On the Mechanics of New-Keynesian Models*

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Abstract

The mechanism of a standard New-Keynesian model is laid out. A particular focus is on capital accumulation, the key ingredient in the transition from the basic framework to medium-scale DSGE models. The widely held view that monetary policy affects output and inflation in these models through a real interest rate channel is misguided. A decline in output and inflation is consistent with a decline, increase, or no change in the real rate. The expected path of Taylor rule shocks and the New-Keynesian Phillips Curve are key for inflation and output; the real rate largely reflects consumption smoothing.

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1 Introduction

The New-Keynesian model—a dynamic stochastic general equilibrium model with sticky prices—has become a workhorse in the analysis of monetary policy. It has grown in popularity at tremendous speed both in academia and at central banks around the world. From a basic framework of a price-setting problem of producers and consumption-labor choice by consumers, it has quickly grown into a model with many different frictions, adjustment costs, and other features. Whereas the basic framework abstracts from capital, the extended model—building on the real business cycle tradition—includes capital and investment as an integral part of the environment. The basic framework is typically used to illustrate optimal monetary policy, whereas the extended model—often referred to as a medium-scale DSGE model—is used for practical monetary policy and forecasting.

Unfortunately, in our view, widespread understanding of the internal mechanism of the New-Keynesian model has been lost along its fast track to popularity. The existing literature does not help. Textbooks covering the New-Keynesian model (such as Walsh, 2010; Gali, 2015) stop at the basic framework and proceed with a discussion of optimal monetary policy, while research based on the medium-scale DSGE models (represented by, e.g., Christiano, Eichenbaum, and Evans, 2005; Smets and Wouters, 2007) starts straight away with the full-blown version. Researchers outside of this field, as well as graduate students, are left on their own to connect the dots, especially when moving from the first to the second strand of the literature.[1] The purpose of this article, therefore, is to carefully lay out the internal mechanism of the New-Keynesian model, demonstrate its behavior under alternative parameterizations, and discuss the role of capital—the key ingredient in the transition from the basic framework to the medium-scale DSGE models. We do this in a step-by-step fashion, starting with the basic framework.

[1] McCandless (2008) comes closest to bridging this gap, but his treatment of New-Keynesian models is carried out in the context of a model with money and a monetary policy rule formulated as a money growth rule, while most of the New-Keynesian literature follows Woodford (2003) by abstracting from money and formulating monetary policy as a nominal interest rate (Taylor) rule. Woodford (2003), while providing a thorough analysis of many aspects of the New-Keynesian framework, discusses the role of capital only briefly.

Our focus is on monetary policy shocks. It was indeed the real effects of monetary policy shocks that motivated the original development of these models. The macroeconomic effects of such shocks stressed by the New-Keynesian literature can be summarized as follows. In response to a positive innovation in a short-term nominal interest rate, (i) output declines, (ii) inflation also declines, but less and more gradually, (iii) the nominal interest rate increases, and (iv) the real interest rate increases.[2]

In our exposition we highlight the following properties of the New-Keynesian model: (i) the widespread interpretation of the model as aggregate supply-demand representation is misleading, (ii) even in the basic model, generating all four desirable impulse-responses depends on parameterization, (iii) consumption smoothing through capital accumulation makes the desirable properties much harder to obtain, and (iv) the real effects of monetary policy in the model with capital have little to do with a popular real interest rate channel; a decline in output and inflation is consistent with a fall, increase, or no change in the ex-ante real interest rate. The expected path of Taylor rule shocks and the New-Keynesian Phillips Curve are key for inflation and output; the real rate largely reflects consumption smoothing.

Point (iii) has a flavor of the well-known finding in the asset pricing literature that desirable asset pricing properties established in economies without capital are much harder to get in economies that allow capital accumulation (e.g., Jermann, 1998). Like in the asset pricing literature, sufficiently high capital adjustment costs help mitigate the effect of capital accumulation on the equilibrium.

The description of the mechanism is based on a linearized version of the model. We express the endogenous variables as linear functions of state variables and obtain analytical solutions for these functions using the method of undetermined coefficients. We then rely on the analytical solutions as much as possible to describe the workings of the model. When insight from the analytical solutions is limited, numerical analysis is used to explore how the equilibrium functions depend on parameter values. We do not discuss optimal policy as this

[2] These properties are based on impulse-responses from structural VARs (see, e.g., Walsh, 2010, Chapter 1, for an overview). Over time, the literature has achieved great success in matching the size and timing of these responses by extending the basic model in several dimensions (Christiano et al., 2005).

is well covered in existing texts. For this reason we are not concerned with output gaps and natural real interest rates.

The paper is divided into two main sections. Section 2 deals with the basic model without capital while Section 3 demonstrates how the key properties of the model change once capital is introduced. Section 4 concludes. Secondary results and derivations are in an Appendix.

2 The basic model without capital

The exposition is based on a common per-period utility function

$$u = \log c - \frac{l^{1+\eta}}{1+\eta}, \quad \eta \geq 0,$$

and the intermediate goods aggregator, in the intermediate-final goods production structure,

$$y = \left[\int y(j)^{\varepsilon} dj \right]^{\frac{1}{\varepsilon}}, \quad \varepsilon \in (0, 1).$$

The starting point of our analysis is the system of equations describing the general equilibrium, with the New-Keynesian Phillips curve (NKPC) already in its linearized form, around zero steady-state inflation rate, a common approximation point. The derivation of this system from first principles is contained, for instance, in Walsh (2010) and Gali (2015). In the spirit of Woodford (2003) and much of the literature that followed, the economy is cashless and monetary policy is formulated as a Taylor-type rule. The general equilibrium is characterized by

$$\frac{w_t}{c_t} = l_t^{\eta}, \tag{1}$$

$$\frac{1}{c_t} = \beta E_t \left(\frac{1}{c_{t+1}} \frac{1+i_t}{1+\pi_{t+1}} \right), \tag{2}$$

$$y_t = l_t, \tag{3}$$

$$\chi_t = w_t, \tag{4}$$

$$\pi_t = -\frac{1}{\phi(\varepsilon - 1)}(\chi_t - \chi) + \beta E_t \pi_{t+1}, \tag{5}$$

$$i_t = i + \nu \pi_t + \xi_t, \tag{6}$$

$$y_t = c_t. \tag{7}$$

Here, c_t is consumption, w_t is a real wage rate, l_t is labor, i_t is a one-period nominal interest rate, π_t is the inflation rate between periods $t-1$ and t, y_t is output, χ_t is the marginal cost, and ξ_t is a mean-zero monetary policy shock. Equation (1) is the consumer's first-order condition for labor, equation (2) is the Euler equation for a one-period nominal bond, equation (3) is a production function, equation (4) gives the marginal cost, equation (5) is the NKPC (for the Rotemberg, 1982, quadratic price adjustment cost specification), equation (6) is the Taylor rule, and equation (7) is the goods market clearing condition. In the NKPC, $\phi \geq 0$ is the Rotemberg cost parameter. Further, $\beta \in (0, 1)$ is a discount factor and $\nu > 1$ is the weight on inflation in the Taylor rule. Variables without a time subscript denote steady-state values (the steady-state value of the inflation rate is equal to zero). For the points made in this paper, the exposition is cleaner when the weight on output in the Taylor rule is set equal to zero, as implicitly assumed in equation (6). The main text also deals only with a Taylor rule that responds to current inflation. The appendix derives results for forward- and backward-looking Taylor rules.[3]

Even though the linearized NKPC is derived for the Rotemberg specification, it is well-known that the same form is obtained also for the Calvo (1983) specification.[4] Namely,

$$\pi_t = \frac{(1-\theta)(1-\theta\beta)}{\theta}(\chi_t - \chi) + \beta E_t \pi_{t+1}, \tag{8}$$

[3]Nothing substantive changes, relative to the main text, under the alternative specifications of the Taylor rules. There are also no substantial differences in the results, relative to the main text, if decreasing returns to scale are assumed (i.e., $y = l^{1-\alpha}, \alpha \in (0, 1)$), instead of the linear production function.

[4]The Calvo specification leads to an aggregation bias that shows up as total factor productivity in the production function (3). This bias, however, disappears once the model is linearized. The Rotemberg specification, on the other hand, leads to a resource cost that shows up in the goods market clearing condition (7). Again, it disappears in a linearized version of the model. For these reasons, the above general equilibrium system abstracts from these two details.

where $\theta \in [0, 1]$ is the fraction of producers not adjusting prices in a given period. The mapping between Rotemberg and Calvo NKPC is thus

$$\frac{(1-\theta)(1-\theta\beta)}{\theta} = -\frac{1}{\phi(\varepsilon - 1)}.$$

The endogenous variables in the system (1)-(7) are c_t, w_t, l_t,i_t, π_t, y_t, and χ_t. The exogenous variable is the monetary policy shock ξ_t. In the model without capital, the shock is the only state variable. In a linear solution, the dynamics of the endogenous variables are thus fully governed by the exogenous process for the shock; the model parameters affect only the sign and size of the responses of the endogenous variables to the shock.[5]

Eliminating equations (3), (4), and (7) by substitutions for c_t, l_t, and χ_t, the system can be reduced to a four-equation system, which when log-linearized around a nonstochastic steady state (with $y = 1$) becomes

$$\widehat{w}_t = (1+\eta)\widehat{y}_t, \tag{9}$$

$$-\widehat{y}_t = -E_t\widehat{y}_{t+1} + \widehat{i}_t - E_t\pi_{t+1}, \tag{10}$$

$$\pi_t = -\frac{1}{\phi(\varepsilon - 1)}\widehat{w}_t + \beta E_t\pi_{t+1}, \tag{11}$$

$$\widehat{i}_t = \nu\pi_t + \xi_t. \tag{12}$$

Here, $\widehat{i}_t \equiv i_t - i$, $\widehat{y}_t \equiv (y_t - y)/y$, and $\widehat{w}_t \equiv (w_t - w)/w$. This system is often reduced further by substituting out w_t from equation (9) and i_t from equation (12), resulting in two equations in two endogenous variables, π_t and y_t,

$$-\widehat{y}_t = -E_t\widehat{y}_{t+1} + \nu\pi_t + \xi_t - E_t\pi_{t+1} \tag{13}$$

[5] As we are not concerned with optimal policy, we do not proceed to further normalize the variables as deviations from flexible-price levels. To study the model dynamics, it is sufficient to work with the deviations from steady state.

$$\pi_t = \Omega \widehat{y}_t + \beta E_t \pi_{t+1}. \tag{14}$$

Here, $\Omega \equiv -(1+\eta)/[\phi(\varepsilon-1)] = (1+\eta)(1-\theta)(1-\theta\beta)/\theta > 0$, depending on whether Rotemberg or Calvo NKPC is used.

2.1 AS-AD representation?

Notice that equation (13) is downward sloping in the $(\widehat{y}_t, \pi_t)$ space, whereas equation (14) is upward sloping in the $(\widehat{y}_t, \pi_t)$ space. For this reason, the first equation is often referred to as the 'aggregate demand curve' and the second equation as the 'aggregate supply curve'. Furthermore, as the monetary policy shock shows up in equation (13), it is sometimes referred to as an 'aggregate demand shock'. These interpretations of the system and the shock, however, are misleading. Supply and demand functions are supposed to determine the market clearing quantity and price. Here, however, the goods market clears along both curves, not just in their intersection—recall that the goods market clearing condition (7) has been used to derive equations (13) and (14). Strictly speaking, equation (13) characterizes the combinations of the inflation rate and *market clearing output* that are consistent with intertemporal optimality of consumers and the monetary policy rule. Equation (14), on the other hand, characterizes, strictly speaking, the combinations of the inflation rate and *market clearing output* that are consistent with the labor market equilibrium and optimal price setting. The intersection satisfies both types of optimality.

What if the market clearing condition is *not* used when deriving the equilibrium system? The system then becomes

$$-\widehat{c}_t = \nu \pi_t - E_t \widehat{c}_{t+1} + \xi_t - E_t \pi_{t+1}, \tag{15}$$

$$\pi_t = -\frac{1}{\phi(\varepsilon-1)}(\eta \widehat{y}_t + \widehat{c}_t) + \beta E_t \pi_{t+1},$$

$$\widehat{c}_t = \widehat{y}_t. \tag{16}$$

Here, in period t, $\widehat{c}_t$ can be interpreted as the amount of goods demanded, while $\widehat{y}_t$ can be interpreted as the amount of goods supplied, given the values of all other variables. Equation (16) clears the market. Eliminating $\widehat{c}_t$ from the NKPC, so as to express the equation only in terms of goods supplied, yields

$$\pi_t = -\frac{1}{\phi(\varepsilon - 1)}(\eta\widehat{y}_t - \nu\pi_t + E_t\widehat{c}_{t+1} - \xi_t + E_t\pi_{t+1}) + \beta E_t\pi_{t+1}. \tag{17}$$

Equation (15) is downward sloping in the $(\widehat{c}_t, \pi_t)$ space, while equation (17) is upward sloping in the $(\widehat{y}_t, \pi_t)$ space. The monetary policy shock, however, ends up shifting both curves.[6]

2.2 Equilibrium output and inflation

The system (13)-(14) can be solved by, for instance, the method of undetermined coefficients. Assume that the equilibrium decision rule and pricing function are linear functions of the state variable

$$\widehat{y}_t = a\xi_t \quad \text{and} \quad \pi_t = b\xi_t,$$

where a and b are unknown. The guesses are linear, rather than affine, functions of the state as the variables are expressed as deviations from steady state. Suppose that the shock follows a stationary AR(1) process

$$\xi_{t+1} = \rho\xi_t + \epsilon_{t+1}, \quad \rho \in [0, 1),$$

where ϵ_{t+1} is an innovation. Substituting the guesses into the system (13)-(14), evaluating the expectations using the AR(1) process, and aligning terms gives unique equilibrium coefficients

$$a = -\frac{1 - \beta\rho}{(1 - \rho)(1 - \beta\rho) + \Omega(\nu - \rho)} < 0, \tag{18}$$

[6]The Appendix shows that when the model is cast into output-real interest rate space, another common 'AS-AD' representation, the monetary policy shock ends up shifting only the AS curve.

$$b = -\frac{1}{(1-\rho)\frac{1-\beta\rho}{\Omega} + (\nu - \rho)} < 0. \tag{19}$$

It is illustrative to consider two extreme cases of price stickiness. First, suppose that prices are fully flexible ($\theta \to 0, \phi \to 0 \Rightarrow \Omega \to \infty$). In this case

$$a \to 0 \quad \text{and} \quad b \to -\frac{1}{\nu - \rho}.$$

Output in this case is unaffected by the monetary policy shock. The coefficient b is greater in absolute value the more persistent is the shock or the smaller is the weight on inflation in the policy rule.[7] Why is the response of inflation to a positive monetary policy shock negative when prices are fully flexible? To answer this question, it is helpful to rewrite the monetary policy rule (6) as

$$i_t = (i + \zeta_t) + \nu(\pi_t - \zeta_t), \tag{20}$$

where we have used $\xi_t \equiv -(\nu - 1)\zeta_t$. The shock ζ_t is just a linear transformation of the original shock ξ_t and thus inherits the persistence of the ξ_t shock. The two shocks, however, are negatively related. When the policy rule is rewritten as equation (20), the shock ζ_t has an interpretation as an inflation target shock. This helps understand why, when ξ_t increases (ζ_t declines), the inflation rate declines.

Second, suppose that prices are completely fixed ($\theta \to 1, \phi \to \infty \Rightarrow \Omega \to 0$). In this case

$$a \to -\frac{1}{1-\rho} \quad \text{and} \quad b \to 0.$$

Now, inflation is unaffected by monetary policy.[8] Why does output decline in response to a positive monetary shock ξ_t? Output is fully determined by the Euler equation and the monetary policy rule (producers find any output level optimal under $\Omega \to 0$; see the NKPC). With the inflation rate equal to the steady-state value of zero, the combination of equations

[7]The equilibrium coefficient for inflation under flexible prices can be alternatively obtained by using, in the method of undetermined coefficients, only equations (10) and (12), with $\widehat{y}_t = 0$.

[8]This solution can be alternatively obtained by using, in the method of undetermined coefficients, only equations (10) and (12), with $\pi_t = 0$.

(10) and (12) yields $E_t\widehat{y}_{t+1} - \widehat{y}_t = \xi_t$. According to this equation, output is expected to be growing as long as the exogenous shock is positive. Because the model is stationary (ξ_t is governed by a stationary AR(1) process), the only way output can grow is if it falls, on the impact of the shock, below its steady state. As the shock converges back to its steady-state level, output gradually grows back to steady state.

Going back to the general solution (18) and (19), observe that Ω can be viewed as a weight shifting the equilibrium coefficients between the fully flexible and completely fixed price solutions. The effects of θ (and thus Ω) on a and b are demonstrated in the upper-left panel of Figure 1 (the other parameters are $\beta = 0.99$, $\eta = 1$, $\nu = 1.5$, and $\rho = 0.5$; all fairly typical values in the literature).[9]

The lower-left panel, instead, demonstrates the effect of ρ (for $\theta = 0.7$, a typical value, implying average price stickiness for 3.3 periods). Notice that as $\rho \to 1$, the equilibrium coefficients converge, approximately, to the flexible-price solution (the convergence can be arbitrarily close the closer is β to one). This is because, in the NKPC, output depends on the expected *change* in the inflation rate. Thus, the more persistent inflation is, the smaller is its period-on-period change and thus the smaller is the effect on output.

2.3 The equilibrium nominal and real interest rates

The equilibrium functions for the nominal and ex-ante real interest rates can be derived, respectively, as

$$\widehat{i}_t = \nu\pi_t + \xi_t = (1 + \nu b)\xi_t,$$

$$E_t\widehat{r}_{t+1} \equiv \widehat{i}_t - E_t\pi_{t+1} = [1 + (\nu - \rho)b]\,\xi_t,$$

where $b < 0$ is given by (19). The equilibrium nominal interest rate consists of two parts: a direct effect of the shock in the Taylor rule and an indirect effect due to the response of the

[9] The advantage of using the Calvo specification for numerical examples is that the literature contains a great amount of estimates of θ. Further, θ can be directly translated into the number of periods prices are on average fixed, as $(1 - \theta)^{-1}$.

equilibrium inflation rate. While the first effect is positive, the second effect is negative. The overall effect of the shock on the nominal interest rate thus depends on the relative strength of the two effects. The equilibrium ex-ante real interest rate depends on three effects. In addition to the above two effects, it also depends on an expected inflation effect. Like the direct effect of the shock, the expected inflation rate effect is positive. By substituting in for b, it is easy to show that the two positive effects always dominate the negative effect.[10]

The right-hand side panels of Figure 1 show the effects of θ and ρ on the equilibrium nominal and ex-ante real interest rates. Notice that for a significant part of the parameter space (lower θ and higher ρ), the equilibrium coefficient of the nominal interest rate is negative. For such values, declines in output and inflation in response to a positive monetary policy shock are accompanied by a decline in the nominal interest rate. This is in contrast to the desired impulse-responses described in the Introduction.

2.4 The Taylor Principle

The Taylor Principle is a theorem according to which $\nu > 1$ is a necessary condition for the existence of a unique stationary equilibrium of the New-Keynesian model (e.g., Bullard and Mitra, 2002). An interpretation of this result goes like this: The nominal interest rate has to respond more than one-for-one to the inflation rate, so as to affect the real interest rate and thus aggregate demand. Through its effect on aggregate demand, it controls by how much producers adjust prices and thus keeps inflation in check. This interpretation, however, seems to be based on partial equilibrium reasoning and is thus misleading. As the upper panels of Figure 1 show, a unique stationary equilibrium, obtained under $\nu > 1$, exists even if the nominal interest rate ends up, in equilibrium, responding less than the inflation rate (for instance, for $\theta = 0.5$, the nominal interest rate does not respond at all while the inflation rate declines by about 0.7 percentage points).

[10] Alternatively, one can see that the ex-ante real rate always responds positively to the ξ_t shock by realizing that output always declines on impact of the shocks (a is always negative) and converges back to its steady state from that point on (i.e., it is growing). This can only happen, according to the Euler equation, if the deviation from steady state of the ex-ante real interest rate is positive.

Rather than to do with suppressing aggregate demand, the Taylor principle in the New-Keynesian model works just like in a flexible-price model: ensuring that a bubble term in the model's asset pricing equation can be excluded (a point made by Cochrane, 2011). In both models, combining equations (10) and (12), the inflation rate has to satisfy

$$\pi_t = \frac{1}{\nu} E_t \pi_{t+1} + \frac{1}{\nu} \left(E_t \widehat{y}_{t+1} - \widehat{y}_t \right) - \frac{\xi_t}{\nu},$$

where the expression in brackets is a deviation of the real interest rate from steady state. Under fully-flexible prices, $\widehat{y}_t = 0$ for all t and the deviation of the real interest rate is equal to zero. Clearly, by forward substitution, $\nu > 1$ is a necessary condition for a unique stationary equilibrium, as $1/\nu < 1$ works as a discount factor of future values of the shock ξ_t, the 'dividends' in the model, and of $E_t \pi_{t+\iota}$, $\iota \to \infty$, the bubble term. When prices are sticky, output (and thus the real interest rate) is endogenous, given by the NKPC

$$\widehat{y}_t = \frac{1}{\Omega} (\pi_t - \beta E_t \pi_{t+1}).$$

But as long as output is bounded (for instance by a time constraint on labor), $\nu > 1$ again ensures that future (stationary) dividends, which now also include $E_t \widehat{y}_{t+1} - \widehat{y}_t$, are discounted and the bubble term can be eliminated.[11]

3 The model with capital

When capital is introduced into the model, the general equilibrium becomes characterized by the following system

$$\frac{w_t}{c_t} = l_t^{\eta}, \tag{21}$$

$$\frac{1}{c_t} = \beta E_t \left[\frac{1}{c_{t+1}} \left(\frac{1 + i_t}{1 + \pi_{t+1}} \right) \right], \tag{22}$$

[11] A formal proof is contained in the Appendix.

$$\frac{1}{c_t} = \beta E_t \left[\frac{1}{c_{t+1}} (1 + r_{t+1} - \delta) \right], \tag{23}$$

$$y_t = k_t^{\alpha} l_t^{1-\alpha}, \tag{24}$$

$$\frac{w_t}{r_t} = \frac{1-\alpha}{\alpha} \left(\frac{k_t}{l_t} \right), \tag{25}$$

$$\chi_t = \left(\frac{r_t}{\alpha} \right)^{\alpha} \left(\frac{w_t}{1-\alpha} \right)^{1-\alpha}, \tag{26}$$

$$\pi_t = \Psi \widehat{\chi}_t + \beta E_t \pi_{t+1}, \tag{27}$$

$$i_t = i + \nu \pi_t + \xi_t, \tag{28}$$

$$y_t = c_t + k_{t+1} - (1-\delta) k_t. \tag{29}$$

Here, k_t is capital, r_t is the capital rental rate, and $\delta \in (0,1)$ is a depreciation rate; investment can be defined as $x_t \equiv k_{t+1} - (1-\delta)k_t$. Further, $\Psi \equiv -\chi/[\phi(\varepsilon-1)] = [(1-\theta)(1-\theta\beta)/\theta]\Theta > 0$, where $\Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha/(1-\varepsilon)}$, and $\widehat{\chi}_t \equiv (\chi_t - \chi)/\chi$. The endogenous variables are c_t, w_t, l_t, i_t, π_t, y_t, χ_t, r_t, k_t; the exogenous variables are ξ_t and k_0. Notice that (21), (22), and (28) are the same as before. Further, (24), (26), and (27) are the same as before for $\alpha = 0$. The truly new equations are equations (23) and (25), which add the two new endogenous variables, k_t and r_t. Equation (23) is the Euler equation for capital and equation (25) is a condition for the optimal mix of capital and labor in production; it equates the marginal rate of technological substitution with the relative factor prices.

Under flexible prices, the model collapses into a real business cycle model with two additions, the Euler equation for bonds and the Taylor rule. To see this, note that under flexible prices, $\Psi \to \infty$. The NKPC (27) then implies $\widehat{\chi}_t = 0$, or $\chi_t = \chi$. If, in addition, $\varepsilon = 1$ (perfect competition), $\chi = 1$; see Gali (2015). This is a standard profit maximization condition under perfect competition, stating that the marginal cost is equal to the good's relative price, which is equal to one, as under perfect competition all goods are prefect substitutes. When this condition is used in equation (26), and the resulting equation is combined with (25), we get the standard conditions equalizing marginal products to factor prices

$$r_t = \alpha k_t^{\alpha-1} l_t^{1-\alpha}, \tag{30}$$

$$w_t = (1-\alpha) k_t^{\alpha} l_t^{-\alpha}. \tag{31}$$

These two conditions replace equations (25) and (26) in the above system. Notice that the system becomes recursive: equations (21), (23), (24), (29), (30), and (31) determine c_t, w_t, l_t, y_t, r_t, and k_t, given k_0, independently of ξ_t (in addition, $\chi_t = 1$ from the NKPC). Equations (22) and (28) then determine i_t and π_t, given stochastic paths of c_t and ξ_t.

When prices are completely fixed, $\Psi \to 0$ and the NKPC implies $\pi_t = 0$. The Euler equation for bonds then determines the growth rate of consumption, given ξ_t, but due to the presence of capital (investment) in equation (29), not the growth rate of output.

In what follows, it will be convenient to substitute in for r_t in equation (26) from (25). The marginal cost then becomes

$$\chi_t = \frac{w_t}{1-\alpha}\left(\frac{y_t}{k_t}\right)^{\frac{\alpha}{1-\alpha}}.$$

For $\alpha = 0$ this expression becomes the same as in the model without capital. Further, substitute in for l_t in equation (21) from the production function (24). This gives the first-order condition for labor as

$$\frac{w_t}{c_t} = \left(\frac{y_t}{k_t^{\alpha}}\right)^{\frac{\eta}{1-\alpha}}.$$

Again, for $\alpha = 0$, this condition is the same as in the model without capital.

With the above two substitutions, we can log-linearize the general equilibrium system to get

$$-\widehat{c}_t + \widehat{w}_t = \frac{\eta}{1-\alpha}\widehat{y}_t - \frac{\alpha\eta}{1-\alpha}\widehat{k}_t$$

$$-\widehat{c}_t = -E_t\widehat{c}_{t+1} + \widehat{i}_t - E_t\pi_{t+1},$$

$$-\widehat{c}_t = -E_t\widehat{c}_{t+1} + E_t\widehat{r}_{t+1},$$

$$\widehat{l}_t = \frac{1}{1-\alpha}\widehat{y}_t - \frac{\alpha}{1-\alpha}\widehat{k}_t,$$

$$\widehat{r}_t = r(\widehat{l}_t - \widehat{k}_t + \widehat{w}_t),$$

$$\widehat{\chi}_t = \widehat{w}_t + \frac{\alpha}{1-\alpha}\widehat{y}_t - \frac{\alpha}{1-\alpha}\widehat{k}_t,$$

$$\pi_t = \Psi\widehat{\chi}_t + \beta E_t \pi_{t+1},$$

$$\widehat{i}_t = \nu\pi_t + \xi_t,$$

$$\widehat{y}_t = \frac{c}{y}\widehat{c}_t + \frac{k}{y}\widehat{k}_{t+1} - (1-\delta)\frac{k}{y}\widehat{k}_t.$$

Here, variables without time subscripts are steady-state values, interest rates are expressed as percentage point deviations from steady state, $\widehat{r}_t \equiv r_t - r$, $\widehat{i}_t \equiv i_t - i$, and all other variables are expressed as percentage deviations from steady state, e.g., $\widehat{c}_t \equiv (c_t - c)/c$. Again, observe that in the absence of capital and its rental rate, and for $\alpha = 0$, the system is the same as in the model without capital. Eliminating $\widehat{r}_t$, $\widehat{\chi}_t$, $\widehat{w}_t$, $\widehat{i}_t$, and $\widehat{l}_t$ we get a final system of four equilibrium first-order difference equations in four endogenous variables $\widehat{c}_t$, $\widehat{y}_t$, $\widehat{k}_t$, and $\widehat{\pi}_t$

$$-\widehat{c}_t = -E_t\widehat{c}_{t+1} + \nu\pi_t - E_t\pi_{t+1} + \xi_t, \tag{32}$$

$$-\widehat{c}_t = -E_t\widehat{c}_{t+1} + rE_t\left(\widehat{c}_{t+1} + \frac{1+\eta}{1-\alpha}\widehat{y}_{t+1} - \frac{1+\alpha\eta}{1-\alpha}\widehat{k}_{t+1}\right), \tag{33}$$

$$\pi_t = \Psi\left[\frac{\eta+\alpha}{1-\alpha}\widehat{y}_t - \frac{\alpha(1+\eta)}{1-\alpha}\widehat{k}_t + \widehat{c}_t\right] + \beta E_t\pi_{t+1}, \tag{34}$$

$$\widehat{y}_t = \frac{c}{y}\widehat{c}_t + \frac{k}{y}\widehat{k}_{t+1} - (1-\delta)\frac{k}{y}\widehat{k}_t. \tag{35}$$

Here, (32) is the same as in the model without capital, (34) is the same as in the model without capital for $\alpha = 0$ and $k = 0$, and (35) is the same as in the model without capital for $k = 0$. Equation (33) is new. Clearly, the presence of capital disrupts the close connection between inflation and output in the basic model, as $\widehat{c}_t \neq \widehat{y}_t$.

3.1 The real effects of monetary policy shocks

In general, how monetary policy shocks affect output in the New-Keynesian model is based on a real interest rate channel. According to this interpretation, a positive shock to the Taylor rule increases the nominal interest rate. Because prices are sticky, the ex-ante real interest rate increases, which suppresses aggregate demand, as consumers postpone consumption (and producers cut down on investment). As producers find it costly to adjust prices in face of a decline in demand, they cut production and aggregate output falls. While this interpretation may be observationally equivalent to the mechanism in the basic model— in which a decline in output is always accompanied by an increase in the ex-ante real rate, though not necessarily the nominal rate—it is hard to justify in the model with capital. Here we take a first look at this issue and revisit it in Sections 3.2 and 3.3.

We base our exposition on the simplifying assumption that the steady-state real interest rate r is small enough to be replaced by zero (under the parameterization below, $r = 0.0351$). Under this assumption, equation (33) implies $\widehat{c}_t = E_t\widehat{c}_{t+1}$; that is, the presence of capital allows perfect consumption smoothing across time. Further, as the model is stationary, it has to be the case that $\widehat{c}_t = E_t\widehat{c}_{t+1} = 0$. Equation (32) then determines the equilibrium inflation rate, which is unaffected by the rest of the model and depends only on the monetary policy shock. The solution for the inflation rate is therefore the same as in the model without capital under flexible prices, $\pi_t = -[1/(\nu - \rho)]\xi_t$. The inflation rate thus falls on the impact of the shock and converges back to zero from below. Along this path, $\pi_t - E_t\pi_{t+1}$ is negative. Thus, for β close to one, equation (34) implies that on the impact of the shock output declines, as both $\widehat{c}_t$ and $\widehat{k}_t$ are equal to zero. From equation (35) then, $\widehat{k}_{t+1}$ has to decline.

While this mechanism provides a simplifying description of the model's dynamics (it's based on $r = 0$ and β close to one), Sections 3.2 and 3.3 show that it is not far fetched.

3.2 Equilibrium functions

The model can be again solved by the method of undetermined coefficients. There are two state variables, k_t and ξ_t. The general solution thus has the form: $\widehat{c}_t = a_0\widehat{k}_t + a_1\xi_t$, $\pi_t = b_0\widehat{k}_t + b_1\xi_t$, $\widehat{y}_t = d_0\widehat{k}_t + d_1\xi_t$, and $\widehat{k}_{t+1} = f_0\widehat{k}_t + f_1\xi_t$. Using these functions in the system (32)-(35) and aligning terms yields a system of eight equations in eight unknowns, a_0,a_1, b_0, b_1, d_0, d_1, f_0, f_1. From equation (32) we get:

$$-a_0 = -a_0 f_0 + \nu b_0 - b_0 f_0, \tag{36}$$

$$-a_1 = -a_0 f_1 - a_1\rho + \nu b_1 - b_0 f_1 - b_1\rho + 1.$$

From equation (33):

$$-a_0 = -(1-r)a_0 f_0 + \frac{r(1+\eta)}{1-\alpha}d_0 f_0 - \frac{r(1+\alpha\eta)}{1-\alpha}f_0, \tag{37}$$

$$-a_1 = -(1-r)a_0 f_1 - (1-r)a_1\rho + \frac{r(1+\eta)}{1-\alpha}d_0 f_1 + \frac{r(1+\eta)}{1-\alpha}d_1\rho - \frac{r(1+\alpha\eta)}{1-\alpha}f_1.$$

From equation (34):

$$b_0 = -\psi\frac{\eta+\alpha}{1-\alpha}d_0 + \psi\frac{\alpha\eta+\alpha}{1-\alpha} - \psi a_0 + \beta b_0 f_0, \tag{38}$$

$$b_1 = -\psi\frac{\eta+\alpha}{1-\alpha}d_1 - \psi a_1 + \beta b_0 f_1 + \beta b_1\rho.$$

And from equation (35):

$$f_0 = \frac{y}{k}d_0 - \frac{c}{y}a_0 + (1-\delta), \tag{39}$$

$$f_1 = \frac{y}{k}d_1 - \frac{c}{y}a_1.$$

The system is recursive, whereby a_0, b_0, d_0, and f_0 can be solved from the four equations (36)-(39), independently of a_1, b_1, d_1, and f_1. The persistence of the shock, ρ, thus has no effect on the equilibrium coefficients loading onto $\widehat{k}_t$. The equilibrium coefficients loading onto $\widehat{k}_t$, however, affect a_1, b_1, d_1, and f_1 and thus the direct responses of the endogenous variables to the monetary policy shock.

The system of equations (36)-(39) can be rewritten as a fourth-order polynomial, which has up to four distinct roots. From here, we proceed numerically, using the following parameter values: $\beta = 0.99$, $\eta = 1$, $\theta = 0.7$, $\nu = 1.5$ (same as before) and $\delta = 0.025$, $\alpha = 0.3$, and $\varepsilon = 0.83$ (fairly standard values).[12] The persistence parameter is treated as a free parameter, exploring the effects of $\rho \in \{0, 0.1, 0.5, 0.95, 0.995\}$. For these parameter values, the polynomial has a single root within the unit circle, and three roots outside of the unit circle, yielding a unique stationary equilibrium. For the baseline value $\rho = 0.5$, the equilibrium functions are

$$\widehat{c}_t = 0.5\widehat{k}_t - 0.5\xi_t,$$

$$\pi_t = -0.057\widehat{k}_t - 1.44\xi_t,$$

$$\widehat{y}_t = 0.13\widehat{k}_t - 11.5\xi_t,$$

$$\widehat{k}_{t+1} = 0.936\widehat{k}_t - 1.56\xi_t.$$

Figures 2-6 display responses to a 1 percentage point increase in ξ_t in period $t = 1$. Interest rates are reported as percentage point deviations from steady state; all other variables as percentage deviations from steady state. The impulse-response functions reveal our conjecture from Section 3.1 that the real effects of monetary policy shock in the model with capital do not propagate through the real interest rate channel. Except for the case of $\rho = 0$, output falls, in response to the positive monetary policy shock, even though the ex-ante real interest rate declines. Consumption and investment also fall, although consumption falls only little, as expected from our discussion in Section 3.1. In all cases, inflation declines (at least on impact), while the nominal interest rate declines in all but the case of no persistence. Experimentation reveals that the nominal interest rate increases, in response to the

[12] The values of k and c are, respectively, 7.0938 and 0.8227. The value of y is again normalized to one. As mentioned above, the value of $r = 1/\beta - 1 + \delta$ is 0.0351.

shock, only for $\rho \in [0, 0.06]$, while both the nominal and real interest rates increase only for $\rho \in [0, 0.04]$.[13] This region is tiny, compared with the model without capital (refer back to the lower panels of Figure 1). As ρ gets closer to one, and inflation becomes more persistent, the response of output converges to close to monetary policy neutrality, as one would expect from the NKPC.

In sum, only when the shock persistence is close to zero, the model produces, at least qualitatively, the empirical responses noted in the Introduction; the presence of capital dramatically reduces the admissible region for the shock persistence generating such responses.

3.3 The effect of capital on the ex-ante real interest rate

Why does the model have such a hard time producing an increase in the ex-ante real interest rate in response to the monetary policy shock? As in Section 2, it is helpful to write out the equilibrium function for the real interest rate

$$
\begin{aligned}
E_t\widehat{r}_{t+1} = \widehat{i}_t - E_t\pi_{t+1} = \nu\pi_t + \xi_t - E_t\pi_{t+1} \\
&= \nu(b_0\widehat{k}_t + b_1\xi_t) + \xi_t - E_t\left(b_0\widehat{k}_{t+1} + b_1\xi_{t+1}\right) \\
&= b_0(\nu - f_0)\widehat{k}_t + \left[1 + (\nu - \rho)b_1 - b_0 f_1\right]\xi_t. \qquad (40)
\end{aligned}
$$

We focus on the discussion of the immediate response (from steady state), setting thus $\widehat{k}_t = 0$.

Observe that the equilibrium coefficient loading onto ξ_t consists of four parts. The first three parts, $1 + \nu b_1 - \rho b_1$, are the same as in the basic model (even though the value of b_1 may be different). These are the direct effect of the shock on the nominal interest rate, the reaction of monetary policy to the response of today's inflation to the shock, and the response of tomorrow's inflation to the expected value of the shock tomorrow. In the model without capital, the equilibrium coefficient on inflation, for the baseline value of $\rho = 0.5$, was

[13] In all cases, the real interest rate increases above its steady-state level several periods after the impact of the shock due to the decline in capital; once the effect of sticky prices (the NKPC) dies off, the dynamics of the real interest rate become governed by the marginal product of capital, as in a real business cycle model.

-0.35 (as can be read off from Figure 1). This resulted in the sum of the three effects being positive (in fact, we were able to prove, using the reduced form solution for the equilibrium coefficient, that the sum of the three effects is always positive). Now, however, $b_1 = -1.44$ (see above) and the sum of the three effects is negative. Furthermore, the fourth effect in the equilibrium coefficient of the real rate, $-b_0 f_1$, makes the equilibrium coefficient even more negative. The fourth effect is related to the role of capital. Specifically, f_1 is the response of $\widehat{k}_{t+1}$ to ξ_t and b_0 gives the response of π_{t+1} to $\widehat{k}_{t+1}$. The product of f_1 and b_0 captures the response of expected inflation to today's monetary policy shock occurring through capital accumulation. As both f_1 and b_0 are negative (see above), the product is positive and the capital accumulation channel increases inflation expectations, in response to a positive ξ_t shock. The capital accumulation channel thus contributes to the decline in the ex-ante real interest rate.[14]

The capital accumulation channel plays a negative role even when the first three effects generate a positive response of the ex-ante real rate. For instance, in the case of $\rho = 0.1$, which yields $b_1 = -0.7$ and thus a positive sum of the first three effects, the fourth effect counterweights them and the real rate declines (Figure 3).

3.4 Adjustment costs

Suppose that whenever the consumer changes the capital stock, he has to incur a resource cost in terms of foregone real income. We assume the simplest possible form of capital adjustment costs

$$-\frac{\kappa}{2}(k_{t+1} - k_t)^2, \quad \kappa \geq 0.$$

In steady state, the adjustment cost is equal to zero. Further, as the adjustment cost is quadratic, it will not affect the resource constraint of the economy in a log-linear approxi-

[14]The intuition for the negative response of k_{t+1} to the shock is based on consumption smoothing. A decline in output (income) induces consumers to reduce capital to keep consumption as smooth as possible. The intuition for the increase in the inflation rate in response to a decline in capital stock relies on the version of the model without price stickiness. A decline in capital increases the marginal product of capital, and thus the real interest rate, which—through equations (22), (23), and (28)—transmits to higher inflation.

mation of the model around a steady state. The Euler equation for capital now becomes

$$1 = \beta E_t \left[\frac{c_t}{c_{t+1}} \left(\frac{r_{t+1} - \delta}{q_t} + \frac{q_{t+1}}{q_t} \right) \right],$$

where

$$q_t \equiv 1 + \kappa(k_{t+1} - k_t).$$

Notice that for $\kappa = 0$, the Euler equation collapses to the Euler equation in the version without adjustment costs. The expression in the round brackets has an interpretation as a sum of a dividend yield and a capital gain. Denote the capital gain by $G_{t+1} \equiv q_{t+1}/q_t$.

The log-linearized system becomes

$$-\widehat{c}_t = -E_t\widehat{c}_{t+1} + \nu\pi_t - E_t\pi_{t+1} + \xi_t, \tag{41}$$

$$-\widehat{c}_t = -E_t\widehat{c}_{t+1} + E_t\widehat{G}_{t+1} + rE_t\left(\widehat{c}_{t+1} + \frac{1+\eta}{1-\alpha}\widehat{y}_{t+1} - \frac{1+\alpha\eta}{1-\alpha}\widehat{k}_{t+1}\right), \tag{42}$$

$$\pi_t = \Psi\left[\frac{\eta+\alpha}{1-\alpha}\widehat{y}_t - \frac{\alpha(1+\eta)}{1-\alpha}\widehat{k}_t + \widehat{c}_t\right] + \beta E_t\pi_{t+1}, \tag{43}$$

$$\widehat{y}_t = \frac{c}{y}\widehat{c}_t + \frac{k}{y}\widehat{k}_{t+1} - (1-\delta)\frac{k}{y}\widehat{k}_t, \tag{44}$$

where

$$\widehat{G}_{t+1} = \widehat{q}_{t+1} - \widehat{q}_t = \overline{\kappa}(\widehat{k}_{t+2} - \widehat{k}_{t+1}) - \overline{\kappa}(\widehat{k}_{t+1} - \widehat{k}_t) \tag{45}$$

is a percentage deviation of capital gains from steady state and $\overline{\kappa} \equiv \kappa k$. Combining equation (41) and (42), the ex-ante real interest rate is equal to the sum of capital gains and expected dividend yield

$$\begin{aligned} i_t - E_t\pi_{t+1} &= \nu\pi_t + \xi_t - E_t\pi_{t+1} \\ &= E_t\widehat{G}_{t+1} + rE_t\left(\widehat{c}_{t+1} + \frac{1+\eta}{1-\alpha}\widehat{y}_{t+1} - \frac{1+\alpha\eta}{1-\alpha}\widehat{k}_{t+1}\right). \end{aligned} \tag{46}$$

Only equation (42) in the above system is different, compared with the system of the previous section. Of course, it coincides with equation (33) if $\kappa = 0$.

The exposition proceeds, as before, under the simplifying assumption that r is close to zero. Under this assumption, the last term in equation (42) again drops out (this is the dividend term). Now, however, the capital gains term does not allow us to conclude that $\widehat{c}_t = E_t\widehat{c}_{t+1} = 0$. Capital adjustment costs prevent perfect consumption smoothing, resulting in $\widehat{c}_t \neq E_t\widehat{c}_{t+1} \neq 0$. Any drop in output dictated by the NKPC and the change in the inflation rate has to be, at least partially, accommodated by a drop in consumption. The higher is κ, the more any given change in the capital stock affects capital gains and thus the expected growth rate of consumption. As a result, the higher is κ the more any given change in output is accounted for by a change in consumption, rather than investment. Increasing κ thus brings the model closer to the version without capital. Further, inflation and real variables are now interconnected. As before, the NKPC (43) provides one such connection. The other connection comes from equations (41) and (42), which relate inflation to capital gains as in equation (46); recall that in the version without adjustment costs, under r close to zero, inflation depends, negatively, only on the monetary policy shock. Here, the monetary policy shock again contributes negatively to current inflation, but the capital gains term contributes positively.

The model can again be solved by the method of undetermined coefficients, guessing $\widehat{c}_t$, π_t, $\widehat{y}_t$, and $\widehat{k}_{t+1}$ as linear functions of $\widehat{k}_t$ and ξ_t. Relative to the system of restrictions in the previous section, only those resulting from equation (42) are different

$$-\overline{\kappa} - a_0 + (1-r)a_0 f_0 - \frac{r(1+\eta)}{1-\alpha} d_0 f_0 + \frac{r(1+\alpha\eta)}{1-\alpha} f_0 + 2\overline{\kappa} f_0 - \overline{\kappa} f_0 = 0,$$

$$-a_1 + (1-r)a_0 f_1 + (1-r)a_1\rho - \frac{r(1+\eta)}{1-\alpha} d_0 f_1 - \frac{r(1+\eta)}{1-\alpha} d_1\rho + \frac{r(1+\alpha\eta)}{1-\alpha} f_1 + 2\overline{\kappa} f_1 - \overline{\kappa} f_0 f_1 - \overline{\kappa} f_1 \rho = 0.$$

Observe that the system preserves the recursive structure of the previous section. The equilibrium function for the ex-ante real rate can again be computed from the equilibrium functions for inflation and capital, using the formula in (40). Capital adjustment costs reduce, in absolute value, both b_1 and the effect due to capital accumulation, making the response of the real interest rate more likely to be positive. The equilibrium function for capital gains can be obtained by substituting the equilibrium functions for capital in equation (45).

Figures 7-9 show the responses of the model under alternative parameterizations of the capital adjustment cost, $\kappa \in \{0.1, 0.2, 0.5\}$. The rest of the parameterization is as usual; specifically, $\theta = 0.7$ and $\rho = 0.5$. Observe that as κ increases, the model starts to produce the empirical responses noted in the Introduction. At $\kappa = 0.1$ the model still suffers from producing a decline in the nominal interest rate and only a gradual increase in the ex-ante real rate. At $\kappa = 0.2$, the ex-ante real rate increases on impact, but the nominal interest rate still falls. At $\kappa = 0.5$, finally, both the ex-ante real rate and the nominal interest rate increase on impact. Throughout these experiments, the increase in the ex-ante real rate occurs exclusively due to expected capital gains.

Sufficiently high capital adjustment costs thus make the model consistent, at least qualitatively, with the empirical impulse-responses. Capital adjustment costs in this model work in the same way as in asset-pricing business cycle models (e.g., Jermann, 1998). They prevent consumption smoothing. Here, consumers want to smooth consumption when income declines, as producers cut output in face of a drop in inflation, as dictated by the NKPC. To prevent consumption smoothing in equilibrium, capital gains has to be sufficiently high. As a result, the ex-ante real interest rate, the real yield on a one-period bond, also has to be high.

4 Conclusions

Appendix

A.1. An alternative 'AS-AD' representation of the basic model

Sometimes, the basic model is interpreted in light of another popular 'AS-AD' representation, a representation in the real interest rate-output space. Here, the real interest rate plays the role of a price. Starting with the system

$$-\widehat{y}_t = -E_t\widehat{y}_{t+1} + \widehat{i}_t - E_t\pi_{t+1},$$

$$\pi_t = \Omega y_t + \beta E_t\pi_{t+1},$$

$$\widehat{i}_t = \nu\pi_t + \xi_t,$$

write the first equation as

$$-\widehat{y}_t = -E_t\widehat{y}_{t+1} + E_t\widehat{r}_{t+1},$$

where

$$E_t\widehat{r}_{t+1} = \widehat{i}_t - E_t\pi_{t+1}.$$

Then eliminate π_t from the NKPC by substitution from the Taylor rule and rearrange terms to get

$$\widehat{i}_t - \nu\beta E_t\pi_{t+1} = \nu\Omega\widehat{y}_t + \xi_t$$

or

$$\widehat{i}_t - E_t\pi_{t+1} + (1-\beta\nu)E_t\pi_{t+1} = \nu\Omega\widehat{y}_t + \xi_t.$$

If $(1-\beta\nu) \approx 0$, we have a system

$$-\widehat{y}_t = -E_t\widehat{y}_{t+1} + E_t\widehat{r}_{t+1}$$

$$E_t\widehat{r}_{t+1} = \nu\Omega\widehat{y}_t + \xi_t.$$

The first equation is downward sloping in the $(E_t\widehat{r}_{t+1}, \widehat{y}_t)$ space, giving rise to an interpretation as an AD curve, whereas the second equation is upward sloping, giving rise to an interpretation as an AS curve. The monetary policy shock, however, shows up in the second equation.

A.2. Proof of the Taylor Principle under sticky prices

Two equations characterize the equilibrium

$$-\widehat{y}_t = -E_t\widehat{y}_{t+1} + \nu\pi_t + \xi_t - E_t\pi_{t+1},$$

$$\pi_t = \Omega\widehat{y}_t + \beta E_t\pi_{t+1}.$$

Substituting out for $\widehat{y}_t$ gives

$$-\frac{1}{\Omega}\pi_t + \frac{\beta}{\Omega}E_t\pi_{t+1} = -E_t\left(\frac{1}{\Omega}\pi_{t+1} - \frac{\beta}{\Omega}E_{t+1}\pi_{t+2}\right) + \nu\pi_t + \xi_t - E_t\pi_{t+1}.$$

To determine the stability of this second-order stochastic differences equation, it is sufficient to study the dynamic properties of its deterministic homogenous part, which can be written as

$$\pi_t - \frac{1+\beta+\Omega}{1+\Omega\nu}\pi_{t+1} + \frac{\beta}{1+\Omega\nu}\pi_{t+2} = 0.$$

Using $\pi_t = L\pi_{t+1}$ and $\pi_t = L^2\pi_{t+2}$, this can be rewritten as

$$L^2 - \frac{1+\beta+\Omega}{1+\Omega\nu}L + \frac{\beta}{1+\Omega\nu} = 0.$$

The roots of this equation satisfy

$$\lambda_1 + \lambda_2 = \frac{1+\beta+\Omega}{1+\Omega\nu} \quad \text{and} \quad \lambda_1\lambda_2 = \frac{\beta}{1+\Omega\nu} > 0 \text{ and } < 1.$$

Based on the second equation, the two roots have the same sign and at least one is less than one. In order to have a unique stationary solution, both roots have to be less than one, so that inflation far in the future matters less and less for current inflation. Thus the sum of the two roots has to be less than two. This requires

$$\frac{1+\Omega}{1+\Omega\nu} < 1,$$

which holds when $\nu > 1$.

A.3. Forward-looking Taylor rule

The monetary policy rule is $\widehat{\imath}_t = \nu E_t \pi_{t+1} + \xi_t$. Under this rule, the equilibrium is characterized by

$$-\widehat{y}_t = -E_t \widehat{y}_{t+1} + \nu E_t \pi_{t+1} + \xi_t - E_t \pi_{t+1},$$

$$\pi_t = \Omega \widehat{y}_t + \beta E_t \pi_{t+1}.$$

Guessing $y_t = a\xi_t$ and $\pi_t = b\xi_t$, yields

$$b = -\frac{1}{(1-\rho)\frac{1-\beta\rho}{\Omega} + (\nu-1)\rho} < 0,$$

which differs only slightly from the solution for the contemporaneous Taylor rule by the second term in the denominator: $(\nu-1)\rho$ in the case of the forward-looking rule vs $(\nu-\rho)$ in the case of the contemporaneous rule.

A.4. Backward-looking Taylor rule

The monetary policy rule is $\widehat{\imath}_t = \nu \pi_{t-1} + \xi_t$. Under this rule, the equilibrium is characterized by

$$-\widehat{y}_t = -E_t \widehat{y}_{t+1} + \nu \pi_{t-1} + \xi_t - E_t \pi_{t+1},$$

$$\pi_t = \Omega \widehat{y}_t + \beta E_t \pi_{t+1}.$$

In this version of the model, there are two state variables: ξ_t and π_{t-1}. Suppose that equilibrium output and the inflation rate are linear functions of the state

$$y_t = a_0 \pi_{t-1} + a_1 \xi_t \quad \text{and} \quad \pi_t = b_0 \pi_{t-1} + b_1 \xi_t.$$

Using again the method of undetermined coefficients yields the following system of four equations in four unknowns

$$-a_0 = -a_0 b_0 + \nu - b_0^2,$$

$$-a_1 = -a_0 b_1 - a_1 \rho + 1 - b_0 b_1 - b_1 \rho,$$

$$b_0 = \Omega a_0 + \beta b_0^2,$$

$$b_1 = \Omega a_1 + \beta b_0 b_1 + \beta b_1 \rho.$$

This system has a recursive structure: the first and third equations determine a_0 and b_0, independently of a_1 and b_1. Combining these two equations yields a third-order polynomial

$$\beta b_0^3 + (1 + \beta + \Omega) b_0^2 + b_0 + \Omega \nu = 0,$$

which has up to three distinct roots. For stability, we require either a unique root $|b_0| < 1$ or one root less than one in absolute value, to load on past inflation, and two roots greater than one in absolute value, to eliminate bubble terms. Once b_0 is determined, the other coefficients are determined uniquely as

$$a_0 = \frac{1}{\Omega}(b_0 - \beta b_0^2)$$

and

$$\begin{bmatrix} a_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} 1-\rho & -(a_0+b_0+\rho) \\ -\Omega & (1-\beta b_0-\beta\rho) \end{bmatrix}^{-1} \begin{bmatrix} -1 \\ 0 \end{bmatrix}.$$

Notice that neither b_0 nor a_0 depend on the persistence of the shock. Thus, the higher the persistence of the shock, the more are the dynamics of the endogenous variables governed by the exogenous shock and the less by past inflation.

For the baseline parameterization $\beta = 0.99$, $\eta = 1$, $\nu = 1.5$, $\theta = 0.7$ (and $\rho = 0.5$), the polynomial has a unique root, equal to -0.1478. Figure A.1 compares the impulse-responses under the backward-looking Taylor rule with those under the contemporaneous Taylor rule. The differences are small.

Figure A.2 plots the equilibrium polynomial for the baseline parameterization, with the unique root equal to -0.1478. For all $\theta \in [0,1]$, a $\nu > 1$ ensures a single root in the interval (-1,1), thus guaranteing a unique stationary equilibrium. For some combinations of $\theta \in [0,1]$ and $\nu > 1$, the polynomial has a unique root, like in the figure. For others (specifically ν close to one or θ small), one or two additional roots, lying outside of the (-1,1) interval exist.

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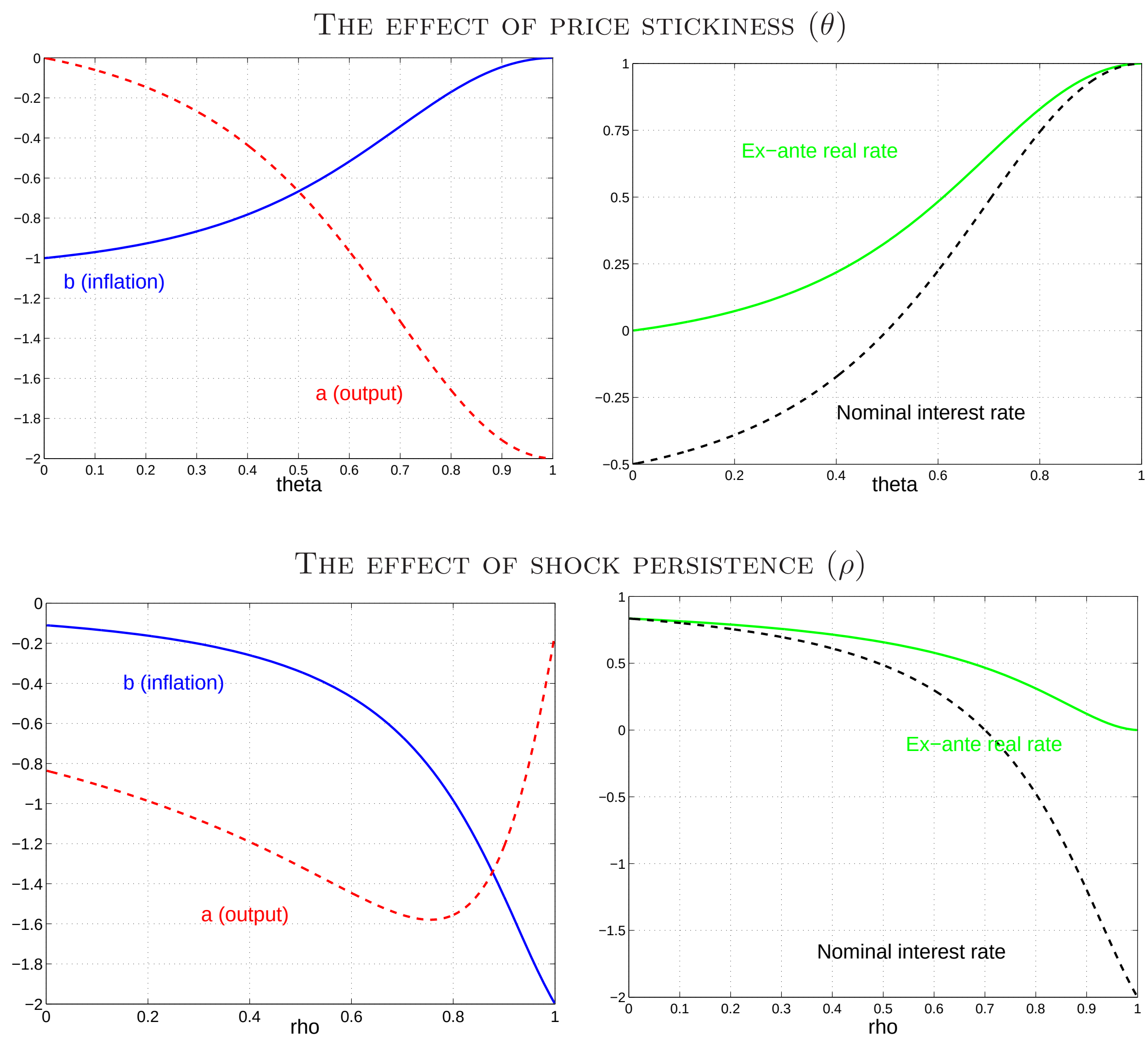

Figure 1: The effect of price stickiness (Calvo parameter θ) and of the persistence of the monetary policy shock (ρ) on the equilibrium coefficient (loading onto ξ_t) in the decision rule for output and the pricing functions for inflation, the nominal interest rate, and the ex-ante real rate. The baseline parameters are $\beta = 0.99$, $\eta = 1$, $\theta = 0.7$, $\nu = 1.5$, and $\rho = 0.5$.

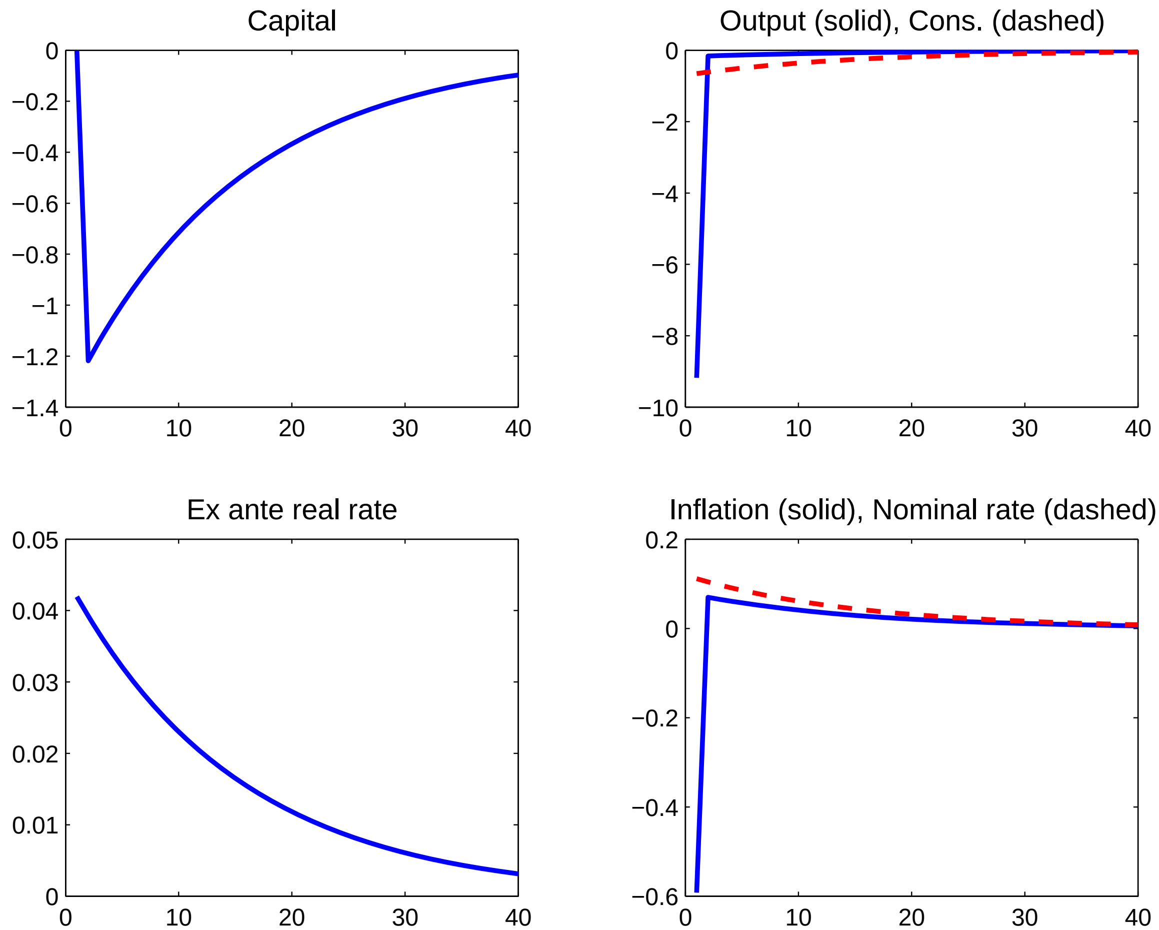

Figure 2: The model with capital, $\rho = 0$. Responses to 1 percentage point increase in ξ_t. The remaining parameterization is: $\beta = 0.99$, $\eta = 1$, $\theta = 0.7$, $\varepsilon = 0.83$, $\nu = 1.5$, $\alpha = 0.3$, and $\delta = 0.025$.

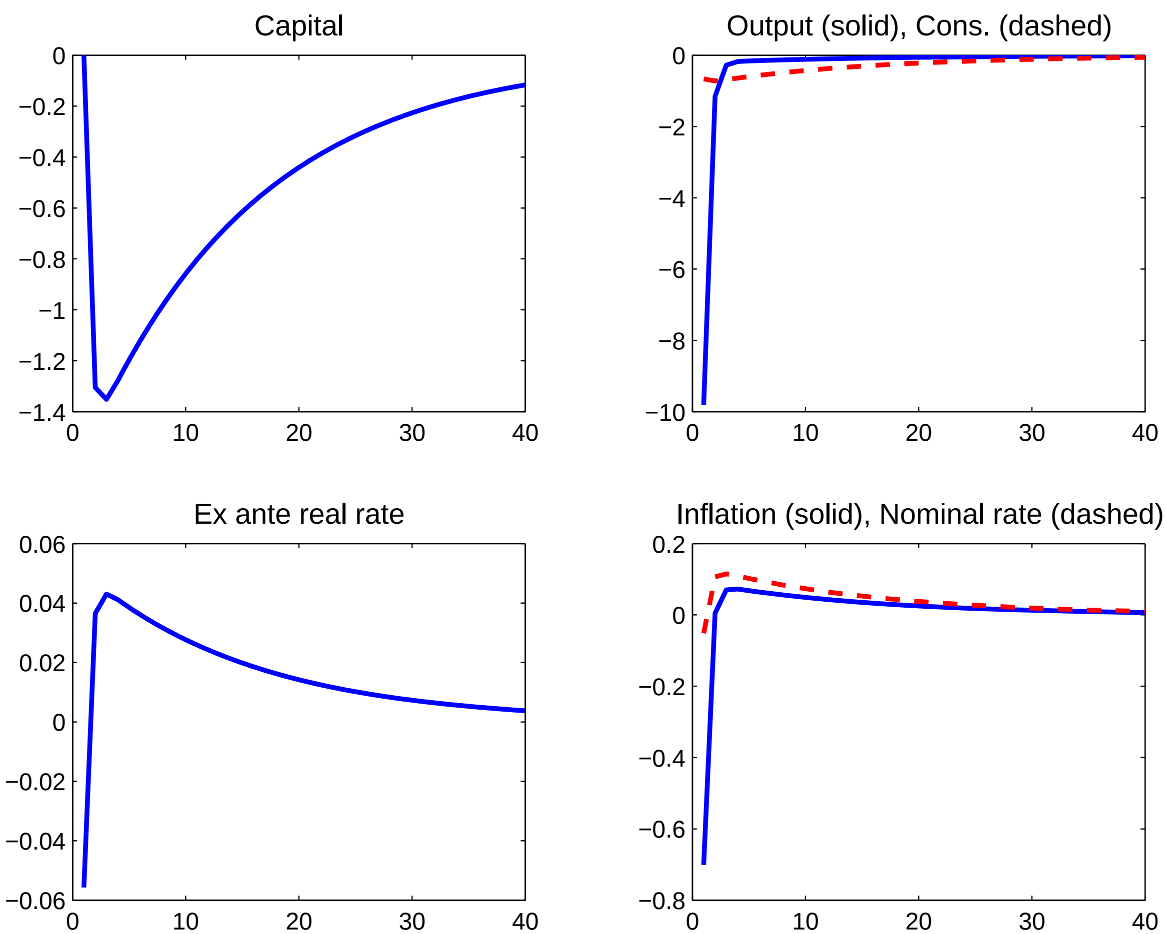

Figure 3: The model with capital, $\rho = 0.1$. Responses to 1 percentage point increase in ξ_t. The remaining parameterization is: $\beta = 0.99$, $\eta = 1$, $\theta = 0.7$, $\varepsilon = 0.83$, $\nu = 1.5$, $\alpha = 0.3$, and $\delta = 0.025$.

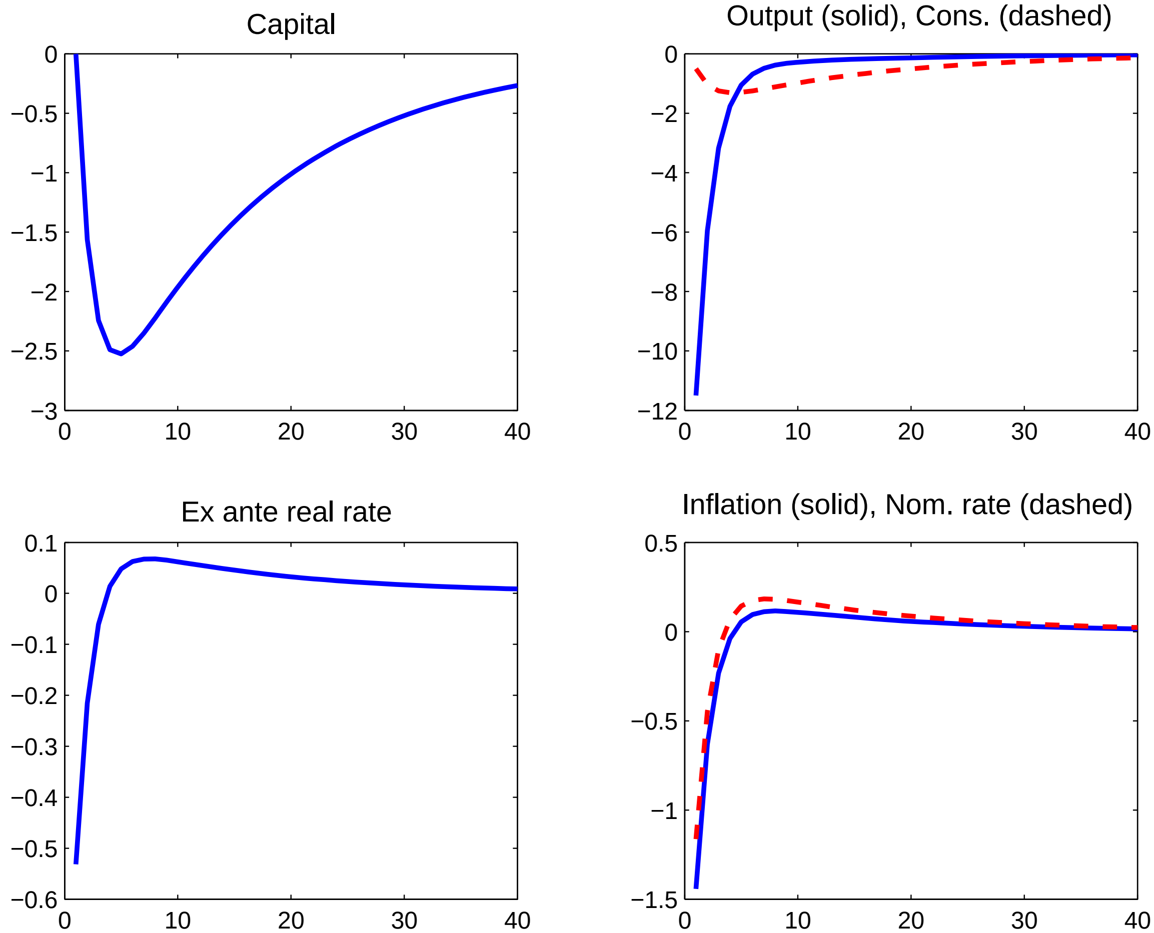

Figure 4: The model with capital, $\rho = 0.5$. Responses to 1 percentage point increase in ξ_t. The remaining parameterization is: $\beta = 0.99$, $\eta = 1$, $\theta = 0.7$, $\varepsilon = 0.83$, $\nu = 1.5$, $\alpha = 0.3$, and $\delta = 0.025$.

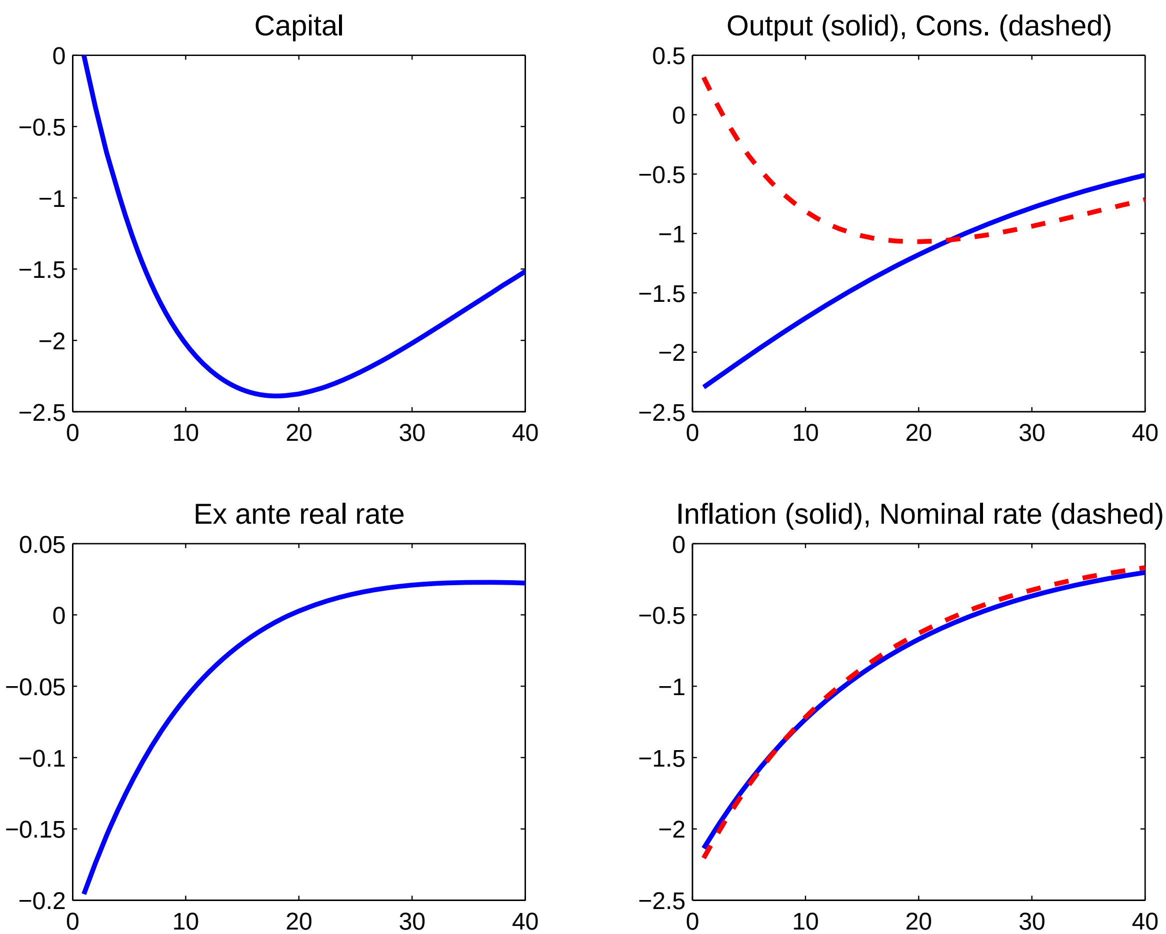

Figure 5: The model with capital, $\rho = 0.95$. Responses to 1 percentage point increase in ξ_t. The remaining parameterization is: $\beta = 0.99$, $\eta = 1$, $\theta = 0.7$, $\varepsilon = 0.83$, $\nu = 1.5$, $\alpha = 0.3$, and $\delta = 0.025$.

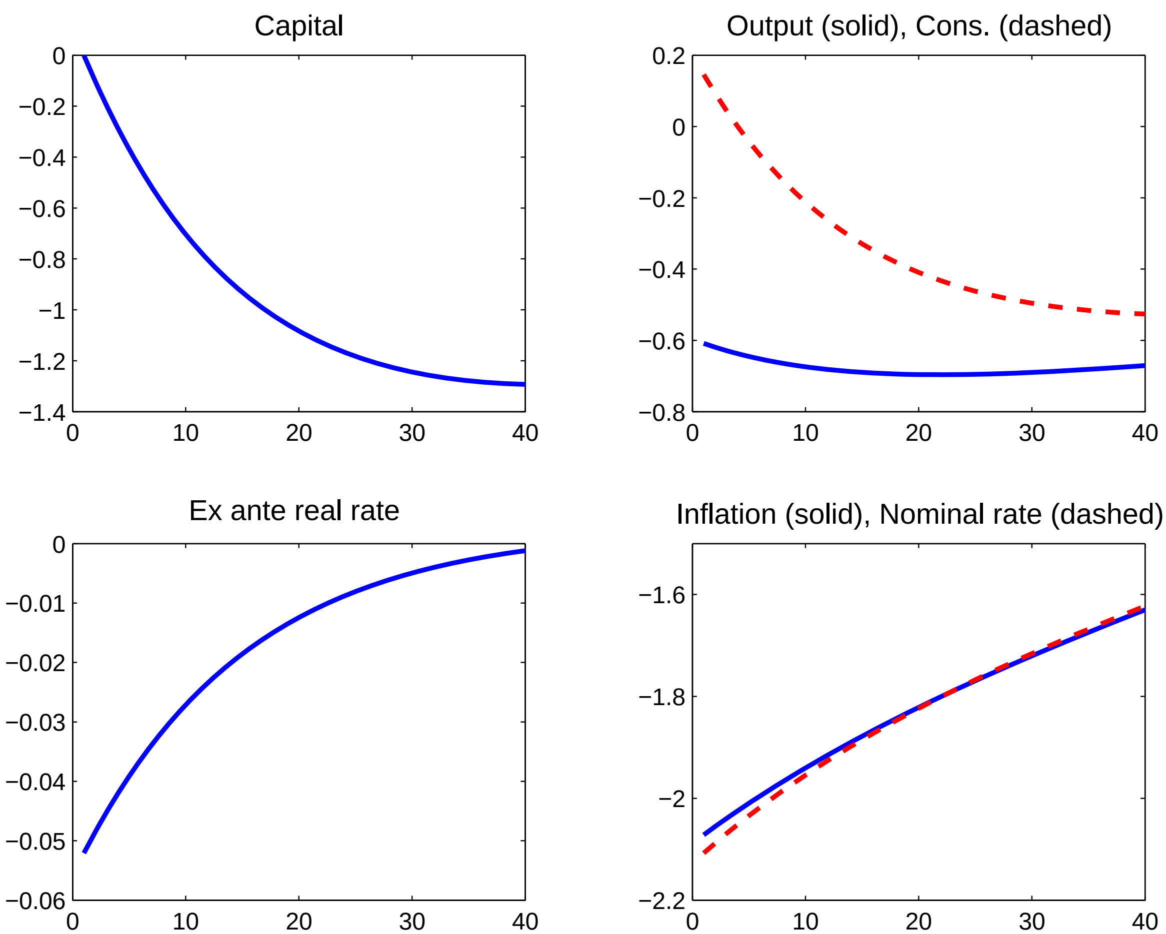

Figure 6: The model with capital, $\rho = 0.995$. Responses to 1 percentage point increase in ξ_t. The remaining parameterization is: $\beta = 0.99$, $\eta = 1$, $\theta = 0.7$, $\varepsilon = 0.83$, $\nu = 1.5$, $\alpha = 0.3$, and $\delta = 0.025$.

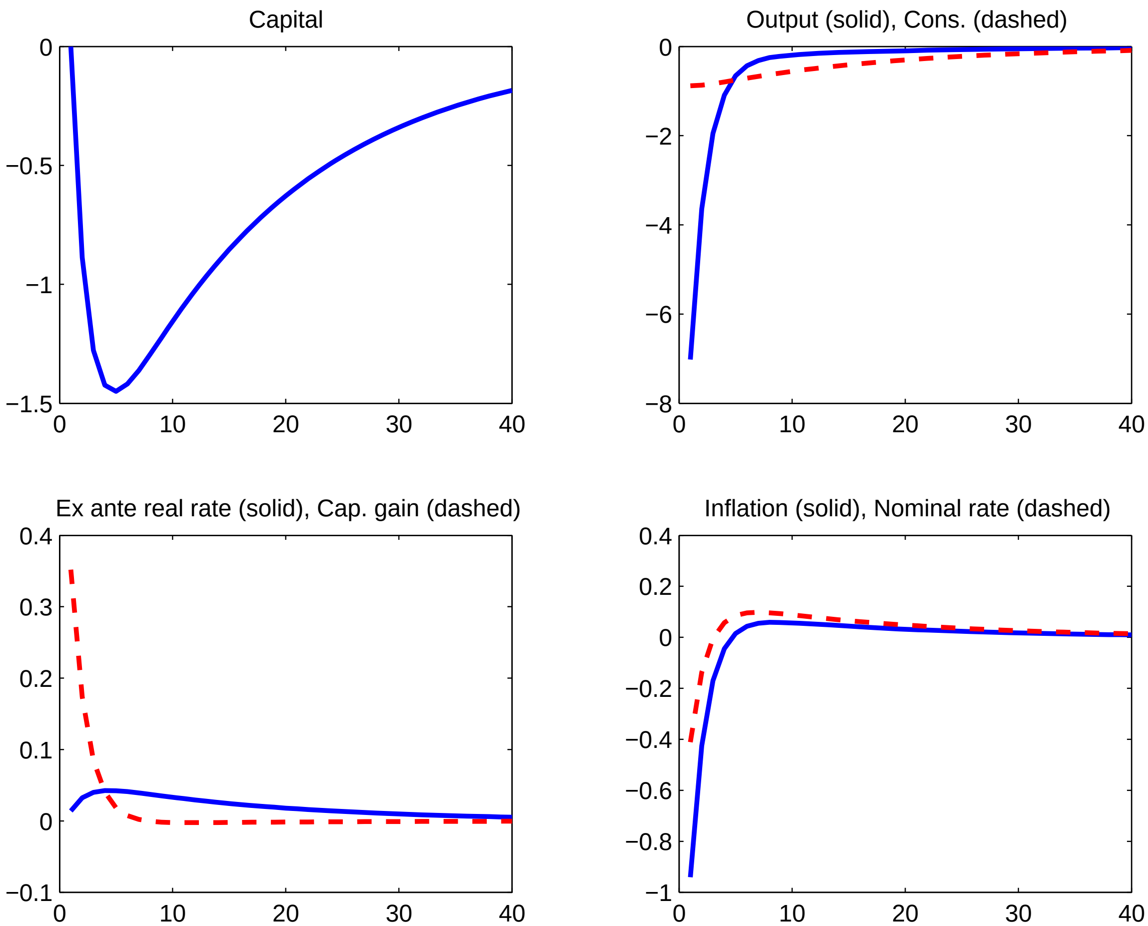

Figure 7: The model with adjustment costs, $\kappa = 0.1$. Responses to 1 percentage point increase in ξ_t. The remaining parameterization is: $\beta = 0.99$, $\eta = 1$, $\theta = 0.7$, $\varepsilon = 0.83$, $\nu = 1.5$, $\alpha = 0.3$, $\delta = 0.025$, and $\rho = 0.5$.

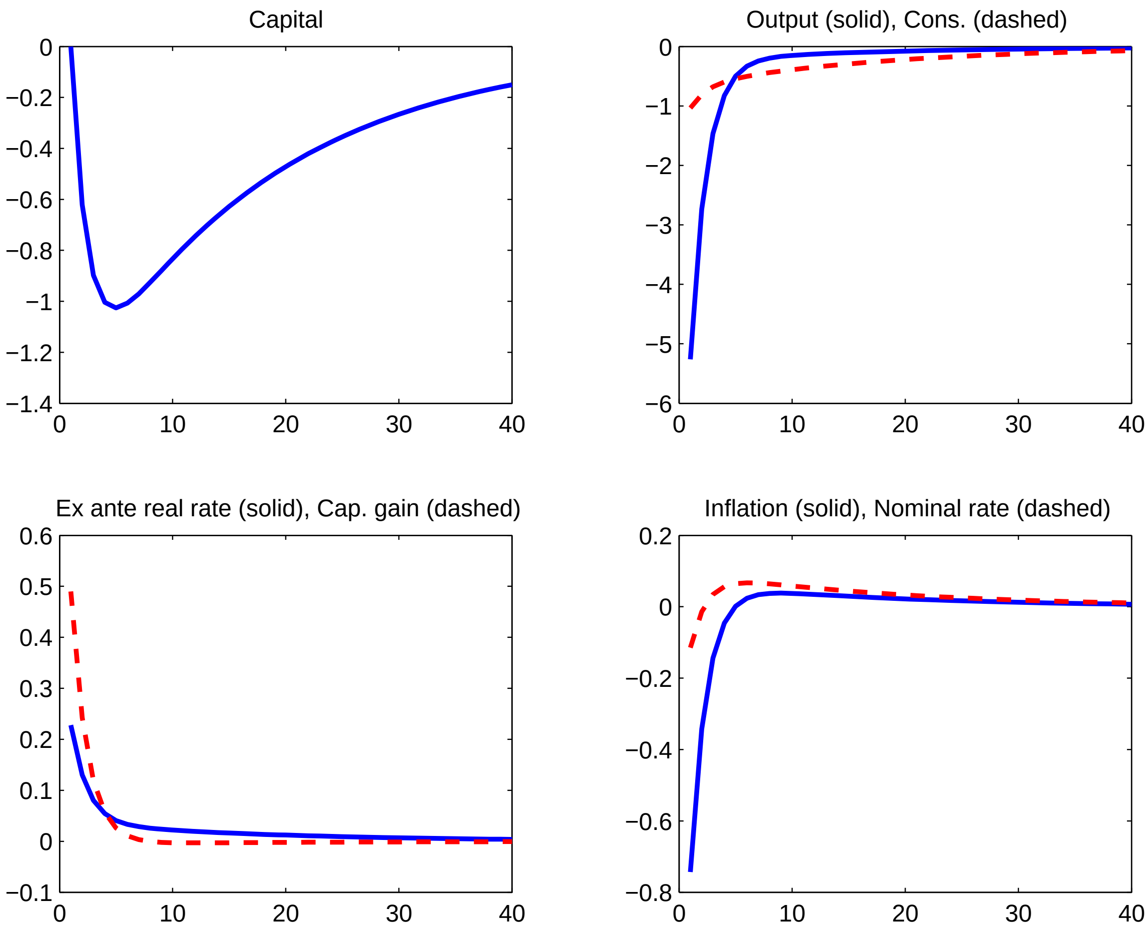

Figure 8: The model with adjustment costs, $\kappa = 0.2$. Responses to 1 percentage point increase in ξ_t. The remaining parameterization is: $\beta = 0.99$, $\eta = 1$, $\theta = 0.7$, $\varepsilon = 0.83$, $\nu = 1.5$, $\alpha = 0.3$, $\delta = 0.025$, and $\rho = 0.5$.

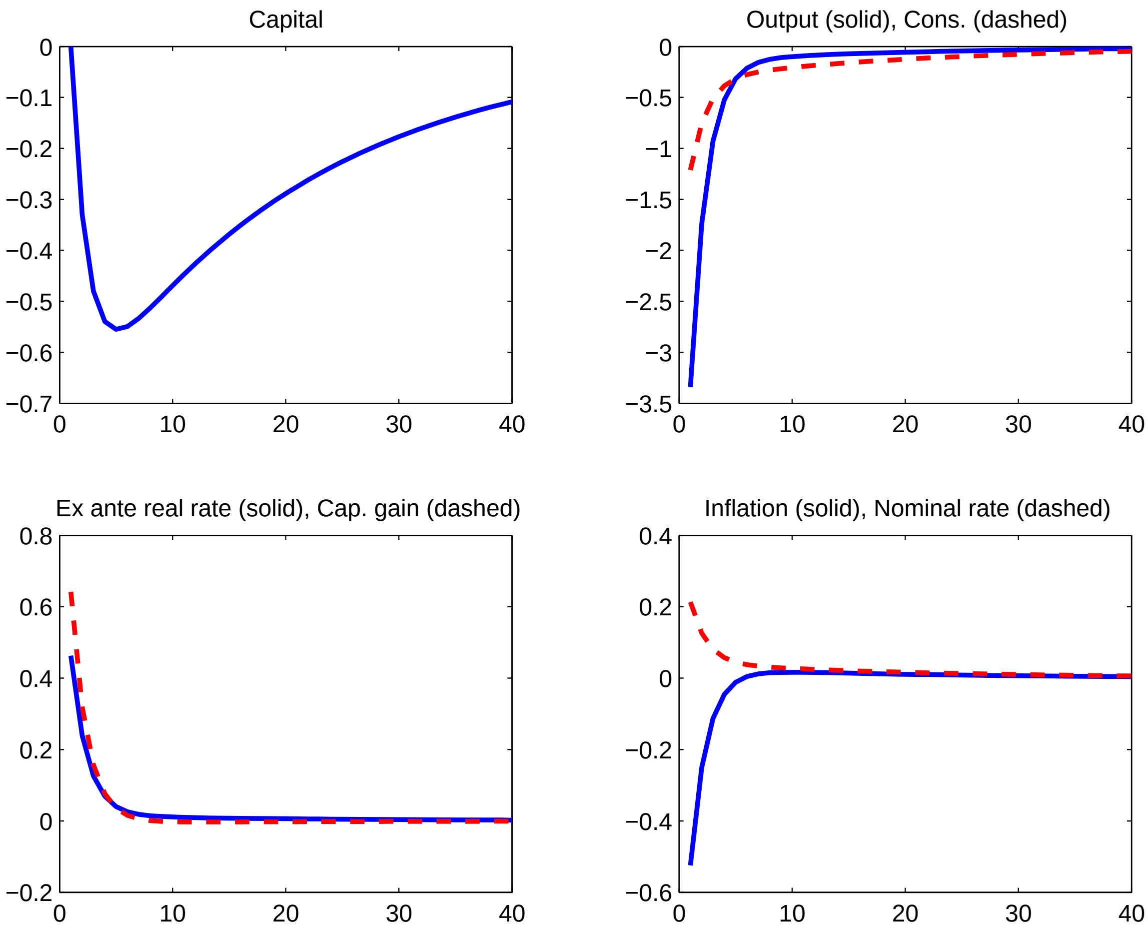

Figure 9: The model with adjustment costs, $\kappa = 0.5$. Responses to 1 percentage point increase in ξ_t. The remaining parameterization is: $\beta = 0.99$, $\eta = 1$, $\theta = 0.7$, $\varepsilon = 0.83$, $\nu = 1.5$, $\alpha = 0.3$, $\delta = 0.025$, and $\rho = 0.5$.

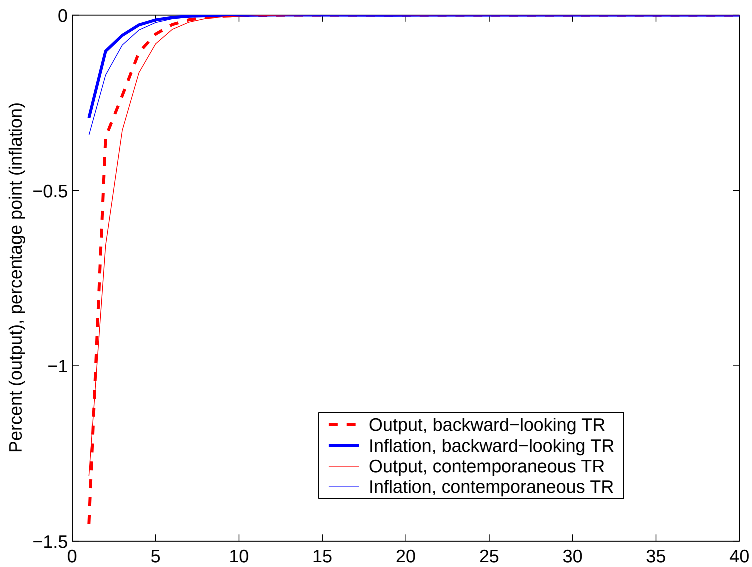

Fig. A.1. Backward-looking vs contemporaneous Taylor rule. Responses to a 1 percentage point increase in the monetary policy shock ξ_t. The parameterization is $\beta = 0.99$, $\eta = 1$, $\theta = 0.7$, $\nu = 1.5$, and $\rho = 0.5$.

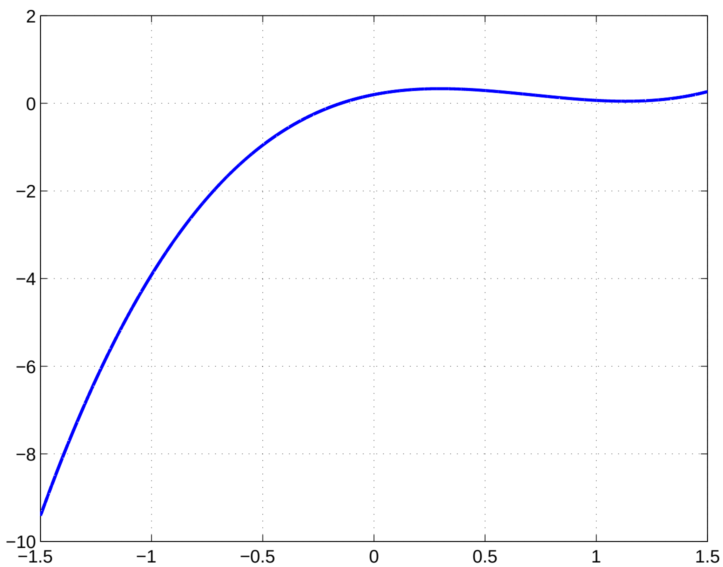

Fig. A.2. Backward-looking Taylor rule. Equilibrium polynomial in b_0. The parameterization is $\beta = 0.99$, $\eta = 1$, $\theta = 0.7$, $\nu = 1.5$, and $\rho = 0.5$.