Domestic and External Debt and Default*

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Abstract

We develop a general equilibrium model with defaultable domestic and external debt. Overlapping generations work, consume, accumulate capital and public debt. Successive, democratically elected governments choose taxes, public goods spending, domestic and external debt issuance and repayment. In Markov perfect equilibrium, political distortions inherent in democratic societies strengthen debt capacity; macroeconomic shocks affect the cost of public funds and debt returns; and debt serves intergenerational risk sharing. Default decisions may or may not be correlated across debt tranches. Minimum debt returns raise the cost of public funds ex post and render default on other tranches more likely; ex ante, they increase the revenue from debt sales but crowd out capital. Political and general equilibrium "wedges" undermine the intertemporal smoothing of the shadow cost of public funds. Under standard functional form assumptions the model is solved in closed form.

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1 Introduction

Public debt takes center stage in many accounts of crises past and present—including the ongoing European debt crisis—and it is often mentioned as a likely contributing factor for future economic problems. Public debt is also the subject of a large body of work in

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the economic literature. Curiously, however, this literature is sharply divided into two strands, one analyzing domestic and the other external public debt.

The domestic debt literature focuses on the crowding out of physical capital and on the (efficiency enhancing) tax smoothing or (redistributive) tax shifting effects of debt.[1] These macroeconomic studies typically assume that the government can commit to servicing its debt even if it may lack the power to commit to other policy instruments.[2] In contrast, the international finance literature analyzes external debt under the assumption that the sovereign cannot commit to debt repayment and it studies the implications of this friction for debt capacity and default.[3]

The dichotomy of theoretical perspectives on government debt is foreign to the public debate and appears overly restrictive, for several reasons. First, the main function of domestic and external debt is identical: to smooth the shadow cost of public funds. It thus seems natural to jointly analyze the two instruments. Second, lack of commitment constrains the use of both types of debt, even in developed economies as the ongoing European debt crisis shows. Again, this renders a joint analysis natural.

Third, Reinhart and Rogoff (2011) and others have argued that external and domestic debt crises often go hand in hand, possibly intermediated by banking crisis, inflation and the uncovering of "hidden" debts. Reinhart and Rogoff (2008) document that domestic debt tends to exceed external debt and they suggest that large domestic debt positions may have affected countries' external default decisions as well as the choice of inflation rates well beyond the seigniorage maximizing levels (see figure 1). These observations call for a unified theory of—defaultable—domestic and external debt.

Finally, it is often argued that the ownership structure of public debt matters—for instance because a large domestic creditor base renders a run on public debt less likely or the cost of financing it lower.[4] Addressing such issues requires a model that features both domestic and external public debt.

In this paper, we develop a model that bridges the gap between the macroeconomic and the international finance literatures. Our model features domestic and external public debt;[5]; public goods spending; a domestic private sector that invests in government bonds and physical capital; external lenders; and political decision makers that are democratically elected in each period. Accordingly, public goods spending, taxes and debt policy is chosen sequentially (Markov perfect equilibrium) and both domestic and external bonds

[1] See Diamond (1965) on crowding out; Barro (1979), Lucas and Stokey (1983) and Aiyagari, Marcet, Sargent and Seppälä (2002) on tax smoothing; and Diamond (1965) and Niepelt (2004) on tax shifting.

[2] For example, Lucas and Stokey (1983) posit that the government can commit to state contingent debt but not to tax rates.

[3] See Eaton and Gersovitz (1981) and Eaton and Fernandez (1995).

[4] Along these lines, it is often argued that debt quotas in Italy and Japan are sustainable in spite of their elevated levels; and that an eventual shift of creditor base due to further deficits in Japan might give rise to a "tipping point."

[5] According to Reinhart and Rogoff (2008), "[d]omestic public debt is issued under home legal jurisdiction. In most countries, over most of their history, it has been denominated in the local currency and held mainly by residents. By the same token, the overwhelming majority of external public debt—debt under the legal jurisdiction of foreign governments—has been denominated in foreign currency and held by foreign residents." Our use of the terms "domestic" and "external" debt corresponds with these historical regularities.

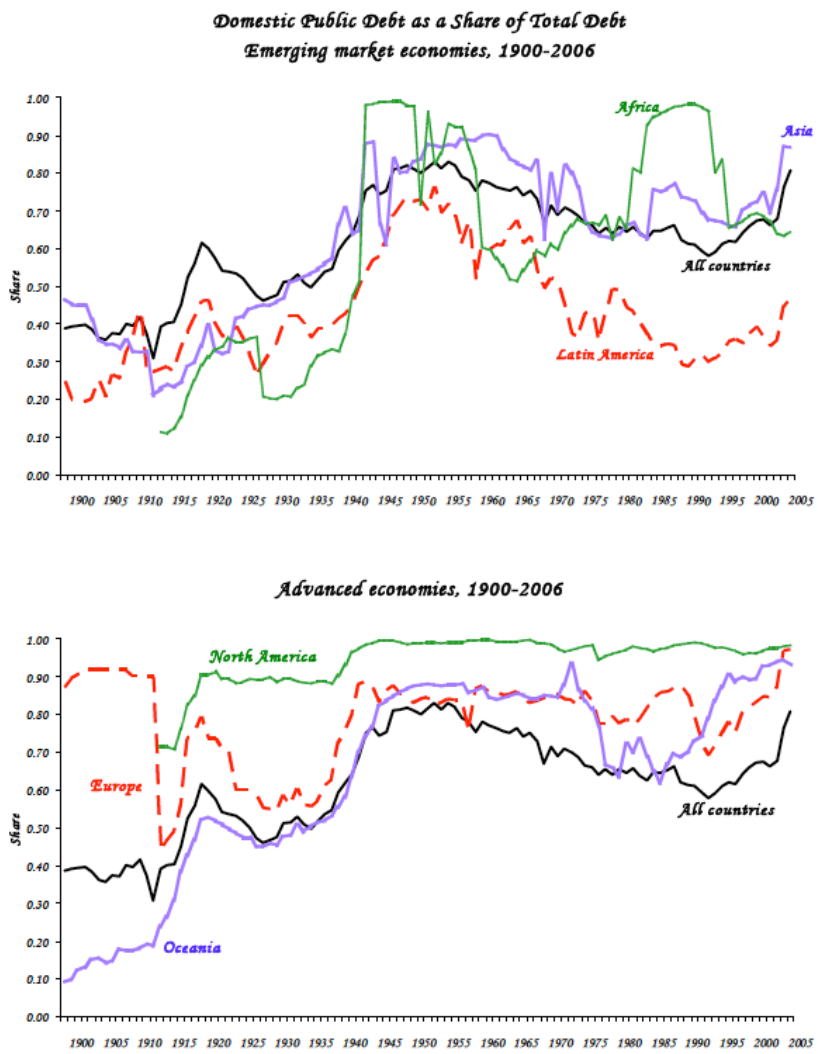

Figure 1: Share of domestic in total government debt by country group over time. Source: Reinhart and Rogoff (2008).

are subject to endogenous default risk. Macroeconomic outcomes reflect this risk as well as shocks to productivity, demographics and other fundamentals.

The government in the model may selectively, and partly default on each tranche of maturing debt. Default directly benefits taxpayers because the government must transfer fewer funds to lenders, and it directly hurts creditors. In the case of domestic debt, the creditors are old, domestic households who invested in government bonds along with physical capital during their youth. In the case of external debt, the creditors are foreigners.

Whether the government deems it advantageous to set a low ex-post repayment rate depends on how attractive political decision makers find it to benefit taxpayers at the cost of creditors. A sufficient statistic for the benefit of defaulting is the shadow cost of public funds—the social cost as perceived by the political process at which government funds may be replenished. Both the shadow cost of public funds and the cost of hurting

creditors are affected by macroeconomic shocks and outcomes; in the case of domestic debt the two are tightly connected, linking government finance and the domestic wealth distribution.

A haircut on external debt triggers deadweight losses, either because renegotiation with external creditors is time consuming and disruptive or because the terms of the debt contract cannot be renegotiated at all and a default triggers social costs. Possible sources of the latter include disruptions in the external or the financial sector; market exclusion; or loss of reputation.[6] We do not take a position on the specific mechanism through which such costs come about and instead opt for generality and tractability.

A default on domestic debt does not trigger deadweight losses because the terms of such debt can easily be renegotiated. Absent deadweight losses of a domestic default the government's incentive to repay domestic liabilities solely derives from the desire to avoid unwarranted redistribution between domestic bond holders and taxpayers.

The assumption that domestic debt can be renegotiated at lower deadweight losses than external debt is not central for the model's results.[7] But it is very realistic. Reneging on the terms of government debt issued under domestic law and held by domestic creditors is relatively straightforward for lawmakers and much less costly than renegotiating the terms of external liabilities. After all, the return on domestic debt held by residents can easily be taxed; financial repression tactics can be employed to lower returns; and since domestic debt typically is denominated in local currency, a government may expropriate by fabricating surprise inflation. If the general public does not understand the causes of the induced debt devaluation the government's actions may not even trigger a public outcry with associated political costs.[8] Moreover, importantly, large parts of domestic government debt take the form of implicit liabilities such as social security promises whose value can be—and often is—renegotiated and reset in the legislative process, for example by changing taxes or wage and price indexation clauses. The largesse of domestic debt service thus reflects the resolution of a distributive conflict between domestic interest groups.

We assume that conflicts about domestic debt repayment as well as other policy choices is resolved through a probabilistic voting game between political candidates.[9] In equilibrium, the government thus implements policies that maximize a weighted average of the welfare of all groups of voters—not only of a hypothetical "median voter." Changes in the economy's demographic structure (which we allow for) lead to gradual changes in policy outcomes, in contrast to what would occur in a median voter environment.

We also consider the consequences of partial commitment—the option for the government to guarantee minimum debt returns in the subsequent period. Exercising this option

[6] See, for example, Reinhart and Rogoff (2004), Sturzenegger and Zettelmeyer (2006, pp. 49–52) or Panizza, Sturzenegger and Zettelmeyer (2009) for a discussion of the costs of sovereign defaults.

[7] Introducing a deadweight loss from domestic default would alter the elasticity of debt returns with respect to shocks.

[8] We do not distinguish between outright and indirect forms of default. Even with the former, defaulting against domestic creditors is less costly than defaulting against foreigners. Reinhart and Rogoff (2008) document the frequency of outright and indirect defaults. Hall and Sargent (2011) document the role of inflation in lowering public debt/GDP ratios in post war US data.

[9] See Lindbeck and Weibull (1987).

allows the government to increase the revenue from debt sales but it reduces the flexibility of the successor government and, in the case of domestic debt, renders old households who hold debt richer than young households who do not. This increases the shadow cost of public funds and induces negative spillover effects onto other debt tranches. Creditors are indifferent as to the extent of partial commitment as they always receive the required rate of return on their bond portfolio. But voters who internalize the general equilibrium consequences of policy face a trade-off. More commitment crowds out capital; this raises interest rates; and the effect on state variables alters future policy and thus, affects the welfare of the current young once they have turned old. The political and general equilibrium "wedges" undermine the inter temporal smoothing of the shadow cost of public funds.

The model generates the following predictions:

i. Propositions. TBC

Under standard functional form assumptions (mainly logarithmic utility and Cobb-Douglas production) the full model can be solved in closed form. We derive the following additional results:

i. Other propositions. TBC

Finally, we study a series of parametric examples and simulations. The following findings emerge:

i. Findings. TBC

Related Literature Aguiar and Amador (2011) develop a model of a small open economy where the incumbent government may expropriate capital in addition to loans extended by foreign investors. In contrast to Aguiar and Amador (2011), we assume that capital is owned by domestic households and we allow for domestic in addition to external debt. Moreover, we analyze the Markov perfect equilibrium while Aguiar and Amador (2011) focus on the best self-enforcing equilibrium.

The choice of repayment rate on domestic debt in our model resembles the resolution of intergenerational conflict in Gonzalez-Eiras and Niepelt's (2008) model of social security.[10]

Di Casola and Sichlimiris (2015) and Paczos and Shakhnov (2014) constitute other recent attempts to integrate the analysis of domestic and external debt.

Outline The remainder of the paper is structured as follows. We layout the model in section 2 and define equilibrium in section 3. Section 4 contains a characterization of equilibrium and a first set of propositions. In section 5, we impose functional form assumptions and establish additional results. Section 6 contains the examples and section 7 concludes.

[10]For an analysis of the equivalence between social security and debt regimes with and without commitment, see Gonzalez-Eiras and Niepelt (2015).

2 The Model

2.1 Structure of the Economy

Time is discrete, indexed by t and extends over an infinite horizon. The economy is inhabited by two-period lived overlapping generations, competitive firms, a government that can be voted out of office in every period, and external lenders. The size of the young cohort in period t relative to the size of the old cohort is given by ν_t.

2.2 Households

Preferences of households born in period t are described by the utility function

$$u(c_t^y) + v_t(g_t) + \delta \mathbb{E}_t[u(c_{t+1}^o) + v_{t+1}(g_{t+1})], \tag{1}$$

where $u(\cdot)$ and $v_t(\cdot)$ denote strictly increasing and concave utility functions; $\delta \in (0,1)$ the discount factor; $\mathbb{E}_t[\cdot]$ the expectation conditional on information available at time t; (c_t^y, c_{t+1}^o) cohort t's consumption path over the life cycle; and g_t government spending for public goods. The preference for public goods may vary over time.

Young households are born without assets and inelastically supply one unit of labor at wage w_t. Their labor income is taxed at rate τ_t. They consume their after-tax labor income or invest it in one of two assets: physical capital, k_{t+1}; or domestic government debt, d_{t+1}. Capital has a price of unity and yields a state-contingent gross return R_{t+1} in the subsequent period. Debt trades at price q_t^d and pays a gross return r_{t+1}^d. Returns are taxed at the same rate as labor income. Consumption in young and old age, respectively, thus equals

$$\begin{aligned} c_t^y &= w_t(1-\tau_t) - k_{t+1} - q_t^d d_{t+1}, & (2)\\ c_{t+1}^o &= (k_{t+1}R_{t+1} + d_{t+1}r_{t+1}^d)(1-\tau_{t+1}). & (3) \end{aligned}$$

2.3 Firms

The aggregate production function is neoclassical and displays constant returns to scale in capital and labor. Output per old household equals $y_t = y_t(k_t, \nu_t)$. The function $y_t(\cdot)$ is indexed by time because parameters of the function may be exogenously time varying. Competitive factor markets imply

$$\begin{aligned} R_t &= \partial y_t(k_t, \nu_t)/\partial k_t, & (4)\\ w_t &= \partial y_t(k_t, \nu_t)/\partial \nu_t. & (5) \end{aligned}$$

2.4 External Lenders

External lenders are competitive. Their asset pricing kernel in period t, m_t, is exogenous from the perspective of the domestic economy. Lenders buy external debt e_{t+1} (per capita

of young households) at price q_t^e from the government and receive a gross return r_{t+1}^e in the subsequent period.[11]

2.5 Government

The government finances public goods spending as well as the domestic and external debt service by levying taxes and issuing new debt. The price of newly issued debt reflects its state contingent return.

The return on debt is chosen ex post and restricted to be weakly positive: $r_t^d, r_t^e \geq 0$. A choice of low repayment rate on domestic debt harms domestic creditors, i.e. the old. Since the old participate in the political process default may be politically costly (see below). We allow for the possibility of *partial commitment*, reflected in a lower bound on the domestic debt return, $\bar{r}_t^d$, chosen in the preceding period.

Default on external debt is costly as well. To formally capture the existence of external default costs we adopt a parsimonious but flexible specification according to which the government's valuation of debt repayment to external lenders equals $\varphi_t(e_t r_t^e)$ and may be constrained by a lower bound on the debt return, $\bar{r}_t^e$, which was chosen in the previous period. Function $\varphi_t(\cdot)$ is weakly increasing and continuously differentiable. It can represent a welfare function—the government (or domestic lobby groups) might value foreign lenders' welfare because it is more difficult to exploit future gains from trade if lenders are disgruntled; or it can represent a cost function that reflects one of the many default or renegotiation costs discussed in the sovereign debt literature.[12]

The budget constraint of the government reads

$$g_t + d_t r_t^d + e_t r_t^e = \tau_t(y_t + d_t r_t^d) + \nu_t q_t^d d_{t+1} + \nu_t q_t^e e_{t+1}. \tag{6}$$

The economy's resource constraint is given by

$$g_t + c_t^o + \nu_t c_t^y + \nu_t k_{t+1} = y_t + \nu_t q_t^e e_{t+1} - e_t r_t^e. \tag{7}$$

2.6 Politics

The government's choice of policy instruments reflects societal preferences as aggregated in the political process. In each period, two candidates compete for office in democratic

[11] External lenders do not buy debt from domestic households. This assumption is without loss of generality when the government can intervene in the capital account. If the "sovereign ceiling" imposed by the government effectively hands control over all external borrowing to the government, private external debt can be replicated by a government tax and debt policy. There exists ample evidence of private sector external debt that was absorbed by the public sector. A recent example are bank liabilities in Euro area countries that were guaranteed or eventually absorbed by national governments during the Euro area debt crisis. See also Reinhart and Rogoff (2011, p. 1692) on the Latin American debt crisis in the 1980s.

[12] For example, a state contingent fixed cost of defaulting can be represented by $\bar{r}_t^e > 0$ and $\arg\max \varphi_t(e_t r_t^e)$ fluctuating between values above and below $\bar{r}_t^e > 0$. Empirical evidence supports the notion of temporary rather than permanent default costs, in particular in a model where one period corresponds to several years. Gelos, Sahay and Sandleris (2011) report that defaults, if resolved quickly, do not preclude sovereigns from re-accessing financial markets.

elections. A candidate represents a policy platform that specifies values for the policy instruments in question as well as an ideological position that is orthogonal to the policy platform.[13] The valuation of the ideological position differs across voters (even if they agree about the preferred policy platform) and is subject to random aggregate shocks which are realized after candidates have proposed their platforms.

In the resulting "probabilistic-voting" Nash equilibrium, the candidates maximize their respective vote shares by both proposing the same policy platform. This equilibrium platform maximizes a convex combination of the welfare of all voters (not only of the majority group) where the weights reflect group size and sensitivity of voting behavior to policy changes.[14] Groups that care more strongly about the policy platform relative to ideology have more political influence since they are more likely to alter their support in response to small changes in the proposed platform. In equilibrium, these "swing voters" thus tilt policy in their own favor. If all voters are equally responsive to changes in the policy platform, electoral competition implements the utilitarian optimum with respect to voters.

Formally, the political process in period t and thus, the government in power in that period maximizes the objective function

$$\omega_t \left(u(c_t^o) + v_t(g_t)\right) + \nu_t(u(c_t^y) + v_t(g_t) + \delta \mathbb{E}_t[u(c_{t+1}^o) + v_{t+1}(g_{t+1})]) + \varphi_t(e_t r_t^e). \qquad (8)$$

The per-capita political weight of old relative to young households, $\omega_t > 0$, reflects whether old voters respond more or less strongly to changes in the policy platform than young voters. [15]

2.7 Timing

Events unfold as follows: At the beginning of period t the exogenous state (including the parameters of the valuation of public goods spending as well as the production and default cost functions) is realized and becomes commonly known,

$$\hat{z}_t \equiv (\nu_t, v_t(\cdot), y_t(\cdot), \varphi_t(\cdot), \omega_t, m_t).$$

Next, a candidate is elected for office. When choosing among the competing candidates, voters take both direct and indirect policy implications for economic outcomes into account. The winning candidate implements her policy platform

$$\pi_t \equiv (g_t, r_t^d, r_t^e, \tau_t, \tilde{d}_{t+1}, \tilde{e}_{t+1}, \bar{r}_{t+1}^d, \bar{r}_{t+1}^e)$$

[13] The ideological position is permanent and cannot be credibly altered in the course of electoral competition.

[14] See Lindbeck and Weibull (1987) and Persson and Tabellini (2000) for discussions of probabilistic voting.

[15] We assume that only the government currently in power values external debt repayment in the period. Alternatively, one could assume that $\varphi_t(\cdot)$ also enters the objective function of the government in period $t-1$ or even the objective functions of all governments prior to period t. Our approach is the simplest. Since $\varphi_t(\cdot)$ is a reduced-form specification, choosing the simplest specification appears adequate.

where $\tilde{d}_{t+1}$ and $\tilde{e}_{t+1}$ denote the supply of debt for the domestic and external market, respectively. Finally, households and external lenders form expectations about future policy choices and a competitive equilibrium results.

The exogenous state $\hat{z}_t$ lies in a finite state space $\hat{Z}$ and follows a Markov process. The complete state in period t is given by

$$z_t \equiv (\hat{z}_t, k_t, \bar{r}_t^d, \bar{r}_t^e, d_t, e_t).$$

We restrict equilibrium policy functions to be functions of the contemporaneous state.[16]

3 Equilibrium

We start by defining the equilibrium in the private sector before turning to politico-economic equilibrium. We denote the state contingent realizations of a generic random variable x_{t+1} and their associated probabilities conditional on information in period t by $\{x_{t+1}\}_{\mathbb{E}_t}$.

3.1 Private Sector Equilibrium

Young households choose consumption and their portfolio holdings in order to maximize (1) subject to their budget constraints, (2) and (3); prices and returns on capital, $(w_t, q_t^d, \{R_{t+1}\}_{\mathbb{E}_t})$; as well as policy, $(\tau_t, \{r_{t+1}^d\}_{\mathbb{E}_t})$. Optimal choices satisfy the first-order conditions[17]

$$u'(c_t^y) = \delta \mathbb{E}_t[u'(c_{t+1}^o) R_{t+1}(1-\tau_{t+1})], \tag{9}$$

$$u'(c_t^y) = \delta \mathbb{E}_t\left[u'(c_{t+1}^o) \frac{r_{t+1}^d}{q_t^d}(1-\tau_{t+1})\right]. \tag{10}$$

The consumption of old households is determined by the budget constraint, (3); their asset holdings, (k_t, d_t); the return on capital, R_t; and policy, (τ_t, r_t^d). Factor prices are determined according to (4) and (5).

Conditional on $\hat{z}_t$ and $\{\pi_{t+1}\}_{\mathbb{E}_t}$, external lenders price the debt to guarantee the required rate of return,

$$q_t^e = \mathbb{E}_t[m_t r_{t+1}^e]. \tag{11}$$

Definition 1. A *private sector equilibrium* in period t conditional on the state, z_t; expected subsequent exogenous states, $\{\hat{z}_{t+1}\}_{\mathbb{E}_t}$; as well as current and expected policy, $(\pi_t, \{\pi_{t+1}\}_{\mathbb{E}_t})$, is a set of

i. factor prices, $(w_t, R_t, \{R_{t+1}\})$;

ii. debt prices, (q_t^d, q_t^e);

[16] For an exposition of Markov perfect equilibrium, see Krusell, Quadrini and Ríos-Rull (1997).

[17] We will adopt assumptions under which private sector equilibrium choices are interior.

iii. consumption levels, $(c_t^y, c_t^o, \{c_{t+1}^o\})$; and

iv. domestic and external portfolio choices, (k_{t+1}, d_{t+1}) and e_{t+1},

such that the following conditions are satisfied:

i. (2), (3), (4), (5), (9), (10) in period t; and (3), (4) in period $t+1$;

ii. the markets for domestic and external debt clear at prices (q_t^d, q_t^e), $d_{t+1} = \tilde{d}_{t+1}$ and $e_{t+1} = \tilde{e}_{t+1}$; and

iii. $\{R_{t+1}\}_{\mathbb{E}_t} = \{R_{t+1}\}$, and condition (11) is satisfied (rational expectations).

For future reference, note that combining the optimality conditions of a young household yields

$$u'(c_t^y)(k_{t+1} + q_t^d d_{t+1}) = \delta \mathbb{E}_t[u'(c_{t+1}^o)(k_{t+1}R_{t+1} + d_{t+1}r_{t+1}^d)(1 - \tau_{t+1})]. \tag{12}$$

3.2 Politico-Economic Equilibrium

Let $\pi(\cdot) \equiv (g(\cdot), r^d(\cdot), r^e(\cdot), \tau(\cdot), \tilde{d}'(\cdot), \tilde{e}'(\cdot), \bar{r}^{d\prime}(\cdot), \bar{r}^{e\prime}(\cdot))$ denote policy functions (of the state) that are expected to govern other governments' policy choices.

The government in period t chooses π_t in order to maximize (8) subject to four constraints. First, that $\pi_{t+1} = \pi(z_{t+1})$ for every possible realization of z_{t+1}. Second, that the government budget constraint, (6), is satisfied. Third, that return floors (if they have been imposed) and non-negativity constraints are respected: $r_t^d \geq \bar{r}_t^d \geq 0$ and $r_t^e \geq \bar{r}_t^e \geq 0$. And fourth, that current and future policy, $(\pi_t, \{\pi_{t+1}\}_{\mathbb{E}_t})$, together with z_t, $\{\hat{z}_{t+1}\}_{\mathbb{E}_t}$ support a private sector equilibrium in period t which in turn may affect z_{t+1}.

Definition 2. A *politico-economic equilibrium* in period t conditional on the state, z_t; expected subsequent exogenous states, $\{\hat{z}_{t+1}\}_{\mathbb{E}_t}$; and policy functions, $\pi(\cdot)$, is a set of

i. policy choices, π_t; and

ii. a private sector equilibrium in period t conditional on z_t, $\{\hat{z}_{t+1}\}_{\mathbb{E}_t}$, and $(\pi_t, \{\pi_{t+1}\}_{\mathbb{E}_t})$,

such that the following conditions are satisfied:

i. the policy choice, π_t, maximizes the objective function (8);

ii. $r_t^d \geq \bar{r}_t^d \geq 0$ and $r_t^e \geq \bar{r}_t^e \geq 0$;

iii. the government budget constraint, (6), is satisfied; and

iv. $\{\pi_{t+1}\}_{\mathbb{E}_t} = \{\pi(z_{t+1})\}_{\mathbb{E}_t}$ is consistent with the law of motion of the state as determined by the private sector equilibrium and the law of motion for the exogenous state (rational expectations).

In a rational expectations equilibrium, the anticipated policy functions are consistent with the actual policy decisions that a government in power takes. Moreover, the sequence of period-t politico-economic equilibria is consistent with the law of motion for the state.

Definition 3. A *politico-economic equilibrium* conditional on the initial state, z_0; and processes for the exogenous state, $\{\{\hat{z}_{t+1}\}_{\mathbb{E}_t}\}_{t\geq 0}$, is a set of

i. policy functions, $\pi(\cdot)$;

ii. a sequence of the state, $\{z_t\}_{t>0}$; and

iii. politico-economic equilibria in periods $t = 0, 1, 2, \ldots$ conditional on z_t, $\{\hat{z}_{t+1}\}_{\mathbb{E}_t}$ and $\pi(\cdot)$,

such that the following conditions are satisfied:

i. in each period t and each state the equilibrium choice of policy, π_t, corresponds with the policy functions, $\pi(\cdot)$, evaluated at the state (rational expectations); and

ii. $\{z_t\}_{t>0}$ is consistent with the law of motion of the state, as determined by policy, the private sector equilibrium and the law of motion for the exogenous state.

We now characterize equilibrium.

4 Characterization of Equilibrium

4.1 Government Program

Since the objective functions and constraints of all agents including the government only involve the products of debt quantities and their returns or prices, debt quantities are not determined in equilibrium: the effect of a change of debt quantity on the equilibrium allocation can be undone by a counteracting change of prices, returns and return commitments.[18] Without loss of generality, the policy functions $\tilde{d}'(\cdot)$ and $\tilde{e}'(\cdot)$ as well as the equilibrium debt levels d_{t+1} and e_{t+1} therefore can be normalized to unity in all periods and histories.

Proposition 1. The policy functions $\tilde{d}'(\cdot)$ and $\tilde{e}'(\cdot)$ satisfy $\tilde{d}'(\cdot) = \tilde{e}'(\cdot) = 1$; and $d_{t+1} = e_{t+1} = 1$ in all periods and histories. Accordingly, we have

$$\pi_t = (g_t, r_t^d, r_t^e, \tau_t, \bar{r}_{t+1}^d, \bar{r}_{t+1}^e), \;\; z_t = (\hat{z}_t, k_t, \bar{r}_t^d, \bar{r}_t^e).$$

[18] This hinges on the fact that old households are homogeneous such that the distribution of debt holdings in the population is trivial. See Gonzalez-Eiras and Niepelt (2015) for a more general discussion and analysis.

From definition 2, the (remaining) policy choices in a politico-economic equilibrium in period t solve the following program, conditional on z_t, $\{\hat{z}_{t+1}\}_{\mathbb{E}_t}$ and $\pi(\cdot)$:

$$
\begin{aligned}
\max \quad & \omega_t \left(u(c_t^o) + v_t(g_t)\right) + \nu_t(u(c_t^y) + v_t(g_t) + \delta \mathbb{E}_t[u(c_{t+1}^o) + v_{t+1}(g(z_{t+1}))]) + \varphi_t(r_t^e) \\
\text{s.t.} \quad & g_t + r_t^d + r_t^e = \tau_t(y_t + r_t^d) + \nu_t q_t^d + \nu_t \mathbb{E}_t[m_t r^e(z_{t+1})], \\
& c_t^o = (k_t R_t + r_t^d)(1 - \tau_t), \\
& c_t^y = w_t(1 - \tau_t) - k_{t+1} - q_t^d, \\
& k_{t+1} = w_t(1 - \tau_t)\kappa_t^1(\zeta_t), \\
& q_t^d = w_t(1 - \tau_t)\kappa_t^2(\zeta_t), \\
& c_{t+1}^o = \left(k_{t+1} R_{t+1}(k_{t+1}) + q_t^d \frac{r^d(z_{t+1})}{q_t^d}\right)(1 - \tau(z_{t+1})), \\
& r_t^d \geq \bar{r}_t^d, \quad r_t^e \geq \bar{r}_t^e.
\end{aligned}
$$

The first constraint reflects the government budget constraint and rational expectations by foreign lenders, conditions (6) and (11) respectively. The second constraint is the budget constraint of old households in period t, (3), and the third constraint the budget constraint of young households, (2). (Recall that debt quantities are normalized to unity.)

Constraints four and five constitute fixed point conditions that pin down young households' asset purchases. The conditions encapsulate the budget constraints, (2) and (3); the equilibrium condition for the interest rate, (4); as well as the Euler equations (9)–(10). The κ_ts denote the shares of workers' disposable income invested in capital and domestic debt, respectively. These shares are functions of $\zeta_t \equiv (w_t(1-\tau_t), \{\bar{r}_{t+1}^d, \bar{r}_{t+1}^e\}, \{\hat{z}_{t+1}\}_{\mathbb{E}_t}, \pi(\cdot))$ that is, of all factors that are exogenous to the portfolio choice problem of young households: disposable income as well as future net asset returns as determined by the equilibrium policy functions, the exogenous state and $(\bar{r}_{t+1}^d, \bar{r}_{t+1}^e)$. The shares do not depend on k_{t+1} (although R_{t+1}, r_{t+1}^d and τ_{t+1}—which affect households' portfolio choices—do depend on k_{t+1}), due to the fixed point.

The sixth constraint combines the budget constraint of old households in period $t+1$, (3); the equilibrium condition for the interest rate, (4); and the policy functions for r_{t+1}^d and τ_{t+1}. Note that the right-hand side of the condition is expressed in a way that clearly distinguishes between the quantities of assets and their rates of return. This distinction is important when we later assess the welfare effects of policy induced changes in asset purchases. The *direct* welfare effects of such changes are of second order. This follows from the usual envelope condition argument: a marginal increase of asset purchases reduces consumption in young age and increases consumption in old age but from the Euler equation, the resulting effect on welfare is zero. The *indirect* welfare effects of changed asset returns due to policy induced changes in asset purchases, however, are of first order.

Finally, the last constraint reflects the return floors.

We now turn to an analysis of the government's policy choices. As is evident from the government's program these choices affect the allocation both directly and indirectly, through induced future policy changes. The latter channel is present either if the government chooses a component of z_{t+1} (that is, $\bar{r}_{t+1}^d$ or $\bar{r}_{t+1}^e$) or if it chooses a component of ζ_t which affects $k_{t+1} \in z_{t+1}$ (that is, τ_t, $\bar{r}_{t+1}^d$ or $\bar{r}_{t+1}^e$).

4.2 Public Goods Spending

Let μ_t denote the shadow cost of the government budget constraint in the program of the government in period t. The first-order condition with respect to g_t,

$$(\omega_t + \nu_t)v_t'(g_t) = \mu_t, \tag{13}$$

states that the social benefit (as perceived in the political process) of higher public goods spending and its budgetary cost are equalized. If government resources are scarce (μ_t is high) then public goods spending is low; this harms households.

4.3 Return on Domestic and External Debt

The equilibrium conditions for r_t^d are given by

$$\omega_t u'(c_t^o) - \mu_t \leq 0, \quad r_t^d \geq \bar{r}_t^d \tag{14}$$

as well as the complementary slackness condition. Raising the repayment rate on domestic debt benefits the old, who hold it, but imposes a burden on the government budget. In equilibrium, marginal costs and benefits are equalized unless the inequality constraint binds.

From (13), a binding inequality constraint implies that $\omega_t u'(c_t^o) < (\omega_t + \nu_t)v'(g_t)$—a waste of resources. Ceteris paribus, public goods spending then is lower and the shadow cost of public funds higher than in the absence of a binding return floor.

The equilibrium conditions for r_t^e are given by

$$\varphi_t'(r_t^e) - \mu_t \leq 0, \quad r_t^e \geq \bar{r}_t^e \tag{15}$$

as well as the complementary slackness condition. They reflect the same type of trade-off as the one governing the choice of r_t^d.

Proposition 2. If the inequality constraints $r_t^d \geq \bar{r}_t^d$ or $r_t^e \geq \bar{r}_t^e$ bind, then the shadow cost of public funds exceeds the social marginal utility of consumption of the old or the social marginal utility of external debt service, respectively. In either case, public goods spending is lower and the shadow cost of public funds higher, conditional on τ_t, than if the constraint(s) were relaxed.

Proof. The first part of the proposition is established in the text. For the second part, suppose that the binding constraint $r_t^d \geq \bar{r}_t^d$ is relaxed. From (14) and holding τ_t fixed, the equilibrium values of r_t^d and/or μ_t then must fall. Suppose r_t^d fell but μ_t remained unchanged. From (13) and (15), equilibrium g_t and r_t^e then would remain unchanged as well and the budget constraint (6) would be violated. Suppose instead that r_t^d remained unchanged but μ_t fell. From (13) and (15), equilibrium g_t and r_t^e (weakly) then would increase and the budget constraint would be violated. We conclude that both r_t^d and μ_t must fall; accordingly, g_t must rise. A parallel argument applies with respect to the other inequality constraint. □

4.4 Taxes

The choice of tax rate differs from the previously discussed government decisions in so far as it affects future state variables. The first-order condition with respect to τ_t reads

$$\omega_t u'(c_t^o)(k_t R_t + r_t^d) + \nu_t \left(u'(c_t^y) w_t - \mathcal{B}_t(\tau_t)\right) = \mu_t \left(y_t + r_t^d + \nu_t \mathcal{Q}_t(\tau_t)\right) \tag{16}$$

where $\mathcal{Q}_t(\tau_t) \equiv \mathrm{d}(q_t^d + q_t^e)/\mathrm{d}\tau_t$ and $q_t^e = \mathbb{E}_t[m_t r^e(z_{t+1})]$. The first two terms on the left-hand side (featuring ω_t) represent the welfare losses from reduced consumption of old households, due to lower disposable income, and the third term represents the corresponding welfare loss of young households. Lower disposable income also affects the savings choices of young households and thus, k_{t+1} and q_t^d. As discussed previously, this has second-order direct welfare consequences but first-order indirect consequences, by changing rates of return and policy in the next period.

The indirect (general equilibrium and political) net benefits of higher taxes for young households are represented by the fourth term on the left-hand side, where

$$\begin{aligned}
\mathcal{B}_t(\tau_t) \equiv \delta \mathbb{E}_t \Bigg[& u'(c_{t+1}^o) \left(k_{t+1} \frac{\mathrm{d}R_{t+1}(k_{t+1})}{\mathrm{d}\tau_t} + q_t^d \frac{\mathrm{d}(r^d(z_{t+1})/q_t^d)}{\mathrm{d}\tau_t} \right) (1 - \tau(z_{t+1})) \\
& + u'(c_{t+1}^o) \left(k_{t+1} R_{t+1}(k_{t+1}) + r^d(z_{t+1}) \right) \frac{\mathrm{d}(1 - \tau(z_{t+1}))}{\mathrm{d}\tau_t} \\
& + v'_{t+1}(g(z_{t+1})) \frac{\mathrm{d}g(z_{t+1})}{\mathrm{d}\tau_t} \Bigg].
\end{aligned}$$

As savers, workers take rates of return and future policy choices as given. As voters, in contrast, they are aware of their collective monopoly power vis-a-vis the subsequent cohort and government. They internalize that lower capital accumulation drives up interest rates (benefitting them) but may also lead to lower public goods spending or higher taxes (harming them).[19]

The terms on the right-hand side of first-order condition (16) represent the budgetary implications of a tax hike. There are two channels: one working through higher income taxes, where the tax base equals $y_t + r_t^d$; and the second through changed purchases of domestic and external debt, with $\nu_t \mathcal{Q}_t(\tau_t)$ representing the change in debt sales.

Equilibrium condition (16) conveys two important lessons. The first concerns $\mathcal{B}_t(\tau_t)$ as well as $\mathcal{Q}_t(\tau_t)$ and their effect on the cost of taxation and debt returns. Ceteris paribus, positive general equilibrium net benefits of taxation, $\mathcal{B}_t(\tau_t)$, reduce the shadow cost of public funds and thus, work in the direction of higher returns on domestic or external debt, see conditions (14) and (15). At its heart, the increased political support for debt service due to a positive $\mathcal{B}_t(\tau_t)$ derives from a transfer of resources from future, yet unborn generations to current taxpayers that lowers the cost of taxation. If $\mathcal{B}_t(\tau_t) > 0$ then workers are partly "compensated" for their tax payments by the prospect of general equilibrium or policy related net benefits. The mirror image costs of these benefits are

[19] See the related discussion in Gonzalez-Eiras and Niepelt (2008). A smaller capital stock in the next period may also affect the choice of $\bar{r}_{t+2}^d$ and $\bar{r}_{t+2}^e$ but this does not affect welfare of households voting in period t.

born by the future young whose interests are not accounted for in the political process. Positive general equilibrium effects on the revenue from debt issuance, $\mathcal{Q}_t(\tau_t)$, also work in the direction of lowering the shadow cost of public funds, negative effects in the opposite direction.

The second, related lesson conveyed by condition (16) concerns the equalization of social marginal utilities of consumption across cohorts. Although taxes constitute a non-distorting instrument for the government to transfer resources from young and old households to the government, and although domestic debt repayment constitutes a non-distorting instrument to transfer resources from the government to old households, social marginal utilities are *not* equalized across cohorts, even if the inequality constraint $r_t^d \geq \bar{r}_t^d$ is slack. As conditions (14) and (16) make clear, this is a consequence of the fact that the presence of $\mathcal{B}_t(\tau_t)$ and $\mathcal{Q}_t(\tau_t)$ drives a wedge between the marginal utility of young households and the shadow cost of public funds.[20]

We summarize the preceding discussion in the following proposition:

Proposition 3. General equilibrium net benefits of taxation, $\mathcal{B}_t(\tau_t)$, and positive general equilibrium effects of taxation on debt sales, $\mathcal{Q}_t(\tau_t)$, reduce the shadow cost of public funds and increase the returns on domestic or external debt. Generically, the presence of $\mathcal{B}_t(\tau_t)$ and $\mathcal{Q}_t(\tau_t)$ drives a wedge between the social marginal utility of young households and the shadow cost of public funds and thus, between the social marginal utilities of young and old households.

The shadow cost of public funds in equation (16) represents a cost-benefit ratio—the marginal social cost of taxation relative to the marginal revenue gain for the government. It is useful to rewrite (16) using two group specific cost-benefit ratios, one for the old and one for the "non-old" (the young and the external lenders). The former is given by

$$\text{cb}_t^o \equiv \frac{\omega_t u'(c_t^o)(k_t R_t + r_t^d)}{k_t R_t + r_t^d}$$

and the latter equals

$$\text{cb}_t^y \equiv \frac{\nu_t \left(u'(c_t^y) w_t - \mathcal{B}_t(\tau_t)\right)}{\nu_t w_t + \nu_t \mathcal{Q}_t(\tau_t)}.$$

Using these definitions, the shadow cost of public funds can be represented as a weighted average of the group specific cost-benefit ratios. Specifically, letting ψ_t denote the olds' share of the revenue base, $\psi_t \equiv (k_t R_t + r_t^d)/(y_t + r_t^d + \nu_t \mathcal{Q}_t(\tau_t))$, we have

$$\mu_t = \psi_t \ \text{cb}_t^o + (1 - \psi_t) \ \text{cb}_t^y. \tag{17}$$

If the government had access to group-specific tax instruments then it could individually target each of the two cost-benefit ratios. It would optimally set them equal to each other and to the shadow cost of public funds. An unrestricted choice of domestic debt service in combination with the choice of tax rate allows the government to achieve the

[20] If $\mathcal{B}_t(\tau_t) + \mathcal{Q}_t(\tau_t) u'(c_t^y) = 0$ then the social marginal utilities of old and young households are equalized unless the inequality constraint $r_t^d \geq \bar{r}_t^d$ binds.

same result. To see this, recall from proposition 2 that the shadow cost of public funds equals the social marginal utility of the old if r_t^d is unconstrained. Note, moreover, that the social marginal utility of the old is identical to cb_t^o because all wealth and income of the old is taxed at rate τ_t. As a consequence, the optimality condition (17) reduces to

$$\mu_t = \text{cb}_t^o = \text{cb}_t^y$$

when r_t^d is unconstrained. We summarize the discussion in the following proposition:

Proposition 4. An interior choice of r_t^d lets the government equalize the cost-benefit ratio of taxation across cohorts. If r_t^d is in a corner then $\text{cb}_t^o < \mu_t < \text{cb}_t^y$.

Importantly, proposition 4 holds independently of whether the external debt service is constrained or not. Constraining the external debt service does not affect the government's ability to efficiently raise revenue; constraining the domestic debt service does.

4.5 Partial Commitment to Debt Service

When a return floor $\bar{r}_{t+1}^d$ or $\bar{r}_{t+1}^e$ will bind in period $t+1$ then raising it further changes two state variables in the subsequent period: the return floor itself, and the capital stock because the binding return floor constitutes a parameter in the household's portfolio choice problem (an element of ζ_t). In particular, domestic debt is crowded out in response to a binding, higher return floor $\bar{r}_{t+1}^d$ because this increases the price at which the debt is sold in period t. By affecting two state variables in the subsequent period, the change of a binding return floor triggers general equilibrium effects and may induce policy responses both of which have first-order welfare consequences.[21]

The equilibrium conditions for the domestic return floor $\bar{r}_{t+1}^d$ *in one particular state* are given by

$$\mu_t \nu_t \mathcal{Q}_t(\bar{r}_{t+1}^d) + \nu_t \mathcal{B}_t(\bar{r}_{t+1}^d) \leq 0, \quad \bar{r}_{t+1}^d \geq 0 \tag{18}$$

and the complementary slackness condition. Here, $\mathcal{Q}_t(\bar{r}_{t+1}^d) \equiv \text{d}(q_t^d + q_t^e)/\text{d}\bar{r}_{t+1}^d$ and

$$\begin{aligned}\mathcal{B}_t(\bar{r}_{t+1}^d) \equiv \delta \mathbb{E}_t \Bigg[& u'(c_{t+1}^o) \left(k_{t+1} \frac{\text{d}R_{t+1}(k_{t+1})}{\text{d}\bar{r}_{t+1}^d} + q_t^d \frac{\text{d}(r^d(z_{t+1})/q_t^d)}{\text{d}\bar{r}_{t+1}^d} \right) (1 - \tau(z_{t+1})) \\ & + u'(c_{t+1}^o) \left(k_{t+1} R_{t+1}(k_{t+1}) + r^d(z_{t+1}) \right) \frac{\text{d}(1 - \tau(z_{t+1}))}{\text{d}\bar{r}_{t+1}^d} \\ & + v'_{t+1}(g(z_{t+1})) \frac{\text{d}g(z_{t+1})}{\text{d}\bar{r}_{t+1}^d} \Bigg].\end{aligned}$$

Note that although the first-order condition reflects the consequences of a change of $\bar{r}_{t+1}^d$ in just one state it nevertheless features an expectation operator. The reason is that the effects of that change may also occur in other states, because of the modified capital accumulation. Also, recall from proposition 4 that a binding $\bar{r}_{t+1}^d$ prevents the government

[21] In contrast, the welfare consequences of the induced portfolio change of savers are of second order.

in period $t+1$ from raising revenue in the most cost efficient way. This (and possibly additional effects) is reflected in the derivatives of the policy functions $\tau(z_{t+1})$ and $g(z_{t+1})$.

Similarly, equilibrium $\bar{r}^e_{t+1}$ in one particular state is characterized by

$$\mu_t \nu_t \mathcal{Q}_t(\bar{r}^e_{t+1}) + \nu_t \mathcal{B}_t(\bar{r}^e_{t+1}) \leq 0, \quad \bar{r}^e_{t+1} \geq 0 \tag{19}$$

and the complementary slackness condition, where $\mathcal{Q}_t(\bar{r}^e_{t+1}) \equiv \mathrm{d}(q^d_t + q^e_t)/\mathrm{d}\bar{r}^e_{t+1}$ and

$$\begin{aligned}\mathcal{B}_t(\bar{r}^e_{t+1}) \equiv \delta \mathbb{E}_t \Bigg[& u'(c^o_{t+1}) \left(k_{t+1} \frac{\mathrm{d}R_{t+1}(k_{t+1})}{\mathrm{d}\bar{r}^e_{t+1}} + q^d_t \frac{\mathrm{d}(r^d(z_{t+1})/q^d_t)}{\mathrm{d}\bar{r}^e_{t+1}} \right) (1-\tau(z_{t+1})) \\ & + u'(c^o_{t+1}) \left(k_{t+1} R_{t+1}(k_{t+1}) + r^d(z_{t+1}) \right) \frac{\mathrm{d}(1-\tau(z_{t+1}))}{\mathrm{d}\bar{r}^e_{t+1}} \\ & + v'_{t+1}(g(z_{t+1})) \frac{\mathrm{d}g(z_{t+1})}{\mathrm{d}\bar{r}^e_{t+1}} \Bigg].\end{aligned}$$

Note that compared with equilibrium taxes (characterized in condition (16)), equilibrium return floors do not directly affect household budget sets. Return floors only have indirect, general equilibrium consequences—which rational voters internalize. We paraphrase this in the following proposition:

Proposition 5. Partial commitment is irrelevant for "myopic" voters that is, for voters who do not internalize the indirect general equilibrium and policy implications of the partial commitment choice.

5 Functional Form Assumptions

To provide as transparent as possible a characterization of politico-economic equilibrium we adopt a series of functional form assumptions and derive closed form solutions.

First, we assume that the utility function of households is logarithmic, $u(c_t) = \ln(c_t)$, and the production function Cobb-Douglas, $y_t = a_t k_t^{\alpha_t} \nu_t^{1-\alpha_t}$ with a_t and α_t representing productivity and the capital share in income respectively.[22] Under these assumptions, we can derive closed form solutions for the private sector equilibrium.

Second, we assume that the valuation function of public goods spending is logarithmic as well, $v_t(g_t) \equiv \gamma_t \ln(g_t)$ with $\gamma_t > 0$; this lets us derive closed form solutions for $\mathcal{B}_t(\cdot)$ and $\mathcal{Q}_t(\cdot)$.

Finally, we assume that $\varphi_t(r^e_t) = \xi_t \ln(r^e_t)$ with $\xi_t > 0$; that external lenders have logarithmic utility and a discount factor β_t; and that their consumption is given by some power function of after tax income in the domestic economy such that $m_t = \beta_t y_t(1-\tau_t)/(y_{t+1}(1-\tau_{t+1}))$. The latter assumption is primarily made for tractability; it could reflect the fact that external lenders are significantly exposed to income streams from the domestic economy or other, similar economies. The last set of assumptions guarantees closed form solutions for the equilibrium policy choices.

We will first solve for politico-economic equilibrium conditional on a given sequence of return floors, $\{\bar{r}^d_t, \bar{r}^e_t\}$ which we specify as $\bar{r}^d_t = \bar{\rho}(\hat{z}_t) y_t$ and $\bar{r}^e_t = \bar{\sigma}(\hat{z}_t) y_t (1-\tau_t)$. Later, we will endogenize the equilibrium functional forms of $\bar{\rho}(\cdot)$ and $\bar{\sigma}(\cdot)$.

[22] This implies $R_t = \alpha_t a_t k_t^{\alpha_t - 1} \nu_t^{1-\alpha_t}$ and $w_t = (1-\alpha_t) a_t k_t^{\alpha_t} \nu_t^{-\alpha_t}$.

5.1 Politico-Economic Equilibrium Conditional on $\bar{\rho}(\cdot)$ and $\bar{\sigma}(\cdot)$

We conjecture that the equilibrium return on domestic and external debt, respectively, is given by $r_t^d = \rho(\hat{z}_t)y_t$ and $r_t^e = \sigma(\hat{z}_t)y_t(1-\tau_t)$ where $\rho(\cdot)$ and $\sigma(\cdot)$ are arbitrary functions of the exogenous state. In appendix A, we derive the private sector optimality conditions and the political equilibrium conditions under the specified functional form assumptions and we verify the conjecture.

The political equilibrium conditions (see conditions (A.3)–(A.6) in appendix A) simplify to

$$g_t = (\omega_t + \nu_t)\gamma_t x(\hat{z}_t) y_t (1-\tau_t), \tag{20}$$

$$x(\hat{z}_t) \equiv \frac{1 + \rho(\hat{z}_t) + \nu_t \tilde{\mathcal{Q}}_t(\tau_t)}{\omega_t + \nu_t \left(1 + \delta - \tilde{\mathcal{B}}_t(\tau_t)\right)}, \tag{21}$$

$$\rho(\hat{z}_t) = \max\left[\bar{\rho}(\hat{z}_t), -\alpha_t + \frac{\omega_t \left(1 - \alpha_t + \nu_t \tilde{\mathcal{Q}}_t(\tau_t)\right)}{\nu_t \left(1 + \delta - \tilde{\mathcal{B}}_t(\tau_t)\right)}\right], \tag{22}$$

$$\sigma(\hat{z}_t) = \max[\bar{\sigma}(\hat{z}_t), \xi_t x(\hat{z}_t)]. \tag{23}$$

The terms $\tilde{\mathcal{B}}_t(\tau_t)$ and $\tilde{\mathcal{Q}}_t(\tau_t)$ are transformations of the general-equilibrium net benefits of taxation and of the effect of taxation on debt sales, respectively. Both terms are functions of $\hat{z}_t$ only (see appendix A).

The tax rate is pinned down by the budget constraint. Using (20)–(23) as well as the function $\kappa^2(\hat{z}_t)$ defined in appendix A, the budget constraint reduces to

$$\begin{aligned}(\omega_t + \nu_t)\gamma_t x(\hat{z}_t) y_t(1-\tau_t) + \rho(\hat{z}_t)y_t + \sigma(\hat{z}_t)y_t(1-\tau_t) = \tau_t y_t(1+\rho(\hat{z}_t)) + \\ (1-\alpha_t)y_t(1-\tau_t)\kappa^2(\hat{z}_t) + \nu_t\beta_t y_t(1-\tau_t)\mathbb{E}_t[\sigma(\hat{z}_{t+1})]\end{aligned}$$

or

$$\frac{\tau_t}{1-\tau_t} = (\omega_t + \nu_t)\gamma_t x(\hat{z}_t) + \rho(\hat{z}_t) + \sigma(\hat{z}_t) - (1-\alpha_t)\kappa^2(\hat{z}_t) - \nu_t\beta_t\mathbb{E}_t[\sigma(\hat{z}_{t+1})]. \tag{24}$$

The last two terms on the right-hand side of (24) reflect domestic and external debt sales, respectively. Note that the equilibrium tax rate is a function of $\hat{z}_t$ only; the elasticity of the equilibrium tax function with respect to the capital stock thus equals zero. From (20), the elasticity of the government spending function with respect to the capital stock therefore equals α_t. Note also that for $\omega_t \to \infty$ the tax rate approaches unity.

We have completely characterized politico-economic equilibrium conditional on the functional forms of the return floors, $\bar{\rho}(\cdot)$ and $\bar{\sigma}(\cdot)$:

i. The values $x(\hat{z}_t)$, $\rho(\hat{z}_t)$ and $\sigma(\hat{z}_t)$ for all $\hat{z}_t \in \hat{Z}$ are pinned down by conditions (21)–(23), see appendix A for details.

ii. The tax rates $\tau(\hat{z}_t)$ then solve condition (24) for all $\hat{z}_t \in \hat{Z}$.

iii. The equilibrium allocation conditional on z_0 is pinned down by condition (20) as well as

$$c_t^o = (\alpha_t + \rho(\hat{z}_t))y_t(1 - \tau(\hat{z}_t)), \tag{25}$$
$$c_t^y = w_t(1 - \tau(\hat{z}_t))/(1 + \delta), \tag{26}$$
$$k_{t+1} = w_t(1 - \tau(\hat{z}_t))\kappa^1(\hat{z}_t), \tag{27}$$
$$q_t^d = w_t(1 - \tau(\hat{z}_t))\kappa^2(\hat{z}_t), \tag{28}$$
$$q_t^e = \beta_t y_t(1 - \tau(\hat{z}_t))\mathbb{E}_t[\sigma_{t+1}(\hat{z}_{t+1})] \tag{29}$$

for all t and all $\hat{z}_t \in \hat{Z}$ where $\kappa^1(\hat{z}_t)$ and $\kappa^2(\hat{z}_t)$ is defined in appendix A.

5.2 Discussion

To understand the expression for the interior value of $\rho(\hat{z}_t)$ (the second argument of the max operator in (22)) suppose that both the general equilibrium net benefits of higher taxes and the effect of higher taxes on debt purchases equalled zero ($\tilde{\mathcal{B}}_t(\tau_t) = \tilde{\mathcal{Q}}_t(\tau_t) = 0$). Equilibrium condition (22) then would reduce to $\alpha_t + \rho(\hat{z}_t) = \omega_t(1 - \alpha_t)/(\nu_t(1 + \delta))$, corresponding to the equalization of social marginal utilities across cohorts.[23] An interior equilibrium value for $\rho(\hat{z}_t)$ exceeds the benchmark value supporting inter-generational consumption smoothing when the general equilibrium net benefits of taxation, $\tilde{\mathcal{B}}_t(\tau_t)$, are "sufficiently positive" and the revenue losses on domestic and external debt issuance, $\tilde{\mathcal{Q}}_t(\tau_t)$, "not too negative."

The variable $x(\hat{z}_t)$ in condition (21) represents a benefit-cost ratio of taxes (see (A.6) and (A.7) in appendix A). Its normalized inverse equals the shadow cost of public funds,

$$x(\hat{z}_t)^{-1} = \mu_t y_t(1 - \tau_t). \tag{30}$$

The numerator of $x(\hat{z}_t)$ represents the marginal benefit of higher taxation, the marginal gain in government revenue. Normalized by output, this gain equals one (for taxes on labor and capital income) plus $\rho(\hat{z}_t)$ (for taxes on domestic debt service) plus $\nu_t\tilde{\mathcal{Q}}_t(\tau_t)$ (for revenue losses on new debt issuance). The costs, reflected in the denominator of $x(\hat{z}_t)$, derive from the income losses inflicted on the old and the young, including the general equilibrium net benefits. If ρ_t is interior then

$$\mu_t = \frac{\omega_t/(1 - \tau_t)}{(\alpha_t + \rho(\hat{z}_t))y_t} = \frac{\nu_t\left(1 + \delta - \tilde{\mathcal{B}}_t(\tau_t)\right)/(1 - \tau_t)}{\left(1 - \alpha_t + \nu_t\tilde{\mathcal{Q}}_t(\tau_t)\right)y_t},$$

corresponding to our earlier finding that $\mu_t = \text{cb}_t^o = \text{cb}_t^y$.

Models in the sovereign debt literature often *assume* that the cost of default is income dependent. Broadly consistent with empirical evidence, these models tend to predict higher average repayment rates on external debt when the domestic economy "booms,"

[23] Equilibrium consumption of old households equals $(\alpha_t + \rho(\hat{z}_t))y_t(1 - \tau_t)$ and that of young households $w_t(1 - \tau_t)/(1 + \delta) = (1 - \alpha_t)y_t/\nu_t(1 - \tau_t)/(1 + \delta)$.

and lower repayment rates during domestic "recessions."[24] The model considered here also generates this prediction, $r_t^e \propto y_t$, although it does *not* assume income dependent default costs. Instead, pro-cyclicality of the external debt return derives from the counter-cyclicality of the shadow cost of public funds: when income is low, the cost-benefit ratio of taxation is high and the politically preferred return on external debt low. In addition, the model also predicts pro-cyclical *domestic* debt returns, $r_t^d \propto y_t$, due to the fact that the domestic debt service moves in tandem with the factor incomes of young and old households.

Proposition 6. Under the specified functional form assumptions, the return on external and domestic debt is proportional to domestic income (although the default cost function $\varphi_t(\cdot)$ is independent of income). Income dependence of debt returns reflects the counter-cyclicality of the shadow cost of public funds.

The effect of a marginal tax hike on debt sales consists of two negative components (see appendix A),

$$\nu_t \tilde{\mathcal{Q}}_t(\tau_t) = -(1-\alpha_t)\kappa^2(\hat{z}_t) - \nu_t \beta_t \mathbb{E}_t \left[\sigma(\hat{z}_{t+1})\alpha_{t+1}\right],$$

reflecting the consequences for the prices of domestic and external debt, respectively. Substituting this relationship as well as condition (21) into the equilibrium expression for taxes, (24), implies that $\rho(\hat{z}_t) - (1-\alpha_t)\kappa^2(\hat{z}_t)$ and thus, the tax rate does not directly depend on the domestic debt return. In fact,

$$1-\tau(\hat{z}_t) = \frac{1}{\sigma(\hat{z}_t) - \nu_t\beta_t\mathbb{E}_t[\sigma(\hat{z}_{t+1})(1-\alpha_{t+1})] + x(\hat{z}_t)((\omega_t+\nu_t)(1+\gamma_t) + \nu_t(\delta - \tilde{\mathcal{B}}_t(\tau_t)))} \tag{31}$$

and domestic debt service thus affects the tax wedge at most through $x(\hat{z}_t)$, $\sigma(\hat{z}_t)$ or $\sigma(\hat{z}_{t+1})$. When the ratio $\sigma(\hat{z}_{t+1})/\sigma(\hat{z}_t)$ is exogenous and $\sigma(\hat{z}_t)$ is interior then the tax wedge is inversely proportional to $x(\hat{z}_t)$, since $\sigma(\hat{z}_t) = \xi_t x(\hat{z}_t)$ (see also the discussion below).

Finally, an interior domestic debt service implies $g_t/c_t^o = (\omega_t+\nu_t)\gamma_t/\omega_t$; an interior external debt service implies $g_t/r_t^e = (\omega_t+\nu_t)\gamma_t/\xi_t$; and interior domestic and external debt service implies $r_t^e/c_t^o = \xi_t/\omega_t$.

5.3 Steady-State Analysis

When the cardinality of the exogenous state space equals unity, $\#\hat{Z} = 1$, such that the policy instruments are time invariant then the economy converges to a steady state.

In this subsection, we assume that the steady-state external debt service is interior. From (31) and using the definition of $\tilde{\mathcal{B}}_t(\tau_t)$, the steady-state tax wedge then equals

$$1-\tau(\hat{z}) = \frac{1}{\xi(1-\nu\beta(1-\alpha)) + (1+\gamma)(\omega+\nu(1+\alpha\delta))}\frac{1}{x(\hat{z})} \tag{32}$$

[24] However, see Tomz and Wright (2007) for a critical discussion.

and the product $x(\hat{z})(1-\tau(\hat{z}))$ is independent of the domestic debt service $\rho(\hat{z})$. The steady-state output shares devoted to both external debt service and public goods spending thus are orthogonal to domestic debt returns as well (since these shares are proportional to $x(\hat{z})(1-\tau(\hat{z}))$) and an increase in $\bar{\rho}(\hat{z})$ is associated with a higher steady-state tax rate and a higher benefit-cost ratio $x(\hat{z})$ but an unchanged normalized shadow cost of public funds, μy.

A higher capital share, α; stronger preference for public goods, γ; more patience, δ; a higher weight attached to the old, ω; and higher world interest rates, a lower β; all reduce $x(\hat{z})(1-\tau(\hat{z}))$ and thus, the steady-state output share devoted to external debt service. The effect of population growth is ambiguous. And the effect of ξ on $x(\hat{z})(1-\tau(\hat{z}))$ is negative but its effect on the share of output for external debt service is positive.[25] The sources of these comparative statics results are as follows:

- A higher capital share renders taxation more costly: On the one hand it makes $\mathrm{d}q^e/\mathrm{d}\tau$ more negative, by increasing the interest rate; and on the other it lowers the general equilibrium net benefit of taxation which depends positively on the elasticity with which a lower capital stock translates into higher interest rates, $1-\alpha$, and negatively on the elasticity with which a lower capital stock translates into lower public goods spending, α.

- A stronger preference for public goods spending raises the political demand for this spending component—which competes for resources with the demand for external debt service—and lowers the general equilibrium net benefit of taxation because it increases the negative impact of taxation on future public goods spending.

- More patience implies that the political process emphasizes the future net cost of taxation more strongly.

- A higher weight attached to the old increases the political demand for public goods spending and the political cost of reducing the disposable income of the old.

- A higher world interest rate reduces the government revenue raised from debt issuance.

- Population growth works, on the one hand, in the same direction as the political weight of the old because the young also care about public goods provision and reduced disposable income. On the other hand, population growth has a positive effect on the steady-state output share of external debt service because it increases the resources raised by new debt issuance.

Since public goods spending relative to external debt service equals $(\omega_t+\nu_t)\gamma_t/\xi_t$, a higher valuation of external debt service, ξ, lowers rather than raises the output share of public-goods spending. Similarly, increases in ω, ν and γ raise the output share of public-goods spending while they lower the output share of external debt service.

We summarize these results in the following proposition:

[25] For $\xi \to \infty$ the output share devoted to external debt service approaches $(1-\nu\beta(1-\alpha))^{-1}$.

Proposition 7. Under the specified functional form assumptions and if $\sigma(\hat{z})$ is interior, the steady-state shares of output devoted to external debt service and public-goods spending are invariant to $\rho(\hat{z})$, and so are $x(\hat{z})(1-\tau(\hat{z}))$ and μy. The steady-state tax rate increases in $\rho(\hat{z})$, as does the steady-state share of domestic relative to external debt service.

Proposition 7 implies that *conditional on output*, higher domestic debt service imposes a burden on taxpayers but not on external lenders or recipients of public goods spending. However, output responds to taxes and a binding commitment to domestic debt service affects steady-state public goods spending and external debt service through its impact on capital accumulation and output.

5.4 Dynamic Analysis

TBC

5.5 Equilibrium Partial Commitment

TBC

6 Examples

6.1 Steady-State Analysis

For a set of parametric examples, we set $a = 1$, $\delta = 0.98^{30}$, $\omega = 1.04$, $\gamma = 0.30$, $\xi = 0.39$, $\alpha = 0.3$ and $\beta = 0.99^{30}$.[26]

Effect of Demographics Figure 2 illustrates the effect of ν in steady state. The solid lines indicate outcomes when $\bar{\rho}(\hat{z}) = \bar{\sigma}(\hat{z}) = 0$ (no return floors) while the dashed and dotted lines, respectively, indicate outcomes with a domestic or external return floor ($\bar{\rho}(\hat{z}) = 0.15$ or $\bar{\sigma}(\hat{z}) = 0.15$).

The two left-most panels in the upper row illustrate that debt returns as a share of output increase as the share of older voters in the electorate rises (lower values of ν). Debt return floors therefore only bind when the population growth rate is sufficiently high. The domestic debt return responds much more elastically to changes in demographics than the external return and thus, is much more constrained for high values of ν when the lower bound $\bar{\rho}(\hat{z})$ is imposed.

The second panel in the second row illustrates the result proved in Proposition **??** that a binding floor for $\rho(\hat{z})$ increases the shadow cost of public funds while a binding floor for $\sigma(\hat{z})$ does not. The government spending quota depicted in the right-most panel

[26]The values for α, β and δ are taken from Gonzalez-Eiras and Niepelt (2012). The values for ω, γ and ξ are chosen such that $\rho \approx \sigma(1-\tau)$, $\tau \approx 0.25$ and $g/y \approx 0.2$ when $\nu = 1.2$ (corresponding to the projected population growth rate over thirty years in Europe in the year 2020).

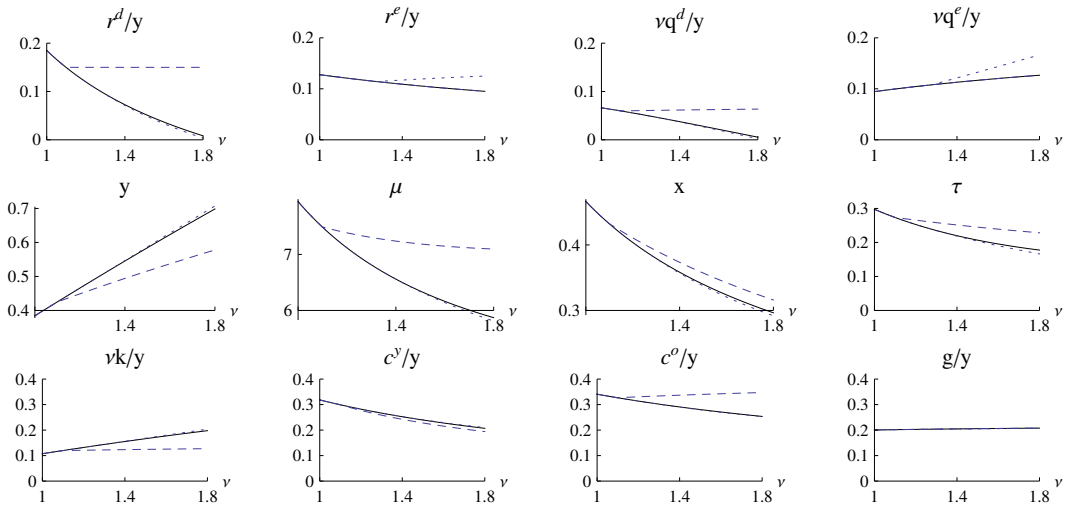

Figure 2: Effect of the population growth rate, ν, in steady state.

in the lower row is unaffected by a binding floor for $\rho(\hat{z})$ but (very slightly) affected by a binding floor for $\sigma(\hat{z})$.

Effect of Factor Share Figure 3 illustrates the effect of α in steady state.

Effect of World Interest Rates Figure 4 illustrates the effect of β in steady state.

6.2 Dynamic Analysis

We start with simple scenarios with two states of nature, $\#\hat{Z} = 2$.

High or Low Capital Income Share Figure 5 illustrates the effect of risky α_t in steady state.

High or Low Population Growth Figure 6 illustrates the effect of risky ν_t in steady state.

High or Low Public Goods Needs Figure 7 illustrates the effect of risky γ_t in steady state.

TBC

7 Conclusions

TBC

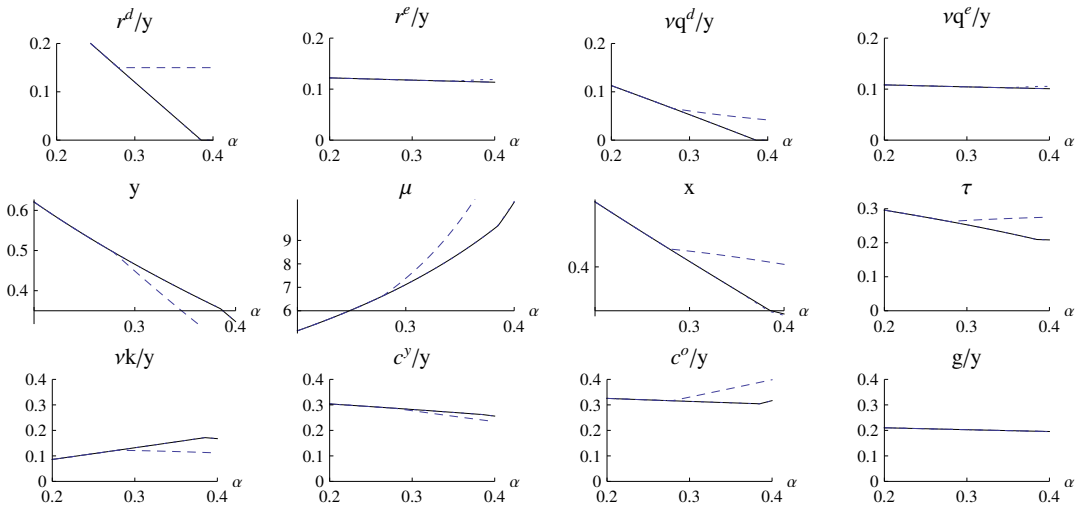

Figure 3: Effect of the capital income share, α, in steady state.

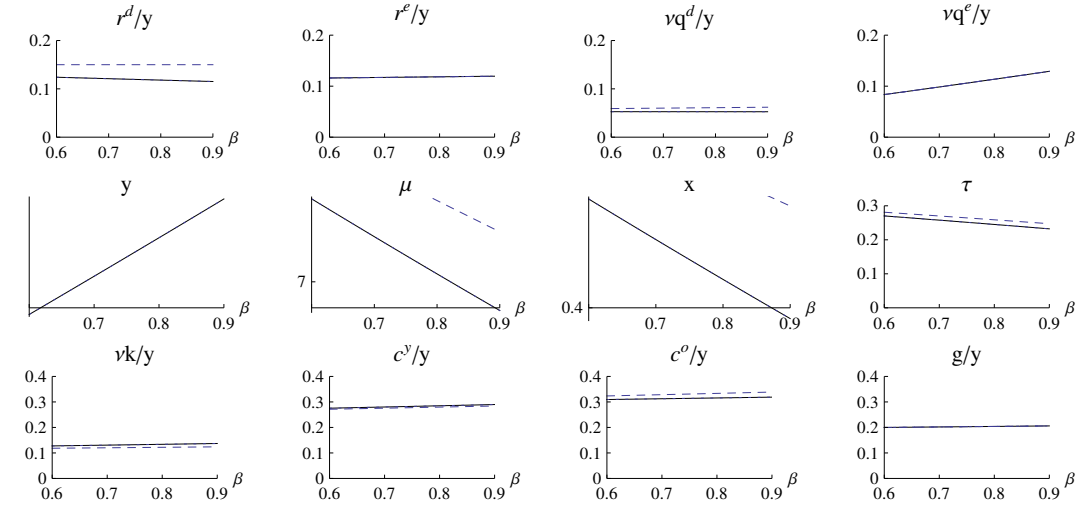

Figure 4: Effect of the inverse world interest rate, β, in steady state.

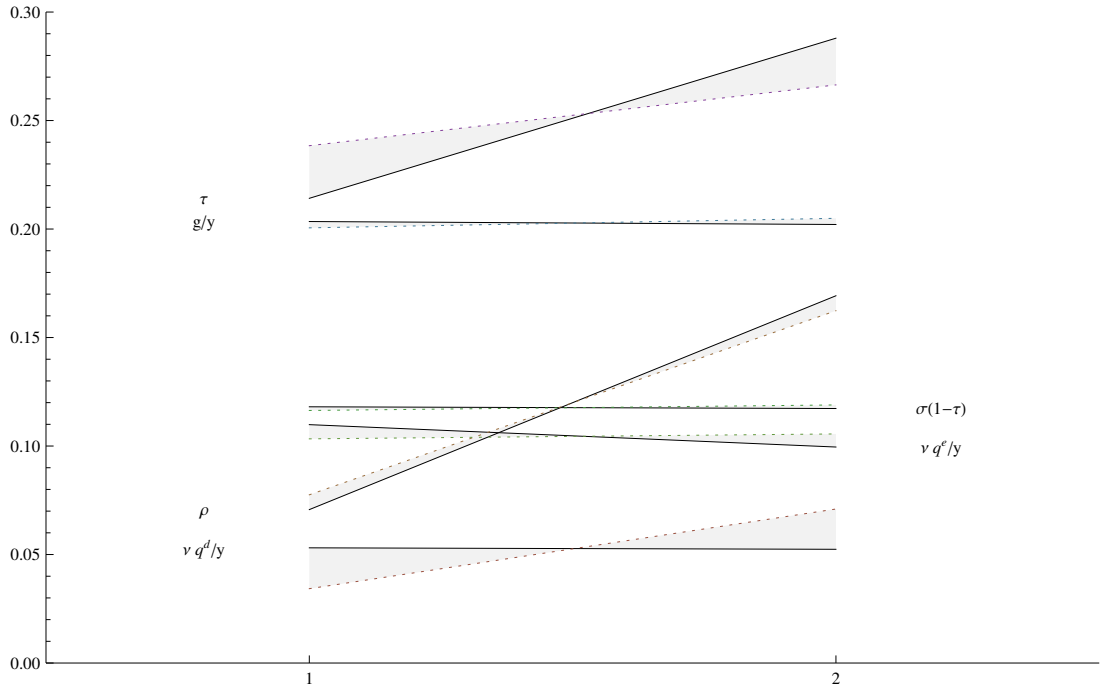

Figure 5: Effect of risky capital income share, α_t.

Note: In addition to the equilibrium outcomes in the two states (connected by solid lines) the figure displays the corresponding steady-state outcomes under the assumption that either state is permanent (connected by dotted lines; the area between pairs of solid and dotted lines is shaded).

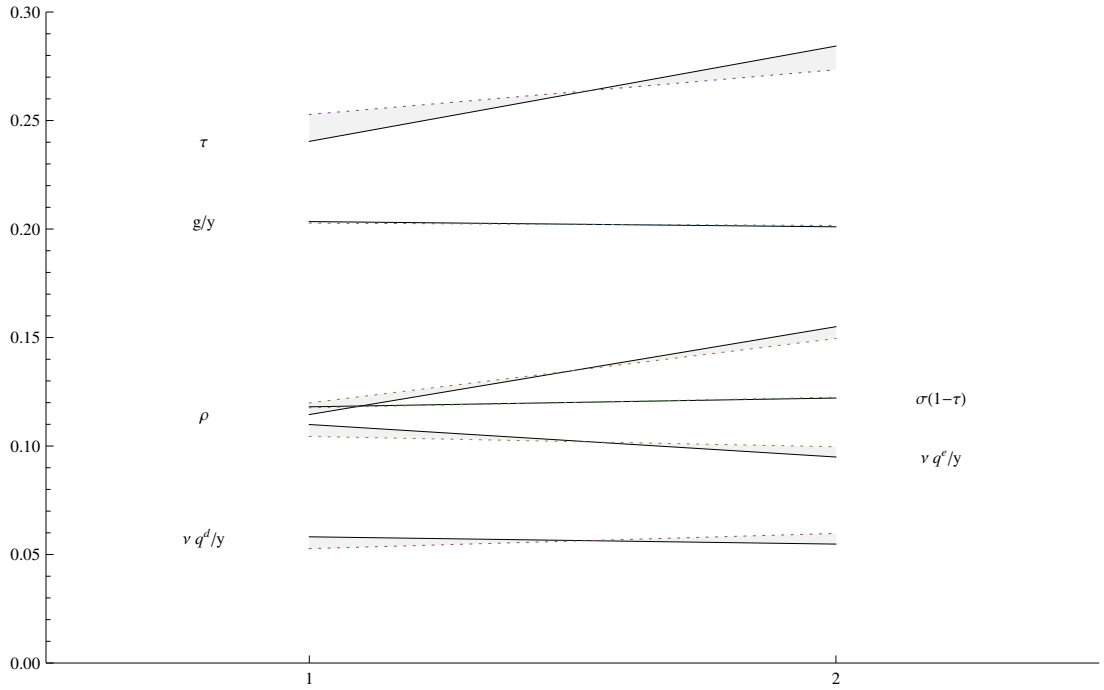

Figure 6: Effect of risky population growth, ν_t.

Note: In addition to the equilibrium outcomes in the two states (connected by solid lines) the figure displays the corresponding steady-state outcomes under the assumption that either state is permanent (connected by dotted lines; the area between pairs of solid and dotted lines is shaded).

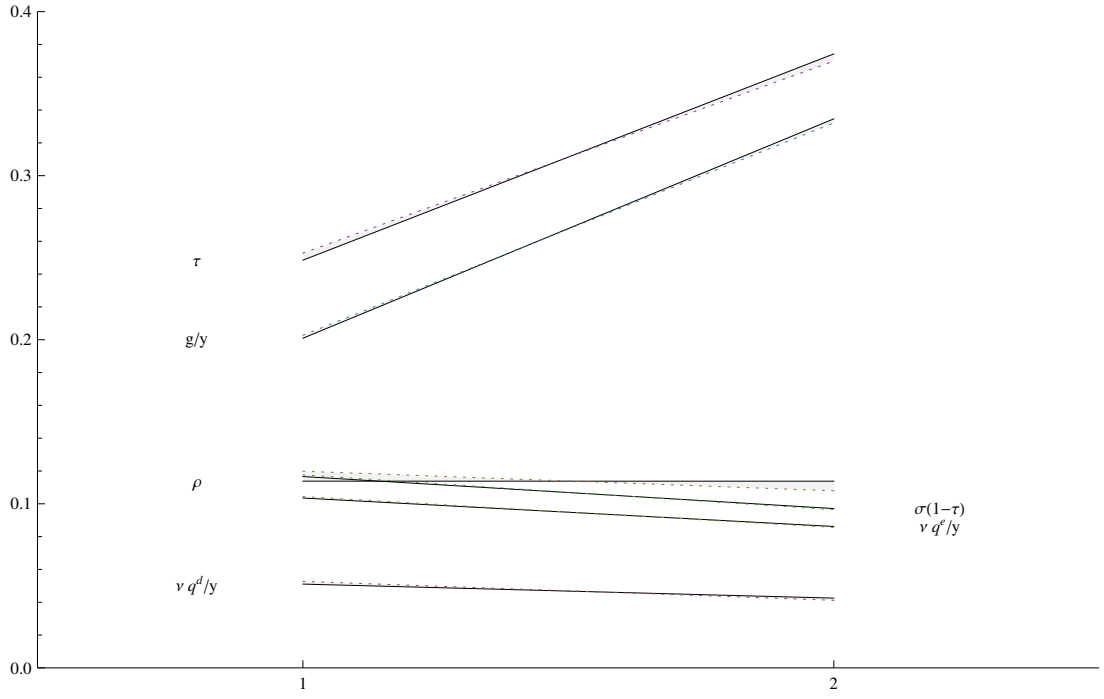

Figure 7: Effect of risky public goods needs, γ_t.

Note: In addition to the equilibrium outcomes in the two states (connected by solid lines) the figure displays the corresponding steady-state outcomes under the assumption that either state is permanent (connected by dotted lines; the area between pairs of solid and dotted lines is shaded).

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