

Risk Diversification and International Trade*

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Abstract

Firms face considerable uncertainty about consumers' demand, arising from the existence of random shocks. In presence of incomplete financial markets, entrepreneurs may not be able to perfectly insure against unexpected demand fluctuations. The key insight of my paper is that firms can reduce demand risk through geographical diversification. I first develop a general equilibrium trade model à la Chaney (2008) characterized by stochastic demand and risk-averse entrepreneurs, who exploit the imperfect correlation of demand across countries to lower the variance of their total sales, in the spirit of modern portfolio analysis. Then I study the implications of this novel source of demand complementarities across markets and show that: i) a firm's exporting decision does not obey a hierarchical structure as in standard models with fixed costs, because it depends on the global diversification strategy of the firm, and ii) the intensity of trade flows to a market are affected by its risk-return profile. To quantify the risk diversification benefits that international trade has for firms and for the aggregate economy, I calibrate the parameters of the model with the Method of Simulated Moments, using Portuguese firm-level data. One of the counterfactual exercises reveals that the welfare gains from trade can be significantly higher than the gains predicted by models which neglect firm level risk. After a trade liberalization, risk-averse firms boost exports to countries that offer better diversification benefits. Hence, in these markets foreign competition is stronger, lowering the price level more. Therefore, "safer" countries gain more from international trade.

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1 Introduction

Firms face substantial uncertainty about consumers' demand. Recent empirical evidence has shown that demand-side shocks account for a large fraction of the total variation of firm sales (see Hottman et al. (2015), Kramarz et al. (2014), Munch and Nguyen (2014), Eaton et al. (2011)).¹ In presence of incomplete financial markets or credit constraints, firms may not be able to perfectly insure against unexpected demand fluctuations.² Therefore, one should expect firms to care about demand risk.

The key idea I put forward in this paper is that firms can hedge demand risk through geographical diversification. The intuition is that selling to markets with imperfectly correlated demand can hedge against idiosyncratic shocks hitting sales. Although this simple insight has always been at the core of the financial economics literature, starting from the seminal works by Markowitz (1952) and Sharpe (1964), the trade literature has so far overlooked the risk diversification potential that international trade has for firms.³

The main contribution of this work is to highlight, both theoretically and empirically, the relevance of demand risk for firms' exporting decisions, and to quantify the risk diversification benefits that international trade has for firms and for the aggregate economy. The main finding of the paper is that the welfare gains from trade can be much higher than the ones predicted by traditional models neglecting firm level risk. These additional gains arise from the fact that firms use international trade not only to increase profits, as in standard models, but also to diversify risk. Therefore when trade barriers go down, firms export more to countries which are a good hedge against demand risk, so markets with either a stable demand or whose demand is negatively correlated with the rest of the world. This increases the entry of foreign firms, which in turn raises the number of varieties available to consumers and lowers prices, leading to higher welfare gains from trade.

In the first tier of my analysis, I develop a general equilibrium trade model with monopolistic competition and fixed costs of production, as in Melitz (2003), and Pareto distributed

¹Hottman et al. (2015) have shown that 50-70 percent of the variance in firm sales can be attributed to differences in firm appeal. Eaton et al. (2011) and Kramarz et al. (2014) with French data and Munch and Nguyen (2014) with Danish data have instead estimated that firm-destination idiosyncratic shocks drive around 40-45% percent of sales variation.

²This may be the case especially in less developed countries (see Jacoby and Skoufias (1997), Greenwood and Smith (1997) and Knight (1998)), and for small-medium firms (see Gertler and Gilchrist (1994) and Hoffmann and Shcherbakova-Stewen (2011)).

³There are some recent exceptions, as Fillat and Garetto (2015) and Riaño (2011). See the discussion below.

firm productivity, as in Chaney (2008) and Arkolakis et al. (2008). The model is characterized by two new elements. First, consumers have a Constant Elasticity of Substitution utility over a continuum of varieties, and demand is subject to country-variety random shocks. In addition, for each variety these demand shocks are imperfectly correlated across countries. This assumption is corroborated by the evidence shown in Section 2 using product-level and firm-level data, that demand is imperfectly correlated across markets. Second, firms are owned by risk-averse entrepreneurs who have mean variance preferences over business profits. This assumption captures the evidence, shown in Section 2, that most firms across several countries are owned by entrepreneurs who do not have well-diversified portfolios, and thus business profits are their main source of income.⁴ Therefore, in my framework, firms do not simply maximize profits as in standard models, but are willing to forgo a portion of the profits in order to lower their variance.

Financial markets are absent, and thus firms cannot hedge unexpected demand fluctuations with financial securities. This assumption captures in an extreme way the incompleteness that characterizes financial markets. Notice that even if there were some financial assets available in the economy, as long as capital markets are incomplete firms would always be subject to a certain degree of demand risk. Shutting down financial markets therefore allows to focus only on international trade as a mechanism firms can use to stabilize their sales.

The entrepreneurs' problem consists of two stages. In the first stage, the entrepreneurs know only the moments of the demand shocks but not their realization. They have to pay workers to perform some marketing activities which allow them to reach a certain number of consumers in each market.⁵ Therefore entrepreneurs, under uncertainty, choose both the extensive (where to sell) and the intensive (how many consumers to sell to) margins. After the marketing and distributional activities are performed, the demand shocks are realized. Firms then make the pricing decisions and produce accordingly, using a production function linear in labor.⁶

The imperfect correlation of the shocks across markets implies that, in the first stage, the extensive margin decision of the firm is a combinatorial problem, hard to solve analytically. I

⁴See Herranz et al. (2015), De Sousa et al. (2015) and Riaño (2011) for models with risk averse firms.

⁵In my model marketing costs are constant, instead of being convex as in Arkolakis (2010). Although the basic idea put forward is the same as in Arkolakis (2010) - firms reach individual consumers rather than the entire market - in my framework choosing the number of consumers allows firms to decide how much to be exposed to a country's demand risk.

⁶The fact that companies cannot change the number of consumers reached after observing the shocks has an intuitive explanation. Investing in marketing activities is an irreversible activity, and thus very costly to adjust after observing the realization of the shocks.

overcome this complication by using the fact that firms maximize mean-variance preferences, and thus solve a quadratic problem whose choice variable is continuous, being the number of consumers to reach in each market. This has the great advantage of allowing firms to simultaneously choose *where to sell* (if n is optimally zero, they do not sell in country j) and *how much to sell* (they can choose to sell to some consumers or to all of them).

Therefore, in the first stage entrepreneurs perform a global diversification strategy, along the lines of the “portfolio analysis” pioneered by Markowitz (1952) and Sharpe (1964). The firms’ problem, however, is more involved than a standard portfolio problem, because it is subject to bounds: the number of consumers reached in a destination can neither be negative nor greater than the size of the population. Using finance jargon, a firm cannot “short” consumers ($n < 0$) or “borrow” them from other countries ($n > 1$). Despite this technical difficulty, I am able to characterize the firm’s optimal solution as a function of the Lagrange multipliers of the inequality constraints, and then derive novel implications.

Specifically, I first show that there is no hierarchical structure of the exporting decision. The fact that a firm with productivity z_1 enters market j does not necessarily imply that a firm with productivity $z_2 > z_1$ will enter j as well. The reason is that the decision to export to a country depends not only on the firm’s productivity, but also on each firm’s optimal diversification strategy. For example, a small firm may enter market j because it provides a good hedge from risk, while a larger firm does not enter j since it prefers to diversify risk by selling to other markets, where the small firm is not able to export. This is a novel feature of my model, and thereby differs from traditional trade models with fixed costs, as Melitz (2003) and Chaney (2008), where the exporting decision is strictly hierarchical.

A further implication of the firm’s diversification problem is that the intensity of the trade flows depend directly on the riskiness of that market. If demand in a country is relatively stable and is negatively correlated with demand in other destinations, then firms may export there to hedge the fluctuations originating in other locations. The better the risk-return trade-off a country offers, the more firms export there, conditional on trade barriers.

In the second tier of my analysis, I calibrate the model to quantify the risk diversification benefits that international trade has for aggregate welfare. The empirical analysis relies on a panel dataset of Portuguese manufacturing firms’ international sales from 1995 to 2005. Portugal is a small and export-intensive country, being at the 72nd percentile worldwide for exports per capita, and therefore can be considered a good laboratory to analyze the implications of my model. Furthermore, 70% of Portuguese exporters in 2005 were small firms, for which the exposure to demand risk is likely to be a first-order concern.

Using the Portuguese firm-level data, I calibrate the parameters of the model with the Method of Simulated Moments. In particular, I calibrate the risk aversion by matching the observed (positive) gradient of the relationship between the mean and the variance of firms' revenues.⁷ The reasoning is straightforward: if firms are risk-averse, they want to be compensated for taking additional risk, and thus higher sales variance must be associated with higher expected revenues. The tractability of the model also allows me to calibrate the cross-country covariance matrix of the demand shocks, as well as the technology parameter of the Pareto distribution, variable and fixed trade costs.

Using the calibrated model, I compare some of my model's predictions with the corresponding features of the data. I first show that only 28% of Portuguese exporters in 2005 were obeying a hierarchical structure in their exporting status, consistent with the prediction of my model. In contrast, standard trade models with fixed costs and risk neutrality, such as Melitz (2003) and Chaney (2008), would predict that all exporters follow a strict sorting into exporting.

I then compare the observed distribution of firm-level exports to a certain destination with the one predicted by my calibrated model. My model outperforms the risk-neutral model in matching the left tail of the distribution. The reason is that some firms, when they are risk averse, optimally choose to reach a small number of consumers in a certain destination, rather than the whole market, and therefore export small amounts of their goods. In the Melitz-Chaney framework, instead, the presence of fixed costs is not compatible with the existence of small exporters, and thus over-predicts their size by some orders of magnitude.

Moreover, I test the prediction that firms' trade flows to a market are decreasing in the market's riskiness. The model endogenously delivers an index of the riskiness of a market, which is a function only of the moments of the demand shocks.⁸ I use the calibrated moments of the shocks to construct this country-level measure, and test how it affects trade flows of Portuguese firms. The findings show that, controlling for firm fixed effects, size of the market and trade barriers, proxied by distance and tariffs, companies export more to safer countries.

Finally, I perform a number of counterfactual simulations. The main policy experiment is to compute the welfare gains from international trade, i.e. from a reduction in trade barriers. My results illustrate that countries providing better risk-return trade-offs to foreign firms benefit more from opening up to trade. These markets have either a relatively stable demand

⁷The CES assumption implies that gross profits are a constant fraction of revenues, so the relationship between the mean and their variance is the same (given that fixed costs are non-stochastic).

⁸As explained earlier, a market is "safe" if it has either a relatively stable demand or is negatively correlated with demand in the rest of the world.

or are negatively correlated with demand in the rest of the world, and thus are a good hedge against risk. The rationale is that firms exploit a trade liberalization not only to increase their profits, but also to diversify their demand risk. This implies that they optimally increase trade flows toward markets that provide better diversification benefits. Consequently, the increase in foreign competition is stronger in these countries, thereby lowering more the price level and expanding the number of varieties available. Therefore, “safer” countries gain more from trade.

In addition, I compare the gains in my model with those predicted by traditional trade models that neglect risk, as in Arkolakis et al. (2012) (ACR henceforth).⁹ My results show that gains from trade are, for the median country, 18% higher than in ACR. While safer countries reap higher welfare gains than in ACR, markets with a worse risk-return profile have lower gains than in ACR, because the competition from foreign firms is weaker. Furthermore, I study the distributional effect of trade across entrepreneurs. I show that a large fraction of the welfare gains from trade accrue to the owners of smaller firms. The reason is that lower trade barriers give an opportunity to small firms to sell to more destinations, and thus lower the variance of their profits. Therefore a trade liberalization is even more beneficial for small firms than for larger firms, which are already able to diversify their risk by selling to many locations, given their size advantage.

This paper relates to the growing literature studying the importance of second order moments for international trade.¹⁰ In Koren (2003), uncertainty on productivity implies that countries do not completely specialize according to their comparative advantage. Rather, the pattern of specialization is pinned down by the representative investor’s portfolio decision. Employing a stylized model based on Helpman and Razin (1978), Di Giovanni and Levchenko (2010) show that when sectors differ in volatility, export patterns are not only conditioned by comparative advantage but also by insurance motives. Allen and Atkin (2015) use a portfolio approach to study the crop choice of Indian farmers under uncertainty. They show that greater trade openness increased farmers’ revenues volatility. Other recent works exploring the link between uncertainty and exporters’ behavior are Riaño (2011), Nguyen (2012), Handley and Limao (2012), Impullitti et al. (2013), Limão and Maggi (2013), Vannoorenberghe (2012), Ramondo et al. (2013), Vannoorenberghe et al. (2014), Novy and Taylor (2014),

⁹The models considered in ACR are characterized by (i) Dixit-Stiglitz preferences; (ii) one factor of production; (iii) linear cost functions; and (iv) perfect or monopolistic competition. Among them, there are the seminal papers by Eaton and Kortum (2002), Melitz (2003) and Chaney (2008).

¹⁰For earlier works, see Helpman and Razin (1978), Kihlstrom and Laffont (1979), Newbery and Stiglitz (1984) and Eaton and Grossman (1985).

Conteduca and Kazakova (2015), Fillat and Garetto (2015) and De Sousa et al. (2015).¹¹

My paper complements the strand of literature that studies the connection between openness to trade and macroeconomic volatility (see Koren and Tenreyro (2007), Di Giovanni and Levchenko (2012)). More recently, Caselli et al. (2012) show that openness to international trade can lower GDP volatility by reducing exposure to domestic shocks and allowing countries to diversify the sources of demand and supply across countries. Kramarz et al. (2014) examine the network structure of an exporter’s portfolio of buyers and argue that co-movements in sales across sellers can generate “granular” fluctuations in aggregate exports. My paper, in contrast, investigates the implications of firm-level demand risk for international trade patterns and aggregate welfare.

There is recent empirical literature showing high and heterogeneous firm-level volatility (Fort et al. (2013), Kramarz et al. (2014)). Recent contributions have underlined the role of demand-related shocks to explain such firm-level sales variation (see Eaton et al. (2011), Bricongne et al. (2012), Nguyen (2012), Munch and Nguyen (2014), Berman et al. (2015), Armenter and Koren (2015), Kramarz et al. (2014), Bernard and Esposito (2015)). Hottman et al. (2015) have shown that 50-70 percent of the variance in firm size can be attributed to differences in firm appeal. Munch and Nguyen (2014) found that firm effects drive around 40 percent of the sales variation of Danish sellers across markets. A similar number has been found by Eaton et al. (2011) with French data.

Finally, my paper connects to the literature that studies the implications of incomplete financial markets for entrepreneurial risk and firms’ behavior and performance. Herranz et al. (2015) show, using data on ownership of US small firms, that entrepreneurs are risk-averse and hedge business risk by adjusting the firm’s capital structure and scale. Other notable contributions to this literature are Kihlstrom and Laffont (1979), Heaton and Lucas (2000), Moskowitz and Vissing-Jorgensen (2002), Roussanov (2010), Luo et al. (2010), Chen et al. (2010), Hoffmann (2014) and Jones and Pratap (2015).

The remainder of the paper is organized as follows. Section 2 presents some stylized facts that corroborate the main assumptions used in the model, presented in Section 3. In Section 4, I study the empirical implications of my model. In Section 5, I perform a number of counterfactual exercises. Section 6 concludes.

¹¹Conteduca and Kazakova (2015) use a portfolio approach to study the decision of the firm to engage in FDI. De Sousa et al. (2015) have a partial equilibrium model where firms are risk-averse, but, in contrast to my model, demand shocks are uncorrelated across countries.

2 Stylized facts

In this section I provide two stylized facts that motivate the main assumptions of the model presented in Section 3. The first is about the level of wealth diversification of entrepreneurs:

Fact 1. *The majority of firms are controlled by imperfectly diversified owners.*

There are many recent works providing this evidence across several countries. The most comprehensive one is Lyandres et al. (2013). Using a dataset about ownership of 162,688 firms in 34 European countries, they show that entrepreneurs' holdings are far from being well-diversified.¹² The median entrepreneur in their sample owns shares of only two firms, and the Herfindhal Index of his holdings is 0.67, a number indicating high concentration of wealth. Furthermore, they show that the owners of large firms have better diversified portfolios than the owners of small firms.¹³

For the US, this evidence is confirmed by the Survey of Small Business Firms (2003). This Survey, administered by Federal Reserve System and the U.S. Small Business Administration, is a cross sectional stratified random sample of about 4,000 non-farm, non-financial, non-real estate small businesses that represent about 5 million firms.¹⁴ According to the survey, a large fraction of small firms' owners invest substantial personal net-worth in their firms: half of them have 20% or more of their net worth invested in one firm, and 87% of them work at their company. Moreover, Moskowitz and Vissing-Jorgensen (2002) estimate that US households with entrepreneurial equity invest on average more than 70 percent of their private holdings in a single private company in which they have an active management interest. Similar evidence that companies are controlled by imperfectly diversified owners has been provided by Benartzi and Thaler (2001), Agnew et al. (2003), Heaton and Lucas (2000), Faccio et al. (2011) and Herranz et al. (2013).

Fact 1 shows that the wealth of many entrepreneurs is non-diversified. Therefore, for most of them, even in a developed country such as the US, the main source of income is their

¹²96% of firms in their sample are privately-held. They use three measures of diversification of entrepreneurs' holdings: i) total number of firms in which the owner holds shares, directly or indirectly; ii) Herfindhal index of firm owner's holdings; iii) the correlation between the mean stock return of public firms in the firm's industry and the shareholder's overall portfolio return.

¹³There is a growing body of theoretical literature that explains this concentration of entrepreneurs' portfolios and thus their exceptional role as owners of equity. See Carroll (2002), Roussanov (2010), Luo et al. (2010) and Chen et al. (2010).

¹⁴All surveys are available at <http://www.federalreserve.gov>.

own company's profits. This suggests that the owners are highly exposed to unexpected demand, and thus profits, fluctuations. In the presence of incomplete financial markets, or when access to financial markets is costly, these entrepreneurs cannot perfectly hedge demand risk using outside instruments.¹⁵ Then, it is reasonable to argue that these owners manage their firms with a certain degree of risk aversion.

The second stylized fact concerns the cross-country correlation of demand for a product:

Fact 2. *Demand is imperfectly correlated across markets.*

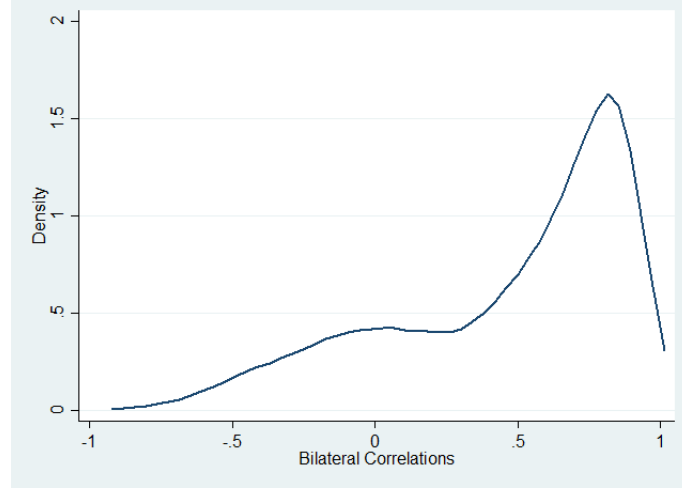
To show Fact 2, I first use data from the BACI dataset, which provides bilateral trade flows at the HS 6-digit product disaggregation for more than 200 countries since 1995.¹⁶ Using trade flows between 1995 and 2005, I first compute the total demand of the biggest 100 countries, for all manufacturing products. Then, I pick the product-destination pairs that had positive demand in all years and, for each product, I compute the matrix of cross-country demand correlations.¹⁷ Figure 1 plots the distribution of these correlations for the most traded product by total sales, automobiles (1500-3000 cc). Interestingly, the Figure shows a substantial variation in the bilateral demand correlations, and that demand is far from being perfectly correlated across countries. This pattern is strikingly similar across all products and sectors, as clearly demonstrated in Figure 2, which plots the distribution of the correlations for *all* products.

¹⁵Furthermore, non-corporate businesses typically have no direct access to capital markets and small business access to bank credit is likely to be limited, in particular during recessions (see Gertler and Gilchrist (1994), Hoffmann and Shcherbakova-Stewen (2011)). This may exacerbate the difficulty of entrepreneurs in hedging business risk.

¹⁶I use the version HS 1992 of the dataset, which is publicly available from the CEPII website.

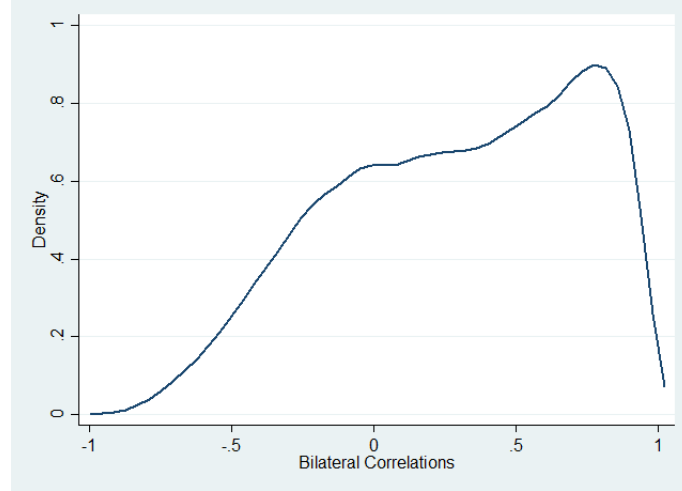
¹⁷There are 932,129 of such pairs, for 982 different products, and there are 83,757 correlations.

Figure 1: Distribution of demand correlations, product level



Notes: The figure shows the distribution of the cross-country correlations of demand for automobiles (spark ignition engine of 1500-3000 cc, HS-6=870323). For each pair of destinations j and s , the correlations $Corr(I_j, I_s)$ are computed using total imports of by country j , I_j , and by country s , I_s , between 1995 and 2005.

Figure 2: Distribution of demand correlations, product level



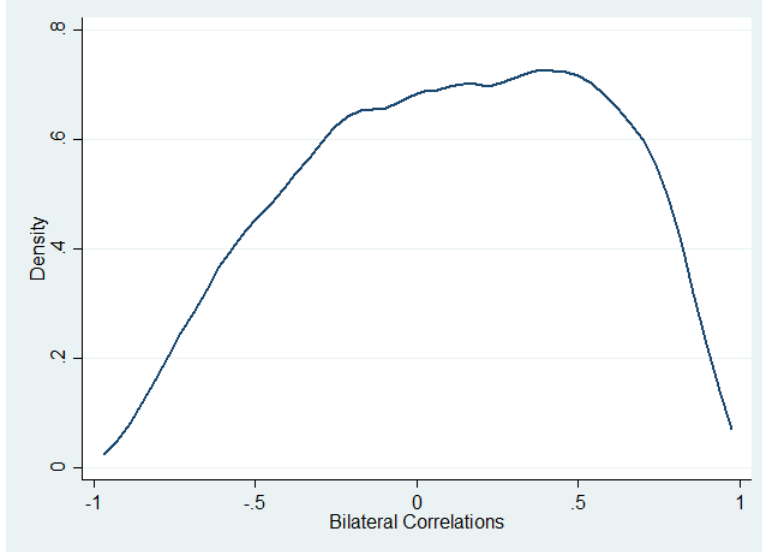
Notes: The figure shows the distribution of the cross-country correlations of demand for all products. For each HS-6 product p and for each pair of destinations j and s , the correlations $Corr(I_j^p, I_s^p)$ are computed using total imports of product p by country j , I_j^p , and by country s , I_s^p , between 1995 and 2005. There are 83,757 of such correlations.

This pattern holds at the firm level as well. I use a panel dataset on international sales of Portuguese manufacturing firms toward 210 countries, between 1995 and 2005.¹⁸ In

¹⁸See Section 4 for more details about the dataset.

particular, I first pick the 5,283 firm-destination pairs that had positive sales in all years. Then, for each firm, I compute the matrix of cross-country correlations using the firm's trade flows between 1995 and 2005. Figure 3 plots the distribution of these bilateral correlations, and shows their considerable heterogeneity.

Figure 3: Distribution of demand correlations, firm level



Notes: The figure shows the distribution of bilateral correlations of firm-level trade flows. For each firm f and for each pair of destinations j and s , the correlations $\text{Corr}(X_j^f, X_s^f)$ are computed using firm's exports to country j , X_j^f , and to country s , X_s^f , between 1995 and 2005. There are 4,064 of such correlations.

The key insight of my paper is that, given the imperfect and heterogeneous cross-country correlations of demand shown above, non-diversified entrepreneurs can use geographical diversification to hedge against idiosyncratic shocks hitting their firm's sales. Selling to markets that have dissimilar demand behavior may insulate the firm's sales from major downswings. In the next section, I lay out a general equilibrium model characterized by stochastic demand and risk-averse entrepreneurs to study the implications of demand risk for firms' production and exporting decision, as well as for aggregate welfare.

3 A trade model with risk-averse entrepreneurs

I consider a static trade model with N countries. Each country j is populated by a continuum

of workers of measure L_j , and a continuum of risk-averse entrepreneurs of measure J_j . The workers' only source of income is labor income, since they are endowed with one unit of labor that they provide, inelastically, to the firms at a given price w . The entrepreneurs' only source of income is the profits from the firm they own. Each firm produces a differentiated variety ω with productivity z under monopolistic competition, as in Melitz (2003) and Chaney (2008). Since there is a one-to-one mapping from the productivity z to the variety produced ω , throughout the rest of the paper I will always use z to identify both. I now describe the optimal consumers' demand.

3.1 Consumers Demand

Both workers and entrepreneurs consume a potentially different set of goods Z_j . Each individual from country j consumes a composite good that is made by combining a continuum of differentiated commodities according to a Constant Elasticity of Substitution (CES) aggregator with elasticity $\sigma > 1$:

$$U_j^c = \left(\int_{Z_j} \alpha_j(z)^{\frac{1}{\sigma}} q_j(z)^{\frac{\sigma-1}{\sigma}} dz \right)^{\frac{\sigma}{\sigma-1}}. \quad (1)$$

The term $\alpha_j(z)$ reflects an exogenous demand shock specific to good z in market j , similarly to Eaton et al. (2011) and Nguyen (2012).¹⁹ Define $\alpha(z) \equiv \alpha_1(z), \dots, \alpha_N(z)$ to be the vector of realizations of the demand shock for variety z , where N is the number of countries. I assume that:

Assumption 1. $\alpha(z) \sim G(\bar{\alpha}, \Sigma)$, i.i.d. across z

Assumption 1 states that the demand shocks are drawn, independently across varieties, from a multivariate distribution characterized by an N -dimensional vector of means $\bar{\alpha}$ and an $N \times N$ variance-covariance matrix Σ . Given the interpretation of $\alpha_j(z)$ as a consumption shifter, I assume that the distribution has support over $\mathbb{R}^+$.²⁰ Note that, by simply specifying a generic covariance matrix Σ , I am not making any restrictions on the cross-country correlations of

¹⁹This may reflect a preference or policy shock. The model could be easily modified to encompass also exchange rate and tariff shocks.

²⁰For example, $G(\bar{\alpha}, \Sigma)$ can be the log-normal distribution, or a normal distribution truncated at zero.

demand, which therefore can range from -1 to 1. Assumption 1 then captures the evidence shown by Fact 2: demand correlations are heterogeneous across countries. Moreover, note that by Assumption 1, the moments of the shocks are the same for all varieties. This somewhat extreme assumption will be useful for the empirical analysis, because I will need to calibrate only one country-level covariance matrix Σ (I will normalize $\bar{\alpha}$ to 1).²¹ It is easy to verify that the optimal demand for each variety is:

$$q_j(z) = \alpha_j(z) \frac{p_j(z)^{-\sigma}}{P_j^{1-\sigma}} n_j(z) Y_j, \quad (2)$$

where $p_j(z)$ is the price of variety z in j and $n_j(z)$ is the fraction of consumers that firm z decides to reach in country j .²² I will show in the following section that $n_j(z)$ can be endogenously zero for some z , limiting the types of varieties available to the consumers. In addition, Y_j is total income spent by consumers in j :

$$Y_j = w_j L_j + \Pi_j \quad (3)$$

where Π_j are the aggregate profits of the entrepreneurs and $w_j L_j$ is the aggregate labor income that the workers earn. The price index P_j is given by:

$$P_j^{1-\sigma} \equiv \int_{Z_j} n_j(z) \alpha_j(z) (p_j(z))^{1-\sigma} dz. \quad (4)$$

The Dixit-Stiglitz price index is interpreted as the endogenous level of competition in market j .²³ The aggregate variables Π_j , w_j and P_j will be determined in general equilibrium, as described in Section 3.3.

3.2 Production

The entrepreneurs are risk-averse individuals and are the only owners and managers of their

²¹Clearly calibrating a covariance matrix for each variety is unfeasible. In Bernard and Esposito (2015), we instead estimate a cross-country covariance matrix of demand shocks for each product at HS-6 level using fixed-effects regressions.

²²The optimal demand in equation (2) is derived by using the fact that there is a continuum number of consumers, as in Arkolakis (2010).

²³ P_j is large enough to be unaffected by the addition or subtraction of any single variety z .

firms.²⁴ This assumption captures in a somewhat extreme way the evidence shown by Fact 1. Many entrepreneurs, and especially owners of small firms, do not have a well-diversified portfolio, and thus the profits coming from their company constitute their main source of income.²⁵ Furthermore, I assume that financial markets are absent.²⁶ The productivity z is drawn from a known distribution independently across countries and firms, and its realization is known by the entrepreneurs.

The entrepreneurs' problem consists of two stages. In the first stage, the realization of the shocks is unknown, and thus business profits are stochastic. Entrepreneurs maximize their utility in real profits, having information only on $G(\alpha)$, the distribution of the demand shocks. In particular, they decide how many consumers to reach in each market j and pay the corresponding marketing costs in domestic labor.²⁷ After the firms allocate labor to the payment of the marketing costs, the demand shocks are realized. In the second stage, when the firms know the realization of the shocks, they make the pricing decision and produce accordingly, using a production function linear in labor. Then, the entrepreneurs allocate the resulting profits (if any) to consumption, using the same CES aggregator as the workers, shown in equation (1).²⁸

In the first stage of production, the entrepreneurs from country i maximize the following mean-variance preferences over real profits $\frac{\pi_i(z)}{P_i}$:

$$\max E \left(\frac{\pi_i(z)}{P_i} \right) - \frac{b}{2} Var \left(\frac{\pi_i(z)}{P_i} \right). \quad (5)$$

Mean-variance preferences have been widely used by the finance literature to model the decision of risk-averse agents allocating their wealth in risky financial assets (see, for example, Markowitz (1952), Sharpe (1964) and Ingersoll (1987)).²⁹ The entrepreneurs perform a global

²⁴Alternatively, we can think of them as the majority shareholders of their firm, with complete power over the firm's production choices.

²⁵I also rule out the possibility of mergers and acquisitions between firms. The possibility of M&A, however, could be an alternative way to diversify risk for the entrepreneurs.

²⁶This assumption captures in an extreme way the incompleteness of financial markets. Notice that even if there were some financial assets available in the economy, as long as capital markets are incomplete firms would always be subject to a certain degree of demand risk. See also Riaño (2011) and Limão and Maggi (2013).

²⁷Estimates of marketing costs (see Barwise and Styler (2003), Butt and Howe (2006) and Arkolakis (2010)) indicate that the amount of marketing spending in a certain market is between 4 to 7.7% of GDP.

²⁸I assume that firms cannot adjust their capacity, i.e. $n_{ij}(z)$, after demand shocks are realized, because these marketing activities are irreversible.

²⁹The mean variance specification in equation 5 can be obtained by assuming that entrepreneurs have CARA utility and shocks are drawn from a multivariate normal distribution truncated at zero (since demand

diversification strategy: they trade off the expected real profits with their variance, the exact slope being governed by the degree of risk aversion $b > 0$.³⁰

It is worth noting that the imperfect correlation of the shocks across markets implies the presence of *demand* complementarities across countries. The trade literature has recently studied the firm's sourcing decisions in the presence of *supply* complementarities (see Antras et al. (2014), Blaum et al. (2015)). The difficulty with these types of models is that the choice of the extensive margin (i.e. either the sourcing or the exporting decisions) appears to be a combinatorial problem very hard to solve. In my model, instead, companies choose the fraction of consumer to reach, which has the great advantage of allowing them to simultaneously choose *where to sell* (if n_{ij} is optimally zero they do not sell in country j) and *how much to sell* (they can choose to sell to some or all consumers). Then, the firm problem is similar to a portfolio optimization problem, where a risk-averse investor maximizes mean-variance preferences and chooses what fraction of wealth to invest in each financial asset.

Therefore, the problem of the firm in the first stage is to choose $n_{ij}(z)$ to maximize:

$$\max_{\{n_{ij}\}} \sum_j E \left(\frac{\pi_{ij}(z)}{P_i} \right) - \frac{b}{2} \sum_j \sum_s Cov \left(\frac{\pi_{ij}(z)}{P_i}, \frac{\pi_{is}(z)}{P_i} \right) \quad (6)$$

$$\text{s. to } 1 \geq n_{ij}(z) \geq 0 \quad (7)$$

where $\pi_{ij}(z)$ are net profits from j :

$$\pi_{ij}(z) = q_{ij}(n_{ij}(z))p_{ij}(z) - q_{ij}(n_{ij}(z)) \frac{\tau_{ij}w_i}{z} - w_i f_{ij}(n_{ij}(z)), \quad (8)$$

and $\tau_{ij} \geq 1$ are iceberg trade costs and f_{ij} are fixed production costs, paid in domestic labor.³¹ Notably, I assume that there is a cost, $f_j > 0$, to reach each consumer in country j , and therefore, total fixed costs are:

$$f_{ij}(n_{ij}(z)) = f_j \tilde{L}_j n_{ij}(z). \quad (9)$$

where $\tilde{L}_j = L_j + J_j$ is the total measure of consumers reached in country j .³² The bounds

cannot be negative). Then, it can be shown (the proof is available upon request) that the entrepreneurs' expected utility can be approximated by equation 5, if the risk aversion is positive but not too high.

³⁰Notice that entrepreneurs deflate the profits using the CES price index, since they will finally consume a CES bundle.

³¹I normalize domestic trade barriers to $\tau_{ii} = 1$.

³²In accordance with Arkolakis (2010), I will make specific assumptions on f_j in the calibration section. However, the fact that f_j does not depend on $n_{ij}(z)$ means that the marginal cost of reaching an additional

on $n_{ij}(z)$ in equation (7) are a resource constraint: the number of consumers reached by a firm cannot be negative and cannot exceed the total size of the population. Using finance jargon, a firm cannot “short” consumers ($n_{ij}(z) < 0$) or “borrow” them from other countries ($n_{ij}(z) > 1$). This makes the maximization problem in (6) quite challenging, because it is subject to $2N$ *inequality* constraints. In finance, it is well known that there is no closed form solution for a portfolio optimization problem with lower and upper bounds (see Jagannathan and Ma (2002), Ingersoll (1987)). In spite of this difficulty, in Proposition 1 I am able to characterize the solution for $n_{ij}(z)$ as a function of the Lagrange multipliers associated with the constraints.

Notice that the variance of global real profits is the sum of the variances of the profits reaped in all potential destinations. In turn, these variances are the sum of the covariances of the profits from j with all markets, including itself. If the demand shocks were not correlated across countries, then the objective function would simply be the sum of the expected profits minus the variances. The assumption that the shocks are independent across a continuum of varieties implies that each firm treats the aggregate variables w and P as non-stochastic. Therefore, plugging into $\pi_{ij}(z)$ the optimal consumers’ demand from equation (2), I can write expected real profits as:

$$E\left(\frac{\pi_{ij}(z)}{P_i}\right) = \bar{\alpha}_j n_{ij}(z) r_{ij}(z) - \frac{w_i f_{ij}(n_{ij}(z))}{P_i}, \quad (10)$$

where $\bar{\alpha}_j$ is the expected value of the demand shock in destination j , and

$$r_{ij}(z) \equiv \frac{p_{ij}(z)^{-\sigma}}{P_i} \frac{Y_j}{P_j^{1-\sigma}} \left(p_{ij}(z) - \frac{\tau_{ij} w_i}{z} \right). \quad (11)$$

Similarly, the covariance between $\frac{\pi_{ij}(z)}{P_i}$ and $\frac{\pi_{is}(z)}{P_i}$ is simply:

$$Cov\left(\frac{\pi_{ij}(z)}{P_i}, \frac{\pi_{is}(z)}{P_i}\right) = n_{ij}(z) r_{ij}(z) n_{is}(z) r_{is}(z) Cov(\alpha_j, \alpha_s), \quad (12)$$

where $Cov(\alpha_j, \alpha_s)$ is the covariance between the shock in country j and in country s .³³

Before going over the solution of the firm’s problem, it is helpful to first look at the firm’s *interior* first order condition:

consumer is constant, which is a special case of Arkolakis (2010).

³³Note that $r_{ij}(z)$ are the real profits that a firm would take in if it sold to all consumers in j . The covariance does not depend on the fixed costs because these are non-stochastic.

$$\underbrace{r_{ij}(z)\bar{\alpha}_j - br_{ij}(z) \sum_s n_{is}(z)r_{is}(z)Cov(\alpha_j, \alpha_s)}_{\text{real marginal benefit}} = \underbrace{w_i f_j \tilde{L}_j / P_i}_{\text{real marginal cost}}. \quad (13)$$

Equation (13) equates the real marginal benefit of adding one consumer to its real marginal cost. While the marginal cost is constant, the marginal benefit is decreasing in $n_{ij}(z)$. In particular, it is equal to the marginal revenues minus a “penalty” for risk, given by the sum of the covariances that destination j has with all other countries (including itself). The higher the covariance of market j with the rest of the world, the smaller the diversification benefit the market provides to a firm exporting from country i .

An additional interpretation is that a market with a high covariance with the rest of the world must have high average real profits to compensate the firm for the additional risk taken: this trade-off between risk and return is determined by the degree of risk aversion. Note the difference in the optimality condition with Arkolakis (2010). In his paper, the marginal benefit of reaching an additional consumer is constant, while the marginal penetration cost is increasing in $n_{ij}(z)$. In my setting, however, the marginal benefit of adding a consumer is decreasing in $n_{ij}(z)$, due to the concavity of the utility function of the entrepreneur, while the marginal cost is constant.

Before describing the result of Proposition 1, I first define the following country-level measure of demand risk:

$$\psi \equiv \Sigma^{-1} \bar{\alpha} \quad (14)$$

The vector ψ is an (inverse) measure of risk since it is equal to the inverse of the covariance matrix times the vector of means. For example, with two countries, ψ_j equals, in country 1:

$$\psi_1 = \frac{\frac{\bar{\alpha}_1}{\sigma_1} - \rho \frac{\bar{\alpha}_2}{\sigma_2}}{\sigma_1(1 - \rho^2)}, \quad (15)$$

where σ_i and $\bar{\alpha}_i$ denote the standard deviation and the mean of the shock in country i , respectively, and ρ is the cross-country correlation of the shocks. Equation (15) shows that ψ_j is the sum of the Sharpe Ratios of the demand shocks, weighted by their cross-country correlation.³⁴ Also, in the general case of N countries, it is easily verifiable that ψ_j is decreasing in σ_j and in the correlation of demand in j with the rest of the world. The more

³⁴Recall that the Sharpe Ratio of a stochastic variable is defined as the ratio of its expected mean (or sometimes its “excess” expected return over the risk-free rate) over its standard deviation.

volatile demand in market j , or the more demand is correlated with the rest the world, the riskier country j and the *lower* ψ_j .

Define $\lambda_{ik}(z)$ to be the Lagrange multiplier associated with the non-negativity constraint of $n_{ij}(z)$, $\mu_{ik}(z)$ to be the Lagrange multiplier associated with the upper bound constraint and C_{jk} to be the (j, k) cofactor of Σ rescaled by the determinant of Σ .³⁵ It is easy to demonstrate that C_{jk} is decreasing in $Cov(\alpha_j, \alpha_k)$, since the cofactors are the elements of the inverse of the covariance matrix. I make only a mild assumption on the covariance matrix, which I assume will hold throughout the paper:

Assumption 2. $\det(\Sigma) > 0$

Assumption 2 is a sufficient condition to have uniqueness of the optimal solution. Since Σ is a covariance matrix, which by definition always has a non-negative determinant, this assumption simply rules out the knife-edge case of a zero determinant.³⁶ In the Appendix, I prove that:

Proposition 1. *When $b > 0$, the optimal $n_{ij}(z)$ satisfies:*

$$n_{ij}(z) = \frac{\psi_j}{r_{ij}(z)b} - \frac{\sum_k C_{jk} \frac{w_i f_k \tilde{L}_k}{r_{ik}(z)}}{r_{ij}(z)b} + \frac{\sum_k \frac{C_{jk}}{r_{ik}(z)} (\lambda_{ik}(z) - \mu_{ik}(z))}{r_{ij}(z)b} \quad (16)$$

Moreover, the price is a constant markup over the marginal cost:

$$p_{ij}(z) = \frac{\sigma}{\sigma - 1} \frac{\tau_{ij} w_i}{z} \quad (17)$$

The fact that entrepreneurs solve a global diversification strategy implies that both the intensive and the extensive margin decisions are not taken for each market in isolation, as in standard trade models, but are interrelated across countries. Specifically, Proposition 1 indicates that there are three channels affecting the optimal decision of the firm under uncertainty.

³⁵The cofactor is defined as $C_{kj} \equiv (-1)^{k+j} M_{kj}$, where M_{kj} is the (k, j) minor of Σ . The minor of a matrix is the determinant of the sub-matrix formed by deleting the k -th row and j -th column.

³⁶A zero determinant would happen only in the case where all pairwise correlations are exactly 1.

First, $n_{ij}(z)$ is increasing in ψ_j : the higher ψ_j , the better the risk-return trade-off, and thus the more the firm exports there, because that country offers good diversification benefits. The second term is the weighted sum of $\frac{w_i f_k \tilde{L}_k}{r_{ik}(z)}$, the fixed cost payments as fraction of the gross profits, where the weights are given by the corresponding cofactor C_{jk} . Thus, $n_{ij}(z)$ also depends on the fixed costs in all other markets, and these fixed costs have a differential impact on n_{ij} depending on the covariance matrix.

For example, say that the fixed cost to pay in market k increases marginally. If the demand in j is negatively correlated with demand in k , so that C_{jk} is positive, then an increase in f_k decreases sales in both k and j , although the fixed cost in j remains the same.³⁷ The reason is that the negative correlation between $q_{ij}(z)$ and $q_{ik}(z)$ implies that the firm optimally uses sales to market j to hedge fluctuations of demand in k . If the cost of selling to k increases, and sales there go down as a result, then there is less need to sell to j to hedge risk, *ceteribus paribus*. Therefore, sales decrease also in j . Conversely, if demands are positively correlated, higher fixed costs in k imply that firm z will shift its sales toward market j , since k is not much needed to diversify risk.

The third component affecting $n_{ij}(z)$ are the Lagrange multipliers of the constraints. Recall that in the standard portfolio optimization problem (see Markowitz (1952), Sharpe (1964)), the optimal solution may entail negative portfolio weights (the investor is shorting an asset) or weights larger than 1 (the investor is borrowing to invest in that asset). In that case the solution is found simply by pre-multiplying the vector of returns by the inverse of the covariance matrix. In the context of my model, this would correspond to the first two terms in equation (16). However, the bounds on $n_{ij}(z)$ constrain the optimal solution to be between 0 and 1, and this forces firms to marginally “re-optimize” their portfolio whenever one of the $2N$ constraints is binding.

For example, say that firm z would like to be heavily exposed to market k (and choose $n_{ik} > 1$) because of either low marketing costs, or high expected profits relative to their variance, or both. If $Cov(\alpha_j, \alpha_k) < 0$, then the firm chooses a high $n_{ij}(z)$ as well, so to hedge any shock to sales in k . However, if the firm is constrained to set $n_{ik} = 1$, then the optimal $n_{ij}(z)$ goes down, because the firm cannot reach its “unconstrained” allocation in k and thus has less need to sell to market j . A reverse mechanism is in play for the lower bound constraint.

Furthermore, in the Appendix, I prove the following Proposition:

³⁷Note that the diagonal cofactor C_{jj} is always positive.

Proposition 2. *There is no hierarchical structure of the exporting decision.*

The fact that a firm with productivity z_1 enters market j , i.e. $n_{ij}(z_1) > 0$, does not necessarily imply that a firm with productivity $z_2 > z_1$ will enter j as well. The reason is that the decision to export to a country depends not only on the firm's productivity, but also on each firm's optimal diversification strategy. For example, a small firm may enter market j because it provides a good hedge from risk, while a larger firm does *not* enter j since it prefers to diversify risk by selling to other markets, where the small firm is not able to export. Another reason for $n_{ij}(z)$ not being monotonic in z is that, when the productivity increases, reaching more consumers does not only raise the expected revenues that can be reaped, but it also raises the variance of the revenues, for a size effect. If the variance effect dominates over the mean effect, then large firms may prefer to reach less consumers than small firms.

This is a novel feature of my model, and it differs from traditional trade models with fixed costs, such as Melitz (2003) and Chaney (2008), where the exporting decision is strictly hierarchical. Recent empirical evidence (see Bernard et al. (2003), Eaton et al. (2011) and Armenter and Koren (2015)) suggests instead that, although exporters are more productive than non-exporters in general, firms do not strictly there exist firms which are more productive than exporters but that still only serve the domestic market. My model is consistent with this evidence and offers an alternative explanation for it.

Although the exporting decision is non-hierarchical, and a less productive firm may enter a market while a more productive one does not, the presence of fixed costs still imposes a constraint on the risk diversification strategy of small firms. This emerges from the optimality condition (13). If a firm wants more exposure in market j and consequently wants to sell to more consumers, it has to consider paying higher marketing costs. Smaller firms have lower gross profits, limiting their ability to be hedged in safer countries. In the quantitative section, I will show using the calibrated parameters that smaller firms indeed have less diversified portfolios and that the data confirm this patterns.

It is worth looking at the optimal solution in the special case of zero risk aversion. In the Appendix I show that, in this case, a firm sells to country j only if its productivity exceeds the following entry cutoff:

$$(\bar{z}_{ij})^{\sigma-1} = \frac{w_i f_j \tilde{L}_j P_j^{1-\sigma} \sigma}{\bar{\alpha}_j \left(\frac{\sigma}{\sigma-1} \tau_{ij} w_i \right)^{1-\sigma} Y_j}, \quad (18)$$

and that, whenever the firm enters a market, it sells to all consumers, so that $n_{ij}(z) = 1$. The similarity of equation (18) with the entry cutoff in trade models with fixed entry costs and risk-neutrality, such as Melitz (2003) and Chaney (2008), is evident: firms enter all profitable locations, i.e. the markets where the revenues are higher than the fixed costs of production. Additionally, upon entry they serve all consumers. Since a risk neutral firm only maximizes the expected profits without taking into account the variance, the entry cutoff in equation (18) corresponds to the condition of positive profits. Then Proposition 2 and equation (18) suggest the possibility that a firm optimally enters a location even if it earns negative *expected* profits, only because that country represents a good hedge for demand risk. This may happen either because demand there is very stable, or because it is negatively correlated with demand in the rest of the world. Conversely, a firm could not enter a market even if it expected positive profits, because that country does not provide enough diversification benefits. This result is a unique feature of my model and offers a new explanation for the pattern of firm entry across countries.

The sales of firm z to country j are:

$$x_{ij}(z) = p_{ij}(z)q_{ij}(z) = \alpha_j(z) \left(\frac{\sigma}{\sigma-1} \frac{\tau_{ij}w_i}{z} \right)^{1-\sigma} \frac{Y_j}{P_j^{1-\sigma}} n_{ij}(z) \quad (19)$$

Using equation (16), it can be seen that firm-level trade flows are increasing in ψ_j , the measure of the risk-return profile of market j . They also depend on the set of destinations where the firm exports.

Moreover, equations (19) and (16) suggest that my model can reconcile the positive relationship between firm entry and market size with the existence of many small exporters in each destination, as shown by Eaton et al. (2011) and Arkolakis (2010). On one hand, upon entry firms can extract higher profits in larger markets. Therefore, more companies enter markets with larger population size. On the other hand, the firms' global diversification strategy may induce them to optimally reach only few consumers, and thus export tiny amounts. In contrast, the standard fixed cost models, such as Melitz (2003) and Chaney (2008), require large fixed costs to explain firm entry patterns, which contradict the existence of many small exporters.

3.3 Trade equilibrium

I now describe the equations that define the trade equilibrium of the model. Following Helpman et al. (2004), Chaney (2008) and Arkolakis et al. (2008), I assume that the productivities are drawn, independently across firms and countries, from a Pareto distribution with density:

$$g(z) = \theta z^{-\theta-1}, \quad z \geq \underline{z}, \quad (20)$$

where $\underline{z} > 0$. The price index is:

$$P_i^{1-\sigma} = \sum_j J_j \int_{\underline{z}}^{\infty} \int_0^{\infty} \alpha_i(z) n_{ji}(z) p_{ji}(z)^{1-\sigma} g_i(\alpha) \theta z^{-\theta-1} d\alpha dz, \quad (21)$$

where $g_i(\alpha)$ is the marginal density function of the demand shock in destination i , and $n_{ji}(z)$ and $p_{ji}(z)$ are given in Proposition 1.³⁸ Since the optimal fraction of consumers reached, $n_{ij}(z)$, is bounded between 0 and 1, a sufficient condition to have a finite integral is that $\theta > \sigma - 1$. As in Chaney (2008), the number of firms is fixed to J , implying that in equilibrium there are business profits, which equal:

$$\Pi_i = J_i \sum_j \left(\int_{\underline{z}}^{\infty} \int_0^{\infty} \alpha_j(z) n_{ij}(z) p_{ij}(z)^{1-\sigma} \frac{Y_j}{P_j^{1-\sigma} \sigma} g_i(\alpha) \theta z^{-\theta-1} d\alpha dz - \int_{\underline{z}}^{\infty} w_i f_j n_{ij}(z) \tilde{L}_j \theta z^{-\theta-1} dz \right). \quad (22)$$

Imposing the current account balance, total income is:

$$Y_i = w_i L_i + \Pi_i. \quad (23)$$

Finally, the labor market clearing condition states that in each country the supply of labor must equal the amount of labor used for production and marketing:

$$J_i \sum_j \int_{\underline{z}}^{\infty} \int_0^{\infty} \alpha_j(z) \frac{\tau_{ij}}{z} n_{ij}(z) \left(\frac{\sigma}{\sigma-1} \frac{\tau_{ij} w_i}{z} \right)^{-\sigma} \frac{Y_j}{P_j^{1-\sigma}} g_i(\alpha) \theta z^{-\theta-1} d\alpha dz + J_i \sum_j \int_{\underline{z}}^{\infty} f_j n_{ij}(z) \tilde{L}_j \theta z^{-\theta-1} dz = L_i, \quad (24)$$

³⁸This marginal density can be found integrating over all the other destinations: $g_i(\alpha) = \int_0^{+\infty} \dots \int_0^{+\infty} G(\alpha_{k=1, \dots, N}) d\alpha$, with $s = 1, \dots, N$ and $s \neq i$, where $G(\alpha_{k=1, \dots, N})$ is the multivariate CDF of the shocks.

Therefore the trade equilibrium in this economy is characterized by a vector of wages $\{w_i\}$, price indexes $\{P_i\}$ and income $\{Y_i\}$ that solve the system of equations (21), (23), (24), where n_{ij} is given by equation (16). It is worth noting that the realization of the demand shocks does not affect the equilibrium wages and prices, because these are set in general equilibrium, and on aggregate the idiosyncratic shocks average out by the Law of Large Numbers.³⁹

3.4 Welfare analysis

Having completed the derivation of the equilibrium, I now evaluate the implications of demand risk for welfare. I define welfare in country i as the equally-weighted sum of the welfare of workers and entrepreneurs:

$$W_i = U_i^w L_i + J_i \int U^e \left(\frac{\pi_i(z)}{P_i} \right) dG(z),$$

where U_i^w is the utility of each worker, while $U^e \left(\frac{\pi_i(z)}{P_i} \right)$ is the utility of each entrepreneur with productivity z . Since workers have a CES utility, their welfare is simply the real wage. Given that business owners have stochastic utility, the correct *money-metric* measure of their welfare is the Certainty Equivalent (see Pratt (1964) and Pope et al. (1983)). The Certainty Equivalent is simply the certain level of wealth for which the decision-maker is indifferent with respect to a risky alternative. The assumption of mean-variance preferences implies that the Certainty Equivalent is:

$$W_i(z) = E \left(\frac{\pi_i(z)}{P_i} \right) - R_i(z),$$

where $R(z)$ is the risk premium:

$$R_i(z) = \frac{b}{2} \text{Var} \left(\frac{\pi_i(z)}{P_i} \right).$$

Given that both workers and entrepreneurs have the same price index, the aggregate welfare is:

³⁹This happens because shocks are i.i.d. across varieties and because there is a continuum number of varieties. Note that my model is not isomorphic to an economy with country-specific shocks because, in that case, the idiosyncratic shocks would not average out since the number of countries is finite.

$$\begin{aligned}
W_i &= \frac{w_i L_i}{P_i} + J_i \int \left(E \left(\frac{\pi_i(z)}{P_i} \right) - R_i(z) \right) dG(z) = \\
&= \frac{w_i L_i}{P_i} + \frac{\Pi_i}{P_i} - R_i,
\end{aligned} \tag{25}$$

where R_i is the aggregate risk premium. Note that when the risk aversion equals zero, the certainty equivalent simply equals the real income produced in the economy, as in all trade models with restricted entry (see Chaney (2008), Arkolakis (2010)).⁴⁰

I now characterize the percentage change in the aggregate certainty equivalent associated with a *small* change in trade costs from τ_{ij} to $\tau'_{ij} < \tau_{ij}$. As is common in the welfare economics literature, welfare changes are analyzed employing measures of willingness to pay (see Pope et al. (1983) for an analysis under risk aversion, ACR under risk neutrality). In particular, using initial welfare as a reference point, willingness to pay is measured with the compensating variation cv , defined as:

$$W_i(\tau_{ij}) = W_i(\tau'_{ij}) - cv_i,$$

where $W_i(\tau'_{ij})$ is the certainty equivalent in the counterfactual equilibrium. Thus, cv_i is the ex-ante sum of money which, if paid in the counterfactual equilibrium, makes all consumers indifferent to a change in trade costs.⁴¹ Expressed in percentage change, welfare changes are:

$$d\ln W_i = \frac{W_i(\tau'_{ij}) - W_i(\tau_{ij})}{W_i(\tau_{ij})}.$$

Using equation (25), I can decompose welfare gains from trade in two terms:

$$\begin{aligned}
d\ln W_i &= \underbrace{\frac{w_i/P_i}{W_i} d\ln \left(\frac{w_i}{P_i} \right)}_{\text{workers' gains}} + \underbrace{\frac{\Pi_i/P_i}{W_i} d\ln \left(\frac{\Pi_i}{P_i} \right) - \frac{R_i}{W_i} d\ln R_i}_{\text{entrepreneurs' gains}}.
\end{aligned} \tag{26}$$

The first term is a standard price effect: lower trade costs imply tougher competition from foreign firms and thus lower prices. These are the gains that are accrued by workers, since

⁴⁰Also note that the sum of the variances of firms' profits is *not* equal to the variance of total profits, because the cross-country covariances of profits are firm-specific.

⁴¹Similarly, the equivalent variation is the sum of money which, when received in the initial equilibrium, makes all consumers indifferent to a change in trade costs.

their utility is given by the real wage they earn. The second term are the welfare gains that flow to the entrepreneurs: they are the sum of a profits effect and a risk effect. The first effect is the change in real profits after the trade shock, weighted by the share of real profits in total welfare. Note that in models with risk neutrality and Pareto distributed productivities, such as Chaney (2008) and Arkolakis et al. (2008), profits are a constant share of total income. Consequently, the sum of workers' gains and the profits effect simply equals $-d\ln P_i$. In my model, in contrast, profits are no longer a constant share of Y_i , as can be gleaned from equation 23.

The second term of entrepreneurial gains is the percentage change in the aggregate risk premium. Given that the variance of the profits increases when trade costs decline, for a size effect, this term tends to decrease the welfare gains from trade because it lowers the utility of the entrepreneurs.

One special case of my model is when the risk aversion equals zero for all firms. In that case, as shown in the Appendix, welfare gains are:

$$d\ln W_i = \frac{1}{\varepsilon} d\ln \lambda_{ii} \quad (27)$$

where λ_{ii} denotes domestic trade shares and $\varepsilon = -\theta$ is the trade elasticity. This result is not surprising, since I have already shown that when $b(z) = 0$, the optimal solution of the firm is the same as in Chaney (2008). Equation (27) suggests that in case of risk neutrality gains from trade correspond to those in a vast class of trade models, as shown in Arkolakis et al. (2012), and can be computed using the observed change in trade shares $d\ln \lambda_{ii}$ and the trade elasticity. This will be an important benchmark for the gains from trade in my model.

In the following section, I will calibrate the model and quantify these gains.

4 Empirical implications

In this section, I use the general equilibrium model laid out in the previous section as a guide through the data. I first use firm level data from Portugal to calibrate the relevant parameters. Then, I test the empirical implications of my model.

4.1 Data

The analysis mostly relies on a panel dataset on international sales of Portuguese firms to 210 countries, between 1995 and 2005. These data come from Statistics Portugal and roughly aggregate to the official total exports of Portugal. I merged this dataset with data on some firm characteristics, such as number of employees, age and equity, which I extracted from a matched employer–employee panel dataset called Quadros de Pessoal.⁴² I describe the two datasets in more detail in Appendix A. Portugal is a small and export-intensive country, being at the 72nd percentile worldwide for exports per capita. Therefore, it can be considered an ideal laboratory to analyze the implications of my model. In the analysis, I consider the 6,387 manufacturing firms that, in 2005, export to at least one of Portugal’s 50 biggest trading partners.⁴³

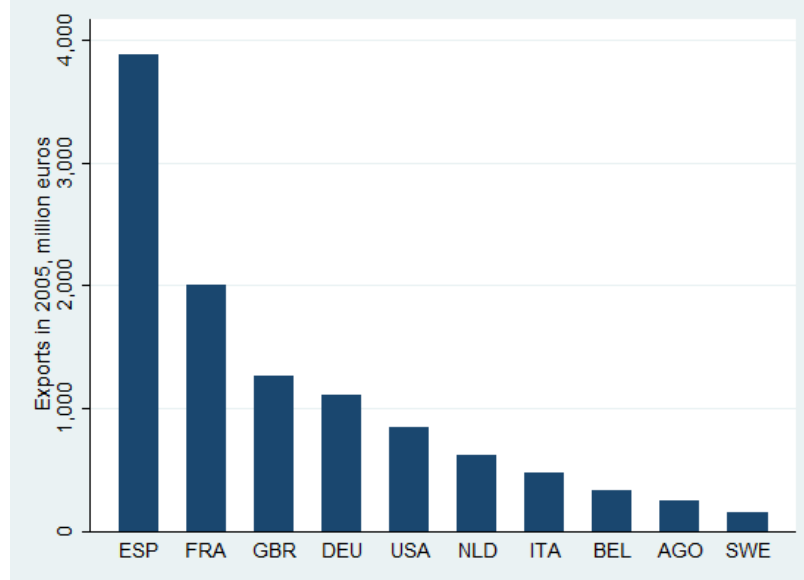
Figure 4 shows Portuguese exports to the top 10 destinations in 2005. Figure 5 plots the distribution of the size of Portuguese exporters.⁴⁴ It is evident that the universe of Portuguese manufacturing exporters is comprised of mostly small firms and fewer large players. In addition, 70% of firms has less than 50 employees, which is the typical threshold to define a small firm in Europe, and the average number of destinations reached is 5. Other empirical studies have revealed similar patterns using data from other countries, such as Bernard et al. (2003) and Eaton et al. (2011).

⁴²I thank the Economic and Research Department of Banco de Portugal for giving me access to these datasets.

⁴³There were 25,769 manufacturing firms in 2005, among which 6,387 exported to the top 50 destinations. Trade flows to these 50 countries accounted for 98% of total manufacturing trade from Portugal in 2005. I include in the analysis all firms that sell domestically and export, as well as the 291 exporters that do not enter the domestic market. Therefore I exclude firms that sell only domestically.

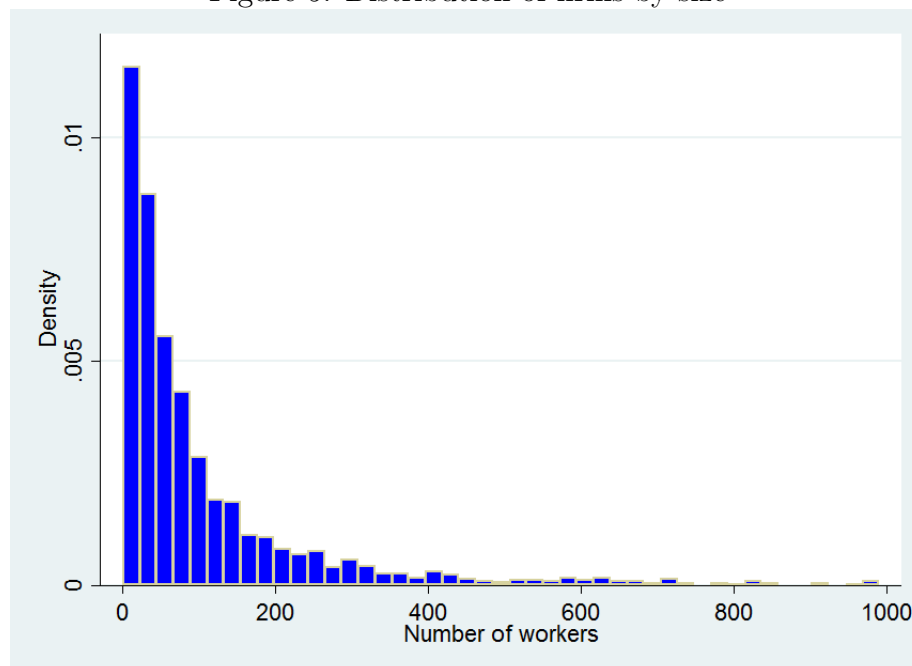
⁴⁴Figure 5 proxies size with the number of workers employed by the firm. The distribution is very similar if instead I proxied size with the revenues or labor productivity.

Figure 4: Portuguese exports in 2005 to top 10 destinations



Notes: The figure shows the exports in 2005 toward the top 10 destinations of Portuguese firms.

Figure 5: Distribution of firms by size



Notes: The figure shows the distribution of the size of Portuguese exporters in 2005, where the size is proxied by the number of workers. Results are very similar if size is proxied with total sales or productivity. For expositional purposes I do not include on the x-axis the few firms with more than 1000 employees.

4.2 Calibration of the model

I use data on manufacturing trade flows in 2005 from the World Input-Output Database as the empirical counterpart of aggregate bilateral trade in the model.⁴⁵ I use firm-level data on sales from Portugal to the rest of the world in 2005 as the empirical counterpart of $x_{Pj}(z)$; from the same dataset, I observe the number of firms selling to a destination j , M_{Pj} , as well as the total number of manufacturers, J_P .⁴⁶ The parametrization strategy is as follows.

Some parameters are directly observable in the data, and thus, I directly assigned values to them. I set $\sigma = 4$ (roughly consistent with an average markup of 33%, as estimated for the manufacturing sector in the US by Christopoulou and Vermeulen (2012)); I proxy L_j with the total number of workers in the manufacturing sector, while J_j is the total number of manufacturing firms. The Appendix describes in detail how I construct these measures. As in Eaton et al. (2011), I assume that $\bar{\alpha}_j = 1$ for all countries. I assume that trade costs have the following functional form:

$$\ln \tau_{ij} = \beta_0 + \beta_1 \ln dist_{ij} + \beta_2 cont_{ij} + \beta_3 lang_{ij}, \quad i \neq j, \quad (28)$$

where $dist_{ij}$ is the geographical distance between countries i and j , $cont_{ij}$ is a dummy equal to 1 if the two countries share a border, and $lang_{ij}$ is a dummy equal to 1 if the two countries share the same language.⁴⁷ I follow Arkolakis (2010) and assume that the per-consumer fixed costs f_j are:

$$f_j = \tilde{f} \left(\tilde{L}_j \right)^{\gamma-1} \quad (29)$$

where $\tilde{f} > 0$.⁴⁸ I set the lower bound of the Pareto distribution to 1.

The remaining parameters are calibrated with the method of simulated moments, so that endogenous outcomes from the model match salient features of the data. These are the

⁴⁵See Costinot and Rodríguez-Clare (2013) for the description of the WIOD database.

⁴⁶Results are robust to the year used for the calibration.

⁴⁷These variables are taken from CEPII.

⁴⁸This functional form has been micro-founded in Arkolakis (2010) as each firm sending costly ads that reach the consumers. Then the number of consumers who see each ad in market j is given by $\tilde{L}_j^{1-\gamma}$. The parameter γ is expected to be between 0 and 1, given the empirical evidence that the cost to reach a certain number of consumers is lower in markets with a larger population (see Mathewson (1972) and Arkolakis (2010)). Assuming that the labor requirement for each ad is $\tilde{f}$, the amount of labor required to reach a fraction $n_{ij}(z)$ of consumers in a market of size $\tilde{L}_j$ is equal to $f_{ij} = \tilde{f} \left(\tilde{L}_j \right)^{\gamma-1} n_{ij}(z) L_j = \tilde{f} \left(\tilde{L}_j \right)^{\gamma} n_{ij}(z)$. Notice that this formulation corresponds to the special case in Arkolakis (2010) where the marginal cost of reaching an additional consumer is constant.

risk aversion b , the technology parameter θ , iceberg trade costs τ_{ij} , fixed costs f_j , and the covariance matrix of the demand shocks Σ . A challenge in the calibration of Σ is that in the data the sales of two different firms to the same country may differ due to variations in both the productivity and the realization of the demand shock. As a result, it is hard to identify the shock separately from the productivity. I overcome this issue by using the assumptions that the moments of the shocks are the same for all varieties, and that the shocks are drawn independently across varieties. This implies that I can compute cross-sectional means, variances and covariances of the sales from Portugal to any pair of countries. The calibration algorithm is as follows:

- 1) Guess a vector $\Theta = \{b, \theta, \beta_{const}, \beta_{dist}, \beta_{cont}, \beta_{lang}, \gamma, \tilde{f}\}$.
- 2) For a given Θ , find the covariance matrix Σ such that the *cross-sectional* covariance between the sales from Portugal to country j and country s matches the cross-sectional covariance observed in the data, for all pairs j, s .⁴⁹ Such covariance in the model is:

$$\begin{aligned} Cov(x_{Pj}, x_{Ps}) &= E[x_{Pj}(z)x_{Ps}(z)] - E[x_{Pj}(z)]E[x_{Ps}(z)] = \\ &= \int_{\underline{z}}^{\infty} x_{Pj}(z)x_{Ps}(z)\theta z^{-\theta-1}dz - \left(\int_{\underline{z}}^{\infty} x_{Pj}(z)\theta z^{-\theta-1}dz\right)\left(\int_{\underline{z}}^{\infty} x_{Ps}(z)\theta z^{-\theta-1}dz\right) \end{aligned} \quad (30)$$

where $x_{Pj}(z)$ is given by equation (19). As stated earlier, the assumptions that the moments of the shocks are the same across all firms and that the shocks are i.i.d. across varieties, implies that I can back out each element of the matrix Σ by simply computing the covariances across all firms that export to both j and s .

- 3) Solve the trade equilibrium using the system of equations (16), (21), (23) and (24).
- 4) Produce four sets of moments:

- **Moment 1.** Aggregate trade flows, X_{ij} , excluding domestic sales. In the model these equal to:

$$X_{ij} = J_i \int_{\underline{z}}^{\infty} \int_0^{\infty} x_{ij}(z)\theta z^{-\theta-1}g(\alpha)d\alpha dz. \quad (31)$$

I stack these trade flows in a $N(N-1)$ -element vector $\hat{m}(1; \Theta)$ and compute the analogous

⁴⁹Note that to find Σ I solve for the trade equilibrium at each iteration, conditional on Θ . See the Appendix for a detailed description of the algorithm used to solve the equilibrium.

moment in the data, $m(1)$. This moment is needed to calibrate the trade costs parameters.

- **Moment 2.** Number of Portuguese exporters M_{Pj} to destination j . Stack M_{Pj} in a N -element vector $\hat{m}(2; \Theta)$, and compute the analogous moment in the data, $m(2)$. This moment is needed to calibrate the marketing costs parameters.
- **Moment 3.** Normalized average sales in Portugal of firms selling to market j , $\bar{X}_{PP,j}/\bar{X}_{PP}$. The numerator of this quantity is the average sales in Portugal of firms selling to location j , computed by integrating the expression for sales in equation 31.⁵⁰ The denominator is instead the average sales in Portugal. This moment is needed to calibrate the technology parameter θ . Indeed, note that θ determines the dispersion of productivities across firms. If θ is very low, then the variability of productivities is higher, and more productive firms have a stronger advantage over the other firms. Since exporters tend to be more productive than non-exporters given the presence of fixed costs of production, I capture this advantage by computing the ratio of domestic sales of exporters, $\bar{X}_{PP,j}$, to domestic sales of all firms, $\bar{X}_{PP}$. I stack $\bar{X}_{PP,j}/\bar{X}_{PP}$ in a N -element vector $\hat{m}(3; \Theta)$ and compute the analogous moment in the data, $m(3)$.
- **Moment 4.** Ratio between average sales and variance of sales in market j , $\frac{\bar{X}_{Pj}}{Var(x_{Pj})}$. The numerator is the average sales of all Portuguese firms that are able to export to country j , $\bar{X}_{Pj} = \frac{X_{Pj}}{M_{Pj}}$, while $Var(x_{Pj})$ is their cross-sectional variance:

$$Var(x_{Pj}) = \int_{\underline{z}}^{\infty} \int_0^{\infty} (x_{Pj}(z))^2 \theta z^{-\theta-1} g(\alpha) d\alpha dz - \left(\int_{\underline{z}}^{\infty} \int_0^{\infty} x_{Pj}(z) \theta z^{-\theta-1} g(\alpha) d\alpha dz \right)^2$$

This mean-variance ratio is used to calibrate the risk aversion. The reason I use this moment is that the risk aversion is the gradient of the relationship between the mean and the variance of each firm's revenues.⁵¹ The higher b , the more firms want to be compensated for taking additional risk, and thus higher variance of sales must be associated with higher average sales.⁵² I stack $\frac{\bar{X}_{Pj}}{Var(x_{Pj})}$ in a N -element vector $\hat{m}(4; \Theta)$ and compute the analogous moment in the data, $m(4)$.

⁵⁰ Arkolakis (2010) does the same using sales of French exporters.

⁵¹ Recall that the CES assumption implies that gross profits are a fraction σ of revenues, so the relationship between the mean and their variance is the same (given that fixed costs are non-stochastic).

⁵² Indeed this ratio can be written as $\frac{\bar{X}_{Pj}}{Var(x_{Pj})} = bf(z, \bar{\alpha}, \Sigma)$.

5) Stacking these four moments gives the following vector of length $2N + N^2 = 2703$:

$$y(\Theta) = \begin{bmatrix} m(1) - \hat{m}(1; \Theta) \\ m(2) - \hat{m}(2; \Theta) \\ m(3) - \hat{m}(3; \Theta) \\ m(4) - \hat{m}(4; \Theta) \end{bmatrix}$$

I iterate over Θ such that the following moment condition holds:

$$E[y(\Theta_0)] = 0$$

where Θ_0 is the true value of Θ . Therefore, I seek a $\hat{\Theta}$ that achieves:

$$\hat{\Theta} = \operatorname{argmin}_{\Theta} \{y(\Theta)' \mathbf{W} y(\Theta)\}$$

where $\mathbf{W}$ is a 2703×2703 weighting matrix. At each function evaluation involving a new value of Θ , I solve the trade equilibrium and construct the moments for them as described above. The weighting matrix is the generalized inverse of the estimated variance-covariance matrix z of the 2703 moments calculated from the data.⁵³

4.2.1 Parameters estimates and discussion

The best fit is achieved with the values shown in Table 1.

Table 1: Calibrated parameters

Parameter	θ	γ	$\tilde{f}$	b	β_0	β_1	β_2	β_3
Value	6.2	0.3232	2.075	1.12	0.0148	0.0989	-0.001	-0.001

⁵³Following Eaton et al. (2011), I calculate z using the following bootstrap procedure: (i) I resample, with replacement, 100,000 firms from the initial data set 2,000 times. (ii) For each resampling b , I calculate m^b , which is given by stacking the moments 2, 3 and 4: $m^b \equiv [m(2); m(3); m(4)]$. (iii) I calculate

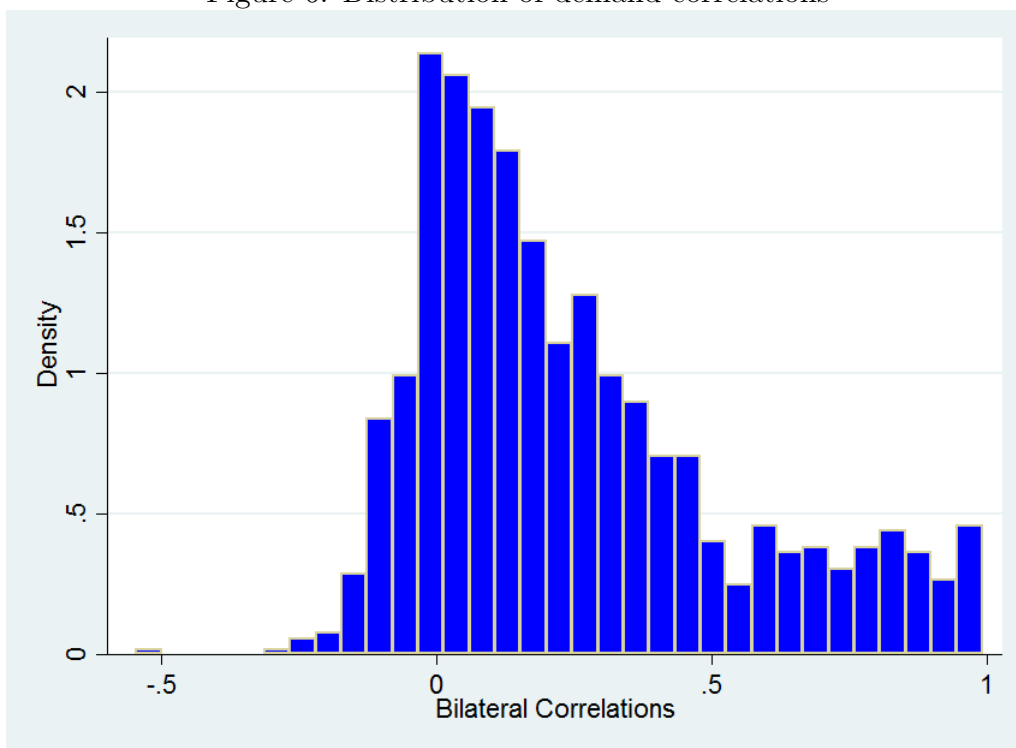
$$z = \frac{1}{2000} \sum_{b=1}^{2000} (m^b - m) (m^b - m)'.$$

Since I do not use firm-level sales for moment 1, I cannot recompute it for each sampling of the firms. Therefore, I assign it zero variance.

The calibrated parameters are consistent with previous estimates in the trade literature. In particular, the technology parameter θ is equal to 6.2, which is in line with the results obtained using different methodologies (see Eaton and Kortum (2002), Bernard et al. (2003), Simonovska and Waugh (2014), Costinot et al. (2012)). Both the elasticity of marketing costs with respect to the size of the market, γ , and the cost of each ad, $\tilde{f}$, correspond with the values estimated in Arkolakis (2010). Using equation (23), these estimates indicate that, in the median country, marketing costs dissipate 40% of gross profits.⁵⁴ Moreover, I found a value for the risk aversion of 1.12. This number is also consistent with findings of the literature. Herranz et al. (2015) calibrate a median risk aversion of 1.55 using data on US small firms' ownership. Allen and Atkin (2015) estimate the risk aversion of Indian farmers to be 1.4.

Figure 6 shows the distribution of the cross-country *correlations* of the demand shocks, computed from the calibrated covariance matrix Σ .

Figure 6: Distribution of demand correlations



Notes: The figure shows the distribution of the cross-country correlations of demand shocks.

The heterogeneity of the correlations, which range from -0.55 (correlation between Chile

⁵⁴Eaton et al (2011) estimate this fraction to be 59 percent.

and Guinea Bissau) to 0.981 (correlation between Taiwan and United Arab Emirates), with a median value of 0.17, is evident. Among European countries the median correlation is 0.25, though it is 0.34 within the Eurozone which is much higher than the overall median. This suggests that the correlation tends to decrease with distance between countries. This intuition is confirmed by Table 2, which shows that, for all countries in the sample, the correlations of demand are inversely related with geographical distance.

Table 2: Correlations and distance

Dep. Var.: Bilateral correlation	(1)
Distance	-0.042*** (0.01)
Dummy for contiguity	0.018 (0.04)
Dummy for common legal origins	0.01 (0.02)
Dummy for RTA	-0.04** (0.02)
Dummy for shared language	-0.01 (0.03)
Constant	0.644
Observations	2068
R-squared	0.012

Notes: The table regresses the calibrated bilateral demand correlations on bilateral distance and other controls, such as dummies for shared language, shared borders, common legal origins, regional trade agreement (all from CEPII). Robust standard errors are shown in parenthesis (*** p<0.01, ** p<0.05, * p<0.1). See Appendix for details.

Finally, using the estimated covariance matrix Σ , I can compute the country-level measure of riskiness, that is ψ_j . Table 3 reports some descriptive statistics of ψ_j , which range between 0.08 and 1.55, with a median value of 0.42.⁵⁵

⁵⁵There is also a mild but positive correlation between ψ_j and the size of a country.

Table 3: Descriptive statistics for ψ_j

Statistics	Value
Mean	0.55
Median	0.42
Std deviation	0.35
Min	0.08
Max	1.55
Mean OECD	0.58
Mean Non-OECD	0.52

In the Appendix I also analyze the robustness of the results to different moments used to calibrate θ , b and Σ , and I show that the parameters estimates are robust to the alternatives explored.

4.3 Testing the model

Using the calibrated model, I compare some of my model's predictions with the corresponding features of the data.

Entry of firms. Proposition 2 shows that in my model there is no strict sorting of firms into markets. An implication of such non-hierarchical structure of the exporting decision is related to the number of entrants to a certain location. First, recall that models characterized by fixed costs and absence of risk, such as Melitz (2003) and Chaney (2008), imply that firms obey a hierarchy: any firm selling to the $k + 1$ st most popular destination necessarily sells to the k th most popular destination as well.⁵⁶ The data however shows a different picture.⁵⁷ Following Eaton et al. (2011), I first list, in Table 4, the top seven destinations from Portugal and the number of Portuguese firms exporting there in 2005. In total, 5201 manufacturers were exporting to at least one of these seven countries.

⁵⁶This is because all firms with $z > z_{ij}^*$ will enter j .

⁵⁷Evidence that exporters and non-exporters are not strictly sorted has been shown also by Eaton et al. (2011) and Armenter and Koren (2015), among others.

Table 4: Firms exporting to top 7 destinations

Export Destination	Number of exporters
Spain (ES)	3176
France (FR)	2591
Germany (GE)	1753
United Kingdom (UK)	1728
Angola (AO)	1342
Belgium (BE)	1283
United Sates (US)	1270
Any among top 7	5201
All exporters	6387

Table 5 instead lists each of the strings of top-seven destinations from Portugal that obey a hierarchical structure, together with the number of Portuguese firms selling to each string (irrespective of their export activity outside the top 7). It can be seen that only 28% of Portuguese exporters were obeying a hierarchical structure in their exporting status. While classical trade models with fixed costs and risk neutrality would predict that all exporters follow a strict sorting into exporting, my model with risk averse firms instead is able to predict fairly well the number of exporters selling to each string of destinations.⁵⁸

Table 5: Firms exporting to strings of top 7 destinations

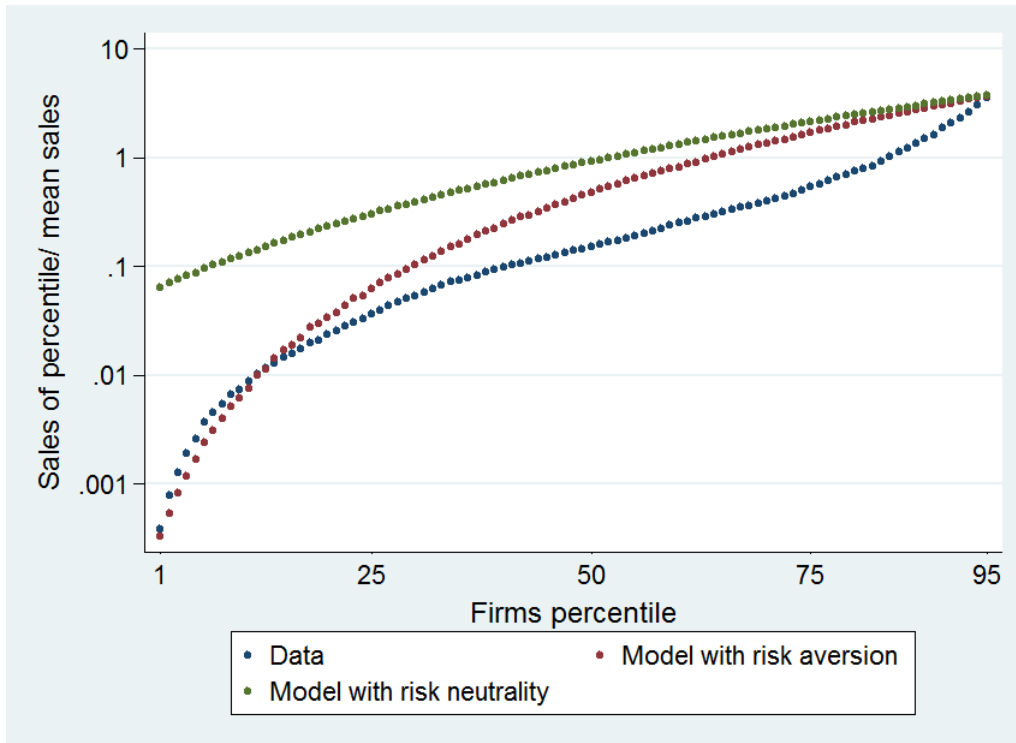
Export string	Number of exporters, data	Number of exporters, model
ES	675	725
ES-FR	318	401
ES-FR-GE	143	181
ES-FR-GE-UK	141	159
ES-FR-GE-UK-AO	18	56
ES-FR-GE-UK-AO-BE	49	74
ES-FR-GE-UK-AO-BE-US	92	104
Total	1436	1700

Distribution of firm-level trade flows. I compare the observed distribution of firm-

⁵⁸Another implication of a non-hierarchical structure of exporting is the possibility that some firms optimally choose to export but not to sell domestically. I verify that this is indeed the case: in 2005, 3.28% of Portuguese manufacturing exporters were *not selling* domestically. This number is consistent with evidence shown in Eaton et al. (2011), which found that 1.5% of French manufacturing exporters were not selling in France.

level exports to a certain destination with the one predicted by my calibrated model. Figure 7 plots these distributions for all Portuguese firms exporting to Spain.⁵⁹ The graph also plots the distribution predicted when I set the risk aversion to zero, which corresponds to the Chaney (2008) model.

Figure 7: Distribution of sales relative to mean sales in calibrated model and in the data



Notes: The figure shows the distribution of sales relative to mean sales from Portugal to Spain in the calibrated model with risk aversion, in the data for 2005, and in the calibrated model with risk neutrality.

We can see that while both models successfully predict the right tail of the distribution, my model outperforms the risk-neutral model in matching the left tail of the distribution. The reason is that some firms, when they are risk averse, optimally choose to reach a small number of consumers in a certain destination, rather than the whole market, and therefore export small amounts of their goods. In the Melitz-Chaney framework, instead, the presence of fixed costs are not compatible with the existence of small exporters, and thus over-predicts their size by many orders of magnitude.

Trade flows and riskiness. Proposition 1 and equation (19) state that firm-level trade

⁵⁹Results look very similar for other destinations.

flows to a market depend crucially on that market's riskiness. I test this prediction in the data by regressing firm level trade flows from Portugal on the proper country-level measure of riskiness delivered by my model, which is ψ_j :

$$\ln(x_{jz}) = \delta_0 + \delta_z + \delta_1\psi_j + \delta'\Gamma + \varepsilon_{jz} \quad (32)$$

where the dependent variable is the log of trade flows of firm z from Portugal to country j in 2005, ψ_j is calculated using the calibrated parameters of the model, δ_z are firm fixed effects and Γ is a vector of controls. The model predicts the coefficient on ψ_j to be positive: if a country's demand is relatively stable and is mildly or negatively correlated with demand in other markets, i.e. ψ_j is high, then that market provides good diversification benefits to firms selling there. As a result, we expect risk averse firms to export more to locations with high ψ_j .

Firm fixed effects control for unobserved heterogeneity specific to the firm. In my model, this is given by both productivity and risk aversion, since the demand shock $\alpha_j(z)$ goes into the error ε_{jz} . Note that the firm fixed effect should also capture the complementarities across countries implied by Proposition 1. In fact, in equilibrium trade flows to market j are affected by the firm's trade flows to all other markets, but this is a firm-specific characteristic, included in δ_z and not in the error ε_{jz} . This would otherwise have lead to a biased and inconsistent estimate of δ_1 .

Since my measure of riskiness is at the country level, I cannot control for destination fixed effects, as econometricians do in standard gravity regressions (there is no need to control for source fixed effects, since all firms are from Portugal). For this reason, I control for distance from Portugal and for the size of the destination, proxied by the log of population.⁶⁰ Finally, I use other standard gravity controls, such as the average bilateral tariff rate and dummies for shared language, shared borders, common legal origins and regional trade agreement.

⁶⁰ Using instead the log of GDP delivers very similar results.

Table 6: Trade flows and riskiness

Dep. Variable:	Log of firms' trade flows	Log of total trade flows
ψ_j	1.267*** (0.06)	1.64** (0.59)
Log of Population	0.1*** (0.02)	0.238** (0.11)
Distance	-0.002*** (0.00)	-0.002*** (0.00)
Constant	5.89	13.53
Firm fixed effects	YES	NO
Observations	34,990	48
R-squared	0.38	0.47

Notes: All data are from 2005. Clustered and robust standard errors are shown in parenthesis (*** p<0.01, ** p<0.05, * p<0.1). Variables omitted in the table are: dummies for shared language, shared borders, common legal origins, regional trade agreement (all from CEPII) and average tariff rate (from TRAINS/WITS and MACMap datasets). See Appendix for details.

Column 1 of Table 6 shows the result of this augmented gravity regression. As the model predicts, firm level trade flows are increasing in ψ_j : controlling for trade barriers, companies export more to safer countries. This relationship holds also in the aggregate, as shown in Column 2, where the dependent variable is instead total trade flows from Portugal.

Geographical diversification and volatility. In the model, entrepreneurs' risk aversion and the imperfect correlation of demand across markets naturally imply that geographical diversification reduces the variance of firms' total sales. This, in turn, suggests that the portfolios of firms selling to more countries are safer, in the sense that they reach a better risk-return trade-off. Since the owners maximize mean-variance preferences, the risk-return profile of each firm's portfolio is given by its Sharpe Ratio:

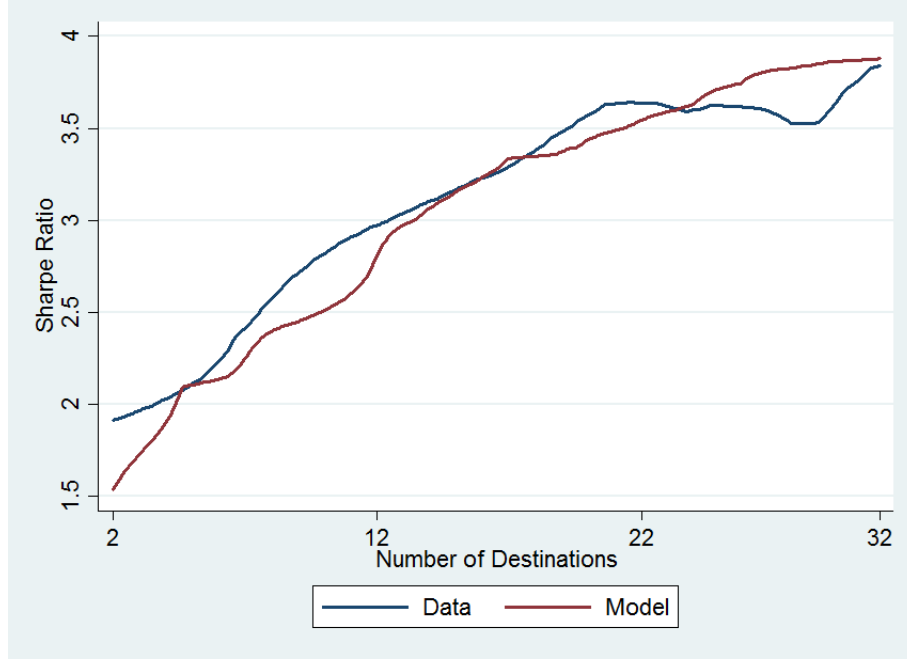
$$S_i(z) \equiv \frac{E(\tilde{\pi}_i(z))}{\sigma(\tilde{\pi}_i(z))} \quad (33)$$

where $E(\tilde{\pi}_i(z))$ is the average of the gross profits of firm z , while $\sigma(\pi_i(z))$ is the standard deviation.⁶¹ Figure 8 plots the Sharpe Ratios of Portuguese firms' portfolios simulated in

⁶¹Since in the data I do not observe fixed costs, I can only compute the Sharpe Ratios of gross profits (obtained by dividing observed total sales by σ). Since fixed costs are non-stochastic, they would only rescale

my model, using the parameters calibrated in the previous section, against the number of destinations reached by the firms. The graph also plots the Sharpe Ratios of Portuguese firms, computed using equation (33) and data on trade flows between 1995 and 2005.

Figure 8: Number of destinations and Sharpe Ratios



Notes: The figure shows the Sharpe Ratio of Portuguese firms' portfolios against the optimal number of destinations to which they were selling. The empirical Sharpe Ratios are computed using sales between 1995 and 2005. The number of destinations are the ones in 1995 (results are very similar if I use average number of destinations across 1995-2005). To compute the Sharpe Ratios in the model, I simulate the model T times. For each simulation, holding fixed the parameters of the model, I draw a vector of demand shocks $\alpha(z) \equiv \alpha_1(z), \dots, \alpha_N(z)$ for each firm and solve for the trade equilibrium. Then for each firm I compute $S_i(z)$ using T observations on $\tilde{\pi}_i(z)$. The plot is obtained by means of an Epanechnikov Kernel-weighted local polynomial smoothing, with parameters: degree = 0, bandwidth = 2.08, pwidth = 3.12.

The model successfully predicts the positive relationship between $S_i(z)$ and the geographical diversification of a firm. Selling to more markets allows country-specific shocks to hedge each other, improving the risk-return trade-off that a firm attains. Consequently, more geographically diversified companies have safer portfolios.⁶²

the numerator of $S_i(z)$.

⁶²Similar evidence has been shown by Kramarz et al. (2014).

5 Counterfactual exercises

In this section I use the calibrated parameters to conduct a number of counterfactual simulations in order to study the aggregate effects of firms' risk-hedging behavior.

5.1 Trade liberalization

In this policy exercise, I quantify the percentage change in welfare after a shock to trade costs τ_{ij} . In addition, I compare the welfare gains in my model with those predicted by models without demand risk. I have shown in this paper that my model converges to Chaney (2008) when the risk aversion is set to zero for all firms. ACR have demonstrated that the welfare gains in Chaney (2008) are the same as in a vast class of trade models, including Melitz (2003) and Eaton and Kortum (2002), and can be computed using only the change in domestic trade shares and the trade elasticity. Therefore, to compare the gains in my model with this class of models, I pursue the following steps:

1. Use the calibrated parameters to solve the trade equilibrium
2. Simulate a reduction in all trade costs by 1% and compute the counterfactual equilibrium
3. Calculate welfare gains
4. Compute the domestic trade shares in both equilibria, as well as the implied trade elasticity
5. Quantify the welfare gains predicted by Chaney (2008) using the ACR formula (shown also in equation (27))

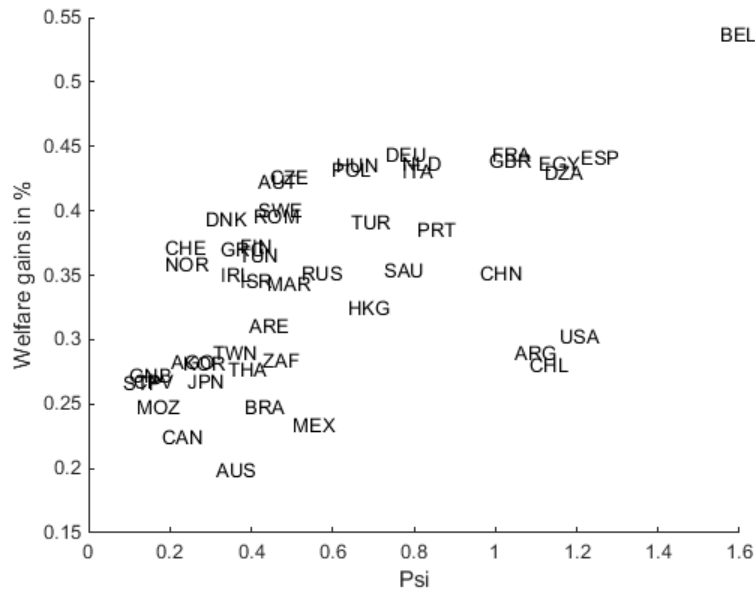
Note that I simulate a 1% reduction in trade costs in order to numerically compute the implied trade elasticity, which by definition holds only for small changes in trade costs.⁶³ Figure 9 illustrates the welfare gains for the 50 countries in the sample, as a function of their measure of risk-return, ψ_j . We can see that the total gains are increasing in ψ_j : countries that provide a better risk-return trade-off to foreign firms benefit more from opening up to trade. Firms exploit a trade liberalization not only to increase their profits, but also to diversify

⁶³See also Edmond et al. (2012) for a similar approach.

their demand risk. This implies that they optimally increase trade flows toward markets that provide better diversification benefits, as shown in the previous section. This also implies that the increase in foreign competition is stronger in these countries, additionally lowering the price level and raising the number of varieties available. Consequently, “safer” countries gain more from trade.⁶⁴

Importantly, this variety effect, i.e. foreign varieties crowding out domestic varieties, is novel compared to existing trade models, because it arises from the diversification strategy of foreign firms. This mechanism is evident in Figure 10, which plots the welfare gains from trade against the percentage change in domestic firms. Countries experiencing a larger drop in the number of inefficient firms gained more from international trade, due to a greater decrease in domestic prices.

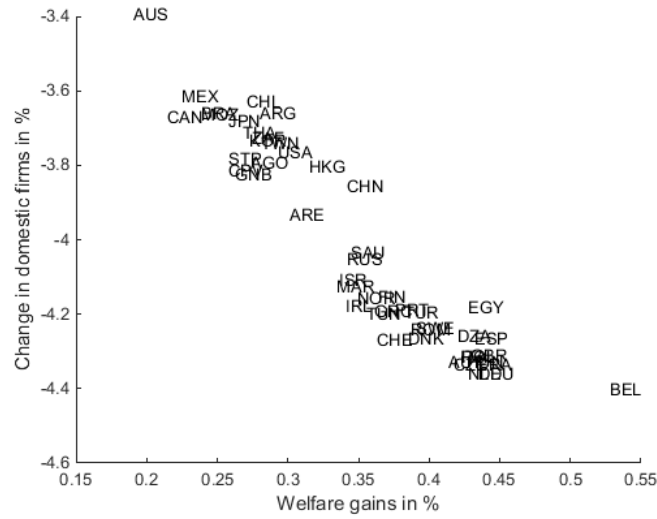
Figure 9: Welfare gains from trade



Notes: The figure plots the percentage change in welfare after a 1% reduction in variable trade costs. The variable on the x-axis is ψ , the country-level measure of risk-return, shown in equation (14).

⁶⁴This result holds also if I perform the slightly different counterfactual exercise of “moving to autarky”. The welfare gains of going from autarky to the observed equilibrium in 2005 are also increasing in ψ_j .

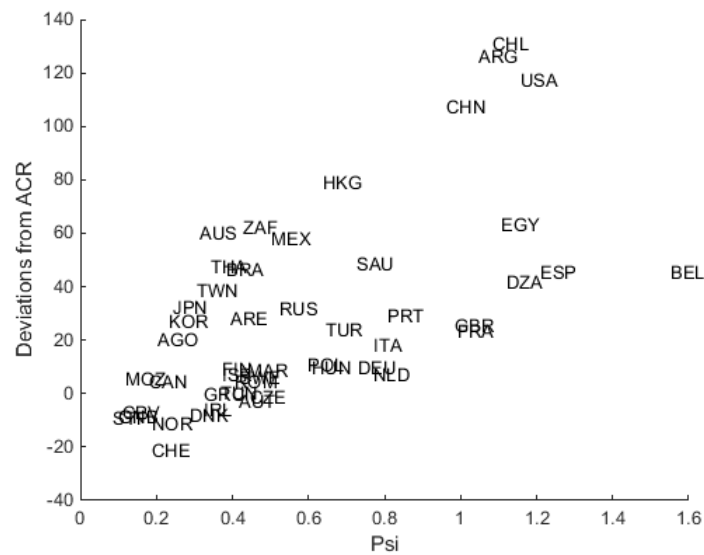
Figure 10: Welfare gains from trade vs exit of firms



Notes: The figure plots the percentage change in domestic firms against the percentage change in welfare, after a 1% reduction in variable trade costs.

Additionally, I compare the gains from trade in my model with the ones predicted by ACR, which encompasses the majority of trade models with risk neutrality. Figure 11 plots the deviations of the welfare gains in my model against those in ACR, as a function of ψ_j . As expected, the gains from trade in “safer” countries are higher than the gains in ACR, while the opposite happens for “riskier” markets. The median deviation from ACR is 18%.

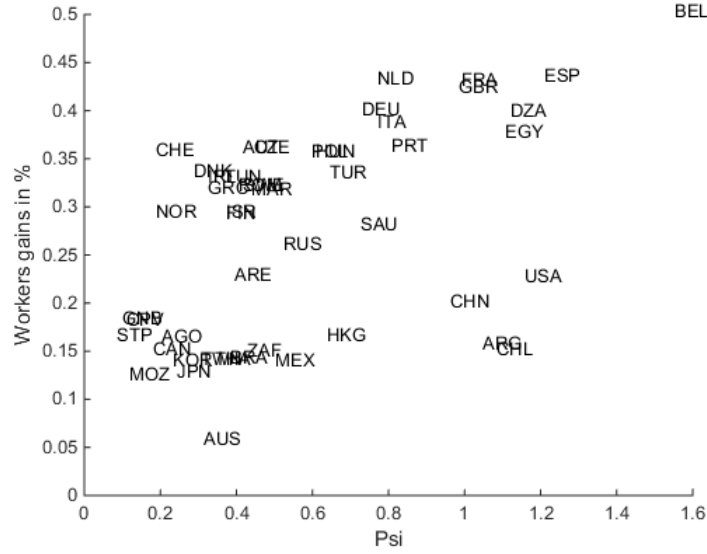
Figure 11: Welfare gains from trade vs ACR



Notes: The figure plots the difference between the welfare gains predicted by my model and those predicted by ACR, after a 1% reduction in variable trade costs. The variable on the x-axis is ψ , the country-level measure of risk-return, shown in equation (14).

Furthermore, using the intuition provided by equation (26), I decompose the welfare changes in workers' gains and entrepreneurial gains.

Figure 12: Workers' welfare gains from trade

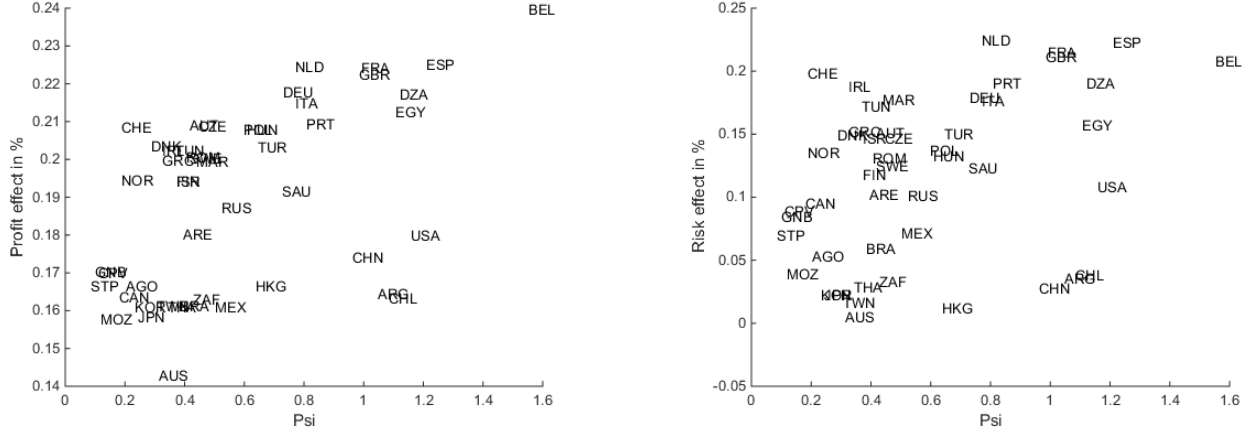


Notes: The figure plots the welfare gains from trade that accrue to workers, after a 1% reduction in variable trade costs. These gains are computed using equation (26). The variable on the x-axis is ψ , the country-level measure of risk-return, shown in equation (14).

Figure 12 shows that workers living in safer nations gained more from international trade. In these countries, consumers benefit more from the decreases in trade barriers, because of the tougher competition among firms which in turn leads to higher real wages. Figure 13 uses equation (26) to decompose the entrepreneurial gains into profit effect and risk effect. The Figure on the left shows that the profit effect, which is the percentage change in real profits, is higher in countries with higher ψ_j . To interpret this result, note that the lower price index that follows a trade liberalization has two opposite effects on entrepreneurs' real profits. First of all, nominal revenues are everywhere lower, but are even lower in safer nations, due to more intense competition from foreign firms. Second, since entrepreneurs are CES consumers themselves, they discount profits with the price index P_j , and thus lower prices improve their purchasing power. Figure 13 suggests that the net effect is positive in all countries. The graph on the right-hand side of Figure 13, instead, shows the risk effect, which is the percentage change in the aggregate risk premium, as described in equation (26). As the profit effect, the risk effect is higher for safer countries. However, the magnitude of this effect is always lower than the profit effect, implying that entrepreneurial gains are positive in all markets.⁶⁵

⁶⁵This result is due to the fact that the risk aversion's decay in the size of the firm is faster than the

Figure 13: Entrepreneurial welfare gains from trade



Notes: The figure on the left shows the percentage change in real profits after a 1% reduction in variable trade costs. The figure on the right, instead, shows the corresponding percentage change in the aggregate risk premium. The variable on the x-axis is ψ , the country-level measure of risk-return, shown in equation (14).

Finally, I study the distributional effect of a trade liberalization across firms. For the median country in my sample, small firms reap about 85% of total entrepreneurial gains from trade. The reason is that smaller firms are in general more constrained than large firms in their diversification strategy, because of the presence of trade barriers. Thus, small firms benefit more from international trade since they can lower the variance of their profits by exporting to more destinations.

5.2 Shock to volatility

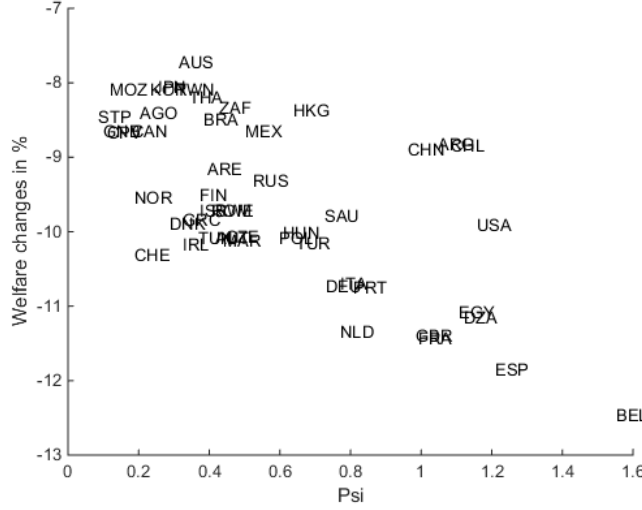
In the second policy exercise, I quantify the percentage change in welfare after a shock to the variance of demand. In particular, I simply assume that the variances in all markets increase by 20%. Interestingly, Figure 14 shows that this generalized increase in uncertainty leads to a decrease in welfare in all markets.

When demand is more volatile everywhere, the world suddenly becomes riskier. This implies that fewer firms enter markets, which in turn softens the competition among companies and raises the price level. Furthermore, safer countries experience a bigger drop in welfare.

corresponding increase of the variance, because the calibrated κ_2 is sufficiently negative. This implies that, aggregating across entrepreneurs, the change in the risk premium is smaller in the change in real profits.

This result can be explained by observing that these markets are losing their “comparative advantage” in risk diversification. Companies no longer view these locations as safe havens in which to diversify their demand risk. Consequently, they reduce the sales to these destinations, thereby lowering the number of varieties available and negatively affecting welfare. It is worth mentioning that simulating, instead, an increase in the cross-country correlations of demand has a very similar effect on welfare.⁶⁶

Figure 14: Welfare changes after a shock to variance



Notes: The figure shows the percentage change in welfare after a 20% increase in the variance of demand in all markets. The variable on the x-axis is ψ , the country-level measure of risk-return, shown in equation (14).

6 Concluding remarks

In this paper, I characterize the link between demand risk, firms’ exporting decisions, and welfare gains from trade. The proposed framework is sufficiently tractable to be estimated using the Method of Moments. Overall, an important message emerges from my analysis: welfare gains from trade significantly differ from trade models that neglect firms’ risk aversion. In addition, I stress the importance of the cross-country covariance of demand in amplifying the impact of a change in trade costs through a simple variety effect.

The main conclusion is that how much a country gains from international trade hinges crucially on its ability to attract foreign firms looking for risk diversification benefits. Policy

⁶⁶I do not show the results of these counterfactual simulations for reasons of brevity, but they are available upon request.

makers should implement policies that stabilize a country's demand, in order to improve its risk-return profile.

Interesting avenues for future research emerge from my study. For example, it would be instructive to extend my model to a dynamic setting, where firms are able to re-optimize their portfolio of destinations over time. Another interesting extension would be to introduce the possibility of mergers and acquisitions among firms or the possibility of holding shares from different companies, as alternative ways to diversify business risk.

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7 Appendix

7.1 Data Appendix

Trade data. Statistics Portugal collects data on export and import transactions by firms that are located in Portugal on a monthly basis. These data include the value and quantity of internationally traded goods (i) between Portugal and other Member States of the EU (intra-EU trade) and (ii) by Portugal with non-EU countries (extra-EU trade). Data on extra-EU trade are collected from customs declarations, while data on intra-EU trade are collected through the Intrastat system, which, in 1993, replaced customs declarations as the

source of trade statistics within the EU. The same information is used for official statistics and, besides small adjustments, the merchandise trade transactions in our dataset aggregate to the official total exports and imports of Portugal. Each transaction record includes, among other information, the firm’s tax identifier, an eight-digit Combined Nomenclature product code, the destination/origin country, the value of the transaction in euros, the quantity (in kilos and, in some case, additional product-specific measuring units) of transacted goods, and the relevant international commercial term (FOB, CIF, FAS, etc.). I use data on export transactions only, aggregated at the firm-HS6 product-destination-year level. I consider only the sales of Portuguese-owned firms, so I neglect the sales of firms that produce in Portugal but are owned by foreign firms.

Matched employer-employee data. The second main data source, Quadros de Pessoal, is a longitudinal dataset matching virtually all firms and workers based in Portugal. Currently, the data set collects data on about 350,000 firms and 3 million employees. As for the trade data, I was able to gain access to information from 1995 to 2005. The data is made available by the Ministry of Employment, drawing on a compulsory annual census of all firms in Portugal that employ at least one worker. Each year, every firm with wage earners is legally obliged to fill in a standardized questionnaire. Reported data cover the firm itself, each of its plants, and each of its workers. Variables available in the dataset include the firm’s location, industry (at 5 digits of NACE rev. 1), total employment, sales, ownership structure (equity breakdown among domestic private, public or foreign), and legal setting. Each firm entering the database is assigned a unique, time-invariant identifying number which I use to follow it over time.⁶⁷

The two datasets are merged by means of the firm identifier. As in Mion and Opromolla (2014) and Cardoso and Portugal (2005), I account for sectoral and geographical specificities of Portugal by restricting the sample to include only firms based in continental Portugal while excluding agriculture and fishery (Nace rev.1, 2-digit industries 1, 2, and 5) as well as minor service activities and extra-territorial activities (Nace rev.1, 2-digit industries 95, 96, 97, and 99). The analysis focuses on manufacturing firms only (Nace rev.1 codes 15 to 37) because of the closer relationship between the export of goods and the industrial activity of the firm. The location of the firm is measured according to the NUTS 3 regional

⁶⁷ The Ministry of Employment implements several checks to ensure that a firm that has already reported to the database is not assigned a different identification number.

disaggregation. In the trade dataset, I restrict the sample to transactions registered as sales as opposed to returns, transfers of goods without transfer of ownership, and work done.

Data on L and J. L_j is the total number of workers in the manufacturing sector, obtained from UNIDO. From the Matched employer-employee dataset I observe the total number of manufacturing firms in Portugal in 2005, which is $J_P = 46,890$. To compute the J for all other countries, I assume that in all countries the ratio J_j/L_j is equal to $J_P/L_P = 46,890/855,779 = 0.055$. Given that I observe L_j for all countries, the number of firms in country j is set to $J_j = 0.055L_j$. Notice that this method assumes that the fraction of manufacturing entrepreneurs is the same for all countries. Although this may not be true in the data (some countries have higher entrepreneurship rates than others), it is a good approximation of the size of the industrial activity of a country.

Tariff data. Bilateral *ad-valorem* tariffs at the HS-6 product level for 2004 were obtained from the MACMap database jointly developed by the ITC (UNCTAD and WTO) and CEPII. It contains bilateral tariff data including ad-valorem equivalents of specific tariffs, bilateral tariff preferences and anti-dumping duties. The methodology and data sources are described in Bouet et al. (2004). I averaged the tariff rates at the (bilateral) country level, using as weights trade flows at the HS-6 level from BACI dataset. Since the regression in equation (32) has only Portugal as source country, I considered only the tariffs applied to Portuguese goods.

7.2 Analytical appendix

Proof of Proposition 1. Since the firm decides the optimal price after the realization of the shock, in the first stage it chooses the optimal fraction of consumers to reach in each market based on the expectation of what the price will be in the second stage. I solve the optimal problem of the firm by backward induction, so starting from the second stage. Since at this stage the shocks are known, any element of uncertainty is eliminated and the firm then can choose the optimal pricing policy that maximizes profits, *given* the optimal $n_{ij}(z, E[p_{ij}(z)])$ decided in the previous stage:

$$\max_{\{p_{ij}\}} \sum_j \alpha_j(z) \frac{p_{ij}(z)^{-\sigma}}{P_j^{1-\sigma}} n_{ij}(z, E[p_{ij}(z)]) Y_j \left(p_{ij}(z) - \frac{\tau_{ij} w_i}{z} \right).$$

It is easy to see that this leads to the standard constant markup over marginal cost:

$$p_{ij}(z) = \frac{\sigma}{\sigma - 1} \frac{\tau_{ij} w_i}{z}. \quad (34)$$

Notice that, given the linearity of profits in $n_{ij}(z, E[p_{ij}(z)])$ and $\alpha_j(z)$, due to the assumptions of CES demand and constant returns to scale in labor, the optimal price does not depend on neither $n_{ij}(z, E[p_{ij}(z)])$ nor α_j . By backward induction, in the first stage the firm can take as given the pricing rule in (34), independently from the realization of the shock, and thus the optimal quantity produced is:

$$q_{ij}(z) = \alpha_j(z) \left(\frac{\sigma}{\sigma - 1} \frac{\tau_{ij} w_i}{z} \right)^{-\sigma} \frac{n_{ij}(z, p_{ij}(z)) Y_j}{P_j^{1-\sigma}}.$$

I now solve the firm problem in the first stage, when there is uncertainty. The maximization problem of firm z is:

$$\max_{\{n_{ij}\}} \sum_j \bar{\alpha}_j n_{ij}(z) r_{ij}(z) - \frac{b}{2} \sum_j \sum_s n_{ij}(z) r_{ij}(z) n_{is}(z) r_{is}(z) \text{Cov}(\alpha_j, \alpha_s) - \sum_j w_i n_{ij}(z) f_j \tilde{L}_j / P_i$$

$$\text{s. to } 1 \geq n_{ij}(z) \geq 0$$

where $r_{ij}(z) \equiv \frac{p_{ij}(z)^{-\sigma}}{P_i} \frac{Y_j}{P_j^{1-\sigma}} \left(p_{ij}(z) - \frac{\tau_{ij} w_i}{z} \right)$. Given the optimal price in (34), this simplifies to:

$$r_{ij}(z) = \left(\frac{\sigma}{\sigma - 1} \frac{\tau_{ij} w_i}{z} \right)^{1-\sigma} \frac{Y_j}{P_j^{1-\sigma} P_i \sigma}$$

The Lagrangian of the problem is:

$$\begin{aligned} \mathcal{L}_i(z) = & \sum_j \bar{\alpha}_j n_{ij}(z) r_{ij}(z) - \frac{b}{2} \sum_j \sum_s n_{ij}(z) r_{ij}(z) n_{is}(z) r_{is}(z) \text{Cov}(\alpha_j, \alpha_s) \\ & - \sum_j w_i n_{ij}(z) f_j \tilde{L}_j / P_i + \sum_j \mu_{ij}(z) (1 - n_{ij}(z)) + \sum_j \lambda_{ij}(z) n_{ij}(z) \end{aligned}$$

where $\lambda_{ij}(z)$ is the Lagrange multiplier associated with the non-negativity constraint, and $\mu_{ij}(z)$ is the Lagrange multiplier associated with the upper bound.

The Kuhn-Tucker Conditions for this problem are, for all j :

$$\bar{\alpha}_j r_{ij}(z) - b \sum_s r_{ij}(z) n_{is}(z) r_{is}(z) Cov(\alpha_j, \alpha_s) - w_i f_j \tilde{L}_j / P_i - \mu_{ij}(z) + \lambda_{ij}(z) = 0$$

$$\mu_{ij}(z)(1 - n_{ij}(z)) = 0$$

$$\lambda_{ij}(z) n_{ij}(z) = 0$$

$$\lambda_{ij}(z) \geq 0$$

$$\mu_{ij}(z) \geq 0$$

Then I can write the solution for $n_{ij}(z)$ in matricial form as:

$$\mathbf{n}_i(z) = \frac{1}{b} \left(\tilde{\Sigma}_i(z) \right)^{-1} \mathbf{r}_i(z), \quad (35)$$

where each element of the N -dimensional vector $\mathbf{r}_i$ equals:

$$\mathbf{r}_i^j(z) \equiv r_{ij}(z) \bar{\alpha}_j - w_i f_j \tilde{L}_j / P_i - \mu_{ij}(z) + \lambda_{ij}(z), \quad (36)$$

and $\tilde{\Sigma}_i$ is a $N \times N$ covariance matrix, whose k, j element is, from equation (12):

$$\tilde{\Sigma}_{i,kj}(z) = r_{ij}(z) r_{ik}(z) Cov(\alpha_j, \alpha_k).$$

The inverse of $\tilde{\Sigma}_i$ is, by the Cramer's rule:

$$\left(\tilde{\Sigma}_i(z) \right)^{-1} = r_i \frac{1}{\det(\Sigma)} C_i r_i, \quad (37)$$

where r_i is the inverse of a diagonal matrix whose j -th element is r_{ij} , and C_i is the (symmetric) matrix of cofactors of Σ .⁶⁸ Since $r_{ij}(z) > 0$ for all i and j , then

$$\det(\Sigma) \neq 0$$

is a sufficient condition to have invertibility of $\tilde{\Sigma}_i$. This is Assumption 1 in the main text.⁶⁹ Replacing equations (37) and (36) into (35), the optimal $n_{ij}(z)$ is:

⁶⁸The cofactor is defined as $C_{kj} \equiv (-1)^{k+j} M_{kj}$, where M_{kj} is the (k, j) minor of Σ . The minor of a matrix is the determinant of the sub-matrix formed by deleting the k -th row and j -th column.

⁶⁹Since Σ is a covariance matrix, its determinant is always non-negative, but?? rules out the possibility that all the correlations are [1].

$$n_{ij}(z) = \frac{\sum_k \frac{C_{jk}}{r_{ik}(z)} \left(r_{ik}(z) \bar{\alpha}_k - w_i f_k \tilde{L}_k - \mu_{ik}(z) + \lambda_{ik}(z) \right)}{br_{ij}(z)},$$

where C_{jk} is the j, k cofactor of Σ , rescaled by $\det(\Sigma)$. I can write the above as:

$$n_{ij}(z) = \frac{\psi_j}{br_{ij}(z)} - \frac{\sum_k \frac{C_{jk}}{r_{ik}(z)} w_i f_k \tilde{L}_k}{br_{ij}(z)} + \frac{\sum_k \frac{C_{jk}}{r_{ik}(z)} (\lambda_{ik}(z) - \mu_{ik}(z))}{br_{ij}(z)},$$

where $\psi_j \equiv \sum_k C_{jk} \bar{\alpha}_k$. Finally, the solution above is a global maximum if i) the constraints are quasi convex and ii) the objective function is concave. The constraints are obviously quasi convex since their are linear. The Hessian matrix of the objective function is:

$$H(z) = \begin{bmatrix} \frac{\partial^2 U}{\partial^2 n_{ij}} & & \frac{\partial^2 U}{\partial n_{ij} \partial n_{iN}} \\ \cdot & \cdot & \\ \cdot & & \cdot \\ \frac{\partial^2 U}{\partial n_{iN} \partial n_{ij}} & & \frac{\partial^2 U}{\partial^2 n_{iN}} \end{bmatrix},$$

where, for all pairs j, k :

$$\frac{\partial^2 U}{\partial n_{ij} \partial n_{ik}} = \frac{\partial^2 U}{\partial n_{ik} \partial n_{ij}} = -b \delta_{ij} \delta_{ik} Cov(\alpha_j, \alpha_k) < 0$$

Given that $\frac{\partial^2 U}{\partial^2 n_{ij}} < 0$, the Hessian is negative semi-definite if and only if its determinant is positive. It is easy to see that the determinant of the Hessian can be written as:

$$\det(H) = \prod_{j=1}^N b \delta_{ij}(z)^2 \det(\Sigma),$$

which is always positive if

$$\det(\Sigma) > 0.$$

Therefore the function is concave and the solution is a global maximum, given the price index P , income Y and wage w . ■

Proof of corollary 1. The derivative of n with respect to z is:

$$\frac{\partial n_{ij}(z)}{\partial z} = \frac{\partial}{\partial z} \sum_k \frac{C_{jk}}{\det(\Sigma)} \left(z^{1-\sigma} \frac{\bar{\alpha}_k}{br_{ij}} - z^{2(1-\sigma)} \frac{w_i f_k \tilde{L}_k}{br_{ij} r_{ik}} - z^{2(1-\sigma)} \frac{\mu_{ik}(z)}{br_{ij} r_{ik}} + z^{2(1-\sigma)} \frac{\lambda_{ik}(z)}{br_{ij} r_{ik}} \right) =$$

where for simplicity r_{ij} is $r_{ij}(z)$ without z . Then:

$$\begin{aligned} \frac{\partial n_{ij}(z)}{\partial z} &= \frac{\partial}{\partial z} \sum_k \frac{C_{jk}}{\det(\Sigma)} \left(z^{1-\sigma} \frac{\bar{\alpha}_k}{\kappa_1 r_{ij}} - z^{2(1-\sigma)} \left(\frac{w_i f_k \tilde{L}_k}{\kappa_1 r_{ij} r_{ik}} + \frac{\mu_{ik}(z)}{\kappa_1 r_{ij} r_{ik}} - \frac{\lambda_{ik}(z)}{\kappa_1 r_{ij} r_{ik}} \right) \right) = \\ &= \sum_k \frac{C_{jk}}{\det(\Sigma)} \left((1-\sigma) z^{-\sigma} \frac{\bar{\alpha}_k}{r_{ij}} + (2(1-\sigma)) z^{\kappa_2+1-2\sigma} \left(\frac{w_i f_k \tilde{L}_k}{r_{ij} r_{ik}} + \frac{\mu_{ik}(z)}{r_{ij} r_{ik}} - \frac{\lambda_{ik}(z)}{r_{ij} r_{ik}} \right) \right) \\ &\quad + \sum_k \frac{C_{jk}}{\det(\Sigma)} \frac{z^{2(1-\sigma)}}{r_{ij} r_{ik}} \left(-\frac{\partial \mu_{ik}(z)}{\partial z} + \frac{\partial \lambda_{ik}(z)}{\partial z} \right) \end{aligned}$$

We can see that the derivative cannot be always positive, and thus n_{ij} is not necessarily monotonically increasing in z . This implies that there is no hierarchical structure of the exporting decision. ■

I now solve the problem of the firm when the risk aversion is zero. In this case the maximization problem of firm z is:

$$\max_{\{n_{ij}\}} \sum_j \bar{\alpha}_j n_{ij}(z) r_{ij}(z) - \sum_j w_i n_{ij}(z) f_j \tilde{L}_j / P_i$$

$$\text{s. to } 1 \geq n_{ij}(z) \geq 0$$

It is evident how the optimal solution is to choose $n_{ij}(z) = 1$ whenever $\bar{\alpha}_j r_{ij}(z) > w_i f_j \tilde{L}_j / P_i$, and $n_{ij}(z) = 0$ whenever $\bar{\alpha}_j r_{ij}(z) \leq w_i f_j \tilde{L}_j / P_i$. Therefore I can find the entry cutoff $\bar{z}_{ij}$ such that $n_{ij}(z) = 0$ if $z \leq \bar{z}_{ij}$:

$$(\bar{z}_{ij})^{\sigma-1} = \frac{w_i f_j \tilde{L}_j P_j^{1-\sigma} \sigma}{\left(\frac{\sigma}{\sigma-1} \tau_{ij} w_i \right)^{1-\sigma} Y_j}$$

Welfare gains under risk neutrality. With risk neutrality, the entry cutoff is given by equation (18). Then I use the equation for trade shares and the fact that the shocks are i.i.d.

across varieties to write:

$$\lambda_{ij} = \frac{J_i \int_{\bar{z}_{ij}}^{\infty} \bar{\alpha}_j p_{ij}(z) q_{ij}(z) g_i(z) dz}{w_j L_j} = \frac{J_i \int_{\bar{z}_{ij}}^{\infty} \bar{\alpha}_j p_{ij}(z)^{1-\sigma} g_i(z) dz}{P_j^{1-\sigma}}$$

Inverting it:

$$\frac{J_i \gamma (\tau_{ij} w_i)^{1-\sigma} (\bar{z}_{ij})^{\sigma-\theta-1}}{\lambda_{ij}} = P_j^{1-\sigma}.$$

Substituting for the cutoff, and using the fact that profits are a constant share of total income, I can write the real wage as a function of trade shares:

$$\left(\frac{w_j}{P_j} \right) = \vartheta \lambda_{jj}^{-\frac{1}{\theta}}.$$

It is easy to verify, from the expenditure minimization problem, that the percentage change in the compensating variation is simply:

$$d \ln W_j = -d \ln P_j$$

and therefore gains from trade are:

$$d \ln W_j = -\frac{1}{\theta} d \ln \lambda_{jj}$$

where $-\theta$ turns out to be also the trade elasticity.

Algorithm used to solve for the equilibrium. Given parameters, I solve for the equilibrium using the following algorithm:

- 1) I set up a grid of 50,000 productivities (or firms) that range from $\underline{z} = 1$ to a sufficiently high number.
- 2) For each z , I numerically solve the firm's maximization problem in equation (6), subject to the constraint (7). Since this is a simple quadratic problem with bounds, it can be quickly solved with a standard software (in Matlab, for example, using the function quadprog.m).
- 3) For each z , I draw a vector of demand shocks $\alpha(z) \equiv \alpha_1(z), \dots, \alpha_N(z)$ from a log-normal distribution with vector of means $\bar{\alpha}$ and covariance matrix Σ .⁷⁰

⁷⁰I also simulate the model using a normal distribution truncated at zero, and results are the same. By the Law of Large Number, what matters for the trade equilibrium are the moments of the demand shocks, not their realization. Note that also Eaton et al. (2011) assume the demand shocks are drawn from a log-normal

- 4) I plug the solution for $n_{ij}(z)$, as well as the demand shocks $\alpha_j(z)$, into equations (21), (23) and (24), which I numerically integrate over the grid of productivities.
- 5) I solve the resulting non-linear system of $3N$ equations with standard solution methods.

7.3 Robustness

7.3.1 Alternative calibration

To test the robustness of my findings, I experiment alternative moments to calibrate the covariance matrix of the shocks Σ and the technological parameter θ .

- Covariance matrix Σ . Instead of using the cross-sectional covariance $Cov(x_{Pj}, x_{Ps})$, as in equation (30), I compute the following statistics, for all $j \neq s$:

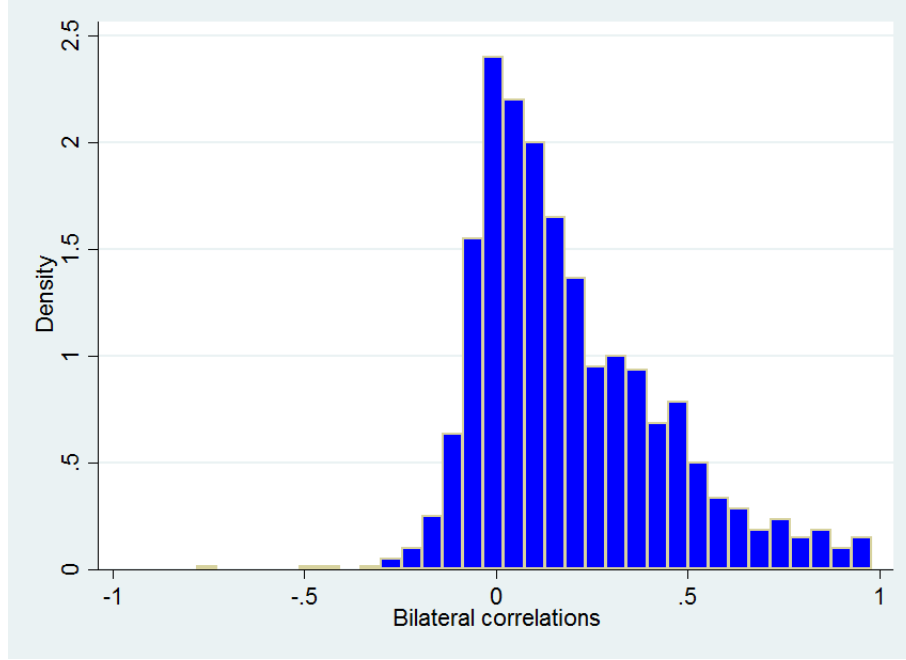
$$Var\left(\frac{x_{Pj}(z)}{x_{Ps}(z)}\right) = \int_{\underline{z}}^{\infty} \left(\frac{x_{Pj}(z)}{x_{Ps}(z)}\right)^2 \theta z^{-\theta-1} dz - \left(\int_{\underline{z}}^{\infty} \frac{x_{Pj}(z)}{x_{Ps}(z)} \theta z^{-\theta-1} dz\right)^2$$

where $x_{Pj}(z)$ are the sales of firm z to country j , given by equation (19). The intuition behind this statistics is simple. The ratio $\frac{x_{Pj}(z)}{x_{Ps}(z)}$ says how much firm z sells more to market j relative to market s . If demand in j is positively correlated with demand in s , then sales in j tend to co-move with sales in s , implying a relatively stable $\frac{x_{Pj}(z)}{x_{Ps}(z)}$ and thus a low variance.⁷¹ Figure 15 shows the calibrated correlation matrix, keeping all the other parameters to their baseline values.

distribution, with the difference that in their setting firms know the realization of the shocks when entering a market.

⁷¹In the extreme case of perfect correlation, then $Var\left(\frac{x_{Pj}(z)}{x_{Ps}(z)}\right)$ would tend to zero with a sufficiently high number of firms.

Figure 15: Distribution of demand correlations, alternative calibration



Notes: The figure shows the distribution of the bilateral correlations of demand shocks, calibrated using alternative moments.

- Technological parameter θ . Following Eaton et al. (2011), I calibrate θ by matching the number of firms that fall into sets of exhaustive and mutually exclusive bins, where the number of firms in each bin is counted in the data and is simulated from the model. The moments I match are:

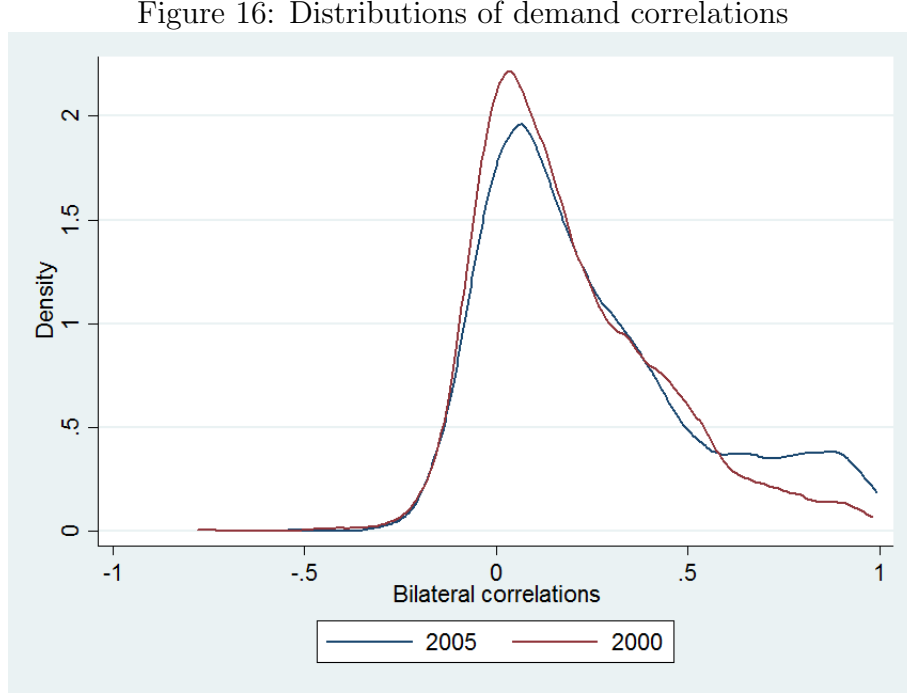
a) The proportion $\hat{m}^k(1; \Theta)$ of simulated exporters selling to each possible combination k of the 7 most popular export destinations. There are 2^7 possible combinations. The corresponding moments from the actual data are simply the proportion $m^k(1)$ of exporters selling to combination k . Stacking these proportions gives us two vectors $\hat{m}(1; \Theta)$ and $m(1)$, each with 128 elements.

b) For firms selling in each of the export destinations j , in the data I compute the q -th percentile sales s_j^q in that market, for $q = 50, 75, 95$. Using these s_j^q , I assign firms that sell in j into four mutually exclusive and exhaustive bins determined by these three sales levels. I compute the proportions $\hat{m}_j(2; \Theta)$ of artificial firms falling into each bin, analogous to the actual proportion $m_j(2)$. Stacking these proportions gives two vectors $\hat{m}(3; \Theta)$ and $m(3)$, each with $4N$ elements (subject to N adding-up constraints).

Keeping all the other parameters to their baseline values, the calibrated θ equals 6.6, close to its baseline estimate.

7.3.2 Stability of correlation matrix

Figure 16 plots the distributions of the correlations calibrated using trade flows data for 2000 and 2005. We can see that there are not substantial differences in the distributions, and thus the correlation matrix is fairly stable over time.



Notes: The figure shows the distribution of the cross-country correlations of demand shocks, calibrated using trade flows data for 2000 and 2005.

7.3.3 Heterogeneous risk aversion

In this section I calibrate the risk aversion relaxing the assumption that the risk aversion is the same across firms. In particular, I assume that b is drawn, for each firm, from a log-normal distribution with CDF $\Phi(\mu_b, \sigma_b)$, as in Herranz et al. (2015). Therefore the parameters I need to calibrate are the mean μ_b and the standard deviation σ_b . I first compute, for each Portuguese firm z in the dataset, the ratio between the mean and the variance of total sales:

$$\hat{\varrho}_P(z) = \frac{E[x_P(z)]}{Var(x_P(z))}$$

using data on trade flows from 1995 to 2005. In the model this ratio is given by:

$$\varrho_P(z) = \frac{E[x_P(z)]}{Var(x_P(z))} = b(z) \frac{\sum_j \bar{\alpha}_j \left(\frac{\sigma}{\sigma-1} \frac{\tau_{Pj} w_P}{z} \right)^{1-\sigma} \frac{Y_j}{P_j^{1-\sigma}} n_{Pj}(z)}{\sum_j \sum_s Cov \left(\left(\frac{\sigma}{\sigma-1} \frac{\tau_{Pj} w_P}{z} \right)^{1-\sigma} \frac{Y_j}{P_j^{1-\sigma}} n_{Pj}(z), \left(\frac{\sigma}{\sigma-1} \frac{\tau_{Ps} w_P}{z} \right)^{1-\sigma} \frac{Y_s}{P_s^{1-\sigma}} n_{Ps}(z) \right)}$$

For a given Θ , I find μ_b and σ_b that match the cumulative distribution function of $\varrho_P(z, b(z))$ in the model with its empirical CDF. I found a mean risk aversion of 1.2 and a standard deviation of 0.9, in line with the baseline estimates used in the counterfactuals.