Financial Regulation in a Quantitative Model of the Modern Banking System*

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Abstract

This paper builds a quantitative general equilibrium model with commercial banks and shadow banks to study the unintended consequences of capital requirements. In particular, we investigate how the shadow banking system responds to capital regulation changes for traditional banks. A key feature of our model are defaultable bank liabilities that provide liquidity services to households. In case of default, commercial bank debt is fully insured and thus provides full liquidity services. In contrast, shadow banks are only randomly bailed out. Thus, shadow banks' liquidity services also depend on their default rate. Commercial banks are subject to a capital requirement. Tightening the requirement from the status quo, leads households to substitute shadow bank liquidity for commercial bank liquidity and therefore to more shadow banking activity in the economy. But this relationship is non-monotonic due to an endogenous leverage constraint on shadow banks that limits their ability to deliver liquidity services. The basic trade-off of a higher requirement is between bank liquidity provision and stability. Calibrating the model to data from the Financial Accounts of the U.S., the optimal capital requirement is around 20%.

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1 Introduction

Unintended consequences are a challenging aspect of macroprudential financial regulation. More stringent regulation of one sector can potentially strengthen its unregulated competitors. For example, Regulation Q posed a cap on deposit rates. During periods of high interest rates, savings were attracted to deposit account alternatives, such as money market mutual funds (Adrian and Ashcraft (2012)). Another example are asset-backed commercial paper conduits, whose existence is justified on grounds of regulatory capital arbitrage (Acharya, Schnabl, and Suarez (2013)). Thus, banking regulation affects not only the targeted institutions but also shadow banks.[1] A tightening of regulatory measures on traditional banks such as capital requirements can shift activity to shadow banks, potentially increasing the fragility of the entire financial system. In this paper, we study and quantify the effects of capital requirements in a general equilibrium model with regulated (commercial banks) and unregulated (shadow banks) financial institutions.

Our general equilibrium model features an endowment economy with households, regulated (commercial) banks, unregulated (shadow) banks, and a regulator. The general equilibrium environment is key to analyze the potential side effects of new policies. The main features of our model are heterogeneous banks that make productive investments, can default on their debt, and differ in their ability to guarantee the safety of their liabilities, as well as household preferences for safe and liquid assets in the form of bank debt whose value to households depends on their safety. While commercial bank debt is insured, shadow bank debt is not. However, with a positive probability shadow banks are bailed by the government. Two trees produce a stochastic endowment. Households have only access to one tree. Shadow

[1] We define shadow banks as financial institutions that share features of depository institutions, either by providing liquidity services such as money market mutual funds or by providing credit either directly (e.g. finance companies) or indirectly (e.g. asset-backed security issuers). At the same time, they are not subject to the same regulatory supervision as traditional banks.

banks and commercial banks compete over shares and intermediate access to the second tree. Matching the model to data from the Flow of Funds and the FDIC, we find an optimal capital requirement that is substantially higher compared to current regulation. The optimal level trades-off the reduction in liquidity provision, against an increase in the safety of the financial system and consumption.

Key to this result is the model's mechanism that pins down the relative size of the two banking sectors. Shadow banks cannot guarantee the safety of their liabilities up to the bailout and therefore face an endogenous leverage constraint. That is, the price at which they issue debt takes into account the default probability implied by their leverage choice. The guarantee on their debt gives commercial banks an advantage in producing liquidity services over shadow banks. However, as long as commercial bank debt and shadow bank debt are imperfect substitutes, shadow banks exists. The relative size is mainly determined by the value of their liquidity services, which does not only depend on the total amount but also on the relative mix between the two types of bank debt providing these services. With imperfect substitutable bank liabilities, households like to consume some fraction of their liquidity services from shadow bank debt as well.

We find that higher capital requirement on regulated banks shifts intermediation activity from regulated to unregulated banks. When banks default, a fraction of the remaining bank value is lost. A higher capital requirement forces commercial banks to lower their leverage, effectively reducing their ability to produce liquidity services. At the same time, they become safer, reducing expected deadweight losses associated with their failure. The reduction of liquidity production by commercial banks increases the attractiveness of shadow bank liabilities. Because shadow banks face an endogenous leverage constraint, they can only produce more liquidity services by lowering their leverage and at the same time increasing their holdings of the intermediated assets. In the process, shadow banks become also safer, reducing expected deadweight losses to a minimum in the economy and thus increasing consumption. Liquidity services are decreasing in the capital requirement because given a particular leverage ratio, shadow banks can never produce the same amount of liquidity services compared to commercial banks as they always take into account their default probability. The optimal requirement finds the welfare maximizing balance between a reduction in liquidity services and the increase in the safety of the financial sector and consumption.

In addition, the simulated dynamic model shows that a greater capital requirement for commercial banks reduces the volatility of asset prices, liquidity provision, and consumption. The reduction in the volatility of consumption is another source of welfare gains to households from increased capital requirements.

The equilibrium in this paper also features excessive liquidity creation by shadow banks and procyclical shadow banking activity. The failure rate of shadow banks affects households' valuation of shadow bank debt. In essence, the riskier shadow banks are, the lower the liquidity services that households can derive from shadow bank debt. Shadow banks do not internalize this effect on the liquidity value of their liabilities for households, which causes them to issue too much debt. It further causes procyclical shadow banking activity: positive aggregate shocks make shadow banks safer and therefore their debt more attractive. This increases the share of shadow banking activity in good states of the world.

We match the model to aggregate data from the Flow of Funds on financial positions of U.S. households and financial institutions, interest rates from FRED, as well as microdata from U.S. depository institutions (FDIC bank call reports). We also use industry reports to obtain estimates for the share of credit by each financial institution type as well as their default and recovery rates on bonds. The key parameters are those that determine the relative size of the two sectors and the spread between their debt securities: the elasticity of substitution between the two types of banking sector debts as well as the parameter that governs the sensitivity of liquidity services to the default rate of shadow banks. It's intuitive to think of commercial bank debt (mostly deposits, but also commercial paper, repo etc) and shadow bank debt (mostly money market mutual fund shares, commercial paper, repo, etc) as being slightly different securities. The elasticity of substitution parameter is pinned down by the relative size of depository institution debt to shadow bank debt from the Flow of Funds. The shadow bank liquidity value parameter in households' utility function is pinned down by the 3-month spread between AA commercial paper for financial institutions and the implied interest rate on depository institution debt from FDIC reports.

2 The main mechanism in a two period model

This section demonstrates the main mechanisms of our dynamic general equilibrium model in a two-period model. We first describe the two-period model and the forces that determine the optimal capital requirement. We discuss the key assumptions of the model in a separate section.

Households maximize utility from consuming goods and liquidity services. The economy receives two endowments of goods: one that households directly receive as income and one that households can only access through financial intermediaries. Two types of intermediaries, C-banks and S-banks, can perform the intermediation. They issue short-term debt to households to fund the intermediated asset. The short term debt of both banks provides households with liquidity services.

Both type of banks can declare bankruptcy and default on their debt. However, the debt of C-banks is riskfree to households since the government provides deposit insurance for these types of banks. In return, C-banks are subject to capital regulation. S-banks, on the other hand, are not subject to regulation that limits their leverage. But their debt is risky for households since debt of defaulting S-banks only pays off a fraction of the face value. S-banks take into account the effect of their leverage choice on the expected payoff of their debt, and hence endogenously choose to limit their leverage.

2.1 Description of the model

Endowments There are two periods. Households receive an income of Y_0 at date 0 and Y_1 at date 1. Further, the intermediated asset is in unit supply and pays off Z at date 1. Y_0 is known at date 0 whereas Y_1 and Z are stochastic. In particular, trees Z and Y_1 have two realizations $G =$ good and $B =$ bad. Banks are exposed to idiosyncratic valuation shocks ρ_i^j that are uniformly distributed on the interval $[\underline{\rho}^j, \overline{\rho}^j]$ for bank types $j = S, C$, respectively. The shocks ρ_i^j are independent of the endowment processes Y_1 and Z.

Shadow banks There is a unit mass of S-banks. S-bank i chooses A_i^S shares of the Z-tree at the beginning of period 0. The shares trade at a market price of p_0. The tree has a stochastic payoff of Z in period 1. To fund their investment, S-banks issue short term debt B_i^S that trades at the price q_i^S. All trades occur in 0, so all prices must be 0 in $t = 1$. At the beginning of period 1, $S-$banks face idiosyncratic valuation risks ρ_i^S that are proportional to their assets, with $\rho_i^S \sim F_\rho^S$, i.i.d. across banks. After a large realization of ρ_i^S, $S-$bank i may find it optimal to declare bankruptcy. In case of a bankruptcy, the bank's equity is wiped out, and its assets are seized by its creditors. In case the bank does not default, it returns the debt it ows and pays out dividends to households.

The problem for shadow bank i is

$$\begin{aligned} V_i^S = \max_{A_i^S, B_i^S} \; & q(A_i^S, B_i^S)B_i^S - p_0 A_i^S \\ & + E_0 \left[M \left(\max\left\{0, ZA_i^S - B_i^S - \rho_i^S A_i^S\right\}\right)\right]. \end{aligned}$$

Since ρ_i^S is uniformly distributed, the probability that bank i with assets A_i^S and debt B_i^S stays in business is given by the probability that $\rho_i^S \leq Z - \frac{B_i^S}{A_i^S}$, i.e.

$$F_\rho^S\left(Z - \frac{B_i^S}{A_i^S}\right) = \frac{Z - \frac{B_i^S}{A_i^S} - \underline{\rho}^S}{\overline{\rho}^S - \underline{\rho}^S}. \tag{1}$$

The bank problem has constant returns to scale in A^S, allowing for direct aggregation. Since there is a unit mass of $S-$ banks, we can directly interpret F^S as the mass of banks that do not fail. To further simplify the problem, we exploit the homogeneity in the scale of the bank A^S by defining leverage per asset $b_i^S = B_i^S/A_i^S$.

Dropping the $i-$subscript, we can hence rewrite the problem as

$$V^S = \max_{A^S \geq 0, b^S} \ A^S v^S(b^S)$$

where[2]

$$v^S(b^S) = q^S(b^S)b^S - p_0 + \mathrm{E}_0\left[M\left(F_\rho^S\left(Z - b^S - \rho_S^-\right)\right)\right]. \tag{2}$$

In the above expression, ρ_S^- is the expected value of ρ_i^S conditional on $S-$ bank's survival and a realization of the random variable Z. We define ρ_S^+ as the expected value of ρ conditional on $S-$bank's failure. Investors who hold debt issued by S-banks that go bankrupt recover a fraction of each bond's value

$$r^S = (1 - \xi_S)\frac{A^S\left(Z - \rho_S^+\right)}{B^S} = (1 - \xi_S)\frac{Z - \rho_S^+}{b^S}. \tag{3}$$

The date-0 dividend payments are equal to the initial equity required by the bank,

$$D_0^S = A^S(p_0 - q^S b^S).$$

and the date-1 dividends are the payout amounts of the non-bankrupt banks:

$$D_1^S = F^S A^S(Z - b^S - \rho_S^-).$$

[2]We describe the derivation in more detail in the appendix section A.1.

Commercial banks The problem of C-banks is similar to the S-bank problem, with the important difference that $C-$banks are subject to a leverage constraint. Moreover, they have to pay regulators the fee κ per unit of debt to fund the insurance. Defining $b^C = B^C/A^C$, we can write the problem for a $C-$bank as

$$V^C = \max_{A^C, b^C} \; A^C v^C(b^C)$$

subject to

$$b^C \leq (1-\theta)p_0,$$

where

$$v^C(b^C) = \left(q^C - \kappa\right) b^C - p_0 + \mathrm{E}_0 \left[M \left(F_\rho^C \left(Z - b^C - \rho_C^- \right) \right) \right]. \tag{4}$$

The term ρ_C^- in the expression above is defined analogously to the $S-$ banks.

Households who invest in C-bank debt always receive the full face value of the bonds. The difference between the face value and the recovery value of the bonds for failing C-banks is provided by the government (deposit insurance). The insurance fund recovers a fraction of the value of the debt of bankrupt C-banks, per bond issued:

$$r^C = (1-\xi_C)\frac{A^C(Z-\rho_C^+)}{B^C} = (1-\xi_C)\frac{Z-\rho_C^+}{b^C}.$$

As $S-$banks, $C-$banks payout D_0^C and D_1^C in period 0 and 1, respectively.

Households In period 0, households have power utility over consumption C_0. In period 1, they have power utility over consumption C_1 and liquidity services H relative to consumption. Thus utility is

$$U(C_0, C_1, N^S, N^C) = \begin{cases} \frac{C_0^{1-\gamma}}{1-\gamma} + \beta\left(\frac{C_1^{1-\gamma}}{1-\gamma} + \psi\frac{(H/C_1)^{1-\eta}}{1-\eta}\right) & \text{for } \eta \text{ ¿ } 1 \\ \frac{C_0^{1-\gamma}}{1-\gamma} + \beta\left(\frac{C_1^{1-\gamma}}{1-\gamma} + \psi\log\left(H/C_1\right)\right) & \text{for } \eta{=}1, \end{cases} \tag{5}$$

with $\gamma > 1$ and $\eta \geq 1$. Liquidity is a composite good (consisting of N^S and N^C) that is produced in equilibrium by both types of intermediaries

$$H\left(N^S, N^C\right) = \begin{cases} \left[(1-\nu)\left(N^S\right)^{\alpha} + \nu\left(N^C\right)^{\alpha}\right]^{1/\alpha} & \text{for } \alpha \neq 1 \\ \left(N^S\right)^{(1-\nu)}\left(N^C\right)^{\nu} & \text{for } \alpha = 1 \end{cases},$$

where α parameterizes the elasticity of substitution between liquidity services of both types of banks. Weight ν determines the quality of liquidity services provided by C-banks relative to S-banks. The weight ν depends on the fraction of defaulting S-banks F^S, i.e. $\nu = 1/(1+(F^S)^{\tilde{\nu}})$, and consequently

$$1 - \nu = \frac{(F^S)^{\tilde{\nu}}}{1 + (F^S)^{\tilde{\nu}}}.$$

Households maximize utility in equation (5) by choosing consumption C_0, C_1, and liquidity services from each bank N^S, N^C, subject to the budget constraints

$$C_0 = Y_0 - \sum_j D_0^j - \sum_j q^j N^j,$$
$$C_1 = Y_1 + \sum_j D_1^j + N^C + N^S\left(\pi_B + (1-\pi_B)\left(F^S + \left(1 - F^S\right) r^s\right)\right) - T.$$

In period 1, households have to pay a lump-sum tax of T that the government raises to make whole the depositors of failed C-banks. Households are the sole owners of all banks and therefore receive the dividend payments both at date 0 and date 1.

Government The government pays out on deposit insurance to reimburse depositors of C-banks. The government covers the deficit by imposing a lump-sum tax on households in period 1. Therefore the amount of tax funds the government raises is

$$T = B^C[(1-F^C)(1-r^C) - \kappa] + \pi_B\left(B^S[(1-F^S)(1-r^S)]\right).$$

Markets Asset market clearing in period zero requires

$$
\begin{aligned}
1 &= A^S + A^C \\
N^S &= B^S \\
N^C &= B^C.
\end{aligned}
$$

This implies for the goods market in period 1

$$
C_1 = Y_1 + Z - \sum_j A^j \left[\mathrm{E}^\rho(\rho^j) + \xi_j (1 - F^j)(Z - \rho_j^+) \right]. \tag{6}
$$

2.2 Discussion of assumptions

The following paragraphs discuss key assumptions of the model.

Banks' Role as Intermediaries In the model, banks are special because they provide liquidity (discussed below) and intermediate assets. The role of banks as intermediaries can be derived from first principles in numerous ways. For example, in models with asymmetric information between borrowers and lenders, lenders with access to a cheaper screening or monitoring technology than other lenders (regular households) become banks (see Freixas and Rochet (1998) for many other examples).

The Role of Banks as Liquidity Providers In this model, households value bank debt because it is liquid[3] and safe, an interpretation of bank debt in Gorton and Pennacchi (1990) and in Gorton et al. (2012). The notion of safe and liquid assets includes bank deposits, money market fund shares, commercial paper, repos, short-term interbank loans, Treasuries, agency and municipal debt, securitized debt, and high-grade financial sector corporate debt.

[3] The idea to view banks as liquidity provider goes back to Diamond and Dybvig (1983). Other work has built upon this idea (e.g. Gorton and Pennacchi (1990))

Aside from the government, commercial banks and shadow banks are the most important providers of these securities.[4] The savings glut hypothesis articulated in Bernanke (2005) and other recent work (e.g. Caballero and Krishnamurthy (2009), Gorton, Lewellen, and Metrick (2012), Gourinchas and Jeanne (2012), and Krishnamurthy and Vissing-Jorgensen (2012)) rests on the notion that there exists a demand for safe and liquid securities. Economic agents demanding these assets are for example households that hold deposits for transaction or liquidity reasons as well as corporations, institutional investors, and high net worth individuals that carry large cash-balances and seek safe and liquid investment vehicles with higher yields than deposits, such as money market mutual funds. Commercial banks provide mostly deposits, but also issue money market fund shares, repos, and commercial paper. Some of these securities that commercial banks hold (most notably deposits) are explicitly insured through deposit insurance. Others, such as money market fund shares and commercial papers are indirectly insured due to government guarantees.[5] Shadow banks generally do not benefit from government guarantees, but nevertheless carry the characteristics of safe and liquid assets during normal times.

In the model, liquidity servives are generated through debt issued by shadow banks and commercial banks. Commercial bank debt represents all commercial bank liabilities precisely because the demand for safe and liquid assets goes beyond merely deposits. We apply the same idea to shadow bank debt with one notable difference: the value of shadow bank liabilities depends on the likelihood at which shadow banks default.

[4]Historically, money market mutual funds (a type of shadow bank) emerged precisely to satisfy demand for safe and liquid assets when Regulation Q imposed a ceiling on deposit rates.

[5]A number of empirical papers presents evidence for market expectations for government guarantees on U.S. banks (see for instance Flannery and Sorescu (1996), Kelly, Lustig, and Van Nieuwerburgh (2011), and Gandhi and Lustig (2013)).

Liquidity services in households preference We capture the idea that bank liabilities provide liquidity services with our utility specification. The households in our model represent a blend of different agents with demands for different types of safe and liquid assets (deposits, money market mutal fund shares, and so forth) provided by all financial institutions. This is why our utility specification aggregates the liquidity services of both bank types.

Commercial bank debt always provides liquidity services no matter their default probability. This is different for shadow banks as the value from their liquidity service depends on their probability of default. This captures the idea that shadow bank debt is only safe as long as shadow banks are safe, that is, not too many of them go bankrupt.

The demand for liquidity services is captured with a money-in-the-utility function specification. Since Sidrauski (1967) money-in-the-utility specifications have been used to capture the benefits from money-like-securities for households in macroeconomic models. Feenstra (1986) proved the functional equivalence of models with money-in-the-utility and models with transaction or liquidity costs. The specific functional form is a version of Poterba and Rotemberg (1986) and Christiano, Motto, and Rostagno (2010).[6] We further assumed ($\eta \geq 1$) complementarity between consumption and liquidity. This is intuitive as higher consumption levels go along with higher transaction volumes and higher savings demand.

2.3 Equilibrium Characterization

Households The stochastic discount factor M_1 equals

$$M = \beta E_0 \left[\frac{C_0}{C_1} \left(1 - \psi \left(\frac{H}{C_1} \right)^{1-\eta} \right) \right] .$$

Its first term is the standard discount factor implied by log utility, while the second term reflects the adjustment to the discount factor resulting from the complementarity between

[6]In Christiano, Motto, and Rostagno (2010), the money and deposit utility parameter relates to the bank debt-consumption elasticity η in the following way: $\sigma_q = 2 - \eta$.

liquidity services and consumption.

$$\mathrm{MRS}^C = \nu \frac{U_1}{U_2}\left(\frac{H}{N^C}\right)^{1-\alpha} \text{ and } \mathrm{MRS}^S = (1-\nu)\frac{U_1}{U_2}\left(\frac{H}{N^S}\right)^{1-\alpha}$$

denote the marginal rates of substitution between consumption and liquidity services by $C-$banks and $S-$banks respectively.

The solution to the HH's problem is characterized by the two FOCs for deposits, which are also the HH's intertemporal Euler equations. The FOC for C-bank deposits N^C is

$$q^C = \mathrm{MRS}^C + \mathrm{E}_0\left[M\right]. \tag{7}$$

The first term on the RHS of equation (7) is the marginal utility from consuming the payoff of 1 per C-bank deposit bond purchased at time 0. MRS^C is the marginal utility from liquidity services.

The FOC for S-bank deposits N^S is

$$q^S = \mathrm{MRS}^S + \mathrm{E}_0\left[M\left(\pi_B + (1-\pi_B)\left(F^S + \left(1-F^S\right)r^s\right)\right)\right] \tag{8}$$

The payoff of one deposit bond issued by S-banks depends on the fraction of surviving banks F^S. For surviving banks, the payoff is simply 1 as for C-bank deposits. For failing banks, it is given by the recovery value per bond r^S. The second term, the marginal utility from liquidity services, is analogous to C-banks.

Bank Portfolio Choices The optimal choice of asset purchases A^j for each bank type $j = C, S$ imply

$$v^j(b^j) = 0, \tag{9}$$

for any $A^j > 0$. We can think of condition (9) as determining the relative size of the j-bank sector. To see this, we show in appendix A that these conditions can be rewritten as

$$p_0 - (q^C - \kappa)b^C = \mathrm{E}_0\left(M\frac{\sqrt{12}}{2}\sigma^C\left(F^C\right)^2\right), \tag{10}$$

$$p_0 - q^S b^S = \mathrm{E}_0\left(M\,\frac{\sqrt{12}}{2}\sigma^S(F^S)^2\right). \tag{11}$$

where σ^j is the standard deviation of ρ^j.

The FOC for leverage per unit of assets b^C is

$$(v^C)'(b^C) = 0.$$

In appendix A, we show that this condition can be expressed as

$$q^C = \lambda^C + \mathrm{E}_0\left(M\,F^C\right). \tag{12}$$

This implies that the Lagrange multiplier on C-banks' leverage constraint is always positive as long as the marginal value of C-bank liquidity is positive.

The positive multiplier implies a binding constraint, i.e.

$$b^C = (1-\theta)p_0.$$

As for C-banks, S-bank leverage per unit of assets is determined trough S-banks' FOC for b^S

$$(v^S)'(b^S) = 0,$$

which can be written as

$$q^S + b^S\frac{\partial q^S(b^S)}{\partial b^S} = \mathrm{E}_0\left(M_1 F^S\right). \tag{13}$$

Comparing the FOC for C-bank leverage in (12) to the S-bank FOC in (13) above reveals the fundamental difference between the two types of banks. While C-banks are subject to a constant leverage constraint that in equilibrium is always binding, S-banks choose to limit their leverage because they internalize the effect of their leverage choice on the price of their deposits, q^S.

Increasing leverage b^S will decrease the survival probability F^S and hence lower the value of S-bank deposits to HH, implying $\frac{\partial q^S(b^S)}{\partial b^S} < 0$, as can be seen in equation (8).

When choosing leverage, we assume that shadow banks take into account that their default risk is priced. In this sense, each shadow bank acts as a monopolist for its own debt. But it does not internalize how its leverage choice and default risk affects the value of liquidity services for households. This is intuitive, as shadow bank bond prices are sensitive to the specific default risk of the issuer. But they also move with changes in aggregate liquidity conditions, which are caused by actions of all shadow banks, but not by any individual bank. Thus, it is best to think of the changes in the value of aggregate liquidity services as an externality arising in general equilibrium.

The following proposition that we prove in the appendix states the resulting endogenous leverage choice of S-banks:

Proposition. *Leverage per units of assets b^S of S-banks is*

$$b^S = E_0 \left(M \, \frac{\sqrt{12}\sigma_S}{\xi_S} \left(1-\nu\right) \left(\frac{1-\psi\left(H/C_1\right)^{1-\eta}}{\psi\left(H/C_1\right)^{-\eta}} \right) \left(\frac{H}{N^S} \right)^{1-\alpha} \right). \tag{14}$$

For any reasonable parameter combinations, the above equilibrium choices of asset purchases and leverage for both types of banks imply that $C-$banks have a dominant position in the intermediated asset ($A^C > A^S$). The intuitive reason for this equilibrium outcome is as follows.

First, C-banks' debt is insured and therefore C-vanks do not internalize the effect of their leverage choice on the price of their debt. Since the marginal liquidity benefit is always positive, C-banks always exhaust their leverage constraint. S-banks, however, do internalize the increase in their default risk. Hence, if C-banks and S-banks are fundamentally equally risky, S-banks choose lower leverage[7]. Required initial equity for C-banks is $p_0 - q^C b^C$. Leverage b^C is a constant fraction the asset price by the collateral constraint and higher than that of S-banks if θ is sufficiently small. Then the bond price q^C must adjust in equilibrium in order for required initial equity to be equal the expected dividend. For reasonable parameter combinations, this in turn means that the marginal benefit of C-bank debt to households must be lower than that of S-bank debt. If debt is further sufficiently substitutable, this means that C-banks must hold a greater share of the intermediated asset in equilibrium[8].

Size of the Banking Sectors & Procyclical Shadow Banking Activity The exact split between both types of banks depends on several parameters, particularly the elasticity of substitution between both kinds of liquidity α.

The relative size of both banking sectors is determined in equilibrium by the marginal benefit of opening each bank in this period and receiving dividends next period. Mathematically, this means that the holdings of the intermediated asset by both types of banks, A^C and A^S, and thus the relative size of both banking sectors, are jointly determined by the FOCs (10) and (11).

[7]This statement of course depends on the parameters of the model, in particular the value of θ, i.e. the tightness of C-banks' leverage constraint. For any values close to the capital requirements of commercial banks, we found this statement to be true.

[8]Even for equal shares of asset holdings ($A^S = A^C$), C-banks will produce $N^C > N^S$ due to their higher leverage. If both types of debt are close to being complements, the higher leverage of C-banks by itself is sufficient to create the lower marginal benefit. Thus the C-bank share is increasing in the elasticity parameter α.

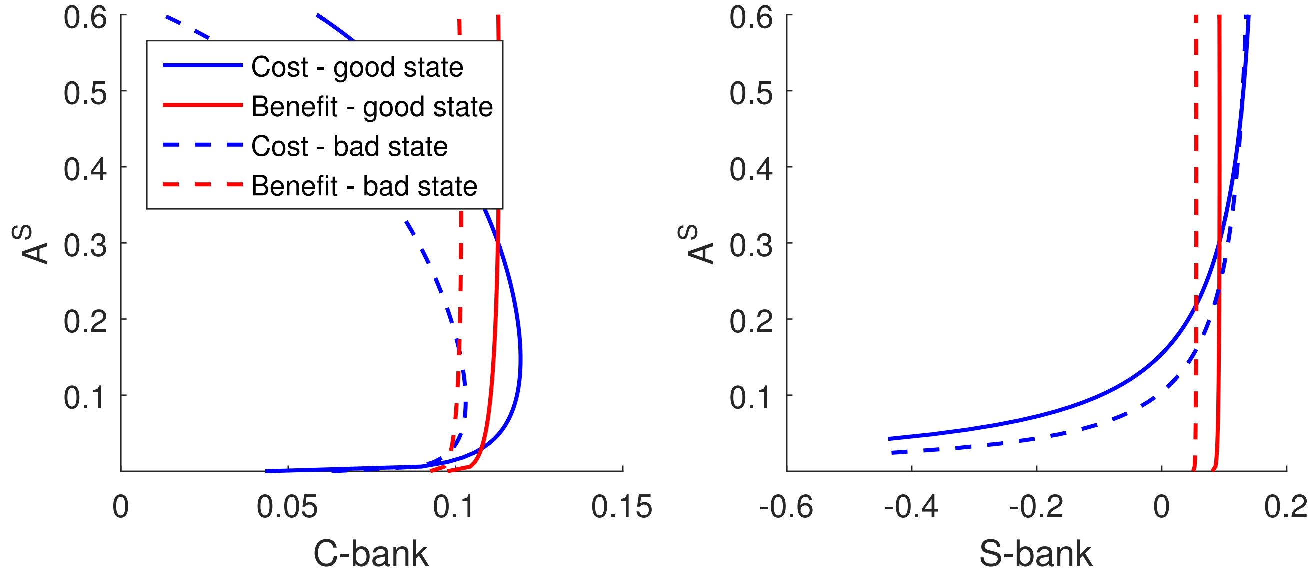

Figure 1: Equilibrium Determination of A^C and A^S for $\alpha = 1/3$

Left-hand side (required equity at time 0, blue line) and right-hand side (expected dividend at time 1, red line) of banks' first-order condition for asset holdings (A^j), while imposing market clearing $1 = A^C + A^S$ and holding fixed all other variables.

Figure (1) shows the LHS and RHS of both equations graphically for the calibrated model, depending on the current state of the economy. We first numerically compute the equilibrium values of all variables. Then we vary the share of S-banks and C-banks, A^S and A^C, while holding all other variables fixed (and imposing market clearing $A^S + A^C = 1$). The blue lines trace the value of the LHS of the first-order conditions (10) and (11) as we vary the shares, $p_0 - q^j b^j$, for $j = C, S$, respectively. Holding constant p_0 and b^j, the only source of variation is through the bond price q^j. Since total debt issued by each type is given by asset share times leverage per unit of assets, $N^j = b^j A^j$, the marginal benefit of deposits of each type changes as we vary the asset shares, as can be seen from the pricing equations for the q^j, (7) and (8). In particular, as we increase the share of type j holding total provided liquidity constant, the marginal liquidity benefit of type j's deposits will decline (for any $\alpha < 1$), and therefore type j's bond price will also decline. This means that the equity required to purchase the bank's initial asset position becomes larger for the same face amount of debt issued.

An opposing effect is that, as long as liquidity services provided by both types of debt are imperfect substitutes, the marginal benefit derived from each type's debt is also affected by the composition of the total debt. When we increase the share of C-banks, we decrease the share of S-banks due to market clearing. Consequently the composition of liquidity services becomes more unequal and the amount of services derived from total debt issued by both banks declines, which leads to a general increase in the marginal benefit of *both kinds* of liquidity. Both effects, the pure effect of an increase in A^C and the equilibrium effect through the implied decrease in A^S can be seen in left panel of (1). Lowering A^S from 0.5 to about 0.15 (= raising A^C from 0.5 to 0.85) causes a decrease in the marginal benefit of C-bank debt, which in turn lowers the bond price q^C and therefore raises the required initial equity of C-banks (the blue line in the graph). By lowering A^S any further, the composition of debt becomes so unequal that the marginal benefit of any liquidity rises again, causing both bond prices (also q^C) to increase again. Hence the required equity of C-banks bends backwards, yielding the overall non-monotonic shape.

The effect on the RHS of equation (11) is depicted by the red lines and quantitatively smaller. The discounted expected dividend at time 1 (per unit of assets) is only indirectly affected by a variation in asset shares through the households' discount factor which depends on consumption C_1. As the share is shifted towards C-banks, consumption C_1 decreases since S-banks have lower leverage and cause lower bankruptcy-induced consumption losses to households. This decrease in C_1 raises the discount factor, leading to a higher present value of the expected dividend.

The share of shadow banking activity also depends on the economic state. In a boom (solid line), shadow banks can issue debt at a higher price because they are less likely to fail next period. Moreover, because the default probability is lower, the expected dividend payment is larger. This means that shadow banking activity is pro-cyclical.

The overall take-away from the left panel of figure (1) is that the FOC of C-banks is satisfied at two points, the actual equilibrium share of $A^C = 0.68$ ($A^S = 0.32$) and an even higher share of $A^C = 0.97$. The unique equilibrium split is then determined by the FOC of S-banks, depicted in the right panel of the figure. The forces determining the shapes of required equity (blue line) and expected dividend (red line) are the same as for C-banks. With respect to required initial equity, both the isolated effect of lowering A^S on the marginal benefit of S-bank debt, the equilibrium effect through debt composition work in the same direction. Thus the required initial equity of S-banks is strictly decreasing as we lower A^S. The unique intersection with the expected discounted dividend of S-banks is at $A^S = 0.32$.

The split between the two types in equilibrium will also depend on other parameters of the model, most importantly the relative quality of C-bank debt compared to S-bank (parameterized by ν).

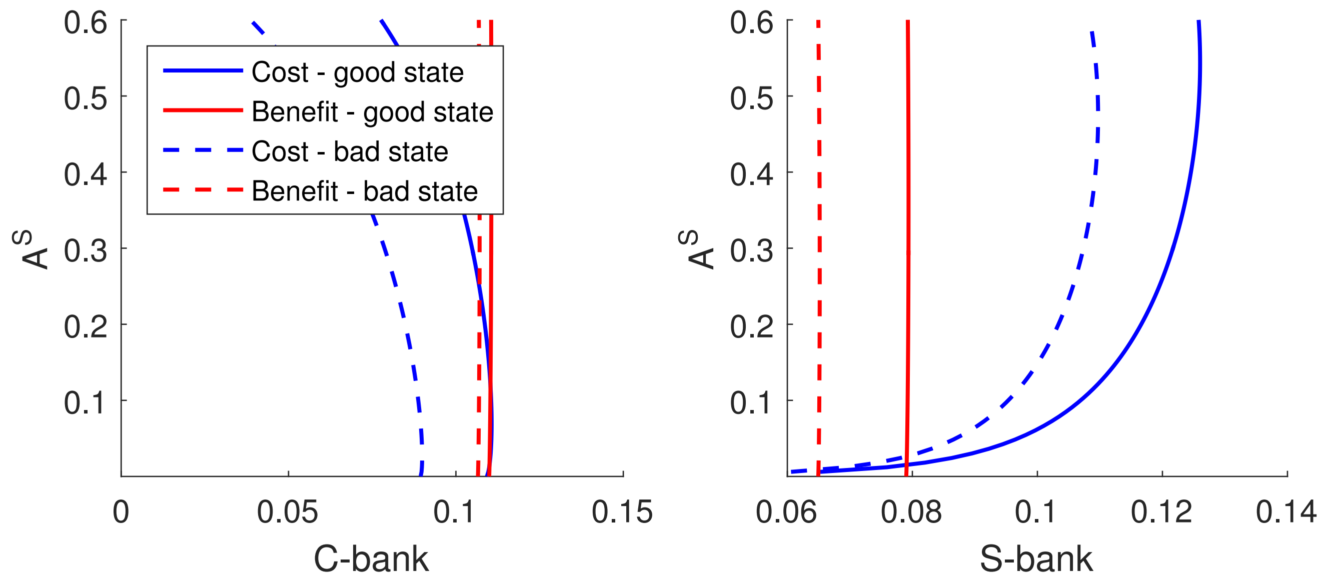

Figure 2: Equilibrium Determination of A^C and A^S for $\alpha = 0.90$

Left-hand side (required equity at time 0, blue line) and right-hand side (expected dividend at time 1, red line) of banks' first-order condition for asset holdings (A^j), while imposing market clearing $1 = A^C + A^S$ and holding fixed all other variables.

Figure (2) shows the equilibrium split for the identical parameters as in figure (1), but with close to perfectly substitutable debt at $\alpha = 0.90$. We can see that for this parameter combination the marginal benefits of liquidity of both types are less responsive to changes in the relative scale, and only at A^S very close to zero does the marginal benefit of S-bank debt increase sufficiently to make required equity equal to expected dividend. Thus as the elasticity of substitution approaches infinity, S-banks' share goes to zero.

Welfare-maximizing Capital Requirement The two-period model delivers a qualitative welfare result with respect to the optimal capital requirement for C-banks, θ. The general conditions for this result to obtain are

C1. C-banks are sufficiently risky such that there exists a range for low values of θ for which some C-banks default, i.e. $F^C < 1$.

C2. S-banks are at least as risky as C-banks; in other words, the standard deviation of their idiosyncratic shocks σ^S is at least as large as that of C-banks.

C3. Households derive a strictly positive utility benefit from liquidity services ($\psi > 0$), and the liquidity services provided by C-banks are at least as good as those of S-banks.

Under these fairly general conditions there exists a trade-off in the model that leads to a unique utility maximum in θ. Figure (3) illustrates the trade-off: increasing the capital requirement leads to an increase in numeraire consumption (top right graph), as fewer C-banks default due to lower leverage and hence bankruptcy losses become smaller. At the same time, decreasing C-bank leverage through tighter capital requirements lowers the amount of liquidity services provided to households (bottom left), as a greater share of the intermediated asset is shifted to S-banks. Since total utility is a weighted sum of both components, household utility is maximized when C-banks have become completely safe ($F^C = 1$) and no further increase in consumption is possible (but liquidity decreases further with higher θ).

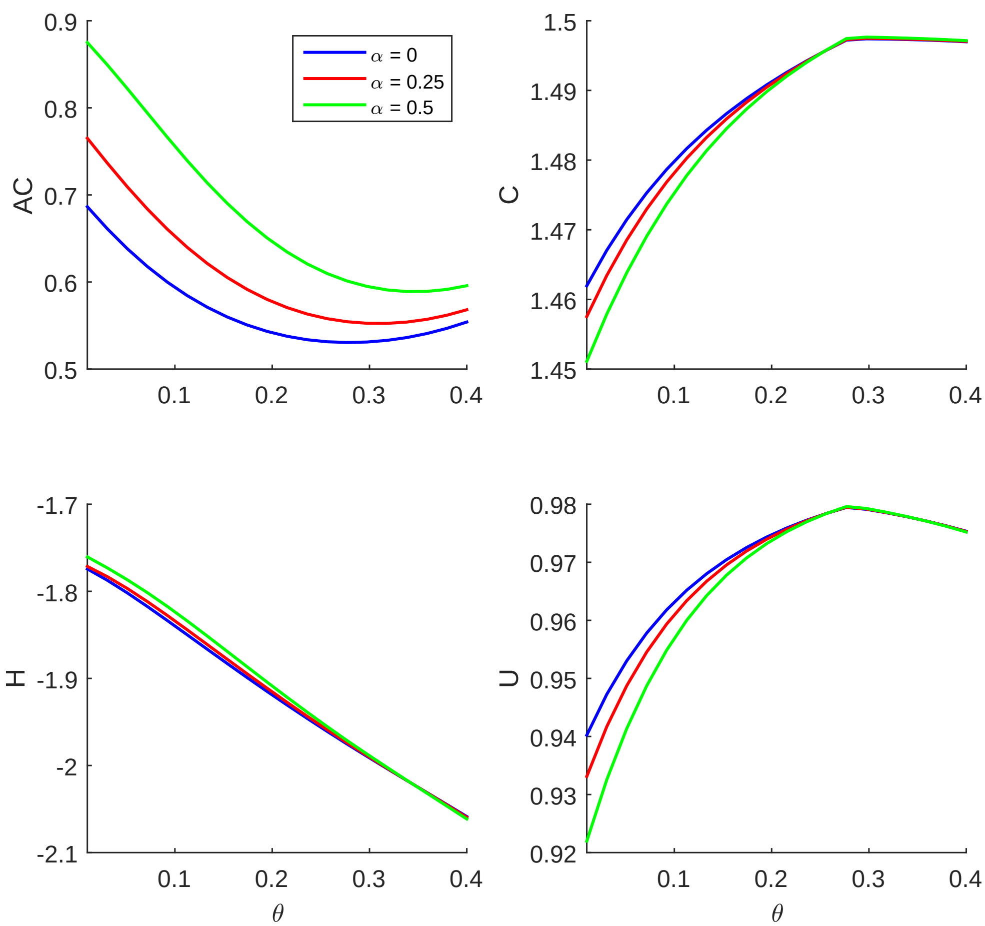

Figure 3: Comparative Statics in Capital Requirement θ

Top left: share of C-banks, top right: consumption at time 1, bottom left: utility from liquidity services, bottom right: total household utility.

3 Dynamic Model

This section presents the quantitative general equilibrium model.

3.1 Model Description

Agents and Environment Time is discrete and infinite. The agent, intermediary, and endowment ownership structure is identical to the two-period model. The two types of intermediaries finance $Z-$tree investments by issuing equity and debt to households. They have limited liability, i.e., they can choose to declare bankruptcy.

S**-Banks** There is a unit mass of S-banks, indexed by i. S-bank i holds $A^S_{t,i}$ shares of the Z-tree at the beginning of period t. The shares trade at a market price of p_t. To fund their investment, S-banks can issue short term debt. The debt of S-bank i trades at the price $q^S_{t,i}$. At the beginning of the period, S-bank i has $B^S_{t,i}$ bonds outstanding.

There is an idiosyncratic component to the payoff of Z-shares when held by bank i. Specifically, the payoff per share held by bank i is Z_t. At the beginning of each period, S-banks can decide to declare bankruptcy. Each period, they face idiosyncratic valuation risks $\rho^S_{t,i}$ that are proportional to its assets. $\rho^S_{t,i}$ is an idiosyncratic loss with $\rho^S_{t,i} \sim F^S_\rho$, i.i.d. across banks and time. In case of a bankruptcy, banks' equity is wiped out, and their assets are seized by their creditors.

Since the idiosyncratic valuation shocks are uncorrelated over time, it is convenient to write bank i's optimization problem after the bankruptcy decision, net of the idiosyncratic payoff. Conditional on asset holdings $A^S_{t,i}$ and debt $B^S_{t,i}$, all banks have the same value and face identical problems:

$$V^S(B^S_t, A^S_t, Z_t) = \max_{A^S_{t+1,i}, B^S_{t+1,i}} (Z_t + p_t - \rho^S_{t,i}) - B^S_{t,i} + q^S_t B^S_{t+1,i} - p_t A^S_{t+1,i}$$
$$+\mathrm{E}_t\left[M_{t,t+1}\tilde{V}^S(A^S_{t+1,i}, B^S_{t+1,i}, Z_{t+1}, \rho^S_{t+1,i})\right].$$

As in the two period model, the problem is again homogenous of degree one and identical for each firm. We can omit subscripts i and define leverage $b^S_t = \frac{B^S_t}{A^S_t}$ and asset growth $a^S_{t+1} = \frac{A^S_{t+1}}{A^S_t}$

and

$$\begin{aligned} v^S(b_t^S, Z_t) &= \frac{V^S(A_{t,i}^S, B_{t,i}^S, Z_t)}{A_{t,i}^S} \\ &= \max_{a_{t+1}^S, b_{t+1}^S} Z_t + p_t - b_t^S + a_{t+1}^S \left(q_t^S b_{t+1}^S - p_t + \mathrm{E}_t \left[M_{t,t+1} \hat{v}^S(b_{t+1}^S, Z_{t+1}) \right] \right). \end{aligned}$$

Using $\rho_{t,i}^S$'s independence of Z_t the continuation value $\hat{v}^S(b_{t+1}^S, Z_{t+1})$ is derived from

$$\begin{aligned} \tilde{V}^S(A_{t,i}^S, B_{t,i}^S, Z_t, \rho_{t,i}^S) &= \max\{0, V(A_{t,i}^S, B_{t,i}^S, Z_t) - \rho_{t,i}^S A_{t,i}^S\} \\ &= 1_{\{\rho_{t,i}^S \leq \frac{V^S(A_{t,i}^S, B_{t,i}^S, Z_t)}{A_{t,i}^S}\}} \left(V^S(A_{t,i}^S, B_{t,i}^S, Z_t) - \rho_{t,i}^S A_{t,i}^S \right). \end{aligned}$$

Taking the expectation with respect to ρ of the last line gives

$$\hat{V}(A_t^S, B_t^S, Z_t) = F_\rho^S \left(\frac{V^S(A_t^S, B_t^S, Z_t)}{A_t^S} \right) V(A_t^S, B_t^S, Z_t) - A_t^S \int_{\underline{\rho}}^{\frac{V(A_t^S, B_t^S, Z_t)}{A_t^S}} \rho \, dF_\rho(\rho).$$

Since we take expectations over the idiosyncratic valuation shock, the problem is identical for each bank and so

$$\hat{v}^S(b_t^S, Z_t) = \frac{\hat{V}^S(A_{t,i}^S, B_{t,i}^S, Z_t)}{A_{t,i}^S}.$$

C-Banks There is a unit mass of C-banks. C-banks are different from S-banks in two ways: (i) they issue short-term debt that is insured and risk free from the perspective of creditors, and (ii) they are subject to regulatory capital requirements. The pay an insurance fee of κ for each bond they issue. The problem of all C banks is identical, thus we can drop the subscript.

We can write the problem of C-bank i as

$$\begin{aligned} v^C(b_t^C, Z_t) &= \frac{V^C(A_t^C, B_t^C, Z_t)}{A_t^C} \\ &= \max_{a_{t+1}^C, b_{t+1}^C} Z_t + p_t - b_t^C + a_{t+1}^C \left(q_t^C b_{t+1}^C - p_t + \mathrm{E}_t \left[M_{t,t+1} \hat{v}^C(b_{t+1}^C, Z_{t+1}) \right] \right). \end{aligned}$$

subject to

$$(1-\theta)\mathrm{E}_t[p_{t+1}] \geq b_t^C.$$

The continuation value $\mathrm{E}_t\left[M_{t,t+1}\hat{v}^C(b_t^C, Z_{t+1})\right]$ is analogous to the continuation value of shadow banks.

Bankruptcy The idiosyncratic asset valuation shock is realized just before the period starts.

If a S-bank declares bankruptcy, its equity becomes worthless, and creditors seize all of the banks assets. Before defining the recovery value, it is useful to define the expectations of idiosyncratic bank losses conditional on survival or bankruptcy as

$$\rho^-(v^j(b_t^j)) = \mathrm{E}_\rho\left[\rho_{t,i}^j \,|\, \rho_{t,i}^j < v^j(b_t^j)\right]$$

for surviving banks, and

$$\rho^+(v^j(b_t^j)) = \mathrm{E}_\rho\left[\rho_{t,i}^j \,|\, \rho_{t,i}^j > v^j(b_t^j)\right]$$

for failing banks, for $j = S, C$ respectively.

Banks that go bankrupt do no pay a dividend and their equity becomes worthless upon bankruptcy. After their restructuring has completed, the bankrupt banks are replaced by new banks who face the same forward-looking portfolio problem as existing banks. The recovery amount per bond issued is hence

$$r^j(b_t^j) = (1 - \xi^j)\frac{Z_t + p_t - \rho^{+j}(v^j(b_t^j))}{b_t^j},$$

with a fraction ξ^j lost in the bankruptcy proceedings. After the bankruptcy proceedings are completed, and new shadow bank is set up to replace the bankrupt one. This bank sells its equity to new owners, and is otherwise identical to an existing shadow bank with zero assets and liabilities.

If a C-bank declares bankruptcy, its equity becomes worthless as well. The bank is then taken over by the government that uses lump sum taxes and revenues from deposit insurance,

κB_{t+1}^C, to pay out the bank's creditors in full. That is, lump sum taxes are defined

$$T_t = (1 - F^C(b_t^C))(1 - r^C(b_t^C))B_t^C - \kappa B_{t+1}^C.$$

The average dividend conditional on survival for S-banks

$$d_t^S = \left(Z_t + p_t - \rho^-(v^S(b_{t+1}^S))\right) - b_t^S + a_{t+1}^S \left(q_t^S b_{t+1}^S - p_t\right),$$

and for C-banks

$$d_t^C = \left(Z_t + p_t - \rho^-(v^C(b_{t+1}^C))\right) - b_t^C + a_{t+1}^C \left(q_t^C b_{t+1}^C - p_t\right).$$

Households Households derive utility from the consumption C_t of the fruit of both trees. Households hold a portfolio of all securities that both types of intermediaries issue. In particular, they buy equity shares of both types of intermediaries, S_t^j, that trade at price of p_t^j, for $j = S, C$ respectively. They further buy the short terms bonds both types issue, N_t^j, trading at prices q_t^j, for $j = S, C$.

Households consume the liquidity services provided by the short term debt they hold at the beginning of the period. This reflects that the liquidity services accrue at the time when the deposits from last period are redeemed. Let $N_t^j = \int_0^1 N_{t,i}^j \, di$, for $j = S, C$. Then the total liquidity services produced are

$$H(N_t^S, N_t^C),$$

and household utility in period t is

$$U(C_t, H(N_t^S, N_t^C)).$$

We specify utility as

$$U(C_t, H\left(N_t^S, N_t^C\right)) = \frac{C_t^{1-\gamma} - 1}{1-\gamma} + \psi \frac{\left(H\left(N_t^S, N_t^C\right)/C_t\right)^{1-\eta}}{1-\eta},$$

with

$$H\left(N_t^S, N_t^C\right) = \left[\Lambda_{S,t}\left(N_t^S\right)^{\alpha} + \Lambda_{C,t}\left(N_t^C\right)^{\alpha}\right]^{1/\alpha}.$$

The elasticity of substitution between the two types of bank liabilities is $1/(1-\alpha)$.

We define the weights on the liquidity services of deposits of commercial banks $\Lambda_{C,t}$ and shadow banks $\Lambda_{S,t}$ as follows:

$$\begin{aligned}
\Lambda_{C,t} &= \frac{1}{1 + F_\rho^S\left(v^S\left(b_t^S\right)\right)^{\nu}} \\
\Lambda_{S,t} &= \frac{F_\rho^S\left(v^S\left(b_t^S\right)\right)^{\nu}}{1 + F_\rho^S\left(v^S\left(b_t^S\right)\right)^{\nu}},
\end{aligned}$$

with $\nu > 0$. The liquidity productivity of shadow banks is lower than that of commercial banks. The discount depends on the fraction of surviving shadow banks. If both bank types are equally safe, that is $F_\rho^S\left(v^S\left(b_t^S\right)\right) = 1$, the weights amount each to $1/2$.

Denoting household wealth at the beginning of the period by W_t, the complete intertemporal problem of households is

$$V^H(W_t, Y_t) = \max_{C_t, N_t^S, N_t^C, S_t^S, S_t^C} U(C_t, H\left(N_t^S, N_t^C\right)) + \beta \, \mathrm{E}_t\left[V(W_{t+1}, Y_{t+1})\right]$$

subject to

$$W_t + Y_t - T_t = C_t + \sum_{j=S,C} p_t^j S_t^j + \sum_{j=S,C} q_t^j N_t^j \tag{15}$$

$$\begin{aligned}
W_{t+1} = &\sum_{j=S,C} F_\rho^j\left(v^j(b_{t+1}^j)\right)(\tilde{D}_{t+1}^j + p_{t+1}^j) S_t^j \\
&+ N_t^S\left[\pi_B + (1-\pi_B)\left(F_\rho^S\left(v^S(b_{t+1}^S)\right) + \left(1 - F_\rho^S\left(v^S(b_{t+1}^S)\right)\right) r^S(b_{t+1}^S)\right)\right] \\
&+ N_t^C.
\end{aligned} \tag{16}$$

The budget constraint in equation (15) shows that households spend their wealth and income on consumption and purchases of equity and debt of both types of intermediaries. The securities issued are the same for all banks, independent of the previous bankruptcy status.

The equity purchases for banks that have gone through bankruptcy at the beginning of period t can be understood as initial equity offerings for these banks, while the purchases of equity of surviving banks are in a secondary market. However, since both new and surviving banks hold identical portfolios, their securities have the same price and there is no need to distinguish primary and secondary markets.[9]

Market Clearing Asset markets

$$A_{t+1}^{S}+A_{t+1}^{C}=1$$
$$B_{t}^{S}=N_{t}^{S}$$
$$B_{t}^{C}=N_{t}^{C}$$
$$S_{t}^{S}=1$$
$$S_{t}^{C}=1.$$

Resource constraint

$$C_t = Y_t + Z_t - \mu_\rho - \sum_{j=S,C} \xi^j (Z_t + p_t - \rho_t^{+,j}) A_t^j \left(1 - F_\rho^j\left(v^j(b_t^j)\right)\right).$$

3.2 Equilibrium Conditions

Household The household's first-order conditions for purchases of bank equity are, for $j=S,C$,

$$p_t^j = \mathrm{E}_t\left[M_{t,t+1} F_\rho^j\left(v^j(b_{t+1}^j)\right)(D_{t+1}^j + p_{t+1}^j)\right],$$

where we have defined the stochastic discount factor

$$M_{t,t+1} = \beta\,\frac{U_1(C_{t+1},H_{t+1})}{U_1(C_t,H_t)}.$$

[9]It is possible to show that the price to an equity claim of bank types j, p_t^j, is equal to the value of that bank's security portfolio, $A_{t+1}^j p_t - q_t^j B_{t+1}^j$.

We further define the intratemporal marginal rate of substitution between consumption and liquidity services

$$Q_t = \frac{U_2(C_t, H_t)}{U_1(C_t, H_t)}.$$

Then the first-order conditions for purchases of bonds of either type of bank are

$$q_t^C = Q_t \Lambda_{C,t} \left(\frac{H_t}{N_t^C} \right)^{1-\alpha} + \mathrm{E}_t \left[M_{t,t+1} \right], \tag{17}$$

$$q_t^S = Q_t \Lambda_{S,t} \left(\frac{H_t}{N_t^S} \right)^{1-\alpha} + E_t \left[M_{t,t+1} \left[F_\rho^S \left(v^S(b_{t+1}^S) \right) + \left(1 - F_\rho^S \left(v^S(b_{t+1}^S) \right) \right) \tilde{r}^S(b_{t+1}^S) \right] \right]. \tag{18}$$

The payoff of commercial bank bonds is 1, whereas the payoff of shadow bank bonds depends on their default probability and recovery value. The first terms in each expression represent the marginal benefit of liquidity services to households.

Banks S-banks are subject to an endogenous borrowing constraint. Each S-banks is a monopolist for its own debt, and hence internalizes the effect of supplying additional bonds on the bond price.

Specifically, each S-bank views the price of its debt as a function of its supply of bonds $q_t^S = q(b_{t+1}^S)$ that is determined by households' first order condition in equation 18.

It follows that the FOC of S-banks for leverage is

$$q(b_{t+1}^S) + b_{t+1}^S \, q'(b_{t+1}^S) = \mathrm{E}_t \left[M_{t,t+1} F_\rho^S \left(v^S \left(b_{t+1}^S, Z_{t+1} \right) \right) \right]. \tag{19}$$

The partial derivative $q'(b_{t+1}^S)$ can be obtained directly from households' FOC for purchases of shadow bank debt. In the appendix we show that differentiating equation (18) yields

$$\begin{aligned} q'(b_{t+1}^S) = -\mathrm{E}_t \Big\{ M_{t,t+1} \Big[& f_\rho \left(v \left(b_{t+1}^S \right) \right) \left(1 - r \left(b_{t+1}^S \right) \right) \\ & + \left(1 - F_\rho^S \left(v \left(b_{t+1}^S \right) \right) \right) \frac{r(b_{t+1}^S)}{b_{t+1}^S} + \frac{1 - \xi_S}{b_{t+1}^S} f_\rho \left(v \left(b_{t+1}^S \right) \right) \left(v \left(b_{t+1}^S \right) - \rho_t^{+,S} \right) \Big] \Big\}. \end{aligned} \tag{20}$$

The RHS is strictly negative, implying that the price of shadow bank debt is decreasing in shadow bank leverage b^S_{t+1}. The first term on the RHS is the loss for lenders from a marginal increase in the probability of default. The second term reflects that the recovery value per bond in case of bankruptcy is marginally decreased if the shadow bank issues more debt. The third term is positive and captures that the conditional expectation of the idiosyncratic losses of bankrupt firms decreases as leverage increases.

The debt price of commercial banks is independent of their leverage choice. Therefore the FOC of C-banks for leverage is

$$q^C_t - \kappa = \frac{\lambda^C_t}{a^C_{t+1}} + \mathrm{E}_t\left[M_{t,t+1}F_\rho\left(v^C(b^C_{t+1}, Z_{t+1})\right)\right].$$

with λ^C_t being the Lagrange multiplier on the leverage constraint. This FOC and the household FOC for purchases of commercial bank debt (17) jointly imply that the Lagrange multiplier is positive and hence the $C-$bank leverage constraint is binding[10], i.e.

$$(1-\theta)\mathrm{E}_t[p_{t+1}] = b^C_t.$$

3.3 Calibration

We match the model to quarterly data.

The stochastic process for the Y-tree is a AR(1) in logs

$$\log(Y_{t+1}) = (1-\rho_Y)\log(\mu_Y) + \rho_Y \log(Y_t) + \epsilon^Y_{t+1},$$

where ϵ^Y_t is i.i.d. with mean zero and volatility σ^Y. To capture the correlation of asset payoffs with fundamental income shocks, we model the payoff of the intermediated asset as

$$Z_t = \phi^Y Y_t \exp(\epsilon^Z_t),$$

[10]The constraint is binding if the marginal liquidity benefit from commercial bank debt is positive and the deposit insurance fee is not too large. This is the case for all relevant parameter combinations.

where ϵ_t^Z is i.i.d. with mean zero and volatility σ^Z, independent of ϵ_t^Y. This structure of the shocks implies that Z_t inherits all stochastic properties of aggregate income Y_t, and is subject to an additional temporary shock that reflects risks specific to intermediated assets, such as credit risk.

We quantify the model with data from the Flow of Funds[11], Compustat, and NIPA. We use quarterly data from 1999 (after the passage of the Gramm-Leach-Bliley Act that revoked parts of the Glass-Steagall Act) until the second quarter of 2015. We choose depository institutions as data counterparts for $C-$banks and shadow bank institutions as data counterparts for $S-$banks. Shadow banks are defined on their asset side as security broker and dealer, finance companies, insurance companies, asset-backed security issuers and so on and on their liability side as money market mutual funds.

We normalize each bank's balance sheet position, as well as consumption and safe asset by total financial assets of each intermediary type. For C-Banks this amounts to dividing by total depository institution financial assets. For S-Banks this means that we need to aggregate total financial assets of all shadow-banking institutions from the Flow of Funds tables.

[11]The Flow of Funds tables are organized according to institutions and instruments. We focus on the balance sheet information on institutions. This is important, as we want to take into account all bank and shadow-bank positions when we quantify the model.

Table 1: Parametrization

Parameters	Function	Value	Target
β	discount rate	0.99	Short rate
α	maps into CES par.	0.5	$S-$bank share / Gallin FOF 30%
ν	liquidity factor	30	spread on $C-$& $S-$bank debt
			CP AA fin.- FDIC r exp, 2.17%
π_S	$S-$ bank bailout prob.	0.1	$S-$bank leverage
ψ	utility weight on safe assets	0.2	share of C to safe & liquid assets
κ	deposit insurance fee	0.0006168	deposit ins. rates(25 bp p.a)
θ	$C-$ bank capital req.	0.10	Effective Tier 1 cap ratio
ξ_ρ^j	bankruptcy loss	0.1	recovery rate (37.1%)
η	compl. bw cons & safe assets	2	vol(consumption/safe assets)
σ_ρ^C	vol of ρ shock	0.25	FDIC default rate 0.04 %
σ_ρ^S	vol of ρ shock	1.8	q^S: FRED CP ON AA fin. sector
μ^Y	mean of Y	0.1	normalization
ρ^Y	persistence of Y	0.75	normalization
σ^Y	vol of Y shock	0.0063	agg. TFP vol
μ^Z	mean of Y	0.007	7% of Y
σ^Z	vol of Z shock	0.0008	normalized fin. sector income vol

The key parameters are those that determine the relative size of the two sectors. These are the elasticity of substitution of HH (α =$1 - 1/$elasticity) between the two liquidity services provided by the two types of banking sector liabilities, and the sensitivity of liquidity services provided by shadow banks to their default rate ($\bar{\nu}$).

It's intuitive to think of commercial bank debt (mostly deposits, but also commercial paper, repo etc) and shadow bank debt (mostly money market mutual fund shares, commercial paper, repo, etc) as being slightly different securities. The elasticity of substitution parameter is pinned down by the relative size of depository institution debt to shadow bank debt from the Flow of Funds. The shadow bank liquidity value parameter in households' utility function is determined by the 3-month spread between AA commercial paper for financial institutions and the implied interest rate on depository institution debt from FDIC reports.

We set κ, the deposit insurance fee to 0.0006168. This is in the range of quarterly FDIC assessment rates. The parameters ξ^S and ξ^C (loss in bankruptcy) are set to match the average recovery value of 30.5 percent from Moody's report.[12] We set σ_ρ^C such that the default probability of bank C equals that of the data. The FDIC publishes[13] estimates for average default probabilities of bank holding companies (0.4175 percent) for the $2006-2008$ period[14]. We set σ_ρ^C such that the probability of default equals 4.2%. We set σ_ρ^S of shadow banks idiosyncratic risk such that the price of shadow banking debt in the model, matches the 3 month AA-rated financial sector commercial paper rate from FRED.

3.4 Results

We solve the dynamic model using second-order approximation around the model's deterministic steady state. We then simulate the model for many periods and compute moments of the simulated series, for different values of the capital requirement θ. Table (2) reports first and second moments for capital requirements ranging from 10% to 25%.

As for the two-period model, there is a unique maximum in aggregate welfare. The welfare maximum for the quantitative model occurs in the interval between 22 and 23 percent. The main forces leading to this result are the same as in the simple model. Increasing θ makes the debt of both types of banks safer, thus reducing bankruptcy losses and increasing aggregate consumption. At the same time, a higher level of θ restricts the amount of liabilities and thus liquidity commercial banks can produce for each unit of assets.

Changing the capital requirement Rows 3 of table (2) shows that an increase in the capital requirement shifts a greater fraction of the intermediated asset to shadow banks. Restricting the liquidity production by commercial banks increases the marginal benefit of

[12]We use Moody's 1984-2004 report. Exhibit 9 in the report presents the recovery rates of defaulted bond for financial institutions. We use the mean for financial institutions over all bonds and preferred stocks.

[13]We take the average default probability from table 1 on the FDIC website.

[14] That is $\frac{0.13+3.21}{8} = 0.4175$

Table 2: Model Moments for Different Capital Requirements

	$\theta = 0.1$		$\theta = 0.15$		$\theta = 0.20$		$\theta = 0.25$	
	mean	stdev	mean	stdev	mean	stdev	mean	stdev
	Exogenous Variables							
Income	0.10	0.004	0.10	0.004	0.10	0.004	0.10	0.004
Asset payoff	0.10	0.005	0.10	0.005	0.10	0.005	0.10	0.005
	Intermediated asset share & price							
S-bank share	0.42	0.053	0.55	0.016	0.51	0.011	0.49	0.011
Asset price	6.96	0.626	8.47	0.582	8.17	0.532	8.19	0.534
	Bank debt & prices							
C-bank debt	3.59	0.186	3.44	0.216	3.34	0.196	3.27	0.186
S-bank debt	0.48	0.136	0.80	0.058	0.73	0.055	0.72	0.054
S debt price	0.989	0.091	0.988	0.045	0.988	0.033	0.988	0.033
C debt price	0.981	0.088	0.994	0.046	0.991	0.035	0.991	0.035
	Consumption & welfare							
Liquidity	1.67	0.134	1.91	0.118	1.82	0.112	1.78	0.108
Consumption	0.190	0.009	0.198	0.009	0.200	0.008	0.200	0.008
Welfare[a]			4.19%	-23.82%	4.72%	-25.82%	4.69%	-25.69%

The table unconditional means and standard deviations of the main outcome variables from a 1,000 period simulation of four different models with different capital requirements.

[a]: Welfare is percentage change of mean and volatility of the household value function relative to the benchmark model with $\theta = 0.1$.

liquidity to households. Shadow banks take advantage of the lower funding costs by increasing their demand for intermediated assets. As for the static model, this effect is non-monotonic and reverses at high levels of θ. Since shadow banks have a higher marginal valuation of the asset than commercial banks, the price of the Z-asset rises by 21% when the capital requirement is increased from 10% to 15%. As the S-bank share declines again for levels of θ greater than 15%, so does the price of the Z-asset.

The price effect is much stronger in the dynamic model than in the two-period model. The large price increase causes the total amount of liquidity to *rise* for the initial increase in θ. This effect illustrates that the amount of total liquidity strongly depends on the market value of the intermediated asset. The $\theta = 15\%$ economy produces more liquidity services (1.91) than the $\theta = 10\%$ economy (1.67). This is because the value of the asset backing the banks' liabilities increases so much that it more than compensates for the fact that commercial banks can now only turn 85% of their asset value into liquidity-producing deposits. The large increase in the asset price allows shadow banks to almost double their deposits (from 0.48 to 0.80), even though their leverage slightly declines (unreported in table) and their asset share only increases by 30%.

Both types of banks become safer as in the two-period model when θ is raised. Bankruptcy losses approach zero around a capital requirement of 18%. Further increases in the requirement only reduce liquidity, but do not yield any additional consumption benefit. Why then is the welfare maximum at a level beyond 20%, where liquidity provision is already decreasing in θ? The reason is that in the dynamic model increasing the requirement yields the additional benefit of lower consumption and liquidity *volatility.* For the risk-averse household, this leads to a further utility gain, albeit a small one. However, the effect of the capital requirement on second moments of consumption and liquidity is an important difference between the two-period and the dynamic model.

Dynamic responses The lower volatility of prices and quantities in the economies with higher capital requirements can be seen clearly in figure (4).

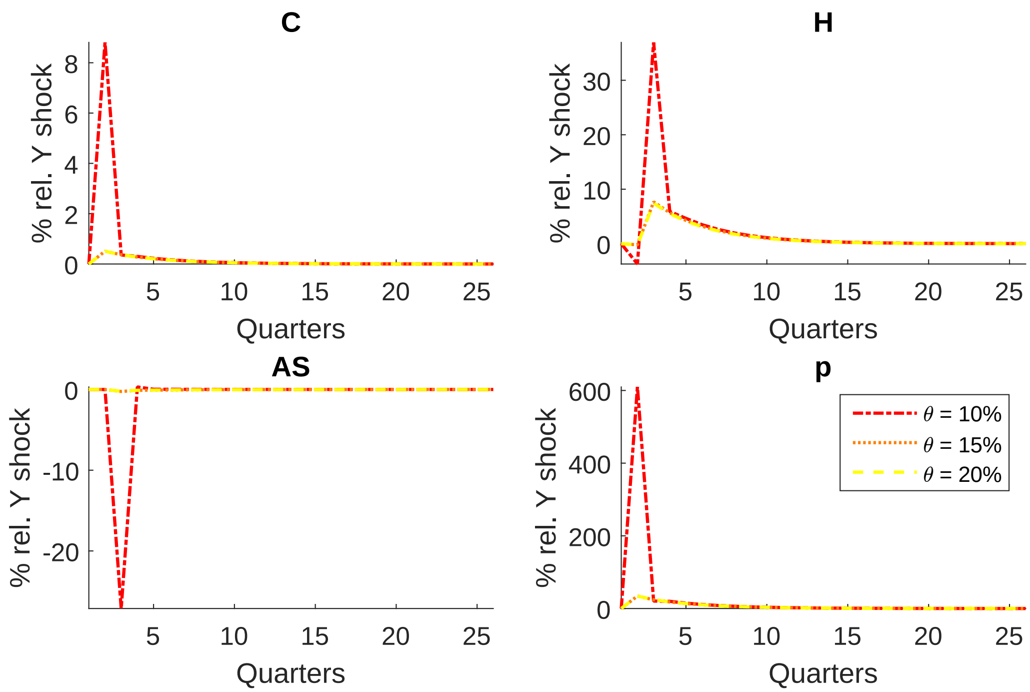

Figure 4: Impulse responses for different levels of θ

Top left: Consumption, Top right: Utility from liquidity services, Bottom left: Share of S-banks, Bottom right: Price of Z-asset.

The dynamic responses of consumption and the price of the intermediated asset to an aggregate income and payoff shock decline in amplitude for higher levels of θ. The responses of all four variables shown in the figure are less persistent. For low values of θ, the shadow bank share is procyclical. For levels of θ that are high enough to make both types of banks safe and hence their liquidity services equally valuable, the asset share hardly responds at all to aggregate shocks. In addition, the cyclicality reverses, and the shadow bank share now becomes slightly countercyclical.

4 Conclusion

This paper proposes a novel model to study the consequences of higher bank capital requirements for the economy. The optimal level of capital regulation trades-off a reduction in liquidity services against an increase in the safety of the banking system and consumption. Increasing the capital requirement for regulated banks leads to more intermediation activity by the shadow banking system and thus a higher valuation for the intermediated asset because households' substitute shadow bank liquidity for commercial bank liquidity. This effect is non-monotonic as shadow banks' endogenous borrowing constraint restricts their ability to provide liquidity services. Moreover, a higher capital requirement makes both bank types safer: they affect commercial banks directly through a mandated reduction in leverage while they make shadow banks indirectly safer through their effect on the intermediated asset valuation.

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A Appendix

A.1 Two-period model

Preliminaries Since ρ_i^j is uniformly distributed, the probability that bank i with assets A_i^j and debt B_i^j stays in business is given by the probability that $\rho_i^j \leq Z - \frac{B_i^j}{A_i^j}$, i.e.

$$F_\rho^j \left(Z - \frac{B_i^j}{A_i^j} \right) = \frac{Z - b_i^j - \underline{\rho}^j}{\overline{\rho}^j - \underline{\rho}^j}. \tag{21}$$

Furthermore, ρ_j^- is the expected value of ρ_i^j conditional on $j-$bank's survival and a realization of the random variable Z:

$$\rho_j^- = \mathrm{E}^\rho[\rho^j \mid \rho^j < Z - b^j] = \frac{1}{2} \left(Z - b^j + \underline{\rho}^j \right). \tag{22}$$

The expected value of ρ conditional on $j-$bank's failure is

$$\rho_j^+ = \mathrm{E}^\rho[\rho^j \mid \rho^j > Z - b^j] = \frac{1}{2} \left(Z - b^j + \overline{\rho}^j \right). \tag{23}$$

First-order conditions with respect to A^j The first-order condition for holdings of the intermediated asset for both types of banks is

$$v^j(b^j) = 0,$$

which implies

$$q^j b^j - p_0 = \mathrm{E}_0 \left[M \left(F_\rho^j (Z - b^S - \rho_j^-) \right) \right]$$

using the expressions for $v^j(b^j)$ defined in equations (4) and (2) for $j = C, S$, respectively. We can now substitute the expressions for F_ρ^j given in equation (21) and for ρ_j^- given in equation (22) into the condition above, which yields

$$q^j b^j - p_0 = \mathrm{E}_0 \left[\frac{M}{2(\overline{\rho}^j - \underline{\rho}^j)} \left(Z - b^j - \underline{\rho}^j \right)^2 \right].$$

Using once more the definition of F_ρ^j this becomes

$$q^j b^j - p_0 = \mathrm{E}_0 \left[\frac{M(\overline{\rho}^j - \underline{\rho}^j)}{2} \left(F_\rho^j\right)^2 \right].$$

Recognizing that the standard deviation of a uniform random variable with support $[\underline{\rho}^j, \overline{\rho}^j]$ is given by

$$\sigma^j = \frac{\overline{\rho}^j - \underline{\rho}^j}{\sqrt{12}}$$

and plugging in gives the result in equations (10) and (11), respectively.

First-order condition for b^C Starting with the definition of $v^C(b^C)$ in equation (4), and substituting the expressions for F_ρ^C and ρ_C^- given in equations (21) and (22), we get

$$v^C(b^C) = (q^C - \kappa)b^C - p_0 - \mathrm{E}_0 \left[\frac{M}{2(\overline{\rho}^C - \underline{\rho}^C)} \left(Z - b^C - \underline{\rho}^C\right)^2 \right].$$

Taking the derivative with respect to b^C (including the leverage constraint $b^C \leq (1-\theta)p_0$) then yields

$$q^C - \kappa = \lambda^C + \mathrm{E}_0 \left[M \left(\frac{Z - b^C - \underline{\rho}^j}{\overline{\rho}^j - \underline{\rho}^j} \right) \right],$$

which gives the result in equation (12).

First-order condition for b^S To derive the FOC of the S-bank, it is helpful to first rewrite several expressions. As for the C-bank, we can write the continuation value as

$$F^S(Z - b^S - \rho_S^-) = \frac{1}{2(\overline{\rho}^S - \underline{\rho}^S)}(Z - b^S - \underline{\rho}^S)^2.$$

Furthermore, noting that $Z - \rho_S^+ = \frac{1}{2}(Z + b^S - \overline{\rho}^S)$, we get for the recovery value on the bonds of bankrupt S-banks

$$\left(1 - F^S\right) r^S = \frac{1 - \xi_S}{2(\overline{\rho}^S - \underline{\rho}^S)} \frac{\left(b^S\right)^2 - \left(Z - \overline{\rho}^S\right)^2}{b^S}.$$

Using these expressions, we can rewrite HH's FOC for N^S (see equation 8) as

$$q^S = \mathrm{E}_0 \left\{ M_1 \frac{1}{\left(\overline{\rho}^S - \underline{\rho}^S\right)} \left[Z - b^S - \underline{\rho}^S + 0.5\left(1 - \xi_S\right) \frac{\left(b^S\right)^2 - \left(Z - \overline{\rho}^S\right)^2}{b^S} + \mathrm{MRS}^S \right] \right\}.$$

Taking the derivative of this equation with respect to b^S

$$\frac{\partial q^S(b^S)}{\partial b^S} = -\mathrm{E}_0 \left\{ M_1 \left(\frac{\xi_S}{\overline{\rho}^S - \underline{\rho}^S} + \left(1 - F^S\right) \frac{r^S}{b^S} \right) \right\}. \tag{24}$$

We can now derive the S-bank's FOC with respect to its leverage ratio b^S

$$q^S + b^S \frac{\partial q^S(b^S)}{\partial b^S} = \mathrm{E}_0 \left[M_1 \frac{Z - b^S - \underline{\rho}^S}{\overline{\rho}^S - \underline{\rho}^S} \right].$$

Plugging in from equation (24), and noting that the last term on the right-hand side equal the survival probability F^S

$$q^S - \mathrm{E}_0 M_1 \left[\frac{b^S \xi_S}{\overline{\rho}^S - \underline{\rho}^S} + (1 - F^S) r^S \right] = \mathrm{E}_0 M_1 F^S,$$

which can be rearranged to yield

$$q^S - \mathrm{E}_0 M_1 \left[F^S + (1 - F^S) r^S \right] = \mathrm{E}_0 M_1 \frac{1}{\overline{\rho}^S - \underline{\rho}^S} \xi_S b^S.$$

Using the HH's FOC for shadow bank debt (8), we can substitute for the difference on the LHS

$$\mathrm{E}_0 \left[M_1 \left(1 - \nu\right) \frac{U_1}{U_2} \left(\frac{H}{N^S} \right)^{1-\alpha} \right] = \mathrm{E}_0 \left[M_1 \frac{1}{\overline{\rho}^S - \underline{\rho}^S} \xi_S b^S \right],$$

and solve for b^S, which yields the expression in equation (14).

Equations and Solution Method The equilibrium of the economy needs to be computed numerically. The equilibrium can be reduced to a system of four nonlinear equations in four unknowns. We numerically find a unique solution for every parameter combination we have tried. The four variables are (p_0, A^S, C_1, b^S). Using the variables, we can first compute the C-bank leverage ratio as

$$b^C = (1 - \theta) p_0$$

Using the market clearing conditions $N^S = b^S A^S$, $N^C = A^C b^C \left(1 - (1 - F^C)(1 - r^C) + \kappa\right)$ and $A^C = 1 - A^S$, we can get the two bond prices (q^S, q^C) from the HH FOCs for deposits. Then the four equations are

$$
\begin{aligned}
p_0 &= (q^C - \kappa)b^C + \mathrm{E}_0\left(M_1 \frac{1}{4\sigma_2\sqrt{3}}(Z - b^C - \underline{\rho}^C)^2\right) \\
p_0 &= q^S b^S + \mathrm{E}_0\left(M_1 \frac{1}{4\sigma_2\sqrt{3}}(Z - b^S - \underline{\rho}^S)^2\right) \\
C_1 &= Y_1 + \sum_j D_1^j + N^C + N^S(F^S + (1 - F^S)r^s) \\
b^S &= \mathrm{E}_0\left(\frac{\sqrt{12}\sigma_S}{\xi_S}(1 - \nu)\left(\frac{1 - \psi\,(H/C_1)^{1-\eta}}{\psi\,(H/C_1)^{-\eta}}\right)\left(\frac{H}{N^S}\right)^{1-\alpha}\right).
\end{aligned}
$$

A.2 Equilibrium Condition Dynamic Model

$$H\left(N_t^S, N_t^C\right) = \left[\frac{F_\rho^S\left(v^S\left(b_t^S\right)\right)^{\nu}}{1 + F_\rho^S\left(v^S\left(b_t^S\right)\right)^{\nu}}\left(N_t^S\right)^{\alpha} + \frac{1}{1 + F_\rho^S\left(v^S\left(b_t^S\right)\right)^{\nu}}\left(N_t^C\right)^{\alpha}\right]^{1/\alpha} \tag{25}$$

$$U_1(C_t, H_t) = C_t^{-\gamma} - \psi H_t^{1-\eta} C_t^{\eta-2} \tag{26}$$

$$U_2(C_t, H_t) = \psi H_t^{-\eta} C_t^{\eta-1} \tag{27}$$

$$MRS_{S,t} = \left(\frac{F_\rho^S\left(v^S\left(b_t^S\right)\right)^{\nu}}{1 + F_\rho^S\left(v^S\left(b_t^S\right)\right)^{\nu}}\right)\left(\frac{U_2(C_t, H_t)}{U_1(C_t, H_t)}\right)\left(\frac{H_t}{N_t^S}\right)^{1-\alpha} \tag{28}$$

$$MRS_{C,t} = \left(\frac{1}{1 + F_\rho^S\left(v^S\left(b_t^S\right)\right)^{\nu}}\right)\left(\frac{U_2(C_t, H_t)}{U_1(C_t, H_t)}\right)\left(\frac{H_t}{N_t^C}\right)^{1-\alpha} \tag{29}$$

$$V^H(W_t, Y_t) = \max_{C_t, N_t^S, N_t^C, S_t^S, S_t^C} U(C_t, H\left(N_t^S, N_t^C\right)) + \beta\,\mathrm{E}_t\left[V(W_{t+1}, Y_{t+1})\right] \tag{30}$$

$$F^j\left(v^j\right) = \Phi\left(\frac{v^j - \mu_{\rho,j}}{\sigma_{\rho,j}}\right) \tag{31}$$

$$f^j\left(v^j\right) = \phi\left(\frac{v^j - \mu_{\rho,j}}{\sigma_{\rho,j}}\right)/\sigma_{\rho,j} \tag{32}$$

$$M_{t,t+1} = \beta \, \frac{U_1(C_{t+1}, H_{t+1})}{U_1(C_t, H_t)} \tag{33}$$

$$W_t + Y_t - T_t = C_t + \sum_{j=S,C} p_t^j S_t^j + \sum_{j=S,C} q_t^j N_t^j \tag{34}$$

$$\begin{aligned} W_{t+1} = & \sum_{j=S,C} \left(\left(F_\rho^j \left(v^j(b_{t+1}^j) \right) \left(D_{t+1}^j + p_{t+1}^j - \mu_{\rho,j} A_t^j \right) + \sigma_{\rho,j}^2 f^j \left(v^j(b_{t+1}^j) \right) A_t^j \right) \right) S_t^j \\ & + N_t^S \left[\pi_B + (1-\pi_B) \left(F_\rho^S \left(v^S(b_{t+1}^S) \right) + \tilde{r}^S(b_{t+1}^S) \right) \right] \\ & + N_t^C. \end{aligned} \tag{35}$$

$$q_t^C = MRS_{C,t} + \mathrm{E}_t \left[M_{t,t+1} \right] \tag{36}$$

$$q_t^S \;\; = \;\; MRS_{S,t} + E_t \left[M_{t,t+1} \left(\pi_B + (1-\pi_B) \left(F_\rho^S \left(v^S(b_{t+1}^S) \right) + \tilde{r}^S \left(b_{t+1}^S \right) \right) \right) \right] \tag{37}$$

$$p_t^j = \mathrm{E}_t \left[M_{t,t+1} \left(F_\rho^j \left(v^j(b_{t+1}^j) \right) \left(D_{t+1}^j + p_{t+1}^j - \mu_{\rho,j} A_t^j \right) + \sigma_{\rho,j}^2 f^j \left(v^j(b_{t+1}^j) \right) A_t^j \right) \right], \text{ for } j = S, C \tag{38}$$

$$v^S(b_t^S, Z_t) = \max_{a_{t+1}^S, b_{t+1}^S} \;\; Z_t + p_t - b_t^S + a_{t+1}^S \left(q_t^S b_{t+1}^S - p_t + \mathrm{E}_t \left[M_{t,t+1} \hat{v}^S(b_{t+1}^S, Z_{t+1}) \right] \right). \tag{39}$$

$$\hat{v}^S(b_t^S, Z_t) = F^S \left(v^S(b_t^S, Z_t) - \mu_{\rho,S} \right) + \sigma_{\rho,S}^2 f^S \left(v^S \right) \tag{40}$$

$$B_t^S = b_t^S A_t^S \tag{41}$$

$$D_t^S = d_t^S A_{t-1}^S \tag{42}$$

$$d_t^S = (Z_t + p_t) - b_t^S + a_{t+1}^S \left(q_t^S b_{t+1}^S - p_t \right) \tag{43}$$

$$A_t^S = a_t^S \, A_{t-1}^S \tag{44}$$

$$q(b_{t+1}^S) + b_{t+1}^S \, q'(b_{t+1}^S) = \mathrm{E}_t \left[M_{t,t+1} F_\rho^S \left(v^S \left(b_{t+1}^S, Z_{t+1} \right) \right) \right] \tag{45}$$

$$q'(b_{t+1}^S) = -\,\mathrm{E}_t \left\{ M_{t,t+1} \, (1-\pi_B) \left[f^S \left(v \left(b_{t+1}^S \right) \right) + \frac{f^S \left(v \left(b_{t+1}^S \right) \right)}{b_{t+1}^S} (1-\xi^S) \left(v \left(b_{t+1}^S \right) - (Z_{t+1} + p_{t+1}) \right) \right. \right. \tag{46}$$

$$\left. \left. + \frac{1}{b_{t+1}^S} \tilde{r}^S(b_{t+1}^S) \right] \right\} \tag{47}$$

$$p_t - b_t^S\, q(b_t^S) = \mathrm{E}_t \left[M_{t,t+1} \hat{v}^S(b_{t+1}^S, Z_{t+1}) \right] \tag{48}$$

$$\tilde{r}^S(b_t^S) = \frac{\left(1-\xi^S\right)}{b_t^S} \left(\left(1 - F_\rho^S \left(v^S(b_t^S)\right)\right)(Z_t + p_t - \mu_{\rho,S}) - \sigma_{\rho,S}^2 f^S \left(v^S(b_t^S)\right) \right) \tag{49}$$

This is the recovery value weighted by the probability of default$\left(1 - F^S\right) r^S$

$$v^C(b_t^C, Z_t) = \max_{a_{t+1}^C, b_{t+1}^C} \quad Z_t + p_t - b_t^C + a_{t+1}^C \left(\left(q_t^C - \kappa\right) b_{t+1}^C - p_t + \mathrm{E}_t \left[M_{t,t+1} \hat{v}^C(b_{t+1}^C, Z_{t+1}) \right] \right). \tag{50}$$

$$\hat{v}^C(b_t^C, Z_t) = F^C v^C(b_t^C) - F^C \mu_{\rho,C} + \sigma_{\rho,C}^2 f^C \left(v^C(b_t^C)\right) \tag{51}$$

$$b_t^C = B_t^C / A_t^C \tag{52}$$

$$D_t^C = d_t^C A_{t-1}^C \tag{53}$$

$$d_t^C = Z_t + p_t - b_t^C + a_{t+1}^C \left(\left(q_t^C - \kappa\right) b_{t+1}^C - p_t \right) \tag{54}$$

$$q_t^C - \kappa = \frac{\lambda_t^C}{a_{t+1}^C} + \mathrm{E}_t \left[M_{t,t+1} F_\rho \left(v^C(b_{t+1}^C, Z_{t+1}) \right) \right] \tag{55}$$

$$p_t - b_{t+1}^C \left(q_t^C - \kappa \right) = \mathrm{E}_t \left[M_{t,t+1} \hat{v}^C(b_{t+1}^C, Z_{t+1}) \right] \tag{56}$$

$$\tilde{r}^C(b_t^C) = \frac{\left(1-\xi^C\right)}{b_t^C} \left(\left(1 - F_\rho^C \left(v^C(b_t^C)\right)\right)(Z_t + p_t - \mu_{\rho,C}) - \sigma_{\rho,C}^2 f^C \left(v^C(b_t^C)\right) \right) \tag{57}$$

This is the recovery value weighted by the probability of default: $(1 - F^j)\, r^j$

$$(1-\theta)\mathrm{E}_t[p_{t+1}] = b_t^C \tag{58}$$

$$A_t^C = a_t^C\, A_{t-1}^C \tag{59}$$

$$A_{t+1}^S + A_{t+1}^C = 1 \tag{60}$$

$$B_t^S = N_t^S \tag{61}$$

$$B_t^C = N_t^C \tag{62}$$

$$T_t = \left(\left(1 - F^C(b_t^C)\right) - \tilde{r}^C(b_t^C)\right) B_t^C + \pi_B \left(\left(1 - F^S(b_t^S)\right) - \tilde{r}^S(b_t^S)\right) B_t^S - \kappa B_{t+1}^C \tag{63}$$

$$C_t = Y_t + Z_t - \mu_\rho - \sum_{j=S,C} \xi^j \left((Z_t + p_t - \mu_{\rho,j}) \left(1 - F_\rho^j \left(v^j(b_t^j)\right)\right) - \sigma_{\rho,j}^2 f^j \left(v^j\right)\right) A_t^j. \tag{64}$$

Exogenous Variables

$$Z_t = \phi^Y Y_t \exp(\epsilon_t^Z) \tag{65}$$

$$\log(Y_{t+1}) = (1 - \rho_Y)\log(\mu_Y) + \rho_Y \log(Y_t) + \epsilon_{t+1}^Y \tag{66}$$

$$Y_t = \exp\left(\log\left(Y_t\right)\right) \tag{67}$$