

Public-Sector Employment in an Equilibrium Search and Matching Model*

James Albrecht (Georgetown University)

Monica Robayo-Abril (Georgetown University)

Susan Vroman (Georgetown University)

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Abstract

In this paper, we extend the Diamond-Mortensen-Pissarides model of equilibrium unemployment to incorporate public-sector employment. We calibrate the model to Colombian data and analyze the effects of public-sector wage and employment policy on the unemployment rate, on the division of employment between the private and public sectors, and on the distributions of wages, productivities and education in the two sectors.

1 Introduction

The public sector accounts for a substantial fraction of employment in both developed and developing economies. Algan et al. (2002) estimates that the public sector accounted for slightly less than 19% of total employment across 17 OECD countries in 2000, and Mizala et al. (2011) estimates that 13% of total urban employment over the period 1996-2007 across eleven Latin American countries was in the public sector.

In this paper, we incorporate a public-sector labor market into an extended version of the Diamond-Mortensen-Pissarides (DMP) search and matching model of equilibrium unemployment (Pissarides 2000). Our model is designed to address distributional questions.

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What types of workers tend to work in the public sector? What types tend to sort into the private sector? How do the size of the public sector and the hiring and wage-setting rules used in that sector affect the overall unemployment rate and the distributions of workers, productivities and wages across the two sectors?

Our extension to the basic DMP model has three key ingredients. First, we assume an exogenous distribution, $Y \sim F(y)$, $\underline{y} \leq y \leq \bar{y}$, of human capital across workers. This makes it possible to address questions about which types of workers tend to work in the two sectors.¹ Second, we allow for *ex post* idiosyncratic match productivity. When a worker of type y meets a prospective employer with a vacancy, the worker draws a match-specific productivity, $X \sim G_s(x|y)$, $\underline{x} \leq x \leq \bar{x}$, where the subscript $s \in \{p, g\}$ indicates whether the job in question is in the private or public (government) sector. To give content to our notion of human capital, we assume first-order stochastic dominance, i.e., $y' > y \Rightarrow G_s(x|y') < G_s(x|y)$. The higher is a worker's level of human capital, the more favorable is that worker's distribution of match-specific productivity, and this is the case in both sectors.² Finally, we take into account that the rules governing public-sector employment and wage determination are in general not the same as those used in the private sector. We assume that the public sector posts an exogenous measure of vacancies, v_g , and that a worker of type y who meets a public-sector vacancy and draws match-specific productivity x is offered the job if and only if that productivity is no less than an exogenous threshold that varies with worker type, that is, if and only if $x \geq R_g(y)$. We also assume that a worker's wage in a public-sector job is determined by an exogenous rule, $w_g(x, y)$, and without loss of generality, we set $w_g(x, y) = 0$ for $x < R_g(y)$. Our combination of these three elements – *ex ante* worker heterogeneity, match-specific productivity with a first-order stochastic dominance assumption, and both private- and public-sector employment – is unique in the search and matching literature.

We calibrate our model using Colombian data and then use the calibrated model to simulate the effects of varying (i) the level of public-sector employment and (ii) the public-sector hiring and wage-setting rules.

Colombia is an interesting case study because its public-sector wage premium is very

¹This feature of our model is taken from Albrecht, Navarro and Vroman (2009). That paper focused on the distribution of worker types across formal employment, informal employment and unemployment.

²This feature of our model is related to Dolado, Jansen and Jimeno (2009), who assume first-order stochastic dominance in conjunction with a two-point distribution for y – “low-skill” and “high-skill” workers. Other papers achieve a similar effect by making a specific functional form assumption, typically that productivity is the product of worker type and an independent match-specific component.

large by international standards. Our baseline calibration indicates that most of this premium is attributable to different distributions of education in the two sectors. While more educated workers are more productive in either sector, we find that more highly educated workers sort into the government sector and this largely accounts for the wage premium. In general, our calibration and our numerical experiments suggest that to understand the differences between public- and private-sector wages and productivity, and, more generally, to understand how the two sectors interact requires explicitly considering worker heterogeneity. This allows us to look not only at the differences in means, but also at the differences in wage and productivity distributions and their effects on the aggregate economy.

In terms of related literature, there are relatively few other papers that incorporate public-sector employment into an equilibrium search and matching framework. Two papers, namely, Burdett (2012) and Bradley, Postel-Vinay and Turon (2014), incorporate a public sector into the Burdett and Mortensen (1998) model of on-the-job search, while four papers, namely, Quadrini and Trigari (2007), Michaillat (2014), Gomes (2015a) and Gomes (2015b), use a DMP framework.³ Among these papers, only Gomes (2015b) allows for worker heterogeneity, but his model differs from ours in several ways. Most importantly, he assumes that workers differ along two dimensions but only in a binary fashion - a worker is either of high or low ability and either has or doesn't have a college degree. As a result, Gomes (2015b) can only address the distributional questions that are the core of our paper in a limited way.⁴

The rest of our paper is organized as follows. In the next section, we lay out our model and establish the existence of equilibrium. In Section 3, we discuss our calibration. Section 4 presents the results of counterfactual experiments, and Section 5 concludes.

³One reason that we use a DMP, rather than a Burdett-Mortensen, approach is that direct transitions from the private to the public sector and vice versa are relatively rare in Colombia. Using the method described in Robayo-Abril (2015), we estimate that on an annual basis, the probability of a direct (i.e., without an intervening spell of unemployment) transition from the public to the private sector is less than .04. Our estimate of the transition probability in the opposite direction is less than one quarter of one percent. Because of time aggregation, these estimates are upper bounds.

⁴In addition to these six completed papers, Navarro and Tejada (work in progress) are applying the approach developed in this paper to analyze the interaction between private- and public-sector labor markets in Chile.

2 Model

We consider a model with search and matching frictions. Only the unemployed search, and their prospects depend on overall labor market tightness, $\theta = (v_p + v_g)/u$, where v_p and v_g are the measures of private- and public-sector vacancies posted at any instant, and u is the fraction of the workforce that is unemployed. Search is random, so conditional on meeting a prospective employer, the probability that the job is in the private sector is $\phi = v_p/(v_p + v_g)$.⁵ Specifically, job seekers meet prospective employers at Poisson rate $m(\theta)$, and employers meet job seekers at rate $m(\theta)/\theta$. Not all meetings lead to matches. In the private sector, a match forms if and only if the realized value of x is high enough so that the match is jointly worthwhile for the worker and firm. The threshold value of x depends in general on the worker's type. That is, a private-sector match forms if and only if $x \geq R_p(y)$, where $R_p(y)$ is a type-specific reservation productivity. In the public sector, a match forms if and only if $x \geq R_g(y)$. The key equilibrium objects are the reservation productivity schedule, $R_p(y)$, overall labor market tightness, θ , and the fraction, ϕ , of vacancy postings that are accounted for by the private sector. These objects are determined in equilibrium by (i) the condition that private-sector matches form if and only if doing so is in the joint interest of the worker and firm, (ii) a free-entry condition for private-sector vacancies, and (iii) steady-state conditions for worker flows into and out of unemployment, private-sector employment and public-sector employment.

2.1 Value Functions, Wages, Reservation Values

We start with the optimization problem for a worker of type y . Let $U(y)$, $N_p(x, y)$, and $N_g(x, y)$ be the values (expected discounted lifetime incomes) associated with unemployment and employment in, respectively, a private-sector job and a public-sector job with match-specific productivity x . The value of unemployment for a worker of type y is defined by

$$rU(y) = z(y) + \phi m(\theta) E \max[N_p(x, y) - U(y), 0] + (1 - \phi) m(\theta) E \max[N_g(x, y) - U(y), 0] \quad (1)$$

⁵As will be seen below, an assumption of sector-specific search, as in Gomes (2015) and Quadrini and Trigari (2007), would give the unrealistic prediction of perfect sorting. That is, all workers above some type y^* would search exclusively in one sector while all workers of type below y^* would search in the other sector.

This expression reflects the following assumptions. Time is continuous, and the worker lives forever, discounting the future at rate r . The worker of type y receives a flow value $z(y)$ while unemployed. Private-sector vacancies are met at rate $\phi m(\theta)$, and public-sector vacancies are met at rate $(1 - \phi)m(\theta)$. When the worker meets a vacancy, a match-specific productivity is realized, and the worker realizes a capital gain, either $N_p(x, y) - U(y)$ or $N_g(x, y) - U(y)$, if the relevant difference is positive; zero otherwise.⁶

The two employment values are defined by

$$rN_p(x, y) = w_p(x, y) + \delta_p(y)(U(y) - N_p(x, y)) \quad (2)$$

$$rN_g(x, y) = w_g(x, y) + \delta_g(y)(U(y) - N_g(x, y)). \quad (3)$$

The private-sector wage is determined by Nash bargaining with an exogenous worker share parameter, as described below, while the public-sector wage schedule is exogenous. Job destruction is assumed to occur at exogenous Poisson rate $\delta_s(y)$, and we allow for the possibility that $\delta_p(y) \neq \delta_g(y)$.

On the private-sector firm side, let $J(x, y)$ be the value (expected discounted profit) associated with a job filled by a worker of type y whose match-specific productivity is x , and let V be the value associated with posting a private-sector vacancy. These values are defined by

$$rJ(x, y) = x - w_p(x, y) + \delta_p(y)(V - J(x, y)) \quad (4)$$

$$rV = -c + \frac{m(\theta)}{\theta} E \max[J(x, y) - V, 0]. \quad (5)$$

The expectation in equation (5) is taken with respect to the joint distribution of (x, y) across the population of unemployed job seekers. A private-sector firm with a vacancy doesn't know what worker type it will meet next nor does it know what match-specific productivity this worker will draw. The firm does know, however, the distribution of worker types among the unemployed and the conditional distribution function $G_p(x|y)$.

We assume that the private-sector wage for a worker of type y with match-specific productivity x is determined via Nash bargaining with exogenous worker share parameter β . Imposing the free-entry condition for private-sector vacancy creation in advance, i.e.,

⁶Note that we allow the flow value of unemployment, z , to vary with worker type. The contact rates, $\phi m(\theta)$ and $(1 - \phi)m(\theta)$ are assumed to be independent of y ; however, the job accession rates in the two sectors do vary with y since not all contacts lead to a match.

$V = 0$, the Nash bargaining solution implies

$$w_p(x, y) = \beta x + (1 - \beta)rU(y); \quad (6)$$

that is, the private-sector wage is a weighted average of the flow productivity of the match, x , and the flow value of the worker's outside option, $rU(y)$.

Substituting equation (6) into equation (2) and assuming that $w_g(x, y)$ is increasing in x for $x \geq R_g(y)$, it is clear that $N_p(x, y)$ and $N_g(x, y)$ are nondecreasing in x for any value of y . Accordingly, reservation productivities can be defined for the type- y worker. The private-sector reservation productivity for a type- y worker, $R_p(y)$, is defined by $N_p(R_p(y), y) = U(y)$. Using equations (2) and (6), $N_p(R_p(y), y) = U(y)$ implies $R_p(y) = rU(y)$. That is, at $x = R_p(y)$ the net surplus associated with the match equals zero. The public-sector reservation productivity for a type- y worker is simply $R_g(y)$. This is equivalent to assuming that, given the public-sector wage schedule, $N_g(R_g(y), y) \geq U(y)$. If $N_g(R_g(y), y) > U(y)$, there is rationing of public-sector jobs for type- y workers. If $N_g(R_g(y), y) = U(y)$, then $R_g(y) = rU(y) = R_p(y)$; that is, the public- and private-sector reservation productivities are equal for the type- y worker. Finally, we could in principle consider the case of $N_g(R_g(y), y) < U(y)$. In this case, however, matches would not form for $x \in [R_g(y), R_p(y))$ because workers would reject them. In this sense, it is without loss of generality to assume $N_g(R_g(y), y) \geq U(y)$.

To further characterize the private-sector reservation productivity, it is useful to rewrite our expression for $rU(y)$. Using equations (2) and (6) and integrating by parts gives

$$E \max[N_p(x, y) - U(y), 0] = \frac{\beta}{r + \delta_p(y)} \int_{R_p(y)}^{\bar{x}} (1 - G_p(x|y)) dx.$$

Similarly, using equation (3) together with $rU(y) = R_p(y)$ gives

$$E \max[N_g(x, y) - U(y), 0] = \frac{1}{r + \delta_g(y)} \int_{R_g(y)}^{\bar{x}} (w_g(x, y) - R_p(y)) dG_g(x|y).$$

Substituting into equation (1) then gives

$$\begin{aligned}
R_p(y) = & z(y) + \phi m(\theta) \frac{\beta}{r + \delta_p(y)} \int_{R_p(y)}^{\bar{x}} (1 - G_p(x|y)) dx \\
& + (1 - \phi) m(\theta) \frac{1}{r + \delta_g(y)} \int_{R_g(y)}^{\bar{x}} (w_g(x, y) - R_p(y)) dG_g(x|y).
\end{aligned} \tag{7}$$

Given overall labor market conditions, i.e., θ and ϕ , and the government's employment and wage-setting policy, equation (7) gives a unique solution for $R_p(y)$ since the RHS of equation (7) is positive at $R_p(y) = 0$, goes to z as $R_p(y) \rightarrow \infty$, and the derivative of the RHS with respect to $R_p(y)$ is negative.

2.2 Free-Entry and Steady State Conditions

The next step is to characterize optimal entry by private-sector firms. Imposing $V = 0$ in advance and using equation (4), we have

$$J(x, y) = \frac{x - w_p(x, y)}{r + \delta_p(y)} = (1 - \beta) \frac{x - R_p(y)}{r + \delta_p(y)}.$$

Letting $F_u(y)$ denote the distribution function of Y among the unemployed, the free-entry condition, i.e., equation (5) with $V = 0$, can be written as

$$\begin{aligned}
c &= \frac{m(\theta)}{\theta} \left(\frac{1 - \beta}{r + \delta_p} \right) \int_{\underline{y}}^{\bar{y}} \int_{R_p(y)}^{\bar{x}} (x - R_p(y)) dG_p(x|y) dF_u(y) \\
&= \frac{m(\theta)}{\theta} \left(\frac{1 - \beta}{r + \delta_p(y)} \right) \int_{\underline{y}}^{\bar{y}} \int_{R_p(y)}^{\bar{x}} (1 - G_p(x|y)) dx dF_u(y),
\end{aligned} \tag{8}$$

where the final equality uses integration by parts.

The only unknown in equation (8) is the contaminated distribution function, $F_u(y)$.

Using Bayes Law, we can write⁷

$$F_u(y) = \frac{u(y)F(y)}{u};$$

that is, the distribution of types among the unemployed, $F_u(y)$, can be written as the type-specific unemployment rate, $u(y)$, times the population distribution function, $F(y)$, normalized by the overall unemployment rate,

$$u = \int_{\underline{y}}^{\bar{y}} u(y)dF(y).$$

To derive the type-specific unemployment rates, $u(y)$, let $n_p(y)$ and $n_g(y)$ be the fractions of time that a type- y worker spends in private-sector and public-sector employment, respectively. In steady state, the following two equations must hold:

$$\delta_p(y)n_p(y) = \phi m(\theta)(1 - G_p(R_p(y)|y))u(y) \quad (9)$$

$$\delta_g(y)n_g(y) = (1 - \phi)m(\theta)(1 - G_g(R_g(y)|y))u(y). \quad (10)$$

The first condition equates the flow from private-sector employment to unemployment and vice versa, and the second condition equates the flow from public-sector employment to unemployment and vice versa. Using

$$u(y) + n_p(y) + n_g(y) = 1,$$

equations (9) and (10) imply

$$\begin{aligned} u(y) &= \frac{\delta_g(y)\delta_p(y)}{\delta_g(y)\delta_p(y) + \delta_g(y)\phi m(\theta)(1 - G_p(R_p(y)|y)) + \delta_p(y)(1 - \phi)m(\theta)(1 - G_g(R_g(y)|y))} \\ n_p(y) &= \frac{\delta_g(y)\phi m(\theta)(1 - G_p(R_p(y)|y))}{\delta_g(y)\delta_p(y) + \delta_g(y)\phi m(\theta)(1 - G_p(R_p(y)|y)) + \delta_p(y)(1 - \phi)m(\theta)(1 - G_g(R_g(y)|y))} \\ n_g(y) &= \frac{\delta_p(y)(1 - \phi)m(\theta)(1 - G_g(R_g(y)|y))}{\delta_g(y)\delta_p(y) + \delta_g(y)\phi m(\theta)(1 - G_p(R_p(y)|y)) + \delta_p(y)(1 - \phi)m(\theta)(1 - G_g(R_g(y)|y))} \end{aligned} \quad (11)$$

⁷ A similar derivation is given in Albrecht, Navarro and Vroman (2009).

Substituting the expression for $u(y)$ into equation (8) completes the characterization of the private-sector free-entry condition.

The final unknown that needs to be characterized is ϕ , the fraction of vacancies that are posted by private-sector firms. To do this, note that since

$$v_p + v_g = \theta u,$$

$$\phi = v_p / (v_p + v_g)$$

implies

$$\phi = \frac{\theta u - v_g}{\theta u}. \quad (12)$$

This closes the model.

2.3 Equilibrium

Definition: A steady-state equilibrium is a function, $R_p(y)$, that satisfies equation (7) for all $y \in [\underline{y}, \bar{y}]$ together with scalars θ and ϕ that satisfy equations (8), (11) and (12).

An equilibrium always exists. First, as noted above, for given values of θ and ϕ , the reservation productivity, $R_p(y)$, is uniquely determined. Second, given any value of ϕ , equation (8) has at least one solution for θ . The argument is standard. The RHS of equation (8) is continuous in θ , it converges to infinity as $\theta \rightarrow 0$, and it goes to zero as $\theta \rightarrow \infty$. Finally, once $R_p(y)$ and θ are determined as functions of ϕ , equation (12) has at least one solution in ϕ . (The complication, of course, is that u depends on ϕ .) Note that we do not claim uniqueness. In equation (8), $f_u(y)$ need not be monotonically decreasing in θ nor is it obvious that equation (12) has a unique solution. Uniqueness depends on the form of $F(y)$, $G_p(x|y)$, $G_g(x|y)$ and public-sector employment policy and needs to be investigated numerically.⁸

Given any parameter configuration and given any assumed public-sector wage and employment policy, once we know $\{R_p(y), \theta, \phi\}$, the model can be solved for the equilibrium distributions of wages, productivities and human capital across the two sectors. This can be done analytically. The model gives us the distribution of Y across the unemployed, namely, $F_u(y)$, and the conditional distributions, $G_p(x|y)$ and $G_g(x|y)$, are given exogenously. Then, using the reservation productivity rules, $R_p(y)$ and $R_g(y)$, together with the contact rates,

⁸The possibility of non-uniqueness of equilibrium is a common feature of DMP models with worker heterogeneity. See, e.g., Albrecht, Navarro and Vroman (2009) and Chéron, Hairault and Langot (2011).

$\phi m(\theta)$ and $(1 - \phi)m(\theta)$, and the job destruction rates, $\delta_p(y)$ and $\delta_g(y)$, we can derive the joint distributions of (X, Y) across the two sectors. Finally, using the Nash bargaining rule for the private sector and the exogenous wage-setting rule, $w_g(x, y)$, for the public sector, we can derive the distributions of wages across the two sectors. Another approach is to find the equilibrium distributions by simulating the model. That is, we feed the assumed distribution of worker types into the model and use the distributions of wages, productivities and human capital across the two sectors that are generated by simulation. This latter approach is the one we use below.

3 Calibration

3.1 Data

To calibrate the model, we use data from the Colombian Household Survey (GEIH) from the second quarter of 2013. These surveys are repeated cross sections that are carried out by the Colombian Statistics Department (DANE) and are administered to a sample of employed and unemployed individuals in thirteen metropolitan areas.⁹ We restrict our sample to male salaried full-time workers, and we exclude workers with less than a completed primary school level of education. We also exclude the self-employed, domestic employees and unpaid family workers. The objective of these exclusions is to construct a sample that is primarily comprised of formal-sector workers.¹⁰ Our sample consists of 8,508 individuals, who represent 2.2 million people.

Our calibration strategy is informed by the quality of data available from the household surveys. The data we work with are as follows. First, we know the number of years of education completed by each individual in the sample. Completed primary corresponds to five years of education, and we index workers by years of education completed, $j = 5, \dots, 20$.¹¹ Second, we know whether each respondent is unemployed, employed in the private sector, or employed in the public sector, that is, we know the distribution of workers across the three labor market states of the model. Third, we observe wages for private- and public-sector employees. More precisely, we observe monthly earnings and weekly hours

⁹These areas include the following cities and their metropolitan areas: Bogotá, Cali, Medellín, Barranquilla, Bucaramanga, Manizales, Pasto, Pereira, Cucuta, Villavicencio, Ibagué, Monteria and Cartagena.

¹⁰For an equilibrium search and matching model of the interplay between the informal and formal labor markets in Colombia, see Robayo-Abril (2014).

¹¹We drop workers with more than 20 years of education because there are too few individuals in this category.

and use these to construct an hourly wage for each employed worker.¹² Wages include tips and commissions. Since we are concerned about measurement error, we trim the top and bottom 1% of wages in each educational class. That is, we trim the top and bottom 1% of wages among workers who have completed 5 years of education, we do the same for workers who have completed 6 years of education, etc. In addition, because we use minimum observed wages in our calibration strategy to estimate reservation productivities (see below), we trim another 9% of wages from the bottom of the distribution within each educational class. In short, we trim the top 1% and the bottom 10% of observed wages within each educational class.¹³

Educational attainment, labor market state and wage all refer to the respondent's situation as of the survey date, so we are reasonably confident in these data. In addition, retrospective data are available on each respondent's labor market state in the previous year and on his elapsed duration in his current labor market state, but we view these data as less reliable. Regarding previous labor market state, the data suffer from the standard time aggregation problem. For example, a respondent who reports himself as unemployed as of the survey date and also reports that he was unemployed one year prior may have had an employment spell (or spells) in the intervening period. The duration data are also problematic. In particular, an individual who is currently employed reports how many months elapsed between the end of his previous job and the start of his current job, but we cannot tell whether he was unemployed or out of the labor force (a state not included in our model) in the intervening period. Accordingly, we primarily rely on the education, labor market state and wage data in our calibration. We do, however, use data on average durations in private- and public-sector employment in one step in our calibration procedure.

3.2 Stylized Facts

We emphasize the following broad facts about the Colombian labor market. First, the level of public sector employment is quite low in Colombia, and the unemployment rate

¹²We express wages in 2008 terms because that is the base year for the Colombian consumer price index.

¹³We have done a partial robustness check with respect to the trimming rule. Specifically, we have looked at some of the implications of trimming the bottom 5% and the bottom 1% of wages in each educational class, and these alternative trimming rules do not change our results qualitatively. We prefer 10% because with this trimming rule less than one percent of public-sector wages are below the legal minimum (\$1.20 per hour in Colombia in 2008 dollars). In principle, no public-sector wages should be less than this value. With a 5% rule, almost 2% of public-sector wages are below the legal minimum; with a 1% rule, almost 4% are below the minimum wage.

is quite high. As can be seen in Table 1, the public sector accounts for 8.3% of total employment, which is quite low by developed and middle-income country standards and is approximately half the level of most Latin American countries.¹⁴ Second, wages in the public sector are considerably higher than in the private sector. Figure 1 shows the kernel density of log wages by sector. As shown in Table 2, the mean log hourly wage in the public sector is \$1.33 as compared to \$0.85 in the private sector, a difference of 0.48 log wage points.¹⁵ This is a large public-sector premium.¹⁶ The degree of wage dispersion is similar in the two sectors – the standard deviation of log hourly wages is \$0.57 in the public versus \$0.60 in the private sector. This is also in contrast to the typical developed and middle-income country pattern, which exhibits a tendency towards wage compression in the public sector. Finally, even though the duration data are less than perfect, it is clear from Table 2 that employment tends to last much longer in the public sector than in the private sector.

The primary observation that motivates our calibration strategy is the fact that wages in the public sector are so much higher than those in the private sector. This fact has several possible explanations. First, there may be a pure public-sector premium; that is, the public-sector wage may simply add a bonus to what an equally qualified and equally productive worker would earn in the private sector. Second, public-sector workers are more highly educated on average than are their private-sector counterparts. Specifically, on average the public-sector workers in our sample have completed 13.9 years of education while the corresponding figure for private-sector workers is 11.6 years of education. Third, the weights on the wage determination rule on productivity versus qualifications may differ between the two sectors. For example, the public-sector wage may give a higher reward to credentials (years of education) with a corresponding lower reward to productivity than is the case in the private sector. Finally, there may be inherent productivity differences between the two sectors; that is, the distributions of productivity conditional on qualifications may not be the same in the private and public sectors. Conditional on our identifying assumptions, we are able to make some progress in our calibration towards distinguishing among these four explanations of the public-sector wage premium.

¹⁴See Table 1 in Mizala et al. (2011). Note that the figures presented there include all urban workers.

¹⁵The corresponding figures in levels are as follows. The mean hourly wage is \$4.50 in the public sector and \$3.03 in the private sector, which corresponds to a 48.5% difference.

¹⁶See Table 2 in Mizala et al. (2011) for Latin American wage gaps. See Borland and Gregory (1999) for a survey of results on the public-sector premium in developed countries.

3.3 Calibration Strategy

Our calibration strategy consists of the following steps.

Step 1: We begin by specifying public-sector wage and employment rules. We assume that wages in the public sector are determined by a surplus splitting ruler and, in addition, we allow for the possibility of a pure public-sector premium. Specifically, we assume

$$w_g(x, y) = \psi + \gamma x + (1 - \gamma)R_p(y).$$

Here ψ represents the pure public-sector premium, γ represents the weight placed on match-specific productivity in public-sector jobs, and $1 - \gamma$ represents the weight placed on “qualifications.” In general, we allow for the possibility that $\gamma \neq \beta$ – for example, the public sector may place a relatively strong emphasis on credentials in its wage setting –that is, $\gamma < \beta$ –, but we set $\gamma = \beta$ in our baseline calibration.

We also need to specify which workers the public sector is willing to hire. We do this by assuming that the public sector hires if and only if $x \geq w_g(x, y)$. That is, when an unemployed worker makes contact with a public-sector vacancy, that contact generates a match if and only if the worker’s productivity is at least as great as the wage he would be paid in the match. This is in the spirit of a basic assumption of the DMP model in the private sector, namely, that a match forms if and only if it is in the joint interest of the worker and employer to do so. Note that setting $R_g(y) = w_g(R_g(y), y)$ implies

$$R_g(y) = \frac{\psi}{1 - \gamma} + R_p(y).$$

The term $\frac{\psi}{1 - \gamma}$ reflects the extent to which public-sector jobs are rationed.¹⁷

Finally, public-sector employment policy is characterized by v_g , the measure of vacancies posted in the public sector. Rather than specifying the level of public-sector vacancy creation exogenously, we estimate v_g as a part of our calibration, as described below.

Step 2: A basic exogenous object in our model is the distribution of human capital, Y , across workers. To estimate this distribution, we equate human capital with completed years of education. We discretize the distribution of Y with p^j estimated as the fraction of the labor force with j years of education for $j = 5, \dots, 20$. We also observe the empirical

¹⁷This discussion implicitly assumes that $\psi \geq 0$, which is what our calibration will indicate.

counterparts of $\{p_p^j\}_{j=5}^{20}$ and $\{p_g^j\}_{j=5}^{20}$, that is, the distributions of educational attainment in private- and public-sector employment. When we assess the goodness of fit of our model, we compare these empirical distributions with the corresponding distributions predicted by our calibrated model.

Step 3: We estimate reservation productivities for each worker type for private- and public-sector employment. Given that private-sector wages are determined by Nash bargaining with exogenous share parameter β , we have

$$w_p^j(x) = \beta x + (1 - \beta)R_p^j; \quad (13)$$

that is, a worker with j years of education with realized productivity x on a private-sector job receives wage $w_p^j(x)$. A private-sector match with a worker of type j forms if and only if $x \geq R_p^j$, and a worker of this type with match-specific productivity R_p^j receives a wage of $w_p^j(R_p^j) = R_p^j$. Accordingly, we use the minimum observed private-sector wage (after trimming) among workers with j years of education to estimate R_p^j . Similarly, we use the minimum observed public-sector wage among workers with j years of education to estimate R_g^j . This procedure gives us estimates $\hat{R}_p^j$ and $\hat{R}_g^j$ for $j = 5, \dots, 20$. We use these estimated reservation productivities to estimate the public-sector rationing factor, namely, $\psi/(1 - \gamma)$, using $\sum_{j=5}^{20} \hat{p}_j (\hat{R}_g^j - \hat{R}_p^j)$. Note that we are imposing the restriction that the public-sector rationing factor is the same for all worker types.

Step 4: Since the private-sector wage for a worker of type j is linear in x and R_p^j , once we have an estimate for R_p^j , we can make assumptions that allow us to use the observed distribution of private-sector wages across workers with j years of education to estimate $G_p^j(x)$, that is, the distribution of private-sector productivity across workers with j years of education. To do this, we assume that $G_p^j(x)$ is a log-normal distribution function with parameters μ_p^j and σ_p^j ; that is, we assume that the log of productivity in potential private-sector jobs across workers having j years of education is normally distributed with mean μ_p^j and standard deviation σ_p^j . Using equation (13), we have

$$\ln x = \ln \left(\frac{w_p^j - (1 - \beta)R_p^j}{\beta} \right).$$

As is typical in models in which the wage is determined by Nash bargaining, β is unidentified. Accordingly, we follow the literature and assume a standard value for the share

parameter, namely, $\beta = 0.5$.¹⁸ Given this assumed value for β , our estimate for R_p^j , and observed wages, we have a set of estimated values for the log productivity of workers of type j who are employed in private-sector jobs. We then use expressions for the mean and variance of a truncated ($\ln x \geq \ln R_p^j$) log-normal distribution to back out estimates of μ_p^j and σ_p^j .

We use an analogous procedure to estimate $G_g^j(x)$, that is, the conditional distribution of public-sector productivity across workers with j years of education. Again, we assume log normality, this time with parameters μ_g^j and σ_g^j . Using

$$w_g^j(x) = \psi + \gamma x + (1 - \gamma)R_p^j$$

or, equivalently,

$$w_g^j(x) = \gamma x + (1 - \gamma)R_g^j$$

gives

$$\ln x = \ln \left(\frac{w_g^j - (1 - \gamma)R_g^j}{\gamma} \right).$$

Since the public-sector share parameter is unidentified, we assume $\gamma = \beta = 0.5$ in our baseline calibration. We then have a set of estimated values for the log productivity for workers with j years education across public sector employees, and, exactly as we did with the private-sector parameters, we use these values to back out estimates of μ_g^j and σ_g^j .

Step 5: Next, we estimate the parameters governing transitions from unemployment to private- and public-sector employment and vice versa. Since the duration information in our dataset is retrospective and subject to a variety of biases (time aggregation, etc.) we want to minimize the extent to which we use duration data in this estimation procedure. Our assumption that workers contact private-sector vacancies at the same rate independent of type, that is, the assumption that $m(\theta)\phi$ does not vary with j , and similarly for the rate at which workers contact public-sector vacancies, helps us achieve this objective.

We proceed as follows. Workers with j years of education move from unemployment to employment at rate $m(\theta)\phi(1 - G_p^j(R_p^j))$, and they flow in the opposite direction at rate δ_p^j ;

¹⁸Employer-side data on productivity are typically required to identify β . Note that we are assuming the same value of β for all worker types, that is, for all j . In order for equation (7) to hold for each worker type, either β or z needs to vary with j . Our choice is to keep β fixed while allowing z to adjust across worker types.

thus, in steady-state,

$$m(\theta)\phi(1 - G_p^j(R_p^j))u^j = \delta_p^j n_p^j. \quad (14)$$

Similarly, the flow of type j workers from unemployment to public-sector employment and vice versa satisfies

$$m(\theta)(1 - \phi)(1 - G_g^j(R_g^j))u^j = \delta_g^j n_g^j. \quad (15)$$

These steady-state equations hold for each worker type. Once we estimate $m(\theta)$ and ϕ , these equations give us estimates of the job destruction rates, $\{\delta_p^j\}_{j=5}^{20}$ and $\{\delta_g^j\}_{j=5}^{20}$.

To estimate $m(\theta)$ and ϕ , we use expressions for the average durations of private- and public-sector employment. The model assumes exponential durations; thus, for example, the expected duration of private-sector employment for a worker with j years of education is $1/\delta_p^j$. The expected duration of private-sector employment averaged across all worker types can therefore be written as

$$E[T_p] = \sum_{j=5}^{20} p_p^j \left(\frac{1}{\delta_p^j} \right).$$

Using equation (14),

$$E[T_p] = \sum_{j=5}^{20} p_p^j \left(\frac{n_p^j}{m(\theta)\phi(1 - G_p^j(R_p^j))u^j} \right). \quad (16)$$

Similarly, the expected duration of public-sector employment across all worker types is

$$E[T_g] = \sum_{j=5}^{20} p_g^j \left(\frac{n_g^j}{m(\theta)(1 - \phi)(1 - G_g^j(R_g^j))u^j} \right). \quad (17)$$

The only “unknowns” on the right-hand sides of equations (16) and (17) are $m(\theta)$ and ϕ . Plugging in the sample counterparts for $E[T_p]$ and $E[T_g]$ together with our already-computed estimates of the various objects on the right-hand sides of equations (16) and (17) gives us estimates of $m(\theta)$ and ϕ .

Step 6: The final step in our calibration procedure assembles a number of loose ends. First, we back out an estimate for θ . To do this, we assume Cobb-Douglas matching, namely,

$$m(\theta) = A\theta^\alpha.$$

Since reliable vacancy data are not available in Colombia, we set values for A and α . Specifically, we choose $\alpha = 0.5$, so the Hosios condition is satisfied, and then set $A = 0.25$. The latter choice is made to be consistent with the literature (e.g., Pissarides and Petrongolo 2001) and to produce a “reasonable” value of θ in the calibration. Given an estimate of $m(\theta)$ from the previous step, we then have an estimate for θ .

Next, we use our estimates of θ and ϕ together with equation (12) to set a value for v_g . We also use our estimates of θ and ϕ to back out an estimate of c , using the free-entry condition for private-sector vacancy creation. To do this, we need to fix a value for the discount rate, and we set $r = 0.0217$.¹⁹

Finally, the last set of parameters that we estimate are the type-specific flow values of leisure, that is, the $\{z^j\}_{j=5}^{20}$. These estimates are backed out from a discretized version of equation (7). As in many other studies, we find negative values.²⁰

3.4 Calibration Results

We use a quarter as the unit of time in our calibration. Tables 4 and 5 show the estimated parameters. Table 4 presents the estimates of parameters that are assumed to be independent of worker type. First, we estimate that the flow cost of posting a private-sector vacancy is $\hat{c} = \$3.66$ per hour, about 16% more than the average wage. Second, we estimate the probability that an unemployed worker makes contact with a job opening within a quarter to be $\widehat{m(\theta)} = 0.214$. Since we observe long average durations of employment, a low contact probability is required to fit the observed unemployment rate of $u = 0.230$. Given our assumptions that $A = 0.25$ and $\alpha = 0.5$, our estimate of $m(\theta)$ implies an estimated labor market tightness of $\hat{\theta} = 0.736$, which is in the usual range of estimates for this variable. Third, we estimate $\hat{\phi} = 0.923$; that is, about 92% of all vacancy postings are made in the private sector.

Table 4 also gives our estimates of the parameters that describe the public-sector wage and employment policy rules. First, we estimate $\psi = \$0.17$; that is, we estimate a pure-public sector premium of \$0.17 per hour, which is a bit more than 5% of the average wage observed in the data. Given our assumption of $\gamma = 0.5$, this implies a public-sector rationing factor of \$0.34 per hour; that is the reservation productivity in the public sector is higher than the corresponding private-sector cutoff value by 34 cents per hour’s worth

¹⁹This is consistent with an annual real interest rate of 8.96%.

²⁰As discussed in Hornstein, Krusell and Violante (2011), see Table 7 in Eckstein and Wolpin (1995) or Table II in the survey paper by Bunzel et al. (2001).

of output. Finally, we use equation (12) to back out $\hat{v}_g = 0.013$. Since the size of the labor force is normalized to one, the interpretation is that at any point in time, slightly more than one public-sector vacancy is posted for every 100 workers in the labor force.

Table 5 presents estimates of the parameters that we allow to vary with worker type, that is, with j . In the second column of the table, note the relatively high weights associated with $j = 5$ (completed primary education), $j = 11$ (completed secondary education), $j = 13$ (2 years of post-secondary education), $j = 14$ (3 years of post-secondary education), and $j = 16$ (university education). Column 3 presents our estimates of the $\{z^j\}_{j=5}^{20}$. These estimates are (i) uniformly negative and (ii) strongly decreasing in j . These estimates lack a natural interpretation. Our general approach in calibrating the model has been to give ourselves sufficient degrees of freedom – in this case, allowing z to vary with j – to allow the model to fit the data well. Starting from a good fit makes it easier to interpret the results of the counterfactual experiments that we present in the next section. As noted above, other studies also find negative z . Columns 4 and 5 present our estimates of the parameters characterizing the conditional distributions $G_p^j(x)$, and columns 6 and 7 present the corresponding estimates of the parameters describing the public-sector distributions. In general, expected productivity increases with education in both sectors, although not perfectly monotonically. Similarly, productivity dispersion increases with j . Comparing the two sectors, our estimates suggest that $\mu_p^j > \mu_g^j$ for very low and very high levels of j . For intermediate levels of j (at least some education beyond basic primary but no more than a university degree), the data suggest the opposite. Comparing columns 5 and 7, there is some tendency for $\sigma_p^j > \sigma_g^j$. The same patterns obtain for the expected values and standard deviations of productivity conditional on $X_s^j \geq R_s^j$ for $s = p, g$. Columns 8 and 9 show the estimated values of the job destruction parameters in the two sectors. These two columns show a very strong tendency for expected duration of employment to increase with years of education in both sectors. In addition, we observe a tendency towards $\delta_g^j > \delta_p^j$ for lower values of j but the opposite (and strongly so) for all levels of education beyond completion of secondary schooling. Finally, the last column of Table 5 shows our estimates of reservation productivities for private-sector employment for each worker type. As expected, these estimated reservation productivities are (almost) monotonically increasing in worker type. Since $R_g^j = 0.34 + R_p^j$, estimates of reservation productivities for public-sector employment by worker type can be found by adding the public-sector rationing factor (0.34) to the corresponding private-sector reservation productivities given in the last column of Table

5.²¹

Table 6 provides a basis for assessing goodness of fit. In this table, using a set of unconditional moments, we compare the predictions of our calibrated model with the corresponding sample moments. The predicted moments are generated by simulating the model with all parameters set to their calibrated values. The model does a perfect job of matching the aggregate distribution of workers across unemployment, private-sector employment and public-sector employment. In terms of wages, our primary focus is on (i) the mean gap in log wages and (ii) the relative dispersion of log wages between the two sectors and (iii) overall log wage dispersion. Here our model provides a close but not perfect fit. We slightly overestimate the mean public-private log wage gap (0.490 versus 0.476) and slightly underestimate the ratio of the standard deviation of public-sector log wages to corresponding figure for private-sector log wages (a ratio of 0.907 versus 0.950). Our model-generated estimate of the overall standard deviation of log wages is essentially equal to what we see in the data. Table 6 also compares the estimated mean and standard deviation of log wages to the corresponding sample moments for each of the sectors. As can be seen, the estimated moments for the private-sector match what we see in the data almost perfectly, while the estimated public-sector moments match the data well but not perfectly. The main factor lying behind our lack of a perfect fit in the public sector is our assumption that the public-sector rationing factor, namely, $\psi/(1 - \gamma)$, is the same for all worker types. We make this assumption – which entails some sacrifice of goodness of fit – because we want our counterfactual experiments which examine the effects of varying the public-sector parameters one by one to be easily interpretable.

Table 6 also shows the mean and standard deviation of educational attainment in the public and private sectors. The key observation is that the mean number of years of education among public-sector employees exceeds the corresponding private-sector mean by more than two years. This is what we observe in the data, and the model captures this sorting almost perfectly. Finally, Table 6 also compares the mean employment duration in the two sectors that we see in the data with the corresponding model prediction. As in the data, the mean duration of employment in the public-sector predicted by the model is

²¹Note that we can estimate the $\{R_g^j\}$ in two different ways. In our counterfactual simulations we use $R_g^j = \psi + R_p^j$ with R_p^j estimated as the minimum observed private-sector wage among type j workers. Alternatively, we can estimate R_g^j as the minimum observed public-sector wage among type j workers. For low values of j , the minimum observed public-sector wage tends to be greater than $R_g^j = 0.34 + R_p^j$; for higher values of j , the inequality is reversed. In general, however, the two estimators coincide reasonably well.

approximately 2.5 times the corresponding figure for the private sector.

We also compare the distributions of worker types across the three labor market states that are generated by the model with the corresponding distributions observed in the data. These distributions (not shown) match up almost perfectly. The distribution of worker types between the private and public sectors together with our estimates of the sector-specific distributions of productivity conditional on worker type allow us to simulate the distributions of productivity in the two sectors. These are shown in Figure 2. The distribution of productivity across public-sector jobs is clearly to the right of the corresponding private-sector distribution. As can be seen in Tables 5 and 6, the reason is not that public-sector jobs are inherently more productive. Rather, the productivity difference between the two sectors has primarily to do with the fact that more highly educated workers – who are in general more productive – are more likely to be found in the public sector than in the private sector. Finally, we compare the private- and public-sector wage densities generated by model simulations with the corresponding data-based kernel densities. This is done in Figure 3, and, as can be seen, the model-generated and estimated densities match reasonably well. We interpret the approximate correspondence between the observed and simulated kernels as an indication that years of education is a good approximation for worker human capital and that our log normality assumption is also a good approximation.

What can we learn from our calibration results? In particular, what accounts for the large difference between public- and private-sector wages in Colombia? Our baseline calibration suggests that most of the wage differential can be ascribed to the different distributions of education between the two sectors. More highly educated workers are more productive, both in the private sector and in the public sector. The sorting of the more highly educated workers into the public sector is not an indication that public-sector jobs are inherently more productive. Indeed, our estimates suggest that the means of the private-sector productivity distributions for highly educated workers are higher than the corresponding public-sector means. Instead, the pattern of educational sorting that we see is likely driven by (i) the (relatively small) pure public-sector premium, ψ , and (ii) the type-specific pattern across private- versus public-sector job destruction rates. The public-sector premium affects the distribution of worker types across the two sectors because it leads to job rationing in the public sector, and this rationing is more binding for the less educated part of the workforce. Regarding job destruction rates, the data indicate that $\delta_g^j > \delta_p^j$ for low values of j , but $\delta_g^j < \delta_p^j$ for high values of j . That is, less educated workers tend to retain their private-sector jobs for a relatively long time, but among more highly

educated workers, the pattern is reversed. The bottom line is that public-sector workers are paid more because they are more highly educated (because job rationing tends to keep the less educated out of the public sector and because once highly educated workers find public-sector employment, they tend to keep their jobs for a long time) and because they are more productive (primarily because more educated workers are more productive, irrespective of where they work).

4 Counterfactual Experiments

We now turn to our counterfactual experiments. In these experiments, we change the parameters that characterize the public-sector wage and employment rules and evaluate the impact of these changes on the unemployment rate, the division of employment between the two sectors, average duration of employment in the two sectors, and on the distributions of education, productivities, and wages across the two sectors. Starting from the baseline case, we change each of the public-sector employment policy parameters, namely, v_g , ψ and γ , in turn while holding all other parameters constant. The objective of these experiments is to shed further light on the source of the large public-sector wage premium in Colombia and, more generally, to get a better understanding of how the private- and public-sector labor markets interact in a search and matching framework. The results of our experiments are presented in Tables 7 and 8 and in Figures 4-6.²²

In the first experiment, we increase the size of the public sector by doubling the measure of government vacancies from $v_g = 0.013$ to $v_g = 0.026$. This policy has a straightforward compositional effect; namely, employment is shifted from the private to the public sector. In addition, the policy change generates a one percentage point increase in the unemployment rate. The reason is that, as higher-paying public-sector jobs become easier to find, private-sector reservation productivities increase, as can be seen in Table 8. Workers are pickier, and, as a result, private-sector firms are less willing to post vacancies; that is, θ falls. The policy change also has distributional effects. As shown in Table 8, the average level of education falls slightly in both the public and in the private sector. This reflects a shift in the incidence of unemployment relative to the baseline case towards more highly educated

²²In Table 8, note that since $R_g^j = \psi + R_p^j$ for all j , the reservation productivity for public-sector jobs averaged across all worker types is the corresponding mean reservation productivity for private-sector jobs plus the public-sector rationing factor (equal to \$0.34 in columns 1, 2 and 4 and equal to zero in column 3). Thus, the standard deviations of reservation productivities for jobs in the two sectors are the same in each column.

workers. Table 8 and Figure 4 show how this change in the educational composition of the public- and private-sector workforces spills over into productivities and wages across the two sectors. Mean productivity falls in both sectors, especially in the public sector. This decrease in average productivity translates to a decrease in the mean wage paid in the public sector. In the private-sector, however, the decrease in average productivity is more than offset by the increase in private-sector reservation productivities, and the mean wage rises. This is why the increase in v_g decreases the mean log wage gap between the two sectors. The policy change also reduces productivity dispersion in both sectors. The shift in employment towards somewhat less educated workers – again, caused by increased pickiness among the more highly educated – coupled with our result that the standard deviation of productivity tends to increase with education is what drives this result. This decrease in productivity dispersion is more pronounced among public-sector workers, and this translates into a reduction in relative log wage dispersion between the two sectors and in overall wage inequality; that is, both $\frac{\sigma(\ln w)_g}{\sigma(\ln w)_p}$ and $\sigma(\ln w)$ fall. Finally, since job destruction rates tend to decrease with education, the policy change leads to a slight decrease in average duration of employment in both sectors.

In the second experiment, we eliminate the pure public-sector wage premium by setting $\psi = 0$, while keeping γ and v_g at their baseline values. This policy change reduces the difference between the mean log wages in the two sectors from 0.490 to 0.428, which corresponds to a decrease of 20 cents in the difference in wages between the two sectors in levels.²³ The interesting result of this experiment is that the spillover effects of this policy change are quite small. With $\psi = 0$, the value of search falls; that is, workers become less picky about which private-sector jobs are acceptable. However, since jobs are scarce – and public-sector jobs are particularly difficult to find – this effect is small; the average value of R_p falls by only 2 cents. Since $R_g^j = \psi + R_p^j$ for all j and since ψ is being reduced from 0.34 to zero, the average value of R_g correspondingly falls by 36 cents. The direct effect of setting $\psi = 0$ thus accounts for the lion’s share of the decrease in the public-sector wage. Almost all of the rest of the effect comes from the relative decrease in average productivities between the two sectors. Since private-sector reservation productivities decrease only slightly, setting $\psi = 0$ has only a small effect on private-sector productivity (about 2 cents per hour on average). The decrease in public-sector reservation productivities is larger and

²³In the baseline case, the average public-sector wage is \$4.50 per hour, and the corresponding figure for private-sector wages is \$3.03. Setting $\psi = 0$ decreases the mean public-sector wage by 22 cents per hour and the mean private-sector wage by 2 cents per hour.

has a correspondingly larger effect on average productivity in government jobs (about 8 cents per hour). This difference in productivity effects between the two sectors accounts for almost all of the remaining change in mean wages between the two sectors. See Figure 5 for the full effect of this policy on the wage distributions in the two sectors. Another interesting aspect of setting $\psi = 0$ is that it has almost no effect on the steady-state distribution of worker types across the two sectors. That is, even though the policy change eliminates public-sector job rationing, which is more of a constraint for less educated workers, we do not see a significant decrease in average years of education in the public sector.

In our third and final experiment, we decrease γ from 0.5 to 0.25. With $\gamma = 0.25$, the public sector puts less weight on productivity (and correspondingly more on formal qualifications) in wage setting than the private sector does. This policy change has a substantial impact on relative wages; in particular, the public-sector premium decreases by 0.31 log-wage points. That is, the reweighting in the public-sector wage-setting rule eliminates almost 2/3 of the public-sector log wage premium. The reason for this effect is that there is more dispersion in match-specific productivity than there is in educational attainment. Since there is a concentration of more highly educated workers in the public sector (both before and after the policy change) and since match-specific productivity tends to increase with education, the lower value of γ decreases the reward that public-sector workers receive for their higher levels of productivity.²⁴ In addition, because a lower weight is being placed on the relatively high variation component of wage determination, the standard deviation of wages in the public sector falls substantially. The decrease in γ also has indirect effects. First, the public-sector rationing constraint becomes less binding, that is, $\psi/(1 - \gamma)$ falls. Public-sector reservation productivities fall accordingly. Second, private-sector reservation productivities fall as a result of the decrease in public-sector wages. This leads in turn to a decrease in private-sector wages, although this decrease is much less pronounced than in the public sector. Figure 6 shows the overall effects of the policy change on the distributions of log wages in the two sectors. The decrease in γ also has some (relatively small) compositional effects. The fall in private-sector reservation productivities and the corresponding decrease in wages makes private-sector vacancy creation more attractive. As a result, θ increases slightly with an attendant small shift of workers from unemployment to private-sector employment.

²⁴ In many countries, the public-sector wage distribution is more compressed than the corresponding private-sector distribution. See, e.g., Gregory and Borland (1999). A higher weight on formal qualifications (and a correspondingly lower weight on productivity) in the the public sector may explain this observation.

The results of our counterfactual experiments refine some of our conclusions about the factors underlying the large gap between public- and private-sector wages that we observe in the data. Starting with the basic conclusion that the public-sector wage premium is primarily driven by the sorting of more educated workers into government jobs, we can see that this pattern is reinforced in Colombia by the weak overall state of the labor market. On average, it takes an unemployed worker over a year to locate a job opportunity ($m(\theta) = 0.214$), and it takes a private-sector firm almost a year to locate a job candidate ($m(\theta)/\theta = 0.291$) for its vacancy. This means that neither workers nor firms can be very selective about which matches they consummate, and, our estimates indeed suggest that a very high percentage of contacts lead to matches. The importance of a weak labor market for our conclusions is illustrated by our first counterfactual experiment. When there are more public-sector vacancies in the market, workers become more selective about which jobs they will accept, and this effect is strongest for the most highly educated. This leads to a decrease in average educational attainment among both private- and public-sector workers, and this in turn reduces the public-private wage differential. Our second experiment helps us better understand why public-sector workers tend to be more highly educated than their private-sector counterparts. The sorting of the more highly educated into the public sector may mean that it is more difficult for the less educated to get jobs in this sector (public-sector job rationing); it may mean that less educated workers find it more difficult to retain their public-sector jobs. The results of our second experiment suggest that the latter factor is the more important one. When we set $\psi = 0$, there is a direct, almost one-for-one, effect on public-sector wages, but the corresponding elimination of public-sector job rationing has virtually no effect on the distribution of education across the two sectors. Finally, our third experiment reinforces our conclusion that the productivity difference between the two sectors is the main driver of the wage differential. When we reduce the weight on productivity (by setting $\gamma = 0.25$) in the public-sector wage-setting rule, almost 2/3 of the difference in mean log wages between the two sectors is eliminated.

More generally, the results of our calibration and of our counterfactual experiments are consistent with an approach that focuses on worker heterogeneity as a key to understanding the interaction between private- and public-sector labor markets. Wage differences between the public and private sector in Colombia are primarily driven by productivity differences between the two sectors, and these productivity differences are in turn primarily driven by the different distributions of educational attainment across the workers in the two sectors. Although there is a (relatively small) pure public-sector wage premium in Colombia

($\psi = 0.17$) and while such pure premia may well exist in other countries, our approach suggests that it is of first-order importance to understand what lies behind the sorting of different worker types into the public versus the private sector. We focused on two potential explanations. First, more highly educated workers may reject private-sector jobs to wait for more attractive public-sector positions. This is more likely to happen when there is a pure public-sector premium; that is, $\psi > 0$ may, in addition to the direct effect of adding a top-up to public-sector wages, have a strong indirect effect by attracting more qualified workers to public-sector employment. This indirect effect is mostly absent in Colombia because job opportunities arrive too infrequently to allow workers to reject many private-sector jobs, but it is likely important in other countries. Second, there may be differences in retention patterns for different worker types between the private and public sector. Jobs in the public sector are typically viewed as “more secure” than private-sector jobs. To the extent that this is particularly true for highly educated workers (as in Colombia), this is a factor that increases the tendency for more highly educated workers to be concentrated in the public sector. Finally, our calibration suggests that the inherent productivity associated with public-sector jobs (in our model, this is captured by the moments of the type-specific conditional productivity distributions) does not differ substantially from the inherent productivity of private-sector jobs. That is, productivity differences between the public and private sectors is likely to be primarily a matter of how worker types are sorted across the two sectors. This seems clearly to be the case in Colombia; we suspect the same is true elsewhere. To the extent that the underlying patterns that we observe in the Colombian data generalize to other countries, a better understanding of public-private wage differences and, more generally, how the public- and private-sector labor markets interact requires explicitly taking worker heterogeneity into account.

5 Conclusion

In this paper, we have developed a search-and-matching model to analyze the interaction between labor markets in the private and public sectors. The focus of our model is on distributional questions. What types of workers sort into the two sectors? How do the size of the public sector and the public sector’s wage and employment policies affect the distribution of wages in the private sector and in the public sector? Given this focus, worker heterogeneity is a key element of our model. We calibrate our model using Colom-

bian data. Colombia is an interesting case study because the wage differential between the public and private sectors there is very large. Our calibration and counterfactual experiments are motivated by a desire to differentiate among various potential explanations of this wage gap. Although there is a pure public-sector premium in Colombia, it is small relative to the differential that needs to be explained. Instead, the primary cause of the public-private wage differential in Colombia is that more highly educated workers, who tend to be more productive regardless of whether they are employed in the private or public sector, get differentially sorted into public-sector employment. A relatively minor aspect of this sorting is that there is rationing of public-sector jobs and that this rationing tends to be more binding for less-educated workers. More importantly, public-sector employment is extremely stable for highly educated workers. Much more so than in the private sector, when a highly educated worker gets a public-sector job, he tends to keep that job for a very long time.

Public sector employment accounts for a significant fraction of employment in most economies, and the effect of public-sector labor market policy on overall labor market performance deserves more attention. The model and the calibration strategy developed in our paper can be applied more generally, and our focus on worker heterogeneity and the sorting of different worker types into the two sectors offers a useful complement to the existing literature.

Tables and Figures

Table 1: Labor Market States

Unemployed (u)	0.230 (.0045)
Employed Private Sector (n_p)	0.705 (.0049)
Employed Public Sector (n_g)	0.064 (.0026)

Author's calculations based on GEIH, Second Quarter 2013, 13 Metropolitan Areas.
Standard Errors in parenthesis

Table 2: Descriptive Statistics for Employed Population

	Employed	Public	Private
Sample Size	6,265	675	5,590
Population	1,685,135	140,656	1,544,479
$E(\ln W_s)$	0.89	1.33	0.85
$SD(\ln W)$	0.62	0.57	0.60
$E(T_{es})$	4.95	11.01	4.40
$SD(T_{es})$	6.62	8.35	6.15

Author's calculations based on GEIH, Second Quarter of 2013, 13 Metropolitan Areas. Sample includes salaried full-time male workers with completed primary education. Self-employed and unpaid family workers are excluded. All statistics weighted using sampling weights. Log earnings in hourly rates, employment duration in months.

Table 3: Fixed Parameters -Based on data or previous micro studies

	Description	Value
PARAMETERS		
r	interest rate	0.02
β	worker's Nash bargaining power	0.50
A	technological parameter, matching function	0.25
α	elasticity matching function	0.50
γ	weight on productivity, public-sector wage rule	0.50

Table 4: Estimated Parameters (1)

	Description	Value
c	cost of posting a vacancy	3.658
$m(\theta)$	offer arrival rate	0.219
ϕ	fraction of private-sector vacancies	0.923
v_g	vacancies, public sector	0.013
ψ	public sector fixed factor, wage setting	0.170

Table 5: Estimated Parameters (2)

j	p^j	z^j	μ_p^j	σ_p^j	μ_g^j	σ_g^j	δ_p^j	δ_g^j	R_p^j
5	0.08	-0.02	0.99	0.38	0.74	0.40	0.10	0.59	0.75
6	0.03	-0.35	1.09	0.50	0.73	0.05	0.12	0.91	0.67
7	0.03	-0.15	1.02	0.36	1.18	0.22	0.10	1.91	0.64
8	0.03	-0.18	0.98	0.49	1.44	0.25	0.09	1.19	0.82
9	0.04	-0.46	1.08	0.41	1.42	0.42	0.09	0.24	0.75
10	0.02	-0.06	0.85	0.49	1.55	0.06	0.11	0.18	0.69
11	0.36	-0.40	0.92	0.46	1.33	0.48	0.06	0.07	1.00
12	0.05	-0.87	1.01	0.51	1.38	0.54	0.06	0.04	1.05
13	0.10	-1.06	0.99	0.59	1.36	0.42	0.05	0.04	1.25
14	0.08	-1.58	1.12	0.62	1.52	0.57	0.05	0.04	1.44
15	0.03	-1.79	1.25	0.67	1.66	0.41	0.06	0.05	1.51
16	0.09	-9.97	2.08	0.82	2.20	0.54	0.05	0.03	1.89
17	0.03	-18.08	2.42	0.60	2.26	0.39	0.03	0.02	2.24
18	0.02	-25.34	2.56	0.83	2.24	0.59	0.02	0.01	2.89
19	0.01	-24.72	2.72	0.70	2.39	0.47	0.03	0.01	2.82
20	0.00	-8.27	2.89	0.82	2.56	0.51	0.09	0.01	5.12

Table 6: Calibration: Model vs. Data

Variable	Model	Data
AGGREGATE UNEMPLOYMENT AND EMPLOYMENT RATES ^a		
u	0.230	0.230
n_P	0.705	0.705
n_G	0.064	0.064
MEASURES OF WAGE DISPERSION		
$\mu(\ln w)_g - \mu(\ln w)_p$	0.490	0.476
$\frac{\sigma(\ln w)_g}{\sigma(\ln w)_p}$	0.907	0.950
$\sigma(\ln w)$	0.618	0.616
LOG WAGE DISTRIBUTION		
$\mu(\ln w)_g$	1.341	1.326
$\mu(\ln w)_p$	0.851	0.850
$\sigma(\ln w)_g$	0.550	0.574
$\sigma(\ln w)_p$	0.607	0.604
HUMAN CAPITAL DISTRIBUTION		
$\mu(y)_g$	13.895	13.897
$\mu(y)_p$	11.637	11.635
$\sigma(y)_g$	3.045	3.019
$\sigma(y)_p$	3.291	3.292
EMPLOYMENT DURATION ^b		
$\mu(t)_p$	4.405	4.405
$\mu(t)_g$	11.027	11.010

^a This unemployment rate represents the number of unemployed as a proportion of the adjusted labor force excluding self-employment, domestic employment and unpaid family workers.

^b Employment durations in years.

Table 7: Aggregate and Distributional Effects of Changes in Public-Sector Size and Wage Rules

Variable	Benchmark	Experiment 1	Experiment 2	Experiment 3
		↑ PS Size	Δ Wage Rule	
		$v_g = 0.0261$	$\psi = 0$	$\gamma = 0.25$
AGGREGATE EFFECTS				
u	0.230	0.243	0.230	0.228
n_P	0.705	0.630	0.705	0.708
n_G	0.064	0.127	0.064	0.064
LM TIGHTNESS & PUBLIC SECTOR SIZE				
θ	0.736	0.674	0.736	0.738
ϕ	0.923	0.840	0.923	0.922
WAGE DISPERSION				
$\mu(\ln w)_g - \mu(\ln w)_p$	0.490	0.472	0.428	0.181
$\frac{\sigma(\ln w)_g}{\sigma(\ln w)_p}$	0.907	0.901	0.962	0.769
$\sigma(\ln w)$	0.618	0.597	0.618	0.603
JOB DURATION ^a				
$\mu(t)_p$	4.405	4.351	4.405	4.406
$\mu(t)_g$	11.027	10.235	11.005	11.058

^a Employment durations in years.

Table 8: Compositional and Distributional Effects of Changes in Public-Sector Size and Wage Rules

Variable	Benchmarck	Experiment 1	Experiment 2	Experiment 3
		$\uparrow$ PS Size	Δ Wage Rule	
		$v_g = 0.0261$	$\psi = 0$	$\gamma = 0.25$
HUMAN CAPITAL				
$\mu(y)_g$	13.895	13.705	13.880	13.894
$\mu(y)_p$	11.635	11.496	11.629	11.642
$\sigma(y)_g$	3.045	2.983	3.045	3.054
$\sigma(y)_p$	3.291	3.273	3.290	3.288
PRODUCTIVITY				
$\mu(x)_g$	7.033	6.861	6.955	6.99
$\mu(x)_p$	4.847	4.744	4.828	4.789
$\sigma(x)_g$	5.367	5.201	5.368	5.369
$\sigma(x)_p$	6.721	6.426	6.760	6.677
RES PRODUCTIVITY				
$\mu(x)_g$	1.5450	1.6063	1.1830	1.3948
$\mu(x)_p$	1.2035	1.2649	1.1830	1.0534
$\sigma(x)_g = \sigma(x)_p$	0.7920	0.8091	0.7867	0.6905
LOG WAGE				
$\mu(\ln w)_g$	1.341	1.334	1.270	0.989
$\mu(\ln w)_p$	0.851	0.862	0.842	0.808
$\sigma(\ln w)_g$	0.550	0.522	0.585	0.470
$\sigma(\ln w)_p$	0.607	0.579	0.608	0.611

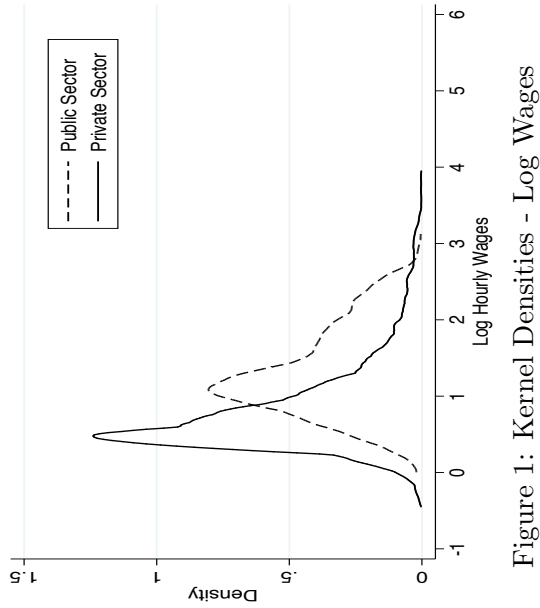


Figure 1: Kernel Densities - Log Wages

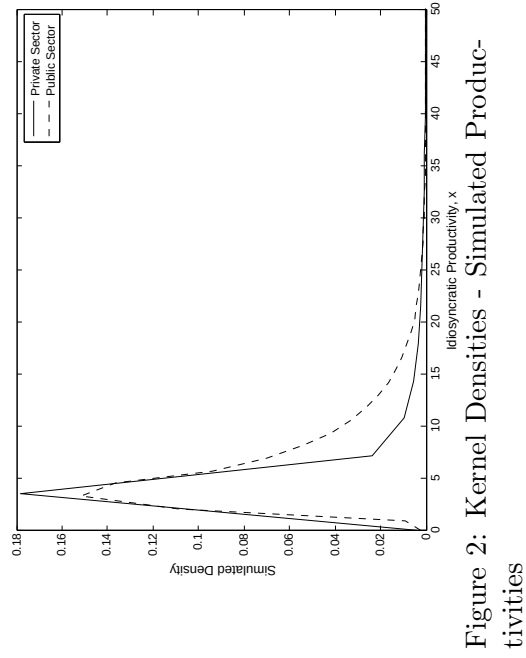


Figure 2: Kernel Densities - Simulated Productivities

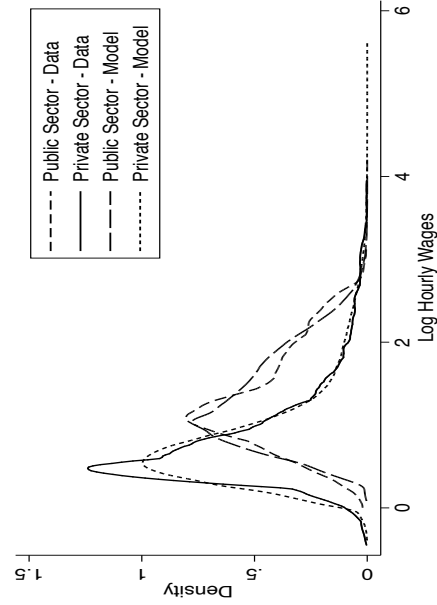


Figure 3: Kernel Densities - Log Wages, by Sector: Model vs. Data

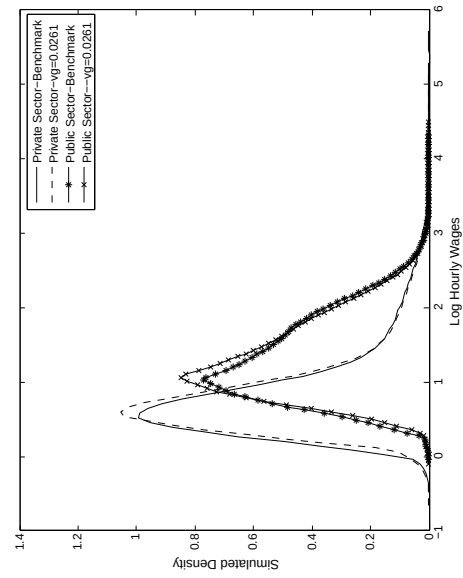


Figure 4: Benchmark vs. Experiment 1

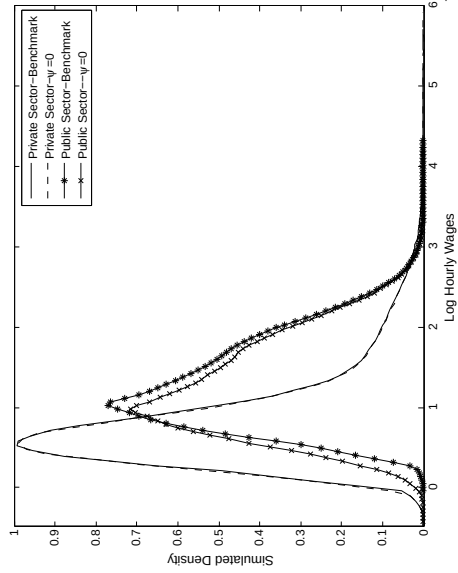


Figure 5: Kernel Densities - Log Wages, by Sector: Benchmark vs. Experiment 2

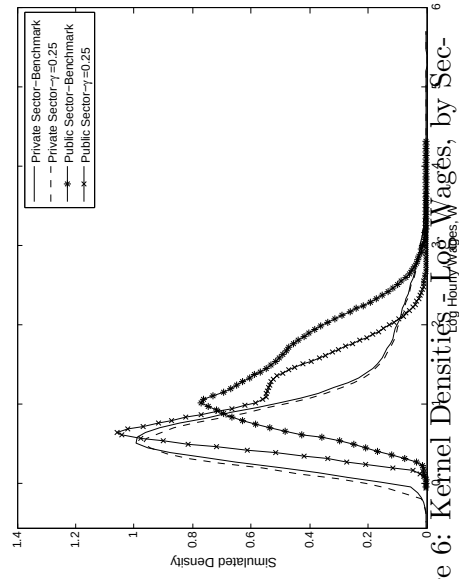


Figure 6: Kernel Densities - Log Wages, by Sector: Benchmark vs. Experiment 3

References

- [1] Albrecht, James, Lucas Navarro and Susan Vroman (2009) “The Effects of Labor Market Policies in an Economy with an Informal Sector,” *Economic Journal*, 119: 1105-29.
- [2] Algan, Yann, Pierre Cahuc and André Zylberberg (2002) “Public Employment and Labour Market Performances”, *Economic Policy*, 34: 9-64.
- [3] Bradley, Jake, Fabien Postel-Vinay, and Hélène Turon (2014) “Public Sector Wage Policy and Labour Market Equilibrium: A Structural Model,” unpublished.
- [4] Bunzel, H., B. J. Christensen, P. Jensen, N. Kiefer, L. Korsholm, L. Muus, G. Neumann and M. Rosholm (2001) “Specification and Estimation of Equilibrium Search Models,” *Review of Economic Dynamics*, 4(1), 90-126.
- [5] Burdett, Kenneth (2011) “Towards a Theory of the Labor Market with a Public Sector,” *Labour Economics*, 19(1): 68-75.
- [6] Burdett, Kenneth and Dale Mortensen (1998) “Wage Differentials, Employer Size and Unemployment” *International Economic Review*, 39: 257-273.
- [7] Chéron, Arnaud, Jean-Olivier Hairault and François Langot (2011) “Age-Dependent Employment Protection Policy,” *Economic Journal*, 121:1477-1504.
- [8] Dolado, Juan, Marcel Jansen, and Juan Jimeno (2009) “A Positive Analysis of Targeted Employment Protection Legislation,” *B.E. Journal of Macroeconomics*, Vol. 7, Issue (1) (Topics), Article 14.
- [9] Eckstein, Zvi and Kenneth Wolpin (1995) “Duration to First Job and the Return to Schooling: Estimates from a Search-Matching Model,” *Review of Economic Studies*, 62(2), 263-86.
- [10] Gomes, Pedro (2015a) “Optimal Public Sector Wages,” *Economic Journal*, forthcoming.
- [11] Gomes, Pedro (2015b) “Heterogeneity and the Public Sector Wage Policy,” mimeo.

- [12] Gregory, Robert and Jeff Borland (1999) “Recent Developments in Public Sector Labor Markets,” in O. Ashenfelter and D. Card (eds), *Handbook of Labor Economics*, vol 3C, North Holland, 3573–3630.
- [13] Hornstein, Andreas, Per Krusell and Giovanni Violante (2011) “Frictional Wage Dispersion in Search Models: A Quantitative Assessment,” *American Economic Review*, 101, 2873-98.
- [14] Michaillat, Pascal (2014) “A Theory of Countercyclical Government Multiplier,” *American Economic Journal: Macroeconomics*, 6(1), 190-217.
- [15] Mizala, Alejandra, Pilar Romaguera and Sebastián Gallegos (2011) “Private-Public Wage Gap in Latin America (1999-2007): A Matching Approach,” *Labour Economics*, 18: S115-S131.
- [16] Navarro, Lucas and Mauricio Tejada (2015), mimeo
- [17] Pissarides, Christopher (2000) *Equilibrium Unemployment Theory* (2nd edition), MIT Press, Cambridge.
- [18] Quadrini, Vincenzo and Antonella Trigari (2007) “Public Employment and the Business Cycle,” *Scandinavian Journal of Economics*, 109: 723-42.
- [19] Robayo-Abril, Monica (2015), “Tax-Transfer Schemes, Informality and Search Frictions in a Small Open Economy,” unpublished