi|n_{-1} = \nu)$, and the distribution of lagged employment conditional on mandated employment, denoted by $\mathcal{H}(\nu|\xi) = \Pr(n_{-1} < \nu|n^* = \xi)$. Of course, the latter are related by Bayes' rule, $\hbar(\nu|\xi)h^*(\xi) = \hbar^*(\xi|\nu)h_{-1}(\nu)$, where lower-case script letters denote associated density functions. However, we preserve separate notation where it aids clarity.

With this notation in hand, we can use the labor demand policy rules—(1) for the fixed costs case, (10) for the linear costs case—to construct a law of motion for the distribution function of actual employment $H(n)$ implied by each type of friction. We then show how these imply the laws of motion for the density $h(n)$ stated in equations (2) and (11) in the main text.

Fixed costs. Consider a point m in the domain of the employment distribution. We wish to derive the flows in and out of the mass $H(m)$. To do this, we first derive the flows for a given lagged employment level n_{-1}. Then inflows into $H(m)$ are summarized as follows:

1) If $m < L(n_{-1})$, or equivalently $n_{-1} > L^{-1}(m)$, then the inflow is equal to $\mathcal{H}^*(m|n_{-1})$.
2) If $m \in [L(n_{-1}), n_{-1})$, or equivalently $n_{-1} \in (m, L^{-1}(m)]$, then the inflow is equal to $\mathcal{H}^*(L(n_{-1})|n_{-1})$.

Likewise, the outflows from $H(m)$ for a given n_{-1} can be evaluated as:

3) If $m \in (n_{-1}, U(n_{-1})]$, or equivalently $n_{-1} \in [U^{-1}(m), m)$, then the outflow is equal to $1 - \mathcal{H}^*(U(n_{-1})|n_{-1})$.
4) If $m > U(n_{-1})$, or equivalently $n_{-1} < U^{-1}(m)$, then the outflows is equal to $1 - \mathcal{H}^*(m|n_{-1})$.

The latter are the flows of mass for a *given* lagged employment, n_{-1}. Integrating with respect to the distribution of lagged employment $H_{-1}(n_{-1})$ recovers the aggregate flows and thereby the law of motion for $H(m)$,

$$\begin{aligned} \Delta H(m) = &\int_{L^{-1}(m)} \mathcal{H}^*(m|n_{-1}) dH_{-1}(n_{-1}) + \int_{m}^{L^{-1}(m)} \mathcal{H}^*(L(n_{-1})|n_{-1}) dH_{-1}(n_{-1}) \\ &- \int_{U^{-1}(m)}^{m} [1 - \mathcal{H}^*(U(n_{-1})|n_{-1})] dH_{-1}(n_{-1}) \\ &- \int^{U^{-1}(m)} [1 - \mathcal{H}^*(m|n_{-1})] dH_{-1}(n_{-1}). \end{aligned} \tag{25}$$

Linear costs. Likewise, one can use the adjustment rule for the linear costs case, (10), to construct an analogous law of motion for H under this friction. Again, we first fix a given level of lagged employment, n_{-1}, and evaluate inflows to, and outflows from, $H(m)$.

These flows are simpler in the linear costs case. Inflows are given by the following case:

1) If $m < n_{-1}$, or equivalently $n_{-1} > m$, then the inflow is equal to $\mathcal{H}^*(l(m)|n_{-1})$.

Similarly, outflows are given by:

2) If $m > n_{-1}$, or equivalently $n_{-1} < m$, then the outflow is equal to $1 - \mathcal{H}^*(u(m)|n_{-1})$.

Following the same logic as in the fixed costs case above, the law of motion for $H(m)$ is thus given by

$$\Delta H(m) = \int_m \mathcal{H}^*(l(m)|n_{-1}) dH_{-1}(n_{-1}) - \int^m [1 - \mathcal{H}^*(u(m)|n_{-1})] dH_{-1}(n_{-1}). \quad (26)$$

Laws of motion for $\boldsymbol{h(n)}$. Differentiating (25) and (26) with respect to m, cancelling terms, and using Bayes' rule to note that $\int_0^\nu \hbar^*(\xi|n_{-1}) h_{-1}(n_{-1}) dn_{-1} = \int_0^\nu \hbar(n_{-1}|\xi) h^*(\xi) dn_{-1}$ yields the simpler laws of motion for the density of employment $h(n)$, equations (2) and (11) in the main text.

B. Proofs of Propositions

Proof of Lemmas 1 and 1′. Here we provide a proof of both versions of Lemma 1 reported in the main text. The proof closely follows results provided in Caballero and Engel (1999). Consider a firm that faces, possibly asymmetric, fixed *and* linear costs of adjustment. The firm's problem is

$$\begin{aligned} \Pi(n_{-1}, x) \equiv \max_n \{ & pxn^\alpha - wn - C^+ \mathbb{I}[n > n_{-1}] - C^- \mathbb{I}[n < n_{-1}] - c^+ \Delta n^+ + c^- \Delta n^- \\ & + \beta \mathbb{E}[\Pi(n, x')|x] \}, \end{aligned} \quad (27)$$

where $\{C^-, C^+\}$ are the fixed costs of adjusting down and up, and $\{c^-, c^+\}$ the analogous linear costs. Frictionless employment solves the first-order condition $px\alpha {n^*}^{\alpha-1} - w = 0$.

Note that, since idiosyncratic shocks follow a geometric random walk, $\ln x' = \ln x + \varepsilon_x'$, so does frictionless employment, $\ln n^{*\prime} = \ln n^* + \varepsilon_{n^*}'$ where $\varepsilon_{n^*}' = \varepsilon_x'/(1-\alpha)$.

Let $C^{-/+} = \Gamma^{-/+} wn^*$ and $c^{-/+} = \gamma^{-/+} w$, define $z = n/n^*$ and $\zeta = n_{-1}/n^*$, and conjecture that $\Pi(n_{-1}, x) = wn^* \tilde{\Pi}(\zeta)$. Under the conjecture, one can write

$$\begin{aligned} \tilde{\Pi}(\zeta) = \max_z \Big\{ & \frac{z^\alpha}{\alpha} - z - \Gamma^+ \mathbb{I}[z > \zeta] - \Gamma^- \mathbb{I}[z < \zeta] - \gamma^+ (z - \zeta)^+ + \gamma^- (z - \zeta)^- \\ & + \beta \mathbb{E}\left[e^{\varepsilon_{n^*}'} \tilde{\Pi}\left(e^{-\varepsilon_{n^*}'} z \right) \right] \Big\}. \end{aligned} \quad (28)$$

We highlight two important aspects of (28). First, the expectation over the forward value is no longer conditional, since it is taken over ε_{n^*}', which is i.i.d. Second, the firm's problem is simplified to the choice of a number $z = n/n^*$ for each realization of the single state variable $\zeta = n_{-1}/n^*$.

An *Ss* policy will thus stipulate that $z = \zeta$ for intermediate values of $\zeta \in [1/U, 1/L]$, and will set $z = 1/u$ whenever $\zeta < 1/U$, and $z = 1/l$ whenever $\zeta > 1/L$. Mapping back into employment terms, we have

$$n = \begin{cases} n^*/u & \text{if} \quad n^* > U \cdot n_{-1}, \\ n_{-1} & \text{if} \quad n^* \in [L \cdot n_{-1}, U \cdot n_{-1}], \\ n^*/l & \text{if} \quad n^* < L \cdot n_{-1}. \end{cases} \tag{29}$$

Noting that the special case of fixed costs implies $u = l$, while pure linear costs imply $l = L < U = u$, establishes part a) of the result.

To establish part b), note that the probability of a desired log employment adjustment of size less than δ can be written, in general, as

$$\Pr(\ln(n^{*\prime}/n) < \delta | n) = \Pr(\varepsilon'_{n^*} < \delta + \ln z \,| n) = \int \Pr(\varepsilon'_{n^*} < \delta + \ln z \,| n, z)\, dZ(z|n), \tag{30}$$

where $Z(z|n)$ denotes the distribution function of z given n. In the context of the canonical model, however, (30) simplifies. First, ε'_{n^*} is independent of n since the former is i.i.d. Second, z is also independent of n. To see this, note first that if a firm adjusts this period, its choice of z is uninformed by n—it sets $z = 1/u$ or $z = 1/l$. If the firm sets $n = n_{-1}$ but adjusted *last* period, then it sets $\ln z = \ln n_{-1} - \ln n^* = \ln z_{-1} - \varepsilon_{n^*}$ and z_{-1} is $1/u$ or $1/l$. Thus, z is again independent of n. More generally, suppose the firm last adjusted T periods ago, that is, $n = n_{-1} = \cdots = n_{-T}$ and $z_{-T} = 1/u$ or $1/l$. Then, $\ln z = \ln n_{-T} - \ln n^* = \ln z_{-T} - \sum_{t=0}^{T-1} \varepsilon_{n^*_{-t}}$. Each term here is independent of $n = n_{-T}$. Equation (30) therefore collapses to

$$\Pr(\ln(n^{*\prime}/n) < \delta | n) = \int \Pr(\varepsilon'_{n^*} < \delta + \ln z \,| z)\, dZ(z), \tag{31}$$

which does not depend on n.

Proof of Proposition 1. Denoting log employment by $\mathcal{n}$, the adjustment rules take the form $L(\mathcal{n}) = \mathcal{n} - \lambda$ and $U(\mathcal{n}) = \mathcal{n} + \upsilon$ for $\lambda > 0$ and $\upsilon > 0$. The flow steady state density of log employment is then defined by

$$\hat{h}(\mathcal{n}) \equiv \frac{1 - \mathcal{H}(\mathcal{n} + \lambda | \mathcal{n}) + \mathcal{H}(\mathcal{n} - \upsilon | \mathcal{n})}{1 - \mathcal{H}^*(\mathcal{n} + \upsilon | \mathcal{n}) + \mathcal{H}^*(\mathcal{n} - \lambda | \mathcal{n})} h^*(\mathcal{n}), \tag{32}$$

where $\mathcal{H}^*(\xi|\nu) \equiv \Pr(\mathcal{n}^* < \xi | \mathcal{n}_{-1} = \nu)$ and $\mathcal{H}(\nu|\xi) \equiv \Pr(\mathcal{n}_{-1} < \nu | \mathcal{n}^* = \xi)$. The property of the canonical model noted in result b) of Lemma 1, that $\mathcal{n}^* - \mathcal{n}_{-1}$ is independent of $\mathcal{n}_{-1}$, implies that

$$\mathcal{H}^*(\xi|\nu) = \Pr(\mathcal{n}^* - \mathcal{n}_{-1} < \xi - \nu) \equiv \widetilde{\mathcal{H}}^*(\xi - \nu). \tag{33}$$

This implies that the probability of adjusting away from $\mathcal{n}$ is independent of $\mathcal{n}$,

$$1 - \mathcal{H}^*(\mathcal{n} + \upsilon | \mathcal{n}) + \mathcal{H}^*(\mathcal{n} - \lambda | \mathcal{n}) = 1 - \int_{-\lambda}^{\upsilon} \tilde{h}^*(z) dz \equiv \phi. \tag{34}$$

Now consider the probability of adjusting to $\mathcal{n}$. Using Bayes' rule, we can write this as

$$
\begin{aligned}
1-\mathcal{H}(n+\lambda|n)+\mathcal{H}(n-v|n) &= 1-\int_{n-v}^{n+\lambda} \hbar^*(n|\nu)\frac{h_{-1}(\nu)}{h^*(n)}d\nu \\
&= 1-\int_{n-v}^{n+\lambda} \tilde{\hbar}^*(n-\nu)\frac{h_{-1}(\nu)}{h^*(n)}d\nu \\
&= 1-\int_{-\lambda}^{v} \tilde{\hbar}^*(z)\frac{h_{-1}(n-z)}{h^*(n)}dz.
\end{aligned}
\tag{35}
$$

Piecing this together, we have

$$
\hat{h}(n)=\frac{h^*(n)-\int_{-\lambda}^{v}\tilde{\hbar}^*(z)h_{-1}(n-z)dz}{1-\int_{-\lambda}^{v}\tilde{\hbar}^*(z)dz}.
\tag{36}
$$

Multiplying both sides by n and integrating yields

$$
\begin{aligned}
\widehat{\mathcal{N}} \equiv \int_{-\infty}^{\infty} n\hat{h}(n)dn &= \frac{\mathcal{N}^*}{\phi}-\frac{1}{\phi}\int_{-\infty}^{\infty}\int_{-\lambda}^{v} n\tilde{\hbar}^*(z)h_{-1}(n-z)dzdn \\
&= \frac{\mathcal{N}^*}{\phi}-\frac{1}{\phi}\int_{-\lambda}^{v}\tilde{\hbar}^*(z)\int_{-\infty}^{\infty} nh_{-1}(n-z)dn\,dz \\
&= \frac{\mathcal{N}^*}{\phi}-\frac{1}{\phi}\int_{-\lambda}^{v}\tilde{\hbar}^*(z)(\mathcal{N}_{-1}+z)dz \\
&= \frac{\mathcal{N}^*}{\phi}-\frac{1-\phi}{\phi}\mathcal{N}_{-1}-\frac{1}{\phi}\int_{-\lambda}^{v} z\tilde{\hbar}^*(z)dz.
\end{aligned}
\tag{37}
$$

Since there is a constant-$\mathcal{N}^*$ state prior to the aggregate shock, aggregate log employment is constant and equal to aggregate flow steady state employment, $\mathcal{N}_{-1}=\mathcal{N}_{-2}=\widehat{\mathcal{N}}_{-1}$. Imposing this and solving for $\widehat{\mathcal{N}}_{-1}$ yields

$$
\widehat{\mathcal{N}}_{-1}=\mathcal{N}_{-1}^*-\int_{-\lambda}^{v} z\tilde{\hbar}_{-1}^*(z)dz.
\tag{38}
$$

Now consider a shock to aggregate log mandated employment, $\Delta\mathcal{N}^*$. On impact this will shift the mean of the distribution of desired employment adjustments, $\tilde{\hbar}^*(\cdot)$, by $\Delta\mathcal{N}^*$. Starting from the prior constant-$\mathcal{N}^*$ state, substitution of (38) into (37) implies

$$
\Delta\widehat{\mathcal{N}}=\frac{\Delta\mathcal{N}^*}{\phi}-\frac{1}{\phi}\int_{-\lambda}^{v} z\Delta\tilde{\hbar}^*(z)dz.
\tag{39}
$$

To a first-order approximation around $\Delta\mathcal{N}^*=0$,

$$
\begin{aligned}
\int_{-\lambda}^{v} z\Delta\tilde{\hbar}^*(z)dz &= \int_{-\lambda}^{v} z\tilde{\hbar}_{-1}^*(z-\Delta\mathcal{N}^*)dz-\int_{-\lambda}^{v} z\tilde{\hbar}_{-1}^*(z)dz \\
&\approx -\int_{-\lambda}^{v} z\tilde{\hbar}_{-1}^{*\prime}(z)dz\,\Delta\mathcal{N}^* \\
&= \left[1-\phi-v\tilde{\hbar}_{-1}^*(v)-\lambda\tilde{\hbar}_{-1}^*(-\lambda)\right]\Delta\mathcal{N}^*.
\end{aligned}
\tag{40}
$$

Since the latter is smaller in magnitude than $\Delta\mathcal{N}^*$ the stated result holds.

Proof of Proposition 2. The proof proceeds in the same way as the proof of Proposition 1 above. The adjustment rules again take the form $l(n) = n - \lambda$ and $u(n) = n + \upsilon$ for $\lambda > 0$ and $\upsilon > 0$. The flow steady state density of log employment is then defined by

$$\hat{h}(n) \equiv \frac{[1 - \mathcal{H}(n|n-\lambda)]h^*(n-\lambda) + \mathcal{H}(n|n+\upsilon)h^*(n+\upsilon)}{1 - \mathcal{H}^*(n+\upsilon|n) + \mathcal{H}^*(n-\lambda|n)}. \tag{41}$$

Since $\mathcal{H}^*(\xi|\nu) = \Pr(n^* - n_{-1} < \xi - \nu) \equiv \tilde{\mathcal{H}}^*(\xi - \nu)$, the probability of adjusting away from n is again independent of n,

$$1 - \mathcal{H}^*(n+\upsilon|n) + \mathcal{H}^*(n-\lambda|n) = 1 - \int_{-\lambda}^{\upsilon} \tilde{\hbar}^*(z)dz \equiv \phi. \tag{42}$$

Now consider the probabilities of adjusting down and up to n. Using Bayes' rule, we can write these as

$$\begin{aligned} 1 - \mathcal{H}(n|n-\lambda) &= \int_n^\infty \hbar(\nu|n-\lambda)d\nu \\ &= \int_n^\infty \hbar^*(n-\lambda|n)\frac{h_{-1}(\nu)}{h^*(n-\lambda)}d\nu \\ &= \int_n^\infty \tilde{\hbar}^*(n-\lambda-\nu)\frac{h_{-1}(\nu)}{h^*(n-\lambda)}d\nu \\ &= \int_{-\infty}^{-\lambda} \tilde{\hbar}^*(z)\frac{h_{-1}(n-\lambda-z)}{h^*(n-\lambda)}dz \text{, and} \end{aligned} \tag{43}$$

$$\begin{aligned} \mathcal{H}(n|n+\upsilon) &= \int_{-\infty}^n \hbar(\nu|n+\upsilon)d\nu \\ &= \int_{-\infty}^n \hbar^*(n+\upsilon|\nu)\frac{h_{-1}(\nu)}{h^*(n+\upsilon)}d\nu \\ &= \int_{-\infty}^n \tilde{\hbar}^*(n+\upsilon-\nu)\frac{h_{-1}(\nu)}{h^*(n+\upsilon)}d\nu \\ &= \int_{\upsilon}^\infty \tilde{\hbar}^*(z)\frac{h_{-1}(n+\upsilon-z)}{h^*(n+\upsilon)}dz. \end{aligned} \tag{44}$$

Piecing this together, we have

$$\hat{h}(n) = \frac{\int_{-\infty}^{-\lambda} \tilde{\hbar}^*(z)h_{-1}(n-\lambda-z)dz + \int_{\upsilon}^\infty \tilde{\hbar}^*(z)h_{-1}(n+\upsilon-z)dz}{1 - \int_{-\lambda}^{\upsilon} \tilde{\hbar}^*(z)dz}. \tag{45}$$

Multiplying both sides by n and integrating yields

$$
\begin{aligned}
\widehat{\mathcal{N}} &= \frac{1}{\phi}\int_{-\infty}^{-\lambda}\tilde{\hbar}^*(z)\int_{-\infty}^{\infty} n h_{-1}(n-\lambda-z)dn\,dz + \frac{1}{\phi}\int_{v}^{\infty}\tilde{\hbar}^*(z)\int_{-\infty}^{\infty} n h_{-1}(n+v-z)dn\,dz \\
&= \frac{1}{\phi}\int_{-\infty}^{-\lambda}\tilde{\hbar}^*(z)(\mathcal{N}_{-1}+\lambda+z)dz + \frac{1}{\phi}\int_{v}^{\infty}\tilde{\hbar}^*(z)(\mathcal{N}_{-1}-v+z)dz \\
&= \mathcal{N}_{-1} + \frac{1}{\phi}\left[\lambda\int_{-\infty}^{-\lambda}\tilde{\hbar}^*(z)dz - v\int_{v}^{\infty}\tilde{\hbar}^*(z)dz\right] + \frac{1}{\phi}\int_{-\infty}^{\infty} z\tilde{\hbar}^*(z)dz - \frac{1}{\phi}\int_{-\lambda}^{v} z\tilde{\hbar}^*(z)dz \\
&= \frac{\mathcal{N}^*}{\phi} - \frac{1-\phi}{\phi}\mathcal{N}_{-1} + \frac{1}{\phi}\left[\lambda\int_{-\infty}^{-\lambda}\tilde{\hbar}^*(z)dz - v\int_{v}^{\infty}\tilde{\hbar}^*(z)dz\right] - \frac{1}{\phi}\int_{-\lambda}^{v} z\tilde{\hbar}^*(z)dz.
\end{aligned}
\tag{46}
$$

Solving for $\widehat{\mathcal{N}}_{-1} = \mathcal{N}_{-1} = \mathcal{N}_{-2}$ in the prior constant-$\mathcal{N}^*$ state yields

$$
\widehat{\mathcal{N}}_{-1} = \mathcal{N}_{-1}^* + \lambda\int_{-\infty}^{-\lambda}\tilde{\hbar}_{-1}^*(z)dz - v\int_{v}^{\infty}\tilde{\hbar}_{-1}^*(z)dz - \int_{-\lambda}^{v} z\tilde{\hbar}_{-1}^*(z)dz. \tag{47}
$$

Substitution of (47) into (46) implies that a shock to aggregate log mandated employment that shifts the mean of $\tilde{\hbar}^*(\cdot)$ by $\Delta\mathcal{N}^*$ will induce a change in $\widehat{\mathcal{N}}$ relative to the prior constant-$\mathcal{N}^*$ state equal to

$$
\Delta\widehat{\mathcal{N}} = \frac{\Delta\mathcal{N}^*}{\phi} + \frac{1}{\phi}\left[\lambda\int_{-\infty}^{-\lambda}\Delta\tilde{\hbar}^*(z)dz - v\int_{v}^{\infty}\Delta\tilde{\hbar}^*(z)dz\right] - \frac{1}{\phi}\int_{-\lambda}^{v} z\Delta\tilde{\hbar}^*(z)dz. \tag{48}
$$

To a first-order approximation around $\Delta\mathcal{N}^* = 0$,

$$
\begin{aligned}
\int_{-\infty}^{-\lambda}\Delta\tilde{\hbar}^*(z)dz &= \int_{-\infty}^{-\lambda}\tilde{\hbar}_{-1}^*(z-\Delta\mathcal{N}^*)dz - \int_{-\infty}^{-\lambda}\tilde{\hbar}_{-1}^*(z)dz \approx -\tilde{\hbar}_{-1}^*(-\lambda)\Delta\mathcal{N}^*, \\
\int_{v}^{\infty}\Delta\tilde{\hbar}^*(z)dz &= \int_{v}^{\infty}\tilde{\hbar}_{-1}^*(z-\Delta\mathcal{N}^*)dz - \int_{v}^{\infty}\tilde{\hbar}_{-1}^*(z)dz \approx \tilde{\hbar}_{-1}^*(v)\Delta\mathcal{N}^*,
\end{aligned}
\tag{49}
$$

and, as in equation (40) above, $\int_{-\lambda}^{v} z\Delta\tilde{\hbar}^*(z)dz \approx \left[1-\phi-v\tilde{\hbar}_{-1}^*(v)-\lambda\tilde{\hbar}_{-1}^*(-\lambda)\right]\Delta\mathcal{N}^*$.

Substitution of these into equation (48) yields the stated result, $\Delta\widehat{\mathcal{N}} \approx \Delta\mathcal{N}^*$.

C. Large-firm canonical search and matching model

The firm's problem for this model combines equations (16) and (17) in the main text to obtain:

$$
\begin{aligned}
&\Pi(n_{-1},x) \equiv \max_n\left\{Apxn^\alpha - (1-\eta)\omega n - \frac{c}{q(\theta)}\Delta n^+ + \beta\mathbb{E}[\Pi(n,x')|x]\right\}, \\
&\text{where } A \equiv \frac{1-\eta}{1-\eta(1-\alpha)}.
\end{aligned}
\tag{50}
$$

To establish Proposition 3 in the main text, it is convenient first to define a notion of *quasi-frictionless* employment. Lemma $1''$ then extends the homogeneity properties of Lemmas 1 and $1'$ to this problem, but with respect to *quasi*-frictionless employment. Using this homogeneous problem and a symmetry result summarized in Lemma 2, we are able to prove that the change in aggregate log flow steady-state employment $\Delta\widehat{\mathcal{N}}$ is approximately equal to the change in aggregate log *quasi*-frictionless employment $\Delta\mathcal{N}^*$. Proposition 3 follows because $\Delta\mathcal{N}^*$ is simply related to the change in aggregate log *frictionless* employment $\Delta\mathcal{N}^{**}$.

Definitions *Quasi-frictionless employment* n^* *solves* $Apx\alpha n^{*\alpha-1} \equiv (1-\eta)\omega$ *; frictionless employment* n^{**} *solves* $px\alpha n^{**\alpha-1} \equiv w^*$*, where* w^* *denotes the frictionless wage.*

Remark *The change in aggregate log quasi-frictionless employment* $\Delta\mathcal{N}^*$ *induced by a change in aggregate productivity* $\Delta \ln p$ *is related to the change in aggregate log frictionless employment* $\Delta\mathcal{N}^{**}$ *according to*

$$\Delta\mathcal{N}^* = \frac{1-\epsilon_\omega}{1-\epsilon_{w^*}}\Delta\mathcal{N}^{**}, \tag{51}$$

where ϵ_ω *and* ϵ_{w^*} *respectively denote the elasticities of the annuitized value of unemployment* ω *and the frictionless wage* w^* *to aggregate productivity* p.

Lemma 1$''$ *If (i)* $\ln x' = \ln x + \varepsilon'_x$ *with* ε'_x *i.i.d., and (ii)* $c \propto \omega$*, then (a) the adjustment triggers take the form in (*10*), are linear,* $l(n) = l \cdot n$ *and* $u(n) = u \cdot n$ *for time-varying* $l < 1 < u$*; and (b) desired (log) employment adjustments,* $\ln(n^*/n_{-1})$*, are independent of initial firm size* n_{-1}.

Proof. If vacancy costs are proportional to the annuitized value of unemployment, say $c = \gamma(1-\eta)\omega$, a conjecture that $\Pi(n_{-1}, x) = (1-\eta)\omega n^* \widetilde{\Pi}(\zeta)$ yields

$$\widetilde{\Pi}(\zeta) \equiv \max_z \left\{\frac{z^\alpha}{\alpha} - z - \frac{\gamma}{q(\theta)}(z-\zeta)^+ + \beta\mathbb{E}\left[e^{\varepsilon'_{n^*}}\widetilde{\Pi}\left(e^{-\varepsilon'_{n^*}}z\right)\right]\right\}. \tag{52}$$

Results (a) and (b) follow from the proof to Lemmas 1 and $1'$ above.

Lemma 2 *If (i) the adjustment triggers are symmetric,* $-\ln l = \ln u \equiv \mu$*, and (ii) the distribution of innovations* ε_{n^*} *is symmetric,* $\mathcal{E}(-\varepsilon_{n^*}) = 1 - \mathcal{E}(\varepsilon_{n^*})$*, then the distribution of desired (log) employment adjustments* $\ln(n^*/n_{-1})$ *is symmetric,* $\widetilde{\mathcal{H}}^*(-\varsigma) = 1 - \widetilde{\mathcal{H}}^*(\varsigma)$.

Proof. Note first that the distribution of the desired log change in employment, $n^* - n_{-1}$, conditional on last period's log gap, $z_{-1} = n_{-1} - n^*_{-1}$, takes the simple form $\Pr(n^* - n_{-1} < \varsigma | z_{-1}) = \mathcal{E}(\varsigma - z_{-1})$, since $\varepsilon_{n^*} \equiv n^* - n^*_{-1}$ is i.i.d. with distribution function $\mathcal{E}(\cdot)$. It follows that the unconditional distribution of $n^* - n_{-1}$ is

$$\widetilde{\mathcal{H}}^*(\varsigma) = \int_{-\mu}^{\mu} \Pr(n^* - n_{-1} < \varsigma | z_{-1})\, g(z_{-1}) \mathrm{d}z_{-1} = \int_{-\mu}^{\mu} \mathcal{E}(\varsigma - z_{-1}) g(z_{-1}) \mathrm{d}z_{-1}, \tag{53}$$

where $g(z_{-1})$ is the ergodic density of z_{-1}. It is simple to verify that $\mathcal{E}(-\varepsilon_{n^*}) = 1 - \mathcal{E}(\varepsilon_{n^*})$ implies $\widetilde{\mathcal{H}}^*(\varsigma) = 1 - \widetilde{\mathcal{H}}^*(-\varsigma)$, provided $g(\cdot)$ also is symmetric, which we now establish.

Our strategy is to conjecture that $g(\cdot)$ is symmetric and verify that this is implied. Consider a firm with an initial $z_{-1} = z - \varepsilon$ such that $z \in (-\mu, \mu)$ lies strictly inside the inaction range. Clearly, this firm migrates to z if it draws ε. Thus, the mass of firms at z this period is given by

$$g(z) = \int_{z-\mu}^{z+\mu} g(z-\varepsilon)\, \mathrm{d}\mathcal{E}(\varepsilon) = \int_{-\mu}^{\mu} g(y)\, \mathrm{d}\mathcal{E}(z-y), \tag{54}$$

where we have used the change of variable $y = z - \varepsilon$. Under the conjecture that $g(y) = g(-y)$, one can confirm $g(z) = g(-z)$. To see this, evaluate $g(\cdot)$ at $-z$, use symmetry of $\mathcal{E}(\cdot)$, a change of variable $\tilde{y} = -y$, and standard rules of calculus to obtain

$$\begin{aligned} g(-z) &= \int_{-\mu}^{\mu} g(y)\,\mathrm{d}\mathcal{E}(-z-y) = \int_{-\mu}^{\mu} g(y)\,\mathrm{d}\mathcal{E}(z+y) = -\int_{\mu}^{-\mu} g(-\tilde{y})\,\mathrm{d}\mathcal{E}(z-\tilde{y}) \\ &= \int_{-\mu}^{\mu} g(-y)\,\mathrm{d}\mathcal{E}(z-y). \end{aligned} \tag{55}$$

Now consider the mass at the lower adjustment barrier, $z = -\mu$. This is comprised of two parts: first, firms that begin at $-\mu$, draw a negative labor demand shock ($\varepsilon < 0$), and adjust to remain at $-\mu$; and second, firms that began away from $-\mu$ and then migrate there. Thus,

$$g(-\mu) = \mathcal{E}(0)g(-\mu) + \int_{0}^{2\mu} g(-\mu+\varepsilon)\mathrm{d}\mathcal{E}(-\varepsilon) = \frac{1}{\mathcal{E}(0)}\int_{0}^{2\mu} g(-\mu+\varepsilon)\mathrm{d}\mathcal{E}(\varepsilon), \tag{56}$$

where the second equality follows from symmetry of $\mathcal{E}(\cdot)$. A similar argument can be used to show that the mass at the upper adjustment barrier $z = \mu$ satisfies

$$g(\mu) = \frac{1}{\mathcal{E}(0)}\int_{0}^{2\mu} g(\mu-\varepsilon)\mathrm{d}\mathcal{E}(\varepsilon). \tag{57}$$

A conjecture of symmetry $g(-\mu+\varepsilon) = g(\mu-\varepsilon)$ is again confirmed, $g(-\mu) = g(\mu)$. It follows that $g(z) = g(-z)$ for all $z \in [-\mu, \mu]$, and symmetry of $\widetilde{\mathcal{H}}^*(\cdot)$ obtains.

Proof of Proposition 3. The first-order conditions that define the triggers for optimal adjustment $z \in \{1/u\,, 1/l\}$ are given by

$$\begin{aligned} l^{1-\alpha} + \beta D(1/l\,;\theta) &\equiv 1, \\ u^{1-\alpha} + \beta D(1/u\,;\theta) &\equiv 1 + \frac{\gamma}{q(\theta)}, \end{aligned} \tag{58}$$

where $D(z;\theta) \equiv \mathbb{E}\left[\widetilde{\Pi}'\left(e^{-\varepsilon'_{n^*}}z\right)\right]$. The latter satisfies the following recursion

$$\begin{aligned} D(z;\theta) = \int_{\ln(lz)}^{\ln(uz)} &\left[e^{(1-\alpha)\varepsilon'_{n^*}}z^{\alpha-1} - 1 + \beta D\left(e^{-\varepsilon'_{n^*}}z;\theta\right)\right] d\mathcal{E}(\varepsilon'_{n^*}) \\ &+ \frac{\gamma}{q(\theta)}[1 - \mathcal{E}(\ln(uz))]. \end{aligned} \tag{59}$$

We first consider a first-order approximation to the firm's optimal policies around $\gamma = 0$.[16] To this end, note first that

[16] Equation (52) has the form $D(z) = \mathcal{C}(D,\gamma)(z)$, where $\mathcal{C}$ is a contraction map on the cross product of the space of bounded and continuous functions (where D "lives") and $[0,\Gamma]$, a closed subinterval of the nonnegative real line from which γ is drawn. By inspection, this map is continuously differentiable with respect to (w.r.t.) $\gamma \in [0,\Gamma]$. It then follows from Lemma 1 of Albrecht, Holmlund, and Lang (1991) that D is continuously differentiable w.r.t. γ and satisfies the recursion, $D_\gamma(z) = \mathcal{C}_\gamma(D,\gamma)(z) + \mathcal{C}_D\left(D_\gamma, D, \gamma\right)(z)$, where $\mathcal{C}_D$ is the Frechet derivative of $\mathcal{C}$. The right side of the latter expression defines a(nother) contraction map on a space of bounded and continuous functions. We have, then, that D_γ is bounded and continuous on $[0,\Gamma]$. Its calculation in (53) follows.

$$D_\gamma(z;\theta) \approx \frac{1}{q(\theta)}[1-\mathcal{E}(\ln(uz))] + \beta\int_{\ln(lz)}^{\ln(uz)} D_\gamma\left(e^{-\varepsilon'_{n^*}}z;\theta\right) d\mathcal{E}(\varepsilon'_{n^*})$$
$$= \frac{1}{q(\theta)}[1-\mathcal{E}(\ln z)] \text{ when } \gamma = 0 \tag{60}$$

Thus we can write $D(z;\theta) \approx \gamma[1-\mathcal{E}(\ln z)]/q(\theta)$. Substituting into the first-order conditions and noting that $l = e^{-\lambda}$ and $u = e^{\upsilon}$ yields

$$e^{-(1-\alpha)\lambda} + \beta\frac{\gamma}{q(\theta)}[1-\mathcal{E}(\lambda)] \approx 1,$$
$$e^{(1-\alpha)\upsilon} + \beta\frac{\gamma}{q(\theta)}[1-\mathcal{E}(-\upsilon)] \approx 1 + \frac{\gamma}{q(\theta)}. \tag{61}$$

Next, linearizing the leading terms around $\lambda = 0$ and $\upsilon = 0$, respectively, leads to

$$-(1-\alpha)\lambda + \beta\frac{\gamma}{q(\theta)}[1-\mathcal{E}(\lambda)] \approx 0,$$
$$(1-\alpha)\upsilon + \beta\frac{\gamma}{q(\theta)}[1-\mathcal{E}(-\upsilon)] \approx \frac{\gamma}{q(\theta)}. \tag{626}$$

Imposing $\beta \approx 1$, and $\mathcal{E}(-\varepsilon) = 1-\mathcal{E}(\varepsilon)$ yields $\lambda \approx \upsilon$.

Now return to the relationship between $\Delta\widehat{\mathcal{N}}$ and $\Delta\mathcal{N}^*$ in equation (48). Time-variation in the adjustment triggers alters the approximations around small aggregate shocks. Specifically, with $\lambda \approx \upsilon \approx \mu$, equation (48) becomes

$$\Delta\widehat{\mathcal{N}} \approx \frac{\Delta\mathcal{N}^*}{\phi} + \frac{1}{\phi}\left[\Delta\left(\mu\int_{-\infty}^{-\mu}\tilde{\hbar}^*(z)dz\right) - \Delta\left(\mu\int_{\mu}^{\infty}\tilde{\hbar}^*(z)dz\right)\right] - \frac{1}{\phi}\Delta\left(\int_{-\mu}^{\mu}z\tilde{\hbar}^*(z)dz\right). \tag{63}$$

Taking first-order approximations around $\Delta\mathcal{N}^* = 0$,

$$\Delta\left(\mu\int_{-\infty}^{-\mu}\tilde{\hbar}^*(z)dz\right) = \mu\int_{-\infty}^{-\mu}\tilde{\hbar}^*_{-1}(z-\Delta\mathcal{N}^*)dz - \mu_{-1}\int_{-\infty}^{-\mu_{-1}}\tilde{\hbar}^*_{-1}(z)dz$$
$$\approx \left[-\mu_{-1}\tilde{\hbar}^*_{-1}(-\mu_{-1}) + \left\{\tilde{\mathcal{H}}^*_{-1}(-\mu_{-1}) - \mu_{-1}\tilde{\hbar}^*_{-1}(-\mu_{-1})\right\}\frac{\partial\mu}{\partial\Delta\mathcal{N}^*}\right]\Delta\mathcal{N}^*; \tag{64}$$

similarly,

$$\Delta\left(\mu\int_{\mu}^{\infty}\tilde{\hbar}^*(z)dz\right) = \mu\int_{\mu}^{\infty}\tilde{\hbar}^*_{-1}(z-\Delta\mathcal{N}^*)dz - \mu_{-1}\int_{\mu_{-1}}^{\infty}\tilde{\hbar}^*_{-1}(z)dz$$
$$\approx \left[\mu_{-1}\tilde{\hbar}^*_{-1}(\mu_{-1}) + \left\{1-\tilde{\mathcal{H}}^*_{-1}(\mu_{-1}) - \mu_{-1}\tilde{\hbar}^*_{-1}(\mu_{-1})\right\}\frac{\partial\mu}{\partial\Delta\mathcal{N}^*}\right]\Delta\mathcal{N}^*; \tag{59}$$

and,

$$\Delta\left(\int_{-\mu}^{\mu}z\tilde{\hbar}^*(z)dz\right) = \int_{-\mu}^{\mu}z\tilde{\hbar}^*_{-1}(z-\Delta\mathcal{N}^*)dz - \int_{-\mu_{-1}}^{\mu_{-1}}z\tilde{\hbar}^*_{-1}(z)dz$$
$$\approx \Big[1-\phi-\mu_{-1}\tilde{\hbar}^*_{-1}(\mu_{-1}) - \mu_{-1}\tilde{\hbar}^*_{-1}(-\mu_{-1})$$
$$-\left(\mu_{-1}\tilde{\hbar}^*_{-1}(-\mu_{-1})\frac{\partial\mu}{\partial\Delta\mathcal{N}^*}\right) + \left(\mu_{-1}\tilde{\hbar}^*_{-1}(\mu_{-1})\frac{\partial\mu}{\partial\Delta\mathcal{N}^*}\right)\Big]\Delta\mathcal{N}^*. \tag{65}$$

Substituting these approximations back into (63) and cancelling yields

$$\Delta\widehat{\mathcal{N}} \approx \Delta\mathcal{N}^* + \frac{1}{\phi}\{\widetilde{\mathcal{H}}^*_{-1}(-\mu_{-1}) - [1 - \widetilde{\mathcal{H}}^*_{-1}(\mu_{-1})]\}\frac{\partial\mu}{\partial\Delta\mathcal{N}^*}\Delta\mathcal{N}^*. \tag{66}$$

To complete the proof, note from Lemma 2 that symmetry of the adjustment barriers, and of $\mathcal{E}(\cdot)$, implies that $\widetilde{\mathcal{H}}^*(\cdot)$ is also symmetric. It follows that $\widetilde{\mathcal{H}}^*_{-1}(-\mu_{-1}) - [1 - \widetilde{\mathcal{H}}^*_{-1}(\mu_{-1})] \approx 0$, and (66) collapses to $\Delta\widehat{\mathcal{N}} \approx \Delta\mathcal{N}^*$. The result then follows from equation (51).

D. Sensitivity analyses

This appendix describes our parameterization of the process for idiosyncratic shocks, contrasts it with alternatives in the literature, and performs sensitivity analyses on our main results to reasonable changes in model parameters.

Idiosyncratic shock process. Recall that we specify the process of idiosyncratic productivity as a geometric AR(1),

$$\ln x_{t+1} = \rho_x \ln x_t + \varepsilon_{xt+1}, \tag{67}$$

where $\varepsilon_{xt+1} \sim N(0, \sigma_x^2)$ and t denotes *quarters*. In what follows, we infer ρ_x and σ_x using estimates of the persistence and volatility of idiosyncratic productivity from data of *different* frequencies.

Annual data. Abraham and White (2006) use annual plant-level data from the U.S. manufacturing sector. Their measure of log idiosyncratic productivity would thus map most naturally to $\ln \sum_{t=1}^4 x_t$, the log of the annual sum of quarterly realizations. For tractability, though, we will interpret the annual data as corresponding to $X_4 \equiv \sum_{t=1}^4 \ln x_t$, and flag this violation of Jensen's inequality as a caveat to our results.

Abraham and White run least squares regressions of X_4 on $X_0 \equiv \sum_{t=-3}^0 \ln x_t$. An average of the estimated slope coefficients from their weighted and unweighted regressions is

$$\frac{cov(X_4, X_0)}{var(X_0)} = 0.39. \tag{68}$$

Under the assumption that the annual data are generated from the quarterly process (67), one can relate the latter covariance and variance terms to ρ_x and σ_x as follows,

$$\begin{aligned} cov(X_4, X_0) &= cov\left(\sum_{t=1}^4 \ln x_t, \sum_{t=-3}^0 \ln x_t\right) \\ &= [\rho_x + 2\rho_x^2 + 3\rho_x^3 + 4\rho_x^4 + 3\rho_x^5 + 2\rho_x^6 + \rho_x^7]\frac{\sigma_x^2}{1-\rho_x^2}, \end{aligned} \tag{69}$$

and,

$$var(X_0) = [4 + 6\rho_x + 4\rho_x^2 + 2\rho_x^3]\frac{\sigma_x^2}{1-\rho_x^2}. \tag{70}$$

Solving for the implied quarterly persistence yields $\rho_x = 0.68$. To infer σ_x we use Abraham and White's estimates to infer that $var(X_0) = 0.21$, which in turn implies that $\sigma_x = 0.1034$.

Quarterly data. Cooper, Haltiwanger, and Willis (2015) estimate a dynamic labor demand model using quarterly data. Like us, they specify a geometric AR(1) for idiosyncratic productivity. They estimate a persistence parameter of $\rho_x = 0.4$ and an innovation standard deviation of $\sigma_x = 0.5$.

Monthly data. Cooper, Haltiwanger, and Willis (2007) estimate a search and matching model using monthly data. They specify an idiosyncratic productivity process that follows a geometric AR(1) at a *monthly* frequency. Their estimates of the monthly analogues to ρ_x and σ_x are $\rho_x^m = 0.395$ and $\sigma_x^m = 0.0449$.[17]

Under the assumption that quarterly data are generated by the monthly AR(1) estimated by Cooper, Haltiwanger and Willis, the above logic implies that ρ_x can be computed as

$$\rho_x = \frac{\rho_x^m + 2(\rho_x^m)^2 + 3(\rho_x^m)^3 + 2(\rho_x^m)^4 + (\rho_x^m)^5}{3 + 4\rho_x^m + 2(\rho_x^m)^2} = 0.194. \tag{71}$$

Likewise, the variance of the innovation to the quarterly process is

$$\sigma_x^2 = (1 - \rho_x^2)\frac{3 + 4\rho_x^m + 2(\rho_x^m)^2}{1 - (\rho_x^m)^2}(\sigma_x^m)^2 = 0.251. \tag{72}$$

This implies a standard deviation $\sigma_x = 0.501$.

[17] These average Cooper et al.'s results for two specifications of adjustment costs that seem most germane to our application: (i) a fixed cost to adjust and linear cost to hire, and (ii) a fixed cost to adjust and linear cost to fire.

E. Additional figures

Figure A. Impulse responses of mandated and frictionless employment (Quarterly inaction rate ≈ 67%)

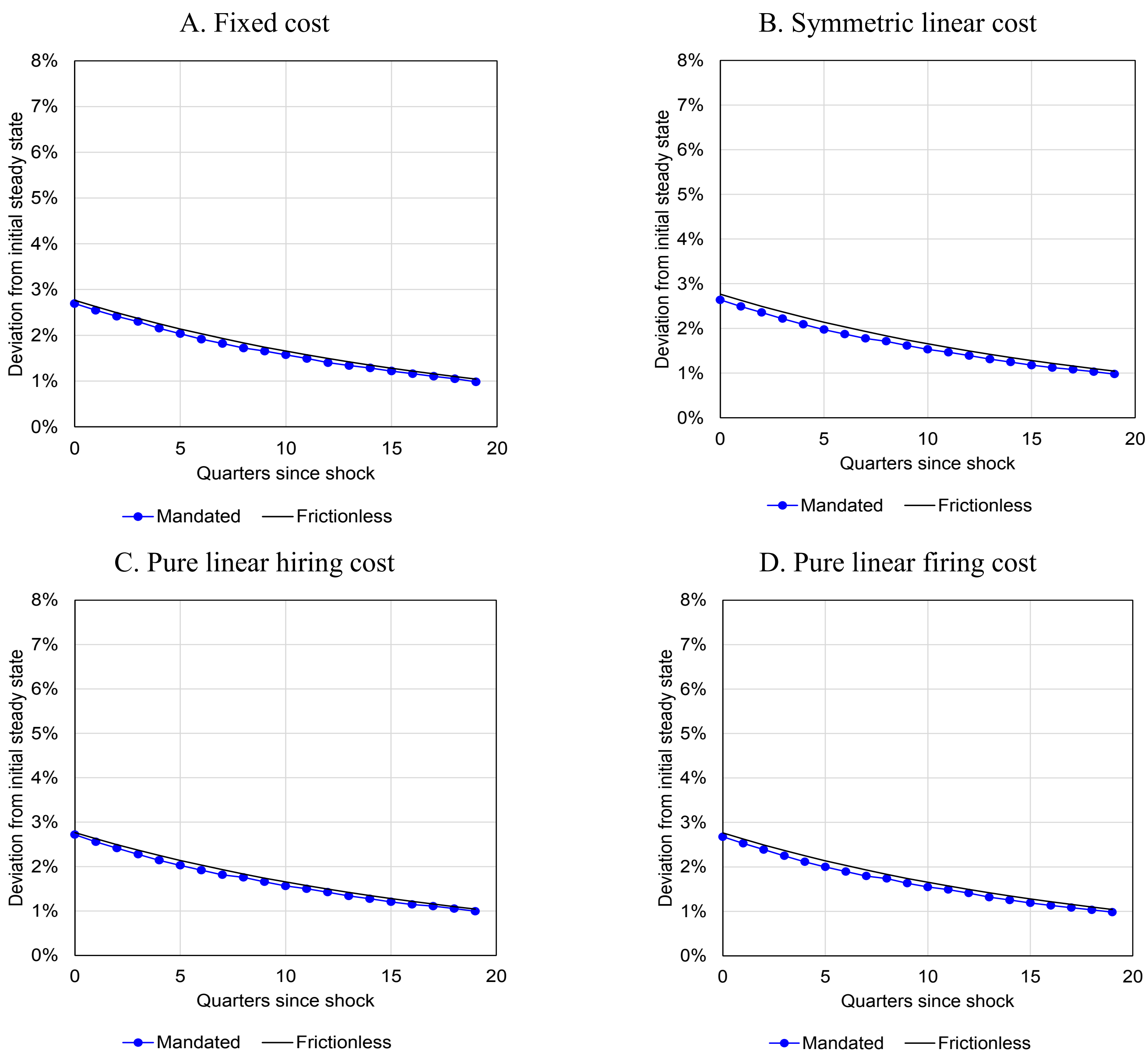
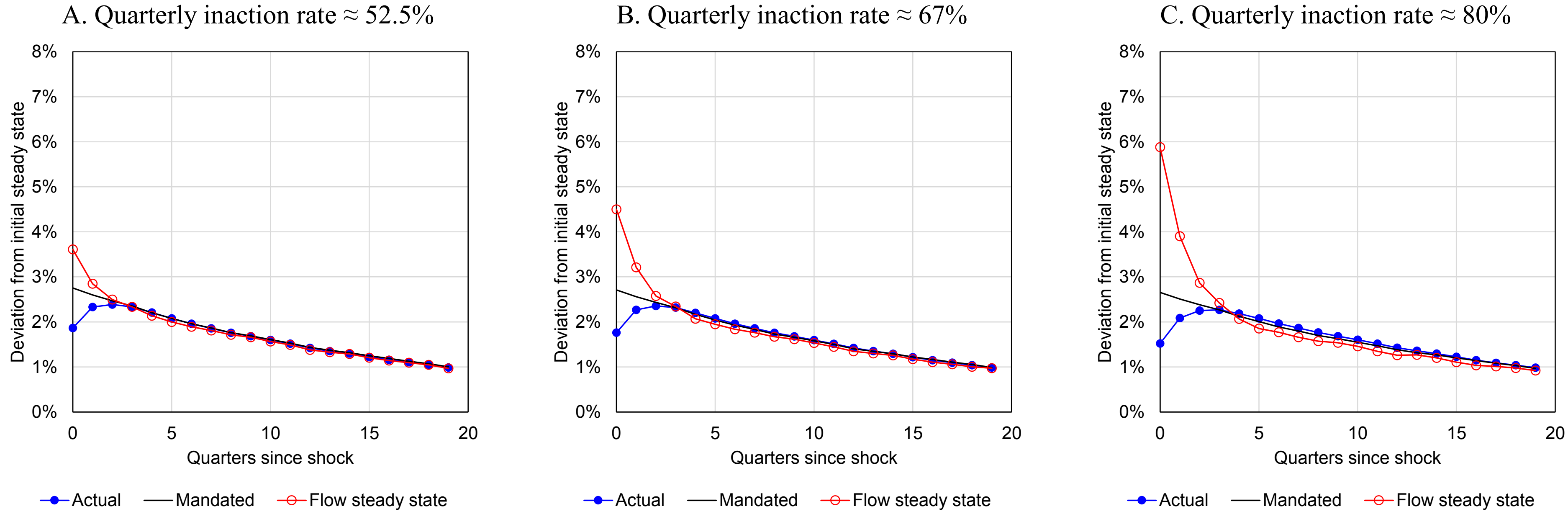


Figure B. Impulse responses of aggregate employment in selected parameterizations: Hybrid fixed and linear costs



Notes: