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Intangible Capital and Measured Productivity

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ABSTRACT

Because firms invest heavily in R&D, software, brands, and other intangible assets—at a rate close to that of tangible assets—changes in measured GDP, which does not include all intangible investments, understate the actual changes in total output. If changes in the labor input are more precisely measured, then it is possible to observe little change in measured total factor productivity (TFP) coincidentally with large changes in hours and investment. This mismeasurement leaves business cycle modelers with large and unexplained labor wedges accounting for most of the fluctuations in aggregate data. In this paper, I incorporate intangible investments into a multi-sector general equilibrium model and parameterize income and cost shares using data from the U.S. input and output tables, with intangible investments added to final goods and services. I use maximum likelihood methods and observations on sectoral gross outputs and per capita hours for the period 1948–2014 to estimate processes for latent sectoral TFPs—that have common and idiosyncratic components—and a time-varying labor wedge. I find that sector-specific shocks and industry linkages play an important role in accounting for the fluctuations in these data, especially during the Great Recession. A large and unexplained labor wedge is not needed to account for the fluctuations.

* The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

1. Introduction

This paper sheds light on a measurement issue that confounds analyses of key macrodata during economic booms and busts. Because firms invest heavily in R&D, software, brands, and other intangible assets—at a rate close to that of tangible assets—changes in GDP, which does not include all intangible investments, understate the actual changes in total output. As a result, it is possible to observe large changes in hours and investment coincidentally with little change in *measured* total factor productivity. In other words, innovation by firms—which is fueled in large part by their intangible investments—may be evident "everywhere but in the productivity statistics."[1]

Here, I use a dynamic multi-sector general equilibrium model and U.S. data from the Bureau of Economic Analysis (BEA) to quantify the impact of updating both theory and the national accounts to include intangible investment. Multiple sectors are included to account for the vast heterogeneity in intangible investment rates across industries. I start with the BEA's 2007 benchmark input-output table, that now includes expenditures on software, R&D, mineral exploration, entertainment, literary, and artistic originals as part of investment rather than as part of intermediate inputs. I additionally reallocate several categories of intermediate inputs that are under consideration for future inclusion in the BEA fixed assets including computer design services, architectural and engineering services, management consulting services, advertising, and marketing research. The revised input and output table is then used to parameterize the model's income and cost shares. From the BEA, I have gross output by industry, which are used to estimate stochastic processes of shocks impacting sector-specific and aggregate total factor productivity.[2] Because the model includes intangible investments, I cannot directly measure industry or aggregate TFP series as has been done in earlier work. (See, for example, Horvath (2000).) Instead, I use maximum likelihood methods to estimate stochastic processes for TFPs—that are assumed to have both sector-specific components and a common component. The estimates can then be used to construct predictions of the model's latent productivity shocks.

Because earlier work finds that a large and unexplained labor wedge is needed to account for

[1] Solow (1987) remarked that the computer age could be seen everywhere but in the economic data.

[2] I also worked with IRS business receipts, which are an important source of information for constructing gross outputs and are available back to the 1920s for many major and minor industries.

observed business cycles, I also include a time-varying wedge as a potential source of fluctuations. (See Chari et al. (2007, 2016).) This wedge, which stands in for unmodeled factors impacting households' labor-leisure decisions, is treated as an exogenous process and serves as a check on the overall model specification. The TFP processes are not correlated with the labor wedge but can be correlated across sectors. I consider two specifications. In the first, I do not restrict the variance-covariance matrix for the sectoral TFP shocks, thus allowing for a common component to have some impact. In the second, I assume that sectoral shocks are the sum of two autoregressive processes: one that is sector-specific and one that is common. The latter specification is somewhat more restrictive, but is more informative about the role of input-output linkages.

Using data for the period 1948–2014, I find that large and unexplained variation in the labor wedge is not needed to account for business cycle fluctuations in U.S. gross output by sector or hours per capita. The main sources of the fluctuations for most of the observables are the sectoral TFP shocks, and industry linkages play an important role in generating realistic cycles. Although I cannot directly measure the sectoral TFPs, I can use the model predictions to infer these latent series. When I compute correlations between these series and typical measures of productivity, such as the aggregate Solow residual or GDP per hour, I find them to be weakly positive or negative for most sectors. Part of the reason for this is the fact that the observed gross output series are not highly correlated with the Solow residual or GDP per hour, especially in the latter part of my sample.

With estimates of the model's latent state variables, I can also decompose the observed time series for gross outputs and per capita hours. I do this during two recessions, one in the early 1980s and the Great Recession starting in 2008. I find important differences between these two episodes, especially with regard to the contribution of the common TFP shock and the sectoral TFP shocks. During the 1980s, the common TFP shock played a larger role in generating business cycle fluctuations, and during the Great Recession, sectoral TFP shocks—especially in manufacturing—played a larger role. During the 1980s, the model's common TFP variable is highly correlated with measured TFP, and during the Great Recession it is not. Thus, the analysis provides a possible resolution of the puzzling decline in the correlation between measured productivity and GDP.

(See McGrattan and Prescott (2014).) If there has been a rise in the importance of intangible investments, the mismeasurement of productivity will have worsened.

Previous theoretical work related to this paper has either abstracted from intangible capital or been more limited in scope. Long and Plosser (1983) analyzed a relatively simple multi-sector model, arguing that firm- and industry-level shocks could generate aggregate fluctuations. Horvath (1998, 2000) and Dupor (1998) extended their model and studied the nature of industry linkages to determine if independent productivity shocks could in fact generate much variation for aggregate variables. Parameterizing the model to match the input-output and capital-use tables for the 1977 BEA benchmark, Horvath (2000) concludes that sectoral shocks *may* have significant aggregate effects, but he does not compute the model's variance decomposition. More recently, Foerster, Sarte, and Watson (2011) do a full structural factor analysis of the errors from the same multi-sector model, but only use data for sectors within manufacturing and mining. Neither Horvath (2000) nor Foerster et al. (2011) distinguish tangible and intangible investments. McGrattan and Prescott (2010) do distinguish the different investments, but focus only on aggregate data for a specific episode, namely the technology boom of the 1990s.

Previous empirical work has documented that intangible investments are large and vary with tangible investments over the business cycle. For example, Corrado, Hulten, and Sichel (2005, 2006) estimate that intangible investments made by businesses are about as large as their tangible investments.[3] McGrattan and Prescott (2014) use firm-level data and show that intangible investments are highly correlated with tangible investments like plant and equipment.

The model is described in Section 2. Parameters of the model are described in Section 3. Section 4 summarizes the results. Section 5 concludes.

2. Model

There is a stand-in household that supplies labor to competitive firms and, as owners of the firms, receives the dividends. There is a government with certain spending obligations that are

[3] For more details on measurement of intangible investments in the national accounts, see recent surveys in the BEA's *Survey of Current Business* (U.S. Department of Commerce, 1929–2013). For more details on measurement of R&D investments, see National Science Foundation (1953–2013). For details on entertainment, literary, and artistic originals, see Soloveichik and Wasshausen (2013).

financed by various taxes on households and firms. Firms produce final goods for households and the government and intermediate inputs for other firms. The sources of fluctuations in the economy are stochastic shocks to firm productivities.[4]

There are J sectors in the economy. Firms in sector j maximize the present value of dividends $\{D_{jt}\}$ paid to their shareholders. I assume that firms in each sector j produce both *tangible* goods and services, Y_j, and *intangible* intangible investment goods and services, X_{Ij}. The technologies available are as follows:

$$Y_{jt} = \left(K^1_{Tjt}\right)^{\theta_j} \left(K_{Ijt}\right)^{\phi_j} \left(\prod_l \left(M^1_{ljt}\right)^{\gamma_{lj}}\right) \left(Z^1_{jt} H^1_{jt}\right)^{1-\theta_j-\phi_j-\gamma_j} \tag{2.1}$$

$$X_{Ijt} = \left(K^2_{Tjt}\right)^{\theta_j} \left(K_{Ijt}\right)^{\phi_j} \left(\prod_l \left(M^2_{ljt}\right)^{\gamma_{lj}}\right) \left(Z^2_{jt} H^2_{jt}\right)^{1-\theta_j-\phi_j-\gamma_j} \tag{2.2}$$

and depend on inputs of tangible capital K^1_{Tj}, K^2_{Tj}, intangible capital K_{Ij}, intermediate inputs $\{M^1_{ljt}\}$, $\{M^2_{ljt}\}$, and hours H^1_j, H^2_j. These production technologies are hit by stochastic technology shocks, Z^1_{jt} and Z^2_{jt}, that could have a common component and sector-specific components. The specific choices for the stochastic processes are discussed below.

Firms in sector j maximize the present value of after-tax dividends on behalf of their owners (households) that discount after-tax future earnings at the rate ϱ_t:

$$\max \; E_0 \sum_{t=0}^{\infty} \left(1 - \tau_{dt}\right) \varrho_t D_{jt},$$

where

$$\begin{aligned}
D_{jt} = {} & P_{jt} Y_{jt} + Q_{jt} X_{Ijt} - W_{jt} H_{jt} - \sum_l P_{lt} M_{ljt} - \sum_l P_{lt} X_{Tljt} - \sum_l Q_{lt} X_{Iljt} \\
& - \tau_{kt} P_{jt} K_{Tjt} - \tau_{xt} \sum_l P_{lt} X_{Tljt} \\
& - \tau_{pt} \{ P_{jt} Y_{jt} + Q_{jt} X_{Ijt} - W_{jt} H_{jt} - \left(\delta_T + \tau_{kt}\right) P_{jt} K_{Tjt} \\
& \quad - \sum_l P_{lt} M_{ljt} - \sum_l Q_{lt} X_{Iljt} \}
\end{aligned} \tag{2.3}$$

$$K_{Tjt+1} = \left(1 - \delta_T\right) K_{Tjt} + \prod_l X^{\zeta_{lj}}_{Tljt} \tag{2.4}$$

$$K_{Ijt+1} = \left(1 - \delta_I\right) K_{Ijt} + \prod_l X^{\nu_{lj}}_{Iljt} \tag{2.5}$$

$$M_{ljt} = M^1_{ljt} + M^2_{ljt}. \tag{2.6}$$

[4] Later, I plan to include shocks to government spending and to tax rates.

Dividends are equal to gross output $P_jY_j + Q_jX_{Ij}$ less wage payments to workers W_jH_j, purchased intermediate goods $\sum_l P_lM_{lj}$, new tangible investments $\sum_l P_lX_{Tlj}$, new intangible investments $\sum_l Q_lX_{Ilj}$, and taxes. New investment goods and services are purchased from other sectors and used to update capital stocks as in (2.4) and (2.5). Taxes are levied on property at rate τ_{kt}, investment at rate τ_{xt} (which could be negative if it is an investment tax credit), and accounting profits at rate τ_{pt}.

Households choose consumption C_t and leisure L_t to maximize expected utility:

$$\max\ E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \left[(C_t/N_t)(L_t/N_t)^{\psi} \right]^{1-\alpha} - 1 \right\} / (1-\alpha) N_t \tag{2.7}$$

with the population equal to $N_t = N_0(1+g_n)^t$. The maximization is subject to the following per-period budget constraint:

$$\begin{aligned}(1+\tau_{ct}) \sum_{j=1}^{J} \left[P_{jt}C_{jt} + V_{jt}(S_{jt+1} - S_{jt}) \right] \\ \leq (1-\tau_{ht}) \sum_{j=1}^{J} W_{jt}H_{jt} + (1-\tau_{dt}) \sum_{j=1}^{J} D_{jt}S_{jt} + \Psi_t,\end{aligned} \tag{2.8}$$

where C_{jt} is consumption of goods made by firms in sector j which are purchased at price P_{jt}, H_{jt} is labor supplied to sector j which is paid W_{jt}, and D_{jt} are dividends paid to the owners of firms in sector j who have S_{jt} outstanding shares that sell at price V_{jt}. Taxes are paid on consumption purchases (τ_{ct}), labor earnings (τ_{ht}) and dividends (τ_{dt}). Any revenues in excess of government purchases of goods and services are lump-sum rebated to the household in the amount Ψ_t.

The composite consumption and leisure that enter the utility function are given by

$$C_t = \left[\sum_j \omega_j C_{jt}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{2.9}$$

$$L_t = N_t - \sum_j H_{jt}. \tag{2.10}$$

Notice that here, I assume CES for consumption and linear for hours. As owners of the firm, the household's discount factor is the relevant measure for ϱ_t in (2.3):

$$\varrho_t = \beta^t U_{ct} / \left[P_t (1+\tau_{ct}) \right]. \tag{2.11}$$

The resource constraints for tangible and intangible goods and services are given as follows:

$$Y_{jt} = C_{jt} + \sum_l X_{Tjlt} + \sum_l M_{jlt} + G_{jt} \tag{2.12}$$

$$X_{Ijt} = \sum_l X_{Ijlt}, \tag{2.13}$$

where Y_j and X_{Ij} are defined in (2.1) and (2.2), respectively. The model economy is closed and, therefore, there is no term for net exports.

Two specifications are considered for the sectoral TFP stochastic processes. In the first,

$$\log Z^i_{jt} = \rho_{ij} \log Z^i_{jt-1} + \eta^i_{jt},$$

with mean zero, serially uncorrelated errors that are correlated across sectors, that is, $E\eta^i_{jt} = 0$, $E\eta^i_{jt}\eta^i_{jt-1} = 0$, and $E\eta^i_{jt}\eta^k_{lt} \neq 0$ for all i, j, k, l. I call this the specification of the model *with an unrestricted variance-covariance matrix.* For the second specification, I assume that the logs of the sectoral TFP processes are equal to the sum of a sector-specific component $\tilde{Z}^i_{jt}$ and a common component Z_t with factor loading φ_j, that is,

$$\log Z^i_{jt} = \log \tilde{Z}^i_{jt} + \varphi_j \log Z_t \tag{2.14}$$

$$\log \tilde{Z}^i_{jt} = \rho_{ij} \log \tilde{Z}^i_{jt-1} + \eta^i_{jt} \tag{2.15}$$

$$\log Z_t = \rho \log Z_{t-1} + \upsilon_t, \tag{2.16}$$

where $E\eta^i_{jt} = 0$, $E\eta^i_{jt}\eta^i_{jt-1} = 0$, $E\eta^i_{jt}\eta^k_{lt} = 0$ for all i, j, k, l except cases with $j = l$, $E\upsilon_t = 0$, $E\upsilon_t\upsilon_{t-1} = 0$, and $E\upsilon_t\eta^i_{jt} = 0$. In other words, the shocks to TFP are correlated within a sector but not across sectors and not with the common TFP component.[5] I call this the specification of the model *with a restricted variance-covariance matrix.*

An approximate equilibrium for the model economy can be found by applying a version of Vaughan's (1970) method to the log-linearized first-order conditions of the household and firm maximization problems. The solution can be summarized as an equilibrium law of motion for the logged and detrended state vector x, namely:

$$x_{t+1} = Ax_t + B\varepsilon_{t+1}, \quad E\varepsilon_t\varepsilon_t' = I, \tag{2.17}$$

[5] One exception is the government sector (NAICS 92). In both specifications of B, I assume that shocks to production in NAICS 92 are independent of all other shocks. If I assume otherwise, then the common shock parameter estimates depend importantly on fluctuations of gross output in this sector during the Great Recession, the source of which is unlikely to be a boom in TFP.

where $x_t = [\vec{k}_{Tt}, \vec{k}_{It}, \vec{z}_{1t}, \vec{z}_{2t}, z_t, \tau_{ht}, 1]'$ is a $(4J{+}3){\times}1$ state vector, $\vec{k}_{Tt}$ is the $J{\times}1$ vector of logged and detrended tangible capital stocks, $\vec{k}_{It}$ is the $J{\times}1$ vector of logged and detrended intangible capital stocks, $\vec{z}_{1t}$ is the $J{\times}1$ vector of logged and detrended sectoral TFPs for production of final goods and services, $\vec{z}_{2t}$ is the $J{\times}1$ vector of logged and detrended sectoral TFPs for production of new intangible investments, and z_t is the logged and detrended common shock. I have also included the tax on labor in the vector x_t, which here serves as the labor wedge; the wedge can be interpreted as a specific form of measurement error in the maximum likelihood estimation.[6] The last element of x_t is a 1, which is used for constant terms. The vector ε_t is a $2J + 2$ vector of normally distributed shocks.

In the model with an unrestricted variance-covariance matrix, I estimate the off-diagonal elements of B relating to cross correlations of $\vec{z}_{1t}$ and $\vec{z}_{2t}$ and set $z_t = 0$. In the model with a restricted variance-covariance matrix, I assume that the only nonzero off-diagonal elements of B are correlations between the tangible and intangible production within a sector. I also estimate, in this case, a stochastic process for the common component, z_t.

In the next section, I use BEA data to parameterize this model economy and to estimate the parameters governing the shock processes in (2.17) using maximum likelihood methods.

3. Parameters

Here, I describe how to parameterize income and cost shares using the 2007 benchmark BEA input-output table and how to estimate processes for components of the sectoral TFPs, namely $\{Z^1_{jt}\}$, $\{Z^2_{jt}\}$, and the labor wedge, τ_{ht}, using observations on sectoral gross outputs and total hours. The remaining parameters, which are also described below, are those related to preferences, growth rates, depreciation, and tax rates and are not critical to the main results.

3.1. Income and Cost Shares

The starting point for my analysis are the input-output tables published by the BEA. In Figure 1, I show an example IO table. The upper left $J \times J$ matrix has intermediate purchases.

[6] Any shocks due to changes in fiscal policy are also picked up. Changes in fiscal policy will be modeled explicitly in a later draft.

The rows are commodities (or inputs) and the columns are the industries using them in production. For the analysis below, I set $J = 15$ and the sectors are the following major industries: (1) agriculture, forestry, fishing, and hunting (NAICS 11); (2) mining (NAICS 21); (3) utilities (NAICS 22); (4) construction (NAICS 23); (5) manufacturing (NAICS 31-33); (6) wholesale trade (NAICS 42); (7) retail trade (NAICS 44-45); (8) transportation and warehousing (NAICS 48-49); (9) information (NAICS 51); (10) finance, insurance, real estate, rental and leasing (NAICS 52-53); (11) professional and business services (NAICS 54-56); (12) educational services, health care, and social assistance (NAICS 61-62); (13) arts, entertainment, recreation, accommodation, and food services (NAICS 71-72); (14) other services except government (81); and (15) public administration (NAICS 92). Before computing intermediate shares, I reallocate intermediate expenses in several categories of professional and business services—categories that national accountants are considering reallocating—to the matrix of intangible investments listed under final uses. Specifically, I move expenses for computer design services, architectural and engineering services, management consulting services, advertising, and marketing research out of the intermediate inputs matrix and into final uses.

In terms of the model, the intermediate purchases that show up in element (l, j) of the matrix are given by $P_l(M^1_{lj} + M^2_{lj})$. I use the relative shares of these purchases to parameterize the intermediate shares, $\{\gamma_{lj}\}$, in (2.1) and (2.2). The actual shares used in the analysis are reported in Table 1. The first panel of the table shows the values of the intermediate shares γ_{lj}. The first row and column headers indicate the commodity and industry NAICS category, respectively, which in turn correspond to the 15 major industries listed above. Notice that most elements are nonzero, indicating that there are many sectoral linkages.

The upper right part of the table in Figure 1 is the final uses of the commodities. The labels on these final uses are not exactly the same as the BEA's because some adjustments need to be made in order for the theory and data to be consistent. Starting with consumption, I include the nondurable goods and services categories from BEA's personal consumption expenditures (PCE). Expenditure shares for these goods and services are governed by the choice of $\{\omega_j\}$ in (2.9), which I set to align the theoretical and empirical shares. These are shown in the final row of Table 1.

The durable goods component of PCE is included with investments. Specifically, durable equipment is assumed to be part of tangible investment, and software and books are assumed to part of intangible investment. Since the tangible and intangible investments, like intermediate purchases, are used by different industries, I need to assign consumer durable purchases to specific elements of the $J \times J$ matrices. In the case of consumer durable equipment, I assume it is a manufactured commodity (commodity 5) used by the real estate industry (industry 10). In the case of software and books, I assume these are information commodities (commodity 9) used by the real estate industry (industry 10). Another adjustment that must me made is to include the durable capital services and depreciation with consumption services. This adjustment also affects incomes, which I describe later.

Detailed investment data are used to fill in elements of the BEA capital flow tables (also referred to as the capital-use tables).[7] The detailed data are broken down by investment category and industries making the investment.[8] I construct two capital flow tables: tangible and intangible. I include fixed investment in equipment and structures—both public and private—and changes in inventories with tangible investment, and I include the new BEA category of *intellectual property* (IP) *products*—both public and private—with intangible investment.[9]

The IP products include expenditures on software, mineral exploration, research and development (R&D), and entertainment, literary, and artistic originals. Some of this spending is done by firms in-house (and is what the BEA calls own-account). For this spending I reassign the commodity source to the own industry, which is more in line with the theory. Once I have the capital flow tables, I can set the parameters ζ_{lj} and ν_{lj} using the spending shares for tangible investment and intangible investment, respectively.

The second panel of Table 1 shows the tangible capital flow shares ζ_{lj}. Notice that many rows of this panel have only zeros because the commodities produced are neither structures nor equipment. Commodities categorized under construction (NAICS 23) and manufacturing (NAICS 31-33) are the main sources of these investment goods. The third panel is the analogous panel for intangible investments. Commodities categorized under information (NAICS 51) and professional

[7] The BEA has not yet published an official capital flow table for the 2007 benchmark IO accounts. I constructed one with detailed investment data available for the BEA fixed asset tables and very useful correspondence with David Wasshausen of the BEA.

[8] Some adjustments need to be made to reallocate from owners to users since these tables record final users of the capital goods.

[9] This category of investment was added in the 2013 comprehensive revision of the accounts.

and business services (NAICS 54-56) are most important in this case. In the BEA data, scientific R&D is listed under NAICS 5417 but much of this is specific to other commodities (e.g., chemical manufacturing) and has been assigned accordingly. For this reason, there are nonzero shares on the diagonal of the 15×15 matrix ν that would be zeros in the BEA's table.

The next columns in the final-use table has purchases of government and the rest of world. I list government purchases as 'government consumption' in the table since government investment is included with the private investments. For all of the simulations below, I also add the government consumption in with private spending and thus the theory assumes zeros for this column. The economy is closed and does not have a rest-of-world sector. Thus, I reallocate net exports to the domestic categories of intermediates, consumption, and investment. I do so in a pro rata way.

The panel below intermediate purchases in Figure 1 shows the categories of value added. The first has industry compensation, which is $W_j H_j$ for all j in the model. The second has business taxes that include consumption and excise taxes $\tau_c C_j$ and property taxes $\tau_k K_{Tj}$. The third category is operating surplus which is the sum of all capital income and capital depreciation (including depreciation of consumer durables) less property taxes. Shares of capital income $\{\theta_j, \phi_j\}$ are set so that the total spending on tangible and intangible investment is equal to that in the U.S. data. These shares are shown in the fourth and fifth panels of Table 1. Adding up the income categories is another way to compute GDP (in addition to adding up expenditures or taking industry outputs and subtracting intermediate purchases).

3.2. Shock Processes

Estimates of the parameters governing the shock processes are found by applying maximum likelihood to the following state space system:

$$x_{t+1} = Ax_t + B\varepsilon_{t+1} \tag{3.1}$$

$$y_t = Cx_t, \tag{3.2}$$

where the elements of x_t are defined above (see (2.17)) and assumed to be unobserved, and y_t are quarterly U.S. data for the period 1948-2014. Given my interest in estimating TFP processes for all sectors and a process for the labor wedge, I use detrended gross outputs by sector and total hours, which I stack in the vector y_t. The sectoral gross outputs are the empirical analogue of

$P_{jt}Y_{jt}+Q_{jt}X_{Ijt}$ in equation (2.3).[10] I use gross outputs, rather than data on value added, because there are no issues with the classification of spending as intermediate or final. Definitions of value added have changed over the postwar period. Hours are included in the set of observables because standard business cycle models that abstract from intangible capital are not able to account for large movements in hours of work. See, for example, Kydland and Prescott (1982).

The model time period is quarterly, but time series on gross outputs by industry are only available annually before 2005. Therefore, before estimating parameters for the shock processes, I use a Kalman filter to compute forecasts of quarterly gross outputs. The idea is to use other available quarterly data by industry and construct quarterly forecasts for the series of interest, namely, gross outputs. Specifically, I use quarterly estimates of BEA's national income by industry, N_{jt}, quarterly estimates of BLS's employment by industry, E_{jt}, and *annual* estimates of BEA's gross outputs, $G_{jt}, t = 4, 8, 12, \ldots$ where $G_{jt} = 0$ for t not divisible by four. Both the national income and gross output data are divided by the GDP deflator. Then all three series are detrended by applying the filter in Hodrick and Prescott (1997) (with a smoothing parameter of 1600 for the quarterly series and 100 for the annual series).

Let $\hat{G}_{jt}$ be the quarterly gross outputs being forecasted. The first step in deriving a forecast is to estimate A_j and B_j of the following state space system via maximum likelihood:

$$x_{jt+1} = A_j x_{jt} + B_j \epsilon_{t+1}$$

$$y_{jt} = C_{jt} x_{jt}$$

where $x_{jt} = [X_{jt}, X_{j,t-1}, X_{j,t-2}, X_{j,t-3}]'$, $X_{jt} = [N_{jt}, E_{jt}, \hat{G}_{jt}]'$, and $y_{jt} = [N_{jt}, E_{jt}, G_{jt}]'$, and

$$A_j = \begin{bmatrix} a_{1j} & a_{2j} & a_{3j} & a_{4j} \\ I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \end{bmatrix}, \quad B_j = \begin{bmatrix} b_j \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C_{jt} = \begin{cases} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/4 & 0 & 0 & 1/4 & 0 & 0 & 1/4 & 0 & 0 & 1/4 \end{bmatrix} & \text{if } t \text{ is 4th quarter} \\ \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} & \text{otherwise.} \end{cases}$$

[10] Both data and model series are deflated before shocks are estimated.

Once I have parameter estimates $\hat{A}_j$ and $\hat{B}_j$, I can construct forecasts of gross outputs for all quarters given the full sample of data, namely $\hat{G}_{jt} = E[G_{jt}|y_{j1}, ..., Y_{jT}]$, by first applying the Kalman filter and then applying the Kalman smoother. (See Harvey (1989) for details.)

I have eighteen series of gross outputs. Fifteen of the series are those of the major industries from the IO table described earlier. Additionally, I include data for chemical manufacturing, broadcasting and communications, and advertising, which are 3-digit industries under manufacturing (industry 5), information (industry 9), and professional and business services (industry 11), respectively. Firms in these minor industries make considerable intangible investments and thus the gross outputs are useful for estimating Z^2_{jt} for the sectors $j = 5, 9, 11$. In addition to series for gross outputs, I have total U.S. hours from the Bureau of Labor Statistics at a quarterly frequency for 1948:1–2014:4.[11]

Once I have constructed the quarterly estimates, I again apply the methods in Harvey (1989) to estimate the parameters of the stochastic processes that appear as coefficients in A and B in (3.1). The estimated stochastic processes are reported in a separate appendix.

3.3. Other parameters

The remaining parameters are those related to preferences, growth in population and technology, depreciation, and taxes.

For preferences, I set $\alpha = 1$, $\psi = 1.2$, and $\beta = 0.995$. Growth in population is 0.25 percent per quarter. Growth in technology is 0.5 percent per quarter. Depreciation is assumed to be the same for all sectors and both types of capital and is set at 0.8 percent per quarter.[12]

Tax rates are based on IRS and national account data and are as follows: $\tau_c = 0.065$, $\tau_d = 0.144$, $\tau_k = 0.003$, $\tau_p = 0.33$ and $\tau_x = 0$. For the results below, these rates are held constant.

[11] The data are available at www.econ.ucsd.edu/∼vramey and frequently updated by Valerie Ramey.

[12] One issue that arises in models with intangible capital is the lack of identification of all parameters. For example, there is insufficient data to estimate both capital shares and depreciation rates, even in the case of R&D assets that are now included in both NIPA and the BEA's fixed asset tables. The BEA uses estimates of intangible depreciation rates to calculate the return to R&D investments and the capital service costs, which are used in capitalizing R&D investments for their fixed asset tables. Unfortunately, as the survey of Li (2012) makes clear, "measuring R&D depreciation rates directly is extremely difficult because both the price and output of R&D capital are generally unobservable." Li discusses different approaches that have been used to estimate industry-specific R&D depreciation rates, finding that there is a wide range of estimates even within narrow categories. She concludes that "the differences in their results cannot be easily reconciled." (See Li, Table 2.)

In the case of the labor tax, I set the mean to 0.382—consistent with IRS data—but I allow the rate τ_{ht} to vary over time. I interpret the variation as fluctuations in the labor wedge since no time series for labor taxes is included in y_t. In some sense, fluctuations in τ_{ht} can be attributed to unexplained variations in y_t.

4. Results

Here, I quantify the contribution of the shocks to TFPs and the labor wedge to fluctuations in my data y_t, and construct estimates of the latent state vector x_t. In particular I am interested in comparing the model predictions with the measure of TFP typically used in macroeconomic analyses and constructing variance decompositions over the whole sample and for two large recessions. I also provide evidence of the model's fit.

4.1. Model Fit

In Figure 2, I plot the actual and predicted series for total gross output (panel A) and hours per capita (panel B), which are two of the 19 series in y_t. The predictions are forecasts based on the sample up to that point, that is $C\hat{x}_t$, with $\hat{x}_t = E[x_t|y_{t-1}, \ldots, y_0, \hat{x}_0]$, and therefore, the difference in the two plotted series is the forecast error that is minimized when maximizing the likelihood function. Although I am not plotting all predictions, I find that the model fits well not only for these two series, but for all nineteen. Here, I show the results for the unrestricted variance-covariance matrix, but the picture is nearly identical if I plot the results for the second specification of the TFP shock processes.

4.2. Correlations of Predicted TFPs and U.S. Data

Next, I report correlations of the model's latent sectoral TFPs and U.S. time series. The latent TFP series are the model predictions for $E[z^i_{jt}|y_0, \ldots, y_T]$ given the data in $\{y_t\}$. The U.S. time series are total gross output, gross output by sector, GDP, GDP per hour, and measured TFP. These series are deflated by the GDP deflator and logged and detrended using the Hodrick and Prescott (1997) filter. The series for measured TFP is the Solow residual namely, log(GDP)

$-.33\log(K) - .67\log(H)$, where K is the total real stock of fixed assets as reported by the BEA and H is total U.S. hours.

In Table 2, I report the correlations for the model with an unrestricted variance-covariance matrix, but the main findings are unchanged if I use the second specification for the TFP processes. The first two columns show correlations between the model's predictions for sectoral TFPs and the U.S. data used when estimating the stochastic processes. Although there are some industries for which the sectoral TFPs are highly correlated with total output, many are only weakly positively correlated. For example, while manufacturing TFP is highly correlated with total gross output, information and professional and business services are not. When I correlate the latent TFPs with output in the own sector, then I find high correlations for almost all industries. Correlations between GDP, which are shown in the next column are, for the most part, even smaller than the correlations with total output and in many cases flip sign when I consider GDP per hour (shown in column 4). Similarly, I find weakly positive or negative correlations between sectoral TFPs and the most commonly used measure of aggregate TFP, the Solow residual. These estimates are shown in the last column of Table 2.

One reason for the relatively weak comovement between measured TFP and the model's sectoral TFPs is that the underlying data used in the estimation, the sectoral gross outputs, are not themselves highly correlated with measured TFP. For example, the correlation between gross output in manufacturing and measured TFP is only 0.4.

Next, I consider the model's predictions for the role of each of the shocks in accounting for the variation of the U.S. data used when estimating the stochastic processes.

4.3. Variance Decompositions

In this section, I report statistics for variance decompositions using the two specifications of the TFP processes. The results of the first case are summarized in Table 3 and the results of the second in Table 4.

In either case, the model prediction for the unconditional variance-covariance matrices of the latent and observed variables are given by

$$V_x = AV_xA' + BB' \tag{4.1}$$

$$V_y = CV_xC' \tag{4.2}$$

where $V_x = Ex_tx_t'$ and $V_y = Ey_ty_t'$ (assuming no additional measurement error for y). With an unrestricted variance covariance structure (that is, with no restrictions on off-diagonal elements of BB') for the TFP shocks, I estimate the variance of y due to the labor wedge shock by replacing BB' in (4.1) with $B\Phi B'$, where Φ is a square matrix of zeros that has a 1 in the diagonal element corresponding to the labor wedge shock.

In the first two columns, I report the estimated decomposition for all TFP shocks and the labor wedge. What is striking is that the labor wedge plays no role for sectoral or aggregate gross output, with the variance nearly 100 percent in all cases. Furthermore, it plays less of a role for hours than is typically found in the business cycle literature. (See Chari et al. (2007, 2016).) Here, close to two-thirds of the variation in per capita hours is due to the productivity shocks with the remainder due to the labor wedge.

Since the off-diagonal elements of BB' are nonzero, there is no way to further decompose the variance in (4.2) without further assumptions. However, I can apply the standard factor analysis with the model's predicted errors, namely,

$$\hat{\epsilon}_t = E\left[x_t|y_0,\ldots,y_T\right] - AE\left[x_{t-1}|y_0,\ldots,y_T\right], \tag{4.3}$$

taken to be my data, as in Roll and Ross (1980) and Foerster et al. (2011). Here, I assume only one factor and estimate factor loadings Λ and variances $\Omega = Ev_tv_t'$ for the following:

$$\hat{\epsilon}_t = \Lambda f_t + v_t$$

where Λ is 17×1 vector, f_t is a latent common factor and Ω is a 17×17 covariance matrix with (i,j) element equal to $Ev_{it}v_{jt}$, where the v_{it} are normally distributed errors that are not correlated with f_t. I assume that the only nonzero off-diagonal elements in Ω are the cross correlation of the shocks to tangible and intangible production within a sector. In order to uniquely identify the factor loadings, I set $Ef_t^2 = 1$ and then apply the Kalman filter as above to estimate the parameters.

In the last two columns of Table 3, I report the estimated factor loadings, which are the elements of Λ, and the contribution of the common factor, which is given by the diagonal elements of $\Lambda\Lambda'$ divided by the diagonal elements of $\Lambda\Lambda' + \Omega$. In all sectors, I find a large fraction of the error variance is due to the idiosyncratic shocks in v_t. For example, in manufacturing, only 16 percent of the error variance is due to the common factor f, with the rest due to variation in idiosyncratic shocks. The common factor has the largest impact on the trade sectors and professional and business services.

In Table 4, I report variance decomposition results for the second specification of the TFP processes given by equations (2.14)–(2.16). Here, given I have specified the shocks to be uncorrelated across sectors, I can provide more detail on the contribution of the different shocks. I am also allowing for serial correlation in the common TFP variable, which implies a more flexible specification and possibly a greater role for the common shock.[13] The estimates for sector-specific shocks have been categorized as due to "own industry" or due to "other industry." More specifically, shocks to $z_{1t}(s)$ or $z_{2t}(s)$ are counted as own-industry shocks in sector s and other-industry shocks for all sectors other than sector s.

There are two noteworthy results in Table 4. First, the impact of the labor wedge is still negligible for gross outputs and slightly higher for hours than in the case of an unrestricted variance-covariance matrix in Table 3.[14] In this case, the labor wedge contributes less than 1 percent for all gross output series and 44 percent for per capita hours. In standard business cycle accounting exercises for the United States, a significant fraction of the variance in output and almost all of the variance in hours is attributable to the unexplained labor wedge. (See Chari et al. (2007, 2016).) Second, the results again indicate that the sector-specific shocks are important, but now we can see that the sectoral linkages play an important role, which is evident from the "other industry" contributions. In the case of total gross output, sector-specific shocks contribute 71 percent to the overall variance, and for most industries the estimate is over 50 percent. In the case of sectoral linkages, only the mining and advertising industries are driven primarily by own-sector shocks.

The large contribution of the sectoral shocks is perhaps not surprising given the earlier results in Table 2. The sectoral TFPs are highly correlated with own-industry gross output, but only weakly correlated, either positively or negatively, with GDP per hour and the Solow residual. These low correlations could be partly due to changes that have occurred over the sample. Consider, for

[13] This extension implies that the two TFP specifications are not nested.

[14] Some of the variation in the labor wedge is due to observed fiscal shocks that will be included in later drafts.

example, the series shown in Figure 3 for two recessions: the early 1980s and the late 2000s. I plot total gross output, measured TFP, and the model's predicted common TFP for the second specification of the TFP processes that has a restricted variance-covariance matrix. In the 1980s, the cyclical patterns and size of changes in these series are similar. In the 2000s, they are not. Gross output falls dramatically between 2008 and 2009 and remains below trend until 2011 whereas measured TFP barely changes between 2008 and 2009 and rises over the next couple of quarters back to trend. The model's common TFP variable Z shows a one quarter delay in its decline relative to U.S. gross output, but falls while measured TFP rises. Clearly, if the measured TFP is fed into the model with no other shocks, the predicted changes in output would be very different than what we observe.

In sum, I find that a large and unexplained labor wedge is not needed to account for business cycle fluctuations in the observed data. Furthermore, sector-specific shocks and industry linkages play an important role in accounting for the fluctuations in these data.

4.4. Decomposition of Gross Output and Hours in Two Recessions

Finally, I decompose the model's predictions of total gross output and per capita hours for the periods 1980–1986 and 2008–2014, when both series plummeted. Specifically, I compute the contributions of capital dynamics and exogenous shock variations in accounting for these observations.

The computations done here use the fact that the observations are the sum of the model's prediction plus a forecast error:

$$\begin{aligned} y_t &= C\hat{x}_t + u_t \\ &= \sum_k C\left(:,k\right)\hat{x}_t\left(k\right) + u_t \end{aligned} \tag{4.4}$$

where $\hat{x}_t(k) = E[x_t(k)|y_{t-1}, \ldots, y_0, \hat{x}_0]$ is the model's prediction of the kth latent state variable and u_t is the vector of forecast errors. Recall that two of the elements of y_t and the model's prediction $C\hat{x}_t$ are displayed in Figure 2 with the difference between them being the forecast errors in u_t. To measure the contribution of the different states to the overall variation in the data, I compute cumulative sums, namely, $\sum_k C(:,k)\hat{x}_t(k)$, first including only the capital stocks in x_t, then I consecutively add the labor wedge, the common TFP shock, and the sectoral TFP shocks.

In all cases I find small variations in the capital dynamics. Thus, I can effectively decompose the movements in y_t into the movements due to the exogenous shocks and a forecast error.

Figure 4 shows the results for the element of y_t corresponding to gross output, with the forecast error is not noted but is the difference between the U.S. data and the cumulated sum. In both the early 1980s and the late 2000s, I find that the labor wedge contributes little to the declines in output and shocks to TFP have a large impact. This finding is consistent with standard business cycle accounting results for the 1980s, but not the Great Recession. For example, using a prototype one-sector growth model without intangible capital, Chari et al. (2016) find that shocks to the efficiency wedge (TFP) accounts for most of the downturn in output in the 1980s, whereas shocks to the labor wedge accounts for most of the downturn in output in the Great Recession. Here, the main difference between the early 1980s and late 2000s is the relative contributions of the common TFP shock and the sectoral TFP shocks, a distinction that is missing in the one-sector growth model. I find that sectoral shocks are far more important in the Great Recession. These shocks in my model with variable intangible investments would show up as unexplained labor wedges in Chari et al.'s prototype model.

Figure 5 shows the same results, except in this case the data are hours per capita. As with output, the capital dynamics have little impact on the variation of hours. The labor wedge does have a big impact in the early 1980s, but less of an impact in the late 2000s. This could be due in part to the fact that there were significant tax changes in the 1980s that would be picked up here by variation in τ_{ht}. (See McGrattan (1994).) On the other hand, there were few tax changes during the Great Recession so something else is driving the decline in the labor wedge, something that occurs roughly a year and a half after the start of the downturn. As in the case of output, the sectoral TFP shocks are more important in the later period, especially in the first year of the downturn. But, if we combine the common and sectoral shocks, we see that hours in 2008 and most of 2009 are falling in response to changes in total factor productivities. If I further decompose the contribution of sectoral TFP shocks, I find that shocks to manufacturing are the main contributor to both downturns.

5. Conclusion

In the recent comprehensive revision of the national accounts, the BEA has greatly expanded its coverage of intellectual property products. In this paper, I use the U.S. data and a multi-sector general equilibrium model to quantify the impact of including these products (which I refer to as intangible investments) in both the theory and the measures of GDP and TFP. I find that updating both—both the theory and the data—is quantitatively important for analyzing U.S. aggregate fluctuations.

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Table 1. Parameters Based on 2007 U.S. Input Output Table

Intermediate goods and services shares (γ_{lj})

NAICS	11	21	22	23	31-33	42	44-45	48-49	51	52-53	54-56	61-62	71-72	81	92
11	.205	.000	.000	.001	.033	.001	.001	.000	.000	.000	.000	.000	.007	.000	.001
21	.003	.069	.107	.005	.037	.000	.000	.003	.000	.001	.000	.000	.001	.001	.004
22	.015	.011	.014	.003	.013	.006	.014	.008	.003	.018	.005	.011	.018	.008	.009
23	.007	.017	.019	.000	.002	.001	.003	.005	.002	.025	.001	.001	.003	.006	.019
31-33	.178	.073	.071	.243	.264	.030	.033	.154	.050	.011	.042	.076	.118	.079	.094
42	.071	.015	.016	.044	.047	.029	.017	.030	.012	.003	.008	.021	.022	.016	.014
44-45	.001	.000	.001	.058	.002	.001	.004	.005	.000	.002	.001	.001	.007	.008	.000
48-49	.033	.023	.067	.018	.022	.047	.053	.123	.015	.007	.015	.010	.014	.009	.018
51	.001	.002	.006	.003	.004	.012	.013	.007	.141	.016	.023	.016	.011	.017	.026
52-53	.045	.032	.052	.023	.015	.086	.126	.093	.050	.212	.088	.136	.097	.159	.040
54-56	.010	.040	.045	.011	.042	.085	.059	.046	.040	.068	.088	.068	.082	.039	.038
61-62	.001	.000	.000	.000	.000	.000	.002	.000	.000	.000	.000	.012	.002	.003	.005
71-72	.001	.002	.010	.002	.003	.005	.003	.004	.021	.010	.018	.011	.025	.006	.009
81	.004	.003	.005	.005	.008	.017	.011	.024	.016	.013	.014	.013	.015	.015	.015
92	.000	.000	.002	.000	.001	.011	.006	.022	.003	.002	.003	.003	.007	.003	.003

Tangible capital flow shares (ζ_{lj})

	11	21	22	23	31-33	42	44-45	48-49	51	52-53	54-56	61-62	71-72	81	92
11	.084	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	.002	.763	.003	.003	.002	.001	.001	.020	.001	.000	.002	.001	.001	.001	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	.154	.054	.431	.058	.165	.228	.477	.261	.320	.329	.205	.430	.574	.496	.699
31-33	.510	.123	.379	.629	.593	.468	.350	.470	.454	.558	.531	.381	.285	.337	.247
42	.129	.031	.096	.160	.124	.191	.089	.119	.115	.016	.135	.097	.072	.086	.040
44-45	.037	.009	.027	.045	.035	.034	.025	.034	.033	.007	.038	.027	.020	.024	0
48-49	.029	.007	.022	.036	.028	.027	.020	.045	.026	.004	.030	.022	.016	.019	.006
51	.008	.002	.006	.009	.007	.007	.005	.007	.008	.001	.008	.006	.004	.005	0
52-53	0	0	0	0	0	0	0	0	0	.066	0	0	0	0	0
54-56	.049	.012	.036	.060	.047	.045	.033	.045	.043	.020	.051	.036	.027	.032	.008
61-62	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
71-72	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
81	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
92	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Intangible capital flow shares (ν_{lj})

	11	21	22	23	31-33	42	44-45	48-49	51	52-53	54-56	61-62	71-72	81	92
11	.028	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	0	.187	0	0	0	0	0	0	0	0	0	0	0	0	0
22	0	0	.115	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	.028	0	0	0	0	0	0	0	0	0	0	0
31-33	0	0	0	0	.725	0	0	0	0	0	0	0	0	0	0
42	0	0	0	0	0	.219	0	0	0	0	0	0	0	0	.003
44-45	0	0	0	0	0	0	.091	0	0	0	0	0	0	0	0
48-49	0	0	0	0	0	0	0	.089	0	0	0	0	0	0	.000
51	.112	.149	.107	.024	.028	.047	.086	.094	.614	.391	.044	.048	.197	.065	.015
52-53	0	0	0	0	0	0	0	0	0	.122	0	0	0	0	0
54-56	.860	.664	.778	.948	.247	.734	.824	.817	.386	.487	.956	.619	.793	.674	.381
61-62	0	0	0	0	0	0	0	0	0	0	0	.333	0	0	0
71-72	0	0	0	0	0	0	0	0	0	0	0	0	.010	0	0
81	0	0	0	0	0	0	0	0	0	0	0	0	0	.261	0
92	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.602

Table 1. Parameters Based on 2007 U.S. Input Output Table (Cont.)

	Tangible capital shares (θ_j)														
NAICS	11	21	22	23	31-33	42	44-45	48-49	51	52-53	54-56	61-62	71-72	81	92
	.301	.558	.384	.167	.165	.127	.136	.132	.201	.408	.059	.076	.142	.130	.102
	Intangible capital shares (ϕ_j)														
	11	21	22	23	31-33	42	44-45	48-49	51	52-53	54-56	61-62	71-72	81	92
	.006	.011	.038	.082	.193	.149	.072	.039	.236	.040	.178	.033	.061	.056	.083
	Consumption shares (ω_j)														
	11	21	22	23	31-33	42	44-45	48-49	51	52-53	54-56	61-62	71-72	81	92
	.005	.000	.021	.000	.118	.038	.089	.022	.033	.202	.018	.163	.068	.031	.193

Table 2. Correlations Between Latent Sectoral TFPs and U.S. Data, 1948:1–2014:4

Latent TFPs, by Sector	Gross Output		GDP		Measured
	Aggregate	Own sector	Aggregate	Per Hour	TFP
Agriculture (11)	.25	.84	.25	.02	.15
Mining (21)	.15	.86	.00	-.29	-.20
Utilities (22)	.01	.71	-.04	-.02	-.05
Construction (23)	.72	.90	.67	.25	.54
Manufacturing (31-33)	.82	.91	.57	-.15	.22
Chemical Manufacturing	.60	.87	.50	.05	.33
Wholesale Trade (42)	.15	.51	-.07	-.12	-.12
Retail Trade (44-45)	.54	.87	.36	.11	.27
Transportation & Warehousing (48-49)	.50	.78	.43	-.22	.09
Information (51)	.08	.44	-.04	-.05	-.05
Broadcasting & Telecommunications	.77	.85	.69	.03	.39
Finance, Insurance & Real Estate (52-53)	.62	.93	.50	.11	.35
Professional & Business Services (54-56)	.14	.44	.19	.08	.13
Advertising	.15	.27	.05	.11	.11
Education, Health & Social Services (61-62)	-.24	.68	-.27	.44	.16
Leisure and Hospitality (71-72)	.46	.71	.43	-.03	.20
Other Services (81)	.20	.60	.22	.14	.20

Note: The statistics are constructed for the model with an unrestricted variance-covariance matrix for the TFP shocks.

Table 3. Variance Decomposition and Predicted Correlations, 1948:1–2014:4
Model with Unrestricted Variance-Covariance Structure for TFP Shocks

Observable	Variance Decomposition: TFP Shocks	Variance Decomposition: Labor Wedge	Factor Analysis: Factor Loadings	Factor Analysis: Common Factor Contribution
Gross Outputs, by sector:				
Agriculture (11)	99.86	0.14	-0.014	1.37
Mining (21)	100.00	0.00	0.051	4.37
Utilities (22)	99.92	0.08	0.015	0.33
Construction (23)	99.74	0.26	-0.005	1.12
Manufacturing (31-33)	99.93	0.07	0.026	15.92
Chemical Manufacturing	99.90	0.10	-0.004	0.83
Wholesale Trade (42)	99.72	0.28	0.029	61.85
Retail Trade (44-45)	99.44	0.56	0.016	38.57
Transportation & Warehousing (48-49)	99.80	0.20	0.010	10.31
Information (51)	99.73	0.27	0.013	2.73
Broadcasting & Telecommunications	99.51	0.49	-0.003	1.28
Finance, Insurance & Real Estate (52-53)	99.90	0.10	0.013	16.46
Professional & Business Services (54-56)	99.39	0.61	-0.036	43.53
Advertising	99.57	0.43	0.033	52.88
Education, Health & Social Services (61-62)	99.30	0.70	0.002	6.15
Leisure and Hospitality (71-72)	99.67	0.33	-0.000	0.01
Other Services (81)	99.37	0.63	0.004	1.78
Total Gross Output	99.79	0.21	–	–
Total Hours	65.01	34.99	–	–

Table 4. Variance Decomposition with Detail for TFP Shocks, 1948:1–2014:4
Model with Restricted Variance-Covariance Structure for TFP Shocks

	TFP Shocks				
		Sector-specific			
Observable	Total	Own Industry	Other Industry	Common Shock	Labor Wedge
Gross Outputs, by sector:					
Agriculture (11)	52.74	3.44	49.30	47.12	0.14
Mining (21)	97.49	95.69	1.80	2.51	0.00
Utilities (22)	78.33	10.61	67.73	21.55	0.12
Construction (23)	81.72	41.76	39.96	17.88	0.40
Manufacturing (31-33)	86.06	69.63	16.43	13.83	0.11
Chemical Manufacturing	80.94	59.32	21.63	18.91	0.15
Wholesale Trade (42)	84.93	62.04	22.89	14.65	0.42
Retail Trade (44-45)	49.63	3.90	45.73	49.79	0.58
Transportation & Warehousing (48-49)	41.32	1.26	40.06	58.32	0.36
Information (51)	41.11	1.13	39.99	58.67	0.22
Broadcasting & Telecommunications	42.61	0.51	42.10	57.01	0.39
Finance, Insurance & Real Estate (52-53)	70.97	1.24	69.73	28.89	0.14
Professional & Business Services (54-56)	73.64	66.90	6.74	25.75	0.61
Advertising	88.66	86.07	2.58	10.90	0.44
Education, Health & Social Services (61-62)	60.69	2.00	58.69	38.48	0.83
Leisure and Hospitality (71-72)	54.44	0.93	53.51	45.20	0.36
Other Services (81)	61.66	2.69	58.97	37.57	0.77
Total Gross Output	71.36	–	–	28.33	0.30
Total hours	42.95	–	–	13.52	43.53

FIGURE 1
INPUT OUTPUT TABLE

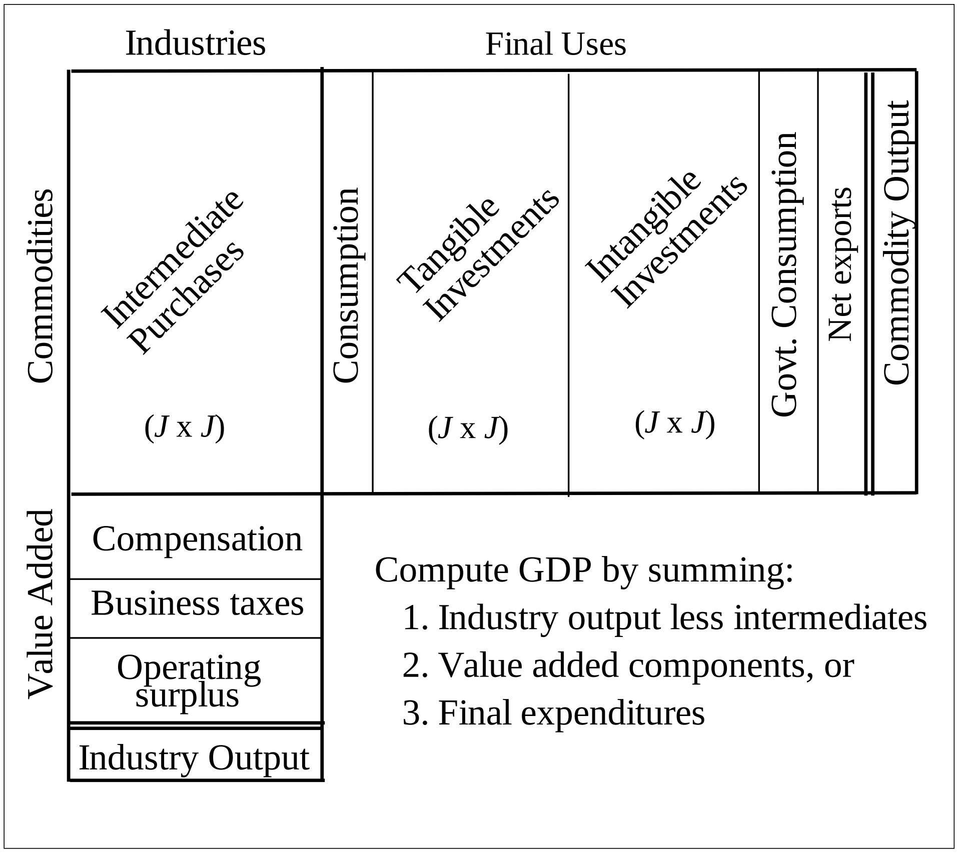
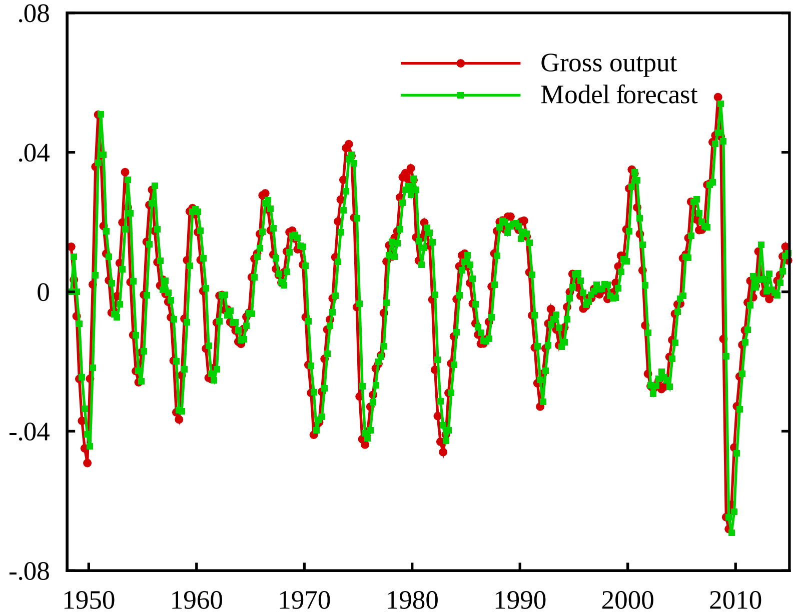
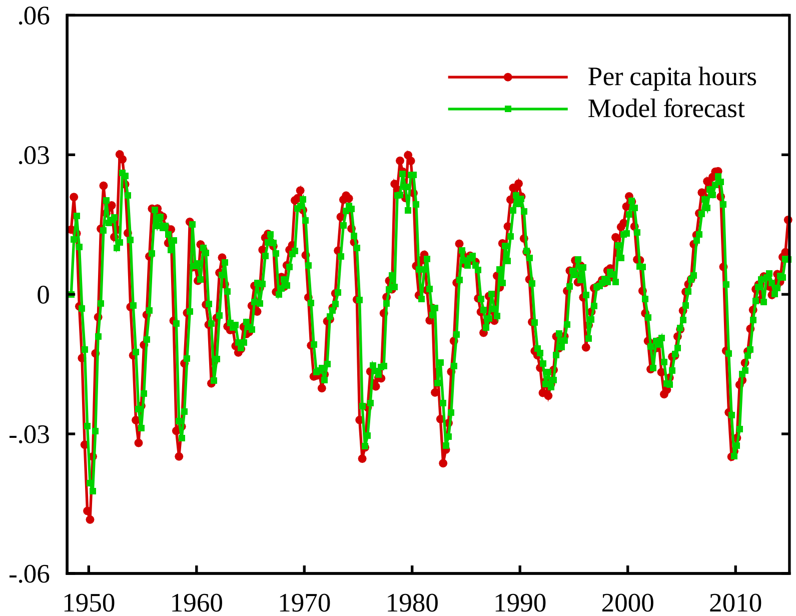


FIGURE 2
U.S. DATA AND MODEL FORECASTS

A. GROSS OUTPUT



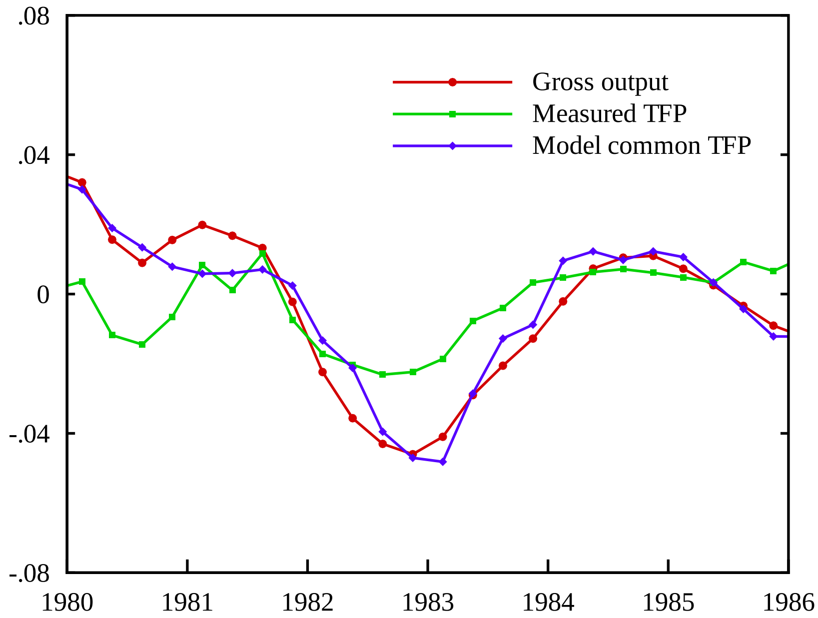
B. HOURS PER CAPITA



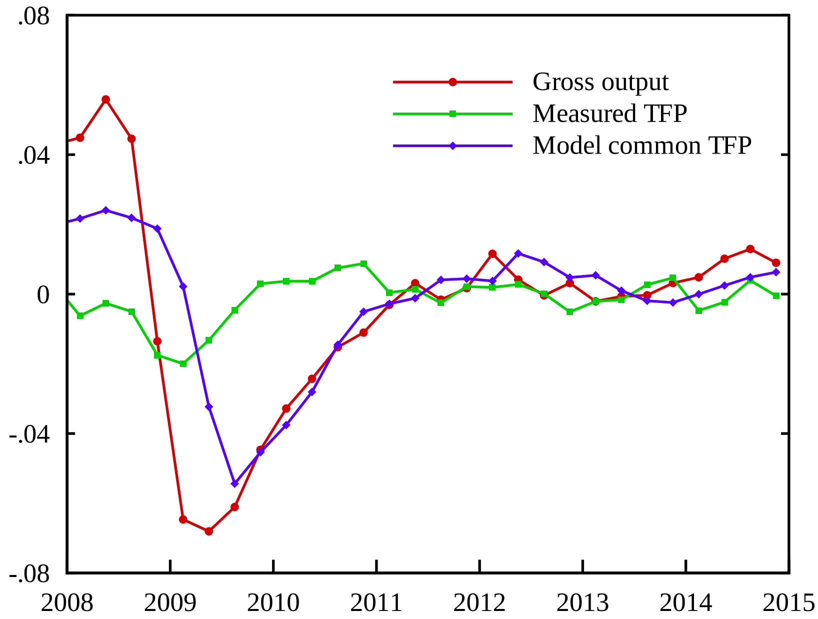
Note: The predicted series is based on the model with an unrestricted variance-covariance matrix for the TFP shocks. The estimates are nearly the same in the model with a restricted variance-covariance structure and are therefore not shown.

Figure 3
U.S. Gross Output and Two Measures of Aggregate TFP

A. 1980:1-1985:4

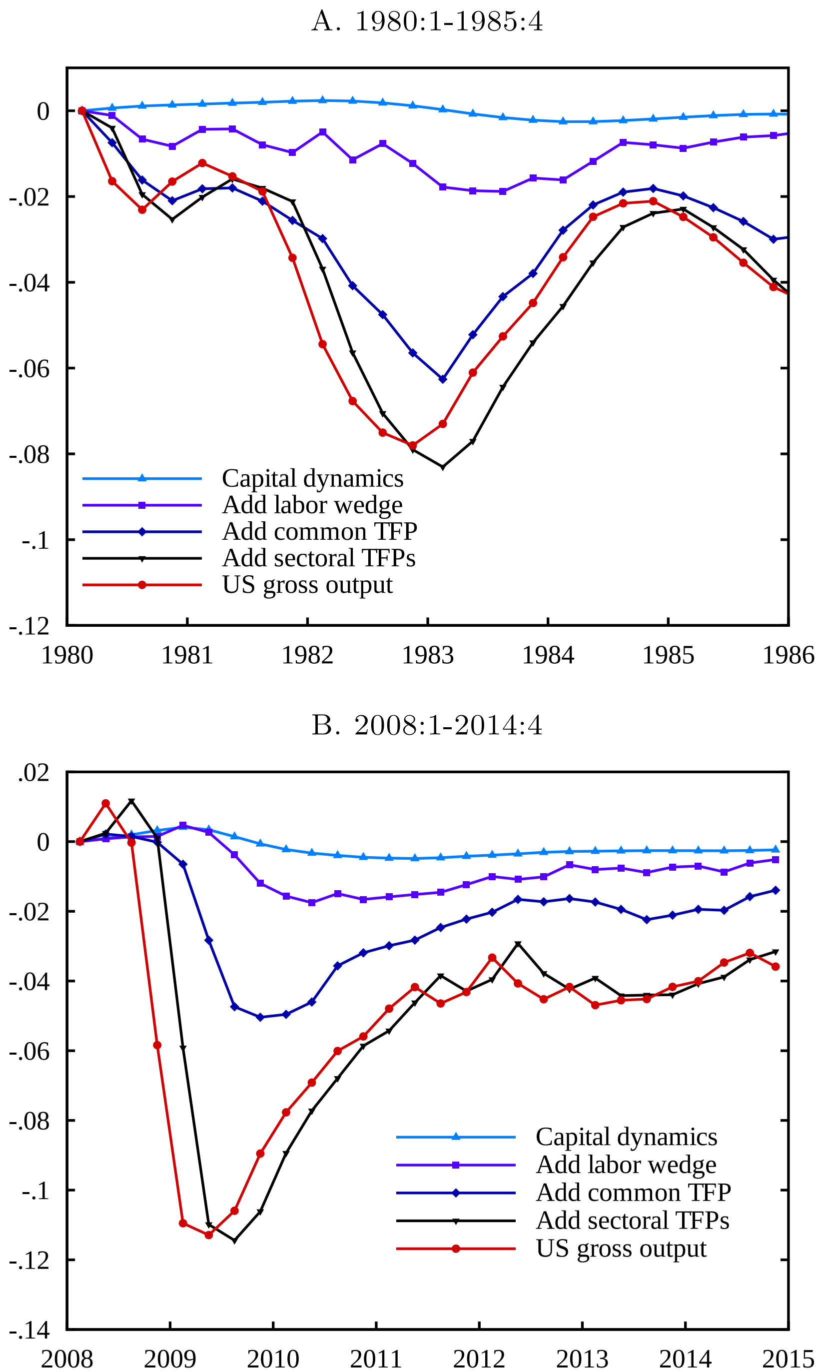


B. 2008:1-2014:4



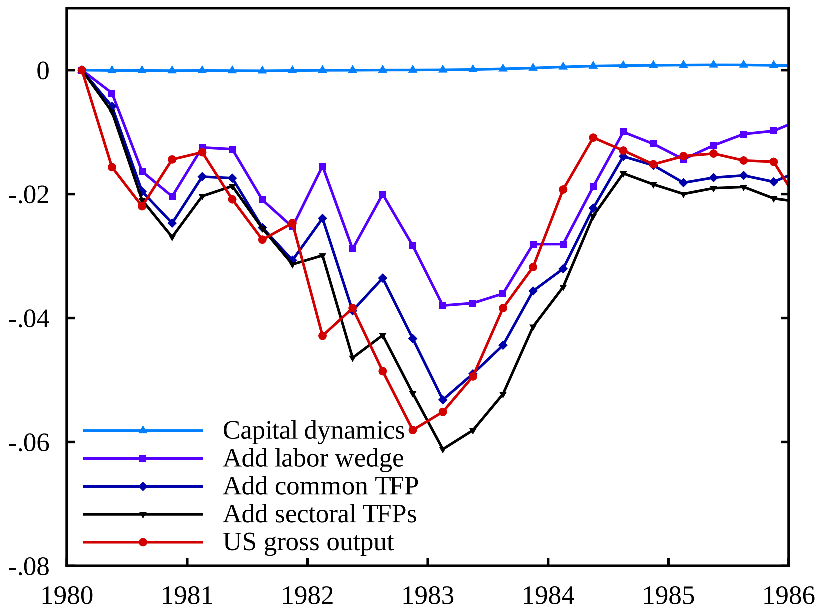
Note: The predicted series is based on the model with a restricted variance-covariance matrix for the TFP shocks.

Figure 4
Decomposition of Total Gross Output

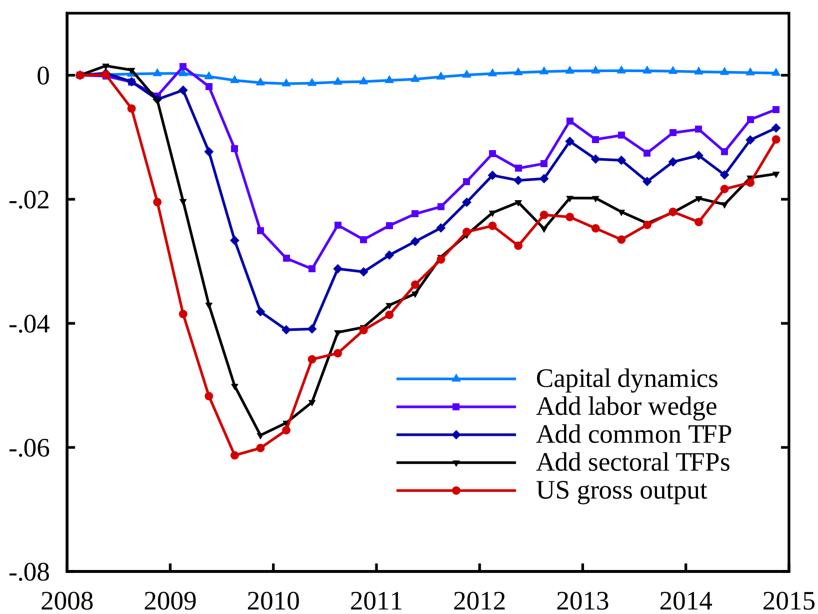


Note: The predicted series is based on the model with a restricted variance-covariance matrix for the TFP shocks.

Figure 5
Decomposition of Hours Per Capita
A. 1980:1-1985:4



B. 2008:1-2014:4



Note: The predicted series is based on the model with a restricted variance-covariance matrix for the TFP shocks.