

Adverse Selection and Self-fulfilling Business Cycles^{*}

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This Version: February 2016

First Version: June 2014

Abstract

We develop a macroeconomic model with adverse selection in credit markets. A continuum of final-goods producers borrow from financial intermediary to purchase intermediate goods as input. The type of producers as borrower is private information. Adverse selection arises here. Higher aggregate supply of credit induces more high-quality borrowers, lowers default risks face by each financial intermediary, and stimulate more individual credit supply. We show that this lending externality can generate multiple equilibria or indeterminacy even when the steady state equilibrium is unique, making self-fulfilling expectation driven business cycles possible.

Keywords: Adverse Selection, Local Indeterminacy, Global Dynamics, Sunspots.

JEL codes: E44, G01, G20.

^{*}We are indebted to Lars Peter Hansen, Alessandro Lizzeri, Jianjun Miao, Venky Venkateswaran, Yi Wen and Tao Zha for very enlightening comments.

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1 Introduction

The seminal work of Wilson (1980) shows that in a static model, adverse selection can generate multiple equilibria because of asymmetric information about product quality. The aim of this paper is to analyze how adverse selection in the credit market can give rise to lending externalities to generate multiple equilibria and indeterminacies in an otherwise standard dynamic general equilibrium model of business cycles.

To make this point, we incorporate a simple type of adverse selection in the credit market into the standard textbook real business cycle model. The model features a continuum of households and a continuum of anonymous producers. These producers use intermediate goods to produce the final goods. They do not have resource to make the up-front payment for intermediate inputs and hence need to borrow from the competitive financial intermediates to finance their working capital. The loan is risky as the borrower may default. We assume that there are two types of borrowers (producers). The honest borrowers always pay back, while the dishonest borrowers always run away with the loan and default. The financial intermediates have no information about the borrowers' honesty. This give rise to adverse selection: given the interest rate, the dishonest borrowers have a stronger incentive to borrow. In such an environment, an optimistic increase in lending from some financial intermediates encourage more honest producers to borrow. The increased quality of borrowers reduce default risk which in turn stimulates more lending from other financial intermediaries. The resulting decline in the interest rate brings down the production cost for all producers/borrowers. This stimulates an expansion in output, which further expands credit supply from the household and generates more lending in the future. In the other words, there exists a lending externality both intratemporally and intertemporally. In our baseline model in Section 2, we show that this lending externality not only generates two steady state equilibria with low and high average default rates, but also gives rise to a continuum of equilibria around one of the steady states.

Adverse selection in the credit market seems to be a realistic feature, both in poor and rich countries (see Sufi (2007) for evidence from syndicated loan in U.S. and Karlan and Zinman (2009) for evidence from field experiment in South African). Our model has several implications that are supported by empirical evidence. First, a large literature has documented that credit risk is countercyclical and have far-reaching macroeconomic consequence. For instance, Gilchrist and Zakrajsek (2012) finds that a shock to credit risk lead to significant declines in consumption, investment, and output. Pintus, Wen and Xing (2015) document that interest

rate faced by the US firms move countercyclically and invertedly leads the business cycle. These facts are consistent with our model's prediction. Second, our model delivers a countercyclical markup, an important empirical regularity well documented in the literature. In our model, because of information asymmetry, dishonest borrowers enjoy an informational rent. But when the average quality of borrowers increases due to higher lending, this informational rent is diluted. So the measured markup declines, which is critical to sustaining indeterminacy by bringing about higher real wages, a positive labor supply response, and a higher output that dominates the income effect on leisure. Third, our extended model in Section ?? can explain the well-known procyclical variation in productivity. The procyclicality of average quality in the credit market implies that resources are reallocated toward producers with lower credit risk when aggregate output increases. The improved resource allocation then raises productivity endogenously. The procyclical endogenous TFP immediately implies that increases in inputs lead to a more than proportional increase in total aggregate output, in other words, aggregate increasing returns. The increasing returns to scale arises only at the aggregate level in our model helps solve the puzzle documented by Basu and Fernald (1997), who find a slightly decreasing returns to scale for a typical two-digit industry in the United States, but strong increasing returns to scale at the aggregate level. Finally, adverse selection is arguably more prevalent in developing countries, possibly due to weak law enforcement and poorly-managed financial intermediates. An important insight of our study is that indeterminacy arises only if adverse selection is severe enough, which then implies that developing countries are more prone to indeterminacy and self-fulfilling expectation-driven business cycles. This also helps to explain another documented fact that developing countries typically exhibit larger output volatility than developed countries (see e.g., Ramey and Ramey (1995) and Easterly, Islam, and Stiglitz (2000)).

In a dynamic setting market forces and competition can mitigate adverse selection through reputation effects that are absent in our baseline model in Section 2. We therefore examine whether indeterminacy is compatible with reputational effects in Section 3. We follow Kehoe and Levine (1993) to assume that if a borrower defaults with some probability, loses reputation, and is excluded from the credit market forever. In this case we show that the steady state equilibrium becomes unique and no default occurs in equilibrium. Nevertheless, perhaps surprisingly, indeterminacy in the form of a continuum of equilibria may continue to exist.

Our paper is closely related to two branches of literature in macroeconomics. First, our paper builds on a large strand literature on the possibility of indeterminacy in RBC models.

Benhabib and Farmer (1994) point out that increasing returns to scale can generate indeterminacy in an RBC model. The required degree of increasing returns to scale for indeterminacy, however, is considered to be implausibly large by empirical evidence (see Basu and Fernald (1995, 1997)). Subsequent work in the literature has introduced additional features to the Benhabib-Farmer model that reduce the degree of increasing returns required for indeterminacy. In an important contribution, Wen (1998) adds variable capacity utilization and shows that indeterminacy can arise with a magnitude of increasing returns similar to that in the data. Gali (1994) and Jaimovich (2007) explore the possibility of indeterminacy via countercyclical markup due to output composition and firm entry respectively. The literature has also shown that models with indeterminacy can replicate many of the standard business cycle moments as the standard RBC model (see Farmer and Guo (1994)). Furthermore, indeterminacy models may outperform the standard RBC models in many other dimensions. For instance, Benhabib and Wen (2004), Wen and Wang (2008), and Benhabib and Wang (2014) show that models with indeterminacy can explain the hump-shaped output dynamics and relative volatility of labor and output, which are challenges for the standard RBC models. Our paper complements this strand of literature by adding adverse selection as a different source of indeterminacy. The adverse selection approach also provides a micro-foundation to the aggregate increasing returns to scale. Indeed, if we specify a Pareto distribution for firm productivity, our model in Section 4 is isomorphic to those that have a representative-firm economy with increasing returns, such as the one studied by Benhabib and Farmer (1994) and Wen (1998). It therefore inherits the ability of reproducing the business cycle features mentioned above without having to rely on increasing returns.¹

Second, our paper is closely related to a small but rapidly growing literature that study the macroeconomic consequences of adverse selection. Kurlat (2013) builds a dynamic general equilibrium with adverse selection in the second-hand market for capital assets. Kurlat (2013) shows that the degree of adverse selection varies countercyclically. Since adverse selection reduces the efficiency of resource allocation, a negative shock that lowers aggregate output will exacerbate adverse selection and worsen resource allocation efficiency. So the impact of the initial shocks on aggregate output is propagated through time. Like Kurlat (2013), Bigio (2014) develops an RBC model with adverse selection in the capital market. As firms must sell the existing capital to finance investment and employment, adverse selection distorts both capital

¹Liu and Wang (2014) provides an alternative mechanism to generate increasing returns via financial constraints.

and labor markets. Bigio (2014) shows that the adverse selection shock widens a dispersion of capital quality, exacerbates the distortion, and leads to a recession with a quantitative pattern similar to that observed during the Great Recession of 2008. Our model generates similar predictions as Kurlat (2013) and Bigio (2014). First, adverse selection is also countercyclical in our model, so the propagation of fundamental shocks via adverse selection highlighted by Kurlat (2013) exist also in our model. Second, adverse selection in the goods market in our model naturally creates the distortions to both capital and labor inputs. A dispersion shock to the quality of products in our extended model in Section ?? aggravates adverse selection, and makes the economy more vulnerable to self-fulfilling expectation-driven fluctuations. While Kurlat (2013) and Bigio (2014) emphasize the role of adverse selection in propagating business cycles shocks, our paper complements their work by showing that adverse selection can generate indeterminacy and hence can be a source of business cycles.² Our extended model in section (3) with reputation effects is also related to that of Chari, Shourideh and Zeltin-Jones (2014), who build a model of a secondary loan market with adverse selection and show how reputation effects can generate persistent adverse selection. Multiple equilibria also arise in their model as in the classic signaling model by Spence (1973). In contrast, multiple equilibria in our model take the form of indeterminacy, and are generated by a different mechanism of endogenously countercyclical markups that translate into aggregate increasing returns.

The rest of the paper is organized as follows. Section 2 describes the baseline model and characterizes the conditions for indeterminacy. Section 3 incorporates warranties and reputation effects into the baseline model and shows indeterminacy may still arise. In Section 4 we introduce a continuous distribution of product quality and show that adverse selection can induce endogenous TFP, amplification, and aggregate increasing returns to scale. Section 4 present an alternative model with adverse selection in the credit market. Section 5 concludes. The appendix collects some of the proofs.

2 The Baseline Model

Time is continuous and proceeds from zero to infinity. There is an infinitely-lived representative household and a continuum of final good producers. The final goods producers purchase the intermediate goods as input to produce final good, which is then sold to household for

²Many other papers have also addressed adverse selection in a dynamic environment. Examples include Eisfeldt (2004), House (2006), Guerrieri, Shimer, and Wright (2010), Chiu and Koepl (2012), Daley and Green (2012), Chang (2014), Camargo and Lester (2014), and Guerrieri and Shimer (2014).

consumption and investment. The intermediate goods is produced by capital and labor in a competitive market. We assume no distortion in the production of intermediate goods. Final goods firms do not have resources to make up-front payments for intermediate good input until production takes place and revenues from sales are realized. They therefore must borrow from the competitive financial intermediates (the lenders) to finance their working capital³. The loan is risky as the final goods producers may default. We assume that there are two types of producers (borrowers): the honest borrowers have the ability to produce and always pay back the loan after the production, while the dishonest borrowers simply run away with the loan. The lenders do not have information about lenders' types. They makes loan to firms by taking the adverse selection problem into consideration. We begin by assuming that all trade is anonymous so we exclude the possibility of reputation effects. We relax this strong assumptions in Section 3, and introduce reputation effects.

2.1 Setup

Households The representative household has a lifetime utility function

$$\int_0^\infty e^{-\rho t} \left[\log(C_t) - \psi \frac{N_t^{1+\gamma}}{1+\gamma} \right] dt \quad (1)$$

where $\rho > 0$ is the subjective discount factor, C_t is the consumption, N_t is the hours worked, $\psi > 0$ is the utility weight for labor, and $\gamma \geq 0$ is the inverse Frisch elasticity of labor supply. The household faces the following budget constraint

$$C_t + I_t \leq R_t u_t K_t + W_t N_t + \Pi_t, \quad (2)$$

where R_t , W_t and Π_t denote respectively the rental price, wage and total profits from all the firms and financial intermediates. In an important contribution, Wen (1998) shows that introducing an endogenous capacity utilization rate u_t makes indeterminacy empirically more plausible in models with production externalities. We will show that capacity utilization serves a similar role in our model. As is standard in the literature, the depreciation rate of capital increases with the capacity utilization rate according to

$$\delta(u_t) = \delta^0 \frac{u_t^{1+\theta}}{1+\theta}, \quad (3)$$

³The financial intermediates provide working capital loans in the form of inside money.

where $\delta^0 > 0$ is a constant and $\theta > 0$.⁴ Finally, the law of motion for capital is governed by

$$\dot{K}_t = -\delta(u_t)K_t + I_t. \quad (4)$$

The households choose a path of consumption X_t , C_t , N_t , u_t , and K_t to maximize the utility function (1), taking R_t , W_t and Π_t as given. The first-order conditions are

$$\frac{1}{C_t}W_t = \psi N_t^\gamma, \quad (5)$$

$$\frac{\dot{C}_t}{C_t} = u_t R_t - \delta(u_t) - \rho, \quad (6)$$

and

$$R_t = \delta^0 u_t^\theta. \quad (7)$$

The left-hand side of Equation (5) is the marginal utility of consumption obtained from an additional unit of work, and the right-hand side is the marginal disutility of a unit of work. Equation (6) is the usual Euler equation. Finally, a one-percent increase in the utilization rate raises the total rent by $R_t K_t$ but also increases total depreciation by $\delta_0 u_t^\theta K_t$, so Equation (7) states that the marginal benefit is equal to the marginal cost of utilization.

Final goods producers There is unit measure of final good producers indexed by i . A fraction π of them are dishonest and a fraction $1 - \pi$ are honest. Each one of the honest producers is endowed with an indivisible project as in Stiglitz and Weiss (1981), which transform Φ units of intermediate goods to Φ units of final goods. Let P_t be price of the intermediate goods input. Each project then requires ΦP_t of working capital. The dishonest producers, however, can claim to be honest and borrow $P_t \Phi$ and then run away with the borrowed funds. They enjoy $P_t \Phi$ profits by doing so. Anticipating this adverse selection problem, the final intermediates will hence charge a gross interest rate $R_{ft} > 1$ to all borrowers. Hence the profit from borrowing and producing for a honest producer is given

$$\Pi_t^h = [1 - R_{ft} P_t] \Phi \quad (8)$$

Denote s_t measure of honest producers invest in their projects, we have

$$s_t = \begin{cases} 1 - \pi & \text{if } R_{ft} < \frac{1}{P_t} \\ \in [0, 1 - \pi) & \text{if } R_{ft} = \frac{1}{P_t} \\ 0 & \text{if } R_{ft} > \frac{1}{P_t} \end{cases}. \quad (9)$$

⁴Dong, Wang, and Wen (2014) develop a search-based theory to offer a micro-foundation for the convex depreciation function.

The total demand for intermediate goods input is hence given by

$$X_t = s_t \Phi, \quad (10)$$

Since each of them also produce Φ unit of final goods, the total final good is hence

$$Y_t = s_t \Phi = X_t \quad (11)$$

Intermediate goods The intermediate goods is produced by capital and labor with the technology

$$X_t = A \tilde{K}_t^\alpha N_t^{1-\alpha}, \quad (12)$$

where $\tilde{K}_t = u_t K_t$ is total capital supply from the households. In a competitive market. The profit is $\Pi_t^x = P_t A \tilde{K}_t^\alpha N_t^{1-\alpha} - W_t N_t - R_t \tilde{K}_t$ The first order conditions are

$$R_t = P_t \alpha \frac{X_t}{\tilde{K}_t} = P_t \alpha \frac{X_t}{u_t K_t}, \quad (13)$$

$$W_t = P_t (1 - \alpha) \frac{X_t}{N_t}. \quad (14)$$

Notice that $\Pi_t^x = 0$, so $W_t N_t + R_t u_t K_t = P_t X_t$.

Financial Intermediates The financial intermediates behave competitive. Anticipating Θ_t fraction of loan will be paid back, the interest rate is then given by

$$R_{ft} = \frac{1}{\Theta_t}. \quad (15)$$

So the financial intermediates earn zero profit. The honest producers borrow total $X_t P_t$ working capital loans and the dishonest producers borrows total $\pi \Phi P_t$ working capital loans. Since only the honest producer pay back their loan, so the average payback rates is

$$\Theta_t = \frac{X_t P_t}{\pi \Phi P_t + X_t P_t} = \frac{X_t}{\pi \Phi + X_t}. \quad (16)$$

2.2 Equilibrium

We focus on an interior solution so $R_{ft} = \frac{1}{P_t}$ ⁵. In equilibrium, the total profits are simply $P_t \Phi$. Hence the total budget constraint becomes

$$C_t + I_t = P_t X_t + P_t \Phi. \quad (17)$$

⁵We assume that Φ is big enough, so $\Phi > A K_t^\alpha N_t^{1-\alpha}$. Notice this is without loss of generality. We can also assume that there are potential infinite measures of honest producers. An free rate entry condition then implies $R_{ft} = \frac{1}{P_t}$.

Since $P_t = \frac{1}{R_{ft}} = \Theta_t$, the above equation can be further reduced to

$$C_t + I_t = P_t X_t + P_t \Phi = X_t = Y_t. \quad (18)$$

We hence obtain the resource constraint

$$C_t + \dot{K}_t = Y_t - \delta(u_t)K_t. \quad (19)$$

The inverse of markup is hence given by:

$$\phi_t \equiv 1 - \frac{\Pi_t}{Y_t} = \Theta_t = P_t.$$

and

$$R_t = \phi_t \cdot \left(\frac{\alpha Y_t}{u_t K_t} \right). \quad (20)$$

Likewise, using Equations (14) and (??), we obtain

$$W_t = \phi_t \cdot \frac{(1 - \alpha) Y_t}{N_t}. \quad (21)$$

Equation (5), (6) and (7) then become

$$\psi N_t^\gamma = \left(\frac{1}{C_t} \right) (1 - \alpha) \phi_t \frac{Y_t}{N_t}, \quad (22)$$

$$\frac{\dot{C}_t}{C_t} = \alpha \phi_t \frac{Y_t}{K_t} - \delta(u_t) - \rho, \quad (23)$$

$$\alpha \phi_t \frac{Y_t}{u_t K_t} = \delta^0 u_t^\theta = (1 + \theta) \frac{\delta(u_t)}{u_t}. \quad (24)$$

Then we have

$$u_t = \left(\frac{\alpha \phi_t Y_t}{\delta^0 K_t} \right)^{\frac{1}{1+\theta}}, \quad (25)$$

and thus

$$\frac{\dot{C}_t}{C_t} = \left(\frac{\theta}{1 + \theta} \right) \alpha \phi_t \frac{Y_t}{K_t} - \rho.$$

Equation (16) then becomes

$$\phi_t = \frac{Y_t}{\pi \Phi + Y_t} \quad (26)$$

Finally the aggregate production function becomes

$$Y_t = A (u_t K_t)^\alpha N_t^{1-\alpha}. \quad (27)$$

In short, the equilibrium can be characterized by equations (22), (23), (24), (27), (19) and (26).

These six equations fully determine the dynamics of the six variables C_t, K_t, Y_t, u_t, N_t and ϕ_t .

Equation (26) implies that ϕ_t increases with aggregate output. Notice that $\frac{1}{\phi_t} = \frac{Y_t}{R_t u_t K_t + W_t N_t}$ is the aggregate markup in our model economy. Therefore the endogenous markup in our model is countercyclical, which is consistent with the empirical regularity well documented in the literature.⁶ The credit spread in our model is $R_{ft} - 1 = \pi\Phi/Y_t$, moving in countercyclical fashion as in the data.

The countercyclical markup has important implications. For example, it can make hours and the real wage move in the same direction. To see this, suppose N_t increases, so output increases. Then according to Equation (26), marginal cost ϕ_t increases as well, which in turn raises the real wage (21). If the markup is a constant, then the real wage would be proportional to the marginal product of labor and would fall when hours increase. Note also that when $\pi = 0$, *i.e.*, there is no adverse selection, Equation (26) implies that $\phi_t = 1$, and our model simply collapses into a standard real business cycle model. The markup is $1/\phi_t > 1$ if and only if lemon producers obtain an information rent arising from the information asymmetry on product quality.

2.3 Steady State

We first study the steady state of the model. We use Z to denote the steady state of variable Z_t . To solve the steady state, we first express all other variables in terms of ϕ and then we solve ϕ as a fixed point problem. Combining Equation (23) and (24) yields

$$\delta^0 u^{\theta+1} - \frac{\delta^0 u^{\theta+1}}{1+\theta} = \rho,$$

or $u = \left[\frac{1}{\delta^0} \frac{\rho}{\theta} (1+\theta) \right]^{1+\theta}$. Notice that u only depends on δ^0 , ρ and θ . Therefore, without loss of generality, we can set $\delta^0 = \frac{\rho}{\theta} (1+\theta)$ so that $u = 1$ at the steady state. The depreciation rate at steady state is then $\delta(u) = \rho/\theta$. Given ϕ , we have

$$\begin{aligned} k_y &= \frac{K}{Y} = \frac{\alpha\phi}{\rho + \rho/\theta} = \frac{\alpha\phi\theta}{\rho(1+\theta)}, \\ c_y &= 1 - \delta k_y = 1 - \frac{\alpha\phi}{1+\theta}, \\ N &= \left(\frac{(1-\alpha)\phi}{1 - \frac{\alpha\phi}{1+\theta}} \frac{1}{\psi} \right)^{\frac{1}{1+\gamma}}, \\ Y &= A^{\frac{1}{1-\alpha}} \left(\frac{\alpha\phi\theta}{\rho(1+\theta)} \right)^{\frac{\alpha}{1-\alpha}} \left(\frac{(1-\alpha)\phi}{1 - \frac{\alpha\phi}{1+\theta}} \frac{1}{\psi} \right)^{\frac{1}{1+\gamma}} \equiv Y(\phi). \end{aligned}$$

⁶See e.g., *Bils (1987) and Rotemberg and Woodford (1999)*.

Then we can use Equation (26) to pin down ϕ from

$$\bar{\Phi} \equiv \pi\Phi = \left(\frac{1-\phi}{\phi}\right) \cdot Y(\phi) \equiv \Psi(\phi), \quad (28)$$

where the left-hand side is the total supply of lemon products and the right hand-side is the maximum amount of lemon that the market can accommodate, given that the average product quality is $q = \phi$. When $\alpha/(1-\alpha) + \frac{1}{1+\gamma} > 1$, $\Psi(\phi)$ is a non-monotonic function of ϕ since $\Psi(0) = 0$ and $\Psi(1) = 0$. On the one hand, if the average quality is 0, the household demand would be zero, and hence no lemon will be needed. On the other hand, if the average quality is one, *i.e.*, $\phi = q = 1$, then by definition no lemon will be sold. So given $\bar{\Phi}$, there may exist two steady state values of ϕ .

Denote $\Psi^* \equiv \max_{0 \leq \phi \leq 1} \Psi(\phi)$, and $\phi^* \equiv \arg \max_{0 \leq \phi \leq 1} \Psi(\phi)$. Then we have the following lemma regarding the possibility of multiple steady state equilibria.

Lemma 1 *When $0 < \bar{\Phi} < \Psi^*$, there exists two steady states ϕ that solve $\bar{\Phi} = \Psi(\phi)$.*

Proof: *The proof is straightforward. First, from the explicit form of $Y(\phi)$, we can easily prove that $\Psi(\phi) \equiv \left(\frac{1-\phi}{\phi}\right) \cdot Y(\phi)$ strictly increases with ϕ when $\phi \in (0, \phi^*)$ but strictly decreases with ϕ when $\phi \in (\phi^*, 1)$. Second, since $\Psi(0) < \bar{\Phi} < \Psi^* = \Psi(\phi^*)$, there exists a unique solution between zero and ϕ^* , denoted by $\bar{\phi}_L$, that solves $\Psi(\phi) = \bar{\Phi}$. Likewise, there also exists a unique solution between ϕ^* and 1, denoted by $\bar{\phi}_H$ that solves $\Psi(\phi) = \bar{\Phi}$. ■*

It is well known that adverse selection can generate multiple equilibria in a static model (see e.g. Wilson (1980)). So it is not surprising that our model has multiple steady state equilibria. The demand for goods depends on the households' expectation of the average quality of the final goods. By Equation (26), however, the average quality of goods depends on the total demand from households, so there is a demand externality. More specifically, if households increase their demand for goods, then the price rises. In turn, more-high quality goods will be produced, and thus the average quality of goods will increase. If the average quality increases faster than the price, each household will then increase their demand as well. We will show that this type of demand externality generates a new type of multiplicity, which shares some similarity with the indeterminacy literature following Benhabib and Farmer (1994).

2.4 Local Dynamics

A number of studies have explored the role of endogenous markup in generating local indeterminacy and endogenous fluctuations (see e.g., Jaimovich (2006) and Benhabib and Wang (2013)). Following the standard practice, we study the local dynamics around the steady state.

Note that at the steady state ϕ and $\bar{\Phi}$ are linked by $\bar{\Phi} = \Psi(\phi)$, so we can parametrize the steady state either by $\bar{\Phi}$ or ϕ . We will use ϕ as it is more convenient for the study of local dynamics. We denote $\hat{x}_t = \log X_t - \log X$ as the percent deviation from its steady state. First, we log-linearize Equation (26) to obtain

$$\hat{\phi}_t = (1 - \phi)\hat{y}_t \equiv \tau\hat{y}_t, \quad (29)$$

which states that the percent deviation of the marginal cost is proportional to output. Log-linearizing Equations (27) and (24) yields

$$\hat{y}_t = \frac{\alpha\theta\hat{k}_t + (1 + \theta)(1 - \alpha)\hat{n}_t}{1 + \theta - (1 + \tau)\alpha} \equiv a\hat{k}_t + b\hat{n}_t, \quad (30)$$

where $a \equiv \frac{\alpha\theta}{1 + \theta - (1 + \tau)\alpha}$ and $b \equiv \frac{(1 + \theta)(1 - \alpha)}{1 + \theta - (1 + \tau)\alpha}$. We assume that $1 + \theta - (1 + \tau)\alpha > 0$, or equivalently $\tau < \frac{1 + \theta}{\alpha} - 1$, to make $a > 0$ and $b > 0$. In general these restrictions are easily satisfied.

It is worth mentioning that $a + b = \frac{1 + \theta - \alpha}{1 + \theta - (1 + \tau)\alpha} = 1$ if $\tau = 0$. Recall that $\tau = 0$ corresponds to the case without adverse selection. Thus endogenous capacity utilization alone does not generate increasing returns to scale at the aggregate level. However, $a + b = \frac{1 + \theta - \alpha}{1 + \theta - (1 + \tau)\alpha} > 1$ if $\tau > 0$; that is, adverse selection combined with endogenous capacity utilization generates increasing returns to scale. Furthermore, if $\tau > \theta$, then $b > 1$. The model then can explain the procyclical movements in labor productivity $\hat{y}_t - \hat{n}_t$ without resorting to exogenous TFP shocks.

We can substitute out $\hat{n}_t$ after log-linearizing equation (22) to express $\hat{y}_t$ as

$$\hat{y}_t = \frac{a(1 + \gamma)}{1 + \gamma - b(1 + \tau)}\hat{k}_t - \frac{b}{1 + \gamma - b(1 + \tau)}\hat{c}_t \equiv \lambda_1\hat{k}_t + \lambda_2\hat{c}_t. \quad (31)$$

According to Equation (30), a one-percent increase in capital directly increases output and the marginal product of labor by a percent and, from Equation (29), reduces the markup by $a\tau$ percent. Thanks to its higher marginal productivity, the labor supply also increases. A one-percent increase in labor supply then increases output by b percent. The precise increase in labor supply depends on the Frisch elasticity γ . This explains why the equilibrium output elasticity with respect to capital, λ_1 , depends on parameters a , b and through them on γ and τ . On the household side, since both leisure and consumption are normal goods, an increase in consumption has a wealth effect on labor supply. The effect of a change in labor supply on output induced by a change in consumption, as seen from Equation (31) obtained after substituting for labor in (30), works through the marginal cost channel, and also depends on

τ . Again since both a and b increase with τ , output elasticities with respect to capital and consumption are increasing functions of τ . In other words, the presence of adverse selection makes equilibrium output more sensitive to changes in capital and to changes in autonomous consumption, and creates an amplification mechanism for business fluctuations.

Using Equation (31) and the log-linearized Equations (23) and (19), we can then characterize the local dynamics as follows:

$$\begin{aligned} \begin{bmatrix} \dot{k}_t \\ \dot{c}_t \end{bmatrix} &= \delta \begin{bmatrix} \left(\frac{1+\theta}{\alpha\phi}\right) \lambda_1 - (1+\tau) \lambda_1 & \left(\frac{1+\theta}{\alpha\phi}\right) (\lambda_2 - 1) + 1 - (1+\tau) \lambda_2 \\ \theta [(1+\tau) \lambda_1 - 1] & \theta (1+\tau) \lambda_2 \end{bmatrix} \begin{bmatrix} \hat{k}_t \\ \hat{c}_t \end{bmatrix} \\ &\equiv J \begin{bmatrix} \hat{k}_t \\ \hat{c}_t \end{bmatrix}, \end{aligned} \quad (32)$$

where $\lambda_1 = \frac{a(1+\gamma)}{1+\gamma-b(1+\tau)}$, $\lambda_2 = -\frac{b}{1+\gamma-b(1+\tau)}$, and $\delta = \rho/\theta$ is the steady state depreciation rate. The local dynamics around the steady state is determined by the roots of J . The model exhibits local indeterminacy if both roots of J are negative. Note that the sum of the roots equals the trace of J , and the product of the roots equals the determinant of J . Thus the sign of the roots of J can be observed from the signal of its trace and determinant of J . The following lemma specifies the sign for the trace and determinant condition for local indeterminacy.

Lemma 2 Denote $\tau_{\min} \equiv \frac{(1+\theta)(1+\gamma)}{(1+\theta)(1-\alpha)+\alpha(1+\gamma)} - 1$ and $\tau_{\max} \equiv 1 - \phi^*$, then $\text{Trace}(J) < 0$ if and only if $\tau > \tau_{\min}$, and $\text{Det}(J) > 0$ if and only if $\tau_{\min} < \tau < \tau_{\max}$.

According to Lemma 2, our baseline model will be indeterminate if and only if $\tau_{\min} < \tau < \tau_{\max}$. In this case, $\text{Trace}(J) < 0$ and $\text{Det}(J) > 0$ jointly imply that both roots of J are negative. We summarize this result in the following proposition.

Proposition 1 The model exhibits local indeterminacy around a particular steady state if and only if

$$\tau_{\min} < \tau < \tau_{\max}. \quad (33)$$

Equivalently, indeterminacy emerges if and only if $\phi \in (\phi_{\min}, \phi_{\max})$, where $\phi_{\min} \equiv 1 - \tau_{\max} = \phi^*$, and $\phi_{\max} \equiv 1 - \tau_{\min}$.⁷

⁷Summary

1. If $\phi < \phi_{\min}$, then $\text{Trace}(J) < 0$, and $\text{Det}(J) < 0$.
2. If $\phi \in (\phi_{\min}, \phi_{\max})$, then $\text{Trace}(J) < 0$, and $\text{Det}(J) > 0$.
3. If $\phi > \phi_{\max}$, then $\text{Trace}(J) > 0$, and $\text{Det}(J) < 0$.

To understand this intuition, first notice that if $\tau > \tau_{\min}$, we have

$$1 + \gamma - b(1 + \tau) < 1 + \gamma - \frac{(1 + \theta)(1 - \alpha)}{1 + \theta - (1 + \tau_{\min})\alpha}(1 + \tau_{\min}) = 0.$$

Then the equilibrium elasticity of output with respect to consumption λ_2 becomes positive, namely, an autonomous change in consumption will lead to an increase in output. Since capital is predetermined, labor must increase by Equation (30). To induce an increase in labor, the real wage must increase enough to overcome the income effect, which is only possible if the increase in markup is large enough. In other words τ in Equation (29) has to be large enough.

We can also understand Proposition 1 from the equilibrium conditions of the goods market. On the one hand, since $P_t = \Theta_t = \phi_t = \frac{1}{R_{ft}}$, Equation (29) can be rewritten as

$$\hat{R}_{ft} = -\tau \hat{y}_t. \quad (\text{S-S})$$

We can treat the above equation as the supply curve in the working capital loan market. When interest rates increase, the average quality of borrowers also increases. If the increase in quality dominates the increase in the interest rate, the effective interest rate received by the lender increases. This explains why the supply curve has a positive slope. On the other hand, combining Equations (22), (27), and (24) yields

$$\hat{R}_{ft} = -\tau_{\min} \cdot \hat{y}_t - \left[\frac{(1 - \alpha)(1 + \theta)}{\alpha(1 + \gamma) + (1 - \alpha)(1 + \theta)} \right] \cdot \hat{c}_t + \left[\frac{\alpha\theta(1 + \gamma)}{\alpha(1 + \gamma) + (1 - \alpha)(1 + \theta)} \right] \cdot \hat{k}_t \quad (\text{D-D})$$

where $\tau_{\min} \equiv \frac{(1 + \theta)(1 + \gamma)}{(1 + \theta)(1 - \alpha) + \alpha(1 + \gamma)} - 1$. We interpret the above equation as the demand curve in the working capital loan market. The firms' production cost increases interest rate R_{ft} , wage W_t and the rental rate R_t . If the production cost increases, firms have incentive to produce less and hence demand less working capital loan. So the demand curve (D-D) slopes down. Interest rate decreases reduces the demand of working capital loan increases, resulted in a reduction in total production. The position of the demand curve depends on $\hat{c}_t$ and $\hat{k}_t$. Everything else being equal, an increase in $\hat{c}_t$ reduces the willingness to work and hence increase wage W_t , resulted in less demand for working capital loan. Hence it shifts the demand curve down. In contrast, an increase in capital will reduce the rental rate R_t , leading to an upward shift of the demand curve.

If $\tau > \tau_{\min}$, the supply curve is steeper than the demand curve, which makes indeterminacy possible. Figure 1 gives a graphic explanation of this. The lines labeled with D and S represent the demand and supply curves respectively. An optimistic belief of higher income induces households to increase their consumption. The wealth effect also creates a press for

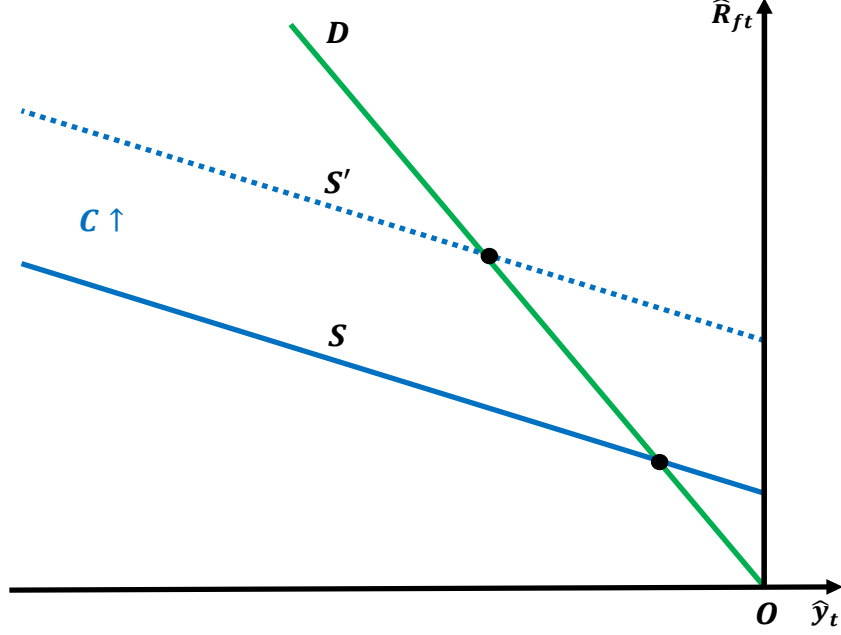


Figure 1: Demand and Supply: Local Indeterminacy

wage increases, shift the demand curve downward. If the supply curve is positively sloped, a downward shift of the demand curve decreases the equilibrium working capital loan and hence output. The realized output and income would then be lower, contradicting the initial optimistic belief. However, if the supply curve slopes downward and is steeper than the demand curve, an downward shift of the demand curve leads to an increase in working capital loan and hence the equilibrium output. Therefore the initial optimistic belief is then consistent with rational expectations.

We have used the mapping between τ and steady state output to characterize the indeterminacy condition in terms of the model's deep parameter values. Notice $\tau_{\max} = 1 - \phi^*$, where $\phi^* \equiv \arg \max_{0 \leq \phi \leq 1} \Psi(\phi)$. Since $1 - \bar{\phi}_L > 1 - \phi^* = \tau_{\max}$, the local dynamics around the steady state associated with $\phi = \bar{\phi}_L$ are determinate according to Proposition 1. Indeterminacy is only possible in the neighborhood of the steady state associated with $\phi = \bar{\phi}_H$. The following corollary formally characterizes the indeterminacy condition in terms of $\bar{\Phi}$.

Corollary 1 *If $\Psi(\phi_{\max}) < \bar{\Phi} < \Psi_{\max}$, the local dynamics around the steady state $\phi = \bar{\phi}_H$ exhibits indeterminacy, while the local dynamics around the steady state $\phi = \bar{\phi}_L$ is a saddle. If $0 < \bar{\Phi} < \Psi(\phi_{\max})$, both steady states are saddles.*

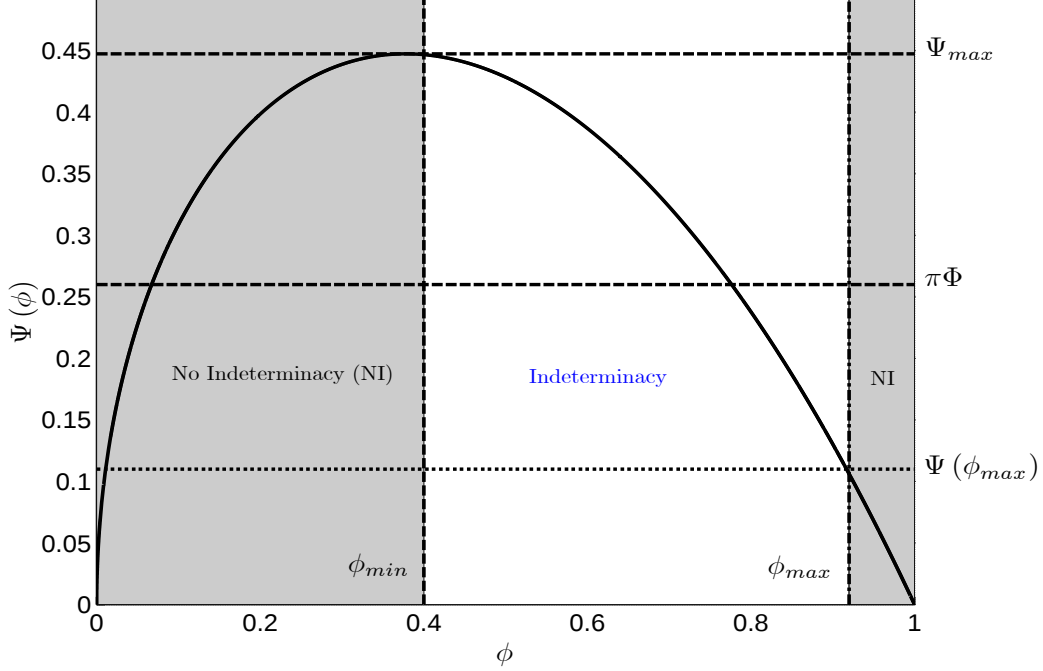


Figure 2: Multiple Steady States and the Indeterminacy Region

When $\Psi(\phi_{\max}) < \bar{\Phi} < \Psi_{\max}$, we have $\phi_{\min} = \phi^* < \bar{\phi}_H < \phi_{\max}$, and $\bar{\phi}_L < \phi_{\min}$. As a result, according to Proposition 1, the steady state $\bar{\phi}_H$ exhibits indeterminacy. And for the steady state $\phi = \bar{\phi}_L$, by Lemma 2, we can conclude that the determinant of J is negative. So the two roots of J must have opposite signs and this implies a saddle. But, if $0 < \bar{\Phi} < \Psi(\phi_{\max})$, we have $\bar{\phi}_H > \phi_{\max}$ and $\bar{\phi}_L < \phi_{\min}$. In this case, the determinants of J at both steady states are negative. So both steady states are a saddle.

The different scenarios are summarized in Figure 2. The inverted U curve illustrates the relationship between ϕ and $\bar{\Phi}$ specified in Equation (28). In Figure 2, ϕ is on the horizontal axis and $\bar{\Phi}$ is on the vertical axis. For a given $\bar{\Phi}$, the two steady states $\bar{\phi}_L$ and $\bar{\phi}_H$ can be located from the intersection between the inverted U curve and a horizontal line through point $(0, \bar{\Phi})$. The two vertical lines passing points $(\phi_{\min}, 0)$ and $(\phi_{\max}, 0)$ divide the diagram into three regions. In the left and right regions, the determinant of the Jacobian matrix J is negative, implying one of the roots is positive and the other is negative. So if a steady state ϕ falls into either of these two regions, it is a saddle. In the middle region, $\det(J) > 0$ and $\text{Trace}(J) < 0$, and thus both roots are negative. So if the steady state ϕ falls into the middle region it is a sink which supports self-fulfilling expectation-driven multiple equilibria, or indeterminacy.

Since $\bar{\Phi} = \pi\Phi$, we can revisit the above corollary in terms of π , the proportion of lemon producers. Without loss of generality, assume Φ is big enough such that $\Phi > \Psi_{\max}$. Denote $\pi_L = \Psi(\phi_{\max})/\Phi$ and $\pi_H = \Psi(\phi_{\min})/\Phi$, and thus $0 < \pi_L < \pi_H < 1$. Then we know that (i) if $\pi \in (0, \pi_L]$, so there are two equilibria, both of which are stable; (ii) if $\pi \in (\pi_L, \pi_H)$, the steady state with $\phi = \bar{\phi}_L$ is a saddle while the steady state with $\phi = \bar{\phi}_H$ is a sink, and (iii) if $\pi \in [\pi_H, 1]$, then there exist no non-degenerate equilibria, and the model economy collapses. The third case is the least interesting, and thus we focus on the scenarios in which $\pi < \pi_H$. Then the model is indeterminate if the adverse selection problem is severe enough, *i.e.*, $\pi > \pi_L$. We summarize the above argument in the following corollary.

Corollary 2 *The likelihood of indeterminacy increases with π , the proportion of firms producing lemons.*

Arguably, adverse selection is more severe in developing countries. Our study then suggests that developing countries are more likely to be subject to self-fulfilling expectation-driven fluctuations and hence exhibit higher economic volatility, which is in line with the empirical regularity emphasized by Ramey and Ramey (1995) and Easterly, Islam, and Stiglitz (2000).

2.5 Empirical Possibility of Indeterminacy

We have proved that our model with adverse selection can generate self-fulfilling equilibria in theory. We now examine the empirical plausibility of self-fulfilling equilibria under calibrated parameter values. The frequency is a quarter. We set $\rho = 0.01$, implying an annual risk-free interest rate of 4%. We set $\theta = 0.3$ so the depreciation rate at steady state is 0.033 and the annualized investment-to-capital ratio is 12% (see Cooper and Haltiwanger (2006)). We set $\alpha = 0.33$ as in standard RBC models. We assume labor supply is elastic, and thus set $\gamma = 0$. We normalize the aggregate productivity $A = 1$. We set $\psi = 1.75$ so that $N = \frac{1}{3}$ in the "good" steady state. We set $\bar{\Phi} = \pi\Phi = 0.13$ so that $\phi = \bar{\phi}_H = 0.9$, which is consistent with average profit rate in the data. The associated $\bar{\phi}_L = 0.011$. If we further set $\pi = 0.1$, *i.e.*, the proportion of dishonest borrower is around 10%, then $\Phi = 1.3^8$. Consequently, based on our calibration and the indeterminacy condition (33), we conclude that our baseline model does generate self-fulfilling equilibria

⁸Only the product $\pi\Phi$ matters.

Parameter	value	Description
ρ	0.01	Discount factor
θ	0.3	Utilization elasticity of depreciation
δ	0.033	Depreciation rate
α	0.33	Capital income share
γ	0	Inverse Frisch elasticity of labor supply
ψ	1.75	Coefficient of labor disutility
π	0.1	Proportion of firms that produce lemons
Φ	1.3	Maximum firm capacity

Table 1: Calibration

Our calibration uses a delinquency rate of about 10%, which of in the same magnitude as in the great recession. but is higher than the average of delinquency rate in the data (the average is 3.73% from period 1985 to 2013). Delinquency rates do vary over time however. For example commercial residential mortgages had high delinquency rates during 2009-2013, which spread panic to financial markets through mortgage-backed securities and other derivatives. Nevertheless we will show in the next subsection, where we introduce reputation effects, that indeterminacy will arise even if there is no equilibrium default.

2.6 Global Dynamics

We have characterized the steady states and the local dynamics around these steady states in our model. We have shown that under some parameters, the equilibrium around the steady state is determined. We now show that gobal indeterminacy always exhit in our model. With some agebra, we can show that obtain

$$C_t = f_0 \cdot g(\phi_t) \cdot h(K_t) \quad (34)$$

by substituting out Y_t, N_t and u_t from equation (22). Here $h(K_t) = K_t^{\frac{\alpha\theta(1+\gamma)}{(1+\theta)(1-\alpha)}}$ and $f_0 = A^{\frac{1+\gamma}{1-\alpha}} \left(\frac{\alpha}{\delta^0}\right)^{\frac{\alpha(1+\gamma)}{(1+\theta)(1-\alpha)}} \left(\frac{1-\alpha}{\psi}\right)$

$$g(\phi_t) = \left[\phi_t^{1-\alpha+\frac{\alpha(1+\gamma)}{1+\theta}} Y(\phi_t)^{1-\alpha-(1-\frac{\alpha}{1+\theta})(1+\gamma)} \right]^{\frac{1}{1-\alpha}},$$

Hence equation (??) hence implicitly determines ϕ_t to be functions of C_t and K_t . We can easily verify $g(0) = g(1) = 0$, $g''(\phi) < 0$, and $g'(\phi_{\max}) = 0$, where $\phi_{\max} = 1 - \tau_{\min}$, and $\tau_{\min}$ is defined in Lemma (2). Therefore we have $\phi_{\max} = \arg \max g(\phi)$. We have the following lemma for equilibrium ϕ_t .

Figure 3:

Lemma 3 *For any K_t and $C_t < f_0 \cdot h(K_t) \cdot g(\phi_{\max})$, there are two possible equilibrium $\phi_t = \phi^+(K_t, C_t) > \phi_{\max}$ and $\phi_t = \phi^-(K_t, C_t) < \phi_{\max}$ that satisfy equation (??).*

As Christiano and Harrison (1999), we refer $\phi_t = \phi^+(K_t, C_t)$ as the upper branch and $\phi_t = \phi^-(K_t, C_t)$ as the lower branch of function g . We illustrate this two possible equilibrium $\phi^+(K_t, C_t)$ and $\phi^-(K_t, C_t)$ graphically below.

The function $g(\phi_t)$ has an inverted U shape. It attains the maximum at $\phi_{\max}$. Notice that $g(0) < C_t/[f_0 \cdot h(K_t)] < g(\phi_{\max})$, and then by the intermediate value theorem, there exist an ϕ_t^- such that $0 < \phi_t^- < \phi_{\max}$ and $g(\phi_t^-) = C_t/[f_0 \cdot h(K_t)]$. As $g'(\phi) > 0$ for $0 < \phi < \phi_{\max}$, ϕ_t^- must be unique. Likewise as $g(1) < C_t/[f_0 \cdot h(K_t)] < g(\phi_{\max})$ and $g'(\phi) < 0$ for $\phi_{\max} < \phi < 1$ there must exist a unique ϕ_t^+ such that $\phi_{\max} < \phi_t^+ < 1$ and $g(\phi_t^+) = C_t/[f_0 \cdot h(K_t)]$.

It now becomes clear that local indeterminacy of the steady state only occur on the lower branch. In such an equilibrium branch, an increase in consumption that will lead to an increase in the quality of borrowers and the lender will be happy to extend more loan even with a decrease in equilibrium interest rate. Also we notice that the steady state with $\phi = \phi_L$ always lies on the lower branch. The steady state with $\phi = \phi_H$ may lie on the lower branch

if $\phi_H < \phi_{\max}$, or it may lie on the upper branch if $\phi_H > \phi_{\max}$. It is also clear that if the equilibrium is on the upper branch, local indeterminacy is not possible. As the above figure shows an increase in consumption would lead a decrease in the borrower's quality, leading a contraction in the total loan supply, which can not support an expansion in equilibrium output.

As Christiano and Harrison (1999) pointed out that model with two branches can display rich global dynamics, regardless the local determinacy. For example, we can construct an equilibrium with regime switch in these branches. For example, we can construct an equilibrium with ϕ_t determined by the lower branch for a period of time, jumps to the upper branch and stay there for sometimes and then jumps back to the lower branch, and so on.

Another

$$\begin{aligned} \frac{C_t}{f_0 h(K_t)} &< g(\phi_{\max}) \\ C_t &= f_0 \cdot g(\phi_t) \cdot h(K_t). \end{aligned} \quad (35)$$

Note that

The global dynamics on (C_t, K_t) is governed by

$$\frac{\dot{C}_t}{C_t} = \rho(u_t^{1+\theta} - 1) \quad (36)$$

$$\dot{K}_t = Y_t - \left(\delta^0 \frac{u_t^{1+\theta}}{1+\theta} \right) K_t - C_t, \quad (37)$$

where

$$u_t^{1+\theta} = \frac{\alpha \phi_t Y_t}{\delta^0 K_t}, \quad (38)$$

$$Y_t = Y(\phi_t) \equiv \left(\frac{\phi_t}{1-\phi_t} \right) \pi \Phi, \quad (39)$$

and

$$\delta(u_t) \equiv \delta^0 \frac{u_t^{1+\theta}}{1+\theta}, \quad (40)$$

where $\delta^0 = \frac{\rho}{\theta}(1+\theta)$ so that $u = 1$ at the steady state.

First, since

$$u_t = \left(\frac{\alpha \phi_t Y_t}{\delta^0 K_t} \right)^{\frac{1}{1+\theta}},$$

we know that

$$N_t^{1-\alpha} = \frac{Y_t}{A u_t^\alpha K_t^\alpha} = \frac{Y_t^{1-\frac{\alpha}{1+\theta}} \phi_t^{-\frac{\alpha}{1+\theta}} K_t^{-\frac{\alpha\theta}{1+\theta}}}{A \left(\frac{\alpha}{\delta^0} \right)^{\frac{\alpha}{1+\theta}}}. \quad (41)$$

Then substituting equation (41) into (??) yields

$$\left[\frac{Y_t^{1-\frac{\alpha}{1+\theta}} \phi_t^{-\frac{\alpha}{1+\theta}} K_t^{-\frac{\alpha\theta}{1+\theta}}}{A \left(\frac{\alpha}{\delta^0} \right)^{\frac{\alpha}{1+\theta}}} \right]^{1+\gamma} = \left[\left(\frac{1}{C_t} \right) \left(\frac{1-\alpha}{\psi} \right) \phi_t Y_t \right]^{1-\alpha}, \quad (42)$$

which can be further simplified as

$$\frac{Y_t^{(1-\frac{\alpha}{1+\theta})(1+\gamma)} \phi_t^{-\frac{\alpha(1+\gamma)}{1+\theta}} K_t^{-\frac{\alpha\theta(1+\gamma)}{1+\theta}}}{A^{1+\gamma} \left(\frac{\alpha}{\delta^0} \right)^{\frac{\alpha(1+\gamma)}{1+\theta}}} = C_t^{-(1-\alpha)} \left(\frac{1-\alpha}{\psi} \right)^{(1-\alpha)} \phi_t^{1-\alpha} Y_t^{1-\alpha}, \quad (43)$$

or equivalently,

$$C_t^{1-\alpha} = A^{1+\gamma} \left(\frac{\alpha}{\delta^0} \right)^{\frac{\alpha(1+\gamma)}{1+\theta}} \left(\frac{1-\alpha}{\psi} \right)^{(1-\alpha)} \phi_t^{1-\alpha+\frac{\alpha(1+\gamma)}{1+\theta}} Y_t^{1-\alpha-(1-\frac{\alpha}{1+\theta})(1+\gamma)} K_t^{\frac{\alpha\theta(1+\gamma)}{1+\theta}}. \quad (44)$$

Substituting equation (39) into (44) yields

$$C_t = C(K_t, \phi_t) \equiv \left[A^{1+\gamma} \left(\frac{\alpha}{\delta^0} \right)^{\frac{\alpha(1+\gamma)}{1+\theta}} \left(\frac{1-\alpha}{\psi} \right)^{(1-\alpha)} \phi_t^{1-\alpha+\frac{\alpha(1+\gamma)}{1+\theta}} Y(\phi_t)^{1-\alpha-(1-\frac{\alpha}{1+\theta})(1+\gamma)} K_t^{\frac{\alpha\theta(1+\gamma)}{1+\theta}} \right]^{\frac{1}{1-\alpha}}, \quad (45)$$

where $Y(\phi_t) \equiv \left(\frac{\phi_t}{1-\phi_t} \right) \pi\Phi$.

Note that

$$u_t = \left(\frac{\alpha}{\delta^0} \frac{\phi_t Y(\phi_t)}{K_t} \right)^{\frac{1}{1+\theta}} \equiv u(K_t, \phi_t).$$

Then we can rewrite equation (36) as

$$\frac{\dot{C}_t}{C_t} = \rho \left(u(K_t, \phi_t)^{1+\theta} - 1 \right). \quad (46)$$

Meanwhile, differentiating both sides of equation (45) yields

$$C_t^{1-\alpha} = A^{1+\gamma} \left(\frac{\alpha}{\delta^0} \right)^{\frac{\alpha(1+\gamma)}{1+\theta}} \left(\frac{1-\alpha}{\psi} \right)^{(1-\alpha)} \phi_t^{1-\alpha+\frac{\alpha(1+\gamma)}{1+\theta}} Y_t^{1-\alpha-(1-\frac{\alpha}{1+\theta})(1+\gamma)} K_t^{\frac{\alpha\theta(1+\gamma)}{1+\theta}}, \quad (47)$$

which can be rewritten as

$$C_t = f_0 \cdot g(\phi_t) \cdot h(K_t).$$

Note that

$$g(\phi_t) = \left[\phi_t^{1-\alpha+\frac{\alpha(1+\gamma)}{1+\theta}} Y(\phi_t)^{1-\alpha-(1-\frac{\alpha}{1+\theta})(1+\gamma)} \right]^{\frac{1}{1-\alpha}}$$

Moreover,

$$(1 - \alpha) \frac{\dot{C}_t}{C_t} = \left(1 - \alpha + \frac{\alpha(1 + \gamma)}{1 + \theta}\right) \frac{\dot{\phi}_t}{\phi_t} + \left(1 - \alpha - \left(1 - \frac{\alpha}{1 + \theta}\right)(1 + \gamma)\right) \frac{\dot{Y}_t}{Y_t} + \left(\frac{\alpha\theta(1 + \gamma)}{1 + \theta}\right) \frac{\dot{K}_t}{K_t} \quad (48)$$

$$= \left(1 - \alpha + \frac{\alpha(1 + \gamma)}{1 + \theta} + \left(1 - \alpha - \left(1 - \frac{\alpha}{1 + \theta}\right)(1 + \gamma)\right) \frac{Y'(\phi_t)\phi_t}{Y(\phi_t)}\right) \frac{\dot{\phi}_t}{\phi_t} + \left(\frac{\alpha\theta(1 + \gamma)}{1 + \theta}\right) \frac{\dot{K}_t}{K_t}$$

$$= \left(1 - \alpha + \frac{\alpha(1 + \gamma)}{1 + \theta} - \left(\left(1 - \frac{\alpha}{1 + \theta}\right)(1 + \gamma) - (1 - \alpha)\right) \left(\frac{1}{1 - \phi_t}\right)\right) \frac{\dot{\phi}_t}{\phi_t} + \left(\frac{\alpha\theta(1 + \gamma)}{1 + \theta}\right) \frac{\dot{K}_t}{K_t} \quad (49)$$

$$= \left(1 - \alpha + \frac{\alpha(1 + \gamma)}{1 + \theta}\right) \left(1 - \left(\frac{1 - \phi_{\max}}{1 - \phi_t}\right)\right) \frac{\dot{\phi}_t}{\phi_t} + \left(\frac{\alpha\theta(1 + \gamma)}{1 + \theta}\right) \frac{\dot{K}_t}{K_t} \quad (50)$$

$$= \left(1 - \alpha + \frac{\alpha(1 + \gamma)}{1 + \theta}\right) \left(\frac{\phi_{\max} - \phi_t}{1 - \phi_t}\right) \frac{\dot{\phi}_t}{\phi_t} + \left(\frac{\alpha\theta(1 + \gamma)}{1 + \theta}\right) \frac{\dot{K}_t}{K_t} \quad (51)$$

where the third equation holds since

$$Y(\phi_t) = \left(\frac{\phi_t}{1 - \phi_t}\right) \pi\Phi, \quad Y'(\phi_t) = \frac{\pi\Phi}{(1 - \phi_t)^2}, \quad \frac{Y'(\phi_t)\phi_t}{Y(\phi_t)} = \frac{1}{1 - \phi_t},$$

and the last equation holds because $\phi_{\max} = 1 - \tau_{\min} = 1 - \left(\frac{(1 + \theta)(1 + \gamma)}{(1 + \theta)(1 - \alpha) + \alpha(1 + \gamma)} - 1\right)$, where $\tau_{\min}$ is defined in Lemma 2.

Note that $\left(1 - \alpha + \frac{\alpha(1 + \gamma)}{1 + \theta}\right) \left(\frac{\phi_{\max} - \phi_t}{1 - \phi_t}\right)$, the coefficient associated with $\frac{\dot{\phi}_t}{\phi_t}$ in equation (51), decreases with ϕ_t , and is non-negative if and only if $\phi_t \leq \phi_{\max}$.

If $\phi_t = \phi_{\max}$ holds for all t , then equation (51) is reduced to

$$\frac{\dot{C}_t}{C_t} = \left(\frac{\alpha\theta(1 + \gamma)}{(1 - \alpha)(1 + \theta)}\right) \frac{\dot{K}_t}{K_t}.$$

That is, $\dot{C}_t = 0$ if and only if $\dot{K}_t = 0$ when $\phi_t = \phi_{\max}$ for all t . Combining $\dot{C}_t = 0$ and equation (46) suggests

$$u(K, \phi) = 1 = \left(\frac{\alpha}{\delta^0} \frac{\phi Y(\phi)}{K}\right)^{\frac{1}{1 + \theta}}.$$

Since $\phi = \phi_{\max}$, we immediately obtain the steady state of K as

$$K = \frac{\alpha}{\delta^0} \phi_{\max} Y(\phi_{\max}). \quad (52)$$

However, $\dot{K}_t = 0$ implies

$$K = \left[\frac{\left(1 - \frac{\alpha\phi_{\max}}{1 + \theta}\right)^{1 - \alpha} Y(\phi_{\max})^{(1 - \frac{\alpha}{1 + \theta})(1 + \gamma)}}{A^{1 + \gamma} \left(\frac{\alpha}{\delta^0}\right)^{\frac{\alpha(1 + \gamma)}{1 + \theta}} \left(\frac{1 - \alpha}{\psi}\right)^{(1 - \alpha)} \phi_{\max}^{1 - \alpha + \frac{\alpha(1 + \gamma)}{1 + \theta}}} \right]^{\frac{1 + \theta}{\alpha\theta(1 + \gamma)}}.$$

Therefore the above two equations conflict with each other. Thus $\phi = \phi_{\max}$ can never be a steady state. However, it may generate chaotics.

Given that $\phi_t \neq \phi_{\max}$, then equation (51) can be rewritten as

$$\begin{aligned} \frac{\dot{\phi}_t}{\phi_t} &= \frac{-\left(\frac{\alpha\theta(1+\gamma)}{1+\theta}\right) \frac{\dot{K}_t}{K_t} + (1-\alpha) \frac{\dot{C}_t}{C_t}}{\left(1-\alpha + \frac{\alpha(1+\gamma)}{1+\theta}\right) \left(\frac{\phi_{\max}-\phi_t}{1-\phi_t}\right)} \\ &= \frac{-\left(\frac{\alpha\theta(1+\gamma)}{1+\theta}\right) \left(\frac{\left(1-\frac{\alpha\phi_t}{1+\theta}\right)Y(\phi_t)-C(K_t,\phi_t)}{K_t}\right) + (1-\alpha) \rho \left(\frac{\alpha}{\delta^0} \frac{\phi_t Y(\phi_t)}{K_t} - 1\right)}{\left(1-\alpha + \frac{\alpha(1+\gamma)}{1+\theta}\right) \left(\frac{\phi_{\max}-\phi_t}{1-\phi_t}\right)} \end{aligned}$$

On the one hand, $\dot{K}_t = 0$ if and only if $\left(1 - \frac{\alpha\phi_t}{1+\theta}\right) Y(\phi_t) - C(K_t, \phi_t) = 0$, which is equivalent to

$$K_t = K(\phi_t) \equiv \left[\frac{\left(1 - \frac{\alpha\phi_t}{1+\theta}\right)^{1-\alpha} Y(\phi_t)^{\left(1-\frac{\alpha}{1+\theta}\right)(1+\gamma)}}{A^{1+\gamma} \left(\frac{\alpha}{\delta^0}\right)^{\frac{\alpha(1+\gamma)}{1+\theta}} \left(\frac{1-\alpha}{\psi}\right)^{(1-\alpha)} \phi_t^{1-\alpha+\frac{\alpha(1+\gamma)}{1+\theta}}} \right]^{\frac{1+\theta}{\alpha\theta(1+\gamma)}}. \quad (53)$$

On the other hand, $\dot{\phi}_t = 0$ when $\phi_t = 0$, or

$$(1-\alpha) \rho \left(\frac{\alpha}{\delta^0} \frac{\phi_t Y(\phi_t)}{K_t} - 1\right) = \left(\frac{\alpha\theta(1+\gamma)}{1+\theta}\right) \left(\frac{\left(1 - \frac{\alpha\phi_t}{1+\theta}\right) Y(\phi_t) - C(K_t, \phi_t)}{K_t}\right), \quad (54)$$

which can be rewritten as

$$K_t = \frac{\alpha}{\delta^0} \phi_t Y(\phi_t) - \left(\frac{\alpha\theta(1+\gamma)}{\rho(1-\alpha)(1+\theta)}\right) \left(\left(1 - \frac{\alpha\phi_t}{1+\theta}\right) Y(\phi_t) - C(K_t, \phi_t)\right). \quad (55)$$

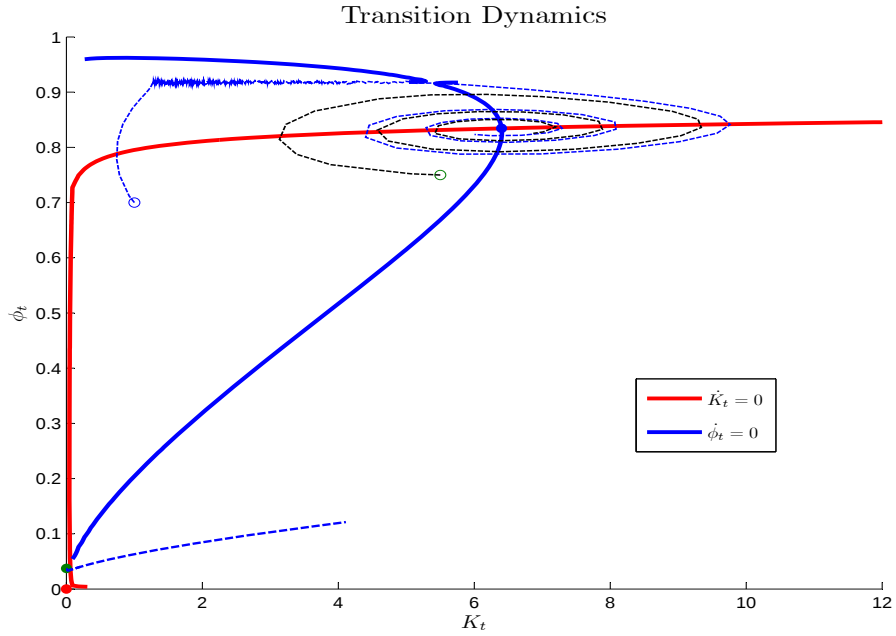
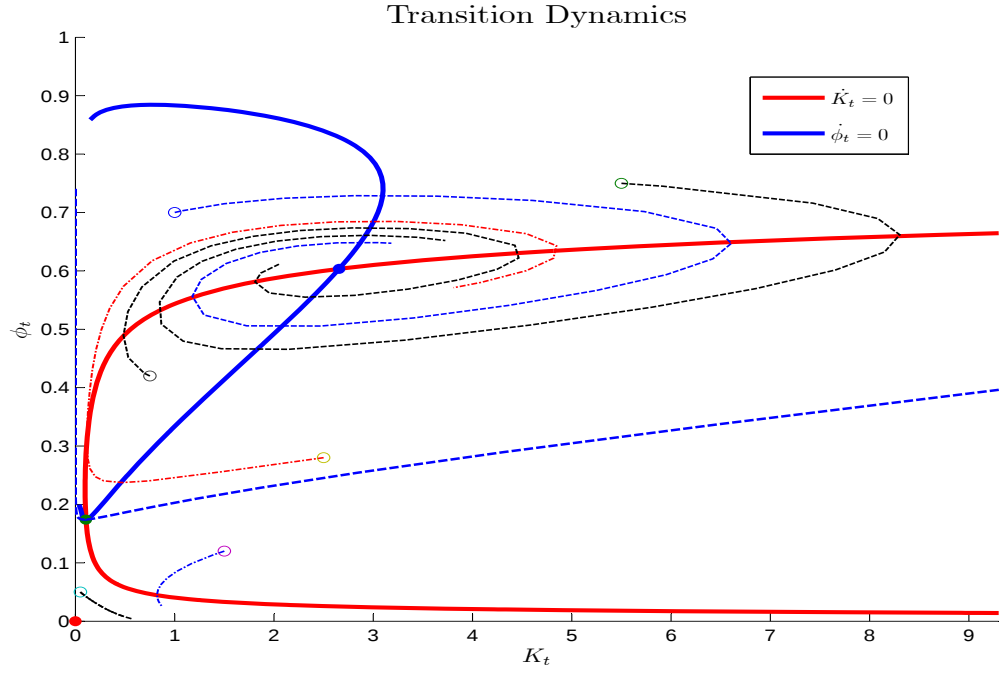
2.7 Global Transition Dynamics

We use the following figures to illustrate the global dynamics developed so far.

Note: we may use our findings to talk about the secular stagnation argued by Larry Summers, and address the short-tem high volatility of interest rate ($1/\phi$) in the credit markets.

3 Reputation

We now study the sensitivity of our indeterminacy results under adverse selection to reputation effects. If firms are not anonymous in the market, they may refrain from defaulting and instead may want to build their reputation. Lenders may also refrain from lending to firms with a bad credit history. Arguably, these market forces can alleviate the asymmetric information



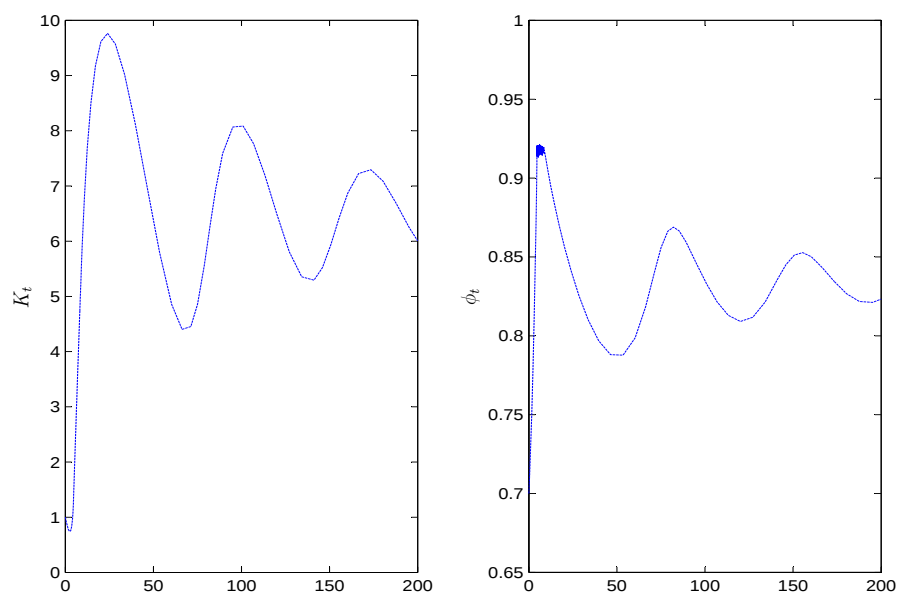


Figure 6: **Global Dynamics with One Saddle (Modest π): High Turbulence in the Short Run (Interest Rate o Right Panel)**

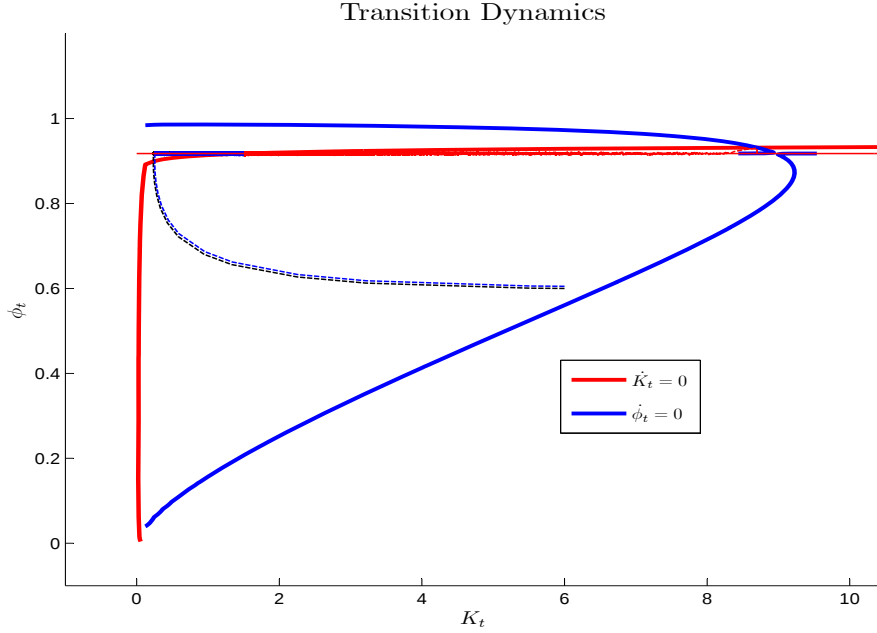


Figure 7: **Global Dynamics with Two Saddle (Low π)**

problem. So we examine whether the indeterminacy results obtained in our baseline model survive if such reputational effects are taken into account.

We follow Kehoe and Levine (1993) closely in modeling reputation. Firms are infinitely-lived, and can choose to default at any time. Firms that default, with some probability, acquire a bad reputation and are excluded from the credit market forever. In equilibrium, the fear of losing all future profits from production discourages firms from defaulting. We will show that self-fulfilling equilibria still exist even if there are no defaults in equilibrium.

To keep the model analytically tractable, we assume that all firms are owned by a representative entrepreneur. The entrepreneur's utility function is given by

$$U(C_{et}) = \int_0^\infty e^{-\rho_e t} \log(C_{et}) dt, \quad (56)$$

where C_{et} is the entrepreneur's consumption and ρ_e her discount factor. For tractability, we follow Liu and Wang (2014) by assuming that $\rho_e \ll \rho$ such that the entrepreneur does not accumulate capital. The entrepreneur's consumption equals the firm's profits.

$$C_{et} = \int_0^1 \Pi_t(i) di \equiv \Pi_t, \quad (57)$$

where $\Pi_t(i)$ denotes the profit of firm i .

Since the only cost of default is the loss of future production opportunities, the price must exceed the marginal cost (also the average cost) of production to be profitable. If the price exceeds the marginal cost, each firm will then have an incentive to produce an infinite amount. To overcome this problem, we will assume firms' production projects are indivisible, as in the benchmark model and that they produce according to the orders that they receive. A production project produces a flow of final goods Φ from intermediate goods. Each unit of the final good requires one unit of intermediate goods for its production. The project will be carried out only if the firms receive a sale order. Denote the total demand for final good as Y_t . Then fraction $\eta_t = Y_t/\Phi$ of firms will receive the sale orders. Again we assume that firms need to borrow to finance their working capital. Denote the intermediate goods' price as P_t , they need to borrow P_t

To illustrate the reputation problem, let us consider a short time interval from t to $t + dt$. We use V_{1t} (V_{0t}) to denote the value of a firm that receives an order (no orders). We can then formulate V_{1t} recursively as

$$V_{1t} = (1 - \phi_t)\Phi dt + e^{-\rho_e dt} \left(\frac{C_{e,t}}{C_{e,t+dt}} \right) [\eta_{t+dt} V_{1t+dt} + (1 - \eta_{t+dt}) V_{0t+dt}]. \quad (58)$$

where $\phi_t = P_t$ is the unit production cost. If $1 > \phi_t$, then the firm receives a positive profit from production. The second term on the right-hand side is the continuation value of the firms. Since firms are owned by the entrepreneur, the future value is discounted by the marginal utility of the entrepreneur. Since there is no default in equilibrium, the gross interest rate for a working capital loan is $R_{ft} = 1$.

The firm can also choose to default on its working capital. By doing so, the firms obtain instantaneous gain of $\Phi\phi_t$. However, default comes with the risk of acquiring a bad reputation. Upon default, the firm acquires a bad reputation in the short time interval between t and $t + dt$ with probability λdt . In that case, the firm will be excluded from production forever. The payoff for defaulting is hence

$$V_t^d = \Phi dt + e^{-\rho_e dt} (1 - \lambda dt) E_t \left(\frac{C_{e,t}}{C_{e,t+dt}} \right) [\eta_{t+dt} V_{1t+dt} + (1 - \eta_{t+dt}) V_{0t+dt}]. \quad (59)$$

The value of a firm that does not receive any order is given by

$$V_{0t} = e^{-\rho_e dt} E_t \left(\frac{C_{e,t}}{C_{e,t+dt}} \right) [\eta_{t+dt} V_{1t+dt} + (1 - \eta_{t+dt}) V_{0t+dt}]. \quad (60)$$

Define $V_t = \eta_t V_{1t} + (1 - \eta_t) V_{0t}$ as the expected value of the firm. The firm has no incentive to produce lemons if and only if $V_{1t} \geq V_t^d$, or

$$\Phi dt \leq (P_t - \phi_t) \Phi dt + \lambda dt e^{-\rho_e dt} \left(\frac{C_{e,t}}{C_{e,t+dt}} \right) V_{t+dt}. \quad (61)$$

In the limit $dt \rightarrow 0$, the incentive compatibility condition becomes $\phi_t \Phi \leq \lambda V_t$.⁹ Then the expected value of the firm is given by the present discounted value of all future profits as

$$V_t = \int_0^\infty e^{-\rho_e s} \frac{C_{et}}{C_{es}} \Pi_s ds. \quad (62)$$

In equilibrium, $\eta_t = \frac{Y_t}{\Phi}$. For simplicity, we assume Φ is big enough such that $\eta_t < 1$ always holds. The average profit is then obtained as $\Pi_t = (1 - \phi_t) Y_t$. In turn, we have

$$V_t = \frac{(P_t - \phi_t) Y_t}{\rho_e}. \quad (63)$$

The households' budget constraint changes to

$$C_t + I_t \leq R_t u_t K_t + W_t N_t = \phi_t Y_t. \quad (64)$$

Then the incentive constraint (61) becomes

$$\phi_t \Phi \leq \lambda \frac{(P_t - \phi_t) Y_t}{\rho_e}. \quad (65)$$

From the household budget constraint (64), we know that household utility increases with ϕ_t and thus the incentive constraint (65) must be binding. Then Equation (65) can be simplified as

$$\phi_t = \frac{Y_t}{\pi \Phi + Y_t} < 1, \quad (66)$$

where now $\pi \equiv \frac{\rho_e}{\lambda}$. Similar to the baseline model, here firms also receive an information rent. However, the rent in the baseline is derived from hidden information while the rent here arises from hidden action. As indicated in Equation (66), ϕ_t is procyclical and hence the markup is countercyclical. When output is high, the total profit from production is high. Therefore the value of a good reputation is high and the opportunity cost of producing a lemon also increases. This then alleviates the moral hazard problem since high output dilutes informational rent.

The cost minimization problem again yields the factor prices given by Equation (20) and (21). Since households do not own firms, their budget constraint is modified as

$$C_t + \dot{K}_t = \phi_t Y_t - \delta(u_t) K_t. \quad (67)$$

⁹Under the incentive compatibility condition we can consider one step deviations since V_{1t} , V_{0t} are then optimal value functions.

The equilibrium system of equations is the same as in the baseline model except that Equation (19) is replaced by Equation (67). The steady state can be computed similarly. The steady state output is given by

$$Y = A^{\frac{1}{1-\alpha}} \left[\frac{\alpha\phi\theta}{\rho(1+\theta)} \right]^{\frac{\alpha}{1-\alpha}} \left[\left(\frac{1-\alpha}{1-\frac{\alpha}{1+\theta}} \right) \cdot \frac{1}{\psi} \right]^{\frac{1}{1+\gamma}} \equiv Y(\phi), \quad (68)$$

and ϕ can be solved from

$$\bar{\Phi} \equiv \pi\Phi \equiv \Psi(\phi) = \left(\frac{1-\phi}{\phi} \right) \cdot Y(\phi). \quad (69)$$

Unlike the baseline model, the steady state equilibrium is unique. We summarize the result in the following lemma.

Lemma 4 *If $\alpha < \frac{1}{2}$, a consistently standard calibrated value of α , then the steady state equilibrium is unique for any $\bar{\Phi} > 0$.*

We can now study the possibility of self-fulfilling equilibria around the steady state. Since ϕ and $\bar{\Phi}$ form a one-to-one mapping, we will treat ϕ as a free parameter in characterizing the indeterminacy condition. We can then use Equation (69) to back out the corresponding value of $\bar{\Phi}$. The following proposition specifies the condition under which self-fulfilling equilibria arises.

Proposition 2 *Let $\tau = 1 - \phi$. Then indeterminacy emerges if and only if*

$$\tau_{\min} < \tau < \min \left\{ \frac{1+\theta}{\alpha} - 1, \tau_H \right\} \equiv \tau_{\max},$$

where $\tau_{\min} \equiv \frac{(1+\theta)(1+\gamma)}{(1+\theta)(1-\alpha)+\alpha(1+\gamma)} - 1$, and τ_H is the positive solution to $A_1\tau^2 - A_2\tau - A_3 = 0$, where

$$\begin{aligned} A_1 &\equiv s(1+\theta)(2+\alpha+\alpha\gamma) \\ A_2 &\equiv (1+\theta)(1+\alpha\gamma) - s[(1+\theta)(1-\alpha)(1-\gamma) + (1+\gamma)\alpha] \\ A_3 &\equiv (1+\theta)(1-\alpha)[s + (1-s)\gamma]. \end{aligned}$$

The necessary condition for indeterminacy turns out to be the same as in our baseline model. It is easy to verify that under $\tau > \tau_{\min}$, the labor demand curve slopes upward and is steeper than the labor supply curve. So the intuition for indeterminacy is similar to that in the baseline. Indeterminacy implies that the model exhibits multiple expectation-driven equilibria

around the steady state. The steady state equilibrium is now unique however, which suggests that the continuum of equilibria implied by indeterminacy cannot be obtained in static models studied the earlier literature. So far, the condition to sustain indeterminacy is given in terms of ϕ and τ . The following lemma specifies the underlying condition in terms of ρ_e , λ and Φ .

Lemma 5 *Indeterminacy emerges if and only if $\frac{\Psi(1-\tau_{\min})}{\Phi} < \frac{\rho_e}{\lambda} < \frac{\Psi(1-\tau_{\max})}{\Phi}$.*

[Given the other parameters, a decrease in ρ_e or an increase in λ increases the steady state ϕ . According to the above lemma, it makes indeterminacy less likely. The intuition is straightforward. A large λ means the opportunity cost of producing lemon increases, as the firm becomes more likely to be excluded from future production. This alleviates the moral hazard problem, which is the source of indeterminacy. Similarly, a decrease of ρ_e means that the entrepreneurs become more patient. So the future profit flow from production is more valuable to them, which again increases the opportunity cost of producing lemons and thus alleviates the moral hazard problem.

4 Adverse Selection and Increasing Returns to Scale

Liu and Wang (2014) shows that credit constraints can generate aggregate increasing returns to scale. We now explore the possibility of increasing returns to scale, we now modify our model in Section 2 as follows. The households' problems as in the benchmark model and thus the first order conditions are still Equations (5), (6) and (7).

We now assume that the risk of lending to final good firms are continuous. We index the final good firms with $j \in [0, 1]$. Again each final good firms have one production project, which requires Φ unit of intermediate goods. The loan is risky as the final goods firms' production may not be successful. More specifically, we assume that final good firm j 's output is governed by

$$y_{jt} = \begin{cases} a_{jt}x_{jt}, & \text{with probability } q_{jt} \\ 0, & \text{with probability } 1 - q_{jt} \end{cases}, \quad (70)$$

where x_{jt} is the intermediate input for firm j and a_{jt} the firm's productivity. We assume q_{jt} is IID drawn from a common distribution function $F(q)$ and $a_{jt} = a_{\min}q_{jt}^{-\tau}$. Notice that expected productivity is given by $q_{jt}a_{jt} = a_{\min}q_{jt}^{1-\tau}$. We assume that $\tau < 1$, i.e., a firm with a higher success probability enjoys a higher expected productivity. Denote P_t as the price of intermediate goods. Then the total borrowing is given by $P_t x_{jt}$. Denote R_{ft} be the gross

interest rate. Then the final good firm j 's profit maximization problem becomes

$$\max_{x_{jt} \in \{0, \Phi\}} q_{jt} [a_{jt} x_{jt} - R_{ft} P_t x_{jt}], \quad (71)$$

Note that, due to limited liability, the final goods firm pays back the working capital loan only if the project is successful. This implies that, given R_{ft} and P_t , the demand for x_{jt} is simply given by

$$x_{jt} = \begin{cases} \Phi & \text{if } a_{jt} > R_{ft} P_t \equiv a_t^* \\ 0 & \text{otherwise} \end{cases}, \quad (72)$$

or equivalently,

$$a_{\min} q_{jt}^{-\tau} > a_t^*, \quad q_{jt} < \left[\frac{a_t^*}{a_{\min}} \right]^{-\frac{1}{\tau}} = q_t^* = \left[\frac{R_{ft} P_t}{a_{\min}} \right]^{-\frac{1}{\tau}}. \quad (73)$$

This establishes that only firms with risky production opportunities will enter the credit markets, which highlight the adverse selection problem in the financial market. Firms with $q_{jt} > q_t^*$ are driven out from the financial market, despite their higher social expected productivity. Since financial intermediaries are assumed to be fully competitive, we have

$$R_{ft} P_t \Phi \int_0^{q_t^*} q dF(q) = P_t \Phi \int_0^{q_t^*} dF(q), \quad (74)$$

where the left-hand side is the actual repayment from the final goods firms, and the right-hand side the actual lending. We obtain

$$R_{ft} = \frac{1}{\int_0^{q_t^*} q dF(q) / \int_0^{q_t^*} dF(q)} = \frac{1}{E(q|q \leq q_t^*)} > 1, \quad (75)$$

where the denominator is average success rate. The above equation says that interest rate decreases with the average success rate.

The total production of final goods is

$$Y_t = \int_0^1 q_j a_{jt} x_{jt} dF(q) = \Phi \int_0^{q_t^*} a_{\min} q^{1-\tau} dF(q). \quad (76)$$

where the second equality follows equation (72). The total production of intermediate goods is

$$X_t = \Phi \int_0^{q_t^*} dF(q). \quad (77)$$

Finally the intermediate goods are produced according to $X_t = A_t (u_t K_t)^\alpha N_t^{1-\alpha}$, where $u_t K_t$ is the capital rented from the households. Equation (76) and then (77) then yields

$$Y_t = \Gamma(q_t^*) A_t (u_t K_t)^\alpha N_t^{1-\alpha}, \quad (78)$$

where $\Gamma(q_t^*) = \left[\int_0^{q_t^*} a_{\min} q^{1-\tau} dF(q) \right] / \int_0^{q_t^*} dF(q)$ depends on the threshold q_t^* and the distribution. The above equation then says the measured TFP is obtained as $TFP_t = \frac{Y_t}{(u_t K_t)^\alpha N_t^{1-\alpha}} = \Gamma(q_t^*) A_t$. Notice

$$\Gamma'(q_t^*) = \frac{a_{\min} f(q_t^*) \int_0^{q_t^*} [q_t^{*1-\tau} - q^{1-\tau}] dF(q)}{\left[\int_0^{q_t^*} dF(q) \right]^2} > 0.$$

So the endogenous TFP increases with the threshold. This is very intuitive as the threshold increases, more firms with high productivity enters the credit market, making resource allocation more efficient. Equation (76) implies that q_t^* increases with Y_t , so we have the following lemma.

Lemma 6 *TFP is endogenous and increase in Y , namely $\frac{\partial \Gamma(q_t^*)}{\partial Y_t} > 0$.*

We have therefore established that the endogenous TFP, $\Gamma(q^*)$, is procyclical. Notice that the procyclicality of endogenous TFP holds generally for continuous distributions. So without loss of generality, we now assume $F(q) = q^\eta$ for tractability. Notice that firm-level measured productivity, $\frac{1}{q}$, then follows a Pareto distribution with the shape parameter of η , which is consistent with the findings of a large literature (see, e.g., Melitz (2003) and references therein). Under the assumption of power distribution, equations (??) and (78) together yield the aggregate output as

$$Y_t = \left(\frac{\eta}{\eta - \tau + 1} \right) a_{\min} \Phi^{-\frac{1-\tau}{\eta}} (A_t u_t^\alpha K_t^\alpha N_t^{1-\alpha})^{1+\frac{1-\tau}{\eta}}. \quad (79)$$

The intuition is as follows. Here a lending externality kicks in because of adverse selection in the credit markets. Suppose that the total lending from financial intermediaries increases. This creates a downward pressure on interest rate R_{ft} , which increases the cutoff q_t^* according to the definition at Equation (73). Firms with higher q have smaller risk of default. A rise in the cutoff q_t^* therefore reduces the average default rate. If it is strong enough, it can in turn stimulate more lending from the financial intermediaries. Since firms with higher q are also more productive on average, the increased efficiency in re-allocating credit implies that resources are better allocated across firms. Notice that the aggregate output again exhibits increasing returns to scale. Equation (79) reveals that the degree of increasing returns to scale clearly depends on the adverse selection problem. The degree of increasing returns to scale decreases with τ and η . When $\eta = \infty$, the firms' product quality is homogeneous. Hence there is no asymmetric information and adverse selection. If $\tau = 1$, firms are equally productive in the sense their expected productivity is the same. It therefore does not matter how credits are

allocated among firms. Given $\tau < 1$, a smaller η implies that firms are more heterogeneous, creating a large asymmetric information problem. Similarly given η , a smaller τ implies that the productivity of firms deteriorates faster with respect to their default risk, making the adverse selection more damaging to resource allocation. We formally state this result in the following proposition.

Proposition 3 *The reduced-form aggregate production in our model exhibits increasing returns to scale if and only if there exists adverse selection, i.e., $\tau < 1$ and $\eta < \infty$.*

In an important contribution, Basu and Fernald (1997) document that increasing returns to scale exist in aggregate production but not at the micro level. In a recent paper, Liu and Wang (2014) show how credit constraints can generate endogenous variation in TFP, and hence aggregate increasing returns. In their model, the less productive firms are driven out from production. Different from Liu and Wang (2014), firms in our model do not suffer from credit constraints; more productive firms in our model are driven out of production due to adverse selection. Arguably, the inefficiency generated by both of these frictions are important in developing countries and contributes to their low TFP.

Also in the benchmark model, both the credit spread, measured by $R_{ft} - 1$, and the expected default risk, $1 - E(q|q \leq q_t^*)$, are countercyclical. These predictions are consistent with the empirical regularities by Gilchrist and Zakrajšek (2012) and many others.

4.1 Indeterminacy

It is straightforward to show $W_t = \phi \frac{(1-\alpha)Y_t}{N_t}$ and $R_t = \phi \frac{\alpha Y_t}{u_t K_t}$ respectively. Here $\phi = \frac{\eta+1-\tau}{\eta+1}$. Together with Equations (5), (6), (7), (79), and (19), we can determine the seven variables, C_t , Y_t , N_t , u_t , K_t , W_t and R_t . The steady state can be obtained as in the baseline model. We can express the other variables in terms of the steady state ϕ . Since ϕ is unique, unlike in the baseline model, the steady state here is unique. We assume that Φ is large enough so that an interior solution to q^* is always guaranteed. We state this result formally in the following corollary.

Corollary 3 *The steady state is unique in the extended model with $F(q) = (q/q_{\max})^\eta$.*

A large literature following Benhabib and Farmer (1994) has shown that increasing returns can give rise to self-fulfilling expectations (see e.g., Wen (1998), Liu and Wang (2014)). The following proposition summarizes the conditions for indeterminacy in this extended model.

Proposition 4 *The model is indeterminate if and only if*

$$\sigma_{\min} < \sigma < \sigma_{\max} \quad (80)$$

$$\text{where } \sigma \equiv \frac{1-\tau}{\eta}, \sigma_{\min} \equiv \left(\frac{1}{\frac{1-\alpha}{1+\gamma} + \frac{\alpha}{1+\theta}} \right) - 1 \text{ and } \sigma_{\max} \equiv \frac{1}{\alpha} - 1.$$

To better understand the proposition, we first consider how output responds to a fundamental shock, such as a change in A , the true TFP. Holding factor inputs constant, we have

$$1 + \tilde{\sigma} \equiv \frac{d \log Y_t}{d \log A} = (1 + \sigma) \left[\frac{1 + \theta}{1 + \theta - \alpha(1 + \sigma)} \right] > 1, \quad (81)$$

The above equations shows that adverse selection and variable capacity utilization can significantly amplify the impact of a TFP shock on output. Let us define $1 + \tilde{\sigma}$ as the multiplier of adverse selection. Note that the necessary condition $\sigma > \sigma_{\min}$ can be written as

$$(1 + \tilde{\sigma})(1 - \alpha) - 1 > \gamma. \quad (82)$$

The model will be indeterminate if the multiplier effect of adverse selection is sufficient large. The restriction $\sigma < \sigma_{\max}$ is typically automatically satisfied. The restriction $\sigma < \frac{1}{\alpha} - 1$ simply requires that $\alpha(1 + \sigma) < 1$, which is the condition to rule out explosive growth in the model.

Whether the model is indeterminate or not, Equation (81) implies that the response of output to TFP shocks will be amplified. In addition, by Proposition 4, the economy will more likely be indeterminate if η is smaller. Our results are hence in the same spirit as those of Kurlat (2013) and Bigio (2014) showing that a dispersion in quality will strengthen the amplification effect of adverse selection.

Empirical Possibility of Indeterminacy To empirically evaluate the possibility of indeterminacy, we set the same value to ρ , θ , δ , α and γ as in Table 1.¹⁰ We have new parameters in this extended model (τ, η). We use two moments to pin them down and set τ and η to match the steady state markup $\frac{\eta+1-\tau}{\eta+1} = 0.9$. Basu and Fernald (1997) estimate aggregate increasing returns to scale for manufacturing to be around 1.1. So we set $\sigma = 0.1$. This leads to $\tau = 0.55$ and $\eta = 4.5$. We have $\sigma_{\min} = 0.083$ and $\sigma_{\max} \equiv 2$, which meet the indeterminacy conditions. Hence, with these parameters the model exhibits self-fulfilling equilibria. Since the steady state equilibrium is unique, such multiple equilibria must come from the dynamic nature of the model.

¹⁰ $q_{\max}$ and Φ do not affect the indeterminacy condition, so we do not need to specify their value.

5 Conclusion

We have argued that in a dynamic general equilibrium model, adverse selection in the goods market can generate a new type of multiplicity of equilibria in the form of indeterminacy, either through endogenous markups or endogenous TFP. Adverse selection can therefore potentially explain some of the excessive output volatility in the absence of fundamental shocks. For example, an RBC model with a negative TFP shock cannot fully explain the increase in labor productivity during the Great Recession (see Ohanian (2010)). However this feature of the Great Recession is consistent with the prediction of our baseline model in Section 2, and is driven by pessimistic beliefs about aggregate output. The pessimistic beliefs reduce aggregate demand and increase the markups, leading to a lower real wage and a lower labor supply. Labor productivity however rises due to decreasing returns to labor.

Appendix

A Proofs

Proof of Lemma 2: Denote φ_1 and φ_2 as the eigenvalues of matrix J so that we have $\varphi_1 + \varphi_2 = \text{Trace}(J)$ and $\varphi_1 \varphi_2 = \text{Det}(J)$. Then the model is indeterminate if the trace of J is negative and the determinate is positive. The trace and the determinant of J are

$$\frac{\text{Trace}(J)}{\delta} = \left(\frac{1+\theta}{\alpha\phi} \right) \lambda_1 - (1+\tau) \lambda_1 + \theta(1+\tau) \lambda_2, \quad (\text{A.1})$$

$$\frac{\text{Det}(J)}{\delta^2\theta} = [(1+\tau) \lambda_1 - 1 + \lambda_2] \left(\frac{1+\theta}{\alpha\phi} - 1 \right) - \tau \lambda_2, \quad (\text{A.2})$$

respectively, where

$$\lambda_1 = \frac{a(1+\gamma)}{1+\gamma-b(1+\tau)}, \text{ and } \lambda_2 = -\frac{b}{1+\gamma-b(1+\tau)}, \quad (\text{A.3})$$

as defined in equation (31).

Substituting out λ_1 and λ_2 we obtain

$$\begin{aligned} \frac{\text{Trace}(J)}{\delta} &= \left[\frac{1}{\gamma+1-(1+\tau)b} \right] \cdot \left[\left(\frac{1+\theta}{\alpha\phi} - 1 - \tau \right) a(1+\gamma) - \theta(1+\tau)b \right] \\ &= \left[\left(\frac{\theta}{\phi} \right) \left(\frac{\alpha(1+\gamma) + (1+\theta)(1-\alpha)}{1+\theta-(1+\tau)\alpha} \right) \right] \cdot \left[\frac{\frac{(1+\gamma)(1+\theta)}{\alpha(1+\gamma)+(1+\theta)(1-\alpha)} - \phi(1+\tau)}{\gamma+1-(1+\tau)b} \right] \\ &= \left[\left(\frac{\theta}{\phi} \right) \left(\frac{\alpha(1+\gamma) + (1+\theta)(1-\alpha)}{1+\theta-(1+\tau)\alpha} \right) \right] \cdot \left[\frac{\frac{(1+\gamma)(1+\theta)}{\alpha(1+\gamma)+(1+\theta)(1-\alpha)} - 1 + \tau^2}{\gamma+1-(1+\tau)b} \right] \end{aligned} \quad (\text{A.4})$$

Notice that $\gamma+1-(1+\tau)b < 0$ is equivalent to

$$\tau > \tau_{\min} \equiv \frac{(1+\gamma)(1+\theta)}{\alpha(1+\gamma) + (1+\theta)(1-\alpha)} - 1.$$

Since $\tau_{\min} > 0$, we know that

$$\frac{(1+\gamma)(1+\theta)}{\alpha(1+\gamma) + (1+\theta)(1-\alpha)} - 1 + \tau^2 > 0.$$

Therefore $\text{Trace}(J) < 0$ if and only if $\tau > \tau_{\min}$. It remains for us to pin down the condition under which $\text{Det}(J) > 0$. Note that $\text{Det}(J)$ can be rewritten as

$$\begin{aligned} \frac{\text{Det}(J)}{\delta^2\theta} &= \left[\frac{1}{\gamma+1-(1+\tau)b} \right] \cdot \left[\left(\frac{1+\theta}{\alpha\phi} - 1 \right) ((1+\gamma)[a(1+\tau)-1] + \tau b) + \tau b \right] \\ &= \frac{1+\theta}{(1+\tau)b - (\gamma+1)} \left\{ (1+\gamma)(1-\alpha) - \left[\frac{(1-\alpha)(1+\theta)}{(1+\theta-\alpha\phi)} + (1+\gamma)\alpha \right] \tau \right\} \end{aligned} \quad (\text{A.5})$$

If $\tau < \tau_{\min}$, then we immediately have $\text{Det}(J) < 0$. Thus to guarantee that $\text{Det}(J) > 0$, we must have $\tau > \tau_{\min}$, which then implies that $(1 + \tau)b - (\gamma + 1) > 0$. As a result, given that $\tau > \tau_{\min}$, $\det(J) > 0$ if and only if

$$(1 + \gamma)(1 - \alpha) - \left[\frac{(1 - \alpha)(1 + \theta)}{1 + \theta - \alpha\phi} + (1 + \gamma)\alpha \right] \tau > 0,$$

which can be further simplified as

$$\tau < \frac{(1 + \gamma)(1 - \alpha)}{\frac{(1 - \alpha)(1 + \theta)}{1 + \theta - \alpha\phi} + (1 + \gamma)\alpha}.$$

Since $\phi = 1 - \tau$, the above inequality can be reformulated as

$$\Delta(\tau) \equiv \alpha^2 \tau^2 + \left[\alpha\theta + \frac{(1 - \alpha)(1 + \theta)}{(1 + \gamma)} \right] \tau - (1 - \alpha)(1 + \theta - \alpha) < 0.$$

Denote $\xi \equiv \alpha\theta + \frac{(1 - \alpha)(1 + \theta)}{(1 + \gamma)}$. Then $\det(J) > 0$ if and only if $\tau > \tau_{\min}$ and

$$\tau < \tau_{\max} \equiv \frac{-\xi + \sqrt{\xi^2 + 4\alpha^2(1 - \alpha)(1 + \theta - \alpha)}}{2\alpha^2}.$$

It remains for us to prove $\tau_H = 1 - \phi^*$, where $\phi^* = \arg \max_{0 \leq \phi \leq 1} \Psi(\phi)$. FOC of $\log \Psi(\phi)$ suggests

$$\left(\frac{1}{1 + \gamma} + \frac{2\alpha - 1}{1 - \alpha} \right) \left(\frac{1}{\phi} \right) + \left(\frac{1}{1 + \gamma} \right) \left(\frac{\alpha}{1 + \theta} \right) \left(\frac{1}{1 - \frac{\alpha\phi}{1 + \theta}} \right) - \frac{1}{1 - \phi} = 0,$$

which is equivalent to

$$\Gamma(\phi) \equiv \alpha^2 \phi^2 - \left[\frac{(1 - \alpha)(1 + \theta)}{1 + \gamma} + \alpha\theta + 2\alpha^2 \right] \phi + \left[\frac{(1 - \alpha)(1 + \theta)}{1 + \gamma} + (2\alpha - 1)(1 + \theta) \right] = 0.$$

Besides, we can easily verify that, for $\phi \in (0, 1)$, it always holds that

$$\frac{d^2}{d\phi^2} (\log \Psi(\phi)) < 0.$$

Since $\tau \equiv 1 - \phi$, we know that $\Delta(1 - \phi) = \Gamma(\phi)$. Denote ϕ_1 and ϕ_2 as the solutions to $\Gamma(\phi) = 0$. Note that $\phi_1 + \phi_2 > 0$, $\phi_1 \cdot \phi_2 > 0$, and $\Gamma(0) > 0$, $\Gamma(1) > 0$. Therefore we know that $0 < \phi_1 < 1 < \phi_2$. Consequently we conclude that

$$\phi^* = \phi_1 = 1 - \tau_{\max} \in (0, 1).$$

Proof of Proposition 1: First, notice that, by definition, $\tau_{\max} = 1 - \phi_{\min}$. Therefore we have $\phi_{\min} = \phi^*$. Then by Lemma 2 immediately we reach the conclusion.

Proof of Corollary 1: First, when adverse selection is severe enough, *i.e.*, $\bar{\Phi} = \pi\Phi \geq \Psi_{\max}$, the economy collapses. The only equilibrium is the trivial case with $\phi = 0$. Given that $\bar{\Phi} < \Psi_{\max}$, Lemma 1 implies that there are two solutions, which are denoted as $(\bar{\phi}_H, \bar{\phi}_L)$. It always holds that $\bar{\phi}_L < \phi^* < \bar{\phi}_H$. Then Lemma 2 immediately suggests that the steady state $\bar{\phi}_L$ is always a saddle. Since $\Psi(\phi)$ decreases with ϕ when $\phi > \phi^*$, as shown by Proposition 1, indeterminacy emerges if and only if $\phi \in (\phi^*, \phi_{\max})$. Therefore the local dynamics around the steady state $\phi = \bar{\phi}_H$ exhibits indeterminacy if and only if $\Psi(\phi_{\max}) < \bar{\Phi} < \Psi_{\max}$.

Proof of Corollary 2: Holding Φ constant, $\bar{\Phi}$ increases with π , the proportion of firms producing lemon products. As is proved in Corollary 1, given $\bar{\Phi} < \Psi_{\max}$, indeterminacy emerges if and only if $\bar{\Phi} > \Psi(\phi_{\max})$. Therefore the likelihood of indeterminacy increases with π .

Proof of Lemma 3: Notice that $\Psi(\phi) = \left(\frac{1-\phi}{\phi}\right) \cdot Y(\phi) \propto (1-\phi)\phi^{\frac{2\alpha-1}{1-\alpha}}$. When $\alpha < \frac{1}{2}$, we know that $(1-\phi)\phi^{\frac{2\alpha-1}{1-\alpha}}$ is decreasing in ϕ . It is easy to check that $\lim_{\phi \rightarrow 0} \Psi(\phi) = \infty$ and $\lim_{\phi \rightarrow 1} \Psi(\phi) = 0$. Hence equation (69) uniquely pins down the steady state ϕ for any $\bar{\Phi} > 0$.

Proof of Proposition 2: The dynamic system of equations is follows:

$$\begin{aligned}
\psi N_t^\gamma &= \frac{1}{C_t}(1-\alpha)\phi_t \frac{Y_t}{N_t}, \\
\frac{\dot{C}_t}{C_t} &= \alpha\phi_t \frac{Y_t}{K_t} - \delta(u_t) - \rho, \\
\alpha\phi_t \frac{Y_t}{u_t K_t} &= \delta^0 u_t^\theta, \\
C_t + \dot{K}_t + C_t^e &= Y_t - \delta(u_t) K_t, \\
Y_t &= A(u_t K_t)^\alpha N_t^{1-\alpha}, \\
\phi_t &= \frac{Y_t}{\pi\Phi + Y_t}, \\
C_t^e &= (1-\phi_t) Y_t,
\end{aligned}$$

where $\pi \equiv \frac{\rho e}{\lambda}$. Denote $s \equiv 1 - \frac{\alpha}{1+\theta}$. Then some of the key ratios in the steady state can be obtained as

$$\begin{aligned}
k_y &= \frac{K}{Y} = \frac{\alpha\phi\theta}{\rho(1+\theta)}, \\
c_y &= \frac{C}{Y} = s\phi = \left(1 - \frac{\alpha}{1+\theta}\right)\phi, \\
N &= \left[\frac{(1-\alpha)\phi}{c_y} \cdot \frac{1}{\psi}\right]^{\frac{1}{1+\gamma}} = \left[\left(\frac{1-\alpha}{1-\frac{\alpha}{1+\theta}}\right) \cdot \frac{1}{\psi}\right]^{\frac{1}{1+\gamma}}, \\
Y &= A^{\frac{1}{1-\alpha}} (k_y)^{\frac{\alpha}{1-\alpha}} N = A^{\frac{1}{1-\alpha}} \left[\frac{\alpha\phi\theta}{\rho(1+\theta)}\right]^{\frac{\alpha}{1-\alpha}} \left[\left(\frac{1-\alpha}{1-\frac{\alpha}{1+\theta}}\right) \cdot \frac{1}{\psi}\right]^{\frac{1}{1+\gamma}}. \quad (\text{A.6})
\end{aligned}$$

We can use Equation (69) to solve for the steady state ϕ and use Equation (A.6) to obtain the steady state Y . Consumption and capital can then be computed by $C = c_y Y$ and $K = k_y Y$, respectively. The log-linearization of the system of equilibrium equations is given by:

$$\begin{aligned}
0 &= \hat{\phi}_t + \hat{y}_t - (1+\gamma)\hat{n}_t - \hat{c}_t, \\
\dot{c}_t &= \rho \left(\hat{\phi}_t + \hat{y}_t - \hat{k}_t \right), \\
\hat{y}_t &= \alpha \left(\hat{u}_t + \hat{k}_t \right) + (1-\alpha)\hat{n}_t, \\
\hat{u}_t &= \frac{1}{1+\theta} (\hat{\phi}_t + \hat{y}_t - \hat{k}_t), \\
\dot{k}_t &= \left(\frac{s\phi}{k_y} \right) \left(\hat{\phi}_t + \hat{y}_t - \hat{k}_t \right) - \left(\frac{c_y}{k_y} \right) (\hat{c}_t - \hat{k}_t), \\
\hat{\phi}_t &= (1-\phi)\hat{y}_t \equiv \tau\hat{y}_t.
\end{aligned}$$

As in the baseline model, we can substitute $\hat{u}_t$ and $\hat{\phi}_t$ to obtain a reduced form of output in terms of capital and labor as

$$\hat{y}_t = \frac{\alpha\theta\hat{k}_t + (1+\theta)(1-\alpha)\hat{n}_t}{1+\theta - (1+\tau)\alpha} \equiv a\hat{k}_t + b\hat{n}_t,$$

where $a \equiv \frac{\alpha\theta}{1+\theta-(1+\tau)\alpha}$ and $b \equiv \frac{(1+\theta)(1-\alpha)}{1+\theta-(1+\tau)\alpha}$. We assume $\tau < \frac{1+\theta}{\alpha} - 1$, which is a reasonable restriction under standard calibration, so that $a > 0$ and $b > 0$. Finally $\hat{n}_t$ can be expressed as a function of $\hat{y}_t$ and $\hat{c}_t$, and thus we have

$$\hat{y}_t = \frac{a(1+\gamma)}{1+\gamma-b(1+\tau)}\hat{k}_t - \frac{b}{1+\gamma-b(1+\tau)}\hat{c}_t \equiv \lambda_1\hat{k}_t + \lambda_2\hat{c}_t,$$

where $\lambda_1 \equiv \frac{\alpha(1+\gamma)}{1+\gamma-b(1+\tau)}$ and $\lambda_2 \equiv -\frac{b}{1+\gamma-b(1+\tau)}$. Consequently the local dynamics is characterized by following differential equations:

$$\begin{aligned} \begin{bmatrix} \dot{k}_t \\ \dot{c}_t \end{bmatrix} &= \delta \begin{bmatrix} \left(\frac{1+\theta}{\alpha\phi}\right) s\phi(1+\tau)\lambda_1 & \left(\frac{1+\theta}{\alpha\phi}\right) [s\phi(1+\tau)\lambda_2 - (1-s\phi)] \\ \theta[(1+\tau)\lambda_1 - 1] & \theta(1+\tau)\lambda_2 \end{bmatrix} \begin{bmatrix} \hat{k}_t \\ \hat{c}_t \end{bmatrix}, \\ &\equiv J \begin{bmatrix} \hat{k}_t \\ \hat{c}_t \end{bmatrix}. \end{aligned} \quad (\text{A.7})$$

where $s \equiv 1 - \frac{\alpha}{1+\theta}$, $c_y = s\phi$, $\delta = \rho/\theta$. The local dynamics around the steady state is determined by the roots of J . Notice that the trace and the determinant of J are

$$\begin{aligned} \frac{\text{Trace}(J)}{\delta} &= \left(\frac{1+\theta}{\alpha}\right) s(1+\tau)\lambda_1 + \theta(1+\tau)\lambda_2 < 0, \\ \frac{\det(J)}{\delta^2\theta\left(\frac{1+\theta}{\alpha\phi}\right)} &= s\phi(1+\tau)\lambda_2 + (1-s\phi)(1+\tau)\lambda_1 - (1-s\phi) > 0. \end{aligned}$$

Similar to the analysis for the indeterminacy of our baseline model, here $\text{Trace}(J) < 0$ if and only if $\tau > \tau_{\min} \equiv \frac{(1+\theta)(1+\gamma)}{(1+\theta)(1-\alpha)+\alpha(1+\gamma)} - 1$. Given that $\tau > \tau_{\min}$, some algebraic manipulation suggests that $\det(J) > 0$ if and only if $\tau < \frac{1+\theta}{\alpha} - 1$, and

$$A_1\tau^2 - A_2\tau - A_3 < 0,$$

where

$$\begin{aligned} A_1 &\equiv s(1+\theta)(2+\alpha+\alpha\gamma) > 0 \\ A_2 &\equiv (1+\theta)(1+\alpha\gamma) - s[(1+\theta)(1-\alpha)(1-\gamma) + (1+\gamma)\alpha] \\ A_3 &\equiv (1+\theta)(1-\alpha)[s + (1-s)\gamma] > 0. \end{aligned}$$

Therefore $A_1\tau^2 - A_2\tau - A_3 < 0$ if and only if $\tau < \tau_H$, where τ_H is the positive solution to $A_1\tau^2 - A_2\tau - A_3 = 0$.

Proof of Lemma 4: Combining Lemma 3 and Proposition 2 immediately yields the desired result.

Proof of Lemma 5: First, using Implicit Function Theorem, Equation (78) suggests that $\frac{\partial q^*}{\partial Y} > 0$. Second, footnote 7 proves that $\frac{\partial TFP}{\partial q^*} > 0$. In turn, the chain rule implies that $\frac{\partial TFP}{\partial Y} = \left(\frac{\partial TFP}{\partial q^*}\right) \left(\frac{\partial q^*}{\partial Y}\right) > 0$.

Proof of Proposition 3: To establish the conditions for indeterminacy, we first log-linearize the equilibrium equations. Substituting $\hat{u}_t$ from the log-linearized Equation (24), we obtain

$$\hat{y}_t = a\hat{k}_t + b\hat{n}_t,$$

where $a = \frac{\theta\alpha(1+\sigma)}{1+\theta-\alpha(1+\sigma)}$ and $b = \frac{(1+\theta)(1-\alpha)(1+\sigma)}{1+\theta-\alpha(1+\sigma)}$. Finally, expressing $\hat{n}_t$ from the log-linearized Equation (22), we obtain

$$\hat{y}_t = \lambda_1\hat{k}_t + \lambda_2\hat{c}_t,$$

where $\lambda_1 \equiv \frac{a(1+\gamma)}{1+\gamma-b}$ and $\lambda_2 \equiv -\frac{a}{1+\gamma-b}$. We hence obtain a two-dimensional system of difference equations

$$\begin{bmatrix} \hat{k}_t \\ \hat{c}_t \end{bmatrix} = \delta \begin{bmatrix} \left(\frac{1+\theta}{\alpha\phi} - 1\right)\lambda_1 & \left(\frac{1+\theta}{\alpha\phi}\right)(\lambda_2 - 1) + 1 - \lambda_2 \\ \theta(\lambda_1 - 1) & \theta\lambda_2 \end{bmatrix} \begin{bmatrix} \hat{k}_t \\ \hat{c}_t \end{bmatrix} \equiv J \begin{bmatrix} \hat{k}_t \\ \hat{c}_t \end{bmatrix}$$

where $\delta = \rho/\theta$. The local dynamics around the steady state is determined by the roots of J . The trace and the determinant of J are

$$\begin{aligned} \frac{\text{Trace}(J)}{\delta} &= \left(\frac{1+\theta}{\alpha\phi} - 1\right)\lambda_1 + \theta\lambda_2 = \frac{\left(\frac{1+\theta}{\alpha\phi} - 1\right)(1+\gamma)a - \theta b}{1+\gamma-b}, \\ \frac{\det(J)}{\delta^2\theta} &= \left(\frac{1+\theta}{\alpha\phi} - 1\right)(\lambda_1 - 1 + \lambda_2) = \left(\frac{1+\theta}{\alpha\phi} - 1\right) \left[\frac{(1+\gamma)(a-1)}{1+\gamma-b} \right]. \end{aligned}$$

Indeterminacy arises if $\text{Trace}(J) < 0$ and $\det(J) > 0$. Under the assumption $a < 1$, or $\alpha(1+\sigma) < 1$, $\det(J) > 0$ is equivalent to $1+\gamma-b > 0$, or $\sigma > \sigma_{\min} \equiv \left(\frac{1}{\frac{1-\alpha}{1+\gamma} + \frac{\alpha}{1+\theta}}\right) - 1$. Then $\text{Trace}(J) < 0$ requires $\left(\frac{1+\theta}{\alpha\phi} - 1\right)(1+\gamma)a > \theta b$. Rearranging terms yields the requirement, $\frac{(1+\sigma)\eta}{1+\eta} < \left(\frac{1}{\frac{1-\alpha}{1+\gamma} + \frac{\alpha}{1+\theta}}\right)$. Recall $\sigma = \frac{1-\chi}{\chi+\eta}$, or $\frac{(1+\sigma)\eta}{1+\eta} = \frac{\eta}{\chi+\eta}$, so that such a requirement is automatically satisfied.

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