

Sovereign Risk and Bank Risk-Taking*

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Abstract

In European countries recently hit by a sovereign debt crisis, the share of domestic sovereign debt held by the national banking system has sharply increased, raising issues in their economic and financial resilience, as well as in policy design. This paper examines these issues by analyzing the banking equilibrium in a model with optimizing banks and depositors. To the extent that sovereign default causes bank losses also independently of their holding of domestic government bonds, undercapitalized banks have an incentive to gamble on these bonds. The optimal reaction by depositors to insolvency risk imposes discipline, but also leaves the economy susceptible to self-fulfilling shifts in sentiments, where sovereign default also causes a banking crisis. Policy interventions face a trade-off between alleviating funding constraints and strengthening incentives to gamble. Subsidized loans to banks, similar to the ECB's non-targeted longer-term refinancing operations (LTRO), may eliminate the good equilibrium when the banking sector is undercapitalized. Targeted interventions have the capacity to eliminate adverse equilibria.

Keywords: Sovereign Debt Crises, Bank Risk-Taking, Financial Constraints, Eurozone

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1 Introduction

Since the eruption of the European debt crisis, the share of domestic sovereign debt held by the national banking system has increased sharply in crisis-hit countries. This created a dangerous nexus between the financial health of banks and sovereigns, and was associated with a rise in bank funding costs and the crowding out of bank lending to the private sector.¹ The high exposure of banks in crisis-hit countries to domestic sovereign debt is indeed considered a key, if not the key, source of instability in the recent European sovereign debt crisis (see e.g. Acharya et al., 2014; Farhi & Tirole, 2015; Brunnermeier et al., 2016). The question is then why have banks in the crisis-hit countries become so highly exposed to domestic sovereign debt.

In this paper, I address this question from a novel angle, calling attention to the interactions between banks and depositors, each optimizing their portfolio strategies vis-à-vis the prospect of a sovereign debt crisis. I develop my analysis specifying a small open economy model with three private agents, households, banks, and non-financial firms, and a government issuing default-risky debt. For the sake of clarity and analytical tractability, I focus on a two period economy. In the first period, banks collect deposits from households and allocate their funds between domestic sovereign bond purchases and lending to firms in need of working capital. In the second period, sovereign default occurs exogenously if fundamentals turn out to be weak.

First, I show that banks face an endogenous incentive to gamble on domestic sovereign debt, resulting from the combination of limited liability and the anticipation of (quantitatively small) losses in the event of sovereign default, that hit banks independently of their sovereign-bond holdings. These balance sheet losses reflect all costs that a domestic sovereign default can impose on banks other than the direct impact of the haircuts on sovereign bonds. By way of example, sovereign default usually leads to a deterioration in the value of illiquid assets, loss of access to foreign financing needed to roll over debt, higher taxes and/or outright expropriation by the defaulting government.

The second and most important finding of the paper pertains to the role of depositors during the crisis. To the extent that deposit insurance is incomplete and/or lacks credibility, the optimal reaction by depositors to insolvency risk has two distinct effects: On the one hand, it imposes discipline on the banks by reducing the temptation to gamble; on the other hand, unless the banks' balance sheets are entirely transparent, it leaves the economy susceptible to self-fulfilling shifts in sentiments.² Expectations may then coordinate on a bad equilibrium where sovereign default also

¹Battistini, Pagano and Simonelli (2013), Broner, Erce, Martin and Ventura (2014) and Acharya and Steffen (2015) document the rise in domestic sovereign debt holdings. Acharya et al. (2015), Altavilla, Pagano and Simonelli (2015) and Ferrando, Popov and Udell (2015) provide evidence on the adverse effects on bank lending while Acharya and Steffen (2013) show that exposure to domestic sovereign debt is associated with an increase in funding costs.

²Deposit insurance schemes typically guarantee deposits only up to a limit (Demirgüç-Kunt et al. 2008). Moreover, recent events in Cyprus and deposit outflows from the periphery show that the credibility of deposit insurance guarantees comes into question during sovereign default episodes. Depositor losses could also stem from a suspension of convertibility and a tax on deposits as in the proposed plan for Cyprus or a currency re-denomination following exit from the Eurozone (Eurogroup 2013a; Reuters 2013).

causes a banking crisis. In this bad equilibrium, shocks to sovereign risk simultaneously raise bank funding costs and drive banks to increase their purchases of domestic debt, at the expense of credit to the private sector.

The model provides a formal framework for policy assessment, calling attention to the trade-off between alleviating the constraint on bank funding and strengthening the incentives to gamble, lying at the core of policy interventions in support of financial intermediaries. As a novel insight, the model suggests that subsidized loans to banks, similar to the ECB’s non-targeted longer-term refinancing operations (LTRO), may actually eliminate the good equilibrium when the banking sector is under-capitalized. On the contrary, targeted interventions have the capacity to overcome the trade-off, and eliminate the bad equilibrium described above at all levels of bank capitalization.

During a sovereign debt crisis, banks may adopt either an ‘efficient’ or a ‘gambling’ strategy. The ‘efficient strategy’ consists of investing in a precautionary manner with the goal of remaining solvent even in the event of a sovereign default; the ‘gambling strategy’ consists of pursuing high exposure to sovereign bonds, and leads to insolvency after sovereign default. Limited liability creates an important asymmetry in the incentives to adopt either strategy. In particular, under-capitalized banks find the gambling strategy more attractive, for well known reasons: if the government does not default *ex post*, domestic sovereign bonds pay a high return driven by the default-risk premium; if the government imposes a haircut on bond holders, banks are shielded from the full consequences of the default by limited liability.³

A discontinuity is present in the optimal deposit supply schedule due to the dependence of bank solvency on deposit repayment obligations. In particular, there is a threshold level of deposits below which depositors anticipate that the bank will remain solvent in case of sovereign default and thus supply their funds at the risk-free interest rate. Above that threshold, depositors anticipate insolvency following sovereign default and require higher interest payments in compensation. The existence of this threshold is what deters banks from following the gambling strategy, because by doing so they find themselves on the high funding costs side of the deposit supply schedule.

Another determinant of a bank’s solvency prospects is its exposure to domestic sovereign debt. The higher this exposure is, the lower the level of deposits at which the bank would become insolvent in case of default. Increasing exposure thus translates into an inward shift of the deposit threshold.

The equilibrium is solved by assuming that, realistically, depositors cannot directly observe sovereign bond exposures. Banks are typically able to obscure the composition of their investment in a variety of ways, including reliance on shell corporations and complex financial instruments.⁴

³I elaborate further on how banks optimize their strategy in Section 4.1. In short, the optimal strategy is the one that yields the highest expected profits while taking the behaviour of the other banks as given. For a strategy to be implemented in equilibrium, it must be feasible and no bank should have an incentive to deviate given that the other banks follow this strategy.

⁴The level of deposits, on the other hand, is public information. Although banks may also raise funds through less transparent methods, this has no impact on the repayment prospects of depositors due to their seniority.

Depositors form expectations about the strategy that a bank follows, and consistently assess its exposure to sovereign debt. I refer to anticipations of an efficient strategy as “positive sentiments”, as opposed to “negative sentiments” associated with anticipation of “gambling.” Since the gambling strategy revolves around higher exposure, negative sentiments result in a tightening of the deposit threshold.

Banks strive to remain within the deposit threshold under an efficient strategy. Any shift to negative sentiments constrains their ability to raise funds and reduces their expected payoff. A shift to negative sentiments, however, does not alter the expected payoff under the gambling strategy. Negative sentiments then become self-fulfilling when the tightening of the deposit threshold makes it optimal for banks to deviate to the gambling strategy.

Solving for a rational expectations equilibrium, which requires that depositor sentiments are confirmed in equilibrium, I find that the type and uniqueness of the resulting equilibria is contingent on the capitalization of the banking sector. When bank capitalization is high, banks adopt an efficient strategy regardless of the location of the deposit threshold and only positive sentiments are confirmed in equilibrium. Conversely, only a gambling strategy may be sustained as an equilibrium with low bank capitalization. Within an intermediate range of capitalization, on the other hand, depositors sentiments become self-fulfilling as described above, and there are multiple equilibria.

Capital injections to the banking sector and moderate amounts of LTRO funding can be effective in eliminating adverse equilibria. However, while the former has a large budgetary cost at a time when the government is likely to be cash-struck, the latter proves to be ineffective when banks are severely under-capitalized and may even eliminate the good equilibrium if employed in excess. Contractionary monetary policy is also capable of shrinking the region of multiplicity, but this comes at a significant cost to the real economy. Strengthening deposit insurance guarantees, on the other hand, reduces bank funding costs, but also gives banks greater incentives to gamble by severing the link between their financial health and borrowing costs.

The main shortcoming of these policy measures is their inability to distinguish between banking strategies. This leads to a trade-off between alleviating funding constraints and strengthening incentives to gamble. It is possible to overcome this trade-off by using the lending requirements of TLTRO to discriminate between banking strategies. Indeed, with the appropriate combination of subsidized funding and working capital lending requirements, it is possible to eliminate adverse equilibria even at very low levels of bank capitalization.

Paradoxically, LTRO and TLTRO remain as off-equilibrium threats when they are successful in eliminating multiplicity. In this case, negative sentiments are no longer validated in equilibrium which leads to an outward shift of the deposit threshold such that banks become indifferent between deposit financing and borrowing through these schemes. Conversely, when the policy interventions are unsuccessful, banks borrow the maximum amount possible through these schemes and use it to gamble on domestic sovereign bonds. Far from assuaging depositor concerns, in this case (T)LTRO provide an additional source of funding for banks to gamble with and facilitate an increase in their

exposure to domestic sovereign debt.

The contribution of this paper is twofold. First, it provides a theoretical explanation for the increase in domestic sovereign bond purchases, the rise in bank funding costs, and the decline in bank lending observed in countries hit by the recent sovereign debt crisis. Second, it sheds new light on the mechanisms through which the sovereign-bank nexus arises, and provides a new perspective for policy evaluation.

This paper is closely related to a growing literature on the consequences of sovereign risk for the domestic the banking sector. Gennaioli, Martin and Rossi (2014) propose that banks hold sovereign bonds as a way to store liquidity. Sovereign default then reduces the liquidity available to the banking sector and leads to a decline in investment. Bocola (2015) couples this with a risk channel whereby the risks associated with lending to the productive sector increase with sovereign risk. According to both channels, banks respond to sovereign risk shocks by reducing their exposure to domestic sovereign bonds. In contrast, the gambling mechanism in this paper suggests that banks respond to sovereign risk by increasing their domestic sovereign bond exposure, in line with empirical evidence (Battistini, Pagano & Simonelli 2013; Acharya & Steffen 2015).

Broner, Erce, Martin and Ventura (2014) reach a similar conclusion with a model of creditor discrimination. In their model, risky sovereign bonds offer a higher expected return to domestic banks due to the anticipation of selective default in their favour. A rise in sovereign risk then leads to the repatriation of sovereign bonds which crowds out bank lending. Farhi and Tirole (2015), on the other hand, suggest that banks retain a high exposure to risky sovereign debt due to the anticipation of a government bailout. The main difference of this paper is that the banking sector is not shielded from the costs of sovereign default through selective default or a government bailout, and may default on depositors as a consequence. Depositors thus optimally react to insolvency risk, which in turn influences banks' gambling incentives in a manner that may create strategic complementarities between the optimal responses of banks and depositors, ultimately leading to multiplicity of equilibria.

Acharya, Dreschler and Schnabl (2014) also develop a model where an incomplete bailout of the banking sector imposes losses on depositors. However, they focus on the interactions between banks and the government and do not investigate potential strategic complementarities. Cooper and Nikolov (2013) and Leonello (2015) consider the adverse feedbacks between bank runs and sovereign default, where the former extend the Calvo (1988) framework and the latter considers rollover crises in sovereign bond markets. In both studies, the strategic complementarities are between the government and bond-holders, as well as across depositors due to the sequential service constraint, rather than between banks and depositors. To my knowledge, this paper is the first to analyze strategic complementarities between the optimal responses of banks and depositors to a sovereign debt crisis.

The remainder of the paper is structured as follows: Section 2 describes the model, Section 3 provides the solution for two benchmark cases and Section 4 describes the generalized solution.

Section 5 explains the calibration of key parameters, Section 6 conducts policy analysis and Section 7 concludes.

2 Model Environment

There are two time periods and two possible states of nature $\{H, L\}$ with high and low fundamentals, which are realized with probabilities $(1 - P)$ and P in the second period. The model features a small open economy with three private agents, households, banks and non-financial firms, and a government. In the first period, banks collect deposits from households and use their funds for sovereign bond purchases and lending to non-financial firms, which in turn produce the consumption good Y . In the remainder of this section, I provide a detailed description of these activities.

2.1 Government

Sovereign default occurs exogenously when there are low fundamentals in the second period with probability P . Thus, domestic sovereign bonds B^G are risky assets with a state-contingent gross return

$$R^G = \left\{ \begin{array}{ll} R^{G,H} & \text{with prob. } 1 - P \\ R^{G,L} & \text{with prob. } P \end{array} \right\}$$

where $R^{G,L}$ is the recovery value following a haircut $\theta \in (0, 1]$ such that $R^{G,L} = (1 - \theta) R^{G,H}$. In case of sovereign default, a lump-sum amount $T > 0$ is also deducted from the net worth N of each domestic bank. While T could reflect all costs that a domestic sovereign default can impose on banks other than the direct impact of the haircuts on sovereign bonds, its important characteristic is that banks cannot take any action to avoid it in the preceding period even though they anticipate it.

Sovereign bonds are internationally traded with deep-pocketed foreign investors as their marginal buyers. Thus, $(R^{G,H}, R^{G,L})$ are priced according to their expected return

$$E[R^G] = (1 - P) R^{G,H} + P R^{G,L} = R^* \quad (2.1)$$

where R^* is the international risk-free rate. In a monetary union setting, R^* can also be considered as the interest rate set by the common central bank.

2.2 Non-Financial Firms

The representative non-financial firm is perfectly competitive and produces consumption goods Y with the use of a Cobb-Douglas production technology $Y = I^a L^{1-a}$ where (I, L) respectively represent working capital investments and labour inputs. Labour is hired from households at a competitive wage w whereas the provision of working capital is subject to specific financial frictions.

Firms need to secure loans in order to fund their working capital investments and households cannot lend directly to them due to information asymmetries (or enforcement problems). Thus, domestic and foreign banks act as financial intermediaries which channel funds to working capital loans (K, K^*) at gross interest rates $(R^K, R^{K,*})$. Although these loans are perfectly substitutable in production with $I = K + K^*$, foreign banks incur an additional cost $\phi(K^*)$ to facilitate each unit of loans due to their disadvantage in resolving information frictions vis-à-vis domestic banks. This creates a wedge between the international risk-free rate R^* and the cost of borrowing from foreign banks such that

$$R^{K,*} = R^* + \phi(K^*) \quad (2.2)$$

where $\phi'(K^*) > 0$ and $\phi(0) = 0$. As I depreciates fully at the end of each period, the representative non-financial firm's first order conditions simply equate $(w, R^K, R^{K,*})$ to their marginal products

$$\begin{aligned} w &= (1 - a)(K + K^*)^a \\ R^{K,*} = R^K &= a(K + K^*)^{a-1} \end{aligned}$$

where labour is provided inelastically by households and normalized to $L = 1$. Combining these first order conditions with (2.2) provides an implicit expression for K^* in terms of K

$$R^* + \phi(K^*) = a(K + K^*)^{a-1} \leftrightarrow K^* = g(K) \quad (2.3)$$

with a strictly negative first derivative $g'(K) < 0$. This can also be used to pin down (w, R^K) for a given K as follows

$$\begin{aligned} w &= (1 - a)(K + g(K))^a \\ R^K &= a(K + g(K))^{a-1} \end{aligned} \quad (2.4)$$

Observe that the returns $(R^K, R^{K,*})$ to working capital loans are completely certain. While this assumption streamlines the representation considerably, the results remain valid under a generalized version of the model with risky returns to working capital lending as long as these returns

covary less strongly with the sovereign default event than the return R^G from sovereign bonds.⁵

2.3 Banks

The domestic banking sector consists of $1/v$ imperfectly competitive banks such that each bank has a market share of $v \in (0, 1]$ within the domestic financial sector. The representative bank is risk-neutral and uses deposits d and its own net worth N to invest in sovereign bonds b and working capital loans k . Thus, its budget constraint can be written as

$$b + k = N + d \quad (2.5)$$

I define $\gamma \in [0, 1]$ as the share of bank funds invested in sovereign debt such that

$$\begin{aligned} b &= \gamma (N + d) \\ k &= (1 - \gamma) (N + d) \end{aligned} \quad (2.6)$$

Then the interim profit of the representative bank is

$$\Pi = \begin{cases} (N + d) [\gamma R^{G,H} + (1 - \gamma) R^K] - Rd & \text{with prob. } 1 - P \\ (N + d) [\gamma R^{G,L} + (1 - \gamma) R^K] - Rd - T & \text{with prob. } P \end{cases} \quad (2.7)$$

where R is the gross return promised to the bank's depositors. Under limited liability, the representative bank's payoff is bounded below at zero and it is declared insolvent when the interim profits become negative. It then reneges on the promised repayments to its depositors and receives a payoff of zero such that its ex-post payoff is

$$\hat{\Pi} = \max[\Pi, 0]$$

⁵In general, returns from lending to non-financial firms tend to be more volatile than returns from sovereign bonds. This is because they respond to various forms of aggregate risk, only a small portion of which is related to sovereign default. As I focus on sovereign default risk, however, the appropriate question is whether they respond with more volatility to a shock realization which leads to sovereign default. The answer depends on the time-frame. In a long time-frame (i.e. a decade), where sovereign default risk is driven by major shocks like the global financial crisis, this may be the case. However, in a short time-frame (i.e. a quarter or a year), sovereign default risk is primarily driven by political events such as the outcome of bailout negotiations, referenda and elections. These political shocks only affect the returns from working capital lending indirectly through their effects on sovereign default. The evidence that output contractions precede the default event suggests that it is the expectations of default that have a negative impact on the real economy rather than the default event itself (Yeyati & Panizza 2011). This is precisely the channel that the model captures and implies that the risks associated with lending to non-financial firms have already been realized in the first period. Consequently, it is plausible that real lending is not as strongly covaried with sovereign default as sovereign bonds payments between $t = 1$ and $t = 2$.

The effective gross return $\hat{R}$ on deposits paid by the bank can then be described as

$$\hat{R} = \begin{cases} R & \text{if } \Pi \geq 0 \\ R^{\min} & \text{otherwise} \end{cases}$$

where $R^{\min} \in [0, R]$ is the amount covered by deposit insurance. This yields the expected gross return

$$E[\hat{R}] = \Pr[\Pi \geq 0] R + (1 - \Pr[\Pi \geq 0]) R^{\min} \quad (2.8)$$

The representative bank always makes a positive profit in state H as it is not subject to the cost T and receives a high return realization $R^{G,H}$ from sovereign bonds. Its solvency prospects in state L , on the other hand, depend on its decisions (d, γ) to leverage and invest in risky sovereign bonds as well as its capitalization N and promised interest payments R to depositors. It is useful to define $\bar{d}$ as the cut-off level of deposits above which the bank is insolvent in case of sovereign default. Using (2.7), it can be defined as follows

$$\begin{aligned} (N + \bar{d}) [\gamma R^{G,L} + (1 - \gamma) R^K] - R\bar{d} - T &= 0 \\ \therefore \bar{d} &= \max \left[\frac{N [\gamma R^{G,L} + (1 - \gamma) R^K] - T}{R - [\gamma R^{G,L} + (1 - \gamma) R^K]}, 0 \right] \end{aligned} \quad (2.9)$$

Observe that $\bar{d}$ is increasing in N and decreasing in (R, γ) . Thus, one can also regard $\bar{d}$ as a function $\bar{d}(R, \gamma, N)$.⁶ When $d \leq \bar{d}(R, \gamma, N)$, the representative bank is solvent in both states of nature such that $\Pr[\Pi \geq 0] = 1$. If we have $d > \bar{d}(R, \gamma, N)$, on the other hand, it becomes insolvent in state L such that $\Pr[\Pi \geq 0] = 1 - P$.

Finally, the relationship between individual and aggregate quantities can be written as follows

$$\begin{bmatrix} d \\ k \\ b \end{bmatrix} = v \begin{bmatrix} D \\ K \\ B \end{bmatrix}$$

where the aggregate variables are in capitals. Note that v is the market share *within the domestic economy*. When markets are internationally integrated, the domestic banking sector has a negligible market share under the small open economy setting and banks behave in a price-taking manner. I elaborate further on this in Section 3.

2.4 Households and the Deposit Supply Schedule

Households may save by depositing an amount D at domestic banks at a potentially state-contingent gross return $\hat{R}$ (as described in Section 2.3) or an amount D^* at foreign banks at a

⁶I alternate between these two notations according to convenience.

safe return R^* . With an inelastic labour supply $L = 1$, the representative household's utility maximization problem can be described as follows⁷

$$\max_{c_1, c_2, D, D^*} E[u(c_1) + \beta u(c_2)]$$

subject to the period budget constraints

$$\begin{aligned} c_1 + D + D^* &= w_1 \\ c_2 &= \hat{R}D + R^*D^* + w_2 \end{aligned} \tag{2.10}$$

where I use a logarithmic utility function $u(c) = \ln(c)$ for simplicity. The first order conditions to this problem take the form of two Euler conditions

$$\begin{aligned} u'(c_1) &= \beta E[\hat{R}u'(c_2)] \\ u'(c_1) &= \beta R^* E[u'(c_2)] \end{aligned}$$

with R^* taken out of the expectations operator as it is a certain return. Combining these conditions and splitting the expectations for $E[\hat{R}u'(c_2)]$ yields the following expression for the risk premium charged by households to domestic banks

$$E[\hat{R}] - R^* = -\frac{Cov(\hat{R}, u'(c_2))}{E[u'(c_2)]} \tag{2.11}$$

where $Cov(\hat{R}, u'(c_2)) < 0$ due to the dependence of c_2 on $\hat{R}$ as depicted by the budget constraint (2.10). Substituting in for $E[\hat{R}]$ using (2.8) provides an expression for the promised return R that the households will require to deposit at domestic banks

$$R = R^* + \frac{1 - \Pr[\Pi \geq 0]}{\Pr[\Pi \geq 0]} (R^* - R^{\min}) - \frac{Cov(\hat{R}, u'(c_2))}{\Pr[\Pi \geq 0] E[u'(c_2)]} \tag{2.12}$$

where the second term reflects the decline in the expected return due to bankruptcy while the final term is the risk premium. As expected, complete deposit insurance $R^{\min} = R$ eliminates both of these terms. When deposit insurance is incomplete with $R^{\min} < R$, however, bank solvency probability $\Pr[\Pi \geq 0]$ becomes relevant to the promised return required by households. As explained in Section 2.3, this depends on the amount of deposits collected by the representative bank such

⁷I assume that there is a unit continuum of symmetric households such that individual households' deposits are identical to the aggregate quantities. With a slight abuse of notation, I use the aggregate terms (D, D^*) while describing the household's problem in order to save on notation and distinguish this from bank values $d = vD$.

that

$$\Pr [\Pi \geq 0] = \begin{cases} 1 & \text{if } d \leq \bar{d}(R, \gamma, N) \\ 1 - P & \text{if } d > \bar{d}(R, \gamma, N) \end{cases}$$

where $\bar{d}(R, \gamma, N)$ is defined by (2.9). Households may observe the amount of deposits d and thus realize that the representative bank will remain solvent in state L when it has $d \leq \bar{d}(R, \gamma, N)$. In this case, the promised return R is certain such that $Cov(\hat{R}, u'(c_2)) = 0$ and (2.12) yields the risk-free rate R^* . When $d > \bar{d}(R, \gamma, N)$, on the other hand, households require a higher promised interest rate $R > R^*$ in compensation for the lower probability of payment and the risk premium due to $Cov(\hat{R}, u'(c_2)) < 0$. Thus, the deposit supply is given by the expression

$$R = \begin{cases} R^* & \text{if } d \leq \bar{d}(R, \gamma, N) \\ R^* + \frac{P}{1-P} (R^* - R^{\min}) - \frac{Cov(\hat{R}, u'(c_2))}{(1-P)E[u'(c_2)]} & \text{if } d > \bar{d}(R, \gamma, N) \end{cases} \quad (2.13)$$

and has a discontinuous jump at $\bar{d}$. Observe also that it is horizontal below $\bar{d}$ but becomes upward-sloping when $d > \bar{d}$ as a rise in d increases the dependence of household income on $\hat{R}$, thus increasing the risk premium whenever $\hat{R}$ is uncertain.

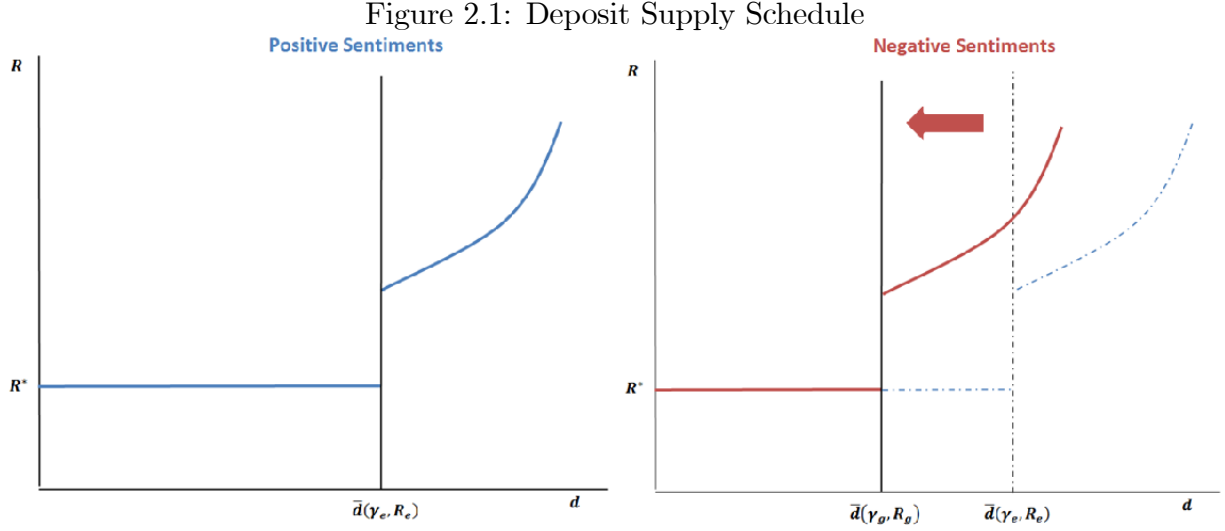
At a first look, the two-way relationship between $\bar{d}$ and R displayed by (2.9) and (2.13) appears to be a source of multiplicity. A high interest rate set by the households may become self-confirming by increasing the banks' borrowing costs to the extent that they become insolvent following sovereign default. As households are atomistic, they may not coordinate on a low interest rate equilibrium.

The problem with this proposed mechanism is that it implicitly assumes that banks are completely passive, while in fact imperfectly competitive banks internalize the deposit supply schedule given by (2.13) along with the discontinuity at $\bar{d}$. Thus, faced with the above scenario, a bank may eliminate multiplicity by reducing its deposits d to a level which ensures that it remains solvent in state L even at high interest rates.

As such, a plausible mechanism for multiplicity must also account for the reaction of banks. To that end, I assume that sovereign bond exposures γ is unobservable which would be the case if banks are able to obscure their investments through the use of shell corporations and/or complex financial instrument. This does not only prevent banks from committing to a γ value, but also creates uncertainty among households about the level of deposits above which banks become insolvent in state L .

Observe from (2.9) that a bank with a smaller share of funds invested in sovereign bonds (i.e. a lower γ value) may remain solvent in state L at higher levels of deposits. Thus, the location of threshold $\bar{d}$ in the deposit supply schedule becomes dependent on household beliefs about the strategy followed by banks. Negative household sentiments in the form of a belief that γ is high may then become self-fulfilling if the resulting inward shift in $\bar{d}$ makes it optimal for banks to adopt such a strategy.

Before I can elaborate further on this, however, it is necessary to provide an explanation of the process through which banks determine their strategy. As a first step, I consider the solutions under two special cases. This serves to provide a benchmark as well as giving some initial intuition about the model without excessive complexity.



3 Solutions for the Special Cases

3.1 Efficient equilibrium

Suppose the representative bank has sufficient capitalization N to avoid bankruptcy after sovereign default. This requires the following restriction

$$(N + d_e) [\gamma_e R^{G,L} + (1 - \gamma_e) R_e^K] - R_e d_e - T \geq 0 \quad (3.1)$$

which also ensures that the representative bank will be solvent in state H as $R^{G,H} > R^{G,L}$. Thus, households treat domestic deposits as safe assets which pay a certain return R_e . Using (2.12) with $\Pr [\Pi_e \geq 0] = 1$ and $Cov(\hat{R}_e, u'(c_{2,e})) = 0$, it is easy to show that domestic banks will be able to borrow at the same safe rate as foreign banks

$$R_e = R^* \quad (3.2)$$

where the subscript e indicates that the representative bank follows an efficient strategy. Under this strategy, the bank anticipates that it will be solvent regardless of the state realization in period 2 and thus internalizes the profit it makes in both of states of nature $\{H, L\}$.⁸ Its profit

⁸I elaborate further on the determination of banking strategies in the next section.

maximization problem can then be described as

$$\max_{d_e, \gamma_e \in [0,1]} E \left[\hat{\Pi}_e \right] = (N + d_e) \left[\gamma_e \left((1 - P) R^{G,H} + P R^{G,L} \right) + (1 - \gamma_e) R_e^K \right] - R_e d_e - PT$$

subject to (2.5), (2.6) and

$$\frac{\partial R_e^K}{\partial k_e} = - \frac{a(1-a)(1+g'(K_e))}{(K_e + g(K_e))^{2-a}}$$

which arises from the bank's price-making power in the market for working capital loans and allows it to internalize the effects of its decisions on R_e^K through (2.4). It is apparent from (2.2) that the bank's market power is proportionate to the additional cost $\phi(K^*)$ faced by foreign banks which diminishes the ability of non-financial firms to substitute domestic and foreign credit.

Without solvency risk, domestic banks lack price-making power in the deposits market. Although they internalize the deposit supply schedule (3.2), it is completely horizontal due to the perfect substitutability between domestic and foreign deposits such that

$$\frac{\partial R_e}{\partial d_e} = 0$$

The first order conditions can then be written as

$$(1 - P) R^{G,H} + P R^{G,L} = R_e \quad (3.3)$$

$$R_e^K = R_e + \mu_k(K_e) \quad (3.4)$$

The first condition equates the expected return of sovereign debt with the return paid on deposits. It is notable that deposit collection is at the efficient level and T , the lump-sum cost contingent on sovereign default, has no effect on banking decisions under this efficient benchmark. However, the second condition indicates that the bank under-provides working capital loans K_e in order to collect an optimal mark-up

$$\mu_k(K_e) \equiv -k_e \frac{\partial R_e^K}{\partial k_e} = va(1-a) \frac{(1+g'(K_e))K_e}{(K_e + g(K_e))^{2-a}} > 0 \quad (3.5)$$

from its lending to non-financial firms.⁹ Combining the first order conditions (3.2), (3.4) and (2.4) of the household, bank and non-financial firm provides an implicit expression for K_e

$$a(K_e + g(K_e))^{a-1} = R^* + va(1-a) \frac{(1+g'(K_e))K_e}{(K_e + g(K_e))^{2-a}} \quad (3.6)$$

and $(R_e^K, \mu_k(K_e))$ follow directly through (2.4) and (3.5). Observe that K_e does not depend on

⁹Clearly, the economy suffers from a monopoly distortion. I use the term "efficient" only in contrast to the gambling equilibrium described in the next section.

sovereign bond purchases b_e . There is no trade-off between sovereign debt purchases and lending to the private sector as banks face a horizontal deposit supply schedule. As such, sovereign bond purchases do not crowd out bank lending in the efficient equilibrium.

Note also that the combination of the first order condition (3.3) with (2.1) indicates that the representative bank is indifferent to the amount of sovereign debt it holds under an efficient strategy. Thus (b_e, γ_e) are indeterminate within the region that satisfies (3.1). This indeterminacy also spills over to deposits which depend on b_e through the budget constraint

$$d_e = b_e + vK_e - N$$

Finally, the expected payoff of the representative bank under the efficient equilibrium is

$$E \left[\hat{\Pi}_e \right] = NR^* + \mu_k(K_e) vK_e - PT \quad (3.7)$$

where the first term reflects the return to bank capital, the second term is the excess profit obtained from lending to non-financial firms and the final term is the non-bond cost imposed in case of sovereign default. Ex-ante there are no expected profits sovereign bond purchases as the bank lacks market power in the internationally integrated markets for deposits. In the next section, I show that the anticipation of bankruptcy under sovereign default changes these results drastically.

3.2 Gambling equilibrium

Suppose that the representative bank's initial capitalization N is so low that it cannot remain solvent in case of sovereign default. This is true under the restriction

$$(N + d_g) (\gamma_g R^{G,L} + (1 - \gamma_g) R_g^K) - R_g d_g - T < 0$$

where the subscript g indicates that the bank follows a gambling strategy based on the anticipation of insolvency in state L . Under limited liability, the representative bank does not internalize its losses in state L . Its optimal strategy is then determined by solving the problem

$$\max_{d_g, \gamma_g \in [0,1]} E \left[\hat{\Pi}_g \right] = (1 - P) [(N + d_g) (\gamma_g R^{G,H} + (1 - \gamma_g) R_g^K) - R_g d_g]$$

subject to (2.5), (2.6) and

$$\frac{\partial R_g^K}{\partial k_g} = - \frac{a(1-a)(1+g'(K_g))}{(K_g + g(K_g))^{2-a}}$$

which reflects its market power vis-à-vis non-financial firms. Unlike the efficient case, however, the bank also has market power in the deposits market. Due to insolvency risk, domestic deposits are considered as risky assets which only pay out with probability $\Pr[\Pi_g \geq 0] = 1 - P$ and become

imperfectly substitutable with safe assets. As per the first order condition (2.12), households require a higher promised interest rate

$$R_g = R^* + \frac{P}{1-P} (R^* - R^{\min}) - \frac{Cov(\hat{R}_g, u'(c_{2,g}))}{(1-P) E[u'(c_{2,g})]} \quad (3.8)$$

in compensation for the decline in payment probability and the risk premium created by the negative covariance between the marginal utility $u'(c_{2,g})$ and the effective return $\hat{R}_g$ from domestic deposits.¹⁰ Indeed, the household budget constraint (2.10) indicates that a rise in D_g increases the dependence of household income on the return from domestic deposits, which in turn increases the magnitude of the covariance term in (3.8). Thus, the risk premium is increasing in d_g such that $\frac{\partial R_g}{\partial d_g} > 0$ and the representative bank faces an upward sloping deposit supply schedule. This gives it an incentive to curtail its deposit demand in order to reduce its borrowing costs. The first order conditions of the representative bank's problem can then be written as

$$R^{G,H} = R_g + \mu_d(D_g) \quad (3.9)$$

$$R_g^K = R_g + \mu_d(D_g) + \mu_k(K_g) \quad (3.10)$$

where the mark-up on working capital lending $\mu_k(K_g)$ is defined in a similar manner to (3.5). Due to limited liability, the representative bank only takes into account the good state return $R^{G,H}$ from sovereign bonds and finds it profitable to increase its deposits d_g to fund additional sovereign debt purchases. $R^{G,H}$ is determined by (2.1) and remains fixed despite the rise in domestic purchases. Thus, d_g is increased until R_g rises to the point where the profit margin $(R^{G,H} - R_g)$ from sovereign debt purchases is reduced to the optimal mark-up

$$\mu_d(D_g) \equiv v D_g \frac{\partial R_g}{\partial d_g} > 0 \quad (3.11)$$

This increases the opportunity cost of providing credit to non-financial firms, which is optimally reduced until R_g^K rises to $R^{G,H} + \mu_k(K_g)$. Consequently, working capital is crowded out and output is reduced compared to the efficient equilibrium such that $(K_g, Y_g) \ll (K_e, Y_e)$. It is the combination of the upward sloping deposit supply schedule and the mispricing of sovereign debt under limited liability that causes this crowding out effect. While the former creates a trade-off between using funds on sovereign bond purchases and working capital loans, the latter generates a risk-shifting incentive in favour of sovereign bond purchases.

As in the previous section, K_g may be pinned down by combining the first order conditions

¹⁰This is under the assumption that deposit insurance is incomplete or insufficiently credible such that $R^{\min} < R$.

(3.9), (3.10) and (2.4) which yield the expression

$$a (K_g + g(K_g))^{a-1} = R^{G,H} + va(1-a) \frac{(1+g'(K_g)) K_g}{(K_g + g(K_g))^{2-a}} \quad (3.12)$$

and $(R_g^K, \mu_k(K_g), Y_g)$ follow directly through (2.4), (3.5) and the production function. Unlike the efficient equilibrium, the budget constraints (2.5), (2.10) and the first order conditions (2.4), (3.8), (3.9) completely pin down the variables $(\gamma_g, K_g, B_g, D_g, R_g)$ so that nothing remains indeterminate. However, the dependence of $\frac{\partial R_g}{\partial d_g}$ on the derivative of the covariance term in (3.8) precludes a closed-form solution. Thus, I obtain a numerical solution for (D_g, D_g^*, R_g) by simultaneously solving (3.9) and the Euler conditions given in Section 2.4. After determining $(d_g, k_g) = v(D_g, K_g)$, it is easy to pin down γ_g and b_g using the bank's budget constraint

$$\begin{aligned} \gamma_g &= 1 - \frac{k_g}{N + d_g} \\ b_g &= N + d_g - k_g \end{aligned} \quad (3.13)$$

An improvement in the representative bank's funding conditions lead to an increase in domestic sovereign bond purchases b_g and exposure γ_g as K_g is independent of (N, d_g, R_g) according to (3.12). Finally, the expected payoff under the gambling equilibrium can be written as

$$E[\hat{\Pi}_g] = (1-P) [NR^{G,H} + v(\mu_k(K_g) K_g + \mu_d(D_g) D_g)] \quad (3.14)$$

where the terms in the square brackets respectively reflect the return made on bank capital and the excess profits stemming from the bank's price-making power in the markets for domestic deposits and working capital lending. Note that the return on bank capital is higher than the efficient case due to the bank's gamble on sovereign debt. However, these returns materialize only in state H when the gamble is successful. In state L , the losses caused by sovereign default render the bank insolvent and it receives zero payoff under limited liability.

In the next section, I relax the restrictions on N such that the representative bank's solvency prospects depend on its decisions (d, γ) to leverage and purchase risky sovereign bonds. This is tantamount to choosing between an efficient and a gambling strategy and yields a complete characterization of the bank's deposit demand schedule. Having determined both the deposit supply and demand schedules, I also provide an elaborate explanation of the multiplicity mechanism described in 2.4.

4 Generalized Solution

4.1 Banks and Strategy Selection

In the generalized setting, the representative bank's problem involves solving the profit maximization problems under efficient and gambling strategies separately and then choosing the strategy that yields the higher expected payoff. As the bank is risk neutral, an efficient strategy which breaches the deposit threshold $\bar{d}$ is always dominated by the gambling strategy.¹¹ Thus, I only consider efficient strategies which remain within the deposit threshold $d_e \leq \bar{d}$ and bring about the risk-free interest rate given by (3.2). The consequent maximization problem is similar to the one described in Section 3.1 but with an additional occasionally binding constraint $d_e \leq \bar{d}$.

$$\max_{d_e, \gamma \in [0,1]} E \left[\hat{\Pi}_e \right] = (N + d_e) \left[\gamma_e \left((1 - P) R^{G,H} + P R^{G,L} \right) + (1 - \gamma_e) R^K \right] - R_e d - PT$$

s.t.

$$\frac{\partial R_e^K}{\partial k_e} = - \frac{a(1-a)(1+g'(K_e))}{(K_e + g(K_e))^{2-a}}$$

$$d_e \leq \bar{d}$$

where $\bar{d}$ is taken as given due to the bank's inability to commit to a γ value. This yields the interior first order conditions

$$(1 - P) R^{G,H} + P R^{G,L} = R_e + \lambda_e \quad (4.1)$$

$$R_e^K = R_e + \mu_k(K_e) + \lambda_e \quad (4.2)$$

$$\lambda_e \geq 0 \quad , \quad \lambda_e (\bar{d} - d_e) = 0 \quad (4.3)$$

where λ_e is the Lagrange multiplier associated with the occasionally binding constraint $d_e \leq \bar{d}$ and (4.3) is the corresponding complementary slackness condition. When this constraint is not binding such that $d_e \leq \bar{d}$, the multiplier λ_e is equal to zero and the resulting equilibrium is identical to the efficient equilibrium described in Section 3.1 with the expected payoff given by (3.7).

I use the subscript c to denote the case when the deposit constraint is binding. In this case, we have $d_c = \bar{d}$ and a positive Lagrange multiplier $\lambda_c > 0$ which can be interpreted as the excess return that stems from banks' inability to collect additional deposits. Note, however that $(1 - P) R^{G,H} + P R^{G,L}$ and R_c are both fixed at R^* by (2.1) and (3.2). Thus, it is not possible for

¹¹For a given borrowing cost R , becoming reliant on limited liability increases the expected payoff of the representative bank due to risk-shifting effects. As such, risk neutral banks have no incentive to follow an efficient strategy unless it leads to lower borrowing costs.

(4.1) to hold with equality when $\lambda_c > 0$ and we have

$$R_c^K - \mu_k(K_c) = R_c + \lambda_c > (1 - P) R^{G,H} + P R^{G,L}$$

which leads to the following proposition.

Proposition 1 *A binding deposit constraint leads to a corner solution where the bank does not purchase any sovereign bonds such that $\gamma_c = b_c = 0$. The Lagrange multiplier λ_c can then be defined as*

$$\lambda_c = \max \left[\frac{a}{(K_c + g(K_c))^{1-a}} \left[1 - v(1-a) \frac{1 + g'(K_c)}{K_c + g(K_c)} K_c \right] - R^*, 0 \right] \quad (4.4)$$

where $K_c = \frac{N + \bar{d}}{v} < K_e$

Proof. Provided in Appendix Section A. ■

This has the immediate implication that all sovereign bonds are purchased by foreign banks such that $B_c^* = \bar{B}$. As before, the solution for R_c^K follows directly from (2.4) as

$$R_c^K = a [K_c + g(K_c)]^{a-1}$$

and the representative bank's expected payoff can be written as

$$\begin{aligned} E[\hat{\Pi}_c] &= R_c^K K_c - R_c \bar{d} - PT \\ &= a \left(\frac{N + \bar{d}}{v} \right) \left[\frac{N + \bar{d}}{v} + g \left(\frac{N + \bar{d}}{v} \right) \right]^{a-1} - R^* \bar{d} - PT \end{aligned} \quad (4.5)$$

The problem for the gambling strategy is identical to Section 3.2 and yields the expected payoff given by (3.14). As such, I proceed to the discussion on strategy selection without elaborating further on this.

It is important to re-iterate that the bank internalizes the consequences of leveraging beyond the threshold $\bar{d}$ on its borrowing costs. Thus, its decision does not depend on a certain borrowing cost R , but on the deposit supply schedule given by (2.13). This schedule contains a discontinuity at the deposit threshold $\bar{d}$ which is taken as given due to the bank's inability to commit. The representative bank finds it optimal to breach this threshold if it can increase its expected payoff by switching to a gambling strategy.

However, the solution is more complicated than simply comparing the payoffs under the gambling and efficient equilibria as this would erroneously assume that an individual bank's decision to gamble triggers the same decision from other banks while in fact these decisions are taken independently. Instead, I evaluate strategy selection in the framework of a simultaneous-move game

between banks.¹² Proposition 2 provides an outline of the conditions under which the game results in an efficient equilibrium.

Proposition 2 *The condition for the efficient equilibrium to be sustainable as a pure strategy Nash equilibrium is contingent on whether the representative bank is deposit constrained under an efficient strategy. It can be written as*

$$\begin{aligned} E \left[\hat{\Pi}_e \right] &\geq E \left[\hat{\Pi}_{g|e} \right] \text{ iff } \lambda_c = 0 \\ E \left[\hat{\Pi}_c \right] &\geq E \left[\hat{\Pi}_{g|c} \right] \text{ iff } \lambda_c > 0 \end{aligned}$$

where $\lambda_c > 0$ indicates that the bank is deposit constrained, $\left(E \left[\hat{\Pi}_e \right], E \left[\hat{\Pi}_c \right] \right)$ are respectively given by (3.7) and (4.5) and $\left(E \left[\hat{\Pi}_{g|e} \right], E \left[\hat{\Pi}_{g|c} \right] \right)$ are the expected payoffs from deviating to a gambling strategy conditional on the other banks remaining at constrained and unconstrained efficient strategies respectively. A definition for $\left(E \left[\hat{\Pi}_{g|e} \right], E \left[\hat{\Pi}_{g|c} \right] \right)$ is provided by (8.3).

Proof. Provided in Appendix Section B. ■

Before deriving the conditions necessary for the existence of multiple equilibria, I provide a brief diagrammatical analysis of the overall model.

4.2 Graphical Analysis

Figure 4.2 provides a graphical representation of the deposit demand and supply schedules as well as the deposit thresholds. The demand and supply schedules are in duplicates with one for the efficient (or constrained) case and another one for the gambling equilibrium. This follows directly from the analysis in the previous sections. Deposit supply is horizontal at $R_e = R^*$ when domestic deposits are perceived to be safe. When the bank is perceived to be gambling, on the other hand, R jumps up discretely due to the fall in expected return and becomes upward sloping as a rise in d increases the risk premium.

Similarly, a quick comparison between (3.3) and (3.9) reveals that the deposit demand schedule is strictly higher when the bank is gambling as it no longer repays depositors in state L . It is also downward sloping due to the bank's market power over the domestic deposit market. Under an efficient strategy, foreign and domestic deposits become perfectly substitutable and the bank loses its market power over the deposit market. When the bank is not constrained, this implies a horizontal deposit demand schedule which overlaps with the supply schedule. When the bank is deposit constrained, on the other hand, the demand schedule retains its downward slope due to

¹²This will be the case when banks cannot alter their strategy after observing the strategies adopted by other banks. Otherwise, the strategy selection process transforms into a sequential game akin to imposing a free entry condition. As (R_e^K, R_g) are increasing in the number of gambling banks, a sequential game invariably results in a separating equilibrium where the portion of gambling banks adjusts to ensure that banks are indifferent between the two strategies. This complicates the solution significantly without providing any additional insights.

the presence of excess returns $\lambda_c > 0$ from working capital lending. If the constraint is relaxed and d rises, these excess returns decrease, leading to a downward sloping schedule until we reach the point $d_e^{\min}$ where $K_c = K_e$ and the bank is no longer constrained.

The dashed lines display the deposit threshold given by (2.9). They are downward sloping due to the deleterious effects of borrowing costs R on the bank's solvency and a rise in domestic sovereign bond purchases γ_g causes a shift to the left. From the representative bank's perspective, however, the threshold $\bar{d}$ is taken as given due to its inability to influence it by committing to a certain γ_g . Thus, the bank perceives the threshold as a vertical bar, which is either at $\bar{d}(\gamma_e, R_e)$ or $\bar{d}(\gamma_g, R_g)$ depending on household sentiment.

Figure 4.1: A Graphical Representation

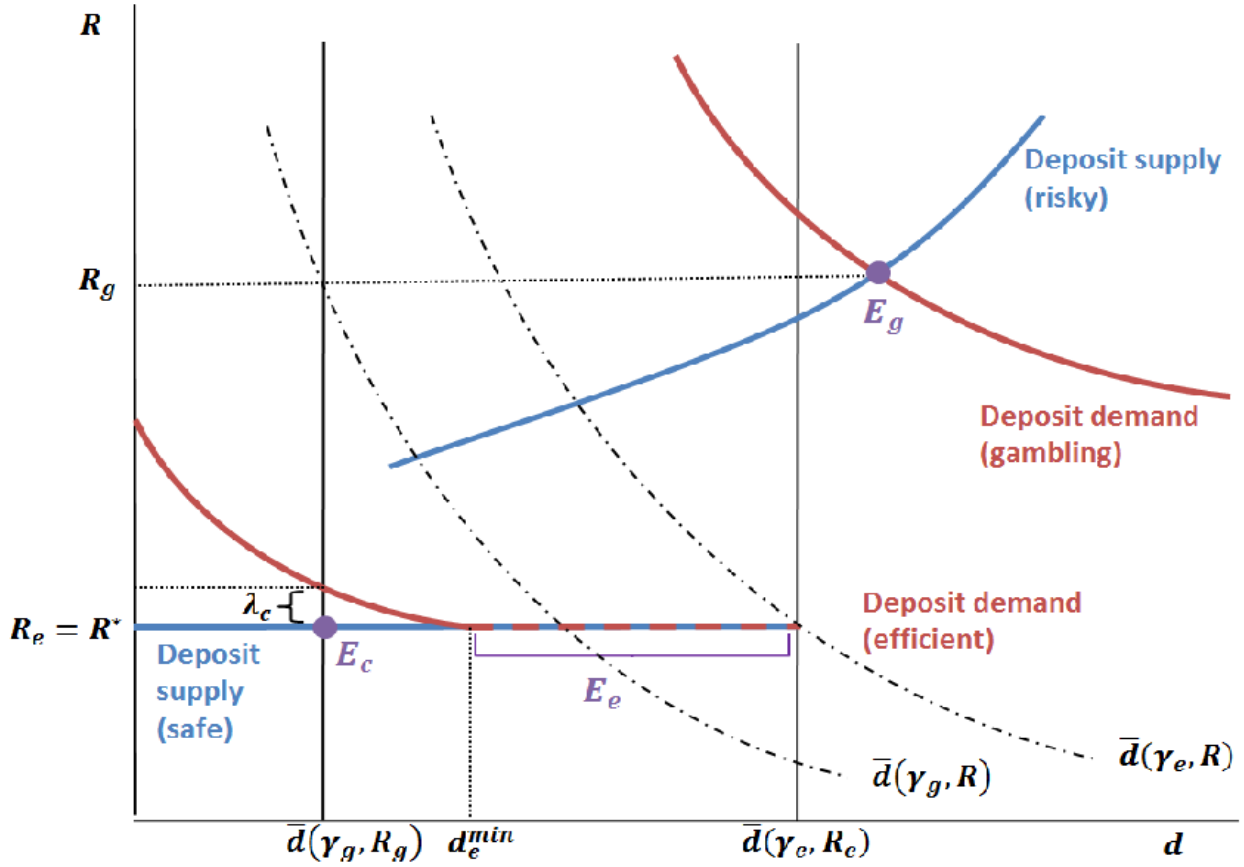


Figure 4.2:

The constrained efficient, unconstrained efficient and gambling equilibria are then respectively labelled as (E_c, E_e, E_g) with E_e referring to a range of values on the x-axis due to the indeterminacy of d_e under the efficient equilibrium. The minimum amount of deposits admissible as an efficient equilibrium is labelled as $d_e^{\min}$. At this level of deposits, a bank following the efficient strategy has just enough funds to exhaust the excess returns such that it does not purchase any sovereign debt.

Thus, $d_e^{\min}$ can be defined as

$$d_e^{\min} = \max [vK_e - N, 0] \quad (4.6)$$

and the bank becomes deposit constrained when $\bar{d} < d_e^{\min}$. As shown in the diagram, negative household sentiments tighten the threshold $\bar{d}(\gamma_g, R_g)$ and move the bank to the constrained equilibrium. When we have

$$\begin{aligned} E \left[\hat{\Pi}_{g|e}(\bar{d}(\gamma_g, R_g)) \right] &\leq E \left[\hat{\Pi}_e \right] \\ E \left[\hat{\Pi}_{g|c}(\bar{d}(\gamma_g, R_g)) \right] &> E \left[\hat{\Pi}_c \right] \end{aligned}$$

the bank deviates to a gambling strategy in response and negative sentiments become self-confirming. This leads to the existence of multiple equilibria. In the next section, I describe the conditions under which multiplicity arises.

4.3 Equilibrium Determination

The equilibrium solution is determined according to the concept of a rational expectations equilibrium which requires that all constraints and first order conditions of banks and households are satisfied and expectations are verified within the equilibrium path.

Firstly, consider the case when household sentiment is positive such that they set a benign deposit threshold $\bar{d}(\gamma_e, R_e)$ consistent with the anticipation of the efficient equilibrium described in Section 3.1. This efficient equilibrium is admissible as a rational expectations equilibrium and verifies the positive sentiments under the following conditions

$$d_e^{\min} \leq \bar{d}(0, R^*) \quad (4.7)$$

$$E \left[\hat{\Pi}_e \right] \geq E \left[\hat{\Pi}_{g|e} \right] \quad (4.8)$$

which respectively ensure that domestic banks remain solvent in state L and have no incentive to deviate from the resulting efficient equilibrium by switching to a gambling strategy.

Now consider the case under negative household sentiments consistent with the anticipation of the gambling equilibrium given in Section 3.2. This drives households to impose a stricter deposit threshold $\bar{d} = \bar{d}(\gamma_c, R_c)$ which may in turn become self-validating by making the gambling equilibrium a rational expectations equilibrium. This requires the following conditions. First, domestic banks must become deposit constrained under negative sentiments such that

$$d_e^{\min} > \bar{d}(\gamma_g, R_g) \quad (4.9)$$

Second, the consequent decline in expected payoffs must give banks an incentive to deviate to a

gambling strategy

$$E \left[\hat{\Pi}_c \right] < E \left[\hat{\Pi}_{g|c} \right] \quad (4.10)$$

and finally, domestic banks must become insolvent following a sovereign default in state L

$$d_g > \bar{d}(\gamma_g, R_g) \quad (4.11)$$

When all of the conditions (4.7)-(4.11) are satisfied, household sentiments become self-validating and there are multiple equilibria. Under positive sentiments, a benign threshold $\bar{d}(\gamma_e, R_e)$ results in an efficient equilibrium with no risk of bankruptcy. An adverse shift in sentiments, on the other hand, tightens the deposit threshold and drives banks to deviate to a gambling strategy, which in turn validates the negative sentiments.

Observe that the elimination of multiplicity is not equivalent to ensuring that the efficient equilibrium is the unique solution. When condition (4.7) or (4.8) is violated, the efficient case ceases to be a rational expectations equilibrium. Thus, the model only admits the combination of the tight threshold $\bar{d}(\gamma_c, R_c)$ with the gambling equilibrium as a rational expectation equilibrium. In contrary, the violation of any of the conditions (4.9)-(4.11) eliminates the gambling equilibrium as a rational expectation equilibrium and leaves the efficient equilibrium as the unique equilibrium.

It is important to note that the conditions related to existence, (4.7), (4.9) and (4.11), take primacy over conditions (4.8) and (4.10) which compare expected payoffs. For example, if (4.9) or (4.11) is violated such that the gambling equilibrium cannot exist, then the profit comparison in (4.8) becomes redundant and vice versa for conditions (4.7) and (4.10).

It is also notable that the constrained equilibrium described in Section 4.1 never emerges as a rational expectation equilibrium. Under positive sentiments, the violation of condition (4.7) also rules out a constrained equilibrium as the efficient equilibrium with $d_e = d_e^{\min}$ maximizes the payoff to the bank in state L ¹³. Thus, if the bank defaults in state L with $d = d_e^{\min}$ as indicated by the violation of (4.7), it also defaults with $d < d_e^{\min}$. Under negative sentiments, on the other hand, the violation of condition (4.10) such that banks do not deviate from a constrained efficient equilibrium means that these sentiments are not verified. Thus, the economy reverts to positive sentiments and the consequent shift out in the deposit threshold to $\bar{d}(\gamma_e, R^*)$ relieves the banks from their deposit constraints.

¹³This can be shown with a simple optimization problem $\max_{\hat{d}} a \left(\frac{(N+d)}{v} + g \left(\frac{N+d}{v} \right) \right)^{a-1} (N + \hat{d}) - \hat{d}R^* - T$ where I have used (3.2), (3.6), (2.4) to substitute in for (R_e, K_e, R_e^K) . This yields the first order condition $a \left(\hat{K} + g \left(\hat{K} \right) \right)^{a-1} = R^* + va(1-a) \frac{(1+g'(\hat{K}))\hat{K}}{(\hat{K}+g(\hat{K}))^{2-a}}$ where $\hat{K} = \frac{N+\hat{d}}{v}$, which is identical to (3.4). Thus $\hat{K} = K_e$ is optimal and re-arranging its definition yields $\hat{d} = d_e^{\min}$.

5 Calibration

Table 5.1 reports the calibrated parameters. The calibration targets the peripheral Euro area countries with sovereign risk related financial distress (specifically Italy, Greece, Ireland, Portugal, Spain and Cyprus) over the period 2008-2014.¹⁴ I use data on 5-bank asset concentration from World Bank's Global Financial Development Database (GFDD) to calibrate the market share parameter v . The combined market share of the five largest banks varies between just below 70% in Italy and nearly 100% in Cyprus. For an individual bank's market share, this indicates a range between 13% and 20% with a cross-country average of 17%. I set a slightly lower value of $v = 0.15$ in order to account for the remainder of the banking sector.

To calibrate the haircut parameter θ , I use data from the sovereign debt restructuring database of Cruces and Trebesch (2013) which yields an average market haircut of $\theta = 0.4$.¹⁵ The sovereign default probability P is calibrated to match the spread between the long-term government bond yields of the distressed countries and Germany (as a benchmark for the safe rate). The spread can be related to the parameters $(R^{G,H}, R^*)$, which are the model counterparts to distressed and safe sovereign bond yields, as follows

$$\hat{S} = \ln(R^{G,H}) - \ln(R^*)$$

Combining this with the definition for the recovery value $R^{G,L} = (1 - \theta) R^{G,H}$ and the sovereign bond pricing equation (2.1) yields the following expression for default probability

$$P = \frac{1}{\hat{\theta}} \left(1 - \frac{1}{\exp(\hat{S})} \right)$$

where $\hat{\theta} = 0.4$ is the calibrated haircut value. This results in $P \approx 0.1$.

The deposit insurance parameter $R^{\min}$ and the risk-free interest rate R^* are policy instruments and Section 6.2 and Section 6.3 provide an extensive evaluation of their comparative statics. As a baseline value, I set $R^{\min} = 0.8$ such that 80% of the base value of deposits is recovered in case of insolvency. The resulting losses are somewhat higher than the stability levy proposed for Cyprus in order to take into account potential losses to depositors which may arise from a suspension of convertibility or a currency re-denomination following an exit from the Euro area. The baseline value for R^* is set to 1.01 in line with a household discount factor of $\beta = 0.99$.

I also consider a broad range of values for bank capital N . T which is the non-bond cost to bank balance sheets in case of sovereign default, is the most difficult parameter to calibrate due to the absence of an empirical estimate. However, Yeyati, Peria and Schmukler (2010) provide evidence from different sovereign default episodes which imply that macroeconomic factors which

¹⁴The inclusion of Cyprus is subject to the availability of data.

¹⁵In this case, international data is used due to the scarcity of historical default episodes pertaining to the listed countries.

Table 5.1: Calibration

Parameter	Calibrated Value	Source
θ	0.40	Cruces & Trebesch (2013)
P	0.10	OECD (2014)
T	0.007	See text
β	0.99	Standard
R^*	1.01	Baseline
$R^{\min}$	0.80	Baseline
v	0.15	World Bank GFDD
a	0.30	Standard
η	0.75	BIS (2014)

would come under the umbrella of T have a significant role in determining bank and depositor behaviour. As such, I set T to a sufficiently high value to influence bank and household choice. As it is in fact the ratio $\frac{N}{T}$ that is significant for the results, fixing T and varying N provides a sensitivity test.

Finally, regarding the non-financial firms, I set a to 0.3 in line with the convention for Cobb-Douglas production functions. For the additional lending cost to foreign banks, I specify a linear specification $\phi(K^*) = \eta K^*$ and calibrate η to match $\frac{I_e}{K_e}$ to the share of domestic credit to the private non-financial sector in the distressed countries. This yields the value $\eta \approx 0.75$.

6 Policy Analysis

In this section, I evaluate the effects of a number of policy measures that have been proposed to re-invigorate bank lending in the Euro area. Firstly, I consider a re-capitalization of the banking sector, a strengthening of deposit insurance and expansionary monetary policy. This consists of comparative statics for the variables $(N, R^{\min}, R^*)$. Secondly, I extend the model to evaluate the effects of two unconventional policy interventions implemented by the European Central Bank, the non-targeted longer-term re-financing operations (LTRO) and their more recent targeted counterpart (TLTRO).

6.1 Bank Recapitalization

The most obvious policy measure to prevent multiplicity is a re-capitalization of the banking sector which leads to a rise in N . It is clear from (3.12) that K_g is determined independently from N . Thus, banks spend the additional funds on risky sovereign debt under a gambling strategy. Nevertheless, this leads to a relaxation of the deposit constraint $\bar{d}(\gamma_g, R_g)$ as long as the recovery value $R^{G,L}$ of sovereign bonds is positive. Moreover, a rise in N directly reduces the reliance of banks on deposit financing as per the negative relation between $d_e^{\min}$ and N given by (4.6). As

shown in Figure 6.1, a sufficiently large intervention can eliminate multiplicity by preventing banks from becoming deposit constrained under an efficient strategy and thus violating condition (4.9).

Figure 6.2 shows that this leads to an efficient equilibrium as $E[\hat{\Pi}_e]$ is higher than the expected payoff from deviating to a gambling equilibrium $E[\hat{\Pi}_{g|e}]$. Indeed, multiplicity is eliminated at a slightly lower level of N than required for banks to become completely unconstrained. The constrained efficient payoff $E[\hat{\Pi}_c]$ overtakes $E[\hat{\Pi}_{g|c}]$ before we reach the level of N required for $d_e^{\min} \leq \bar{d}(\gamma_g, R_g)$. As such, banks do not deviate to a gambling strategy and the negative sentiments cease to be self-validating. This violates condition (4.10) and ensures that a tight deposit threshold $\bar{d}(\gamma_g, R_g)$ does not arise in equilibrium.

Although capital injections to the banking sector are a potent way of bringing about an efficient equilibrium, they require a significant transfer of resources at a time when the government is cash-struck. Thus, I also consider other policy measures ranging from conventional monetary policy to unconventional measures such as LTRO.

Figure 6.1: Banking Sector Recapitalization (1)

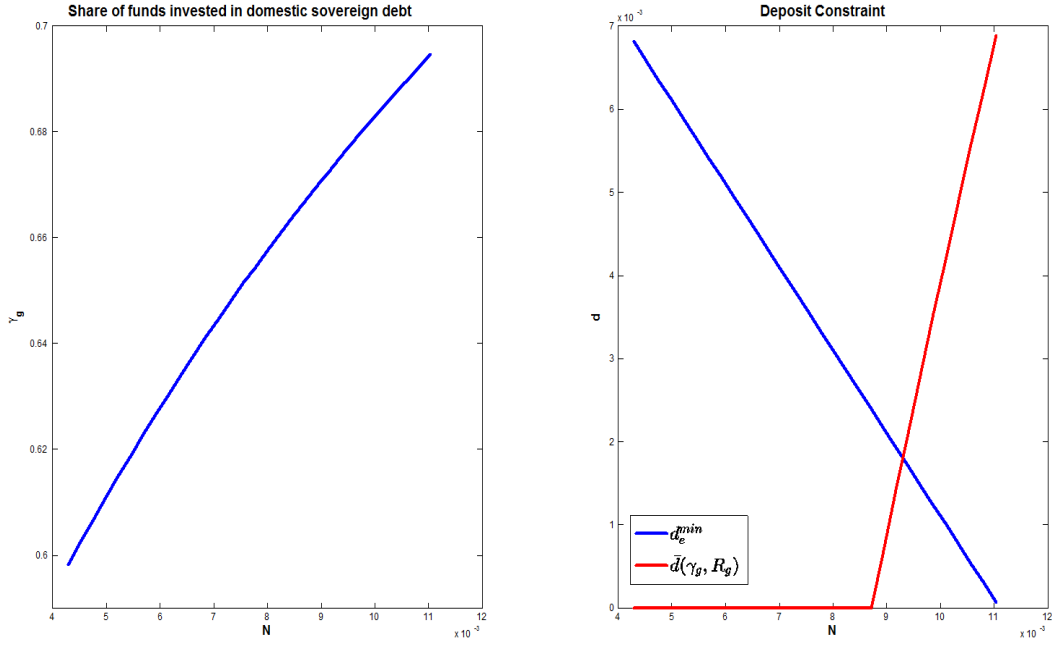
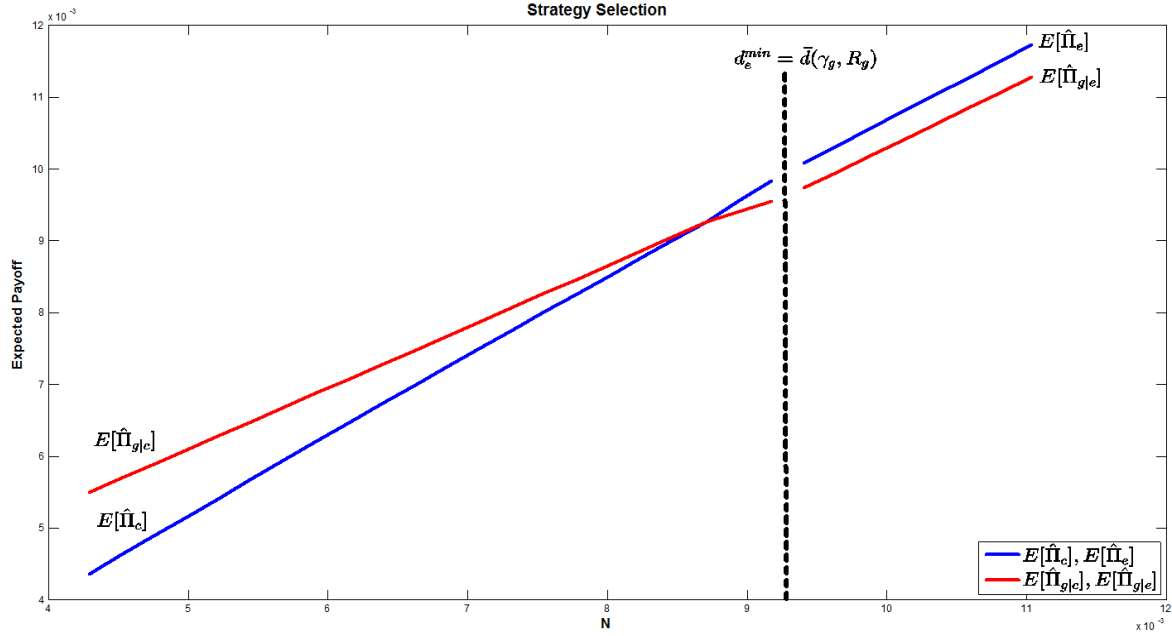


Figure 6.2: Banking Sector Recapitalization (2)



6.2 Monetary Policy

In a monetary union setting, monetary policy takes the form of a change in R^* . In Figure 6.3, I map the equilibrium outcomes under different combinations of (N, R^*) . The mapping suggests that expansionary monetary policy is ineffective in preventing a gambling equilibrium at very low levels of capitalization and has the adverse effect of slightly expanding the region with multiplicity at intermediate levels of N .

I choose a level of capitalization at the boundary of the region of multiplicity in order to analyze the transmission of monetary policy. This demonstrates the importance of accounting for the reaction of banks as argued in Section 2.4. According to (3.8), a fall in R^* reduces the borrowing costs of risky banks. If the banks remain passive, this improves their solvency prospects and this helps shrink the multiplicity region by relaxing the deposit threshold $\bar{d}(\gamma_g, R_g)$ under negative sentiments. However, the banks react to the decline in R^* actively and with important implications. The following expression is attained by combining (2.1) with the definition for the recovery value $R^{G,L}$

$$R^{G,H} = \frac{R^*}{1 - P\theta}$$

$$\therefore \frac{\partial R^{G,H}}{\partial R^*} = \frac{1}{1 - P\theta}$$

A quick comparison with (3.8) reveals that a fall in R^* reduces R_g more than $R^{G,H}$. The first order

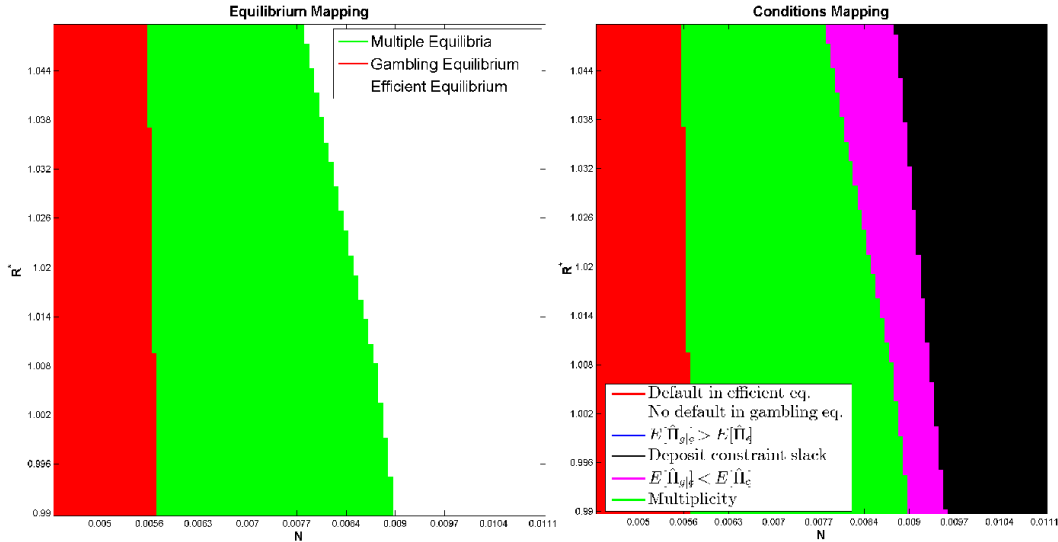
condition (3.9) then suggests that the optimal response under a gambling strategy is to increase deposit collection (d_g) and the portion of funds spent on sovereign debt purchases (γ_g). As shown in Figure 6.4, this mitigates the positive effect of lower borrowing costs on the deposit threshold $\bar{d}(\gamma_g, R_g)$.

Moreover, a fall in R^* increases lending by foreign banks to non-financial firms. When domestic banks are deposit constrained under negative sentiments, they increase their lending and thus experience a fall in expected payoff due to an erosion of their mark-up. This increases the incentive to deviate to a gambling strategy, the expected payoff from which remains largely unchanged. The implication is that the efficient equilibrium is easier to achieve under contractionary monetary policy.

This does not preclude expansionary monetary policy from expanding output. Figure 6.5 plots (Y_g, Y_e) across a range of R^* values.¹⁶ The lines are only drawn when the corresponding equilibrium exists, so the overlapping area corresponds to the region of multiplicity. Although output is lower under the gambling equilibrium due to the crowding out of bank lending, both Y_g and Y_e increase significantly in response to a fall in R^* .

Nevertheless, the capacity of high interest rates to eliminate multiplicity leads to important non-linearities under negative sentiments. Perversely, a marginal hike in the interest rates that triggers a switch from Y_g to Y_e by eliminating the gambling equilibrium causes a rise in output equivalent to an interest rate cut of 2%.

Figure 6.3: Monetary Policy (1)



¹⁶The y-axis is scaled so that $Y_e = 1$ when $R^* = 1.01$ equals 1. The figure implies that the effects on output are quantitatively small. However, the stylized model lacks several frictions (financial and otherwise) which would inevitably amplify these effects. As such, I find it more appropriate to focus on qualitative comparisons and describe the effects of the switch to an efficient equilibrium in relation to the effects of an interest rate cut.

Figure 6.4: Monetary Policy (2)

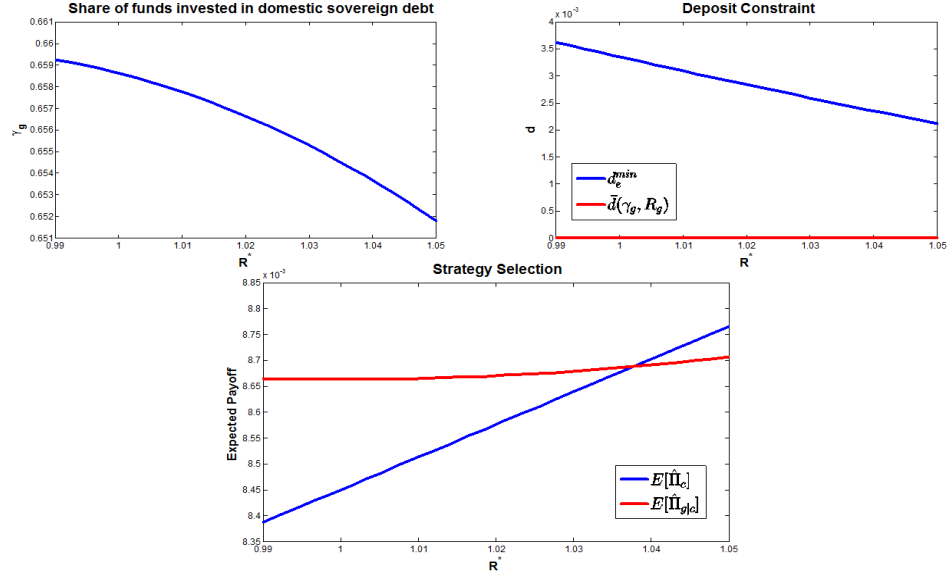
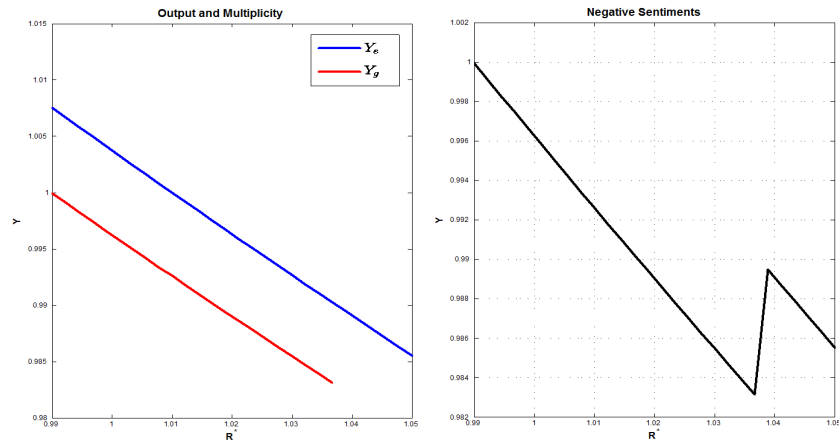


Figure 6.5: Monetary Policy (3)



6.3 Deposit Insurance

A strengthening of deposit insurance guarantees takes the form of a rise in $R^{\min}$, with a complete bailout corresponding to $R^{\min} = R^*$. I focus solely on the effects of deposit insurance on the banking sector and abstain from its implications on sovereign risk through the government's deposit insurance liabilities. This roughly corresponds to the proposals for a common European deposit insurance mechanism under a banking union. Figure 6.6 shows that deposit insurance significantly expands the region of multiplicity and even results in a unique gambling equilibrium at very high levels.

As in the previous section, the explanation lies in the reaction of banks under a gambling strategy. The direct effect of a rise in $R^{\min}$ is to flatten the portion of the deposit supply schedule that lies beyond the threshold $\bar{d}$. Although this is successful in reducing bank funding costs, it also weakens the negative relationship between D_g and the deposit market mark-up $\mu_d(D_g)$ which allows gambling banks to increase their deposit collection further without eroding their mark-up. As K_g is independent of $R^{\min}$ and D_g , the additional funds are spent on sovereign debt purchases resulting in a rise in γ_g and a tightening of the deposit threshold.

The consequences are displayed in Figure 6.7. As the deposit constraint becomes binding, the expected payoff from following an efficient strategy declines. In contrast, the expected payoff from deviating to a gambling strategy increases as the deposit supply schedule becomes flatter. Even at intermediate levels of $R^{\min}$, conditions (4.9) and (4.10) are satisfied such that negative sentiments become self-fulfilling and there is multiplicity.

At very high levels, however, deposit insurance eliminates multiplicity. Setting $R^{\min} = R^*$ makes the households indifferent to the solvency prospects of banks and the deposit supply schedule becomes completely horizontal. This ensures that banks are no longer constrained by the deposit threshold and insulates the banking sector from shifts in sentiments. However, it also leads to a drastic rise in the expected payoff from gambling as shown in the final plot of Figure 6.7. Given that the expected payoff under an efficient strategy is not affected by deposit insurance (as this strategy does not result in insolvency), banks find it optimal to deviate to a gambling strategy even under positive sentiments and condition (4.8) fails. Thus, near-complete levels of deposit insurance eliminate the efficient equilibrium rather than the gambling equilibrium.

While the finding that deposit insurance creates a risk-taking incentive in the absence of regulation dates back to Kareken and Wallace (1978), it becomes particularly important when domestic sovereign bonds are perceived as correlated risk due to the zero risk-weight attached to them by regulators.

Figure 6.6: Deposit Insurance (1)

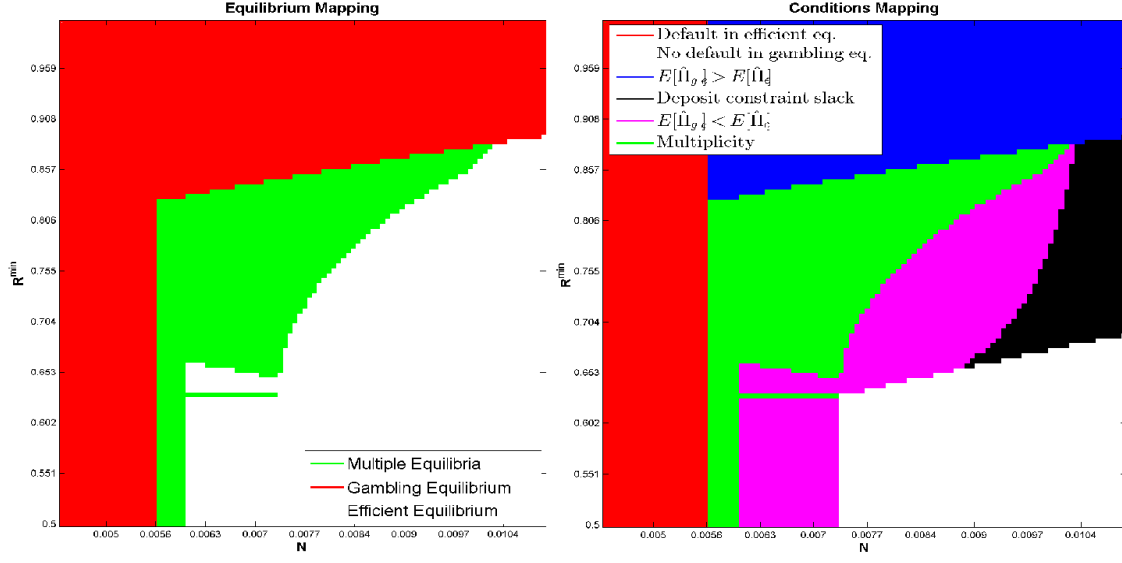
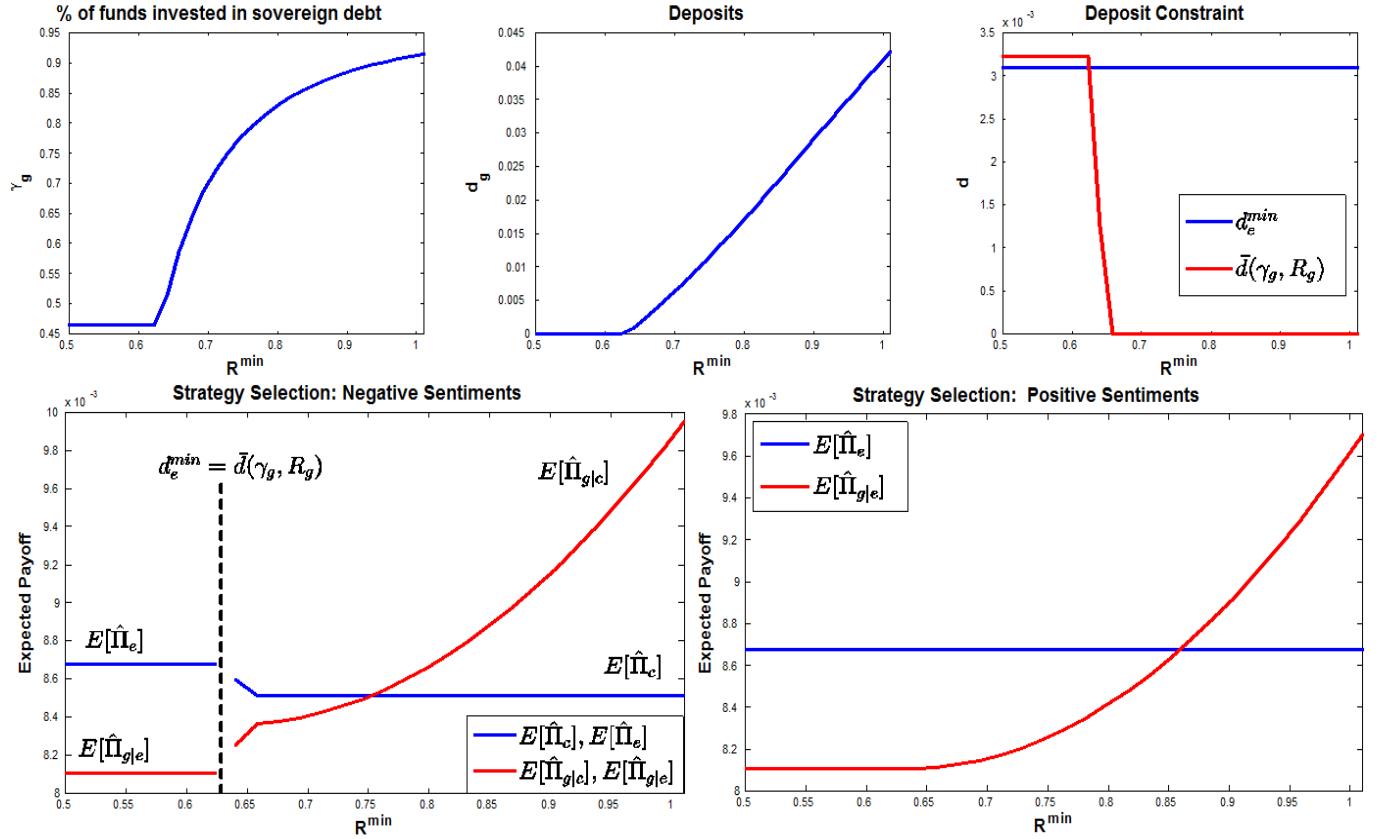


Figure 6.7: Deposit Insurance (2)



6.4 LTRO

6.4.1 Extended Model

I incorporate LTRO into the model by allowing banks to borrow up to an amount $\bar{L}$ from the central bank at a safe interest rate R^* .¹⁷ Thus, the representative bank's budget constraint becomes

$$b + k = N + d + L$$

and (b, k) are re-defined as

$$\begin{aligned} b &= \gamma (N + d + L) \\ k &= (1 - \gamma) (N + d + L) \end{aligned}$$

where $L \in [0, \bar{L}]$ is the amount of funds borrowed by the bank under LTRO. I assume that official lending has priority over depositors such that the deposit threshold $\bar{d}(\gamma, R)$ becomes¹⁸

$$\begin{aligned} (N + \bar{d} + L) [\gamma R^{G,L} + (1 - \gamma) R^K] - R^* L - R \bar{d} - T &= 0 \\ \therefore \bar{d}(\gamma, R) &= \frac{(N + \bar{L}) [\gamma R^{G,L} + (1 - \gamma) R^K] - R^* L - T}{R - \gamma R^{G,L} - (1 - \gamma) R^K} \end{aligned} \quad (6.1)$$

and the effects of LTRO lending on $\bar{d}(\gamma_g, R_g)$ depend on the reaction of γ_g . Access to LTRO funding creates another choice variable L for the representative bank, but does not change the first order conditions for (d, γ) under any strategy. Under an efficient strategy, the representative bank can collect deposits at a safe interest rate of R^* . Thus, unless it is deposit constrained, it remains indifferent to the amount of LTRO loans it takes such that L_e is indeterminate in the region $L_e \in [0, \bar{L}]$. As such, the efficient equilibrium described in Section (3.1) is completely unaffected by LTRO.

When the representative bank is deposit constrained, on the other hand, it follows directly from Proposition 1 that it will borrow up to the upper bound $\bar{L}$ unless it becomes unconstrained as a result of LTRO funding, in which case it becomes indifferent. Thus, I set $L_c = \bar{L}$ and adjust $d_e^{\min}$ to account for the possibility that it becomes unconstrained.

$$d_e^{\min} = vK_e - N - \bar{L}$$

When the representative bank remains constrained despite fully using the LTRO funding, it spends

¹⁷Although collateral is required for these loans, this does not prevent the form of gambling considered here due to the ECB's decision to suspend collateral eligibility requirements for sovereign debt issued by distressed Euro area countries (ECB, 2012).

¹⁸This assumption is in line with historical precedent, which was also upheld during the recent bail-in of the Cypriot banking sector (Eurogroup, 2013b)

all of the additional funding on working capital lending such that $\gamma_c = 0$ and K_c can be written as

$$K_c = \frac{N + \bar{d} + \bar{L}}{v}$$

The solutions for $(R_c^K, \mu_k(K_c), \lambda_c)$ can then be obtained using (2.4), (3.5) and (4.4). The expected payoff then becomes

$$E[\hat{\Pi}_c] = NR_c^K + (R_c^K - R^*)(\bar{L} + \bar{d}) - PT$$

Under the gambling strategy, the representative bank always finds it profitable to borrow at a low interest rate through LTRO and invest it in sovereign bonds which pay a return of $R^{G,H} > R^*$ in the good state where the bank is solvent. As such, it always borrows the highest possible amount $L_g = \bar{L}$. Given that its first order conditions (3.9)-(3.10) remain the same, $(K_g, \mu_k(K_g), R_g, D_g, \mu_d(D_g))$ are also unaffected by LTRO. Thus, all of the additional LTRO funding is spent on sovereign bond purchases such that γ_g increases to

$$\gamma_g = 1 - \frac{vK_g}{N + d_g + \bar{L}}$$

and the expected payoff rises to

$$E[\hat{\Pi}_g] = (1 - P)[NR^{G,H} + v(\mu_k(K_g)K_g + \mu_d(D_g)D_g) + (R^{G,H} - R^*)\bar{L}]$$

where $(R^{G,H} - R^*)\bar{L}$ reflects the additional profits from investing LTRO funding in domestic sovereign bond purchases. Due to the seniority of official lending over depositors, this leads to a tightening of the deposit threshold $\bar{d}(\gamma_g, R_g)$ under negative sentiments. Finally, the expected payoffs from deviating to a gambling equilibrium are derived in the same manner as Proposition 2 and can be written as

$$\begin{aligned} E[\hat{\Pi}_{g|e}] &= (1 - P)[NR^{G,H} + \mu_{d|e}(D_{g|e})D_{g|e} + [K_{g|e} - (1 - v)K_e]\mu_k(K_{g|e}) + (R^{G,H} - R^*)\bar{L}] \\ E[\hat{\Pi}_{g|c}] &= (1 - P)[NR^{G,H} + \mu_{d|c}(D_{g|c})D_{g|c} + [K_{g|c} - (1 - v)K_c]\mu_k(K_{g|c}) + (R^{G,H} - R^*)\bar{L}] \end{aligned}$$

where $(K_{g|i}, \mu_k(K_{g|i}), D_{g|i}, \mu_{d|i}(D_{g|i}))_{i=\{e,c\}}$ are given by the solution method in Proposition 2 but with the use of the relevant $(K_{g|e}, K_{g|c})$ values.

Observe that LTRO leads to a number of alterations in the equilibrium determination conditions described in Section 4.3 with $(E[\hat{\Pi}_c], E[\hat{\Pi}_{g|c}], E[\hat{\Pi}_{g|e}], d_e^{\min})$ now described as above and $\bar{d}(\gamma_g, R_g)$ defined according to (6.1) and the new (γ_g, R_g) values. The next section provides a numerical evaluation of the implications of these changes.

6.4.2 Numerical Analysis

Figure 6.8 provides a map of the equilibrium outcomes for combinations of $(N, \bar{L})$. The mapping indicates that intermediate amounts of LTRO funding expand the region with a unique efficient equilibrium whereas excessively high amounts result in a unique gambling equilibrium.

In order to evaluate the transmission of LTRO, I choose a boundary level of capitalization at $N = 0.008$. As predicted, Figure 6.9 shows that LTRO funding leads to a rise in γ_g . Combined with the seniority of official lending over depositors, this ensures that LTRO does not relax the deposit threshold $\bar{d}(\gamma_g, R_g)$ under negative sentiments. Nevertheless, it alleviates the deposit constraint by providing banks with an alternative source of funding. As $\bar{d}(\gamma_g, R_g) = 0$ at this level of capitalization, banks become unconstrained when $\bar{L}$ is sufficiently high to bring about $d_e^{\min} = 0$.

Observe that a lower level of LTRO funding is sufficient to ensure that $E[\hat{\Pi}_c] \geq E[\hat{\Pi}_{g|c}]$ such that banks do not deviate to a gambling strategy despite their constraints. This prevents multiplicity by ensuring that condition (4.10) is violated, in which case negative sentiments cease to be self-fulfilling. Thus, neither the gambling strategy nor the deposit constraint can exist in a rational expectations equilibrium such that we revert to the efficient equilibrium described in Section 3.1.

It is easy to see how LTRO funding leads to a rise in the constrained efficient expected payoff $E[\hat{\Pi}_c]$. It permits banks to increase K_c and hence capture a portion of the excess return λ_c . In contrast, the expected payoff from deviating to a gambling strategy $E[\hat{\Pi}_{g|c}]$ has a negative relationship with $\bar{L}$. As explained above, a rise in $\bar{L}$ leads to increased working capital lending from the other banks which reduces the mark-up $\mu_k(K_{g|c})$. The bottom plot of Figure 6.9 shows that this leads to a decline in $E[\hat{\Pi}_{g|c}]$ despite the additional profits $(R^{G,H} - R^*)\bar{L}$ from investing the LTRO funds in domestic sovereign debt. This is precisely the reason why $E[\hat{\Pi}_c]$ overtakes $E[\hat{\Pi}_{g|c}]$ at a relatively low level of $\bar{L}$.

Once $\bar{L}$ is sufficiently high to make the deposit constraint slack, however, these effects are reversed. Given the ability of unconstrained banks to collect deposits at the safe interest rate R^* , LTRO funding has no effect on K_e or $E[\hat{\Pi}_e]$. Moreover, as K_e remains fixed in response to a rise in $\bar{L}$, there are no negative effects associated with the erosion of the mark-up for $E[\hat{\Pi}_{g|e}]$ and the incentive to deviate to a gambling strategy increases due to the term $(R^{G,H} - R^*)\bar{L}$. At a sufficiently high level of $\bar{L}$, this leads to $E[\hat{\Pi}_{g|e}] > E[\hat{\Pi}_e]$ such that condition (4.8) is violated and only a gambling equilibrium is admissible as a rational expectations equilibrium.

Such an equilibrium is characterized by significant deposit outflows and high interest rates R_g paid to depositors by risky domestic banks, combined with a high LTRO take up $L_g = \bar{L}$, which is then channelled into domestic sovereign debt purchases rather than lending to non-financial firms.

Finally, observe from Figure 6.8 that LTRO is unable to ensure that there is an efficient equilibrium unless it is coupled with an intermediate level of bank capital N . As N decreases, the

range of $\bar{L}$ under which there is a unique efficient equilibrium shrinks and then disappears. This stems from the inability of LTRO to distinguish between banking strategies which leads to a trade-off between alleviating deposit constraints and creating stronger incentives to follow a gambling strategy.

At low levels of N , greater amounts of LTRO funding is required to alleviate deposit constraints. However, this also increases the expected payoff $E[\hat{\Pi}_{g|e}]$ from deviating to a gambling strategy through the term $(R^{G,H} - R^*)\bar{L}$. When N is very low, $E[\hat{\Pi}_{g|e}]$ overtakes $E[\hat{\Pi}_e]$ such that there is a unique gambling equilibrium before the deposit constraint can be relaxed enough to ensure $E[\hat{\Pi}_c] \geq E[\hat{\Pi}_{g|c}]$. In the next section, I show that TLTRO improves significantly upon the outcome under LTRO by overcoming this trade-off.

Figure 6.8: LTRO (1)

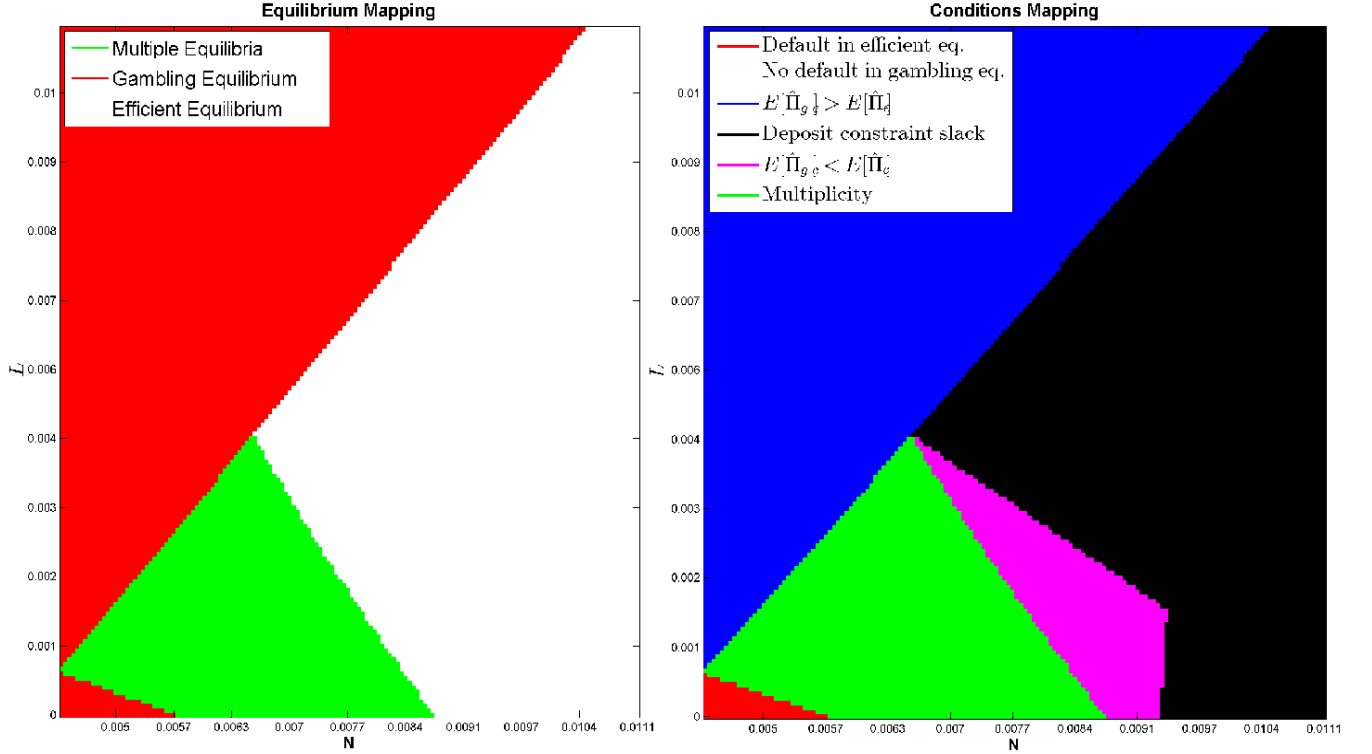
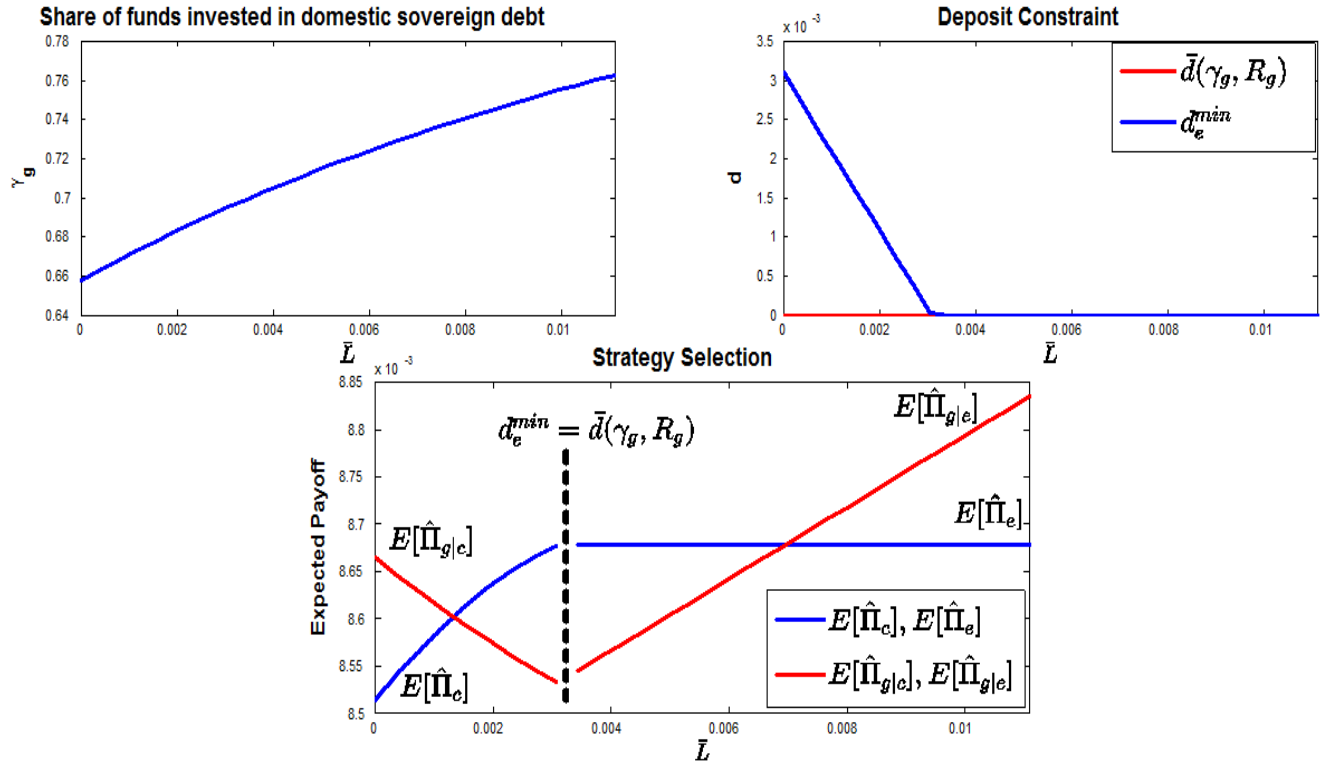


Figure 6.9: LTRO (2)



6.5 TLTRO

6.5.1 Extended Model

Like its non-targeted counterpart, TLTRO provides low interest rate loans to the banking sector but in addition it also place conditionalities on the provision of credit to the private sector by participating banks. Participating banks are monitored over time and an early repayment is required in case they fail to meet the lending requirements (ECB, 2014).

In a two-period setting, early repayment is equivalent to non-participation. Thus, I incorporate TLTRO into the model as the option to borrow up to $\bar{L}$ from the central bank at a safe interest rate R^* with a minimum lending requirement of $\bar{k}$.¹⁹

As with LTRO, if the representative bank is following an efficient strategy and faces no binding deposit constraints, it does not attach any additional value to the TLTRO loan and remains indifferent such that $L_{e,t} \in [0, \bar{L}]$ is indeterminate, where the additional subscript t reflects participation in the TLTRO. When the deposit constraint is binding, on the other hand, the representative bank always finds it optimal to participate in the TLTRO, but can only satisfy the lending requirement under the following condition

$$\bar{k} \leq N + \bar{d} + \bar{L} \quad (6.2)$$

When this condition is satisfied, TLTRO alleviates the deposit constraint by reducing the bank's dependence on deposit funding $d_{e,t}^{\min}$. If the representative bank becomes unconstrained as a result, it becomes indifferent to TLTRO funding after borrowing a minimum amount $L_{c,t} = vK_e - N - \bar{d}$ which is sufficient to achieve $d_{e,t}^{\min} = \bar{d}$. If it remains constrained, on the other hand, the maximum amount of TLTRO funding $\bar{L}$ is used and lending to non-financial firms can be pinned down as

$$K_{c,t} = \frac{N + \bar{d} + \bar{L}}{v}$$

where $(K_{c,t}^*, R_{c,t}^K, \mu_k(K_{c,t}), \lambda_{c,t})$ follow from (2.3), (2.4), (3.5), (4.4) and Proposition 1 remains valid with $\gamma_{c,t} = 0$. The representative bank's expected payoff can then be written as

$$E \left[\hat{\Pi}_{c,t} \right] = NR_{c,t}^K + (R_{c,t}^K - R^*) (\bar{L} + \bar{d}) - PT$$

Under the gambling strategy, the representative bank has the ability to raise additional deposits to satisfy the lending requirement but may not always be willing to. To begin with, the outcome is identical to LTRO when the lending requirement is slack such that

$$\bar{k} \leq vK_g \quad (6.3)$$

¹⁹Since the central bank does not rely on TLTRO to address monopoly distortions, the lending requirement is restricted to $\bar{k} \in [0, vK_e]$.

where K_g is given by (3.12). When the lending requirement binds, on the other hand, it is necessary to consider the outcome under participation in order to determine the incentive compatibility condition for gambling banks to participate. Conditional on participation, the representative bank borrows the highest possible amount of TLTRO loans and extends just enough working capital lending to fulfil the lending requirement such that

$$K_{g,t} = \frac{\bar{k}}{v} \quad (6.4)$$

$$\gamma_{g,t} = 1 - \frac{\bar{k}}{N + d_{g,t} + \bar{L}} \quad (6.5)$$

Its profit maximization problem can then be written as

$$\max_{d_{g,t}} E \left[\hat{\Pi}_{g,t} \right] = (1 - P) \left[(N + d_{g,t} + \bar{L} - \bar{k}) R^{G,H} + R_{g,t}^K \bar{k} - R^* \bar{L} - R_{g,t} d_{g,t} \right]$$

where I have used (6.5) to substitute for $\gamma_{g,t}$ and $R_{g,t}^K$ follows from combining (6.4) with (2.3) and (2.4). As working capital lending is determined by the binding lending requirement, there is a single first order condition

$$R^{G,H} = R_{g,t} + \mu_d(D_{g,t}) \quad (6.6)$$

with the deposit market mark-up $\mu_d(Dt)$ defined by (3.11). As in the previous sections, the numerical solutions for $(R_{g,t}, D_{g,t}, D_{g,t}^*, \mu_d(D_{g,t}))$ can be obtained by jointly solving (6.6) and the representative household's Euler conditions given in Section 2.4.²⁰ The representative bank's expected payoff under a gambling strategy with TLTRO participation can then be written as

$$E \left[\hat{\Pi}_{g,t} \right] = (1 - P) \left[N R^{G,H} + \mu_d(D_{g,t}) v D_{g,t} + (R^{G,H} - R^*) \bar{L} - (R^{G,H} - R_{g,t}^K) \bar{k} \right]$$

Observe that the additional profit $(R^{G,H} - R^*) \bar{L}$ from investing the TLTRO funds in domestic sovereign bonds is partially offset by $-(R^{G,H} - R_{g,t}^K) \bar{k}$ which reflects the above optimal level of working capital lending dictated by the binding lending requirement. Although the deposit threshold $\bar{d}(\gamma_{g,t}, R_{g,t})$ is still defined according to (6.1), participation in TLTRO may now shift it outwards if the ratio $\bar{k}/\bar{L}$ is sufficiently large. However, I show below that the incentive compatibility condition for participation places an upper bound on this ratio.

²⁰There is also a boundary restriction $d_{g,t} \geq \bar{k} - \bar{L} - N$ which requires that the representative bank raises sufficient deposits to satisfy the lending requirement. If this is binding, (6.6) no longer holds with equality and it is replaced by $d_{g,t} = \bar{k} - \bar{L} - N$ in the joint solution, which also implies that $\gamma_{g,t} = 0$. Although my computations account for the possibility of this case, it occurs only when N is very low, $R^{\min}$ is close to zero and $\bar{k}$ is near its upper bound.

Proposition 3 *TLTRO participation under a gambling strategy can only be sustained as a pure strategy Nash equilibrium if there is no incentive to deviate to non-participation given that the remaining banks participate. This leads to the incentive compatibility condition*

$$E \left[\hat{\Pi}_{g,t} \right] \geq E \left[\hat{\Pi}_{g|g,t} \right] \quad (6.7)$$

where $E \left[\hat{\Pi}_{g|g,t} \right]$, the expected payoff from deviation to non-participation, is defined as

$$E \left[\hat{\Pi}_{g|g,t} \right] = (1 - P) \left[NR^{G,H} + \mu_{d|g,t} (D_{g|g,t}) D_{g|g,t} + [K_{g|g,t} - (1 - v) K_{g,t}] \mu_k (K_{g|g,t}) \right]$$

with $(K_{g|g,t}, \mu_k (K_{g|g,t}), D_{g|g,t}, \mu_{d|g,t} (D_{g|g,t}))$ derived in the same manner as Proposition 2 but with the use of $K_{g,t}$ as the level of working capital lending by the other banks instead of K_e .

Proof. This is a corollary to Proposition 2. ■

When the incentive compatibility condition is satisfied, gambling banks participate in TLTRO even with a binding lending requirement and the outcome is as described above. Otherwise, there is no participation (unless the lending requirement is slack) and the outcome under the gambling strategy is similar to the baseline case.

The equilibrium determination conditions in Section 4.3 also change accordingly. Firstly, when (6.2) is satisfied such that the representative bank is capable of fulfilling the lending requirement under deposit constraints, $d_e^{\min}$ in conditions (4.7) and (4.9) is defined as $d_e^{\min} = vK_e - N - \bar{L}$ while $E \left[\hat{\Pi}_{c,t} \right]$ is used in (4.10) instead of $E \left[\hat{\Pi}_c \right]$.

Similarly, when (6.3) is true such that the lending requirement is slack under a gambling strategy, the deposit threshold in (4.9) and (4.11) is defined according to the LTRO outcome described in Section 6.4. If the lending requirement is binding and the incentive compatibility condition (6.7) is fulfilled, on the other hand, the deposit threshold is defined according to (6.1) and $(\gamma_{g,t}, R_{g,t})$ instead of (γ_g, R_g) . When both (6.3) and (6.7) fail such that the lending requirement is binding and not incentive compatible, the representative bank returns to the baseline case under a gambling strategy.

Finally, the expected payoffs from deviating to a gambling strategy given in the right hand sides of conditions (4.8) and (4.10) depend on a combination of these factors. To begin with, $E \left[\hat{\Pi}_{g|c} \right]$ is conditional on $K_{c,t}$ when (6.2) is satisfied such that banks participate in TLTRO under deposit constraints. If the lending requirement is slack under a gambling strategy such that (6.3) is satisfied, $(E \left[\hat{\Pi}_{g|e} \right], E \left[\hat{\Pi}_{g|c} \right])$ are calculated in a similar manner as in Section 6.4 with the additional profit $(R^{G,H} - R^*) \bar{L}$ from investing TLTRO funds into sovereign bond purchases. If the lending requirement is binding and the incentive compatibility condition (6.7) is satisfied, on the other hand, banks anticipate that they will set their working capital lending to $\bar{k}$ after deviating

and $\left(E \left[\hat{\Pi}_{g|e} \right], E \left[\hat{\Pi}_{g|c} \right] \right)$ become

$$\begin{aligned} E \left[\hat{\Pi}_{g,t|e} \right] &= (1 - P) \left[N R^{G,H} + \mu_{d|e} (D_{g,t|e}) D_{g,t|e} + (R^{G,H} - R^*) \bar{L} - (R^{G,H} - R_{g,t|e}^K) \bar{k} \right] \\ E \left[\hat{\Pi}_{g,t|c} \right] &= (1 - P) \left[N R^{G,H} + \mu_{d|c} (D_{g,t|c}) D_{g,t|c} + (R^{G,H} - R^*) \bar{L} - (R^{G,H} - R_{g,t|c}^K) \bar{k} \right] \end{aligned}$$

where $(R_{g,t|e}^K, R_{g,t|c}^K)$ are calculated using (2.4), (2.3) and the working capital lending level

$$K_{g,t|i} = \bar{k} + (1 - v) K_i \quad \forall i \in \{e, c\}$$

where the deposits $(D_{g,t|e}, D_{g,t|c})$ and mark-ups $(\mu_{d|e} (D_{g,t|e}), \mu_{d|c} (D_{g,t|c}))$ are calculated as in Proposition 2 and $K_{c,t}$ is used instead of K_c if (6.2) is satisfied.

As the outcome under TLTRO changes according the conditions (6.2), (6.3) and (6.7), there may be several alternative transmission mechanisms for its effects on the economy. Thus, I conduct a numerical analysis which displays the effects of TLTRO under alternative combinations of $(\bar{L}, \bar{k})$ in the next section which serves to provide additional intuition about the effective transmission mechanism.

6.5.2 Numerical Analysis

Figure 6.10 provides a map of the equilibrium outcomes under TLTRO for different combinations of $(N, \bar{L})$. The lending requirement $\bar{k}$ is fixed at a value just above vK_g to ensure that it is binding under the gambling strategy. According to the bottom plots in the figure, gambling banks never participate in TLTRO whereas constrained banks participate when $(N, \bar{L})$ are sufficiently high to allow them to fulfil the lending requirement.

The non-participation of gambling banks allows TLTRO to overcome the trade-off between alleviating deposit constraints and creating stronger incentives to gamble. As such, the main benefit from the lending requirement $\bar{k}$ is in its use as a mechanism to reveal banking strategies rather than as a way to increase working capital lending under a given strategy. Thus, in contrast with LTRO, TLTRO allows the central bank to provide higher levels of funding $\bar{L}$ without triggering a deviation to the gambling strategy. Indeed, the top plots in Figure 6.10 show that a sufficiently high $\bar{L}$ value can make the efficient equilibrium unique even at low levels of bank capital N .

Figure 6.11 provides additional intuition about the transmission mechanism by plotting key variables across $\bar{L}$ values at a boundary level of capitalization $N = 0.0057$. As before, the vertical line marked with $d_e^{\min} = \bar{d}(\gamma_g, R_g)$ shows the point where the deposit constraint becomes slack while the line under $N + \bar{d} + \bar{L} = \bar{k}$ marks the point where (6.2) is satisfied such that constrained banks can participate in TLTRO. The top right plot confirms the non-participation of gambling banks by showing that the incentive compatibility condition (6.7) is not fulfilled at any $\bar{L}$ value.

Consequently, (γ_g, R_g) do not change across $\bar{L}$ values and the deposit threshold $\bar{d}(\gamma_g, R_g)$

remains the same. The top left plot shows that TLTRO instead alleviates deposit constraints by reducing $d_e^{\min}$, the reliance of banks on deposit funding under an efficient strategy. Observe that $d_e^{\min}$ jumps down discretely and becomes downward sloping in $\bar{L}$ upon the participation of constrained banks in TLTRO.

Finally, the bottom plot shows the evolution of expected payoffs under negative sentiments. Note that the participation of constrained banks in TLTRO does not only increase their expected payoff from $E[\hat{\Pi}_c]$ to $E[\hat{\Pi}_{c,t}]$ but also reduces the incentive to deviate. The explanation is simple: As in Section 6.4.2, the increase in working capital lending K_c following TLTRO participation erodes the mark-up following a deviation. Note also that the $\bar{L}$ value necessary to eliminate multiplicity is slightly lower than the amount that completely offsets deposit constraints as banks no longer find it optimal to deviate to a gambling strategy. In other words, equilibrium determination condition (4.10) is violated before (4.9).

In order to gain more intuition about the role of the lending requirement $\bar{k}$, I fix the available funding at $\bar{L} = 0.0049$ and consider the outcome under different $\bar{k}$ values in Figure 6.12. The bottom plots show that gambling banks opt out of TLTRO as soon as the lending requirement becomes binding with $\bar{k} > vK_g$ and further increases in $\bar{k}$ only serve to expand the region of non-participation for constrained banks. Thus, it is optimal to set $\bar{k}$ just above vK_g as in Figure 6.10 to achieve the most favourable conditions for constrained banks to benefit from TLTRO while deterring the use of TLTRO funds under a gambling strategy.

The mapping of outcomes allocations shown in the top plots changes accordingly. At low levels of $\bar{k} < vK_g$, the lending requirement is slack and the representative bank has a strong incentive to gamble by investing TLTRO funds in domestic sovereign debt purchases. This leads to a unique gambling equilibrium as in the case with excessively high amounts of funding under LTRO. At very high levels of $\bar{k} > N + \bar{d} + \bar{L}$, on the other hand, constrained banks are unable to participate in TLTRO and the policy is completely ineffective. As such, TLTRO is effective in reducing multiplicity when $\bar{k}$ is in the intermediate region $vK_g < \bar{k} < N + \bar{d} + \bar{L}$ which allows banks to participate when they are deposit constrained but deters them when they switch to a gambling strategy.

The bottom left plot in Figure 6.13 demonstrates the transmission mechanism behind this. The expected payoff from deviating to a gambling strategy declines when the lending requirement becomes binding but rises again when $\bar{k}$ is large enough to hinder participation under deposit constraints. Thus, (4.10) is only violated in the intermediate region.

Moreover, the bottom right plot shows that the representative bank prefers to deviate to a gambling strategy even under positive sentiments when the lending requirement is not binding. This brings about the region with a unique gambling equilibrium shown in Figure 6.12. Regarding the deposit constraint, although there is a slight decline in γ_g when gambling banks opt out of TLTRO, this is insufficient to cause a noticeable shift in the deposit threshold. Nevertheless, the deposit constraint is alleviated as long as the constrained banks can participate in TLTRO since

this reduces their reliance on deposit funding $d_e^{\min}$.

Overall, I find that TLTRO can improve upon the outcome under LTRO significantly such that the appropriate combination of $(\bar{L}, \bar{k})$ brings about a unique efficient equilibrium at all levels of capitalization N . The improvement stems from ‘participation effects’ whereby the lending requirement $\bar{k}$ hinders the participation of gambling banks rather than ‘incentive effects’ associated with reductions in γ_g , the share of funds spent on sovereign debt purchases under a gambling strategy. The ability of TLTRO to indirectly discriminate between strategies then allows a rise in the amount of funding $\bar{L}$ to a level that is sufficient to prevent negative sentiments from becoming self-fulfilling without creating incentives to deviate to a gambling strategy.

Figure 6.10: TLTRO (1)

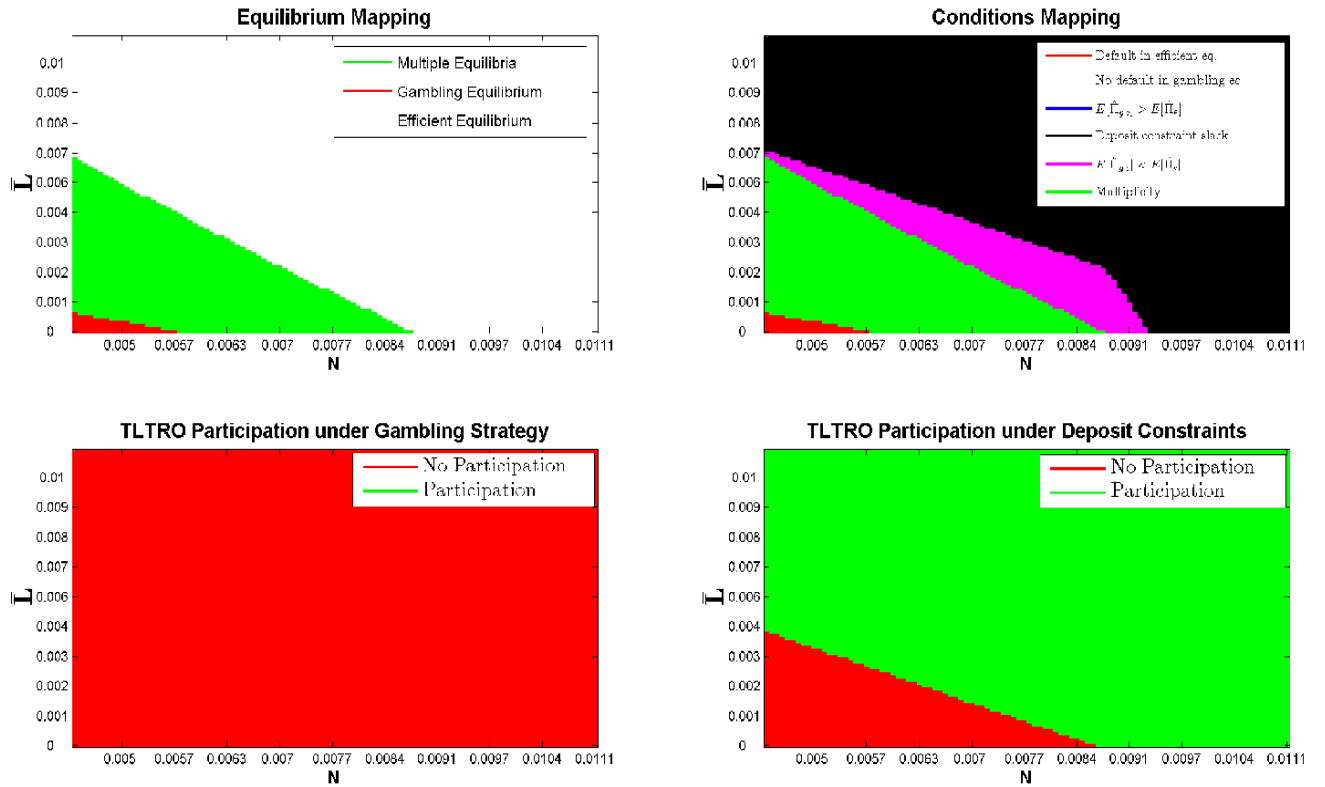


Figure 6.11: TLTRO (2)

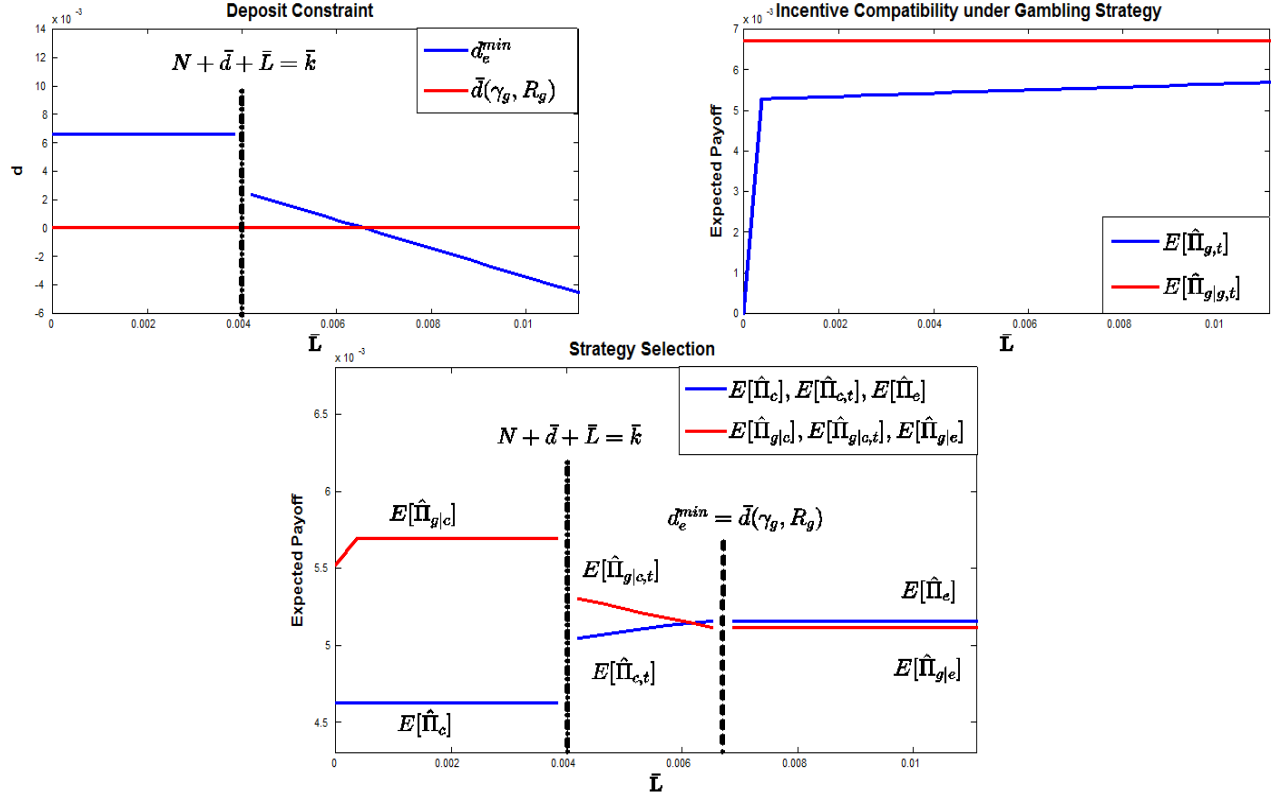


Figure 6.12: TLTRO (3)

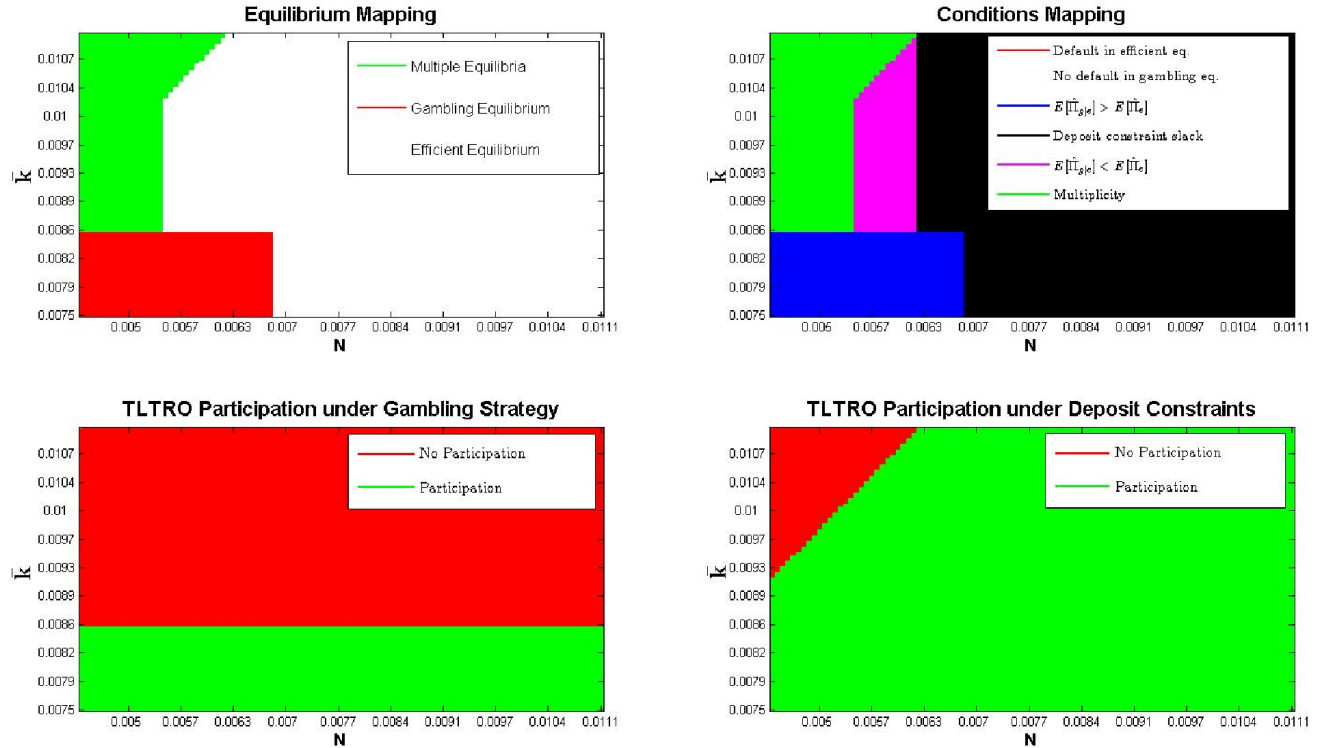
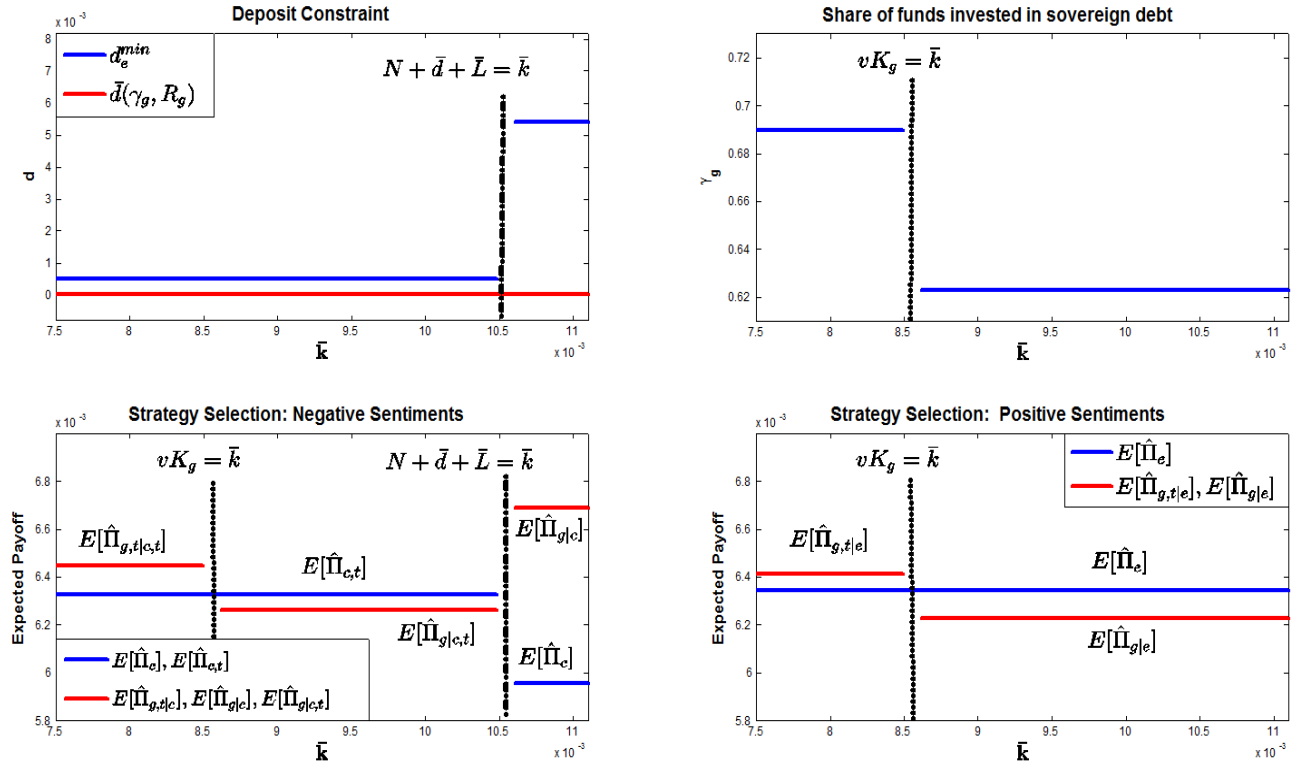


Figure 6.13: TLTRO (4)



6.6 Paradox of Observation

An important insight that emerges from sections 6.4 and 6.5 pertains to the take-up of LTRO and TLTRO lending by banks. If these policies are successful in making the efficient equilibrium unique, negative sentiments cease to be self-validating and the deposit constraint does not bind in a rational expectations equilibrium. Thus, in equilibrium, banks become indifferent between deposit financing and borrowing from the central bank via (T)LTRO.

If these policies fail to eliminate adverse equilibria, on the other hand, negative sentiments remain self-confirming such that banks deviate to a gambling strategy under tight deposit constraints. As the constrained efficient outcome never occurs in equilibrium, any strict preference for raising funds through (T)LTRO stems from a gambling strategy which invests these funds in domestic sovereign debt. This is also true when the outcome is a unique gambling equilibrium.

Paradoxically, this implies that LTRO and TLTRO are only successful when they remain as off-equilibrium threats. The observation of a strict preference by banks towards raising funds through these schemes implies that there is either a unique gambling equilibrium or multiple equilibria with negative sentiments. Far from assuaging depositor concerns, in this case (T)LTRO provides an additional source of funding for banks to gamble with and facilitates an increase in their exposure to domestic sovereign debt.

7 Conclusion

Recent times have seen a sharp increase in the share of domestic sovereign debt held by the national banking system in European countries hit by the sovereign debt crisis. In this paper, I propose a model with optimizing banks and depositors for analysing the implications for economic vulnerability to crisis and policy design.

Two important insights emerge as a consequence. First, undercapitalized banks have an incentive to gamble on domestic sovereign bonds when they expect to suffer from non-bond losses in the aftermath of sovereign default. Second, optimal depositor reactions to insolvency risk impose discipline on banks but also leave the economy susceptible to self-fulfilling shifts in sentiments when bank balance sheets are intransparent. In the adverse equilibrium, banks become heavily exposed to domestic sovereign bonds, leading to a rise in bank funding costs and the crowding out of bank lending to the private sector, and sovereign default also leads to a banking crisis.

The model also provides a useful framework for the evaluation of recent and proposed policy interventions. While the most obvious policy remedy is the recapitalization of the banking sector, this requires a significant resource transfer at a time when the government is cash-struck. Contractionary monetary policy may also eliminate the gambling equilibrium, but this comes at a significant cost to the real economy. Strengthening of deposit insurance, on the other hand, reduces bank funding costs but gives banks greater incentive to gamble by severing the link between their

solvency prospects and funding costs.

I also evaluate the implications of two subsidized lending schemes undertaken by the European Central Bank; LTRO and its targeted counterpart TLTRO. I find that a limited amount of LTRO funding can eliminate the gambling equilibrium provided that there is an intermediate level of bank capitalization, but excessive levels of LTRO funding create strong incentives to gamble and may instead eliminate the efficient equilibrium. This stems from LTRO's inability to distinguish between banking strategies which creates a trade-off between alleviating funding constraints and strengthening incentives to gamble. It is possible to overcome this trade-off by using the lending requirement of TLTRO as an indirect mechanism to reveal banking strategies. When implemented appropriately, I show that TLTRO can bring about a unique efficient equilibrium at all levels of bank capitalization.

The mechanisms described in this paper can be interpreted in a broader context than a sovereign debt crisis. They are relevant, for example, to a wide range of assets with payoffs correlated to aggregate risk and can also play an important role when banks have a large prior exposure to a relatively illiquid asset. They are particularly strong in the case of domestic sovereign bonds due to the triple coincidence of the high correlation of sovereign default with aggregate risk, the zero risk-weight in regulation for domestic sovereign bonds and the prospect of a bail-in in the aftermath of sovereign default.

An interesting extension to this line of research would be the introduction of endogenous sovereign default. A framework with endogenous sovereign default may present an additional layer of multiplicity as the decline in bank lending in a gambling equilibrium reduces tax revenues. This would yield additional insights for vulnerability to crises and policy design.

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8 Appendix

A Proof of Proposition 1

Suppose the representative bank allocates a positive share of its funds to sovereign bond purchases such that $\gamma_c > 0$. Then we will have

$$R_c^K - \mu_k(K_c) = R^* + \lambda_c > R^* = (1 - P) R^{G,H} + P R^{G,L}$$

where the equalities are respectively due to (4.2) and (2.1) and the inequality stems from the positive multiplier $\lambda_c > 0$ under the constrained equilibrium. Intuitively, the inequality is driven by the pricing of sovereign debt by foreign banks: the expected return from sovereign bonds remains fixed as b_c is reduced below b_e while a similar fall in k_c triggers a rise in R_c^K .

As long as $R_c^K - \mu_k(K_c) > R^*$, the bank will find it profitable to reduce γ_c by re-allocating funds from sovereign bond purchases to working capital loans. If the consequent rise in K_c allows working capital to reach its unconstrained level K_e before the lower bound of γ_c becomes binding, we will have

$$\begin{aligned} R_c^K - \mu_k(K_c) &= R^* \\ \lambda_c &= 0 \end{aligned}$$

Recall from Section 3.1 that (b_e, γ_e, d_e) are indeterminate in the efficient equilibrium. Then this solution is admissible as an efficient equilibrium with $d_e = \bar{d}$ and yields the same level of working capital, output and expected payoff. As such, there may only be a constrained equilibrium if γ_c is constrained by its lower bound at zero while $K_c < K_e$. In other words, there must be an opportunity to increase profits by lending additional working capital that banks are unable to exploit due to their deposit constraints. A constrained equilibrium will then be characterized by

$$\begin{aligned} \gamma_c &= b_c = 0 \\ k_c &= N + \bar{d} \\ R_c^K - \mu_k(K_c) &= (1 - P) R^{G,H} + P R^{G,L} + \lambda_c \end{aligned}$$

where the final equation implies that $K_c < K_e$. Indeed, combining this equation with (2.4) and (4.1) yields an expression for λ_c

$$\begin{aligned} \lambda_c &= \max [R_c^K - \mu_k(K_c) - R^*, 0] \\ &= \max \left[\frac{a}{(K_c + g(K_c))^{1-a}} \left[1 - v(1 - a) \frac{1 + g'(K_c)}{K_c + g(K_c)} K_c \right] - R^*, 0 \right] \end{aligned}$$

B Proof of Proposition 2

The payoff matrix of an individual bank takes the form

Figure 8.1: Payoff Matrix

		<i>Strategy taken by other banks</i>	
		Efficient	Gambling
<i>Bank's own strategy</i>	Efficient	$E[\hat{\Pi}_e]$	$E[\hat{\Pi}_{e g}]$
	Gambling	$E[\hat{\Pi}_{g e}]$	$E[\hat{\Pi}_g]$

where $E[\hat{\Pi}_{g|e}]$ refers to the payoff from a gambling strategy conditional on the other banks following an efficient strategy and vice versa for $E[\hat{\Pi}_{e|g}]$. It is notable that the condition for the gambling equilibrium to be a Nash equilibrium $E[\hat{\Pi}_g] \geq E[\hat{\Pi}_{e|g}]$ differs from the condition that rules out an efficient equilibrium, $E[\hat{\Pi}_{g|e}] > E[\hat{\Pi}_e]$. The game also accommodates a range of mixed strategy Nash equilibria and separating equilibria where a portion of banks gamble. Considering these equilibria complicates the model significantly while adding little to its intuition. Moreover, it is not clear that these equilibria are better than the gambling equilibrium from a social welfare perspective. Thus, I focus on the existence of an efficient equilibrium and use the condition $E[\hat{\Pi}_{g|e}] > E[\hat{\Pi}_e]$ to evaluate whether there is any incentive to deviate to a gambling strategy.

In order to determine $E[\hat{\Pi}_{g|e}]$, I evaluate the outcome of a gambling strategy when the other banks follow an efficient strategy (γ_e, d_e) . The problem is identical to Section 3.2 but with the definitions for $(K_{g|e}, \mu_{d|e}(D_{g|e}))$ altered to account for the change in the behaviour of other banks.

$$K_{g|e} = k_g + (1 - v) K_e \quad (8.1)$$

$$\mu_{d|e}(D_{g|e}) = \frac{\partial R_{g|e}}{\partial d_{g|e}} d_{g|e} \quad (8.2)$$

where K_e is given by (3.6) and the new definition for $\mu_{d|e}(D_{g|e})$ reflects that the bank now acts as a monopoly in the market for risky deposits as the only bank to follow a gambling strategy. By combining the new definition for $K_{g|e}$ with (3.5), I obtain the bank's mark-up in the working capital market

$$\mu_k(K_{g|e}) = \frac{a(1-a)(1+g'(K_{g|e}))}{(K_{g|e} + g(K_{g|e}))^{2-a}} (K_{g|e} - (1-v)K_e)$$

As before, combining this with (3.9), (3.10) and (2.4) yields an implicit solution for $K_{g|e}$

$$a \left(K_{g|e} + g \left(K_{g|e} \right) \right)^{a-1} = R^{G,H} + \frac{a(1-a) \left(1 + g' \left(K_{g|e} \right) \right)}{\left(K_{g|e} + g \left(K_{g|e} \right) \right)^{2-a}} \left(K_{g|e} - (1-v) K_e \right)$$

and $(R_{g|e}, D_{g|e})$ may be determined by numerically solving the set of simultaneous equations given by (3.9), (8.2) and the representative household's Euler conditions. The expected payoff from deviating to a gambling strategy can then be written as

$$E \left[\hat{\Pi}_{g|e} \right] = (1-P) \left[N R^{G,H} + \mu_{d|e} \left(D_{g|e} \right) D_{g|e} + \left[K_{g|e} - (1-v) K_e \right] \mu_k \left(K_{g|e} \right) \right] \quad (8.3)$$

The precise condition for strategy selection depends on whether the banks are deposit constrained under an efficient strategy. If the representative bank is not deposit constrained such that $\lambda_c = 0$, the condition for an efficient equilibrium to be sustainable as a Nash equilibrium is

$$E \left[\hat{\Pi}_e \right] \geq E \left[\hat{\Pi}_{g|e} \right]$$

where $E \left[\hat{\Pi}_e \right]$ is given by (3.7). When the representative bank is deposit constrained (such that $\lambda_c > 0$), on the other hand, the relevant condition becomes

$$E \left[\hat{\Pi}_c \right] \geq E \left[\hat{\Pi}_{g|c} \right]$$

with the constrained payoff $E \left[\hat{\Pi}_c \right]$ given by (4.5) and $E \left[\hat{\Pi}_{g|c} \right]$ obtained in the same manner as $E \left[\hat{\Pi}_{g|e} \right]$ but with the use of $K_{g|c} = k_g + (1-v) K_c$ rather than $K_{g|e}$. Note that a tightening of the deposit threshold in the form of a fall in $\bar{d}$ reduces $E \left[\hat{\Pi}_c \right]$ and makes it harder for an efficient equilibrium to be sustained.