

MONETARY POLICY AND DURABLE GOODS*

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Abstract

We analyze monetary policy in a New Keynesian model with durable and non-durable goods, each with a separate degree of price rigidity. Durability has profound implications for the business cycle properties of the model and its response to interest rate interventions. Since utility depends on the service flow from the stock of durables, the flow demand for new durables is inherently sensitive to temporary changes in the relevant real interest rate. For a sufficiently long-lived “ideal” durable, we obtain an intriguing variant of the well-known “divine coincidence –in this case, the output gap depends only on inflation in the durable goods sector. We use numerical methods to verify the robustness of this analytical result for a broader class of model parameterizations. We then analyze the optimal Taylor rule for this economy. If the monetary authority places a high weight on stabilizing aggregate inflation then it is optimal to respond to sectoral inflation in direct proportion to the sectoral shares of economic activity. However, if the monetary authority wants to stabilize the aggregate output gap, it puts disproportionate weight on inflation in durable goods prices.

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1 Introduction

Monetary policy is often cast in terms of output and inflation targeting. If the Federal Reserve targets headline inflation then it implicitly weighs different sectoral (or regional) inflation measures according to their overall share of Gross Domestic Product. While weights proportional to sector size appear natural, it is not clear that the central bank should treat inflation in different sectors symmetrically. We argue in this paper that optimal monetary policies should often place greater weight on inflation in durable goods sectors. In earlier work (Barsky, House and Kimball 2007, hereafter BHK), three of us demonstrated that the reaction of New Keynesian models with durable and non-durable goods depended importantly on whether durable goods had flexible prices or sticky prices. In the special case in which durable goods prices were fully flexible, aggregate employment and output would not react to changes in the money supply. Put differently, money was neutral with respect to aggregate output.

In the present paper, we extend the earlier analysis in two important dimensions. First, we focus on the model in which both durables and non-durable goods have sticky prices whereas the earlier paper focused primarily on the case in which durable goods have flexible prices. Second, as in Carlstrom and Fuerst [2006] and Monacelli [2009], we consider interest rate rules rather than money supply rules. These extensions lead to several key results. First, we obtain two Phillips Curves – one for the durable goods sector and one for the non-durable goods sector. Using an approximation technique, we then show that the output gap (i.e., GDP gap) depends only on inflation in the durables sector. Hence, the central bank can ensure that actual output equals potential output by stabilizing durable goods prices. Second, expected inflation in the durable goods sector closely tracks the nominal interest rate. As a result, setting the nominal interest rate is essentially equivalent to setting inflation expectations for the durable goods sector. Third, we derive a utility-based second-order approximate welfare function for the model. The objective includes the output gaps and inflation measures of both sectors separately.

We then consider optimal monetary policy in the model with durable and non-durable goods. We consider both ad hoc objectives as well as the utility-based criterion. For each criterion, we analyze the optimal Taylor rule as in Boehm and House [2014] (the optimal Taylor rule is characterized by the coefficients that maximize the objective function subject to the constraint that the monetary authority follows an interest rate rule of the Taylor form). For the ad hoc objectives, the monetary authority minimizes a weighted sum of the output gap

and total inflation. The more the monetary authority cares about output stabilization, the more emphasis the bank will place on durable goods inflation. In contrast, if the monetary authority cares primarily about stabilizing aggregate inflation then it is optimal to weigh sectoral inflation in proportion to each sector's share in GDP.

The remainder of the paper is set out as follows. Section 2 reviews the literature. Section 3 presents the basic model and performs several numerical illustrations of the basic mechanisms at work. Section 4 presents the welfare objectives and analyzes the optimal Taylor rule. Section 5 concludes.

2 Related Literature

Our paper relates to two bodies of work. The first is a growing literature on multisectoral New Keynesian models. The second is the literature on inflation targeting and optimal policy.

The canonical New Keynesian model includes only a single nondurables sector and abstracts from differences in price rigidity across goods.¹ Recently, many researchers have turned their attention to studying multi-sector New Keynesian models. Early contributions to this literature include Ohanian and Stockman [1994], Ohanian *et al.* (1995), Aoki [2001] and Barsky *et al.* [2003].

Aoki [2001] considers a two sector model in which one sector has sticky prices while the other has flexible prices. Aoki's analysis shows that monetary policy should target inflation in the sector with sticky prices. Aoki's result is quite natural and anticipates the analysis in Carvalho [2006] who shows that in models with many different degrees of price rigidity, the sectors with the greatest price rigidity tend to have a much larger influence on the equilibrium behavior than the sectors with more flexible prices. This effect is particularly pronounced if there are strong strategic complementarities across firms.

Barsky *et al.* [2007] (BHK) show that flexibly priced durable goods have a strong tendency to generate negative comovement in response to monetary policy shocks. Carlstrom and Fuerst [2006] demonstrate that sectoral comovement in New Keynesian models can be substantially strengthened by including wage rigidities, credit constraints and investment adjustment costs. Carlstrom *et al.* [2006] analyze a two-sector New Keynesian model to see whether equilibrium determinacy depends on the inflation rate the monetary authority targets. As we do in our

¹See Woodford [2003] and Gali [2008] for comprehensive presentations of the standard New Keynesian model.

model, they allow for the possibility that price rigidity can differ across the sectors. They show that the well-known Taylor principle, which requires that the central bank reacts more than one-for-one to aggregate inflation, can be weakened in a multi-sector model. Specifically, if the central bank reacts more than one-for-one to *any* individual sectoral inflation rate then the equilibrium will be determinate.

Bils, Klenow and Malin [2012] analyze a multi-sector DSGE model that shares many features of our model and use it as a basis for testing what they refer to as “Keynesian Labor Demand.” Unlike our framework, they limit the degree to which factors can move between sectors. This implies that marginal costs will differ by sector. As in our model, sectors with greater durability are much more sensitive to relative price shocks in their setting. The authors show that markups in durables sectors appear substantially more cyclical than markups in non-durable sectors.

Much of the literature on inflation targeting focuses on the difference between “headline” inflation and “core” inflation. Because headline inflation includes energy prices while core inflation does not, energy price shocks will cause the two series to differ.² Bodenstein *et al.* [2008] use a New Keynesian DSGE model to show that policy rules that respond to headline inflation are associated with significantly different outcomes than policies that respond to core inflation. In a related study, Huang and Liu [2005] use a DSGE model to analyze whether the central bank should target the Consumer Price Index or the Producer Price Index. Their analysis suggests that the central bank should include both measures to maximize welfare.

Using a factor-augmented vector autoregression (FAVAR), Boivin *et al.* [2009] find that sectoral prices are sticky with respect to aggregate shocks even though they are quite responsive to sector-specific shocks. They write “(t)he picture that emerges is thus one in which many prices fluctuate considerably in response to sector-specific shocks, but they respond only sluggishly to aggregate macroeconomic shocks [...] (A)t the disaggregated level, individual prices are found to adjust relatively frequently, while estimates of the degree of price rigidity are much higher when based on aggregate data” (See Boivin *et al.* 2009, p. 352). Balke and Wynne [2007] study the reaction of relative prices underlying the Producer Price Index in response to a variety of measures of changes in monetary policy. They argue that, empirically, monetary policy systematically alters real relative prices suggesting that differential price rigidity is an important feature of the economies reaction to monetary policy. Leith

²Bodenstein *et al.* [2008] note that many central banks differ as to which inflation rate they use for their inflation target.

and Malley [2007] estimate New Keynesian Phillips curves for different industries within U.S. manufacturing. They find evidence of substantial variation across industries in price rigidity. Some producers reset prices once every 8 months. At the other end of the spectrum, some producers reset prices once every 24 months. Similarly, Imbs *et al.* [2011] use sectoral data on production and prices in France to estimate structural parameters of New Keynesian Phillips Curves. Like the Leith and Malley study, Imbs *et al.* find substantial variation in price rigidity across sectors.

While there is work documenting differences in price adjustment across products, there has been less work focusing on price rigidity for durable goods. Some prominent studies have data that indicates that prices are fairly rigid for both durable and non-durable goods. In the Bils and Klenow [2004] study, the authors reported that durables and non-durables have comparable frequencies of price adjustment (the monthly frequency is roughly 30 percent per month for both durable and non-durable goods; quoted prices for services change less frequently). In a follow-up study to the Bils and Klenow, Nakamura and Steinsson [2008] report evidence for a limited number of durable goods suggesting that durable good prices adjust relatively infrequently and when they do adjust, the price changes are often motivated by sales events or coinciding with product substitutions.

3 Model

The model extends the two-sector environment in BHK to include several important features. First, unlike the model in BHK, monetary policy is cast in terms of a Taylor rule. The Taylor rule we consider stipulates a nominal interest rate as a function of output gaps (the difference between actual output and the level of output that would prevail if all prices were flexible) and inflation. Because the model has two sectors, our “Taylor” rule can respond differentially to inflation rates in the durable and non-durable sectors. Second, in keeping with the established New Keynesian literature, we include both shocks to the natural level of output (modelled as shocks to sectoral productivity) and “cost-push” shocks – shocks to sectoral Phillips Curves. Finally, as in Boehm and House [2014] we assume that the output gap is measured with error. Measurement error shocks are not only realistic (see e.g., the references in Boehm and House [2014]) but they also serve to naturally temper the reaction of the monetary authority to fluctuations in economic activity. If the central bank does not completely trust its current measures of economic performance then it will be optimal to under-react to measured changes

in inflation and output.

Below we present the key structural equations governing the model. Additional model details as well as computer files are available from the authors by request.

3.1 Households

The representative household receives flow utility from consumption of a non-durable good C_t , a stock of durable goods D_t and real money balances M_t/P_t . The household receives disutility from labor N_t . Households discount future utility flows at the subjective time discount factor β . We assume that the utility function takes the following semi-parametric form

$$E_t \sum_{j=0}^{\infty} \beta^j \left\{ U(C_{t+j}, D_{t+j}) - \psi_n \frac{\eta}{\eta+1} N_{t+j}^{\frac{\eta+1}{\eta}} + \Lambda\left(\frac{M_{t+j}}{P_{t+j}}\right) \right\}. \quad (1)$$

As in BHK, we will take the flow utility function $U(\cdot)$ to be given by

$$U(C_{t+j}, D_{t+j}) = \frac{\sigma}{\sigma-1} \left[\left(\psi_c C_{t+j}^{\frac{\rho-1}{\rho}} + (1-\psi_c) D_{t+j}^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}} \right]^{\frac{\sigma-1}{\sigma}}.$$

Here σ is the intertemporal elasticity of substitution, ρ is the elasticity of substitution between durable and non-durable consumption. The parameter η represents the Frisch labor supply elasticity and the function $\Lambda(\cdot)$ describes the utility benefit of real money holdings. We assume that $\Lambda' > 0$ and $\Lambda'' < 0$. Since we focus on interest rate rules in our analysis, the precise nature of the function $\Lambda(\cdot)$ is irrelevant. The household seeks to maximize (1) subject to the nominal budget constraint

$$P_t^c C_t + P_t^x X_t + S_t + M_t = W_t N_t + R_t K + S_{t-1} (1 + i_{t-1}) + M_{t-1} + \Pi_t \quad (2)$$

and the law of motion for durable goods

$$D_t = D_{t-1} (1 - \delta) + X_t. \quad (3)$$

Each period, the household earns labor income $W_t N_t$ and capital income $R_t K$. As in Barsky *et al.* [2007] we assume that the stock of productive capital K is exogenously fixed.³ Here, X_t denotes date t purchases of new durables and δ denotes the depreciation rate of the durable.

³This assumption is innocuous. See Barsky *et al.* [2007] for further discussion of this point.

Note that durables and non-durables have different nominal prices P_t^x and P_t^c . Finally, S_t is nominal saving (which will be zero in equilibrium) and Π_t denotes nominal profits and other lump-sum transfers returned to the household. i_t is the nominal interest rate.

For simplicity we let $U_{C,t}$ and $U_{D,t}$ denote the flow marginal utility of non-durable consumption and the flow marginal utility associated with a given durable stock. Let q_t be the shadow value (in utility units) of an additional unit of the durable – i.e., the Lagrange multiplier associated with the constraint (3). The first order conditions for N_t , C_t , X_t and S_t require the following optimality conditions,

$$\frac{U_{C,t}}{q_t} = \frac{P_t^c}{P_t^x}, \quad (4)$$

$$\psi_n N_t^{\frac{1}{\eta}} = \frac{W_t}{P_t^c} U_{C,t} = \frac{W_t}{P_t^x} q_t, \quad (5)$$

$$q_t = U_{D,t} + \beta (1 - \delta) E_t [q_{t+1}] \quad (6)$$

and

$$U_{C,t} \frac{1}{P_t^c} = \beta (1 + i_t) E_t \left[U_{C,t+1} \frac{1}{P_{t+1}^c} \right]. \quad (7)$$

Condition (4) is the intratemporal optimality condition characterizing the optimal mix of durable and non-durable goods consumption by the household. The household's optimal labor supply is characterized by conditions (5). Equation (6) relates the shadow value of additional durables to the discounted flow utility of the durable $U_{D,t}$. Finally, (7) is a standard Euler equation for non-durable consumption goods. Note that, by combining (7) with (4), we have an Euler equation for durable goods purchases,

$$q_t = (1 + i_t) \beta E_t \left[\frac{P_t^x}{P_{t+1}^x} q_{t+1} \right]. \quad (8)$$

The Fisher equations for the durable and non-durable goods give the ex post real rate of return for good $j = C, X$ as

$$1 + r_{t+1}^j = (1 + i_t) \frac{P_t^j}{P_{t+1}^j} \quad (9)$$

3.2 Firms and Pricing

We model price rigidity using a standard Calvo mechanism for two sectors. Final goods in both the durable and non-durable sectors are produced from intermediate goods according to

the CES production functions

$$X_t = \left[\int_0^1 x_t(s)^{\frac{\varepsilon_t^x - 1}{\varepsilon_t^x}} ds \right]^{\frac{\varepsilon_t^x}{\varepsilon_t^x - 1}} \quad (10)$$

and

$$C_t = \left[\int_0^1 c_t(s)^{\frac{\varepsilon_t^c - 1}{\varepsilon_t^c}} ds \right]^{\frac{\varepsilon_t^c}{\varepsilon_t^c - 1}} \quad (11)$$

where $\varepsilon_t^j > 1$ for $j = C, X$. The elasticity parameters ε_t^j are time-varying components of the model. Below we explicitly consider exogenous shocks to these parameters as a way of generating exogenous fluctuations in desired markups and thus introducing cost-push shocks to inflation. Final goods producers are perfectly competitive and take the final goods prices P_t^j and intermediate goods prices $p_t^j(s)$ as given. It is straightforward to show that demand for each intermediate good has an isoelastic form

$$x_t(s) = X_t \left(\frac{p_t^x(s)}{P_t^x} \right)^{-\varepsilon_t^x} \quad (12)$$

and

$$c_t(s) = C_t \left(\frac{p_t^c(s)}{P_t^c} \right)^{-\varepsilon_t^c}. \quad (13)$$

Competition among final goods producers ensures that the final goods nominal prices are combinations of the nominal prices of the intermediate goods used in production,

$$P_t^x = \left[\int_0^1 p_t^x(s)^{1-\varepsilon_t^x} ds \right]^{\frac{1}{1-\varepsilon_t^x}} \quad (14)$$

$$P_t^c = \left[\int_0^1 p_t^c(s)^{1-\varepsilon_t^c} ds \right]^{\frac{1}{1-\varepsilon_t^c}}. \quad (15)$$

Intermediate goods are produced by monopolistically competitive firms who take the demand curves (12) and (13) as given when they set their prices. Each intermediate goods firm in each sector has a Cobb-Douglas production function

$$x_t(s) = Z_t^x [k_t^x(s)]^\alpha [n_t^x(s)]^{1-\alpha} \quad (16)$$

$$c_t(s) = Z_t^c [k_t^c(s)]^\alpha [n_t^c(s)]^{1-\alpha} \quad (17)$$

Here Z^x and Z^c are sector-specific productivity shocks. The intermediate goods firms take the nominal input prices W_t and R_t as given. Cost minimization implies that within either sector firms choose the same capital-to-labor ratio,

$$\frac{k_t^j(s)}{n_t^j(s)} = \frac{K_t^j}{N_t^j} = \frac{K}{N_t} = \frac{\alpha}{1-\alpha} \frac{W_t}{R_t}$$

where $K_t^j = \int k_t^j(s) ds$ and $N_t^j = \int n_t^j(s) ds$ are capital and labor used in sector $j = C, X$. Because the production functions exhibit constant returns to scale, and because factors can freely move from one sector to another, the relative marginal costs of production are given simply by the ratio of the productivity terms. Specifically, the nominal date t marginal cost in sector j is

$$MC_t^j = \frac{W_t}{Z_t^j} \frac{1}{1-\alpha} \left(\frac{K}{N_t} \right)^{-\alpha} = \frac{W_t}{Z_t^j} f(N_t) \quad (18)$$

where $f(N) = \frac{1}{1-\alpha} (N_t/K)^\alpha$ (keep in mind our assumption that the stock of productive capital is exogenously fixed). As a result, the relative marginal cost of production is $MC_t^c/MC_t^x = Z_t^x/Z_t^c$. Notice that in equilibrium, the nominal marginal costs are functions of aggregate employment N_t rather than sectoral employment N_t^x and N_t^c . Nominal marginal costs can alternatively be expressed in terms of the underlying nominal input prices W_t and R_t as

$$MC_t^j = \frac{W_t^{1-\alpha} R_t^\alpha}{Z_t^j} \left(\frac{1}{1-\alpha} \right)^{1-\alpha} \left(\frac{1}{\alpha} \right)^\alpha \text{ for } j = X, C.$$

As in the standard New Keynesian model, prices for each intermediate good producer are governed by a Calvo mechanism. Importantly we allow for different degrees of price rigidity in each sector. Let θ^j be the probability a price is stuck for firms in sector j . Thus, each period in sector j , the fraction $1 - \theta^j$ of firms reset their prices. Let $p_{j,t}^*$ denote the optimal reset price for an intermediate goods firm that receives the Calvo draw in period t . The optimal reset prices in each sector are

$$p_{c,t}^* = \mu_{c,t} \frac{\sum_{j=0}^{\infty} (\theta^c \beta)^j E_t \left[U_{C,t+j} (P_{t+j}^c)^{\varepsilon_t^c - 1} MC_{t+j} C_{t+j} \right]}{\sum_{j=0}^{\infty} (\theta^c \beta)^j E_t \left[U_{C,t+j} (P_{t+j}^c)^{\varepsilon_t^c - 1} C_{t+j} \right]} \quad (19)$$

$$p_{x,t}^* = \mu_{x,t} \frac{\sum_{j=0}^{\infty} (\theta^x \beta)^j E_t \left[q_{t+j} (P_{t+j}^x)^{\varepsilon_t^x - 1} MC_{t+j} X_{t+j} \right]}{\sum_{j=0}^{\infty} (\theta^x \beta)^j E_t \left[q_{t+j} (P_{t+j}^x)^{\varepsilon_t^x - 1} X_{t+j} \right]} \quad (20)$$

where $\mu_{j,t} \equiv \frac{\varepsilon_t^j}{\varepsilon_t^j - 1}$ is the date t desired markup. Final goods prices evolve according to

$$P_t^c = \left[\theta^c (P_{t-1}^c)^{1-\varepsilon_t^c} + (1 - \theta^c) (p_{c,t}^*)^{1-\varepsilon_t^c} \right]^{\frac{1}{1-\varepsilon_t^c}} \quad (21)$$

$$P_t^x = \left[\theta^x (P_{t-1}^x)^{1-\varepsilon_t^x} + (1 - \theta^x) (p_{x,t}^*)^{1-\varepsilon_t^x} \right]^{\frac{1}{1-\varepsilon_t^x}} \quad (22)$$

3.3 GDP, Market Clearing and Monetary Policy

Nominal GDP for this model is naturally the total dollar value of all final goods and services produced in a given period,

$$P_t^Y Y_t = P_t^x X_t + P_t^c C_t.$$

Real GDP is computed as the Bureau of Economic Analysis does for the actual data – namely by fixing a set of constant base-year prices $\bar{P}^x$ and $\bar{P}^c$. We take the base year prices to be 1 so that real GDP is simply

$$Y_t = \bar{P}^x X_t + \bar{P}^c C_t = X_t + C_t.$$

Finally, the implicit aggregate price deflator is computed as the ratio of nominal GDP to real GDP,

$$P_t^Y = \frac{P_t^x X_t + P_t^c C_t}{X_t + C_t}.$$

The aggregate rate of inflation is simply the percent change in the GDP deflator, $1 + \pi_t = \frac{P_t^Y}{P_{t-1}^Y}$.

The labor market and the market for productive capital must be in equilibrium in each period. This requires

$$K = K_t^x + K_t^c$$

and

$$N_t = N_t^x + N_t^c.$$

To close the model, we assume that the monetary authority sets the nominal interest rate according to a generalized Taylor rule. In particular, we consider Taylor rules in the family

$$i_t = \bar{i} + \phi_Y Y_t^m + \phi_{\pi,x} \pi_t^x + \phi_{\pi,c} \pi_t^c \quad (23)$$

Here the notation Y_t^m indicates that the central bank is reacting to the measured output gap. We assume that the measured output gap is equal to the actual output gap plus a mean zero

i.i.d. measurement error m_t^Y , that is,

$$Y_t^m = Y_t + m_t^Y.$$

The reader will notice that this specification nests the standard Taylor rule for appropriate choices of the reaction parameters ϕ . It is well known that interest rate rules of the form (23) may imply indeterminate equilibria for certain parameter values. We do not discuss the issue of indeterminacy in this paper. Instead we appeal to the analysis in Carlstrom *et al.* [2006] who show that equilibria in multisectoral models is determinate provided that the central bank reacts sufficiently strongly to any one component of inflation. Thus, in what follows, we assume that $\phi_{\pi,x} \geq 0$, $\phi_{\pi,c} \geq 0$ and $\phi_{\pi,x} + \phi_{\pi,c} > 1$.

3.4 Forcing Variables

There are several exogenous forcing variables in the model. We allow for separate productivity shocks to both the durable and non-durable sectors. These shocks move the flex-price equilibrium or the natural level of output however they do not shift the Phillips Curves separately from their influence on the marginal cost. We also consider shocks to the desired markups $\mu_{c,t}$ and $\mu_{x,t}$ in each sector (these shocks arise from shocks to the elasticity parameters ε_t^j). Because these shocks affect the desired markups for the price setters but not the natural level of output, they are conventionally referred to as “cost-push” shocks. We denote the cost-push shocks in each sector as u_t^j . The sectoral productivity shocks and cost push shocks follow simple autoregressive processes

$$Z_t^j = (1 - \rho_{z,j}) + \rho_{z,j} Z_{t-1}^j + e_t^{z,j},$$

and

$$u_t^j = (1 - \rho_{u,j}) + \rho_{u,j} u_t^j + e_t^{u,j}$$

for $j = X, C$.⁴ In addition, the model also includes the measurement error shock m_t^Y . This shocks induces movements in the real variables by prompting undesirable changes in monetary policy. m_t^Y is assumed to be serially uncorrelated and mean zero.

⁴The cost-push shocks u_t are slightly different from the desired markups themselves. In particular one can show that $u_t^j = \mu_{j,t} (1 - \theta^j) (1 - \rho_{u,j} \beta \theta^j) (\theta^j)^{-1}$

4 Analysis

In this section we analyze the model's equilibrium and study the properties of optimal Taylor rules. We begin by drawing out several key properties of the model that hold for low depreciation rates. We then turn our attention to a special case of the model in which the equilibrium and the associated optimal Taylor Rule can be solved analytically. This limiting case reveals several properties of optimal monetary policy which carry over to more general versions of the model. Finally, we use numerical methods to analyze more realistic versions of the model.

4.1 Key Properties of the Model

Here we point out several important properties of the model equilibrium behavior. We derive two New Keynesian Phillips Curves – one for each sector. We then find expressions for the real GDP gap and the real employment gap. Finally, we solve for a second-order accurate approximation to the utility of the household which can be expressed in terms of sectoral inflation and the sectoral output gaps.

Sectoral Phillips Curves. Following the convention in the New Keynesian literature, we define real marginal cost in each sector as the ratio of nominal marginal cost to the sectoral final price. That is, $mc_t^x = \frac{MC_t^x}{P_t^x}$, and $mc_t^c = \frac{MC_t^c}{P_t^c}$. Using the definitions of real marginal cost together with the log-linear versions of (19), (20), (21) and (22) straightforward algebra shows that the model implies two (sectoral) New Keynesian Phillips curves,

$$\tilde{\pi}_t^x = \lambda^x \widetilde{mc}_t^x + \beta E_t [\tilde{\pi}_{t+1}^x] + u_t^x \quad (24)$$

$$\tilde{\pi}_t^c = \lambda^c \widetilde{mc}_t^c + \beta E_t [\tilde{\pi}_{t+1}^c] + u_t^c \quad (25)$$

where

$$\lambda^c = \frac{(1 - \theta^c)(1 - \theta^c \beta)}{\theta^c}, \text{ and } \lambda^x = \frac{(1 - \theta^x)(1 - \theta^x \beta)}{\theta^x}$$

are parameters describing the frequency of price adjustment in each sector. These coefficients are sometimes referred to as the “microeconomic” rates of price adjustment.⁵ We use the standard notation of $\tilde{v}$ to denote the percent change in the variable v , that is $\tilde{v} = dv/\bar{v}$ where $dv = v - \bar{v}$ and $\bar{v}$ is the non-stochastic steady state value of v .⁶

⁵See Leith and Malley [2007] for a detailed empirical analysis of sectoral Phillips Curves across U.S. manufacturing.

⁶With some abuse of notation we also write $\tilde{\pi}_t \equiv \pi_t - \bar{\pi}$ and $\tilde{i}_t \equiv i_t - \bar{i}$.

Real GDP Gap and the Aggregate Employment Gap. For any variable v_t we define the “gap” as the log difference between the equilibrium value of the variable and the value that would be obtained if prices were perfectly flexible. We denote the gap by $\hat{v}_t = \tilde{v}_t - \tilde{v}_t^{\text{Flex}}$ where $\tilde{v}_t^{\text{Flex}}$ is the flex-price value of the variable. We show next that the real GDP gap is approximately only a function of the gap in aggregate employment.

Recall that each intermediate goods firm chooses the same capital to labor ratio, $\frac{k}{n} = \frac{K}{N_t}$. Then according to (12) and (16), output for any single intermediate producer of new durables is (the calculations for the non-durables are the same),

$$x_t(s) = X_t \left(\frac{p_t^x(s)}{P_t^x} \right)^{-\varepsilon} = Z_t^x \left(\frac{K}{N_t} \right)^\alpha n_t^x(s)$$

Integrating over durable goods intermediate firms and using $N_t^x = \int_0^1 n_t^x(s) ds$ gives

$$X_t \xi_t^x = Z_t^x \left(\frac{K}{N_t} \right)^\alpha N_t^x$$

where $\xi_t^x \equiv \int_0^1 \left(\frac{p_t^x(s)}{P_t^x} \right)^{-\varepsilon} ds$ is a measure of (inefficient) price and output variation across firms in the durable goods sector. (Similarly, for the non-durable goods sector we have price variation given by a term ξ_t^c .) As shown in Galí [2008], in a neighborhood of the zero inflation steady state, ξ_t^x is 1 to a first-order approximation (see e.g., Galí 2008, p. 62). This implies that real GDP is approximately

$$Y_t \approx \left(\frac{K}{N_t} \right)^\alpha [Z_t^c N_t^c + Z_t^x N_t^x].$$

or in log deviations,

$$\tilde{Y}_t \approx \tilde{Z}_t + (1 - \alpha) \tilde{N}_t$$

where the aggregate productivity term is a weighted average of sectoral productivity $\tilde{Z}_t = \tilde{Z}_t^c \frac{N^c}{N} + \tilde{Z}_t^x \frac{N^x}{N}$. The GDP gap $\hat{Y}_t = \tilde{Y}_t - \tilde{Y}_t^{\text{Flex}}$ is then

$$\hat{Y}_t \approx (1 - \alpha) \hat{N}_t. \tag{26}$$

Approximate Welfare Objective. For the welfare analysis of this paper we consider two alternative approaches. In line with the New Keynesian literature, we first derive a second order

approximation of the representative consumer's utility function. Among others, Rotemberg and Woodford [1999], Woodford [2003], and Coibion, Gorodnichenko, and Wieland [2012] have used this approach to study optimal policy. Second, we use an ad hoc welfare criterion which places greater importance on stabilizing output than utility-based criteria suggest.

The following proposition gives the approximate welfare objective for a social planner who tries to maximize the utility of the representative household. All proofs are in the appendix.

Proposition 1. *The unconditional expectation of the household's (scaled) per period utility function is*

$$\begin{aligned} \mathbb{E} \left[2 \frac{U(C_t, D_t) - \psi_n \frac{\eta}{\eta+1} N_t^{\frac{\eta+1}{\eta}}}{U_C C} \right] &= -\sigma_C^{-1} \mathbb{V} [\hat{C}_t] - \sigma_D^{-1} \frac{U_D D}{U_C C} \mathbb{V} [\hat{D}_t] + 2 \frac{U_{DC} D}{U_C} \text{Cov} (\hat{D}_t, \hat{C}_t) \\ &\quad - \frac{N}{N_C} \frac{\alpha + \eta^{-1}}{1 - \alpha} \mathbb{V} [\hat{Y}_t] - \varepsilon \lambda^c \mathbb{V} [\tilde{\pi}_{C,t}] - \frac{N_X}{N_C} \varepsilon \lambda^x \mathbb{V} [\tilde{\pi}_{X,t}] + \text{t.i.p.} + \text{h.o.t.} \end{aligned}$$

where

$$\sigma_C = -\frac{U_C}{C \cdot U_{CC}} \text{ and } \sigma_D = -\frac{U_D}{D \cdot U_{DD}}$$

and *t.i.p.* stands for terms independent of policy and *h.o.t.* for higher order terms.

Since consumers are risk averse, greater variances of nondurable and durable consumption decrease welfare. Consistent with intuition, the covariance between durables and nondurables consumption impacts welfare with a coefficient that rises in the degree of complementarity between the two. Next, welfare decreases in output volatility and this disutility from output volatility falls with the labor supply elasticity η . Finally, inflation volatility in both sectors reduces welfare. This loss depends on the relative sector size, the elasticity of substitution ε and the microeconomic rates of price adjustments λ^c and λ^x .

4.2 Ideal Durables

We next turn to a limiting case of “ideal durable goods” in which discount rates and depreciation rates are near zero. In this low-depreciation limit, the durable good survives for an arbitrarily long time period and the household does not discount the future relative to the present. Following BHK, we argue that the shadow value q_t for such long-lived durable goods is essentially constant. This near constancy of q_t is equivalent to saying that the intertemporal elasticity of substitution for purchases of durables is extremely high. In this case, the output

of the durable goods sector responds sharply to changes in intertemporal relative prices. Our limiting approximation implies that, for sufficiently low depreciation and discount rates, the intertemporal elasticity of substitution for purchases of durable goods is infinite. For goods with realistic depreciation rates, the approximation will be somewhat less accurate. We check this accuracy numerically and report the results below.

The shadow value of the durable good is the present value of marginal utilities of the service flow of the durable, discounted at the subjective rate of time preference and the rate of economic depreciation. That is,

$$q_t = E_t \left[\sum_{j=0}^{\infty} [\beta(1-\delta)]^j U_{D,t+j} \right]. \quad (27)$$

Two features guarantee that, for long-lived durables, q_t will be approximately invariant to transitory shocks. First, durables with low depreciation rates have high stock-flow ratios. In our model, the steady state stock-flow ratio is $1/\delta$. A high stock-flow ratio implies that even relatively large changes in durable goods production have only small effects on the total stock in the short run. That is, for transitory shocks, we can appeal to the approximation $D_t \approx D$. Because the stock changes only slightly, equilibrium changes in the production of durable goods entail only minor changes in marginal service flows $U_{D,t+j}$.

Second, if δ is sufficiently low, expression (27) is dominated by the marginal service flows in the distant future. Because the effects of the shock are temporary, and because $D_t \approx D$, the future terms in (27) remain close to their steady state values. Thus, even if there were significant changes in the first few terms of the expansion, these effects would have a small percentage effect on the present value as a whole. This implies that the model can feature service flows that change substantially over time due to complementarities with other variables that fluctuate in the short run and still imply a nearly invariant shadow value.

Together, these two observations suggest that it is reasonable to treat the shadow value of sufficiently long-lived durables as roughly constant in the face of a monetary disturbance (or indeed any short-lived shock). Thus, for a long-lived durable, and for plausible half-lives of price rigidity we can take $D_t \approx D$ and $q_t \approx q$.⁷

⁷The idealized durable goods setting also allows us to simplify the utility structure. In the households objective function, the marginal utility of consumption $U_{C,t+j}$ is in principle a function of both non-durables C_t and durables D_t . However, since the percent changes in D_t are negligible, we can write $U_{C,t+j} = \frac{\partial u(C_t, D_t)}{\partial C_t} \approx \frac{\partial u(C_t, \bar{D})}{\partial C_t}$ so the marginal utility of non-durable consumption is approximately a function of C_t alone.

Implications. There are several important consequences for the equilibrium in the low-depreciation limit. First, regardless of price rigidity in the durable and/or non-durable sector, the labor supply curve (5) implies that, for sufficiently transitory shocks, equilibrium employment is governed only by changes in the real product wage in the durable goods sector. Using (5) and $q_t \approx \bar{q}$ we have

$$\psi_n N_t^{\frac{1}{\eta}} \approx \frac{W_t}{P_t^x} \bar{q}.$$

Since q_t is approximately, constant, movements in N_t are determined solely by changes in the real product wage W_t/P_t^x . Consider the flex-price equilibrium. In this case, the nominal price of new durable goods will simply be the desired markup μ^x over the nominal marginal cost. That is, $P_t^x = \mu^x MC_t^x$ so that $mc_t^x = (\mu^x)^{-1}$. We can then use the approximate relationship above together with the nominal marginal cost of durables (18) to get an expression for the flexible price employment level. We then have

$$(\mu^x)^{-1} = \frac{MC_t^{x,\text{Flex}}}{P_t^{x,\text{Flex}}} \approx \frac{\psi_n (N_t^{\text{Flex}})^{\frac{1}{\eta}}}{\bar{q}} \frac{f(N_t^{\text{Flex}})}{Z_t^x}.$$

or, using the form of $f(\cdot)$, in log deviations near the zero inflation steady state,

$$\tilde{N}_t^{\text{Flex}} \approx \frac{1}{\frac{1}{\eta} + \alpha} \cdot \tilde{Z}_t^x \quad (28)$$

This is a single equation in the variables N_t^{Flex} and Z_t^x . Thus, the efficient aggregate employment level is governed solely by the productivity disturbance in the durable goods sector. (This result is reminiscent of a similar result in Kimball [1994].)

Second, nondurable consumption (and production) moves one-for-one with changes in the real relative price of nondurables / durables. Using condition (4) and $q_t \approx \bar{q}$ we have

$$U_{C,t} \approx \frac{P_t^c}{P_t^x} \bar{q}.$$

Third, the equilibrium nominal interest rate is a direct reflection of expected inflation in the durable goods sector. That is, for durable goods prices, there is a pure Fisher effect. To see this, note that the Euler equation for the durable good requires

$$q_t = (1 + i_t) \beta E_t \left[\frac{P_t^x}{P_{t+1}^x} q_{t+1} \right].$$

Again, because the shocks are assumed to be short-lived, we can use $q_t \approx q_{t+1} \approx \bar{q}$ to immediately get

$$\frac{1}{1+i_t} \approx \beta E_t \left[\frac{1}{1+\pi_{t+1}^x} \right]$$

So, to a first order approximation $E_t [\tilde{\pi}_{t+1}^x] \approx \beta \tilde{\pi}_t$. This relationship shows that a monetary policy rule for setting the nominal interest rate i_t is tantamount to a rule that specifies a target for durable goods inflation.

Using the expression for the nominal marginal cost of durables (18) together with the labor supply condition (5) we can write the “real marginal cost” for the durable good mc_t^x as

$$mc_t^x \approx \frac{\psi_n N_t^{\frac{1}{\eta}}}{\bar{q}} \frac{f(N_t)}{Z_t^x}.$$

Again, using the form for $f(\cdot)$ together with (26) and (28) we can express the real marginal cost in the durable goods sector as a function of the output gap $\hat{Y}_t$,

$$\widetilde{mc}_t^x \approx \zeta \hat{Y}_t,$$

where $\zeta = \left(\frac{1}{\eta} + \alpha\right) \left(\frac{1}{1-\alpha}\right)$ (the real rigidity derivative emphasized by Ball and Romer 1990). Substituting this expression into the Phillips curve for the durable goods sector (24) we have

$$\tilde{\pi}_t^x \approx \lambda^x \zeta \hat{Y}_t + \beta E_t [\tilde{\pi}_{t+1}^x] + u_t^x. \quad (29)$$

Notice that only the price rigidity of the durable good matters for influencing the output gap.⁸

In the absence of a cost-push shock in the durables sector ($u_t^x = 0$), this version of the Phillips curve implies that to stabilize the output gap (or equivalently to stabilize employment), it is necessary and sufficient to stabilize inflation in the durable goods sector π_t^x . This is a special instance of a “divine coincidence” for durable goods (see Blanchard and Galí, 2007).

⁸BHK (2008) analyzed a special case in which prices for long-lived durable goods were perfectly flexible. They showed that in that case, monetary policy was approximately neutral. The model here extends this result and shows that if durable goods have sticky prices, then it is only the price rigidity for the durable goods that matters for the dynamics of aggregate output and employment.

4.3 A Useful Special Case

To gain insight into the behavior of the model, we next consider an instance of the model that permits an approximate analytical solution. The special case requires both δ and r near 0 as well as large price rigidity. We assume that the structural shocks are short-lived and so present only transitory changes to the equilibrium. These assumptions imply that we are considering a case of an “ideal durable” as discussed above and ensure that we can use the approximations $q_t \approx q$ and $D_t \approx D$ in constructing the equilibrium. To further simplify the analysis, we impose the additional assumption that $\rho = \sigma$ which implies that the marginal utility of non-durable consumption is simply $U_{C,t} = C_t^{-\frac{1}{\sigma}}$.

The Taylor rule we consider limits attention to inflation rates in the two sectors. That is, we restrict monetary policy to reaction functions of the form

$$i_t = \bar{i} + \phi_{\pi,x} \pi_t^x + \phi_{\pi,c}^c \pi_t^c \quad (30)$$

where $\phi_{\pi,x}$ and $\phi_{\pi,c}$ are the monetary authorities’ reactions to changes in inflation in the two sectors separately.

4.3.1 Equilibrium

The New Keynesian Phillips Curve for the durable sector is given by equation (29). To find the approximate Phillips Curve for the non-durable goods sector we use (18) to write the real marginal cost of non-durables as

$$mc_t^c = mc_t^x \frac{Z_t^x}{Z_t^c} \frac{P_t^x}{P_t^c} \approx mc_t^x \frac{Z_t^x}{Z_t^c} q C_t^{\frac{1}{\sigma}}$$

where the approximation uses the intratemporal efficiency condition (4) together with $q_t \approx q$. We can now write the New Keynesian Phillips Curve for the durable sector as⁹

$$\tilde{\pi}_t^c = \lambda^c \left[\frac{1}{\sigma} \tilde{C}_t + \tilde{Z}_t^x - \tilde{Z}_t^c + \frac{1}{1-\alpha} \left(\frac{1}{\eta} + \alpha \right) \hat{Y}_t \right] + \beta E_t [\tilde{\pi}_{t+1}^c] + u_t^c \quad (31)$$

⁹In the Flex-Price equilibrium $mc_t^c = mc_t^x = 1$ so $(C_t^{\text{Flex}})^{-\frac{1}{\sigma}} \approx q \frac{Z_t^x}{Z_t^c}$ and thus, we can equivalently express the non-durables Phillips Curves in terms of only the gaps $\hat{C}_t$ and $\hat{Y}_t$

$$\tilde{\pi}_t^c = \lambda^c \left[\frac{1}{\sigma} \hat{C}_t + \frac{1}{1-\alpha} \left(\frac{1}{\eta} + \alpha \right) \hat{Y}_t \right] + \beta E_t [\tilde{\pi}_{t+1}^c] + u_t^c$$

The system is now reduced to the two approximate Phillips Curves (29) and (31), the two Euler equations (7) and (8), and the Taylor rule (30). The log-linear versions of these last conditions are

$$\begin{aligned}\tilde{y}_t &= \phi_{\pi,x} \tilde{\pi}_t^x + \phi_{\pi,c} \tilde{\pi}_t^c \\ -\frac{1}{\sigma} \tilde{C}_t &= \beta \tilde{y}_t - \frac{1}{\sigma} E_t [\tilde{C}_{t+1}] - E_t [\tilde{\pi}_{t+1}^c]\end{aligned}$$

and, again using the approximation $q_t \approx q$

$$0 = \beta \tilde{y}_t - E_t [\tilde{\pi}_{t+1}^x].$$

This system of five equations is still complicated by the presence of the expectation terms above. To suppress this complexity we assume that the shocks are sufficiently transitory so that we can treat $E_t [\tilde{\pi}_{t+1}^c] = E_t [\tilde{\pi}_{t+1}^x] = 0$.¹⁰ Using the last two conditions, we immediately have $\tilde{C}_t \approx \tilde{y}_t \approx 0$ in equilibrium. This conclusion may seem unusual but it actually has a fairly direct interpretation in the context of traditional IS/LM models. The near constancy of q is similar to a perfectly elastic IS-curve. As a result, changes in monetary policy have no effect on the real interest rate. If future inflation expectations are anchored by long-run factors then the nominal interest rate will be unchanged. Since there are no changes in the nominal interest rate in equilibrium we have

$$\tilde{\pi}_t^x = -\frac{\phi_{\pi,c}}{\phi_{\pi,x}} \tilde{\pi}_t^c.$$

Assuming that the Taylor Rule coefficients are both positive, this condition implies perfectly negatively correlated inflation rates across sectors regardless of sectoral price rigidity and the type of shocks. Additionally, only the relative response to inflation (i.e. the ratio $\frac{\phi_{\pi,c}}{\phi_{\pi,x}}$) matters for the equilibrium.

Solving for the output gap and inflation in terms of the structural shocks gives the approximate equilibrium.

Proposition 2. *For the model described in this section with long-lived durables ($\delta, r \rightarrow 0$) sufficiently short-lived shocks, $\sigma = \rho$, and zero measurement error ($V[m_t^Y] = 0$), the*

¹⁰This assumption has two parts. First, we assume that productivity and cost-push shocks are uncorrelated (have no persistence). Second, we ignore the relative price between the two sectors as a state variable. Since inflation rates are quite small in equilibrium, the latter assumption is reasonable for short-lived shocks. We discuss the accuracy of this approximation below.

approximate equilibrium is given by the following equations

(i.)

$$\tilde{\pi}_t^c = -\lambda^c S^x \tilde{Z}_t^c + \lambda^c S^x \tilde{Z}_t^x + S^x u_t^c - \frac{\lambda^c}{\lambda^x} S^x u_t^x$$

(ii.)

$$\tilde{\pi}_t^x = -\lambda^x S^c \tilde{Z}_t^x + \lambda^x S^c \tilde{Z}_t^c + S^c u_t^x - \frac{\lambda^x}{\lambda^c} S^c u_t^c$$

(iii.)

$$\hat{Y}_t = \frac{1 - \alpha}{\left(\frac{1}{\eta} + \alpha\right)} \left\{ -S^c \tilde{Z}_t^x + S^c \tilde{Z}_t^c + \frac{1}{\lambda^x} (1 - S^c) u_t^x - \frac{1}{\lambda^c} S^c u_t^c \right\}$$

where $S^x = \frac{\phi_{\pi,x} \lambda^x}{\phi_{\pi,x} \lambda^x + \phi_{\pi,c} \lambda^c}$ and $S^c = 1 - S^x$.

To check the accuracy of the approximations $q_t \approx q$ and $D_t \approx D$, Table 1 compares the impact responses implied by the approximations in Proposition 2 to the exact (linearized) responses for several realistic depreciation rates. The table reports the impact reaction of employment, the output gap, and durable and non-durable inflation to the two productivity shocks and the two cost-push shocks. While the approximation does fairly well for small values of δ , it gradually breaks down as δ rises. (See Table 2 for the parameter values used to compute this special case.)

It is straight-forward to find expressions for the approximate variances of the model variables in the special case. These calculation are fairly tedious and so we summarize the expressions for the variance of aggregate inflation $\tilde{\Pi}_t = \frac{N_x}{N} \tilde{\pi}_t^x + \frac{N_c}{N} \tilde{\pi}_t^c$, sectoral inflation rates and the variance of the output gap in the following corollary.

Corollary. *The unconditional variances of aggregate inflation, sectoral inflation and the output gap are given by*

(i.)

$$\mathbb{V} [\tilde{\Pi}_t] = \left(\frac{N_x}{N} \lambda^x S^c - \frac{N_c}{N} \lambda^c S^x \right)^2 \Psi \quad (32)$$

(ii.)

$$\mathbb{V} [\tilde{\pi}_t^x] = (\lambda^x S^c)^2 \Psi \text{ and } \mathbb{V} [\tilde{\pi}_t^c] = (\lambda^c S^x)^2 \Psi$$

(iii.)

$$\mathbb{V} [\hat{Y}_t] = \left(\frac{1 - \alpha}{\frac{1}{\eta} + \alpha} \right)^2 \left\{ (S^c)^2 \Psi + \left(\frac{1}{\lambda^x} \right)^2 (S^x - S^c) \mathbb{V} [u_t^x] \right\}$$

where $\Psi \equiv V [\tilde{Z}_t^x] + V [\tilde{Z}_t^c] + \left(\frac{1}{\lambda^x}\right)^2 V [u_t^x] + \left(\frac{1}{\lambda^c}\right)^2 V [u_t^c]$.

Notice that because the inflation rates are perfectly negatively correlated in equilibrium, the variance of aggregate inflation is not simply the share-weighted average of sectoral inflation variances. Table 1 includes a comparison of the standard deviation of inflation and the output gap implied by the Corollary above with the exact standard deviations. Again, the approximation does a fairly good job, particularly for low depreciation rates.

4.3.2 The Optimal Taylor Rule in the Special Case

We are now in a position to consider the optimal Taylor Rule for the special case above. For expositional purposes we will assume that the objective of the monetary authority takes the simple ad hoc form

$$L = \mathbb{V} \left[\hat{Y}_t \right] + W_\pi \mathbb{V} \left[\tilde{\Pi}_t \right] \quad (33)$$

where W_π gives the weight of inflation relative to output stabilization in the monetary authority's objective. As in Woodford [2003] we express the objective as a “timeless” one in which the monetary authority does not incorporate the initial position of the economy but rather simply chooses a policy to minimize the unconditional weighted variance in (33). The following proposition provides an expression for the optimal Taylor Rule coefficient.

Proposition 3. *For the special case with long-lived durables ($\delta, r \rightarrow 0$) sufficiently short-lived shocks, $\sigma = \rho$, and zero measurement error, the optimal Taylor Rule requires*

$$\frac{\phi_{\pi,x}}{\phi_{\pi,c}} = \frac{\lambda^c}{\lambda^x} \left(\frac{\zeta^2 - \left(\frac{\zeta}{\lambda^x}\right)^2 \frac{\mathbb{V}[u_t^x]}{\Psi} + \frac{N_x}{N} \lambda^x W_\pi \left[\frac{N_x}{N} \lambda^x + \frac{N_c}{N} \lambda^c \right]}{\left(\frac{\zeta}{\lambda^x}\right)^2 \frac{\mathbb{V}[u_t^x]}{\Psi} + \frac{N_c}{N} \lambda^c W_\pi \left[\frac{N_x}{N} \lambda^x + \frac{N_c}{N} \lambda^c \right]} \right)$$

where $\zeta = \frac{1-\alpha}{\frac{1}{\eta}+\alpha}$.

The expression in Proposition 3 offers several insights into the optimal policy. First, consider the special case in which the monetary authority cares only about stabilizing inflation, that is, $W_\pi \rightarrow \infty$. Since sectoral inflation rates are negatively correlated, aggregate inflation variance can be completely eliminated in the model by choosing Taylor Rule coefficients in proportion to the size of each sector, that is, $\frac{\phi_{\pi,x}}{\phi_{\pi,c}} = \frac{N_x}{N_c}$. In this case, it is easy to show that the squared term in (32) is zero so $\mathbb{V} \left[\tilde{\Pi}_t \right] = 0$. This might be viewed as the most natural way for a monetary authority to react to inflation in different sectors. Intuitively, larger sectors receive greater weight than smaller sectors.

Second, suppose that $W_\pi \ll \infty$ so the monetary authority values both output and inflation

stability but that there are no cost push shocks ($\mathbb{V}[u_t^x] = \mathbb{V}[u_t^c] = 0$). In this case, the optimal ratio of Taylor coefficients is

$$\frac{\phi_{\pi,x}}{\phi_{\pi,c}} = \frac{1}{\lambda^x} \left(\frac{\zeta^2}{\frac{N_c}{N} W_\pi \left[\frac{N_x}{N} \lambda^x + \frac{N_c}{N} \lambda^c \right]} \right) + \frac{N_x}{N_c}.$$

This Taylor rule places more emphasis on durable goods inflation relative to the sectoral weights. The deviation from sectoral weights $\frac{N_x}{N_c}$ depends positively on ζ and negatively on λ^x and negatively on the average price rigidity parameters $\frac{N_x}{N} \lambda^x + \frac{N_c}{N} \lambda^c$.

Third, if $W_\pi = 0$ so that the central bank cares only about output stabilization, then the optimal ratio of the Taylor coefficients is

$$\frac{\phi_{\pi,x}}{\phi_{\pi,c}} = \frac{\lambda^c}{\lambda^x} \left((\lambda^x)^2 \frac{\Psi}{\mathbb{V}[u_t^x]} - 1 \right).$$

Note first that the definition of the aggregate variance parameter Ψ implies that $(\lambda^x)^2 \frac{\Psi}{\mathbb{V}[u_t^x]} > 1$. As $\mathbb{V}[u_t^x] \rightarrow 0$ the central bank optimally responds infinitely strongly to durable goods inflation. The cost push shocks in the durable goods sector are the only impediment to achieving complete output stabilization. In the absence of these cost push shocks, the central bank can achieve its goals simply by targeting durable goods inflation. If instead $\mathbb{V}[u_t^x] \rightarrow \infty$ then the optimal response places less and less weight on the durable goods sector. In the more realistic case in which the variances of both cost-push shocks become arbitrarily large, the ratio $\frac{\phi_{\pi,x}}{\phi_{\pi,c}}$ approaches $\frac{\lambda^c}{\lambda^x}$ (a result reminiscent of Aoki 2001).

These limiting results are natural in light of the close connection between inflation stabilization and output stabilization. In the limiting special case considered here, the monetary authority optimally reacts more to durable goods inflation if it cares more about stabilizing the output gap and/or if the variance of durable goods cost-push shocks is lowered. In contrast, if the variance of durable goods cost push shocks is high, it is optimally to react to sectoral inflation in accordance with their share in GDP.

Of course, these results depend critically on being in the low-depreciation (i.e., ideal durable) limit and the other assumptions underlying Propositions 2 and 3. To check the accuracy of the predictions, we calculate the impact responses to sectoral productivity shocks Z_t^x and Z_t^c , and to sectoral cost-push shocks u_t^x and u_t^c for the special case for several different depreciation rates. Table 1 reports the variances of employment, the output gap, durable goods inflation and non-durable goods inflation, as well as the ratio of optimal Taylor rule

coefficients for both the approximation (the first column) and the exact numerical solutions. According to the table, the approximation is surprisingly accurate for low depreciation rates.

4.4 Quantitative Analysis

In this section we use calibrated versions of the DSGE model to analyze the performance of different Taylor rules. We begin by specifying a baseline calibration of the model and then compute the optimal Taylor rule associated with various model specifications. An optimal Taylor rule maximizes a given criterion function subject to the constraint that monetary policy adhere to a specification in the family given by (23).¹¹ In this section, motivated in part by the results in the special case above, we impose an additional restriction on the rule. The special case above showed that the equilibrium depended only on the ratio of the Taylor coefficients $\frac{\phi_{\pi,x}}{\phi_{\pi,c}}$. This result approximately carries over to many parametric settings and, in such cases, leads to numerical instability of the optimal Taylor rule coefficients. We place additional limitations on the Taylor rule by further restricting the family to

$$i_t = \bar{i} + \phi_Y Y_t^m + \Phi_\Pi (w_x \pi_t^x + (1 - w_x) \pi_t^c) \quad (34)$$

where Φ_Π is a fixed number which normalizes the central bank's reaction to a measure of inflation (below we select $\phi_\Pi = 2$). In terms of the earlier analysis, the ratio of the coefficients is simply $\frac{\phi_{\pi,x}}{\phi_{\pi,c}} = \frac{w_x}{1-w_x}$. If the weights w_x and $(1 - w_x)$ are equal to the share of durable goods production and non-durable goods production in the economy, then this rule reacts to standard aggregate inflation. For fixed ϕ_Π , the optimal Taylor rule in this setup is one in which ϕ_Y and the weight w_x maximize the objective function.

4.4.1 Baseline Calibration

We calibrate the model for ease of exposition of our results rather than for empirical plausibility and illustrate the robustness of the conclusions. We set the annual time discount rate to imply a subjective time discount factor of two percent. The intertemporal elasticity of substitution (σ) is 0.5. This is somewhat higher than conventional estimates (e.g., Hall 1988) but lower than log utility. The Frisch labor supply elasticity (η) is set to 1.0. We set the elasticity of substitution between durable and non-durable consumption (ρ) of 0.8 and we choose the

¹¹See Boehm and House [2014] and the references therein for additional discussion of optimal Taylor Rules.

weight on nondurable consumption (ψ_c) to imply $\frac{C}{Y} = .75$. The elasticity in the two sectors' aggregators (ε) is 7. The capital share parameter (α) is 0.35. We choose the Calvo parameters θ^x and θ^c to imply a 6-month half-life of nominal rigidity.

The persistence of the productivity and cost-push shocks is set to 0.5 annually. Measurement error shocks have no persistence. Innovations of the productivity shocks are assumed to have an annual standard deviation of 1 percent. The annual standard deviations of cost-push shocks are 0.1. Hence, the baseline calibration places substantially greater weight on productivity shocks than the cost-push shocks.

We consider several different depreciation rates (δ). The baseline depreciation rate is set to 0.05 annually. This is somewhat lower than the standard calibration of 10 percent annually but somewhat higher than the depreciation rate for structures. See Fraumeni [1998] for a detailed discussion of the various depreciation rates corresponding to a wide variety of goods in the economy. The baseline calibration is summarized in Table 2.

4.4.2 Numerical Illustrations

Table 3 shows the optimal weight on durables inflation w_x and the optimal response to the output gap ϕ_Y for different calibrations and welfare objectives. Calibration (i) is the baseline setting. The remaining calibrations (labeled (ii.), ... (vi.) in the table) have parameter values equal to the baseline calibration except for a single parametric variation. Calibration (ii) features substantially less measurement error in the output gap than in the baseline calibration. Specifically, we set the (annual) standard deviation of measurement error to $\sqrt{0.25}$ rather than 1.00. Calibration (iii) has a high annual depreciation rate ($\delta = 0.20$). Calibrations (iv) and (v) have asymmetric price rigidity across sectors; (iv) has a half-life of price rigidity in the non-durables sector of 1 quarter while (v) has the reverse (a half-life of durable good price rigidity of 1 quarter). Calibration (vi) has an annual standard deviation of the cost-push shock innovations of 0.2 rather than 0.1.

In the baseline calibration, w_x always exceeds a quarter – the weight durable goods inflation receives in overall CPI inflation. Importantly, w_x exceeds a quarter regardless of the objective. When the central bank cares equally about inflation and the output gap it is optimal to place a weight of almost 72 percent on durable goods inflation. Under the assumption that the central bank minimizes the utility-based loss function, w_x is still about 0.50. Interestingly, it is optimal to place substantial weight on durables inflation even if the central bank does not value low output volatility. When the central bank only values output stability a weight of

almost 80 percent is optimal.

The remaining columns of Table 3 illustrate other calibrations. Perhaps not surprisingly, if measurement error is fairly small, the central bank responds stronger to the output gap (calibration ii.). In this case w_x falls somewhat but still remains well above one quarter. Some of this difference is due to the “substitutability” between responding to the output gap and responding to durables inflation. Reflecting on the previous discussion in Section 4.1, in the absence of strong cost push shocks, output stabilization and stabilization of the durable goods sector are identical. As a result, if the output gap is poorly measured, it is preferable to avoid strong responses to the gap and to instead target durables inflation.

Calibration (iii.) shows that the results continue to hold even when the depreciation rate is as large as 20 percent annually. While w_x barely changes relative to the baseline calibration, it falls to one quarter as the depreciation rate approaches one. Figure 1 shows the optimal weight w_x for the four different welfare objectives in Table 3. The figure shows that for low depreciation rates, the central bank reacts more to durable goods inflation relative to its share in GDP (the dashed line at 0.25 in the figure). Not surprisingly, as the depreciation rate approaches 1.00, the weight on durable goods inflation approaches its share in GDP 0.25. Columns (iv.), (v.), and (vi.) show that our results continue to hold for asymmetric sectoral price rigidity or when the cost-push shocks have a greater variance than in the baseline calibration.

Figures 2 to 5 show impulse response functions. We contrast three Taylor rules: (a) a Taylor rule in which the coefficients are chosen to maximize the ad-hoc objective with $W_\pi = 1$, (b) a Taylor rule which is optimal for the utility-based objective, and (c) a Taylor rule that uses the weight $w_x = 0.25$ as CPI inflation would suggest and an output gap response that is optimal for the ad-hoc objective with $W_\pi = 1$. Across all four figures, the Taylor rule that is optimal for the utility-based objective (full black line) produces the smallest inflation responses. In contrast, the Taylor rule that is optimal for the ad-hoc objective (dashed line) leads to the smallest output gap. The Taylor rule with sub-optimal coefficient $w_x = 0.25$ produces a substantially larger output gap – except for the cost-push shock in the durables sector.

5 Conclusion

Inflation targeting has become a standard operating procedure for many modern central banks. In part this emphasis is motivated by modern New Keynesian theory which argues that there is a direct connection between inflation stabilization and output stabilization. The “divine coincidence” says that output stabilization and inflation stabilization are equivalent. However, most of the analysis providing the basic rationale for inflation targeting is based on models with a single good and thus a single rate of inflation. Compared to the volume of work focusing on one-good models, much less attention has been devoted to the study of which inflation rate central banks should target when there are non-trivial differences in inflation across sectors.

This study considers whether durable goods inflation should be overweighted relative to its share in GDP. We find that often it is indeed preferable to place greater emphasis on stabilizing durable goods inflation. Durable goods have much higher interest elasticities of demand than non-durables and thus should be much more sensitive to interest rate changes (and thus the specification of the Taylor rule). In a limiting case, we obtain a divine coincidence for durable goods inflation. In the absence of cost-push shocks in the durable goods sector, stabilizing inflation in the durable goods sector is tantamount to stabilizing aggregate output and employment. These results suggest that greater emphasis should be placed on accurately measuring and monitoring durable goods price changes over the monetary business cycle.

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Appendix: Proofs of the Propositions

This appendix provides abbreviated proofs of the propositions. Detailed proofs as well as computer programs used in the paper, are available from the authors upon request.

Lemma 1. The efficient allocation must satisfy the following conditions

$$U_{C,t}(C_t^F, D_t^F) \alpha \frac{C_t^F}{K_{C,t}^F} = \gamma_t^F \alpha \frac{X_t^F}{K_{X,t}^F} \quad (\text{A1})$$

$$U_C(C_t^F, D_t^F) = \frac{v'(N_t^F)}{(1-\alpha) \frac{C_t^F}{N_{C,t}^F}} \quad (\text{A2})$$

$$\gamma_t^F = \frac{v'(N_t^F)}{(1-\alpha) A_{x,t} (K_{X,t}^F)^\alpha (N_{X,t}^F)^{-\alpha}} \quad (\text{A3})$$

$$\gamma_t^F = U_D(C_t^F, D_t^F) + \beta(1-\delta) E_t[\gamma_{t+1}^F] \quad (\text{A4})$$

Proof of Lemma 1: The efficient (flexible price) allocation (indexed with superscript F) solves the following planning problem

$$\max E_0 \sum_{t=0}^{\infty} \beta^j (U(C_t^F, D_t^F) - v(N_t^F))$$

subject to

$$\begin{aligned} D_t^F &= X_t^F + D_{t-1}^F (1-\delta) \\ N_{C,t}^F + N_{X,t}^F &= N_t^F \text{ and } K_{C,t}^F + K_{X,t}^F = K_t^F \\ X_t^F &= A_{x,t} (K_{X,t}^F)^\alpha (N_{X,t}^F)^{1-\alpha} \text{ and } C_t^F = A_{c,t} (K_{C,t}^F)^\alpha (N_{C,t}^F)^{1-\alpha} \end{aligned}$$

The conditions above are the associated first order conditions for this optimization problem. (It is well-known (e.g. Gali 2008, or Woodford 2003) that this allocation can be decentralized by subsidizing sales of monopolistic firms in order to restore the efficient level of production. We assume that such a subsidy is in place.) $\square$

Lemma 2. Let Z be a random variable and $g(Z)$ a twice differentiable function, then

(i.) the variance of $g(Z)$ approximated around constant Z_0 is

$$\mathbb{V}[g(Z)] = (g'(Z_0))^2 \mathbb{V}[Z] + o(\|\text{d}\Omega_t\|^3). \quad (\text{A5})$$

(ii.) the expectation of $g(Z)$ approximated around its mean $E[Z]$ is

$$\mathbb{E}[g(Z)] = g(\mathbb{E}[Z]) + \frac{g''(\mathbb{E}[Z])}{2} \mathbb{V}[Z] + o(\|\text{d}\Omega_t\|^3). \quad (\text{A6})$$

(iii.)

$$Z_t - Z_t^F = Z_t^F \hat{Z}_t + \frac{1}{2} Z_t^F (\hat{Z}_t)^2 + o(\|\text{d}\Omega_t\|^3) \quad (\text{A7})$$

Proof of Lemma 2: For (i) note that the second order Taylor expansion around Z_0 is

$$g(Z) = g(Z_0) + g'(Z_0)(Z - Z_0) + \frac{g''(Z_0)}{2} (Z - Z_0)^2 + o(\|\text{d}\Omega_t\|^3).$$

Then

$$\begin{aligned}\mathbb{V}[g(Z)] &= \mathbb{V}\left[g(Z_0) + g'(Z_0)(Z - Z_0) + \frac{g''(Z_0)}{2}(Z - Z_0)^2 + o(\|d\Omega_t\|^3)\right] \\ &= (g'(Z_0))^2 \mathbb{V}[Z] + o(\|d\Omega_t\|^3).\end{aligned}$$

The proof of (ii.) is similar to (i.) and is therefore omitted. To prove (iii.) notice that for any variable Z_t

$$\frac{Z_t - Z_t^F}{Z_t^F} = \exp\left(\ln\left(\frac{Z_t}{Z_t^F}\right)\right) - 1 = \ln\left(\frac{Z_t}{Z_t^F}\right) + \frac{1}{2}\left(\ln\left(\frac{Z_t}{Z_t^F}\right)\right)^2 + o(\|d\Omega_t\|^3).$$

(iii.) then follows immediately. $\square$

Proof of Proposition 1: We denote the log difference between a generic variable X_t and its flexible price counterpart X_t^F as $\hat{X}_t = \ln X_t - \ln X_t^F$. Denote differences in levels as $dX_t = X_t - X_t^F$. We adopt the following notation for dealing with the orders of magnitude in the approximation. Let Ω_t be the vector of all state variables. We denote any first-order function of the difference $d\Omega_t$ as

$$\Gamma_1(d\Omega_t) = o(\|d\Omega_t\|^1),$$

if $\Gamma_1(d\Omega_t) \rightarrow 0$ as $\|d\Omega_t\| \rightarrow 0$. Here, $\|\cdot\|$ is the Euclidean norm. Similarly, a function Γ_2 is of second order, written as $\Gamma_2(d\Omega_t) = o(\|d\Omega_t\|^2)$, if $\frac{\Gamma_2(d\Omega_t)}{\|d\Omega_t\|^2} \rightarrow 0$ as $\|d\Omega_t\| \rightarrow 0$. Higher orders are defined analogously.

We construct a second order approximation to the unconditional expectation of the households' utility function, $(1 - \beta) \mathbb{E}[\sum_{t=0}^{\infty} \beta^t U_t]$ in the neighborhood of the flexible price allocation. We write the households' flow-utility as $\mathcal{U}_t = U(C_t, D_t) - v(N_t)$. Using Lemma 2,

$$\begin{aligned}U(C_t, D_t) &= U_t^F + U_{C,t}^F(C_t - C_t^F) + U_{D,t}^F(D_t - D_t^F) + \frac{1}{2}U_{CC,t}^F(C_t - C_t^F)^2 \\ &\quad + \frac{1}{2}U_{DD,t}^F(D_t - D_t^F)^2 + U_{CD,t}^F(C_t - C_t^F)(D_t - D_t^F) + o(\|d\Omega_t\|^3)\end{aligned}\tag{A8}$$

$$v(N_t) = v_t^F + v_{N,t}^F(N_t - N_t^F) + \frac{1}{2}v_{NN,t}^F(N_t - N_t^F)^2 + o(\|d\Omega_t\|^3)\tag{A9}$$

where $U_t^F = U(C_t^F, D_t^F)$, $v_t^F = v(N_t^F)$, $U_{C,t}^F = \frac{\partial U(C_t, D_t)}{\partial C_t}$ evaluated at (C_t^F, D_t^F) , etc. Next, notice that

$$\sum_{t=0}^{\infty} \beta^t U_{D,t}^F(D_t - D_t^F) = \sum_{t=0}^{\infty} \beta^t U_{D,t}^F \sum_{s=0}^t (1 - \delta)^s (X_{t-s} - X_{t-s}^F) + \sum_{t=0}^{\infty} \beta^t U_{D,t}^F (1 - \delta)^{t+1} (D_{-1} - D_{-1}^F)$$

The first term can be written as

$$\begin{aligned}\sum_{t=0}^{\infty} \beta^t U_{D,t}^F \sum_{s=0}^t (1 - \delta)^s (X_{t-s} - X_{t-s}^F) &= U_{D,0}^F (X_0 - X_0^F) + \beta U_{D,1}^F (X_1 - X_1^F + (1 - \delta)(X_0 - X_0^F)) \\ &\quad + \beta^2 U_{D,2}^F (X_2 - X_2^F + (1 - \delta)(X_1 - X_1^F) + (1 - \delta)^2 (X_0 - X_0^F)) + \dots \\ &= \sum_{t=0}^{\infty} \beta^t (X_t - X_t^F) \sum_{s=0}^{\infty} [\beta(1 - \delta)]^s U_{D,t+s}^F\end{aligned}$$

Next, iterate forward equation (A4) to get

$$\gamma_t^F = \mathbb{E}_t \sum_{s=0}^{\infty} [\beta(1 - \delta)]^s U_{D,t+s}^F$$

Hence, we have that

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U_{D,t}^F \sum_{s=0}^t (1-\delta)^s (X_{t-s} - X_{t-s}^F) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (X_t - X_t^F) \gamma_t^F$$

and then

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U_{D,t}^F (D_t - D_t^F) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (X_t - X_t^F) \gamma_t^F + \text{t.i.p.}$$

where t.i.p. stands for terms independent of policy such as $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U_{D,t}^F (1-\delta)^{t+1} (D_{-1} - D_{-1}^F)$.

It then follows that

$$\begin{aligned} \mathbb{E} \sum_{t=0}^{\infty} \beta^t u(C_t, D_t) &= \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[u_t^F + u_{C,t}^F (C_t - C_t^F) + \gamma_t^F (X_t - X_t^F) + \frac{1}{2} u_{CC,t}^F (C_t - C_t^F)^2 \right. \\ &\quad \left. + \frac{1}{2} u_{DD,t}^F (D_t - D_t^F)^2 + u_{CD,t}^F (C_t - C_t^F) (D_t - D_t^F) + \text{t.i.p.} + o(\|\text{d}\Omega_t\|^3) \right] \end{aligned}$$

We will (somewhat imprecisely) continue to refer to the term in square brackets as $u(C_t, D_t)$.

Next, use (A7) to get

$$\begin{aligned} U(C_t, D_t) &= U_t^F + U_{C,t}^F C_t^F \left(\hat{C}_t + \frac{1}{2} \left(1 + \frac{u_{CC,t}^F C_t^F}{u_{C,t}^F} \right) (\hat{C}_t)^2 \right) + \gamma_t^F \left(X_t^F \hat{X}_t + \frac{1}{2} X_t^F (\hat{X}_t)^2 \right) \\ &\quad + \frac{1}{2} U_{DD,t}^F (D_t^F \hat{D}_t)^2 + U_{CD,t}^F (C_t^F \hat{C}_t) (D_t^F \hat{D}_t) + \text{t.i.p.} + o(\|\text{d}\Omega_t\|^3) \end{aligned}$$

Define σ_C as $\sigma_C = -\frac{U_C}{U_{CC}}$ and note that

$$\frac{U_{CC,t}^F C_t^F}{U_{C,t}^F} = \frac{U_{CC} C}{U_C} + o(\|\text{d}\Omega_t\|^1) = -\sigma_C^{-1} + o(\|\text{d}\Omega_t\|^1)$$

Since $\frac{U_{CC,t}^F C_t^F}{U_{C,t}^F}$ is multiplied only by second order terms, it can be replaced with $-\sigma_C^{-1}$. Then

$$\begin{aligned} U(C_t, D_t) &= U_t^F + U_{C,t}^F C_t^F \left(\hat{C}_t + \frac{1}{2} (1 - \sigma_C^{-1}) (\hat{C}_t)^2 \right) + \gamma_t^F \left(X_t^F \hat{X}_t + \frac{1}{2} X_t^F (\hat{X}_t)^2 \right) \\ &\quad + \frac{1}{2} U_{DD,t}^F (D_t^F \hat{D}_t)^2 + U_{CD,t}^F (C_t^F \hat{C}_t) (D_t^F \hat{D}_t) + \text{t.i.p.} + o(\|\text{d}\Omega_t\|^3) \end{aligned}$$

Use equation (A7) together with the disutility of labor (A9) and rearrange to get

$$v(N_t) = v_t^F + v_{N,t}^F N_t^F \left(\hat{N}_t + \frac{1}{2} \left(1 + \frac{v_{NN,t}^F N_t^F}{v_{N,t}^F} \right) (\hat{N}_t)^2 \right) + o(\|\text{d}\Omega_t\|^3)$$

Let $\eta = \frac{v_N}{v_{NN}}$. Then $\frac{v_{NN,t}^F N_t^F}{v_{N,t}^F} = \eta^{-1} + o(\|\text{d}\Omega_t\|^1)$ so that

$$v(N_t) = v_t^F + v_{t,1}^F N_t^F \left(\hat{N}_t + \frac{1}{2} (1 + \eta^{-1}) (\hat{N}_t)^2 \right) + o(\|\text{d}\Omega_t\|^3).$$

Taken together, we get

$$\begin{aligned}
\mathcal{U}_t - \mathcal{U}_t^F &= U_{C,t}^F C_t^F \left(\hat{C}_t + \frac{1}{2} (1 - \sigma_C^{-1}) (\hat{C}_t)^2 \right) + \gamma_t^F \left(X_t^F \hat{X}_t + \frac{1}{2} X_t^F (\hat{X}_t)^2 \right) \\
&\quad + \frac{1}{2} U_{DD,t}^F (D_t^F \hat{D}_t)^2 + U_{CD,t}^F (C_t^F \hat{C}_t) (D_t^F \hat{D}_t) \\
&\quad - v_{t,1}^F N_t^F \left(\hat{N}_t + \frac{1}{2} (1 + \eta^{-1}) (\hat{N}_t)^2 \right) + \text{t.i.p.} + o(\|\text{d}\Omega_t\|^3).
\end{aligned} \tag{A10}$$

We now construct second order approximations of $\hat{N}_{X,t}$, $\hat{N}_{C,t}$ and $\hat{N}_t$. Notice that $N_{X,t} = \int_0^1 n_{x,t}(s) ds$. Together with the production function $x_t(s) = A_{x,t} (k_{x,t}(s))^\alpha (n_{x,t}(s))^{1-\alpha}$ and the equilibrium condition $\frac{k_{x,t}(s)}{n_{x,t}(s)} = \frac{k_{c,t}(s)}{n_{c,t}(s)} = \frac{K}{N_t}$ we obtain $n_{x,t}(s) = \frac{x_t(s)}{A_{x,t} \left(\frac{K}{N_t}\right)^\alpha}$. Combining this with the demand function $x_t(s) = X_t \left(\frac{p_{x,t}(s)}{P_{X,t}}\right)^{-\varepsilon}$ we have

$$N_{X,t} = \frac{1}{A_{x,t} \left(\frac{K}{N_t}\right)^\alpha} X_t (P_{X,t})^\varepsilon \int_0^1 (p_{x,t}(s))^{-\varepsilon} ds$$

or in logs,

$$\ln(N_{X,t}) = -\ln(A_{x,t}) + \alpha \ln(N_t) - \alpha \ln(K) + \ln(X_t) + \varepsilon \ln(P_{X,t}) + \ln \int_0^1 (p_{x,t}(s))^{-\varepsilon} ds.$$

while in the flex-price economy we have $\ln(N_{X,t}^F) = \ln(X_t^F) - \ln(A_{x,t}) - \alpha \ln(K) + \alpha \ln(N_t^F)$. Subtracting $\ln(N_{X,t}^F)$ from $\ln(N_{X,t})$ then yields

$$\hat{N}_{X,t} = \alpha \hat{N}_t + \hat{X}_t + \varepsilon \ln(P_{X,t}) + \ln \int_0^1 (p_{x,t}(s))^{-\varepsilon} ds. \tag{A11}$$

To approximate the log of the integral, use (A6) for $Z = (p_{x,t}(s))^{-\varepsilon}$ and the transformation $g(Z) = \ln(Z)$, to get

$$\ln \int_0^1 (p_{x,t}(s))^{-\varepsilon} ds = -\varepsilon \mathbb{E}[\ln(p_{x,t}(s))] + \frac{1}{2} \frac{1}{\left\{ \mathbb{E}[(p_{x,t}(s))^{-\varepsilon}] \right\}^2} \mathbb{V}[(p_{x,t}(s))^{-\varepsilon}] + o(\|\text{d}\Omega_t\|^3) \tag{A12}$$

Similar steps lead to

$$E[(p_{x,t}(s))^{-\varepsilon}] = \exp[-\varepsilon E[\ln(p_{x,t}(s))]] \left(1 + \frac{\varepsilon^2}{2} \mathbb{V}[\ln(p_{x,t}(s))] \right) + o(\|\text{d}\Omega_t\|^3)$$

and

$$\mathbb{V}[(p_{x,t}(s))^{-\varepsilon}] = \varepsilon^2 (\exp[-\varepsilon E[\ln(p_{x,t}(s))]])^2 \mathbb{V}[\ln(p_{x,t}(s))] + o(\|\text{d}\Omega_t\|^3)$$

We can now write

$$\ln \int_0^1 (p_{x,t}(s))^{-\varepsilon} ds = -\varepsilon \mathbb{E}[\ln(p_{x,t}(s))] + \frac{1}{2} \varepsilon^2 \frac{\mathbb{V}[\ln(p_{x,t}(s))]}{\left\{ \left(1 + \frac{\varepsilon^2}{2} \mathbb{V}[\ln(p_{x,t}(s))] \right) \right\}^2} + o(\|\text{d}\Omega_t\|^3)$$

Now let $\bar{P}_{X,t} = \mathbb{E}[\ln(p_{x,t}(s))]$ and $\Delta_{X,t} = \mathbb{V}[\ln(p_{x,t}(s))]$ so that

$$\ln \int_0^1 (p_{x,t}(s))^{-\varepsilon} ds = -\varepsilon \bar{P}_{X,t} + \frac{1}{2} \varepsilon^2 \frac{\Delta_{X,t}}{\left\{ 1 + \frac{\varepsilon^2}{2} \Delta_{X,t} \right\}^2} + o(\|\text{d}\Omega_t\|^3)$$

We now approximate the term $\Delta_{X,t} \left\{ 1 + \frac{\varepsilon^2}{2} \Delta_{X,t} \right\}^{-2}$ around $\Delta_X = 0$, the steady state (and flex-price)

level of price dispersion. Since

$$\Delta_{X,t} \left\{ 1 + \frac{\varepsilon^2}{2} \Delta_{X,t} \right\}^{-2} = \Delta_{X,t} + o(\|\mathrm{d}\Omega_t\|^3)$$

we get

$$\ln \int_0^1 (p_{x,t}(s))^{-\varepsilon} ds = -\varepsilon \bar{P}_{X,t} + \frac{1}{2} \varepsilon^2 \Delta_{X,t} + o(\|\mathrm{d}\Omega_t\|^3)$$

Together with (A11) it follows that

$$\hat{N}_{X,t} = \alpha \hat{N}_t + \hat{X}_t + \varepsilon (\ln(P_{X,t}) - \bar{P}_{X,t}) + \frac{1}{2} \varepsilon^2 \Delta_{X,t} + o(\|\mathrm{d}\Omega_t\|^3) \quad (\text{A13})$$

We next turn to the price index which, in logs, is

$$\ln(P_{X,t}) = \frac{1}{1-\varepsilon} \ln \left(\int_0^1 [P_{X,t}(i)]^{1-\varepsilon} di \right)$$

Applying equation (A6) with $g(z) = \ln(Z)$ and $Z = [P_{X,t}(i)]^{1-\varepsilon}$ gives

$$\ln \left(\int_0^1 [P_{X,t}(i)]^{1-\varepsilon} di \right) = (1-\varepsilon) (\mathbb{E} [\ln(P_{X,t}(i))]) + \frac{1}{2} \frac{1}{\left(\mathbb{E} [[P_{X,t}(i)]^{1-\varepsilon}] \right)^2} \mathbb{V} [[P_{X,t}(i)]^{1-\varepsilon}] + o(\|\mathrm{d}\Omega_t\|^3) \quad (\text{A14})$$

The terms $\mathbb{E} [[P_{X,t}(i)]^{1-\varepsilon}]$ and $\mathbb{V} [[P_{X,t}(i)]^{1-\varepsilon}]$ can be expressed as

$$\mathbb{E} [[P_{X,t}(i)]^{1-\varepsilon}] = \exp((1-\varepsilon) \mathbb{E} [\ln(P_{X,t}(i))]) \left(1 + \frac{(1-\varepsilon)^2}{2} \mathbb{V} [\ln(P_{X,t}(i))] \right) + o(\|\mathrm{d}\Omega_t\|^3)$$

and

$$\mathbb{V} [(P_{X,t}(i))^{1-\varepsilon}] = ((1-\varepsilon) \exp((1-\varepsilon) \mathbb{E} [\ln(P_{X,t}(i))]))^2 \mathbb{V} [\ln(P_{X,t}(i))] + o(\|\mathrm{d}\Omega_t\|^3)$$

so that (A14) yields

$$\ln \left(\int_0^1 [P_{X,t}(i)]^{1-\varepsilon} di \right) = (1-\varepsilon) (\mathbb{E} [\ln(P_{X,t}(i))]) + \frac{(1-\varepsilon)^2}{2} \frac{\mathbb{V} [\ln(P_{X,t}(i))]}{\left(1 + \frac{(1-\varepsilon)^2}{2} \mathbb{V} [\ln(P_{X,t}(i))] \right)^2} + o(\|\mathrm{d}\Omega_t\|^3)$$

Recalling that $\Delta_{X,t} = \mathbb{V} [\ln(P_{X,t}(i))]$ and approximating $\Delta_{X,t} \left(1 + \frac{(1-\varepsilon)^2}{2} \Delta_{X,t} \right)^{-2}$ around $\Delta_X = 0$ yields

$$\frac{\mathbb{V} [\ln(P_{X,t}(i))]}{\left(1 + \frac{(1-\varepsilon)^2}{2} \mathbb{V} [\ln(P_{X,t}(i))] \right)^2} = \Delta_{X,t} + o(\|\mathrm{d}\Omega_t\|^3)$$

Since $\bar{P}_{X,t} = \mathbb{E} [\ln(p_{x,t}(s))]$, we can then write

$$\ln \left(\int_0^1 [P_{X,t}(i)]^{1-\varepsilon} di \right) = (1-\varepsilon) \bar{P}_{X,t} + \frac{(1-\varepsilon)^2}{2} \Delta_{X,t} + o(\|\mathrm{d}\Omega_t\|^3)$$

and hence

$$\ln(P_{X,t}) = \bar{P}_{X,t} + \frac{1-\varepsilon}{2} \Delta_{X,t} + o(\|\mathrm{d}\Omega_t\|^3)$$

Plugging this into equation (A13) gives

$$\hat{N}_{X,t} = \alpha \hat{N}_t + \hat{X}_t + \frac{\varepsilon}{2} \Delta_{X,t} + o(\|\mathrm{d}\Omega_t\|^3) \quad (\text{A15})$$

Since production in the nondurables sector is symmetric, we obtain an analogous equation for C (steps omitted)

Next apply equation (A7) to

$$N_t - N_t^F = (N_{X,t} - N_{X,t}^F) + (N_{C,t} - N_{C,t}^F)$$

to obtain

$$N_t^F \hat{N}_t + \frac{1}{2} N_t^F (\hat{N}_t)^2 = N_{X,t}^F \hat{N}_{X,t} + \frac{1}{2} N_{X,t}^F (\hat{N}_{X,t})^2 + N_{C,t}^F \hat{N}_{C,t} + \frac{1}{2} N_{C,t}^F (\hat{N}_{C,t})^2 + o(\|d\Omega_t\|^3)$$

Plugging in equation (A15) and its analogue for the nondurables sector yields

$$\begin{aligned} N_t^F \hat{N}_t + \frac{1}{2} N_t^F (\hat{N}_t)^2 &= N_{X,t}^F \left(\alpha \hat{N}_t + \hat{X}_t + \frac{1}{2} \varepsilon \Delta_{X,t} \right) + \frac{1}{2} N_{X,t}^F (\alpha \hat{N}_t + \hat{X}_t)^2 \\ &\quad + N_{C,t}^F \left(\alpha \hat{N}_t + \hat{C}_t + \frac{1}{2} \varepsilon \Delta_{C,t} \right) + \frac{1}{2} N_{C,t}^F (\alpha \hat{N}_t + \hat{C}_t)^2 + o(\|d\Omega_t\|^3) \end{aligned}$$

To solve this equation for $\hat{N}_t$ we guess the relationship

$$\hat{N}_t = \Phi_{C,t}^F \hat{C}_t + \Phi_{X,t}^F \hat{X}_t + \Phi_{CC,t}^F (\hat{C}_t)^2 + \Phi_{XX,t}^F (\hat{X}_t)^2 + \Phi_{CX,t}^F \hat{C}_t \hat{X}_t + \Phi_{\Delta C,t}^F \Delta_{C,t} + \Phi_{\Delta X,t}^F \Delta_{X,t} + o(\|d\Omega_t\|^3)$$

and verify this guess. In particular, we obtain

$$\begin{aligned} \Phi_{C,t}^F &= \frac{1}{1-\alpha} \frac{N_{C,t}^F}{N_t^F}, \quad \Phi_{X,t}^F = \frac{1}{1-\alpha} \frac{N_{X,t}^F}{N_t^F} \\ \Phi_{\Delta C,t}^F &= \frac{1}{2} \varepsilon \frac{1}{1-\alpha} \frac{N_{C,t}^F}{N_t^F}, \quad \Phi_{\Delta X,t}^F = \frac{1}{2} \varepsilon \frac{1}{1-\alpha} \frac{N_{X,t}^F}{N_t^F} \\ \Phi_{CC,t}^F &= \frac{1}{2} \frac{1}{1-\alpha} \frac{N_{C,t}^F}{N_t^F} - \frac{1}{2} \frac{1}{1-\alpha} \left(\frac{N_{C,t}^F}{N_t^F} \right)^2, \quad \Phi_{XX,t}^F = \frac{1}{2} \frac{1}{1-\alpha} \frac{N_{X,t}^F}{N_t^F} - \frac{1}{2} \frac{1}{1-\alpha} \left(\frac{N_{X,t}^F}{N_t^F} \right)^2 \\ \Phi_{CX,t}^F &= -\frac{1}{1-\alpha} \frac{N_{X,t}^F}{N_t^F} \frac{N_{C,t}^F}{N_t^F} \end{aligned}$$

Plugging these into the guess yields an expression for $\hat{N}_t$ which we use to compute $\hat{N}_t + \frac{1}{2} (1 + \eta^{-1}) (\hat{N}_t)^2$. After some algebra, we obtain

$$\begin{aligned} \hat{N}_t + \frac{1}{2} (1 + \eta^{-1}) (\hat{N}_t)^2 &= \frac{1}{1-\alpha} \frac{N_{C,t}^F}{N_t^F} \hat{C}_t + \frac{1}{1-\alpha} \frac{N_{X,t}^F}{N_t^F} \hat{X}_t + \frac{1}{2} \frac{1}{1-\alpha} \frac{N_{C,t}^F}{N_t^F} (\hat{C}_t)^2 + \frac{1}{2} \frac{1}{1-\alpha} \frac{N_{X,t}^F}{N_t^F} (\hat{X}_t)^2 \\ &\quad + \frac{1}{2} (\alpha + \eta^{-1}) \left(\frac{1}{1-\alpha} \right)^2 \left[\frac{N_{C,t}^F}{N_t^F} \hat{C}_t + \frac{N_{X,t}^F}{N_t^F} \hat{X}_t \right]^2 \\ &\quad + \frac{1}{2} \varepsilon \frac{1}{1-\alpha} \frac{N_{C,t}^F}{N_t^F} \Delta_{C,t} + \frac{1}{2} \varepsilon \frac{1}{1-\alpha} \frac{N_{X,t}^F}{N_t^F} \Delta_{X,t} + o(\|d\Omega_t\|^3) \end{aligned} \tag{A16}$$

Using (A16), (A10), (A1) and (A4) we get

$$\begin{aligned} \frac{U(C_t, D_t) - v(N_t)}{U_{C,t}^F C_t^{\frac{1}{2}}} &= -\sigma_C^{-1} (\hat{C}_t)^2 - \frac{N_t^F}{N_{C,t}^F} \frac{\alpha + \eta^{-1}}{1-\alpha} \left[\frac{N_{C,t}^F}{N_t^F} \hat{C}_t + \frac{N_{X,t}^F}{N_t^F} \hat{X}_t \right]^2 - \varepsilon \Delta_{C,t} - \frac{N_{X,t}^F}{N_{C,t}^F} \varepsilon \Delta_{X,t} \\ &\quad + \frac{U_{DD,t}^F (D_t^F)^2}{U_{C,t}^F C_t^F} (\hat{D}_t)^2 + 2 \frac{U_{DC,t}^F D_t^F}{U_{C,t}^F} (\hat{D}_t \hat{C}_t) + \text{t.i.p.} + o(\|d\Omega_t\|^3) \end{aligned} \tag{A17}$$

We next show that $\frac{N_{C,t}^F}{N_t^F} \hat{C}_t + \frac{N_{X,t}^F}{N_t^F} \hat{X}_t$ is a first order approximation to GDP. Notice that a first order approximation is enough here because this term is squared later. By definition, real GDP is $Y_t = P_C C_t + P_X X_t$. Then to a first order

$$Y_t - Y_t^F = P_C (C_t - C_t^F) + P_X (X_t - X_t^F) + o(\|d\Omega_t\|^2)$$

Using (A7) gives

$$\hat{Y}_t = \frac{P_C C_t^F}{P_C C_t^F + P_X X_t^F} \hat{C}_t + \frac{P_X X_t^F}{P_C C_t^F + P_X X_t^F} \hat{X}_t + o(\|d\Omega_t\|^2)$$

We assume that $P_C = P_X$ in the steady state. Furthermore, $C_t^F = A_{c,t} \left(\frac{K}{N_t^F}\right)^\alpha N_{C,t}^F$ and $X_t^F = A_{x,t} \left(\frac{K}{N_t^F}\right)^\alpha N_{X,t}^F$ which implies

$$\hat{Y}_t = \frac{A_{c,t} N_{C,t}^F}{A_{c,t} N_{C,t}^F + A_{x,t} N_{X,t}^F} \hat{C}_t + \frac{A_{x,t} N_{X,t}^F}{A_{c,t} N_{C,t}^F + A_{x,t} N_{X,t}^F} \hat{X}_t + o(\|d\Omega_t\|^2)$$

Next notice that

$$\frac{A_{c,t} N_{C,t}^F}{A_{c,t} N_{C,t}^F + A_{x,t} N_{X,t}^F} = \frac{N_{C,t}^F}{N_t^F} + o(\|d\Omega_t\|^1)$$

where we assumed here that $A_c = A_x$. A similar relationship holds for $\frac{A_{x,t} N_{X,t}^F}{A_{c,t} N_{C,t}^F + A_{x,t} N_{X,t}^F}$. We then obtain

$$\hat{Y}_t = \frac{N_{C,t}^F}{N_t^F} \hat{C}_t + \frac{N_{X,t}^F}{N_t^F} \hat{X}_t + o(\|d\Omega_t\|^2)$$

Plugging this into (A17) gives

$$\begin{aligned} \frac{U(C_t, D_t) - v(N_t)}{u_{C,t}^F C_t^{F\frac{1}{2}}} &= -\sigma_C^{-1} (\hat{C}_t)^2 - \frac{N_t^F}{N_{C,t}^F} \frac{\alpha + \eta^{-1}}{1 - \alpha} (\hat{Y}_t)^2 - \varepsilon \Delta_{C,t} - \frac{N_{X,t}^F}{N_{C,t}^F} \varepsilon \Delta_{X,t} \\ &\quad + \frac{U_{DD,t}^F (D_t^F)^2}{U_{C,t}^F C_t^F} (\hat{D}_t)^2 + 2 \frac{U_{DC,t}^F D_t^F}{U_{C,t}^F} (\hat{D}_t \hat{C}_t) + \text{t.i.p.} + o(\|d\Omega_t\|^3) \end{aligned}$$

The remaining terms are second order. Since the flex-price level differs only to a first order from the steady state, we can replace flex-price levels by steady state levels. Then

$$\begin{aligned} \frac{U(C_t, D_t) - v(N_t)}{U_C C^{\frac{1}{2}}} &= -\sigma_C^{-1} (\hat{C}_t)^2 - \frac{N}{N_C} \left[\frac{\alpha + \eta^{-1}}{1 - \alpha} (\hat{Y}_t)^2 \right] - \varepsilon \Delta_{C,t} - \frac{N_X}{N_C} \varepsilon \Delta_{X,t} \\ &\quad + \frac{U_{DD}(D)^2}{U_C C} (\hat{D}_t)^2 + 2 \frac{U_{DC} D}{U_C} (\hat{D}_t \hat{C}_t) + \text{t.i.p.} + o(\|d\Omega_t\|^3) \end{aligned}$$

Finally, following Woodford (2003, pp.399-400) we have

$$\sum_{t=0}^{\infty} \beta^t \Delta_{X,t} = \frac{\theta_X}{(1 - \theta_X)(1 - \beta\theta_X)} \sum_{t=0}^{\infty} \beta^t \pi_{X,t}^2 + \text{t.i.p.} + o(\|d\Omega_t\|^3)$$

and we can write the welfare function (up to a multiplicative constant) as

$$(1 - \beta) \mathbb{E} \left[\sum_{t=0}^{\infty} \beta^t \frac{U(C_t, D_t) - v(N_t)}{U_C C^{\frac{1}{2}}} \right] = \mathbb{E} \left[\frac{U(C_t, D_t) - v(N_t)}{U_C C^{\frac{1}{2}}} \right]$$

where

$$\begin{aligned} \frac{U(C_t, D_t) - v(N_t)}{U_C C^{\frac{1}{2}}} &= -\sigma_C^{-1} (\hat{C}_t)^2 - \frac{N}{N_C} \frac{\alpha + \eta^{-1}}{1 - \alpha} (\hat{Y}_t)^2 - \frac{\varepsilon \theta_C}{(1 - \theta_C)(1 - \beta \theta_C)} \pi_{C,t}^2 - \frac{N_X}{N_C} \frac{\varepsilon \theta_X}{(1 - \theta_X)(1 - \beta \theta_X)} \pi_{X,t}^2 \\ &\quad + \frac{U_{DD}(D)^2}{U_C C} (\hat{D}_t)^2 + 2 \frac{U_{DC} D}{U_C} (\hat{D}_t \hat{C}_t) + \text{t.i.p.} + o(\|\text{d}\Omega_t\|^3). \end{aligned}$$

Taking the unconditional expectation gives the result. $\square$

Proof of Proposition 2. Under the assumptions of the approximation, the model reduces to

$$\begin{aligned} \tilde{\pi}_t^c &= \lambda^c \left[\left(\frac{1}{\eta} + \alpha \right) \tilde{N}_t - \tilde{Z}_t^c \right] + u_t^c \\ \tilde{\pi}_t^x &= \lambda^x \left[\left(\frac{1}{\eta} + \alpha \right) \tilde{N}_t - \tilde{Z}_t^x \right] + u_t^x \\ 0 &= \phi_{\pi,x} \pi_t^x + \phi_{\pi,c} \pi_t^c \end{aligned}$$

which is a system of three equations and three endogenous variables. Using the method of undetermined coefficients, this system is easily solved. It is then simple to find the expression for the output gap using the approximate Phillips curve in the durables sector $\tilde{\pi}_t^x = \lambda^x \zeta \hat{Y}_t + u_t^x$. $\square$

Proof of the Corollary to Proposition 2: The results follow after straightforward algebra from Proposition 2. $\square$

Proof of Proposition 3: Using the Corollary of Proposition 2, it is straightforward to write

$$\begin{aligned} \mathbb{V}[\hat{Y}_t] &= \left(\frac{\zeta \lambda^c}{\lambda^x \Phi + \lambda^c} \right)^2 \mathbb{V}[\tilde{Z}_t^c] + \left(\frac{\zeta \lambda^c}{\Phi \lambda^x + \lambda^c} \right)^2 \mathbb{V}[\tilde{Z}_t^x] + \left(\frac{\zeta}{\Phi \lambda^x + \lambda^c} \right)^2 \mathbb{V}[u_t^c] + \left(\frac{\zeta \Phi}{\Phi \lambda^x + \lambda^c} \right)^2 \mathbb{V}[u_t^x] \\ \mathbb{V}[\tilde{\pi}_t] &= \left(\frac{N_X}{N} - \frac{N_C}{N} \Phi \right)^2 \left(\frac{\lambda^x \lambda^c}{\lambda^x \Phi + \lambda^c} \right)^2 \Psi \end{aligned}$$

where $\Psi = \left[\mathbb{V}[\tilde{Z}_t^c] + \mathbb{V}[\tilde{Z}_t^x] + (\lambda^c)^{-2} \mathbb{V}[u_t^c] + (\lambda^x)^{-2} \mathbb{V}[u_t^x] \right]$ and $\Phi = \frac{\phi_{\pi,x}}{\phi_{\pi,c}}$.

Recall that the loss function is $L = V[\hat{Y}_t] + W_\pi V[\tilde{\pi}_t]$. Plugging in the above variances gives, after simplification,

$$\begin{aligned} L &= (\lambda^x \Phi + \lambda^c)^{-2} \left\{ (\zeta \lambda^c)^2 \mathbb{V}[\tilde{Z}_t^c] + (\zeta \lambda^c)^2 \mathbb{V}[\tilde{Z}_t^x] + (\zeta)^2 \mathbb{V}[u_t^c] + (\zeta \Phi)^2 \mathbb{V}[u_t^x] \right. \\ &\quad \left. + W_\pi \left(\frac{N_X}{N} \right)^2 (\lambda^x \lambda^c)^2 \Psi - 2W_\pi \frac{N_X}{N} \frac{N_C}{N} \Phi (\lambda^x \lambda^c)^2 \Psi + W_\pi \left(\frac{N_C}{N} \Phi \right)^2 (\lambda^x \lambda^c)^2 \Psi \right\} \end{aligned}$$

Simplification of the first order condition

$$\begin{aligned} 0 &= -2(\lambda^x \Phi + \lambda^c)^{-3} \lambda^x \left\{ (\zeta \lambda^c)^2 \mathbb{V}[\tilde{Z}_t^c] + (\zeta \lambda^c)^2 \mathbb{V}[\tilde{Z}_t^x] + (\zeta)^2 \mathbb{V}[u_t^c] + (\zeta \Phi)^2 \mathbb{V}[u_t^x] \right. \\ &\quad \left. + W_\pi \left(\frac{N_X}{N} \right)^2 (\lambda^x \lambda^c)^2 \Psi - 2W_\pi \frac{N_X}{N} \frac{N_C}{N} \Phi (\lambda^x \lambda^c)^2 \Psi + W_\pi \left(\frac{N_C}{N} \Phi \right)^2 (\lambda^x \lambda^c)^2 \Psi \right\} \\ &\quad + (\lambda^x \Phi + \lambda^c)^{-2} \left\{ 2\Phi (\zeta)^2 \mathbb{V}[u_t^x] - 2W_\pi \frac{N_X}{N} \frac{N_C}{N} (\lambda^x \lambda^c)^2 \Psi + 2W_\pi \Phi \left(\frac{N_C}{N} \right)^2 (\lambda^x \lambda^c)^2 \Psi \right\} \end{aligned}$$

gives the desired result. $\square$

TABLE 1: COMPARISON OF APPROXIMATION AND EXACT SOLUTION

	Approx.	$\delta = .01$	$\delta = .03$	$\delta = .05$	$\delta = .10$	$\delta = .20$
<i>Reaction to a shock to Z_t^x</i>						
Employment	0.189	0.191	0.190	0.187	0.181	0.154
Output gap	-0.118	-0.115	-0.114	-0.113	-0.111	-0.116
Durables inflation	-0.014	-0.011	-0.011	-0.012	-0.014	-0.022
Nondur. inflation	0.014	0.014	0.014	0.013	0.011	0.003
<i>Reaction to a shock to Z_t^c</i>						
Employment	0.182	0.175	0.172	0.167	0.154	0.096
Output gap	0.118	0.115	0.114	0.112	0.106	0.075
Durables inflation	0.014	0.011	0.010	0.009	0.005	-0.012
Nondur. inflation	-0.014	-0.014	-0.014	-0.016	-0.020	-0.037
<i>Reaction to a shock to u_t^x</i>						
Employment	-1.357	-1.380	-1.374	-1.365	-1.344	-1.254
Output gap	-0.882	-0.897	-0.893	-0.887	-0.873	-0.815
Durables inflation	0.098	0.079	0.079	0.081	0.088	0.117
Nondur. inflation	-0.102	-0.100	-0.099	-0.097	-0.090	-0.061
<i>Reaction to a shock to u_t^c</i>						
Employment	-1.308	-1.274	-1.270	-1.261	-1.237	-1.128
Output gap	-0.850	-0.828	-0.825	-0.820	-0.804	-0.733
Durables inflation	-0.098	-0.078	-0.077	-0.075	-0.067	-0.038
Nondur. inflation	0.102	0.100	0.101	0.103	0.112	0.140
<i>Standard deviation</i>						
Aggregate inflation	0.074	0.077	0.078	0.080	0.092	0.136
Output gap	1.236	1.233	1.229	1.221	1.202	1.125
<i>Optimal Taylor rule</i>						
$\phi_{\pi,x}/\phi_{\pi,c}$	1.038	0.998	0.999	0.999	0.997	0.983

Notes: This table compares the impact responses of employment, the output gap, durable and nondurable goods inflation for the approximate solution as described in Proposition 2 with the exact solution. The calibration for this limiting case is given in Table 2.

TABLE 2: MODEL CALIBRATIONS

Parameter	Special Case	Baseline calibration	Notes for baseline calibration
β	0.9951	0.9951	Corresponds to annual discount rate of 2 percent
δ	(See Table 1)	0.0127	Corresponds to annual depreciation rate of 5 percent
θ^x, θ^c	0.8909 ^a	0.7071	Corresponds to half-life of prices of 6 months
σ	0.5	0.5	Intertemporal elasticity of substitution
η	1	1	Frisch labor supply elasticity
ρ	0.8	0.8	Elasticity of substitution between durable and nondurable goods consumption
ε	7	7	Elasticity of substitution in CES aggregators
α	0.35	0.35	Capital share
$\rho_{z,c} = \rho_{z,x} = \rho_{u,c} = \rho_{u,x}$	0	0.5	Shock persistence (annual autocorrelation)
$\sigma_{z,x} = \sigma_{z,c}$	1	1	Standard deviation of productivity shocks (annual)
$\sigma_{u,x} = \sigma_{u,c}$	0.1	0.1	Standard deviation of cost-push shocks (annual)
σ_m	1	1	Standard deviation of measurement error of the output gap (annual)
ψ_c	n.a.	0.4896 ^b	Weight on durable goods utility. Calibrated to imply a consumption to output ratio of 0.75

Notes:

^a Corresponds to a half-life of exogenous price rigidity of 18 months.^b This parameter choice depends on other parameter settings. It is always set to guarantee that one quarter of value added in the economy is generated by durable goods production.

TABLE 3: OPTIMAL TAYLOR RULES FOR VARIOUS CALIBRATIONS AND OBJECTIVES

	(i.)	(ii.)	(iii.)	(iv.)	(v.)	(vi.)
	Baseline calibration	Low measure- ment error	High Depreciation	Flexible nondurable goods prices	Flexible durable goods prices	High cost- push shock variance
<i>Modified Parameter</i>	n.a.	$\sigma_m = .25$	$\delta = 0.2$	$\theta^c = 0.5^a$	$\theta^x = 0.5^b$	$\sigma_u = 0.2^c$
<i>Ad-hoc objective; $W_{\Pi} = 1$</i>						
w_X	0.716	0.664	0.699	0.683	0.619	0.458
ϕ_Y	0.417	2.025	0.316	0.524	0.000	2.618
<i>Utility-based objective</i>						
w_X	0.498	0.493	0.467	0.618	0.387	0.474
ϕ_Y	0.085	0.137	0.132	0.192	0.000	0.008
<i>Ad-hoc objective; $W_{\Pi} = 1,000,000$</i>						
w_X	0.466	0.465	0.436	0.448	0.447	0.459
ϕ_Y	0.004	0.005	0.039	0.027	0.000	0.133
<i>Ad-hoc objective; $W_{\Pi} = 0$</i>						
w_X	0.796	0.665	0.796	0.853	0.976	1.000
ϕ_Y	0.505	37.790	0.307	0.768	0.739	225.484

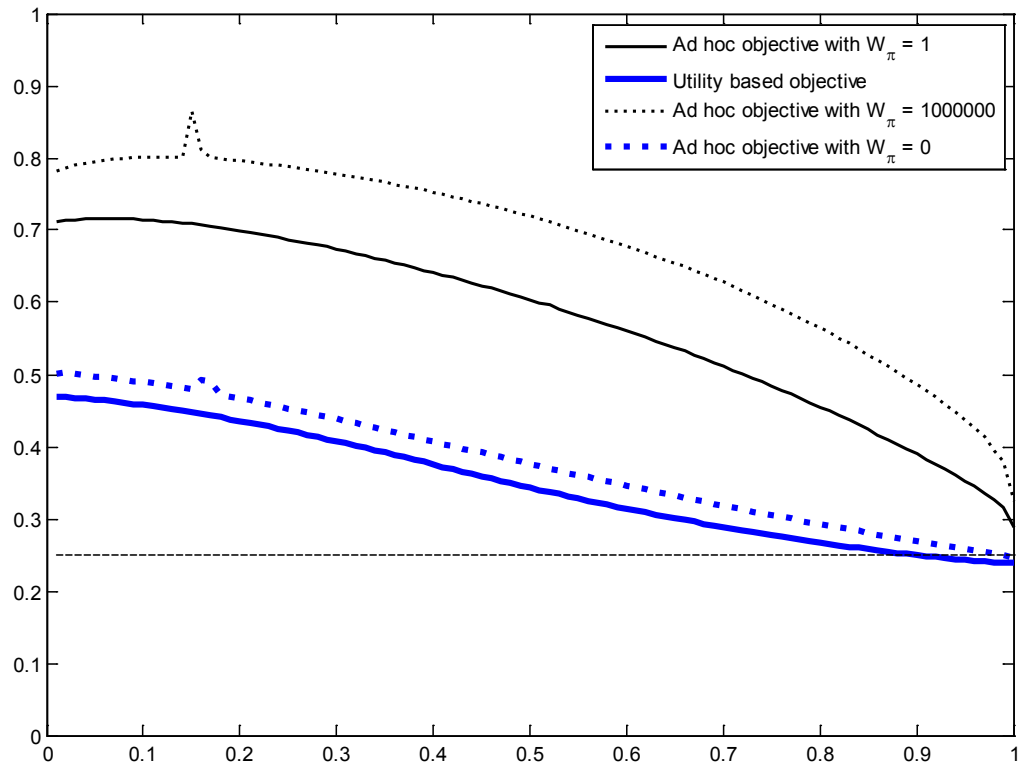
Notes: All columns start with the baseline calibration and change a single parameter. The column “low measurement error” reduces measurement error of the output gap to a quarter of that in the baseline calibration. In the column “high depreciation” the depreciation rate is increased to correspond to an annual rate of 0.2. In the next column, “flexible nondurables” the half-life of prices in the durable goods sector is reduced to 3 months. We set the standard deviation of innovations to cost-push shocks to 1 (like that of productivity shocks) in last column “high cost-push shock variances”.

^a Corresponds to a half-life of exogenous price rigidity of one quarter.

^b Corresponds to a half-life of exogenous price rigidity of one quarter.

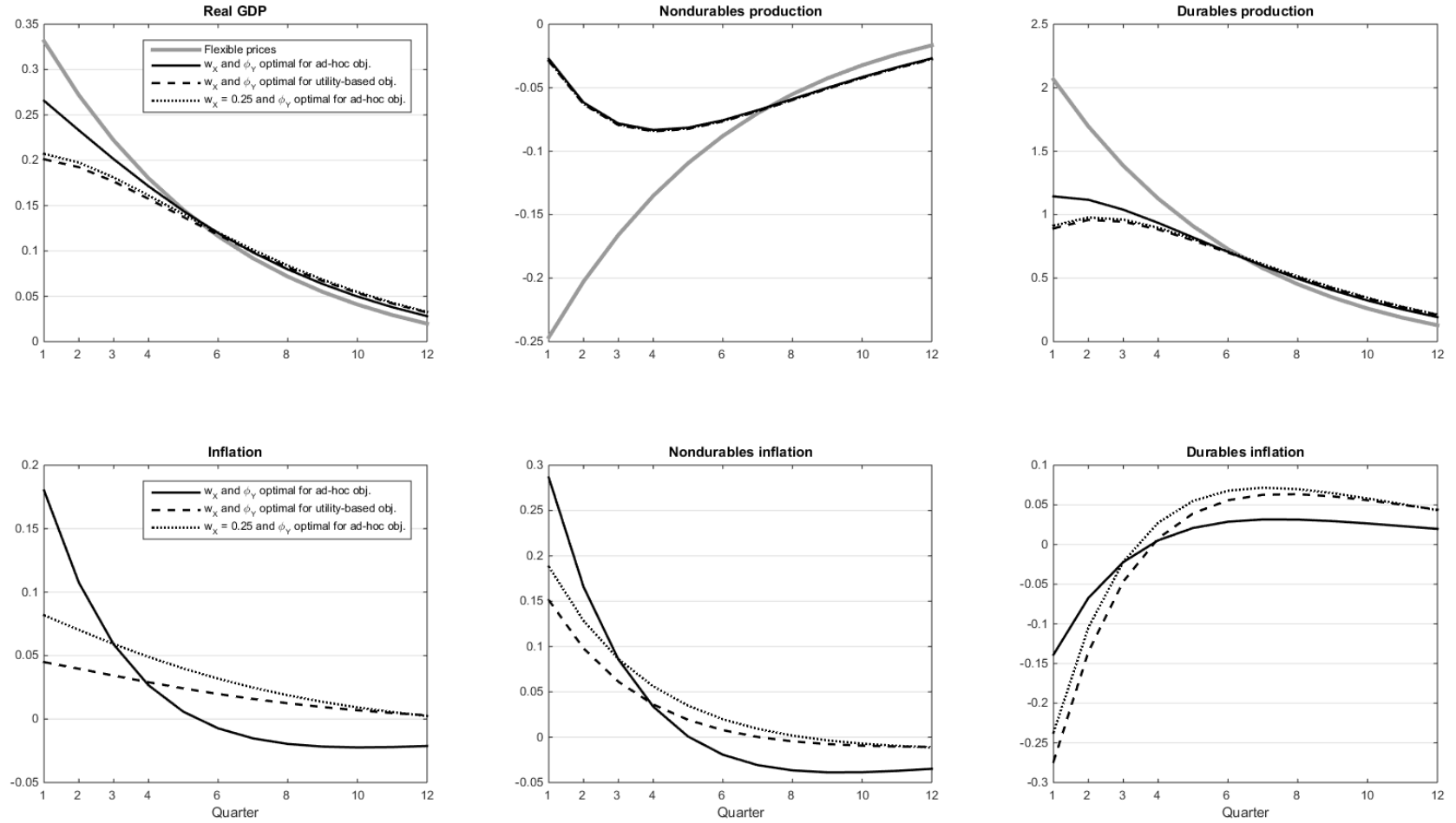
^c The variance of cost-push shocks is the same for both sectors.

FIGURE 1: OPTIMAL WEIGHT ON DURABLE GOODS VS. DEPRECIATION



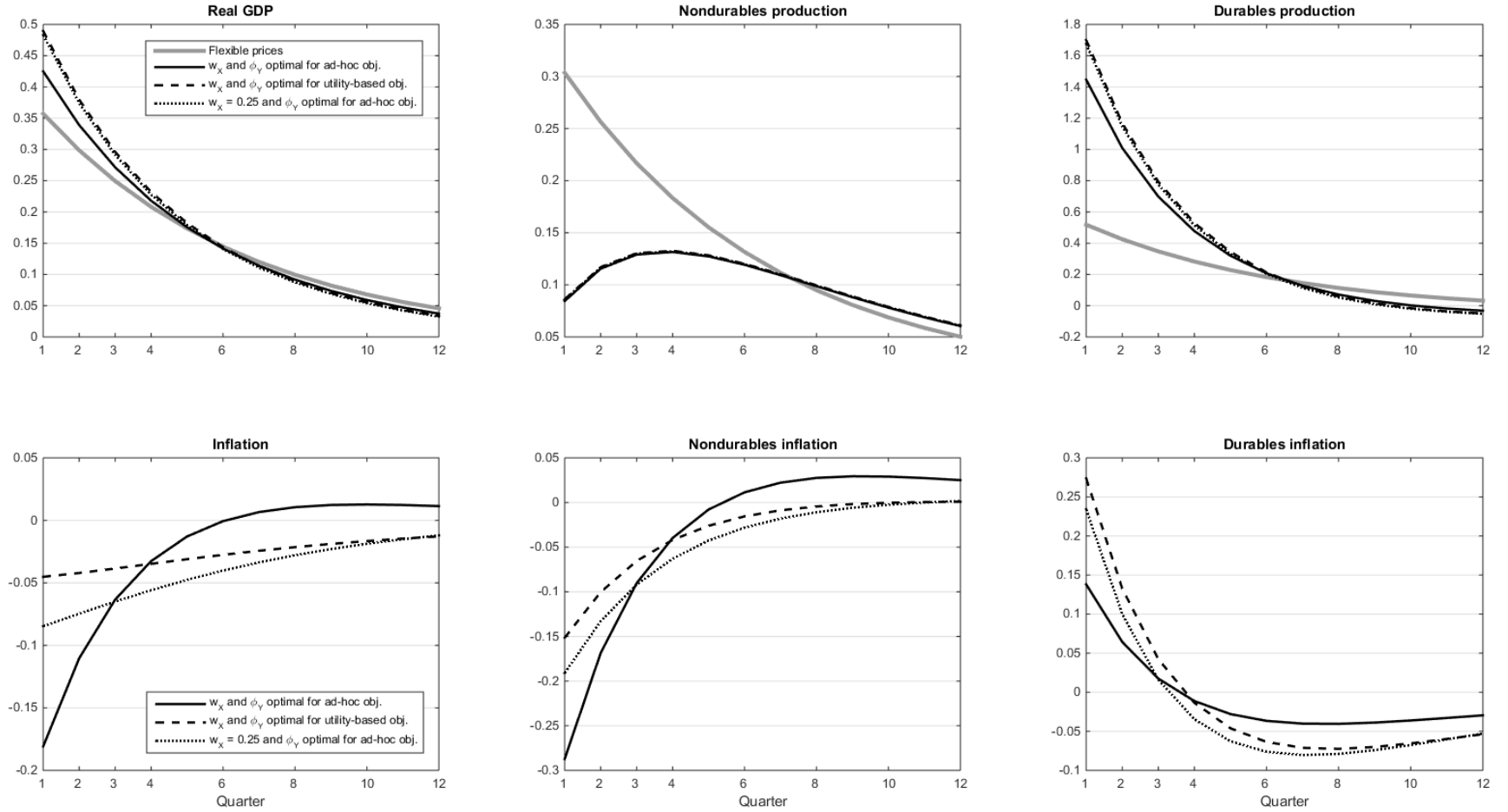
Notes: The figure shows the optimal value for w_x for the four different objectives listed plotted against various depreciation rates.

FIGURE 2: IMPULSE RESPONSE TO A SHOCK TO Z^x



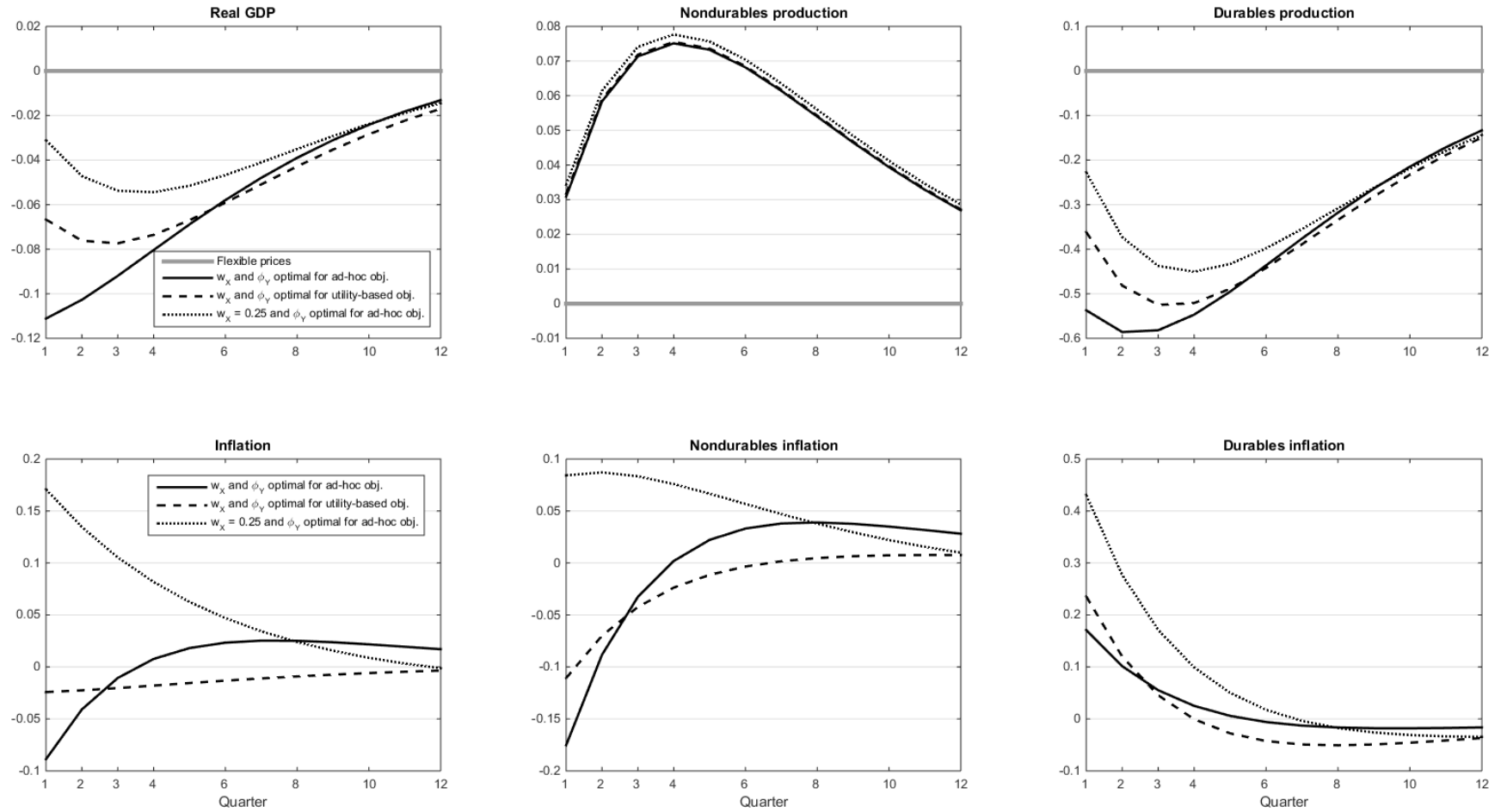
Notes: The figure shows the model's response to a one standard deviation innovation in durable goods productivity Z^x . See Table 1 for the parameter values used in the calculation.

FIGURE 3: IMPULSE RESPONSE TO A SHOCK TO Z^c



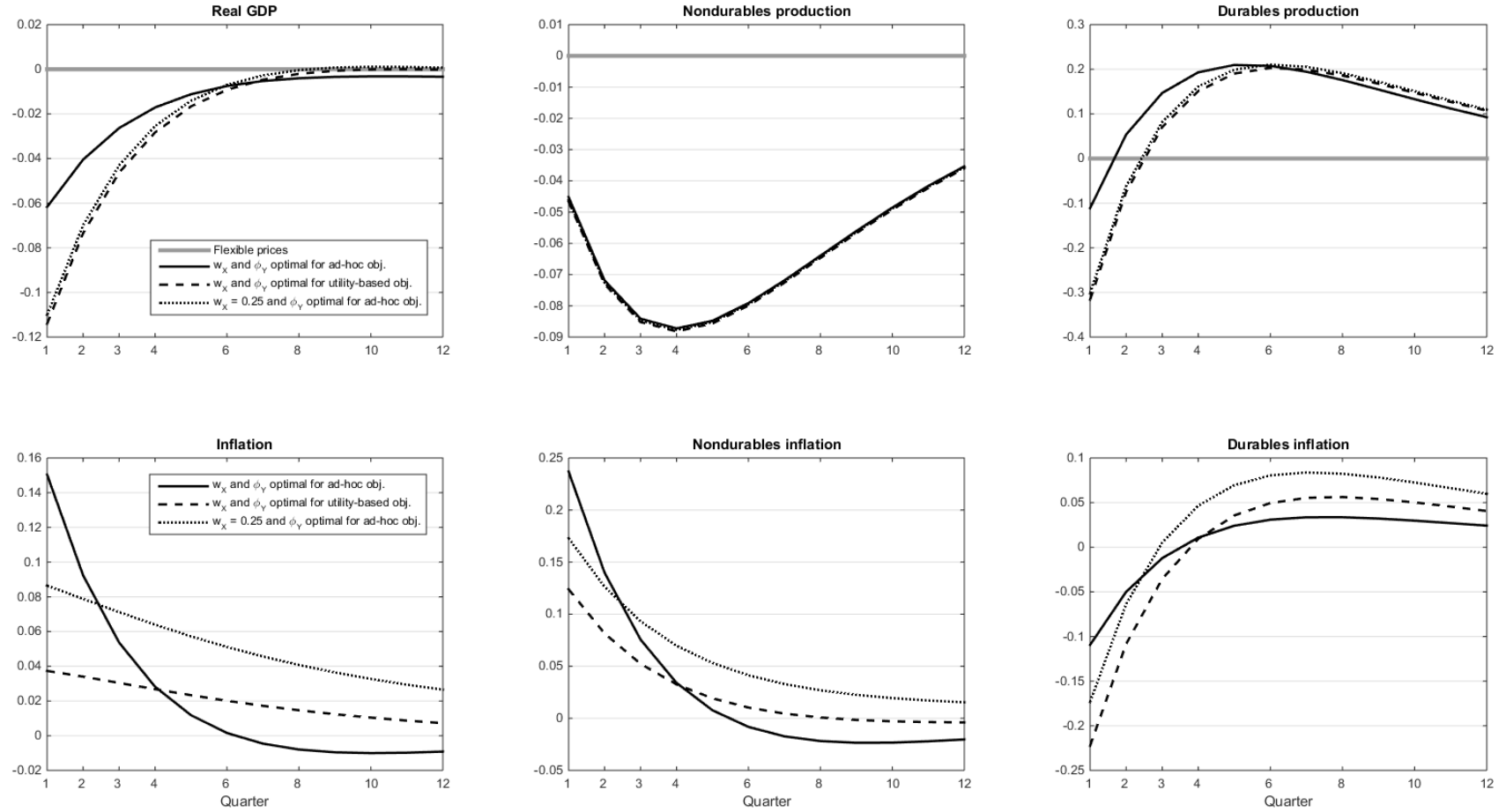
Notes: The figure shows the model's response to a one standard deviation innovation in non-durable goods productivity Z^c . See Table 1 for the parameter values used in the calculation.

FIGURE 4: IMPULSE RESPONSE TO A SHOCK TO u^x



Notes: The figure shows the model's response to a one standard deviation innovation to a cost-push shock in the durable goods sector u^x . See Table 1 for the parameter values used in the calculation.

FIGURE 5: IMPULSE RESPONSE TO A SHOCK TO u^c



Notes: The figure shows the model's response to a one standard deviation innovation to a cost-push shock in the non-durable goods sector u^c . See Table 1 for the parameter values used in the calculation.