Inclusive versus Exclusive Markets: Search Frictions and Competing Mechanisms*

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January 14, 2016

Abstract

In a market in which sellers compete for heterogeneous buyers by posting mechanisms, we analyze how the properties of the meeting technology affect the allocation of buyers to sellers. We show that *exclusive* markets (i.e. a separate submarket for each type of buyer) are the efficient outcome if and only if meetings are bilateral. In contrast, an *inclusive* market (i.e. a single market in which all buyer types pool) is optimal if and only if the meeting technology satisfies a novel condition, which we call "joint concavity." Both outcomes can be decentralized by sellers posting auctions combined with a fee that is paid by (or to) all buyers with whom the seller meets. Finally, we compare joint concavity to two other properties of meeting technologies, invariance and non-rivalry, and explain the differences.

JEL classification: C78, D44, D83.

Keywords: search frictions, matching function, meeting technology, competing mechanisms, heterogeneity.

*The idea for this paper grew out of earlier fruitful collaboration with James Albrecht, Ben Lester, Ludo Visschers and Susan Vroman. We thank Philipp Kircher, Guido Menzio, Michael Peters, Gábor Virág, and various seminar and conference participants for valuable comments. Ronald Wolthoff gratefully acknowledges financial support from the Connaught Fund at the University of Toronto.

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1 Introduction

One of the fundamental questions in the economic literature concerns the choice of a trading mechanism by a seller who wishes to sell a good. A large literature in mechanism design analyzes this question in the context of a monopolistic seller and often finds that auctions dominate prices. Recent work by Eeckhout and Kircher (2010) however points out that in a market setting in which sellers compete for buyers with private valuations by posting mechanisms, the process that governs meetings between buyers and sellers—i.e., the *meeting technology*—is crucially important for the choice of mechanism. If buyers randomly select a seller among the ones that maximize their expected payoff and sellers are unconstrained in the number of buyers that they can meet (urn-ball meetings), then auctions are indeed useful instruments to identify the buyer with the highest valuation. The efficient equilibrium in this case consists of a single market in which all sellers post auctions and all buyer types pool, as this maximally spreads high-type buyers across sellers. However, if sellers can only meet one buyer at a time (bilateral meetings), low-type buyers may crowd out high-type buyers. In that case, sellers prefer to post prices, which induces perfect separation of buyers into homogeneous submarkets.

Although the assumption of either bilateral or urn-ball meetings is nearly universal in the search literature[1], neither technology is necessarily an adequate description of real-life markets. In many cases, it might be more realistic to consider a technology which allows a seller to meet and learn the type of multiple but not all buyers who are interested in matching with him.[2] This raises the question how robust the above outcomes are. In this paper, we answer this question by deriving necessary and sufficient conditions on the meeting technology under which it is optimal to have (i) a separate market for each type of buyer (*exclusive* markets), or (ii) a single market in which all buyer types pool (*inclusive* market), for all type distributions.[3]

To do so, we analyze an environment based on Eeckhout and Kircher (2010), in which a continuum of buyers and sellers try to trade subject to the frictions generated by an arbitrary meeting technology. Unlike Eeckhout and Kircher (2010), who restrict attention to two types of buyers, we allow for arbitrary distributions of valuations. After describing this environment in detail in section 2, we start our analysis in section 3 by considering the trade-off of a social planner between the desire to spread high-type buyers as much as possible and the risk of them being crowded out by low-type buyers.

[1]Bilateral meetings can be found in e.g. Moen (1997), Guerrieri et al. (2010), and Menzio and Shi (2011). Urn-ball is used in e.g. Peters (1997), Burdett et al. (2001), Shimer (2005), and Albrecht et al. (2014).

[2]See Fraja and Sákovics (2001), Lester and Wolthoff (2014), and Wolthoff (2015) for models of the labor market in which screening costs prevent firms from learning the type of all their applicants.

[3]Eeckhout and Kircher (2010) move beyond bilateral and urn-ball by considering general meeting technologies, but stop short of characterizing necessary and sufficient conditions for pooling or separating equilibria. We discuss the connection with their work in detail in section 4. Lester et al. (2015a) and Cai (2015) also consider general meeting technologies, but assume homogeneous buyers and random search, respectively, and are therefore silent on the optimality of inclusive or exclusive markets.

Our first result concerns the optimality of exclusive markets. We find that this outcome is not very robust: bilateral meetings are not only sufficient, but also necessary. That is, if one moves away from bilateral meetings by allowing a seller to meet multiple buyers (potentially with arbitrary small probability), then there exist distributions of buyer valuations for which perfect separation is no longer efficient. Intuitively, full separation does not exploit the efficiency gains that arise from sellers ranking multiple buyers: with homogenous submarkets, any meetings beyond the first are irrelevant since they always present the seller with a clone of the buyer that he has already met.

Although the necessity of bilateral meetings for exclusive markets is a new result in the literature, it is perhaps not very surprising. Most of our attention therefore goes out to the optimality of an inclusive market. We show that this outcome is more robust. To be precise, an inclusive market is the efficient outcome if and only if the meeting technology satisfies a novel condition which we call "joint concavity." Loosely speaking, this condition guarantees that social surplus can be increased by merging any two submarkets, irrespective of their composition. Joint concavity is satisfied by the urn-ball meeting technology, which explains why pooling is the efficient outcome in e.g. Peters and Severinov (1997), Shi (2006) and Albrecht et al. (2014), but we also describe a number of other meeting technologies that exhibit this property.

In the second half of section 3, we discuss how both the inclusive and the exclusive outcome can be decentralized by each seller posting a second-price auction, combined with a meeting fee to be paid by (or to) each buyer meeting him.[4] Intuitively, in a large market, sellers take buyers' equilibrium payoffs as given, making sellers the residual claimant on any extra surplus that they create and providing them with an incentive to post efficient mechanisms. Auctions guarantee that the good is allocated efficiently, while the meeting fees price any positive or negative externalities in the meeting process, providing all agents with a payoff equal to their social contribution.

In a companion paper (Cai et al., 2015), we derive an alternative representation of the meeting technology based on a transformation of the probability generating function. We show that if (i) the meeting technology has constant returns to scale and (ii) buyers are identical in terms of how they meet with sellers and how they affect meetings between sellers and other buyers, that the queues in a submarket only depend on the buyer seller ratio and the probability that a seller meets at least one buyer from some arbitrary subset. In (Cai et al., 2015) we also show existence of an equilibrium for meeting technologies that yield neither perfect separation nor perfect pooling and we derive conditions for which this equilibrium is unique and socially efficient. In the present paper we use this new representation of the meeting function extensively. First, joint concavity is defined on this alternative representation. Second, we use it to unify the existing literature on meeting technologies in section 4. In particular, we discuss how joint

[4]As we explain in more detail below, this mechanism reduces to posted prices when meetings are bilateral.

concavity relates to two other properties of meeting technologies described in the literature: (i) invariance as introduced by Lester et al. (2015a), and (ii) non-rivalry as introduced by Eeckhout and Kircher (2010). We show that invariance is a sufficient (but not a necessary) condition for joint concavity, while non-rivalry is a necessary (but not a sufficient) condition, and we explain why this is the case. Finally, the appendix contains all proofs and a number of additional results.

2 Environment

Agents and Preferences. A static economy is populated by a measure 1 of sellers, indexed by $j \in [0, 1]$, and a measure $\Lambda > 0$ of buyers. Both types of agents are risk-neutral. Each seller possesses a single unit of an indivisible good, for which each buyer has unit demand.[5] All sellers have the same valuation for their good, which we normalize to zero. A buyer's valuation is an independent draw from a distribution $F(x)$ with $0 \leq x \leq 1$.[6] We impose no additional structure on $F(x)$, although we will sometimes pretend that buyers have either a low valuation x_1 or a high valuation x_2 when describing the intuition behind our results.[7] Buyers observe their valuation before making any decisions. An agent's payoff is the sum of (i) his monetary transfers and (ii) his valuation if he possesses the good at the end of the period (and zero otherwise).

Search. In order to attract buyers, each seller posts and commits to a direct mechanism.[8] A direct mechanism specifies an extensive form game that determines for each buyer i a probability of trade and an expected payment as a function of: (i) the total number n of buyers that meet with the seller; (ii) the valuation x_i that buyer i reports; and (iii) the valuations x_{-i} reported by the $n - 1$ other buyers.[9]

All identical mechanisms are treated symmetrically by buyers and are therefore said to form a *submarket*. After observing all submarkets, each buyer chooses the one in which he wishes

[5] Although we analyze a goods market, it is straightforward to cast our model in a labor market setting in which homogeneous firms post menus of wages to attract workers who differ in their productivity, as in Shi (2006). All our results carry over to such an environment.

[6] The assumption that all buyers have a (weakly) higher valuation than the seller is standard as well as innocuous. Buyers with lower valuations would simply never trade.

[7] It is worth highlighting that none of our results are driven by the requirement that they should hold for all $F(x)$. That is, our results remain the same if we consider the *weaker* requirement that they should hold for all $F(x)$ with only two points of support.

[8] The assumption that the homogeneous side of the market posts mechanisms is in line with the work by e.g. Peters and Severinov (1997), Shi (2006), Eeckhout and Kircher (2010) and Albrecht et al. (2014). A natural alternative is to study environments in which the heterogeneous side of the market posts, as in e.g. Delacroix and Shi (2013) and Albrecht et al. (2016), which may change the nature of the equilibrium.

[9] In line with most of the literature (e.g. Peters, 1997; Eeckhout and Kircher, 2010; Lester et al., 2015a,b), we abstract from mechanisms that condition on other mechanisms present in the market. See Epstein and Peters (1999) for a detailed discussion of such mechanisms.

to attempt to match.[10] As standard in the literature (see e.g. Shimer, 2005), we capture the anonymity of the large market by assuming that: i) sellers can condition their strategies on the actions of buyers but not on their identities, and ii) identical buyers must use identical mixed strategies in equilibrium.

Meeting Technology. Conditional upon the choice of a submarket, meetings between buyers and sellers are governed by a frictional process, the *meeting technology*. To introduce this process, suppose a submarket is visited by a measure b of buyers and a measure s of sellers. Defining $\lambda = b/s$ as the *queue length* (or the inverse of the market tightness) in this submarket, we then follow Eeckhout and Kircher (2010) and Lester et al. (2015a) by specifying that a seller meets $n \in \mathbb{N}_0 = \{0, 1, 2, \ldots\}$ buyers with probability $P_n(\lambda)$.

Assumptions. We impose a number of restrictions on the meeting technology. First, it must satisfy $\sum_{n=0}^{\infty} nP_n(\lambda) \leq \lambda$, because the number of meetings cannot exceed the number of buyers in the submarket. Second, we assume that $P_n(\lambda)$ is twice-continuously differentiable. Finally, we maintain the assumption of Eeckhout and Kircher (2010) that, within each submarket, the meeting technology allocates buyers to sellers in a way that is independent of types.[11] In other words, if a measure $\mu \in [0, \lambda]$ of the buyers in the submarket are labeled "blue," then the probability for a seller to meet i blue buyers and $n - i$ other buyers equals

$$P_n(\lambda)\binom{n}{i}\left(\frac{\mu}{\lambda}\right)^i\left(1-\frac{\mu}{\lambda}\right)^{n-i}.$$

Alternative Representation. Cai et al. (2015) show that the analysis of arbitrary meeting technologies is often greatly simplified by using an alternative representation of $P_n(\lambda)$. This alternative representation is the probability $\phi(\mu, \lambda)$ that a seller with a queue μ of blue buyers and a queue $\lambda - \mu$ of other buyers meets at least one blue buyer. We follow this approach here. Given the assumption regarding type-independent allocation of buyers, $\phi(\mu, \lambda)$ equals

$$\phi(\mu, \lambda) = 1 - \sum_{n=0}^{\infty} P_n(\lambda)\left(1-\frac{\mu}{\lambda}\right)^n. \tag{1}$$

To simplify notation, we will often omit the arguments of ϕ and use subscripts to indicate its partial derivatives.[12]

[10] The assumption that a buyer can meet only one seller per period is standard in the directed search literature. See Albrecht et al. (2006), Galenianos and Kircher (2009), Kircher (2009), Gautier and Holzner (2014) and Wolthoff (2015) for papers that relax this assumption.

[11] Of course, the equilibrium (or planner's) allocation of buyers to *submarkets* can depend on types.

[12] Cai et al. (2015) establish that the relation between $\phi(\mu, \lambda)$ and $P_n(\lambda)$ is one-to-one using the probability-generating function $m(z, \lambda) \equiv \sum_{n=0}^{\infty} P_n(\lambda) z^n = 1 - \phi(\lambda(1-z), \lambda)$. In particular, it follows from the properties

Examples of Meeting Technologies. For future reference, it will be useful to formally define a few examples of meeting technologies that satisfy all our assumptions.

1. *Urn-Ball.* First explored by Butters (1977) and Hall (1977), this technology specifies that—within a submarket—each buyer is randomly allocated to one of the sellers. As a result, the number of buyers that meet a particular seller follows a Poisson distribution with mean equal to the queue length λ. That is, $P_n(\lambda) = e^{-\lambda}\frac{\lambda^n}{n!}$, which yields $\phi(\mu,\lambda) = 1 - e^{-\mu}$.[13]

2. *Bilateral.* With this technology, each seller meets at most one buyer, i.e. $P_0(\lambda)+P_1(\lambda) = 1$ or $\phi(\mu,\lambda) = P_1(\lambda)\frac{\mu}{\lambda}$, with $P_0(\lambda)$ strictly convex. A potential micro-foundation consists of randomly pairing agents and keeping only pairs that consist of one buyer and one seller, yielding $P_1(\lambda) = \frac{\lambda}{1+\lambda}$.[14]

3. *Pairwise Urn-Ball.* This technology, described by Lester et al. (2015a), is a variation on the urn-ball technology. Buyers first form pairs, after which each pair is randomly assigned to a seller in the submarket. That is, $P_n(\lambda) = 0$ for $n \in \{1,3,5,\ldots\}$ and $P_n(\lambda) = e^{-\lambda/2}\frac{(\lambda/2)^{n/2}}{(n/2)!}$ for $n \in \{0,2,4,\ldots\}$, which implies $\phi(\mu,\lambda) = 1 - e^{-\mu\left(1-\frac{1}{2}\frac{\mu}{\lambda}\right)}$.

4. *Multi-Platform.* This technology consists of two platforms or rounds. In the first round, all b buyers and a fraction $0 < \alpha < 1$ of the s sellers in a submarket attempt to meet according to the random-pairing bilateral technology described above. The $\frac{b}{b+\alpha s}b = \frac{\lambda}{\lambda+\alpha}b$ buyers who fail to meet a seller then participate in the second round, in which they meet the remaining $(1-\alpha)s$ sellers according to an urn-ball process. That is,

$$P_n(\lambda) = \begin{cases} \alpha\frac{\alpha}{\lambda+\alpha} + (1-\alpha)e^{-\xi} & \text{if } n = 0 \\ \alpha\frac{\lambda}{\lambda+\alpha} + (1-\alpha)\xi e^{-\xi} & \text{if } n = 1 \\ (1-\alpha)\frac{\xi^n e^{-\xi}}{n!} & \text{if } n \in \{2,3,\ldots\}, \end{cases}$$

   where $\xi = \frac{\lambda^2}{(1-\alpha)(\lambda+\alpha)}$ is the queue length in the second round. This yields $\phi(\mu,\lambda) = \alpha\frac{\mu}{\lambda+\alpha} + (1-\alpha)\left(1 - e^{-\frac{\lambda\mu}{(1-\alpha)(\lambda+\alpha)}}\right)$.[15]

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of probability-generating functions that $P_n(\lambda) = \frac{1}{n!}\frac{\partial^n}{\partial z^n}m(z,\lambda)\big|_{z=0} = \frac{(-\lambda)^n}{n!}\frac{\partial^n}{\partial\mu^n}(1-\phi(\mu,\lambda))\big|_{\mu=\lambda}$.

[13] To keep the exposition concise, we omit the (straightforward) derivation of $\phi(\mu,\lambda)$ for each example.

[14] This micro-foundation can be found in the money search literature (see e.g. Kiyotaki and Wright, 1993). Some papers in the labor search literature provide an alternative, consisting of an urn-ball process augmented with the constraint that each seller can only contact one random buyer among the ones that wish to meet him, such that $P_1(\lambda) = 1 - e^{-\lambda}$ (see e.g. Albrecht et al., 2006; Galenianos and Kircher, 2009; Gautier and Wolthoff, 2009; Gautier et al., 2015).

[15] While this technology may seem more involved than the other examples, the two-round structure actually resembles the meeting process in various real-life markets: (i) buyers who cannot find a product at the local

3 Planner's Problem and Market Equilibrium

We start by analyzing the problem of a social planner whose objective is to maximize social surplus, while being subject to the frictions generated by the meeting technology. To keep the exposition as simple as possible, we initially assume that the planner knows buyers' valuations, allowing him to provide different types of buyers with different instructions. After deriving the necessary and sufficient conditions for perfect separation and perfect pooling, we then establish that a planner who does not know buyers' valuations can implement the same solution. We do so by showing that this solution can be decentralized as a directed search equilibrium.

3.1 Planner's Problem

The planner's problem consists of two parts. First, the planner has to allocate buyers and sellers to submarkets, creating a queue length and a distribution of buyer types at each seller. Second, the planner has to specify how trade will take place after buyers arrive at sellers. We solve these stages in reverse order.

Trading Rule. Once a number of buyers $n \in \mathbb{N}_1 \equiv \{1, 2, 3, \ldots\}$ has arrived at a seller, surplus is clearly maximized by allocating the good to the buyer with the highest valuation. The following lemma establishes the expected surplus generated by this trading rule.

Lemma 1. *The surplus created by a seller with a queue λ of buyers whose types are distributed according to the distribution $G(x)$ equals*

$$S(\lambda, G) = \int_0^1 \phi\left(\lambda\left(1 - G(x)\right), \lambda\right) dx.$$

Allocation of Buyers. Now consider the allocation of buyers to sellers. For each seller $j \in [0, 1]$, the planner chooses—with a slight abuse of notation—a queue length $\lambda(j)$ and a distribution of buyer types $G(j, x)$ to maximize total surplus $\int_0^1 S\left(\lambda(j), G(j, x)\right) dj$. Of course, the planner cannot allocate more buyers of a certain type than are available. Formally, $\int_0^1 \lambda(j)\nu(j, B)\, dj \leq \Lambda\nu_F(B)$ for any Borel-measurable set B, where ν_F is the measure associated with F and $\nu(j, \cdot)$ is the measure associated with $G(j, \cdot)$.

Exclusive Markets. We first establish that bilateral meetings are a necessary and sufficient condition for the optimality of exclusive markets.

bazaar may subsequently submit a bid at an online auction site; (ii) workers who have trouble finding a job are often put in touch with firms by a public employment agency; and (iii) singles who fail to meet someone in a bar may subscribe to a dating website. Admittedly, the analogy is not perfect because terms of trade may differ across platforms in reality, which is ruled out here by the definition of a submarket.

Proposition 1. *Bilateral meetings are a necessary and sufficient condition for the planner to create a separate submarket for each type of buyer under any type distribution $F(x)$.*

Sufficiency of bilateral meetings for full separation is a well-known result in the literature: a separate submarket for each active buyer type avoids the high degree of crowding-out that arises if high-type and low-type buyers visit the same submarket and sellers meet one of both at random.[16]

Necessity is however—to the best of our knowledge—a new result. To understand the intuition, suppose that a seller can meet two or more buyers with positive probability. With full separation, any meetings beyond the first are irrelevant—as a seller will always meet a clone of the first buyer—and the gain in surplus relative to a bilateral technology is zero. Letting one high-type and one low-type buyer swap submarket, however, provides a way to increase surplus. After all, there is a positive probability that both these buyers meet sellers who meet other buyers as well. In that case, the buyers' joint contribution to surplus was 0 before the swap, but $x_2 - x_1$ after the swap (generated by the high-type buyer; the low-type buyer still contributes 0). Of course, this argument is complete only if a buyer's probability to meet a seller is the same in both submarkets, or otherwise the change in surplus associated with changing the buyers' meeting probabilities has to be taken into account. Necessity therefore follows from the case in which x_1 and x_2 are arbitrarily close. In contrast, if types are discretely different, a meeting technology may give rise to perfect separation even though it is not strictly bilateral (see Eeckhout and Kircher, 2010, for a detailed discussion).

Inclusive Market. To state our main result regarding the optimality of an inclusive market, we define a novel property of meeting technologies, which we call "joint concavity."

Definition 1. *A meeting technology exhibits* joint concavity *if and only if $\phi(\mu, \lambda)$ is concave in (μ, λ), i.e.*

$$\phi_{\mu\mu}\phi_{\lambda\lambda} \geq \phi_{\mu\lambda}^2, \tag{2}$$

for all $0 \leq \mu \leq \lambda < \infty$.[17]

The following proposition then establishes that joint concavity is closely related to the optimality of an inclusive market.

Proposition 2. *Joint concavity is a necessary and sufficient condition for the planner to send all agents to the same market under any type distribution $F(x)$.*

[16] A planner may of course keep the lowest types out of the market altogether if a marginal seller can generate more surplus in a different submarket (see e.g. Eeckhout and Kircher, 2010).

[17] Condition (2) is necessary and sufficient for concavity, since $\phi_{\mu\mu} < 0$ for all non-bilateral technologies.

The intuition for this result is straightforward.[18] Joint concavity implies that by merging two submarkets, the probability $\phi(\lambda(1-F(x)),\lambda)$ of a seller meeting at least one buyer with a valuation higher than x will increase. Since the surplus created by a submarket is linear in ϕ, pooling all agents into one submarket is optimal. In contrast, if joint concavity fails, there exist particular distributions of buyers that support multiple submarkets as the optimal allocation, i.e. partial or complete separation is optimal. We provide a detailed discussion of joint concavity in section 4, after considering decentralization first.

3.2 Market Equilibrium

The previous subsection analyzed the problem of a planner who knows buyers' valuations. This raises the question whether his solution would be different without that knowledge. In this section, we establish that this is not case by demonstrating that the solution can be decentralized as a directed search equilibrium in which each seller posts a second-price auction combined with a meeting fee τ to be paid by each buyer meeting him.[19] As the argument resembles Cai et al. (2015), we keep the exposition brief.

Equilibrium Definition. To define equilibrium, let $R(\tau,\lambda,G)$ denote the expected payoff of a seller who posts a second-price auction with a meeting fee τ and attracts a queue of buyers (λ,G). Further, let $U(x,\tau,\lambda,G)$ denote the expected payoff of a buyer with valuation x who visits this seller. Each seller aims to maximize his revenue R, but must take into account that his queue (λ,G) is endogenously determined and depends on the fee τ that he posts. To formalize this, let $\overline{U}(x)$ denote the *market utility* of a buyer with valuation x, i.e. the highest expected payoff that this buyer can obtain in equilibrium. In line with the literature, we then impose that a seller expects to be visited by buyers of type x with positive probability only if he offers them their market utility. For many meeting technologies, the market utility assumption uniquely determines the queue. In case of multiplicity, we follow Eeckhout and Kircher (2010) and assume that sellers believe that they will attract the queue that is most favorable to them.[20] We can now define an equilibrium as follows.

Definition 2. *A directed search equilibrium is a second-price auction, a meeting fee $\tau(j)$ and a queue $(\lambda(j),G(j,x))$ for each seller $j\in[0,1]$, combined with a market utility $\overline{U}(x)$ for each type of buyer x, such that ...*

1. *each tuple $(\tau(j),\lambda(j),G(j,x))$ maximizes $R(\tau,\lambda,G)$ subject to $U(x,\tau,\lambda,G)\leq\overline{U}(x)$, with equality for each x in the support of G;*

[18] Here the advantage of using ϕ becomes apparent; the equivalent condition in terms of P_n, which we derive in the online appendix **??**, is far less simple and intuitive.

[19] The meeting fee can be negative in equilibrium, turning it into a meeting subsidy paid *to* each buyer.

[20] We discuss relaxation of this assumption in detail in Cai et al. (2015).

2. aggregating queues across sellers does not exceed the total measure of buyers of each type.

Decentralization. The following proposition establishes that the solution to the planner's problem characterized in the previous subsection can always be decentralized as a directed search equilibrium.

Proposition 3. *For any meeting technology, the planner's solution* $\{\lambda(j), G(j,x)\}$ *can be decentralized as a directed search equilibrium in which seller* $j \in [0,1]$ *posts a second-price auction and a meeting fee equal to*

$$\tau(j) = -\frac{\int_0^1 \phi_\lambda(\lambda(j)(1-G(j,x)), \lambda(j))\, dx}{\phi_\mu(0,\lambda(j))}. \tag{3}$$

The intuition for this result is similar to the intuition for efficiency in many other directed search models. Since sellers take buyers' equilibrium payoffs as given, they are the residual claimant on any surplus that they create. This provides them with an incentive to post mechanisms that decentralize the planner's solution, which requires efficiency along two margins: (i) the allocation of buyers to sellers, and (ii) the allocation of the good given a queue of buyers.

The second-price auction fulfills the second requirement and provides each buyer with a payoff equal to the extra surplus that he creates when he has the highest valuation. To satisfy the first requirement however, each buyer must receive an expected payoff exactly equal to his marginal contribution to social surplus, which includes the externality that he may impose during the meeting process (e.g. by preventing a buyer with a higher valuation from meeting the seller). Because this externality is type-independent, it can be priced by the meeting fee (3), which equals the (negative of) the spillovers that a buyer imposes on other buyers (the numerator) conditional on the event that he meets a seller (the probability of which is given by the denominator).[21]

Posted Prices versus Auctions. It is worth highlighting that the equilibrium mechanism nests two popular trading mechanisms as special cases. As we discuss in more detail in the next section, technologies that exhibit joint concavity give rise to meeting fees that are non-positive. For a subset of those technologies, the equilibrium meeting fee is exactly zero, reducing the equilibrium mechanism to a *standard auction.* In contrast, when meetings are bilateral, the

[21] The equilibrium is of course not unique. Because of risk neutrality and revenue equivalence, a seller could replace the second-price by a first-price auction, in which buyers may either know or not know the number of competing bidders (see e.g. McAfee and McMillan, 1987). Likewise, the seller could condition the meeting fee on the number of buyers that shows up. However, all equilibrium mechanisms give rise to the same *expected* payoffs. See e.g. Peters and Severinov (1997), Albrecht et al. (2014), Lester et al. (2015a) for detailed discussions of efficiency in related models.

second-price auction does not generate any revenue and the meeting fees, which are then strictly positive, act as *posted prices*.

4 Categorization of Meeting Technologies

Bilateral meetings are well understood, but joint concavity is a novel condition and warrants discussion. To better understand this condition, we compare it in this section to two other properties of meeting technologies described in the literature, *invariance* and *non-rivalry*. We show that invariance is a sufficient (but not a necessary) condition, while non-rivalry is a necessary (but not a sufficient) condition. Figure 1 summarizes this discussion.

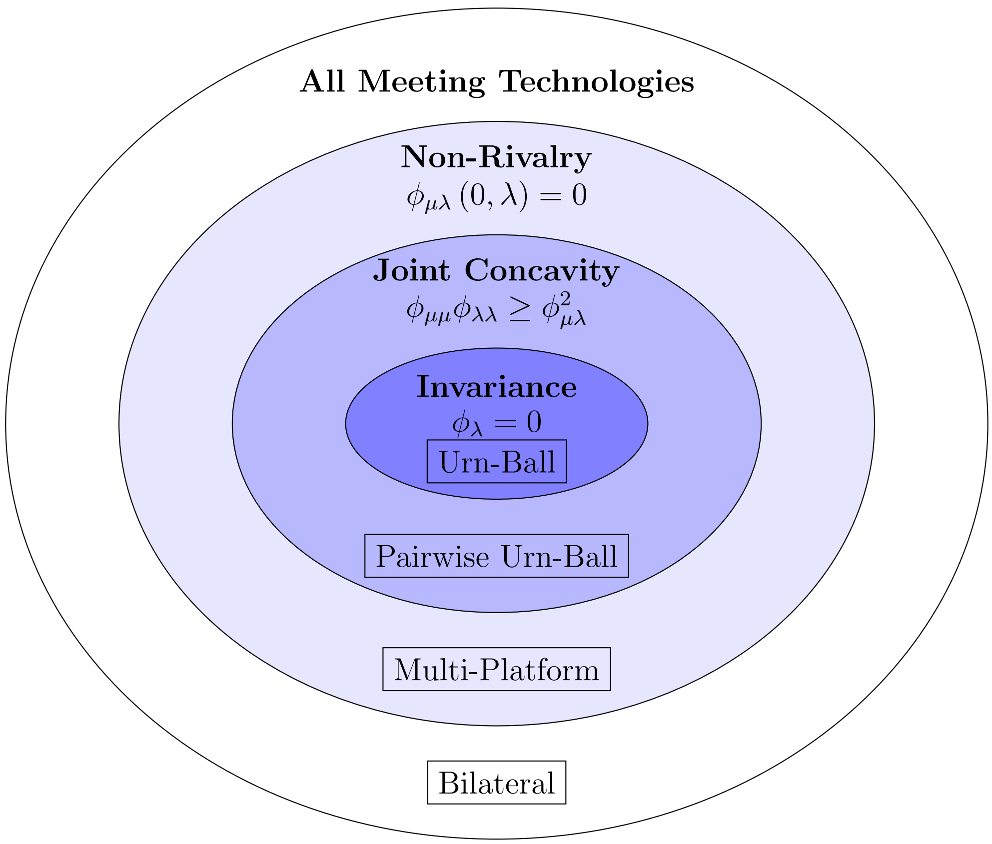

Figure 1: Classification of Meeting Technologies

Invariance. Introduced by Lester et al. (2015a), an invariant technology is one in which the queue of blue buyers μ at a seller is a sufficient statistic for the distribution of the number of meetings between blue buyers and that seller.[22] Formally,

[22]Cai (2015) shows that this property drives the efficiency of vacancy creation with wage posting if unemployed and employed workers search equally efficient in the model of Gautier et al. (2010).

$$\sum_{N=n}^{\infty} P_N(\lambda) \binom{N}{n} \left(\frac{\mu}{\lambda}\right)^n \left(1-\frac{\mu}{\lambda}\right)^{N-n} = P_n(\mu), \tag{4}$$

for all $0 \leq \mu \leq \lambda < \infty$ and $n \in \mathbb{N}_0$. Perhaps the best-known example of an invariant technology is the urn-ball technology.[23] In Lemma 2, we establish that if (4) holds for $n = 0$, then it holds for all n. That is, invariance can alternatively be defined as the condition that the probability that a seller meets *at least one* of the μ blue buyers is independent of the number of other buyers visiting the same submarket.

Lemma 2. *A meeting technology is invariant if and only if* $\phi_\lambda(\mu, \lambda) = 0$ *for all* $0 \leq \mu \leq \lambda < \infty$.

Using this lemma, it is easy to establish that invariance is a sufficient condition for joint concavity: if $\phi_\lambda = 0$, then $\phi_{\mu\lambda} = \phi_{\lambda\lambda} = 0$ and joint concavity is (weakly) satisfied. In words, invariance implies that meetings between blue buyers and sellers are unaffected by the presence of other buyers in the submarket. Joint concavity therefore reduces to concavity in the measure of blue buyers, which is always satisfied.

In contrast, invariance is not necessary for joint concavity; the condition for joint concavity can hold without ϕ_λ, $\phi_{\mu\lambda}$ or $\phi_{\lambda\lambda}$ being zero. We prove this using the pairwise urn-ball technology: although this technology exhibits joint concavity, it is not invariant, as explained by Lester et al. (2015a). Intuitively, when there are very few other buyers in the submarket, most buyer pairs consist of two blue types, making it likely that a seller will meet an even number of blue buyers. Adding additional other buyers to this submarket increases the probability that a buyer pair will consist of one blue and one other type, and that a seller will meet an odd number of blue buyers. This makes it more likely that a seller will meet at least one blue buyer, i.e. $\phi_\lambda > 0$, violating invariance.[24] The following proposition formalizes this.

Proposition 4. *Invariance implies joint concavity, but joint concavity does not imply invariance.*

Non-Rivalry. Eeckhout and Kircher (2010) define a (purely) non-rival technology as one in which a buyer's probability to meet one of the sellers is not affected by the presence of other buyers in the market. We first establish that their definition is equivalent to $\phi_{\mu\lambda}(0, \lambda) = 0$ for all $0 \leq \lambda < \infty$.[25]

Lemma 3. *A meeting technology is non-rival if and only if* $\phi_{\mu\lambda}(0, \lambda) = 0$ *for all* $0 \leq \lambda < \infty$.

[23]Recall that urn-ball implies $\phi(\mu, \lambda) = 1 - e^{-\mu}$. As discussed in Lester et al. (2015a), a second example of an invariant technology is the geometric distribution $P_n(\lambda) = \frac{\lambda^n}{(1+\lambda)^{n+1}}$, which yields $\phi(\mu, \lambda) = \frac{\mu}{1+\mu}$.

[24]This may raise the question how $\phi_\lambda \geq 0$ relates to joint concavity. We prove in the online appendix **??** that it is a necessary but not a sufficient condition.

[25]Note that non-rivalry uses the point of view of a buyer. It is therefore *not* equivalent to a seller's probability to meet at least one buyer, which is $1 - P_0(\lambda)$, being independent of λ.

To understand this expression, recall that $\phi(\mu, \lambda)$ represents the probability that a seller meets at least one blue buyer, which is clearly zero if $\mu = 0$. The partial derivative $\phi_\mu(0, \lambda)$ captures how this changes if a single buyer (or more precisely, an arbitrarily small measure of buyers) in the queue becomes blue and must therefore equal the probability that this blue buyer succeeds in meeting the seller. Since meetings are type-independent, the same expression applies to all λ buyers in the queue, irrespective of how many of them are blue. Non-rivalry then says that this meeting probability should be independent of λ.

It is easy to verify that the above examples of technologies that exhibit joint concavity, i.e. urn-ball and pairwise urn-ball, both satisfy non-rivalry. This is not a coincidence. As the following proposition establishes, all technologies that exhibit joint concavity are non-rival. However, not all non-rival technologies exhibit joint concavity.

Proposition 5. *Joint concavity implies non-rivalry, but non-rivalry does not imply joint concavity.*

To understand why non-rivalry is a necessary condition for joint concavity, consider a submarket with an arbitrarily small measure of blue buyers, i.e. $\mu \to 0$. A seller's probability of meeting at least one blue buyer in this submarket then goes to zero, $\phi(0, \lambda) = 0$. Since this is the case irrespective of the queue λ of other buyers, the derivatives of all orders of $\phi(0, \lambda)$ with respect to λ are zero as well. By (2), joint concavity then requires $\phi_{\mu\lambda}(0, \lambda) = 0$, which is exactly the condition for non-rivalry.

More generally, however, joint concavity requires that condition (2) is satisfied for any $0 \leq \mu \leq \lambda$ and not just for $\mu = 0$. Hence, non-rivalry is not sufficient. The multi-platform technology is a good example. This technology is non-rival, because every buyer meets a seller with probability 1. However, this technology clearly does not satisfy joint concavity for sufficiently large α, as it converges to a bilateral technology for $\alpha \to 1$. The proof of proposition 5 formalizes this and establishes that joint concavity is in fact violated for any $\alpha > 0$. Hence, non-rivalry does not imply joint concavity.[26]

5 Conclusion

We study an environment in which sellers compete for heterogeneous buyers by posting mechanisms. Buyers can direct their search to the mechanism that maximizes their expected payoff, but may experience frictions in meeting a particular seller. We derive necessary and sufficient

[26]Joint with proposition 2, this result contradicts proposition 5 in Eeckhout and Kircher (2010) which states that non-rivalry is a sufficient condition for a single market. The discrepancy originates in the fact that the proof of their proposition implicitly assumes invariance rather than non-rivalry when treating the trading probability for high-type buyers as independent of the queue of low-type buyers.

conditions on the technology that governs these meetings under which either exclusive markets (a separate submarket for each type of buyer) or an inclusive market (a single market in which all buyers pool) are optimal. We find that exclusive markets are the efficient equilibrium outcome if and only if meetings are bilateral, while an inclusive market arises if the meeting technology satisfies a novel property, which we call "joint concavity."

Appendix

Proof of Lemma 1. The maximum valuation at a seller who meets $n \in \mathbb{N}_1$ buyers is an order statistic, distributed according to $G^n(x)$. Taking the expectation over x and n, followed by integration by parts and using the Dominated Convergence Theorem to interchange summation and integration, yields

$$S(\lambda, G) = \sum_{n=1}^{\infty} P_n(\lambda) \int_0^1 x \, dG^n(x) = \int_0^1 \left(1 - \sum_{n=0}^{\infty} P_n(\lambda) G^n(x)\right) dx.$$

The result then follows because the rightmost integrand equals $\phi(\lambda(1 - G(x)), \lambda)$. $\square$

Proof of Proposition 1. **Part 1** (bilateral meetings imply full separation): Eeckhout and Kircher (2010) demonstrate sufficiency for two types of buyers; we extend their result to arbitrary distributions. Suppose that there exists a submarket with a measure s of sellers and a queue (λ, G). Because of lemma 1 and the fact that meetings are bilateral, the surplus created in this submarket equals

$$sS(\lambda, G) = sP_1(\lambda) \int_0^1 (1 - G(x)) \, dx. \tag{5}$$

Now suppose the planner would decompose this submarket into a separate submarket for each type of buyer x, allocating the s sellers according to a distribution $H(x)$. Let $\widehat{\lambda}(x) = \lambda \frac{dG(x)}{dH(x)}$ denote the queue length in submarket x. A seller in this submarket then creates a surplus $P_1\left(\widehat{\lambda}(x)\right) x$, such that surplus across all submarkets equals

$$s \int_0^1 P_1\left(\widehat{\lambda}(x)\right) x \, dH(x). \tag{6}$$

Clearly, if the planner chooses $H(x) = G(x)$, then $\widehat{\lambda}(x) = \lambda$ for all x and surpluses (5) and (6) are equal to each other. In that case, the marginal value of a seller in submarket x equals $(P_1(\lambda) - \lambda P_1'(\lambda)) x$. This value is increasing in x, which means that the allocation of sellers is suboptimal and surplus (6) can be increased by sending some sellers to different submarkets.[27]

Part 2 (full separation implies bilateral meetings): We prove this result by showing that if meetings are not bilateral for some $\Lambda > 0$, i.e. $P_0(\Lambda) + P_1(\Lambda) < 1$, then there exists a two-type distribution of buyers such that full separation is not optimal.[28] To do so, suppose the market is populated by a measure 1 of sellers, a measure b_1 of buyers with valuation x_1, and a measure b_2 of buyers with valuation x_2, satisfying $b_1 + b_2 = \Lambda$ and $x_2 > x_1$.

Suppose the planner fully separates the two types of buyers and optimally allocates s_i sellers to the submarket for valuation x_i, where $s_1 + s_2 = 1$. Define queue lengths $\lambda_i = \frac{b_i}{s_i}$. Clearly, if $x_1 \to x_2$, then $s_1 \to \frac{b_1}{b_1+b_2}$ and $s_2 \to \frac{b_2}{b_1+b_2}$, which implies $\lambda_1 \to \Lambda$ and $\lambda_2 \to \Lambda$.

Let now a measure ε of buyers with valuation x_1 and an equally large measure of buyers with valuation x_2 swap submarket, such that—in both submarkets—the queue lengths stay the same, but the composition of types becomes marginally more diverse. Again by lemma 1, social surplus of this new allocation equals

$$\begin{aligned}\mathcal{S}(\varepsilon) &= s_2\left[(x_2 - x_1)\,\phi\left(\frac{b_2-\varepsilon}{s_2}, \lambda_2\right) + x_1\phi(\lambda_2, \lambda_2)\right] \\ &\quad + s_1\left[(x_2 - x_1)\,\phi\left(\frac{\varepsilon}{s_1}, \lambda_1\right) + x_1\phi(\lambda_1, \lambda_1)\right].\end{aligned}$$

Clearly, $\varepsilon = 0$ corresponds to full separation. To analyze whether this is the optimal outcome, consider the change in surplus associated with a marginal increase in ε, i.e.

$$\mathcal{S}'(0) = (x_2 - x_1)\left(\phi_\mu(0, \lambda_1) - \phi_\mu(\lambda_2, \lambda_2)\right) \tag{7}$$

Note that

$$\phi_\mu(\mu, \lambda) = \frac{P_1(\lambda)}{\lambda} + \frac{1}{\lambda}\sum_{n=2}^{\infty} nP_n(\lambda)\left(1 - \frac{\mu}{\lambda}\right)^{n-1},$$

which implies that $\phi_\mu(0, \lambda) \geq \frac{P_1(\lambda)}{\lambda} + \frac{2(1-P_0(\lambda)-P_1(\lambda))}{\lambda}$ and $\phi_\mu(\lambda, \lambda) = \frac{P_1(\lambda)}{\lambda}$. Substitution into (7) yields

$$\frac{1}{x_2 - x_1}\mathcal{S}'(0) \geq \frac{P_1(\lambda_1)}{\lambda_1} + \frac{2(1 - P_0(\lambda_1) - P_1(\lambda_1))}{\lambda_1} - \frac{P_1(\lambda_2)}{\lambda_2}.$$

Let $x_1 \to x_2$, such that $\lambda_1 \to \Lambda$ and $\lambda_2 \to \Lambda$. Then

$$\frac{1}{x_2 - x_1}\mathcal{S}'(0) \to \frac{2(1 - P_0(\Lambda) - P_1(\Lambda))}{\Lambda} > 0.$$

[27] For example, if $P_1(\lambda)$ is concave, $P_1(\lambda) - \lambda P_1'(\lambda)$ is increasing in λ, and the planner can increase surplus by allocating relatively more sellers to the submarkets in which buyers have high valuations.

[28] Of course, if $P_0(\Lambda) + P_1(\Lambda) < 1$, then—by continuity—there exists a small neighborhood of Λ for which $P_0 + P_1 < 1$.

Hence, full separation is not optimal and social surplus can be increased by slightly mixing the submarkets. □

Proof of Proposition 2. **Part 1** (joint concavity implies perfect pooling): To prove this result, suppose that there are two submarkets, indexed by $i \in \{1, 2\}$, consisting of $s_i > 0$ sellers who each have a queue (λ_i, G_i). By lemma 1, total surplus across the two submarkets is equal to

$$s_1 \int_0^1 \phi\left(\lambda_1\left(1 - G_1\left(x\right)\right), \lambda_1\right)\, dx + s_2 \int_0^1 \phi\left(\lambda_2\left(1 - G_2\left(x\right)\right), \lambda_2\right)\, dx. \tag{8}$$

We show a higher surplus can be generated by merging the two submarkets, creating one market with $s_0 = s_1 + s_2$ sellers, each with a queue $\lambda_0 = \frac{s_1\lambda_1 + s_2\lambda_2}{s_1+s_2}$ of buyers whose valuations are distributed according to

$$G_0\left(x\right) = \frac{s_1\lambda_1 G_1\left(x\right) + s_2\lambda_2 G_2\left(x\right)}{s_1\lambda_1 + s_2\lambda_2}.$$

Again by lemma 1, this combined market will create a surplus $s_0 \int_0^1 \phi\left(\lambda_0\left(1 - G_0\left(x\right)\right), \lambda_0\right)\, dx$, which is larger than (8) because concavity of $\phi\left(\mu, \lambda\right)$ implies that

$$s_1\phi\left(\mu_1, \lambda_1\right) + s_2\phi\left(\mu_2, \lambda_2\right) \leq s_0\phi\left(\frac{s_1\mu_1 + s_2\mu_2}{s_1 + s_2}, \frac{s_1\lambda_1 + s_2\lambda_2}{s_1 + s_2}\right).$$

Hence, a single market is optimal for technologies that exhibit joint concavity.

Part 2 (perfect pooling implies joint concavity): We prove this result by showing that if ϕ is not concave, there exists a two-type distribution of buyers such that one market is not optimal. Note that if ϕ is not concave, then—by the definition of concavity—there exist values α, μ_1, μ_2, λ_1 and λ_2, such that

$$\alpha\phi(\mu_1, \lambda_1) + (1 - \alpha)\phi(\mu_2, \lambda_2) > \phi(\mu_0, \lambda_0), \tag{9}$$

where $\mu_0 = \alpha\mu_1 + (1 - \alpha)\mu_2$ and $\lambda_0 = \alpha\lambda_1 + (1 - \alpha)\lambda_2$.

Consider now a market in which buyers' valuations are either x_1 or x_2, with $0 < x_1 < x_2$. Set the measure of high-type buyers equal to μ_0 and the measure of low-type buyers equal to $\lambda_0 - \mu_0$, while maintaining the assumption that the measure of sellers equals 1. Then by Lemma 1, the social surplus of creating a single market is $\mathcal{S}_1 = (x_2 - x_1)\phi(\mu_0, \lambda_0) + x_1\phi(\lambda_0, \lambda_0)$.

Now, decompose the single market into two submarkets A and B, with seller measures α and $1 - \alpha$, total queue lengths λ_1 and λ_2, and high-type queue lengths μ_1 and μ_2, respectively.[29]

[29]This is possible because $\mu_0 = \alpha\mu_1 + (1 - \alpha)\mu_2$ and $\lambda_0 = \alpha\lambda_1 + (1 - \alpha)\lambda_2$.

The social surplus per seller for the two submarkets is

$$S_2^A = (x_2 - x_1)\phi(\mu_1, \lambda_1) + x_1\phi(\lambda_1, \lambda_1)$$
$$S_2^B = (x_2 - x_1)\phi(\mu_2, \lambda_2) + x_1\phi(\lambda_2, \lambda_2)$$

and total surplus across the two submarkets equals $\mathcal{S}_2 = \alpha S_2^A + (1-\alpha)S_2^B$.

In the limit $x_1 \to 0$, the two submarkets create more surplus than the single market, i.e. $\mathcal{S}_2 > \mathcal{S}_1$, if and only if

$$x_2\left(\alpha\phi(\mu_1, \lambda_1) + (1-\alpha)\phi(\mu_2, \lambda_2)\right) > x_2\phi(\mu_0, \lambda_0),$$

which holds because it is exactly equation (9). Hence, joint concavity is a necessary condition for a single market. $\square$

Proof of Proposition 3. This result follows directly from Cai et al. (2015). For completeness, we also give a short proof here. This proof consists of two parts. First, we consider a seller who can choose the length and composition of his queue directly in a competitive market ("relaxed maximization problem"). By the first welfare theorem, the equilibrium in this market is Pareto optimal, which necessarily implies that it maximizes social net output as there is only one consumption good. Subsequently, we establish that a seller who posts the proposed equilibrium mechanism to attract an endogenously determined queue of buyers ("constrained maximization problem") implements the same solution.

Part 1 (relaxed maximization problem): For a given market utility function $\overline{U}(x)$, a seller chooses the queue (λ, G) that maximizes his expected payoff, which equals the difference between surplus $S(\lambda, G)$ and the expected payoff that the seller has to offer to each of the buyers. That is,

$$\int_0^1 \phi\left(\lambda\left(1 - G(z)\right), \lambda\right)\, dz - \int_0^1 \overline{U}(z)\, d\lambda G(z).$$

Because the seller takes the market utility function as given, he is a residual claimant on any extra surplus that he creates. Hence, the seller will compare the marginal cost $\overline{U}(x)$ of attracting a buyer with valuation x to this buyer's marginal contribution to surplus $T(x)$. To calculate $T(x)$, increase the measure of buyers with values around x, formally $[x, x + \Delta x]$, by ε and denote the new market tightness and buyer value distribution as λ' and G' respectively. That is, $\lambda' = \lambda + \varepsilon$, while $\lambda'(1 - G'(z)) = \lambda(1 - G(z))$ for $z > x$ and $\lambda'(1 - G'(z)) = \lambda(1 - G(z)) + \varepsilon$

for $z < x$. By Lemma 1, the average contribution to surplus by buyers with values around x is

$$\frac{S(\lambda', G') - S(\lambda, G)}{\varepsilon} = \frac{1}{\varepsilon}\left(\int_0^x \phi(\lambda(1-G(x)) + \varepsilon, \lambda + \varepsilon) - \phi(\lambda(1-G(x)), \lambda)\right) + \frac{1}{\varepsilon}\left(\int_x^1 \phi(\lambda(1-G(x)), \lambda + \varepsilon) - \phi(\lambda(1-G(x)), \lambda)\right)$$

Let $\varepsilon \to 0$, then the above equation converges to

$$T(x) = \int_0^1 \phi_\lambda\left(\lambda\left(1 - G\left(z\right)\right), \lambda\right)\, dz + \int_0^x \phi_\mu\left(\lambda\left(1 - G\left(z\right)\right), \lambda\right)\, dz. \tag{10}$$

The solution to the relaxed maximization problem must therefore satisfy

$$\overline{U}(x) \geq T(x) \text{ for all } x \text{, with equality for all } x \in \operatorname{supp} G \tag{11}$$

Part 2 (constrained maximization problem): Consider now a seller who posts a second-price auction and a meeting fee τ, attracting a queue (λ, G). A buyer with valuation x in the support of G meets the seller together with $n-1$ other buyers with probability $\frac{nP_n(\lambda)}{\lambda}$.[30] Hence, he pays the meeting fee τ with probability $\frac{1}{\lambda}\sum_{n=1}^{\infty} nP_n\left(\lambda\right) = \phi_\mu\left(0, \lambda\right)$ and trades with probability $\frac{1}{\lambda}\sum_{n=1}^{\infty} nP_n\left(\lambda\right) G(x)^{n-1} = \phi_\mu\left(\lambda\left(1 - G\left(x\right)\right), \lambda\right)$. As a result, his expected payoff is

$$U\left(x, \tau, \lambda, G\right) = -\phi_\mu\left(0, \lambda\right)\tau + \int_0^x \phi_\mu\left(\lambda\left(1 - G\left(y\right)\right), \lambda\right)\, dy, \tag{12}$$

where the second term is the payoff from the auction, which—by standard results in auction theory—equals the integral over the trading probabilities (see e.g. Peters, 2013). A queue (λ, G) is therefore compatible with an auction with fee τ if and only if

$$\overline{U}(x) \geq U(x, \tau, \lambda, G) \text{ for all } x \text{, with equality for all } x \in \operatorname{supp} G \tag{13}$$

Clearly, if a queue (λ, G) satisfies (11), then by setting the entry fee τ in equation (12) equal to

$$\tau = -\frac{\int_0^1 \phi_\lambda\left(\lambda\left(1 - G\left(x\right)\right), \lambda\right)\, dx}{\phi_\mu\left(0, \lambda\right)},$$

it also satisfies (13). Therefore, any queue chosen by an unconstrained seller who can buy queues directly at a price $\overline{U}(x)$ is also compatible with an auction with an entry fee. □

[30] See Eeckhout and Kircher (2010) or Lester et al. (2015a).

Proof of Lemma 2. **Part 1** (invariance implies $\phi_\lambda = 0$): Evaluating the definition of invariance (4) in $n = 0$ yields

$$\sum_{N=0}^{\infty} P_N(\lambda)\left(1-\frac{\mu}{\lambda}\right)^N = P_0(\mu). \tag{14}$$

The left-hand side of this equation is $1-\phi(\mu,\lambda)$ and the right-hand side is independent of λ. Hence, $\phi_\lambda(\mu,\lambda) = 0$ for all $0 \leq \mu \leq \lambda < \infty$.

Part 2 ($\phi_\lambda = 0$ implies invariance): Note that $\phi_\lambda(\mu,\lambda) = 0$ for all $0 \leq \mu \leq \lambda < \infty$ implies that $\phi(\mu,\lambda) = \phi(\mu,\mu)$. By equation (1), $\phi(\mu,\mu) = 1 - P_0(\mu)$. Consequently, equation (14) must hold for all $0 \leq \mu \leq \lambda < \infty$. By standard results from analytic function theory (see e.g. Ahlfors, 1979, p.32), we can differentiate both sides of this equation n times with respect to μ, which yields

$$\sum_{N=n}^{\infty} \frac{N!}{(N-n)!} P_N(\lambda)\left(-\frac{1}{\lambda}\right)^n \left(1-\frac{\mu}{\lambda}\right)^{N-n} = P_0^{(n)}(\mu), \tag{15}$$

for all $0 \leq \mu \leq \lambda < \infty$. For $\mu = \lambda$, this gives $P_0^{(n)}(\mu) = \frac{n!}{(-\mu)^n} P_n(\mu)$. Substitute this into the right hand side of equation (15) and rearrange the term $\frac{n!}{(-\mu)^n}$ to the left hand side gives (4). Hence, $\phi_\lambda = 0$ implies invariance. $\square$

Proof of Proposition 4. **Part 1** (invariance implies joint concavity): This result follows immediately from lemma 2: $\phi_\lambda(\mu,\lambda) = 0$ for all $0 \leq \mu \leq \lambda < \infty$ implies that $\phi_{\lambda\lambda}(\mu,\lambda) = \phi_{\mu\lambda}(\mu,\lambda) = 0$ for all $0 \leq \mu \leq \lambda < \infty$, which in turn implies that equation (2) is satisfied.

Part 2 (joint concavity does not imply invariance): Consider the pairwise urn-ball technology, which satisfies $\phi(\mu,\lambda) = 1 - e^{-\mu\left(1-\frac{1}{2}\frac{\mu}{\lambda}\right)}$. Since $1-e^{-y}$ is an increasing, concave function, a sufficient condition for $\phi(\mu,\lambda)$ to be concave is that the map $(\mu,\lambda) \to \mu(1-\frac{1}{2}\frac{\mu}{\lambda})$ is concave.[31] The Hessian of this map is indeed negative semi-definite. However, the technology is not invariant, as

$$\phi_\lambda(\mu,\lambda) = \frac{1}{2}\frac{\mu^2}{\lambda^2} e^{-\mu\left(1-\frac{1}{2}\frac{\mu}{\lambda}\right)} > 0.$$

Hence, joint concavity does not imply invariance. $\square$

Proof of Lemma 3. As shown by Lester et al. (2015a), non-rivalry is satisfied if and only if $\frac{\partial}{\partial\lambda}\frac{1}{\lambda}\sum_{n=1}^{\infty} nP_n(\lambda) = 0$, for all $0 \leq \lambda < \infty$. The desired result then follows directly from observing that $\phi_\mu(0,\lambda) = \frac{1}{\lambda}\sum_{n=1}^{\infty} nP_n(\lambda)$. $\square$

Proof of Proposition 5. **Part 1** (joint concavity implies non-rivalry): For any meeting technology, $\phi(0,\lambda) = 0$ for all $0 \leq \lambda < \infty$, i.e. the probability of meeting a blue buyer is zero if $\mu = 0$, irrespective of the value of λ. This implies that $\frac{\partial^n}{\partial\lambda^n}\phi(0,\lambda) = 0$ for all $n \in \mathbb{N}_1$.

[31] See, for example, Theorem 5.1 in Rockafellar (1970, p.32).

Suppose now that a technology does not satisfy non-rivalry, i.e. $\phi_{\mu\lambda}(0,\lambda) \neq 0$. Then

$$\phi_{\mu\mu}(0,\lambda)\,\phi_{\lambda\lambda}(0,\lambda) - \phi_{\mu\lambda}^2(0,\lambda) < \phi_{\mu\mu}(0,\lambda)\,\phi_{\lambda\lambda}(0,\lambda) = 0,$$

i.e. ϕ does not exhibit joint concavity. Hence, joint concavity implies non-rivalry.

Part 2 (non-rivalry does not imply joint concavity): Consider the multi-platform technology. Starting from the expression for $\phi(\mu,\lambda)$ for this technology, one can derive

$$\phi_{\mu\lambda} = -\left(1 - e^{-\frac{\lambda\mu}{(1-\alpha)(\lambda+\alpha)}} - \frac{\lambda\mu}{(1-\alpha)(\lambda+\alpha)} e^{-\frac{\lambda\mu}{(1-\alpha)(\lambda+\alpha)}}\right) \frac{\alpha}{(\lambda+\alpha)^2} \leq 0,$$

which equals 0 (only) for $\mu = 0$. Hence, the multi-platform technology is non-rival.

Further, we get

$$\phi_\lambda = -\left(1 - e^{-\frac{\lambda\mu}{(1-\alpha)(\lambda+\alpha)}}\right) \frac{\mu\alpha}{(\lambda+\alpha)^2},$$

which is strictly negative for all $0 < \mu \leq \lambda < \infty$ and $\alpha > 0$. As we show in the online appendix, $\phi_\lambda \geq 0$ is a necessary condition for joint concavity. Hence, the multi-platform technology does not exhibit this property. $\square$

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