

Monetary Policy and Sovereign Debt Vulnerability*

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Abstract

We investigate the trade-offs between price stability and sovereign debt sustainability, in a small-open-economy model where the government issues nominal debt without committing not to default or inflate. Inflation allows to absorb the effect of aggregate shocks on the debt ratio, which improves sovereign debt sustainability. But the government incurs an ‘inflationary bias’: it creates (costly) inflation even when default is distant. For plausible calibrations, we find that abandoning the ability to inflate debt away raises welfare, even when the economy is close to default: the benefits from eliminating the inflationary bias dominate the costs from losing inflation’s debt-stabilizing role.

Keywords: monetary and fiscal policy, discretion, sovereign default, continuous time, optimal stopping.

JEL codes: E5, E62,F34

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1 Introduction

One of the main legacies of the 2007-9 financial crisis and the subsequent recession has been the emergence of large fiscal deficits across the industrialized world, resulting in government debt-to-GDP ratios near or above record levels in countries such as the United States, the United Kingdom, or the Euro area periphery (e.g. Greece, Ireland or Portugal). Before the summer of 2012, the peripheral Euro area economies experienced dramatic spikes in their sovereign default premia, whereas other highly indebted countries such as the US and the UK did not. Many observers emphasized that a key difference between both groups of countries was that, whereas the US and the UK controlled the supply of the currency in which they issued their debt and hence had the option to reduce its real value by creating inflation, such an option was not available to the peripheral Euro area economies. On the other hand, the experience of many developing countries, in which large portions of sovereign debt are issued directly in foreign currency, illustrates situations in which governments sometimes renounce the possibility of inflating away their debt, despite the presumable increase in its vulnerability. These developments raise the question as to whether or not an economy may benefit from retaining the ability to stabilize its debt by inflating it away.

In this paper, we try to shed light on the above question by studying the trade-offs between price stability and sovereign debt sustainability when the government cannot commit not to default explicitly on its debt, but also not to reduce its real value through inflation. With this purpose, we build a general equilibrium, continuous-time model of a small open economy in which a benevolent government issues long-term sovereign nominal bonds to foreign investors. At any time, the government may default if it finds it optimal to do so. Default produces some costs due to temporary exclusion from capital markets and a drop in the output endowment. We show that the default decision is characterized by an optimal default threshold for the model's single state variable, the debt-to-GDP ratio. In addition, the government chooses fiscal and monetary policy optimally under discretion, i.e. without commitments on the future path of primary deficit and inflation.

When choosing inflation, the government trades off benefits and costs. On the one hand, inflation reduces the real value of debt; *ceteris paribus*, this improves sovereign debt sustainability by making default a less likely outcome. On the other hand, inflation entails welfare costs due to nominal price rigidities. We show that discretionary optimal monetary policy features an 'inflationary bias': the government chooses positive inflation as long as outstanding debt is positive, even in the absence of aggregate uncertainty. We refer to this baseline scenario as the 'inflationary regime'.

We calibrate our model to capture some salient features of the EMU periphery economies, including their observed inflation record prior to joining the euro. Under our baseline calibration, the

optimal inflation policy function increases roughly linearly with the debt ratio, and then increases steeply as the latter approaches the optimal default threshold. Importantly, the government allows for relatively high inflation rates at debt ratios for which default is still perceived as rather distant by investors; i.e. the inflationary bias mentioned before is quantitatively relevant.

As explained above, our focus is on the welfare consequences of abandoning the possibility of inflating debt away. With this purpose, we compare the baseline inflationary regime with a scenario in which inflation is zero at all times. This 'no inflation' regime can be interpreted as a situation in which the government directly issues foreign currency debt, or in which it joins a monetary union with a very strong and credible anti-inflationary stance. We find that welfare in the no-inflation regime is *higher* at any debt ratio, even close to the default threshold.¹

To understand this key result, it is useful to focus on the costs and benefits of discretionary inflation. On the one hand, the inflationary regime features the inflationary bias discussed before. This inflation bias entails direct welfare costs. Moreover, expectations of future inflation give rise to an inflation premium that raises bond yields *ceteris paribus*, which damages consumption utility by making (external) primary deficits more costly to finance. On the other hand, the fact that optimal inflation depends positively on the debt ratio allows it to partially absorb the effects of output shocks on such debt ratio, which favors the sustainability of sovereign debt by making default (and its associated welfare costs) less likely. The no-inflation regime eliminates the inflationary bias altogether, but also the shock-absorbing role of state-contingent inflation.

Quantitatively, we find that the first effect dominates. That is, renouncing the use of discretionary inflation avoids the welfare costs of the inflationary bias while barely worsening the sustainability of sovereign debt. This is manifested in equilibrium bond yields, which are lower in the no-inflation regime at all debt ratios, reflecting the fact that the elimination of the inflation premium dominates the (small) increase in default premia *vis-à-vis* the inflationary regime. Indeed, default is perceived as rather distant by investors at all debt ratios except for those very close to default. Moreover, even at debt ratios close to default, the beneficial effects of inflation on debt sustainability are largely undone by the increase in nominal yields.

Having characterized equilibrium in both regimes at each point of the state space, we then compute the stationary distribution of the main variables so as to analyze the average performance of both regimes. Since value functions in both regimes are strictly decreasing in debt ratios, the inflationary regime could in principle yield higher average welfare by delivering sufficiently lower debt most of the time. We find that the inflationary regime indeed shifts the distribution of the debt ratio to the left *vis-à-vis* the no-inflation one, reflecting the stabilizing role of unanticipated inflation. However, this shift is too small to overturn the state-by-state dominance of the no infla-

¹We also find that optimal default thresholds are nearly identical in both regimes, such that they both share essentially the same equilibrium no-default region.

tion regime. Therefore, on average the welfare gains from eliminating the inflation bias continue to dominate the costs from renouncing state-contingent inflation. We show that, for our baseline calibration, the average welfare gains from the no-inflation regime are first-order in magnitude.

We show that our findings are robust to a wide range of alternative parameter values. We also find that, if fluctuations in output growth are large enough, then for debt ratios sufficiently close to default the inflationary regime may outperform the no inflation one, for it is then that the beneficial effects of state-contingent inflation as a shock absorber outweigh the costs from the inflationary bias. However, the necessary level of output growth volatility is too high to be of much practical importance, even for emerging market economies. We conclude that, for empirically plausible calibrations, our main results on the desirability of relinquishing discretionary inflation policies remain robust.

Finally, as an alternative to giving up the debt inflation margin altogether, we investigate an intermediate arrangement in which the government delegates monetary policy to an independent central banker with a greater distaste for inflation than society as a whole. We find that delegating monetary policy to such a 'conservative' central banker allows achieving superior welfare outcomes vis-à-vis the baseline inflationary regime, in which the benevolent government chooses inflation discretionarily. As it turns out, however, average welfare never reaches that of the 'no inflation' regime: it increases monotonically with the central banker's distaste for inflation, converging asymptotically to its level under the latter regime.

Taken together, our results offer an important qualification of the conventional wisdom that individual countries may benefit from retaining the option to inflate away their sovereign debt. In particular, our analysis suggests that such countries may actually be better off by renouncing such a tool if their governments are unable to make credible commitments about their future inflation policy. Our findings may also rationalize why a number of developing countries with limited inflation credibility typically resort to issuing debt in terms of a hard foreign currency.

Literature review. Our paper relates to a recent theoretical literature that analyzes the link between sovereign debt vulnerability and monetary policy. One strand of this literature considers models of self-fulfilling debt crises, typically along the lines of Calvo (1988) or Cole and Kehoe (2000); see e.g. Aguiar et al. (2013, 2015), Reis (2013), Corsetti and Dedola (2014), Camous and Cooper (2014), and Bacchetta, Perazzi and van Wincoop (2015). We complement this literature by considering a framework in which sovereign default is instead an optimal government decision based on fundamentals, in the tradition of Eaton and Gersovitz (1981).² Also, most of the above contributions are qualitative, working in environments with two periods or two-period-lived agents (Corsetti and Dedola, 2014; Camous and Cooper, 2014) or without fundamental

²In Corsetti and Dedola (2014) default crisis can also be due to weak fundamentals.

uncertainty (Aguiar et al., 2013).³ By contrast, we adopt a fully dynamic, stochastic approach. On the normative front, aggregate fundamental uncertainty introduces a key role for unanticipated inflation in partially absorbing the effects of negative shocks on the sustainability of sovereign debt, as discussed above. On the positive front, our approach makes our model potentially useful for quantitative analysis. In particular, we show that our model can replicate well average sovereign yields and default premia in the data, while also matching average external sovereign debt stocks. We also show that our model can rationalize the reduction in sovereign bond yields across EMU periphery countries relative to the pre-EMU period, which suggests that investors perceived the reduction in inflation expectations as more important than the presumable increase in default risk.

In modelling the discretionary choice of inflation as a trade-off between the *ceteris paribus* reduction in the real debt burden and the utility costs of inflation, our model bears some resemblance with Aguiar et al. (2013). Apart from this aspect, both papers differ notably in modelling and focus. Aguiar et al. (2013) study the effects of inflation credibility (defined in their framework as the relative weight on inflation disutility in government preferences) on the potential for self-fulfilling debt crises, in a model without fundamental uncertainty where failure by investors to roll over the debt may lead the government to choose outright default over full principal repayment.⁴ By contrast, we evaluate the welfare costs and benefits of optimal discretionary inflation, *vis-à-vis* the abandonment of such policy tool, in a stochastic framework where the government may choose to default if fundamentals are bad enough. As explained above, aggregate uncertainty introduces a key role for unanticipated inflation in partially absorbing the effects of negative shocks on the sustainability of sovereign debt. We find that, for realistic levels of aggregate volatility, the debt-stabilizing benefits from discretionary inflation are outweighed by its costs.

In modeling optimal default à la Eaton-Gersovitz in a quantitative framework, our model is more in line with the literature on quantitative sovereign default models initiated by Aguiar and Gopinath (2006) and Arellano (2008).⁵ We build on this literature by introducing nominal bonds

³An exception is Bachetta et al. (2014), who explore quantitatively a monetary version of Lorenzoni and Werning's (2014) 'slow-moving' debt crisis model, which in turn extends the Calvo (1988) framework to a dynamic setting. In their model, fundamental uncertainty is about the value of primary surpluses from some future date onwards. The connection between inflation and the possibility of self-fulfilling debt crisis in fully dynamic, stochastic frameworks is also analyzed in Da Rocha et al. (2013) and Araujo et al. (2013). Da Rocha et al. (2013) analyze optimal debt and exchange rate policy in a model with foreign currency debt where the government is exposed to both self-fulfilling defaults and devaluations. Araujo et al. (2013) consider the welfare gains or losses from issuing debt in local versus foreign currency, in a framework where the costs of local currency debt are due to an exogenous inflation shock. In a real model, Roch and Uhlig (2015) analyze the role of bailout policies in eliminating self-fulfilling debt crises.

⁴Aguiar et al. (2013) find that the 'safety zone' (the debt region in which the economy is not vulnerable to a rollover crisis) is maximized for intermediate levels of inflation credibility.

⁵Other notable contributions to this literature include Arellano and Ramanarayanan (2012), Benjamin and Wright (2009), Chatterjee and Eyigungor (2012), Hatchondo and Martínez (2009) and Yue (2010). Mendoza and Yue (2012) integrate an optimal sovereign default model into a standard real business cycle framework with endogenous production.

and studying the optimal inflation policy when the government cannot commit not to inflate in the future. This allows us to address the trade-offs between price stability and sovereign debt vulnerability in a unified framework.⁶

More broadly, our paper relates to a literature that analyzes optimal monetary and fiscal policy under commitment in models where the government issues noncontingent nominal debt, but in which government default is not considered (Chari, Christiano and Kehoe, 1991; Schmitt-Grohé and Uribe, 2004; Siu, 2004; Faraglia et al., 2013). Here, we study instead optimal monetary and fiscal policy when the government cannot commit not to default on its nominal debt but also not to inflate it away, in a model where sovereign default may thus occur in equilibrium.⁷

In studying the effects of delegating monetary policy to an independent, conservative central banker, our analysis revisits an old theme initiated by Rogoff (1985) and further discussed e.g. in Clarida, Galí and Gertler (1999), although it does so in a very different context. In particular, we explore the effects of delegation in a framework in which the benefit of allowing for inflation is not to exploit a short-run output/inflation trade-off, as in the mainstream New Keynesian literature, but to make sovereign debt more sustainable. Contrary to the linear(ized) frameworks typically used in the New Keynesian literature, our framework takes full account of the strong non-linearities that emerge in the presence of equilibrium sovereign default. As in that literature, we find that there are welfare gains from delegating discretionary monetary policy to an independent authority with a greater distaste for inflation than that of society. What is perhaps more striking is that, whereas the optimal 'delegated' inflation distaste in the above literature is relatively large but finite, in our framework welfare is maximized when such distaste is arbitrarily large, i.e. when the government completely abandons the option of adjusting inflation.

Finally, we make a technical contribution by laying out a quantitative optimal sovereign default model in continuous time and introducing a new numerical method to compute the equilibrium.⁸

⁶Other papers introducing nominal debt and monetary policy in sovereign default models à la Aguiar-Gopinath (2006) and Arellano (2008) are Sunder-Plassmann (2014) and Du and Schreger (2015). Sunder-Plassmann (2014) studies how the denomination of sovereign debt (nominal vs. real/foreign currency) affects the government's incentives to inflate or default on its debt, in a model with monetary frictions. Du and Schreger (2015) analyze how the share of local vs. foreign currency debt in overall corporate debt determines the sovereign's incentive to inflate or default on its own debt when the latter is denominated in local currency, in a framework where firms face borrowing constraints and a currency mismatch between revenues and liabilities.

⁷Arellano and Heathcote (2010) study the costs and benefits of dollarization, vis-à-vis retaining an autonomous monetary policy, in a model in which government default is possible but does not occur in equilibrium due to endogenous borrowing constraints.

⁸Achdou et al. (2015) and Nuño and Moll (2015) also analyze numerical methods applied to stochastic control in continuous-time economies. Achdou et al. (2015) introduce an algorithm to compute competitive equilibria with a continuum of heterogeneous agents. Nuño and Moll (2015) propose an algorithm to solve planning problems in which the state variable is a continuous distribution. Both papers analyze coupled systems of differential equations composed by an Hamilton-Jacobi-Bellman (HJB) and a Kolmogorov-Forward equation. Here, by contrast, we focus on a two-player environment in which the coupled system is composed of the government's HJB equation and investors' bond pricing equation.

Compared to discrete-time methods, working in continuous time has several advantages. First, the computational burden is reduced: while solving the discrete-time Bellman equation requires computation of expectations over all possible future states, in the continuous-time Hamilton-Jacobi-Bellman equation (HJB) expectations are replaced by the first- and second-order derivatives of the value function.⁹ Second, in (a sequence of) one-dimensional optimal stopping problems, such as the one presented here, the optimal default threshold is determined by the so-called 'value matching' and 'smooth pasting' conditions, which can be easily incorporated in the numerical finite-difference scheme as boundary conditions.¹⁰ Third, ergodic distributions can be efficiently computed using the Kolmogorov Forward (KF) equation (also known as Fokker-Planck equation), thus making it unnecessary to use more time-consuming and less precise methods such as Monte Carlo simulation, as typically done in discrete-time models.¹¹ Fourth, our setting, while featuring long-term debt, does not suffer from the convergence problems of discrete-time sovereign default models with long-term debt identified by Chatterjee and Eyigungor (2012).¹²

The structure of the paper is as follows. In section 2 we introduce the model. Section 3 provides the main results. In Section 4 we perform some robustness analyses. Section 5 introduces monetary policy delegation. Section 6 concludes.

2 Model

We consider a continuous-time model of a small open economy.

2.1 Output, price level and sovereign debt

Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, \mathbb{P})$ be a filtered probability space. There is a single, freely-traded consumption good which has an international price normalized to one. The economy is endowed with Y_t units

⁹Doraszelski and Judd (2012) analyze how this reduction in the computational burden makes continuous-time more suitable than discrete-time for analyzing dynamic stochastic games with a finite number of states.

¹⁰In state spaces with more than one dimension, optimal stopping problems are typically solved using optimal splitting methods, as in Barles, Daher and Romano (1995). These methods are closer to the value function comparisons used in discrete-time problems.

¹¹Achdou et al. (2015) show that the solution of the stationary KF equation reduces to the inversion of the matrix characterizing the infinitesimal generator of the stochastic process. This is related to the discrete-time transition-matrix approach but the extreme sparseness of the transition matrix from the KF equation makes it an optimal candidate to be inverted using efficient numerical techniques.

¹²These authors discuss how, in order to ensure convergence of the numerical algorithm, it is convenient to extend the standard model (which typically features discrete-valued output processes) by introducing an additional i.i.d. output shock drawn from a continuous distribution. Here, instead, no additional shock is needed as the original output shock follows a Brownian motion.

of the good each period (real GDP). The evolution of Y_t is given by

$$dY_t = \mu Y_t dt + \sigma Y_t dW_t, \quad (1)$$

where W_t is a $\mathcal{F}_t$ -Brownian motion, $\mu \in \mathbb{R}$ is the drift parameter and $\sigma \in \mathbb{R}_+$ is the volatility. The local currency price relative to the World price at time t is denoted P_t . It evolves according to

$$dP_t = \pi_t P_t dt, \quad (2)$$

where π_t is the instantaneous inflation rate.

The government trades a nominal non-contingent bond with risk-neutral competitive foreign investors.¹³ Let B_t denote the outstanding stock of nominal government bonds; assuming that each bond has a nominal value of one unit of domestic currency, B_t also represents the total nominal value of outstanding debt. We assume that outstanding debt is amortized at rate $\lambda > 0$ per unit of time. The nominal value of outstanding debt thus evolves as follows,

$$dB_t = B_t^{new} dt - \lambda dt B_t,$$

where B_t^{new} is the flow of new debt issued at time t . The nominal market price of government bonds at time t is Q_t . Each bond pays a proportional coupon δ per unit of time. Also, the government incurs a nominal primary deficit $P_t (C_t - Y_t)$, where C_t is aggregate consumption.¹⁴ The government's flow of funds constraint is then

$$Q_t B_t^{new} = (\lambda + \delta) B_t + P_t (C_t - Y_t).$$

That is, the proceeds from issuance of new bonds must cover amortization and coupon payments plus the primary deficit. Combining the last two equations, we obtain the following dynamics for nominal debt outstanding,

$$dB_t = \left[\left(\frac{\lambda + \delta}{Q_t} - \lambda \right) B_t + \frac{P_t}{Q_t} (C_t - Y_t) \right] dt. \quad (3)$$

We define the debt-to-GDP ratio as $b_t \equiv B_t / (P_t Y_t)$. Its dynamics are obtained by applying Itô's

¹³The assumption that government debt is fully held abroad is a good approximation for emerging market economies. It is also a reasonable approximation for peripheral EMU economies. For instance, in 2012 the fraction of Greece's sovereign debt held abroad was 86%.

¹⁴As in Arellano (2008), we assume that the government rebates back to households all the net proceedings from its international credit operations (i.e. its primary deficit) in a lump-sum fashion. Denoting by T_t the primary deficit, we thus have $P_t C_t = P_t Y_t + T_t$. This implies $T_t = P_t (C_t - Y_t)$.

lemma to equations (1)-(3),

$$db_t = \left[\left(\frac{\lambda + \delta}{Q_t} - \lambda + \sigma^2 - \mu - \pi_t \right) b_t + \frac{c_t}{Q_t} \right] dt - \sigma b_t dW_t, \quad (4)$$

where $c_t \equiv (C_t - Y_t)/Y_t$ is the primary deficit-to-GDP ratio. Equation (4) describes the evolution of the debt-to-GDP ratio as a function of the primary deficit ratio, inflation and the bond price. In particular, *ceteris paribus* inflation π_t allows to reduce the debt ratio by reducing the real value of nominal debt. We also impose a non-negativity constraint on debt: $b_t \geq 0$.

2.2 Preferences

The representative household has preferences over paths for consumption and domestic inflation given by

$$U_0 \equiv \mathbb{E}_0 \left[\int_0^\infty e^{-\rho t} u(C_t, \pi_t) dt \right]. \quad (5)$$

Instantaneous utility takes the form

$$u(C_t, \pi_t) = \log(C_t) - \frac{\psi}{2} \pi_t^2, \quad (6)$$

where $\psi > 0$. The functional form for the utility costs of inflation, $\psi \pi_t^2/2$, can be justified on the grounds of costly price adjustment by firms. In particular, in Appendix A we lay out an economy where firms are explicitly modelled, and where a subset of them are price-setters but incur a standard quadratic cost of price adjustment *à la* Rotemberg (1982). As we show there, social welfare in such an economy can be expressed as in equations (5) and (6), and the equilibrium conditions are identical to those in the simple model described here.¹⁵

Using $C_t = (1 + c_t) Y_t$, we can express welfare in terms of the primary deficit ratio c_t as follows,

$$\begin{aligned} U_0 &= \mathbb{E}_0 \left[\int_0^\infty e^{-\rho t} \left(\log(1 + c_t) + \log(Y_t) - \frac{\psi}{2} \pi_t^2 \right) dt \right] \\ &= \mathbb{E}_0 \left[\int_0^\infty e^{-\rho t} \left(\log(1 + c_t) - \frac{\psi}{2} \pi_t^2 \right) dt \right] + V_0^{aut}, \end{aligned} \quad (7)$$

where

$$V_0^{aut} \equiv \mathbb{E}_0 \left[\int_0^\infty e^{-\rho t} \log(Y_t) dt \right] = \frac{\log(Y_0)}{\rho} + \frac{\mu - \sigma^2/2}{\rho^2} \quad (8)$$

¹⁵To be precise, the quadratic utility cost $\psi \pi_t^2/2$ is an approximation to the exact utility cost of inflation in the model of Appendix A; see the latter appendix for further details. In Appendix I, we simulate the model using the exact inflation utility cost and show that the results are virtually identical to those in the paper.

is the (exogenous) value at time $t = 0$ of being in autarky forever.¹⁶ Thus, welfare increases with the primary deficit ratio c_t , as this allows households to consume more for a given exogenous output.

2.3 Fiscal and monetary policy

The government chooses fiscal policy at each point in time along two dimensions: it sets optimally the primary surplus ratio c_t , and it chooses whether to continue honoring debt repayments or else to default. In addition, the government implements monetary policy by choosing the inflation rate π_t at each point in time. We now present the sovereign default scenario, which affects the boundary conditions of the general optimization problem.

2.3.1 The default scenario

Following most of the literature on quantitative sovereign default models (e.g. Aguiar and Gopinath, 2006; Arellano, 2008), we assume that a default entails two types of costs. First, the government is excluded from international capital markets temporarily. The duration of this exclusion period, τ , is random and follows an exponential distribution with average duration $1/\chi$. Second, during the exclusion period the country's output endowment declines. Suppose the government defaults at an arbitrary debt ratio b . Then during the exclusion period the country's output endowment is given by $Y_t^{def} = Y_t \exp[-\epsilon \max\{0, b - \hat{b}\}]$, with $\epsilon, \hat{b} > 0$, such that the loss in (log)output equals $\epsilon \max\{0, b - \hat{b}\}$. Therefore, the country suffers an output loss only if it defaults at a debt ratio higher than a threshold $\hat{b}$. This specification of output loss is similar to the one in Arellano (2008), except that in our case it depends on the debt ratio at the time of default as opposed to output at the time of default.¹⁷ During the exclusion phase, households simply consume the output endowment, $C_t = Y_t^{def}$, which implies

$$\log(C_t) = \log(Y_t) - \epsilon \max\{0, b - \hat{b}\}.$$

¹⁶Notice that (1) and Itô's Lemma imply $d \log Y_t = (\mu - \sigma^2/2) dt + \sigma dW_t$. Solving for $\log Y_t$ and taking time-0 conditional expectations yields $\mathbb{E}_0(\log Y_t) = \log Y_0 + (\mu - \sigma^2/2)t$, which combined with the definition of V_0^{aut} gives us the right-hand side of (8).

¹⁷In Arellano (2008), the output loss following default equals $Y_t - Y_t^{def} = \max\{0, Y_t - \hat{Y}\}$, for some threshold output level $\hat{Y}$. Specifying our output loss function in terms of b_t (as opposed to Y_t) allows us to retain the convenient model feature that b_t is the only relevant state variable. One possible rationale for making output losses dependent on the government debt ratio is to think of a setup where firms use government debt as collateral in order to obtain funding for their activities. In such a scenario, the higher the debt ratio upon which the government defaults, the larger the destruction of collateral and hence the larger the contraction in credit and in economic activity in relation to aggregate GDP. See Mendoza and Yue (2012) for a model that endogenizes the output costs of default in a quantitative sovereign default model with endogenous production.

In Section 4.2, we explore the robustness of our results to alternative functional forms for the output costs of default.

The main benefit of defaulting is of course the possibility of reducing the debt burden. During the exclusion period, which we may interpret as a renegotiation process between the government and the investors, the latter receive no repayments. Let $\tilde{t}$ denote the time of the most recent default. We assume that at the end of the exclusion period, i.e. at time $\tilde{t} + \tau$, both parties reach an agreement by which investors recover a fraction $\theta Y_{\tilde{t}+\tau} P_{\tilde{t}+\tau} / (Y_{\tilde{t}} P_{\tilde{t}})$ of the nominal value of outstanding bonds at the time of default, for some parameter $\theta > 0$. This specification captures in reduced form the idea that the terms of the debt restructuring agreement are somehow sensitive to the country's macroeconomic performance.¹⁸ Importantly, it allows us to keep the set of state variables restricted to the debt ratio only. To see this, notice that upon regaining access to capital markets, the debt ratio is

$$b_{\tilde{t}+\tau} = \left(\theta \frac{Y_{\tilde{t}+\tau} P_{\tilde{t}+\tau}}{Y_{\tilde{t}} P_{\tilde{t}}} \right) B_{\tilde{t}} \frac{1}{Y_{\tilde{t}+\tau} P_{\tilde{t}+\tau}} = \theta b_{\tilde{t}},$$

where $b_{\tilde{t}} = B_{\tilde{t}} / Y_{\tilde{t}} P_{\tilde{t}}$ is the debt ratio at the time of default. Therefore, the government reenters capital markets with a debt ratio that is a fraction θ of the ratio at which it defaulted. It follows that the government has no incentive to create inflation during the exclusion period, as that would generate direct welfare costs while not reducing the debt ratio upon reentry; we thus have $\pi_t = 0$ for $t \in (\tilde{t}, \tilde{t} + \tau)$.

Taking all these elements together, we can express the value of defaulting at $\tilde{t} = 0$ as $U_0^{def} = V_0^{def} + V_0^{aut}$, where V_0^{aut} is the autarky value as defined in (8), and $V_0^{def} \equiv V_{def}(b_0)$ is the value of defaulting net of the autarky value, given by

$$\begin{aligned} V_{def}(b) &= \mathbb{E} \left\{ - \int_0^\tau e^{-\rho t} \epsilon \max\{0, b - \hat{b}\} dt + e^{-\rho \tau} V(\theta b) \right\} \\ &= \int_0^\infty \chi e^{-\chi \tau} \left(- \int_0^\tau e^{-\rho t} \epsilon \max\{0, b - \hat{b}\} dt + e^{-\rho \tau} V(\theta b) \right) d\tau \\ &= - \frac{\epsilon \max\{0, b - \hat{b}\}}{\rho + \chi} + \frac{\chi}{\rho + \chi} V(\theta b), \end{aligned} \tag{9}$$

where in the second equality we use our assumption that τ is exponentially distributed, and where $V(\cdot)$ is the value function during repayment spells, to be defined later. For future reference, the slope of the default value function is

$$V'_{def}(b) = - \frac{\epsilon}{\rho + \chi} \mathbf{1}(b > \hat{b}) + \frac{\chi}{\rho + \chi} V'(\theta b) \theta, \tag{10}$$

for all $b \in [0, \hat{b}) \cup (\hat{b}, \infty)$, where $\mathbf{1}(\cdot)$ is the indicator function.

¹⁸See Benjamin and Wright (2009) and Yue (2010) for studies that endogenize the recovery rate upon default, in models with explicit renegotiation between the government and its creditors.

2.3.2 The general problem

As mentioned before, at every point in time the government decides optimally whether to default or not, in addition to choosing the primary deficit ratio and the inflation rate. Following a default, and once the government regains access to capital markets, it starts accumulating debt and is confronted again with the choice of defaulting. This is a sequence of *optimal stopping* problems, as one of the policy instruments is a sequence of stopping times. The solution to this problem will be characterized by an optimal *default threshold* for the debt ratio, which we denote by b^* . This threshold defines an “inaction region” of the state space, $[0, b^*)$, in which the government chooses not to default, and a region $[b^*, \infty)$ in which the government defaults. We denote by $T(b^*)$ the *time to default*. The latter is a stopping time with respect to the filtration $\{\mathcal{F}_t\}$, defined as the smallest time t' such that $b_{t+t'} = b^*$, i.e. $T(b^*) = \min\{t' : b_{t+t'} = b^*\}$.¹⁹ The government maximizes social welfare under discretion. When doing so, it takes as given the bond price schedule $Q(b)$, which determines how investors price government bonds in each state and which is characterized in section 2.4. The value function of the government (net of the exogenous autarky value) at time $t = 0$ can then be expressed as

$$V(b) = \max_{b^*, \{c_t, \pi_t\}} \mathbb{E}_0 \left\{ \int_0^{T(b^*)} e^{-\rho t} \left(\log(1 + c_t) - \frac{\psi}{2} \pi_t^2 \right) dt + e^{-\rho T(b^*)} V_{def}(b^*) \mid b_0 = b \right\}, \quad (11)$$

subject to the law of motion of the debt ratio, equation (4). The optimal default threshold b^* must satisfy the following two conditions,

$$V(b^*) = V_{def}(b^*), \quad (12)$$

$$V'(b^*) = V'_{def}(b^*), \quad (13)$$

where $V_{def}(b^*)$ and $V'_{def}(b^*)$ are given respectively by equations (9) and (10) evaluated at $b = b^*$.²⁰ Equation (12) is the *value matching* condition and it requires that, at the default threshold, the value of honoring debt repayments equals the value of defaulting. Equation (13) is the *smooth pasting* condition, and it requires that there is no kink at the optimal default threshold.²¹ Both are standard conditions in optimal stopping problems; see e.g. Dixit and Pindyck (1994), Øksendal and Sulem (2007) and Stokey (2009). These conditions imply that the value function is continuous

¹⁹Therefore, the time of default in absolute time is $\tilde{t} = t + T(b^*)$, i.e. $T(b^*) = \tilde{t} - t$.

²⁰In addition to these two boundary conditions, there exists the constraint $b \geq 0$ introduced before. This is a *state constraint boundary condition*, as explained in Achdou et al. (2015). This boundary condition implies that $s(0) \geq 0$. A more precise formulation requires the use of *viscosity solutions* as in Soner (1986a,b), but they are not required here due to the smoothness of the government problem.

²¹To be precise, the smooth pasting condition holds as long as $b^* \neq \hat{b}$ as the continuation value $V_{def}(b)$ is not differentiable at $\hat{b}$.

and continuously differentiable: $V \in C^1([0, \infty))$.

The solution of this problem must satisfy the Hamilton-Jacobi-Bellman (HJB) equation,

$$\rho V(b) = \max_{c, \pi} \left\{ \log(1 + c) - \frac{\psi}{2} \pi^2 + s(b, c, \pi) V'(b) + \frac{(\sigma b)^2}{2} V''(b) \right\}, \quad (14)$$

$\forall b \in [0, b^*)$, where

$$s(b, c, \pi) = \left(\frac{\lambda + \delta}{Q(b)} - \lambda + \sigma^2 - \mu - \pi \right) b + \frac{c}{Q(b)} \quad (15)$$

is the *drift* of the state variable (see equation 4); together with the boundary conditions (12) and (13).²² The first order conditions of this problem imply the following policy functions for the primary deficit ratio and inflation,

$$c(b) = \frac{Q(b)}{-V'(b)} - 1, \quad (16)$$

$$\pi(b) = -\frac{b}{\psi} V'(b). \quad (17)$$

Therefore, the optimal primary deficit ratio increases with bond prices and decreases with the slope of the value function (in absolute value).²³ The intuition is straightforward. Higher bond prices (equivalently, lower bond yields) make it cheaper for the government to finance primary deficits. Likewise, a steeper value function makes it more costly to increase the debt burden by incurring primary deficits. As regards optimal inflation, the latter increases both with the debt ratio and the slope (in absolute value) of the value function. Intuitively, the higher the debt ratio the larger the reduction in the debt burden that can be achieved through a marginal increase in inflation. Similarly, a steeper value function increases the incentive to use inflation so as to reduce the debt burden.

²² Obviously, $\forall b \in [b^*, \infty)$, $V(b) = V_{def}(b)$.

²³ As noted above, when making its policy decisions the government takes as given the bond price function $Q(b)$ and therefore internalizes how such decisions affect bond prices through their effect on the state b . To see this explicitly, consider for simplicity the case with no uncertainty, $\sigma = 0$. Combining (i) the 'envelope condition' of the government's problem (obtained by differentiating the HJB equation with respect to b), (ii) the first-order condition for c (eq. 16), and (iii) the equation that results from differentiating the latter condition with respect to b , one obtains the following Euler equation for c (an analogous equation can be obtained for π),

$$\rho = s_b(b, c, \pi) + s(b, c, \pi) \left[\frac{Q'(b)}{Q(b)} - \frac{c'(b)}{1 + c(b)} \right],$$

where $s_b(b, \cdot)$ is the derivative of the drift with respect to b . In the above equation, the term $Q'(b)$ captures the effect on bond prices of a marginal increase in the debt ratio. The derivation of the Euler equations under uncertainty, while more complex, follows the same logic and is available upon request.

2.3.3 The 'no inflation' regime: renouncing debt inflation

So far we have analyzed the decision problem of a benevolent government that cannot make credible commitments about its future fiscal policy (including the possibility of defaulting) and monetary policy. In particular, the inability to commit *not* to use inflation in the future so as to inflate debt away implies that the government is unable to steer investor's inflation expectations in a way that favors welfare outcomes. While lacking commitment, however, we can think of situations in which the government effectively relinquishes the ability to inflate debt away. Formally, we may consider a monetary regime in which inflation is zero in all states: $\pi = 0, \forall b \in [0, b^*)$. The government's problem is given by (14) with $\pi = 0$ replacing the optimal inflation choice, and with boundary conditions given again by (12) and (13). We may denote the value function in the 'no inflation' regime as $V_{\pi=0}(b)$.

We may interpret such a 'no inflation' scenario in alternative ways. One can first think of a situation in which the government appoints an independent central banker with a strong, in fact arbitrarily great, distaste for inflation. Even under discretion, such a central banker would always choose $\pi = 0$. One problem with this interpretation, though, is that it is unlikely that a government that cannot make credible commitments would appoint (much less keep in place) a central banker with such extreme preferences towards inflation.²⁴

A second, perhaps more plausible interpretation is that the government directly issues bonds denominated in *foreign currency*. In that case, the possibility of inflating debt away simply disappears, and with it the only benefit of inflating in this model. As a result, optimal inflation is always zero in such a scenario.

Finally, we may think of a situation in which the government joins a *monetary union* in which the common monetary authority has a strong and credible anti-inflationary mandate. If the costs of exiting the monetary union are very high, then joining it signals a credible anti-inflationary commitment.

In what follows, we will simply refer to this scenario as the 'no inflation regime', keeping in mind that such scenario admits several interpretations along the lines just discussed.

2.4 Foreign investors and bond pricing

When choosing fiscal and monetary policy, the government takes as given the mapping between the debt ratio and the nominal price of bonds, $Q(b)$. We now characterize such bond price function. The government sells bonds to competitive risk-neutral foreign investors that can invest elsewhere at the risk-free real rate $\bar{r}$. As explained before, during repayment spells bonds pay a coupon rate

²⁴In section 5 we will consider a more general scenario in which the government appoints a conservative central banker whose distaste for inflation is greater than that of society, but not so extreme as to imply zero inflation at all times.

δ and are amortized at rate λ . But following a default (at some time $\tilde{t}$), and during the exclusion period of the government, investors receive no payments. Once the exclusion/renegotiation period ends (at time $\tilde{t} + \tau$), investors recover a fraction $\theta P_{\tilde{t}+\tau} Y_{\tilde{t}+\tau} / (P_{\tilde{t}} Y_{\tilde{t}}) = \theta Y_{\tilde{t}+\tau} / Y_{\tilde{t}}$ of the nominal value of each bond, where we have used the fact that optimal inflation is zero during the exclusion period, such that $P_{\tilde{t}+\tau} = P_{\tilde{t}}$.²⁵ They also anticipate that the government's debt ratio at the time of reentering financial markets will be θb^* , such that their outstanding bonds will carry a market price $Q(\theta b^*)$. Finally, investors discount future nominal payoffs with the accumulated inflation between the time of the bond purchase (say, $t = 0$) and the time such payoffs accrue: $\int_0^t \pi_s ds$, where $\pi_s = \pi(b_s)$. Taking all these elements together, the nominal price of the bond at time $t = 0$ for a current debt ratio $b \leq b^*$ is given by

$$Q(b) = \mathbb{E}_0 \left[\begin{aligned} & \int_0^{T(b^*)} e^{-(\bar{r}+\lambda)t - \int_0^t \pi_s ds} (\lambda + \delta) dt \\ & + e^{-\bar{r}[T(b^*)+\tau] - \lambda T(b^*) - \int_0^{T(b^*)} \pi_s ds} \theta \frac{Y_{T(b^*)+\tau}}{Y_{T(b^*)}} Q(\theta b^*) \end{aligned} \mid b_0 = b \right], \quad (18)$$

where again $T(b^*)$ denotes the time to default and b follows the law of motion (4).²⁶ Applying the Feynman-Kac formula, we obtain the following recursive representation,

$$Q(b) (\bar{r} + \pi(b) + \lambda) = (\lambda + \delta) + s(b, c(b), \pi(b)) Q'(b) + \frac{(\sigma b)^2}{2} Q''(b), \quad (19)$$

for all $b \in [0, b^*)$, where the drift function $s(b, c(b), \pi(b))$ is given by (15). To determine the boundary condition for $Q(b)$, we calculate the expected value of outstanding bonds at the time of default ($T(b^*) = 0$),

$$\begin{aligned} Q(b^*) &= \mathbb{E}_0 \left[e^{-\bar{r}\tau} \theta \frac{Y_\tau}{Y_0} Q(\theta b^*) \right] = \int_0^\infty \chi e^{-\chi\tau} \mathbb{E}_0 \left[e^{-\bar{r}\tau} \theta \frac{Y_\tau}{Y_0} Q(\theta b^*) \mid \tau \right] d\tau \\ &= \int_0^\infty \chi e^{-(\bar{r}+\chi-\mu)\tau} \theta Q(\theta b^*) d\tau = \frac{\chi}{\bar{r} + \chi - \mu} \theta Q(\theta b^*), \end{aligned} \quad (20)$$

where in the third equality we have used $\mathbb{E}_0 \left[\frac{Y_\tau}{Y_0} \mid \tau \right] = e^{\mu\tau}$. The partial differential equation (19), together with the boundary condition (20), provide the risk-neutral pricing of the nominal defaultable sovereign bond.²⁷

²⁵The average recovery rate equals $\mathbb{E} [\theta Y_{\tilde{t}+\tau} / Y_{\tilde{t}}] = \theta \mathbb{E}_\tau \{ \mathbb{E} [Y_{\tilde{t}+\tau} / Y_{\tilde{t}} \mid \tau] \} = \theta \int_0^\infty \chi e^{-\chi\tau} e^{\mu\tau} d\tau = \theta \chi / (\chi - \mu)$, where we have used $\mathbb{E} [Y_{\tilde{t}+\tau} / Y_{\tilde{t}} \mid \tau] = \exp(\mu\tau)$ and the fact that τ is exponentially distributed.

²⁶Notice that the recovery payoff $\theta (Y_{T(b^*)+\tau} / Y_{T(b^*)}) Q(\theta b^*)$ is discounted by $\exp\{-\lambda T(b^*) - \int_0^{T(b^*)} \pi_s ds\}$, as opposed to $\exp\{-\lambda [T(b^*) + \tau] - \int_0^{T(b^*)+\tau} \pi_s ds\}$, because no principal is repaid and no inflation is created during the exclusion period (of length τ).

²⁷Again, there also exists the state constraint $b \geq 0$.

2.5 Some definitions

Given a current nominal bond price $Q(b)$, the implicit nominal *bond yield* $r(b)$ is the discount rate for which the discounted future promised cash flows from the bond equal its price. The discounted future promised payments are $\int_0^\infty e^{-(r(b)+\lambda)t} (\lambda + \delta) dt = \frac{\lambda+\delta}{r(b)+\lambda}$. Therefore, the bond yield function is

$$r(b) = \frac{\lambda + \delta}{Q(b)} - \lambda. \quad (21)$$

The gap between the nominal yield $r(b)$ and the riskless real rate $\bar{r}$ reflects both (a) the risk of sovereign default, i.e. a *default premium*, and (b) the anticipation of inflation during the life of the bond, i.e. an *inflation premium*. In order to disentangle both factors, we define a notional *riskless nominal yield* $\tilde{r}(b) = \frac{\lambda+\delta}{\tilde{Q}(b)} - \lambda$, where $\tilde{Q}(b)$ is the price that the investor would pay for a riskless nominal bond with the same promised cash flows as the risky nominal bond. Appendix D defines $\tilde{Q}(b)$ and explains how to solve for it. We then express $r(b) - \bar{r}$ as

$$r(b) - \bar{r} = [r(b) - \tilde{r}(b)] + [\tilde{r}(b) - \bar{r}],$$

where $r(b) - \tilde{r}(b)$ is the *default premium*, and $\tilde{r}(b) - \bar{r}$ is the *inflation premium*. In the no inflation regime, the riskless rate is simply $\tilde{r}(b) = \bar{r}$, the inflation premium is zero, and the default premium is $r(b) - \bar{r}$.

Given the definition of the bond yield, the drift function (eq. 15) can be expressed as

$$s(b, c(b), \pi(b)) = (r(b) - \pi(b) - \mu + \sigma^2) b + \frac{c(b)}{Q(b)} \equiv s(b), \quad (22)$$

with a slight abuse of notation in the last definition. Therefore, together with the deficit ratio and bond prices $(c(b), Q(b))$, an important determinant of the drift is the difference between the nominal bond yield and the instantaneous inflation rate, $r(b) - \pi(b)$. We may refer to the latter as the *ex-post real interest rate*, or simply the real interest rate.

Finally, we define the *expected time to default*, given a current debt ratio b , as

$$T^e(b) \equiv \mathbb{E}_0 [T(b^*) | b_0 = b] = \mathbb{E}_0 \left[\int_0^{T(b^*)} 1 dt \mid b_0 = b \right]. \quad (23)$$

Appendix E shows how to compute $T^e(b)$ numerically.

2.6 Equilibrium

We define our equilibrium concept:

Definition 1 A Markov Perfect Equilibrium is an interval $\Phi = [0, b^*)$, a value function $V : \Phi \rightarrow \mathbb{R}$, a pair of policy functions $c, \pi : \Phi \rightarrow \mathbb{R}$ and a bond price function $Q : \Phi \rightarrow \mathbb{R}_+$ such that:

1. Given prices Q , for any initial debt $b_0 \in \Phi$ the value function V solves the government problem (14), with boundary conditions (12) and (13); the optimal inflation is π , the optimal deficit ratio is c , and the optimal debt threshold is b^* .
2. Given the optimal inflation π , deficit ratio c and the interval Φ , bond prices satisfy the pricing equation (19).

The government takes the bond price function as given and chooses inflation and deficit (continuous policies) and whether to default or not (stopping policy) to maximize its value function. The investors take these policies as given and price government bonds accordingly. Equilibrium in the no inflation regime is defined analogously, with $\pi = 0$ at all $b \in \Phi$ replacing the inflation policy function.

The definition of Markov Perfect Equilibrium (MPE) is a particular case of a Markov equilibrium in continuous-time games. It is composed by a coupled system of two ordinary differential equations (ODEs): the HJB equation and the bond pricing equation.²⁸

2.7 Some analytical results

Analytical solutions are seldom found in Markov Perfect equilibrium models, not even in the deterministic case.²⁹ This is why most continuous-time games are analyzed assuming commitment (technically, *open-loop* Nash equilibrium). Under certain simplifying assumptions, deterministic versions of these games can be solved analytically, or at least characterized tightly, by employing the Pontryagin maximum principle. This approach is not available here as we focus on the Markov Perfect solution, i.e. without commitment.³⁰

Even if a complete analytical characterization of equilibrium is out of our reach, it is worthwhile to provide some analytical insights before moving to the numerical analysis in the following sections.

Proposition 1 (Inflation bias) *In the inflationary regime, inflation is always positive at positive debt ratios: $\pi(b) > 0, \forall b \in (0, b^*)$.*

²⁸See Başar and Oldser (1999) or Dockner et al. (2000) for references on continuous-time (also known as differential) game theory.

²⁹In fact, in the deterministic case of Markov Perfect Equilibrium, even existence of a solution is not guaranteed in most cases, as discussed in Bressan (2010, Section 5). The stochastic case typically has a solution, but very restrictive assumptions (e.g. linear-quadratic structures) need to be imposed in order to be able to find it analytically; see e.g. the examples in Dockner et al. (2000).

³⁰In a more stylized model of optimal default, for instance, Bressan and Nguyen (2015) are able to prove the existence of a solution in the open-loop Nash case but not in the Markov Perfect one. In the latter case they can only show that, if a smooth solution exists, it should satisfy a nonlinear partial differential equation, a result analogous to the definition of equilibrium in our model.

Proof. The policy function for the primary deficit ratio (equation 16) and the fact that $C = (1 + c)Y$ imply that consumption utility equals $\log(C) = \log\left(\frac{Q(b)}{-V'(b)}\right)$ plus exogenous (log)output. Given that $Q(b) > 0$, consumption utility is well-defined and finite only if $-V'(b) > 0$. Using this in the inflation policy function (equation 17), we have $\pi(b) = -V'(b)\frac{b}{\psi} > 0$ for all $b \in (0, b^*)$. ■

Notice that Proposition 1 holds regardless of the degree of aggregate uncertainty. Therefore, under discretion the government has an incentive to create inflation also in the deterministic case ($\sigma = 0$). The result in Proposition 1 is thus reminiscent of the classical ‘inflationary bias’ of discretionary monetary policy originally emphasized by Kydland and Prescott (1977) and Barro and Gordon (1983). In those papers, the source of the inflation bias is a persistent attempt by the monetary authority to raise output above its natural level. Here, by contrast, it arises from the existence of a positive stock of non-contingent nominal sovereign debt (such that $b > 0$) and from the welfare gains that can be achieved by reducing the real value of such nominal debt ($-V'(b) > 0$).

As noted above, an analytical solution of our model is rather elusive even in the deterministic limit. It is possible however to obtain some results regarding the nature of the deterministic equilibrium in ‘no inflation’ regime. First we make the following assumption.

Condition 1 Assume $\rho > \bar{r}$ and $\mu \geq 0$.

The condition is very mild, as it requires that households’ subjective discount rate ρ is higher than the world real interest rate $\bar{r}$, and that the long-run growth rate is not negative. One can then prove (see Appendix B) that there is no deterministic steady-state with positive debt, i.e. in the deterministic limit there is no equilibrium with $db/dt = s(b) = 0$.

Proposition 2 (Lack of a deterministic steady-state in the no inflation regime) *In the deterministic limit ($\sigma = 0$) of the no inflation regime,*

1. *There is no deterministic steady-state with positive debt: $\nexists b \in (0, b^*) : s(b) = 0$.*
2. *The drift is positive: $s(b) \geq 0, \forall b \in \Phi$.*

Provided that an equilibrium exists, then starting from any debt ratio $0 < b_0 < b^*$ the drift is strictly positive and the government defaults periodically with a fixed frequency. As discussed in the next section, in the stochastic case ($\sigma > 0$) the duration of repayment spells is no longer deterministic, and a stochastic steady state exists for the assumed parameter values.³¹

³¹The presence of aggregate uncertainty creates a precautionary savings motive for the government to reduce primary deficit. This shifts the drift function $s(b)$ downwards sufficiently to ensure the existence of a debt ratio b_{ss} for which $s(b_{ss}) = 0$, i.e. a stochastic steady-state.

3 Quantitative analysis

Having laid out our theoretical model, we now use it in order to analyze the trade-off between price stability and the sustainability of sovereign debt. We next describe our numerical solution algorithm.

3.1 Computational algorithm

Here we propose a computational algorithm aimed at finding the equilibrium. The structure of the model complicates its solution as it comprises a pair of coupled ordinary difference equations (ODEs): the HJB equation (14) and the bond pricing equation (19). The policies obtained from the HJB are necessary to compute the bond prices and, simultaneously, bond prices are necessary to compute the drift in the HJB equation.

In order to solve the HJB and bond pricing equations, we employ an *upwind finite difference* method.³² It approximates the value function $V(b)$ and the bond price function $Q(b)$ on a finite grid with steps Δb : $b \in \{b_1, \dots, b_I\}$, where $b_i = b_{i-1} + \Delta b = b_1 + (i-1)\Delta b$ for $2 \leq i \leq I$, with bounds $b_1 = 0$ and $b_{I+1} = b^*$.³³ We use the notation $V_i^{(n)} \equiv V^{(n)}(b_i)$, $i = 1, \dots, I$, where $n = 0, 1, 2, \dots$ is the iteration counter, and analogously for $Q_i^{(n)}$.

In order to compute the numerical solution to the equilibrium we proceed in three steps. We consider an initial guess of the bond price function, $Q^{(0)} \equiv \{Q_i^{(0)}\}_{i=1}^I$, and the default threshold, $b_{(0)}^*$. Set $n = 1$. Then:

Step 1: Government problem. Given $Q^{(n-1)}$ and $b_{(n-1)}^*$, we solve the optimal stopping problem with variable controls. This means solving the HJB equation (14) in the domain $[0, b_{(n-1)}^*]$ imposing the smooth pasting condition (13) (but not the value matching condition) to obtain an estimate of the value function $V^{(n)} \equiv \{V_i^{(n)}\}_{i=1}^I$ and of primary deficit and inflation, $(c^{(n)}, \pi^{(n)}) \equiv \{c_i^{(n)}, \pi_i^{(n)}\}_{i=1}^I$.

Step 2: Investors problem. Given $c^{(n)}$, $\pi^{(n)}$ and $b_{(n-1)}^*$, solve the bond pricing equation (19) and obtain $Q^{(n)}$ in the domain $[0, b_{(n-1)}^*]$. Then iterate again on steps 1 and 2 until both the value and bond price functions converge for given $b_{(n-1)}^*$.

Step 3: Optimal boundary. Given $V^{(n)}$ from step 2, we check whether the value matching condition (12) is satisfied. We compute $V^{(n)}(b_{(n-1)}^*) = V_{I+1}^{(n)}$ and $V_{def}^{(n)}(b_{(n-1)}^*) = -\frac{\epsilon \max\{0, b_{(n-1)}^* - \hat{b}\}}{\rho + \chi} + \frac{\chi}{\rho + \chi} V_{\theta I}^{(n)}$. If $V^{(n)}(b_{(n-1)}^*) > V_{def}^{(n)}(b_{(n-1)}^*)$, then increase the threshold to a new value $b_{(n)}^*$. If

³²Barles and Souganidis (1991) have proved how this method converges to the unique *viscosity solution* of the problem. The latter is the appropriate concept of a general solution for stochastic optimal control problems (Crandall and Lions, 1983; Crandall, Ishii and Lions, 1992).

³³We thus have $\Delta b = b^*/I$. We use $I = 800$ grid points in all our simulations.

$V^{(n)}(b_{(n-1)}^*) < V_{def}^{(n)}(b_{(n-1)}^*)$, then decrease the threshold. Set $n := n + 1$. Proceed again to steps 1 and 2 until the value matching condition $V(b^*) = V_{def}(b^*)$ is satisfied.

Appendix C provides further details on these steps. The idea of the algorithm is to find the equilibrium numerically by moving the default threshold b^* and solving the HJB and bond pricing equations. The algorithm stops when the *value matching* condition (12) is satisfied.

3.2 Calibration

Let the unit of time be 1 year, such that all rates are in annual terms. Most papers in the literature on quantitative optimal sovereign default models set the world riskless real interest rate and the subjective discount rate to 1% and 5% per quarter, respectively.³⁴ We thus set $\bar{r} = 0.04$ and $\rho = 0.20$ per year.

In order to calibrate the drift and volatility of the exogenous output process, we use annual GDP growth data for the EMU periphery countries over the period 1995-2012.³⁵ Averaging the mean and standard deviation of GDP growth across these countries, we obtain $\mu = 0.022$ and $\sigma = 0.032$.

The bond amortization rate λ is such that the average Macaulay bond duration, $1/(\lambda + \bar{r})$, is 5 years, which is broadly consistent with international evidence on bond duration (see e.g. Cruces et al. 2002). We set the coupon rate δ equal to $\bar{r}$, such that the price of a riskless real bond, $(\delta + \lambda) / (\bar{r} + \lambda)$, is normalized to 1.

We set χ such that the average duration of the exclusion period is $1/\chi = 3$ years, consistently with international evidence on exclusion periods in Dias and Richmond (2007). The bond recovery rate parameter, θ , is set such that the mean recovery rate, $\theta\chi/(\chi - \mu)$, is 60%, consistent with the evidence in Benjamin and Wright (2009) and Cruces and Trebesch (2011).

The parameters determining the output loss during the exclusion period, $\hat{b}$ and ε , are set in order for the model with zero inflation to replicate (i) the average ratio of external public debt over GDP across EMU periphery economies in our sample period (35.6%) and (ii) an output decline of 6% following default.³⁶ Regarding the latter, the literature offers a broad range of values, from 2%

³⁴The world interest rate is set to 1% per quarter in Aguiar and Gopinath (2006), Benjamin and Wright (2009), Hatchondo and Martinez (2009), Yue (2010), Mendoza and Yue (2012) and Chatterjee and Eyigungor (2012). The subjective discount rate is set equal to or close to 5% per quarter in Arellano (2008), Benjamin and Wright (2009), Hatchondo and Martinez (2009), and Chatterjee and Eyigungor (2012), and in Aguiar and Gopinath's (2006) model extension with bailouts.

³⁵In particular, we consider Greece, Italy, Ireland, Portugal and Spain. See Appendix G for data sources and treatment. We could have alternatively calibrated our model to an individual country, such as Greece, which displayed somewhat higher growth volatility than the average peripheral economy. As we show in Section 4.1, our results are robust to higher levels of output growth volatility.

³⁶We use the no-inflation scenario as the model counterpart of our sample region and period (the average EMU peripheral economy in the euro period). First, as discussed in section 2.3.3, the no-inflation regime can be interpreted

(Aguilar and Gopinath, 2006) to 13-14% (Mendoza and Yue, 2012; Arellano, 2008). The midpoint of this range would be 8%. We target a more conservative output loss of 6%.

Finally, in order to calibrate the scale of inflation utility costs, ψ , we turn to the inflationary model regime and target an average inflation rate of 3.2%. The latter corresponds to the average CPI inflation differential between the EMU periphery economies and the US during the period 1987-1997.³⁷ We thus use observed inflation differentials in the years *before* the creation of EMU in order to back up the cost of inflation in such countries at a time when they were able to issue debt in their own currency and inflate it away at discretion. Table 1 summarizes the calibration.

Table 1. Baseline calibration

Parameter	Value	Description	Source/Target
$\bar{r}$	0.04	world real interest rate	standard
ρ	0.20	subjective discount rate	standard
μ	0.022	drift output growth	average growth EMU periphery
σ	0.032	diffusion output growth	growth volatility EMU periphery
λ	0.16	bond amortization rate	Macauley duration = 5 years
δ	0.04	bond coupon rate	price of riskless real bond = 1
χ	0.33	reentry rate	mean duration of exclusion = 3 years
θ	0.56	recovery rate parameter	mean recovery rate = 60%
ϵ	1.50	default cost parameter	output loss during exclusion = 6%
$\hat{b}$	0.332	default cost parameter	average external debt/GDP ratio (35.6%)
ψ	9.15	inflation disutility parameter	mean inflation rate (1987-1997) = 3.2%

3.3 Equilibrium

Inflationary equilibrium. The green dotted lines in Figure 1 show the equilibrium in the 'inflationary regime'. As shown by the upper left subplot, the value function declines gently and almost linearly with the country's debt burden, except for debt ratios very close to default when the slope increases sharply. The optimal default threshold equals $b^* = 37.0\%$ and is marked by a green

as an (anti-inflationary) monetary union. Second, as we explain below, we choose the US CPI as the empirical proxy for the 'World price' in the model, which is furthermore normalized to 1. We thus use CPI inflation differentials (rather than levels) relative to the US as the relevant empirical counterpart for inflation in the model. As we show in section 3.4, the average inflation differential across EMU peripheral economies was close to zero (0.4% annual) in our sample period, such that the no-inflation regime provides a good approximation for observed inflation differentials in our sample.

³⁷We thus take the US CPI as the proxy for the 'World price' in the model. Notice also that, since the latter is normalized to 1, we target inflation *differentials* as opposed to inflation levels.

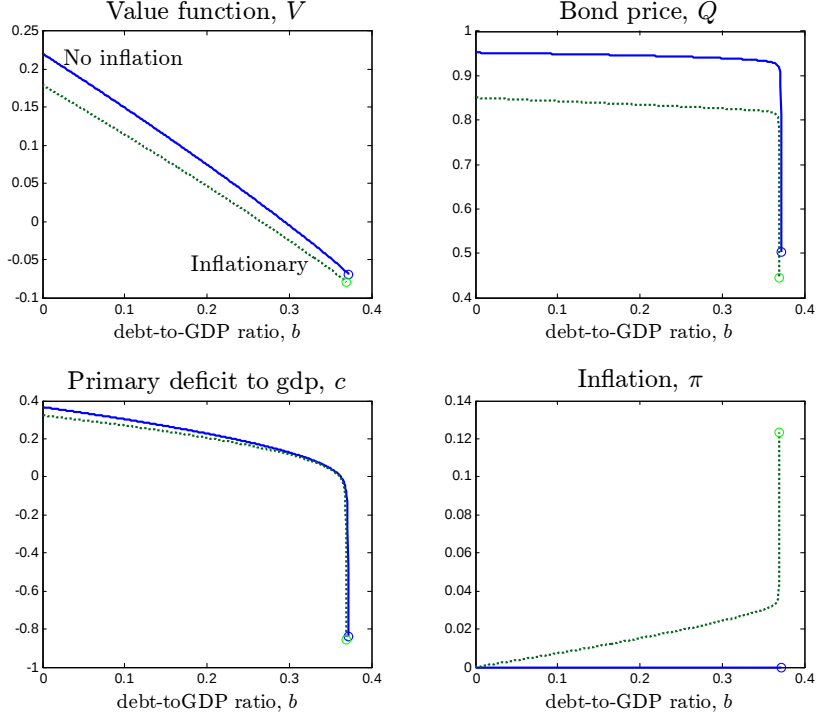


Figure 1: Equilibrium value function, bond price and policy functions.

circle. At that point, the government defaults. Following the exclusion period, it reenters capital markets with a debt ratio $\theta b^* = 20.7\%$.

As regards nominal bond prices, $Q(b)$, their gap with respect to the price of a riskless real bond (normalized to 1) reflects mainly expected inflation during the life of the bond as opposed to default risk, except for debt ratios close to default. This can be seen more clearly in the second line of Figure 2, which displays how the gap between the nominal yield, $r(b) = (\delta + \lambda)/Q(b) - \lambda$, and the riskless real rate $\bar{r}$ is decomposed between the default and inflation premia, as defined in section 2.5. Indeed, for all b except those very close to b^* , bond yields reflect mostly the inflation premium, rather than the default premium, because default is still perceived as a very distant outcome, as reflected by an expected time-to-default of around 40 years. It is only as debt approaches the default threshold that investors start perceiving default as rather imminent, demanding higher and higher default premia, which leads to the collapse of bond prices to their boundary value ($Q(b^*) = 0.44$).

The value function and bond prices, together with the state b , determine in turn the policy functions for inflation and primary deficit, as described in equations (16) and (17). Regarding inflation, the government's incentive to inflate debt away increases approximately linearly with the debt ratio. This is because the value function is approximately linear, such that the welfare gain

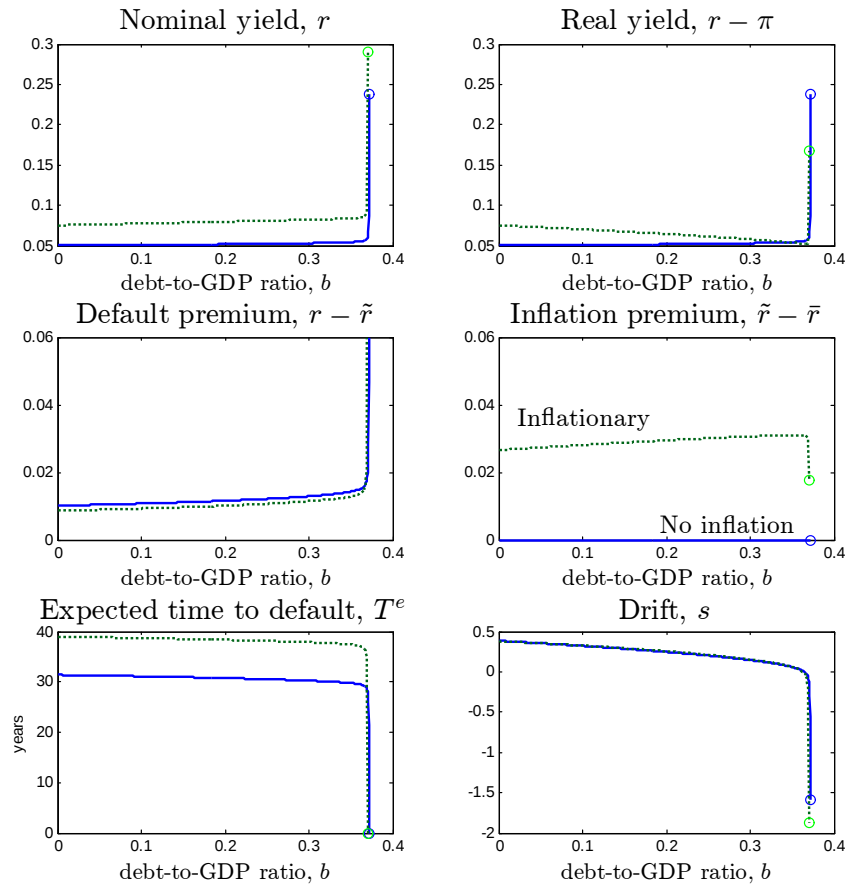


Figure 2: Equilibrium yields, default and inflation premia, expected time-to-default and drift.

per unit of debt reduction is roughly constant. However, in the vicinity of the default threshold, the value function starts declining more and more steeply, such that a marginal reduction in the debt ratio yields a higher and higher marginal gain in welfare.³⁸ As a result, optimal inflation increases steeply until reaching about 12% at the default threshold. Therefore, under discretion, the optimal trade-off between price stability and sovereign debt sustainability prescribes a roughly linear increase in inflation for moderate debt levels, and a strong increase as the economy approaches default.

Finally, the primary deficit ratio declines too in an almost linear fashion, reflecting the gentle decline in bond prices and the nearly constant slope of the value function. As debt approaches the default threshold, however, the sharp decline in bond prices leads the government to drastically reduce its primary deficit, which actually turns to surplus once the economy gets sufficiently close to default.

No-inflation equilibrium. Consider now the equilibrium in the 'no-inflation regime', depicted by the solid blue lines in Figure 1. As explained in section 2.3.3, this scenario can be interpreted as issuing foreign currency debt or joining a monetary union with a very strong anti-inflationary commitment. Notice first that the optimal default threshold ($b_{\pi=0}^* = 37.2\%$; see blue circles) is essentially the same as in the baseline inflationary regime, for reasons that will become clear later. This means that the equilibrium range of debt ratios is basically the same in both regimes. The first subplot of Figure 1 reveals our first main result: the value function is *higher* under no inflation for *any* debt ratio, even when the economy is close to default. A first and obvious reason is that the no inflation regime avoids the utility costs of inflation, $(\psi/2)\pi^2$. A second reason is that the no inflation regime raises bond prices relative to the inflationary regime. Higher bond prices in turn lead the government to choose (slightly) higher primary deficits (see eq. 16), thus allowing for higher consumption given the exogenous output flow.

To understand why the no inflation regime delivers higher bond prices, or equivalently lower bond yields, we show in Figure 2 how the latter are decomposed between default and inflation premia. The no inflation regime raises default premia *vis-à-vis* the inflationary regime. This reflects the fact that default becomes more likely when the government gives up the ability to use inflation so as to stabilize its debt. However, the increase in default premia is very small compared with the reduction in inflation premia (to zero), which results in lower bond yields. The reason for such a small increase in default premia is that, for all debt ratios except those very close to b^* , default is still perceived as rather distant, as reflected by an expected time to default of about 30 years. As a result, the fact that investors expect default to happen somewhat sooner than in the inflationary regime (by about 8 years for most of the state space) is not enough to raise default

³⁸The increase in the slope (in absolute value) of the value function is hard to appreciate in Figure 1, because it only takes place at debt ratios very close to b^* . Zooming in the $V(b)$ plot in the neighborhood of b^* reveals clearly such increase in the slope. The latter plot is available upon request.

premia significantly.

In order to gain further understanding of why social welfare is higher under zero inflation for any debt ratio, we decompose the value function into two components: $V(b) = V_c(b) + V_\pi(b)$, where $V_c(b)$ and $V_\pi(b)$ capture the contribution of consumption utility and inflation disutility, respectively, to overall welfare (net of the autarky value). Notice that consumption utility (net of exogenous (log)output) equals $\log(1 + c(b))$ when the country is in good credit standing and $-\epsilon \max\{0, b - \hat{b}\}$ while in exclusion. Likewise, inflation disutility equals $-\psi \pi(b)^2/2$ while in good credit and zero during exclusion. We thus have

$$\rho \begin{bmatrix} V_c(b) \\ V_\pi(b) \end{bmatrix} = \begin{bmatrix} \log(1 + c(b)) \\ -\frac{\psi}{2} \pi(b)^2 \end{bmatrix} + s(b) \begin{bmatrix} V'_c(b) \\ V'_\pi(b) \end{bmatrix} + \frac{(\sigma b)^2}{2} \begin{bmatrix} V''_c(b) \\ V''_\pi(b) \end{bmatrix},$$

for $b < b^*$, with respective boundary conditions

$$V_c(b^*) = -\frac{\epsilon \max\{0, b^* - \hat{b}\}}{\rho + \chi} + \frac{\chi}{\rho + \chi} V_c(\theta b^*), \quad V_\pi(b^*) = \frac{\chi}{\rho + \chi} V_\pi(\theta b^*) \quad (24)$$

Both value functions can be solved using numerical methods similar to those described in Appendix C.³⁹ Figure 3 shows the contribution of each component to overall welfare in each monetary regime. The reduction in inflation disutility from giving up debt inflation is relatively large and increases slightly with the debt ratio. As regards consumption utility, it contributes positively to the welfare gap between the no-inflation and the inflationary regime at relatively low debt ratios, and negatively in the vicinity of the default threshold. The reason is that, in the inflationary case, the ex-post real yield $r(b) - \pi(b)$ decreases with b (as yields increase less than one-for-one with current inflation) and eventually falls below the no-inflation real rate as the economy approaches default; see Figure 2. *Ceteris paribus*, this permits a lower drift and hence a slower debt accumulation *vis-à-vis* the no-inflation regime, which favors debt sustainability. However, the aforementioned reduction in the real rate is fairly small, as inflation is largely compensated by higher nominal yields. As a result, the contribution of consumption utility is relatively similar in both regimes even when close to default. Likewise, the fact that debt sustainability is barely favored by inflation helps explain why default thresholds are so similar in both regimes.

Figure 3 also reveals the following: the fact that the value function at the default threshold is lower in the inflationary regime is due entirely to the expected inflation costs to be incurred once the government reenters capital markets, $V_\pi(b^*)$. In both monetary regimes, the value function at

³⁹Notice that, since we have already solved for the optimal default threshold b^* , we do not need to impose smooth pasting conditions in order to solve for $V_c(b)$ and $V_\pi(b)$.

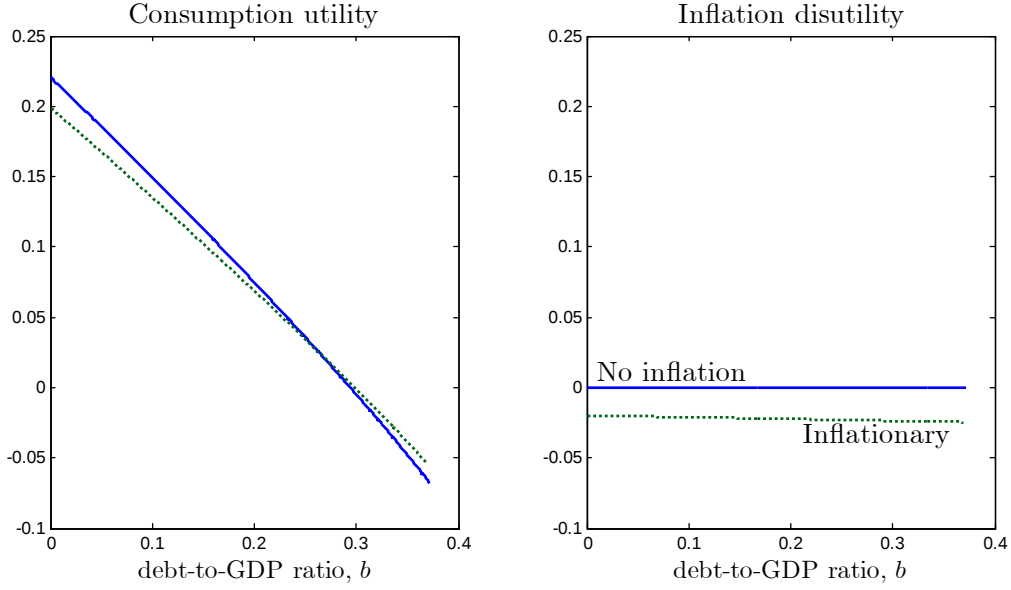


Figure 3: Welfare decomposition.

default equals

$$V(b^*) = V_{def}(b^*) = -\frac{\epsilon \max\{0, b^* - \hat{b}\}}{\rho + \chi} + \frac{\chi}{\rho + \chi} V(\theta b^*),$$

which is precisely the sum of the two terminal values in (24). The fact that b^* is very similar in both cases implies that so is the output loss from default, $\epsilon \max\{0, b^* - \hat{b}\}$, as is the debt ratio at which the government reenters capital markets following the exclusion period ($\theta b^* = 20.7\%$, versus $\theta b_{\pi=0}^* = 20.8\%$).⁴⁰ However, at such reentry ratio the value function is higher in the no-inflation regime, precisely because it avoids the welfare costs of inflation.

To summarize the previous discussion, the no-inflation regime achieves superior welfare outcomes at any debt ratio. It does so by avoiding the temptation to inflate at points of the state space where default is still perceived as rather distant, and hence where the stabilizing benefits from inflating debt away are relatively minor. Even at debt ratios relatively close to default, the gains from inflating debt away discretionarily are rather small, because by then bond yields in both regimes are dominated by the large increase in default premia, reflecting the imminence of default. Therefore, abandoning the ability to inflate debt away eliminates the inflationary bias altogether while barely worsening the sustainability of sovereign debt.

⁴⁰Notice that $b^* > \hat{b} = 0.332$ in both regimes. Thus, the loss in (log)output from default equals $\epsilon(b^* - \hat{b})$ in both cases.

3.4 Average performance

So far we have analyzed the equilibrium value function and policy functions, i.e. the optimal choices of fiscal and monetary policy and the associated welfare at each point of the state space. The main result from the previous section is that the no-inflation regime yields higher welfare at any debt ratio, including at the respective default thresholds. This does not guarantee however that *average* welfare would be higher too. Since value functions are monotonically decreasing in both regimes, if the inflationary regime delivered sufficiently lower debt ratios most of the time, it could also achieve higher average welfare.

In order to compute unconditional averages of welfare and other variables, we thus need to solve for the stationary distribution of the state variable, the debt ratio. For this purpose, it is useful to distinguish between (a) 'normal' times in which the country is in good credit standing and (b) the exclusion periods that follow each default. The stationary distribution conditional on being in 'normal' times, which we may denote by $f(b)$, satisfies the following *Kolmogorov Forward Equation* (KFE),

$$0 = -\frac{d}{db} \{s(b) f(b)\} + \frac{1}{2} \frac{d^2}{db^2} [(\sigma b)^2 f(b)] + \chi h \delta(b - \theta b^*) - \chi h \delta(b - b^*), \quad (25)$$

with the constraint $1 = \int_0^{b^*} f(b) db$. In equation (25), the term $\chi h \delta(b - \theta b^*)$ reflects the fact that, following an exclusion spell, the government reenters the capital market at a debt ratio $b = \theta b^*$, where $\delta(\cdot)$ is the Dirac 'delta' and h is a function of the average time spent in exclusion. Likewise, $\chi h \delta(b - b^*)$ captures the fact that at $b = b^*$ the government defaults and hence exits the conditional distribution $f(b)$. Appendix F provides further details on how to obtain equation (25) and shows how to compute $f(b)$ numerically, using an upwind finite difference scheme similar to the one employed to solve for the value and bond price functions. Figure 4 displays the stationary distributions of the debt ratio for both the inflationary and the no-inflation regimes, conditional on being in normal times. In the baseline regime, the possibility of using inflation to inflate debt away allows the government to shift the debt distribution slightly to the left *vis-à-vis* the no-inflation regime.

Conditional on being in an exclusion period, we have already seen that primary deficit and inflation are both zero, $c_t = \pi_t = 0$. Since the rate at which the country reenters capital markets is constant at χ and hence independent of the time elapsed since default, we have that the value function and bond price are equal to their boundary values: $V_t = V(b^*) = V_{def}(b^*)$, $Q_t = Q(b^*)$. Finally, we assume for simplicity that during the exclusion period the debt ratio is equal to b^* , i.e. the ratio at which the country defaults.⁴¹

⁴¹We are thus assuming that during the exclusion/renegotiation period nominal debt outstanding is adjusted at each point in time to changes in the output endowment, such that the debt ratio is kept constant at b^* . We could

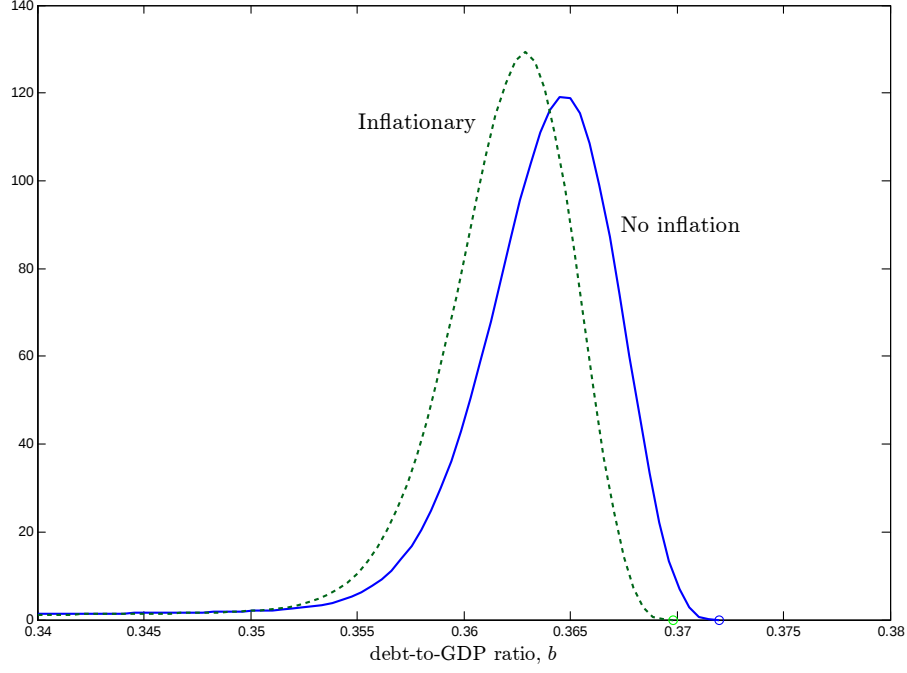


Figure 4: Stationary distribution of debt ratio, $f(b)$.

We can now compute the unconditional mean of each variable as the weighted average of the conditional means, using as weights the average time spent in normal and exclusion periods. It is relatively straightforward to show that the stationary probability of being in normal times and in exclusion periods equal $\mathbb{P}[b_t < b^*] = \frac{T^e(\theta b^*)}{1/\chi + T^e(\theta b^*)}$ and $\mathbb{P}[b_t = b^*] = \frac{1/\chi}{1/\chi + T^e(\theta b^*)}$, respectively. Thus, the unconditional mean of a variable x_t equals

$$\begin{aligned} \mathbb{E}[x_t] &= \mathbb{P}[b_t < b^*] \mathbb{E}[x_t | b_t < b^*] + \mathbb{P}[b_t = b^*] x^* \\ &= \frac{T^e(\theta b^*)}{1/\chi + T^e(\theta b^*)} \int_0^{b^*} x(b) f(b) db + \frac{1/\chi}{1/\chi + T^e(\theta b^*)} x^*, \end{aligned}$$

where x^* is the value of x_t during the exclusion period.⁴²

Table 2 displays average values of key model variables for both monetary regimes, as well as their corresponding empirical counterparts across EMU periphery countries.⁴³ Notice first that,

alternatively assume that, during the exclusion period, nominal debt outstanding is kept constant at its value at the time of default (B_t^*), such that the debt ratio changes with the output endowment. This would complicate the analysis while barely affecting the numerical results, given the relatively short average duration of the exclusion period.

⁴²As explained above, $c^* = 0$, $\pi^* = 0$, $V^* = V(b^*)$, and $Q^* = Q(b^*)$.

⁴³All data are annual except bond yields and default premia which are quarterly and annualized. We stop the sample for yields and default premia in 2012:Q2 (included) in order to isolate our analysis from the effects of the announcement by the European Central Bank of the Outright Monetary Transactions (OMT) programme in the

remarkably, the no inflation regime (our model counterpart for the EMU period) replicates almost exactly the average bond default premium (154 bp) conditional on being in good credit standing ($b < b^*$). In the inflationary regime, average bond yields (net of $\bar{r} = 400$ bp) during repayment spells equal 446 bp, which reflects mostly average inflation premia (309 bp) rather than average default premia (137 bp). They are also significantly higher than average yields under no inflation.

Table 2. Averages values

		Data	Model			
		1995-2012	No inflation		Inflationary	
	units		$b < b^*$	uncond.	$b < b^*$	uncond.
debt-to-GDP, b	%	35.6	35.6	35.8	35.6	35.7
primary deficit ratio, c	%	4.1	0.5	0.5	0.4	0.4
inflation, π	%	0.4	0	0	3.2	2.9
bond yields (net of $\bar{r}$), $r - \bar{r}$	bp	187	153	315	446	596
default premium, $r - \tilde{r}$	bp	154	153	315	137	297
inflation premium, $\tilde{r} - \bar{r}$	bp	33	0	0	309	299
Exp. time to default, T^e	years	-	29.5	-	37.1	-
Welfare loss, $V - V_{\pi=0}$	% cons.	-	0	0	-0.27	-0.26

Note: Data from IMF, national accounts, and Bloomberg. All data are annual except bond yields and default premia which are quarterly (annualized) and run through 2012:Q2. Inflation is relative to the US. See Data Appendix for details. The German 10-year bond yield is used as empirical proxy for the riskless bond yield, $\tilde{r}$. The column labelled ' $b < b^*$ ' displays results conditional on not being in exclusion, the column labelled 'uncond.' displays fully unconditional results. Welfare losses are relative to the no-inflation regime and are expressed in % of permanent consumption.

Interestingly, the fact that the no-inflation regime delivers lower average yields than the inflationary regime rationalizes the observed reduction in average sovereign yields across the EMU periphery brought about by the creation of the eurozone, if one interprets both regimes as the model counterparts of the EMU and pre-EMU periods respectively. Indeed, average yields on 10-year peripheral bonds decreased from 12.84% in the period 1987-94 to 5.87% in 1995-2012.⁴⁴ Viewed through the lens of our model, this suggests that, when these countries decided to renounce the ability to inflate away their debts by joining EMU, the reduction in inflation expectations was a more important factor in investors' pricing of the new euro-denominated bonds than the pre-sustainable increase in default risk.

summer of 2012.

⁴⁴Notice that $r = 5.87\% = (r - \tilde{r}) + (\tilde{r} - \bar{r}) + \bar{r} = 1.54\% + 0.33\% + 4\%$.

From a welfare perspective, we find that average welfare is *lower* in the inflationary regime, i.e. when the government inflates away its debt at discretion. The average welfare losses *vis-à-vis* the no-inflation regime are equivalent to a reduction in permanent consumption of almost 0.3%. Therefore, the leftward shift in the debt distribution shown in Figure 4 is not sufficient to compensate for the fact that the value function is lower at any debt ratio. Such a small shift in the debt distribution reflects the low effectiveness of discretionary debt inflation policies in our framework. Notice that optimal instantaneous inflation is relatively high in the range where the debt distribution accumulates more density, i.e. for $b > 0.35$.⁴⁵ However, this is largely undone by the increase in nominal yields that goes along with higher inflation expectations. As a result, in the relevant debt range the inflationary regime achieves only marginally lower real interest rates, and hence only marginally slower debt accumulation, relative to the no inflation regime.

4 Robustness

We now evaluate the robustness of our main results to alternative calibrations of key model parameters. As we show next, our main results on the welfare ranking between the no inflation and the inflationary regime (both state by state and on average) continue to hold for a wide range of parameter values. The only exception is the case of very high output growth volatility (σ), which we analyze in a separate subsection below.

Bond duration. The amortization rate λ determines the average Macaulay bond duration, $1/(\lambda + \bar{r})$, for given riskless real return $\bar{r}$. Table 3 displays averages of a number of key variables for bond durations of 3 and 7 years, both for the no-inflation and the baseline inflationary regimes. For comparison, it also displays the same statistics for the benchmark calibration, with a 5-year bond duration. We find that average welfare continues to be higher in the no-inflation regime. The welfare loss from using discretionary inflation decreases with bond duration. Intuitively, longer bond durations give more stability to the debt ratio, thus reducing the need to use debt inflation. This allows to reduce inflation premia in bond yields and direct utility costs, and hence the welfare loss relative to the no-inflation case.

Bond recovery rate. The bond recovery parameter, θ , controls the average bond recovery rate after default, $\theta\chi/(\chi - \mu)$, for given reentry and trend growth rates (χ, μ) . Table 3 displays results for average recovery rates of 50% and 70% (the benchmark calibration is 60%). Again, average welfare is higher if the government renounces the possibility to inflate debt away. In this case, the welfare gains are fairly similar across different calibrations. As in the baseline calibration, the reduction in average inflation premia from giving up debt inflation clearly dominates the increase in average default premia.

⁴⁵As shown in Figure 1, for $b > 0.35$ inflation is above 3%, with a maximum of around 12% in the limit as $b \rightarrow b^*$.

Output loss from default. Parameter $\hat{b}$ controls the loss in (log-)output following default, $\epsilon \max\{0, b^* - \hat{b}\}$, for given scale parameter ϵ and equilibrium default threshold b^* . We consider values of $\hat{b}$ such that, in equilibrium, output declines by 3.5% and 7% upon default (compared to the benchmark 6% loss). In this case, the welfare gains from not inflating debt away, while positive, seem more sensitive to the size of output losses associated to default. The reason is the following. In our model, a positive relationship exists between $\hat{b}$ and average debt ratios. Therefore, higher values of $\hat{b}$ imply higher debt on average and therefore a stronger incentive to inflate the latter away. Higher average inflation in turn raises inflation premia and direct utility costs, thus increasing the welfare gap with respect to the no-inflation scenario.

Household discount rate. In our baseline calibration, the household discount rate ρ was set to 20% per annum. As discussed in the calibration section, the latter value is widely used in the quantitative sovereign default literature. However, one may wonder how robust our results are to making households, and hence the (benevolent) government, more patient. Table 3 shows the effects of considering discount rates of 10% and 6%.⁴⁶ We find that, as households become more patient, the average welfare gain from renouncing the use of inflation increases. Intuitively, the more patient households are, the less they discount the utility costs that they may incur in the future when, following a default, their government reenters capital markets and starts inflating again.

Inflation disutility-average inflation. In our baseline calibration exercise, we set the scale parameter of inflation disutility, ψ , in order for the inflationary model regime to replicate the inflation record in our target economies (see section 3.2). This delivered a baseline value of $\psi = 9.15$, which produced an average inflation of 3.2% conditional on being in good credit standing, and 2.9% including exclusion spells. We now consider values of ψ that deliver average inflation rates of 2% and 4%. We find that renouncing debt inflation continues to dominate in welfare terms, with the welfare gap decreasing with the scale of inflation disutility. Intuitively, higher inflation costs lowers the incentive to inflate debt in the inflationary regime, thus reducing the direct welfare costs and the increase in nominal yields associated to the latter regime.

Duration of exclusion period. The reentry rate χ determines the average duration of the exclusion periods following default, $1/\chi$, set to 3 years in our benchmark calibration. We study the effects of considering average durations of 2 and 5 years. In addition, we consider the relatively extreme case of a 40-year duration, which approximates the case of a permanent autarky following default. Table 3 reveals that our benchmark results remain mostly unchanged, even for quasi-permanent exclusion from capital markets. As we saw in the previous section, the welfare gap between both monetary regimes depends not so much on what happens in exclusion (during which

⁴⁶Our algorithm fails to converge in the inflationary case for discount rates ρ below 5.7%, i.e. for gaps between ρ and investor's discount rate $\bar{r}$ below 1.7%.

output losses are fairly similar and inflation is zero in both cases), but on what happens while in good credit standing. As a result, changes in duration of the exclusion period do not significantly alter the welfare gap between both regimes.

Table 3. Robustness analysis

	Welfare % cons.	Prim. deficit ratio, %	Inflation %	Nominal yield net of $\bar{r}$, bp	Premia (bp)		Exp. time to default, years
					Default	Inflation	
Benchmark							
No inflation	-	0.5	0	153	153	0	29.5
Inflationary	-0.26	0.4	2.9	446	137	309	37.1
Average bond duration = 3 years							
No inflation	-	0.2	0	111	111	0	40.4
Inflationary	-0.43	0.2	3.3	448	106	342	47.4
Average bond duration = 7 years							
No inflation	-	0.6	0	173	173	0	26.0
Inflationary	-0.15	0.5	2.7	433	148	286	34.7
Bond recovery rate = 50%							
No inflation	-	0.6	0	174	174	0	30.8
Inflationary	-0.25	0.5	3.0	466	154	312	38.6
Bond recovery rate = 70%							
No inflation	-	0.3	0	129	129	0	28.3
Inflationary	-0.27	0.2	2.9	425	118	307	35.8
Default costs = 3.5% of GDP							
No inflation	-	0.6	0	152	152	0	29.8
Inflationary	-0.09	0.5	1.8	329	142	187	34.5
Default costs = 7% of GDP							
No inflation	-	0.4	0	152	152	0	29.6
Inflationary	-0.35	0.3	3.4	491	135	356	38.4
Household discount rate = 10%							
No inflation	-	-0.6	0	53	53	0	87.9
Inflationary	-0.70	-0.6	3.2	374	46	328	115.2
Household discount rate = 6%							
No inflation	-	-0.7	0	22	22	0	209.5
Inflationary	-1.26	-0.6	3.3	351	18	333	294.2

Note: Welfare is calculated with respect to the corresponding no-inflation scenario and is expressed in % of

permanent consumption. Average values are unconditional for welfare, deficit and inflation; for all other variables, averages are conditional on not being in exclusion ($b < b^*$). Benchmark calibration: average bond duration = 5 years, bond recovery rate = 60%, default cost = 6% of GDP, discount rate = 20%

Table 3 (cont'd). Robustness analysis

	Welfare % cons.	Prim. deficit ratio, %	Inflation %	Nominal yield net of $\bar{r}$, bp	Premia (bp)		Exp. Time to default, years
					Default	Inflation	
Scale inflation disutility $\psi = 6.40$ (average inflation = 4%)							
No inflation	-	0.5	0	153	153	0	29.5
Inflationary	-0.34	0.4	4.0	551	133	419	40.0
Scale inflation disutility $\psi = 13.99$ (average inflation = 2%)							
No inflation	-	0.5	0	153	153	0	29.5
Inflationary	-0.18	0.4	2.0	352	142	210	34.6
Average duration of exclusion period = 2 years							
No inflation	-	0.5	0	181	181	0	23.6
Inflationary	-0.25	0.4	3.0	477	160	316	29.6
Average duration of exclusion period = 5 years							
No inflation	-	0.4	0	127	127	0	39.0
Inflationary	-0.27	0.3	2.8	420	116	304	49.2
Average duration of exclusion period = 40 years							
No inflation	-	0.6	0	126	126	0	74.3
Inflationary	-0.25	0.5	2.1	409	115	294	93.2
Volatility output growth = 10%							
No inflation	-	-0.7	0	285	285	0	15.7
Inflationary	-0.08	-0.6	2.6	539	263	275	19.0
Volatility output growth = 20%							
No inflation	-	-1.2	0	279	279	0	16.2
Inflationary	0.07	-1.1	2.4	519	262	257	19.1

Note: Welfare is calculated with respect to the corresponding no-inflation scenario and is expressed in % of permanent consumption. Average values are unconditional for welfare, deficit and inflation; for all other variables, averages are conditional on not being in exclusion ($b < b^*$). Benchmark calibration: scale inflation disutility $\psi = 9.15$, average duration of exclusion period = 3 years, output growth volatility = 3.2%

Our robustness analysis thus far shows a positive *average* welfare gap between the no-inflation and the inflationary regime for all the cases considered. Importantly, as in the case of the baseline calibration, such positive welfare gap takes place not only on average, but also *state by state*.

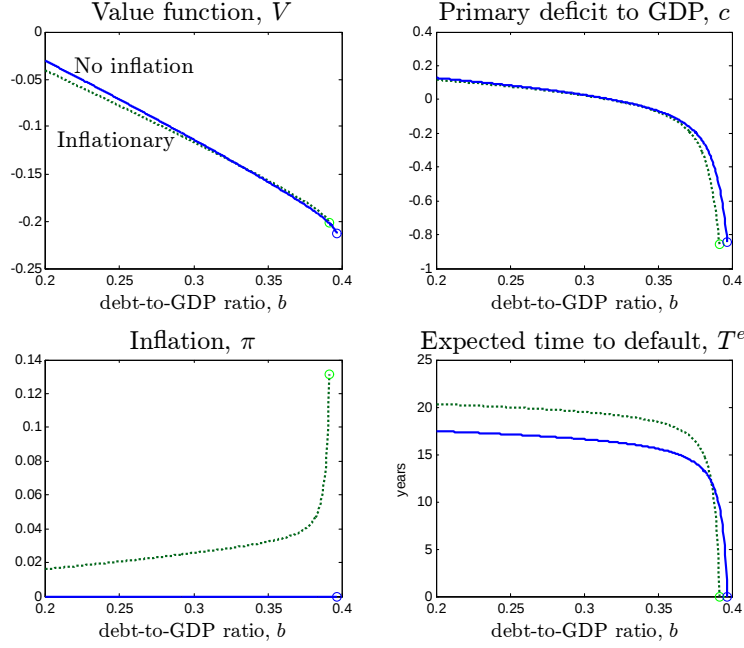


Figure 5: Equilibrium with $\sigma = 20\%$.

Appendix H displays the value functions in both regimes for each of the alternative calibrations considered thus far. As we show there, both value functions never cross for any of the cases considered.

4.1 Volatility of output shocks

As discussed in the calibration section, our baseline value for the volatility of shocks to output growth, $\sigma = 3.2\%$, is calibrated to match the average growth volatility across peripheral EMU economies since the creation of the euro. However, to the extent that one wants to extrapolate the present analysis to other geographical contexts, such as emerging market economies, one may want to consider higher levels of output growth volatility. The last panel of Table 3 shows the effects of raising σ to 10%; for the purpose of illustration, we also consider an extremely large volatility of 20%. The first message is that the average welfare gap between both regimes decreases with output volatility. Moreover, for output volatilities as high as 20%, the inflationary regime actually achieves *higher* average welfare relative to the no inflation regime, even though the difference is of second order. Figure 5 displays the value and policy functions for $\sigma = 20\%$. As shown by the upper left panel, while the no inflation regime continues to dominate the inflationary one for most debt ratios, the opposite is true for *debt ratios close to default*.

To understand the above results, notice that in our model inflation acts as a sort of state-

contingent fiscal instrument that allows the government to (partially) absorb the effect of output shocks on the debt ratio, and hence on the probability of defaulting within a certain period.⁴⁷ When the debt ratio is relatively far away from the default threshold, such an insurance role is relatively unimportant, even when output shocks are very large. As a result, renouncing such an insurance tool is relatively costless in terms of welfare, and the no inflation regime continues to outperform the inflationary one. But when the economy is relatively close to default and output growth is very volatile, the probability of defaulting within a relatively short period of time becomes much higher. This becomes apparent by comparing the expected time-to-default function in Figure 5 with that under the baseline calibration (Figure 2): with higher growth volatility, not only are expected times-to-default lower for all debt levels, but they also start decreasing towards zero at lower debt ratios. As a consequence, for debt ratios sufficiently close to default, the shock-absorbing role of state-contingent inflation becomes important enough for the inflationary regime to actually deliver higher welfare.⁴⁸ Moreover, because the economy defaults (and hence incurs the consumption losses associated to default) sufficiently less often under the inflationary regime, the latter is able to achieve higher welfare also on average.

In any case, we emphasize that the degree of output growth volatility required in order for the inflationary regime to deliver higher welfare is high enough to be of little practical importance, even for emerging market economies.⁴⁹ We thus conclude that, for realistic levels of output growth volatility, our main results regarding the desirability of renouncing the ability to conduct discretionary inflation policies remain robust.

4.2 Default costs

In our baseline model, we assumed that defaulting at some debt ratio b entails a loss in (log)output given by $\epsilon \max\{0, b - \hat{b}\}$.⁵⁰ We now check the robustness of our results to alternative specifications of default costs. In particular, we consider the following functional form: $\kappa(e^{\alpha b} - 1) \equiv \Theta(b)$. Contrary to the baseline function, which has a kink at $\hat{b}$ and increases linearly thereafter, this

⁴⁷For analyses of the shock-absorbing role of state-contingent inflation in an economy with nominal non-contingent government debt and nominal rigidities but no equilibrium default, see Siu (2004) and Schmitt-Grohé and Uribe (2004).

⁴⁸Notice that this is despite the fact that the two arguments of the utility function, the primary deficit ratio and inflation, continue to work in favor of the no inflation regime, as shown by Figure 5. Also, default thresholds b^* , and hence output costs from default, are again very similar in both regimes. Therefore, the welfare advantage of the inflationary regime at debt ratios close to default must be due to lower default probabilities.

⁴⁹We have computed the threshold values of σ above which the inflationary regime delivers higher welfare (i) on average and (ii) for at least some debt ratios, holding all other parameters at their baseline values. The thresholds are (i) 15.1% and (ii) 15.8%, respectively.

⁵⁰As discussed in section 2.3.1, having the output costs of default increase with the debt ratio can be seen as a reduced form for a setup in which firms use government debt as collateral in order to obtain funding for their activities.

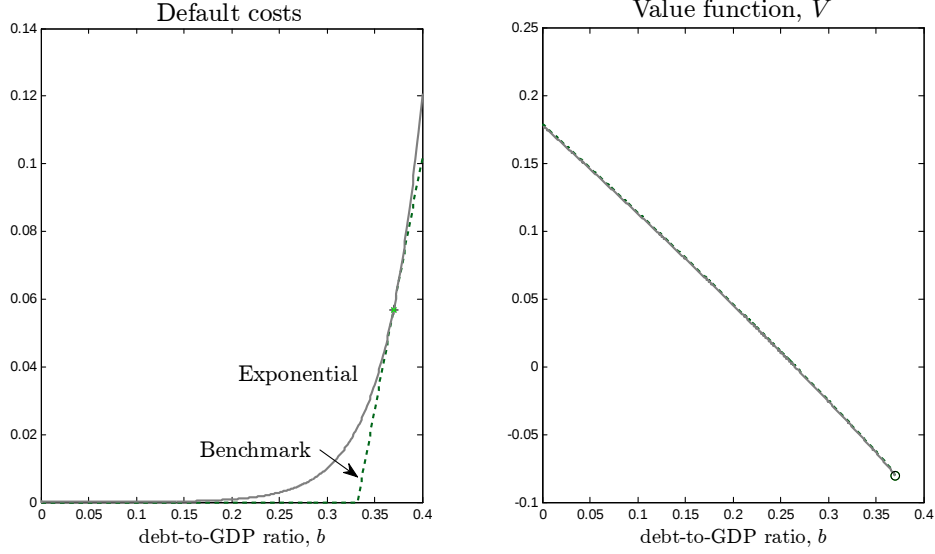


Figure 6: Default costs and value function in the inflationary case for alternative default cost specifications.

specification is a smooth, convex function of b . It shares however a number of plausible properties. First, $\Theta(0) = 0$, i.e. there is no cost of default if there is no debt to default upon. Second, $\Theta(b) \geq 0$, i.e. default *is* costly. Third, $\Theta'(b) \geq 0$, i.e. the cost should be increasing in the debt ratio that is defaulted upon. Finally, it only depends on two parameters (κ, α) , and is thus equally parsimonious.

As in our baseline calibration exercise, we choose κ and α such that the no inflation regime replicates (i) the average ratio of external government debt over GDP across the EMU periphery economies (35.6%) and (ii) an output decline of 6% following default. Since default costs affect the equilibrium only through the value matching and smooth pasting conditions (equations 12 and 13), this is equivalent to imposing that the new cost function has the same value and slope at the default threshold $b_{\pi=0}^*$.⁵¹ Therefore, both cost specifications yield the exact same equilibrium in the no-inflation case by construction. However, since the default threshold is endogenous, both specifications may yield different equilibrium outcomes in the inflationary regime.

The baseline and alternative default cost function are displayed in the left panel of Figure 6. As shown by the right panel, both specifications result in an almost identical value function *also*

⁵¹That is, α and κ must satisfy

$$\begin{aligned} \epsilon \left(b_{\pi=0}^* - \hat{b} \right) &= \kappa \left(e^{\alpha b_{\pi=0}^*} - 1 \right), \\ \epsilon &= \kappa \alpha e^{\alpha b_{\pi=0}^*}, \end{aligned}$$

where we have used the fact that, in the equilibrium with the baseline cost formulation, $b_{\pi=0}^* > \hat{b}$. For the values of ϵ and $\hat{b}$ in Table 1, we obtain $\alpha = 25.02$ and $\kappa = 5.44 \cdot 10^{-6}$.

in the inflationary case.⁵² The reason is that, once again, the default threshold in the inflationary case ($b^* = 36.98\%$) is very close to that in the no-inflation regime ($b_{\pi=0}^* = 37.20\%$), and hence so is the slope of the default costs at both thresholds, thus making the linear function $\epsilon(b - \hat{b})$ a good approximation of $\kappa(e^{\alpha b} - 1)$ around $b_{\pi=0}^*$. This conclusion also holds for other plausible default cost functions that satisfy the above properties, such as a 2-parameter polynomial approximation. This makes us confident about the robustness of our results to alternative functional forms.

5 Monetary policy delegation

As explained in section 2.3.3, the no-inflation regime can be interpreted as the government issuing foreign currency debt, or joining a monetary union with a very strong anti-inflationary stance. We also argued that one could view the 'no inflation' regime as a situation in which the government appoints an independent central banker with an extremely great distaste for inflation.

In this section, we consider an intermediate arrangement by which the government delegates (discretionary) monetary policy to an independent central banker whose distaste for inflation is greater than that of society, but not so extreme as to imply zero inflation at all times. The question here is whether one can find intermediate preferences towards inflation that achieve better welfare outcomes than the two regimes considered thus far.

Formally, our maximization problem is modified as follows. On the one hand, the benevolent government retains the primary deficit and default decisions, taking as given the inflation policy function of the independent monetary authority, which we denote by $\tilde{\pi}(b)$. With a slight abuse of notation, let $V(b)$ denote the value function of the government when the latter no longer chooses inflation. The corresponding HJB equation is

$$\rho V(b) = \max_{c, b^*} \left\{ \log(1 + c) - \frac{\psi}{2} \tilde{\pi}(b)^2 + s(b, c, \tilde{\pi}(b)) V'(b) + \frac{(\sigma b)^2}{2} V''(b) \right\}, \quad (26)$$

where the value matching and smooth pasting conditions are given again by equations (12) and (13), respectively. The optimal primary deficit ratio is given again by equation (16). Investors' bond pricing schedule $Q(b)$ (which affects the drift s) is determined exactly as before.

The monetary authority chooses inflation taking as given the government's primary deficit policy, $c(b)$, and optimal default threshold, b^* . Letting $\tilde{V}(b)$ denote the monetary authority's

⁵²We only show the value function but the two models also produce identical policies and bond prices. Results are available upon request.

value function, the latter satisfies the following HJB equation,

$$\rho \tilde{V}(b) = \max_{\pi} \left\{ \log(1 + c(b)) - \frac{\tilde{\psi}}{2} \pi^2 + s(b, c(b), \pi) \tilde{V}'(b) + \frac{(\sigma b)^2}{2} \tilde{V}''(b) \right\}, \quad (27)$$

where $\tilde{\psi} \geq \psi$ captures the central banker's distaste for inflation. $\tilde{V}$ also satisfies a value matching condition analogous to (12). The optimal inflation decision is given by equation (17) with $\tilde{\psi}$ and $\tilde{V}'$ replacing ψ and V' , which defines the new inflation policy function $\tilde{\pi}(b)$. Notice that $\lim_{\tilde{\psi} \rightarrow \infty} \tilde{\pi}(b) = 0$ for all b . Thus, as argued in section 2.3.3, the 'no inflation' regime can be viewed as an extreme case of the independent central banker problem laid out here, in which the latter has an arbitrarily great distaste for inflation.

In order to solve this problem we need to extend the numerical algorithm introduced in section 3.1. In particular, we replace the government problem (step 1) by:

Step 1a: Government problem. Given $Q^{(n-1)}$, $\pi^{(n-1)}$ and $b_{(n-1)}^*$, we solve the HJB equation (26) in the domain $[0, b_{(n-1)}^*]$ imposing the smooth pasting condition (13) to obtain an estimate of the government's value function $V^{(n)}$ and of primary deficit $c^{(n)}$.

Step 1b: Central bank problem. Given $Q^{(n-1)}$, $c^{(n)}$ and $b_{(n-1)}^*$, we solve the HJB equation (27) in the domain $[0, b_{(n-1)}^*]$ imposing the value matching condition (12) to obtain an estimate of the central bank's value function $\tilde{V}^{(n)}$ and of inflation $\pi^{(n)}$.

Figure 7 displays the unconditional means of social welfare and other relevant variables as we vary the conservative central banker's distaste for inflation, $\tilde{\psi}$.⁵³ The main message is that average social welfare increases monotonically with the inflation conservatism of the delegated monetary authority, but it is always *lower* than that achieved in the no-inflation regime, which is reached only asymptotically ($\tilde{\psi} \rightarrow \infty$).

To understand this result, the figure also displays the contribution of consumption utility and inflation disutility to average welfare. On the one hand, central bank conservativeness reduces the average welfare due to consumption. Intuitively, the economy is more likely to default and hence to incur output losses when the monetary authority is more focused on stabilizing inflation. On the other hand, central bank conservativeness also reduces the average welfare costs of inflation. Quantitatively, the latter effect outweighs the first one, which explains the increase in overall welfare. Interestingly, we also find that a more conservative monetary authority reduces average nominal interest rates, reflecting once again the fact the ensuing reduction in average inflation

⁵³To facilitate interpretation, the x-axes in Figure 7 display the central banker's distaste for inflation relative to that of society, $\tilde{\psi}/\psi$, where ψ is held constant at its calibrated value (see Table 1). Thus $\tilde{\psi}/\psi = 1$ represents our baseline 'inflationary regime' in which inflation is chosen by the benevolent government.

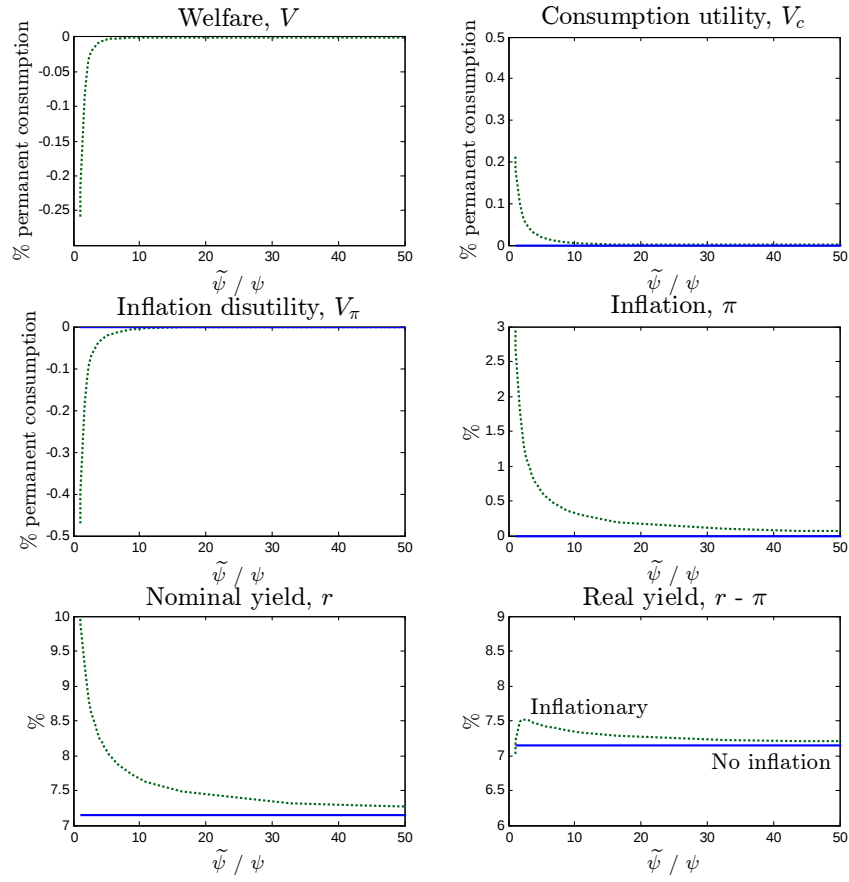


Figure 7: Effect of central bank conservatism under monetary policy delegation (average values).

premia dominates the increase in average default premia. Beyond a certain degree of conservatism, it also reduces average *real* interest rates, as the reduction in average yields outpaces that in average inflation.

To summarize our results in this section, we find that if the government is unable to make credible commitments, delegating monetary policy to an independent, relatively conservative central banker achieves better welfare outcomes by reducing current and expected inflation. However, such an institutional solution continues to be dominated by a scenario in which the government fully renounces the ability to inflate debt away, as would be exemplified e.g. by issuing foreign currency debt or joining a monetary union with a very strong and credible anti-inflationary mandate.

6 Conclusions

We have analyzed the trade-offs between price stability and the sustainability of sovereign debt, in the context of a small open economy model where a benevolent government issues nominal debt but cannot commit not to default on it and not to inflate it away. Our main focus has been to compare this scenario with an alternative regime in which the government effectively renounces the option to inflate debt away, e.g. by issuing foreign currency debt or joining an anti-inflationary monetary union.

We have found that giving up the option to inflate debt away achieves higher welfare, both at any debt ratio and on average. The reason lies in the costs and benefits of optimal discretionary inflation. On the one hand, inflation increases with the debt ratio (the only state variable in our model); thus, unanticipated inflation partially absorbs the effects of output growth shocks on such debt ratio, which favors sovereign debt sustainability. On the other hand, discretionary monetary policy features an inflation bias, by which the government chooses positive inflation even when the economy is far away from default. This bias entails direct welfare costs, but also raises bond yields *ceteris paribus*, which makes (external) primary deficits more costly to finance and hence lowers consumption for given output. Abandoning the ability to inflate debt away eliminates the inflationary bias altogether, but also the shock-absorbing role of state-contingent inflation. We have found that, for plausible calibrations, the first effect dominates: the welfare benefits from eliminating the inflationary bias outweigh the costs from losing the debt-stabilizing role of inflation.

Our results thus qualify the conventional wisdom according to which individual countries should benefit from retaining the possibility of inflating away their sovereign debt, in the sense that such a benefit may not materialize if governments in those countries cannot make commitments about its future monetary policy. Our findings may also rationalize why some emerging market economies issue their sovereign debt in a hard, foreign currency, thus renouncing the ability to inflate it away despite the presumable increase in its vulnerability.

Looking ahead, we note that we have analyzed the problem of a single government in a small open economy setup. Given our interest in recent developments in the euro area, we believe that extending the analysis presented here to the case of a monetary union with a common monetary authority and many national fiscal authorities that differ in their outstanding sovereign debt levels is of great importance. We leave this task for future research.

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Appendix

A. An economy with costly price adjustment

In this appendix, we lay out a model economy with the following characteristics: (i) firms are explicitly modelled, (ii) a subset of them are price-setters but incur a convex cost for changing their nominal price, and (iii) the social welfare function and the equilibrium conditions are the same as in the model economy in the main text.

Final good producer

In the model laid out in the main text, we assumed that output of the single consumption good Y_t is exogenous. Consider now an alternative setup in which the single consumption good is produced by a representative, perfectly competitive final good producer with the following Dixit-Stiglitz technology,

$$Y_t = \left(\int_0^1 y_{it}^{(\varepsilon-1)/\varepsilon} di \right)^{\varepsilon/(\varepsilon-1)}, \quad (28)$$

where $\{y_{it}\}$ is a continuum of intermediate goods and $\varepsilon > 1$. Let P_{it} denote the nominal price of intermediate good $i \in [0, 1]$. The firm chooses $\{y_{it}\}$ to maximize profits, $P_t Y_t - \int_0^1 P_{it} y_{it} di$, subject to (28). The first order conditions are

$$y_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} Y_t, \quad (29)$$

for each $i \in [0, 1]$. Assuming free entry, the zero profit condition and equations (29) imply $P_t = (\int_0^1 P_{it}^{1-\varepsilon} di)^{1/(1-\varepsilon)}$.

Intermediate goods producers

Each intermediate good i is produced by a monopolistically competitive intermediate-good producer, which we will refer to as 'firm i ' henceforth for brevity. Firm i operates a linear production technology,

$$y_{it} = Z_t n_{it}, \quad (30)$$

where n_{it} is labor input and Z_t is productivity. The latter is assumed to follow a geometric Brownian motion,

$$dZ_t = \mu Z_t dt + \sigma Z_t dW_t. \quad (31)$$

At each point in time, firms can change the price of their product but face quadratic price adjustment cost as in Rotemberg (1982). Letting $\dot{P}_{it} \equiv dP_{it}/dt$ denote the change in the firm's price,

price adjustment costs in units of the final good are given by

$$\Psi_t \left(\frac{\dot{P}_{it}}{P_{it}} \right) \equiv \frac{\psi}{2} \left(\frac{\dot{P}_{it}}{P_{it}} \right)^2 \tilde{C}_t, \quad (32)$$

where $\tilde{C}_t$ is aggregate consumption. Let $\pi_{it} \equiv \dot{P}_{it}/P_{it}$ denote the rate of increase in the firm's price. The instantaneous profit function in units of the final good is given by

$$\begin{aligned} \Pi_{it} &= \frac{P_{it}}{P_t} y_{it} - w_t n_{it} - \Psi_t(\pi_{it}) \\ &= \left(\frac{P_{it}}{P_t} - \frac{w_t}{Z_t} \right) \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} Y_t - \Psi_t(\pi_{it}), \end{aligned} \quad (33)$$

where w_t is the perfectly competitive real wage and in the second equality we have used (29) and (30). Without loss of generality, firms are assumed to be risk neutral and have the same discount factor as households, ρ . Then firm i 's objective function is

$$\mathbb{E}_0 \int_0^\infty e^{-\rho t} \Pi_{it} dt,$$

with Π_{it} given by (33). Notice that the firm's optimization *problem* is not affected by sovereign defaults, although of course default does affect the aggregate variables that enter the firm's problem (Y_t, P_t , etc.). The state variable specific to firm i , P_{it} , evolves according to $dP_{it} = \pi_{it} P_{it} dt$. We conjecture that the aggregate state relevant to the firm's decisions can be summarized by $(b_t, Z_t, P_t) \equiv S_t$.⁵⁴ Then firm i 's *value function* $J(P_{it}, S_t)$ must satisfy the following Hamilton-Jacobi-Bellman (HJB) equation,

$$\begin{aligned} \rho J(P_i, S) &= \max_{\pi_i} \left\{ \left(\frac{P_i}{P} - \frac{w}{Z} \right) \left(\frac{P_i}{P} \right)^{-\varepsilon} Y - \Psi(\pi_i) + \pi_i P_i \frac{\partial J}{\partial P_i}(P_i, S) \right\} \\ &\quad + \mu'_S(S) D_S J(P_i, S) + \frac{1}{2} \sigma'_S(S) (D_{SS} J(P_i, S)) \sigma_S(S), \end{aligned}$$

where the vectors $(\mu_S(S), \sigma_S(S))$ collect the drift and diffusion terms, respectively, of the aggregate states S , and (D_S, D_{SS}) are the gradient and Hessian operators, respectively, with respect to S .⁵⁵ The first order and envelope conditions of this problem are (we omit the arguments of J to

⁵⁴In particular, we later show that in equilibrium $Y_t = Z_t$, whereas w_t and $\tilde{C}_t$ are also functions of (b_t, Z_t, P_t) . The states P_t and b_t follow the same laws of motion as in the main text, equations (2) and (4) respectively, whereas Z_t follows equation (31).

⁵⁵In particular, $\mu_S(S) = [s(b), \mu Z, \pi P]'$, where $s(b)$ is the drift of the debt ratio b as defined in section 2.5 of the main text; and $\sigma_S(S) = [-\sigma b, \sigma Z, 0]'$.

ease the notation),

$$\psi \pi_i \tilde{C} = P_i \frac{\partial J}{\partial P_i},$$

$$\begin{aligned} \rho \frac{\partial J}{\partial P_i} = & \left[\varepsilon \frac{w}{Z} - (\varepsilon - 1) \frac{P_i}{P} \right] \left(\frac{P_i}{P} \right)^{-\varepsilon} \frac{Y}{P_i} + \pi_i \left(\frac{\partial J}{\partial P_i} + P_i \frac{\partial^2 J}{\partial P_i^2} \right) \\ & + \frac{\partial}{\partial P_i} \left[\mu'_S(S) D_S J + \frac{1}{2} \sigma'_S(S) (D_{SS} J) \sigma_S(S) \right]. \end{aligned}$$

In what follows, we will consider a symmetric equilibrium in which all firms choose the same price: $P_i = P, \pi_i = \pi$ for all i . After some algebra, it can be shown that the above conditions imply the following pricing Euler equation,⁵⁶

$$\begin{aligned} \left(\rho - \frac{\tilde{C}_b(b, Z) s(b) + \tilde{C}_Z(b, Z) \mu Z}{\tilde{C}(b, Z)} \right) \pi(b) = & \frac{\varepsilon - 1}{\psi} \left(\frac{\varepsilon}{\varepsilon - 1} \frac{w}{Z} - 1 \right) \frac{Z}{\tilde{C}(b, Z)} + s(b) \pi'(b) \\ & + \sigma^2 F(S). \end{aligned} \quad (34)$$

where $\tilde{C}(b, Z)$ and $\pi(b)$ denote the equilibrium policy functions for total spending and inflation, and $F(S)$ is a function of the aggregate state; in particular, the term $\sigma^2 F(S)$ captures the effect of aggregate uncertainty on firms' pricing decision.⁵⁷ Equation (34) determines the market clearing wage w as a function of S .

Households

The representative household's preferences are given by

$$\mathbb{E}_0 \int_0^\infty e^{-\rho t} \log(\tilde{C}_t) dt,$$

⁵⁶The proof is available upon request.

⁵⁷In the special case with no aggregate uncertainty, $\sigma = 0$, equation (34) simplifies to

$$\left(\rho - \frac{\dot{\tilde{C}}}{\tilde{C}} \right) \pi = \frac{\varepsilon - 1}{\psi} \left(\frac{\varepsilon}{\varepsilon - 1} \frac{w}{Z} - 1 \right) \frac{Z}{\tilde{C}} + \dot{\pi},$$

where we have used the fact, in this case, $s(b) = db/dt$ and $\mu Z = dZ/dt$, which implies $\tilde{C}_b s(b) + \tilde{C}_Z \mu Z = d\tilde{C}/dt \equiv \dot{\tilde{C}}$ and $\pi'(b) s(b) = d\pi/dt \equiv \dot{\pi}$.

where $\tilde{C}_t$ is household consumption of the final good. Define *total real spending* as the sum of household consumption and price adjustment costs,

$$\begin{aligned} C_t &\equiv \tilde{C}_t + \int_0^1 \Psi_t(\pi_{it}) di \\ &= \tilde{C}_t + \frac{\psi}{2} \pi_t^2 \tilde{C}_t, \end{aligned} \quad (35)$$

where in the second equality we have used the definition of Ψ_t (eq. 32) and the symmetry across firms in equilibrium. Instantaneous utility can then be expressed as

$$\begin{aligned} \log(\tilde{C}_t) &= \log(C_t) - \log\left(1 + \frac{\psi}{2} \pi_t^2\right) \\ &= \log(C_t) - \frac{\psi}{2} \pi_t^2 + O\left(\left\|\frac{\psi}{2} \pi_t^2\right\|^2\right), \end{aligned} \quad (36)$$

where $O(\|x\|^2)$ denotes terms of order second and higher in x . Expression (36) is the same as the utility function in the main text (eq. 6), up to a first order approximation of $\log(1 + x)$ around $x = 0$, where $x \equiv \frac{\psi}{2} \pi^2$ represents the percentage of aggregate spending that is lost to price adjustment. For our baseline calibration ($\psi = 9.15$), the latter object is relatively small even for relatively high inflation rates, and therefore so is the error in computing the utility losses from price adjustment.⁵⁸ Therefore, the utility function used in the main text provides a fairly accurate approximation of the welfare losses caused by inflation in the economy with costly price adjustment described here. In Appendix I, we solve the equilibrium implied by the exact inflation disutility function in the first line of equation (36) and show that the results are virtually identical to those in the main text.

As in the model in the main text, the government rebates to the household all the net proceedings from its international credit operations, denoted by T_t in nominal terms. We assume that the household supplies one unit of labor input inelastically: $n_t = 1$. It also receives firms' profits in a lump-sum manner. Thus the household's nominal budget constraint is

$$P_t \tilde{C}_t = P_t w_t + P_t \int_0^1 \Pi_{it} di + T_t.$$

In the symmetric equilibrium, each firm's labor demand is $n_{it} = y_{it}/Z_t = Y_t/Z_t$. Since labor supply

⁵⁸For instance, for an inflation rate as high as $\pi = 12\%$ (the maximum equilibrium inflation rate obtained in the inflationary regime), the exact and the approximated price adjustment cost are $\log(1 + \frac{\psi}{2} \pi^2) = 6.38\%$ and $\frac{\psi}{2} \pi^2 = 6.59\%$ of aggregate spending, respectively.

equals one, labor market clearing requires

$$\int_0^1 n_{it} di = Y_t / Z_t = 1 \Leftrightarrow Y_t = Z_t.$$

Therefore, in equilibrium output is simply equal to exogenous productivity Z_t . Each firm's real profits equal $\Pi_{it} = Y_t - w_t - \frac{\psi}{2} \pi_t^2 \tilde{C}_t$. Using this in the household's budget constraint, we obtain

$$T_t = P_t \left(\tilde{C}_t + \frac{\psi}{2} \pi_t^2 \tilde{C}_t - Y_t \right) = P_t (C_t - Y_t),$$

where in the second equality we have used (35). Therefore, the *primary deficit ratio* is

$$c_t \equiv \frac{T_t}{P_t Y_t} = \frac{C_t - Y_t}{Y_t},$$

as in the main text. It follows that $\log(C_t) = \log(1 + c_t) + \log(Y_t)$, such that household welfare can be expressed as a function of the policy variables (c_t, π_t) as in equation (7) in the main text.

Fiscal and monetary policy

The government maximizes household welfare subject to the laws of motion of the aggregate state variables. The default scenario is the same as in the main text, with one qualification: upon default at a debt ratio b_t , and during the subsequent exclusion period, productivity equals $Z_t \exp[-\epsilon \max\{0, b_t - \hat{b}\}]$. This, together with the fact that in equilibrium $Y_t = Z_t$, implies that the default scenario is exactly as in the main text. It is then trivial to show that the government's maximization problem is exactly the same as in the main text, once we take into account that (i) the welfare criterion is the same (equation 11), and (ii) the law of motion of the debt ratio is the same (equation 4). As a result, the policy functions for inflation and primary deficit ratio will also be the same: $\pi_t = \pi(b_t)$, $c_t = c(b_t)$.

Notice finally that, since $Y_t = Z_t$, in equilibrium we have $C_t = (1 + c(b_t)) Z_t \equiv C(b_t, Z_t)$, and therefore $\tilde{C}_t = C(b_t, Z_t) / [1 + \frac{\psi}{2} \pi(b_t)^2] \equiv \tilde{C}(b_t, Z_t)$. Likewise, the pricing Euler equation derived above (equation 34) determines the market clearing wage given the aggregate state: $w_t = w(Z_t, b_t, P_t)$. We thus verify our previous conjecture that (b_t, Z_t, P_t) are the relevant aggregate states for firms.

B. Proof of Proposition 2

Consider the no-inflation regime with $\sigma = 0$. We first conjecture that there is a stable steady state, that is, a point $b_{ss} \in [0, b^*)$ such that $s(b_{ss}) = 0$ and $s'(b_{ss}) < 0$. Then, there is an interval around

b_{ss} with radius $\varepsilon > 0$ such that, for any initial value $b_0 \in (b_{ss} - \varepsilon, b_{ss} + \varepsilon)$, b_t converges to b_{ss} and remains there forever.⁵⁹ In that interval default cannot happen and therefore the price of the bond is constant at $Q(b) = \frac{\lambda + \delta}{\lambda + \bar{r}}$, which implies $Q'(b) = 0$ for all $b \in (b_{ss} - \varepsilon, b_{ss} + \varepsilon)$. The envelope condition of the HJB equation is

$$\rho V'(b) = \left\{ \left(\frac{\lambda + \delta}{Q(b)} - \lambda - \mu \right) - [(\lambda + \delta)b + c] \frac{Q'(b)}{Q^2(b)} \right\} V'(b) + s(b) V''(b). \quad (37)$$

In the stable steady state, the above condition simplifies to

$$(\rho - \bar{r} + \mu) V'(b_{ss}) = 0,$$

where we have used the fact that $s(b_{ss}) = 0$, $Q(b_{ss}) = \frac{\lambda + \delta}{\lambda + \bar{r}}$ and $Q'(b_{ss}) = 0$. From the Proof of Proposition 1 in the main text, $V'(b)$ cannot be zero in equilibrium for any $b \in [0, b^*)$. Also, Condition 1 implies $(\rho - \bar{r} + \mu) > 0$. Therefore, the envelope condition (37) is *not* satisfied at the conjectured stable steady-state ratio b_{ss} . Therefore the initial conjecture is false: there is no stable steady-state.⁶⁰

Assume instead that there is an unstable steady state $b_{ss} \in (0, b^*)$ with $s(b_{ss}) = 0$ and $s'(b_{ss}) > 0$. Since we have already proved that there is no stable steady state, it must be the case that, for any initial $b_0 < b_{ss}$, $s(b_0) < 0$ and therefore b_t converges to zero. However, at $b = 0$ the state constraint $b \geq 0$ implies $s(0) \geq 0$. If $s(0)$ is non negative, $b = 0$ is not a stable steady state ($s'(0) \geq 0$) and $s(b)$ does not cross zero until $b = b_{ss}$ then $s(b_0)$ cannot be negative for any $b_0 \in (0, b_{ss})$.⁶¹ Hence there can be no unstable steady-state.⁶² Notice that $b_{ss} = 0$ is not considered, as we cannot rule out the case $s(0) = 0$ and $s'(0) > 0$.

Assume that there is a steady-state $s(b_{ss}) = 0$ with $s'(b_{ss}) = 0$ and $s''(b_{ss}) \neq 0$. In this case, depending on the concavity of $s(b)$ there will be either a left- or a right-interval including b_{ss} such that, for any initial value b_0 in that interval, b_t converges to b_{ss} and default cannot happen. If we take the limit as $b \rightarrow b_{ss}$ of (37) in this half-interval the envelope condition is again not satisfied and thus we can also rule out this possibility.⁶³

Finally, if $s(b_{ss}) = s'(b_{ss}) = s''(b_{ss}) = 0$ there are two possible cases. Provided that $s'''(b_{ss}) < 0$

⁵⁹In the case of $b_{ss} = 0$, this is an interval $[0, \varepsilon)$.

⁶⁰This result also holds if the drift $s(b)$ is discontinuous around b_{ss} but $s(b_{ss} - \varepsilon) > 0$ and $s(b_{ss} + \varepsilon) < 0$ as the price of the bond is constant in the interval: $Q(b_{ss}) = \frac{\lambda + \delta}{\lambda + \bar{r}}$ and $Q'(b_{ss}) = 0$ and the envelope condition (37) is not satisfied.

⁶¹This implicitly assumes that $s(b)$ is continuous in the subinterval. If this were not the case, that is, if there was a discontinuity at some $b_1 \in (0, b_{ss})$ such that $s(b)$ jumps from the positive to the negative region at b_1 , then b_1 would be a stable steady state, a possibility that had already been discarded.

⁶²Again, the proof is similar in the case of a discontinuity at b_{ss} with the drift jumping from the negative to the positive region.

⁶³The result also holds if the drift $s(b)$ is discontinuous around b_{ss} but $s(b_{ss} - \varepsilon) < 0$ and $s(b_{ss} + \varepsilon) > 0$ following the same line of reasoning as above.

then the steady state is stable and we are back in the first part of the proof. If $s'''(b_{ss}) > 0$ then the steady-state is unstable and we are in the second part of the proof. In any case no steady-state is possible. We can proceed with higher order derivatives, but the proof will always fall in any of the two previous cases.

The conclusion is that, given Condition 1 and the envelope condition (37), there is no steady-state with positive debt, $s(b) \neq 0 \forall b \in (0, b^*)$. And since $s(0) \geq 0$, we have $s(b) > 0 \forall b \in (0, b^*)$. QED.

C. Numerical algorithm

We describe the numerical algorithm used to jointly solve for the equilibrium value function, $V(b)$, and bond price function, $Q(b)$. The algorithm proceeds in 3 steps. We describe each step in turn.

Step 1: Solution to the Hamilton-Jacobi-Bellman equation

The HJB equation (14) is solved using an *upwind finite difference* scheme following Achdou et al. (2015). It approximates the value function $V(b)$ on a finite grid with step $\Delta b : b \in \{b_1, \dots, b_I\}$, where $b_i = b_{i-1} + \Delta b = b_1 + (i-1)\Delta b$ for $2 \leq i \leq I$. The bounds are $b_1 = 0$ and $b_I = b^* - \Delta b$, such that $\Delta b = b^*/I$. We choose θ such that $\theta(I+1) \in \mathbb{N}$. We use the notation $V_i \equiv V(b_i)$, $i = 1, \dots, I$, and similarly for the bond price function Q_i and the policy functions (π_i, c_i) .

Notice first that the HJB equation involves first and second derivatives of the value function, $V'(b)$ and $V''(b)$. At each point of the grid, the first derivative can be approximated with a forward (F) or a backward (B) approximation,

$$V'(b_i) \approx \partial_F V_i \equiv \frac{V_{i+1} - V_i}{\Delta b}, \quad (38)$$

$$V'(b_i) \approx \partial_B V_i \equiv \frac{V_i - V_{i-1}}{\Delta b}, \quad (39)$$

whereas the second derivative is approximated by

$$V''(b_i) \approx \partial_{bb} V_i \equiv \frac{V_{i+1} + V_{i-1} - 2V_i}{(\Delta b)^2}. \quad (40)$$

In an upwind scheme, the choice of forward or backward derivative depends on the sign of the *drift function* for the state variable, given by

$$s(b) \equiv \left(\frac{\lambda + \delta}{Q(b)} + \sigma^2 - \mu - \lambda - \pi(b) \right) b + \frac{c(b)}{Q(b)}, \quad (41)$$

for $b \leq b^*$, where

$$\begin{aligned} c(b) &= -\frac{Q(b)}{V'(b)} - 1, \\ \pi(\pi) &= -\frac{b}{\psi} V'(b) = \frac{bQ(b)}{\psi(1+c)}. \end{aligned}$$

Let superscript n denote the iteration counter. The HJB equation is approximated by the following upwind scheme,

$$\frac{V_i^{n+1} - V_i^n}{\Delta} + \rho V_i^{n+1} = \log(c_i^n + 1) - \frac{\psi}{2} (\pi_i^n)^2 + \partial_F V_i^{n+1} s_{i,F}^n \mathbf{1}_{s_{i,F}^n > 0} + \partial_B V_i^{n+1} s_{i,B}^n \mathbf{1}_{s_{i,B}^n < 0} + \frac{(\sigma b_i)^2}{2} \partial_{bb} V_i^{n+1},$$

for $i = 1, \dots, I$, where $\mathbf{1}(\cdot)$ is the indicator function and

$$\begin{aligned} s_{i,F}^n &= \left(\frac{\lambda + \delta}{Q_i} + \sigma^2 - \mu - \lambda + \frac{b_i}{\psi} \partial_F V_i^n \right) b_i - \left(\frac{1}{\partial_F V_i^n} + \frac{1}{Q_i} \right), \\ s_{i,B}^n &= \left(\frac{\lambda + \delta}{Q_i} + \sigma^2 - \mu - \lambda + \frac{b_i}{\psi} \partial_B V_i^n \right) b_i - \left(\frac{1}{\partial_B V_i^n} + \frac{1}{Q_i} \right). \end{aligned}$$

Therefore, when the drift is positive ($s_{i,F}^n > 0$) we employ a forward approximation of the derivative, $\partial_F V_i^{n+1}$; when it is negative ($s_{i,B}^n < 0$) we employ a backward approximation, $\partial_B V_i^{n+1}$. The term $\frac{V_i^{n+1} - V_i^n}{\Delta} \rightarrow 0$ as $V_i^{n+1} \rightarrow V_i^n$. Moving all terms involving V^{n+1} to the left hand side and the rest to the right hand side, we obtain

$$V_{i-1}^{n+1} \alpha_i^n + V_i^{n+1} \beta_i^n + V_{i+1}^{n+1} \xi_i^n = \log(c_i^n + 1) - \frac{\psi}{2} (\pi_i^n)^2 + \frac{V_i^n}{\Delta}, \quad (42)$$

where

$$\begin{aligned} \alpha_i^n &\equiv \frac{s_{i,B}^n \mathbf{1}_{s_{i,B}^n < 0}}{\Delta b} - \frac{(\sigma b_i)^2}{2(\Delta b)^2}, \\ \beta_i^n &\equiv \frac{1}{\Delta} + \rho + \frac{s_{i,F}^n \mathbf{1}_{s_{i,F}^n > 0}}{\Delta b} - \frac{s_{i,B}^n \mathbf{1}_{s_{i,B}^n < 0}}{\Delta b} + \frac{(\sigma b_i)^2}{(\Delta b)^2}, \\ \xi_i^n &\equiv -\frac{s_{i,F}^n \mathbf{1}_{s_{i,F}^n > 0}}{\Delta b} - \frac{(\sigma b_i)^2}{2(\Delta b)^2}, \end{aligned}$$

for $i = 1, \dots, I$. Notice that the state constraint $b \geq 0$ means that $s_{1,B}^n = 0$, which together with $b_1 = 0$ implies $\alpha_1^n = 0$. In equation (42), the optimal primary deficit ratio is set to

$$c_i^n = \left(\frac{-Q_i^n}{\partial V_i^n} - 1 \right), \quad (43)$$

where

$$\partial V_i = \partial_F V_i \mathbf{1}_{s_{i,F}^n > 0} + \partial_B V_i \mathbf{1}_{s_{i,B}^n < 0} - \frac{Q(b_i)}{1 + \bar{c}_i} \mathbf{1}_{s_{i,F}^n \leq 0} \mathbf{1}_{s_{i,B}^n \geq 0}.$$

In the above expression, $\bar{c}_i$ is the consumption level such that $s(b_i) \equiv s_i^n = 0$, i.e. it solves

$$\left(\frac{\lambda + \delta}{Q_i} + \sigma^2 - \mu - \lambda - \frac{b_i Q_i}{\psi(1 + \bar{c}_i)} \right) b_i + \frac{\bar{c}_i}{Q_i} = 0.$$

The solution is the higher root of the above equation,

$$\bar{c}_i = \frac{-(1 + \Gamma_i Q_i) + \sqrt{(1 + \Gamma_i Q_i)^2 - 4 \left[\Gamma_i Q_i - \frac{b_i^2 Q_i^2}{\psi} \right]}}{2},$$

where $\Gamma_i \equiv \left(\frac{\lambda + \delta}{Q(b_i)} + \sigma^2 - \mu - \lambda \right) b_i$. Given c_i^n , the optimal inflation rate is

$$\pi_i^n = \frac{b Q_i}{\psi(1 + c_i^n)}. \quad (44)$$

The smooth pasting boundary condition (equation 13) can be approximated by⁶⁴

$$\frac{V_{I+1}^{n+1} - V_I^{n+1}}{\Delta b} = -\frac{\epsilon}{\chi + \rho} + \frac{\chi}{\chi + \rho} \theta \partial_F V_{\theta(I+1)}^n \Rightarrow V_{I+1}^{n+1} = V_I^{n+1} + \left(-\frac{\epsilon}{\chi + \rho} + \frac{\chi}{\chi + \rho} \theta \partial_F V_{\theta(I+1)}^n \right) \Delta b. \quad (45)$$

Equation (42) is a system of I linear equations which can be written in matrix notation as:

$$\mathbf{A}^n \mathbf{V}^{n+1} = \mathbf{d}^n, \quad (46)$$

where the matrix $\mathbf{A}^n$ and the vectors $\mathbf{V}^{n+1}$ and $\mathbf{d}^n$ are defined by

$$\mathbf{A}^n = \begin{bmatrix} \beta_1^n & \xi_1^n & 0 & 0 & \cdots & 0 \\ \alpha_2^n & \beta_2^n & \xi_2^n & 0 & \cdots & 0 \\ 0 & \alpha_3^n & \beta_3^n & \xi_3^n & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots \\ 0 & 0 & \cdots & \alpha_{I-1}^n & \beta_{I-1}^n & \xi_{I-1}^n \\ 0 & 0 & \cdots & 0 & \alpha_I^n & \beta_I^n + \xi_I^n \end{bmatrix}, \quad \mathbf{V}^{n+1} = \begin{bmatrix} V_1^{n+1} \\ V_2^{n+1} \\ V_3^{n+1} \\ \vdots \\ V_{I-1}^{n+1} \\ V_I^{n+1} \end{bmatrix} \quad (47)$$

⁶⁴Notice that we solve for the value function under the guess that the optimal default threshold satisfies $b^* > \hat{b}$, such that $\max\{0, b^* - \hat{b}\} = b^* - \hat{b}$. We verify that our guess is satisfied in equilibrium in all our simulations.

$$\mathbf{d}^n = \begin{bmatrix} \log(c_1^n + 1) - \frac{\psi}{2} (\pi_1^n)^2 + \frac{V_1^n}{\Delta} \\ \log(c_2^n + 1) - \frac{\psi}{2} (\pi_2^n)^2 + \frac{V_2^n}{\Delta} \\ \log(c_3^n + 1) - \frac{\psi}{2} (\pi_3^n)^2 + \frac{V_3^n}{\Delta} \\ \vdots \\ \log(c_{I-1}^n + 1) - \frac{\psi}{2} (\pi_{I-1}^n)^2 + \frac{V_{I-1}^n}{\Delta} \\ \log(c_I^n + 1) - \frac{\psi}{2} (\pi_I^n)^2 + \frac{V_I^n}{\Delta} + \xi_I^n \left(\frac{\epsilon}{\chi + \rho} - \frac{\chi}{\chi + \rho} \theta \partial_F V_{\theta(I+1)}^n \right) \Delta b \end{bmatrix}.$$

Notice that the element (I, I) in A is $\beta_I^n + \xi_I^n$ due to the smooth pasting condition (45).

The algorithm to solve the HJB equation runs as follows. Begin with an initial guess $V_i^0 = -b_i$, $i = 1, \dots, I$. Set $n = 0$. Then:

1. Compute $\partial_F V_i^n$, $\partial_B V_i^n$ and $\partial_{bb} V_i^n$ using (38)-(40).
2. Compute c_i^n and π_i^n using (43) and (44).
3. Find V_i^{n+1} solving the linear system of equations (46).
4. If V_i^{n+1} is close enough to V_i^n , stop. If not set $n := n + 1$ and go to 1.

Step 2: Solution to the Bond Pricing Equation

The pricing equation (19) is also solved using an upwind finite difference scheme. The equation in this case is

$$Q(b) (\bar{r} + \pi(b) + \lambda) = (\lambda + \delta) + \left[\left(\frac{\lambda + \delta}{Q(b)} + \sigma^2 - \mu - \lambda - \pi(b) \right) b + \frac{c(b)}{Q(b)} \right] Q'(b) + \frac{(\sigma b)^2}{2} Q''(b),$$

with a boundary condition

$$Q(b^*) = \frac{\chi}{\bar{r} - \mu + \chi} \theta Q(\theta b^*).$$

This case is similar to the HJB equation. Using the notation $Q_i = Q(b_i)$, the equation can be expressed as

$$\frac{Q_i^{n+1} - Q_i^n}{\Delta} + Q_i^{n+1} (\bar{r} + \pi_i + \lambda) = \lambda + \delta + \partial_F Q_i^{n+1} s_{i,F}^n \mathbf{1}_{s_{i,F}^n > 0} + \partial_B Q_i^{n+1} s_{i,B}^n \mathbf{1}_{s_{i,B}^n < 0} + \frac{(\sigma b_i)^2}{2} \partial_{bb} Q_i^{n+1}, \quad (48)$$

where:

$$\begin{aligned} Q'(b_i) &\approx \partial_F Q_i \equiv \frac{Q_{i+1} - Q_i}{\Delta b}, \\ Q'(b_i) &\approx \partial_B Q_i \equiv \frac{Q_i - Q_{i-1}}{\Delta b}, \\ Q''(b_i) &\approx \partial_{bb} Q_i \equiv \frac{Q_{i+1} + Q_{i-1} - 2Q_i}{(\Delta b)^2} \end{aligned}$$

and rearranging terms

$$\begin{aligned} Q_{i-1}^{n+1} \alpha_i^n + Q_i^{n+1} (\beta_i^n + \bar{r} + \pi_i + \lambda - \rho) + Q_{i+1}^{n+1} \xi_i^n &= \lambda + \delta + \frac{Q_i^n}{\Delta}, \quad \forall i < I + 1, \\ Q_{I+1} &= \frac{\chi}{\rho - \mu + \chi} \theta Q(\theta(I + 1)). \end{aligned}$$

Notice the abuse of notation, as

$$s_{i,F}^n = s_{i,B}^n = s_i^n = \left(\frac{\lambda + \delta}{Q^n(b_i)} + \sigma^2 - \mu - \lambda - \pi_i \right) b_i + \frac{c_i}{Q^n(b_i)}.$$

Equation (48) is again a system of I linear equations which can be written in matrix notation as:

$$\mathbf{F}^n \mathbf{Q}^{n+1} = \mathbf{q}^n, \quad (49)$$

where the matrix $\mathbf{F}^n$ and the vectors $\mathbf{Q}^{n+1}$ and $\mathbf{f}^n$ are defined by:

$$\mathbf{F}^n = \begin{bmatrix} (\beta_1^n + \pi_1 + \lambda + \bar{r} - \rho) & \xi_1^n & 0 & \dots & 0 \\ \alpha_2^n & (\beta_2^n + \pi_2 + \lambda + \bar{r} - \rho) & \xi_2^n & \dots & 0 \\ 0 & \alpha_3^n & (\beta_3^n + \pi_3 + \lambda + \bar{r} - \rho) & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots \\ 0 & 0 & \dots & (\beta_{I-1}^n + \pi_{I-1} + \lambda + \bar{r} - \rho) & \xi_{I-1}^n \\ 0 & 0 & \dots & \alpha_I^n & (\beta_I^n + \pi_I + \lambda + \bar{r} - \rho) \end{bmatrix},$$

$$\mathbf{Q}^{n+1} = \begin{bmatrix} Q_1^{n+1} \\ Q_{2,1}^{n+1} \\ Q_{3,1}^{n+1} \\ \vdots \\ Q_{I-1}^{n+1} \\ Q_I^{n+1} \end{bmatrix}, \quad \mathbf{q}^n = \begin{bmatrix} \lambda + \delta + \frac{Q_1^n}{\Delta} \\ \lambda + \delta + \frac{Q_2^n}{\Delta} \\ \lambda + \delta + \frac{Q_3^n}{\Delta} \\ \vdots \\ \lambda + \delta + \frac{Q_{I-1}^n}{\Delta} \\ \lambda + \delta + \frac{Q_I^n}{\Delta} - \xi_I^n \frac{\chi}{\rho - \mu + \chi} \theta Q_{\theta(I+1)}^n \end{bmatrix}.$$

The algorithm to solve the bond pricing equation is similar to the HJB. Begin with an initial guess $Q_i^0 = \frac{\lambda + \delta}{\bar{r} + \lambda}$, set $n = 0$. Then:

1. Find Q_i^{n+1} solving the linear system of equations (49).
2. If Q_i^{n+1} is close enough to Q_i^n , stop. If not set $n := n + 1$ and go to 1.

Step 3: Value Matching

Finally, we iterate until the value matching condition (12) is satisfied:

$$V_{I+1} = -\frac{\max\{0, b_{I+1} - \hat{b}\}}{\rho + \chi} + \frac{\chi}{\chi + \rho} V_{\theta(I+1)}. \quad (50)$$

Taking into account (45), condition (50) can be rewritten as

$$V_I + \left(-\frac{\epsilon}{\chi + \rho} + \frac{\chi}{\chi + \rho} \theta \partial_F V_{\theta(I+1)} \right) \Delta b + \frac{\max\{0, b_{I+1} - \hat{b}\}}{\rho + \chi} - \frac{\chi}{\chi + \rho} V_{\theta(I+1)} = 0.$$

D. The riskless nominal bond

We define a new instrument, a riskless nominal bond. This is a non-defaultable bond issued in the domestic currency and with the same promised payoffs as the defaultable bond. In this case, the nominal price of the bond for a current debt ratio $b \leq b^*$ is given by

$$\tilde{Q}(b) = \mathbb{E} \left[\int_0^{T(b^*)} e^{-(\bar{r} + \lambda)t - \int_0^t \pi_s ds} (\lambda + \delta) dt + \int_{T(b^*)}^{T(b^*) + \tau} e^{-(\bar{r} + \lambda)t - \int_0^{T(b^*)} \pi_s ds} (\lambda + \delta) dt + e^{-(\bar{r} + \lambda)(T(b^*) + \tau) - \int_0^{T(b^*)} \pi_s ds} \tilde{Q}(\theta b^*) \mid b_0 = b \right].$$

where $\pi_s = \pi(b_s)$ and we have used the fact that $\pi_s = 0$ for $s \in (T(b^*), T(b^*) + \tau)$. Applying again the Feynman-Kac formula, we obtain

$$\tilde{Q}(b) (\bar{r} + \pi(b) + \lambda) = (\lambda + \delta) + \left[\left(\frac{\lambda + \delta}{Q(b)} + \sigma^2 - \mu - \lambda - \pi \right) b + \frac{c(b)}{Q(b)} \right] \tilde{Q}'(b) + \frac{(\sigma b)^2}{2} \tilde{Q}''(b),$$

for all $b \in [0, b^*)$. The boundary condition for $\tilde{Q}(b)$ is given by

$$\begin{aligned}\tilde{Q}(b^*) &= \mathbb{E} \left[\int_0^\tau e^{-(\bar{r}+\lambda)t} (\lambda + \delta) dt + e^{-(\bar{r}+\lambda)\tau} \tilde{Q}(\theta b^*) \right] \\ &= \int_0^\infty \chi e^{-\chi\tau} \left[(1 - e^{-(\bar{r}+\lambda)\tau}) \frac{\lambda + \delta}{\bar{r} + \lambda} dt + e^{-(\bar{r}+\lambda)\tau} \tilde{Q}(\theta b^*) \right] d\tau \\ &= \frac{\lambda + \delta}{\bar{r} + \chi + \lambda} + \frac{\chi}{\bar{r} + \chi + \lambda} \tilde{Q}(\theta b^*).\end{aligned}$$

Given the equilibrium default threshold b^* , we solve for the riskless bond price function $\tilde{Q}(b)$ using a finite difference scheme similar to the one used to solve for $Q(b)$ in Step 2 of the general algorithm (see Appendix C).

E. Computing the expected time-to-default

Given the definition of the expected time to default (23), applying the Feynman-Kac formula we obtain

$$1 + \left[\left(\frac{\lambda + \delta}{Q(b)} + \sigma^2 - \mu - \lambda - \pi \right) b + \frac{c}{Q(b)} \right] T^{el}(b) + \frac{(\sigma b)^2}{2} T^{e''}(b) = 0,$$

with a boundary condition

$$T^e(b^*) = 0.$$

This can be solved using a finite difference scheme similar to the one described for the bond price in Appendix C.

F. Solution to the Kolmogorov Forward equation

Let $\tilde{f}(b)$ denote the stationary share of time spent at debt ratio b while in good credit standing. It satisfies the following Kolmogorov Forward equation:

$$0 = -\frac{d}{db} [s(b)\tilde{f}(b)] + \frac{1}{2} \frac{d^2}{db^2} [(\sigma b)^2 \tilde{f}(b)] + \chi \tilde{h} \delta(b - \theta b^*) - \chi \tilde{h} \delta(b - b^*), \quad (51)$$

subject to

$$1 = \int_0^{b^*} \tilde{f}(b) db + \tilde{h},$$

where $s(b)$ is the drift function given by (41), $\tilde{h}$ is the stationary share of time spent in exclusion,

$$\tilde{h} \equiv \frac{\mathbb{E}_0[\tau]}{\mathbb{E}_0[\tau + T(b^*)|b_0 = \theta b^*]} = \frac{1/\chi}{1/\chi + T^e(\theta b^*)},$$

and $\delta(\cdot)$ is the Dirac 'delta'.⁶⁵ In equation (51), the term $\chi\tilde{h}\delta(b - \theta b^*)$ reflects the fact that the government reenters capital markets at $b = \theta b^*$ following exclusion spells, on which it spends a fraction $\tilde{h}$ of time and from which it exits at rate χ . Similarly, the term $\chi\tilde{h}\delta(b - b^*)$ captures the fact that at $b = b^*$ the government defaults and hence exits the repayment spell; in the latter term, we use the fact that, in the ergodic limit, the flow of transitions from repayment to exclusion spells must equal the flow of transitions from exclusion to repayments spells, $\chi\tilde{h}$. We define $f(b) \equiv \frac{\tilde{f}(b)}{1-\tilde{h}}$, which denotes the distribution of the debt ratio conditional on being in good credit standing. We also define $h \equiv \frac{\tilde{h}}{1-\tilde{h}}$. Equation (51) can then be written as,

$$0 = -\frac{d}{db} \left\{ \left[\left(\frac{\lambda + \delta}{Q(b)} + \sigma^2 - \mu - \lambda - \pi(b) \right) b + \frac{c(b)}{Q(b)} \right] f(b) \right\} + \frac{1}{2} \frac{d^2}{db^2} [(\sigma b)^2 f(b)] + \chi h \delta(b - \theta b^*) - \chi h \delta(b - b^*),$$

where now

$$1 = \int_0^{b^*} f(b) db.$$

We solve the above equation using an upwind finite difference scheme as in Achdou et al. (2015) or Nuño and Moll (2015). We use the notation $f_i \equiv f(b_i)$. The system can be now expressed as

$$0 = -\frac{f_i s_{i,F} \mathbf{1}_{s_{i,F}^n > 0} - f_{i-1} s_{i-1,F} \mathbf{1}_{s_{i-1,F}^n > 0}}{\Delta b} - \frac{f_{i+1} s_{i+1,B} \mathbf{1}_{s_{i+1,B}^n < 0} - f_i s_{i,B} \mathbf{1}_{s_{i,B}^n < 0}}{\Delta b} + \frac{f_{i+1} (\sigma b_{i+1})^2 + f_{i-1} (\sigma b_{i-1})^2 - 2f_i (\sigma b_i)^2}{2(\Delta b)^2} + \chi h \mathbf{1}_{\theta(I+1)} - \chi h \mathbf{1}_{I+1},$$

or equivalently

$$f_{i-1} \xi_{i-1} + f_{i+1} \alpha_{i+1} + f_i \left(\beta_i - \frac{1}{\Delta} - \rho \right) = 0. \quad (52)$$

The boundary conditions are $b \geq 0$ and $f(b^*) = 0$. Therefore, (52) is also a system of I linear equations which can be written in matrix notation as:

$$\left(\mathbf{A} - \left(\frac{1}{\Delta} + \rho \right) \mathbf{I} - \xi_I \mathbf{1}_I \right)^{\mathbf{T}} \mathbf{f} = \mathbf{h}, \quad (53)$$

⁶⁵The *Dirac delta* is a distribution or generalized function such that $\delta[f] = \int_{-\varepsilon}^{\varepsilon} f(x) \delta(x) dx = f(0)$, $\forall \varepsilon > 0$, $f \in L^1(-\varepsilon, \varepsilon)$. A heuristic characterization is the following:

$$\int_{-\infty}^{\infty} \delta(x) dx = 1, \\ \delta(x) = \begin{cases} \infty, & x = 0, \\ 0, & x \neq 0. \end{cases}$$

where $(\mathbf{A} - (\frac{1}{\Delta} + \rho) \mathbf{I} - \xi_I \mathbf{1}_I)^\mathbf{T}$ is the transpose of $(\mathbf{A} - (\frac{1}{\Delta} + \rho) \mathbf{I} - \xi_I \mathbf{1}_I) = \lim_{n \rightarrow \infty} (\mathbf{A}^n - (\frac{1}{\Delta} + \rho) \mathbf{I} - \xi_I^n \mathbf{1}_I)$, and $\mathbf{A}^n$ was defined in (47). $\mathbf{h} = -\mathbf{1}_{\theta(I+1)}$ is a vector of zeros with a -1 at position $\theta(I+1)$.⁶⁶

We solve the system (53) and obtain a solution $\hat{\mathbf{f}}$. Then we renormalize as

$$f_i = \frac{\hat{f}_i}{\sum_{i=1}^I \hat{f}_i \Delta b}.$$

G. Data Appendix

Data on GDP, inflation and primary deficit for the five EMU periphery countries (Greece, Italy, Ireland, Portugal and Spain), and inflation for the United States, come from the IMF's World Economic Outlook database.⁶⁷ The inflation differential is computed as the difference between the average inflation in the EMU periphery and that of the United States for the period 1987-1997.

External public debt is "General Government Gross consolidated Debt held by non-residents of the Member State" and is taken from each country's national accounts. Sovereign default premia (spreads) are the difference between the average yield on 10-year bonds of EMU periphery countries and that of German bonds, taken from Bloomberg. We use the yield on the German 10-year bond (also from Bloomberg) as the empirical proxy for the model's riskless yield, $\tilde{r}_t$. Bond yields for the pre-EMU period are annual and are taken from the European Commission's macroeconomic database (AMECO).

All data are annual except bond yields and default premia which are quarterly. We stop the sample for yields and default premia in 2012:Q2 (included) in order to isolate our analysis from the effects of the announcement by the European Central Bank of the Outright Monetary Transactions (OMT) programme in the summer of 2012.

H. Value functions for alternative calibrations

See Figure 8.

I. Exact utility cost of inflation

As explained in Appendix A, the quadratic utility cost of inflation in the model, $\frac{\psi}{2}\pi^2$, is an approximation to the exact utility cost that arises in a model where firms face a quadratic cost of price adjustment and households have log preferences over consumption. Here we perform our

⁶⁶Notice how we do not include the term χh due to the linearity and the rescaling of the system.

⁶⁷We use the current account balance as our measure of primary deficit. As explained in main text, in our model sovereign debt is fully held abroad, such that the external primary deficit (i.e. the current account balance) is equivalent to the fiscal primary deficit.

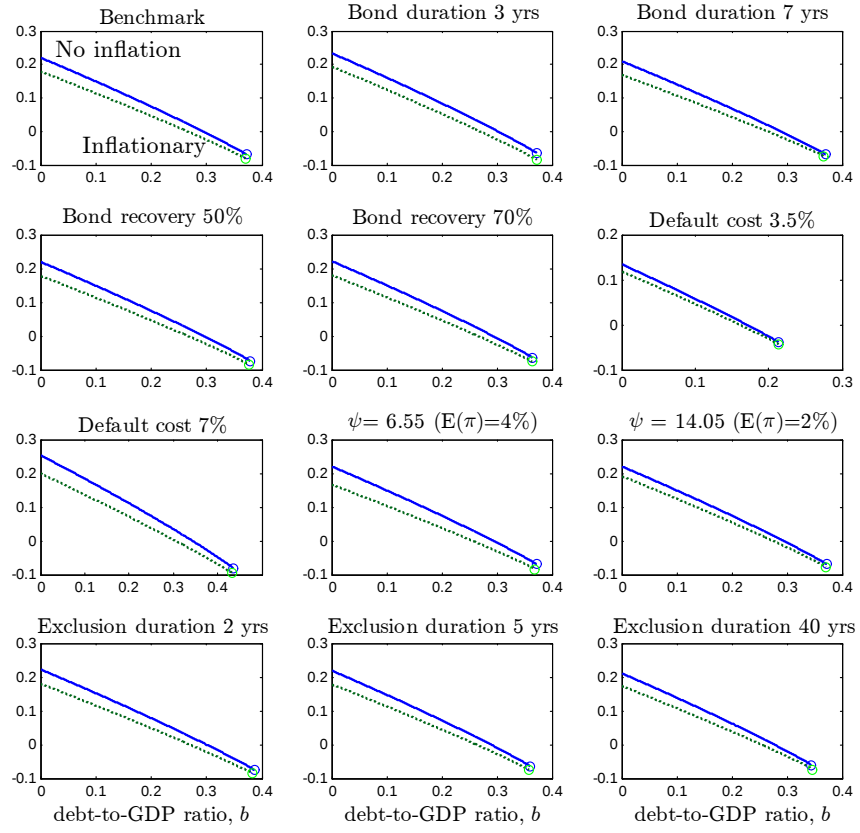


Figure 8: Value functions for alternative calibrations

baseline simulations using the exact instantaneous utility cost of inflation derived in Appendix A, given by $\log(1 + \frac{\psi}{2}\pi^2)$. The value function is now given by

$$\rho V(b) = \max_{c, \pi} \left\{ \log(1 + c) - \log\left(1 + \frac{\psi}{2}\pi^2\right) + s(b, c, \pi) V'(b) + \frac{(\sigma b)^2}{2} V''(b) \right\}, \quad (54)$$

where the drift function $s(b, c, \pi)$ is given again by equation (15) in the main text. All equilibrium conditions are as in the main text, except for the first order condition for inflation, which is now given by

$$\frac{\psi\pi(b)}{1 + \frac{\psi}{2}\pi(b)^2} = -bV'(b). \quad (55)$$

This optimal inflation decision differs from the one in the main text (eq. 17) due to the presence of the term $\frac{\psi}{2}\pi(b)^2$ in the left hand side. In particular, optimal inflation is given by the *lower* root of (55),⁶⁸

$$\pi(b) = \frac{1 - \sqrt{1 - \frac{2}{\psi}(bV'(b))^2}}{-bV'(b)}.$$

Figure 9 shows, for the inflationary regime, the value and inflation policy functions consistent with (a) the exact inflation disutility function (red dashed lines) and (b) the approximated inflation disutility function used in the paper (green solid lines). As shown by the left panel, both value functions are indistinguishable from each other. As shown by the right panel, so are the inflation policy functions, except for the fact that optimal inflation at default, $\pi(b^*)$, is slightly higher when exact inflation costs are used (red dot). But since this level of inflation happens only exactly at the time of default, it does not affect the social welfare at default. The figure also displays the corresponding functions in the no inflation regime (blue solid lines), which are obviously unaffected by the approximation of inflation disutility. Therefore, we conclude that our results on the welfare ranking between both regimes are not affected by our approximation of the exact inflation utility costs.⁶⁹

⁶⁸Notice that the marginal cost of inflation, $\psi\pi/(1 + \frac{\psi}{2}\pi^2)$, is strictly increasing up to $\pi = \sqrt{2/\psi}$, where it reaches a maximum, and strictly decreasing thereafter. Therefore, provided the marginal benefit $b(-) V'(b)$ is less than the maximum marginal cost, equation (55) admits two solutions. However, the higher root implies a *decreasing* marginal cost and therefore it cannot be an optimum. Optimal inflation is therefore given by the lower root of equation (55).

⁶⁹The distribution of the debt ratio $f(b)$ implied by the exact inflation disutility function is also indistinguishable from the one in the paper. Therefore, the average values for the inflationary regime computed in Section 3.4 are virtually identical. These results are available upon request.

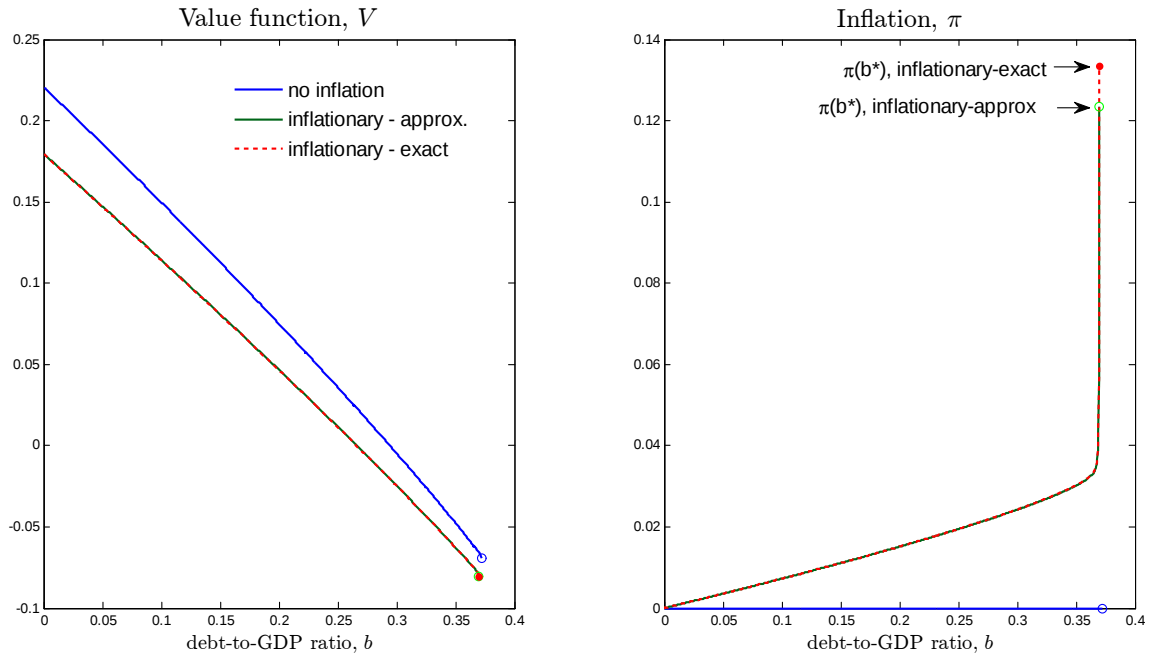


Figure 9: Equilibrium using exact vs. approximated inflation utility costs.