Money, Inflation and Inequality: THANK U (Tractable Heterogenous Agent New Keynesian Model with Unemployment Risk)

Florin O. Bilbiie*
Paris School of Economics,
U. Paris 1 Panthéon-Sorbonne
and CEPR

Xavier Ragot
Paris School of Economics,
CNRS, OFCE

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Abstract

We put together a model where heterogenous households hold money because they particpate infrequently in financial markets, while firms face nominal frictions, as in the New Keynesian literature. We build a tractable framework whereby we characterize analytically the effect of money injections on inflation, economic activity, and inequality. Our framework gives rise, inter alia, to endogenous persistence in response to transitory shocks. Monetary policy is intimately linked to inequality as the nominal interest rate is a relevant price for holding money for self-insurance reasons.

1 Introduction

The rapid expansion of central bank balance sheets after the recent quantitative easing programs brought back the question of the effect of money injections on economic activity, and eventually, on inflation. Recent monetary policy analysis has followed two different routes to investigate this link. At one extreme, models in the "New Keynesian" tradition carefully model inflation dynamics and optimal interest rate policy, but introduce several shortcuts to model money demand. Money

*URL: http://florin.bilbiie.googlepages.com.

either enters the utlity function or, equivalently, is modelled by a cash-in-advance constraint in a representative agent framework (as illustrated i.a. by the textbooks by Woodford or Gali). These approaches of modelling money demand are not consistent with recent advances in money theory, which exhibit money holdings heterogeneity and thus possible redistributive effect of monetary policy across agents. In the opposite corner, models providing a microfoundation of money demand rely on price mechanisms which abstract from nominal frictions (Bewely 1980; Alvarez and Lippi 2009; Gu, Mattesini, and Wright, 2015 among many others)–whereas it is widely accepted that nominal frictions are necessary to obtain realistic inflation dynamics, interactions with real activity, and a meaningful role for monetary policy in influencing real activity.

In this paper, we present a microfounded model of money demand with nominal frictions to identify the real and nominal effects of money injections. This framework is thus an extension of the New Keynesian framework to include recent developments on money demand. Indeed, empirical analyses show that households' money demand is best understood when one introduces a friction that generates limited participation in financial markets. In this case, agents use money to smooth consumption between periods at which they adjust their financial portfolio. The distribution of money across households generated by this friction is much more similar to the data than the distribution generated by alternative money demand approaches (Alvarez and Lippi 2009; Cao et al 2012; Ragot 2014). The initial idea for this foundation for money demand dates back to Baumol and Tobin's seminal contributions, and it has been further developed in the limited participation literature in monetary economics.

This paper studies the positive implications of money creation in a model where money demand is based on limited participation, and where nominal frictions generate slow nominal adjustment, as in the New Keynesian literature. This friction generates a new role for monetary policy in the business cycle. It is already known that limited participation generates some relevant sort-run effects, such as the liquidity effect of money injection: An increase in the quantity of money decreases the nominal interest rate, as only a part of the population must absorb the new money created (Lucas 1990; Alvarez, Atkeson and Edmond 2009, among others). Nevertheless, this promising literature has faced difficulties in dealing with agents' heterogeneity (see the literature review below), which have prevented the introduction of additional features which are important for understanding the business cycle, such as nominal frictions.

To generate a micro-founded money demand, we introduce frictions in financial markets, which generate a consumption-smoothing role for money. Those frictions rely on both uninsurable idiosyncratic risks and on infrequent participation in financial markets. In the general case, such frictions generate a large amount of heterogeneity and the economy is characterized by a continuous distribution of wealth, which is very hard to study with aggregate shocks and with sticky prices (see Bewley (1983) for a formulation of incomplete insurance-market economies; the case with limited participation is studied in Ragot (2014)). For this reason, we use tools developed in the incomplete-market literature to reduce the amount of heterogeneity, while retaining the essence of both intertemporal trade-offs and redistributive effects of monetary policy in general equilibrium. The gain of this modeling strategy is that standard tools used in New-Keynesian economics can be used. In particular, we can use standard perturbation methods (loglinearization) and solve the model analytically in several special cases. Our analytical framework consisting of a three-equation model that is very close to the standard New Keynesian model, may be of independent interest to researchers looking for a unifying view of limited-participation frameworks for money demand and the New Keynesian framework for monetary policy analysis.

The main difference with the New Keynsian model is captured by an intertemporal equation governing money demand for the self-insurance motive. This key equation in our framework, combined with an otherwise standard Euler equation linking consumption growth of participating agents to the ral interest rate, introduces a deep link between the nominal interest rate (which is also the opportunity cost of holding money to self-insure against unemployment risk) and the (expected) consumption differential between participating-employed and nonparticipating-unemployed agents. Thus, our model features a deep link between monetary policy (summarized by the nominal interest rate) and inequality: a looser monetary policy leads to an decrease in inequality, the intuition being that inflation erodes the real value of money balances and redistributes wealth from participants to non-participants.

2 Preview of results

We start by studying the model under the assumption that the central bank follows a money-growth rule, and study the effects of a (helicopter-drop) transitory increase in money growth and of a transitory increase in total factor productivity. The equilibrium is always (locally) determinate under a money-growth rule, just like in a standard New Keynesian model.

A monetary expansion leads to an increase in aggregate consumption and inflation. The expansion is driven partly by non-participating agents (whom, being endogenously constrained, spend all the extra income) and partly by participating agents (who increase their money holdings).

Inequality increases on impact, as the interest rate increases (no liquidity effect under sticky prices, but this is an artefact of looking at a helicopter drop); furthermore, inequality increases in the future too as participating/employed agents accumulate real money balances.

Indeed, a key feature of our framework is endogenous persistence in response to purely transitory shocks. Two ingredients are key for this result: elastic labor of participating/employed agents, and sticky prices. We characterize analytically the effects of these parameters, as well as of the job finding rate and the separation rate, on persistence and on the monetary multiplier.

Similar results hold in response to transitory technology shocks: they are expansionary, and have persistent inflationary effects because of the expansion in money (even though on impact they are, of course, deflationary). Inequality also increase in response to technology shocks, since the interest rate increases.

We then study a different monetary policy setup, namely interest rate rules and endogenous money. We show that the requirement for equilibrium determinacy is much more stringent in this framework than the Taylor principle. Nevertheless, we show analytically that the monetary authority can restore determinacy by responding to measures of real activity, and most notably to a measure of inequality.

When the equilibrium is determinate under an interest rate rule, we solve analytically for the effect of a monetary shock and describe how they depend on deep parameters. Endogeous persistence does not obtain under an interest rate rule, but a liquidity effect now occurs naturally. Furthermore, a decrease in inequality is still associated with a decrease in interest rates, but that decrease in interest rates is now associated with an increase in money holdings.

3 Economy

3.1 Households

There is a mass 1 of households, indexed by $i \in [0, 1]$. Households have access to three assets claims on the profits of the firms, money, which has a zero nominal return, public debt which as a nominal returns[1] $i_t > 0$. Money is thus a dominated asset. To generate a micro-founded money demand, we introduce frictions in financial markets, which generate a consumption-smoothing role for money. Those frictions rely on both uninsurable idiosyncratic risks and on infrequent participation in financial markets. In the general case, it is known that such frictions generate a large amount of heterogeneity and that the economy is characterized by a continuous distribution of wealth, which is very hard to study with aggregate shocks and with sticky prices (see Bewley (1983) for a formulation of incomplete insurance-market economies the case with limited participation is studied in Ragot (2014)). For this

[1] In this article, we do not consider a binding zero lower bound to first investigate the properties of the model in general setup. We leave such an analysis for future work.

reason, we use some tools developed in the incomplete market literature to reduce the amount of heterogeneity while keeping the essence of both intertemporal trade-offs and redistributive effects of monetary policy in general equilibrium. The gain of this modeling strategy is that standard tools used in New-Keynesian economics can be used.

More precisely, households participate infrequently in financial markets. When households participate in financial markets, they can freely adjust their financial portfolio and they receive the dividends paid by firms..When they do not participate, they can only use money to smooth consumption.

It is assumed that when they participate, in period t the probability that they participate in period $t+1$ is α (and the probability that they do not participate next period is $1-\alpha$). When they do not participate in period t the probability that they do not participate in period $t+1$ is ρ (and the probability that they participate is $1-\rho$). The fraction of participating households is $n = (1-\rho) / (2-\alpha-\rho)$, and the fraction $1-n = (1-\alpha) / (2-\alpha-\rho)$ does not participate.

In addition, it is assumed that households belong to a family, and that the family head maximizes the intertemporal welfare of family members using a utilitarian welfare criteria (all households equally weighted) but face some limits to the amount of risk sharing it can implement. Indeed, households can be in two locations or "islands". All households who are participating in financial markets are on the same island, denoted as island P. All households who are not participating in financial markets are on the same island, denoted as island N. The family head can transfer resources across households within the island, but cannot transfer some resources between islands.

Additionally, it is assumed that households in the participating island can work at a real wage rate w_t, whereas households in the non-participating island don't work but get an amount δ generated by home production. Note that we assume financial risks (participating or not) and labor market risks (working or not) are perfectly correlated, to simplify the exposition. As an extension, we consider a more general framework where all employed and unemployed can participate or not. This complicates the algebra without any additional insight, as the main risk is not participating in financial market when the income is low.

The timing is the following. At the beginning of the period the family head pools resources within the island. The aggregate shocks are revealed and the family head determines the consumption/saving choice for each household in each island. Then, households learn their next period participation status and have to move to the corresponding island accordingly, taking their assets with them. The key assumption is that the family head can not make transfers to households after the idiosyncratic shock is revealed and it will take this constraint for the consumption/saving choice.

Flows across islands. The flows across islands are the following. The total measure of households leaving the N island each period is the number of households who participate next period: $(1-n)(1-\rho)$. The measure of households staying on the island is thus $(1-n)\rho$. In addition, a measure $(1-\alpha)n$ leaves the P island for the N island at the end of each period.

Total welfare maximization implies that the family head pool resources at the beginning of the period in each island and implement symmetric consumption/saving choice for all households in each island. Denote as p_t the price of unit of the consumption good in period t after the money shock. Denote as b_{t+1}^P and M_{t+1}^P, the per capita period t capital, bonds and money balances respectively, in the P island, after the consumption-saving choice. The real money balances are $m_{t+1}^P = M_{t+1}^P/p_t$. The end-of-period per capita real values (after the consumption/saving choice but before agents move across islands) are $\tilde{b}_{t+1}^P$ and $\tilde{m}_{t+1}^P$..Denote as m_t^N, the per capita beginning-of-period capital money in the N island (where the only asset is money). The end-of-period values (before agents move across islands) are $\tilde{m}_{t+1}^N$. We have the following relations:

$$nm_{t+1}^P = \alpha n\tilde{m}_{t+1}^P + (1-\rho)(1-n)\tilde{m}_{t+1}^N$$
$$(1-n)m_{t+1}^N = (1-\alpha)n\tilde{m}_{t+1}^P + \rho(1-n)\tilde{m}_{t+1}^N$$

Thus

$$m_{t+1}^P = \alpha\tilde{m}_{t+1}^P + (1-n)\tilde{m}_{t+1}^N \tag{1}$$
$$m_{t+1}^N = (1-\rho)\tilde{m}_{t+1}^P + \rho\tilde{m}_{t+1}^N \tag{2}$$

Finally, as bonds do not leave the P island, we have

$$b_{t+1}^P = \tilde{b}_{t+1}^P.$$

Finally, the net inflation rate is denoted $\pi_t = (p_t - p_{t-1})/p_{t-1}$ and all households pay et taxes τ_t in all periods. These lump-sum taxes include any new money created or destroyed.

Program of the family head.

The program of the family head can be written compactly, as

$$W\left(b_t^P, m_t^P, m_t^N, X_t\right) = \max_{\{c_t^P, \tilde{b}_{t+1}^P, \tilde{m}_{t+1}^P, \tilde{m}_{t+1}^N, c_t^N\}} n\left[u\left(c_t^P\right) - \frac{\left(l_t^P\right)^{1+\varphi}}{1+\varphi}\right] + (1-n)u\left(c_t^N\right) + \beta EW\left(b_{t+1}^P, m_{t+1}^P, m_{t+1}^N, X\right.$$

subject to

$$c_t^P + \tilde{b}_{t+1}^P + \tilde{m}_{t+1}^P = w_t + \tau_t \tag{3}$$
$$+ \frac{1+i_{t-1}}{1+\pi_t}b_t^P + \frac{m_t^P}{1+\pi_t} + \Pi_t,$$
$$\tilde{m}_{t+1}^N + c_t^N = \kappa w_t + \tau_t + \frac{m_t^N}{1+\pi_t} \tag{4}$$
$$\tilde{m}_{t+1}^{PA}, \tilde{m}_{t+1}^N \geq 0 \tag{5}$$

and the laws of motion (1) and (2). Equation (3) is the per capita budget constraint in the P island. Note that P-households receive total profits Π_t as they own the firms. Equation (4) is the budget constraint in the N island. Finally (5) are positive constraints on money holdinds. As discussed below, these constraints play the role of credit constraint in the heterogeneous agent literature. The variable X_t in the value functions refer to all relevant period t information necessary to form rational expectations.

Using the first-order and envelope conditions, one finds for the P island.

$$u'\left(c_t^P\right) \geq \beta E \frac{1+i_t}{1+\pi_{t+1}} u'\left(c_{t+1}^P\right) \text{ or } \tilde{b}_{t+1}^P = 0 \tag{6}$$

$$u'\left(c_t^P\right) \geq \beta E\left[\alpha u'\left(c_{t+1}^P\right) + (1-\alpha) u'\left(c_{t+1}^N\right)\right] \frac{1}{1+\pi_{t+1}} \text{ or } \tilde{m}_{t+1}^P = 0 \tag{7}$$

$$u'\left(c_t^N\right) \geq \beta E\left[(1-\rho) u'\left(c_{t+1}^P\right) + \rho u'\left(c_{t+1}^N\right)\right] \frac{1}{1+\pi_{t+1}} \text{ or } \tilde{m}_{t+1}^N = 0 \tag{8}$$

The first equation is the choice of bonds. As households in the A island cannot bring their bonds to other islands, there is no self-insurance motive for these assets. As a consequence, the Euler equations for bonds is the same as the one of a representative agent. This, again, will simplify the structure of the equilibrium. The second equation (7) determines the money choice of agents in the A island, which takes into account the fact that money can be used by households moving to the other island. The last equation (8) determines the money choice of agents in the PU islands.

The important implication of this market structure is that the previous Euler equations (7) and (8) have the same expression as in a full-fledged incomplete market model of the Bewely-Huggett-Aiyagari literature. In particular, the probability $1-\alpha$ measures uninsurable risk to swith to low income next period, for each only money is available to self-insure. As a consequence, although money is a dominated assets, it will be held for self-insurance purpose.

3.2 Production and Price setting

3.2.1 Final goods.

The final good is produced by a firm facing which uses intermediate goods as inputs. The final sector production function is $Y_t = \left(\int_0^1 \left(y_t(i)\right)^{1-\frac{1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}}$, where y_t^i is the amount of type i intermediate good used in production. Denote as $p_t(i)$ the price of intermediate goods i. Profit maximization yields the standard exresion

$$y_t(i) = \left(\frac{p_t(i)}{P_t}\right)^{-\varepsilon} Y_t \tag{9}$$

with $P_t = \left(\int_0^1 p_t(i)^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}$.

3.2.2 Intermediate goods

The differentiated intermediate goods are produced with the linear production function $y_t(i) = A_t n_t(i)$. Firms receive a production subsidy σ per unit produced (to avoid steady state distortions), and face quadratic adjustment costs, as in Rotemberg (1982). The program of the firm is

$$\max_{p_t(i)} E_t \sum_{s=t}^{\infty} \Lambda_{t,s} \left[\left(\frac{(1+\sigma) p_t(i)}{P_t} - \frac{w_t}{A_t} \right) y_t(i) - \frac{\nu}{2} \left(\frac{p_t(i)}{p_{t-1}(i)} - 1 \right)^2 \right]$$

subject to (9). The stochastic discount factor used by firms is

$$\Lambda_{t,s} = \beta^{s-t} \frac{u'(c_s^p)}{u'(c_t^p)}$$

The solution is known to be

$$\begin{aligned} \pi_t (1+\pi_t) = \beta \mathbb{E}_t \left[\left(\frac{c_{P,t}}{c_{P,t+1}} \right)^{\gamma} \frac{Y_{t+1}}{Y_t} \pi_{t+1} (1+\pi_{t+1}) \right] \\ + \frac{\varepsilon - 1}{\nu} \left[\frac{\varepsilon}{\varepsilon - 1} \frac{W_t}{A_t P_t} - (1+\sigma) \right] \end{aligned} \quad (10)$$

3.3 Money creation, budget of the state and market equilibria

The new money is created lump-sum transfers to all households in the baseline specification. We consider open-market operations as an extension. Denote as M_t^{tot} the total nominal quantity of money in circulation at the end of each period. As a consequence, $M_t^{tot} = M_{t-1}^{tot} + P_t \tau_t^m$. Or in real terms

$$m_t^{tot} = \frac{m_{t-1}^{tot}}{1+\pi_t} + \tau_t \quad (11)$$

In this paper it is assumed that bonds are in zero net supply. The only transfer received by households are thus only the new money created by the State[2]. The market for bonds is thus $b_{t+1}^p = 0$. The money market is

$$m_t^{tot} = (1-n) \tilde{m}_{t+1}^N + n \tilde{m}_{t+1}^P$$

and the goods market equilibrium is $c_t = Y_t + (1-n)\delta$ where c_t is total consumption

$$c_t \equiv n c_t^P + (1-n) c_t^N$$

[2] Change in public debt has interesting effect on output, money and inflation in this non-Ricardian framework, that we leave for future research.

4 Structure of the equilibrium

We now assume that nominal bonds are in zero net supply, and we guess-an-verify the structure of the equilibrium. Guess that $\tilde{m}_t^N = 0$, *i.e.* non-participating households never hold money at the end of the period. The model equations are given by the 6 following equations:

$$
\begin{aligned}
u'\left(c_t^P\right) &= \beta E \frac{1+i_t}{1+\pi_{t+1}} u'\left(c_{t+1}^P\right) \\
u'\left(c_t^P\right) &= \beta E\left[\alpha u'\left(c_{t+1}^P\right) + (1-\alpha) u'\left(c_{t+1}^N\right)\right] \frac{1}{1+\pi_{t+1}} \\
c_t^P + \tilde{m}_{t+1}^P &= w_t - \tau_t + \frac{\alpha}{1+\pi_t} \tilde{m}_t^P \\
c_t^N &= \delta - \tau_t + \frac{1-\rho}{1+\pi_t} \tilde{m}_t^P \\
\pi_t \left(1+\pi_t\right) &= \beta \mathbb{E}_t \left[\left(\frac{c_t^P}{c_{t+1}^P}\right)^\gamma \frac{Y_{t+1}}{Y_t} \pi_{t+1} \left(1+\pi_{t+1}\right)\right] + \frac{\varepsilon - 1}{\nu}\left[\frac{\varepsilon}{\varepsilon-1} \frac{w_t}{A_t} - (1+\sigma)\right] \\
Y_t &= n c_t^P + (1-n) c_t^N - (1-n)\delta \\
\tilde{m}_t^P &= \frac{\tilde{m}_{t-1}^P}{1+\pi_t} + \frac{1}{n}\tau_t
\end{aligned}
$$

in the six equations $(c_t^p, i_t, \pi_t, c_t^N, \tilde{m}_{t+1}^P, w_t, Y_t)$ where τ_t is a monetary shock and A_t is a technology shock. The conditions for households in the $N-$island not to hold money is

$$
u'\left(c_t^N\right) > \beta E\left[(1-\rho) u'\left(c_{t+1}^P\right) + \rho u'\left(c_{t+1}^N\right)\right] \frac{1}{1+\pi_{t+1}}
$$

We will have to check that the previous inequality holds in any equilibrium we consider.

5 Steady state

The steady-state of the model can be easily defined. One finds

$$
\begin{aligned}
1+i &= \frac{1+\pi}{\beta} \\
\frac{u'\left(c^N\right)}{u'\left(c^P\right)} &= \frac{\frac{1+\pi}{\beta} - \alpha}{1-\alpha} \geq 1
\end{aligned}
$$

With a CRRA utility function, we obtain

$$
\frac{c^P}{c^N} = \left(\frac{\frac{1+\pi}{\beta} - \alpha}{1-\alpha}\right)^{\frac{1}{\gamma}}
$$

In this paper, we focus on a zero-inflation steady state for simplicity; in such an equilibrium, and assuming that a subsidy induces marginal-cost pricing in steady state, $\sigma = 1/(\varepsilon - 1)$, we further obtain, where p and h are the steady-state consumption shares of each agent, respectively:

$$h \equiv \frac{c^N}{c} = \frac{1}{1+n(F-1)}; p \equiv \frac{c^P}{c} = \frac{F}{1+n(F-1)}$$

$$\frac{m^{tot}}{c^N} \equiv \frac{\mu}{h} = \frac{n}{1-\rho}\left(\frac{1}{1+n(F-1)} - \frac{\delta}{c}\right)(1+n(F-1)) \tag{12}$$

$$= \frac{n}{1-\rho}\left\{1 - \frac{\delta}{c}[1+n(F-1)]\right\} \tag{13}$$

where the share of unemployment benefits (home production) in consumption $\frac{\delta}{c}$ will be a calibrated parameter, and

$$F = \left(\frac{1-\alpha\beta}{\beta(1-\alpha)}\right)^{\frac{1}{\gamma}}$$

6 Loglinearized equilibrium

We solve the model by taking a loglinear approximation around the steady state defined above, and restricting attention to the parameter configurations for which the relevant constraints keep binding (in particular, ρ, α and δ are such that money demand is strictly positive). Table 1 summarizes the original equilibrium conditions, loglinearized, where we denote by $x_t \equiv -\frac{1}{m^{tot}}(\tau_t - \tau)$ the monetary transfer. Under money growth rule, this transfer is exogenous and the interest rate is determined endogenously; under an interest rate rule, we append one equation (a Taylor rule) and the transfer x_t becomes endogenous.

Table 1: Summary of loglinearized equilibrium conditions

Euler P	$\hat{c}^P_t = E_t\hat{c}^P_{t+1} - \gamma^{-1}(i_t - E_t\hat{\pi}_{t+1})$
Self-insurance/money demand	$\hat{c}^P_t = \alpha\beta E_t\hat{c}^P_{t+1} + (1-\alpha\beta)E_t\hat{c}^N_{t+1} + \gamma^{-1}E_t\hat{\pi}_{t+1}$
Budget constraint N	$\hat{c}^N_t = \frac{\mu}{h}\left[\frac{1-\rho}{n}(\hat{m}^{tot}_t - \hat{\pi}_t) + x_t\right]$
Ec res cons (replace P's BC)	$\hat{c}_t = \left(1-(1-n)\frac{\delta}{c}\right)\hat{l}_t$
Aggregate labor	$\hat{l}_t = \hat{l}^P_t$
Labor supply	$\varphi\hat{l}^P_t = \hat{w}_t - \gamma\hat{c}^P_t$
Aggregate C	$\hat{c}^P_t = \frac{1}{np}\hat{c}_t - \frac{(1-n)h}{np}\hat{c}^N_t$
Money creation	$x_t = \hat{m}^{tot}_{t+1} - \hat{m}^{tot}_t + \hat{\pi}_t$
Phillips curve	$\pi_t = \beta\mathbb{E}_t\pi_{t+1} + \frac{\varepsilon-1}{\nu}\hat{w}_t$

Note, importantly, that combining the first two equations we obtain a core summary equation of our framework, that captures the intimate link between inequality and interest rates – and hence monetary policy:

$$E_t\hat{c}^P_{t+1} - E_t\hat{c}^N_{t+1} = \frac{\gamma^{-1}}{1-\alpha\beta}i_t \tag{14}$$

7 Money growth rules

Under a policy of choosing the money growth rule, the equilibrium conditions of our model boil down to (where we use the extra notation $\frac{\varphi}{1-(1-n)\frac{\delta}{c}} = \bar{\varphi}; \frac{\varepsilon-1}{\nu} = \psi$)

$$\hat{c}_t^N = \frac{\mu}{h}\frac{1-\alpha}{1-n}\left(\hat{m}_t^{tot} - \hat{\pi}_t\right) + \frac{\mu}{h}x_t \tag{15}$$

Money accumulation, Euler equation (money demand) and Phillips curve

$$m_{t+1} = m_t - \hat{\pi}_t + x_t \tag{16}$$

$$\hat{c}_t^P = \alpha\beta E_t\hat{c}_{t+1}^P + (1-\alpha\beta) E_t\hat{c}_{t+1}^N + \gamma^{-1}E_t\hat{\pi}_{t+1} \tag{17}$$

$$\pi_t = \beta E_t\pi_{t+1} + \psi\left(\bar{\varphi}np + \gamma\right)\hat{c}_t^P + \psi\bar{\varphi}\left(1-n\right)h\hat{c}_t^N \tag{18}$$

while the interest rate is determined by 14. Using the first (static) equation 15 and replacing it into the next three, 16, 17 and 18, this can be reduced to a 3-by-3 system in $\hat{c}_t^P, \pi_t, m_{t+1}$, with two forward-looking and one predetermined variable. x_t is an exogenous process (AR1 for instace).

Even though the model looks disarmingly simple, obtaining an analytical solution in the general case proves very challenging–which is hardly surprising, given that even the simplest monetary model with MIU and sticky prices (see e.g. Gali's textbook, Chapter 3) cannot be solved analytically. In order to help intuition for our results in this richer framework, we solve the model in three special cases: flexible prices, fixed prices, and sticky prices but infinitely elastic labor (and myopic firms, or a static Phillips curve). We then provide a numerical solution for a more general case.

7.1 Flexible prices

Under $\nu = 0$, we denote variables with a star and rewrite in terms of aggregate consumption, the Phillips curve becomes

$$c_t^{N*} = \frac{\bar{\varphi}\gamma^{-1}np + 1}{1-n}\hat{c}_t^*$$

After repeated substitutions, one can show that the model boils down to one difference equation

$$\hat{c}_t^* = \left(1 - \frac{\frac{\gamma\bar{\varphi}^{-1}+1}{(1-n)h}\left(1-\alpha\beta\right)}{1 + \frac{1}{\mu}\frac{\gamma^{-1}np+\bar{\varphi}^{-1}}{1-\alpha}}\right)E_t\hat{c}_{t+1}^* - \bar{\varphi}^{-1}\frac{\left(1-\frac{n}{1-\rho}\right)}{1+\frac{1}{\mu}\frac{\gamma^{-1}np+\bar{\varphi}^{-1}}{1-\alpha}}x_t - \bar{\varphi}^{-1}\frac{\frac{n}{1-\rho}}{1+\frac{1}{\mu}\frac{\gamma^{-1}np+\bar{\varphi}^{-1}}{1-\alpha}}E_tx_{t+1}$$

It can be shown that there always exists a unique equilibrium (coefficient on expected consumption is smaller than one). The solution is, for AR1 x with coefficient θ, $E_t x_{t+1} = \theta x_t$

$$\hat{c}_t^* = \bar{\varphi}^{-1} \frac{\frac{n}{1-\rho}(1-\theta) - 1}{1 + \frac{1}{\mu}\frac{\gamma^{-1}np + \bar{\varphi}^{-1}}{1-\alpha}} \sum_{j=0}^{\infty} \left(1 - \frac{\frac{\gamma\bar{\varphi}^{-1}+1}{(1-n)h}(1-\alpha\beta)}{1 + \frac{1}{\mu}\frac{\gamma^{-1}np + \bar{\varphi}^{-1}}{1-\alpha}} \right)^j x_{t+j}$$

Monetary expansion generates a boom as long as

$$\alpha + \rho > 1 + \theta$$

i.e. as long as the shock is not too persistent. The full solution is

$$\hat{c}_t^* = \bar{\varphi}^{-1} \frac{\frac{n}{1-\rho}(1-\theta) - 1}{\left(1 + \frac{1}{\mu}\frac{\bar{\gamma}^{-1} + \bar{\varphi}^{-1}}{1-\alpha}\right)(1-\theta) + \theta \frac{\gamma\bar{\varphi}^{-1}+1}{(1-n)\omega^N}(1-\alpha\beta)} x_t$$

The effect on interest rates is given by

$$\begin{aligned} i_t &= \frac{1-\alpha\beta}{\bar{\gamma}^{-1}} E_t \hat{c}_{t+1} - \frac{1-\alpha\beta}{\bar{\gamma}^{-1}\omega^N} E_t c_{t+1}^N \\ &= -(1-\alpha\beta)\frac{\bar{\varphi}+\gamma}{(1-n)\,\omega^N} E_t \hat{c}_{t+1}^* \end{aligned}$$

If shock is persistent, and expansion condition holds, there is a liquidity effect:

$$i_t = -\frac{\left(1+\gamma\bar{\varphi}^{-1}\right)(1-\alpha\beta)}{(1-n)\,h} \frac{\frac{n}{1-\rho}(1-\theta) - 1}{\left(1 + \frac{1}{\mu}\frac{\bar{\gamma}^{-1} + \bar{\varphi}^{-1}}{1-\alpha}\right)(1-\theta) + \theta \frac{\gamma\bar{\varphi}^{-1}+1}{(1-n)h}(1-\alpha\beta)} \theta x_t$$

The response of inflation is, considering iid shocks for simplicity:

$$\hat{\pi}_t^* = \hat{m}_t^* + \frac{\frac{n}{1-\rho} + \frac{1}{\mu}\frac{\bar{\gamma}^{-1}+\bar{\varphi}^{-1}}{1-\alpha}}{1 + \frac{1}{\mu}\frac{\bar{\gamma}^{-1}+\bar{\varphi}^{-1}}{1-\alpha}} x_t$$

which is more than 1-to1 since

$$\alpha + \rho > 1$$

Defining a measure of inequality (in units of SS aggregate consumption) as

$$q_t = c_t^P - c_t^N = \frac{1}{n}\left(\hat{c}_t - c_t^N\right) = -\frac{1 + p\bar{\varphi}\gamma^{-1}}{1-n}\hat{c}_t^*,$$

the following proposition immediately follows

Proposition 1 *Insofar as a monetary expansion creates a consumption boom, it always decreases inequality.*

Finally, notice that there is no endogenous persistence: an iid shock has iid effect under flexible prices.

7.2 Fixed prices

Take the other extreme, that of fixed prices $\psi = 0$. The Phillips curve implies

$$\pi_t = 0 \tag{19}$$

It follows immediately that money has a unit root

$$m_{t+1} = m_t + x_t, \tag{20}$$

and hence iid shocks will in fact have permanent effects moving the economy to a new steady state. To anticipate, for intermediary levels of stickiness we will obtain (by continuity) endogenous persistence. Consumption of nonparticipants is

$$\hat{c}_t^N = \frac{\mu}{h}\frac{1-\alpha}{1-n}\hat{m}_t^{tot} + \frac{\mu}{h}x_t \tag{21}$$

And the model boils down again to one difference equation

$$\hat{c}_t^P = \alpha\beta E_t\hat{c}_{t+1}^P + (1-\alpha\beta)\frac{\mu}{h}\frac{1-\alpha}{1-n}m_t + (1-\alpha\beta)\frac{\mu}{h}\left(\frac{1-\alpha}{1-n}+\theta\right)x_t \tag{22}$$

that can be solved to deliver

$$\begin{aligned} c_t^P &= \frac{\mu}{h}\frac{1-\alpha}{1-n}m_t + \Omega_{Px}x_t, \\ \Omega_{Px} &= \frac{2-\alpha-\rho+(1-\alpha\beta)\theta}{1-\alpha\beta\theta}\frac{\mu}{h} \end{aligned}$$

Therefore for aggregate consumption

$$\begin{aligned} c_t &= \frac{\mu}{h}\frac{1-\alpha}{1-n}m_t + \Omega_{cx}x_t, \\ \Omega_{cx} &= \frac{1+np(1-\alpha-\rho)+(np-\alpha\beta)\theta}{1-\alpha\beta\theta}\frac{\mu}{h} \end{aligned}$$

The equilibrium interest rate is

$$i_t = -\theta\frac{\mu}{h}\frac{1-\alpha\beta}{\gamma^{-1}}\frac{\alpha+\rho-1-\theta}{1-\alpha\beta\theta}x_t,$$

which shows that if the shock is persistent, there is a liquidity effect (if it is not not "too persistent", i.e. as long as $\theta < \alpha+\rho-1$). It follows that inequality is reduced whenever the liquidity effect is operating.

7.3 Sticky prices, special case

In order to obtain analytical results, we simplify the model further in the case of arbitrary price stickiness: in particular, we assume that the discount factor of price-setter is zero, that is that the Phillips curve is static. Since the role of the forward-looking Phillips curve is well udnerstood and our moedl has a very rich intertemporal structure on the demand side, we fel this is a small sacrifice to make–which we nevertheless correct when solving the model numerically in the next section. In addition, we assume that labor supply is infinitely elastic–again, something we relax in the next section.

Under these assumptions, the Phillips curve is:

$$\pi_t = \psi\gamma\hat{c}_t^P \tag{23}$$

while the other equations are unchanged.

Letting $A \equiv (1-\alpha\beta)\frac{\mu}{h}\frac{1-\alpha}{1-n} = (1-\alpha\beta)\left\{1-\frac{\delta}{c}\left[1+n\frac{1-\beta}{\beta(1-\alpha)}\right]\right\}$, we have, eliminating inflation:

$$m_t = (1+\psi\gamma A)\, m_{t+1} + \psi\gamma\,(\alpha\beta+\psi-A\psi\gamma)\, E_t\hat{c}_{t+1}^P + \psi\gamma\,(1-\alpha\beta)\,\frac{\mu}{h}\theta x_t - x_t \tag{24}$$

$$\hat{c}_t^P = (\alpha\beta+\psi-A\psi\gamma)\, E_t\hat{c}_{t+1}^P + Am_{t+1} + (1-\alpha\beta)\,\frac{\mu}{h}\theta x_t \tag{25}$$

It can be easily shown that the transition matrix has one eigenvalue inside and one outside the unit circle, ensuring a unique stable equilibrium.

In order to solve the model, we focus on iid shocks $\theta = 0$ – unde rwhich assumption the aggregate state of this model is simply $m_t + x_t$, and the policy rules are of the form

$$m_{t+1} = \Omega_m\,(m_t + x_t)\,;\, c_t^P = \Omega_P\,(m_t + x_t)$$

The method of undetermined coefficient readily delivers (choosing the appropriate roots of a quadratic):

$$\Omega_m = \frac{(1+\alpha\beta+\psi) - \sqrt{(1+\alpha\beta+\psi)^2 - 4\,(\alpha\beta+\psi-A\psi\gamma)}}{2\,(\alpha\beta+\psi-A\psi\gamma)}$$

$$\Omega_P = \frac{\alpha\beta+\psi-2A\psi\gamma-1+\sqrt{(1+\alpha\beta+\psi)^2 - 4\,(\alpha\beta+\psi-A\psi\gamma)}}{2\psi\gamma\,(\alpha\beta+\psi-A\psi\gamma)}$$

Notice that Ω_m is a measure of persistence for this model, and it is increasing with price stickiness.

TO BE COMPLETED

7.4 General case

Solving the model numerically for forward-looking Phillips curve with slope $\psi = 0.05$ and $\varphi = 2$, effects of iid monetary shocks and technology shocks.

FIGURES

8 Interest rate rules

Using the same notation as previously but considering that transfers x_t now adjust endogenously, we have that under an interest rate rule equilibrium is determined by

$$\hat{c}_t^N = \frac{\mu}{h}\left[\hat{m}_{t+1}^{tot} - (\alpha + \rho - 1)\left(\hat{m}_t^{tot} - \hat{\pi}_t\right)\right]$$

$$\pi_t = \beta E_t \pi_{t+1} + \psi\left(np\bar{\varphi} + \gamma\right)\hat{c}_t^P + \psi\bar{\varphi}\left(1 - n\right)h\hat{c}_t^N$$

$$\hat{c}_t^P = E_t \hat{c}_{t+1}^P - \gamma^{-1}\left(i_t - E_t\hat{\pi}_{t+1}\right)$$

plus

$$E_t\hat{c}_{t+1}^P - E_t\hat{c}_{t+1}^N = \frac{\gamma^{-1}}{1 - \alpha\beta} i_t,$$

and a Taylor rule. Notice that a Taylor rule that responds to expected inflation $i_t = \phi E_t \hat{\pi}_{t+1} + \varepsilon_t$ always generates indeterminacy because of our new equation – it does not pin down actual values, but only expectations. Therefore, we concentrate on rules that respond to realized inflation.

For such rules, we show that a stronger version of the Taylor principle holds.

TBC

Solution – no endogenous persistence. Liquidity effect?

9 References

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