College Pricing and Income Inequality

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PRELIMINARY AND INCOMPLETE

1 Introduction

Rising college tuition costs have become a major concern for households with children. Policymakers worry that rising tuition costs may put a college education out of reach for high ability children from low income households. Before considering possible policy interventions, it is important to understand what is driving tuition up. In this paper we evaluate the hypothesis that increases in the price of college are driven by rising income inequality. Because a college degree is a big lumpy investment, the top of the income distribution is where customers for college are disproportionately located. Thus, rapid income gains at the top of the income distribution and corresponding growth in demand for high quality schools would appear to be a plausible mechanism for driving up college tuition.

While the average cost of college has been rising much faster than general inflation for decades, it has not risen uniformly across different colleges and different students. First, tuition dispersion across schools has been increasing. For example, tuition at private four year schools has risen much faster than tuition at community colleges. Second, tuition dispersion within schools has also risen dramatically. In particular, the gap between sticker price tuition and average tuition actually paid has risen, with applicants that colleges are especially keen to admit receiving large discounts. Finally,

while the college wage premium has widened over time, college enrollment has remained relatively stagnant since the mid 1990s. We will argue that widening income inequality is the single common cause underlying all of these trends.

In our model we will take the view that colleges operate as profit-maximizing businesses. Profit maximization will be a natural objective in our model because we model the college market as competitive with free entry: any objective besides profit maximization would lead to operational losses. In practice there is a large public presence in higher education, and subsidies to this sector potentially allow it to pursue additional objectives besides profit-maxization. Still, the higher education market is becoming increasingly competitive, and thus the maximally competitive environment is a natural benchmark.

To understand how college education is priced, we need a model of what a college education is. We follow the existing literature that recognizes two determining factors in the quality of a college education: the physical resources spent on each student (faculty, classrooms, computers), and the average ability of the student body, which could be interpreted as capturing average IQ or college preparedness. There are two reasons why schools with higher average test scores might be more attractive to college applicants: (i) they offer better prospects of learning from peer students, and (ii) they offer social and professional connections to people who are likely to be succesful post-graduation. One clue that average ability is quantitatively important is that the influential US News ranking of college quality puts a large weight on the average SAT / ACT scores of incoming students.

We therefore construct a model in which households differ with respect to household income and the ability of the household child. For simplicity, we assume the latter can only take two values. Colleges can observe both income and ability (e.g. by observing test scores) and are allowed to price discriminate in both dimensions. Households observe tuition schedules for colleges of different quality levels, and decide whether to send their child to college, and if so, to which quality level of college. They value standard consumption and get direct utility from college quality.

On the supply side, there is free entry. Colleges can enter at any quality level, using a constant returns to scale technology to deliver college spaces. Each college slot requires a fixed expenditure on resources (room and board). Colleges seek to deliver any chosen quality level most cheaply by choosing the optimal mix of variable resources versus average student ability.

The model equilibrium has the property that holding fixed ability, chosen college quality is increasing in income, and holding fixed income, chosen quality is increasing in ability. Price discrimination by income does not survive in a competitive environment, but colleges do offer lower tuition to high ability students, reflecting the fact that they effectively make the college more atttractive for other students.

As in other club-good models, solving for competitive equilibrium is more complicated than in models where the quality of the product sold is independent of the attributes of the pool of customers. In particular, for each quality level, the number of college places demanded by both high and low ability students must equate to the corresponding numbers of students that colleges at that quality level want to employ as quality-producing inputs. This imples that tuition is not a linear function of quality, but depends on the shape of the income distribution. Moreover the tuition-quality profile can move in interesting ways with changes in the income distribution.

We first describe how to compute equilibrium in this environment. We then calibrate the model parameters to replicate the observed distribution of income, and to replicate a set of facts about college attendance and college tuition.

Our key experiments involve simulating the impact of an exogenous change in the distribution of income resembing the change in the household income distribution observed in the United States over the past 30 years. Given the new income distribution we conduct two sets of simulations. In the first, short-run experiment, we hold fixed the distribution of college quality, and compute new profit-maximizing tuition schedules, assuming college spots cannot go unfilled. In the second, long-run experiment, we allow for entry and exit, and solve for the new distribution of college quality at which all operating colleges make exactly zero profits.

2 Model Details

- The model is static

2.1 Households

- Each household contains a parent and a college-age child. The parent makes all decisions and maximizes its own welfare
- Households differ in two dimensions: parent income y and student ability a
- There are two ability levels, $a \in \{a_l, a_h\}$ where a_l is low ability and a_h is high ability. We will normalize $a_h = 1$.
- There is a continuous distribution for income $y \in [y_{\min}, y_{\max}]$ where $y_{\max}$ might be infinity. Let $F(y; a)$ denote the distribution for income conditional on ability a.
- Colleges differ by quality q. Households derive utility from non-durable consumption c and from the quality of their child's education
- Households enjoy utility $u(c, q)$ from consumption c and college quality q. The particular utility function we will assume is

$$u(c, q) = \log c + \varphi \log(\kappa + q)$$

  An alternative assumption to making utility a function of q is to instead posit that utility is a function of the child's human capital, where the child's human capital depends on both q and a.
- The household's problem is to choose what quality college to attend among the set Q of qualities offered. College tuition $t^a(q)$ is potentially

a function of ability.

$$\begin{aligned} & \max_{q \in Q} \{u(c,q)\} \\ & s.t. \\ c &= y - t^a(q) \end{aligned}$$

- Let $q^*(a,y)$ denote the solution to this problem. In case of indifference, we will assume the household chooses the better quality school.

2.2 Colleges

- Each college of a given type q offers the same quality level to all admitted students.
- The technology for producing college slots is constant returns to scale.
- The quality per student depends on the average ability of the student body a and the quantity of variable resources e devoted

$$\begin{aligned} q &= a^\theta e^{1-\theta} \\ a &= \eta a_h + (1-\eta) a_l \end{aligned}$$

  where η is the fraction of the student body that is high ability
- Each student admitted requires a fixed resource cost ϕ (room and board and basic facilities).
- Colleges profit maximize. In equilibrium, the market for college is perfectly competitive, and colleges take tuition functions $t^a(q)$ as given.
- Perfect competition means that colleges cannot price discriminate based on income (a college profitably charging higher income students more than low income students of the same ability would be undercut)
- Given a target quality level $\hat{q}$, a college seeks to maximize per student

net profit, by solving

$$\begin{aligned} & \max_{\eta,e} \left\{ p^h(\hat{q})\eta + p^l(\hat{q})\left(1-\eta\right) - p_e e - \phi \right\} \\ & s.t. \\ \hat{q} \quad = \quad & \left(\eta a_h + \left(1-\eta\right) a_l\right)^{\theta} e^{1-\theta} \end{aligned}$$

where p_e is the price of the variable resource input in units of the consumption good.

- For any potential quality level q, let $\eta(q)$ and $e(q)$ denote the values for η and e that solve the firms problem
- Let $\pi(q)$ denote profit per student for colleges of quality q

$$\pi(q) = \eta(q)t^h(q) + (1-\eta(q))t^l(q) - p_e e(q) - \phi$$

- For most of our analysis we will assume free entry in the college business, which implies $\pi(q) = 0$ for all q.
- Let $\chi(q)$ denote the measure of college places in colleges of quality q. Because the college production function is CRS, the size distribution of colleges at any given quality level is indeterminate and irrelevant.
- We label $q = 0$ (a zero price option) as a choice not to go to college.
- There are two reasons households might choose not to go to college.

    1. (i) Even if they don't go to college, with $\kappa > 0$ their marginal utility from quality is bounded.

    (ii) Because there is a fixed per student cost to creating a college spot, even the cheapest college might be outside the budget set of poor agents.

- Let $\chi(0)$ denote the measure of individuals choosing not to go to college.

2.3 Definition of Equilibrium

- Assume the total measure of households is one. The measure of high (low) ability households is $\frac{1}{2}$.
- An equilibrium is a set of functions $\chi(q)$, $\eta(q)$, $t^a(q)$, $q(y,a)$, $e(q)$ such that

  1. Given $t^a(q)$, $q(y,a)$ solves the household's problem
  2. Given $t^a(q)$, $\eta(q)$ and $e(q)$ solve the college's problem
  3. For all q s.t. $\chi(q) > 0$, $\pi(q) = 0$. For all q s.t. $\chi(q) = 0$, $\pi(q) \leq 0$. All colleges make zero profits at quality levels that are offered in equilibrium, and if a marginal new college were to enter at any quality level not offered, that college would make negative profits
  4. The number of high and low ability students wanting to purchase college spots at each quality level q is equal to the number of spots for high and low ability students that colleges are creating at that quality level, i.e., for all q

$$\begin{aligned} \frac{1}{2}\int 1_{q^*(y,a^h)=q} dF^{a^h} y &= \eta(q)\chi(q) \\ \frac{1}{2}\int 1_{q^*(y,a^l)=q} dF^{a^l} y &= (1-\eta(q))\chi(q) \end{aligned}$$

2.4 Equilibrium Characterization

We conjecture that there exists an equilibrium with the following properties.

(i) Tuition, conditional on ability, is strictly increasing in quality q (otherwise no-one would choose the worse college).

(ii) College quality choices, conditional on ability, are (weakly) increasing in household income y

Given the second property, let $\bar{y}(a, \tilde{q}; t^a)$ denote the maximum value for income such that $q^*(a, y; t^a) \leq \tilde{q}$. We can write equilibrium condition 4 as

follows. For all $\tilde{q}$

$$\begin{aligned}\frac{F\left(\bar{y}(a^h,\tilde{q};t^h);a^h\right)}{2} &= \int_0^{\tilde{q}}\eta(q)\chi(q)dq \\ \frac{F\left(\bar{y}(a^l,\tilde{q};t^l);a^l\right)}{2} &= \int_0^{\tilde{q}}(1-\eta(q))\chi(q)dq\end{aligned}$$

The LHS of this expression is the measure of high ability households wanting to buy colleges of quality less than or equal to $\tilde{q}$. The RHS of this expression is the number of college spots created for high ability households at colleges of quality less than or equal to $\tilde{q}$

It is possible we can write this in a different way if in equilibrium there is a compact set of values $Q = [q_{\min}, q_{\max}]$ s.t. for all $q \in Q$, $\chi(q) > 0$, and, conditional on an ability level a, there is a unique value for income y s.t. $q^*(a, y) = q$. Then we can write

$$\begin{aligned}\frac{F\left(\bar{y}(a^h,q_{\min};t^h);a^h\right)}{2} &= \eta(0)\chi(0) \\ \frac{F\left(\bar{y}(a^h,q_{\min};t^h);a^h\right)}{2} &= (1-\eta(0))\chi(0)\end{aligned}$$

and for all $q > q_{\min}$

$$\begin{aligned}\frac{f\left(\bar{y}(a^h,\tilde{q};t^h);a^h\right)}{2} &= \eta(\tilde{q})\chi(\tilde{q}) \\ \frac{F\left(\bar{y}(a^l,\tilde{q};t^l);a^l\right)}{2} &= (1-\eta(\tilde{q}))\chi(\tilde{q})\end{aligned}$$

3 Understanding College Pricing

The first part of the paper is about describing how colleges end up being priced, and how students get allocated to colleges in equilibrium. Holding fixed ability, higher income students go to better colleges, and holding fixed income, higher ability students go to better colleges. Pricing is such that all colleges – at every quality level that is offered in equilibrium – make zero profits, and there is no profitable entry at quality levels that are not offered

in equilibrium. It is also such that the masses of high and low ability students who optimally chose any particular quality level offered in equilibrium are equal to the masses of spots for the two ability levels at that quality level.

We contrast the equilibrium price schedules with those that would obtain if college was not a club good (e.g. if college was like fish, so that people could freely separate the quantity of fish they supply to the market from the amount that they buy from the market)

We explain that we cannot have lots of high ability students at low quality schools, and the way prices adjust to ensure that this doesn't happen in equilibrium is by making low quality education expensive for high ability students.

But because low quality education must be expensive for high ability students it must also (by free entry) be cheap for low quality students. So, relative to a linear (non-club) pricing model, high ability students over-pay for low quality education. For high quality education, pricing must be such that low quality students choose not to buy. This might dictate high prices for low quality students, but high prices might not be necessary given that the low quality schools are cheap and relatively attractive for the low ability types.

4 The Impact of Rising Income Inequality

The second part of the paper is about understanding what happens when income inequality increases.

We distinguish between short and long run responses: in the short run, the distribution of college qualities is held fixed, in the long run free entry means the equilibrium distribution changes

We first try to prove that for any specific fixed distribution of college quality, if income inequality changes in a particular way – eg if income rises for every student attending every equilibrium quality level – then average tuition will go up, and the price-quality profile will get steeper.

We then do a calibrated numerical exericse.

5 Theoretical Results In Short-Run Equilibrium

In a short-run equilibrium where the distribution of college is held exogenously, it is possible to derive an analytical expression for equilibrium price.

5.1 Case 1: continuous college distribution

Assumptions we make are as follows:

1. $\theta = 0$; quality of the college only depends on student ability. This assumption is important for the analytical result and has been used in the literature
2. Log utility as defined.
3. $\chi(q)$ has a convex support. i.e. there exists an interval $[q_{\min}, q_{\max}]$ such that for any $q \in [q_{\min}, q_{\max}], \chi(q) > 0$
4. $y^a(i)$ are differentiable functions. This is a technical assumption that allows us to use integration by parts.

The first theoretical result is the following:

If assumption 1 and 2 holds, then for any $\chi(q)$ that satisfies assumption 3 and $y^a(i)$ that satisfies assumption 4:

$$t^a(q) = y^a(i^a(q)) - (\kappa+q)^{-\varphi}\int_{i_a^*}^{i^a(q)} (\kappa + q^a(j))^{\varphi}\, dy^a(j) - \frac{\kappa^{\varphi}}{(\kappa+q)^{\varphi}} y(i_a^*)$$

Where $i^a(q), q^a(i), i_a^*$ are endogenous objects defined solely as functions of $\chi(q)$. see notes Theoretical_proof.tex for details.

To conduct comparative statics of how the tuition function varies with the variance parameter, we need to make further assumptions on income distribution $y^a(i)$ and college distribution $\chi(q)$. The general statement is:

Denote v^y the second moment of $y^a(i)$. if $y^a(i)$ satisfies some properties and $\chi(q)$ satisfies some properties.

$$\begin{aligned}
\frac{\partial t^a(q)}{\partial v^y} &> 0 \text{ [Tuition goes up]} \\
\frac{\partial t^{a\prime}(q)}{\partial v^y} &> 0 \text{ [Tuition schedule becomes steeper]} \\
\frac{\partial \left[t^l(q) - t^h(q)\right]}{\partial v^y} &> 0 \text{ [Tuition becomes more expensive for low-ability within the same college]}
\end{aligned}$$

5.2 Case 2: discrete college distribution

In this case, sharper results can be obtained for arbitrary $\chi(q)$ and assuming that income is either pareto or uninform. See notes Theoretical_proof.tex

6 Calibration

We assume that half households are high ability, and half are low ability. For the cost of college, we focus on targets for tuition plus room and board actually paid (not sticker price, and not the actual resource cost of providing the education). We take enrollment targets from all 4 year colleges, but focus on tuition numbers for private 4 year colleges. We effectively assume the entire sector is private and abstract from the interaction between the private and public sectors.

1. The income distribution parameters (exogenous, from data). We use an EMG distribution. We normalize mean household income to one. The variance of the normal component is 0.4117 and the exponential parameter is 2.2.(both from the SCF). The mean parameter differs across the high and low ability types. The mean parameter is set so that the ratio of income of high versus low ability is consistent with the evidence on average family income conditional on the child's AFQT score being above or below the median, from the 1997 NLSY. Gianluca says the average household income is \$56,000, and the average income for those below the median is \$45,000, so the average income for those

above is $67,000. We also truncate the EMG distribution so that the richest household of a given type makes 40 times average income for that type.

2. The ability gap between high and low ability students. We target the average differential between sticker price and net price paid for tuition plus room and board. We assume that sticker price is the price paid by the low ability type, and the net price is the average price paid across the two types. The data here is from the college board, Figure 13, year 2013-2014 (CollegeFinance_Final.xls). Sticker Tuition + RB at private nonprofit 4 year schools is 41,770. Net tuition + RB was 22,900. Of the $18,870 difference, $12,380 was institutional aid, while the rest was other aid. We assume that average net tuition + RB is equal to

$$\int \chi(q)(\eta(q)t_1(q) + (1 - \eta(q)t_2(q))dq = \$22,900$$

We assume that average sticker tuition plus RB is

$$\int \chi(q)t_2(q)dq = \$41,770 - (18,870 - 12,380)$$

Note that there is no $(1 - \eta(q))$ here. We assume all the institutional aid goes to the high ability students, while everyone gets the other aid. We compute the analogous statistic in the model.

3. The preference parameters φ and κ, and the technology parameters θ and ϕ.

   (a) We set $\theta = 0.5$ (Caucutt, IER). This is also broadly consistent with how US News ranks colleges. Plus we will experiment with alternatives.

   (b) That leaves 3 parameters, for which we have 3 targets: aggregate enrollment, the share of aggregate consumption devoted to 4 year colleges, and the ratio of tuition to room and board payments,

which we identify with fixed and variable costs of education.

(c) We take aggregate enrollment from the CPS ASEC, Educational Attainment in the US, which is 31.9%. This is the fraction of the US population aged 25 or higher in 2014 with at least 4 years of college. The total breakdown is: <HS 15.8%, HS 27.1%, <4yrC 25.3%, 4yrC 15.1%, >4yrC 16.8%.

(d) From the College Board, in 2014-2015, average net tuition and fees is \$12,360 for net tuition and average net room and board is \$11,190. So out of total spending 52.5% (12360/(12360+11190) represents variable costs (E) and 47.5% fixed costs (ϕ).

(e) What share of expenditure goes on college? Ours is a static model. We proceed as follows. Average tuition plus RB in 2014-15 is \$23,550. GDP per capita in 2014 was \$54,630 (2013 was \$52,980). In 2012, 10.34 million people were attending 4 year college. This is 10.34 / 314.1 = 3.29% of the US population. C in 2014 is 11,866/17,348 = 68.4% of GDP. So total resources devoted to 4 year college, as a share of total consumption is (23,550*0.0329/52,980)/0.684 = 2.14%.

7 Longer Run Project

Make the model dynamic, which college quality + ability determining future income.

We could consider a shock such that college quality comes to have a larger impact on income, so that income inequality increases endogenously, and evolves dynamically over time across generations.

This dynamic model needs some more ingredients.

First, an exogenous process defining the inter-generational transmission of ability. For example, we could assume that the probability the child is high (low) ability given the parent is high (low) ability is $\pi > 0.5$.

Second, a mapping from ability and quality into income, $y'(a, q)$. For example $y' = a^{\theta} q^{1-\theta}$.

To simulate this model, we would (i) start with an initial distribution over (a, y), (ii) solve for equilibrium $q(a, y)$ using our current static model, (iii) Use the mapping $y'(a, q)$ and the process for a to compute a new distribution over (a, y). Presumably, this joint distribution will converge to something.

Then, given that ergodic distribution, we could consider an exogenous permanent shock to θ, such that college quality becomes more important and raw ability less important in determining next generation income. We simulate the model to compute the new stationary distribution for college quality and for income.

There are lots of potentially interesting questions we can ask here: Do we get more income inequality in the long run? Do we get less inter-generational income mobility? The answer to the second may be "yes" because parental income y has an impact on q, which now has a larger impact on y'.

8 To Do List

1. Read the theoretical club literature. Sort of done. First welfare theorem would seem to apply to our economy. There would likely be welfare gains from introducing lotteries.

2. Try to get some theoretical results that apply with a general distribution of college quality.

3. Understand better the price differences between the linear pricing model, and the club-good pricing model.