

Identifying ambiguity shocks in business cycle models using survey data*

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Abstract

We develop a macroeconomic framework with agents facing time-varying concerns for model misspecification. These concerns lead agents to interpret the economy through the lens of a pessimistically biased ‘worst-case’ model. We use survey data to identify exogenous fluctuations in the worst-case model. In an estimated New-Keynesian business cycle model with frictional labor markets, these ambiguity shocks explain a substantial portion of the variation in labor market quantities.

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1 Introduction

Equilibrium outcomes in the macroeconomy depend on the belief formation mechanism of economic agents. While the rational expectations assumption is in many cases a fruitful benchmark that allows transparent estimation and testing of economic models using time-series data, there is extensive empirical evidence against this assumption. However, if we are to dispense with rational expectations, we need to replace them with a belief formation framework that preserves structural integrity and testability, and allows us to understand how deviations of agents' subjective beliefs interact with economic dynamics.

In this paper, we provide a tightly specified framework that links agents' decisions and beliefs with observable economic outcomes and survey data. The theoretical foundation of the belief formation mechanism is our extension of the robust preference model of Hansen and Sargent (2001a,b). Agents endowed with robust preferences are concerned that the particular model they view as their 'benchmark' model of the economy may be misspecified. Instead of only using only the benchmark model, they consider a whole set of models that are statistically hard to distinguish from the benchmark model. The concerns for model misspecification lead them to choose the model from this set that delivers the lowest utility. This 'worst-case' model is then the basis for their decisions, akin the utility-minimizing prior in the multiple prior framework of Gilboa and Schmeidler (1989) and Epstein and Schneider (2003). The robust preference framework thus delivers a specific form of ambiguity aversion.

We extend this robust preference framework to allow the agents to be exposed to shocks to their ambiguity aversion. The time-variation in ambiguity aversion induces fluctuations in agents' worst-case beliefs and endogenously affects equilibrium dynamics. While our extension delivers a more flexible specification of the time-variation in the worst-case model, it still tightly restricts the beliefs across alternative states in a given period. Agents fear outcomes with adverse utility consequences and overweigh their probabilities in a specific way.

In order to identify the variation in the worst-case model empirically, we assume that agents' forecasts in the survey data are based on their worst-case model. Our theoretical model yields directly testable predictions about the comovement of these forecasts under the worst-case model. We show that household forecasts for key macroeconomic variables in the University of Michigan Surveys of Consumers are indeed significantly pessimistically biased, with a discernible business cycle component. We start by estimating a vector-autoregression (VAR) that embeds household survey data, explicitly restricting the belief distortion (or wedge) between the worst-case model and the data-generating probability measure. A common component of these belief distortions in different survey answers identifies a latent factor that captures the time-variation in the worst-case model, and its impact on observable macroeconomic quantities.

We then combine the robust preference framework and the survey data in a dynamic stochastic general equilibrium model with frictional labor markets, sticky prices and a monetary authority that follows an interest rate rule. We estimate this model using Bayesian methods and study the quantitative role of the ambiguity shocks in the dynamics of the labor market, identification of the

monetary policy rule and the comovement of macroeconomic variables.

The results from the reduced-form and structural models show a common pattern. The worst-case belief is identified from the common variation of the biases in survey answers, and it explains a substantial amount of variation in these biases, in particular in the households' forecasts of unemployment and GDP growth. Ambiguity-averse households interpret high unemployment and low GDP growth states as particularly adverse to their utility, and overweight worst-case probabilities of those states substantially.

An adverse ambiguity shock also has significant contractionary effects, propagated particularly strongly through the labor market. In the labor market with search and matching, creation of new matches and hiring depend on the assessment of the future surpluses generated in a new match. An increase in ambiguity leads to a more pessimistic evaluation of future surpluses and therefore to lower match creation, which increases unemployment and decreases output.

The paper contributes to the growing literature that quantitatively assesses the role of ambiguity aversion in the macroeconomy, building on alternative decision-theoretical foundations by [Gilboa and Schmeidler \(1989\)](#), [Epstein and Schneider \(2003\)](#), [Klibanoff et al. \(2005, 2009\)](#), [Hansen and Sargent \(2001a,b\)](#), [Strzalecki \(2011\)](#) and others. Applications to macroeconomic models include [Cagetti et al. \(2002\)](#) and [Bidder and Smith \(2012\)](#). For a survey of applications in finance, see [Epstein and Schneider \(2010\)](#).

Perhaps the closest to our paper is the work by [Ilut and Schneider \(2014\)](#) and [Bianchi et al. \(2014\)](#) who utilize the recursive multiple-prior preferences of [Epstein and Schneider \(2003\)](#). The first crucial difference lies in the fact that the multiple-prior framework does not impose tight a priori restrictions on the relative distortions of individual shocks under the worst-case model, and thus introduces a heavier burden on identification through observable data. We rely much more strongly on the restrictions on shock distortions implied by the robust preference framework. Second, we use data on cross-sectional average distortions measured in household survey answers, for which our theory has direct quantitative predictions, as a source of identification of the ambiguity shocks. [Ilut and Schneider \(2014\)](#) instead use the forecast *dispersion* as a proxy for confidence and show an empirically plausible relation of this measure to the notion of ambiguity aversion. Despite the differences, both these approaches should be viewed as complementary.

2 Survey expectations

We analyze the data on households' expectations from the University of Michigan Surveys of Consumers (Michigan Survey). These surveys collect answers to questions about the households' own economic situation as well as their forecasts about the future state of the economy. Specifically, we focus on the forecasts of future inflation, unemployment rate and the expected index of consumer sentiment, which we use as a proxy for GDP growth.

We are interested in deviations in these survey answers from rational expectations forecasts. This necessarily requires taking a stand on how to determine the probability measure that generates

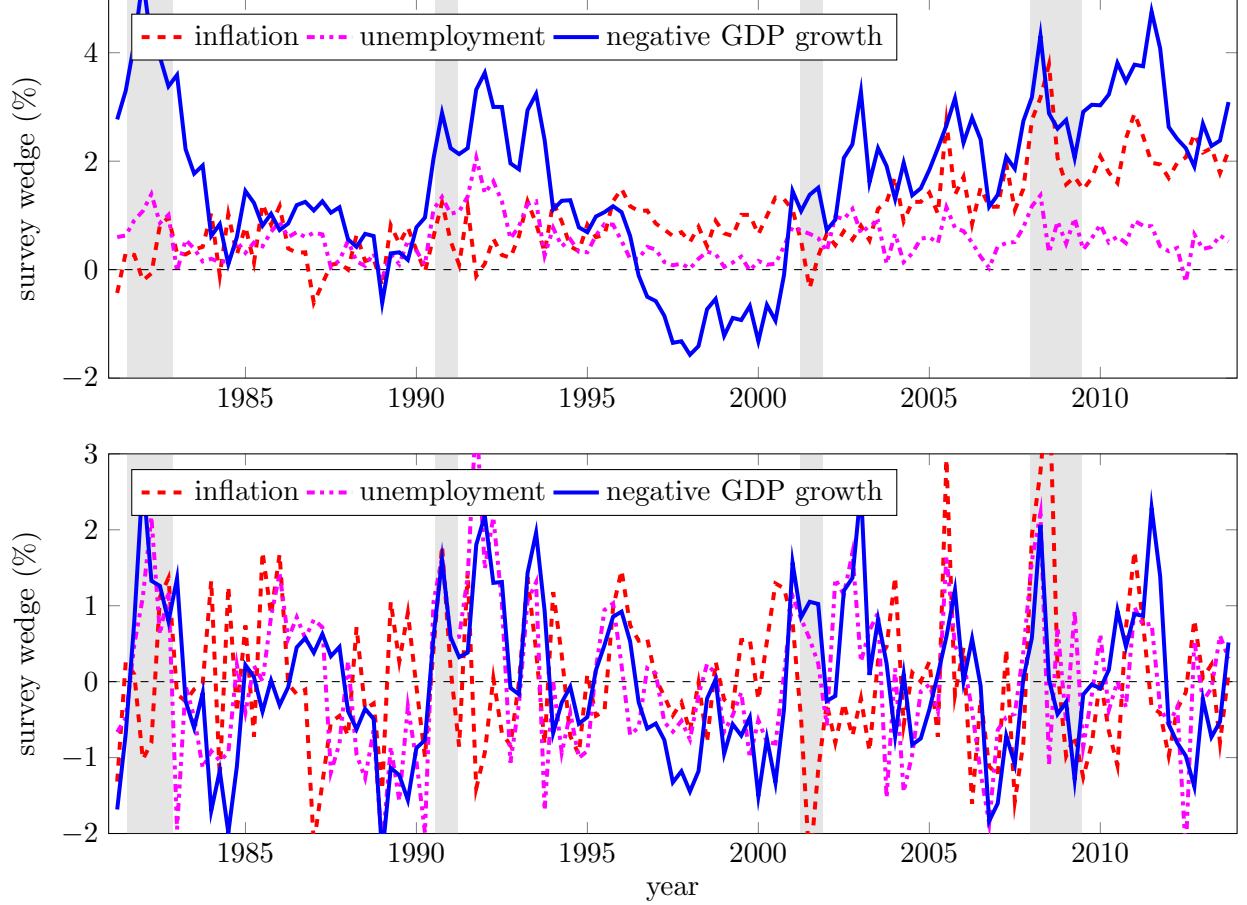


Figure 1: Difference in survey expectations between the Michigan Survey and Survey of Professional Forecasters. Top panel original data, bottom panel HP-filtered and standardized. GDP growth for the Michigan Survey proxied by the expected consumer confidence index. Details on the construction of the data series are in Appendix C. NBER recessions shaded.

the data. Here, we assume that the Survey of Professional Forecasters provides unbiased estimates for the variables we study. In Sections 3 and 5, we also contrast these household survey answers with predictions obtained from VAR and structural models.

Figure 1 shows the differences in survey expectations between the Michigan Survey and the Survey of Professional Forecasters for inflation, unemployment and GDP growth. The survey expectations are mean one-year ahead expectations in the survey samples. The Michigan Survey does not contain a question about GDP growth, and we therefore proxy it by projecting GDP growth on the survey answer on expected consumer sentiment. We detail the construction of the time series in Appendix C.

The top panel of Figure 1 reveals that households' expectations are systematically pessimistically biased — relative to professional forecasters, households overpredict future unemployment and inflation, and underpredict GDP growth (with the exception of the boom period during the late 1990s). Moreover, despite a substantial amount of noise, the three time series for the belief

wedges have a common business cycle component and are statistically significantly correlated. The correlation coefficient for the unemployment and negative GDP growth wedges is 0.52, while the correlation between the inflation and negative GDP growth wedges is 0.31, both with a standard error of 0.07. The comovement over the business cycle can be visually confirmed in the bottom panel of Figure 1 that plots HP-filtered and standardized data.

Our theoretical framework formalizes the notion of pessimistic belief distortions through the structure of the robust preference model. The common component of the three belief wedges from Figure 1 identifies the fluctuations in the worst-case model of economic agents. We embed the belief distortions in a representative agent framework, which provides a justification for using average forecasts as a measure of subjective expectations in the model.

Mankiw et al. (2003), Bachmann et al. (2012) and others use measures of cross-sectional forecast dispersion as a proxy for economic uncertainty. This proxy is typically based on the presumption that a higher dispersion is indicative of more difficulty in estimating the forecast distribution. The model we develop in this paper does not feature heterogeneity in individual forecasts, and therefore yields no predictions about forecast dispersion measures. However, it is possible to extend the framework by introducing heterogeneity in agents' concerns for uncertainty. Agents with differing degrees of ambiguity aversion deduce alternative worst-case models from observable data, which generates dispersion in forecasts in the model. While conceptually interesting, this extension is beyond the scope of this paper.

3 A one-factor model of distorted beliefs

We want to formalize the empirical facts that we established in the previous section. We start with a statistical model that describes the joint dynamics of macroeconomic variables and households' expectations. In this model, households' expectations are allowed to differ from the expectations implied by the distribution of the data-generating process. The underlying idea is to extract a common component in the variation of the belief wedges, and study its impact on the dynamics of the macroeconomic variables.

While we specify a flexible, reduced-form specification for the dynamics of observable variables, we impose tight restrictions on the households' expectations. These restrictions reflect those implied by our structural model of robust preferences that we introduce in Section 4.

We specify a $(k - 1) \times 1$ vector of observable economic variables y_t and an unobservable scalar latent process f_t . In particular, consider the model

$$\begin{aligned} y_{t+1} &= \psi_y y_t + \psi_{yf} f_{t+1} + \psi_{yw} w_{t+1}^y \\ f_{t+1} &= \psi_f f_t + \psi_{fw} w_{t+1}^f \end{aligned}$$

where $w'_{t+1} = \left((w_{t+1}^y)' , w_{t+1}^f \right) \sim N(0, I_k)$ is a $k \times 1$ vector of normally distributed iid shocks. We

can rewrite these equations, expressing the joint process $(y'_t, f_t)'$ as follows:

$$\begin{pmatrix} y_{t+1} \\ f_{t+1} \end{pmatrix} = \begin{pmatrix} \psi_y & \psi_{yf}\psi_f \\ 0 & \psi_f \end{pmatrix} \begin{pmatrix} y_t \\ f_t \end{pmatrix} + \begin{pmatrix} \psi_{yw} & \psi_{yf}\psi_{fw} \\ 0 & \psi_{fw} \end{pmatrix} \begin{pmatrix} w_{t+1}^y \\ w_{t+1}^f \end{pmatrix}. \quad (1)$$

This process generates a filtered probability space $(\Omega, \{\mathcal{F}_t\}_{t=0}^\infty, P)$ where P is the objective, data-generating probability measure. The factor f_t is exogenous to the dynamics of macroeconomic variables and will serve as a source of common variation in the dynamics of the macroeconomy and households' belief wedges.

Households' expectations are represented by a subjective probability measure $\tilde{P}$ that can differ from P . In Section 4, we derive a formal structural model for $\tilde{P}$. Here, we focus on imposing restrictions on $\tilde{P}$ that are consistent with the structural model and that allow us to identify $\tilde{P}$ using household survey data.

Let z_t be a subset of observable variables y_t for which survey data are available. We define the τ -period belief wedge $\Delta_t^{(\tau)}$ as the difference between the τ -period forecasts under the beliefs of the households and under objective expectations:

$$\Delta_t^{(\tau)} \doteq \tilde{E}_t z_{t+\tau} - E_t z_{t+\tau}$$

where $\tilde{E}_t z_{t+\tau}$ is the time- t expectation of $z_{t+\tau}$ under the subjective probability measure of the households. In addition we define the τ -period average belief wedge $\overline{\Delta}_t^{(\tau)}$ as the the average difference in forecasts under the beliefs of the households and under objective expectations:

$$\overline{\Delta}_t^{(\tau)} \doteq \frac{1}{\tau} \sum_{s=1}^{\tau} \Delta_t^{(s)}$$

We impose that the dynamics of belief wedges $\Delta_t^{(\tau)}$ and $\overline{\Delta}_t^{(\tau)}$ can be summarized using the scalar factor

$$\theta_t = (F_y, F_f) \begin{pmatrix} y_t \\ f_t \end{pmatrix}. \quad (2)$$

Individual one-period forecasts of the innovation means under the households' expectations are then represented by a vector of factor loadings H :

$$\tilde{E}_t [w_{t+1}] = H\theta_t. \quad (3)$$

Applying the law of iterated expectations, belief wedges for the τ -period forecasts can be written as

$$\Delta_t^{(\tau)} = G_x^{(\tau)} x_t + G_0^{(\tau)}$$

where the coefficients $G_x^{(\tau)}$ and $G_0^{(\tau)}$ are derived in Appendix A. The model (2)–(3) thus implies a one-factor structure of belief wedges where θ_t captures the common comovement in the belief

wedges. In this reduced form model, we interpret θ_t as the time-varying measure of pessimism among the households reflected in the survey data that impacts the dynamics of macroeconomic variables. In Section 5, this one-factor structure is derived from the decision problem of the household endowed with robust preferences, where θ_t reflects the time-variation in households' ambiguity concerns.

3.1 Data and estimation

Data on macroeconomic variables are obtained from the Federal Reserve Bank of St. Louis database (FRED), at quarterly frequency. The vector y_t includes real log GDP growth, log CPI inflation, the unemployment rate, and the Federal Funds rate. We include three belief wedges from Figure 1 in the vector $\bar{\Delta}_t^{(4)}$, constructed as 4-quarter ahead average belief wedges between the Michigan Survey and SPF forecasts for log GDP growth, the unemployment rate and log inflation. Appendix C provides details on the construction of the data, presented in Section 2. The data for y_t covers the period 1951Q2–2013Q3. The belief wedges for the unemployment rate, GDP growth and inflation cover the periods 1977Q4–2013Q3, 1968Q4–2013Q3 and 1981Q2–2013Q3 respectively.

In order to keep the estimation and interpretation of the model transparent, we restrict the dynamics of beliefs and set $F_y = 0$, thereby setting $\theta_t = f_t$. This implies that fluctuations in the belief wedges are driven purely by the belief factor f_t , and not directly by the dynamics of endogenous macroeconomic variables y_t . In addition, we normalize $F_f = 1$ and set the element of H corresponding to the GDP growth shock to be -1 in order to identify the sign and scale of θ_t .

The shock exposure matrix ψ_{yw} is only identified as the covariance matrix $\psi_{yw}\psi'_{yw}$. For the purpose of estimation, we shall impose a recursive identification scheme for ψ_{yw} . However, ψ_{yw} only appears as $\psi_{yw}\psi'_{yw}$ in the formulas for the belief wedges. Therefore, given an estimate of $\psi_{yw}\psi'_{yw}$, the identification of ψ_{yw} does not play a role in the estimation of the factor shocks w_t^f .

More specifically, we estimate the model (1) together with a vector of observation equations for the wedges

$$\bar{\Delta}_{t+1}^{(4)} = \psi_{\Delta f} f_{t+1} + \psi_{\Delta \varepsilon} \varepsilon_{t+1}^{\Delta}$$

where $\psi_{\Delta \varepsilon}$ is diagonal and $\varepsilon_{t+1}^{\Delta} \sim N(0, I)$ is a vector of normally distributed iid measurement errors. y_t and $\bar{\Delta}_t^{(4)}$ are demeaned. We seek estimates for the parameters

$$\{\psi_y, \psi_{yf}, \psi_{yw}, \psi_f, \psi_{fw}, \psi_{\Delta f}, \psi_{\Delta \varepsilon}\}$$

and the belief factor $\theta_t = f_t$.

We estimate the model using Bayesian methods. Further details, including the imposed priors and estimated posteriors are summarized in Appendix D.

3.2 Results

A variance decomposition at the modal parameter estimate, summarized in Table 4 in Appendix D, reveals that the factor shock explains 60%, 48%, and 7% of the variation in the output wedge,

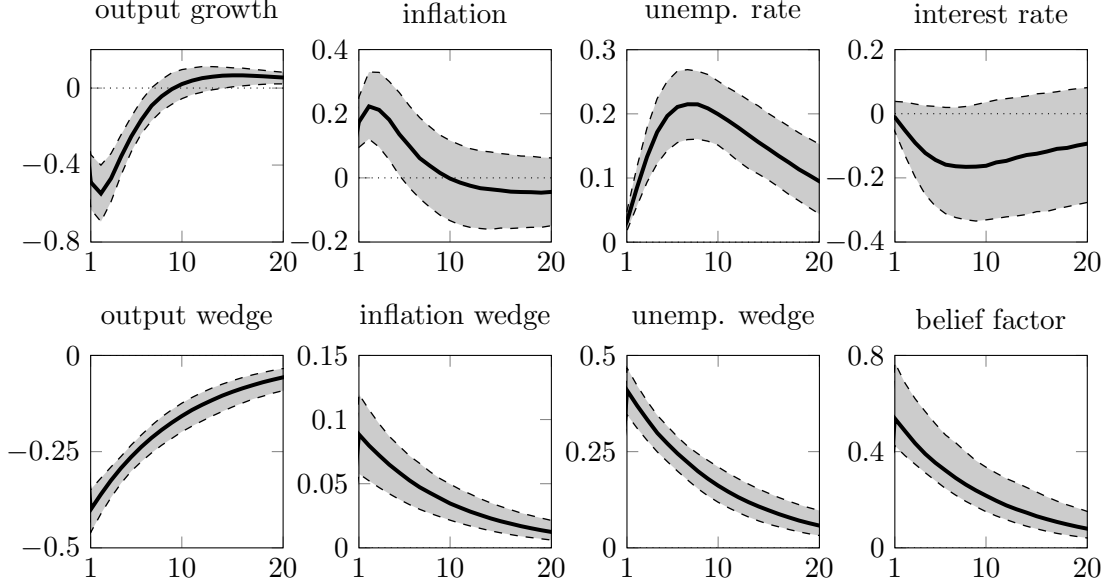


Figure 2: Bayesian impulse response functions in the one-factor model to the belief shock w^f . The solid lines indicate median estimates, while the dashed line correspond to 10th and 90th percentile error bands. Output growth, inflation, and interest rate are annualized and in percentage deviations. The unemployment rate is in percentage points. The output wedge and inflation wedge are scaled to correspond to the belief wedges of annualized output growth and annualized inflation. The horizontal axis is in quarters.

unemployment wedge, and inflation wedge respectively. These results confirm the strong correlation between the belief wedges that concern real quantities. The fact that a sizeable fraction of variation in the wedges is explained by θ_t supports the single factor model. Moreover, the posterior estimates shown in Table 4 in Appendix D reveal a very tightly identified persistence ψ_f of this factor with posterior mean of 0.9 at the quarterly frequency.

Figure 2 plots the impulse response functions of y_t and $\bar{\Delta}_t^{(4)}$ to a positive one standard deviation shock w_t^f to $\theta_t = f_t$, with factor response plotted in the bottom right panel. We find a negative impulse response for the output belief wedge and a positive impulse response for the unemployment and inflation wedges in response to a positive shock to θ_t . An increase in θ_t leads household forecasts for GDP growth to be biased further downward relative to the SPF forecasts, while the biases in the household forecasts for unemployment and inflation increase relative to the SPF forecasts. The impulse responses of the belief wedges are consistent with the correlations and average signs of the wedges described in Section 2.

These results are consistent with the interpretation of θ_t as a time-varying measure of the level of pessimism among households. From the perspective of the robust preference model that we develop in the next section, households are concerned about a future path that exhibits low GDP growth, a high unemployment rate and high inflation. An increase in θ_t makes these concerns stronger, biasing households' beliefs more strongly in this direction.

The belief shock also has real effects. In response to a positive shock to θ_t , GDP growth falls

and unemployment rises. The impulse response for inflation is positive for the first year and close to zero subsequently. Interest rates exhibit a negative median response with large error bands that contain zero. At the modal parameter estimate, θ_t explains 8%, 32%, 2% and 3% of the movements in GDP growth, unemployment, inflation and interest rates, respectively. Our estimates therefore suggest that a rise in pessimism has contractionary effects, and we emphasize the especially large adverse response of unemployment.

In Section 5, we develop and estimate a structural macroeconomic model with a frictional labor market and ambiguity averse agents and revisit these empirical findings. In line with the results from the factor model, the ambiguity shock in the structural model generates nontrivial recessionary responses, with a particularly pronounced response in the labor market.

4 Robust preferences

Motivated by the empirical results from Sections 2 and 3, we now introduce a preference model that generates endogenous deviations of agents' beliefs from the data-generating probability measure. This model extends the robust preference framework of Hansen and Sargent (2001a,b) to allow for more flexible form of belief distortions, similar to Hansen and Sargent (2015). The flexibility allows for time-variation in the degree of agents' pessimism over time, which we identify from survey data, while tightly restricting the structure of pessimistic distortions across individual states, linking them to agents' preferences.

We consider a class of Markov models for the equilibrium dynamics

$$x_{t+1} = \psi(x_t, w_{t+1}) \quad (4)$$

where x_t is an $n \times 1$ vector of stationary economic variables and $w_{t+1} \sim N(0_k, I_{k \times k})$ an iid vector of normally distributed shocks under the data-generating probability measure P . Agents are endowed with a version of robust preferences that satisfy the continuation value recursion

$$V_t = \min_{\substack{m_{t+1} > 0 \\ E_t[m_{t+1}] = 1}} u(x_t) + \beta E_t[m_{t+1} V_{t+1}] + \frac{\beta}{\theta_t} E_t[m_{t+1} \log m_{t+1}] \quad (5)$$

with period utility $u(x_t)$. These preferences have been formulated by Hansen and Sargent (2001a,b) as a way of introducing concerns for model misspecification on the side of the agents. The agent treats model (4) as an approximating or benchmark model and considers potential stochastic deviations from this model, represented by the strictly positive, mean-one random variable m_{t+1} . The minimization problem in (5) captures the search for a 'worst-case' model that serves as a basis for the agent's decisions. The models that are considered by the agent are difficult to distinguish statistically from the benchmark model, and the degree of statistical similarity is controlled by the entropy penalty $E_t[m_{t+1} \log m_{t+1}]$, scaled by the penalty parameter θ_t . More pronounced statistical deviations that are easier to detect are represented by random variables m_{t+1} with a large

dispersion that yields a large entropy.

The preferences considered by [Hansen and Sargent \(2001a,b\)](#) impose a constant parameter $\theta > 0$. As $\theta \searrow 0$, the penalty for deviating from the benchmark model becomes more severe, and the resulting preferences are closer to a utility-maximizing household with rational expectations.

We are interested in a specification that permits more flexibility in the set of models that the households views as plausible. In particular, we envision the time-varying model

$$\theta_t = \bar{\theta} x_t. \quad (6)$$

where $\bar{\theta}$ is a $1 \times n$ vector of parameters. It is well-known that the worst-case model distortion relative to the benchmark model given by the solution of (5) satisfies

$$m_{t+1} = \frac{\exp(-\theta_t V_{t+1})}{E_t[\exp(-\theta_t V_{t+1})]}. \quad (7)$$

The variation in θ_t thus implies a time-varying model for the worst-case distortion. The chained sequence of random variables m_{t+1} specifies a strictly positive martingale M recursively as $M_{t+1} = m_{t+1} M_t$ with $M_0 = 1$ that defines a probability measure $\tilde{P}$ with conditional expectations

$$\tilde{E}_t[x_{t+1}] \doteq E_t[m_{t+1} x_{t+1}].$$

Consequently, the wedge between the one-period forecasts of x_{t+1} under the worst-case and benchmark models is given by

$$\Delta_t \doteq \tilde{E}_t[x_{t+1}] - E_t[x_{t+1}]. \quad (8)$$

Notice that the distortion (7) implies a large value of m_{t+1} for low realizations of the continuation value V_{t+1} . The worst-case model, represented by the probability measure $\tilde{P}$, thus overweighs adverse states as ranked by the preferences of the agent. In this way, the preference model implies tightly restricted endogenous pessimism on the side of the agents, generated by concerns for model misspecification. The degree of pessimism is controlled by the evolution of θ_t .

4.1 A linear approximation

We are interested in deriving a tractable approximation of the equilibrium dynamics (4) and the worst-case biases Δ_t in (8). Assuming that the function $\psi(x, w)$ is sufficiently smooth, we combine the series expansion method of [Holmes \(1995\)](#) and [Lombardo \(2010\)](#) with an extension of the worst-case model approximation used in [Borovička and Hansen \(2013, 2014\)](#). The method, outlined in detail in Appendix B, approximates the dynamics in the neighborhood of the deterministic steady state $\bar{x}$ that is given by the solution to $\bar{x} = \psi(\bar{x}, 0)$. The dynamics of the state vector x_t can be approximated as

$$x_t \approx \bar{x} + \mathbf{q} x_{1t}$$

where q is a perturbation parameter. The law of motion for the ‘first-derivative’ process x_{1t} that represents the local dynamics in the neighborhood of the steady state can be derived from the linear approximation of (4):

$$x_{1t+1} = \psi_q + \psi_x x_{1t} + \psi_w w_{t+1} \quad (9)$$

where ψ_q , ψ_x and ψ_w are conforming coefficient matrices. Similarly, we can construct a linear approximation of the continuation value (5) where the deviation of the continuation value from its steady state satisfies

$$V_{1t} = V_x x_{1t} + V_q.$$

We show in Appendix B that under the household’s worst-case model $\tilde{P}$, the innovations w_{t+1} are distributed as

$$w_{t+1} \sim N \left(-\bar{\theta} (\bar{x} + x_{1t}) (V_x \psi_w)' , I_{k \times k} \right).$$

Instead of facing a vector of zero-mean shocks w_{t+1} , the agent perceives these shocks as having a time-varying drift. The time-variation is determined by a linear approximation to θ_t from equation (6), given by $\bar{\theta} (\bar{x} + x_{1t})$. The relative magnitudes of the distortions of individual shocks are given by the sensitivity of the continuation value to the dynamics of the state vector, V_x , and the loadings of the state vector on individual shocks, ψ_w . The agent perceives larger distortions during periods when θ_t is large, and distorts relative more the shocks which impact the continuation value more strongly.

Consequently, the dynamics of the model (9) under the agents’ worst-case beliefs satisfy

$$\begin{aligned} x_{1t+1} &= [\psi_q - \psi_w \psi_w' V_x' \bar{\theta} \bar{x}] + [\psi_x - \psi_w \psi_w' V_x' \bar{\theta}] x_{1t} + \psi_w \tilde{w}_{t+1} \\ &= \tilde{\psi}_q + \tilde{\psi}_x x_{1t} + \psi_w \tilde{w}_{t+1}. \end{aligned}$$

The worst-case model alters both the conditional mean and the persistence of economic shocks. Moreover, variables that tend to move ambiguity and the continuation value in opposite directions tend to exhibit a higher persistence under the worst-case model.¹

4.2 Worst-case model and survey responses

In Section 3, we estimated a one-factor model of biases embedded in survey responses on household expectations of future economic variables. The extracted belief biases indicated that households substantially overweigh states which can be viewed as adverse, and that these biases exhibit a non-negligible variation over the business cycle. We also extracted a one-factor structure underlying these belief biases.

The preference framework introduced in this section implies that agents’ actions are based on forecasts under the worst-case probability distribution $\tilde{P}$. We connect the empirical observations on survey responses and the theoretical predictions on decisions under robust preferences and

¹This statement is precisely correct in the scalar case, when $\psi_x^2 V_x \bar{\theta} < 0$.

hypothesize that surveyed households provide answers regarding economic forecasts having in mind the same probability distribution $\tilde{P}$.

Using the survey data and the rational forecasts from the linearized model (9), we identify the belief wedges (8) as

$$\Delta_t = \psi_x \tilde{E}_t [w_{t+1}] = -\bar{\theta} (\bar{x} + x_{1t}) (\psi_w \psi'_w) V'_x.$$

The one-factor structure in survey answers is driven by the time-variation in $\bar{\theta} (\bar{x} + x_{1t})$, with the constant vector of loadings $-(\psi_w \psi'_w) V'_x$.

Observe that this specification of belief wedges is a restricted case of the reduced-form model (1)–(3). Ignoring the constant term in the belief wedge,² we have

$$F = \bar{\theta}, \quad H = -(\psi_w \psi'_w) V'_x.$$

The terms $\bar{\theta}$, ψ_w , V_x are functions of structural parameters in the model.

5 A structural monetary policy model

In this section we introduce the robust preference framework from Section 4 into a dynamic stochastic general equilibrium model of the macroeconomy. For this we use a simplified version of the New-Keynesian framework with a frictional labor market introduced in [Ravenna and Walsh \(2008\)](#) and [Christiano et al. \(2015\)](#). The frictional labor market with search and matching features and nominal rigidities provides a well-defined notion of unemployment and inflation which directly map to the survey questions.

In Section 3, we used a reduced form VAR specification with a one-factor structure in beliefs to extract a latent component that accounts for the co-movement between the several belief wedges in the data. In this section, our strategy is to use an estimated version of the structural model to quantify the role and channels through which ambiguity shocks affect the dynamics of realized outcomes and associated belief wedges.

5.1 The model

The representative household is endowed with robust preferences given by the recursion (5) with period utility over aggregate consumption C_t ,

$$u(x_t) = \log C_t.$$

In line with our factor model specification from Section 3, we assume that the stochastic process for the robust concerns is given by $\theta_t = \bar{\theta} x_t \doteq f_t$ where f_t follows an AR(1) process

$$f_{t+1} = (1 - \rho_f) \bar{f} + \rho_f f_t + \sigma_f w_{t+1}^f. \quad (10)$$

²The constant term can be matched by introducing a constant element into the vector of observables in (1).

The worst-case belief of the household is

$$m_{t+1} = \frac{\exp(-\theta_t V_{t+1})}{E_t[\exp(-\theta_t V_{t+1})]}.$$

As outlined in Section 4, the endogenous loadings of V_t on the shock processes will be key in determining the how Radon-Nikodým derivative m_{t+1} distorts the households' expectations.

5.1.1 Labor markets

The household consists of a unit mass of workers who perfectly share consumption risk. Fraction l_t is employed and earns a wage w_t . Fraction $u_t = 1 - l_t$ is unemployed and collect unemployment benefits b financed through lump sum taxes. At the end of period t , employed workers separate with probability $1 - \rho$ and join the pool of unemployed who search for jobs at the beginning of period $t + 1$. The total number of searchers at the beginning of period $t + 1$ therefore is $1 - \rho l_t$ and these searchers face a job finding probability j_{t+1} . The law of motion for employed workers thus is

$$l_{t+1} = \rho l_t + (1 - \rho l_t) j_{t+1} = (\rho + x_{t+1}) l_t$$

where

$$x_{t+1} = \frac{j_{t+1} (1 - \rho l_t)}{l_{t-1}}$$

is the hiring rate. The value of an employed worker is

$$W_t = \xi_t + \tilde{E}_t \left[\frac{S_{t+1}}{S_t} ((\rho + (1 - \rho) j_{t+1}) V_{t+1} + (1 - \rho) (1 - j_{t+1}) U_{t+1}) \right]$$

where U_{t+1} is the value of being unemployed next period, given by the recursion

$$U_t = b + \tilde{E}_t \left[\frac{S_{t+1}}{S_t} (j_{t+1} W_{t+1} + (1 - j_{t+1}) U_{t+1}) \right].$$

Denote ϑ_t the real marginal revenue in period t from hiring an additional worker. The value of the worker to a firm is given by the revenue generated in the match net of the wages paid,

$$J_t = \vartheta_t - \xi_t + \rho \tilde{E}_t \left[\frac{S_{t+1}}{S_t} J_{t+1} \right].$$

We assume free entry of firms, so that in equilibrium, $J_t = \kappa$ where κ is the fixed cost of hiring a worker. The expectations operators in the recursions indicate that both the workers and the firms evaluate the distribution of future quantities under the worst-case measure $\tilde{P}$.

What remains to be determined is the split of the surplus from a match between the firm's surplus, J_t , and the worker's surplus, $W_t - U_t$. As in Hall and Milgrom (2008) and Christiano et al. (2015), we adopt the alternating offer bargaining protocol of Rubinstein (1982) and Binmore et al. (1986). The details of the bargaining protocol are outlined in Appendix E where we show that the

outcome of this bargaining mechanism is

$$J_t = \beta_0 + \beta_1 (W_t - U_t) + \beta_2 (\vartheta_t - b)$$

with parameters β_i , $i = 0, 1, 2$ that depend on the parameters of the bargaining problem. Notice that when $\beta_0 = \beta_2 = 0$, we obtain the Nash bargaining solution with workers' share $\eta = (1 + \beta_1)^{-1}$. Relative to the Nash bargaining solution, the alternative offer bargaining makes the firms' surplus more procyclical, leading to smoother wages and more procyclical hiring patterns over the business cycle.

5.1.2 Production and equilibrium

The frictional labor market is embedded in a New-Keynesian framework with [Calvo \(1983\)](#) price setting. A homogeneous final good Y_t with price P_t is produced in a competitive market using the production technology

$$Y_t = \left[\int_0^1 (Y_{i,t})^{\frac{1}{\lambda}} di \right]^\lambda, \quad \lambda > 1.$$

where $Y_{i,t}$ are specialized inputs with prices $P_{i,t}$. Final good producers solve the static competitive problem

$$\max_{Y_{i,t}} P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} di,$$

leading to the first-order conditions

$$Y_{i,t} = \left(\frac{P_t}{P_{i,t}} \right)^{\frac{\lambda}{\lambda-1}} Y_t, \quad i \in [0, 1].$$

Specialized inputs are produced by monopolist retailers indexed by i , using the production technology

$$Y_{i,t} = \exp(a_t) h_{i,t}$$

where $h_{i,t}$ is the quantity of intermediate goods purchased in a competitive market and a_t aggregate productivity. The retailer purchases intermediate goods at price $(1 - \nu) P_t^h$ in a competitive market where ν is a subsidy that eliminates steady state monopoly distortions. Finally, he is subject to the sticky price friction, implying that every period he is allowed to reset the price with probability $1 - \chi$.

Intermediate goods are produced by wholesalers using a technology that turns one unit of labor into one unit of capital. Therefore

$$\int_0^1 h_{i,t} di = h_t = l_t.$$

The household collects wages, unemployment benefits and profits from monopolistic firms net

of taxes paid as subsidies, T_t . This yields the nominal budget constraint

$$P_t C_t + B_{t+1} \leq \xi_t l_t + (1 - l_t) P_t b + R_{t-1} B_t - T_t$$

where B_{t+1} are one-period nominal bonds in zero net supply issued at par at time t and earning interest rate R_t . The aggregate resource constraint is given by

$$C_t + \kappa x_t l_{t-1} = Y_t = \exp(a_t) l_t$$

i.e., output is split between consumption and cost of hiring.

5.1.3 Shock structure and monetary policy

We assume that the monetary authority follows the interest rate policy

$$\log(R_t/\bar{R}) = \rho_R \ln(R_{t-1}/\bar{R}) + (1 - \rho_R) [r_w \log(\pi_t/\bar{\pi}) + r_y \log(l_t/\bar{l})] + \sigma_R w_t^R$$

where w_t^R is a monetary policy shock. The two other sources of exogenous variation is the productivity shock that follows an AR(1) process

$$a_t = \rho_a a_{t-1} + \sigma_a w_t^a$$

and the ambiguity shock process (10).

5.2 Estimation

We are interested in studying the quantitative role of ambiguity shocks in the joint dynamics of output, unemployment, inflation and interest rates through the lens of the structural model introduced above. The impact of these shocks on the economy is restricted through the structure of the model, and we use survey data as a new source of information to aid identification.

As in the reduced form analysis in Section 3, we use data on the unemployment rate, federal funds rate, inflation rate, inflation wedge, unemployment wedge and output wedge with iid measurement errors on the three wedges.³ We estimate the model using Bayesian methods. In order to make the estimation tractable and transparent, we calibrate a subset of parameters to values listed in the bottom part of Table 1, and focus our estimation on the parameter vector ζ consisting of parameters associated with the monetary policy rule and the underlying shock processes,

$$\zeta = (\rho_R, r_w, r_y, \sigma_R, \rho_a, \sigma_a, \rho_f, \sigma_f, \sigma_{m,\pi}, \sigma_{m,u}, \sigma_{m,y}).$$

The last three parameters in ζ are the standard deviations on the measurement errors. Our priors for the Taylor rule coefficients and stochastic processes for productivity and monetary policy shocks

³The details of the data construction are in Appendix C.

Parameter		Prior D(Mean,Std)	Posterior Mode, Std
Monetary Policy			
ρ_R	Taylor Rule: Smoothing	$\mathcal{B}(0.7, 0.05)$	0.64, 0.03
r_π	Taylor Rule: Inflation	$\mathcal{G}(1.7, 0.05)$	1.51, 0.06
r_l	Taylor Rule: Employment	$\mathcal{G}(0.1, 0.05)$	0.1, 0.01
Shock Processes			
σ_R	Std. Monetary Policy	$\mathcal{G}(0.001, 0.01)$	0.0012, 0.0002
σ_a	Std. Productivity Shock	$\mathcal{G}(0.01, 0.01)$	0.012, 0.001
σ_θ	Std. Ambiguity Shock	$\mathcal{G}(1, 0.05)$	0.76, 0.04
ρ_a	AR(1) Productivity Shock	$\mathcal{B}(0.8, 0.05)$	0.88, 0.02
ρ_θ	AR(1) Ambiguity Shock	$\mathcal{B}(0.8, 0.01)$	0.84, 0.006
Measurement errors			
$\sigma_{m,y}$	output wedge	$\mathcal{G}(0.001, 0.01)$	0.017, 0.001
$\sigma_{m,\pi}$	inflation wedge	$\mathcal{G}(0.001, 0.01)$	0.008, 0.0005
$\sigma_{m,u}$	unemployment wedge	$\mathcal{G}(0.001, 0.01)$	0.002, 0.0001
Calibrated parameters			
β	Discount factor	$1.03^{-0.25}$	-
χ	Calvo price stickiness	0.66	-
λ	Price Markup	1.2	-
ρ	Job survival probability	0.9	-
M	Max. bargaining rounds per quarter	60	-

Table 1: Estimated and calibrated parameters. The priors $\mathcal{G}(\mu, \sigma)$, $\mathcal{B}(\mu, \sigma)$ denote Gamma and Beta distributions with mean μ and standard deviation σ .

are similar to [Christiano et al. \(2015\)](#).

The first part of Table 1 summarizes the results of our estimation. The posterior distributions are plotted in Appendix E. Table 2 provides the variance decomposition for key macroeconomic variables and the belief wedges. At the posterior modes, the factor explains about a third of the joint movement in all the wedges with 20% of the variance of employment and inflation and 5% of output.

5.3 Understanding the role of ambiguity shocks

Figure 3 depicts the impulse responses for the ambiguity shock w_t^f . A one-standard deviation increase in ambiguity leads to a fall of about 1% in output and employment and makes the households more pessimistic about future output and employment.⁴ Relative to the rational forecast, house-

⁴The magnitude is comparable to the response of employment to a unit standard deviation fall in the productivity shock.

Variable		w^a	w^R	w^f	meas. error
$\log Y_t$	Output	91.5	3.9	4.6	-
u_t	Unemployment	65.3	16.0	18.7	-
π_t	Inflation	72.8	1.0	26.3	-
R_t	Interest rate	65.6	9.5	24.8	-
ξ_t	Real wages	86.0	1.0	13.0	-
x_t	Hiring rate	9.9	42.2	48.0	-
j_t	Job finding rate	52.8	22.0	25.2	-
$\overline{\Delta}_t^{(4)}(\log Y)$	Output wedge	-	-	34.0	66.0
$\overline{\Delta}_t^{(4)}(u)$	Unemployment wedge	-	-	29.8	70.2
$\overline{\Delta}_t^{(4)}(\pi)$	Inflation wedge	-	-	25.2	74.8

Table 2: Variance decomposition at the posterior modes. All values are in percent.

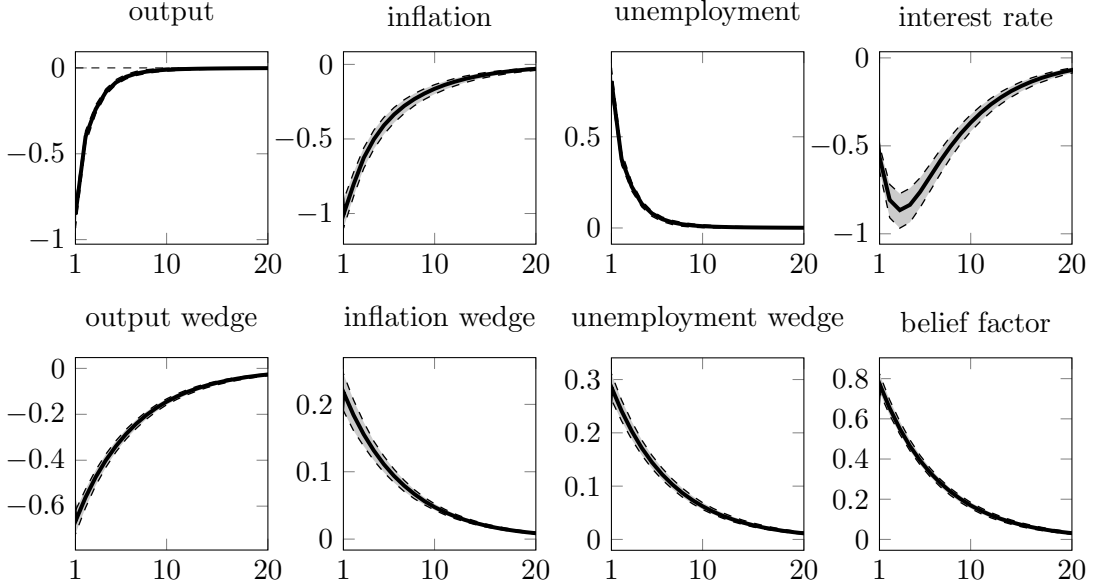


Figure 3: Bayesian impulse response functions to the belief shock w^f . Output in percentage deviations, inflation and interest rates in annualized percentage points, and employment in percentage points.

holds expect output and employment to be about 0.5% lower on impact and this wedge persists for about 15 quarters. Realized inflation is lower by 1% on impact and the households' worst-case expectation of the inflation rate is 0.2% higher relative to the rational forecast.

In the structural model, the effects of ambiguity are transmitted through the households' continuation values and the equilibrium policy rules for employment, output, and inflation. As explained in Section 4, the endogenous exposure of the continuation values to the three fundamental shocks (productivity, monetary policy, and ambiguity) pins down the relative magnitudes of the distortions to each of the shocks. The households compute their worst-case expectations using these distortions which then determine current outcomes through the forward-looking behavior.

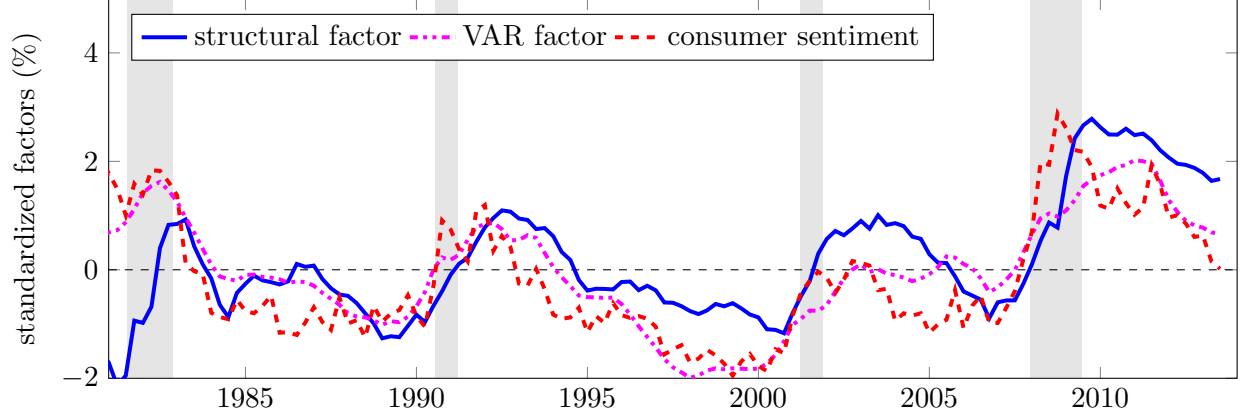


Figure 4: A comparison of the extracted ambiguity factor with the Michigan Survey measure of contemporaneous Consumer Sentiment. The solid blue line is the smoothed factor from the structural model, the purple dash-dotted line the smoothed factor from the reduced-form VAR model. All data series are standardized.

The household's continuation value V_t loads positively on the innovation to the productivity shock and negatively on the innovations to the monetary policy and the ambiguity shocks. In terms of magnitudes, the loadings on productivity and ambiguity are of similar proportion and the loading on the monetary policy shock is about 10 times lower. Households perceive an increase in ambiguity as a news about lower productivity and tighter monetary policy. Furthermore, the persistence of this adverse news under the worst-case model is higher, with a half-life of about 11 quarters as compared to the dynamics of the factor under the objective measure, which at our current estimates has a half-life of 4 quarters.

These pessimistic expectations interact in crucial ways through the frictional labor market. With search and matching rigidities, hiring and bargaining decisions are based on the value of the discounted future surplus generated by a match. Both firms and workers inherit the representative household's beliefs to make future forecasts when they compute their respective continuation values. Lower expected productivity and higher expected interest rates lower the value of the match from the perspective of the worst-case beliefs shared by the worker-firm pair. This lowers equilibrium hiring rates, and lower employment also implies lower output. This channel induced by fluctuations in household's ambiguity concerns is a novel and potent source of fluctuations in the labor market. The variance decomposition in Table 2 reveals that the ambiguity shocks drives a substantial portion of the overall variation in the labor market variables, for instance almost one half of the variation in the hiring rate.

The effect on the inflation rate comes from a balance of two forces. Lower contemporaneous aggregate demand pushes the intermediate goods producers that change prices to set them to lower levels. At the same time, expectations of lower productivity imply higher marginal costs and this pushes current and future prices upwards. At our current estimates, the net effect of an increase in ambiguity is a lower equilibrium inflation rate. However, the response of the inflation wedge is

positive, indicating that the worst-case model is biased toward a higher inflation rate in the future.

In Figure 4 we plot the extracted series for the ambiguity factor obtained from the reduced-form and structural models, along with the Consumer Sentiment index reported by the Michigan Survey. All three series are highly correlated and attest to a consistent narrative of how ambiguity affects business cycle dynamics.

6 Conclusion

We develop a framework in which time-variation in ambiguity perceived by households generates fluctuations in aggregate dynamics of the macroeconomy. The framework is based on an extension of the robust preference model that introduces shocks to agents' concerns about model misspecification. We identify these *ambiguity shocks* using survey data from the University of Michigan Surveys of Consumers and the Survey of Professional Forecasters. We show that in the data and in an estimated business cycle model, the ambiguity shocks are a potent source of variation in labor market variables.

The structural interpretation of ambiguity shocks identified in our framework opens new directions for policy analysis under ambiguity. In parallel work, we study the implications of this framework for optimal monetary policy. A monetary authority facing households endowed with robust preferences infers that policy changes lead to endogenous changes in the worst-case model. The choice of optimal policy therefore involves explicit management of households' expectations by the monetary authority.

Appendix

A Distorted beliefs in the one-factor model

Let $(\Omega, \{\mathcal{F}_t\}_{t=0}^\infty, P)$ be the probability space generated by the innovations of model (1). The subjective probability measure $\tilde{P}$ is formally defined by specifying a strictly positive martingale M with one-period increment

$$m_{t+1} = \frac{M_{t+1}}{M_t} = \exp\left(-\frac{1}{2}|k_t|^2 + k_t' w_{t+1}\right).$$

We then have $\tilde{E}_t[w_{t+1}] = k_t$. Factor structure (2)–(3) of households' expectations is obtained by imposing the restriction

$$k_t = HFx_t$$

where H is a $k \times 1$ vector. Let $z_t = Zy_t$ where Z is a matrix that selects observables with data on households' expectations. Expressing (1) in a concise form for $x_t = (y_t', f_t')'$ as

$$x_{t+1} = \psi_x x_t + \psi_w w_{t+1},$$

we can write

$$\Delta_t^{(1)} = \tilde{E}_t[z_{t+1}] - E_t[z_{t+1}] = Z\psi_w HFx_t$$

and thus $G = Z\psi_w H$. The n -period forecasts under the data-generating and subjective probability measures are given by

$$\begin{aligned} E_t[z_{t+\tau}] &= Z(\psi_x)^\tau y_t + Z \sum_{i=0}^{\tau-1} (\psi_x)^i \psi_q \\ \tilde{E}_t[z_{t+\tau}] &= Z(\psi_x + \psi_w HF)^\tau x_t + Z \sum_{i=0}^{\tau-1} (\psi_x + \psi_w HF)^i \psi_q \end{aligned}$$

and thus

$$\begin{aligned} \Delta_t^{(\tau)} &= Z[(\psi_x + \psi_w HF)^\tau - (\psi_x)^\tau] y_t + \\ &\quad + Z\left[(I - (\psi_x + \psi_w HF))^{-1}(I - (\psi_x + \psi_w HF)^\tau) - (I - \psi_x)^{-1}(I - (\psi_x)^\tau)\right] \psi_q \\ &= G_y^{(\tau)} x_t + G_0^{(\tau)} \end{aligned}$$

B Series expansion of the worst-case model

The linear approximation in this paper is an extension of the series expansion method used in [Holmes \(1995\)](#) or [Lombardo \(2010\)](#). [Borovička and Hansen \(2013, 2014\)](#) adapt the series expansion method to an approximation of models with robust preferences. Here, we further extend this methodology to derive a linear solution that allows for endogenously determined time-varying belief distortions. The critical step in the expansion lies in the joint perturbation of the shock vector w_t and the penalty process θ_t .

B.1 Law of motion

We start with the approximation of the model for the law of motion (4). We consider a class of models indexed by the scalar perturbation parameter $\mathbf{q}$ that scales the volatility of the shock vector w_t

$$x_t(\mathbf{q}) = \psi(x_{t-1}(\mathbf{q}), \mathbf{q}w_t, \mathbf{q}) \quad (11)$$

and assume that there exists a series expansion of x_t around $\mathbf{q} = 0$:

$$x_t(\mathbf{q}) \approx \bar{x} + \mathbf{q}x_{1t} + \frac{\mathbf{q}^2}{2}x_{2t} + \dots$$

The processes x_{jt} , $j = 0, 1, \dots$ can be viewed as derivatives of x_t with respect to the perturbation parameter, and their laws of motion can be inferred by differentiating (11) j times and evaluating the derivatives at $\mathbf{q} = 0$, assuming that ψ is sufficiently smooth. Here, we focus only on the approximation up to the first order:

$$\begin{aligned} \bar{x} &= \psi(\bar{x}, 0, 0) \\ x_{1t} &= \psi_x x_{1t-1} + \psi_w w_t + \psi_q \end{aligned} \quad (12)$$

B.2 Continuation values

We now focus on the expansion on the continuation value recursion. Substituting the worst-case belief distortion (7) into the recursion (5) yields

$$V_t = u(x_t) - \frac{\beta}{\theta_t} \log E_t[\exp(-\theta_t V_{t+1})].$$

We are looking for an expansion of the continuation value

$$V_t(\mathbf{q}) \approx \bar{V} + \mathbf{q}V_{1t}. \quad (13)$$

In order to derive the solution of the continuation value, we are interested in expanding the following perturbation of the recursion:

$$V_t(\mathbf{q}) = u(x_t(\mathbf{q}), \mathbf{q}) - \beta \frac{\mathbf{q}}{\bar{\theta}(\bar{x} + x_{1t})} \log E_t \left[\exp \left(-\frac{\bar{\theta}(\bar{x} + x_{1t})}{\mathbf{q}} V_{t+1}(\mathbf{q}) \right) \right]. \quad (14)$$

Here, we utilized the fact that $\theta_t = \bar{\theta}x_t \approx \bar{\theta}(\bar{x} + x_{1t})$. More importantly, the perturbation scales jointly the volatility of the stochastic processes for V_t and $u(x_t)$ with the magnitude of the penalty parameter θ_t . In particular, the penalty parameter in the perturbation of equation (5) becomes $\mathbf{q}/\theta_t$ and decreases jointly with the volatility of the shock process. This assumption will imply that the benchmark and worst-case models do not converge as $\mathbf{q} \rightarrow 0$, and the linear approximation around a deterministic steady state yields a nontrivial contribution of the worst-case dynamics.

Using the expansion of the period utility function

$$u(x_t(\mathbf{q}), \mathbf{q}) \approx \bar{u} + \mathbf{q}u_{1t} = \bar{u} + \mathbf{q}(u_x x_{1t} + u_q)$$

we get the deterministic steady state (zero-th order) term by setting $\mathbf{q} = 0$:

$$\bar{V} = (1 - \beta)^{-1} \bar{u}.$$

The first-order term in the expansion is derived by differentiating (14) with respect to $\mathbf{q}$ and is given by the recursion

$$V_{1t} = u_{1t} - \beta \frac{1}{\bar{\theta}(\bar{x} + x_{1t})} \log E_t [\exp(-\bar{\theta}(\bar{x} + x_{1t}) V_{1t+1})] \quad (15)$$

Since $\bar{x}$ is constant and x_{1t} has linear dynamics (12), we hope to find linear dynamics for V_{1t} as well. Since $u_t = u(x_t)$, we can make the guess that $V_t^i(\mathbf{q}) = V^i(x_t(\mathbf{q}), \mathbf{q})$ which leads to the following expressions for the derivative of V_t :

$$V_{1t} = V_x x_{1t} + V_q.$$

Using this guess and comparing coefficients, equation (15) leads to a pair of algebraic equations for the unknown coefficients V_x and V_q :

$$\begin{aligned} V_x &= u_x - \frac{\beta}{2} V_x \psi_w \psi'_w V'_x \bar{\theta} + \beta V_x \psi_x \\ V_q &= u_q - \frac{\beta}{2} \bar{\theta} \bar{x} V_x \psi_w \psi'_w V'_x + \beta V_x \psi_q + \beta V_q \end{aligned}$$

The first from this pair of equations is a Riccati equation for V_x , which can be solved for given coefficients ψ_x and ψ_w .

B.3 Distortions

With the approximation of the continuation value at hand, we can derive the expansion of the one-period belief distortion m_{t+1} that defines the worst-case model relative to the benchmark model. As in (14), we scale the penalty parameter θ_t jointly with the volatility of the underlying shocks:

$$m_{t+1}(\mathbf{q}) = \frac{\exp\left(-\frac{1}{\mathbf{q}} \theta_t V_{t+1}(\mathbf{q})\right)}{E_t \left[\exp\left(-\frac{1}{\mathbf{q}} \theta_t V_{t+1}(\mathbf{q})\right) \right]} \approx m_{0,t+1} + \mathbf{q} m_{1,t+1}.$$

It turns out that in order to derive the correct first-order expansion, we are required to consider a second-order expansion of the continuation value

$$V_t(\mathbf{q}) \approx \bar{V} + \mathbf{q} V_{1t} + \frac{\mathbf{q}^2}{2} V_{2t},$$

although the term V_{2t} will be inconsequential for subsequent analysis. Substituting in expression (13) and noting that $\bar{V}$ is a deterministic term, we can approximate m_{t+1} with

$$m_{t+1}(\mathbf{q}) \approx \frac{\exp(-\bar{\theta}(\bar{x} + x_{1t}) (V_{1t+1} + \frac{\mathbf{q}}{2} V_{2t+1}))}{E_t [\exp(-\bar{\theta}(\bar{x} + x_{1t}) (V_{1t+1} + \frac{\mathbf{q}}{2} V_{2t+1}))]}$$

Differentiating with respect to $\mathbf{q}$ and evaluating at $\mathbf{q} = 0$, we obtain the expansion

$$\begin{aligned}
m_{0t+1} &= \frac{\exp(-\bar{\theta}(\bar{x} + x_{1t})V_{1t+1})}{E_t[\exp(-\bar{\theta}(\bar{x} + x_{1t})V_{1t+1})]} \\
m_{1t+1} &= -\frac{1}{2\bar{\theta}(\bar{x} + x_{1t})}M_{0t+1}[V_{2t+1} - E_t[M_{0t+1}V_{2t+1}]]
\end{aligned} \tag{16}$$

This expansion is distinctly different from the standard polynomial expansion familiar from the perturbation literature. First, observe that m_{0t+1} is not constant, as one would expect for a zeroth-order term, but nonlinear in V_{1t+1} . However, since $E_t[m_{0t+1}] = 1$ we can thus treat M_{0t+1} as a change of measure that will adjust the distribution of shocks that are correlated with m_{0t+1} . We will show that with Gaussian shocks, we can still preserve tractability. Further notice that $E_t[m_{1t+1}] = 0$.

The linear structure of V_{1t} also has an important implication for the worst-case distortion constructed from m_{0t+1} . Substituting into (16) yields

$$m_{0t+1} = \frac{\exp(-\bar{\theta}(\bar{x} + x_{1t})V_x\psi_w w_{t+1})}{E_t[\exp(-\bar{\theta}(\bar{x} + x_{1t})V_x\psi_w w_{t+1})]}.$$

This implies that for a function $f(w_{t+1})$ with a shock vector $w_{t+1} \sim N(0, I)$,

$$E_t[m_{0t+1}f(w_{t+1})] \approx E_t[m_{t+1}f(w_{t+1})] = \tilde{E}_t[f(w_{t+1})]$$

where, under the $\tilde{P}$ (worst-case) measure, the vector w_{t+1} has the following distribution:

$$w_{t+1} \sim N(-\bar{\theta}(\bar{x} + x_{1t})(V_x\psi_w)', I_k). \tag{17}$$

the mean of the shock is therefore time-varying and depends on the linear process x_{1t} .

B.4 Equilibrium conditions

We assume that equilibrium conditions in our framework can be written as

$$0 = E_t[\tilde{g}(x_{t+1}, x_t, x_{t-1}, w_{t+1}, w_t)] \tag{18}$$

where $\tilde{g}$ is an $n \times 1$ vector function and the dynamics for x_t is implied by (4). This vector of equations includes expectational equations like Euler equations of the robust household, which can be represented using worst-case belief distortions m_{t+1} . We therefore assume that we can write the j -th component of $\tilde{g}$ as

$$\tilde{g}^j(x_{t+1}, x_t, x_{t-1}, w_{t+1}, w_t) = m_{t+1}^{\sigma_j} g^j(x_{t+1}, x_t, x_{t-1}, w_{t+1}, w_t).$$

where $\sigma_j \in \{0, 1\}$ captures whether the expectation in the j -th equation is under the household's worst-case model.⁵ In particular, all nonexpectational equations and all equations not involving agents' preferences have $\sigma_j = 0$. System (18) can then be written as

$$0 = E_t[\mathbb{M}_{t+1}g(x_{t+1}, x_t, x_{t-1}, w_{t+1}, w_t)]$$

⁵The generalization to multiple agents with potentially heterogeneous concerns for robustness is straightforward, see the construction in [Borovička and Hansen \(2013\)](#).

where $\mathbb{M}_{t+1} = \text{diag} \{m_{t+1}^{\sigma_1}, \dots, m_{t+1}^{\sigma_n}\}$ is a diagonal matrix of the belief distortions, and g is independent of the robustness parameter θ_t . As in [Borovička and Hansen \(2013\)](#), the zero-th and first-order expansions are

$$\begin{aligned} 0 &= E_t [\mathbb{M}_{0t+1} g_{0t+1}] = g_{0t+1} \\ 0 &= E_t [\mathbb{M}_{0t+1} g_{1t+1}] + E_t [\mathbb{M}_{1t+1} g_{0t+1}] = E_t [\mathbb{M}_{0t+1} g_{1t+1}] \end{aligned}$$

where the last equality follows from $E_t [m_{1t+1}] = 0$.

For the first-order derivative of the equilibrium conditions, we have

$$0 = E_t [\mathbb{M}_{0t+1} g_{1t+1}] \quad (19)$$

The first-order term in the expansion of g_{t+1} is given by

$$\begin{aligned} g_{1t+1} &= g_{x+} x_{1t+1} + g_x x_{1t} + g_{x-} x_{1t-1} + g_{w+} w_{t+1} + g_w w_t + g_q = \\ &= [(g_{x+} \psi_x + g_x) \psi_x + g_{x-}] x_{1t-1} + [(g_{x+} \psi_x + g_x) \psi_w + g_w] w_t + \\ &\quad + (g_{x+} \psi_x + g_{x+} + g_x) \psi_q + g_q + (g_{x+} \psi_w + g_{w+}) w_{t+1} \end{aligned} \quad (20)$$

where symbols x_+, x, x_-, w_+, w, q represent partial derivatives with respect to $x_{t+1}, x_t, x_{t-1}, w_{t+1}, w_t$ and q , respectively. Given the worst-case distribution of the shock vector (17), we can write

$$\tilde{E}_t [w_{t+1}] = - (V_x \psi_w)' \bar{\theta} [(\bar{x} + \psi_q) + \psi_x x_{1t-1} + \psi_w w_t]$$

Let $[A]^i$ denote the i -th row of matrix A . Notice that

$$[g_{x+} \psi_w + g_{w+}]^i (V_x \psi_w)' \bar{\theta}$$

is a $1 \times n$ vector. Construct the $n \times n$ matrix $\mathbb{E}$ by stacking these row vectors for all equations $i = 1, \dots, n$:

$$\mathbb{E} = \text{stack} \left\{ \sigma_i [g_{x+} \psi_w + g_{w+}]^i (V_x \psi_w)' \bar{\theta} \right\}$$

which contains non-zero rows for expectational equations under the worst-case model. Using matrix $\mathbb{E}$, we construct the conditional expectation of the last term in g_{1t+1} in (20). In particular

$$\begin{aligned} 0 &= E_t [\mathbb{M}_{0t+1} g_{1t+1}] = \\ &= [(g_{x+} \psi_x + g_x) \psi_x + g_{x-}] x_{1t-1} + [(g_{x+} \psi_x + g_x) \psi_w + g_w] w_t + \\ &\quad + (g_{x+} \psi_x + g_{x+} + g_x) \psi_q + g_q - \mathbb{E} [(\bar{x} + \psi_q) + \psi_x x_{1t-1} + \psi_w w_t] \end{aligned}$$

Equation (19) is thus a system of linear second-order stochastic difference equations. There are well-known results that discuss the conditions under which there exists a unique stable equilibrium path to this system ([Blanchard and Kahn \(1980\)](#), [Sims \(2002\)](#)). We assume that such conditions are satisfied. Comparing coefficients on x_{1t-1}, w_t and the constant term implies that

$$0 = (g_{x+} \psi_x + g_x - \mathbb{E}) \psi_x + g_{x-} \quad (21)$$

$$0 = (g_{x+} \psi_x + g_x - \mathbb{E}) \psi_w + g_w \quad (22)$$

$$0 = (g_{x+} \psi_x + g_{x+} + g_x) \psi_q + g_q - \mathbb{E} (\bar{x} + \psi_q) \quad (23)$$

These equations need to be solved for ψ_x , ψ_w , ψ_q and V_x where

$$V_x = u_x - \frac{\beta}{2} V_x \psi_w \psi'_w V'_x \bar{\theta} + \beta V_x \psi_x$$

and

$$\mathbb{E} = \text{stack} \left\{ \sigma_i [g_{x+} \psi_w + g_{w+}]^i (V_x \psi_w)' \bar{\theta} \right\}. \quad (24)$$

B.5 Special case: θ_t is an exogenous AR(1) process

In the application, we consider a special case that restricts θ_t to be an exogenous AR(1) process. With a slight abuse in notation, this restriction can be implemented by replacing the vector of variables x_t with $(x'_t, f'_t)'$ where f_t is a scalar AR(1) process representing the time-variation in the concerns for robustness as an exogenously specified ‘belief’ shock:

$$f_{t+1} = (1 - \rho_f) \bar{f} + \rho_f f_t + \sigma_f w_{t+1}^f. \quad (25)$$

The dynamics of the model then satisfies

$$x_t = \psi(x_{t-1}, w_t, f_t) \quad (26)$$

with steady state $(\bar{x}', \bar{f})'$. The vector $\bar{\theta}$ in (6) is then partitioned as $\bar{\theta}' = (\bar{\theta}'_x, \bar{\theta}'_f) = (0_{1 \times n-1}, 1)$ and thus $\theta_t = f_t$. Constructing the first-order series expansion of (26), we obtain

$$\begin{pmatrix} x_{1t+1} \\ f_{1t+1} \end{pmatrix} = \begin{pmatrix} \psi_q \\ 0 \end{pmatrix} + \begin{pmatrix} \psi_x & \rho_f \psi_{xf} \\ 0 & \rho_f \end{pmatrix} \begin{pmatrix} x_{1t} \\ f_{1t} \end{pmatrix} + \begin{pmatrix} \psi_w & \sigma_f \psi_{xf} \\ 0 & \sigma_f \end{pmatrix} \begin{pmatrix} w_{t+1} \\ w_{t+1}^f \end{pmatrix}$$

where w_{t+1} and w_{t+1}^f are uncorrelated innovations. The matrices ψ_x and ψ_w thus do not involve any direct impact of the dynamics of the belief shock f_{1t} and the matrix ψ_{xf} captures how the dynamics of f_{1t} influences the dynamics of endogenous state variables.

Let us further assume that the system (18) represents the equilibrium restrictions of the model *except* equation (25). In this case, the function g does not directly depend on f . Repeating the expansion of the equilibrium conditions from Section B.4 and comparing coefficients on x_{t-1} , f_{t-1} , w_t and the constant term yields the set of conditions for matrices ψ_x , ψ_w , ψ_{xf} and ψ_q :

$$0 = (g_{x+} \psi_x + g_x) \psi_x + g_{x-} \quad (27)$$

$$0 = (g_{x+} \rho_f \psi_{xf} - \mathbb{E}) + (g_{x+} \psi_x + g_x) \psi_{xf} \quad (28)$$

$$0 = (g_{x+} \psi_x + g_x) \psi_w + g_w \quad (29)$$

$$0 = (g_{x+} \psi_x + g_{x+} + g_x) \psi_q + g_q - \mathbb{E} \bar{f} \quad (30)$$

with

$$V_x = u_x + \beta V_x \psi_x \quad (31)$$

$$V_f = u_f - \frac{\beta \bar{\theta}}{2} (V_f^2 \sigma_f^2 + 2 V_x \psi_{xf} \sigma_f^2 V_f + V_x (\sigma_f^2 \psi_{xf} \psi'_{xf} + \psi_w \psi'_w) V'_x) + \beta (V_f \rho_f + V_x \psi_{xf} \rho_f) \quad (32)$$

$$\mathbb{E} = \text{stack} \left\{ \sigma^i [g_{x+} \psi_{xf} \sigma_f^2 (V_f + V_x \psi_{xf}) + (g_{x+} \psi_w + g_{w+}) \psi'_w V'_x]^i \right\} \bar{\theta}. \quad (33)$$

This set of equations is the counterpart of equations (21)–(24) and can be solved sequentially. First, notice that equations (27) and (29) can be solved for ψ_x and ψ_w , and these coefficients are not impacted by the dynamics of f_t . But the equilibrium dynamics of x_t is affected by movements in f_t through the coefficient ψ_{xf} . The coefficient $\rho_f \psi_{xf}$ introduces an additional component in the time-varying drift of x_t , while $\sigma_f \psi_{xf}$ is an additional source of volatility arising from the shocks to household's concerns for robustness.

We solve this set of equations by backward induction. First, we use (21), (24) and (31) to find the no-ambiguity solution for ψ_x, ψ_w, V_x . Then we postulate that (26) is in fact a time-dependent law of motion

$$x_t = \psi^t(x_{t-1}, w_t, f_t)$$

with terminal condition at a distant date T

$$x_T = \psi^T(x_{T-1}, w_T, 0).$$

This corresponds to assuming that starting from date T , ambiguity is absent in the model. Plugging this guess to the set of equilibrium conditions, we obtain the set of algebraic equations

$$0 = (g_{x+} \psi_{xf}^{t+1} \rho_f - \mathbb{E}^{t+1}) + (g_{x+} \psi_x + g_x) \psi_{xf}^t \quad (34)$$

$$V_f^t = u_f - \frac{\beta \bar{\theta}}{2} \left((V_f^{t+1} \sigma_f)^2 + 2V_x \psi_{xf}^{t+1} \sigma_f^2 V_f^{t+1} + V_x \left(\sigma_f^2 \psi_{xf}^{t+1} (\psi_{xf}^{t+1})' + \psi_w \psi_w' \right) V_x' \right) \\ + \beta \rho_f (V_f^{t+1} + V_x \psi_{xf}^{t+1}) \quad (35)$$

$$\mathbb{E}^{t+1} = \left[g_{x+} \psi_{xf}^{t+1} (V_f^{t+1} + V_x \psi_{xf}^{t+1}) \sigma_f^2 + (g_{x+} \psi_w + g_{w+}) \psi_w' V_x' \right] \bar{\theta}. \quad (36)$$

Equation (34) can then be solved for

$$\psi_{xf}^t = (g_{x+} \psi_x + g_x)^{-1} (\mathbb{E}^{t+1} - g_{x+} \psi_{xf}^{t+1}) \rho_f \quad (37)$$

Iterating backwards on equations (35)–(37) backward until convergence yields the stationary solution of the economy with ambiguity as a long-horizon limit of an economy where ambiguity vanishes at a distant T . The system converges as long as its dynamics are stationary under the worst-case model. Once we find the limit $\lim_{t \rightarrow -\infty} \mathbb{E}^t = \mathbb{E}$, we can also determine

$$\psi_q = (g_{x+} \psi_x + g_{x+} + g_x)^{-1} (\mathbb{E} \bar{f} - g_q).$$

C Data

To be written.

D Estimation of the one-factor model

Recall that we estimate the model

$$\begin{pmatrix} y_{t+1} \\ f_{t+1} \end{pmatrix} = \begin{pmatrix} \psi_y & \psi_{yf} \psi_f \\ 0 & \psi_f \end{pmatrix} \begin{pmatrix} y_t \\ f_t \end{pmatrix} + \begin{pmatrix} \psi_{yw} & \psi_{yf} \psi_{fw} \\ 0 & \psi_{fw} \end{pmatrix} \begin{pmatrix} w_{t+1}^y \\ w_{t+1}^f \end{pmatrix} \\ \Delta_{t+1}^{(4)} = \psi_{\Delta f} f_{t+1} + \psi_{\Delta w} \varepsilon_{t+1}^{\Delta}$$

Parameter	Prior			Posterior	
	distribution	mean	s.d.	mean	s.d.
ψ_y (diagonal)	Beta	0.7	0.2		
ψ_y (off-diagonal)	Normal	0	2		
ψ_{yw} (diagonal)	Inverse-Gamma	1	0.5		
ψ_{yw} (off-diagonal)	Normal	0	2		
ψ_f	Beta	0.5	0.1	0.90	0.02
ψ_{fw}	Inverse-Gamma	0.5	0.2	0.57	0.23
$H_{unemployment}$	Normal	0	1	-2.48	0.50
$H_{inflation}$	Normal	0	1	1.45	0.39
$H_{interest}$	Normal	0	1	-0.14	1.02
H_f	Normal	0	1	-1.75	0.36

Table 3: Priors and posteriors for factor model estimation.

	GDP growth	unemployment	inflation	interest rate
realized	7.7%	32.2%	2.4%	2.5%
wedge	60.5%	48.0%	7.5%	

Table 4: Contributions of w_t^f from variance decomposition at modal estimate.

We estimate the model using a Metropolis–Hastings algorithm. We take five chains with different initial draws and make 10,000 draws in each chain. The first 5,000 draws of each chain are dropped. The priors and posterior parameter estimates are reported in Table 3. The contribution of w_t^f to the variation in y_t and $\overline{\Delta}_t^{(4)}$ based on a variance decomposition at the estimated mode of the parameters is reported in Table 4.

E Details on the New-Keynesian model

To be written.

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